9/18/17

Recitation 2

1. Consider the systems S_1 and S_2 whose IP/OP relationships are given below:

S_1:
$$y[n] = \begin{cases} x\left[\frac{3n}{2}\right], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

S_2:
$$y[n] = x[2n]$$

a) Let S_a be the following cascade system:

$$\longrightarrow \boxed{S_1} \longrightarrow \boxed{S_2} \longrightarrow$$

Is S_a invertible? Justify your answer.

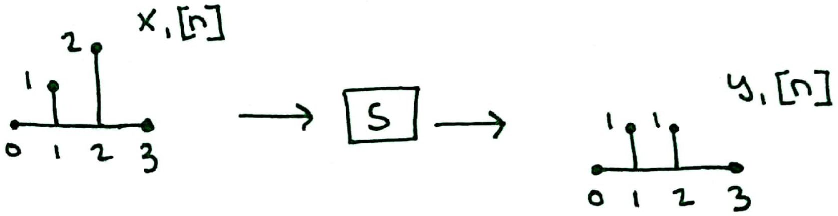
b) Let S_b be the following cascade system:

$$\longrightarrow \boxed{S_2} \longrightarrow \boxed{S_1} \longrightarrow$$

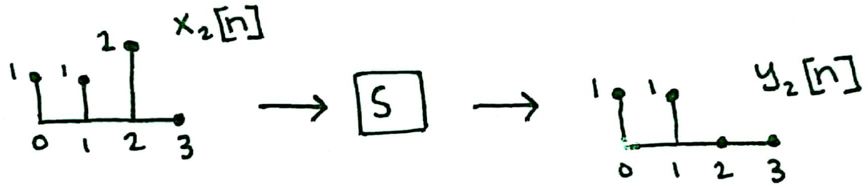
Is S_b invertible? Justify your answer.

2. Below are three IP/OP pairs of some system S. The signals are null where unspecified.

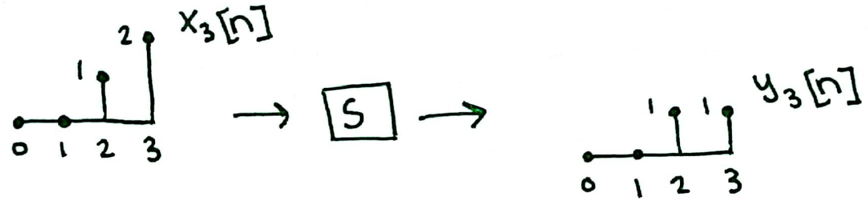
(i)



(ii)



(iii)



a) Is this system TI?

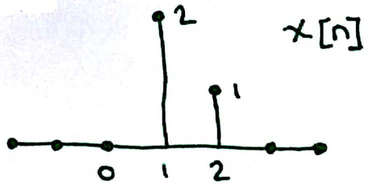
Soln: It could be, but we need to see all possible IP/OP pairs to know for sure that it is.

b) Suppose S is TI. Is it invertible?

Soln: No. Note that by time-invariance, we have $x_2[n-1] \rightarrow y_2[n-1] = y_1[n]$.

But $x_1[n] \neq x_2[n-1]$, and so the system S is not one-to-one, and therefore cannot have an inverse.

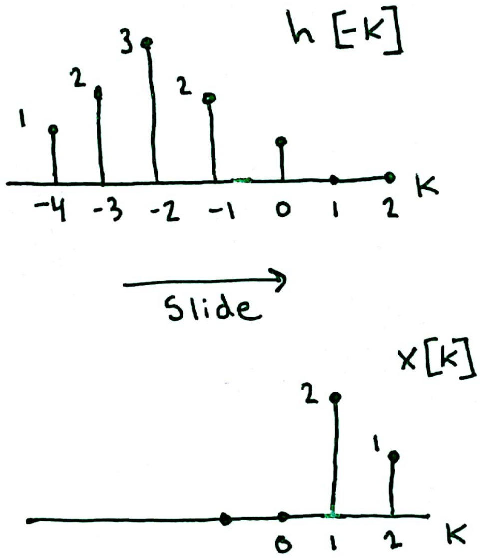
3. Consider the signals $x[n]$ and $h[n]$ given below. Assume they are null where unspecified.



Find the convolution $y[n] = x[n] * h[n]$

Soln:

$$\sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$



$y[0] = 0$
$y[1] = 2$
$y[2] = 5$
$y[3] = 8$
$y[4] = 7$
$y[5] = 4$
$y[6] = 1$
$y[7] = 0$

Alternatively, solve using distributive property of convolution:

Define $x_1[n] = \{\ \ldots 0\ 2\ 0\ 0 \ldots\}$ so that $x[n] = x_1[n] + x_2[n]$
$x_2[n] = \{\ \ldots 0\ 0\ 1\ 0 \ldots\}$

$$x[n] * h[n] = (x_1[n] + x_2[n]) * h[n] = x_1[n] * h[n] + x_2[n] * h[n]$$

(By the sifting property of $\delta[n]$)
$$= 2\,\delta[n-1] * h[n] + \delta[n-2] * h[n]$$
$$= 2\,h[n-1] + h[n-2]$$

4. Find the convolution of $x[n]$ and $h[n]$ below.

$$x[n] = n(u[n-1] - u[n-5])$$
$$h[n] = u[n+1] - u[n-2]$$

Soln: $x[n] = x[n]$

$$h[n] = \delta[n+1] + \delta[n] + \delta[n-1]$$

$$x[n] * h[n] = x[n+1] + x[n] + x[n-1]$$