University of Liège
Bachelor in Applied Sciences - 3^{rd} year
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ELEN0071-1 : Applied digital signal processing
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Homework 1 : Report

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1 Magnitude response of a filter.

We consider a filter with the following system function

$$H(z) = \frac{b_0}{\left[1 - 2rcos(\omega_0)z^{-1} + r^2 z^{-2}\right]^K}$$

which can be written like

$$H(z) = \left[\frac{b_0^{\frac{1}{K}}}{1 - 2rcos(\omega_0)z^{-1} + r^2 z^{-2}}\right]^K$$

With this new formulation, we can consider our filter $H(z)$ as a cascade of K second-order filters. So that we can compute and plot the magnitude response $\left|H\left(e^{j\omega}\right)\right|$ in dB.

In addition, let us consider the following data

- $K = 8$
- $r = 0.9$
- $b_0 = 5.3936 \cdot 10^{-7}$

(a) Magnitude response for $\omega_0 = \frac{\pi}{3}$

Firstly, we use the value : $\omega_0 = \frac{\pi}{3}$. With this following value, we compute and plot the magnitude response of $H(z)$ according to the normalized frequency (i.e. in $\frac{\pi \text{ rad}}{\text{sample}}$). The corresponding magnitude response of $H(z)$ is shown in the figure 1 below.

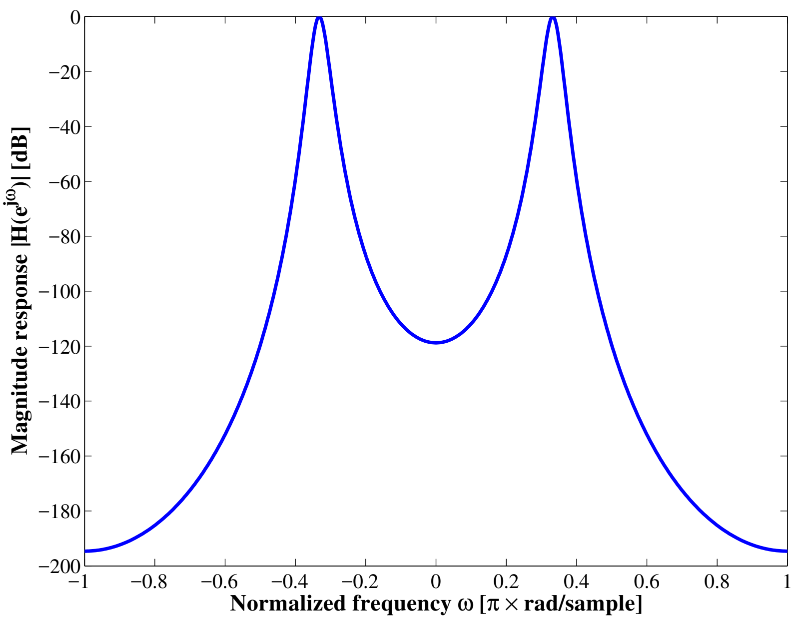

FIGURE 1 – Magnitude response function for $\omega_0 = \frac{\pi}{3}$

(b) Magnitude response for $\omega_0 = \frac{2\pi}{3}$

Secondly, we do the same with the value : $\omega_0 = \frac{2\pi}{3}$. Likewise, we compute and plot this magnitude response of $H(z)$ according to the normalized frequency. The corresponding magnitude response of $H(z)$ is shown in the figure 2 instead.

