Intermediate Programming

Day 15

Outline

- Exercise 14
- Numerical representation
- Casting
- Review questions

Exercise 14

Convert **char *** message to **int** number

encrypt.c

```
...
int str_to_int( char msg[] , int len )
{
	int v = 0;
	if( len>32 )
	{
		fprintf( stderr , "[WARNING] Not enough bits, overflow\n" );
		len = 32;
	}
	for( int i=0 ; i<len ; i++ ) if( msg[len-i-1]=='1' ) v += pow( 2 , i );
	return v;
}
...
```

Exercise 14

Convert **char *** message to **int** number

- Note that 2^i = 1<<i

encrypt.c

```
...
int str_to_int( char msg[] , int len )
{
	int v = 0;
	if( len>32 )
	{
		fprintf( stderr , "[WARNING] Not enough bits, overflow\n" );
		len = 32;
	}
	for( int i=0 ; i<len ; i++ ) if( msg[len-i-1]=='1' ) v += 1<<i;
	return v;
}
...
```

Exercise 14

Convert `char *` message to `int` number

- Note that 2^i = `1<<i`
- Note that if `i` and `j` are variables with no common bits turned on (`i&j==0`) then `i+j = i|j`

encrypt.c

```
...
int str_to_int( char msg[] , int len )
{
	int v = 0;
	if( len>32 )
	{
		fprintf( stderr , "[WARNING] Not enough bits, overflow\n" );
		len = 32;
	}
	for( int i=0 ; i<len ; i++ ) if( msg[len-i-1]=='1' ) v |= 1<<i;
	return v;
}
...
```

Exercise 14

Convert **int** number to **char *** message

encrypt.c

```
...
void int_to_str( int num_encrypted , char msg_encrypted[] , int len )
{
	for( int i=0 ; i<len ; i++ )
	{
		if( num_encrypted&1 ) msg_encrypted[len-i-1] = '1';
		else                  msg_encrypted[len-i-1] = '0';
		num_encrypted >>= 1;
	}
	if( num_encrypted )
		fprintf( stderr , "[WARNING] Not enough bits, overflow\n"
	);
}
...
```

Exercise 14

Compute the encrypted message by repeatedly left-shifting the message by 1 and XORing.

encrypt.c

```
…
int main( void )
{
	int num_msg = 0;
	char msg[33] = {'\0'};
	int n = -1; // n used in encryption
	…

	int num_encrypted = 0;

	for( int i=0 ; i<n ; i++ ) num_encrypted ^= num_msg<<i;
	…

	return 0;
}
```

Outline

- Exercise 14
- Numerical representation
- Casting
- Review questions

Arithmetic

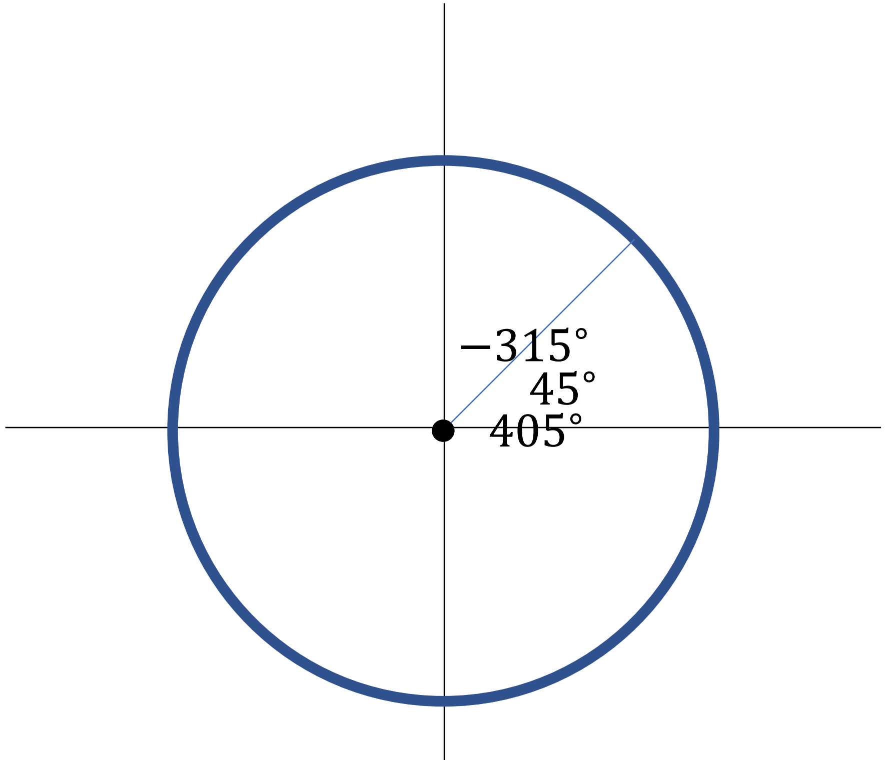
- The sets of integers is a set of numbers
- It has an addition operator, $+$, that takes a pair of integers and returns an integer
  - It contains a zero element, 0, with the property that adding zero to any integer gives back that integer:
  $$a + 0 = a$$
  - Every integer a has an inverse $-a$ such that the sum of the two is zero:
  $$a + (-a) = a - a = 0$$

Modular arithmetic

- Given a positive integer, M, we say that two integers a and b <u>are equivalent modulo M</u>, if there is exists some integer k such that:

$$a \equiv b + k \cdot M$$

  - Degrees in a circle (mod $360°$)
  - Hours on a clock (mod 12)



Modular arithmetic

- Given a positive integer, M, we say that two integers a and b <u>are equivalent modulo M</u>, if there is exists some integer k such that:
$$a \equiv b + k \cdot M$$

- We can represent integers mod M using values in the range $[0, M)$
  - While an integer is bigger than or equal to M, repeatedly subtract M
  - While an integer is less than zero, repeatedly add M

Modular arithmetic

- Given a positive integer, M, we say that two integers a and b <u>are equivalent modulo M</u>, if there is exists some integer k such that:

$$a \equiv b + k \cdot M$$

- We can represent integers mod M using values in the range $[0, M)$
- Or, we can represent integers mod M using the range $[-10, M - 10)$
- Or, we can represent integers mod M using the range $\left[-\frac{M}{2}, \frac{M}{2}\right)$

