



COMP 412
FALL 2010

*Context-sensitive Analysis
or
Semantic Elaboration
Comp 412*

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Beyond Syntax

There is a level of correctness that is deeper than grammar

```
fie(a,b,c,d) {  
    int a, b, c, d;  
    ...  
}  
fee() {  
    int f[3],g[0], h, i, j, k;  
    char *p;  
    fie(h,i,"ab",j, k);  
    k = f * i + j;  
    h = g[17];  
    printf("<%s,%s>.\n",p,q);  
    p = 10;  
}
```

What is wrong with this program?
(let me count the ways ...)

- number of args to fie()
- declared g[0], used g[17]
- "ab" is not an int
- wrong dimension on use of f
- undeclared variable q
- 10 is not a character string

All of these are
"deeper than syntax"

To generate code, we need to understand its meaning !



Beyond Syntax

To generate code, the compiler needs to answer many questions

- Is "x" a scalar, an array, or a function? Is "x" declared?
- Are there names that are not declared? Declared but not used?
- Which declaration of "x" does a given use reference?
- Is the expression "x * y + z" type-consistent?
- In "a[i,j,k]", does a have three dimensions?
- Where can "z" be stored? (*register, local, global, heap, static*)
- In "f ← 15", how should 15 be represented?
- How many arguments does "fie()" take? What about "printf ()" ?
- Does "*p" reference the result of a "malloc()" ?
- Do "p" & "q" refer to the same memory location?
- Is "x" defined before it is used?

These are beyond the expressive power of a CFG



Beyond Syntax

These questions are part of context-sensitive analysis

- Answers depend on values, not parts of speech
- Questions & answers involve non-local information
- Answers may involve computation

How can we answer these questions?

- Use formal methods
 - Context-sensitive grammars?
 - Attribute grammars?
- Use *ad-hoc* techniques
 - Symbol tables
 - *Ad-hoc* code

(*attributed grammars?*)

(*action routines*)

In parsing, formalisms won.

In context-sensitive analysis, ad-hoc techniques dominate practice.



Beyond Syntax

Telling the story

- We will study the formalism — an attribute grammar
 - Clarify many issues in a succinct and immediate way
 - Separate analysis problems from their implementations
- We will see that the problems with attribute grammars motivate actual, *ad-hoc* practice
 - Non-local computation
 - Need for centralized information
- Some folks still argue for attribute grammars
 - Knowledge is power
 - Information is immunization

We will cover attribute grammars, then move on to *ad-hoc* ideas



Attribute Grammars

What is an attribute grammar?

- A context-free grammar augmented with a set of rules
- Each symbol in the derivation (or parse tree) has a set of named values, or *attributes*
- The rules specify how to compute a value for each attribute
 - Attribution rules are functional; they uniquely define the value

Example grammar

1	<i>Number</i>	→	<i>Sign List</i>
2	<i>Sign</i>	→	+
3			-
4	<i>List</i>	→	<i>List Bit</i>
5			<i>Bit</i>
6	<i>Bit</i>	→	0
7			1

This grammar describes signed binary numbers

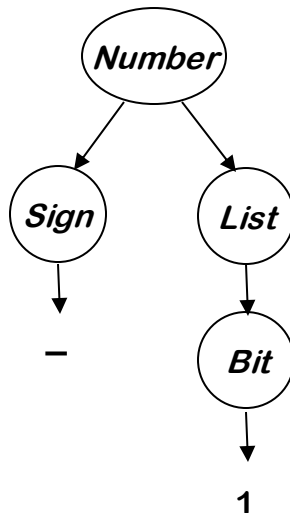
We would like to augment it with rules that compute the decimal value of each valid input string



Examples

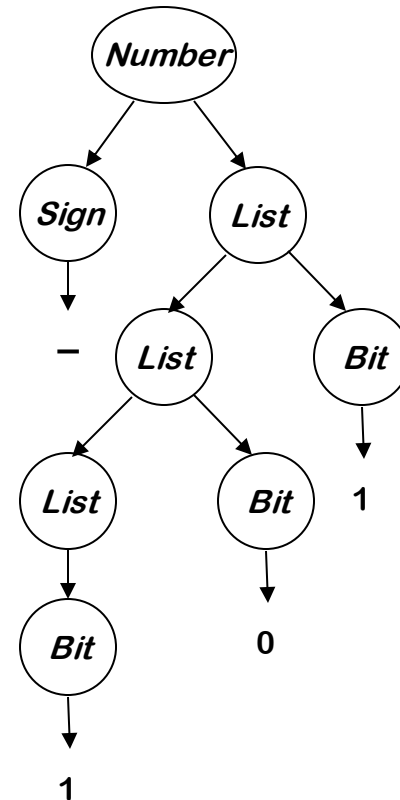
For “-1”

Number → *Sign List*
→ *Sign Bit*
→ *Sign 1*
→ - 1



For “-101”

Number → *Sign List*
→ *Sign List Bit*
→ *Sign List 1*
→ *Sign List Bit 1*
→ *Sign List 0 1*
→ *Sign Bit 0 1*
→ *Sign 1 0 1*
→ - 101



We will use these two examples throughout the lecture



Attribute Grammars

Add rules to compute the decimal value of a signed binary number

Productions		Attribution Rules
<i>Number</i>	\rightarrow <i>Sign List</i>	$List.pos \leftarrow 0$ if <i>Sign.neg</i> then $Number.val \leftarrow - List.val$ else $Number.val \leftarrow List.val$
<i>Sign</i>	\rightarrow +	$Sign.neg \leftarrow false$
	-	$Sign.neg \leftarrow true$
<i>List</i> ₀	\rightarrow <i>List</i> ₁ <i>Bit</i>	$List_1.pos \leftarrow List_0.pos + 1$ $Bit.pos \leftarrow List_0.pos$ $List_0.val \leftarrow List_1.val + Bit.val$
	<i>Bit</i>	$Bit.pos \leftarrow List.pos$ $List.val \leftarrow Bit.val$
<i>Bit</i>	\rightarrow 0	$Bit.val \leftarrow 0$
	1	$Bit.val \leftarrow 2^{Bit.pos}$

Symbol	Attributes
<i>Number</i>	val
<i>Sign</i>	neg
<i>List</i>	pos, val
<i>Bit</i>	pos, val

