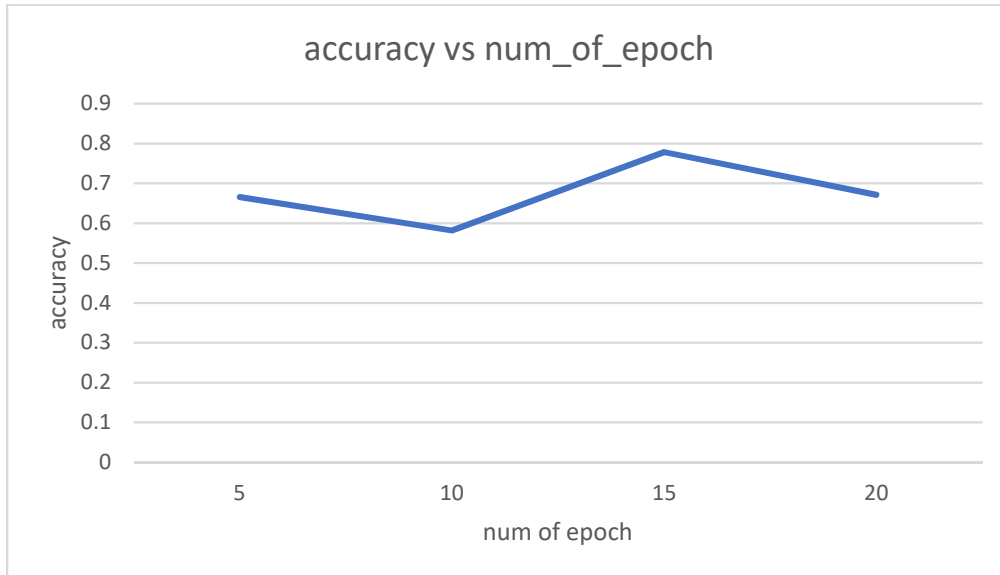
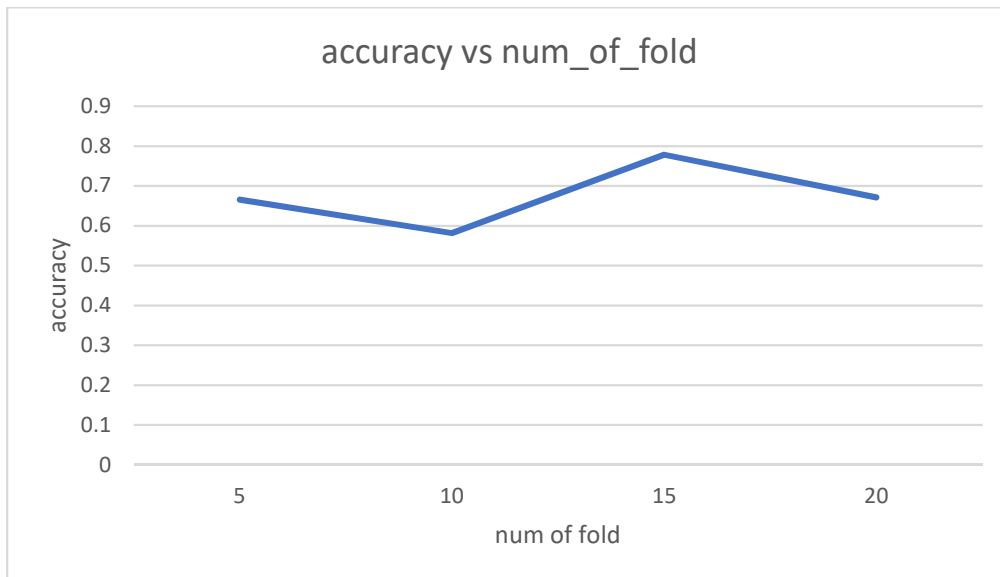