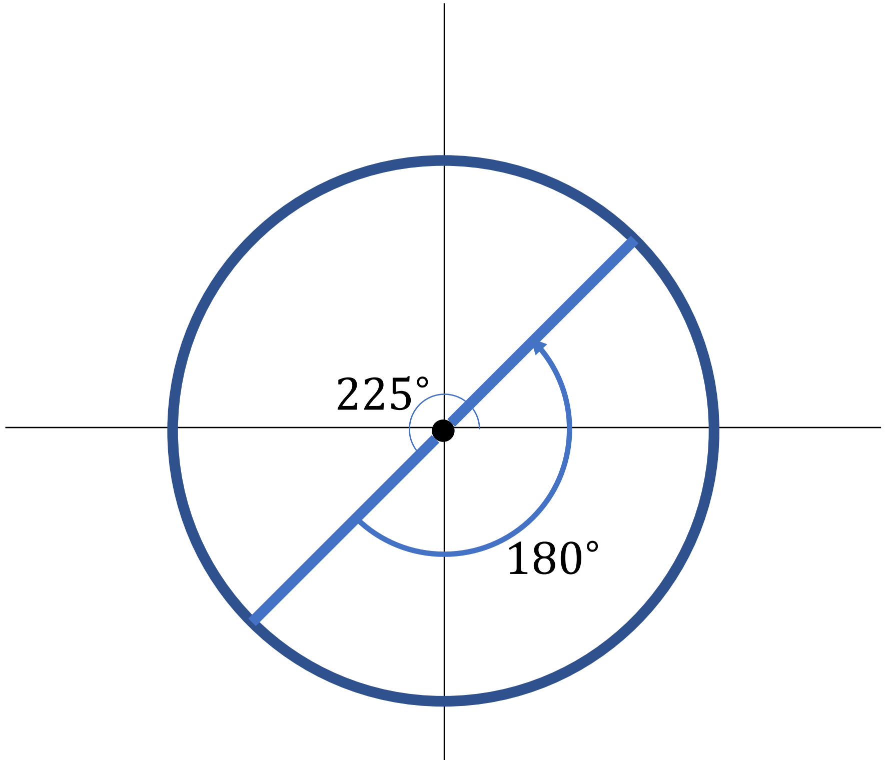
Modular arithmetic

- Given a positive integer, M, we say that two integers a and b <u>are equivalent modulo M</u>, if there is exists some integer k such that:

$$a \equiv b + k \cdot M$$

- We can add numbers modulo M:

$$225^\circ + 180^\circ = 405^\circ$$



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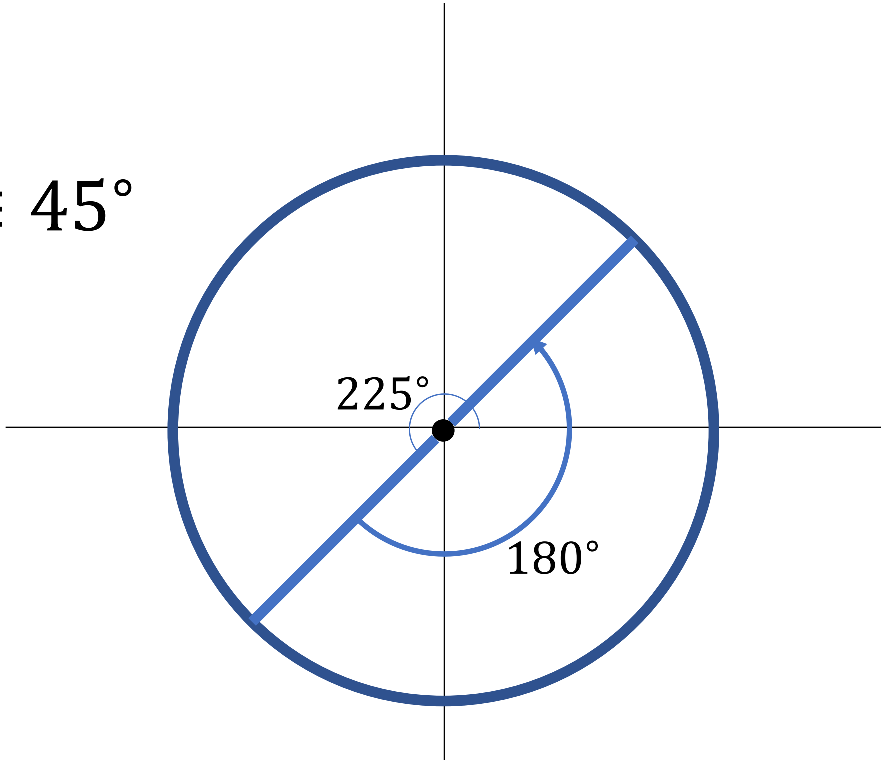

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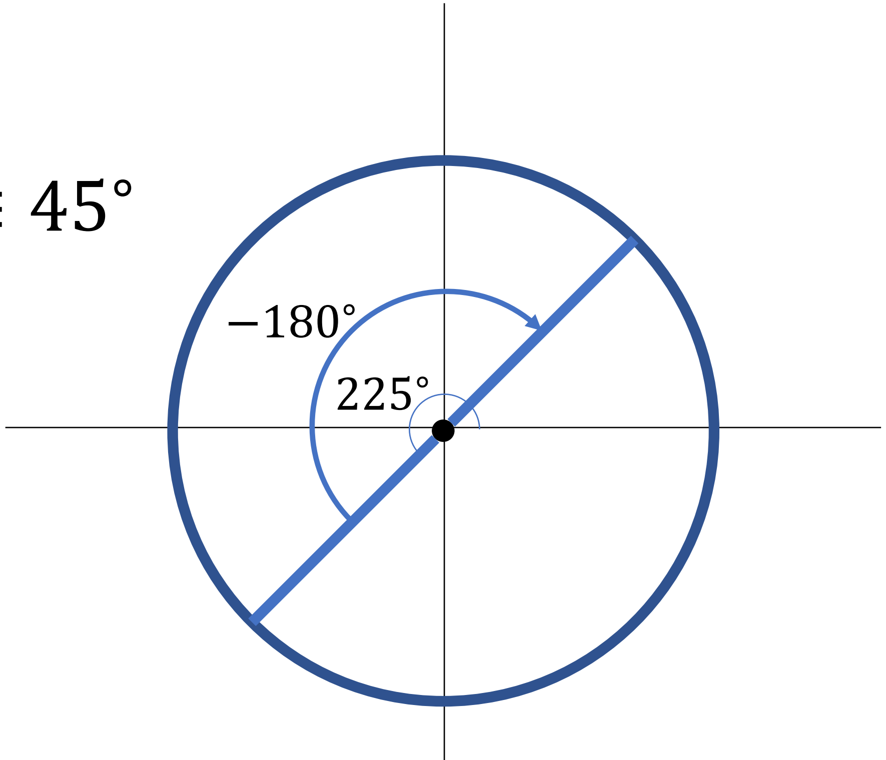
- We can add numbers modulo M:

$$225° - 180° \equiv 225° + 180° = 405° \equiv 45°$$



Modular arithmetic

- Given a positive integer, M, we say that two integers a and b <u>are equivalent modulo M</u>, if there is exists some integer k such that:

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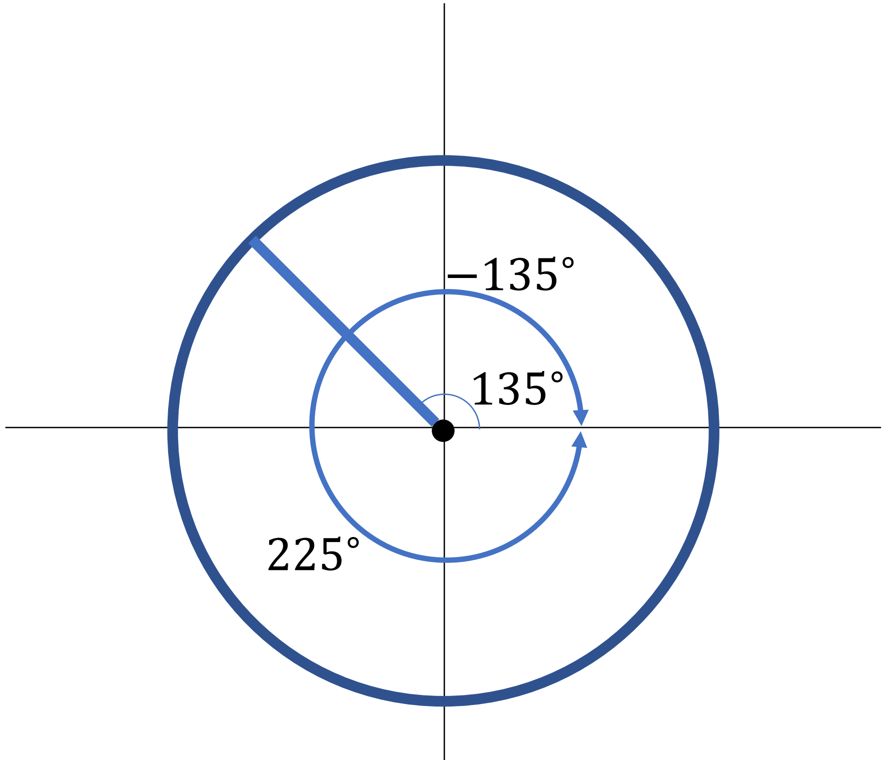
- We can add numbers modulo M

