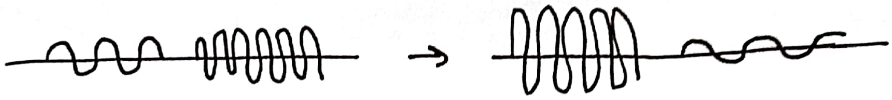
Recitation 9

FIR discrete time filters

$$y[n] = \sum_{k=0}^{N} b_k x[n-k] \qquad \text{causal}$$

Sometime we want linear phase

- All frequencies get delayed same amount
- Achieve this by imposing symmetry on the b_ks



Filter shuffled our signal

- Not linear phase

How to choose the b_ks? Filter design.

- windowing
    - design an IIR filter and truncate it by multiplying by a window
- use optimization techniques
    - Define error as difference between desired and actual
    - use convex optimization to pick the b_ks (decision variables)
- use an adaptive algorithm with a gradient update to converge to desired frequency response

Why FIR filters?

- Simple to implement on computer
    - Finite summation
- Easy to ensure linear phase
    - Impose symmetry (odd/even)

Problems. $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$ $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

2. $$X(j\omega) = \frac{\omega}{1+\omega^2} = \omega \cdot \frac{1}{1-j\omega}\cdot\frac{1}{1+j\omega} = \frac{A_1}{1-j\omega} + \frac{A_2}{1+j\omega}$$

$$A_2 = \frac{j}{1-j(j)} = \frac{j}{2} \qquad A_1 = \frac{-j}{2}$$

$$X(j\omega) = \frac{-j}{2(1-j\omega)} + \frac{j}{2(1+j\omega)} = \frac{j}{2}\left[\frac{1}{1+j\omega} - \frac{1}{1-j\omega}\right]$$ ← Time/freq. reversal

$$\Rightarrow \quad x(t) = \frac{j}{2}\left[e^{-t}u(t) - e^{t}u(-t)\right]$$

using $x(t) = e^{-at}u(t) \leftrightarrow \frac{1}{a+j\omega}$ for $Re\{a\} > 0$

1. Find FT of $\frac{d}{dt}\left[u(-t-2) + u(t-2)\right] = -\delta(t+2) + \delta(t-2)$

$$\leftrightarrow \quad -e^{j2\omega} + e^{-j2\omega} = -\sin(2\omega)\cdot 2j$$

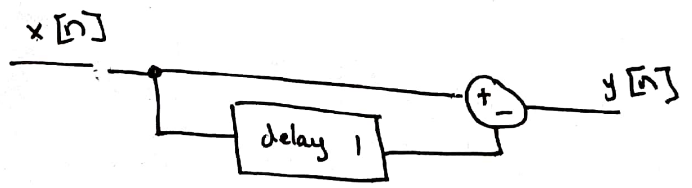
3. Let $x[k]$ be a real periodic signal with period N even. Show that $a_{N/2}$ is real. $a_k = \frac{1}{N}\sum_{\langle N\rangle = n} x[n]e^{-jk\frac{2\pi}{N}n}$

Soln: Let $N = 2M$.

$$a_{N/2} = a_M = \frac{1}{2M}\sum_{k=0}^{N-1} x[k]e^{-jk\frac{2\pi}{N}\cdot M} = \frac{1}{2M}\sum_{k=0}^{2M-1} x[k]e^{-j\pi k} = \frac{1}{2M}\sum_k x[k](-1)^k$$

4. MATLAB demo

5. write block diagram for $y[n] = x[n] - x[n-1]$



write block diagram for $y[n] - y[n-1] + y[n-2] = 2x[n]$