Distributed Systems
600.437
Communication and Knowledge

Department of Computer Science
The Johns Hopkins University

Communication
and
Knowledge
Lecture 12

Common Knowledge

A group has common knowledge of a fact *p* if they all know *p*, they all know that they all know *p*, they all know that they .... and so on, infinite number of times.


Communication Knowledge and Action

- In many practical systems, common knowledge is not attainable.
- Sometimes, (remember the generals), a lack of common knowledge means inability to act.
- Sometimes, it does not.

Communication

Action   **Knowledge**

The Cheating Husbands Puzzle

- Once upon a time, there was a matriarchal city.
- The women had to pass a logic exam before being allowed to marry.
- The Queen was not required to take that exam, but it was common knowledge that she was truthful.
- The women were obedient to the queen.
- The city was small, so all women heard every shot fired in the city.
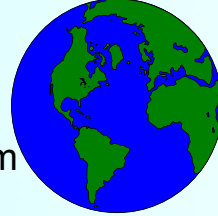
Basic Puzzle

Henrietta I:

- All the women gathered at the city square.
- The queen read the following:
  - **There are (one or more) unfaithful husbands in the city.**

Basic Puzzle (cont.)

- **Although none of you knew before this gathering whether your own husband was faithful, each of you knows which of the other husbands are unfaithful.**
- **I forbid you to discuss it.**
- **Should you discover that your husband is unfaithful, you must shoot him on the midnight of the day you find out about it.**
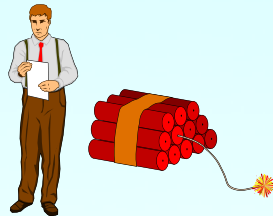
Basic Puzzle (cont..)

Theorem 1.

If there had been n unfaithful husbands, they would all have been shot on the midnight of the n'th day.

Basic Puzzle (cont..)

Theorem 1.

> If there had been n unfaithful husbands, they would all have been shot on the midnight of the n'th day.

- Henrietta I was highly regarded for her wisdom. She ordered her successors to continue the moral fight against male infidelity.

Asynchronous Communication

Henrietta II:

- To avoid the need for gathering, a mail system was installed.
- The first letter informed all the women that every letter is guaranteed to eventually arrive.
- The second letter was an exact copy of Henrietta I 's original message.
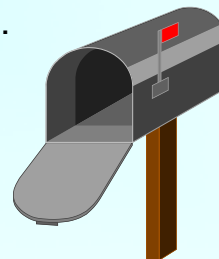
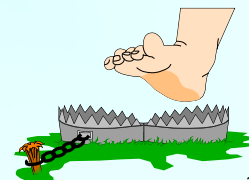# Asynchronous Communication (cont.)

Theorem 2:

If there is more than one unfaithful husband, and an asynchronous channel is used, then no unfaithful husbands are shot.

Asynchronous Communication (cont.)

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If there is more than one unfaithful husband, and an asynchronous channel is used, then no unfaithful husbands are shot.

- Henrietta II suffered great disgrace and died in despair. She ordered her successors not to repeat her mistake.

Weakly Synchronous Communication

Henrietta III:

- The mail system was upgraded.
- The first letter informed all the women that every letter is guaranteed to arrive no later than b days after it is sent.
- The second letter was an exact copy of Henrietta I 's original message.
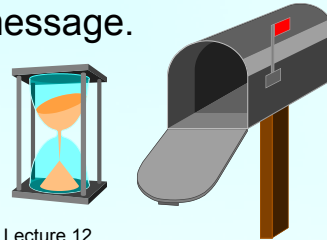
Weakly Synchronous Communication (cont.)

Theorem 3:

A wife that knows of exactly k unfaithful husbands will know that her own husband is unfaithful once kb silent nights pass after the day she receives the letter.

Weakly Synchronous Communication (cont.)

Theorem 3:

A wife that knows of exactly k unfaithful husbands will know that her own husband is unfaithful once kb silent nights pass after the day she receives the letter.

Theorem 4:

The first cheated wives to receive the letter shoot their husbands. All other cheated wives remain forever in doubt.


The Problem

- A letter arrives in no more than 2 days (b=2).
- Susan receives the letter on Monday
- Susan knows that Mary's husband is unfaithful and hears a shot on Tuesday night.
- Susan cannot tell whether:
  - **Mary received the letter on Sunday.**
 - **Mary received the letter on Tuesday.**

The Problem

- A letter arrives in no more than 2 days (b=2).
- Susan receives the letter on Monday
- Susan knows that Mary's husband is unfaithful and hears a shot on Tuesday night.
- Susan cannot tell whether:
  - **Mary received the letter on Sunday.**
  - **Mary received the letter on Tuesday.**

If only she told
them to wait for a while...

