Beginning Algebra

MATH 100B

Preface

This math book has been created by the BYU-Idaho Math Study Center for the college student who needs an introduction to Algebra. This book is the product of many years of implementation of an extremely successful Beginning Algebra program and includes perspectives and tips from experienced instructors and tutors.

Videos of instruction and solutions can be found at the following url: https://youtu.be/YKgyvSq42j8

We hope that it will be helpful to you as you take Algebra this semester.

The BYU-Idaho Math Study Center
April 2025 Edition

Contents

Section R-1 Factoring and Least Common Multiple

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

LCM and Factoring	Find Factors	Find LCM
Fractions	Addition/ Subtraction	Multiplication/ Division
Decimals	Addition/ Subtraction	Multiplication/ Division
Rounding	Nearest Place Value	
Percents	Change fraction and decimals to percents	Find percents of totals
Exponents, Roots	Expand and evaluate exponentials	Find roots that are whole numbers
Order of Operations	Use the order of operations correctly	
Variables and Formulas	Translate words to variables	Replace numbers and use formulas

Factoring

Main Topics	Examples
Find all Factors Steps: 1. Start with 1 and move up finding numbers that are factors. 2. List the numbers you have found. These are all the factors.	**Example 1: Find all factors of 48**

$1 \times 48 = 48$ $2 \times 24 = 48$ $3 \times 16 = 48$ $4 \times 12 = 48$ $6 \times 8 = 48$ $8 \times 6 = 48$	Step 1: Find all factors that multiply to be the product 48, starting with the number 1 and 48, and moving up the number line
	Note: In the last row we see $8 \times 6 = 48$. The 8 has already been used in the factors, so we know that all of the factors have been found.
1, 48, 2, 24, 3, 16, 4, 12, 6, 8	Step 2: Now we will list all the numbers we used until we saw the repeated number, and these will be our factors
1, 2, 3, 4, 6, 8, 12, 16, 24, 48	To make them a little easier to see, we can put them in numerical order from smallest to largest

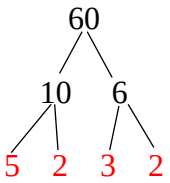
Prime Factorize

Prime: A number with exactly two factors.

Steps

1. Find a factor, break the number up.
2. Repeat until all factors are prime.

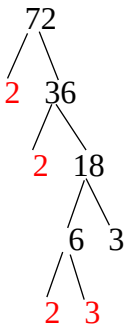
Example 2: Find the prime factorization for 60 and 72.



$60 = 2 \times 2 \times 3 \times 5$

Step 1: Break up the number into two factors.

Step 2: Repeat for each factor until prime numbers are reached.



$72 = 2 \times 2 \times 2 \times 3 \times 3$

Least Common Multiple

Main Topics	Examples
Find the LCM (Observation) Steps: 1. Write out the multiples for 4 and 5. 2. The first number that both multiples hit is 20. 3. The LCM of 4 and 5 is 20.	**Example 3: Find the LCM of 4 and 5** 4: 4, 8, 12, 16, 20, 24, 28, 32 5: 5, 10, 15, 20, 25, 30, 35, 40
Find the LCM (Prime Factorization) Steps: 1. Prime Factorize 2. Write the smallest number that contains all of the numbers. 3. Multiply it out = LCM. Note: the LCM contains the largest set of each prime factor.	**Example 4:** Prime factorization of 4: $4 = 2 \times 2$ Prime factorization of 6: $6 = 2 \times 3$ $2 \times 2 \times 3$ $2 \times 2 \times 3 = 12$ 12 is the LCM of 4 and 6

Example 5:

Prime factorization of 40

$40 = 2 \times 2 \times 2 \times 5$

Prime factorization of 36

$36 = 2 \times 2 \times 3 \times 3$

$2 \times 2 \times 2 \times 3 \times 3 \times 5$

$2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

360 is the LCM of 40 and 36

Section R-1 Exercises

Find the factors for each number

1. 30 **2.** 150 **3.** 37

4. 12 **5.** 75 **6.** 81

Find the prime factorization of each number.

7. 50 **8.** 16 **9.** 27

10. 100 **11.** 99 **12.** 13

Find the least common multiple (LCM) of each set of numbers.

13. $3, 8$ **14.** $20, 75$ **15.** $5, 7$ **16.** $12, 120$

17. $4, 16$ **18.** $8, 12$ **19.** $6, 14$ **20.** $17, 10$

21. $12, 15$ **22.** $3, 5, 10$ **23.** $2, 6, 9$ **24.** $4, 8, 10$

Preparation.

After reading some of section R.2, find:

25. $\frac{3}{5} + \frac{1}{10}$

26. $2.38 \div .75$

Section R-1 Answers

1. $1, 2, 3, 5, 6, 10, 15, 30$

2. $1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150$

3. $1, 37$

4. $1, 2, 3, 4, 6, 12$

5. $1, 3, 5, 15, 25, 75$

6. $1, 3, 9, 27, 81$

7. $2 \times 5 \times 5$

8. $2 \times 2 \times 2 \times 2$

9. $3 \times 3 \times 3$

10. $2 \times 2 \times 5 \times 5$

11. $3 \times 3 \times 11$

12. 13

13. 24

14. 300

15. 35

16. 120

17. 16

18. 24

19. 42

20. 170

21. 60

22. 30

23. 18

24. 40

25. In Class.

26. In Class.

Section R-2 Fractions and Decimals

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LCM and Factoring	Find Factors	Find LCM
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Arithmetic of Fractions

Main Topics	Examples
Key Terms Denominator Numerator Simplify One Common Denominators Least Common Denominator (LCD)	**Denominator** - The bottom of the fraction. To denominate means to name or label. **Numerator** – The top of a fraction. A numerator counts how many of the denominators there are. $\frac{5}{7}$ is read five sevenths. **Simplify** - Fractions are simplified when the numerator and denominator have no factors in common. You can also say that the fraction is ***reduced.*** **One** - Any number divided by itself is 1. $\frac{5}{5}, \frac{7}{7}, \frac{120}{120}$ **Common Denominators** - Addition and subtraction require like things. In the case of fractions, "like things" means common denominators. **Least Common Denominator (LCD)** - As the name indicates, this is the lowest possible common denominator between two or more fractions. There are an infinite number of possible common denominators, but usually the easiest one to choose is the lowest or least one. *Finding the LCD is the same process as finding the LCM

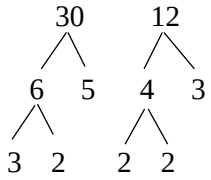
Addition and Subtraction of Fractions

Steps

1. Get a common denominator between the fractions.
2. Add or subtract the numerators.
3. Simplify.

Example 1: Add $\frac{13}{30} + \frac{7}{12}$

$$\frac{13}{30} + \frac{7}{12}$$



$\frac{13}{30} \times \frac{2}{2} = \frac{26}{60}$ $\quad \frac{7}{12} \times \frac{5}{5} = \frac{35}{60}$

$$\frac{26}{60} + \frac{35}{60} = 61$$

$$\frac{26}{60} + \frac{35}{60} = \frac{61}{60}$$

Answer: $\frac{61}{60}$

Step 1: Common denominator. Let's use prime factorization to find the LCD.

Prime factorization of 30:

$$2 \times 3 \times 5$$

Prime factorization of 12:

$$2 \times 2 \times 3$$

We need a number whose factors include each of these:

$$2 \times 2 \times 3 \times 5 = 60$$

Step 2: Add the numerators

Example 2: Subtract $\frac{5}{9} - \frac{1}{3}$

$$\frac{5}{9} - \frac{1}{3}$$

$$\frac{5}{9} - \frac{1}{3} \begin{matrix} \\ \times 3 \end{matrix}$$
$$\times 3$$

$$\frac{5}{9} - \frac{3}{9} = \frac{2}{9}$$

Answer: $\frac{2}{9}$

Step 1: The common denominator is 9, so change the $\frac{1}{3}$ to a $\frac{3}{9}$.

Step 2: Subtract the numerators

Multiplication of Fractions

Steps:

1. No common denominators.
2. Multiply numerators.
3. Multiply denominators.
4. Simplify.

Note: three expressions of multiplication:

- Times × $5 \times 3 = 15$
- Dot · $5 \cdot 3 = 15$
- Next to $5(3) = 15$

Example 3: Multiply $\frac{5}{6} \times \frac{1}{3}$

$\frac{5}{6} \times \frac{1}{3}$

Step 1: For multiplication, don't worry about getting common denominators

$\frac{5}{6} \times \frac{1}{3} = \frac{5}{?}$

Step 2: Multiply the numerators straight across

$\frac{5}{6} \times \frac{1}{3} = \frac{5}{18}$

Step 3: Multiply the denominators straight across

Answer: $\frac{5}{18}$

Division of Fractions

Steps:

1. Change any fractions into improper fractions.
2. Keep it, change it, flip it.
3. Multiply straight across.
4. Simplify

Example 4: Divide $3\frac{4}{7} \div \frac{2}{3}$

$\left(\frac{21}{7} + \frac{4}{7}\right) \div \frac{2}{3}$

$\frac{25}{7} \div \frac{2}{3}$

Step 1: Change the first term into an improper fraction. (Whole numbers just get common denominators like others.)

$\frac{25}{7} \times \frac{3}{2}$

Step 2: Keep the first fraction the same, change the division sign to multiplication, and flip the second fraction

$\frac{25 \to 3}{7 \to 2} = \frac{75}{14}$

Step 3: Multiply the numerators and the denominators straight across

Answer: $\frac{75}{14}$

Step 4: Simplify (if necessary)

	Example 5: Divide $\frac{2}{5} \div 1\frac{3}{4}$ $\frac{2}{5} \div \left(\frac{4}{4} + \frac{3}{4}\right)$ $\frac{2}{5} \div \frac{7}{4}$ $\frac{2}{5} \cdot \frac{4}{7}$ $\frac{2 \cdot 4}{5 \cdot 7} = \frac{8}{35}$ **Answer: $\frac{8}{35}$**	Step 1: Change the first term into an improper fraction. (Whole numbers just get common denominators like others.) Step 2: Keep the first fraction the same, change the division sign to multiplication, and flip the second fraction's numerator and denominator Step 3: Multiply the numerator across and the denominator across Step 4: Simplify (if necessary)

Using a Calculator

After learning to do these by hand, and practicing a few of them, you should learn to use a calculator to do the problems as well. Most scientific calculators have a key that will allow you to input fractions as well as receive the answer as a fraction as well. Have your teacher or a tutor point out which buttons on your calculator are for fractions. Then practice it.

Arithmetic of Decimals

Main Topics	Examples
Key Terms	
Place Values	Place Values - Every place on the left or right of the decimal holds a certain value. To the left of the decimal, the values are ones, tens, hundreds, thousands, and so forth. On the right of the decimal, the place values are tenths, hundredths, thousands, and so forth. 3 , 4 5 1 . 9 7 2 thousands, hundreds, tens, ones, tenths, hundredths, thousandths
Decimal	Decimal - Deci- is a prefix meaning 10. Since every place value is either 10 times larger or smaller than the place next to it, we call each place a decimal place.

Process:	Use your calculator! If you need help using your calculator, contact your tutor and/or instructor.

Turn a Fraction into a Decimal

Main Topics	Examples
Key Terms	A fraction bar, $\frac{a}{b}$ and a division sign $\div$ are the same thing. **Example 6:** Write $\frac{3}{8}$ as a decimal. $\frac{3}{8}$ is the same as $3 \div 8 = 0.375$ **Answer: 0.375** **Example 7:** Write $\frac{5}{11}$ as a decimal. $\frac{5}{11}$ or $5/11$ is the same as $5 \div 11 = 0.45\overline{45}$ **Answer: $\mathbf{0.45\overline{45}}$** This bar signifies a repeating pattern

Section R-2 Exercises

R-1

Find Factors.

1. 16 **2.** 48 **3.** 110

Find the prime factorization.

4. 60 **5.** 630 **6.** 225

Find the least common multiple (LCM).

7. 3 & 13 **8.** 8 & 22 **9.** 35 & 21

10. 108 & 32 **11.** 1500 & 180

12. If two planets are aligned with the sun and one planet goes around the sun every 12 years and the other planet takes 22 years, how long will it be before they are in alignment again?

R-2

Perform the operations by hand. Simplify.

13. $\frac{1}{6}+\frac{1}{3}$ **14.** $8+\frac{2}{3}$ **15.** $\frac{5}{8}-\frac{1}{2}$

16. $3\frac{6}{7}-1\frac{2}{3}$ **17.** $3\times\frac{1}{12}$ **18.** $\frac{4}{5}\times\frac{1}{6}$

19. $\frac{5}{12}\div\frac{1}{3}$ **20.** $\frac{6}{9}\div 6$

Compute with calculator.

21. $\frac{4}{7}+\frac{1}{9}$ **22.** $\frac{14}{19}+\frac{2}{17}$ **23.** $\frac{8}{13}-\frac{11}{26}$

24. $\frac{11}{12}-\frac{1}{21}$ **25.** $\frac{45}{3}\times\frac{4}{19}$ **26.** $\frac{15}{23}\times\frac{11}{9}$

27. $\frac{34}{37}\div\frac{2}{7}$ **28.** $\frac{22}{33}\div\frac{17}{21}$

Perform the indicated operation (with or without a calculator).

29. $186.4+57.06$ **30.** $58.93-17.986$ **31.** $2{,}578+389.4$

32. 365.8×0.5 **33.** $5{,}968.4\div 9$ **34.** $0.07\div 0.006$

Write each fraction as a decimal.

35. $\frac{4}{11}$ **36.** $\frac{7}{2}$ **37.** $\frac{7}{9}$

Preparation.

After reading some of section R.3, find the following:

38. What percent did Jotham get if he had 7 out of 8 questions correct on his quiz?

39. Evaluate $3+8\times 2^4$

Section R-2 Answers

1. $1, 2, 4, 8, 16$

2. $1, 2, 3, 4, 6, 8, 12, 16, 24, 48$

3. $1, 2, 5, 10, 11, 22, 55, 110$

4. $2 \times 2 \times 3 \times 5$

5. $2 \times 3 \times 3 \times 5 \times 7$

6. $3 \times 3 \times 5 \times 5$

7. 39

8. 88

9. 105

10. 864

11. 4500

12. 132 years

13. $\frac{1}{2}$

14. $\frac{26}{3}$ *or* $8\frac{2}{3}$

15. $\frac{1}{8}$

16. $2\frac{4}{21}$ *or* $\frac{46}{21}$

17. $\frac{1}{4}$

18. $\frac{2}{15}$

19. $\frac{5}{4}$ *or* $1\frac{1}{4}$

20. $\frac{1}{9}$

21. $\frac{43}{63}$

22. $\frac{276}{323}$

23. $\frac{5}{26}$

24. $\frac{73}{84}$

25. $\frac{60}{19}$ *or* $3\frac{3}{19}$

26. $\frac{55}{69}$

27. $\frac{119}{37}$ *or* $3\frac{8}{37}$

28. $\frac{14}{17}$

29. 243.46

30. 40.944

31. $2{,}967.4$

32. 182.9

33. $663.1\overline{5}$ *or* 663.156

34. $11.\overline{6}$ *or* 11.667

35. $.\overline{36}$

36. 3.5

37. $.\overline{7}$

38. In Class.

39. In Class.

Section R-3 Rounding, Percents, Exponents, Roots, and Order of Operations

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Rounding

Main Topics	Examples
Key Terms Rounding	In rounding, we decide to not keep the exact number. For example: If I have $528.37 in the bank, I might easily say that I have about $500. I have just rounded to the nearest hundred. On the other hand, I might be a little more specific and say that I have about (still not exact) $530. I have just rounded to the nearest ten.
Steps: 1. Look at the digit to the right of where the rounding is. 2. Less than 5 goes down. 5 or greater goes up.	**Example 1: Round $4,278.23 to the nearest hundred** $4,278.23 → $4,300.00 or $4,200.00 Decide if our number is closer to the nearest hundred above the number or below the number Since 7 is greater than 5, we round the 2 up to a 3. $4,278.23 ≈ $4,300.00 **Answer: $4,300.00**

Percents

Main Topics	Examples
Key Terms Percent - The word "percent" comes from two words: PER means divide, and CENT means 100. When we divide by 100 and move from a percent to a decimal, the decimal moves two places to the left. Decimal to percent moves the decimal two place to the right.	**Example 2: Turn the following fractions into decimals and percents.** Divide → Two decimal places Fraction | Decimal/Number | Percent (rounded) $\frac{3}{8}$ | .375 | 37.5% $\frac{7}{10}$ | .7 | 70% $2\frac{1}{2}$ | 2.5 | 250% $\frac{15}{18}$ | $.833\overline{3}$ | 83.3% $\frac{5}{11}$ | $.45\overline{45}$ | 45.5% 8 | 8 | 800% $\frac{51}{73}$ | .698630137 . . . | 69.9% **Turn the following percents into decimals and fractions:** x% ⟺ Two decimals ⟺ place value $23\% = .23 = \frac{23}{100}$ $40\% = .4 = \frac{4}{10} = \frac{2}{5}$ $532.7\% = 5.327 = \frac{5327}{1000}$
Percent of something - A percent doesn't represent an amount when it is alone. "Of" means to multiply Steps 1. Change to decimal. 2. Multiply. Note: three expressions of multiplication: • Times × $5 \times 3 = 15$ • Dot · $5 \cdot 3 = 15$ • Next to $5(3) = 15$	**Example 3:** $20\% \ of \ 358$ $0.2(358) = ?$ $0.2(358) = 71.6$ **Example 4:** $7.2\% \ of \ 500$ $0.072(500) = ?$ $0.072(500) = 36$ **Example 5:** If sales tax of 7% was charged on a $25 purchase, what is the amount of sales tax? $0.07 \times 25 = \$1.75$ **Answer: $1.75** **Example 6:** 550 people attended a meeting. If 26% of them were driving green cars how many people drove green cars? $26\% \ of \ 550$ $0.26(550) = ?$ $0.26(550) = 143$ **Answer: 143 cars**

Exponents, Roots

Main Topics | Examples

The Three E's of Exponents

Exponential Notation	Expanded Notation	Evaluated Notation
2^1	2	2
2^2	2×2	4
2^3	$2 \times 2 \times 2$	8
2^4	$2 \times 2 \times 2 \times 2$	16
2^5	$2 \times 2 \times 2 \times 2 \times 2$	32

Example 7:

Write 4^3 in expanded form
Answer: $4 \times 4 \times 4$

Example 8:

Write $2 \times 2 \times 2 \times 2$ in exponential form
Answer: 2^4

Note: three expressions of multiplication:

- Times × $5 \times 3 = 15$
- Dot · $5 \cdot 3 = 15$
- Next to $5(3) = 15$

Evaluate.

Example 9:	**Example 10:**	**Example 11:**
4^3	2^5	m^3 where $m = -3$
$4 \times 4 \times 4 = 64$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$	$(-3)(-3)(-3) = -27$
Answer: 64	**Answer: 32**	**Answer: -27**

The Anatomy (parts) of an Exponent

2^4

The little number up high is the exponent.

The bigger number being repeatedly multiplied is the base. The base can consist of numbers and/or variables.

Evaluating Exponents

Common Calculator Buttons for exponents:

- x^y
- ^
- x
- $x \wedge y$

Example 12: Evaluate 8^4

$8^4 = 8 \cdot 8 \cdot 8 \cdot 8$ $= 64 \cdot 8 \cdot 8$ $= 512 \cdot 8$ $= 4096$ **Answer: 4096**	Set up the bases, and then multiply.

Common Mistakes

1. Mistaking exponents for multiplication	Incorrect: $2^4 = 2 \times 4 = 8$
	Correct: $2^4 = 2 \times 2 \times 2 \times 2 = 16$
2. When we say exponential notation out loud: 2^4	Incorrect: "Two four"
	Correct: "Two to the fourth"
3. Exponential notation can only be used in multiplication of terms with the same base, not in addition.	Incorrect: $2 + 2 + 2 + 2 = 2^4$
	Correct: $2 \times 2 \times 2 \times 2 = 2^4$

Evaluating Roots

A square root is the opposite of squaring (raising to the 2nd power).

Example 13: Evaluate $\sqrt{196}$

$\sqrt{196}$	Either recognize that $14 \times 14 = 196$ or find the $\sqrt{x}$ button on your calculator
Answer: 14	

Order of Operations

Main Topics	Examples
Getting the right order Steps: **P**arentheses (Grouping) **E**xponents (Roots) **M**ultiplication **D**ivision **A**ddition **S**ubtraction	**Example 14: Simplify $2 + 3 \cdot 4 - 5$**

$2 + 3 \cdot 4 - 5$	There are no parentheses or exponents, so first we multiply
$2 + 12 - 5$	Because addition and subtraction are on the same level, we do them from left to right.
$14 - 5$	
9	
Answer: 9	

Example 15: Simplify $4 \times 3^2 - 7 \times 2 + 4$

$4 \times 3^2 - 7 \times 2 + 4$	Because there are no parentheses, we first do exponents
$4 \times 9 - 7 \times 2 + 4$	Next, we do multiplication
$36 - 14 + 4$	Add and subtract left to right
$22 + 4$	
Answer: 26	

Note: a fraction bar groups like parentheses:

$$\frac{5+7}{3} = \frac{12}{3} = 4$$

Common Mistakes

Remember that multiplication and division are on the same level, so when you see both of them, compute from left to right. Similarly, add and subtract left to right.

Section R-3 Exercises

Perform the indicated operation.

R-1

1. Find the prime factorization of 216.

2. LCM (75, 90)

3. In a distant solar system, two planets are aligned. One has a 15-year orbit and the other has a 54-year orbit. How many years until they are aligned again?

R-2

4. $\frac{6}{25}+\frac{5}{8}$

5. $\frac{3}{5}-\frac{8}{120}$

6. $\frac{4}{25}\times\frac{5}{8}$

7. $\frac{1}{3}\times\frac{2}{12}$

Perform the indicated operation (round to three decimal places when needed).

8. $9.83-5.635$

9. $.18\times.63$

Convert fraction to decimal.

10. $\frac{5}{16}$

11. $\frac{24}{5}$

R-3

Round to the nearest tenth.

12. 42.142956

13. .47937

14. 13,693.639

15. 284.359432

Round to the nearest hundred.

16. 23,978.74

17. 5674.90

18. 149.99

19. 3,499,599.99

Write each fraction as a percent.

20. $\frac{22}{25}$

21. $\frac{23}{30}$

22. $\frac{3}{50}$

Write each percent as a fraction. Simplify.

23. 175%

24. 28%

25. 60%

Find the following:

26. 24% of 92

27. 17% of 85

28. .3% of 365

29. The amount of a 7% tax on pants that cost $25

30. The amount of a 15% tip on a $36 meal

31. The amount saved with a 30% discount of a coat with a cost of $85

Write the following exponents in expanded notation and evaluate.

32. 4^3 **33.** 45^2 **34.** 7^4 **35.** 2^5

Find the roots.

36. $\sqrt{36}$ **37.** $\sqrt{3025}$ **38.** $\sqrt{256}$ **39.** $\sqrt{9216}$

Follow order of operations to evaluate.

40. $216 \cdot 6^3 \div 6^2$ **41.** $\frac{30+18\div 3}{3}$ **42.** $5^2 + (11 - 6) \cdot 7$

43. $26 - 11 + 27 \div 3$ **44.** $\frac{6}{8} \cdot \frac{8}{3} + 2$ **45.** $3^3 - 5 \cdot 3 + 8 \cdot 10/2$

46. $8 \div 4 + 35 - (23 - 16) \times 4$ **47.** $1 + 1 + 1 + 1 + 1 \cdot 0$

Preparation.

After reading some of section R.4, find the following if $x = 7$ *and* $a = 2$:

48. $3x + a^3$

Section R-3 Answers

1. $2 \times 2 \times 2 \times 3 \times 3 \times 3$ *or* $2^3 \times 3^3$

2. 450

3. 270 *years*

4. $\frac{173}{200}$

5. $\frac{8}{15}$

6. $\frac{1}{10}$

7. $\frac{1}{18}$

8. 4.195

9. .113

10. .3125

11. 4.8

12. 42.1

13. .5

14. 13,693.6

15. 284.4

16. 24,000

17. 5700

18. 100

19. 3,499,600

20. 88%

21. $76.\bar{6}\%$ *or* 76.7%

22. 6%

23. $1\frac{3}{4}$ *or* $\frac{7}{4}$

24. $\frac{7}{25}$

25. $\frac{3}{5}$

26. 22.08

27. 14.45

28. 1.095

29. $1.75

30. $5.40

31. $25.50

32. $4 \cdot 4 \cdot 4$; 64

33. $45(45)$; 2025

34. $7 \times 7 \times 7 \times 7$; 2401

35. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$; 32

36. 6

37. 55

38. 16

39. 96

40. 1296

41. 12

42. 60

43. 24

44. 4

45. 52

46. 9

47. 4

48. In class.

Section R-4 Variables and Formulas

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

LCM and Factoring	Find Factors	Find LCM
Fractions	Addition/ Subtraction	Multiplication/ Division
Decimals	Addition/ Subtraction	Multiplication/ Division
Rounding	Nearest Place Value	
Percents	Change fraction and decimals to percents	Find percents of totals
Exponents, Roots	Expand and evaluate exponentials	Find roots that are whole numbers
Order of Operations	Use the order of operations correctly	
Variables and Formulas	Translate words to variables	Replace numbers and use formulas

Variables

Main Topics	Examples
Key Terms Variables Substitution Note: two expressions of multiplication with variables: Dot: $5 \cdot m$ Next to: $5m$	Variables - These symbols or letters, represent numbers, but the numbers can change from time to time, or vary. Thus, they are called variables. Substitution - Stick in the number for the letter. Then do the operations. **Example 1:** Find $7b$ if $b=3$ $7b =$ $7(3) = 21$ **Example 2:** Find $7b$ if $b = 9$ $7b =$ $7 \cdot 9 = 63$ **Example 3:** Find $7b$ if $b = 13$ $7b =$ $7(13) = 91$ **Example 4:** Evaluate $5t + s$ if $t = 3$ and $s = 36$ $5t + s =$ $5 \cdot 3 + 36 =$ $15 + 36 =$ 51

Example 5:

Evaluate $6x - 2y$ if $x = 9$ and $y = 11$
$6x - 2y =$ $6(9) - 2(11) =$ $54 - 22 =$ 32

Translation – Language Dictionary

+	-	·	÷	=
plus	minus	times	divide	totals
add	subtract	twice	half	is
bigger than	smaller than (switch)	double	third	will be
more than	less than (switch)	triple	out of	am
increase	decrease	of	**quotient**	are
warmer	colder	percent of		**equals**
interest	discount	**product**		
gained	sale			
tip	**difference**			
sum				

Note: Switch
2 less than 20 is 18

$20 - 2 = 18$

Example 6:

The cost is 5 less than the product of the width and length

$$C = wl - 5$$

Example 7:
25% of the sum of the warmest and coldest temperatures is 5 times the quotient of my height and 4.

$$.25(w + c) = 5 \cdot \frac{h}{4}$$

Formulas

Main Topics	Examples
Key Terms Formulas	Formulas - These are patterns in the form of equations and variables, often with numbers, which help us find something we want to know.

Formula	Practical Use
$x = vt + x_o$	Physics – finding position
$P = 4v^2$	Medicine – pressure in the heart
$A = P\left(1 + \frac{r}{n}\right)^{nt}$	Finances – bank account balance with compound interest

Distance, Rate, and Time

Formula:
$d = rt$
r = rate, *t* = time, *d* = distance

Example 8: Stacey traveled 3 hours while going 27 mph. using the formula $rt = d$ determine the distance that she traveled.

$d = rt$ $t = 3$ hours $r = 27$ mph	Write down the information
$d = ?$	What are we trying to find?
$d = (27)(3)$	Plug in what is known
$d = 81$	Simplify for what we are looking for.
Answer: 81 miles	

Calculating Taxes and Discounts

Formulas:
Tax
$T = rP$

T=Tax
r = rate
P = Original Price

Example 9: If you want to buy a $759 computer with 8% sales tax, how much tax will you end up paying?

$T = rP$ $r = 8\%$ $P = \$759$	Write down the information.
$T = ?$	What are we trying to find?
$T = (0.08)(759)$	Plug in what is known.
$T = 60.72$	Simplify for what we are looking for.
Answer: $60.72 in sales tax	

Simple Interest

$I = Prt$

I=interest, *P*=Principal amount, *r* = rate, *t* = time

Example 11: Mindy sets up a savings plan that gives her simple interest of 7% per year. If she invests $750, how much interest will she earn in 10 years?

$I = Prt$ $r = 7\%$ $P = \$750$ $t = 10$ years $I = ?$ $D = (750)(0.07)(10)$ $T = 525$ **Answer: Mindy will earn $525**	Write down the information. What are we trying to find? Plug in what is known. Simplify for what we are looking for.

Temperature Conversions of Celsius and Fahrenheit

Fahrenheit to Celsius

$$C = \frac{5}{9}(F - 32)$$

C= Celsius, F=Fahrenheit

Example 12: If your thermometer in your car says it is 94º Fahrenheit, what is that temperature in Celsius?

$C = \frac{5}{9}(F - 32)$ $F = 94$ $C = ?$ $C = \frac{5}{9}(94 - 32)$ $C = \frac{5}{9}(62)$ $C = 34.4$ **Answer: 34.4° Celsius**	Write down the information. What are we trying to find? Plug in what is known. Simplify for what we are looking for.

Celsius to Fahrenheit

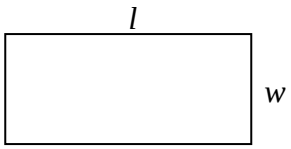
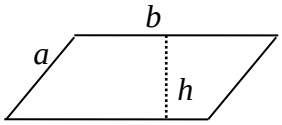
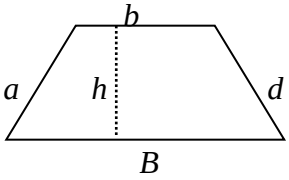
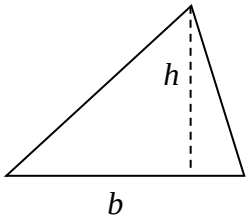
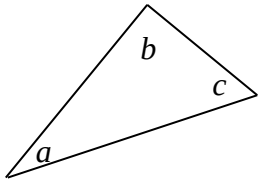
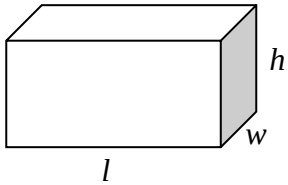
$$F = \frac{9}{5}C + 32$$

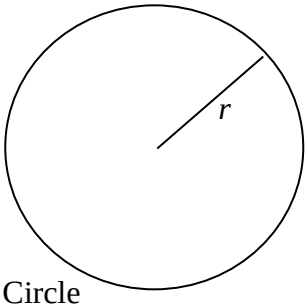
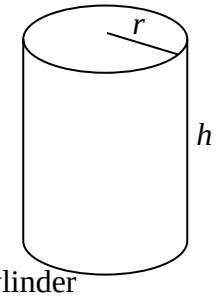
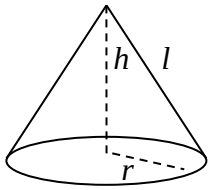
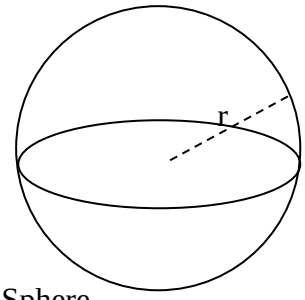
F=Fahrenheit, C=Celsius

Example 13: If it is 4º Celsius outside, what is the temperature in Fahrenheit?