- For any integer a, the negative of a modulo M can be represented by $M - a$



Modular arithmetic

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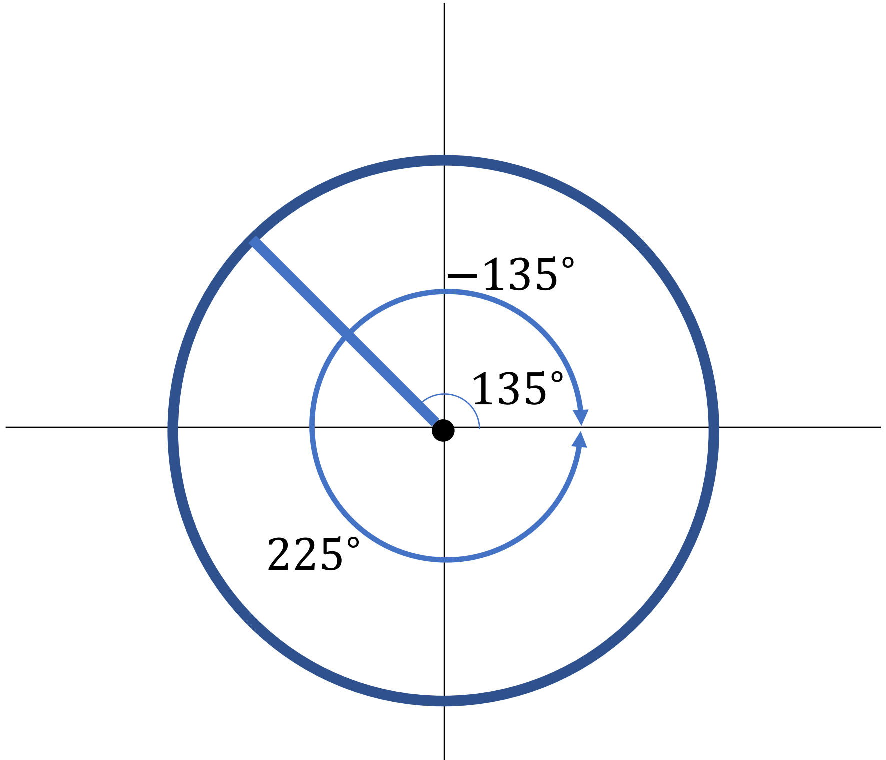
- We can add numbers modulo M

- For any integer a, the negative of a modulo M can be represented by $M - a$:
$$a + (M - a) = (a - a) + M$$



Modular arithmetic

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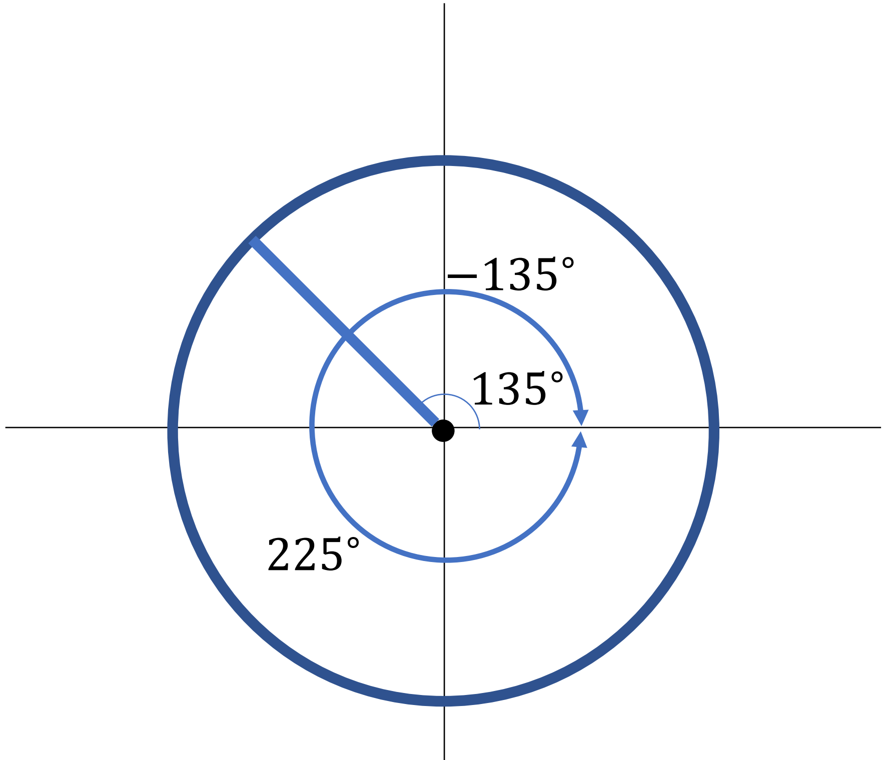
$$a \equiv b + k \cdot M$$

- We can add numbers modulo M

- For any integer a, the negative of a modulo M can be represented by $M - a$:
$$a + (M - a) = (a - a) + M = M$$