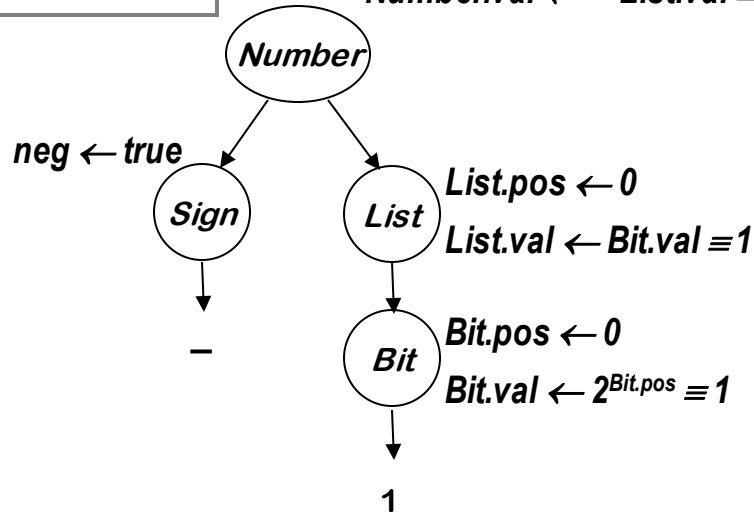
Back to the Examples

Rules + parse tree imply an attribute dependence graph



For “-1”

$Number.val \leftarrow -List.val \equiv -1$



One possible evaluation order:

- 1 List.pos
- 2 Sign.neg
- 3 Bit.pos
- 4 Bit.val
- 5 List.val
- 6 Number.val

Other orders are possible

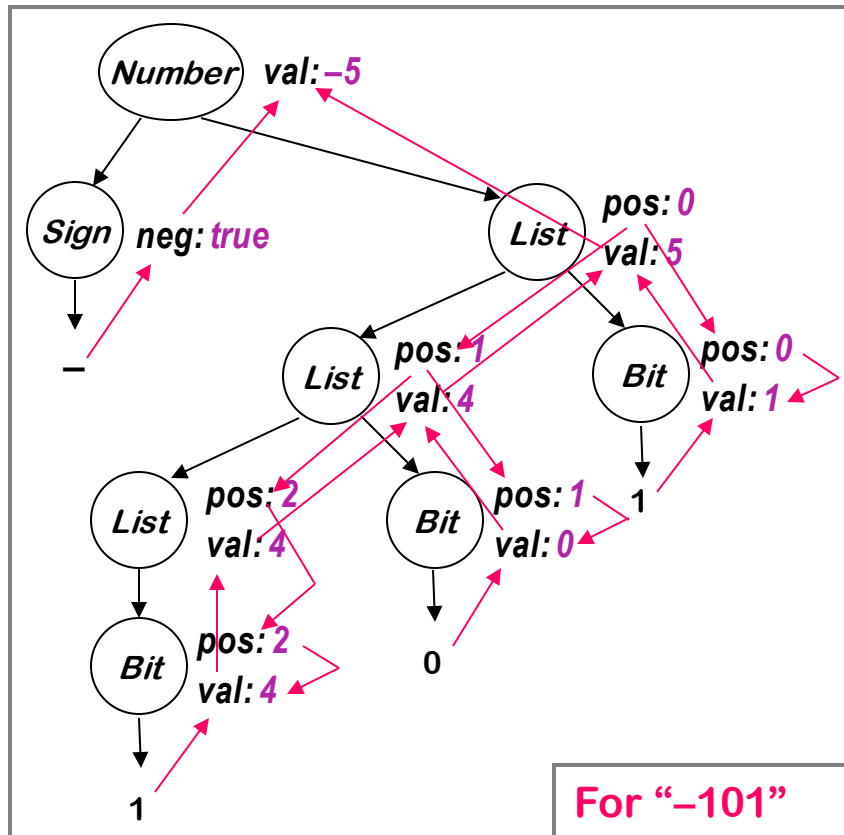
Knuth suggested a data-flow model for evaluation

- Independent attributes first
- Others in order as input values become available

Evaluation order must be consistent with the attribute dependence graph



Back to the Examples



This is the complete attribute dependence graph for "-101".

It shows the flow of *all* attribute values in the example.

Some flow downward
→ **inherited attributes**

Some flow upward
→ **synthesized attributes**

A rule may use attributes in the parent, children, or siblings of a node



The Rules of the Game

- Attributes associated with nodes in parse tree
- Rules are value assignments associated with productions
- Attribute is defined once, using local information
- Label identical terms in production for uniqueness
- Rules & parse tree define an attribute dependence graph
 - Graph must be non-circular

This produces a high-level, functional specification

Synthesized attribute

- Depends on values from children

Inherited attribute

- Depends on values from siblings & parent

N.B.: AG is a specification
for the computation, not an
algorithm



Using Attribute Grammars

Attribute grammars can specify context-sensitive actions

- Take values from syntax
- Perform computations with values
- Insert tests, logic, ...

Synthesized Attributes

- Use values from children & from constants
- S-attributed grammars
- Evaluate in a single bottom-up pass

Good match to LR parsing

Inherited Attributes

- Use values from parent, constants, & siblings
- Directly express context
- Can rewrite to avoid them
- Thought to be more *natural*

Not easily done at parse time

We want to use both kinds of attributes



Evaluation Methods

Dynamic, dependence-based methods

- Build the parse tree
- Build the dependence graph
- Topological sort the dependence graph
- Define attributes in topological order

Rule-based methods

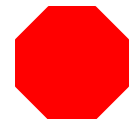
(treewalk)

- Analyze rules at compiler-generation time
- Determine a fixed (static) ordering
- Evaluate nodes in that order

