ELEN0071

Applied digital signal processing

P. Sacré - P. Embrahimbabaie

Homework 3 : Distortion of the filtered signals

Beaulieu Guillaume

Custinne François

Escalona Saul

3rd BAC Civil Engineering

Faculty of Applied Sciences

Academic years 2018-2019

1 Problem 1 : Noise Filtering

The main goal of this homework was to design a filter to remove an arbitrary noise $v[n]$ from a noisy signal $x_{ns}[n]$. This signal could be represented like :

$$x_{ns}[n] = x[n] + v[n] \tag{1}$$

where $x[n]$ is the original signal one such as :

$$x[n] = \cos(20\pi t) + 0.5 \cdot \cos(40\pi t + 1.4) + 0.8 \cdot \cos(120\pi t + 0.7) \tag{2}$$

The design of this filter should pay attention to remove the noise from the signal $x_{ns}[n]$ without any distortions. In other words, the filtered signal should have the same "shape" of the noisy one.

(a) Plots of signals $x_{ns}[n]$ and $x[n]$

The two signals $x_{ns}[n]$ and $x[n]$ are given by the MATLAB dataset `Signal_plus_Noise.mat`. These two signals have been sampled by a sampling frequency $F_s = 1000\,Hz$. With this information, we could deduce the sampling period of these two signals which is $T_s = \frac{1}{F_s} = 0.001\,sec$. In addition the length of these signals have been found numerically which is $N = 7000$. Thus, the maximum time of the signal could be calculated like :

$$T_{max} = (N-1) \cdot T_s \tag{3}$$
$$\Leftrightarrow T_{max} = 6,999\,sec \tag{4}$$

By this way, each sample was considered between 0 and this maximum time T_{max} with a constant time step T_s.

Finally, as asked, the following plot range was considered to represented both signals :

$$\left[\frac{N}{2} - 200 \quad ; \quad \frac{N}{2} + 200\right] \tag{5}$$
$$= [3300 \quad ; \quad 3700] \tag{6}$$

Which correspond in time $[sec]$ units with

$$[3.3\,sec \quad ; \quad 3.7\,sec] \tag{7}$$

So, the signals $x_{ns}[n]$ and $x[n]$ were plotted in the same frame. The corresponding plot is shown in FIGURE 1. As it can be seen in this FIGURE 1, the signal orange (noised signal $x_{ns}[n]$) is completely different from the original signal($x[n]$).

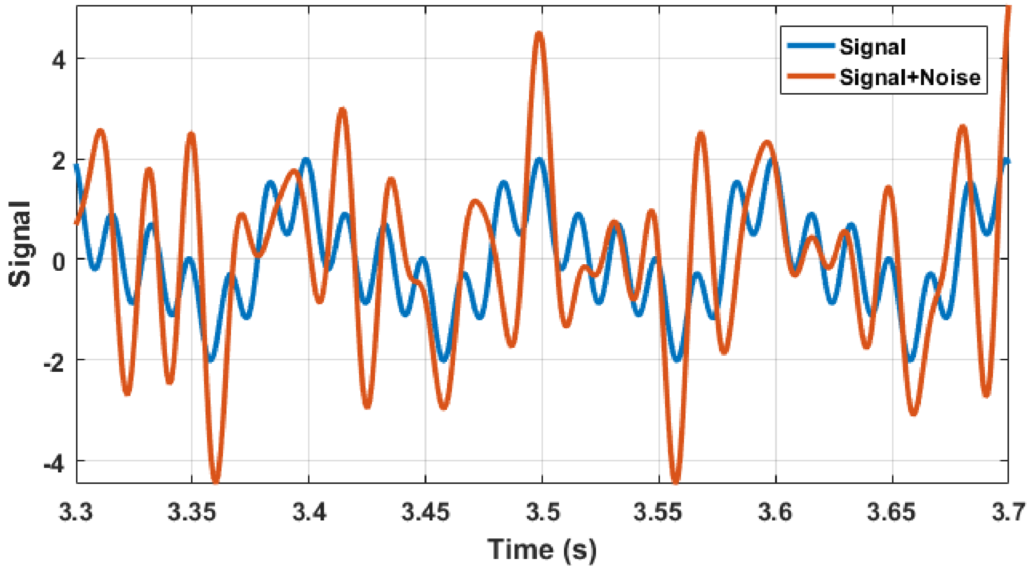

FIGURE 1 – Graphs of the original signal (without noises) $x[n]$ and the noise-corrupted signal $x_{ns}[n]$