Part B

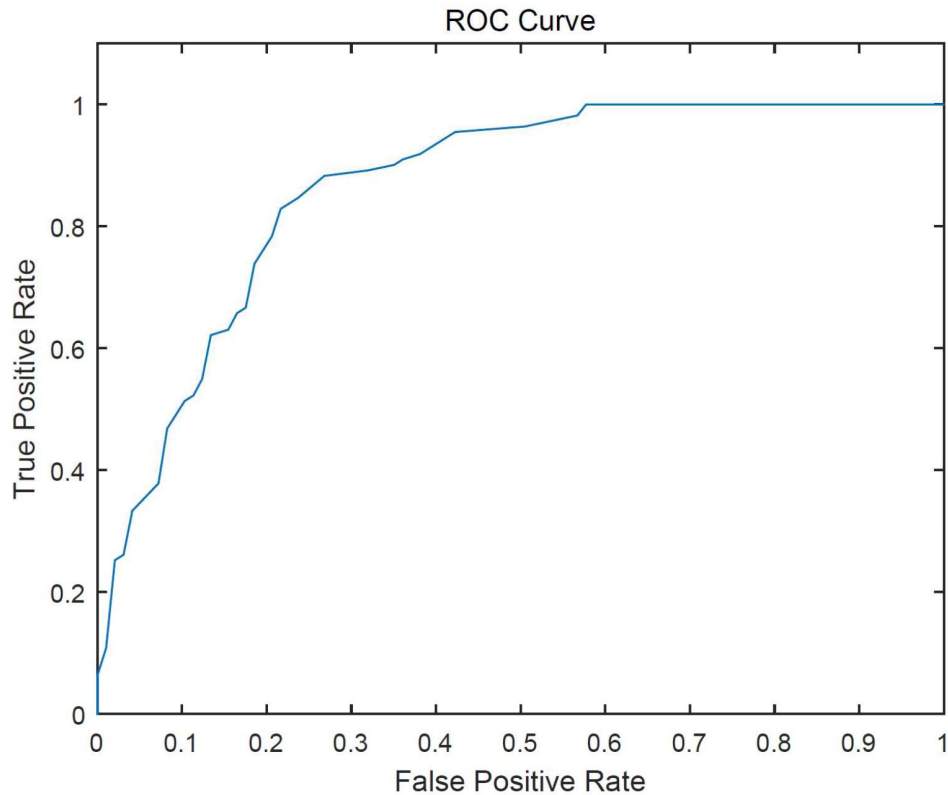
1.



2.



3.



Part C

1. Support : Binary vectors of length K

$$x \in \{0, 1\}^K$$

$$Y \sim \text{Bernoulli}(\theta)$$

$$X_k \sim \text{Bernoulli}(\theta_{k,Y}) \quad \forall k \in \{1, \dots, K\}$$

$$\text{Model: } p_{\theta, \theta}(x, y) = (\theta)^y (1 - \theta)^{(1-y)}$$

$$\theta = \frac{\sum_{i=1}^N \prod(y^{(i)} = 1)}{N}$$

$$\theta_{k,0} = \frac{\sum_{i=1}^N \prod(y^{(i)} = 0 \wedge x_k^{(i)} = 0)}{\sum_{i=1}^N \prod(y^{(i)} = 0)}$$

$$\theta_{k,1} = \frac{\sum_{i=1}^N \prod(y^{(i)} = 1 \wedge x_k^{(i)} = 1)}{\sum_{i=1}^N \prod(y^{(i)} = 1)}$$

$$\forall k \in \{1, \dots, K\}$$

2.

According to the information showed in the problem. We can define

$$P(Y = y_k | X) \propto \exp(w_{k0} + \sum_{i=1}^d w_{ki} X_i) \quad \text{for } k = 1, \dots, K-1$$

Since all probabilities must sum to 1, we should have

$$P(Y = y_k | X) = 1 - \sum_{k=1}^K P(Y = y_k | X)$$

In binary classification, we define

$$P(Y = y_k|X) = \frac{1}{1 + \sum_{k=1}^K \exp(w_{k0} + \sum_{i=1}^d w_{ki}X_i)}$$

and for $k = 1, \dots, K-1$

$$P(Y = y_k|X) = \frac{\exp(w_{k0} + \sum_{i=1}^d w_{ki}X_i)}{1 + \sum_{k=1}^K \exp(w_{k0} + \sum_{i=1}^d w_{ki}X_i)}$$

Hence, it is clearly that $p(y = i|x)$ is a softmax