Oblivious methods

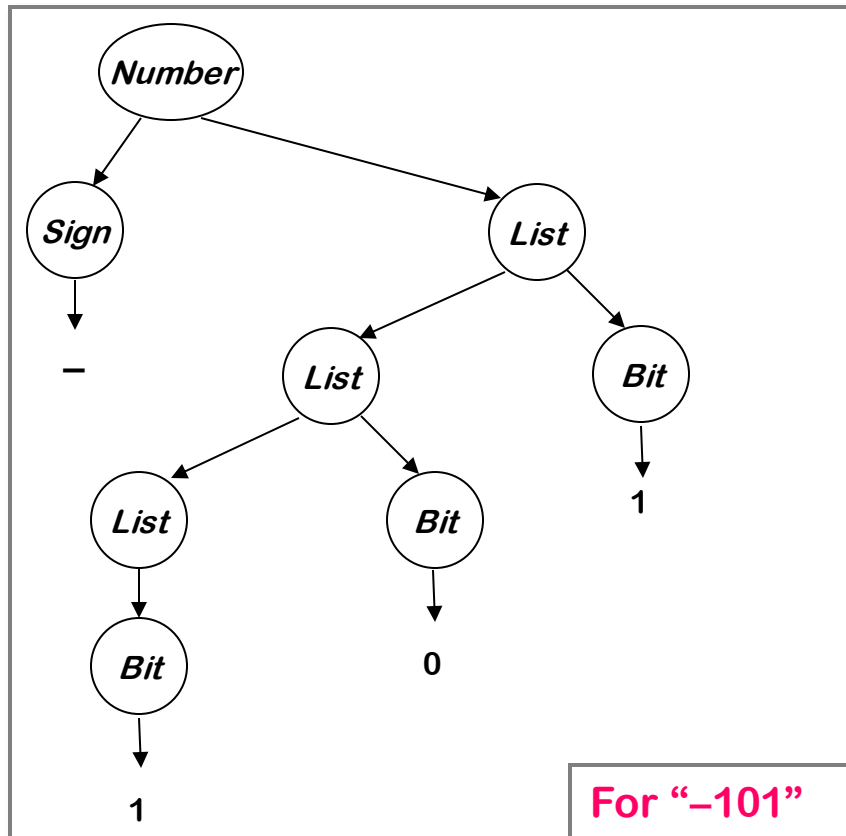
(passes, dataflow)

- Ignore rules & parse tree
- Pick a convenient order (at design time) & use it





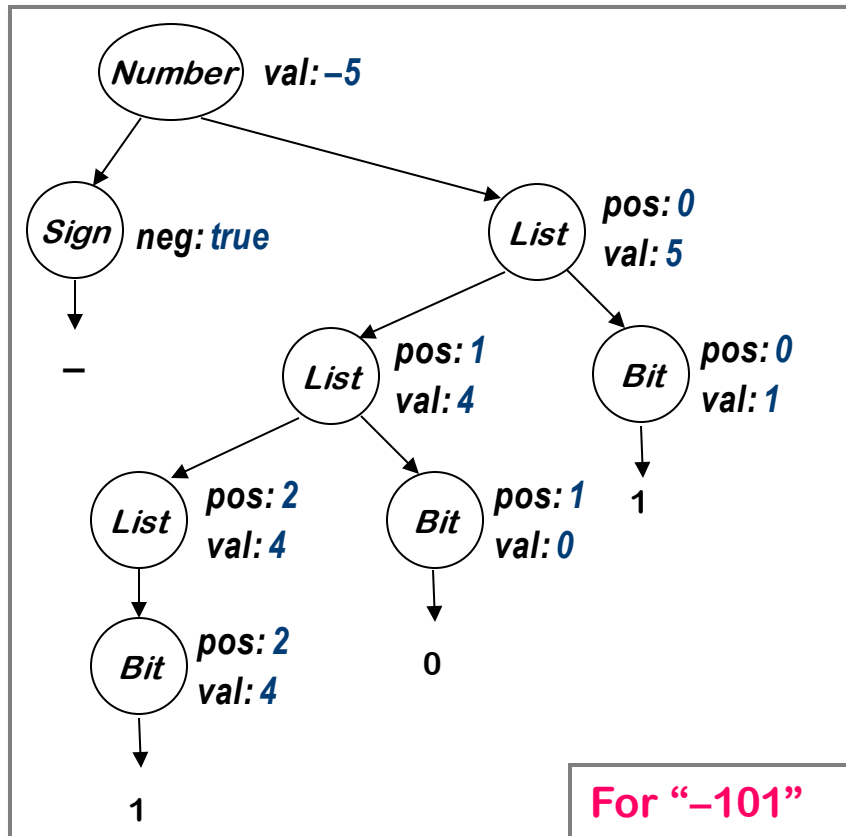
Back to the Example



Syntax Tree



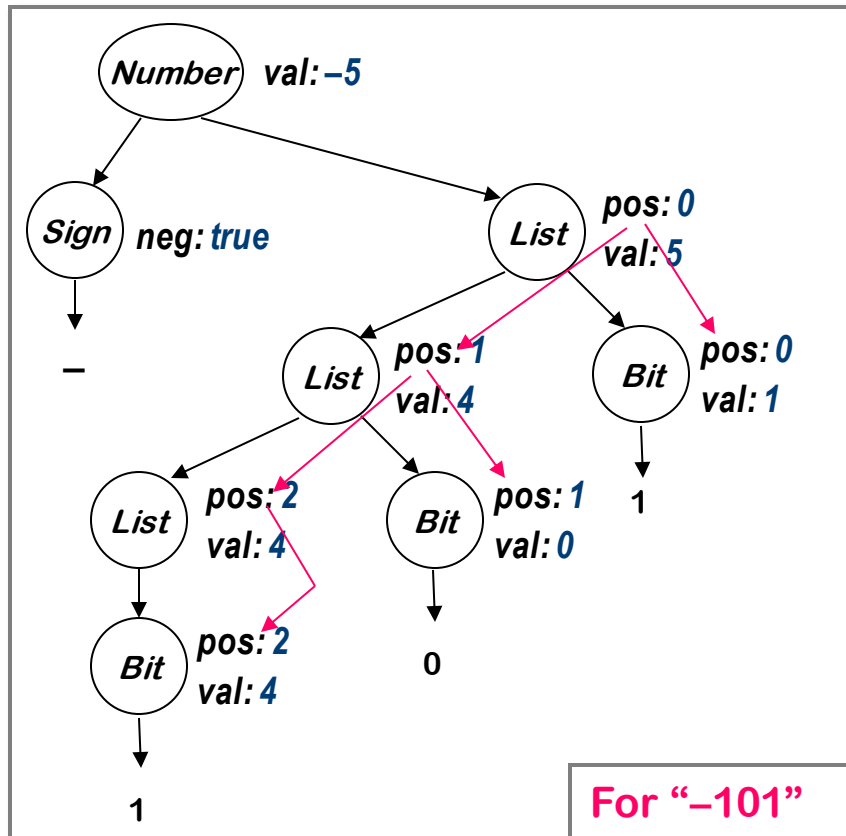
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Attributed Syntax Tree



