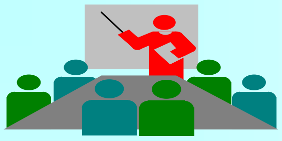
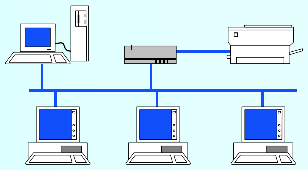



Distributed Systems
600.437
Synchronous Models for Consensus

Department of Computer Science
The Johns Hopkins University

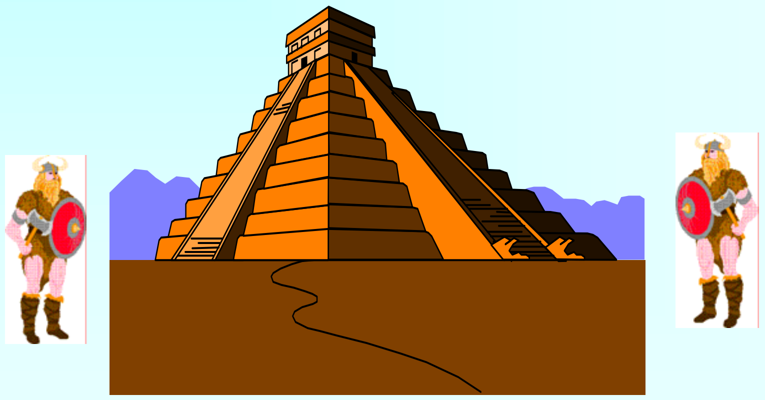
Synchronous Models For Consensus

Lecture 2

Further reading:

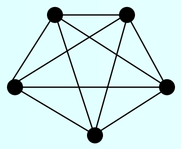
Distributed Algorithms
Nancy Lynch,
Morgan Kaufmann Publishers, 1996.

Distributed Consensus



No Faults
Problem Description

Assumptions:



- *n* processes connected by a full graph.
- Each process starts with an initial value {0, 1}.
- Synchronous setting - solution is required within *r* rounds for some fixed *r*.

No Faults
Problem Description (cont.)

Requirements:

- **Agreement:** All processes decide on the same value.
- **Validity:** If a process decides on a value, there was a process that started with that value.

What if we eliminate the validity requirement ?

No Faults
Problem Description (cont.)

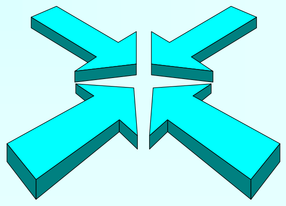
Requirements:

- **Agreement:** All processes decide on the same value.
- **Validity:** If a process decides on a value, there was a process that started with that value.

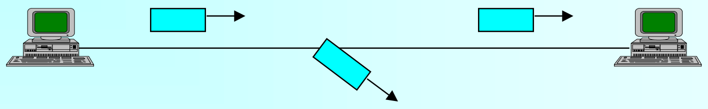
The validity requirement eliminates trivial meaningless solutions.

No Faults
One-Round Algorithm

- Send your value to all the processes.
- If all the values you have (including your own) are 1 then decide 1. Otherwise decide 0.



Message Omissions
Problem Description



Assumptions:

- *n* processes connected by a full graph.
- Each process starts with an initial value {0, 1}.
- Synchronous setting - solution is required within *r* rounds for some fixed *r*.
- Any number of messages might be lost.

Message Omissions Problem Description (cont.)

Requirements:

- **Agreement:** All processes decide on the same value.
- **Validity:** If all processes start with 0, then the decision value is 0; if all processes start with 1 and no message is lost, then the decision value is 1.

Notice that the validity requirement is **weaker** then the original validity requirement.

Message Omissions Consensus is Not Solvable!

Theorem: There is no algorithm that solves the consensus problem for even 2 processes.

Definition: Execution α is **indistinguishable** from execution β with respect to process *p* if in both α and β, *p* has the same initial state and receives exactly the same messages at the same rounds.

$$\alpha \overset{p}{\sim} \beta$$

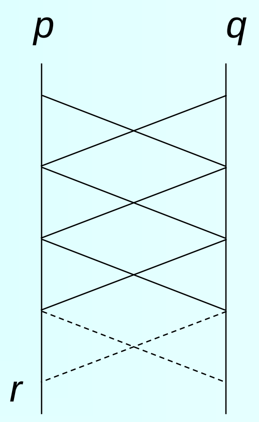
Proof

$\alpha 1$: Both processes start with 1 and no message is lost.

$\alpha 2$: Similar to $\alpha 1$ except that the last message from *p* to *q* is lost.

$\alpha 3$: Similar to $\alpha 2$ except that the last message from *q* to *p* is lost.

