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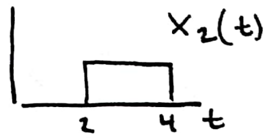
# Recitation 3
## ESE 325

1. $x_1(t) = r(t) - r(t-1) - u(t-1)$

$x_2(t) = u(t-2) - u(t-4)$

Find $x_1(t) * x_2(t)$

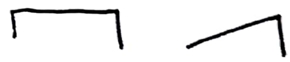




$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau = \int_0^1 \tau\, x_2(t-\tau)\, d\tau$$

Cases:

$t - \tau < 2$

$\Rightarrow t < 2 + \tau$ No overlap



$\Rightarrow t < 2$

$t - \tau > 4$

$\Rightarrow t > 4 + \tau$ No overlap

$\Rightarrow t > 5$

$2 \le t - \tau < 3$ overlap

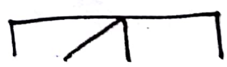
$\Rightarrow t \ge 2 + \tau$

$t < 3 + \tau$

$t \ge 2$

$t < 3$

$3 \le t - \tau < 4$ Envelope



$\Rightarrow t \ge 3 + \tau$

$t < 4 + \tau$

$t \ge 3$

$t < 4$

$4 \le t - \tau < 5$

overlap

$\Rightarrow \quad t \ge 4 + \tau$

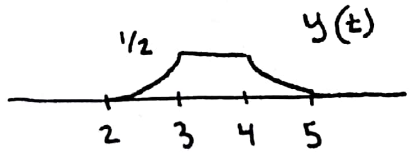
$\qquad t < 5 + \tau$

$\qquad t \ge 4$

$\qquad t < 5$

$y(t) = x_1(t) * x_2(t)$

$$= \begin{cases} 0 & , \quad t < 2 \\ \int_0^{t-2} \tau \, d\tau & , \quad 2 \le t < 3 \\ 1/2 & , \quad 3 \le t < 4 \\ \int_{t-4}^{1} \tau \, d\tau & , \quad 4 \le t < 5 \\ 0 & , \quad 5 \le t \end{cases} = \begin{cases} 0 & , \quad t < 2 \\ \frac{(t-2)^2}{2} & , \quad 2 \le t < 3 \\ 1/2 & , \quad 3 \le t < 4 \\ \frac{1-(t-4)^2}{2} & , \quad 4 \le t < 5 \\ 0 & , \quad 5 \le t \end{cases}$$



2.

$$S_1: \quad y[n] = \begin{cases} x\left[\frac{n}{2}\right], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

$$S_2: \quad y[n] = x[2n]$$

$S_1$:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| x | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| y | -4 | | -2 | | 0 | | 2 | | 4 |

$S_2$:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| x | ~~-3~~ | -2 | ~~-1~~ | 0 | ~~1~~ | 2 | ~~3~~ | 4 |
| y | -3 | -2 | -1 | 0 | 1 | 2 | 3 | |

$S_2$ discards all the odd samples!

| $n$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $x[n]$ | 1 | 0 | 0 | 1 | 1 | 0 | 1 |

$S_1 \downarrow$

| $n$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $y_1[n]$ | 0 | 0 | 1 | 0 | 1 | 0 | 0 |

$\downarrow$ $S_2$

| $n$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $y_2[n]$ | | | 0 | 1 | 1 | 0 | |

we recover the original signal.

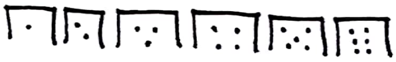
# Convolution

What kind of operation is convolution?

- It's a smoothing operation
- A way to multiply two functions

It turns, out, convolution is closely related to the CLT

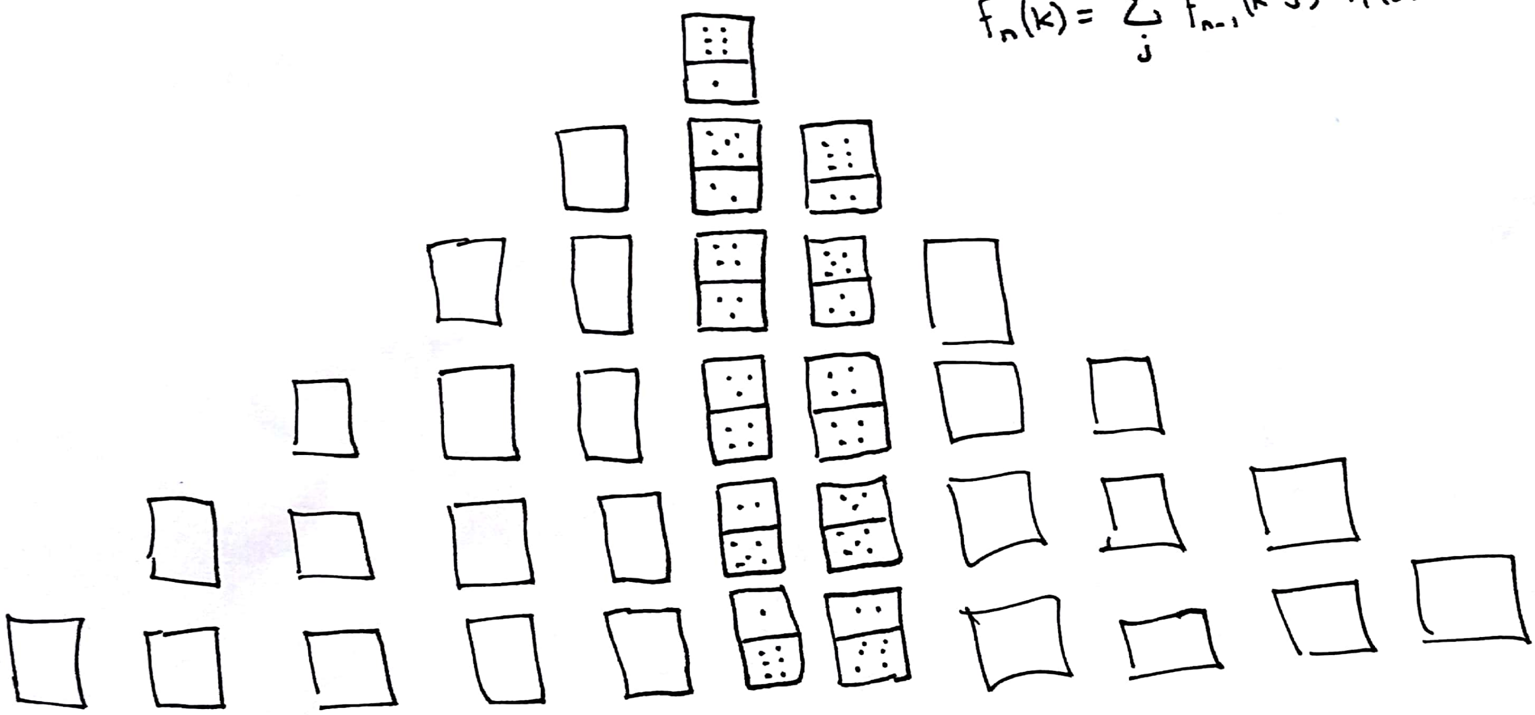
- PDF of a sum of ind. r.v.s is conv. of the two PDFs
- Face of a die:



square pulse

- Sum of two dice:

$$f_n(k) = \sum_j f_{n-1}(k-j)\, f_1(j)$$



MATLAB example