HW #5
Math/CS 471, Fall 2021

Michael Sands
John Tran

November 17, 2021

Contents

NOTE: When we refer to a forcing term being zero or non-zero, we mean the force either has no effect on the given bird or some measurable effect on the given bird, respectively. Since we are discussing the location of the birds in 2D space, the forcing terms are consequently vectors that have two components (i.e., $x-$ and $y-$components).

1 Problem to be Solved

Derive a system of ODE's capable of modeling a system of birds that flock around a leader and who's leader is attracted to a food source. Include a repelling force that repels the five closest birds at each iteration. As said in the homework spec., there may be other forces to consider (e.g., predator avoidance in the extra credit), but we are mainly concerned with four forcing terms, described in the section below.

This code should function as a general model that can be used to describe other systems such as particle motion and charged particle interactions in future applications.

2 Our Approach

We developed a numerical simulation that plots the location of birds over time depending on four separate forces acting upon them: the following force (F^{follow}), the food force (F^{food}), the flocking force (F^{fl}), and the repelling force (F^{rep}). The interaction of these forces is described in Equation 1. In this simulation we used the parameters `food flag`, `alpha`, `gamma_1`, `gamma_2`, `kappa`, `rho`, and `delta` to uniquely tailor the simulation to most realistically model the movement of birds flocking around a leader.

In the extra credit, we also considered `kappa_smelly`, `rho_smelly`, and `rho_pred` to describe the unique interactions of the smelly bird and external predator with the rest of the flock.

3 Explanation of Forcing Terms:

There are four forcing terms in our simulation. The first force is the food force in Equation 2. This force is a non-zero magnitude vector for the leader bird and a zero magnitude vector for all other birds in the flock. This force is responsible for the movement of the leader bird. As all other birds follow the lead bird, the food force implicitly effects all birds in the flock. The second force is the follow force in Equation 3. This force is a non-zero magnitude vector for all non-leader birds and is a zero magnitude vector for the leader bird. This force enables all non-leader birds to follow the leader as the leader follows the food. The third force is the flocking force in Equation 4 which is a non-zero magnitude vector for all non-leader birds and is a zero magnitude vector for the leader bird. This force attracts all non-leader birds to the mass center of the *entire* flock. Finally the repelling force in Equation 5 is the opposite of the flocking force, repelling the nearest five neighbors of a given bird. This force avoids unrealistic scenarios such as overlap of birds in space.

$$f(t, B(t)) = F^{follow}(t, B(t)) + F^{food}(t, B(t)) + F^{fl}(t, B(t)) + F^{rep}(t, B(t)) \quad (1)$$

$$F^{food}(t, B(t)) = \gamma_1(C(t) - B_1(t)) \quad (2)$$

$$F^{follow}(t, B(t)) = \gamma_2(B_1(t) - B_k(t)), k = 1, 2, \ldots N \quad (3)$$

$$F^{fl}(t, B(t)) = \kappa(\bar{B}(t) - B_k(t)), k = 2, 3, \ldots N \quad (4)$$

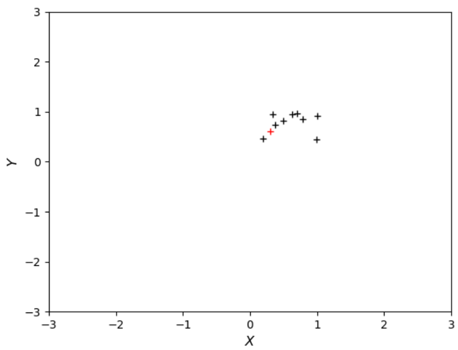
$$F^{rep}(t, B(t)) = \sum_{i=1}^{5} \rho \frac{(B_{k,j}(t) - B_{l_k^i,j}(t))}{(B_{k,j}(t) - B_{l_k^i,j}(t))^2 + \delta}, k = 2, 3, \ldots N \quad (5)$$

4 Explanation of Parameters:

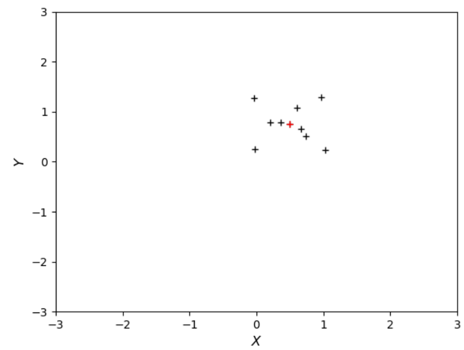
NOTE: The default values given to us in the code skeleton were used to generate the movie frames below.