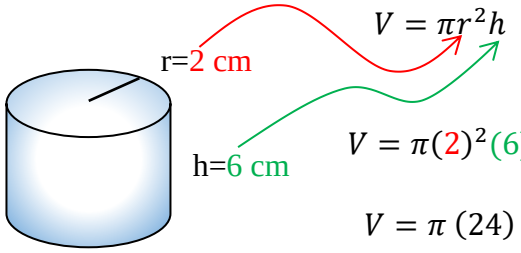
$F = \frac{9}{5}C + 32$	Write down the information.
$C = 4^o$	
$F = ?$	What are we trying to find?
$F = \frac{9}{5}(4) + 32$	Plug in what is known.
$F = 7.2 + 32$	Simplify for what we are looking for.
$F = 39.2$	
Answer: 39.2° F	

Common Geometry Formulas

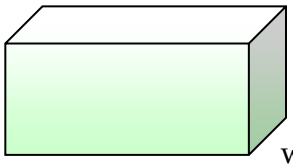
 Rectangle	$P = 2l + 2w$ $A = lw$	***P*** is the Perimeter ***l*** is the length ***w*** is the width ***A*** is the Area
 Parallelogram	$P = 2a + 2b$ $A = bh$	***P*** is the Perimeter ***a*** is a side length ***b*** is the other side length ***h*** is height ***A*** is the Area
 Trapezoid	$P = b+a+B+d$ $A = \frac{1}{2}h(B+b)$	***P*** is Perimeter ***b*** is the little base ***B*** is the big Base ***a*** is a leg ***h*** is height ***d*** is a leg ***A*** is the Area
 Triangle	$P = s_1 + s_2 + s_3$ $A = \frac{1}{2}bh$	***P*** is the Perimeter ***h*** is height ***b*** is base ***A*** is the Area s_1 is one side s_2 is a second side s_3 is the third side
 Triangle	$a + b + c = 180$	***a*** is one angle ***b*** is another angle ***c*** is another angle
 Rectangular Solid	$SA = 2lw+2wh+2lh$ $V = lwh$	***l*** is the length ***h*** is the height ***w*** is the width ***SA*** is the Surface Area ***V*** is Volume

 Circle	$C = 2\pi r$ $A = \pi r^2$	***C*** is the Circumference or perimeter ***π*** is a number, about 3.14159 . . . (it has a button on your calculator) ***r*** is the radius of the circle ***A*** is the area inside the circle.
 Cylinder	$LSA = 2\pi rh$ $SA = 2\pi rh + 2\pi r^2$ $V = \pi r^2 h$	***LSA*** is Lateral Surface Area = Area just on the sides ***h*** is the height ***SA*** is total Surface Area ***π*** is a number, about 3.14159 . . . (it has a button on your calculator) ***r*** is the radius of the circle ***V*** is Volume
 Cone	$LSA = \pi rl$ $SA = \pi r^2 + \pi rl$ $V = \frac{1}{3}\pi r^2 h$	***h*** is the height ***r*** is the radius of the circle ***l*** is the slant height ***π*** is a number, about 3.14159 . . . (it has a button on your calculator) ***SA*** is total Surface Area ***LSA*** is Lateral Surface Area = Area just on the sides ***V*** is Volume
 Sphere	$SA = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$	***r*** is the radius ***SA*** is the Surface Area ***V*** is the Volume

Note: *LSA* and *SA* are single quantities and do <u>not</u> represent L·S·A or S·A (multiplication of individual variables). The designations in the third column identify what quantities are represented.

Main Topics	Examples
Steps: 1. Draw a picture of the situation and label. 2. Write out the equation. 3. Substitute. 4. Evaluate.	**Example 12:** What is the volume of a cylinder when the height is 6 cm and the radius is 2 cm? Remember that $V = \pi r^2 h$.  $V = \pi r^2 h$ — write out formula $V = \pi(2)^2(6)$ — substitute for r and substitute for h $V = \pi\,(24)$ — Evaluate **Answer: 75.4 cm^3** **Example 13:** What is the area of your lawn if the length is 25 feet longer than the width and the width is 35 feet?  width = 35 ft length = width + 25 $A = l \cdot w$ $l = w + 25$ — Write the formula for the area of the lawn; and translate the information about the length. $A = (width + 25)(35)$ — Substitute for *w* and substitute for *l* $A = (35 + 25)(35)$ $A = (60)(35)$ — Simplify **Answer: 2100 ft^2**

Example 14: The depth of a rectangular pool is 17 feet less than half of the length and the width is 20 feet less than the length. If the pool is 54 feet long, how much water would you need to fill up a rectangular pool? Remember that $V = lwh$.



depth = $\frac{1}{2}l - 17$

width = $l - 20$

length = 54 feet

$$V = l(l-20)\left(\frac{1}{2}\cdot l - 17\right)$$

$$V = 54(54-20)\left(\frac{1}{2}\cdot 54 - 17\right)$$

$$V = 54(34)(10)$$

- Write out an equation of the volume in terms of what you know - in our case "l."
- Substitute for l.
- Evaluate.

Answer: 18360ft^3

Units for Geometry Answers

1 – Dimensional	2 – Dimensional	3 – Dimensional
length, width, radius, height, distance	*Area*	*Volume*
ft, m, yd, mi, in, cm, mm, km, etc.	ft^2, m^2, yd^2, mi^2, in^2, cm^2, mm^2, km^2, etc.	ft^3, m^3, yd^3, mi^3, in^3, cm^3, mm^3, km^3, etc.

Section R-4 Exercises

Perform the indicated operation.

R-2

1. $7.2 + 13.258$ **2.** $237.58 - 18.6794$ **3.** $.298 \times 1.4$

Perform the indicated operation.

4. $\frac{4}{5} \times \frac{15}{16}$ **5.** $\frac{15}{21} \div \frac{5}{42}$ **6.** $\frac{7}{120} \div \frac{21}{40}$

Evaluate.

R-3

7. $\sqrt{196}$ **8.** 65^3 **9.** $\sqrt{7^6}$

Convert to decimal notation (round to four decimal places) and then to a percent.

10. $\frac{6}{19}$ **11.** $\frac{15}{4}$ **12.** $\frac{126}{3150}$

Round to the nearest hundredth.

13. 163.69387 **14.** .01982465

Round to the nearest thousand.

15. 235,724.98 **16.** 98,482.994

Evaluate.

17. $3^4 + (5 \times (8 \div 4) - 3)$ **18.** $4 \times (8 + 15 \div (26 - 23) \times 5)$

Evaluate the expression with the given variables.

R-4

19. $4x + t$: when x = 4 and t = 16 **20.** $19x + 47y$: when x = 4 and y = 3

21. $x + 2y - z$: when x =18; y = 3; z = 20 **22.** $\frac{4}{5}x + \frac{2}{7}y$: when x = 5 and y = 7

Translate the following into math.

23. John is 5 years older than Maria.
24. Kris is 17 years older than twice Charlotte's age.
25. The radius is 5 less than 4 times the height.
26. Twice the number of nickels is equal to 3 less than the number of pennies.
27. There are four times as many horses as cows.
28. The sum of the numbers of pigs and chickens is equal to 17.
29. A number increased by 30% of the number is equal to 75.
30. Twice the difference between Mark's height and Nick's is equal to 38.

Find the missing variable. (Note: If you don't use the π button on your calculator for the formulas that use it, your answer may differ slightly)

31. For a triangle
b = 4 in
h = 7 in
A = ?

32. For a cone
r = 3.8 m
l = 5.1 m
SA = ?

33. For a sphere
r = 16 cm
V = ?

34. I have a rectangular sand box whose length is 4 more than its width. If the width is 12 ft, what is the perimeter of the sand box?

35. What is the volume of a cylinder whose height is 3 cm less than twice its radius? The radius is 4 cm.

Find the missing variable.

36. Distance
r = 75 mph
t = 5 hrs
d = ?

37. Tax
r = 6%
P = $29.95
T = ?

38. Discount
r = 30%
P = $48
D = ?

39. Simple Interest
P = $2500
r = 3.5%
t = 2 years
I = ?

40. Temperature
F = 88°
C = ?

41. Temperature
C = 12°
F = ?

Section R-4 Answers

1. 20.458

2. 218.9006

3. 0.4172

4. $\frac{3}{4}$

5. 6

6. $\frac{1}{9}$

7. 14

8. 274,625

9. 343

10. .3158, 31.58%

11. 3.75, 375%

12. .04, 4%

13. 163.69

14. .02

15. 236,000

16. 98,000

17. 88

18. 132

19. 32

20. 217

21. 4

22. 6

23. $J = 5 + M$

24. $K = 17 + 2C$

25. $r = 4h - 5$

26. $2n = p - 3$

27. $h = 4c$

28. $p + c = 17$

29. $n + .3n = 75$

30. $2(M - N) = 38$

31. 14 in^2

32. 106.25 m^2

33. 17157.28 cm^3

34. 56 ft

35. 251.33 cm^3

36. 375 miles

37. \$1.80

38. \$14.40

39. \$175

40. 31.11° C

41. 53.6 ° F

Chapter R Review Exercises

1. Create a visual chart of the methods, formulas, and examples from studying how to evaluate and simplify the operations used in this chapter. (Video instruction and example)

R-1

2. Find the prime factorization of 132.

3. Find the LCM of 18 and 24.

4. In a distant solar system three planets are lined up. Their orbits are 12 years, 25 years, and 30 years. How long until they are lined up again?

Perform the indicated operations.

R-2

5. Simplify $\frac{27}{45}$

6. $\frac{3}{8}+\frac{1}{6}$

7. $\frac{9}{10}-\frac{4}{13}$

8. $\frac{4}{25}\div\frac{3}{5}$

9. $\frac{13}{22}+\frac{2}{11}$

10. $\frac{5}{7}\cdot\frac{4}{3}$

Perform the indicated operations.

11.
$$\begin{array}{r} 241.32 \\ +\ 413.86 \\ \hline \end{array}$$

12. $24 \div (0.8)$

13.
$$\begin{array}{r} 112.3 \\ \times\ 12.1 \\ \hline \end{array}$$

14. $58.46 - 2.974$

15. $4 \div .0002$

16. $3.6(1.4)$

Convert the following fractions to decimals.

17. $\frac{15}{24}$

18. $\frac{16}{33}$

19. $\frac{87}{25}$

20. Convert 0.323 to a fraction.

21. Convert 115% to decimal notation.

R-3

22. Round 385.241 to the nearest hundredth.

23. Round 385.241 to the nearest hundred.

Change the following to percents.

24. $\frac{12}{15}$

25. 0.021

26. 2.16

Write in exponential notation and evaluate.

27. 7·7·7·7

28. 8·8·8

Evaluate.

29. 38% of 72

30. 95% of 643

Evaluate.

31. $34 \cdot 2 + 12 \div 2 + \frac{55-13}{3}$

32. $4^2 + 3 \cdot (2 + 4) - 2 \times 7$

R-4

33. A large rug has a width of 6 ft, and its length is 1 ft less than twice the width. What is the area of the rug?

34. A fence has to go around the perimeter of a rectangular area of my lawn that measures $20\ ft\ by\ 28\ ft$. If fencing costs $5.10 per foot, how much will it cost to fence this part of my lawn?

35. The weather forecast in Brazil is 33° Celsius. How warm is that in degrees Fahrenheit?

Evaluate the expression with the given variable(s).

36. $2\pi rh:\ when\ r = 3, h = 6$

37. $\sqrt{a^2 + b^2}: when\ a = 5, b = 12$

Translate the following into math.

38. Bethany will be 12 years older than twice Richard's age.

39. There are twice as many students as tutors.

40. A price decreased by 19% of the price is equal to 250.

Chapter R Review Answers

1. Grid format, one side, full of steps and examples from this chapter. Submit.

2. $2 \cdot 2 \cdot 3 \cdot 11$

3. 72

4. 300 years

5. $\frac{3}{5}$

6. $\frac{13}{24}$

7. $\frac{77}{130}$

8. $\frac{4}{15}$

9. $\frac{17}{22}$

10. $\frac{20}{21}$

11. 655.18

12. 30

13. 1358.83

14. 55.486

15. 20,000

16. 5.04

17. .625

18. $.48\overline{48}$

19. 3.48

20. $\frac{323}{1000}$

21. 1.15

22. 385.24

23. 400

24. 80%

25. 2.1%

26. 216%

27. 7^4, 2401

28. 8^3, 512

29. 27.36

30. 610.85

31. 88

32. 20

33. $66\ ft^2$

34. \$489.60

35. 91.4° F

36. 113.1

37. 13

38. $B = 12 + 2R$

39. $s = 2t$

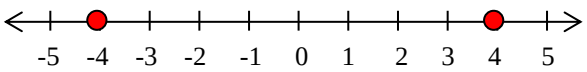
40. $P - .19P = 250$

Section 1-1 Negatives, Inequalities, Addition, Subtraction

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

Number Line	Positives	Negatives
Inequality	Greater than	Less than
Negatives	Add/Subtract	Absolute Value
Negatives	Multiplication	Division
Laws of Simplifying	Combining Like Terms	Identity/Inverse
	Associative/Commutative	Distributive

Number Line

Main Topics	Examples
Key Terms Negative	the negative sign means "opposite direction," as seen on the number line below.  Example : - $\frac{7}{8}$ is just $\frac{7}{8}$ in the opposite direction. Addition goes to the right Subtraction goes to the left A negative sign means the other direction **Example 1: Graph the points -5, 7, $-\frac{2}{3}$, 3.7, and $\sqrt{28}$ on a number line.** 

Inequalities

Main Topics	Examples	
Symbols of Inequalities The line underneath the symbol represents when quantities could be equal or greater/less than.	< **Less than**	> **Greater than**
	≤ **Less than or equal to**	≥ **Greater than or equal to**

Extra help:
The symbol always points to the smaller number.

The lines are farther apart on the BIGGER side

"The alligator eats the bigger number."

Mneumonic helps:
<ess G>reater

smaller < **BIGGER**

You can always flip the sign as long as the numbers on either side are flipped too! So:

$7 > -8$ is the SAME as $-8 < 7$

Example 2:
Write an inequality that has the same meaning as $38 > 14$.

$38 > 14$	
$14 < 38$	flip the sign and the numbers; 38 stays bigger
Answer: $14 < 38$	

Absolute Value

Main Topics	Examples
Key Terms Absolute Value Symbol is | |.	Absolute Value - Means to find how far away a number is from zero, like on a number line. **Find the absolute value of the following**

Example 3:

$\|8\| = 8$
Answer: 8

Example 4:

$\|-16\| = 16$
Answer: 16

Example 5:

$\|5 - 18\| = \|-13\| = 13$
Answer:13

Adding/Subtracting Negatives

<table>
<tr><td>Main Topics</td><td>Examples</td></tr>
<tr><td>Opposite Signs Subtract and the Stronger Wins

Note: Subtraction is the same as adding a negative.</td><td>
<table>
<tr><td>Example 6:</td><td>Example 7:</td><td>Example 8:</td></tr>
<tr><td>$7 - 11$</td><td>$-3 + 8$</td><td>$-12 + 3$</td></tr>
<tr><td>$7 - 11 = -4$</td><td>$-3 + 8 = 5$</td><td>$-12 + 3 = -9$</td></tr>
<tr><td>Subtract to get 4 and the negative is stronger</td><td>Subtract to get 5 and the positive is stronger</td><td>Subtract to get 9 and the negative is stronger</td></tr>
</table>
</td></tr>
<tr><td>Numbers with Same Signs will add in that same direction</td><td>
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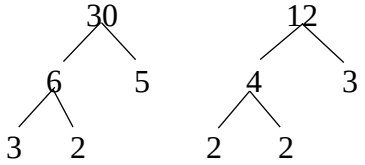
<tr><td>Example 9:</td><td>Example 10:</td><td>Example 11:</td></tr>
<tr><td>$-7 - 11$</td><td>$-3 - 8$</td><td>$12 + 3$</td></tr>
<tr><td>$-7 - 11 = -18$</td><td>$-3 - 8 = -11$</td><td>$12 + 3 = 15$</td></tr>
<tr><td>Add in the negative direction</td><td>Add in the negative direction</td><td>Add in the positive direction</td></tr>
</table>
</td></tr>
<tr><td>Subtracting a Negative is Addition</td><td>
<table>
<tr><td>Example 12:</td><td>Example 13:</td><td>Example 14:</td></tr>
<tr><td>$-7 - (-11)$</td><td>$-3 - (-8)$</td><td>$12 - (-3)$</td></tr>
<tr><td>$-7 - (-11) =$</td><td>$-3 - (-8) =$</td><td>$12 - (-3) =$</td></tr>
<tr><td>$-7 + 11 = 4$</td><td>$-3 + 8 = 5$</td><td>$12 + 3 = 15$</td></tr>
<tr><td>Change to a plus, then subtract</td><td>Change to a plus, then subtract</td><td>Change to a plus, then add</td></tr>
</table>
</td></tr>
</table>

Examples with Fractions

Example 15: Add $\frac{13}{30}+\left(-\frac{7}{12}\right)$

$$\frac{13}{30}+\left(-\frac{7}{12}\right)$$
$$\frac{13}{30}-\frac{7}{12}$$

Step 1: Common denominator. Let's use prime factorization to find the LCD.



Prime factorization of 30: 2×3×5
Prime factorization of 24: 2×2×3
We need a number whose factors include each of these:

$$2\times2\times3\times5=60$$

$$\frac{13}{30}\times\frac{2}{2}=\frac{\mathbf{26}}{\mathbf{60}} \qquad \frac{7}{12}\times\frac{5}{5}=\frac{\mathbf{35}}{\mathbf{60}}$$

$$\frac{26}{60}-\frac{35}{60}=-9$$

Step 2: Now that the denominators are the same, subtract the numerators.

Step 3: Keep the denominator

$$\frac{26}{60}-\frac{35}{60}=\frac{-9}{60}$$

$$\frac{-9}{60}\div\frac{3}{3}=\frac{-3}{20}$$

Step 4: Simplify.

Answer: $\mathbf{-\frac{3}{20}}$

Example 16: Subtract $-\frac{5}{9}-\frac{1}{3}$

$$-\frac{5}{9}-\frac{1}{3}$$

Common denominator. Change the $\frac{1}{3}$ to have a denominator of 9 by multiplying by $\frac{3}{3}$.

$$-\frac{5}{9}-\frac{1}{3}\left(\frac{3}{3}\right)$$

The common denominator is now 9

$$-\frac{5}{9}-\frac{3}{9}$$

Add the numerators; keep the denominator.

$$\frac{-5}{9}+\frac{-3}{9}=\frac{-8}{9}$$

Simplify if necessary.

Answer: $\mathbf{-\frac{8}{9}}$

Common Mistakes

Do two negatives make a positive?

False in Addition and Subtraction - With addition and subtraction negatives and positives work against each other in a tug of war. Whichever one is stronger will win.

Examples:

Debt is negative and income is positive. If there is more debt than income, the net result is debt.	
If we are \$77 in debt and get income of \$66 then we have a net debt of \$11: $-77 + 66 = -11$	If we have \$77 and \$66 of debt, then the net is a positive \$11: $77 - 66 = 11$

Falling is negative and rising is positive.	
An airplane rises 307 feet and then falls 23 feet, then the result is a rise of 284 feet: $307 - 23 = 284$	If, however, the airplane falls 307 feet and then rises 23 feet, then the result is a fall of 284 feet: $-307 + 23 = 284$

Section 1-1 Exercises

Perform the indicated operation.

R-3

1. $3^3 - 2 \cdot 4 + \sqrt{81} \cdot 10 \div 2$

2. 17% of 84

Evaluate each formula with the given variables.

R-4

3. ***Evaluate*** $\dfrac{5x+2}{t}$ when $x = 6$ *and* $t = 4$

4. **For a cone**
r = 4.6 m
l = 5.3 m
SA = ?

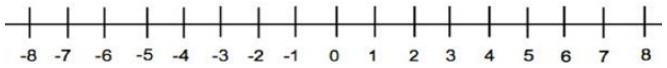
5. I have a rectangular sand box whose length is 4 more than three times its width. If the width is 13 ft, what is the area covered by the sand box?

Translate into math.

6. Bill is 5 years older than twice the sum of Jenny's and Penny's ages.
7. Dave's income is 25 dollars less than 3 times Rebecca's.
8. There are 8 times as many bunnies as turtles.

1-1

9. Locate 7, -2.3, 4, -8, π, $\frac{9}{5}$, and $-\frac{3}{4}$ on a number line.



For each pair of numbers write the correct inequality between them.

10. $-2 \_\_\_ 1.5$

11. $-3 \_\_\_\_ -7$

12. $27 \_\_\_\_ 13$

Write an inequality that has the same meaning.

13. $6 \geq 1.5$

14. $2{,}349 < 4{,}991$

15 $-16 > x$

Find the absolute value.

16. $|-17|$

17. $|8 - 14|$

18. $|3(4-2)|$

Perform the indicated operation by hand, and then check your answers with your calculator.

19. $-3 + 5$

20. $-4 - 7$

21. $5 - 18 - 3$

22. $5 - (-18) + (-17)$

23. $6 + (-15) - 12 - (-5)$

24. $-\frac{5}{8} + \frac{1}{4}$

25. $-\frac{3}{20} - \frac{13}{16}$

26. $-5.7 - (-14.8)$

27. $15 - 18.4$

28. At 6:00am in Rexburg it was $-13° F$. By the warmest part of the day, the temperature had risen 38 degrees. By 8:00pm it has cooled down 12 degrees. What was the temperature at 8:00pm?

29. Clifford is on a bridge 47 feet above the Salmon River. The fish are 15 feet below the surface of the water. How much fishing line does he need to let out to reach them?

Preparation.

30. Read some of 1.2 and then evaluate $-2(-3)(-4)$

Section 1-1 Answers

1. 64

2. 14.28

3. 8

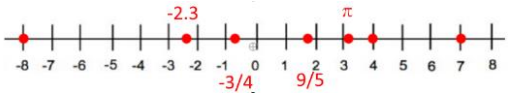
4. $143.07\ m^2$

5. $559\ ft^2$

6. $B = 5 + 2(J + P)$

7. $D = 3R - 25$

8. $b = 8t$

9. 

10. $-2 < 1.5$

11. $-3 > -7$

12. $27 > 13$

13. $1.5 \leq 6$

14. $4{,}991 > 2{,}349$

15. $x < -16$

16. 17

17. 6

18. 6

19. 2

20. -11

21. -16

22. 6

23. -16

24. $-\frac{3}{8}$

25. $-\frac{77}{80}$

26. 9.1

27. -3.4

28. $13°\ F$

29. $62\ ft$

30. In class

Section 1-2 Negatives in Multiplication and Division

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

Number Line	Positives	Negatives
Inequality	Greater than	Less than
Negatives	Add/Subtract	Absolute Value
Negatives	Multiplication	Division
Laws of Simplifying	Combining Like Terms	Identity/Inverse
	Associative/Commutative	Distributive

Main Topics	Examples
Multiplying or Dividing Opposite Signs Gives a Negative	**Example 1:** $7(-11)$ → $7(-11) = -77$ **Example 2:** $-3 \cdot 8$ → $-3 \cdot 8 = -24$ **Example 3:** $\frac{12}{7} \times \left(-\frac{3}{5}\right)$ → $\frac{12}{7} \times \left(-\frac{3}{5}\right) = -\frac{36}{35}$ **Example 4:** $33 \div (-11)$ → $33 \div (-11) = -3$ **Example 5:** $-32 \div 8$ → $-32 \div 8 = -4$ **Example 6:** $\frac{3}{8} \div \left(-\frac{1}{6}\right)$ → $\frac{3}{8} \cdot \left(-\frac{6}{1}\right) = -\frac{18}{8} = -\frac{9}{4}$
Multiplying or Dividing two negatives will be positive	**Example 7:** $-7(-11)$ → $-7(-11) = 77$ **Example 8:** $-3 \cdot (-8)$ → $-3 \cdot (-8) = 24$ **Example 9:** $-\frac{12}{7} \times \left(-\frac{3}{5}\right)$ → $-\frac{12}{7} \times \left(-\frac{3}{5}\right) = \frac{36}{35}$ **Example 10:** $-33 \div (-11)$ → $-33 \div (-11) = 3$ **Example 11:** $-32 \div -8$ → $-32 \div -8 = 4$ **Example 12:** $-\frac{3}{8} \div \left(-\frac{1}{6}\right)$ → $-\frac{3}{8} \times \left(-\frac{6}{1}\right) = \frac{9}{4}$
Division by Zero is impossible (undefined)	**Example 13:** $\frac{0}{10} = 0$; $-\frac{0}{15} = 0$; $\frac{13}{0}$ is undefined; $-\frac{3}{0}$ is undefined **Example 14:**

Common Mistakes

Main Topics	Examples
Common Mistakes – Two negatives make a positive.	**True in Multiplication and Division** - Since a negative sign simply means other direction, when we switch direction twice, we are headed back the way we started.

Example 15:
$-(-5) = 5$
Example 16:
$-(-2)(-1)(-3)(-5) = -----30 = -30$
Example 17:
$-(-40 \div -8) = -(--5) = -5$

False in Addition and Subtraction - With addition and subtraction negatives and positives work against each other in a sort of tug of war. Whichever one is stronger will win.

Falling is negative and rising is positive.	
An airplane rises 307 feet and then falls 23 feet, then the result is a rise of 284 feet: $307 - 23 = 284$	If, however, the airplane falls 307 feet and then rises 23 feet, then the result is a fall of 284 feet: $-307 + 23 = -284$

Other Examples:
Debt is negative and income is positive.
Discount is negative and markup or sales tax is positive.
Warmer is positive and colder is negative.

Whichever is greater will give you the sign of the net result.

Section 1-2 Exercises

Evaluate each formula with the given variables.

R-4

1. $\boldsymbol{Evaluate\ \dfrac{3m^2+2}{n}}$ when $m = -4\ and\ n = 10$

2. For a cylinder
r = 4 m
h = 5.7 m
SA = ?

3. I have a rectangular sandbox whose length is 2 more than three times its width. If the width is 13 ft, what is the area of the sand box?

Translate into math.

4. Chelsea is 5 years older than twice the difference between Kaitlyn's and Becca's ages.

5. Dave's speed is 15 miles per hour less than 4 times Rebecca's.

6. A population increased by 23% of the population to a level of 13,204.

Find the absolute value.

1-1

7. $|-27|$ **8.** $|18-14|$ **9.** $|80 \div (4-12)|$

Perform the indicated operation.

10. $-4-17$ **11.** $5+18-3$ **12.** $-6+(-15)+12-(-5)$

13. $-\frac{5}{9}-\frac{1}{3}$ **14.** $-5.7-(-24.8)$ **15.** $-25-18.4$

16. Rick went bungee jumping. After jumping off the bridge, he fell 83 feet before the bungee cords pulled him back up. On the first recoil he ascended 42 feet before starting to fall again. On his final bounce, he finally came to rest 14 feet lower than that. How far below the bridge did he come to rest?

17. A kite is flying above a tree when the string of the kite gets caught on a branch of the tree. There is 15 feet of string below where the string is caught and 37 feet from the tree branch up to the kite. How many total feet of string is extended?

Perform the indicated operation by hand, and then check your answers with your calculator.

1-2

18. $-5(3)$ **19.** $3 \cdot (-11)$ **20.** $(-42) \div 6$ **21.** $-28 \div (-7)$

22. $-(6 \cdot 4)$ **23.** $-16 \div (-2)$ **24.** $-7(-3) \cdot (-1)$ **25.** $6-5(-9.7)$

26. $-(-8)(-6)$ **27.** $-\frac{5}{8} \cdot \left(-\frac{1}{3}\right)$ **28.** $\frac{7}{40} \div \left(-\frac{3}{10}\right)$ **29.** $-\frac{5}{44} \div \left(-\frac{1}{4}\right)$

30. $0 \div 7$ **31.** $-\frac{13}{0}$ **32.** $15 \div 0$ **33.** $\frac{0}{296}$

Preparation.

34. Read some of 1.3 and then simplify the following: **a)** 2x + 4x **b)** 8 – 4 + 3y +8y

Section 1-2 Answers

1. 5

2. $243.79\ m^2$

3. $533\ ft^2$

4. $C = 5 + 2(K - B)$

5. $D = 4R - 15$

6. $p + .23p = 13{,}204$

7. 27

8. 4

9. 10

10. -21

11. 20

12. -4

13. $-\frac{8}{9}$

14. 19.1

15. -43.4

16. $-55ft \quad or \quad 55ft\ below\ the\ bridge$

17. $52\ ft$

18. -15

19. -33

20. -7

21. 4

22. -24

23. 8

24. -21

25. 54.5

26. -48

27. $\frac{5}{24}$

28. $-\frac{7}{12}$

29. $\frac{5}{11}$

30. 0

31. Undefined

32. Undefined

33. 0

34. In class.

Section 1-3 Laws of Simplifying

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

Number Line	Positives	Negatives
Inequality	Greater than	Less than
Negatives	Add/Subtract	Absolute Value
Negatives	Multiplication	Division
Laws of Simplifying	Combining Like Terms	Identity/Inverse
	Associative/Commutative	Distributive

Laws of Simplifying

Main Topics	Examples
Key Terms Simplify Solve	**Simplify** – No "=" signs (or >, <, etc) -To change the form of a number to the standard way that one usually accepts and uses numbers **Solve** – Uses "=" signs (or >, <, etc), find out what *x* equals

Name	**What it does**	**Operation**	**Examples**
Commutative	Switch order	*Addition*	$5 + 7 = 7 + 5$ $2x + 3y = 3y + 2x$
		Multiplication	$7xtzy = x7tzy$
Associative	Move parentheses.	*Addition*	$7 + (5 + 1) = (7 + 5) + 1$ $(x + 5) + 9 = x + (5 + 9)$
		Multiplication	$7 \cdot (3 \cdot 2) = (7 \cdot 3) \cdot 2$
Associative and Commutative together allow us to move stuff around and (if we take care of multiplication before addition) add things up in any order we desire.			
Identity	The invisible number	*Additive:* 0	$6 + 0 = 6$ $x + 0 = x$
		Multiplicative: 1 $\frac{7}{7}, \frac{3}{3}, and\ \frac{x}{x}$ are all examples of 1.	$6 \cdot 1 = 6$ $57y \cdot 1 = 57y$ $\frac{3}{8} \cdot \frac{x}{x} = \frac{3x}{8x}$

Inverse	Cancels, or undoes the number	*Additive Inverse:* opposite	$6+(-6)=0$ $3t+(-3t)=0$ $-17+17=0$
		Multiplicative Inverse: reciprocal	$5\cdot\frac{1}{5}=1$ $-17p\cdot\frac{1}{-17p}=1$ $\frac{2}{3}\cdot\frac{3}{2}=1$
Distributive	Jump numbers into parentheses	*Both*	$6(43) = 6(40+3) = 6(40) + 6(3)$ 240 $7(2x-5) = 14x-35$

Factoring

Main Topics	Examples
Key Terms Common Factor	Common Factor – a factor that two or more terms have in common **Example 1:**Two terms: 24 and 36. Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24 Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36 The numbers in boxes are the common factors between 24 and 36
Greatest Common Factor	Greatest Common Factor - This is the biggest factor that all terms share in common. **Example 2:** The Greatest Common Factor of 24 and 36 is **12**, because it is a factor that they share and it is also the greatest
Factor	To factor an expression just means to pull out the greatest common factor of each term in the expression. **It's like the distributive property in reverse**.

Steps to Factor:

1. Identify the factors of the terms (and their coefficients).
2. Choose the greatest common factor.
3. Rewrite the expression having factored.

Example 3: Factor 20x+48

Factors of 20: 1,2,4,5,10,20
Factors of 48: 1,2,3,4,6,8,12,16,24,48

List the factors of each number

Factors of 20: 1,2,4,5,10,20
Factors of 48: 1,2,3,4,6,8,12,16,24,48

Identify the factors in common, and choose the greatest

$20x + 48$

Undistribute the 4

$4(5x + 12)$

Answer: 4(5x+12)

Example 4: Factor 3x+6

Factors of 3: 1,3
Factors of 6: 1,2,3,6

-Find the greatest common factor

$\frac{3x}{3} + \frac{6}{3}$

-Divide each term by the greatest common factor

3(x+2)

-Write the expression with the common factor on the outside of the parentheses and the "left-overs" inside

Answer: 3(x+2)

Simplifying with Variables

Key Terms

Like things

Like things – in addition and subtraction we must only deal with like things.
Adding and subtracting like terms works the same way with variables as it does with sheep, penguins, fractions with common denominators, and other quantities with like terms.

Example 5: 5 sheep + 2 sheep =7 sheep.

Example 6: 5 sheep + 2 penguins = ?
We really can't add them together, because they aren't like things.

Example 7:
$\frac{1}{3}$ cups of flour +$\frac{2}{3}$ cups of flour = $\frac{3}{3}$ or 1 cup of flour.

Example 8: Simplify 3x+9x-7y

$3x + 9x - 7y$

Combine like terms

$12x - 7y$

Answer: $12x - 7y$

Example 9: Simplify 3(2a-4b) +5(2b)

$3(2a - 4b) + 5(2b)$

Distributive property and multiplication

$6a - 12b + 10b$

$6a - 2b$

Combine like terms

Answer: $6a - 2b$

BIG Example 10:

Simplify

$\mathbf{2\{2 - [3(6x - 2) + 2] + [2(6 + x) - 3x]\}}$

Distribute the parentheses

$2\{2 - [18x - 6 + 2] + [12 + 2x - 3x]\}$

Next, distribute to clear the brackets

$2\{2 - 18x + 6 - 2 + 12 + 2x - 3x\}$

Now distribute the braces

$4 - 36x + 12 - 4 + 24 + 4x - 6x$

Combine like terms

$-38x + 36$

Section 1-3 Exercises

Translate into math.

R-4

1. The number of quarters plus twice the number of nickels is equal to 4 less than the number of dimes.

Find the absolute value.

1-1

2. $|28 - 34|$

3. The elevation of Jericho is -846 ft. Jerusalem has an elevation of 2428 ft. If Ben travels from Jericho to Jerusalem, how much total vertical elevation will he have gained?

Perform the indicated operation.

1-2

4. $-9 - 15 + 12 - (-5)$ **5.** $-\frac{7}{12} \div \left(-\frac{3}{8}\right)$

Use the commutative properties of addition and multiplication to rewrite each expression.

1-3

6. $3 + x$ **7.** $27 \cdot 36$ **8.** $x \cdot 15$ **9.** $17x - 23y$

Use the associative properties of addition and multiplication to rewrite each expression.

10. $(5 + 2t) + 7p$ **11.** $5(3x)$

Create each fraction with a denominator of 15x.