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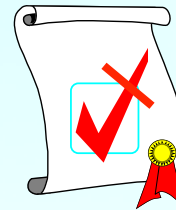
Bribing the Postman

Theorem 7:

A wife that bribes the postman into telling her when the queen had sent the letter, does eventually know whether her own husband is faithful.
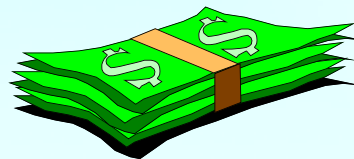
Synchronous Communication

Henrietta IV:

- The Queen gathered everyone and announced that, from this day on, every letter will be stamped with the mailing date and will arrive no later than b days after it is sent.
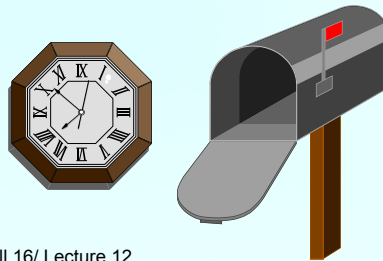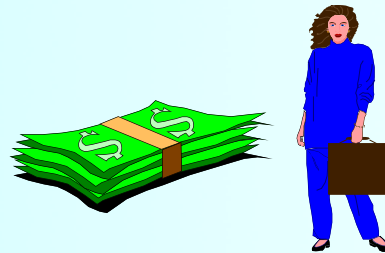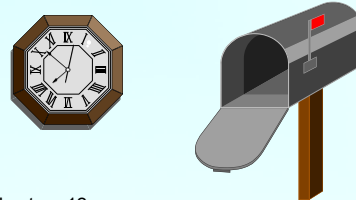

Comparison

Henrietta III:

It takes (n-1)b+1
to eliminate n>1
cheating husbands

Henrietta IV:
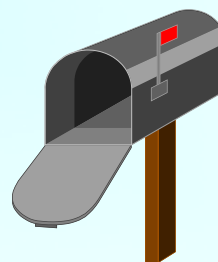
It takes (n-1)+b
to eliminate n>1
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Faulty Behavior

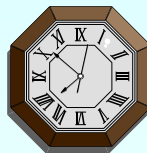Henrietta IV:

- However, due to Henrietta III's injustice, it was no longer clear that the women will obey the Queen.

Faulty Behavior (cont.)

Theorem 8:

If all the cheated wives are disobedient, then all the other wives are in danger of shooting their husbands.

Faulty Behavior (cont.)

Theorem 9:

If it is common knowledge that there is at least one obedient cheated wife, then all the obedient cheated wives will shoot their husbands.

Building on Common Knowledge

Margaret:

- An express mail system was installed. This system guaranteed that each letter will arrive on the same day it is sent.

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- An express mail system was installed. This system guaranteed that each letter will arrive on the same day it is sent.

- Clearly, sending Henrietta I 's original message solves the problem, however, there were too many unfaithful husbands and Queen Margaret did not want to wait...
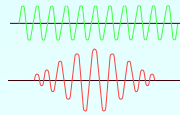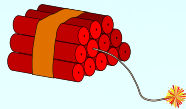
Building on Common Knowledge (cont.)

Theorem 11:

There is a protocol that allows shooting in the air at midnight, in which the cheating husbands are all shot by the third night.

Can you think of it?

Building on Common Knowledge (cont.)

Theorem 11:

There is a protocol that allows shooting in the air at midnight, in which the cheating husbands are all shot by the third night.

Can you think
of it?
- E-mail a solution to cs437-help@dsn.jhu.edu
  by 11:59pm tomorrow
- **Do it by yourself without outside resources of
  any kind and without consulting any other person**
- **State the above red sentence in your e-mail**

A Four Nights Protocol

- If you know of k = 0 (mod 3) cheating husbands, shoot in the air on the first night
- If you know of k = 1 (mod 3) cheating husbands, shoot in the air on the second night
- If you know of k = 2 (mod 3) cheating husbands, shoot in the air on the third night
- If your husband cheats, shoot him on the forth night. (There is at least one cheating husband. So, if shooting occurs only on one of the first 3 nights, all of the husbands cheat.)

Based on their existing communication system, could any of the previous queens implement such a protocol ?