In the figures below, we present a series of still frames that act as a representation of the flock movement for $N = 10$, 30, and 100. Their corresponding movies can be found in the git repository:

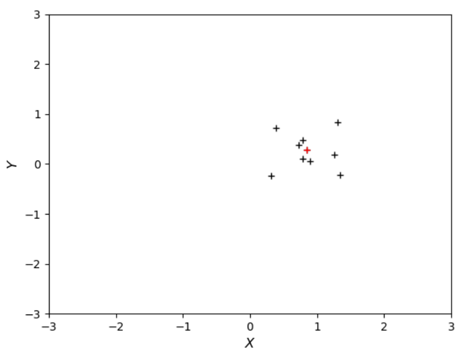
2020_fall_math471_sands/homeworks/hw5/code_output/normal_movie



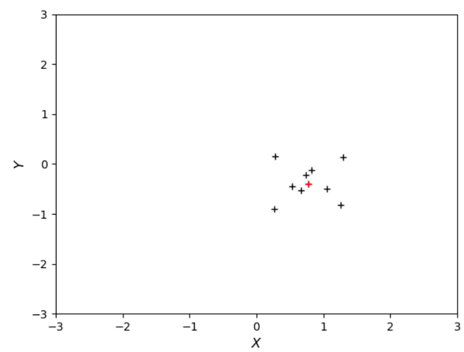
(a) Beginning Frame



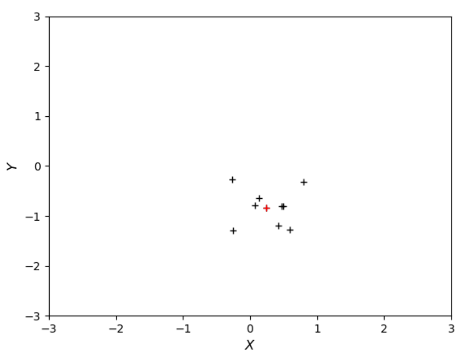
(b) Middle Frame 1



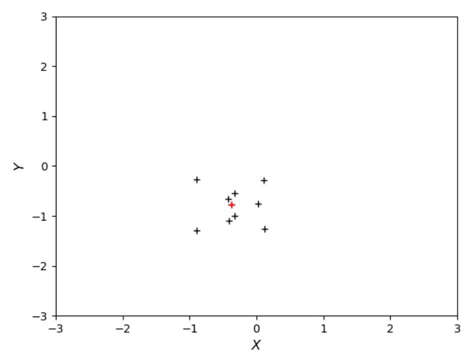
(c) Middle Frame 2



(d) Middle Frame 3

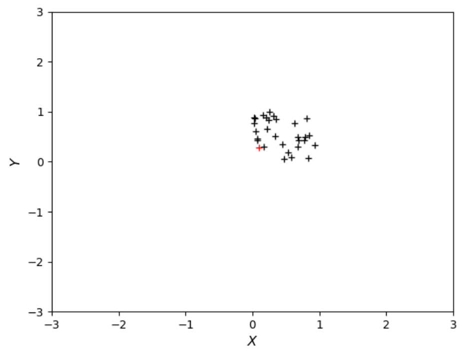


(e) Middle Frame 4

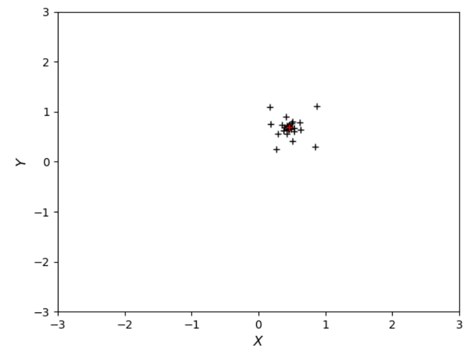


(f) End Frame

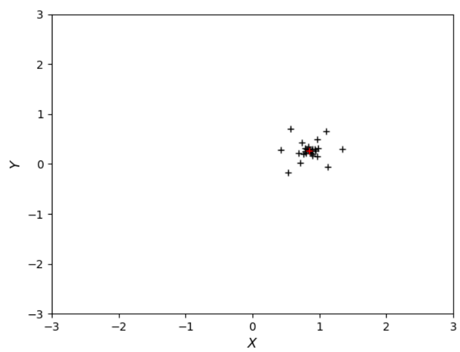
Figure 1: Example plot for $N = 10$



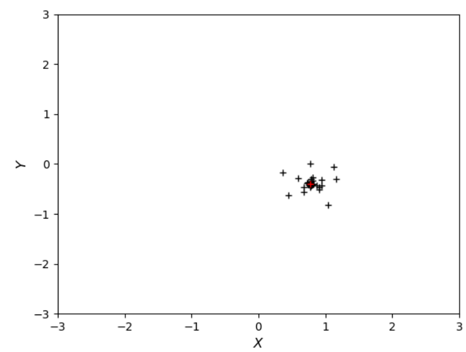
(a) Beginning Frame



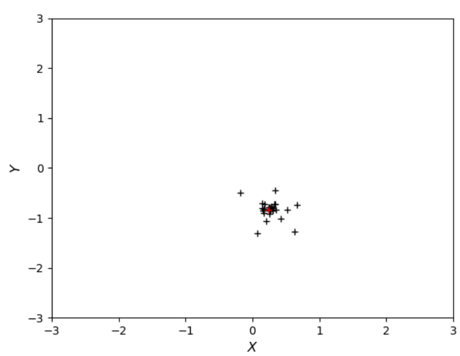
(b) Middle Frame 1



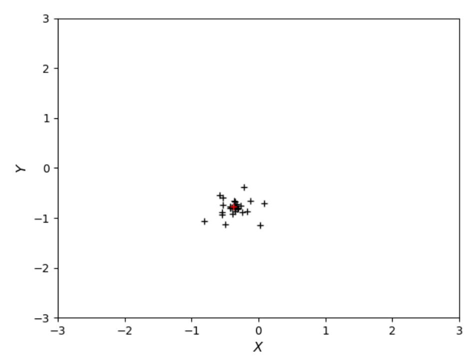
(c) Middle Frame 2



(d) Middle Frame 3

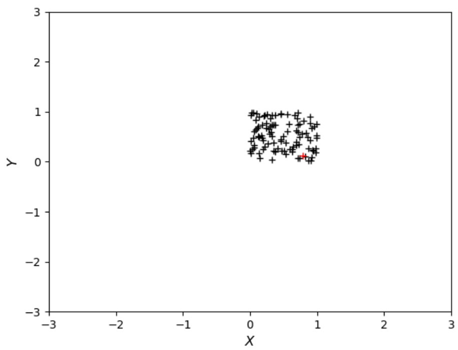


(e) Middle Frame 4

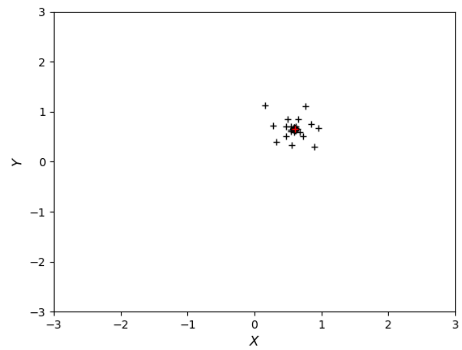


(f) End Frame

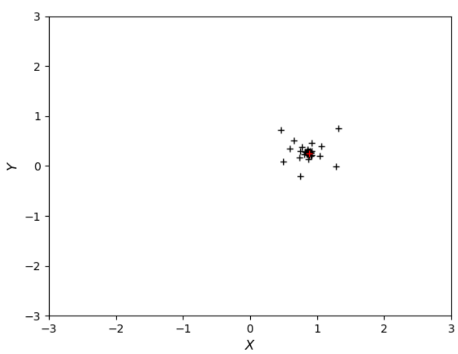
Figure 2: Example plot for $N = 30$