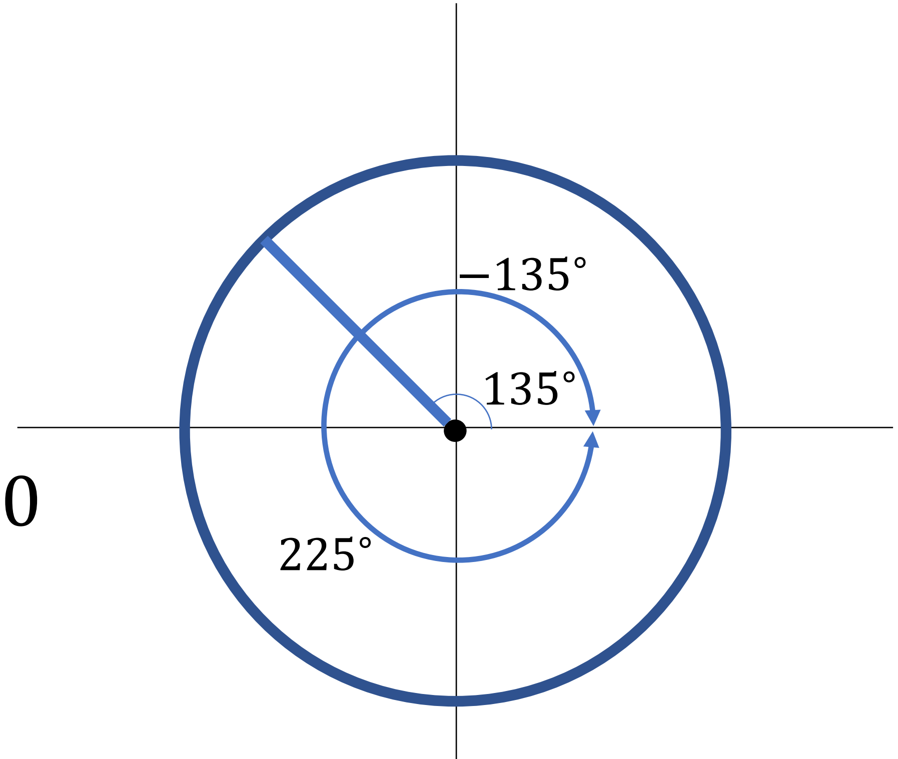


Modular arithmetic

- Given a positive integer, M, we say that two integers a and b <u>are equivalent modulo M</u>, if there is exists some integer k such that:
$$a \equiv b + k \cdot M$$

- We can add numbers modulo M

- For any integer a, the negative of a modulo M can be represented by $M - a$:
$$a + (M - a) = (a - a) + M = M \equiv 0$$



Bases (decimal)

- When we write out an integer in decimal notation, we are representing it as a sum of "one"s, "ten"s, "hundred"s, etc.

$$\begin{aligned} 365 &= 3 \times 100 + 6 \times 10 + 5 \times 1 \\ &= 3 \times 10^2 + 6 \times 10^1 + 5 \times 10^0 \end{aligned}$$

- This is unique because each digit is in the range 0 to 9, written $[0,10)$

Bases (decimal)

- We add two numbers by adding the digits from smallest to largest
  - If the sum of digits falls outside the range $[0,10)$ we carry

$$
\begin{array}{cccc}
 & 3 & 6 & 5 \\
+ & 6 & 7 & 3 \\
\hline
\end{array}
$$

Bases (decimal)

- We add two numbers by adding the digits from smallest to largest
  - If the sum of digits falls outside the range $[0,10)$ we <u>carry</u>

$$\begin{array}{cccc} & 3 & 6 & 5 \\ + & 6 & 7 & 3 \\ \hline & & & 8 \end{array}$$

Bases (decimal)

- We add two numbers by adding the digits from smallest to largest
  - If the sum of digits falls outside the range $[0,10)$ we <u>carry</u>

$$
\begin{array}{cccc}
 & 1 & & \\
 & 3 & 6 & 5 \\
+ & 6 & 7 & 3 \\
\hline
 & & 3 & 8
\end{array}
$$

Bases (decimal)

- We add two numbers by adding the digits from smallest to largest
  - If the sum of digits falls outside the range $[0,10)$ we carry

$$
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1 & 1 & & \\
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\hline
 & 0 & 3 & 8
\end{array}
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$$\begin{array}{cccc} 1 & 1 & & \\ & 3 & 6 & 5 \\ + & 6 & 7 & 3 \\ \hline 1 & 0 & 3 & 8 \end{array}$$

Bases (decimal)

Q: If we use three digits, how many numbers can we represent?

A: $1000 = 10^3$ (including zero)

<u>Note</u>:

- The sum of two numbers represented using three digits may require four digits to store:

$$\begin{array}{cccc} & 3 & 6 & 5 \\ + & 6 & 7 & 3 \\ \hline 1 & 0 & 3 & 8 \end{array}$$

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Q: If we use three digits, how many numbers can we represent?

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Note:

- The sum of two numbers represented using three digits may require four digits to store:

$$\begin{array}{cccc} & 3 & 6 & 5 \\ + & 6 & 7 & 3 \\ \hline & 0 & 3 & 8 \end{array}$$

- If we only use three digits, we lose the leading digit to overflow
- This is the same as the number mod 10^3

Bases (binary)

- We can also write out numbers in base two

$$\begin{aligned}(s_3 s_2 s_1 s_0)_2 &= s_3 \times 8 \;\; + s_2 \times 4 \;\; + s_1 \times 2 \;\; + s_0 \times 1 \\ &= s_3 \times 2^3 + s_2 \times 2^2 + s_1 \times 2^1 + s_0 \times 2^0\end{aligned}$$

where s_0, s_1, s_2, s_3 are either 0 or 1.

Bases (binary)

- As before, we add two numbers by adding the digits from smallest to largest, carrying if the sum is larger than 1:

$$\begin{array}{rccc} & (1 & 1 & 0)_2 \\ + & (0 & 1 & 1)_2 \\ \hline (& & &)_2 \end{array}$$

Bases (binary)

- As before, we add two numbers by adding the digits from smallest to largest, carrying if the sum is larger than 1:

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\begin{array}{ccccc}
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\hline
(& & & 1)_2
\end{array}
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+ & (0 & 1 & 1)_2 \\
\hline
(& 0 & 0 & 1)_2
\end{array}
$$

Bases (binary)

- As before, we add two numbers by adding the digits from smallest to largest, carrying if the sum is larger than 1:

$$
\begin{array}{cccc}
1 & 1 & & \\
 & (1 & 1 & 0)_2 \\
+ & (0 & 1 & 1)_2 \\
\hline
(1 & 0 & 0 & 1)_2
\end{array}
$$

Bases (binary)

Q: Using three digits in base two, how many numbers can we represent?

A: 8 (including zero)

Note:

- As before, the sum of two numbers represented using three digits may require four digits to store:

$$\begin{array}{ccccc} & 1 & 1 & & \\ & & (1 & 1 & 0)_2 \\ + & & (0 & 1 & 1)_2 \\ \hline (1 & & 0 & 0 & 1)_2 \end{array}$$

Bases (general)

Q: Using three digits in base two, how many numbers can we represent?

A: 8 (including zero)

<u>Note</u>:

- As before, the sum of two numbers represented using three digits may require four digits to store:

$$\begin{array}{rccc} & (1 & 1 & 0)_2 \\ + & (0 & 1 & 1)_2 \\ \hline & (0 & 0 & 1)_2 \end{array}$$

- If we only use three digits, we lose the leading digit to <u>overflow</u>
- This is the same as the number mod 8.

Bases (decimal)

- Given a number in base 10:

$$16{,}384 = 1 \times 10^4 + \ 6 \times 10^3 + 3 \times 10^2 + 8 \times 10^1 + 4 \times 10^0$$

we can get an expression of the number in base 100 by grouping digits:

$$16{,}384 = 1 \times 100^2 + \ 63 \times 100^1 + 84 \times 100^0$$

Similarly, we can get an expression of the number in base 1000, etc.

Bases (two)

- Similarly, given a number in base two:
$$(s_3 s_2 s_1 s_0)_2 = s_3 \times 8 + s_2 \times 4 + s_1 \times 2 + s_0 \times 1$$
we can get an expression of the number in base 4 by grouping digits:
$$(s_3 s_2 s_1 s_0)_2 = (s_3 \times 2 + s_2) \times 4 + (s_1 \times 2 + s_0) \times 1$$
Similarly, we can get an expression of the number in base 8, or base 16, or ...

Bases (examples)

What is this value in base 10?

- $(1101)_2 =$

Bases (examples)

What is this value in base 10?