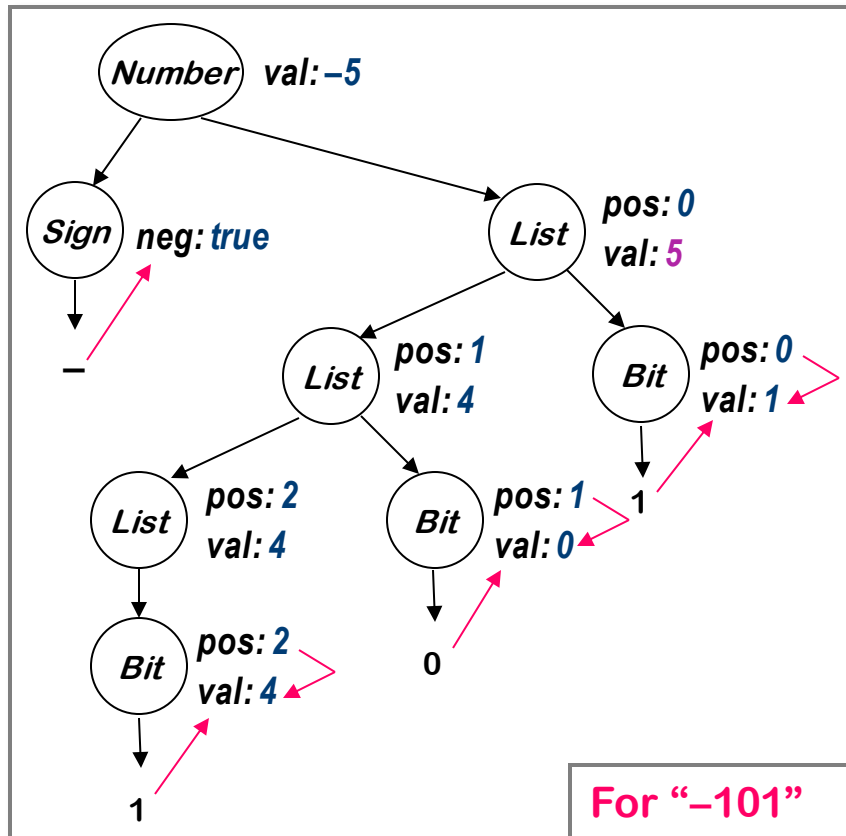
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Inherited Attributes



Back to the Example

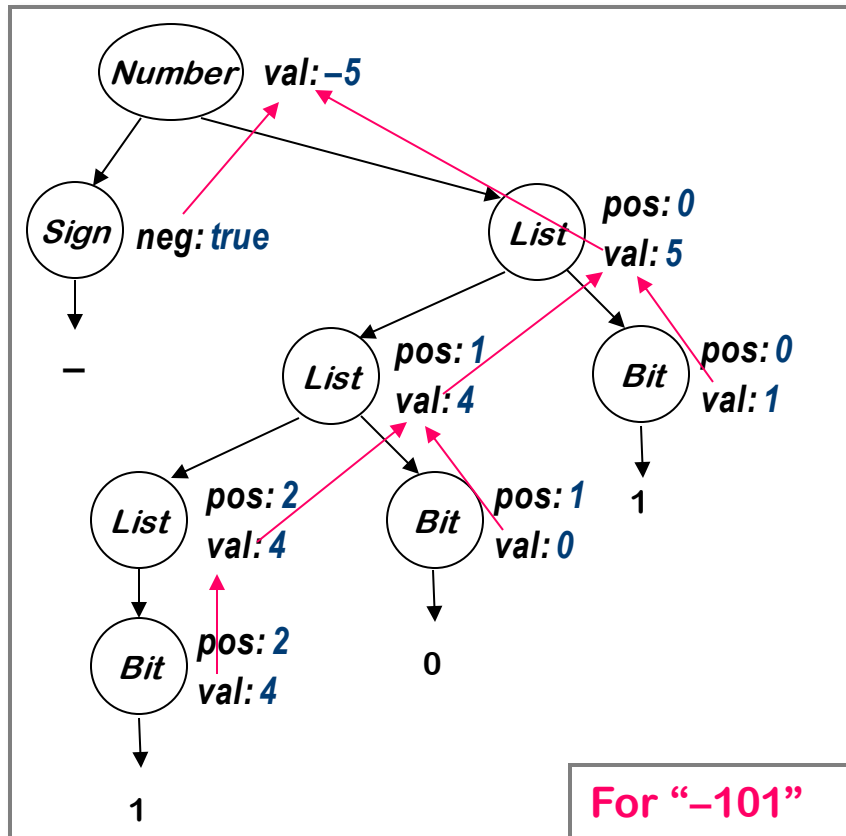


Synthesized attributes

Val draws from children & the same node.



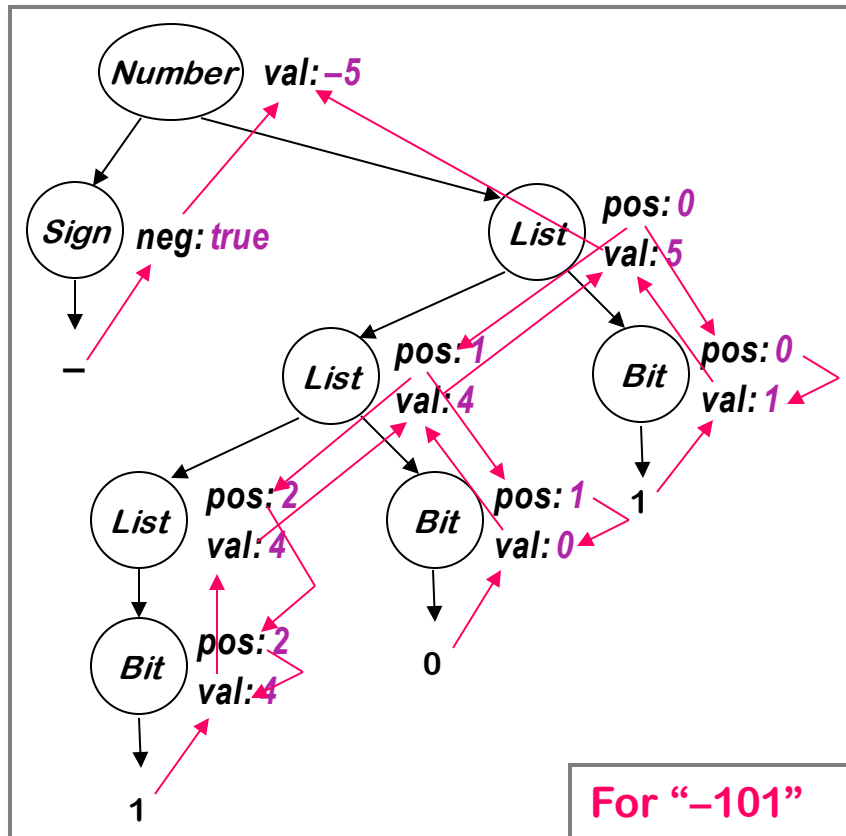
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More Synthesized attributes