12. $\frac{4}{5x}$ **13.** $\frac{7}{15}$ **14.** $-\frac{2}{3}$

Find the additive inverses of each number.

15. 7 **16.** -4 **17.** $-\frac{7}{3}$ **18** $.05$

Find the multiplicative inverses of each number.

19. 7 **20.** -4 **21.** $-\frac{7}{3}$ **22.** $.05$

Use the distributive property.

23. $3(x + 7)$ **24.** $-2(x + 5)$ **25.** $4(2x - 7y + 3)$

Factor.

26. $10y + 5$ **27.** $20t - 24s$ **28.** $4a - 6b + 12c$

Combine like terms and simplify.

29. $8a + 25a$

30. $21m^2 + 85 - 15m^2 + 16$

31. $x - 37y + 16x + 13y$

32. $2(5x + 6) + 3x$

33. $2(11z - 2a) + 27a - 3z$

34. $-\left(\frac{1}{3}a + \frac{2}{5}\right) + 2$

35. $2^3 - 5(3x + 8) - 10$

36. $23 + 5t + 7y - t - y - 27$

37. $2\{[6 - 3(2x - 3)] - [2(-x + 1) - 3(-5)]\}$

38. $-4\{[3(x - 2) + 7] - [4(3x + 2) + 3]\}$

39. $7\{2 - [3(11 - 2x) + 1] - 8(2x - 4)\}$

Section 1-3 Answers

1. $q + 2n = d - 4$

2. 6

3. 3274 ft

4. -7

5. $\frac{14}{9}$ *or* $1.\bar{5}$

6. $x + 3$

7. $36 \cdot 27$

8. $15x$

9. $-23y + 17x$

10. $5 + (2t + 7p)$

11. $(5 \cdot 3)x$ *or* $15x$

12. $\frac{12}{15x}$

13. $\frac{7x}{15x}$

14. $-\frac{10x}{15x}$

15. -7

16. 4

17. $\frac{7}{3}$

18. $-.05$

19. $\frac{1}{7}$

20. $-\frac{1}{4}$

21. $-\frac{3}{7}$

22. 20

23. $3x + 21$

24. $-2x - 10$

25. $8x - 28y + 12$

26. $5(2y + 1)$

27. $4(5t - 6s)$

28. $2(2a - 3b + 6c)$

29. $33a$

30. $6m^2 + 101$

31. $17x - 24y$

32. $13x + 12$

33. $23a + 19z$

34. $-\frac{1}{3}a + \frac{8}{5}$

35. $-15x - 42$

36. $4t + 6y - 4$

37. $-8x - 4$

38. $36x + 40$

39. $-70x$

Chapter 1 Review Exercises

1. Make a Visual Chart of the rules, processes, and topics of Chapter 1. It should be in a grid format on one side of a page and have plenty of illustrative examples.

Evaluate.

R-3

2. $15 \div 3 + (7 - 3 \times 6)$

3. $(3^4 - 27 \div 3) \div 2 + 4 \cdot (-3 \cdot 2)$

Evaluate the expression with the given variable(s).

4. $5r + 7r^2$; when $r = -3$

5. $\frac{6a}{b}$; *when* $a = 3,\ b = 4$

6. Roy earns $\boldsymbol{m}$ amount of money per month. Jessica earns $\boldsymbol{2m - 360}$. How much does Jessica earn if Roy earns $700 per month?

Translate the following into math.

R-4

7. Frank is 23 years less than twice Julie's age.

8. There are three times as many kittens as puppies.

9. A price increased by 7% of the price is equal to $363.80.

Change these fractions into decimals.

10. $-\frac{7}{3}$

11. $\frac{-1}{-20}$

Write an inequality that has the same meaning.

1-1

12. $-1 \leq 5.9$

13. $m < 99$

14. $-16 > -120$

Find the absolute value.

15. $|14|$

16. $|5 - 7.3|$

17. $|3(1 - 6)|$

Perform the indicated operations.

1-2

18. $5 - (-3) + (-17)$

19. $3 + (-15) - 12 - (-5)$

20. $-\frac{1}{8} + \frac{3}{4}$

21. $-\frac{2}{7} - \frac{3}{14}$

22. $-4.21 - 3.2$

23. $8.1 - (-9.1)$

24. $-3.8(-4) \cdot 7$

25. $\frac{3^2 - (4+9) \cdot 3}{2(3-8)}$

26. $-27 \div (-.003)$

27. One of the stock market indices started the week at 2,901 points. During Monday, it lost 130 points. Tuesday is gained 57 points. Wednesday it gained 110. How many total points did it have after closing on Wednesday?

28. An anchor dropped from a large cruise ship starts at 22 feet above sea level and the ocean floor is 57 feet below sea level, how much chain must be let out?

Write an equivalent fraction for each of the following with a denominator of 36x.

29. $\frac{5}{36}$ **30.** $\frac{5}{12}$ **31.** $\frac{2}{3x}$

Multiply.

1-3

32. $5(x-2y)$ **33.** $m(3+5t)$

Factor.

34. $3ty-2t$ **35.** $20+5b$ +15c

Simplify by collecting like terms.

36. $4t-3[2(8-t)+5]$ **37.** $-12(x-5)+7(4x+3)$

38. $2\{[3(x+2)+4x]-[5+2(x-4)]\}$ **39.** $7\{2m+3[5+3(m-7)]\}$

Chapter 1 Review Answers

1. Submit the complete, one-page chart.

2. -6

3. 12

4. 48

5. $\frac{9}{2}$ *or* 4.5

6. $1040

7. $F = 2J - 23$

8. $k = 3p$

9. $P + .07P = 363.80$

10. $-2.\overline{3}$

11. .05

12. $5.9 \geq -1$

13. $99 > m$

14. $-120 < -16$

15. 14

16. 2.3

17. 15

18. -9

19. -19

20. $\frac{5}{8}$

21. $-\frac{1}{2}$

22. -7.41

23. 17.2

24. 106.4

25. 3

26. 9,000

27. 2,938

28. 79 ft

29. $\frac{5x}{36x}$

30. $\frac{15x}{36x}$

31. $\frac{24}{36x}$

32. $5x - 10y$

33. $3m + 5mt$

34. $t(3y - 2)$

35. $5(4 + b + 3c)$

36. $10t - 63$

37. $16x + 81$

38. $10x + 18$

39. $77m - 336$

Section 2-1 The 3-Step Process to Solving

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

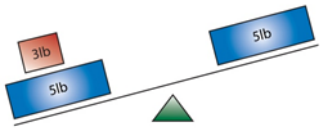
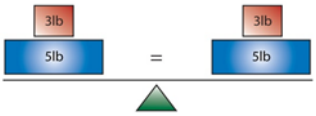
	GOAL: Get x alone (x will represent any variable) 1. SIMPLIFY
2.3	A) Distribute Across ( )
2.5	B) Get rid of fractions (multiply all by LCD)
2.3	C) Combine "Like terms" and get all x's on the same side
2.1	2. ADDITION PRINCIPLE
2.1	3. MULTIPLICATION PRINCIPLE
2.2	Applications: Shapes, Formulas, Solving for a Variable
2.4	Applications: Substitution and Percents
2.6	Inequalities
2.7	Inequalities and Applications

Basics

Definitions & Basics	Examples
Algebra: Two types of Problems Simplify & Solve	**Simplify:** No "=" signs (or >, <, etc.) Combine like terms Example: 2x+3x ***Answer***: $\mathbf{5x}$ **Solve:** Uses "=" signs (or >,<, etc.) $3x = 15$ Example: 3x=15 Find out what x equals $\frac{3x}{3} = \frac{15}{3}$ Divide each side by 3 $x = 5$ ***Answer***: $\boldsymbol{x = 5}$
Equation: Solution: Solve:	A mathematical sentence. It must have an equal sign and an expression on each side. $1 + 3 = 4;\ x = 5;\ or\ x + 7 = 10$ Any number when replaced for the variable that makes an equation true. Example: $x = 3$ is the "SOLUTION" for the equation $x+7 = 10$ Find all the "SOLUTIONS" for an equation.

Checking a solution	**Example 1:**
1. Plug into the equation. 2. Simplify. 3. If the result is a true statement, the number is a solution.	Is $x = 5$ a solution to the equation: $x - 15 = -10$? $5 - 15 = -10$ Plug in 5 into the equation $5 + (-15) = -10$ Simplify $-10 = -10$ It is a true statement **Answer: $x = 5$ is a solution** **Example 2:** Is $x = -4$ a solution to the equation: $5x + 23 = 43$? $5(-4) + 23 = 43$ Plug -4 into the equation. $-20 + 23 = 43$ Simplify combining like terms. $3 = 43$ It is a false statement. **Answer: -4 is not a solution.**

The Balance Rule

Balance Rule of Solving	Examples
Whatever I do to one side of the equal sign, I must do the exact same to the other side to maintain equality. Since an equals sign indicates that two sides are balanced, we can ADD, SUBTRACT, MULTIPLY, or DIVIDE anything on both sides, and it will remain balanced.	 Here we have a scale with a 5 lb. weight on each side. Obviously it is balanced or "equal."  What would happen if I added a 3 lb. weight to one side? It obviously would drop, or lose its equality.  However, if I do the same to the other side of the scale (add 3 lbs.), it again is balanced or "equal." **Example 3:**

$23 = 23$	
$2(23) = 2(23)$ $46 = 46$	Multiplying ***each side*** by 2.
$46 - 4 = 46 - 4$ $42 = 42$	Subtract 4 from ***each side.***
$42 \div 7 = 42 \div 7$ $6 = 6$	Divide ***each side*** by 7. The equation stays balanced.

Example 4: Use the BALANCE RULE to keep the equations below equivalent or the SAME.

$x - 7 = 14$ $+7$	$+7$ is added to the right side
$x - 7 = 14$ $+7 \quad +7$	We must do the same thing (in blue) to the other side
$x = 21$	The equation is balanced

Example 5: Use the BALANCE RULE to keep the equations below equivalent or the SAME.

$\frac{7x}{7} = \frac{21}{}$	7 is divided to the left side
$\frac{7x}{7} = \frac{21}{7}$	We must do the same thing (in blue) to the other side
$x = 3$	The equation is balanced

3-Step Process to solving: The Addition Principle

The Addition Principle	Examples
The GOLDEN Directions:	**3-Step Process to Solving** GOAL: Get x alone (x will represent any variable) 1. SIMPLIFY A) Distribute Across ( ). B) Get rid of fractions (multiply all by LCD). C) Combine "Like terms" and get all x's on the same side. 2. ADDITION PRINCIPLE. 3. MULTIPLICATION PRINCIPLE.
To Get *x* alone… 2) Identify what is being added or subtracted to your variable. 3) Do the opposite (operation) to **both sides** of the equation.	**Example 6:** Solve $x + 5 = 15$ $x + 5 = 15$ — 5 is being added to x $x + 5 = 15$ $-5 \quad -5$ — Do the opposite (subtract 5) to both sides. $x + 0 = 10$ $10 + 5 = 15$ — Check, is 10 the solution to the equation: $x + 5 = 15$? Plug in the answer into the original equation $15 = 15$ — True, 10 is a Solution **Answer: $x = 10$**

Note: Some students (especially international ones) may be familiar with moving the number to the opposite side of the equation or "passing it over" with the inverse operation. This is called "transposition" and was used in the U.S. for decades. It is equivalent to the balance rule being taught here.

Example 7: Solve $-4.3 = y - 7.7$

$-4.3 = y - 7.7$	7.7 is being subtracted from y
$-4.3 = y - 7.7$ $+7.7 \quad +7.7$	Do the opposite (add 7.7) to both sides.
$3.4 = y + 0$ $3.4 = y$	Check, is 3.4 the solution to the equation: $-4.3 = y - 7.7$?
$-4.3 = 3.4 - 7.7$	Plug in the answer into the original equation
$-4.3 = -4.3$	True, 3.4 is a Solution
Answer: $y = 3.4$	

Example 8: Solve $-\frac{3}{4} + x = \frac{5}{8}$

$-\frac{3}{4} + x = \frac{5}{8}$	$-\frac{3}{4}$ is being added to x
$-\frac{3}{4} + x = \frac{5}{8}$ $+\frac{3}{4} \quad +\frac{3}{4}$	Do the opposite (subtract) to both sides. $-\left(-\frac{3}{4}\right) = +\frac{3}{4}$
$0 + x = \frac{11}{8}$	Simplify
$x = \frac{11}{8}$	Check, is $\frac{11}{8}$ the solution to the equation $-\frac{3}{4} + x = \frac{5}{8}$?
$-\frac{3}{4} + \frac{11}{8} = \frac{5}{8}$	Plug in the answer into the original equation
$\frac{5}{8} = \frac{5}{8}$	True, $\frac{11}{8}$ is a Solution
Answer: $x = \frac{11}{8}$	

3-Step Process to solving: The Multiplication Principle

Multiplication Principle	Examples
To Get *x* alone… 2) Identify what is being multiplied or divided to your variable. 3) Do the opposite (operation) to **both sides** of the equation.	**Example 9:** Solve $3x = 15$

$3x = 15$	3 is being multiplied to x
$\frac{3x}{3} = \frac{15}{3}$	Do the opposite (divide) to both sides.
$x = 5$	Simplify
Answer: $x = 5$	

Example 10: Solve $\frac{x}{-4} = -12$

$\frac{x}{-4} = -12$	-4 is being divided into x
$-4\left(\frac{x}{-4}\right) = -4(-12)$	Do the opposite (multiply) to both sides.
$-4\left(\frac{x}{-4}\right) = -4(-12)$	Simplify
Answer: $x = 48$	

Example 11: Solve $\frac{2}{5} = \frac{4}{15}b$

Note: The cancelling seen here can also be done by multiplying numerators and denominators, then simplifying.

$\frac{2}{5} = \frac{4}{15}b$	$\frac{4}{15}$ is being multiplied to b
$\frac{15}{4}\cdot\frac{2}{5} = \frac{15}{4}\cdot\frac{4}{15}b$	Do the opposite (divide) to both sides. $\frac{4}{15} \div \frac{4}{15} = \frac{4}{15}\cdot\frac{15}{4}$
$\frac{3\,15}{2\,4}\cdot\frac{2}{5} = \frac{15}{4}\cdot\frac{4}{15}b$	Simplify
Answer: $x = \frac{3}{2}$	

SOLVING: Using Both Addition and Multiplication Principles Together

Addition & Multiplication Principles	Examples
GOAL: Get *x* alone (*x* will represent any variable) **1. SIMPLIFY** **A)** Distribute ( ). **B)** Get rid of Fractions (multiply all by LCD). **C)** Combine Like Terms (L.T.) and Get all *x*'s to 1 side. **2. ADDITION PRINCIPLE.** **3. MULTIPLICATION PRINCIPLE.**	**Example 12:** Solve $3x + 10 = 22$ $3x + 10 = 22$ — Use the Addition Principle. 10 is being added to x $3x + 10 = 22$ $-10 \quad -10$ — Do the opposite (subtract 10) to both sides. $3x = 12$ — Simplify $3x = 12$ — Use the Multiplication Principle. 3 is being multiplied to x $\frac{3x}{3} = \frac{12}{3}$ — Do the opposite (divide 3) to both sides. $x = 4$ — Simplify **Answer: $x = 4$** **Example 13:** Solve $12.4 - 4.5b = -16.4$ $12.4 - 4.5b = -16.4$ — Use the Addition Principle. 12.4 is being added to b $12.4 - 4.5b = -16.4$ $-12.4 \quad -12.4$ — Do the opposite (subtract 12.4) to both sides. $-4.5b = -28.8$ — Simplify $\frac{-4.5b}{-4.5} = \frac{-28.8}{-4.5}$ — Use the Multiplication Principle $b = 6.4$ **Answer: $b = 6.4$**

Section 2-1 Exercises

Find the Volume of a rectangular solid when the width, height and length are given.
Formula is $V = lwh$

R-4

1. l = 4 in
w = 2.5 in
h = 3 in
V =

2. l = 7 ft
w = 4 ft
h = 2.8 ft
V =

3. l = 7.2 m
w = 9 m
h = 3 m
V =

Find the Area of a trapezoid when the bases and height are given.
Formula is A = $\frac{1}{2}$ h(B+b)

4. B = 15
b = 10
h = 7
A=

5. B = 21
b = 11
h = 3
A=

6. B = 19
b = 6
h = 10
A=

Identify the property that is illustrated by each statement.

1-3

7. $(8 + 5) + 3 = 3 + (8 + 5)$ **8.** $(3xy)7x = (3yx)7x$ **9.** $(8ab)7c = 8(ab7)c$

Simplify.

10. $2s(t - 7) - 6t(s + 3)$ **11.** $3(x^2 - 5n) + 3n - 7x^2$ **12.** $6kj - 7k + 8kj + 11$

2-1 **Check to see if the specified number is a solution for the given equation.**

13. 13; $y + 24 = 37$
Is 13 a solution for $y + 24 = 37$?

14. 19; $p + 14 = 32$
Is 19 a solution for $p + 14 = 32$?

15. 24; $t - 34 = 58$

16. 45; $x - 21 = 24$

Solve.

17. $x + 4 = 13$

18. $13 + t = 27$

19. $y + 17 = -12$

20. $y + \frac{2}{7} = 6$

21. $x + \frac{9}{2} = 4$

22. $8 = x - \frac{5}{8}$

23. $p - 16.2 = 11.2$

24. $-6.1 + x = -6.7$

25. $-4.2 + z = -3.1$

26. $-y = 15$

27. $45 = -x$

28. $-p = -34$

29. $\frac{8}{3}y = 16$

30. $-\frac{x}{4} = \frac{1}{6}$

31. $\frac{7}{4} = -\frac{x}{5}$

32. $\frac{4}{5}p = -5.6$

33. $-\frac{4}{3}z = -15.3$

34. $\frac{-x}{14} = 6$

Use both the addition and multiplication principles together to solve the following.

35.	$12x + 7 = 31$	**36.**	$4y + 18 = 30$	**37.**	$5z + 21 = 56$
38.	$5x - 5 = 20$	**39.**	$3y - 7 = 27$	**40.**	$-8x - 10 = 62$
41.	$-4x - 12 = 18$	**42.**	$2.7m + 12.13 = 20.5$	**43.**	$-3.5x + 2.4 = 24.1$

Preparation: Read some of 2.2 and then

44. Solve for p:

$$3p + 7 = 15$$

45. Solve for p:

$$mp + t = q$$

Section 2-1 Answers

1. 30 in^3

2. 78.4 ft^3

3. 194.4 m^3

4. 87.5

5. 48

6. 125

7. Commutative property of addition

8. Commutative property of multiplication

9. Associative property of multiplication

10. $-4st - 14s - 18t$

11. $-4x^2 - 12n$

12. $14kj - 7k + 11$

13. Yes

14. No

15. No

16. Yes

17. $x = 9$

18. $t = 14$

19. $y = -29$

20. $y = 5\frac{5}{7}\ or\ \frac{40}{7}$

21. $x = -\frac{1}{2}\ or -.5$

22. $x = 8\frac{5}{8}\ or \frac{69}{8}\ or\ 8.625$

23. $p = 27.4$

24. $x = -0.6$

25. $z = 1.1$

26. $y = -15$

27. $x = -45$

28. $p = 34$

29. $y = 6$

30. $x = -\frac{2}{3}$

31. $x = -\frac{35}{4}\ or -8\frac{3}{4}$

32. $p = -7$

33. $z = 11.475$

34. $x = -84$

35. $x = 2$

36. $y = 3$

37. $z = 7$

38. $x = 5$

39. $y = \frac{34}{3}\ or\ 11\frac{1}{3}$

40. $x = -9$

41. $x = -7.5\ or\ -\frac{15}{2}$

42. $m = 3.1$

43. $x = -6.2$

44. In class

45. In class.

Section 2-2 Applications and Formulas

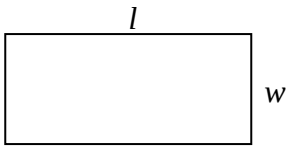
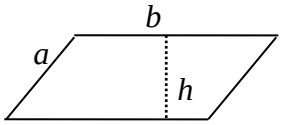
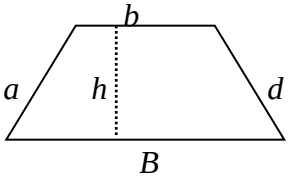
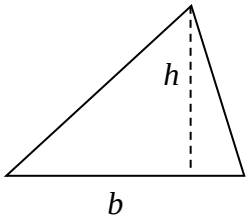
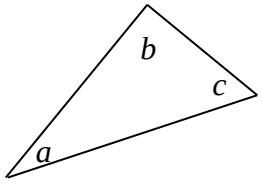
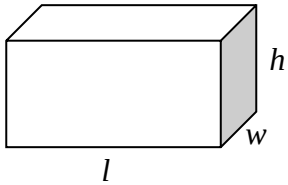
CHAPTER OVERVIEW (Video Instruction and Solutions Link)

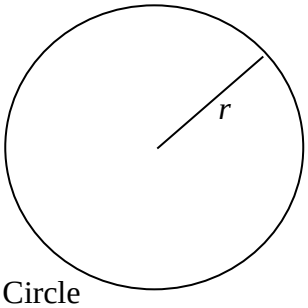
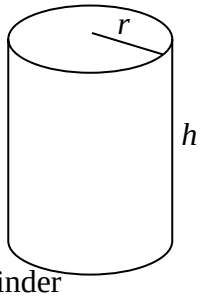
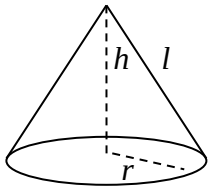
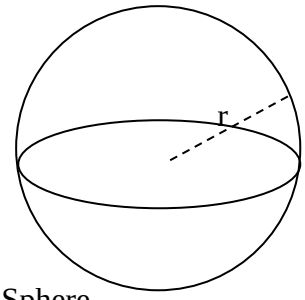
2.3 2.5 2.3 2.1 2.1	GOAL: Get x alone (x will represent any variable) 1. SIMPLIFY A) Distribute Across ( ) B) Get rid of fractions (multiply all by LCD) C) Combine "Like terms" and get all x's on the same side 2. ADDITION PRINCIPLE 3. MULTIPLICATION PRINCIPLE
2.2	Applications: Shapes, Formulas, Solving for a Variable
2.4	Applications: Substitution and Percents
2.6	Inequalities
2.7	Inequalities and Applications

D.U.P.E. Process for word problems with translation & formulas

Main Topics	Examples
D. U. P. E.	**D**ata – Find the information (numbers, formulas, relationships, etc.). **U**nknown - What value are you finding? Assign it a **variable**. **P**lan – Think: "How can I use the data to make an equation?" **E**quation – Make an equation from your plan. Then solve it!
Goal: Solve for m D- Data. U-Unknown. P- Plan (translating). E-Equation.	**Example 1:** Seven less than 3 times what number is 41? 7, 3, 41 are the numbers involved. Let m be the number we don't know. Seven less than 3 times what number is 41? $3m - 7 = 41$ $3m - 7 = 41$ $3m = 48$ $m = 16$ **Answer:16**
D- Data. U-Unknown. P- Plan (use formula). E-Equation.	**Example 2:** Stacey traveled 81 miles while going 27 mph. Using the formula $\boldsymbol{rt=d}$, determine the time that she traveled. $d=81\ miles$ $r=27\ mph$ $t=?$ $rt=d$ $(27)t = (81)$ $\div 27 \quad \div 27$ $t = 3$ **Answer: 3 hours**

Common Geometry Formulas

 Rectangle	$P = 2l + 2w$ $A = lw$	***P*** is the Perimeter ***l*** is the length ***w*** is the width ***A*** is the Area
 Parallelogram	$P = 2a + 2b$ $A = bh$	***P*** is the Perimeter ***a*** is a side length ***b*** is the other side length ***h*** is height ***A*** is the Area
 Trapezoid	$P = b+a+B+d$ $A = \frac{1}{2}h(B+b)$	***P*** is Perimeter ***b*** is the little base ***B*** is the big Base ***a*** is a leg ***h*** is height ***d*** is a leg ***A*** is the Area
 Triangle	$P = s_1 + s_2 + s_3$ $A = \frac{1}{2}bh$	***P*** is the Perimeter ***h*** is height ***b*** is base ***A*** is the Area s_1 is one side s_2 is a second side s_3 is the third side
 Triangle	$a + b + c = 180$	***a*** is one angle ***b*** is another angle ***c*** is another angle
 Rectangular Solid	$SA = 2lw+2wh+2lh$ $V = lwh$	***l*** is the length ***h*** is the height ***w*** is the width ***SA*** is the Surface Area ***V*** is Volume

 Circle	$C = 2\pi r$ $A = \pi r^2$	***C*** is the Circumference or perimeter ***π*** is a number, about 3.14159 . . . (it has a button on your calculator) ***r*** is the radius of the circle ***A*** is the area inside the circle.
 Cylinder	$LSA = 2\pi rh$ $SA = 2\pi rh + 2\pi r^2$ $V = \pi r^2 h$	***LSA*** is Lateral Surface Area = Area just on the sides ***h*** is the height ***SA*** is total Surface Area ***π*** is a number, about 3.14159 . . . (it has a button on your calculator) ***r*** is the radius of the circle ***V*** is Volume
 Cone	$LSA = \pi rl$ $SA = \pi r^2 + \pi rl$ $V = \frac{1}{3}\pi r^2 h$	***h*** is the height ***r*** is the radius of the circle ***l*** is the slant height ***π*** is a number, about 3.14159 . . . (it has a button on your calculator) ***SA*** is total Surface Area ***LSA*** is Lateral Surface Area = Area just on the sides ***V*** is Volume
 Sphere	$SA = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$	***r*** is the radius ***SA*** is the Surface Area ***V*** is the Volume

Main Topics	Examples
D- Data. U-Unknown. P- Plan(use the formula for a triangle). E-Equation. Solve as before.	**Example 3:** If the angles of a triangle are 66°, x, and 2x, solve for x. 66° is the angle we know x = what we are trying to find $angles\ add\ to\ 180$ $66 + x + 2x = 180$ $66 + 3x = 180$ $66 + 3x = 180$ $3x = 114$ $x = 38$ If $x = 38, then\ 2x = 78.$ **Answer:** The three angles are 66°, 38°, and 78°.
D- Data. U-Unknown. P- Plan(use the formula for volume). E-Equation. Solve as before.	**Example 4: Using the formulas for a cylinder find the missing variable:** $r = 9cm \quad h = ? \quad V = 356\ cm^3$ $r\ = 9cm$ $V\ = 356\ cm^3$ h = height that we are trying to find $V = \pi r^2 h$ $356 = \pi(81)h$ $356 = 254.47h$ $\frac{356}{254.47} = \frac{254.47h}{254.47}$ $1.4 = h$ **Answer: height is 1.4 cm**

Solving for Variable

Steps	Examples
Original Problem Divide both sides by r	**Example 5:** Solve ***rt=d*** for *t* $rt = d$ for t $\frac{rt}{r} = \frac{d}{r}$ $t = \frac{d}{r}$ **Answer:** $t = \frac{d}{r}$
Original Problem Subtract " c" from both sides Divide both sides by " b"	**Example 6 :** Solve $\boldsymbol{y - c = bx}$ for *x* $y - c = bx$ $\frac{y - c}{b} = \frac{bx}{b}$ $\frac{y - c}{b} = x$ **Answer:** $x = \frac{y-c}{b}$ Note that the answer could also be written as $x = \frac{-c+y}{b}$.
Original Problem Add (4pt) to both sides Divide both sides by -3	**Example 7:** Solve $\boldsymbol{-3m - 4pt = 7}$ *for m* $-3m - 4pt = 7$ for m $-3m = 7 + 4pt$ $\frac{-3m}{-3} = \frac{7 + 4pt}{-3}$ $m = \frac{7+4pt}{-3}$ **Answer:** $m = \frac{7+4pt}{-3}$ Note that the answer could also be written as $m = -\frac{7+4pt}{3}$ or $m = \frac{-7-4pt}{3}$ or $m = -\frac{7}{3} - \frac{4pt}{3}$

Section 2-2 Exercises

Check to see if the specified number is a solution for the given equation.

2-1

1. 4; $7y + 13 = 15$ **2.** 9; $99 - 3p = 72$ **3.** -21; $y + 4 = y + 4$

Solve.

4. $t - 15 = 43$ **5.** $y - 22 = 23$ **6.** $p - 12 = -21$

7. $\frac{8}{9} + y = \frac{13}{6}$ **8.** $8.1 = 4.2 + x$ **9.** $12.6 = z - 13.3$

10. $\frac{2}{5}x = \frac{1}{10}$ **11.** $-\frac{4}{9}y = \frac{1}{3}$ **12.** $-\frac{5}{7} = -\frac{15}{14}z$

13. $3.6y = 18$ **14.** $94.8 = 23.7x$ **15.** $-2.1z = 12.6$

16. $7y + 7 = 35$ **17.** $2z + 13 = 3$ **18.** $4y + 25 = 13$

19. $21 - x = 13$ **20.** $9 - 5y = 27$ **21.** $-14 - 6y = 17$

2-2

22. 27 is 6 more than 3 times a number. What is the number?

23. 18 less than 5 times a number is 52. What is the number?

24. A triangle has angles that measure x, 3x, and 72°. Solve for x and find the angle measures.

25. If a cone has a Lateral Surface Area of 250 ft^2, a radius of 8ft, what is the slant height of the cone?

26. If a cylinder has a volume of 538 cm^3 and a radius of 6 cm, how tall is it?

27. Find the missing variable for a rectangle:

P = 39 ft
w = 7.2 ft
l =

28. Find the missing variable for a cylinder:

SA = 800 in^2
h =
r = 9 in

Solve for the specified variable.

29. $y = mx + b$ for b

30. $5m - 7 = r$ for m

31. $A = 2\pi rh$ for h

32. $A = \frac{1}{2}bh$ for b

33. $3m - 8qt = 14$ for m

34. $19 = 3pqr$ for r

35. $C = \frac{5}{9}(F - 32)$ for F

36. $V = \frac{1}{3}\pi r^2 h$ for h

Preparation.

37. After reading some from Section 2.3, try to solve this equation for x.

$x +$  $= 9 +$ 

38. Solve the following for *x*:

$5x + 9y + 10p = 9y + 15 + 10p$

Section 2-2 Answers

1. No

2. Yes

3. Yes

4. $t = 58$

5. $y = 45$

6. $p = -9$

7. $y = \frac{23}{18}$

8. $x = 3.9$

9. $z = 25.9$

10. $x = \frac{1}{4}$

11. $y = -\frac{3}{4}$

12. $z = \frac{2}{3}$

13. $y = 5$

14. $x = 4$

15. $z = -6$

16. $y = 4$

17. $z = -5$

18. $y = -3$

19. $x = 8$

20. $y = -\frac{18}{5}$

21. $y = -\frac{31}{6}$ or $-5\frac{1}{6}$ or $-5.1\overline{6}$

22. 7

23. 14

24. x = 27; angles 27°, 81°, 72°

25. 9.95 ft

26. 4.76 cm

27. 12.3 ft

28. 5.15 in

29. $b = y - mx$

30. $m = \frac{r+7}{5}$

31. $h = \frac{A}{2\pi r}$

32. $b = \frac{2A}{h}$

33. $m = \frac{14+8qt}{3}$

34. $r = \frac{19}{3pq}$

35. $F = \frac{9}{5}C + 32$

36. $h = \frac{3V}{\pi r^2}$

37. In class.

38. In class.

Answers for numbers 29-36 are acceptable if expressed in a different but equivalent form.