- $$\begin{aligned}(1101)_2 &= 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 \\ &= 8 + 4 + 1 \\ &= 13\end{aligned}$$

Bases (examples)

What is the value in base 2?

- $27 =$

Bases (examples)

What is the value in base 2?

- $$\begin{aligned} 27 &= 16 + 8 + 4 + 2 + 1 \\ &= 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1 \\ &= (11011)_2 \end{aligned}$$

Bases (examples)

What is the value in base 2?

- $$\begin{aligned} 27 &= 16 + 8 + 4 + 2 + 1 \\ &= 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1 \\ &= (11011)_2 \end{aligned}$$

What is the value in base 4?

- $27 =$

Bases (examples)

What is the value in base 2?

- $27 = 16 + 8 + 4 + 2 + 1$
  $= 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1$
  $= (11011)_2$

What is the value in base 4?

- $27 = 16 + 8 + 3$
  $= 1 \times 16 + 2 \times 4 + 3 \times 1$
  $= (123)_4$

Bases (in the wild)

- Decimal (base 10)
  - We have ten fingers

- Sexagesimal (base 60):
  - Minutes / seconds
  - Easy to tell if a number is divisible by 2, 3, 4, 5, 6 , 10, 12, 15, or 30
  - Dates back to the Babylonians

Bases (in the wild)

- Binary (base 2)
  - Numbers in a computer

- Hexadecimal a.k.a. hex (base 16)
  - Numbers in a computer ($16 = 2^4$)
    - We can easily convert binary to hex by grouping sets of four digits
    - We get a more compact representation, replacing 4 digits with 1

Bases (in the wild)

- Binary (base 2)
  - Numbers in a computer

- Hexadecimal a.k.a. hex (base 16)

  Q: How should we separate the digits? $(115)_{16}$
  - $(115)_{16} = 1 \times 16^2 + 1 \times 16^1 + \quad 5 \times 16^0$
  - $(115)_{16} = \qquad 1 \times 16^1 + 15 \times 16^0$
  - $(115)_{16} = \qquad 11 \times 16^1 + \quad 5 \times 16^0$

Bases (in the wild)

- Binary (base 2)
  - Numbers in a computer

- Hexadecimal a.k.a. hex (base 16)

  Q: How should we separate the digits? $(115)_{16}$

  A: Use numbers and letters:
  - $\{0,1,2,3,4,5,6,7,8,9\}$ to represent numbers in the range $[0,10)$
  - $\{a, b, c, d, e, f\}$ to represent values in the range $[10,16)$:
    - $(115)_{16} = 1 \times 16^2 + 1 \times 16^1 + \ \ 5 \times 16^0$
    - $(1f)_{16} \quad = \qquad\qquad\quad 1 \times 16^1 + 15 \times 16^0$
    - $(b5)_{16} \quad = \qquad\qquad\quad 11 \times 16^1 + \ \ 5 \times 16^0$

Representing integers

- On most machines, [unsigned] ints are represented using 4 bytes*
  - Each byte is composed of 8 bits

  ⇒ An [unsigned] int is represented by 32 bits
  - Each bit can be either “on” or “off”

  ⇒ An [unsigned] int is represented in binary using 32 digits with values 0 or 1

  ⇒ An [unsigned] int can have one of 2^{32} values

*“[...]” notation indicates an optional argument

Representing integers

- On most machines, [unsigned] ints are represented using 4 bytes
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  ⇒ An [unsigned] int can have one of 2^{32} values

```c
#include <stdio.h>
int main( void )
{
        int a = 30;
        printf( "%d\n" , a );
        return 0;
}
```

```
>> ./a.out
30
>>
```

On the machine, a is assigned the value:

$$a \leftarrow (\,00000000\ 00000000\ 00000000\ 00011110\,)_2$$

$$a \leftarrow (\,00\ 00\ 00\ 1e)_{16}$$

Representing integers

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  ⇒ An [unsigned] int is represented in binary using 32 digits with values 0 or 1

  ⇒ An [unsigned] int can have one of 2^{32} values

```
#include <stdio.h>
int main( void )
{
        int a = 0x1e;
        printf( "%d\n" , a );
        return 0;
}
```

```
>> ./a.out
30
>>
```

On the machine, a is assigned the value:

$$a \leftarrow (\,00000000\ 00000000\ 00000000\ 00011110\,)_2$$

$$a \leftarrow (\,00\ 00\ 00\ 1e)_{16}$$

- You can assign using base 16 by preceding the number with 0x to indicate hex

Representing integers

- On most machines, [unsigned] ints are represented using 4 bytes
  - Each byte is composed of 8 bits

  ⇒ An [unsigned] int is represented by 32 bits
  - Each bit can be either "on" or "off"

  ⇒ An [unsigned] int is represented in binary using 32 digits with values 0 or 1

  ⇒ An [unsigned] int can have one of 2^{32} values

```c
#include <stdio.h>
int main( void )
{
	int a = 30;
	printf( "%x\n" , a );
	return 0;
}
```

```
>> ./a.out
1e
>>
```

On the machine, a is assigned the value:

$$\mathrm{a} \leftarrow (\,00000000\ 00000000\ 00000000\ 00011110\,)_2$$

$$\mathrm{a} \leftarrow (\,00\ 00\ 00\ 1e)_{16}$$

- You can assign using base 16 by preceding the number with 0x to indicate hex
- You can print the base 16 representation by using %x for formatting

Representing integers

- On most machines, [unsigned] chars are represented using 1 byte
  - ⇒ A [unsigned] char can have one of 2^8 values
- On most machines, [unsigned] long ints are represented using 8 bytes
  - A [unsigned] long int can have one of 2^{64} values

Representing integers*

⇒ An [unsigned] char can have one of $2^8 = 256$ values

⇒ [unsigned] chars are integer values mod 2^8

- unsigned char: We will use the range $[0,256)$ to represent integers
- char: We will use the range $[-128,128)$ to represent integers

Q: What's the difference?
Integers mod 2^8 are integers mod 2^8, regardless of the representation!!!

*For simplicity the following discussion will focus on chars, though it holds for other integer representations (e.g. ints and long ints)

Representing integers

- unsigned char: We will use the range $[0,256)$ to represent integers
- char: We will use the range $[-128,128)$ to represent integers

Q: What's the difference?

A: Is $125 < 129 \bmod 256$?

Since $129 \equiv -127 \bmod 256$,
it depends on the range we use

```
#include <stdio.h>
int main( void )
{
        unsigned char c1 = 125 , c2 = 129;
        printf( "%d\n" , c1<c2 );
        return 0;
}
```

```
>> ./a.out
1
>>
```

Representing integers

- unsigned char: We will use the range $[0,256)$ to represent integers
- char: We will use the range $[-128,128)$ to represent integers

Q: What's the difference?

A: Is $125 < 129 \bmod 256$?

Since $129 \equiv -127 \bmod 256$,
it depends on the range we use

```
#include <stdio.h>
int main( void )
{
        char c1 = 125 , c2 = 129;
        printf( "%d\n" , c1<c2 );
        return 0;
}
```

```
>> ./a.out
0
>>
```

Representing integers

- Addition:
  We add two numbers, $a + b$, by adding the digits from smallest to largest
  - We carry as necessary
  - And we cut off at 8 bits

$$
\begin{array}{r}
1\,1\quad\;\; 11 \\
(11010011)_2 \\
+(01000110)_2 \\
\hline
(1\,00011001)_2 \\
=(00011001)_2
\end{array}
$$

Representing integers

- Addition:
  We add two numbers, $a + b$. by adding the digits from smallest to largest
  - We carry as necessary
  - And we cut off at 8 bits

$$
\begin{array}{r}
1\,1\quad\;\; 11\;\;\;\;\;\;\; \\
(11010011)_2 \\
+(01000110)_2 \\
\hline
(00011001)_2
\end{array}
$$

Q: What about subtraction, $a - b$?