(b) Single-sided amplitude spectrum of the noisy signal $x_{ns}[n]$

Then, it was asked to represent the single-sided magnitude spectrum of the noisy signal $x_{ns}[n]$. In order to do that, the *Fast Fourier Transform* (*i.e.* the *FFT*) of the noisy signal $x_{ns}[n]$ has been computed. Then, the corresponding result has been divided by the signal's length N in order to scale it. The absolute value of it has been taken to obtain the magnitude response. Indeed, because it was asked to plot the single-sided spectrum, only the positive part of the result is needed. Thus, this result had to be multiplied by 2 (excepted the DC part of the signal which corresponded to a frequency $f = 0\,Hz$) in order to keep the same amount of energy and to compensate the fact that only one half of the spectrum has been considered.

The Single-sided magnitude spectrum of the noisy signal $x_{ns}[n]$ is given in FIGURE 2 below.

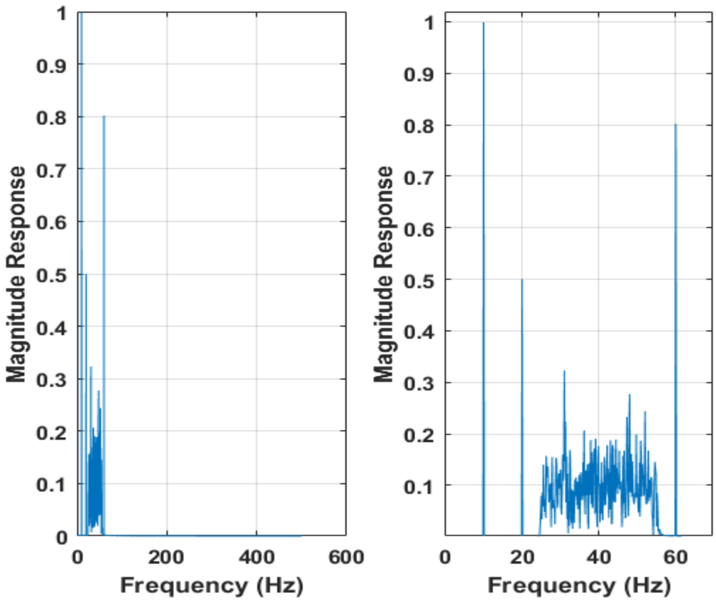

FIGURE 2 – Graph of the Single-sided magnitude spectrum of the noisy signal $x_{ns}[n]$ according to frequency and a zoom in the noisy zone

(c) Approximate frequency range of the noise $v[n]$

Thanks to the previous section, it can be seen in FIGURE 2 that the approximate noisy frequency range is between :

$$25 \quad \text{and} \quad 57 \qquad [Hz] \tag{8}$$

(d) Design of the filter

The goal of this fourth point was to remove the noises characterized by the noises' frequency range (8) found in the previous section. Furthermore, as previously said, the filtered signal must not have distortion (*i.e.* it had to have the same "shape" as the original signal $x[n]$). This distortionless signal which is named $y[n]$ could be mathematically defined like :

$$y[n] = G \cdot x[n - n_d] \tag{9}$$

where $x[n]$ is the original signal, G is the gain and n_d is the delay.

In order to design such a filter, a finite impulse response filter (i.e. *FIR* filter) has been chosen. Indeed, this last one is always characterised by a linear phase, which will allow us to not modify the shape of the signal after filtering.

The filter design procedure was the following. The *FIR* filter has been constructed by using the *Matlab - Apps - Filter Design and Analysis* window and the following characteristics have been chosen :

— *Response type* : Bandstop

— *Design method* : FIR - Window

— *Filter order* : Minimum order

— *Options* : Window - Kaiser

— *Frequency specifications* : $Fs = 1000Hz$, $F_{pass1} = 24Hz$, $F_{stop1} = 25Hz$, $F_{pass2} = 56Hz$, $F_{stop2} = 57Hz$

— *Magnitude specifications* : $A_{pass1} = 1dB$, $A_{stop} = 20dB$, $A_{pass2} = 1dB$

So, by using the FIR filter designed above, the filtered signal $x_{filt}[n]$ has been obtained by using the MATLAB function `filter()` with the FIR filter designed and the noise-corrupted signal $x_{ns}[n]$ as arguments. The magnitude response of the filter is plotted using `fvtool()` function, the result is shown in FIGURE 3.