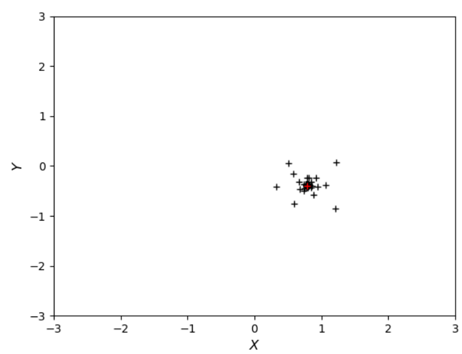
(a) Beginning Frame



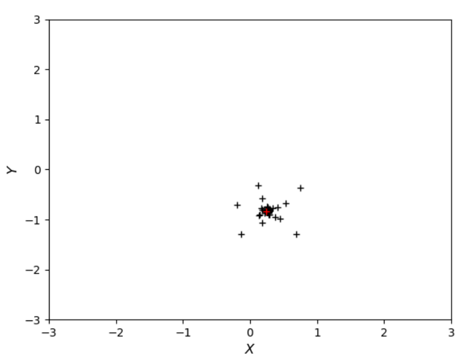
(b) Middle Frame 1



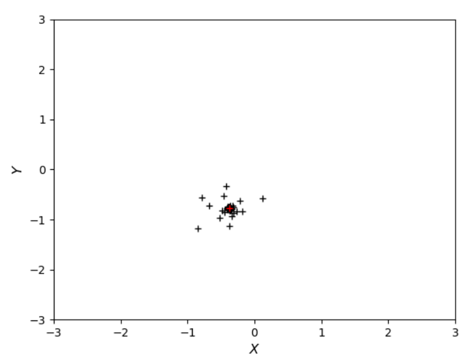
(c) Middle Frame 2



(d) Middle Frame 3



(e) Middle Frame 4



(f) End Frame

Figure 3: Example plot for $N = 100$

There are a variety of parameters that affect the distribution and movement of birds in our simulation. The food flag is either 0 or 1. If the food flag is 0, the location of the food is stationary (set to the origin $(0,0)$) and the leader bird does not move much from its original position. The rest of the birds are attracted to the leader bird by forces described below, so if the food is stationary, the flock is more or less stationary as well. If food flag is 1, the food moves in a circle described by $C(t)$ in Equation 6, and the leader bird follows it. In this case, the flock as a whole follows the food in a circular motion. The α parameter in Equation 6 describes the period of the circle in which the food moves. As this parameter increases, the period of the movement of the flock decreases, and the flock moves faster as the food moves faster. α is always a positive non-zero scalar. γ_1 in Equation 2 describes the magnitude of the force dependent on how "good the food tastes." In terms of the flock, this parameter describes how strongly the lead bird is attracted to and follows the food. γ_1 is always a positive non-zero scalar. γ_2 in Equation 3 describes the magnitude of the follow force and is always a positive, non-zero scalar. In terms of the flock, this force describes the force of the non-lead birds to follow the leader bird as it follows the food. κ in Equation 4 describes the magnitude of the flocking force. Being closer to the center of the flock is safer for the birds, so as this parameter increases, the diameter of the flock decreases. ρ in Equation 5 describes the magnitude of the repelling force on the five closest neighbors. As this repelling force increases, the flock diameter increases as well. Finally, δ in Equation 5 acts as a damping constant where large values of δ reduces the effectiveness of the repelling force in relation to the five nearest neighbors.