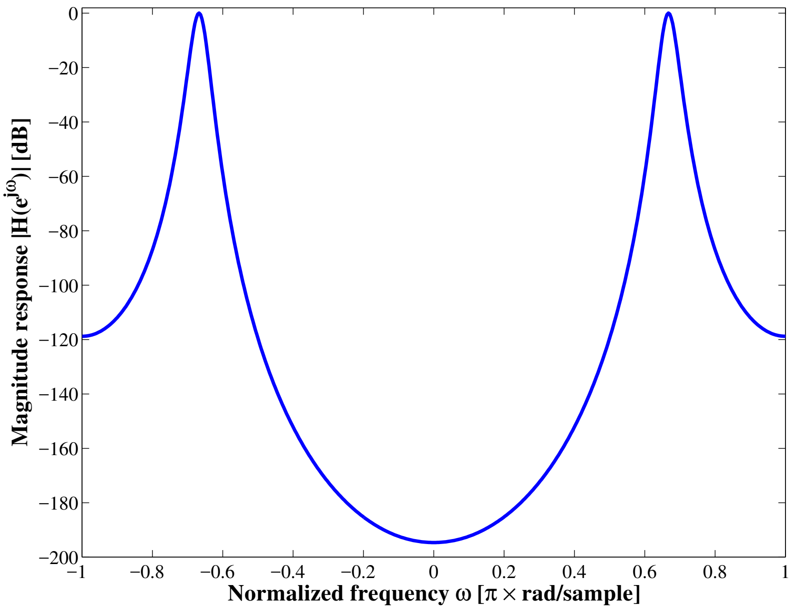

FIGURE 2 – Magnitude response function for $\omega_0 = \frac{2\pi}{3}$

(c) Main effect of the variation of ω_0

As seen in the figure 1 and 2, graphs have both two zeros (represented by the two peaks on these ones). Moreover, we note that both graphs have the same magnitude response (something near zero for us) for their correspond value of ω_0.

2 Autocorrelation of a single echo.

To develop an expression for the autocorrelation $r_y[l]$ in terms of the autocorrelation $r_x[l]$, D and a being such that

$$y[n] = x[n] + a\ x[n-D] \tag{1}$$

Where $a \in [-1; 1]$ and D, both constant.

We take the definition of the autocorrelation

$$r_y[l] = \sum_{n=-\infty}^{+\infty} y[n] * y[n-l] \qquad \text{for } l \in \mathbb{Z}. \tag{2}$$

Replacing our function $y[n]$ by its expression (1) in the following equation (2), we obtain the development

$$
\begin{aligned}
r_y[l] &= \sum_{n\in\mathbb{Z}} \big(x[n] + ax[n-D]\big) * \big(x[n-l] + ax[n-D-l]\big) \\
\Leftrightarrow r_y[l] &= \sum_{n\in\mathbb{Z}} \big(x[n]*x[n-l] + a\ x[n-D]*x[n-l] + a\ x[n]*x[n-D-l] + a^2\ x[n-D]*x[n-l-D]\big) \\
\Leftrightarrow r_y[l] &= \underbrace{\sum_{n\in\mathbb{Z}} \big(x[n]*x[n-l]\big)}_{(\blacklozenge)} + a \underbrace{\sum_{n\in\mathbb{Z}} \big(x[n-D]*x[n-l]\big)}_{(\bigstar)} + a \underbrace{\sum_{n\in\mathbb{Z}} \big(x[n]*x[n-D-l]\big)}_{(\clubsuit)} \\
&\quad + a^2 \underbrace{\sum_{n\in\mathbb{Z}} \big(x[n-D]*x[n-l-D]\big)}_{(\spadesuit)}
\end{aligned}
$$

By using the autocorrelation definition on expressions $(\blacklozenge)$, $(\bigstar)$, $(\clubsuit)$ and $(\spadesuit)$ we find that

$$
(\blacklozenge) = r_x[l]
$$

$$
\begin{aligned}
(\bigstar) &= \sum_{n\in\mathbb{Z}} \big(x[n-D]*x[n-l+D-D]\big) \\
&= \sum_{n\in\mathbb{Z}} \big(x[n-D]*x[(n-D)-(l-D)]\big) \\
&= r_x[l-D]
\end{aligned}
$$

$$
\begin{aligned}
(\clubsuit) &= \sum_{n\in\mathbb{Z}} \big(x[n]*x[n-(D+l)]\big) \\
&= r_x[D+l]
\end{aligned}
$$

$$
\begin{aligned}
(\spadesuit) &= \sum_{n\in\mathbb{Z}} \big(x[n-D]*x[(n-D)-l]\big) \\
&= r_x[l]
\end{aligned}
$$

So we obtain the final expression

$$
r_y[l] = \big(1+a^2\big)\ r_x[l] + a\ r_x[l-D] + a\ r_x[L+D] \tag{3}
$$

Which is an expression of the autocorrelation $r_y[l]$ in term of $r_x[l]$, D and a.

3 Echo cancellation.

(a) Autocorrelation function

Thanks to the function *xcorr()* defined on MATLAB, we can easily plot the autocorrelation of this echoed signal y such that

$$
y[n] = x[n] + a\ x[n-D] \tag{4}
$$

where x is the initial signal, D the delay in samples and a the variation of the reflected sound amplitudes.