Representing integers

- <u>Addition</u>:
  We add two numbers, $a + b$. by adding the digits from smallest to largest
  - We carry as necessary
  - And we cut off at 8 bits

$$
\begin{array}{r}
1\,1\quad\;\;\, 11 \\
(11010011)_2 \\
+(01000110)_2 \\
\hline
(00011001)_2
\end{array}
$$

Q: What about subtraction, $a - b = a + (-b)$?

Equivalently, how do we define the negative of a number?

Negation

- Recall:
  The negative of an integer is the number we would have to add to get back zero.
- Defining negative one:
  - Mod 256, we have $-1 \equiv 255 = (11111111)_2$

Negation

- <u>Recall</u>:
  The negative of an integer is the number we would have to add to get back zero.
- <u>Defining negatives in general</u>:
  1. Given a binary value in 8 bits:
     $$(10011101)_2$$
  2. We can flip the bits:
     $$(01100010)_2$$
  3. Adding the two values we get $255 \equiv -1$:
     $$(11111111)_2$$
  4. Adding one to that we get 0

Negation

- Recall:
  The negative of an integer is the number we would have to add to get back zero.
- 2’s complement:
  To get the binary representation of the negative of a number
  1. Flip the bits
  2. Add 1

Floating point value representation

$$\pm b \times 2^e$$

- On most machines, `float`s are represented using 4 bytes (32 bits)
  - These are (roughly) used to encode:
    - The sign ($\pm$): 1 bit
    - The signed (integer) exponent (e): 8 bits*
    - The unsigned (integer) base (b): 23 bits

*This is what makes the point float

Floating point value representation

[WARNING]:

- Adding floating point values requires aligning their precisions first*

$$b_1 \times 2^{e_1} + b_2 \times 2^{e_2}$$
$$\Downarrow$$
$$(b_1 \times 2^{e_1 - e_2}) \times 2^{e_2} + b_2 \times 2^{e_2}$$
$$\Downarrow$$
$$(b_1 / 2^{e_2 - e_1} + b_2) \times 2^{e_2}$$

⇒ If b_1 is less than $2^{e_2 - e_1}$, then $b_1 / 2^{e_2 - e_1}$ will become zero

⇒ Addition of floating points may not be associative:

$$(a + b) + c \,!= a + (b + c)$$

*Assume $e_2 > e_1$

Floating point value representation

[WARNING]:

- Adding floating point values requires aligning their precisions first*

```
#include <stdio.h>
int main( void )
{
	float a = 1e-4f , b = 1e+4f , c = -b;
	printf( "%.3e %.3e %.3e\n" , a , b , c );
	printf( "%.3e %.3e\n" , ( a + b ) + c , a + ( b + c ) );
	return 0;
}
```

```
>> ./a.out
1.000e-04 1.000e+04 -1.000e+04
0.000e+00 1.000e-04
>>
```

⇒ If $b_1 \times$ … zero

⇒ Additio…

… $+ c)$

*Assume $e_2 > e_1$

Floating point value representation

$$\pm b \times 2^{e}$$

- On most machines, `double`s are represented using 8 bytes (64 bits)
  - These are (roughly) used to encode:
    - The sign ($\pm$): 1 bit
    - The signed (integer) exponent (e): 11 bits
    - The unsigned (integer) base (b): 52 bits

Bit-wise operations: **Integer types only**

Recall:

- We can determine if a bit is on or off using << and &

```
#include <stdio.h>
int main( void )
{
	char a = 5;        // (00000101)_2
	char mask = 1<<2;// (00000100)_2
	char c = a & mask;
	printf( "%d %d\n" , c , c!=
	return 0;
}
```

```
>> ./a.out
4 1
>>
```

Bit-wise operations: **Integer types only**

Recall:

- We can determine if a bit is on or off using << and &
- Or we can use >> and &

```
#include <stdio.h>
int main( void )
{
	char a = 5;		// (00000101)_2
	char b = a>>2;		// (00000001)_2
	char c = b & 1;
	printf( "%d %d\n" , c , c!
	return 0;
}
```

```
>> ./a.out
1 1
>>
```

Bit-wise operations: **Integer types only**

Recall:

- We can determine if a bit is on or off using << and &
- Or we can use >> and &

Note:

Integers in $[0,128)$ all have a binary representation of the form:

$$(0\ *******)$$

Integers in $[128,256) \equiv [-128,0)$ all have a binary representation of the form:

$$(1\ *******)$$

Bit-wise operations: Integer types only

Recall:

- We can determine if a bit is on or off using << and &
- Or we can use >> and &
- We can determine the sign by testing the highest (a.k.a. most significant) bit

```
#include <stdio.h>
int main( void )
{
	char a = 5;		// (00000101)_2
	char b = -4;		// (11111100)_2
	char mask = 1<<7;// (10000000)_2
	printf( "%d %d\n" , ( a & mask )!=0 , ( b & mask )!=0 );
	return 0;
}
```

```
>> ./a.out
0 1
>>
```

Bit-wise operations: Integer types only

Recall:

- We can (
- Or we ca
- We can (ificant) bit*

Note:
We set the mask to 1<<7 because chars are 8 bits long.
For ints we would use a mask of 1<<31.
Etc.

```
#include <stdio.h>
int main( void )
{
        char a = 5;          // (00000101)_2
        char b = -4;         // (11111100)_2
        char mask = 1<<7;// (10000000)_2
        printf( "%d %d\n" , ( a & mask )!=0 , ( b & mask )!=0 );
        return 0;
}
```

```
>> ./a.out
```

*This assumes that the most significant bit is on the left.
It's true for most (big-endian) machines but should not be assumed.

Outline

- Exercise 14
- Numerical representation
- Casting
- Review questions

Casting between types (numbers)

When you assign a value to a variable, the right-hand-side (RHS) is implicitly converted (a.k.a. cast) to the type of the left-hand-side (LHS)

```
<type-1> lhs;
<type-2> rhs;
lhs = rhs;
```

Casting between types (numbers)

When you assign a value to a variable, the right-hand-side (RHS) is implicitly converted (a.k.a. cast) to the type of the left-hand-side (LHS)

- If both are integers and sizeof(LHS)>=sizeof(RHS)
  ⇒ the conversion happens without loss of information

```
#include <stdio.h>
int main( void )
{
    char c = 'a';
    int i = c;
    printf( "%d -> %d\n" , c , i );
    return 0;
}
```

```
>> ./a.out
97 -> 97
>>
```

Casting between types (numbers)

When you assign a value to a variable, the right-hand-side (RHS) is implicitly converted (a.k.a. cast) to the type of the left-hand-side (LHS)

- If both are integers and sizeof(LHS)<sizeof(RHS)
  ⇒ an implicit "modulo" operation is performed
  (modulo 2^b where b is the number of bits in the LHS)

```
#include <stdio.h>
int main( void )
{
    int i = 511;
    char c = i;
    printf( "%d -> %d\n" , i , c );
    return 0;
}
```

```
>> ./a.out
511 -> -1
>>
```