Section 2-3 3-Step Process to Solving

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

	GOAL: Get x alone (x will represent any variable) 1. SIMPLIFY
2.3	A) Distribute Across ( )
2.5	B) Get rid of fractions (multiply all by LCD)
2.3	C) Combine "Like terms" and get all x's on the same side
2.1	2. ADDITION PRINCIPLE
2.1	3. MULTIPLICATION PRINCIPLE
2.2	Applications: Shapes, Formulas, Solving for a Variable
2.4	Applications: Substitution and Percents
2.6	Inequalities
2.7	Inequalities and Applications

Combine "like terms" and get all x's on one side

Main Topics	Examples
GOAL: Get *x* alone (*x* will represent any variable) 1. Simplify A) Distribute across ( ). B) Get rid of fractions (multiply all by LCD). C) Combine "like terms" and get all *x*'s on the same side. 2. Addition Principle. 3. Multiplication Principle.	**Example 1:** Solve $9(x+2)-4x=28+x+2$

$9(x+2)-4x=28+x+2$ $9x+18-4x=28+x+2$	First distribute into the parentheses
$9x+18-4x=28+x+2$ $5x+18=30+x$	Combine all of the like terms on each side of the equation
$5x+18=30+x$ $-x \quad -x$ $4x+18=30$	Get all of the x's on one side of the equation
$4x+18=30$ $-18 \quad -18$ $4x=12$	Subtract 18 from both sides
$4x=12$ $\div 4 \quad \div 4$ $x=3$	Divide both sides of the equation by 4 to get the x alone
Answer: $x=3$	

Example 2: Solve $24 - 2(3x - 4) = -4$

$24 - 2(3x - 4) = -4$ $24 - 6x + 8 = -4$	First distribute into the parentheses
$24 - 6x + 8 = -4$ $32 - 6x = -4$	Combine all of the like terms on each side All terms with x-s are already on the same side.
$32 - 6x = -4$ $(-32) \quad (-32)$ $-6x = -36$	Subtract 32 from both sides.
$-6x = -36$ $\div(-6) \quad \div(-6)$ $x = 6$	Divide both sides of the equation to get the x alone
Answer: $x = 6$	

Distribute across Parentheses

Main Topics	Examples
1. Simplify. A) Distribute across ( ). B) Get rid of fractions (multiply all by LCD). C) Combine "like terms" and get all x's on the same side. 2. Addition Principle. 3. Multiplication Principle.	**Example 3:** Simplify: $4[6(1 + x) - 3x] = 6 - 2(5 - x)$

$4[6(1 + x) - 3x] = 6 - 2(5 - x)$ $4[6 + 6x - 3x] = 6 - 10 + 2x$	Distribute into the parenthesis Distribute into the brackets
$24 + 24x - 12x = 6 - 10 + 2x$ $24 + 12x = -4 + 2x$	Combine like terms
$24 + 12x = -4 + 2x$ $-2x \quad -2x$ $10x + 24 = -4$	Get all of the x's on one side
$10x + 24 = -4$ $-24 \quad -24$	Subtract 24 from both sides
$10x = -28$ $\div 10 \quad \div 10$ $x = -2.8$	Divide by the number attached to the x
Answer: $x = -2.8$	

Special Cases

Main Topics	Examples
If equation becomes a statement that is true all the time, the answer is all real numbers. There are an infinite number of solutions.	**Example 4:** Solve $2x + 1 = 2x + 1$ $2x + 1 = 2x + 1$ (Get all x's on one side) $-2x \quad -2x$ $1 = 1$ (The x's all vanished!) Solution is **all real numbers** if you get something like: 0 = 0 5 = 5 -3 = -3 **Answer: All Real Numbers**
If the equation is an untrue statement then the answer is no solution.	**Example 5:** Solve $2x + 1 = 2x - 5$ $2x + 1 = 2x - 5$ (Get all x's on one side) $-2x \quad -2x$ $1 = -5$ (Again, the x's all vanished.) There is **no solution** if you get something like: $0 = 1$ $5 = 7$ $-3 = 2$ **Answer: No Solution**

Section 2-3 Exercises

Solve.
Solve for specified variable.

1. $\frac{x}{-6} = -3$

2. $13.7 - 3.4t = -18.9$

3. $-17 - 7m = -18$

4. $\frac{3}{7}t + 1 = -11$

5. $9 = 3x + 17$

6. $\frac{5x+7}{4} = 13$

7. $8t + 3t + 14t - 17 = -17$

8. $94.8 = 23.7x - 13.5$

9. $p = fx + bn$ for f

10. $F = \frac{xf-xz}{2}$ for f

11. $M = 5t - 3p$ for t

12. $LSA = \pi rl$ for r

13. $E = Q - \frac{T_1}{T_2}$ for Q

14. $\frac{3s-4g}{7} = c$ for g

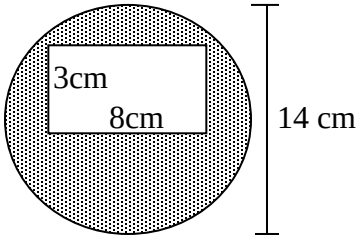
15. 48 is 9 more than 3 times a number. What is the number?

16. 18 less than 7 times a number is 80. What is the number?

17. If two angles of a triangle are 70° and 48°, what is the measure of the third angle?

18. What is the width of a rectangle that has an area of 390in ^{2} and a length of 20in?

19. Find the area of the shaded region:



20. What is the slant height of a cone that has radius of 7m and a surface area of 700m^2?

21. What is the width of a rectangular solid that has a volume of 238mm^3, a length of 17mm and a height of 2mm?

22. If a cone has a volume of 338 cm^3 and a radius of 6 cm, how tall is it?

23. Find the missing variable for a parallelogram:

A = 64 in^2
h =
b = 12.6 in

Solve.

24. $5p + 12 = 33 - p$

25. $7n + 18 = 5(n - 2)$

26. $5x - 10 = 5x + 7$

27. $x - 7 = 15x$

28. $2x - 4(x - 3) = -2x + 12$

29. $.07x = 13 - .12x$

30. $.7(3x - 2) = 3.5x + 1$

31. $.3x - 9 + 2x = 4x - 3$

32. $.4y = 78 + .4y$

33. $7(x - 5) - 3x = 4x - 35$

34. $9x - 4(x - 3) = 15x$

35. $2x - 3x + 7x = 9x + 8x$

Preparation.

36. Find the final price of an object that is $200 but has 15% off.

37. Find the final amount of a savings account that as $170 and then has 15% added to it.

38. After reading some of Section 2-4, try to find out what the original price of an object was if the final price after 15% off was $85.

Section 2-3 Answers

1. $x = 18$

2. $t = 9.59$

3. $m = \frac{1}{7}$

4. $t = -28$

5. $x = -\frac{8}{3}$

6. $x = 9$

7. $t = 0$

8. $x = 4.57$

9. $f = \frac{p-bn}{x}$

10. $f = \frac{2F+xz}{x}$

11. $t = \frac{M+3p}{5}$

12. $r = \frac{LSA}{\pi l}$

13. $Q = E + \frac{T_1}{T_2}$

14. $g = \frac{7c-3s}{-4}$ or $\frac{3s-7c}{4}$

15. $x = 13$

16. $x = 14$

17. $62°$

18. $19.5\ in$

19. $129.9\ cm^2$

20. $24.83\ m$

21. $7\ mm$

22. $h = 8.97\ cm$

23. $h = 5.08\ in$

24. $p = \frac{7}{2}$ or 3.5

25. $n = -14$

26. No solution

27. $x = -\frac{1}{2}$

28. All real numbers

29. 68.42

30. $x = -\frac{12}{7}$ or -1.71

31. $x = -3.53$

32. No solution

33. All real numbers

34. $x = \frac{6}{5}$ or 1.2

35. $x = 0$

36. In class.

37. In class.

38. In class.

Section 2-4 Applications: Substitution and Percents

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

2.3 2.5 2.3 2.1 2.1	GOAL: Get x alone (x will represent any variable) 1. SIMPLIFY A) Distribute Across ( ) B) Get rid of fractions (multiply all by LCD) C) Combine "Like terms" and get all x's on the same side 2. ADDITION PRINCIPLE 3. MULTIPLICATION PRINCIPLE
2.2	Applications: Shapes, Formulas, Solving for a Variable
2.4	Applications: Substitution and Percents
2.6	Inequalities
2.7	Inequalities and Applications

D.U.P.E. Process for word problems with substitution

Main Topics	Examples
D. U. P. E.	**D**ata – Find the information (numbers, formulas, relationships, etc.). **U**nknown - What value are you finding? Assign it a **variable**. **P**lan – Think: "How can I use the data to make an equation?" **E**quation – Make an equation from your plan. Then solve it!
Steps 1. D -What numbers are important? 2. U- What variables will we use? 3. P- Substitute. 4. E - Write the equation to solve.	**Example 1:** Two numbers add to 15, and the second is 7 bigger than the first. What are the two numbers?

$f + s = 15$ $s = f + 7$	1. The two numbers both add to 15, and one is 7 bigger than the other 2. f will represent the first number and s will represent the second number
$f + s = 15$ $s = f + 7$	3. Substitute $f + 7$ in for s
$f + f + 7 = 15$	4. Write the equation to solve
$f + f + 7 = 15$ $2f + 7 = 15$ $2f = 8$ $f = 4$	Solve for f
$s = f + 7$ $s = 4 + 7$ $s = 11$	Now that you know the first number f, you can find the second number s using the equations you wrote at the very beginning.
Answer: 4 and 11	

Steps

1. D -What numbers are important?
2. U- What variables will we use?
3. P- Substitute.
4. E - Write the equation to solve.

Example 2: A man cuts a 65-inch board so that one piece is four times bigger than the other. What are the lengths of the two pieces?

$x + y = 65$ $y = 4x$	1. They equal 65 inches total, and one is 4 times the other 2. x will represent the one piece, and y will represent the other piece
$x + y = 65$ $y = 4x$	3. Substitute $4x$ in for y
$x + 4x = 65$	4. Write the equation to solve
$x + 4x = 65$ $5x = 65$ $x = 13$	Solve the equation
$x + y = 65$ $13 + y = 65$ $y = 52$	Now that you know how long one piece x is, you can solve for the other piece y by using the equations you wrote at the very beginning.
Answer: 13 inches & 52 inches	

Example 3: If a rectangle's length is 5 more than 3 times the width, and the perimeter is 58 mm, what are the dimensions of the rectangle?

$2w + 2l = 58$ $l = 3w + 5$	1. l will represent the length, and w will represent the width 2. 58 millimeters total, and the length is 3 times the width plus 5
$2w + 2l = 58$ $l = 3w + 5$	3. Substitute $3w + 5$ in for l
$2w+2(3w+5)=58$	4. Write the equation to solve
$2w+6w+10 = 58$ $8w + 10 = 58$ $8w = 48$ $w = 6$	Solve the equation
$2w + 2l = 58$ $2(6)+ 2l = 58$ $12 + 2l = 58$ $2l = 46$ $l = 23$	Solve for the other variable
Answer: width is 6 mm, length is 23 mm	

Steps

1. D -What numbers are important?

2.U- What variables will we use?

3.P- Substitute.

4.E - Write the equation to solve.

Example 4: I have created a triangular garden such that the largest side is 8m less than twice the smallest side, and the medium side is 12m longer than the smallest side. If the total perimeter of the garden is 104m, what are the lengths of the three sides?

$s + m + l = 104$ $l = 2s - 8$ $m = s + 12$	1. 104m total 2. l for largest, m for medium, and s for smallest
$s + m + l = 104$ $m = (s + 12)$ $l = (2s - 8)$	3. Substitute the values for l and m
$s + (s + 12) + (2s - 8) = 104$	4. Write the equation to solve
$s+(s+12)+(2s-8) =104$ $4s + 4 = 104$ $4s = 100$ $s = 25$	Solve the equation
$m = s + 12$ $m = 25 + 12$ $m = 37$ $l = 2s - 8$ $l = 2(25) - 8$ $l = 50 - 8$ $l = 42$	Plug in to find the other variables
Answer: smallest is 25m, medium is 37m, largest is 42m	

Example 5: Three consecutive integers add to 39. What are they?

Steps

1.D -What numbers are important?

2.U- What variables will we use?

3.P- Substitute.

4.E - Write the equation to solve.

$x + y + z = 39$ $y = x + 1$ $z = x + 2$	1. The three numbers add to 39. The word *consecutive* tells us that each number is 1 more than the one before it, which is where the other two equations come from 2. x is the first number, y is the second, z is the third
$x + y + z = 39$ $y = (x + 1)$ $z = (x + 2)$	3. Substitute the values for y and z
$x+(x+1)+(x+2)=39$	4. Write the equation to solve
$x + x+1 + x+2= 39$ $3x + 3 = 39$ $3x = 36$ $x = 12$	Solve the equation
Answer: 12, 13, & 14	

Steps

1.D -What numbers are important?

2.U- What variables will we use?

3.P- Substitute.

4.E - Write the equation to solve.

Example 6: Three consecutive even integers add to 72.
What are they?

$x + y + z = 72$ $y = x + 2$ $z = x + 4$	1. The three numbers add to 72. The words *consecutive even* tells us that each number is 2 more than the one before it, which is where the other two equations come from 2. x is the first number, y is the second, z is the third
$x + y + z = 72$ $y = x + 2$ $z = x + 4$	3. Substitute the values for y and z
$x+(x+2)+(x+4)=72$	4. Write the equation to solve
$x + x+2 + x+4= 72$ $3x + 6 = 72$ $3x = 66$ $x = 22$	Solve the equation
Answer: 22, 24, 26	

Simple Percent Problems

Main Topics	Examples
Remember this: "of" means "times". "what" means "x". "is" means "=".	**Percent:** per = divide, cent = 100 0.73 = 73% 0.2 = 20% 1 = 100% 2.3 = 230% 2.14 = 214%
	Example 7: What is 26% of $40? What is 26% of $40 Set up an equation. Remember "what" means "x", "is" means "=" and "of" means times $x = .26 \cdot 40$ $x = .26 \times 40$ Solve the equation $x = 10.4$ **Answer: 10.4** **Example 8:** 118.08 is what percent of 246? 118.08 is what percent of 246 Set up an equation. Remember "what" means "x", "is" means "=" and "of" means times $118.08 = x \cdot 246$ $118.08 = x \cdot 246$ Solve the equation $\div 246 \quad \div 246$ $.48 = x$ $x = .48$ Turn the answer into a percent by moving the decimal two places to the right **Answer: 48%**

	Example 9: 136 is 16% of what? 136 is 16% of what — Set up an equation. Remember "what" means "x", "is" means "=" and "of" means times $136 = .16 \cdot x$ $136 = .16 \cdot x$ — Solve the equation $\div .16 \quad \div .16$ $850 = x$ **Answer: 850**

Forward Percent Problems

Main Topics	Examples
Percent Change Formula $P \pm rP = F$ or $P(1 \pm r) = F$ P = Principal (original) amount r = rate (percent as a decimal) F = (Final amount) $\pm$ means that you use + for percent increases and – for percent decreases.	**Example 10:** If you want to buy a \$759 computer with 8% sales tax, what is the final price? $r = 8\%$ — D – Data $P = \$759$ $F = ?$ — U – Unknown $P + rP = F$ — P – Plan (tax is added) $759 + (.08)(759) = F$ — E – Equation $F = 819.72$ — Solve as before **Answer: Final Price is \$819.72** **Example 11:** How much will Alice pay for a pair of shoes that are worth \$92 but are on sale for 20% off? $r = 20\%$ — D – Data $P = \$92$ $F = ?$ — U – Unknown $P - rP = F$ — P – Plan (discount is subtracted) $92 - (.20)(92) = F$ — E – Equation $F = 73.6$ — Solve as before **Answer: Final Price is \$73.60** **Example 12:** The original price of a TV was \$75, and it has a 6% sales tax. What is the final price of the TV? $P = 75$ — D – Data $r = .06$ $F = ?$ — U – Unknown $F = P + rP$ — P – Plan (tax is added) $F = 75 + .06(75)$ — E – Equation $F = 75 + 4.5$ — Solve as before $F = 79.5$ **Answer: Final Price is \$79.50**

Backward Percent Problems

Main Topics	Examples
Percent Change Formula $P \pm rP = F$ or $P(1 \pm r) = F$ P = Principal (original) amount r = rate (percent as a decimal) F = (Final amount) $\pm$ means that you use + for percent <u>increases</u> and – for percent <u>decreases.</u>	**Example 13:** Henry purchased a picture frame. After a 15% discount its cost was \$30.60. What was the original price? $P - .15P = 30.60$ r = .15, P is what we don't know, F=30.60 $P - .15P = 30.60$ Solve the equation for P $.85P = 30.60$ Combine like terms $.85P = 30.60$ $\div .85 \quad \div .85$ Divide both sides by .85 $x = 36$ **Answer: Original cost was \$36.** **Example 14:** How many people lived in a town last year if 19,980 people live there this year and it grew 8% from last year? $x + .08x = 19{,}980$ Add the amount of growth to last year's population (x) and set it equal to this year's population $x + .08x = 19{,}980$ Solve the equation for x $1.08x = 19{,}980$ $\div 1.08 \quad \div 1.08$ $x = 18{,}500$ **Answer: Last year the town's population was 18,500**

Reverse Averages

Main Topics	Examples
Average: Add up then divide by number of entries.	**Example 15:** Mindy has earned 79, 85, 92, and 88 on her first four tests. What will she need to get on her 5th test in order to have an average of 87?

$79, 85, 92, 88$ — Write down what information we have been given—scores of her first four tests.

$T = ?$ — Pick a variable for the score on the 5th test.

$$\frac{79 + 85 + 92 + 88 + T}{5} = 87$$

Plug all the information that we already know.

$$\frac{344 + T}{5} = 87$$

Simplify

$$5 \cdot \frac{344 + T}{5} = 87 \cdot 5$$

Multiply both sides by 5 to clear out the division.

$$344 + T = 435$$

$$T = 91$$

Answer: Mindy needs a score of 91 to get an average of 87.

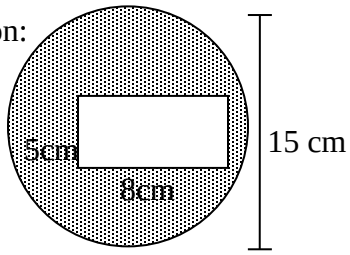
Section 2-4 Exercises

2-2

1. 45 is 12 more than 3 times a number. What is the number?

2. 25 less than 7 times a number is 108. What is the number?

3. Find the area of the shaded region:



4. If a parallelogram has an area of 258.9 cm^2 and a base of 23.2 cm, how tall is it?

5. Find the missing variable for a trapezoid:

A = 68 ft^2
b =
h= 4ft
B = 21ft

2-3

Solve.

6. $7p + 13 = 33 - 4p$

7. $5n + 48 = 7n - 2(n - 2)$

8. $5x - 10 = 7(x - 2)$

9. $3x - 7 = 12x$

10. $5x - 7(x + 3) = -2x - 21$

11. $.06x = 15 - .18x$

12. $.8(7m - 2) = 9.5m + 1$

13. $.2q - 7 + 2q = 3q - 5$

14. $12t = 45 + .4t$

15. $6(x - 5) - x = 5x - 20$

16. $9x - 2(x - 3) = 15x + 7$

17. $5x - 13x + x = 7x + 8x$

2-4

18. Two numbers add to 251 and the second is 41 bigger than the first. What are the two numbers?

19. Jack earned 68, 75, and 82 on his first three tests. What does he need on his 4th test to average 79?

20. I have created a triangular garden such that the largest side is 8m less than twice the smallest and the medium side is 12m larger than the smallest side. If the total perimeter of the garden is 108m, what are the lengths of the three sides?

21. If a rectangle's length is 5 more than 3 times the width and the perimeter is 58 mm what are the dimensions of the rectangle?

22. Two consecutive integers add to 123, what are they?

23. Three consecutive odd integers add to 93. What are they?

24. Cindy has 7 quiz scores of 87, 76, 88, 92, 93, 88, and 85. What does she need to get on her 8^{th} quiz to get an average of 88?

25. 18 is what percent of 58?

26. What is 87% of 54?

27. 34 is 56% of what?

28. 119 is 8% of what?

29. 23 is what percent of 74?

30. Original Price:$92.56
Tax: 7.3%
Final Price:

31. Original Price:
Discount: 40%
Final Price: $43.90

32. Original Price:
Tax: 5%
Final Price: $237.50

33. Original Price: $58.50
Discount: 30%
Final Price:

34. If the population of a town grew 21% up to 15,049 people, what was the population last year?

35. If the price of an object dropped 25% down to $101.25, what was the original price?

Preparation.

36. After reading some from Section 2.5, Try to solve this equation.

$$\frac{x}{7}+\frac{13}{7}=\frac{15}{7}-\frac{2x}{7}$$

37. Solve.

$$\frac{x}{3}+\frac{13}{3}=\frac{15}{3}-\frac{2x}{3}$$

Section 2-4 Answers

1. $x = 11$

2. $x = 19$

3. 136.71 cm^2

4. 11.16 cm

5. 13ft $= b$

6. $p = \frac{20}{11}$

7. No solution

8. $x = 2$

9. $x = -\frac{7}{9}$

10. All numbers

11. $x = 62.5$

12. $m = -0.667$ or $m = -\frac{2}{3}$

13. $q = -2.5$

14. $t = 3.879$

15. No solution

16. $x = -\frac{1}{8}$

17. $x = 0$

18. 105, 146

19. 91

20. 26m, 38m, 44m

21. $w = 6\text{mm},\ l = 23\text{mm}$

22. 61 & 62

23. 29, 31, & 33

24. 95

25. 31%

26. 46.98

27. 60.7

28. 1487.5

29. 31%

30. $99.32

31. $73.17

32. $226.19

33. $40.95

34. 12,437

35. $135

36. In class

37. In class

Section 2-5 The 3-Step Process to Solving

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

	GOAL: Get x alone (x will represent any variable) 1. SIMPLIFY
2.3	A) Distribute Across ( )
2.5	B) Get rid of fractions (multiply all by LCD)
2.3	C) Combine "Like terms" and get all x's on the same side
2.1	2. ADDITION PRINCIPLE
2.1	3. MULTIPLICATION PRINCIPLE
2.2	Applications: Shapes, Formulas, Solving for a Variable
2.4	Applications: Substitution and Percents
2.6	Inequalities
2.7	Inequalities and Applications

STEP 1—Simplify the Equation: Fractions, Parentheses, Like Terms

Main Topic	Examples
Getting Rid of Fractions 1) Determine the LCD. 2) Multiply everything by the LCD to remove fractions.	**Example 1:** Solve $\frac{5}{6}x+\frac{1}{4}=\frac{11}{3}$

$\frac{5}{6}x+\frac{1}{4}=\frac{11}{3}$	First find the LCD.
6 4 3 / \ / \ / \ 2 3 2 2 3 1	(2)(2)(3) = **12, LCD**
$12\cdot\frac{5}{6}x+\frac{1}{4}\cdot 12=\frac{11}{3}\cdot 12$	Multiply everything by 12. Simplify.
$10x+3=44$ $-3 \quad -3$ $10x=41$ $/10 \quad /10$ $x=\frac{41}{10}$	Subtract 3 from both sides Divide by 10.
Answer: $\boldsymbol{x=\frac{41}{10}}$	

GOAL: Get x alone (x will represent any variable)

1. SIMPLIFY
   A) Distribute Across ( )
   B) Get rid of fractions (multiply all by LCD)
   C) Combine "Like terms" and get all x's on the same side

2. ADDITION PRINCIPLE

3. MULTIPLICATION PRINCIPLE

Example 2: Solve $\frac{2}{7}-\frac{3}{4}x=\frac{1}{2}$

7 4 2 | ⋀ | 7 2 2 2 $2\times 2\times 7=28$	Find the LCD by making a factor tree of the denominators.
$\frac{2}{7}\times 28-\frac{3}{4}x\times 28=\frac{1}{2}\times 28$ $8-21x=14$	Multiply everything by 28. Simplify. Shazaam! No more fractions.
$8-21x=14$ $-8 \quad -8$	Subtract 8 from both sides.
$-21x=6$ $/-21 \quad /-21$	Divide both sides by -21
$x=-\frac{6}{21}$	
$x=-\frac{2}{7}$	Simplify
Answer: $x=-\frac{2}{7}$	

Example 3: Solve $\frac{2}{3}x-2=\frac{1}{2}$

$\frac{2}{3}x\times 6-2\times 6=\frac{1}{2}\times 6$	Multiply every by LCD of 6.
$4x-12=3$ $+12 \quad +12$	Shazaam! No fractions. Add 12 to both sides.
$4x=15$ $/4 \quad /4$	Divide both sides 4.
$x=\frac{15}{4}$	
Answer: $x=\frac{15}{4}$	

GOAL: Get x alone (x will represent any variable)

1. SIMPLIFY
   A) Get rid of fractions (multiply all by LCD)
   B) Distribute Across ( )
   C) Combine "Like terms" and get all x's on the same side
2. ADDITION PRINCIPLE
3. MULTIPLICATION PRINCIPLE

Example 4: Solve $\frac{3}{10}x-\frac{2}{5}(x-3)=\frac{3}{2}x+3$

$\frac{3}{10}x-\frac{2}{5}(x-3)=\frac{3}{2}x+3$	Distribute across parentheses.
$\overset{(10)}{\frac{3}{10}}x-\overset{(10)}{\frac{2}{5}}x+\overset{(10)}{\frac{6}{5}}=\overset{(10)}{\frac{3}{2}}x+3$	Multiply every by LCD of 10.
$3x-4x+12=15x+30$	Simplify. No fractions 😊.
$-x+12=15x+30$ $+x \quad +x$	Combine like terms on both sides. Add x to get the x-s on one side.
$12=16x+30$ $-30 \quad -30$	Subtract 30 from both sides.
$-18=16x$ $/16 \quad /16$ $x=-\frac{9}{8}$	Divide both sides by 16.
Answer: $\boldsymbol{x=-\frac{9}{8}}$	

Example 5: Solve $\frac{2x-5}{3}=\frac{4x-1}{2}$

$\frac{2x}{3}-\frac{5}{3}=\frac{4x}{2}-\frac{1}{2}$	Grouping over a fraction acts as a parentheses. Break up into individual fractions. Same as distributing.
$\overset{(6)}{\frac{2x}{3}}-\overset{(6)}{\frac{5}{3}}=\overset{(6)}{\frac{4x}{2}}-\overset{(6)}{\frac{1}{2}}$ $4x-10=12x-3$	Multiply every by LCD of 6. Simplify. No fractions 😊.
$4x-10=12x-3$ $-4x \quad -4x$	Combine like terms on both sides. Subtract 4x to get the x-s on one side.
$-10=8x-3$ $+3 \quad +3$	Add 3 to both sides.
$-7=8x$ $-\frac{7}{8}=x$	Divide both sides by 8.
Answer: $\boldsymbol{x=-\frac{7}{8}}$	

Section 2-5 Exercises

2-2

1. 35 less than 7 times a number is 98. What is the number?

2. The perimeter of a rectangle is 702 cm. The length is 71 cm longer than the width. What are the dimensions?

2-3

Solve.

3. $7p + 12 = 33 - 4p$

4. $3n + 48 = 7 - 2(n - 2)$

5. $5x - 10 = 5(x - 2)$

6. $3x - 7 = 15x$

7. $5x - 7(x + 3) = -2x + 12$

8. $.09x = 13 - .18x$

9. $.8(3x - 2) = 9.5x + 1$

10. $.2x - 7 + 2x = 3x - 5$

11. $12m = 70 + .4m$

12. $5(x - 5) - x = 4x - 20$

13. $9x - 4(x - 3) = 15x + 7$

14. $8x - 12x + x = 9x + 8x$

2-4

15. I have three colors of paint: blue, green, and yellow. The number of gallons of blue paint is 5 more than twice the number of green. The number of gallons of yellow paint is 3 less than 7 times the number of green. All together I have 82 gallons of paint. How many of each color are there?

16. 85 is what percent of 39?

17. 85 is 54% of what?

18. What is 19% of 2,340?

19. What is 23% of 79?

20. 119 is 18% of what?

21. 43 is what percent of 174?

22. Original Price:$72.56
Tax: 7.3%
Final Price:

23. Original Price:
Discount: 30%
Final Price: $49.70

24. Original Price:
Tax: 5%
Final Price: $339.50

25. Original Price: $55.50
Discount: 40%
Final Price:

26. If the population of a town grew 31% up to 17,049. What was the population last year?

27. If the price of an object dropped 35% down to $101.25, what was the original price?

Solve.

2-5

28. $\frac{4}{5}x = 2x - \frac{5}{3}$

29. $\frac{2}{3}x - 6 = 3 + \frac{1}{2}x$

30*. $7/3t - 5 = 19$

31. $-\frac{3}{8}(x - 7) = 5 + 3x$

32. $\frac{3}{5}x - \frac{2}{5}(x - 3) = \frac{1}{5}x + 3$

33. $\frac{3x+2}{7} = \frac{4x-1}{5}$

34. $.9(-4x - 5) = 2.5x + 6$

35. $.0005x + .0045 = .004x$

36. $\frac{x+7}{4} = 8 - \frac{5}{6}x$

Preparation.

37. Solve.

a. 3x – 7 = 17 b. 3x – 7 < 17 c. 3x – 7 > 17

*Remember that the "/" means the same as the ÷ symbol.

Section 2-5 Answers

1. 19

2. w = 140cm, l = 211cm

3. $p = \frac{21}{11}$

4. $n = -\frac{37}{5}$ or -7.4

5. All numbers

6. $x = -\frac{7}{12}$

7. no solution

8. x = 48.15

9. x = -.366

10. x = -2.5

11. m = 6.03

12. no solution

13. $x = \frac{1}{2}$

14. x = 0

15. 21 gal – blue, 8gal – green, 53gal – yellow

16. 218%

17. 157.4

18. 444.6

19. 18.17

20. 661.1

21. 24.7%

22. $77.86

23. $71.00

24. $323.33

25. $33.30

26. 13,015

27. $155.77

28. $x = \frac{25}{18}$

29. x = 54

30. $t = \frac{72}{7}$

31. $x = -\frac{19}{27}$

32. No solution

33. $x = \frac{17}{13}$

34. $x = -\frac{105}{61}$

35. $x = \frac{9}{7}$

36. $x = \frac{75}{13}$

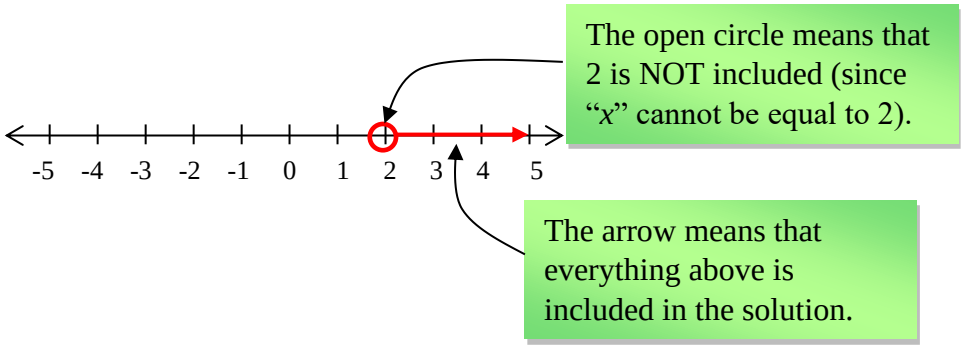
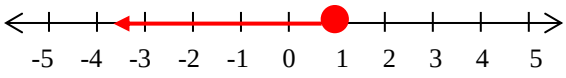
37. In class.

Section 2-6 Inequalities

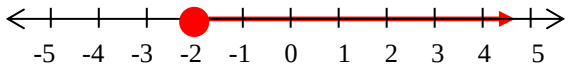
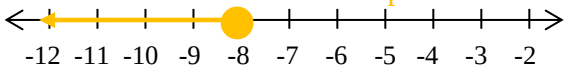
CHAPTER OVERVIEW (Video Instruction and Solutions Link)

2.3 2.5 2.3 2.1 2.1	GOAL: Get x alone (x will represent any variable) 1. SIMPLIFY A) Distribute Across ( ) B) Get rid of fractions (multiply all by LCD) C) Combine "Like terms" and get all x's on the same side 2. ADDITION PRINCIPLE 3. MULTIPLICATION PRINCIPLE
2.2	Applications: Shapes, Formulas, Solving for a Variable
2.4	Applications: Substitution and Percents
2.6	Inequalities
2.7	Inequalities and Applications

Graphing Inequalities

Main Topic	Examples
> or < are open circles. ≤ or ≥ are closed circles. Note: $1 \geq x$ has the same graph as Example 2.	**Example 1** $Graph \quad x > 2$  **Example 2:** $Graph \quad x \leq 1$  Notice that the circle is filled in because "x" can be equal to 1.

Solving Inequalities

Main Topic	Examples
1. Solve like equations (3-Step Process). 2. Switch direction if multiply or divide by negative.	**Example 3:** Solve and graph $5x - 18 \geq -28$ $5x - 18 \geq -28$ — No parentheses, no fractions, only one term with x. $5x - 18 \geq -28$ $+18 \quad +18$ — Add 18 to both sides. $5x \geq -10$ $5x \geq -10$ — Divide to get x all alone. $/5 \quad /5$ $x \geq -2$ — Graph the solution.  **Example 4:** Solve and graph $4x + 2 - 5x \geq 10$ $4x + 2 - 5x \geq 10$ — Combine all like terms $2 - x \geq 10$ $2 - x \geq 10$ $-2 \quad -2$ — Subtract 2 from both sides. $-x \geq 8$ $-x \geq 8$ — Divide to get x all alone. $\div(-1) \quad \div(-1)$ **Switch the direction of the inequality sign if we divide by a negative number.** $x \leq -8$ Graph the answer. 

Example 5: Solve and graph $2(3+4y)-9\geq 45$

$2(3+4y)-9\geq 45$ — Distribute into the parentheses.

$6+8y-9\geq 45$

$6+8y-9\geq 45$ — Combine all like terms.

$8y-3\geq 45$

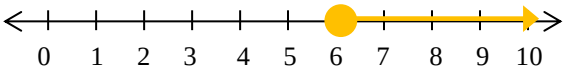
$8y-3\geq 45$ — Add 3 to both sides.

$+3\quad +3$

$8y\geq 48$ — Divide to get y all alone.

$\div 8\quad \div 8$

$y\geq 6$ — ***Don't*** switch the sign because we ***did not*** divide by a negative number.



Graph the answer.

Example 6: Solve and graph $\frac{3}{5}(x+4)<\frac{7}{2}x+1$

$\frac{3}{5}(x+4)<\frac{7}{2}x+1$ — Distribute into the parentheses.

