MultiPath Fitter

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MultiPath Fitter (MPW) Kalpana/Jie

The Multi-path Water Fitter (MPW) is an alternative fitter developed for the SNO+ water phase, based on the Multi-path Fitter developed for the partial water fill.

The MPW fitter currently fits for position, time and direction of a water phase event. The fitter uses prompt light and straight line paths for likelihood calculations. Then it utilizes the Multi-path Fitter to maximize the likelihood functions and find the best-fit values. The concept of this fitter is the same of the QSNO fitter in SNO time.

MPW Fitter Structure

The MPW fitter consists of:

- Fitter Data : Includes physics constants, set-values and pdfs for the MPW fitter.

  - Water reflection index (water_RI, or n_{water}), used for group velocity (v_g =c/n_{water}) calculation.

  The MPW fitter currently uses one fixed number for n_{water}, rather than a function depending on wavelength. This value is different for Monte Carlo simulations and real water data. For the Monte Carlo, $n_{water} = 1.40$ is an optimized value from tuning the simulations. For the water data, $n_{water} = 1.38486$ is obtained from analyzing the ^{16}N data reconstructed by the MPW fitter.

  - Constants for fit setting: Includes the fitter tolerance, the maximum iterations for the Multi-path Fitter to converge, time offset, radius cut for position vertex, fitting bin-width and steps.

  - Other physics constants: air reflection index (air_RI), psup radius.

  - PMT response time (timing) pdf for the position reconstruction, as shown in 1. It is the measured PMT response time distribution from SNO time and the late light response is forced to be de-weighted.

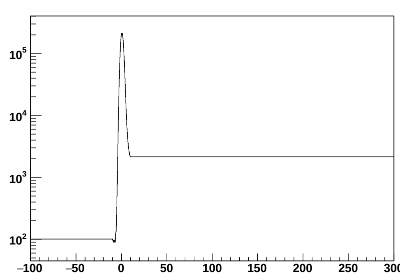

Figure 1: PMT response time as the timing pdf.

  - PMT angular response pdf for the direction reconstruction, as shown in 2. It is taken from the Monte Carlo simulation of 5 MeV electrons traverse in the AV with one direction.

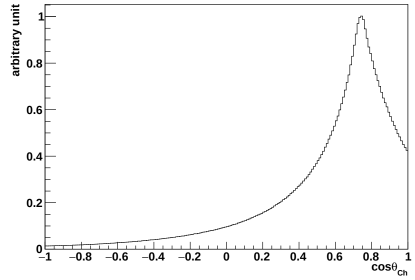

Figure 2: PMT angular distribution as the angular response pdf.

- Fit the position, time and direction.

  - Likelihood Calculation Classes: Constructs likelihood functions, calculates likelihoods and their derivatives. There are two classes: FitterWaterPosition for position reconstruction and FitterWaterDirection for direction reconstruction.

  - Multi-path Fitter: Maximizes the likelihood functions and finds the best-fit position and direction by using the Levenberg-Marquardt (MRQ) method. It takes the number of fitting parameters, the calculated likelihoods and derivatives for fitting while does not care about how the likelihood functions are constructed and how the likelihoods are calculated. It does not depend on the type of the fitter.

  - Dump Likelihood: Stores the likelihood surfaces and their derivatives from the fitting of the Multi-path Fitter to check whether the fitter finds global or local maximum of some interested events. This is used for a detailed study of the reconstruction performances.

  - Multi-path Processor: Processes the MPW fitter. It is a general processor and is shared with the fitters using the Multi-path Fitter, including the MPW fitter, air-water (AW) fitter and wavelength-shifter (WLS) fitter. It processes a certain fitter by being assigned the fitter name in macro. It processes the fitter event by event: for every triggered event, it extracts the hit time and positions of hit normal PMTs selected by the PMT selector and sends these values to the FitterWaterPosition and FitterWaterDirection for likelihood calculations. Then it calls the Multi-path Fitter to maximize the likelihoods and find the best-fit. If the likelihood maxima is found 5 times for any position and direction then values are returned as the fitted position and direction.

Position and Direction Reconstructions of the MPW Fitter

For the position reconstruction of the MPW fitter, the likelihood function simply calculates the likelihood assuming straight line paths of prompt light from a position vertex $\vec{X}_0$ (fVertex) and a starting time offset t_0 to each of the hit PMTs.

The time residue (t_{res}) is taken as the fitting parameter of the likelihood function for position reconstruction. The t_{res} of the i-th hit PMT is calculated as $t^i_{res} = t^i_{\mathrm{PMT}} - |\vec{X}_0 - \vec{X}^i_{\mathrm{PMT}}|/v_g - t_0$, where t^i_{PMT} is the hit time of the i-th hit PMT and $\vec{X}^i_{\mathrm{PMT}}$ (fPosition) is the position of the i-th hit PMT. Then the likelihood function for position reconstruction is constructed as:

$$L(\vec{x}_0, t_0) = \sum_{i=1}^{\mathrm{Nhits}} L_i(t^i_{res})$$

We define the position difference $\vec{X}_{\mathrm{diffCh}} = \vec{X}_0 - \vec{X}_{\mathrm{PMT}}$, then the time of flight for prompt light is $t_{\mathrm{Ch}} = |\vec{X}_{\mathrm{diffCh}}|/v_g$ and $L_{\mathrm{Ch}} = L(t_{\mathrm{Ch}})$.