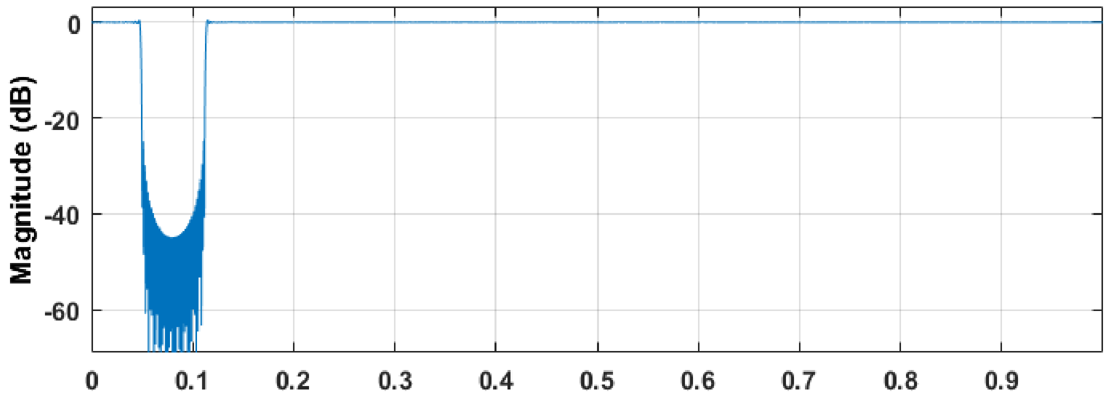

FIGURE 3 – Graph of the Magnitude Response of the filter

(e) Single-sided amplitude spectrum of the filtered signal $x_{filt}[n]$

The single-sided magnitude spectrum of the filtered signal has been computed using the same procedure described in the SECTION (b). The only one difference is that this procedure has been applied on the filtered signal $x_{filt}[n]$ instead of the noisy signal $x_{ns}[n]$ one.

So, the corresponding result according to the frequency is shown in FIGURE 4. By analysing this graph and comparing to the graph of FIGURE 2, it can be seen that the noise frequencies have been filtered.

