10/16/17

Recitation 6

1. Suppose an LTI system S has step response $s[n]$. What is its impulse response? Is S invertible? Find its inverse if it exists.

Soln:
$$u[n] \rightarrow s[n]$$
$$\underbrace{u[n] - u[n-1]}_{\delta[n]} \rightarrow \underbrace{s[n] - s[n-1]}_{h[n]} \quad \text{(by LTI)}$$

$$y[n] = x[n] - x[n-1] \quad \text{(first backward difference)}$$

Let $h_1[n] = s[n] - s[n-1]$

For inverse to exist, we must have:

$$h_1[n] * h_2[n] = \delta[n] \quad \text{(cascade is the identity system)}$$

$$\Rightarrow \quad h_2[n] - h_2[n-1] = \delta[n]$$

$$\Rightarrow \quad h_2[0] - h_2[-1] = 1$$

and $h_2[n] = h_2[n-1]$ for $n \neq 0$

one choice then is $h_2[0] = 1$, $h_2[n] = 1$ for $n \geq 1$,

and $h_2[n] = 0$ for $n < 0$.

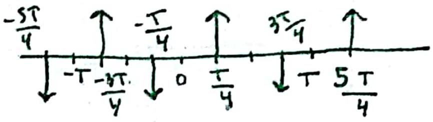
And so $h_2[n] = u[n] = \sum_{k=-\infty}^{n} \delta[k]$

(Anti-causal $h_2 = -u[-n-1]$ solution also exists)

2.

$$x_1(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) \quad \overset{F.S.}{\longleftrightarrow} \quad a_k \quad , \quad T > 0$$

$x_2(t)$



Express b_k as a function of a_k

$$x_2(t) = x_1(t - T/4) - x_1(t + T/4)$$

$$b_k = a_k\left(e^{-jk\omega_0 \frac{T}{4} t} - e^{jk\omega_0 \frac{T}{4} t}\right) \qquad \omega_0 = \frac{2\pi}{T}$$

$$= a_k\left(-2j \sin\left(k \frac{\pi t}{2}\right)\right)$$

3. Let $x(t)$ be a real and even periodic signal with fundamental frequency ω_0. Show a representation $x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_0 k t)$ where $A_k \in \mathbb{R}$.

$$x(t) = \sum_{K=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad = \sum_{k=1}^{\infty} a_k e^{jk\omega_0 t} + \sum_{k=1}^{\infty} a_{-k} e^{j(-k)\omega_0 t} + a_0$$

$$= \sum_{K=-\infty}^{\infty} a_k e^{jk\omega_0(-t)} \qquad = \sum_{k=1}^{\infty} a_k \left(e^{jk\omega_0 t} + e^{-jk\omega_0 t}\right) + a_0$$

$$= \sum_{K=-\infty}^{\infty} a_{-k} e^{jk\omega_0 t} \qquad = \sum_{k=1}^{\infty} a_k \cdot 2\cos(k\omega_0 t) + a_0 = \sum_{k=0}^{\infty} A_k \cos(k\omega_0 t)$$

$$\Rightarrow \quad a_k = a_{-k}$$

And so $A_k = 2a_k$ for $k \neq 0$

$A_0 = a_0$