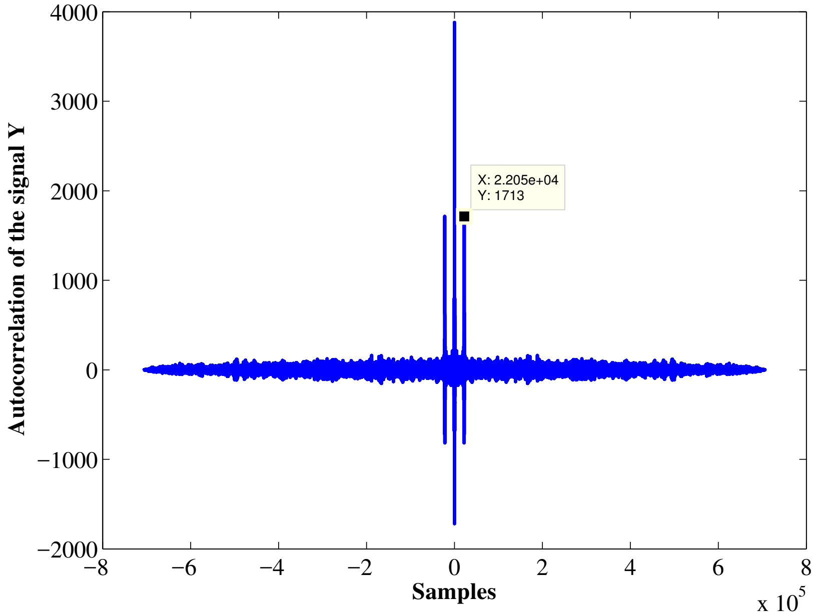

FIGURE 3 – Autocorrelation function of the echoed signal y.

The delay can be found with the graph. As can be seen on the figure 3, the number of sample on the peak is $D = 2205$ samples. The sampling frequency F_s is found by the function *audioread* and is equal to 44100 samples per second. So, the delay in time can be computed as

$$\tau = \frac{D}{F_s} = \frac{2205}{44100} = 0.5 \text{ second}$$

(b) Design procedure of the filter

In order to delete the following echo, we design a filter such that it is the inverse transfer function $H(z)^{-1}$ of the computed z-transform of our equation (4). By this way, we have

$$Y(z) = X(z) + a\ X(z)\ z^{-D} = X(z)\ \left(1 + a\ z^{-D}\right) \tag{5}$$

so that,

$$H(z)^{-1} = \frac{1}{1 + a\ z^{-D}} \tag{6}$$

We assume that $a = 0.6$ such that the amplitude of the reflected signal is sixty percent of the emitted one.

By using the MATLAB function *filter()* which takes the inverse transfer function in argument, we can find the initial signal without an echo. As the figure 4 shown, the filtered signal is not correlated anymore.

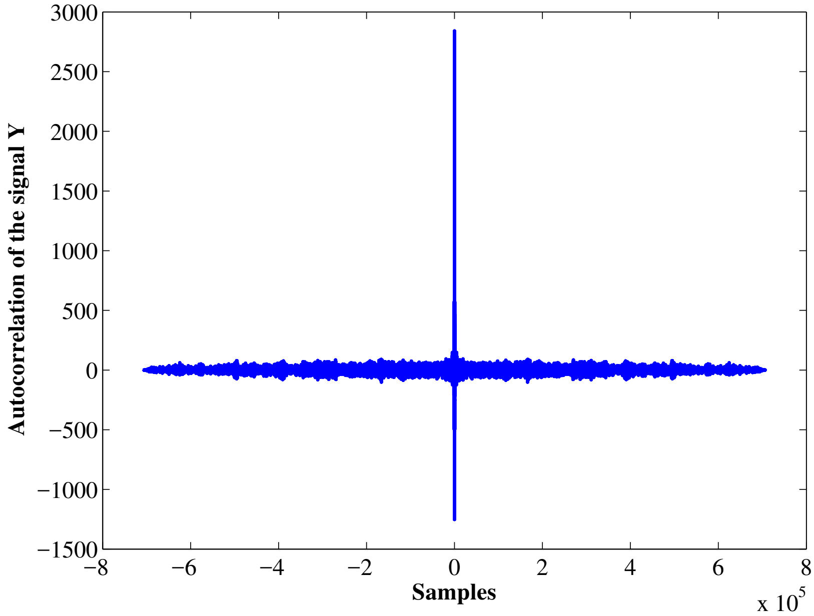

FIGURE 4 – Autocorrelation function of the filtered signal y.