$\frac{3}{5}x+\frac{12}{5}<\frac{7}{2}x+1$

$(10)\quad (10)\quad (10)\quad (10)$

$\frac{3}{5}x+\frac{12}{5}<\frac{7}{2}x+1$ — Multiply by LCD of 10 to clear fractions.

$6x+24<35x+10$

$-6x\qquad -6x$ — Get all of the x-s on one side.

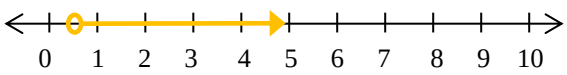
$24<29x+10$ — Subtract 10 from both sides.

$-10\qquad -10$

$14<29x$

$/29\quad /29$ — Divide to get y all alone.

$\frac{14}{29}<x$ — ***Don't*** switch the sign because we ***did not*** divide by a negative number.



Graph the answer.

Section 2-6 Exercises

Solve for the specified variable.

2-2

1. $\frac{2s-at^2}{2t} = V$ for s

2. $r = \frac{I}{pt}$ for p

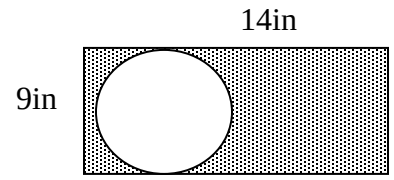
3. $d = \frac{LP}{F}$ for F

4. $\frac{9s-5g}{11} = c$ for s

5. 84 is 6 more than 3 times a number. What is the number?

6. A stick that is 438cm long is cut into two pieces. The first is 74 bigger than the second. What are the lengths of the two pieces?

7. Find the area of the shaded region:



8. If a rectangle's length is 7 more than 4 times the width and the perimeter is 194 mm, what are the dimensions of the rectangle?

9. Find the missing variable for a rectangle:

P = 48.3 ft
w = 7.2 ft
l =

10. Find the missing variable for a circle:

C = 800 in
r =

Solve.

2-3

11. $7p + 12 = 13 - 7p$

12. $4n + 68 = 7 - 2(n - 2)$

13. $7x - 10 = 5(x - 2)$

14. $9x - 4 = 15x$

15. $8x - 7(x + 3) = x - 21$

16. $.18x = 13 - .20x$

2-4

17. 219 is 28% of what?

18. 27 is what percent of 74?

19. Original Price:$192.56
Tax: 7.3%
Final Price:

20. Original Price:
Discount: 35%
Final Price: $43.90

21. If the price of a meal after a 20% tip was $28.80? What was the price of the meal before the tip was added?

22. If the price of an object dropped 15% down to $59.50, what was the original price?

Solve.

2-5

23. $\frac{7}{3}t - 2 = 19 + 5t$

24. $-\frac{3}{4}(x - 4) = 5 + 2x$

25. $\frac{1}{6}x - 4 = 3 + \frac{3}{10}x$

26. $\frac{5}{2}(-4x - 2) = \frac{3}{4}x + 6$

27. $\frac{x-5}{3} = \frac{5x+8}{6}$

28. $\frac{x+7}{14} = 6 - \frac{3}{7}x$

Solve and graph.

2-6

29. $3t + 5 > 12$

30. $4m + 2 \leq -18$

31. $3(x + 4) - 6x \geq 5(x - 2)$

32. $-7p + 3 < 24$

33. $\frac{3}{5}(\mathrm{n} + 4) - 2 \geq \frac{3}{2}n$

34. $3m < -21$

35. $\frac{3}{4}(\mathrm{x} - 5) \geq \frac{7}{2}\mathrm{x} + 1$

36. $3a + 5a \leq 7a - 8a$

37. $5y - 7 \geq \frac{2}{3}y + 4$

Preparation.
Solve the following.
38. At a family reunion, Logan reserves a table at a dinner and a show event. There is a $50 reservation fee for the show, plus a fee of $15 per person for the dinner. If he has a budget of $450, how many people can come to the dinner?

39. On his first two tests, Josh received scores of 85 and 89. If he wants at least a 90 for the average of his first three tests, what possible scores could he get on his third test?

Section 2-6 Answers

1. $s = \frac{2Vt+at^2}{2}$

2. $p = \frac{I}{rt}$

3. $F = \frac{LP}{d}$

4. $s = \frac{11c+5g}{9}$

5. 26

6. 182cm, 256cm

7. 62.38 in^2

8. 18mm X 79mm

9. l = 16.95 ft

10. 127.3 *in*

11. $p = \frac{1}{14}$

12. n = -9.5

13. x = 0

14. $x = -\frac{2}{3}$

15. All numbers

16. x = 34.21

17. 782.1

18. 36.5%

19. $206.62

20. $67.54

21. $24

22. $70

23. $t = -\frac{63}{8}$ or -7.875

24. $x = -\frac{8}{11}$ or -0.73

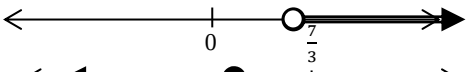
25. x = -52.5

26. $x = -\frac{44}{43}$ or -1.02

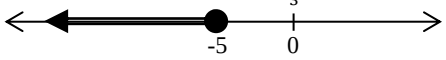
27. x = -6

28. x = 11

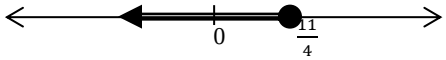
29. $t > \frac{7}{3}$



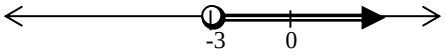
30. $m \leq$ -5



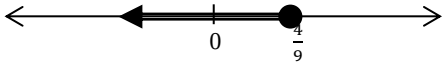
31. $x \leq \frac{11}{4}$



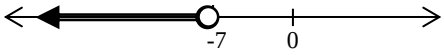
32. $p >$ -3



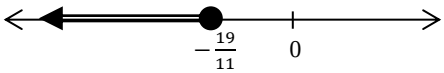
33. $n \leq \frac{4}{9}$



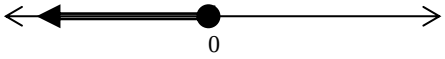
34. $m <$ -7



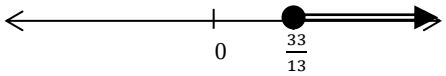
35. $x \leq -\frac{19}{11}$



36. $a \leq 0$



37. $y \geq \frac{33}{13}$



38. In class.

39. In class.

Section 2-7 Word Problems: Inequalities

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

2.5 2.3 2.3 2.1 2.1	GOAL: Get x alone (x will represent any variable) 1. SIMPLIFY A) Get rid of fractions (multiply all by LCD) B) Distribute Across ( ) C) Combine "Like terms" and get all x's on the same side 2. ADDITION PRINCIPLE 3. MULTIPLICATION PRINCIPLE
2.2	Applications: Shapes, Formulas, Solving for a Variable
2.4	Applications: Substitution and Percents
2.6	Inequalities
2.7	Inequalities and Applications

INEQUALITY WORD PROBLEMS: TRANSLATION

COMMON PHRASES	EXAMPLE	TRANSLATION
at least / minimum	The minimum speed is 35 mph.	$S \geq 35$
at most	You can have at most 15 in the group.	$P \leq 15$
cannot exceed / maximum	The weight cannot exceed 2000lbs.	$W \leq 2000$
must exceed	The cost must exceed \$10	$C > 10$
less than	He has less than a 95%	$x < 95\%$
more than	We have more than \$1000 in our account.	$M > \$1000$
between	His age is somewhere between 25 and 30 years old.	$25 < A < 30$
no more than	There can be no more than10 people on the boat.	$P \leq 10$
no less than	You should walk no less than 5 miles a day.	$d \geq 5$

Main Topics	Examples
Power of "at least" 1horsepower. Power $\geq$ 10 horsepower	**Example 1:** In order for my model rocket to work it needs to put out a power of at least 10 horsepower Determine the key phrase that will help us to figure out which inequality sign to use Translate it into an inequality sign **Answer: Power $\geq$ 10 horsepower**

Thickness of penny is "between" 1.8 and 2mm.

$1.8mm < T < 2mm$

Note that it could also be

$1.8mm \leq T \leq 2mm$

Example 2: The required thickness of penny is between1.8 and 2mm.

Determine the key phrase that will help us to figure out which inequality sign to use
Translate it into an inequality sign
Answer: 1.8mm < T < 2mm

	Examples
Speed "**limit**" of 45mph. Speed $\leq$ 45mph	**Example 3:** Glen Road has a speed limit of 45mph.
SOLVING PROBLEMS	**Example 4:** A box that weighs 5 lbs can hold up to 25 books that each weighs 2.5 lbs. Due to recent back surgery I can only carry at most 48 lbs. If I want to move the box, how many books can the box have in it?

Determine the key phrase that will help us to figure out which inequality sign to use
Translate it into an inequality sign
Answer: speed $\leq$ 45mph

The weight of the box (5lbs) plus 2.5 lbs for every book (x) can be at most 48 lbs.
Set up an inequality with the information we are given. $5 + 2.5x \leq 48$ $5 + 2.5x \leq 48$ $-5 \quad -5$
Solve the equation we just came up with $2.5x \leq 43$ $\div 2.5 \quad \div 2.5$
Check to see if the answer makes sense Can we have 17.2 books? No, so we need to change it to 17 $x \leq 17$ We also note that a negative number of books would make no sense, so we have to have at least 0.
Answer: 0 $\leq$ Books $\leq$ 17

Set up an inequality with the information we are given.

Solve the inequality we just came up with

Check to see if the answer makes sense

Note that solving these problems using equations and then adding the inequality sign when the answer is obtained is acceptable.

Example 5: In order for David to reach his saving goal he needs to earn at least $109,200 in commission this year. He earns 15% commission from all of his sales. If he has already earned $89,700 in commission this year, how much more in sales does he need this year in order to reach his goal?

David's sales amount last year ($89,700) needs to increase by 15% of x to be at least $109,200
$89{,}700 + .15x \geq 109{,}200$
$89{,}700 + .15x \geq 109{,}200$ $-89{,}700 \quad -89{,}700$
$.15x \geq 19{,}500$ $\div .15 \quad \div .15$ $x \geq 130{,}000$
Can we have 130,000 dollars? Yes, our answer works $x \geq 130{,}000$
Answer: sales $\geq \$130{,}000$

Main Topics	Examples
Set Notation **{ } means "set of".** **x | means "x such that".** $\{x \mid x > 5\}$ **The set of all x such that x > 5.**	

Inequality	Set Notation
$x > 5$	$\{x \mid x > 5\}$
$x \leq -7$	$\{x \mid x \leq -7\}$
$-2 < x \leq \frac{7}{2}$	$\left\{x \mid -2 < x \leq \frac{7}{2}\right\}$

Section 2-7 Exercises

Translate the following statements into an equation and solve.

2-5

1. 56 is what percent of 448?

2. What is 15% of 0.0012?

3. 421 is 105.25% of what?

4. While studying the weather patterns in Omaha, Jackson recorded that between the months of March and May the average temperature highs rose by 25%. If the average temperature in May is 78°F, what was the average temperature in March?

5. While running her latest marathon, Erika lost 2% of her body weight in sweat. After drinking water after the race, she regained 90% of the weight that she lost. If she originally weighed 120 pounds, how much did she weigh after drinking?

Solve the following inequalities, and write the answer in set notation.

2-6

6. $14h + (-7.2) < -220$

7. $8k - \frac{3}{4} \geq 10k$

8. $0.5(8r + 22) \leq 2r + 11$

9. $\frac{75}{3} > \frac{15b-30}{3}$

10. $12.7p + 4.5 + 1.3p \geq -31.9$

Write the following statements as an inequality and graph.

2-7

11. It is at most 5° outside.

12. The show will begin in less than 5 minutes.

13. 60 inches is the minimum height to enter.

14. The road is between 5 and 8 miles away.

15. The speed limit is 45 miles per hour.

16. I have more than 3 years until graduation.

Solve the following word problems by writing them in an inequality.

17. On his first two tests, Josh received scores of 88 and 92. If he wants at least an average of 93, what does his score on the third test have to be?

18. An elevator can hold up to 3,300 pounds. If each person on the elevator weighs an average of 165 pounds, how many people can ride at one time? (When you state your answer, remember that we can't have a negative number of people.)

19. At a family reunion, Logan reserved a table at a dinner and a show event. There was a $50 reservation fee for the show, plus a fee of $15 per person for the dinner. If he had a budget of $450, how many people can come to the dinner? (When you state your answer, remember that we can't have a negative number of people.)

20. In order to qualify for financial aid, Sheyla needs to take at least 30 credits combined between two semesters. If she took 16 credits last semester, how many more credits does she need to take this semester to qualify for aid?

21. Patty wants to know how long she can talk to her grandma on a long-distance phone call with the $2.20 she has. If it costs $0.50 to place a call and $0.10 per minute, how long can she talk?

22. The width of a rectangle is fixed at 6 meters. For what lengths will the area be more than $96m^2$?

Section 2-7 Answers

1. 12.5%

2. 0.00018

3. 400

4. 62.4° F

5. 119.76 pounds

6. $\{h|h < -15.2\}$

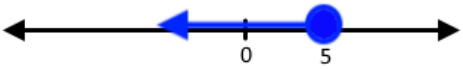
7. $\{k|k \leq -0.375\}$

8. $\{r|r \leq 0\}$

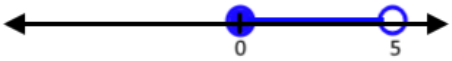
9. $\{b|b < 7\}$

10. $\{p|p \geq -2.6\}$

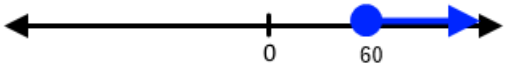
11. $Temperature \leq 5°$



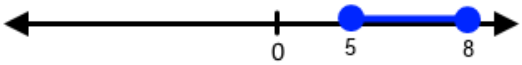
12. $0 \leq Show\ begins < 5\ min.$



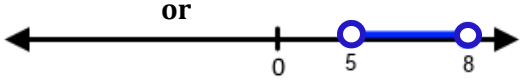
13. $height \geq 60\ inches$



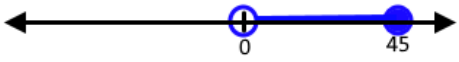
14. $5\ mi \leq road \leq 8\ mi$
or $5\ mi < road < 8\ mi$



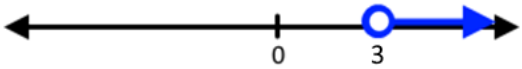
or



15. $0 < speed \leq 45\ mph$



16. $years > 3$



17. $grade \geq 99$

18. $0 \leq people \leq 20$

19. $0 \leq people \leq 26$ or $0 \leq P < 27$

20. $credits \geq 14$

21. $0 < time \leq 17\ minutes$

22. $length > 16\ meters$

Chapter 2 Review Exercises

1. Create a visual chart of all of the methods, formulas, and examples from studying how to solve these linear equations and inequalities.

Solve.

2-1 **2.** $-\frac{2}{9}m = 24$ **3.** $9\left(\frac{8-6x}{4}+6\right)+5=-31$ **4.** $\frac{8x-5}{3}=33$

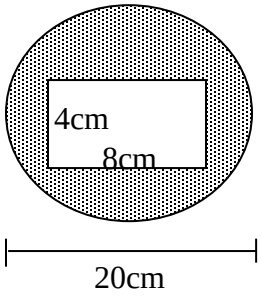
Solve for the specified variable.

5. $Rd = pM(f+t)$ for t **6.** $d=\frac{pM(f-t)}{R}$ for R

2-2 **7.** 13.2 less than 7 times a number is 18.8. What is the number?

8. Two numbers add to 336 and the first is 24 bigger than the second. What are the two numbers?

9. Find the area of the shaded region:



10. I have created a triangular garden such that the largest side is 6m less than twice the smallest and the medium side is 15m larger than the smallest side. If the total perimeter of the garden is 105m, what are the lengths of the three sides?

11. If a parallelogram has an area of 158.9 cm^2 and a base of 23.2 cm, how tall is it?

12. A triangle has angles of $g-3, 2g+8, and\ 3g-17$. Solve for g and find the measures of the angles.

Solve.

2-3 **13.** $7p+12=12+7p$ **14.** $9n+48=7n-2(n-2)$ **15.** $7x+18=9(x-3)$

16. Rick has taken 4 tests. His scores are 83, 92, 94, and 85. What does he need to get on his 5th test for his average to be 90?

2-4

17. 45 is what percent of 39?

18. 25 is 44% of what?

19. What is 59% of 2,340?

20. Original Price:
Tax: 5%
Final Price: $359.50

21. Original Price: $55.50
Discount: 20%
Final Price:

22. If the population of a town grew 11% up to 17,046. What was the population last year?

23. If the price of an object dropped 15% down to $62.90, what was the original price?

Solve.

2-5

24. $\frac{7}{3}t - 8 = 4 + 7t$

25. $-\frac{3}{7}(m - 12) = 3m + 6$

26. $\frac{5}{6}x - 8 = 7 + \frac{7}{8}x$

27. $.13(-2x + 2) = .05x + 7$

28. $\frac{x-7}{4} = \frac{5x+3}{10}$

Solve and graph.

2-6

29. $3t + 5 > 15$

30. $4m + 30 \leq -18$

31. $3(x + 2) - 6x \geq 5(x - 2)$

32. $-7p + 3 < -10$

33. $\frac{3}{5}(\mathrm{n} + 6) - 2 \geq \frac{3}{2}n$

34. $7m < -21$

35. $\frac{3}{4}(\mathrm{x} - 5) \geq \frac{7}{2}\mathrm{x} + 15$

36. $3a + 5a > 7a - 8a$

37. $5y - 15 \geq \frac{2}{3}y + 4$

Solve. Write your answer as an inequality. Then graph the solution.

2-7

38. A copy job to run a pamphlet costs $7 for a setup fee and then $2.21 for each copy. How many copies can be run if the budget is $175? (Include in the answer the idea that a negative number of copies cannot be made.)

39. An envelope has to have a maximum area of 18 in^2. What can the length be if the width is $3\frac{1}{4}$ in? (Include the idea that a length cannot be negative.)

Chapter 2 Review Answers

1. It better be good.

2. $m = -108$

3. $x = 8$

4. $x = 13$

5. $t = \frac{Rd - pMf}{pM}$ or $t = \frac{Rd}{pM} - f$

6. $R = \frac{pM(f-t)}{d}$ *or* $R = \frac{pMf - pMt}{d}$

7. $\frac{32}{7}$ or 4.57

8. 156, 180

9. 282.16cm2

10. 24m, 39m, 42m

11. 6.85cm

12. $g = 32$ *angles are* $29°, 72°, 79°$

13. All numbers

14. n = -11

15. $x = \frac{45}{2}$ or 22.5

16. 96

17. 115.4%

18. 56.82

19. 1380.6

20. $342.38

21. $44.40

22. 15,357

23. $74

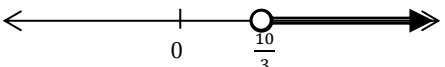
24. $t = -\frac{18}{7}$

25. $m = -\frac{1}{4}$

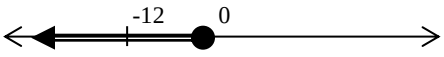
26. $x = -360$

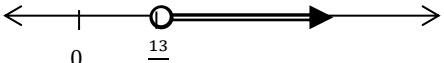
27. $x = -21.74$

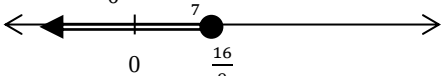
28. $x = -\frac{41}{5}$ or -8.2

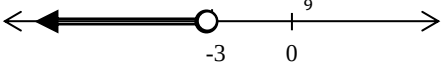
29. $\left\{t \middle| t > \frac{10}{3}\right\}$ 

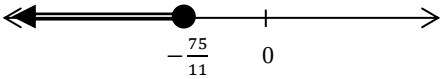
30. $\{m|m \leq -12\}$ 

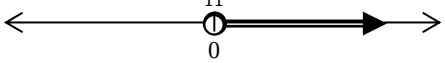
31. $\{x|x \leq 2\}$ 

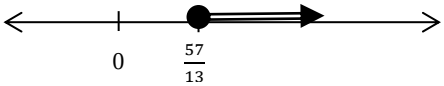
32. $\left\{p \middle| p > \frac{13}{7}\right\}$ 

33. $\left\{n \middle| n \leq \frac{16}{9}\right\}$ 

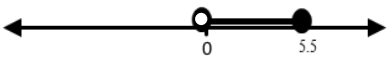
34. $\{m|m < -3\}$ 

35. $\left\{x \middle| x \leq -\frac{75}{11}\right\}$ 

36. $\{a|a > 0\}$ 

37. $\left\{y|y \geq \frac{57}{13}\right\}$ 

38. $0 \leq$ pamphlets ≤ 76 

39. $0 <$ length ≤ 5.5 

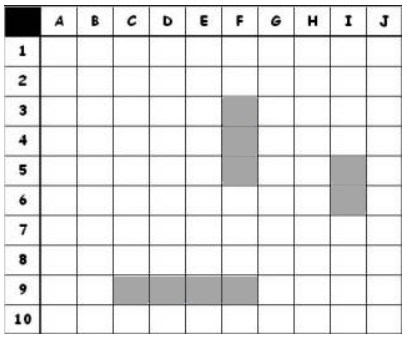
Section 3-1 Coordinates and Graphing Lines

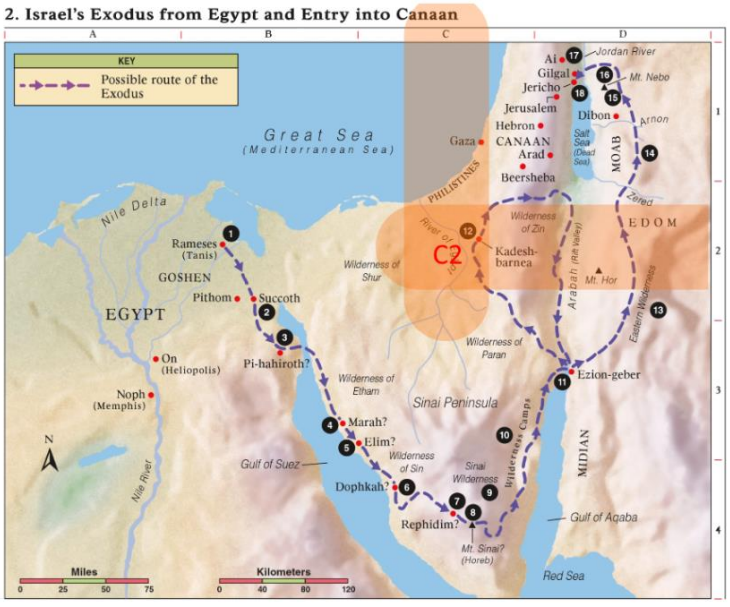
CHAPTER OVERVIEW (Video Instruction and Solutions Link)

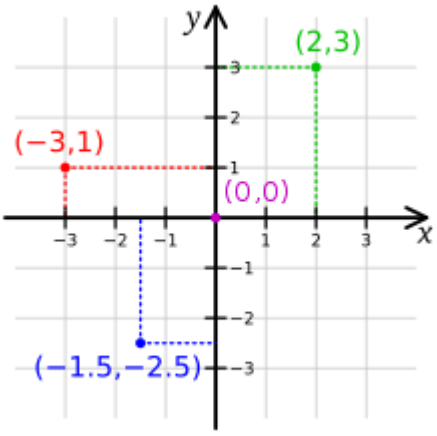
3-1: Coordinates	Graphing Lines
3-2: Intercepts	Vertical and Horizontal Lines
3-3: Slope	
3-4: Graphing with slope	Equation shortcuts
3-5: Writing Equations of Lines	Prediction

Graphs

Main Topics	Examples
Coordinate Axes Ordinates	Alphabetical Ordinates *A, B, C, D, E …* Numerical Ordinates *1, 2, 3, 4, 5…*
Games	Battleship: Coordinates C9, D9, E9, F9 indicate the placement of a ship.
Maps	Maps: Kadesh-Barnea is found at C2 on this LDS Study Helps





Cartesian Coordinates	In 1637, Rene Descartes published the idea of using pairs of numerical ordinates in equations with variables to describe geometric objects such as lines and circles. The first ordinate x and the second ordinate y give us (x, y). We now call this the Cartesian (named after Descartes) Coordinate System. 

Finding Points

Main Topics	Examples
Two Points	One point can have many lines going through it, but once you find a 2nd point, there is only one line.
Pick 'n Stick To organize the points we make, we can outline the points on the graph by using a table. Solution:	**Example 1: Fill in the table for** $y = \frac{1}{4}x - 2$ x | y 4 | 0 | | 0 | 3 when x = 4 we have $y = \frac{1}{4}(4) - 2$ which means y = -1. when x = 0 we have $y = \frac{1}{4}(0) - 2$ which means y = -2. when y = 0 we have $0 = \frac{1}{4}(x) - 2$ which means x = 8. when y = 3 we have $3 = \frac{1}{4}(x) - 2$ which means x = 20. Now fill in the table with each value. x | y 4 | -1 0 | -2 8 | 0 20 | 3

Pick any number for either variable, substitute that number in, and solve for the other variable. Though you only need two, you could get billions. Here are just a few.

Example 2:

$3x + 4y = 7$

x	y
2	
	0
	1
	5
0	

3x + 4y = 7

x	y
2	
	0
	1
	5
0	

$3(2) + 4y = 7$
$4y = 1$
$y = \frac{1}{4}$

$3(0) + 4y = 7$
$4y = 7$
$y = \frac{7}{4}$

$3x + 4(0) = 7$
$3x = 7$
$x = \frac{7}{3}$

$3x + 4(1) = 7$
$3x = 3$
$x = 1$

$3x + 4(5) = 7$
$3x = -13$
$x = -\frac{13}{3}$

Solution:

x	y
2	$\frac{1}{4}$
$\frac{7}{3}$	0
1	1
$-\frac{13}{3}$	5
0	$\frac{7}{4}$

Graphing

1. Pick a number.
2. Stick in (substitute) for x or y.
3. Solve for the partner.
4. Connect points.

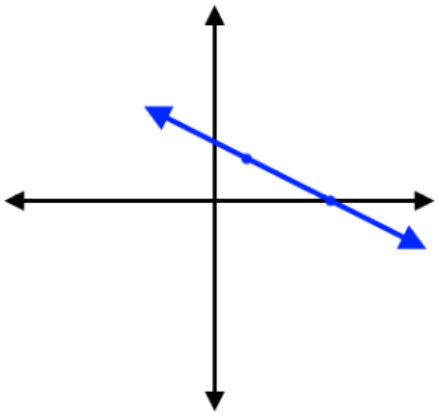
Example 2: Graph $2x + 3y = 7$

x	y
2	
	0

$2(2) + 3y = 7$

$4 + 3y = 7$
$3y = 3$
$y = 1$
$(2,1)$

$2x + 3(0) = 7$
$2x = 7$
$x = \frac{7}{2}$
$\left(\frac{7}{2}, 0\right)$



Example 3: Graph $y = \frac{2}{3}x - 4$

x	y
0	
5	

$y = \frac{2}{3}(0) - 4$

$y = 0 - 4$

$y = -4$

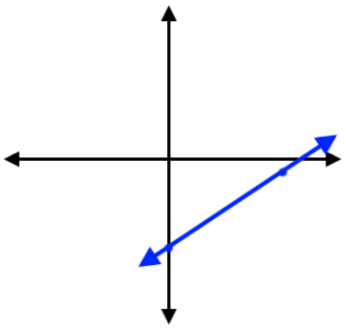
$(0, -4)$

$y = \frac{2}{3}(5) - 4$

$y = \frac{10}{3} - 4$

$y = -\frac{2}{3}$

$\left(5, -\frac{2}{3}\right)$



Section 3-1 Exercises

2-4

1. Three types of bears are in a national park. The number of grizzly bears is 4 more than twice the number of black bears, and the number of panda bears is 50 more than the number of black bears. There are a total of 874 bears in the park. How many of each kind are there?

2. An international phone call costs 35¢ to connect and 12¢ for every minute of the call. How long can a person talk for $3.60?

3. A 52m rope is cut so that one piece is 18m longer than the other. What are the lengths of the pieces?

4. Original Price:$292.50
Discount:20%
Final Price:

5. Original Price:
Discount: 40%
Final Price: $73.90

Solve and graph (on a number line).

2-7

6. $5(x-2) > 7x + 8$

3-1

Fill out the table for each of the following:

7. $x + y = 9$

x	y
5	
-4	
	3
	0
	7

8. $2x - y = 5$

x	y
2	
0	
-1	
	0
	4

9. $5x + 4y = 9$

x	y
1	
0	
-3	
	0
	5

10. $x - 7y = 13$

x	y
	1
	3
2	
0	
	-1

Graph the following lines, and label three points (your points may be different than mine).

11. $3x + y = 10$

12. $y = 2x$

13. $x - 4y = 7$

14. $2x + y = 3$

15. $y = -\frac{3}{7}x + 4$

16. $6x - 5y = 12$

17. $y = \frac{1}{2}x - 4$

18. $5x + 2y = 6$

Preparation.

19. After reading a bit of section 3.2, try to find the x-intercept and y-intercept of $y = \frac{2}{3}x + 5$.

20. Solve for y in each of these equations:

a) $2x + y = 7$ b) $5x + 3y = 6$ c) $2x - 7y = 11$

Section 3-1 Answers

1. 205 Black, 414 Grizzly, 255 Panda

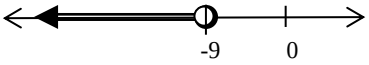
2. 27 minutes

3. 17m, 35m

4. $234

5. $123.17

6. $x < -9$



7.

x	y
5	**4**
-4	**13**
6	3
9	0
2	7

8.

x	y
2	**-1**
0	**-5**
-1	**-7**
$\frac{5}{2}$	0
$\frac{9}{2}$	4

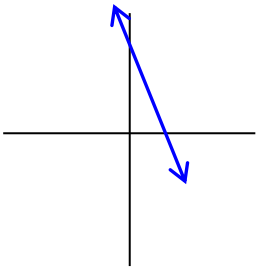
9.

x	y
1	**1**
0	$\frac{9}{4}$
-3	**6**
$\frac{9}{5}$	0
$-\frac{11}{5}$	5

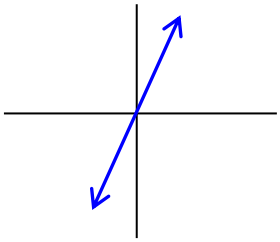
10.

x	y
20	1
34	3
2	$-\frac{11}{7}$
0	$-\frac{13}{7}$
6	-1

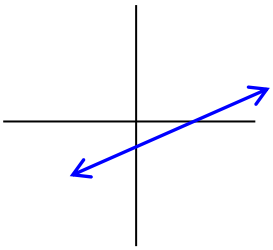
11. (0,10) (3,1) (-1,13)



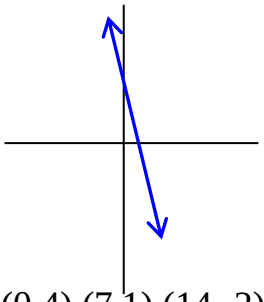
12. (0,0) (1,2) (2,4)



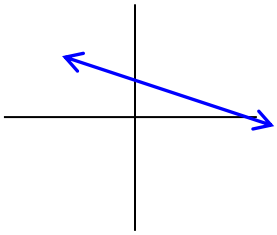
13. (7,0) (3,-1) $(0,-\frac{7}{4})$



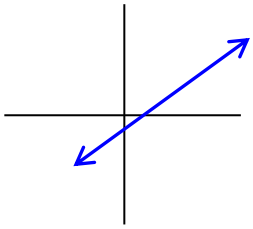
14. (0,3) (1,1) (2,-1)



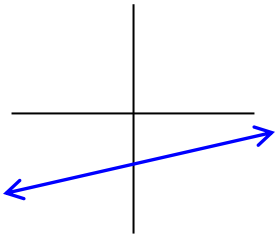
15. (0,4) (7,1) (14,-2)



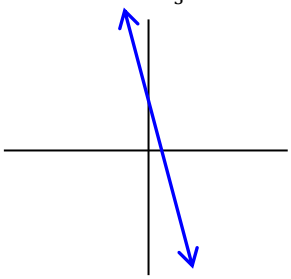
16. (2,0) $(0,-\frac{12}{5})$ (7,6)



17. (0,-4) (2,-3) (4,-2)



18. (0,3) (2,-2) $(\frac{6}{5},0)$



19. In class.

20. In class.

Section 3-2 Intercepts, Vertical, Horizontal

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

3-1: Coordinates	Graphing Lines
3-2: Intercepts	Vertical and Horizontal Lines
3-3: Slope	Parallel and Perpendicular
3-4: Graphing with slope	Slope Intercept and equation shortcuts
3-5: Writing Equations of Lines	Prediction

Intercepts

Main Topics

Intercepts

$x-intercept\ when\ y\ is\ 0.$
$(\quad ,0)$

$y-intercept\ when\ x\ is\ 0.$
$(0,\quad)$

Why do we call them intercepts?
In math, the word "intercept" means "to cross". A point (0,) is on the y-axis.