The derivatives of the likelihood function can be calculated from explicit mathematical forms as:

$$\frac{dL}{dt_0} = \frac{dL_{\mathrm{Ch}}}{dt_{\mathrm{Ch}}},$$

$$\frac{dL}{dx} = \frac{dL_{\mathrm{Ch}}}{dt_{\mathrm{Ch}}}\frac{dt_{\mathrm{Ch}}}{dx} = -\frac{dL_{\mathrm{Ch}}}{dt_{\mathrm{Ch}}}\frac{X_{\mathrm{diffCh}}}{|\vec{X}_{\mathrm{diffCh}}| \cdot v_g},$$

$$\frac{dL}{dy} = -\frac{dL_{\text{Ch}}}{dt_{\text{Ch}}} \frac{Y_{\text{diffCh}}}{|\vec{X}_{\text{diffCh}}| \cdot v_g},$$
$$\frac{dL}{dz} = -\frac{dL_{\text{Ch}}}{dt_{\text{Ch}}} \frac{Z_{\text{diffCh}}}{|\vec{X}_{\text{diffCh}}| \cdot v_g}$$

, where $\frac{dL_{\text{Ch}}}{dt_{\text{Ch}}}$ can be calculated numerically from the timing pdf.

In the FitterWaterPosition class, it starts with a random $(\vec{x}_0, t_0)$ as seed and calculates the likelihoods and their derivatives for various paths. These values are sent to the Multi-path Fitter, which is fitting 4 parameters: x, y, z, t and to maximize the likelihood function through the MRQ method and to find the best-fit positions.

For the direction reconstruction, the direction vertex $\vec{u}_0 = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$ (fDirection), where the θ is zenith angle and ϕ the azimuth. $\cos\theta_{\text{Ch}}$ is the angle between $\vec{u}_0$ and $\vec{X}_{\text{diffCh}}$, which is taken as the fitting parameter of the likelihood function for the direction reconstruction. For the i-th hit PMT, $\cos\theta^i_{\text{Ch}} = \vec{u}_0 \cdot \frac{\vec{X}^i_{\text{diffCh}}}{|\vec{X}^i_{\text{diffCh}}|}$, then the likelihood function is:

$$L(\vec{u}_0) = \sum_{i=1}^{\text{Nhits}} L_i(\cos\theta^i_{\text{Ch}})$$

The derivatives have explicit mathematical forms:

$$\frac{dL}{d\theta} = \frac{dL_{\text{Ch}}}{d\cos\theta_{\text{Ch}}} \frac{d\cos\theta_{\text{Ch}}}{d\theta} = \frac{dL}{d\cos\theta_{\text{Ch}}} \frac{\vec{u}_0}{d\theta} \cdot \frac{\vec{X}_{\text{diffCh}}}{|\vec{X}_{\text{diffCh}}|}$$

, where $d\vec{u}_0/d\theta = (\cos\phi\cos\theta, \sin\phi\cos\theta, -\sin\theta)$ and

$$\frac{dL}{d\phi} = \frac{dL_{\text{Ch}}}{d\cos\theta_{\text{Ch}}} \frac{d\cos\theta_{\text{Ch}}}{d\phi} = \frac{dL}{d\cos\theta_{\text{Ch}}} \frac{\vec{u}_0}{d\phi} \cdot \frac{\vec{X}_{\text{diffCh}}}{|\vec{X}_{\text{diffCh}}|}$$

, where $d\vec{u}_0/d\phi = (-\sin\phi\sin\theta, \cos\phi\sin\theta, 0)$. $\frac{dL_{\text{Ch}}}{d\cos\theta_{\text{Ch}}}$ can be calculated numerically from the PMT angular response pdf.

In the FitterWaterDirection class, it starts with a random (θ_0, ϕ_0) as seed and calculates the likelihoods and their derivatives for various paths. These values are sent to the Multi-path Fitter, which is now fitting 2 parameters: (θ, ϕ) and to maximize the likelihood function through the MRQ method and to find the best-fit directions.

Drive Correction for MPW fitter

Once the MPW fitter obtains reconstruction results, a drive correction is applied on the fitted position by $\vec{X}_{\text{corrected}} = p_0\vec{X}_{fit} + p_1\vec{u}_{fit}$, where p_0 and p_1 are the correction parameters.

To obtain the values of p_0 and p_1, we generated electron events distributed isotropically inside the AV. The simulations of 2, 3, 4, ... ,10 MeV electrons are produced. Then the MPW fitter is applied on each simulations and returns the results of $\vec{X}_{fit}$ and $\vec{u}_{fit}$. Take the Monte Carlo generated positions $\vec{X}_{MC}$ as true positions, for all the fitted events, a χ^2 function is calculated by:

$$\chi^2 = \sum_{i=1}^{N_{\text{events}}} [\vec{X}^i_{MC} - (p_0\vec{X}^i_{fit} + p_1\vec{u}^i_{fit})]^2$$

Then p_0 and p_1 are obtained by minimizing the χ^2 function. When doing the χ^2 calculation, the fitted events of $|\vec{X}_{fit} - \vec{X}_{MC}| > 3\ m$ are thrown away (do a cut of 3 m) to improve the χ^2 minimization results.

For the 2 to 10 MeV electrons simulations, we find different values of p_0 and p_1, as listed in Table. These values are energy or Nhit dependent, however it does not improve the results very much after introducing the Nhit dependent functions. Finally we take the average values from the 5 to 10 MeV electrons simulations and the drive correction is set as $\vec{X}_{\text{corrected}} = 0.997143\vec{X}_{fit} + -64.2801\vec{u}_{fit}$.

By fitting the simulations of 5 MeV electrons generated at the detector center and travelling along +X direction, the drive of the MPW fitter causes a ~50 mm biases from the detector center along +X axis. The drive correction reduces this drive bias to ~0.2 mm.