Casting between types (numbers)

When you assign a value to a variable, the right-hand-side (RHS) is implicitly converted (a.k.a. cast) to the type of the left-hand-side (LHS)

- If both are floats and sizeof(LHS)>=sizeof(RHS)
  ⇒ the conversion happens without loss of information

```
#include <stdio.h>
int main( void )
{
    float f = 1.5;
    double d = f;
    printf( "%.8f -> %.8f\n" , f , d );
    return 0;
}
```

```
>> ./a.out
1.50000000 -> 1.50000000
>>
```

Casting between types (numbers)

When you assign a value to a variable, the right-hand-side (RHS) is implicitly converted (a.k.a. cast) to the type of the left-hand-side (LHS)

- If both are floats and `sizeof( LHS )<sizeof( RHS )`
  ⇒ rounding is performed

```
#include <stdio.h>
int main( void )
{
    double d = 1.7;
    float f = d;
    printf( "%.8f -> %.8f\n" , d , f );
    return 0;
}
```

```
>> ./a.out
1.70000000 -> 1.70000005
>>
```

Casting between types (numbers)

When you assign a value to a variable, the right-hand-side (RHS) is implicitly converted (a.k.a. cast) to the type of the left-hand-side (LHS)

- If the LHS is an integer and the RHS is a floating point value
  ⇒ the fractional part is discarded

```
#include <stdio.h>
int main( void )
{
    double d = -3.6;
    int i = d;
    printf( "%.8f -> %d\n" , d , i );
    return 0;
}
```

```
>> ./a.out
-3.60000000 -> -3
>>
```

Note that this is not the same thing as rounding down to the nearest integer

Casting between types (numbers)

When you assign a value to a variable, the right-hand-side (RHS) is implicitly converted (a.k.a. cast) to the type of the left-hand-side (LHS)

- If the LHS is a floating point value and the RHS is an integer
  ⇒ the closest floating point representation is used

```
#include <stdio.h>
int main( void )
{
    int i = 123456789;
    float f = i;
    printf( "%d -> %.0f\n" , i , f );
    return 0;
}
```

```
>> ./a.out
123456789 -> 123456792
>>
```

Casting between types (numbers)

When you assign a value to a variable, the right-hand-side (RHS) is implicitly converted (a.k.a. cast) to the type of the left-hand-side (LHS)

The same rules apply when passing values to/from a function

```
#include <stdio.h>
char foo( unsigned char c ){ return c; }
int main( void )
{
    double d = 511.5;
    float f = foo( d );
    printf( "%g -> %g\n" , d , f );
    return 0;
}
```

```
>> ./a.out
511.5 -> -1
>>
```

Casting between types (numbers)

When you assign a value to a variable, the right-hand-side (RHS) is implicitly converted (a.k.a. cast) to the type of the left-hand-side (LHS)

The same rules apply when passing values to/from a function

- double → unsigned char:
  511.5 → 511 → 255

```
#include <stdio.h>
char foo( unsigned char c ){ return c; }
int main( void )
{
    double d = 511.5;
    float f = foo( d );
    printf( "%g -> %g\n" , d , f );
    return 0;
}
```

```
>> ./a.out
511.5 -> -1
>>
```

Casting between types (numbers)

When you assign a value to a variable, the right-hand-side (RHS) is implicitly converted (a.k.a. cast) to the type of the left-hand-side (LHS)

The same rules apply when passing values to/from a function

- double → unsigned char:
  511.5 → 511 → 255
- unsigned char → char:
  255 → -1

```
#include <stdio.h>
char foo( unsigned char c ){ return c; }
int main( void )
{
    double d = 511.5;
    float f = foo( d );
    printf( "%g -> %g\n" , d , f );
    return 0;
}
```

```
>> ./a.out
511.5 -> -1
>>
```

Casting between types (numbers)

When you assign a value to a variable, the right-hand-side (RHS) is implicitly converted (a.k.a. cast) to the type of the left-hand-side (LHS)

The same rules apply when passing values to/from a function

- double → unsigned char:
  511.5 → 511 → 255
- unsigned char → char:
  255 → -1
- char → float:
  -1 → -1.f

```
#include <stdio.h>
char foo( unsigned char c ){ return c; }
int main( void )
{
    double d = 511.5;
    float f = foo( d );
    printf( "%g -> %g\n" , d , f );
    return 0;
}
```

```
>> ./a.out
511.5 -> -1
>>
```

Casting between types (numbers)

When casting, the types are ranked:

- Larger size integers/floats are “higher rank”

  char < int < long
  unsigned char < unsigned int < unsigned long
  float < double

- Unsigned integers are “higher rank” than signed integers

  char < unsigned char < int < unsigned int < long < unsigned long
  float < double

- Floating point values are “higher rank” than integers

  char < unsigned char < int < unsigned int < long < unsigned long < float < double

Casting between types (numbers)

When casting, the types are ranked:

char < unsigned char < int < unsigned int < long < unsigned long < float < double

When we cast from lower rank to higher rank, we are **promoting**.

When we cast from higher rank to lower rank, we are **narrowing**.

Casting between types (numbers)

When performing a binary operation (arithmetic or comparison) the "lower rank" operand is implicitly promoted:

char < unsigned char < int < unsigned int < long < unsigned long < float < double

```
#include <stdio.h>
int main( void )
{
    int i = -1;
    unsigned int ui = 1;
    if( i<ui ) printf( "hi\n" );
    else printf( "bye\n" );
    return 0;
}
```

```
>> ./a.out
bye
>>
```

Casting between types (numbers)

When performing a binary operation (arithmetic or comparison) with different types the "lower rank" operand is implicitly promoted:

char < unsigned char < int < unsigned int < long < unsigned long < float < double

```
#include <stdio.h>
int main( void )
{
    int i = 2;
    double d = 2.5;
    i = i * d;
    printf( "%d\n" , i );
    return 0;
}
```

```
>> ./a.out
5
>>
```

Casting between types (numbers)

When performing a binary operation (arithmetic or comparison) with different types the "lower rank" operand is implicitly promoted:

char < unsigned char < int < unsigned int < long < unsigned long < float < double

```
#include <stdio.h>
int main( void )
{
    int i = 2;
    double d = 2.5;
    i *= d;
    printf( "%d\n" , i );
    return 0;
}
```

```
>> ./a.out
5
>>
```

Casting between types (numbers)

When performing a binary operation (arithmetic or comparison) with different types the “lower rank” operand is implicitly promoted:

char < unsigned char < int < unsigned int < long < unsigned long < float < double

```
#include <stdio.h>
int main( void )
{
    int one = 1;
    int four = 4;
    int i = one / four * four;
    printf( "%d\n" , i );
    return 0;
}
```

```
>> ./a.out
0
>>
```

Casting between types (numbers)

When performing a binary operation (arithmetic or comparison) with different types the "lower rank" operand is implicitly promoted:

char < unsigned char < int < unsigned int < long < unsigned long < float < double

```
#include <stdio.h>
int main( void )
{
    double one = 1;
    int four = 4;
    int i = one / four * four;
    printf( "%d\n" , i );
    return 0;
}
```

```
>> ./a.out
1
>>
```

Casting between types (numbers)