Back to the Example

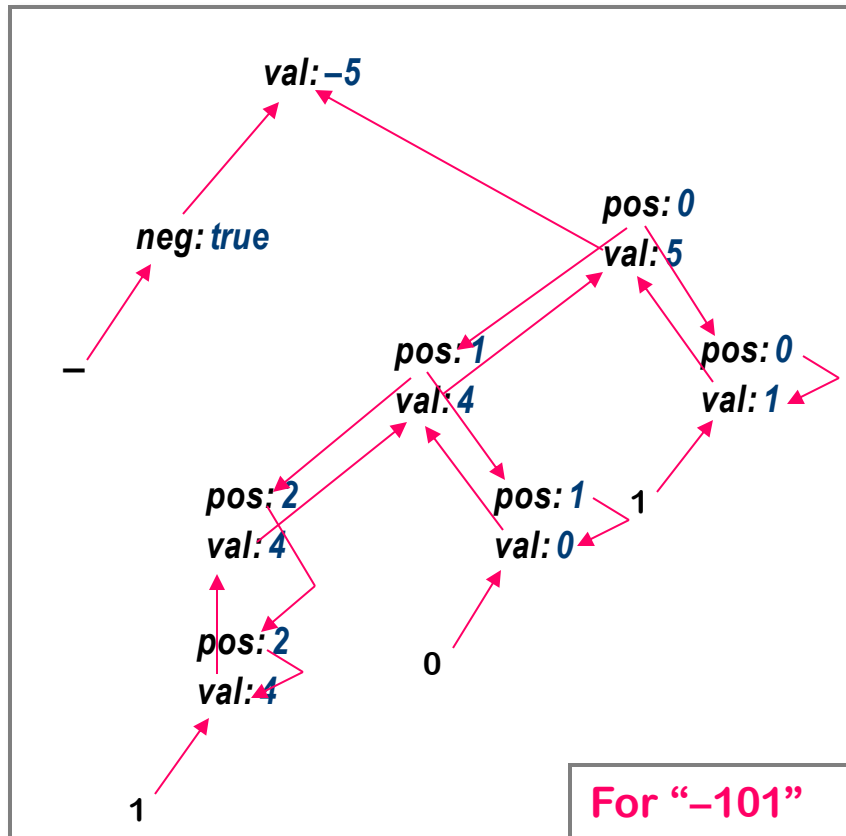


If we show the computation ...

& then peel away the parse tree ...



Back to the Example



All that is left is the **attribute dependence graph**.

This succinctly represents the flow of values in the problem instance.

The dynamic methods sort this graph to find independent values, then work along graph edges.

The rule-based methods try to discover "good" orders by analyzing the rules.

The oblivious methods ignore the structure of this graph.

The dependence graph must be acyclic



Circularity

We can only evaluate acyclic instances

- **General circularity testing** problem is inherently exponential!
- We can prove that some grammars can only generate instances with acyclic dependence graphs
 - Largest such class is “strongly non-circular” grammars (*SNC*)
 - *SNC* grammars can be tested in polynomial time
 - Failing the *SNC* test is not conclusive

Many evaluation methods discover circularity dynamically

⇒ Bad property for a compiler to have



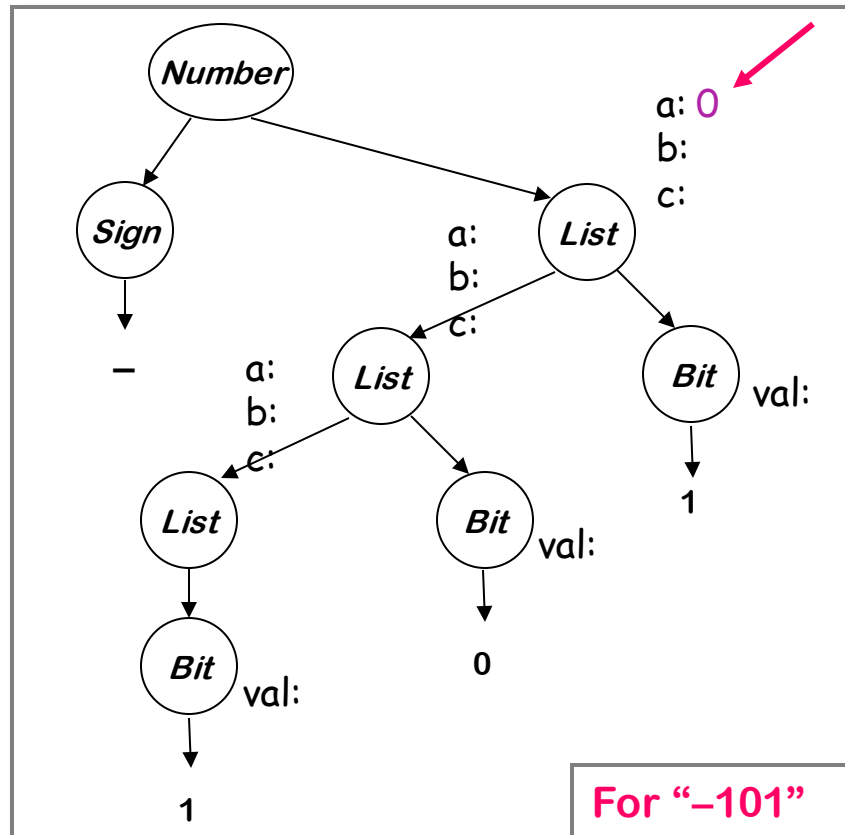
A Circular Attribute Grammar

Productions		Attribution Rules
<i>Number</i>	\rightarrow <i>List</i>	$List.a \leftarrow 0$
$List_0$	\rightarrow <i>List</i> ₁ <i>Bit</i>	$List_1.a \leftarrow List_0.a + 1$ $List_0.b \leftarrow List_1.b$ $List_1.c \leftarrow List_1.b + Bit.val$
	$ $ <i>Bit</i>	$List_0.b \leftarrow List_0.a + List_0.c + Bit.val$
<i>Bit</i>	\rightarrow <i>0</i>	$Bit.val \leftarrow 0$
	$ $ <i>1</i>	$Bit.val \leftarrow 1$