Examples

Example 1: Find the x-intercept of $2x-7y=12$

$Stick\ 0\ in\ for\ y$
$2x-7\cdot 0=12$

$2x=12$

$x=6$
The x-intercept is $(6,0)$

Example 2: Find the y-intercept of $2x-7y=12$

$Stick\ 0\ in\ for\ x$
$2\cdot 0-7y=12$

$-7y=12$

$y=-\frac{12}{7}$
The y-intercept is $\left(0,-\frac{12}{7}\right)$

Example 3: Graph the following line and label the x- and y- intercept.
$y=\frac{2}{3}x-5$

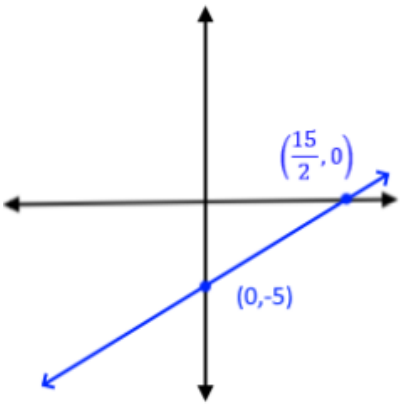
y-intercept: *stick in 0 for x*
$y=\frac{2}{3}(0)-5$
$=-5$

$(0,-5)$

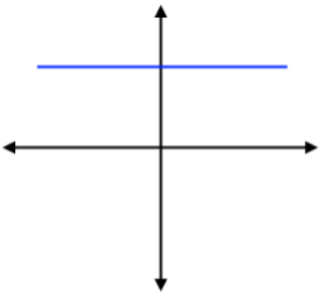
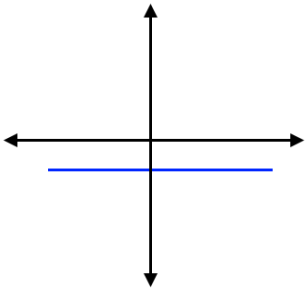
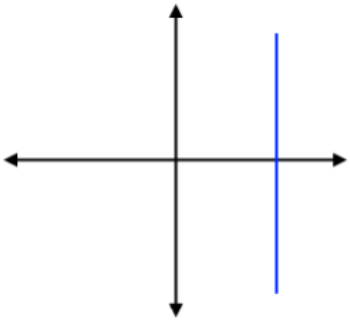
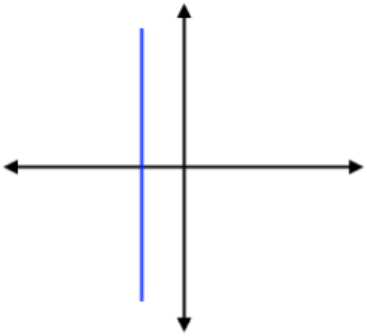
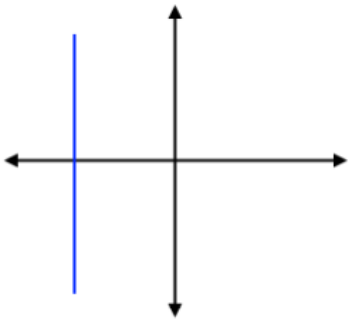
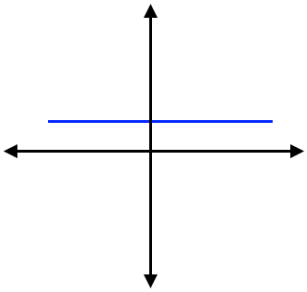
x- intercept: *stick in 0 for y*
$0=\frac{2}{3}x-5$

$5=\frac{2}{3}x$

$\frac{15}{2}=x\quad \left(\frac{15}{2},0\right)$



Vertical and Horizontal Lines

Main Topics	Examples
Horizontal All points (including the y-intercept) have the same y value.	**Example 3:** $y = 7$  **Example 4:** $y = -2$ 
Vertical All points (including the x-intercept) have the same x value.	**Example 5:** $x = 7$  **Example 6:** $x = -2$ 
Some require you to solve the equation, then graph.	**Example 7:** $-2x = 12$ $\boldsymbol{x = -6}$  **Example 8:** $5y + 6 = 16$ $5y = 10$ $\boldsymbol{y = 2}$ 

Section 3-2 Exercises

1. Three types of trees are in a local park. The number of aspens is 5 more than three times the number of oaks, and the number of maples is 20 less than the number of oaks. There are a total of 850 trees in the park. How many of each kind are there?

2. Original Price:$49.50
Discount:20%
Final Price:

3. Original Price:
Discount: 40%
Final Price: $53.70

Solve and graph (on a number line).

2-6

4. $6(x-2) > 7x + 8$

Fill out the table for each of the following:

3-1

5. $2x + y = 9$

x	y
5	
-4	
	3
	0
	7

6. $2x - 3y = 5$

x	y
2	
0	
-1	
	0
	4

Graph the following lines, and label three points (your points may be different than mine).

7. $y = -\frac{3}{7}x + 4$

8. $6x - 5y = 12$

Find the x and y intercepts for each line. Then graph.

3-2

9. $5x + 2y = 20$

10. $y = 2x - 10$

11. $4x - y = 8$

12. $x + y = 7$

13. $y = -\frac{3}{7}x - 6$

14. $2x - 6y = 18$

Graph the following lines, and label three points (your points may be different than mine).

15. $x = 5$

16. $y = 4$

17. $3x = -6$

18. $2y + 1 = -15$

Preparation.

19. After reading a bit of section 3-3, try to find the slope between (4,1) and (7,11).

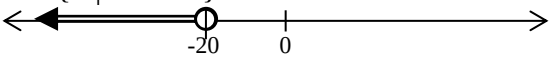
Section 3-2 Answers

1. 173 Oaks, 524 Aspen,153 Maple

2. $39.60

3. $89.50

4. {x | x < -20}



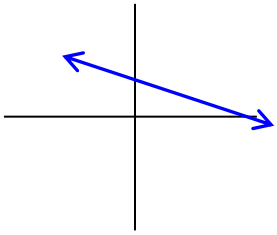
5.

x	y
5	**-1**
-4	**17**
3	3
$\frac{9}{2}$	0
1	7

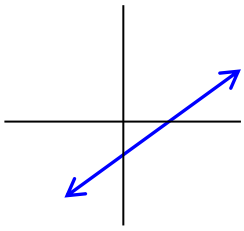
6.

x	y
2	$-\frac{1}{3}$
0	$-\frac{5}{3}$
-1	$-\frac{7}{3}$
$\frac{5}{2}$	0
$\frac{17}{2}$	4

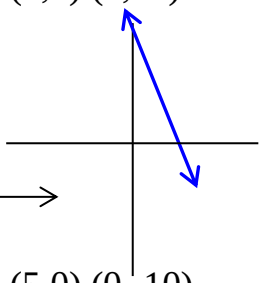
7. (0,4) (7,1) (14,-2)



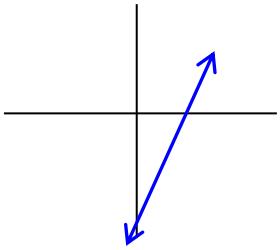
8. (2,0) $(0,-\frac{12}{5})$ (7,6)



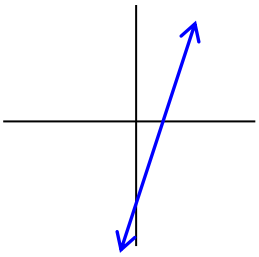
9. (4,0) (0,10)



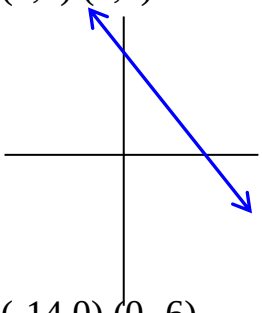
10. (5,0) (0,-10)



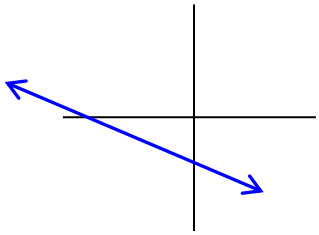
11. (2,0) (0,-8)



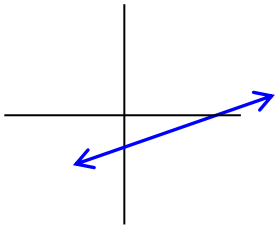
12. (7,0) (0,7)



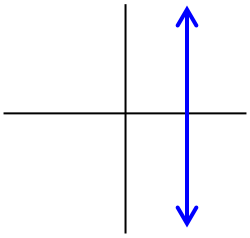
13. (-14,0) (0,-6)



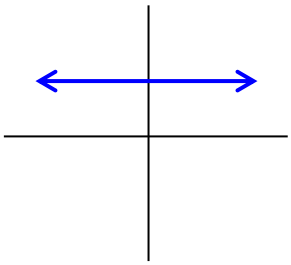
14. (9,0) (0,-3)



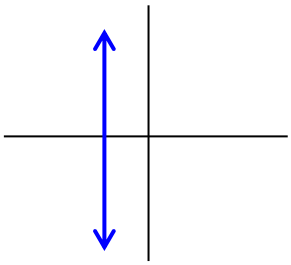
15. (5,0) (5,1) (5,-4)



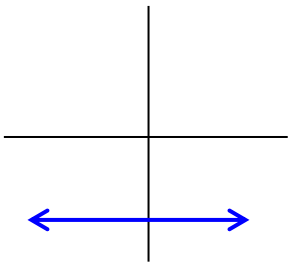
16. (0,4) (2,4) (7,4)



17. (-2,0) (-2,2) (-2,-8)



18. (0,-8) (5,-8) (-11,-8)



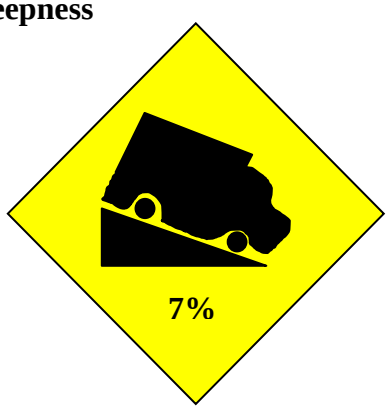
19. In class.

Section 3-3 Slope

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

3-1: Coordinates	Graphing Lines
3-2: Intercepts	Vertical and Horizontal Lines
3-3: Slope	Parallel and Perpendicular
3-4: Graphing with slope	Slope Intercept and equation shortcuts
3-5: Writing Equations of Lines	Prediction

Definitions

Main topic:	Example:
SLOPE: Represented by the letter "m". It is referred to as the **rate of change.** $\textbf{Slope} = \frac{\textbf{Vertical Change}}{\textbf{Horizontal Change}}$	**Example 1: Road Grade or Steepness** $7\% = \frac{7}{100} = \frac{7\ feet\ of\ rise}{100\ feet\ of\ run}$ 

CALCULATING SLOPE

Main Topics	Examples
Formula for slope: $m = \frac{y_1 - y_2}{x_1 - x_2}$	**Find the slope between each set of points:** **Example 2:** $(4,3)$ & $(-2,1)$ $\quad \frac{3-1}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$ **Example 3:** $(1,7)$ & $(-2,1)$ $\quad \frac{7-1}{1-(-2)} = 2$ **Example 4**: $(5,-3)$ & $(-2,1)$ $\quad \frac{-3-1}{5-(-2)} = -\frac{4}{7}$

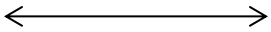
Example 5:

$(5,-3)$ & $(5,1)$ $\quad \frac{-3-1}{5-5} = -\frac{4}{0}$ $\quad$ ***undefined***

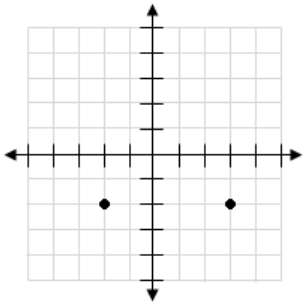
Example 6:

$(1,7)$ & $(-2,7)$ $\quad \frac{7-7}{1-(-2)} = \frac{0}{3} = \mathbf{0}$

All horizontal lines will have a slope of 0.



Example 7:



Slope of the line

$y = -2$

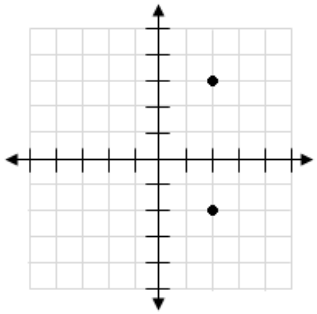
Use points: (3,-2) and (-2,-2).

Plug in the formula: $m = \frac{y_1 - y_2}{x_1 - x_2}$

$$m = \frac{-2-(-2)}{3-(-2)} = \frac{0}{5} = 0$$

All vertical lines have undefined slope

Example 8:



Slope of the line

$x = 2$

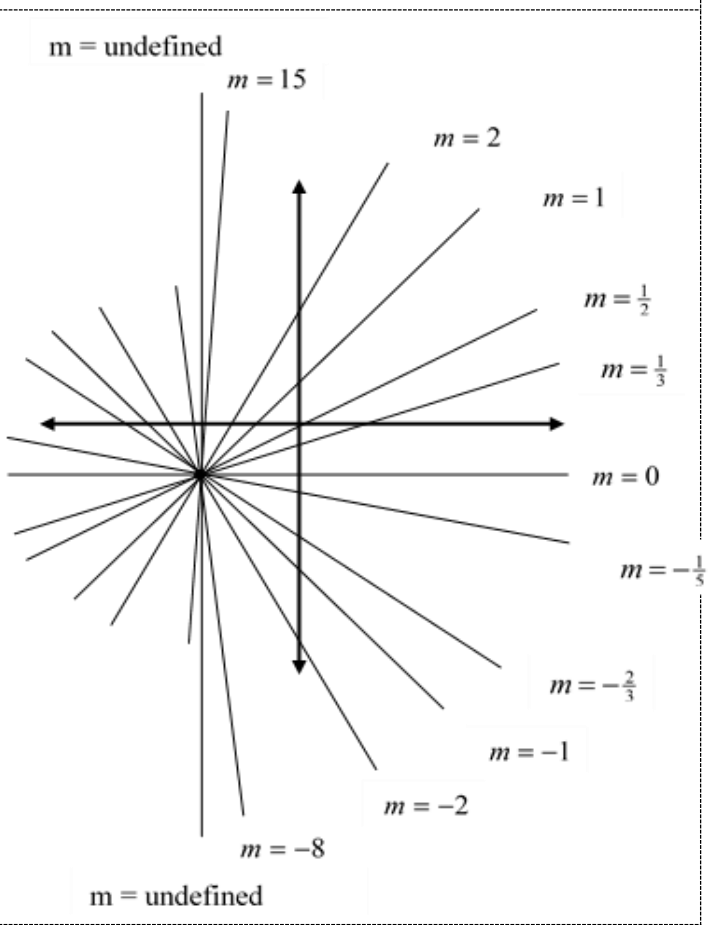
Use points: (2,-2) and (2,3).

Plug in the formula: $m = \frac{y_1 - y_2}{x_1 - x_2}$

$$m = \frac{-2-3}{2-2} = \frac{-5}{0} = \text{UNDEFINED}$$

(Division by zero is undefined.)

Trends with Slope

Main Topics	Examples
1. **Bigger numbers** for slope correspond to **steeper lines**. 2. **Positive slopes head up** as you go to the right. 3. **Negative slopes will head down** as you go to the right. Common Mistake: A vertical line and horizontal line are often mistaken as having no slope. A **vertical line** has **undefined slope** and a **horizontal line** has a **slope of zero.**	

Special slope relationships

Main Topics	Examples
Parallel Slope: When lines have the **same slope, or steepness**, then they **never intersect**. They are parallel with each other. Perpendicular Slope: When 2 lines **intersect each other at a 90° angle**. The **slopes of the two** lines are always **negative reciprocals.**	**Example 9:** Slope: $m = \frac{3}{4}$ Parallel Slope: $\frac{3}{4}$ → they have the **same slope.** Perpendicular Slope: $-\frac{4}{3}$ → the slopes are **negative reciprocals.** **Example 10:** Slope: $m = 1$ Parallel Slope: 1 Perpendicular Slope: -1 **Example 11:** Slope: $m = \frac{5}{8}$ Parallel Slope: $\frac{5}{8}$ Perpendicular Slope: $-\frac{8}{5}$ **Example 12:** Slope: $m = 0$ Parallel Slope: 0 Perpendicular Slope: undefined

Section 3-3 Exercises

2-4

1. Solve for m: $-5x + 2m = 13$

2. Solve for y: $3x - 11y = 5$

3. If grading in a class is set up so that 10% is attendance, 10% is tutoring, 30% homework, and 50% tests, what is a student's grade if he has 80% attendance, 50% tutoring, 50% homework and 80% on tests?

4. Original Price:$392.50
Discount:20%
Final Price:

5. Original Price:
Discount: 45%
Final Price: $73.90

6. Four consecutive odd integers add up to 328. What are the four numbers?

Fill out the table for each of the following:

3-1

7. $2x + y = 9$

x	y
5	
-4	
	3
	0
	7

8. $y = 5x + 2$

x	y
2	
0	
-1	
	0
	4

Find the x and y intercepts. Then graph.

3-2

9. $3x + 2y = 10$ **10.** $y = 2x - 7$ **11.** $y = \frac{1}{2}x$

12. $4x + y = 8$ **13.** $y = -\frac{3}{7}x - 9$ **14.** $2x - 5y = 12$

Graph the following lines, and label three points (your points may be different than mine).

15. $x = 4$ **16.** $y = 3$

Find the slope between each pair of points.

3-3

17. $(5,-2)$ $(7,3)$ **18.** $(4,1)$ $(-5,6)$ **19.** $(5,-1)$ $(-3,-8)$

20. $(7,3)$ $(-2,3)$ **21.** $(-5,2)$ $(4,-3)$ **22.** $(-6,1)$ $(-6,5)$

23. A road rises 400 feet over a horizontal distance of 6,000 feet. What is the slope (or grade) of the road?

For each slope, write down the parallel and perpendicular slopes.

24. $m = 4$ **25.** $m = -\frac{3}{8}$

26. Explain the difference between a slope of zero and an undefined slope.

Preparation. 27. Find two points of each line and then use those points to find the slope.

$y = \frac{7}{3}x - 8$ $y = \frac{3}{5}x + 4$

Section 3-3 Answers

1. $m = \frac{13+5x}{2}$ *or* $m = \frac{5}{2}x + \frac{13}{2}$

2. $y = \frac{3}{11}x - \frac{5}{11}$ or $y = \frac{5-3x}{-11}$

3. 68%

4. $314

5. $134.36

6. 79, 81, 83, 85

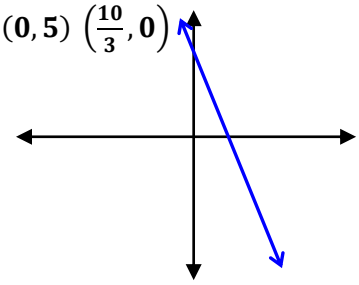
7.

x	y
5	**-1**
-4	**17**
3	3
$\frac{9}{2}$	0
1	7

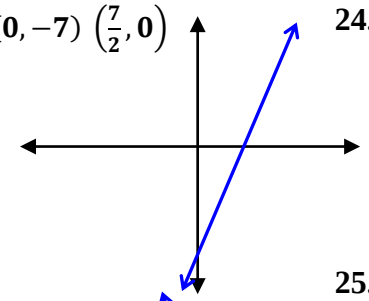
8.

x	y
2	**12**
0	**2**
-1	**-3**
$-\frac{2}{5}$	0
$\frac{2}{5}$	4

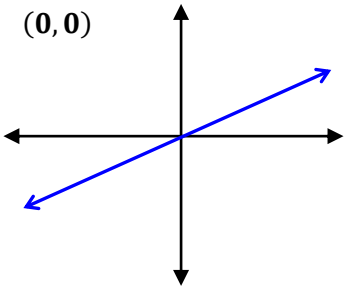
9. $(0, 5)$ $\left(\frac{10}{3}, 0\right)$



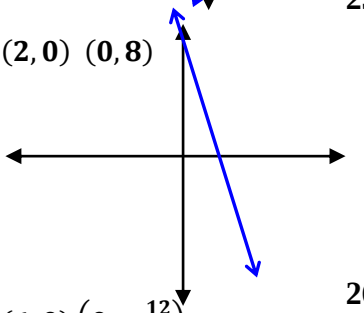
10. $(0, -7)$ $\left(\frac{7}{2}, 0\right)$



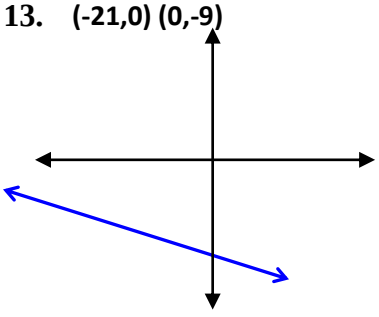
11. $(0, 0)$



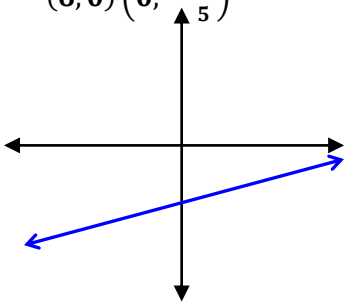
12. $(2, 0)$ $(0, 8)$



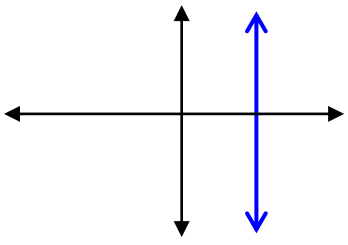
13. **(-21,0) (0,-9)**



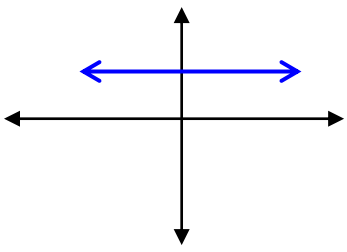
14. $(6, 0)\left(0, -\frac{12}{5}\right)$



15. $(4, 0)(4, -5)(4, 1)$



16. $(-1, 3)$ $(0, 3)(7, 3)$



17. $m = \frac{5}{2}$

18. $m = -\frac{5}{9}$

19. $m = \frac{7}{8}$

20. $m = 0$

21. $m = -\frac{5}{9}$

22. $m = undefined$

23. $m = \frac{1}{15}$ *or* 6.7% *road grade*

24. Parallel: $m = 4$
Perpendicular: $m = -\frac{1}{4}$

25. Parallel: $m = -\frac{3}{8}$
Perpendicular: $m = \frac{8}{3}$

26. Undefined is vertical
Straight up and down
0 is horizontal
Straight across

27. In class

Section 3-4 Slope-Intercept Form

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

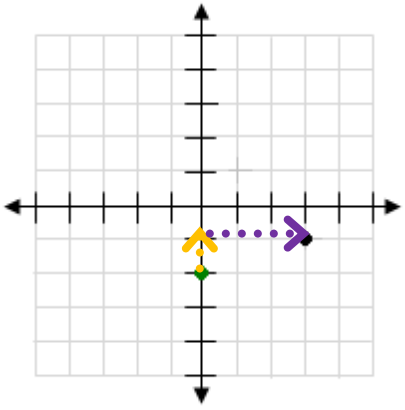
3-1: Coordinates	Graphing Lines
3-2: Intercepts	Vertical and Horizontal Lines
3-3: Slope	Parallel and Perpendicular
3-4: Graphing with slope	Slope Intercept and equation shortcuts
3-5: Writing Equations of Lines	Prediction

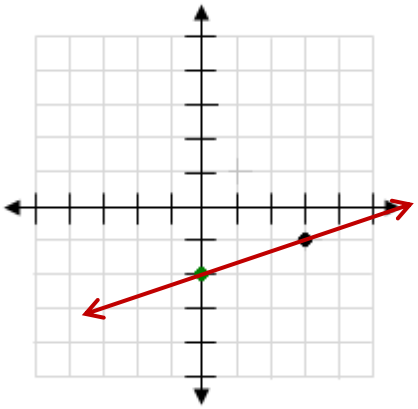
Slope from Slope-Intercept Form

Main Topics	Examples
Key Terms Slope	Slope $m = \frac{y_1 - y_2}{x_1 - x_2}$ $m = \frac{y\ change}{x\ change}$ $m = \frac{rise}{run}$ $m = \frac{\Delta y}{\Delta x}$
Slope-intercept form	Slope-Intercept Form $y = mx + b.$ m is the slope b is the y-intercept
Steps to convert into slope-intercept form: 1. Get the y's alone on one side. 2. Divide every term by the number attached to the y.	**Example 1:** Convert this equation into slope-intercept form: $3x + 4y = 8$ $3x + 4y = 8$ Get the y's alone on one side. $-3x \quad -3x$ $4y = -3x + 8$ $4y = -3x + 8$ Divide every term by the number attached to the y. $\div 4 \quad \div 4 \quad \div 4$ $y = -\frac{3}{4}x + 2$ **Answer:** $\boldsymbol{y = -\frac{3}{4}x + 2}$

	Example 2: Convert this equation into slope-intercept form: $x - 2y = 3$ $x - 2y = 3$ $-x \quad -x$ $-2y = -x + 3$ $-2y = -x + 3$ $(\div -2) \quad (\div -2)(\div -2)$ $y = \frac{1}{2}x - \frac{3}{2}$ **Answer:** $y = \frac{1}{2}x - \frac{3}{2}$ Get the y's alone on one side. Divide every term by the number attached to the y.

Graphing with Slope and Y-intercept

Main Topics	Examples
	Example 3: Graph the equation: $y = \frac{1}{3}x - 2$ $m = \frac{1}{3}$ $y\ intercept = (0, -2)$  Determine the slope and the y intercept from the equation. Plot the y intercept . Trace out the next point by following the slope. In this example go up 1 and over 3.



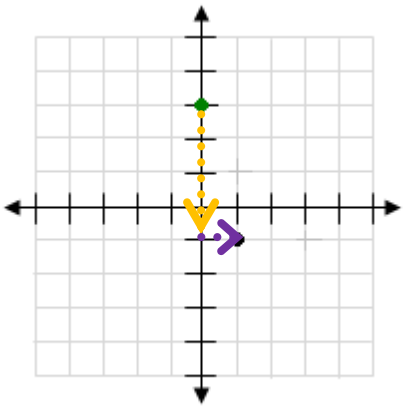
Connect the two dots we just made.

Example 4: Graph the equation: $4x + y = 3$

$y = -4x + 3$ Solve for y.

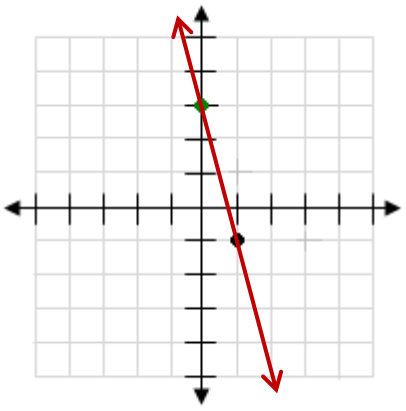
$m = -4$
$y\ intercept = (0,3)$

Determine the slope and the y intercept from the equation.



Plot the y intercept.

Trace out the next point by following the slope. In this example go down 4 and over 1.



Connect the two dots we just made.

Section 3-4 Exercises

1. Three types of horses are in a local ranch. The number of Arabians is 8 more than twice the number of quarter-horses, and the number of Clydsdales is 50 more than the number of Quarter-horses. There are a total of 282 horses at the ranch. How many of each kind are there?

2. What is the radius of a cone that has Lateral Surface Area of 197.92 in^2 and a slant height of 9 in?

3. Solve and graph the solution: $3x - 1 > \frac{5}{2}x + 9$

4. Original Price:$392.50
Tax: 6%
Final Price:

5. Original Price:
Tax: 7%
Final Price: $73.90

Fill out the table for each of the following:

3-1

6. $2x - 3y = 9$

x	y
5	
-4	
	3
	0
	7

7. $y = \frac{7}{2}x+2$

x	y
2	
0	
-1	
	0
	4

Graph the following lines, and label x and y intercepts (need the same points as my answers).

3-2

8. $5x + 2y = 10$

9. $y = \frac{4}{7}x - 6$

10. $y = \frac{8}{3}x + 1$

11. $x = 10$

12. $y = -\frac{3}{7}x + 4$

13. $7x - y = 14$

Find the slope between each pair of points.

3-3

14. (8,-2) (7,3)

15. (8,1) (-5,6)

16. (-3,-1) (-3,-8)

17. (7,9) (-2,3)

18. (-5,6) (4,6)

19. If a road rises 300 feet over a horizontal distance of 3600 feet, what is the road grade?

Find the slope and the y-intercept. Then graph the lines.

3-4

20. $y = 6x + 10$

21. $y = 4x + 3$

22. $y = \frac{1}{2}x - 4$

23. $x = -6$

24. $y = -\frac{3}{7}x - 2$

25. $3x - 4y = 12$

26. $y = -\frac{5}{3}x + \frac{10}{3}$

27. $x + 4y = -9$

28. $y = 7$

29. What is the slope of a line parallel to $\boldsymbol{y = \frac{5}{3}x + 2}$ **?**

30. What is the slope of a line perpendicular to $y = \mathbf{5}$ **?**

Preparation

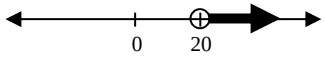
31. Write the equations of three lines that have a slope of $\frac{2}{7}$.

Section 3-4 Answers

1. 56 Quarter-horses, 106 Clydesdales, 120 Arabian

2. radius = 7 in

3. x > 20



4. $416.05

5. $69.07

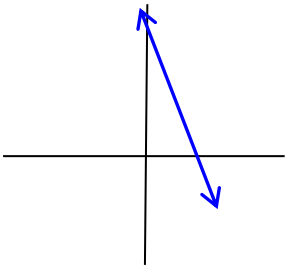
6.

x	y
5	$\frac{1}{3}$
-4	$-\frac{17}{3}$
9	3
$\frac{9}{2}$	0
15	7

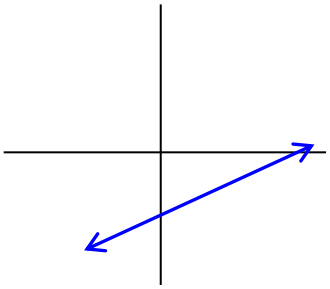
7.

x	y
2	**9**
0	**2**
-1	$-\frac{3}{2}$
$-\frac{4}{7}$	0
$\frac{4}{7}$	4

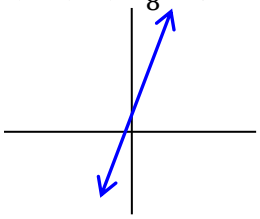
8. (0,5) (2,0)



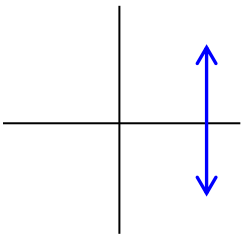
9. (0,-6) ($\frac{21}{2}$,0)



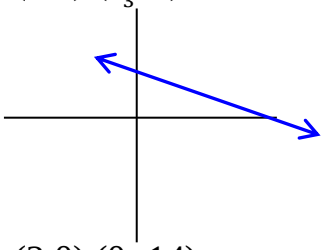
10. (0,1) $(-\frac{3}{8}, 0)$



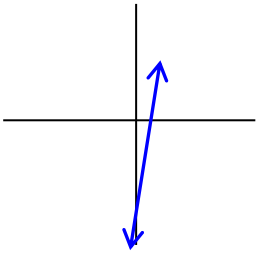
11. (10,0) no y-int



12. (0,4) ($\frac{28}{3}$,0)



13. (2,0) (0,-14)



14. m = -5

15. $m = -\frac{5}{13}$

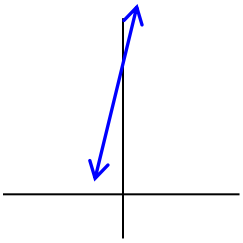
16. m = undefined

17. $m = \frac{2}{3}$

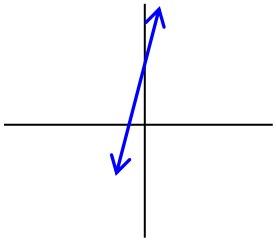
18. m = 0

19. $m = \frac{1}{12}$ or 8.3%

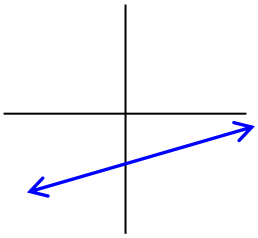
20. (0,10); m = 6



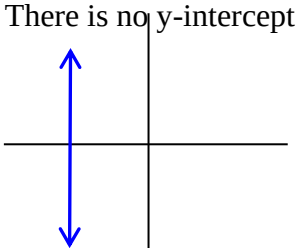
21. (0,3); m = 4



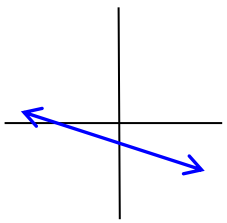
22. (0,-4); m = $\frac{1}{2}$



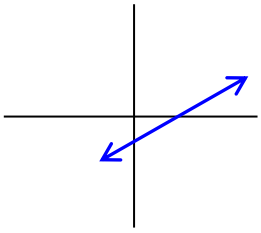
23. (-6,0); m = undefined
There is no y-intercept



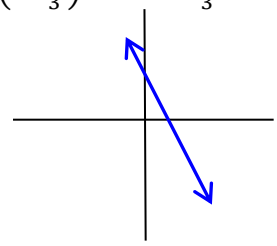
24. (0,-2); m = $-\frac{3}{7}$



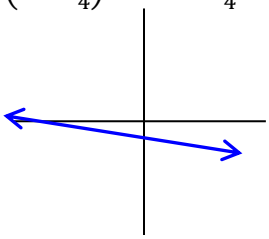
25. (0, -3); m = $\frac{3}{4}$



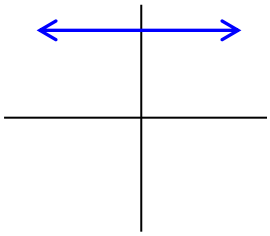
26. $\left(0, \frac{10}{3}\right)$ $m = -\frac{5}{3}$



27. $\left(0, -\frac{9}{4}\right); m = -\frac{1}{4}$



28. (0,7); m = 0



29. $m = \frac{5}{3}$

30. Slope is undefined.

31. In class

Section 3-5 Writing Equations

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

3-1: Coordinates	Graphing Lines
3-2: Intercepts	Vertical and Horizontal Lines
3-3: Slope	Parallel and Perpendicular
3-4: Graphing with slope	Slope Intercept and equation shortcuts
3-5: Writing Equations of Lines	Prediction

Writing Equations

Given slope and one point

Steps:

1. Write in slope-intercept form
   a. Fill in slope (m) in $y = mx + b$.
   b. Fill in x and y using one point.
   c. Solve for b.
   d. Re-write with only x and y as variables.