$$C(t) = sin(\alpha t)cos(\alpha t) \tag{6}$$

5 Explanation of Flock Diameter in Different Scenarios

The flock diameter describes the distance between the two furthest birds in the flock at each time step. In our code, we implemented this by looping through all the birds (outer loop) and then looping through all the other birds (i.e., not including the current bird) to find the Euclidean distance between the current bird and the other bird. Then we kept track of a max of the diameter as we went though the two nested loops, treating it as the official diameter value for any given time step. We expect that the flock diameter will eventually reach an equilibrium between the flocking and repelling forces over time. In the figures below, the flock diameter is seen to remain relatively constant, within 0.2 of an average diameter across the time spectrum. We also note that because the initial coordinates of each bird is random, the change in flock diameter after the first time step is expected to be the most dramatic. The flock diameter for $N = 10$ appears to be the most stable. We propose that this is due to a smaller number of birds that the forces are acting on. The repelling force acts on the five nearest neighbors for each bird. For $N = 10$, the repelling force acts on a relatively larger portion of the flock compared to the repelling force in the case of $N = 100$. This may lead to more chaos in larger N values as there will be more birds to repel in a more confined space. In our example plots below for $N = 10$ and $N = 30$ (Figures 4 and 5), an equilibrium appears to have been reached as the flock diameter appears to oscillate about 1.4 and 1.05, respectively.