Remember, the circularity is in the attribution rules, not the underlying CFG



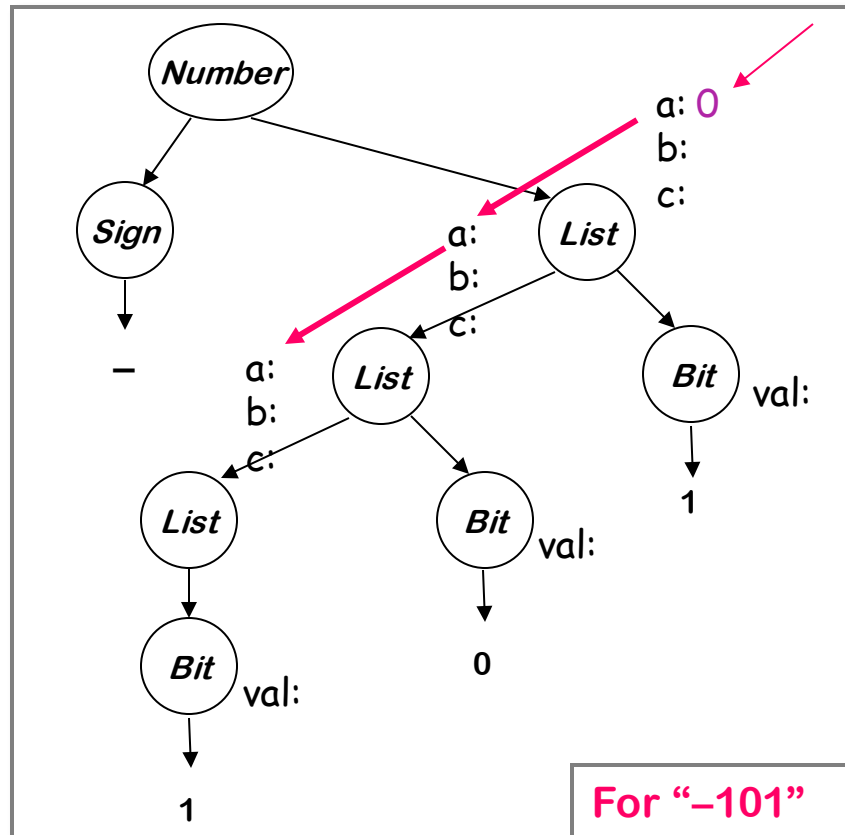
Circular Grammar Example



Productions		Attribution Rules
<i>Number</i>	\rightarrow <i>List</i>	$List.a \leftarrow 0$
<i>List</i> ₀	\rightarrow <i>List</i> ₁	$List_1.a \leftarrow List_0.a + 1$
	<i>Bit</i>	$List_0.b \leftarrow List_1.b$
		$List_1.c \leftarrow List_1.b + Bit.val$
	$ $ <i>Bit</i>	$List_0.b \leftarrow List_0.a + List_0.c + Bit.val$
<i>Bit</i>	\rightarrow 0	$Bit.val \leftarrow 0$
	$ $ 1	$Bit.val \leftarrow 1$



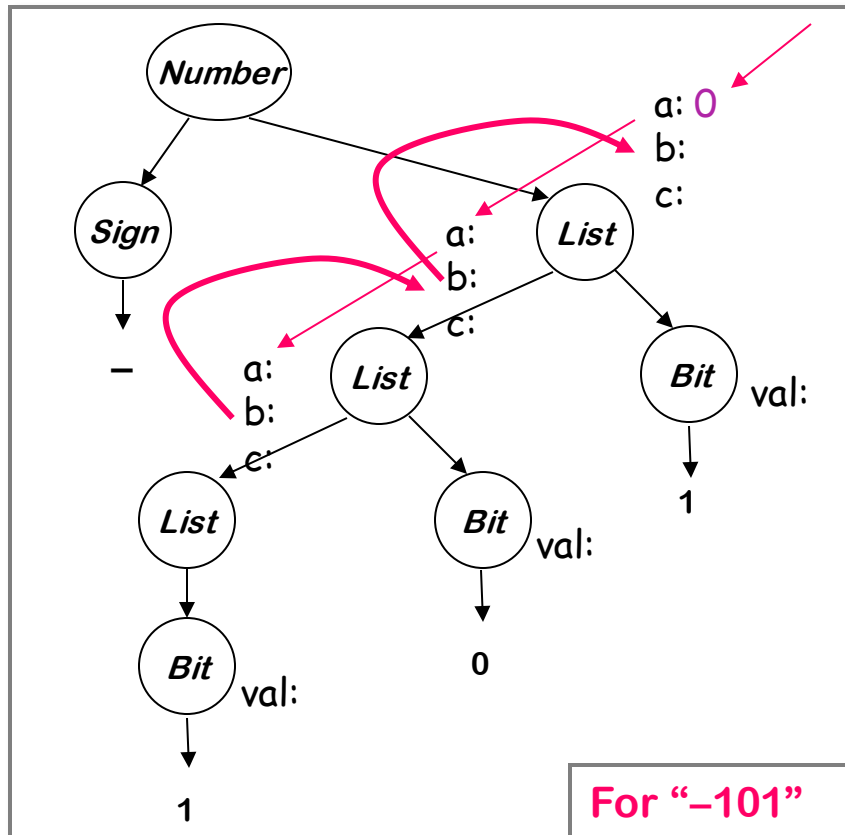
Circular Grammar Example



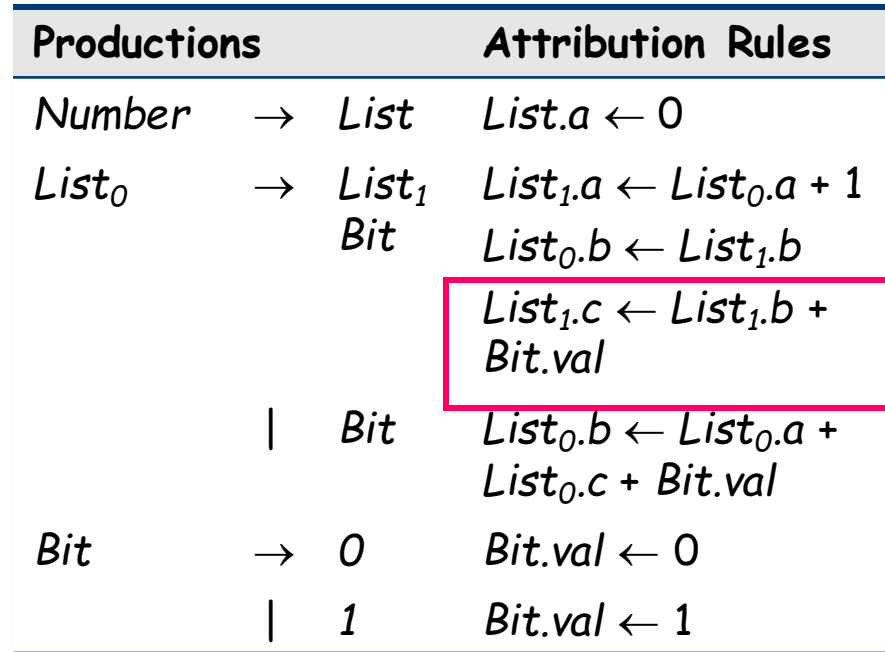
Productions		Attribution Rules
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<i>Bit</i>	\rightarrow <i>0</i>	$Bit.val \leftarrow 0$
	$ $ <i>1</i>	$Bit.val \leftarrow 1$



