9/11/17

Recitation 1

1. Let S be a time invariant system. Not necessarily linear.

$$x(t) \rightarrow \boxed{S} \rightarrow y(t)$$

Suppose $x(t)$ produces output $y(t)$ when sent through S. Further suppose $x(t)$ is periodic with period $T>0$. Is $y(t)$ periodic in general? Prove that it is or provide a counterexample.

Soln:

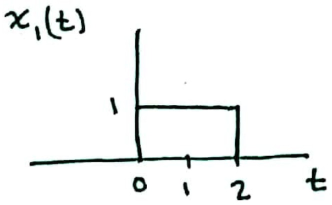
$x(t) \rightarrow y(t)$ (given)

$x(t+T) \rightarrow y(t+T)$ (time invariance)

But $x(t) = x(t+T)$ for all T (periodicity)

And so must have $y(t) = y(t+T)$

2. Let S be an LTI system. Suppose $x_1(t) \rightarrow y_1(t)$.



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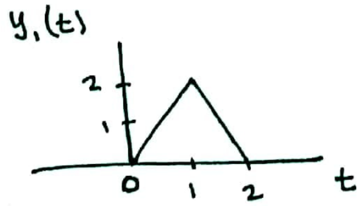
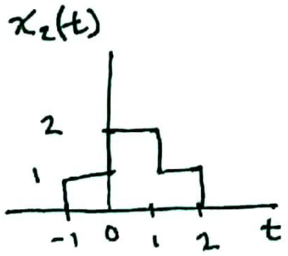


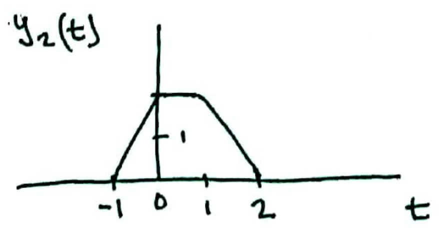
What is the output of S when the input is $x_2(t)$?



Soln:

$x_2(t) = x_1(t) + x_1(t+1)$

$y_2(t) = y_1(t) + y_1(t+1)$



3. Recall the convolution sum $x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

Let $x[n] = \alpha^n u[n]$
and $h[n] = \beta^n u[n]$ and suppose $\alpha \neq \beta$

Compute $x[n] * h[n]$

Soln: $x[n] * h[n] = \sum_{k=-\infty}^{\infty} \alpha^k u[k]\, \beta^{n-k} u[n-k]$

$$= \sum_{k=0}^{n} \alpha^k \beta^{n-k} = \beta^n \sum_{k=0}^{n} (\alpha/\beta)^k = \beta^n \cdot \frac{1-(\alpha/\beta)^{n+1}}{1-\alpha/\beta}$$

$$= \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha}$$ (but this is only true for $n \geq 0$)

$$= \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha}\, u[n]$$

4. Solve Problem 3 for the case when $\alpha = \beta$.

$\beta^n \sum_{k=0}^{n} (\alpha/\beta)^k = \beta^n \sum_{k=0}^{n} 1 = \beta^n (n+1)$ (again only true for $n \geq 0$)

$$= \beta^n (n+1) \cdot u[n]$$

5. Consider a system with I/O relationship:

$$y[n] = x[n] \cdot x[n-2]$$

What is the output to $\delta[n]$?

Soln: $y[n] = \delta[n] \cdot \delta[n-2] = 0$ for all n

6. What is the output of the above system when the IP is $x[n] = \delta[n] + \delta[n+2]$?

Soln: Look at the "critical" points $n = -2, 0,$ and 2. The deltas are zero elsewhere.

$$y[n] = \Big(\underset{\substack{\uparrow \\ n=0}}{\delta[n]} + \underset{\substack{\uparrow \\ n=-2}}{\delta[n+2]}\Big)\Big(\underset{\substack{\uparrow \\ n=2}}{\delta[n-2]} + \underset{\substack{\uparrow \\ n=0}}{\delta[n]}\Big)$$

$$y[n] = \begin{cases} 0, & n = -2 \\ 1, & n = 0 \\ 0, & n = 2 \\ 0 & \text{o.w.} \end{cases}$$

$$= \delta[n]$$