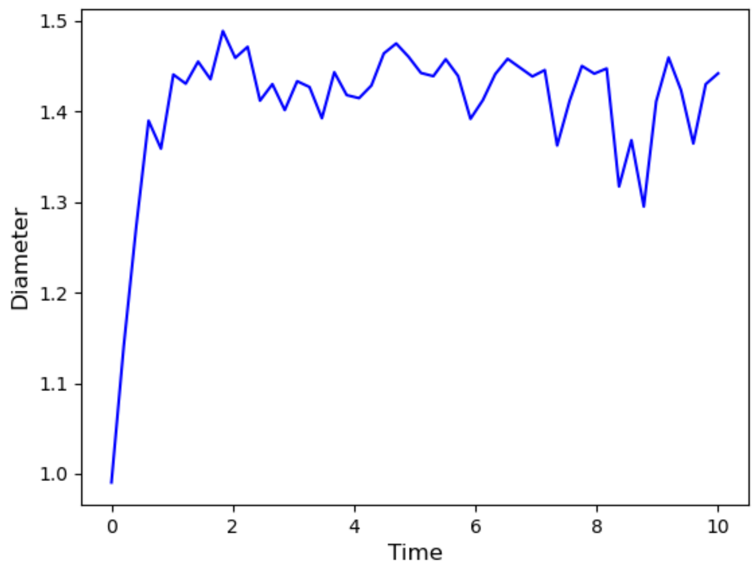


Figure 4: Max. Flock Diameter Plot with N = 10

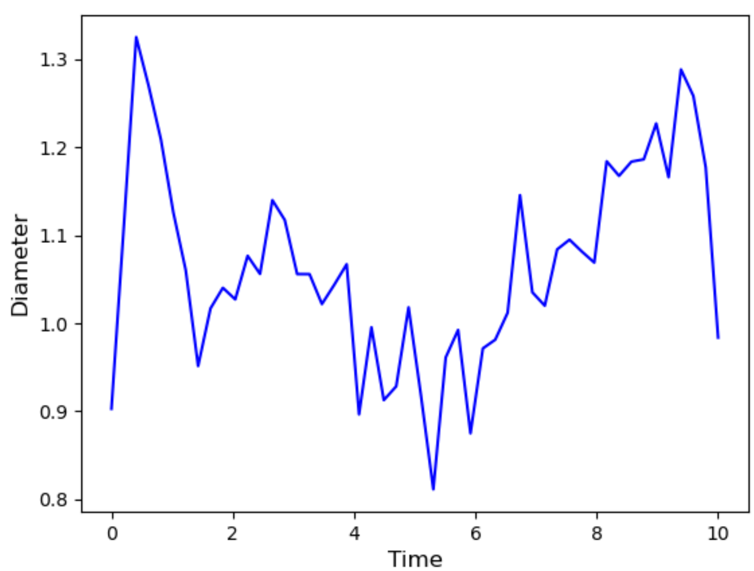


Figure 5: Max. Flock Diameter Plot with N = 30

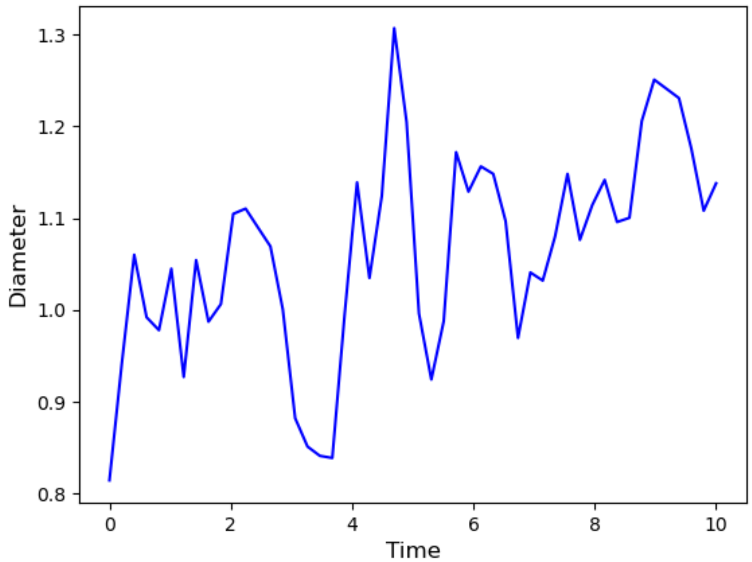


Figure 6: Max. Flock Diameter Plot with N = 100

6 Smelly Bird

We implemented a yellow smelly bird to our code with a κ value of 4 and a ρ value of 3. All the other birds are repelled by it, but the smelly bird tries to stay in the center of the flock. Another note we found was that the smelly bird (yellow cross) seems to lag behind the other birds, which makes sense since the other birds will try to stay away from the smelly bird as it tries to reach closer to the center of the flock.

To implement the smelly bird, we used a similar approach to the flocking force for the portion where the smelly bird wants to be at the center of the flock. As for the birds being repelled by the smelly bird, we used a similar approach to the repelling force with a ρ that is greater in magnitude. We also replaced the summation over the 5 nearest neighbors with just the position of the smelly bird relative to the current bird we are calculating the force for. As mentioned above, the values for κ and ρ are unique and (necessarily) different in the smelly bird case.

NOTE: For our implementation, we let $k = 2$ be the designated index for the smelly bird, so that B_{smelly} is just B_2. All previous forces were also applied for the smelly bird. We just added *extra* forcing components to the smelly bird that amplified the flocking force and the repelling force in the ways mentioned above. The repelling force on the other birds by the smelly bird (Equation 8) will run from $k = 1$, skipping 2, and continuing from 3 up to N due to our definition of B_2 as the smelly bird. Also, we used the same δ parameter from the original repelling force equation (4) for the smelly bird version to keep the forces consistent.

$$F^{fl_s melly}(t, B(t)) = \kappa_s melly(\bar{B}(t) - B_{smelly}(t)), k = 2 \tag{7}$$

$$F^{rep_s melly}(t, B(t)) = \rho_{smelly}\frac{(B_k(t) - B_{smelly}(t))}{(B_k(t) - B_{smelly}(t))^2 + \delta}, k = 1, 3, 4, \ldots N \tag{8}$$

The movie for the smelly bird is located in the git repository:
2020_fall_math471_sands/homeworks/hw5/code_output/smelly_bird_movie

7 Blue Predator

We also implemented a blue predator which is fixed at a random point in space and has a repelling force greater than that of the all the other birds, including the smelly bird and the food source. To implement this, we randomly generated a point in the coordinate space. Then we described a new repelling force, Equation 9, to repel all birds from the predator.

$$F^{rep_p redator}(t, B(t)) = \rho_{pred}\frac{(B_k(t) - B_{pred}(t))}{(B_k(t) - B_{pred}(t))^2 + \delta}, k = 1, 2, 3, 4, \ldots N \tag{9}$$

The movie for the predator is located in the git repository:
2020_fall_math471_sands/homeworks/hw5/code_output/predator_movie