Recitation 4

10/2/17

1. Let $x(t) = \cos(\omega_1 t) - 2\sin(\omega_2 t)$,

where $\omega_1 = \frac{\pi}{7}$ and $\omega_2 = \frac{3\pi}{2}$

a) Find the fundamental period T of $x(t)$.

Soln:

$$\frac{\pi}{7} \cdot T_1 = 2\pi \quad \Rightarrow \quad T_1 = 14$$

$$\frac{3\pi}{2} \cdot T_2 = 2\pi \quad \Rightarrow \quad T_2 = \frac{4}{3}$$

$$LCM(T_1, T_2) = 28$$

And so $x(t)$ is periodic with period $T = 28$.

b) Find the F.S. coefficients a_k of $x(t)$.

Soln:

$$x(t) = \frac{1}{2}\left[e^{j\omega_1 t} + e^{-j\omega_1 t}\right] - \left(e^{j\omega_2 t} - e^{-j\omega_2 t}\right) \cdot \frac{1}{j}$$

$$= \frac{1}{2}\left[e^{j\frac{2\pi}{28}(2)t} + e^{-j\frac{2\pi}{28}(2)t}\right] - \left[e^{j\frac{2\pi}{28}(21)t} - e^{-j\frac{2\pi}{28}(21)t}\right] \cdot \frac{1}{j}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}kt}$$

And by inspection: $a_2 = 1/2$, $a_{-2} = 1/2$, $a_{21} = \frac{-1}{j}$, $a_{-21} = \frac{1}{j}$

4. Suppose a system S has step response $e^t \sin(t) u(t)$. Find the impulse response $h(t)$ of S.

Soln: $\delta(t) = \frac{d}{dt}\left[u(t)\right]$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

$$\frac{dy}{dt} = \frac{d}{dt}\left[\int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau\right] = \int \frac{d}{dt}\left[x(t-\tau)\right] h(\tau)\, d\tau$$

$$\Rightarrow \quad y'(t) = x'(t) * h(t)$$

Now let $x(t) = u(t)$

so that $x'(t) = \delta(t)$

and we see that $y'(t) = \delta(t) * h(t)$

$$= h(t) = \frac{d}{dt}\left[e^t \sin t \, u(t)\right]$$

$$= e^t \sin(t) u(t) + e^t \cos(t) u(t) + e^0 \sin(0)\, \delta(t)$$

$$= e^t (\sin(t) + \cos(t))\, u(t)$$