Example 1: Write the equation of a line that goes through the point (-3, 4) and has a slope of $-\frac{2}{3}$.

$y = mx + b$

$4 = \left(-\frac{2}{3}\right)(-3) + b$ — Plug the point and the slope into equation.

Solve for b.

$4 = 2 + b$

$b = 2$

$y = -\frac{2}{3}x + 2$ — Write the final equation with slope and with y-intercept.

Answer: $\boldsymbol{y = -\frac{2}{3}x + 2}$

Given two points

Steps:

1. Use the points to find the slope.
2. Write in slope-intercept form
   a. Fill in slope (m) in $y = mx + b$.
   b. Fill in x and y using one point.
   c. Solve for b.
   d. Re-write with only x and y as variables.

Example 2: Write the equation of a line that goes through the points: (-2, -2) and (8, 4).

$m = \frac{-2-4}{-2-8} = \frac{-6}{-10} = \frac{3}{5}$ — Find slope.

Remember: $m = \frac{y_1 - y_2}{x_1 - x_2}$

$m = \frac{3}{5}$

$y = mx + b$

$4 = \left(\frac{3}{5}\right)(8) + b$ — Plug one of the points and the slope into equation.

Solve for b.

$4 = \frac{24}{5} + b$

$b = -\frac{4}{5}$

$y = \frac{3}{5}x + \frac{-4}{5}$ — Write the final equation with slope and with y-intercept.

Answer: $\boldsymbol{y = \frac{3}{5}x - \frac{4}{5}}$

Writing Equations of Horizontal Lines

Slick note: All horizontal lines have equations that look like

$$y =$$

And since the point in Example 3 has a y value of 5, the whole line must be

$$\boldsymbol{y = 5}$$

Example 3:
Write the equation of the line with slope, $m = 0$ (horizontal), that goes through the point $(-3,5)$.

y =mx+b — Plug one of the points and the slope into equation.

$5 = (0)(-3) + b$

$5 = 0 + b$ — Solve for b.

$b = 5$

$y = 0x + 5$ — Write the final equation with slope and with y-intercept.

Answer: $y = 5$

Writing Equations of Vertical Lines

Slick note: All vertical lines have equations that look like

$$x =$$

And since the point in Example 4 has an x value of -3, the whole line must be

$$\boldsymbol{x = -3}$$

Example 4:
Write the equation of the line with undefined slope (vertical), and goes through the point $(-3,5)$.

y =mx+b — Plug one of the points and the slope into equation.

$5 = (undefined)(-3) + b$

We are stuck! And we cannot complete this problem with $y = mx + b$.

Predict Values

Main Topics	Examples
Steps: 1. Use the points to write the equation. 2. Plug in the new value.	**Example 5:** Two points on a line are (0, 7) and (6, 2). Follow this same pattern and predict the y-value when x=9. $m = \frac{7-2}{0-6} = -\frac{5}{6}$ — Calculate the slope from the two given points. Remember: $m = \frac{y_1 - y_2}{x_1 - x_2}$ $y = mx+b$ $7 = \left(-\frac{5}{6}\right)(0) + b$ — Plug one of the points and the slope into equation. $7 = b$ — Solve for b. $y = -\frac{5}{6}x + 7$ — Write the final equation with slope and with y-intercept. $y = -\frac{5}{6}(9) + 7$ $y = -\frac{15}{2} + 7 = -\frac{1}{2}$ — Plug in x = 9 **Answer:** $\left(\mathbf{9}, -\frac{\mathbf{1}}{\mathbf{2}}\right)$ **Example 6:** The target heart rate of a person 20 years old is 150 beats per minute (20,150). The target heart rate of an 80-year old is 105 beats per minute (80,110). Use this information to write an equation of a line and predict the target heart rate for a 30-year old. $m = \frac{150-110}{20-80} = -\frac{40}{60} = -\frac{2}{3}$ — Calculate the slope from the two given points. Remember: $m = \frac{y_1 - y_2}{x_1 - x_2}$ $y = mx+b$ $150 = \left(-\frac{2}{3}\right)(20) + b$ — Plug one of the points we have been given. $163.3 = b$ — Solve for b. $y = -\frac{2}{3}x + 163.3$ — Write out the final equation with the slope and with the y-intercept filled in. $y = -\frac{2}{3}(30) + 163.3$ $y = -20 + 163.3 = 143.3$ — Plug in x = 30 **Answer: 143.3 beats per minute**

Section 3-5 Exercises

Fill out the table for each of the following:

3-1

1. 2x - 5y = 11

x	y
5	
-4	
	3
	0
	7

2. $y = \frac{7}{2}x+6$

x	y
2	
0	
-1	
	0
	4

Graph the following lines, and label x and y intercepts.

3-2

3. 4x - 2y = 10

4. $y = -\frac{5}{3}x - 6$

5. y = 5x

Find the slope between each pair of points.

3-3

6. (3,-2) (7,3)

7. (9,1) (-7,6)

8. (5,-1) (-3,-8)

9. (-2,9) (-2,3)

10. (-5,2) (5,6)

11. (19,1) (6,1)

12. Explain the difference between a slope of zero and an undefined slope.

Graph the following lines giving one point and the slope (your point may be different than mine).

3-4

13. -3x + 4y = 10

14. y = 2x - 7

15. $y = \frac{2}{5}x - 4$

16. y = 17

17. $y = -\frac{3}{7}x - 2$

18. 2x – 6y = 12

19. Fill out the Slope Monster (on the next page). Record the time it takes you to complete it.

Write the equations of the lines with the slopes and points:

3-5

20. Write an equation of the line that has slope $m = -3$ and goes through the point (-4,6).

21. Write an equation of the line that at has slope $m = \frac{5}{8}$ and goes through the point (3,6).

22. Write an equation of the line that has slope $m = 0$ and goes through the point (1,-3).

23. Write an equation of the line that is vertical and goes through the point $(2, -4)$

24. Write an equation of the line that goes through (0,1) and (5,-3).

25. Write an equation of the line that goes through (1,7) and (3,11).

26. Two points on a line are (4,7) and (1,-2). Write the equation of the line and then find the y value when x = 9?

27. The number of lung transplants in year 2 after they were possible was 113. In year 12 after they were possible, there were 248. Using the points (2,113) and (12,248). Find the equation of the line and predict how many were performed in year 16.

Slope Monster

Instructions: For each equation, fill in the slope of that equation and either the slope that is perpendicular to that line or parallel to it. Time yourself and write down how long it took you to complete the whole chart.

Equation	Slope	Perpendicular Slope		Equation	Slope	Parallel Slope
$\boldsymbol{2x + 5y = 7}$				$4x - y = 7$		
$\boldsymbol{y = \frac{5}{9}x - 4}$				$y = 2.387x - 4$		
$\boldsymbol{5x - 3y = 7}$				$8x - 3y = 12$		
$\boldsymbol{y = \frac{5}{3}x + 4}$				$-4x + 7y = 19$		
$\boldsymbol{x = 13}$				$x = -19$		
$\boldsymbol{y = \frac{8}{3}x - 8}$				$y = \frac{8}{7}x - 4$		
$\boldsymbol{y = 5x - 8}$				$y = -3x - 8$		
$\boldsymbol{y = \frac{7}{9}x + 4}$				$-10x + 6y = 4$		
$\boldsymbol{y = -3}$				$y = 15$		
$\boldsymbol{y = -\frac{3}{11}x - 4}$				$y = \frac{6}{11}x - 4$		
$\boldsymbol{7x - 3y = 7}$				$2x - 8y = 17$		
$\boldsymbol{y = \frac{2}{9}x - 4}$				$y = \frac{5}{2}x + 6$		
$\boldsymbol{5x - 3y = 7}$				$y = -.06x + 4$		
$\boldsymbol{4x + 7y = 19}$				$2x - 9y = 19$		
$\boldsymbol{x = -3}$				$x = 7$		

*Time to complete:*_____

Section 3-5 Answers

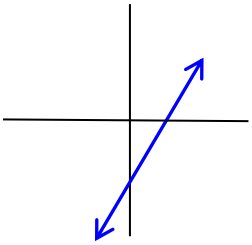
1.

x	y
5	$-\frac{1}{5}$
-4	$-\frac{19}{5}$
13	3
$\frac{11}{2}$	0
23	7

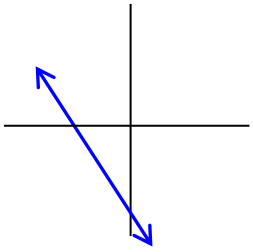
2.

x	y
2	**13**
0	**6**
-1	$\frac{5}{2}$
$-\frac{12}{7}$	0
$-\frac{4}{7}$	4

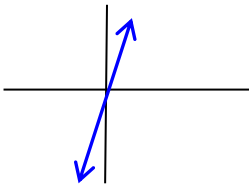
3. (0,-5) $(\frac{5}{2},0)$



4. (0,-6) $(-\frac{18}{5},0)$



5. (0,0) is both intercepts.
Graph any other point.
(1,5)



6. $m = \frac{5}{4}$

7. $m = -\frac{5}{16}$

8. $m = \frac{7}{8}$

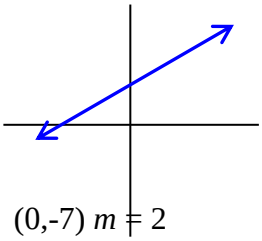
9. m = undefined

10. $m = \frac{2}{5}$

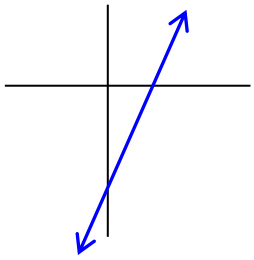
11. $m = 0$

12. Undefined is vertical
0 is horizontal

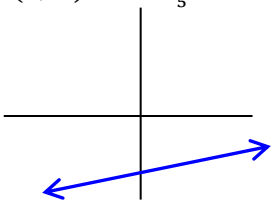
13. $(0,\frac{5}{2})$ $m = \frac{3}{4}$



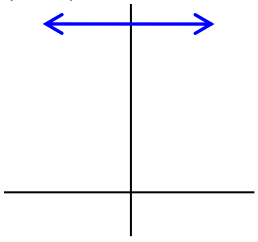
14. (0,-7) $m = 2$



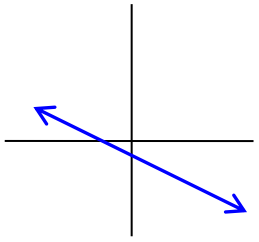
15. (0,-4) $m = \frac{2}{5}$



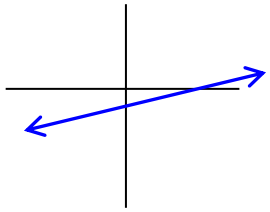
16. (0,17) m = 0



17. (0,-2) $m = -\frac{3}{7}$



18. (0, -2) $m = \frac{1}{3}$



19. Correct it in class.

20. $y = -3x - 6$

21. $y = \frac{5}{8}x + \frac{33}{8}$

22. $y = -3$

23. $x = 2$

24. $y = -\frac{4}{5}x + 1$

25. $y = 2x + 5$

26. $y = 3x - 5$
$y = 22$ (9,22)

27. $y = 13.5x + 86$
302 transplants $(16, 302)$

Chapter 3 Review Exercises

1. Create a visual chart of all of the methods, formulas, and examples of finding points and intercepts, graphing lines, finding slope, and writing equations of lines.

Fill out the table for each of the following.

3-1

2. $2x + 3y = 4$

x	y
	4
	$\frac{10}{3}$
-1	
2	
	-2

3. $9x - 5y = -160$

x	y
-40	
	32
	71.78
90	
	212

Find the *x*-intercept and *y*-intercept of each of the lines.

3-2

4. $y = .25x - 4$

5. $7x - 2y = -3$

6. $x - y = -2$

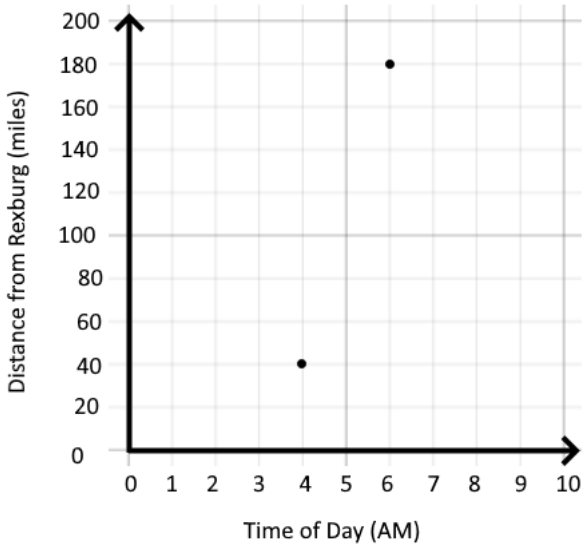
Graph each line and find the slope.

7. $y = -3$

8. $2x + 5 = 12$

3-3

9. The following graph shows two points for an early morning car trip from Rexburg to Salt Lake. Using the two points, find the average rate of speed of the car.



Find the slope between each pair of points.

10. $\left(-3, \frac{2}{3}\right)$ $\left(0, \frac{2}{3}\right)$

11. (2,-3) (-6,-7)

Find the slope and y-intercept for each line. Then graph the line.

3-4

12. $y = \frac{4}{3}x + 1$

13. $y = \frac{-7}{6}x - 2$

14. $4x - 3y = 2$

15. $5x - 9y = -18$

16. $y = 3x + 4$

17. $y = \frac{-9}{10}x - 1$

Write the equation of the line with the given characteristics.

3-5

18. $m = \frac{5}{6}$, $goes\ through\ (1,3)$

19. $m = -5, goes\ through\ (2,8)$

20. $m\ =\ 0, goes\ through\ (2,-4)$

21. $Parallel\ to\ y = -2x + 11,$ $goes\ through\ \ (-4,2)$

22. $Perpendicular\ to\ y = \frac{3}{2}x - 11,$ $goes\ through\ (7,8)$

23. $vertical\ line\ going\ through\ \ (4,2)$

24. $contains\ (0,7)\ and\ (3,8)$

25. $contains (8,3)\ \ and\ \ (-6,3)$

26. Write the equation of the line that represents the target heart rate (y) when compared to age (x) given the points (12, 156) and (48, 129). Then use that line to predict the target heart rate for someone age 72.

Chapter 3 Review Answers

1. Make it good.

2.

x	y
-4	4
-3	$\frac{10}{3}$
-1	**2**
2	**0**
5	-2

3.

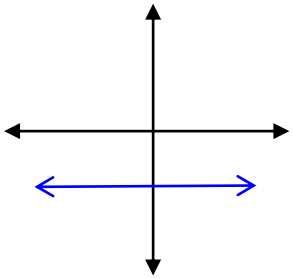
x	y
-40	**-40**
0	32
22.1	71.78
90	**194**
100	212

4. $(16,0)\ (0,-4)$

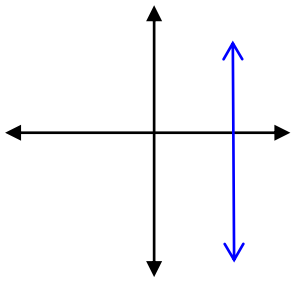
5. $\left(-\frac{3}{7},0\right)\ \left(0,\frac{3}{2}\right)$

6. $(-2,0)\ (0,2)$

7. $m = 0$



8. $m\ is\ undefined$

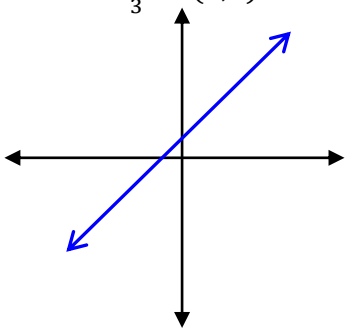


9. $70\ mph$

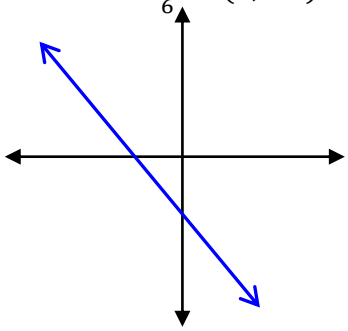
10. $m = 0$

11. $m = \frac{1}{2}$

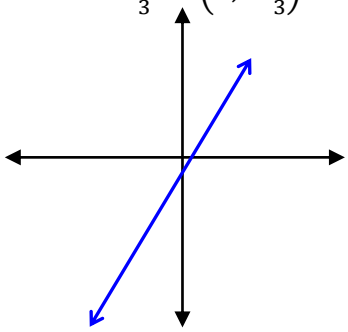
12. $m = \frac{4}{3}$ $(0,1)$



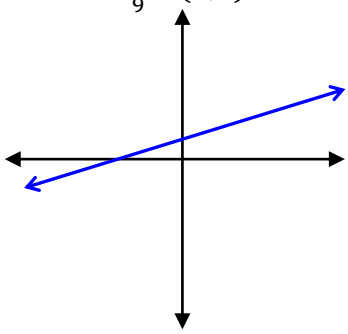
13. $m = -\frac{7}{6}$ $(0,-2)$



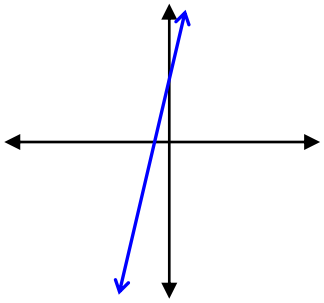
14. $m = \frac{4}{3}$ $\left(0,-\frac{2}{3}\right)$



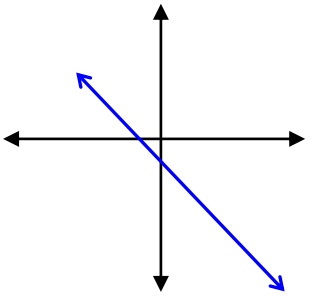
15. $m = \frac{5}{9}$ $(0,2)$



16. m = 3 (0,4)



17. $m = -\frac{9}{10}$ $(0,-1)$



18. $y = \frac{5}{6}x + \frac{13}{6}$

19. $y = -5x + 18$

20. $y = -4$

21. $y = -2x - 6$

22. $y = -\frac{2}{3}x + \frac{38}{3}$

23. $x = 4$

24. $y = \frac{1}{3}x + 7$

25. $y = 3$

26. $y = -\frac{3}{4}x + 165$

$111\ beats\ per\ minute$

Section 4-1 Exponents and Rules

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

4-1: Exponents and Rules	Scientific Notation
4-2: Polynomial Intro	Term, Coefficient, Degree,
Polynomial Addition	Polynomial Subtraction
4-3: Polynomial Multiplication	
4-4: Polynomial Division	

Exponent Review

Main Topics	Examples
EXPONENTS	2^4 ← The little number up high is the exponent. The bigger portion down low is the base. The base can consist of numbers and/or variables.

Example: $\underbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}_{7} = 128$ is the same as $2^7 = 128$

Example 12: Evaluate 7^4

$7^4 = 7 \times 7 \times 7 \times 7$ $= 49 \times 7 \times 7$ $= 343 \times 7$ $= 2401$ **Answer: 2401**	Set up the bases, and then multiply each couple in turn

Example: We know that $2^7 = 128$, so to work it backwards, we say:

ROOTS

$2 = \sqrt[7]{128}$ or if 128 was cut into 7 equal parts multiplied together, then those parts would all be 2's.

Example 13: Evaluate $\sqrt{196}$

$\sqrt{196}$	Either recognize that $14 \times 14 = 196$ or
Answer: 14	find the $\sqrt{x}$ button on your calculator

Exponents

Rule	Examples
One Anything to the power of one = itself $a^1 = a$	$7^1 = 7$ $x^1 = x$
Multiplication When expressions with the same base are multiplied, the exponents add. $a^m \cdot a^n = a^{m+n}$	$x^2 \cdot x^3 = x^5$ $x^3 x^7 = x^{10}$
Power When we raise a power to a power, the exponents multiply. $(a^m)^n = a^{m \cdot n}$	$(x^2)^3 = x^6$
Division Exponents being divided with the same base are subtracted. (if a ≠ 0) $\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^5}{x^3} = x^{5-3} = x^2$
Negative All negative exponents can be converted to a positive exponent by simply taking the reciprocal. (if a ≠ 0) $a^{-m} = \frac{1}{a^m}$ AND $\frac{1}{a^{-m}} = a^m$	$3^{-4} = \frac{1}{3^4} = \frac{1}{81}$ $4x^{-3} = \frac{4}{x^3}$ $\frac{7}{y^{-4}} = \frac{7}{1/y^4} = 7y^4$
Zero exponent Anything to the power of zero = 1 (if a ≠ 0) $a^0 = 1$	$7^0 = 1$ $\frac{x^3}{x^3} = x^{3-3} = x^0 = 1$
Parentheses Exponent applies to everything inside $(ab)^n = a^n b^n \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$(2x^4)^5 = 32x^{20}$ $\left(\frac{3}{y}\right)^4 = \frac{81}{y^4}$

Examples:

Using the Laws of Exponents, simplify the following:

PROBLEM	**SOLUTION**	**LAW(S) USED**
5^1	5	One Rule
x^1	x	One Rule
$(2x)^0$	1	Zero Rule
$8^3 \cdot 8^{19}$	$8^{3+19} = 8^{22}$	Multiplication Rule
$x^2 \cdot x^7$	$x^{2+7} = x^9$	Multiplication Rule
4^{-2}	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$	Negative Exponent Rule
$d^{-3} \cdot d^8$	$d^{-3+8} = d^5$ OR $\frac{d^8}{d^3} = d^{8-3} = d^5$	Multiplication Rule or Negative Exponent and Division Rules
$\frac{x^{16}}{x^{16}}$	$x^{16-16} = x^0 = 1$	Division & Zero Rules
$\frac{b^7}{b^5}$	$b^{7-5} = b^2$	Division Rule
$\frac{p^{-2}}{p^{-5}}$	$p^{-2-5} = p^3$ OR $\frac{p^5}{p^2} = p^{5-2} = p^3$	Division Rule or Negative Exponent Rule, then Division Rule
$\frac{x^2}{x^3}$	$\frac{1}{x^{3-2}} = \frac{1}{x^1} = \frac{1}{x}$ OR $x^{2-3} = x^{-1} = \frac{1}{x}$	Division & One Rule or Division, Negative Exponent, & One Rules
$(3x^3)^2$	$3^{1\cdot2}x^{3\cdot2} = 3^2x^6 = 9x^6$	Parentheses & Power Rule
$(2x^2y^3z^4)^3$	$2^{1\cdot3}x^{2\cdot3}y^{3\cdot3}z^{4\cdot3} =$ $2^3x^6y^9z^{12} = 8x^6y^9z^{12}$	Parentheses & Power Rule
$(5x^2y^{-3})^2$	$5^{1\cdot2}x^{2\cdot2}y^{-3\cdot2} =$ $5^2x^4y^{-6} = \frac{25x^4}{y^6}$	Power Rule & Negative Exponent Rule

Scientific Notation

Main Topics	Examples
Sci. Not. ⇒ Decimal Multiply or divide by 10.	**Example 1:** $2.53\ X\ 10^{8}$ $move\ decimal\ 8\ places$ 253,000,000 **Example 2:** $4.6\ X\ 10^{-7}\ move\ 7\ places\ other\ direction$ .00000046
Standard Scientific Notation leaves one digit left of the decimal.	All these are the same number: $253\ X\ 10^{6}$ $25.3\ X\ 10^{7}$ $2.53X\ 10^{8}$ $.253\ X\ 10^{9}$ Note that you can always move the decimal ***left*** if you move the exponent ***up*** and the number will remain unchanged.
Multiplying and Dividing Scientific Notation by hand • Powers of 10 follow the rules of exponents.	**Example 3:** $3.1 \times 10^{4} \cdot 4 \times 10^{8}$ $= 12.4 \times 10^{12}$ Add exponents for powers of 10 $= 1.24 \times 10^{13}$ Move decimal left for standard notation **Example 4:** $\frac{7 \times 10^{4}}{3.5 \times 10^{8}}$ $= 2 \times 10^{-4}$ Subtract exponents for powers of 10
Calculator Common buttons for scientific notation are • × 10^ • EE	Become familiar with how your calculator displays and inputs numbers in scientific notation. Most calculators will allow you to change the mode, so that all answers, even something like $3 \times 5 = 15$, will be given in scientific notation: $3 \times 5 = 1.5 \times 10^{1}$.

Section 4-1 Exercises

Find the equation of the given line.

3-5

1. $m = -\frac{3}{4}$ and goes through (0,2) **2.** $m = \frac{3}{5}$ and goes through (2,5)

Use positive exponents to express the following.

4-1

3. $\frac{1}{k^{-9}}$ **4.** $a^5b^{-3}c^{-2}$ **5.** $\frac{z^{-1}}{x^{-3}y^{-12}}$

Use negative exponents to express the following.

6. $\frac{1}{x^2}$ **7.** $\frac{3}{B^5}$ **8.** $\frac{1}{4^2}$

Simplify each expression.

9. $t^4 \cdot t^5$ **10.** $\frac{p^8}{p^3}$ **11.** x^9x^{-2}

12. $(y^5)^6$ **13.** $(5r^3)^2$ **14.** $2a^4 \cdot 9a^2$

15. $(g^{-8})^7$ **16.** $\frac{Q^{-8}}{Q^{-12}}$ **17.** $\frac{m^5}{m^{-2}}$

18. $\frac{8x^7}{2x^{10}}$ **19.** $\frac{8x^{10}}{2x^7}$ **20.** $3^5 \cdot 3^3$

21. $\left(\frac{2}{c^8}\right)^{-3}$ **22.** $\left(\frac{m^4}{n^{-3}}\right)^2$ **23.** $\left(\frac{6k^{-5}j^3}{k^5j^2}\right)^{-4}$

24. 5^{-2} **25.** $\frac{17^{38}}{17^{40}}$ **26.** $\left(\frac{5x^3y^8}{x^{14}y^{-3}}\right)^{-2}$

Convert into decimal notation.

27. 5.30×10^9 **28.** 3.14×10^{-11}

Convert into scientific notation.

29. 2,000,000 **30.** .000082

Multiply. Write the answer in scientific notation.

31. $4.2 \times 10^{12} \cdot 1.8 \times 10^{-4}$ **32.** $3.02 \times 10^{-8} \cdot 7.3 \times 10^{-5}$

Divide. Write the answer in scientific notation.

33. $\frac{8 \times 10^7}{4 \times 10^5}$ **34.** $\frac{5.7 \times 10^{-7}}{3 \times 10^4}$ **35.** $\frac{2.7 \times 10^{13}}{9 \times 10^{-5}}$

Preparation

36. After reading some of 4.2, classify each of the following as a monomial, binomial, trinomial.

a) $x + 3m$ **b)** $x + 2y + z$ **c)** $5x^2yz$

Section 4-1 Answers

1. $y = -\frac{3}{4}x + 2$ or 3x + 4y = 8

2. $y = \frac{3}{5}x + \frac{19}{5}$ or 3x – 5y = -19

3. k^9

4. $\frac{a^5}{b^3c^2}$

5. $\frac{x^3y^{12}}{z}$

6. x^{-2}

7. $3B^{-5}$

8. 4^{-2}

9. t^9

10. p^5

11. x^7

12. y^{30}

13. $25r^6$

14. $18a^6$

15. $\frac{1}{g^{56}}$

16. Q^4

17. m^7

18. $\frac{4}{x^3}$

19. $4x^3$

20. 3^8 *or* 6561

21. $\frac{c^{24}}{8}$

22. m^8n^6

23. $\frac{k^{40}}{1296j^4}$ *or* $\frac{k^{40}}{6^4j^4}$

24. $\frac{1}{25}$ or .04

25. $\frac{1}{289}$

26. $\frac{x^{22}}{25y^{22}}$

27. 5,300,000,000

28. .000 000 000 031 4

29. 2×10^6

30. 8.2×10^{-5}

31. 7.56×10^8

32. 2.2046×10^{-12}

33. 2×10^2

34. 1.9×10^{-11}

35. 3×10^{17}

36. In class.

Section 4-2 Introduction to Polynomials, Add and Subtract

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

4-1: Exponents and Rules	Scientific Notation
4-2: Polynomial Intro	Term, Coefficient, Degree,
Polynomial Addition	Polynomial Subtraction
4-3: Polynomial Multiplication	
4-4: Polynomial Division	

Polynomials and Terms

Main Topics	Examples
Polynomial	"Poly" means many, and "nomial" means number or term, so a polynomial is something with many terms.
Term	Terms are separated by plus (+) and minus (-) signs. Hence $7x - 5y$ has two terms, and $x^2 + 5x - 6$ has three terms The sign before the term always goes with it. Hence the two terms in $7x - 5y$ are $7x$ and -5, and the three terms in $x^2 + 5x - 6$ are x^2 , $5x$, and -6.

The family of polynomials includes the monomials, binomials, and trinomials. Monomials, binomials, and trinomials are all polynomials with either one, two, or three terms respectively:

MONOMIAL	BINOMIAL	TRINOMIAL
Mono = One Nomial = Term(s) "One Term"	Bi = Two Nomial = Term(s) "Two Terms"	Tri = Three Nomial = Term(s) "Three Terms"

Polynomials

Main Topics	Examples
Polynomial Parts Coefficient: Number in each term. Degree of Term: Exponent (total of exponents) on variable of the term.	**Example 1:** **List the terms, coefficients and degrees of each term. Then find the degree of the polynomial.** $-3x^5y^2 + 2x^4 + 5$ $Three\ terms\ =\ trinomial$ Leading term: $-3x^5y^2$ • Leading coefficient: -3 • Degree: 7

Degree of Polynomial: Largest degree of all terms.	Second term: $2x^4$ • Coefficient: 2 • Degree: 4 Last term (Constant term): 5 • Coefficient: 5 • Degree: 0 Degree of polynomial: 7

Polynomial Etiquette: Descending Order

Main Topics	Examples
Descending Order largest to smallest degree.	It is common practice, though not right or wrong, to write all answers to polynomial problems in descending order. **Example 2:** **Write in descending order.** $4x^5 - 3x - 10 + 9xy^2 - 7xy + 2x^7$ **Answer:** $2x^7 + 4x^5 + 9xy^2 - 7xy - 3x - 10$ **Example 3:** **Write in descending order.** $2x^3 - 12x + 9x^4 - 7$ **Answer:** $9x^4 + 2x^3 - 12x - 7$

Evaluating Polynomials

Main Topics	Examples
Everywhere there's an x you'll **substitute** in a -1… You'll now **simplify** by applying the order of operations:	**Example 4:** Evaluate the polynomial $2x^2 - 3x + 5$, when $x = -1$ $2x^2 - 3x + 5$ $2(-1)^2 - 3(-1) + 5$ $2(1) - 3(-1) + 5$ $2 + 3 + 5$ 10

Example 5:

Evaluate the polynomial $\boldsymbol{x^2 - 2xy + 3y^2}$, when $\boldsymbol{x = 2}$ and $\boldsymbol{y = -1}$

$$x^2 - 2xy + 3y^2$$
$$(2)^2 - 2(2)(-1) + 3(-1)^2$$
$$4 - 2(2)(-1) + 3(1)$$
$$4 + 4 + 3$$
$$11$$

LIKE TERMS

Example 6:

Simplify: $\boldsymbol{5x^2 + 11x - 7 - 4x + 12 + 3x^2}$

Identify.

$$\boxed{5x^2} + \bigcirc{11x} - 7 - \bigcirc{4x} + 12 + \boxed{3x^2}$$

Collect.

$$\boxed{5x^2} + \boxed{3x^2} + \bigcirc{11x} - \bigcirc{4x} - 7 + 12$$

Combine.

$$8x^2 + 7x + 5$$

Example 7:

Simplify: $\boldsymbol{2xy^2 - 5xy + 3y^2 + 7xy^2 + 8y^2 - 2xy}$

$$\boxed{2xy^2} - 5xy + \bigcirc{3y^2} + \boxed{7xy^2} + \bigcirc{8y^2} - 2xy$$

$$\boxed{2xy^2} + \boxed{7xy^2} - 5xy - 2xy + \bigcirc{3y^2} + \bigcirc{8y^2}$$

$$9xy^2 - 7xy + 11y^2$$

Polynomial Addition and Subtraction

Main Topics	Examples
Addition Combine like terms.	**Example 8**: $(\boldsymbol{5x^4 - 3x^2 + 10}) + (\boldsymbol{9x^4 + 4x^2 - 3})$ $5x^4 - 3x^2 + 10 + 9x^4 + 4x^2 - 3$ $5x^4 \boxed{- 3x^2} \bigcirc{+ 10} + 9x^4 \boxed{+ 4x^2} \bigcirc{- 3}$ $14x^4 + x^2 + 7$

	Example 9: $(3q^2 + 7q^3 - q) + (8q - 6q^3)$ $q^3 + 3q^2 + 7q$
Subtraction Distribute the negative. Combine like terms.	**Example 10:** $(\mathbf{5x^4 - 3x^2 - 10}) - (\mathbf{9x^4 + 4x^2 - 3})$ $5x^4 - 3x^2 - 10 - 9x^4 - 4x^2 + 3$ $5x^4 - 3x^2 - 10 - 9x^4 - 4x^2 + 3$ $-4x^4 - 7x^2 - 7$ **Example 11:** $(5p^3 - 7p + 2) - (5p - 9p^3 - 3)$ $5p^3 - 7p + 2 - 5p + 9p^3 + 3$ $14p^3 - 12p + 5$
Common Mistake Confusing the rules of exponents with combing like terms.	**Example 12:** $3x^6 + 5x^6 = 8x^6$ Correct. *not* ~~$3x^6 + 5x^6 = 8x^{12}$~~ Incorrect.

