Recitation 8

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\left(\frac{2\pi}{N}\right)n} \qquad a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, e^{-jk\left(\frac{2\pi}{N}\right)n}$$

1. Let $h[n] = \left(\frac{1}{2}\right)^{|n|}$ and $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$

Find the F.S. coeff's of $y[n] = x[n] * h[n]$

Soln: $y[n] = \sum_k \sum_\ell \delta[k-4\ell]\left(\frac{1}{2}\right)^{|n-k|}$

only look at $k = \ldots, -4, 0, 4, \ldots$

$$= \sum_k \frac{1}{2}^{|n-4k|}$$

$=$

$n=0$ we get 4δ (from $m=0$)

$n=1$ we get 8δ (from $m=0$)

$n=2$ 0

$n=3$ 0 $N=4$

$n=4$ we get 4δ (from $m=1$)

$n=5$ we get 8δ (from $m=1$)

$$a_k = \frac{1}{4}\sum_n x[n] e^{-jk\frac{2\pi}{4}n}$$

$$= \frac{1}{4}\left[4 + 8e^{-jk\frac{2\pi}{4}}\right] = 1 + 2e^{-jk\frac{2\pi}{4}}$$

$a_0 = 3$, $a_1 = 1+2e^{-j\frac{2\pi}{4}}$, $a_2 = -1$, $a_3 = 1+2j$

1.

Let $x[n] = 1 + \cos\left(\frac{2\pi}{6} n\right)$

$y[n] = \sin\left(\frac{2\pi}{6} n + \frac{\pi}{4}\right)$

Q: Find FS coefficients of x and y.

Soln: $x[n] = e^0 + \frac{1}{2}\left(e^{j\frac{2\pi n}{6}} + e^{-j\frac{2\pi n}{6}}\right)$

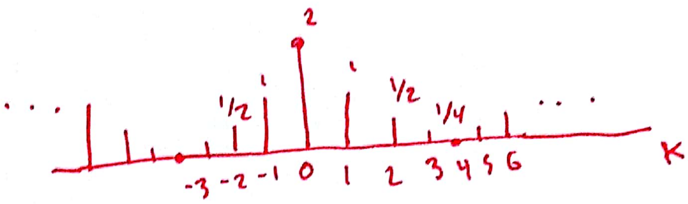
By inspection $a_0 = 1$, $a_1 = \frac{1}{2}$, $a_{-1} = \frac{1}{2}$, $a_k = 0$ otherwise

$$y[n] = \frac{1}{2j}\left(e^{j\left(\frac{2\pi n}{6} + \frac{\pi}{4}\right)} - e^{-j\left(\frac{2\pi n}{6} + \frac{\pi}{4}\right)}\right)$$

$$= \frac{1}{2j} e^{j\pi/4} e^{\cdots} - \frac{1}{2j} e^{-j\pi/4} e^{\cdots}$$

By inspection $b_1 = \frac{e^{j\pi/4}}{2j}$ and $b_{-1} = \frac{-e^{-j\pi/4}}{2j}$

3. Let a_k be drawn below:



Find $x[n]$:

Soln: $x[n] = 2 + 2\cos(\pi n/4) + \cos(\pi n/2) + \frac{1}{2}\cos\left(\frac{3\pi n}{4}\right)$

4. Let $x[n] = \sum^{\infty} 4\,\delta[n-4m] + 8\,\delta[n-1-4m]$.

Find the F.S. coefficients of $x[n]$.

Soln:

$n = 0 \rightarrow 4\delta$ (from $m=0$)

$= 1 \rightarrow 8\delta$ (from $m=0$)

$= 2 \rightarrow 0$

$= 3 \rightarrow 0$

$= 4 \rightarrow 4\delta$ (from $m=1$)

$= 5 \rightarrow 8\delta$ (from $m=1$)

And so $x[0] = 4$, $x[1] = 8$, $x[2] = 0$, $x[3] = 0$, $N = 4$

$$a_k = \frac{1}{4}\sum_{n=0}^{3} x[n]\, e^{-jk\frac{2\pi n}{4}}$$

$$= \frac{1}{4}\left[4 + 8e^{-jk\frac{2\pi}{4}}\right] = 1 + 2e^{-jk\frac{\pi}{2}}$$

$a_0 = 3$, $a_1 = 1 + 2e^{-j\frac{2\pi}{4}}$, $a_2 = -1$, $a_3 = 1 + 2j$