When performing a binary operation (arithmetic or comparison) with different types the “lower rank” operand is implicitly promoted:

char < unsigned char < int < unsigned int < long < unsigned long < float < double

```
#include <stdio.h>
int main( void )
{
    int one = 1 , four = 4;
    float f = one / four;
    printf( "%g\n" , f );
    return 0;
}
```

```
>> ./a.out
0
>>
```

Casting between types (numbers)

- Since evaluation precedes assignment, we may get truncated results even when the LHS doesn't require it
- The desired behavior can be forced with explicit casting:
  - Preceding the variable name with `(<type-name>)` converts the variable to type `<type-name>`
  - Since casting takes precedence over arithmetic operations:
    1. We convert `one` to a `float`
    2. And then divide a `float` by an `int`
       a. This implicitly promotes `four` to a `float`
       b. And then performs `float` by `float` division

```
#include <stdio.h>
int main( void )
{
    int one = 1 , four = 4;
    float f = (float)one / four;
    printf( "%g\n" , f );
    return 0;
}
```

```
>> ./a.out
0.25
>>
```

Casting between types (pointers)

- Since pointers represent locations in memory (independent of type)
  - We can cast between pointer types
    - This needs to be done explicitly

```
#include <stdio.h>
int main( void )
{
	…
	int i = 1;
	int* ip = &i;
	float *fp= (float*)ip;
	…
}
```

Casting between types (pointers)

- Since pointers represent locations in memory (independent of type)
  - We can cast between pointer types
    - This needs to be done explicitly
    - Unless one of them has type void*

```
…
void * malloc( size_t );
…
```

```
#include <stdio.h>
int main( void )
{
    …
    float* a = malloc( 10 * sizeof( float ) );
    …
}
```

Casting between types (pointers)

- Since pointers represent locations in memory (independent of type)
  - We can cast between pointer types
    - This needs to be done explicitly
    - Unless one of them has type `void*`
  - We can also explicitly cast between pointers and integers
    - This needs to be done with care since a pointer can have different sizes on different machines:
      - 4 bytes on a 32-bit machine
      - 8 bytes on a 64-bit machine
    - The `size_t` type is guaranteed to always have the size of a pointer

```
#include <stdio.h>
int main( void )
{
    int i = 100;
    int* ip = &i;
    size_t addr = (size_t)ip;
    printf( "Address is: %zu\n" , addr );
    return 0;
}
```

Casting between types

Three types of casting:

1. Nothing changes in the binary representation
    - pointers ↔ pointers
        - A memory address is a memory address
        - The compiler needs to know the type to transform element offsets into byte offsets
    - unsigned integers ↔ signed integers
        - Different representations of numbers modulo M still represent the same number
        - The compiler needs to know the type for comparisons

Casting between types

Three types of casting:

1. Nothing changes in the bi
   - pointers ↔ pointers
     - A memory address is a memo
     - The compiler needs to know t
   - unsigned integers ↔ signed
     - Different representations of n
     - The compiler needs to know t

```
#include <stdio.h>

void PrintBinary( const void* mem , size_t sz ){ ... }
int main( void )
{
	int iArray[] = { 1 , 2 , 3 , 4 };
	int* iPtr = iArray;
	char* cPtr = (char*)iPtr;
	PrintBinary( iPtr , sizeof(iPtr) );
	PrintBinary( cPtr , sizeof(cPtr) );
	return 0;
```

```
>> ./a.out
00100000 01001001 11010100 11001000 11111100 01111111 00000000 00000000
00100000 01001001 11010100 11001000 11111100 01111111 00000000 00000000
>>
```

Casting between types

Three types of casting:

1. Nothing changes in the b
   - pointers ↔ pointers
     - A memory address is a memo
     - The compiler needs to know
   - unsigned integers ↔ signed
     - Different representations of r
     - The compiler needs to know

```
#include <stdio.h>

void PrintBinary( const void* mem , size_t sz ){ ... }
int main( void )
{
        unsigned int ui =(1<<31)|1;
        int i = ui;
        printf( " %u = " , ui ) ; PrintBinary( &ui , sizeof(ui) );
        printf( "%d = " , i ) ; PrintBinary( &i , sizeof(i) );
        return 0;
}
```

```
>> ./a.out
 2147483649 = 10000000 00000000 00000000 00000001
-2147483647 = 10000000 00000000 00000000 00000001
>>
```

Casting between types

Three types of casting:

1. Nothing changes in the binary representation
2. Binary representations are truncated/expanded
   - integers ↔ integers (of different sizes)

Casting between types

Three types of casting:

1. Nothing changes in the bin
2. Binary representations are
   - integers ↔ integers (of differe

```
#include <stdio.h>

void PrintBinary( const void* mem , size_t sz ){ ... }
int main( void )
{
	int i = 254;
	unsigned char c = i;
	printf( "%d = " , i ) ; PrintBinary( &i , sizeof(i) );
	printf( "%d = " , c ) ; PrintBinary( &c , sizeof(c) );
	return 0;
}
```

```
>> ./a.out
254 = 00000000 00000000 00000000 11111110
254 = 11111110
>>
```

Casting between types

Three types of casting:

1. Nothing changes in the binary representation
2. Binary representations are truncated/expanded
3. Binary representations are completely different
    - integers ↔ floating point values
    - floating point values ↔ floating point values (of different sizes)

Casting between types

Three types of casting:

1. Nothing changes in the bi
2. Binary representations are
3. Binary representations are
    - integers ↔ floating point valu
    - floating point values ↔ floati

```
#include <stdio.h>

void PrintBinary( const void* mem , size_t sz ){ … }
int main( void )
{
        int i = 1;
        float f = i;
        printf( "%d   = " , i ) ; PrintBinary( &i , sizeof(i) );
        printf( "%.1f = " , f ) ; PrintBinary( &f , sizeof(f) );
        return 0;
}
```

```
>> ./a.out
1   = 00000000 00000000 00000000 00000001
1.0 = 00111111 10000000 00000000 00000000
>>
```

Outline

- Exercise 14
- Numerical representation
- Casting
- Review questions

Review questions

1. What is *two's complement* representation?

It is a signed integer representation. The negative of a number is obtained by flipping the bits and adding one.

Review questions

2. How does representation of integers and floating-point values differ in C?

Integers are simply converted to binary.

Floating-point values are stored in two parts - mantissa and exponent.

Review questions

3. What is *type narrowing*?

Converting a "larger" data type into a "smaller" one, like `float` to `int`

`char < unsigned char < int < unsigned int < long < unsigned long < float < double`

Review questions

4. What is *type promotion*?

Converting a "smaller" data type into a "larger" one, like `char` to `int`

`char < unsigned char < int < unsigned int < long < unsigned long < float < double`

Review questions

5. What is *type casting*?

Explicitly or implicitly converting a value from one type to another

Review questions

5. What is the output of:

```
int n = 32065;
float x = 24.79;
printf( "int n = %d but (char)n = %c\n" , n , (char)n );
printf( "float x = %f but (long)x = %ld\n" , x , (long)x );
```

In binary, we have:

32065 = (00000000 00000000 01111101 01000001)_2

Casting to a char we get:

(01000001)_2 = 65 -> 'A'

```
int n = 32065 but (char)n = A
float x = 24.790001 but (long)x = 24
```

Exercise 14

- Website -> Course Materials -> Exercise 14