Circular Grammar Example

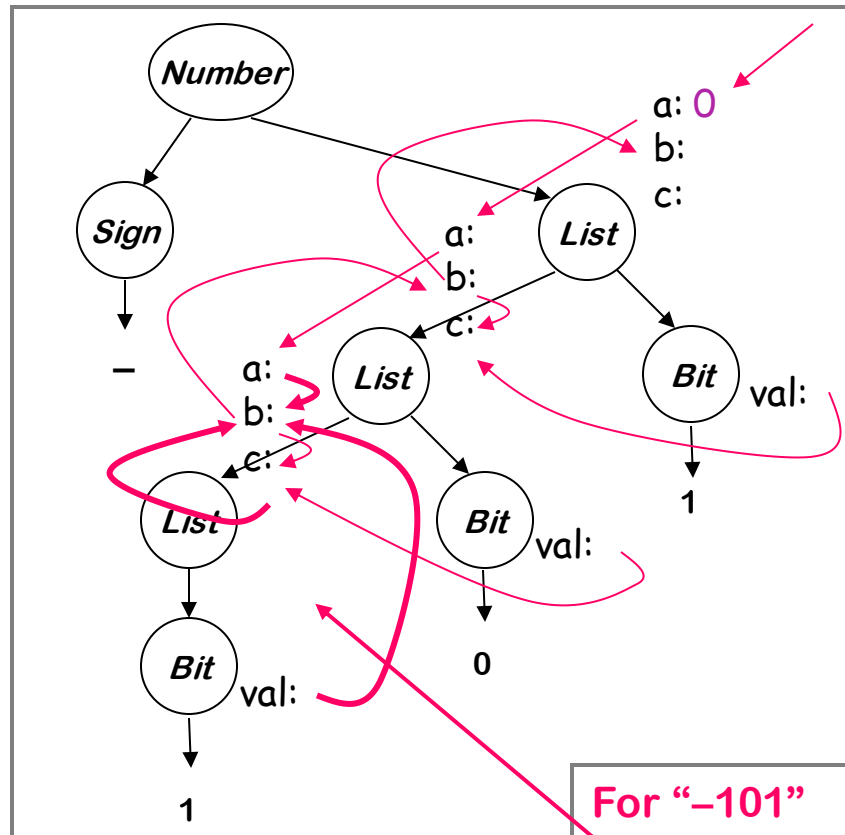


Productions		Attribution Rules
<i>Number</i>	\rightarrow <i>List</i>	$List.a \leftarrow 0$
<i>List</i> ₀	\rightarrow <i>List</i> ₁	$List_1.a \leftarrow List_0.a + 1$
	<i>Bit</i>	$List_0.b \leftarrow List_1.b$
		$List_1.c \leftarrow List_1.b + Bit.val$
	\mid <i>Bit</i>	$List_0.b \leftarrow List_0.a + List_0.c + Bit.val$
<i>Bit</i>	\rightarrow 0	$Bit.val \leftarrow 0$
	\mid 1	$Bit.val \leftarrow 1$





Circular Grammar Example

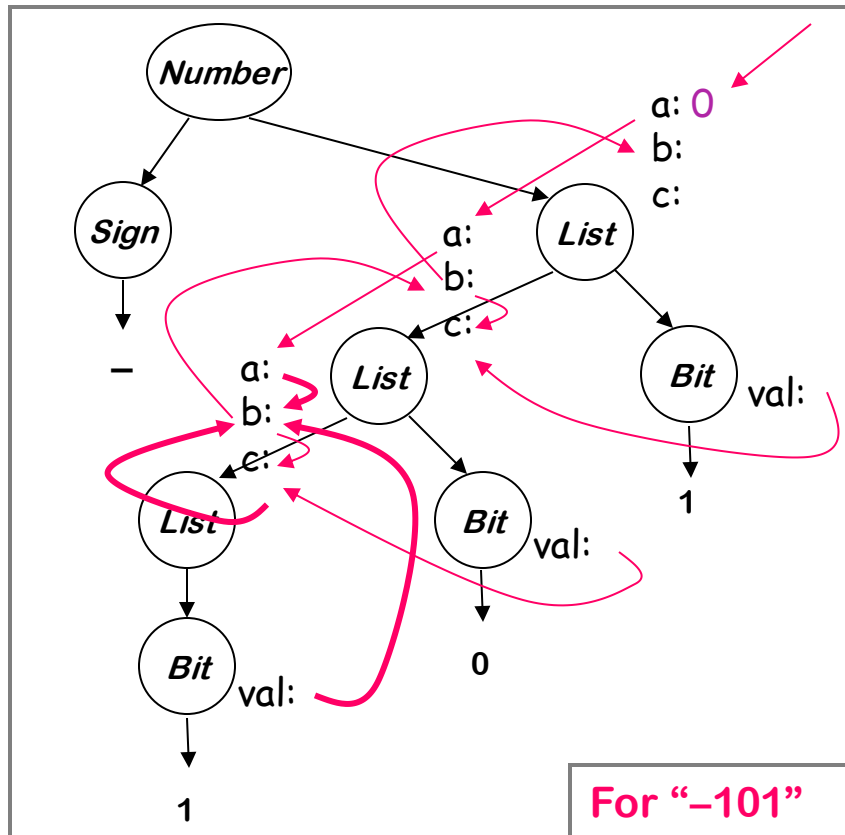


Productions		Attribution Rules
<i>Number</i>	\rightarrow <i>List</i>	$List.a \leftarrow 0$
<i>List</i> ₀	\rightarrow <i>List</i> ₁	$List_1.a \leftarrow List_0.a + 1$
	<i>Bit</i>	$List_0.b \leftarrow List_1.b$
		$List_1.c \leftarrow List_1.b + Bit.val$
		$Bit.val$
	$ $ <i>Bit</i>	$List_0.b \leftarrow List_0.a + List_0.c + Bit.val$
<i>Bit</i>	\rightarrow 0	$Bit.val \leftarrow 0$
	$ $ 1	$Bit.val \leftarrow 1$

Here is the circularity ...