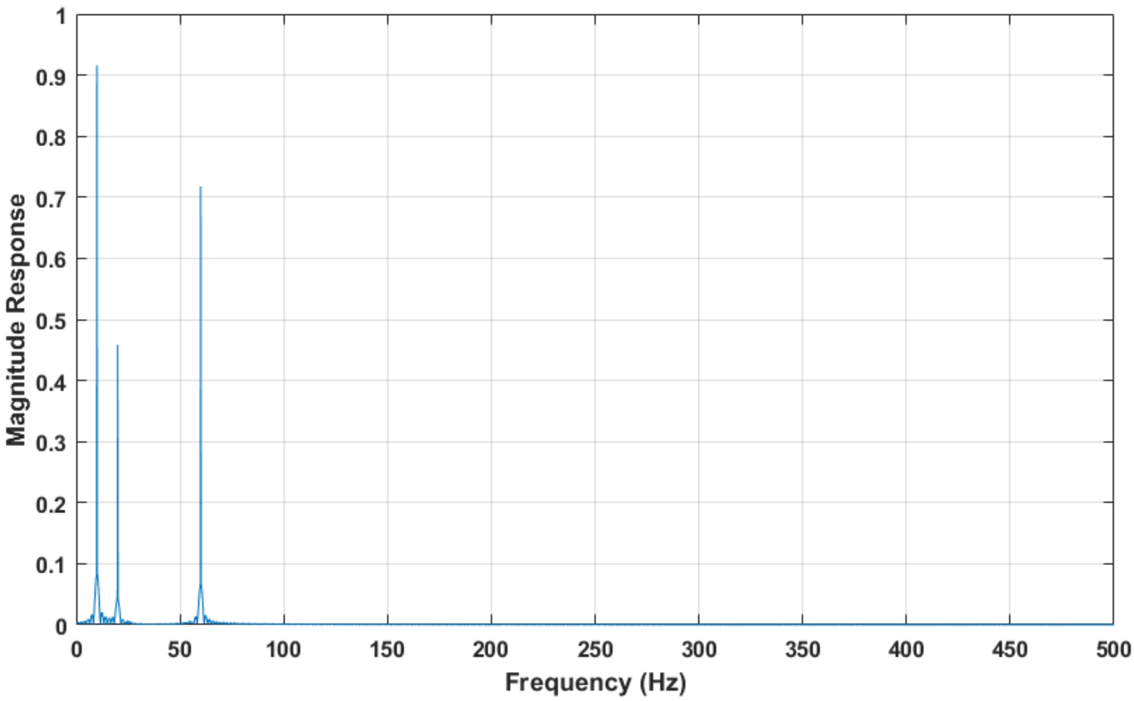

FIGURE 4 – Graph of the single-sided magnitude spectrum of the filtered signal $x_{filt}[n]$.

(f) Plots of signals $x_{filt}[n]$ and $x[n]$

Finally, it was asked to plot the two signals $x_{filt}[n]$ and $x[n]$, which corresponded to the filtered signal and the original one, on a single frame. Also in this representation, the plot range previously defined at (5) and (7) has been used. So, this plot is given in FIGURE 5.

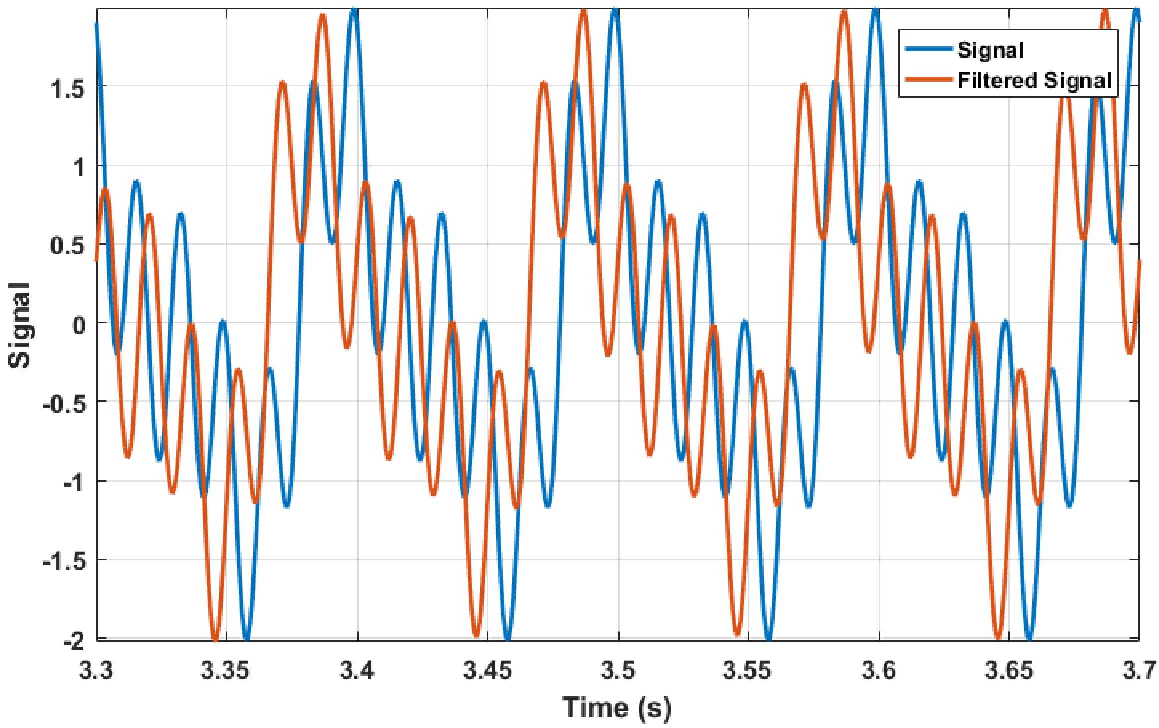

FIGURE 5 – Graphs of the original signal (without noises) $x[n]$ and the filtered signal $x_{filt}[n]$

As it can be seen on the graph given in FIGURE 5, the filtered signal corresponds to the original signal, but shifted. Therefore, the noise has been removed and the filtered signal as the same initial shape.