$\alpha 1 \overset{p}{\sim} \alpha 2 \quad \alpha 2 \overset{q}{\sim} \alpha 3$



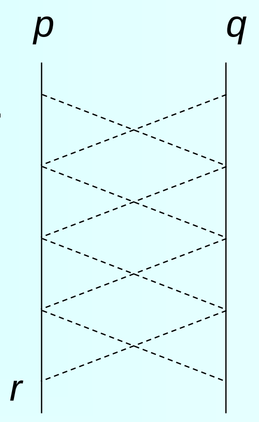
Proof (cont.)

αx: Both processes start with 1 and all messages are lost.

βx: Similar to αx except that *q* starts with 0.

βy: Similar to βx except that *p* starts with 0.

$\alpha x \overset{p}{\sim} \beta x \quad \beta x \overset{q}{\sim} \beta y$



Contradiction

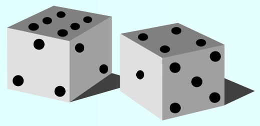
Message Omissions Randomized Consensus

An Adversary is an arbitrary choice of:

- Initial values for all processes.
- Subset of { (*p*1, *p*2, *i*) } where *p*1, *p*2 are processes and *i* is a round number.

The subset represents which messages are lost.

Message Omissions Randomized Solution



Requirements:

- **Agreement:** For any adversary A:

  The probability that some process decides 0 and some process decides 1 is less or equal to ε.

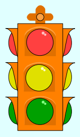
- **Validity:** If all processes start with 0, then the decision value is 0; if all processes start with 1 and no message is lost, then the decision value is 1.

Message Omissions
A Randomized Algorithm

At initialization one specific process, *p*, chooses a *key* at **random**, uniformly from the range [1..*r*].

At each round the processes send the following:

- Initial value.
- *key* (for process *p* only).
- *color*

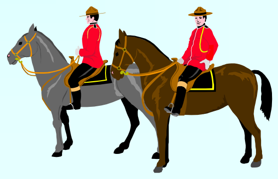


Each process holds a variable *color* initialized to **green**. If red message was received, or a message was missed, the process sets *color* to red.

Message Omissions
A Randomized Algorithm (cont.)

Decision Rule:

A process decides 1 after *r* rounds if it knows that at least one process started with 1, it knows the value of *key*, and it has received all the messages in all the first *key* rounds and all of them were **green**. Otherwise, it decides 0.



Correctness Proof

Set *r* to be an integer that is bigger or equal to the desired 1/ε. The algorithm satisfies the agreement and validity requirements because **for any adversary**:

- If no message is lost the processes all decisions will be identical.
- Look at the first message omitted by the adversary: only if this message is omitted at round *key* there might be disagreement.
- Remember that *key* is selected uniformly at random from the range [1..*r*].

Fail-Stop Faults
Problem Description

Assumptions:

- *n* processes connected by a full graph.
- Each process starts with an initial value {0, 1}.
- Synchronous setting - solution is required within *r* rounds for some fixed *r*.
- The number of Fail-Stop faults is bounded in advance to *f.* A process might fail in the middle of message sending at some round. Once a process fails, it never recovers.
- No omission failures.

Fail-Stop Faults
Problem Description (cont.)

Requirements:

- **Agreement:** All correct processes do decide on the same value.
- **Validity:** If a correct process decides on a value, there was a process that started with that value.

Fail-Stop Faults
f+1 Rounds Algorithm

Each process maintains a vector containing a value for each process {0,1, u}. u = undefined.

round $\left\{ \begin{array}{l} \text{• Send your vector to all processes.} \\ \text{• Update local vector according to the received vectors (in case local vector has a “u”, and any of received vectors contain “0” or “1”).} \end{array} \right.$

- After *f+1* rounds decide according to the local vector. If you have 1 in the vector then decide 1, otherwise decide 0.

Submission: Wednesday, October 1 at the beginning of class.
Homework solution must be typed. You must work on this by yourself – no collaboration is allowed for this.

Homework

Both 337 and 437

-Show a scenario that does not work with only f rounds, for any n processes n>f.

-Prove that the previous algorithm is correct.

Hints:

- First, list what properties are needed to be proven.
- Remember: no process decides before the *f*+1 round.
- The values of the local vectors are identical at all the processes that make it to the *f*+1 round (why?) .

Submission: Wednesday, October 1 at the beginning of class.
Homework solution must be typed. You must work on this by yourself – no collaboration is allowed for this.

Homework (cont.)

Only 437

-Suppose that n=4, and at most one process may lie (say whatever it wants, and maybe different things to different members). Construct the simplest algorithm that solves the consensus problem in this case.

Validity in this case: If a correct process decides on a value, there was a correct process that started with that value.