Circular Grammar Example



Productions	Attribution Rules
$Number \rightarrow List$	$List.a \leftarrow 0$
$List_0 \rightarrow List_1$	$List_1.a \leftarrow List_0.a + 1$
Bit	$List_0.b \leftarrow List_1.b$
	$List_1.c \leftarrow List_1.b + Bit.val$
	$List_0.b \leftarrow List_0.a + List_0.c + Bit.val$
$Bit \rightarrow 0$	$Bit.val \leftarrow 0$
$Bit \rightarrow 1$	$Bit.val \leftarrow 1$

Here is the circularity ...



Circularity — The Point

- Circular grammars have indeterminate values
 - Algorithmic evaluators will fail
- Noncircular grammars evaluate to a unique set of values
- Circular grammar might give rise to noncircular instance
 - Probably shouldn't bet the compiler on it ...

⇒ Should (*undoubtedly*) use provably noncircular grammars

Remember, we are studying AGs to gain insight

- We should avoid circular, indeterminate computations
- If we stick to provably noncircular schemes, evaluation should be easier



An Extended Attribute Grammar Example

Grammar for a basic block

(§ 4.3.3)

1	$Block_0$	\rightarrow	$Block_1$ Assign
2			Assign
3	$Assign_0$	\rightarrow	Ident = Expr ;
4	$Expr_0$	\rightarrow	$Expr_1 + Term$
5			$Expr_1 - Term$
6			Term
7	$Term_0$	\rightarrow	$Term_1 * Factor$
8			$Term_1 / Factor$
9			Factor
10	Factor	\rightarrow	(Expr)
11			Number
12			Ident

Let's estimate cycle counts

- Each operation has a COST
- Add them, bottom up
- Assume a load per value
- Assume no reuse

Simple problem for an AG

Hey, that is a practical application!

An Extended Example

(continued)



1	$Block_0 \rightarrow Block_1 \text{ Assign}$	$Block_0.cost \leftarrow Block_1.cost + Assign.cost$
2	$Assign$	$Block_0.cost \leftarrow Assign.cost$
3	$Assign_0 \rightarrow Ident = Expr ;$	$Assign.cost \leftarrow COST(store) + Expr.cost$
4	$Expr_0 \rightarrow Expr_1 + Term$	$Expr_0.cost \leftarrow Expr_1.cost + COST(add) + Term.cost$
5	$Expr_1 - Term$	$Expr_0.cost \leftarrow Expr_1.cost + COST(sub) + Term.cost$
6	$Term$	$Expr_0.cost \leftarrow Term.cost$
7	$Term_0 \rightarrow Term_1 * Factor$	$Term_0.cost \leftarrow Term_1.cost + COST(mult) + Factor.cost$
8	$Term_1 / Factor$	$Term_0.cost \leftarrow Term_1.cost + COST(div) + Factor.cost$
9	$Factor$	$Term_0.cost \leftarrow Factor.cost$
10	$Factor \rightarrow (Expr)$	$Factor.cost \leftarrow Expr.cost$
11	$Number$	$Factor.cost \leftarrow COST(loadI)$
12	$Ident$	$Factor.cost \leftarrow COST(load)$

These are all synthesized attributes !

Values flow from rhs to lhs in prod'ns



An Extended Example

(continued)

Properties of the example grammar

- All attributes are synthesized \Rightarrow S-attributed grammar
- Rules can be evaluated bottom-up in a single pass
 - Good fit to bottom-up, shift/reduce parser
- Easily understood solution
- Seems to fit the problem well

What about an improvement?

- Values are loaded only once per block (not at each use)
- Need to track which values have been already loaded



A Better Execution Model

Adding load tracking

- Need sets *Before* and *After* for each production
- Must be initialized, updated, and passed around the tree

10	<i>Factor</i> \rightarrow (<i>Expr</i>)	<i>Factor.cost</i> \leftarrow <i>Expr.cost</i> <i>Expr.before</i> \leftarrow <i>Factor.before</i> <i>Factor.after</i> \leftarrow <i>Expr.after</i>
11	<i>Number</i>	<i>Factor.cost</i> \leftarrow <i>COST</i> (loadI) <i>Factor.after</i> \leftarrow <i>Factor.before</i>
12	<i>Ident</i>	If (<i>Ident.name</i> \notin <i>Factor.before</i>) then <i>Factor.cost</i> \leftarrow <i>COST</i> (load) <i>Factor.after</i> \leftarrow <i>Factor.before</i> \cup { <i>Ident.name</i> } else <i>Factor.cost</i> \leftarrow 0 <i>Factor.after</i> \leftarrow <i>Factor.before</i>

This version is much more complex



A Better Execution Model

- Load tracking adds complexity
- But, most of it is in the "copy rules"
- Every production needs rules to copy *Before & After*

A sample production

4	$Expr_0 \rightarrow Expr_1 + Term$	$Expr_0.cost \leftarrow Expr_1.cost +$ $COST(add) + Term.cost$ $Expr_1.before \leftarrow Expr_0.before$ $Term.before \leftarrow Expr_1.before$ $Expr_0.after \leftarrow Term.after$
---	------------------------------------	---

These copy rules multiply rapidly

Each creates an instance of the set

Lots of work, lots of space, lots of rules to write



An Even Better Model

What about accounting for finite register sets?

- *Before & After* must be of limited size
- Adds complexity to *Factor*→*Identifier*
- Requires more complex initialization

Jump from tracking loads to tracking registers is small

- Copy rules are already in place
- Some local code to perform the allocation



And Its Extensions

Tracking loads

- Introduced *Before* and *After* sets to record loads
- Added ≥ 2 copy rules per production
 - Serialized evaluation into execution order
- Made the whole attribute grammar large & cumbersome

Finite register set

- Complicated one production (*Factor* \rightarrow Identifier)
- Needed a little fancier initialization
- Changes were quite limited

Why is one change hard and the other easy?



The Moral of the Story

- Non-local computation needed lots of supporting rules
- Complex local computation was relatively easy

The Problems

- Copy rules increase cognitive overhead
- Copy rules increase space requirements
 - Need copies of attributes
 - Can use pointers, for even more cognitive overhead
- Result is an attributed tree
 - Must build the parse tree
 - Either search tree for answers or copy them to the root

A good rule of thumb is that the compiler touches all the space it allocates, usually multiple times

(somewhat subtle points)