Section 4-2 Exercises

Simplify.

4-1

1. $(5a)^{-3}$ **2.** $(2x^{-3}y^{-8})^{6}$ **3.** $\left(\frac{9k^{-5}k^{3}}{h^{7}}\right)^{-2}$

4. Express in scientific notation: 15,966,000,000,000

5. Express in decimal form: 2.97×10^{-9}

Perform the indicated operation.

6. $5.3 \times 10^{-4} \cdot 1.01 \times 10^{8}$ **7.** $\frac{1.8\times10^{-8}}{6\times10^{-6}}$ **8.** $\frac{9.9\times10^{-7}}{3.3\times10^{4}}$

Classify the following as monomials, binomials, trinomials.(If none of them, write "polynomial")

4-2

9. $x^2 - 10x + 25$ **10.** $18a^4b^3yz^9$

11. $-2a(b^6c) - xy$ **12.** $a^4 + a^2b^2 + ab^2 - b^3 + 5$

Identify each term. Name the coefficient and degree of each term, as well as the degree of the polynomial.

13. 9 + 3k **14.** $x^2 + 8x + 16$ **15.** $13x^3 + x^2 + 5x + 3$ **16.** $-14b - 2b^7 + 3$

Write in descending order.

17. $s + 7 + 3s^2$ **18.** $3x^2 + 5 + x^4 + 2x^3 + 4x$

Evaluate.

19. $x^2 - 10x + 25$
when $x = 4$

20. $3a^4 + 4a^2 - 10a - 19$
when $a = -5$

21. $a^4 + a^2b^2 + ab^2 - b^3 + 5$
when $a = -1$ and $b = 3$

22. $x^3 - 3x^2y + xy^2 + y$
when $x = 2$ and $y = -4$

Simplify. Write your answer in descending order.

23. $5x^2 + 3x + x - 9$ **24.** $4x - 6x^5 + 17x + 15x^5 + 3x - x^3$

25. $b^{19} - 4b^{14} + 5b^{20} - 2b^{14}$ **26.** $(4k - 12k) + 5k^2 - 4$

27. $-x^8 - 5xy + 4xy^2 - 9x^3y + x^8$

28. $\frac{7}{2}y^2 + x^4 - \frac{3}{2}y^2 + \frac{1}{3}x^3 + 7y$

Add the polynomials.

29. $(3x - 2) + (x + 5)$

30. $(4a) + (2a - 5)$

31. $(-7x^2 + 5y - 17) + (3x^2 - 4x + 12y)$

32. $(5x^4 - x^3 + 3x^2 - 5) + (4x^4 + 4x^3 + x)$

Subtract the polynomials.

33. $(5x + 2) - (4x + 3)$

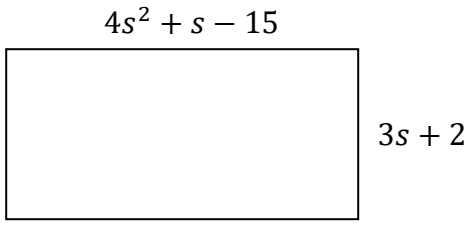
34. $(3x^2 - x + 7) - (9x^2 + x + 8)$

35. $(4y^7 + x^2 + 6y) - (6y^7 - 5y^5 + 11x^2 - y + 17)$

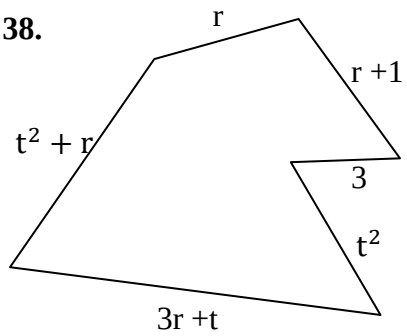
36. $(6a^3 - b^3 + b^2) - (-a^3 + 2b^3 - b^2)$

Find a polynomial that describes the perimeter of these shapes.

37.



38.



Preparation:

1-3

39. Match the following three equations with the property that is being used.

a. $9(x - 2) = 9x - 18$	Commutative Addition
b. $3ab + 4t = 4t + 3ab$	Associative Multiplication
c. $2(x5) = (2x)5$	Distributive

Section 4-2 Answers

1. $\frac{1}{125a^3}$

2. $\frac{64}{x^{18}y^{48}}$

3. $\frac{h^{14}k^4}{81}$

4. 1.5966×10^{13}

5. .00000000297

6. 5.353×10^4

7. 3×10^{-3}

8. 3×10^{-11}

9. Trinomial

10. Monomial

11. Binomial

12. Polynomial

13. 9: coefficient = 9, degree = 0

3*k*: coefficient = 3, degree = 1

degree of polynomial = 1

14. x^2: c = 1, d = 2

8*x*: c = 8, d = 1

16: c =16, d = 0

degree of polynomial = 2

15. $13x^3$: c = 13, d = 3

x^2: c = 1, d = 2

5*x*: c = 5, d = 1

3: c = 3, d = 0

degree of polynomial = 3

16. $-2b^7$: c = –2, d = 7

$-14b$: c = –14, d = 1

3: c = 3, d = 0

degree of polynomial = 7

17. $3s^2 + s + 7$

18. $x^4 + 2x^3 + 3x^2 + 4x + 5$

19. 1

20. 2006

21. -21

22. 84

23. $5x^2 + 4x - 9$

24. $9x^5 - x^3 + 24x$

25. $5b^{20} + b^{19} - 6b^{14}$

26. $5k^2 - 8k - 4$

27. $-9x^3y + 4xy^2 - 5xy$

28. $x^4 + \frac{1}{3}x^3 + 2y^2 + 7y$

29. $4x + 3$

30. $6a - 5$

31. $-4x^2 - 4x + 17y - 17$

32. $9x^4 + 3x^3 + 3x^2 + x - 5$

33. $x - 1$

34. $-6x^2 - 2x - 1$

35. $-2y^7 + 5y^5 - 10x^2 + 7y - 17$

36. $7a^3 - 3b^3 + 2b^2$

37. $8s^2 + 8s - 26$

38. $2t^2 + t + 6r + 4$

39. In class.

Section 4-3 Polynomial Multiplication

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

4-1: Exponents and Rules	Scientific Notation
4-2: Polynomial Intro	Term, Coefficient, Degree,
Polynomial Addition	Polynomial Subtraction
4-3: Polynomial Multiplication	
4-4: Polynomial Division	

Type 1: Monomials × Monomials

Main Topics	Examples
Steps 1. Multiply numbers with numbers (coefficients). 2. Multiply like variables with like variables (x's with x's, y's with y's).	**Example 1:** Multiply $(4x)(5x)$ $4 \cdot x \cdot 5 \cdot x$ $4 \cdot 5 \cdot x \cdot x$ $20x^2$ **Example 2:** Multiply $(-4x^5)(6x^2)$ $(-4)(6)\ \ x^5 \cdot x^2$ $-24x^{5+2}$ $-24x^7$ **Answer:** $\boldsymbol{-24x^7}$ **Example 3:** Multiply $(4xy^2)(-2x^3y)$ $(4)(-2)\ \ x \cdot x^3\ \ y^2 \cdot y$ $-8x^{1+3}y^{2+1}$ $-8x^4y^3$ **Answer:** $\boldsymbol{-8x^4y^3}$ **Example 4:** Multiply $(-2y^2)(6y^3)(-4y^5)$ $(-2)(6)(-4)\ \ y^2 \cdot y^3 \cdot y^5$ $48y^{2+3+5}$ $48y^{10}$ **Answer:** $\boldsymbol{48y^{10}}$

Type 2: Monomials × Polynomial

Main Topics	Examples

Steps:

1. Multiply the monomial (the distributor) times each term in the polynomial.
2. Format it such that you have: (**mono**)×(**term**) + (**mono**)×(**term**) + … keeping the sign of each term with it in the parentheses.
3. Combine like terms and simplify.

Example 5: Multiply $3x(5x-4)$

$3x(5x-4)$	Distribute the monomial to the polynomial.
$(3x)(5x)+(3x)(-4)$	Multiply $3x$ to each term in the polynomial.
$3\cdot 5\ x\cdot x+3(-4)\ x$	Now handle each (mono)×(term). Simplify if needed.
$15x^2-12x$	
Answer: $15x^2-12x$	

Example 6: Multiply $-4x(-x-4)$

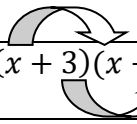
$-4x(-x-4)$	Distribute the $(-4x)$, not just the $(4x)$.
$(-4x)(-x)+(-4x)(-4)$	
$(-4)(-1)x\cdot x+(-4)(-4)\ x$	Multiply numbers times numbers and like terms times like terms.
$4x^2+16x$	
Answer: $4x^2+16x$	

Example 7: Multiply $2x^2(3x^2-6x+8)$

$2x^2(3x^2-6x+8)$	mono × (any polynomial)
$(2x^2)(3x^2)+(2x^2)(-6x)+(2x^2)(8)$	
$(2)(3)\ x^2\cdot x^2+(2)(-6)\ x^2\cdot x+(2)(8)\ x^2$	Multiply numbers times numbers and like terms times like terms.
$6x^4-12x^3+16x^2$	
Answer: $6x^4-12x^3+16x^2$	

Type 3: Binomials × Binomials

Main Topics	Examples
Steps: 1. SUPER Distribute.	**Example 8:** Multiply $(x+3)(x-7)$



$(x+3)(x-7)$	binomial × binomial
$x(x-7)+3(x-7)$	Multiply each term in the 1st set of parentheses by each term in the 2nd set of parentheses.
$(x)(x)+(x)(-7)+(3)(x)+(3)(-7)$	Format as (mono)×(term) + (mono)×(term) + …
$x^2-7x+3x-21$	Multiply each term.
$x^2-4x-21$	Combine like terms to simplify.

Example 9: Multiply $(4x-2)(2x-3)$

$(4x-2)(2x-3)$	
$4x(2x-3)-2(2x-3)$	Everything in first set of parentheses goes to everything in the second set
$8x^2-12x-4x+6$	Simplify
$8x^2-16x+6$	
Answer: $8x^2-16x+6$	

Extra Information

FOIL

The F.O.I.L. acronym is a mnemonic device (tool for memorization).
It is no different than the rule: **everything in the 1st parentheses goes to everything in the 2nd parentheses** where

$$(a+b)(c+d)=a(c+d)+b(c+d)=\ ac+ad+bc+bd$$

F = First, O = Outside, I = Inside, L = Last

First terms $=a\cdot c$, Outside terms $=a\cdot d$, Inside terms $=\ b\cdot c$, and Last terms $=b\cdot d$

Hence, $ac+ad+bc+bd$.

BOX

Polynomial multiplication can also be visualized by a **box** with one polynomial on one side and the other on another side. The area is the multiplication.

Example 9 (again):

	$4x$	-2
$2x$	$8x^2$	$-4x$
-3	$-12x$	$+6$

And combine like terms to get $\mathbf{8x^2-16x+6}$.

Type 4: Any Polynomial × Any Polynomial

Main Topics	Examples
SUPER Distribute	**Example 10:** Multiply

$$(x-2)(x^2-5x+6)$$
$$x(x^2-5x+6) \quad -2(x^2-5x+6)$$
$$(x)(x^2)+(x)(-5x)+(x)(6)+(-2)(x^2)+(-2)(-5x)+(-2)(6)$$
$$x^3-5x^2+6x-2x^2+10x-12$$
$$x^3-7x^2+16x-12$$

Answer: $\boldsymbol{x^3-7x^2+16x-12}$

Everything in the **1st ( )** goes to
Everything in the **2nd ( )**
Follow (**mono**)×(**term**)
+…format
Multiply the (**mono**)×(**mono**)'s
Use symbol method for like terms
Combine like terms

Slick Note: You could stack them so the like terms are above each other:

$$\begin{array}{r} 2x^4+10x^3-2x^2 \\ -5x^3-25x^2+5x \\ +x^2+5x-1 \\ \hline 2x^4+5x^3-26x^2+10x-1 \end{array}$$

Example 11:

$$(2x^2-5x+1)(x^2+5x-1)$$
$$(2x^2-5x+1)(x^2+5x-1)$$
$$2x^2(x^2+5x-1)-5x(x^2+5x-1)+1(x^2+5x-1)$$
$$2x^4+10x^3-2x^2 \quad -5x^3-25x^2+5x \quad +x^2+5x-1$$
$$2x^4+5x^3-26x^2+10x-1$$

Answer: $\boldsymbol{2x^4+5x^3-26x^2+10x-1}$

$(Binomial)^2$

Example 12:

$$(x+3)^2$$
$$(x+3)(x+3)$$
$$x^2+3x+3x+9$$
$$x^2+6x+9$$

Example 13:

$$(3x-y)^2$$
$$(3x-y)(3x-y)$$
$$9x^2-3xy-3xy+y^2$$
$$9x^2-6xy+y^2$$

Common Mistake This mistake is called the *Freshman's Dream*.	<u>People often take $(x+3)^2$ and incorrectly say the answer is x^2+9</u> 1. They apply the Law of Exponent Power Rule from Section 4.1 which states that everything in the parentheses receives the power. This is only true for monomials, and we have a binomial. 2. When they do this they just square the x and square the 3 to get $\boldsymbol{x^2+9}$ **---<u>an incorrect answer</u>** 3. It is critical to remember that anything being squared is really that thing times itself, i.e. $$(x+3)^2 = (x+3)(x+3)$$ Even the Power Rule is just a short cut around this fact, i.e. $(3x)^2 = (3x)(3x)$.

Section 4-3 Exercises

Simplify.

4-1

1. $\frac{kx^2gf^{-1}}{x^{-3}f^2}$ **2.** $\left(\frac{1}{4a^2b^{-3}}\right)^{-1}$

Perform the indicated operation. Write your answer in both scientific notation and decimal form.

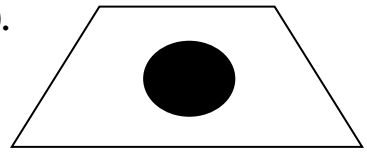
3. $1.23 \times 10^{-3} \cdot 4.36 \times 10^{4}$ **4.** $\frac{4.36\times10^{3}}{5.02\times10^{-2}}$ **5.** $\frac{6.02\times10^{23}}{4.8\times10^{25}}$

Identify each term. Name the coefficient and degree of each term, as well as the degree of the polynomial.

6. $4q^3 - 2q^2 + 3q - 2$ **7.** $3p^2 + 4p$ **8.** $3j^3 - 5$

The area of the black circle is $6y^2 - 2y + 3$. The area of the trapezoid is $-2y^2 + 4y + 11$. Find the area of the white section between the black circle and the outer trapezoid.

9.



Perform the indicated operations.

4-2

10. $-2x + (3x - 4)$ **11.** $-(3x + 4) - (2x^2 + x)$

12. $3z^3 - (2z^2 + 7z^3) + \frac{1}{2}z^2$ **13.** $3m(k^2 - 2m^2 + 1)$

4-3

14. $(3s - 1)(s + 4)$ **15.** $(s^2 - 2)(s + 2)$

16. $(a + b)(2a^2 + a - 3)$ **17.** $(c^2 - 2c + 1)(2c^2 + c - 3)$

Perform the indicated operations.

18. $(x - 3)(x + 3)$ **19.** $(2x + 1)(2x - 1)$

20. $(3mn - 1)(3mn + 1)$ **21.** $(3a + 4b)(3a - 4b)$

22. $(k^3 - 3)(k^3 + 3)$ **23.** $(3x + 1)^2$

24. $(2x - 1)^2$ **25.** $(k + 2)^2$

26. $(z^2 - 1)^2$ **27.** $(k^3 + 2m)^2$

Preparation: After reading some of 4.4, simplify the following.

28. $\frac{(8y^2+4)}{2}$ **29.** $(6x^3 - 2x^2 + x) \div x$

Section 4-3 Answers

1. $\frac{kx^5g}{f^3}$

2. $\frac{4a^2}{b^3}$

3. 5.3628×10^1, 53.628

4. 8.685×10^4, 86852.59

5. 1.254×10^{-2}, .01254

6. $4q^3$: coefficient = 4, degree = 3;
$-2q^2$: coefficient = -2, degree = 2
$3q$: coefficient = 3, degree = 1
-2: coefficient = -2, degree = 0
degree of polynomial = 3

7. $3p^2$: c = 3, d = 2
$4p$: c = 4, d = 1
degree of polynomial = 2

8. $3j^3$: c = 3, d = 3
-5: c = -5, d = 0
degree of polynomial = 3

9. $-8y^2 + 6y + 8$

10. $x - 4$

11. $-2x^2 - 4x - 4$

12. $-4z^3 - \frac{3}{2}z^2$

13. $-6m^3 + 3mk^2 + 3m$

14. $3s^2 + 11s - 4$

15. $s^3 + 2s^2 - 2s - 4$

16. $2a^3 + a^2 - 3a + 2a^2b + ab - 3b$

17. $2c^4 - 3c^3 - 3c^2 + 7c - 3$

18. $x^2 - 9$

19. $4x^2 - 1$

20. $9m^2n^2 - 1$

21. $9a^2 - 16b^2$

22. $k^6 - 9$

23. $9x^2 + 6x + 1$

24. $4x^2 - 4x + 1$

25. $k^2 + 4k + 4$

26. $z^4 - 2z^2 + 1$

27. $k^6 + 4k^3\,m + 4m^2$

28. In class.

29. In class.

Section 4-4 Polynomial Division

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

1: Exponents and Rules	Scientific Notation
2: Polynomial Intro	Term, Coefficient, Degree,
Polynomial Addition	Polynomial Subtraction
3: Polynomial Multiplication	
4: Polynomial Division	

Type 1: Polynomials ÷ Monomials

Main Topics

Steps:

1. Place the monomial under each term in the polynomial.
2. Simplify each new term of $\left(\frac{monomial}{monomial}\right)$ dividing numbers with numbers and like variables with like variables. Use the law of exponents.

Examples

Example 1: Divide $\frac{35x^5-20x^3}{5x^3}$

$\frac{35x^5}{5x^3}-\frac{20x^3}{5x^3}$	Place the monomial under each term in the polynomial
$\frac{35}{5}x^{5-3}-\frac{20}{5}x^{3-3}$	Simplify each new term. Divide numbers with numbers and like variables with like variables.
$7x^2-4x^0$	Using the quotient rule (4.1) we subtract top-bottom exponents
$7x^2-4$	The zero rule says $x^0=1$ and we know anything times 1 =’s itself
Answer: $7x^2-4$	

Example 2: Divide: $(24a^6-48a^5+10a^4)\div(4a^4)$

$\frac{24a^6}{4a^4}-\frac{48a^5}{4a^4}+\frac{10a^4}{4a^4}$	Place the monomial under each term in the polynomial.
$\frac{24}{4}a^{6-4}-\frac{48}{4}a^{5-4}+\frac{10}{4}a^{4-4}$	Simplify each new term.
$6a^2-12a^1+\frac{5}{2}a^0$	Reduce them as much as possible.
$6a^2-12a+\frac{5}{2}$	In simplifying, use the zero and one rule for exponents if needed.
Answer: $6a^2-12a+\frac{5}{2}$	

Example 3: Divide $\frac{15x^{10}y^7-8x^6y^3+18x^4y-3x^2y}{3x^2y}$

$\frac{15x^{10}y^7}{3x^2y}-\frac{8x^6y^3}{3x^2y}+\frac{18x^4y}{3x^2y}-\frac{3x^2y}{3x^2y}$	Place the monomial under each term in the polynomial.
$\frac{15}{3}x^{10-2}y^{7-1}-\frac{8}{3}x^{6-2}y^{3-1}+\frac{18}{3}x^{4-2}y^{1-1}-\frac{3}{3}x^{2-2}y^{1-1}$	Simplify each new term.
$5x^8y^6-\frac{8}{3}x^4y^2+6x^2y^0-1x^0y^0$	Not all fractions reduce completely
$5x^8y^6-\frac{8}{3}x^4y^2+6x^2-1$	Use the Zero and One Rule
Answer: $\mathbf{5x^8y^6-\frac{8}{3}x^4y^2+6x^2-1}$	

Long Division (for more than 1 term)

Main Topics	Examples
Long-hand	**Example 5:** Divide $(8x^3-34x^2+43x-77)\div(2x-7)$ $4x^2-3x+11$ $2x-7\ \overline{)\ 8x^3-34x^2+43x-77}$ $8x^3-28x^2$ $-6x^2+43x$ $-6x^2+21x$ $22x-77$ $22x-77$ 0 **Answer:** $\mathbf{4x^2-3x+11}$
Special Consideration Add in missing terms	**Example 6:** Divide $(12x^3-5x-33)\div(2x-3)$ $6x^2+9x+11$ $2x-3\ \overline{)\ 12x^3+0x^2-5x-33}$ $12x^3-18x^2$ $18x^2-5x$ $18x^2-27x$ $22x-33$ $22x-33$ 0 **Answer:** $\mathbf{6x^2+9x+11}$

Section 4-4 Exercises

Simplify.

4-1

1. $a^2e^3i^{-2}o^0u^{-2}a^3e^{-4}i^{-2}o^{-2}u^4$ **2.** $e^2i^{-3}e^4i^{-2}o^3$ **3.** $\frac{r^2a^{-3}ce^2}{c^4ar^{-1}}$

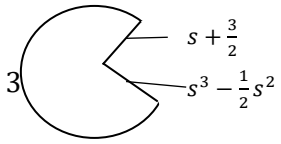
Simplify. Write your answer in descending order.

4. $\frac{3}{(s+1)^{-2}}$ **5.** $-j+4j^2-3j-1+\frac{1}{j^{-1}}$

Find a polynomial that describes the perimeter of the shape.

4-2

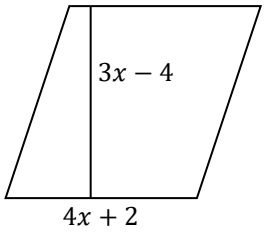
6.



Find a polynomial that describes the area of these shapes.

4-3

7.



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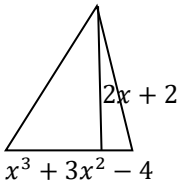
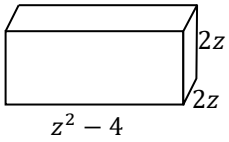


Find a polynomial that describes the volume of these shapes.

9.



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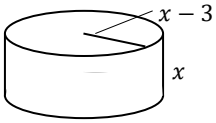


Perform the indicated operations

11. $4x+2x^2+x-2x^2+5x-3x^2$

12. $z^3+4z^2-12z-(-z^3+z-3z^2)$

13. $4y+3y^2x+y(4yx+5)$

14. $-12m(6m^3-8m^2+m-12)$

15. $8g(3h^3+9h-12)$

16. $(3p+4)(p-4)$

17. $(7a^2+3)(2a^2+3a+6)$

18. $\left(\frac{1}{2}b+1\right)\left(\frac{1}{2}b-1\right)$

19. $(8f^2+6)(8f^2-6)$

20. $(m^2n^2+1)(m^2n^2-1)$

21. $(7x+1)^2$

22. $(w^2x^2-y^2z^2)^2$

Perform the indicated operations.

4-4

23. $9m^4\div(3m)$

24. $(6x^2+12x)\div(3x)$

25. $(2x^2y+3xy+y^2-2)\div(2x)$

26. $(-56y^4+44y^3+64y^2-16y)\div(8y)$

27. $(4b^{13}-9b^8-3b^5)\div(3b^3)$

28. $(-6y^5-3y^3+y)\div(2y)$

29. $(18x^2-23x-6)\div(2x-3)$

30. $(18x^3-5x+2)\div(3x+2)$

Section 4-4 Answers

1. $\frac{a^5u^2}{ei^4o^2}$

2. $\frac{e^6o^3}{i^5}$

3. $\frac{r^3e^2}{a^4c^3}$

4. $3s^2 + 6s + 3$

5. $4j^2 - 3j - 1$

6. $s^3 - \frac{1}{2}s^2 + s + \frac{9}{2}$

7. $12x^2 - 10x - 8$

8. $x^4 + 4x^3 + 3x^2 - 4x - 4$

9. $4z^4 - 16z^2$

10. $\pi(x^3 - 6x^2 + 9x)$ *or*

$\pi x^3 - 6\pi x^2 + 9\pi x$

11. $-3x^2 + 10x$

12. $2z^3 + 7z^2 - 13z$

13. $7y^2x + 9y$

14. $-72m^4 + 96m^3 - 12m^2 + 144m$

15. $24gh^3 + 72gh - 96g$

16. $3p^2 - 8p - 16$

17. $14a^4 + 21a^3 + 48a^2 + 9a + 18$

18. $\frac{1}{4}b^2 - 1$

19. $64f^4 - 36$

20. $m^4n^4 - 1$

21. $49x^2 + 14x + 1$

22. $w^4x^4 - 2w^2x^2y^2z^2 + y^4z^4$

23. $3m^3$

24. $2x + 4$

25. $xy + \frac{3}{2}y + \frac{y^2}{2x} - \frac{1}{x}$

26. $-7y^3 + \frac{11}{2}y^2 + 8y - 2$

27. $\frac{4}{3}b^{10} - 3b^5 - b^2$

28. $-3y^4 - \frac{3}{2}y^2 + \frac{1}{2}$

29. $9x + 2$

30. $6x^2 - 4x + 1$

Chapter 4 Review Exercises

1. Create a visual chart of all the methods, formulas, and examples of how to work with exponents and polynomials.

Simplify or evaluate.

4-1

2. $3^2 \cdot 3^{-4}$

3. $\left(\frac{2}{3}\right)^2$

4. $\frac{m^{-2}a^2t^{-3}h^{-3}}{h^2a^{-7}p^2p^{-3}y^2}$

5. $\frac{g^4m^3}{(a^{-2}gm^2)^{-1}}$

6. $\left(\frac{4xy^3}{3x^2y^{-5}z^3}\right)^{-2}$

7. $\left(\frac{2a^{-5}b^4c^{-1}}{a^{-2}b^9}\right)^4$

Perform the operation and write your answer in scientific form. Round to three decimal places.

8. $\frac{6.3781\times10^3}{1.989\times10^{30}}$

9. $\frac{1.51\times10^{-7}}{5.002\times10^{-5}}$

Evaluate the polynomials at the given values.

4-2

10. $a^2 - 3a + 2$, for $a = 5$

11. $x^3 + 2x^2 - 4x$, for $x = -3$

Add or subtract the polynomials. Write answer in descending order.

12. $(-3y^2 + 7) + (y^2 + 2y - 6)$

13. $(4x^3 - 1 + x + 3x^2) + (x^2 - x + 5)$

14. $(3j^4 - j + 2j^2) - (-j^2 + 4j + j^4)$

15. $(-3p - 9p^2) - (-12p^2 - 5p + 4)$

Perform the indicated operations.

4-3

16. $(3x)(x - 5)$

17. $\left(-\frac{1}{2}a\right)(4a^2 + 6a - 2)$

18. $(5d^2 - 1)(3d + 1)$

19. $(6e + 4)(-5e + 3)$

20. $(g - 7)(g + 6)$

21. $(hi + 2)(i - 3h)$

22. $(j + 2k)(k^2 - 2j)$

23. $(2l - 3)(2l + 3)$

24. $(4m^3 - 2)(4m^3 + 2)$

25. $(n^6 + 3y^3)(n^6 - 3y^3)$

26. $(5p^2 + 1)^2$

27. $(-q + 2)^2$

28. $(2r + 2s)^2$

29. $(t + 3)(t^2 - 3t - 4)$

30. $(u^2 + u^3)(u^3 - u^2 + u - 1)$

31. $(2x^4 + 7x^3 - x)(x^2 + 3x + 2)$

Divide the polynomials.

4-4

32. $(-80w^6 + 35w^5 - 50w^4) \div (10w^4)$

33. $(33x^3 - 18x^2 + 3x) \div (3x)$

34. $(24y^3 - 2y^2) \div (2y)$

35. $(8abz^3 - 2jz^4 + z^5) \div (z^3)$

Chapter 4 Review Answers

1. Make it neat, thorough, and organized.

2. $\frac{1}{9}$

3. $\frac{4}{9}$

4. $\frac{a^9 p}{m^2 t^3 h^5 y^2}$

5. $\frac{g^5 m^5}{a^2}$

6. $\frac{9x^2 z^6}{16y^{16}}$

7. $\frac{16}{a^{12} b^{20} c^4}$

8. 3.207×10^{-27}

9. 3.019×10^{-3}

10. 12

11. 3

12. $-2y^2 + 2y + 1$

13. $4x^3 + 4x^2 + 4$

14. $2j^4 + 3j^2 - 5j$

15. $3p^2 + 2p - 4$

16. $3x^2 - 15x$

17. $-2a^3 - 3a^2 + a$

18. $15d^3 + 5d^2 - 3d - 1$

19. $-30e^2 - 2e + 12$

20. $g^2 - g - 42$

21. $hi^2 - 3h^2 i + 2i - 6h$

22. $2k^3 + jk^2 - 4jk - 2j^2$

23. $4l^2 - 9$

24. $16m^6 - 4$

25. $n^{12} - 9y^6$

26. $25p^4 + 10p^2 + 1$

27. $q^2 - 4q + 4$

28. $4r^2 + 8rs + 4s^2$

29. $t^3 - 13t - 12$

30. $u^6 - u^2$

31. $2x^6 + 13x^5 + 25x^4 + 13x^3 - 3x^2 - 2x$

32. $-8w^2 + \frac{7}{2}w - 5$

33. $11x^2 - 6x + 1$

34. $12y^2 - y$

35. $8ab - 2jz + z^2$

Section 5-1 Factoring: Method 1 and 2 (GCF and Grouping)

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

THE 5 METHODS of FACTORING	
Method	**Type of Polynomial used on:**
1: Greatest Common Factor (GCF)	All
2: Grouping	4 Terms
3: $ax^2 + bx + c$, where $a = 1$	Trinomials, where $a = 1$
4: $ax^2 + bx + c$, where $a \neq 1$ $ac - method$	Trinomials, where $a \neq 1$
5: Special Cases	Binomials & Perfect Squares

WHEN TO TRY EACH METHOD:

Polynomial	Factoring Methods Possible
$y^3 + 4y^2 + 2y + 8$	Method 1: Greatest Common Factor Method 2: Grouping
$x^2 + 6x + 9$	Method 1: Greatest Common Factor Method 3: $ax^2 + bx + c$, where $a = 1$
$16x^2 - 16x + 4$	Method 1: Greatest Common Factor Method 4: $ax^2 + bx + c$, where $a \neq 1$
$54x^3 - 6x$	Method 1: Greatest Common Factor Method 5: Special Cases (because it's a binomial)
$x^2 - 5x + 6$	Method 1: Greatest Common Factor Method 3: $ax^2 + bx + c$, where $a = 1$
$2c^3 + 8c^2 - 6c - 12$	Method 1: Greatest Common Factor Method 2: Grouping
$36a^2 - 25$	Method 1: Greatest Common Factor Method 5: Special Cases (because it's a binomial)
$35x^3 + 42x^2 - 14x - 77xy - 14y + 7$	Method 1: Greatest Common Factor (only method because it has 6 terms)
$6y^2 + 25y + 25$	Method 1: Greatest Common Factor Method 4: $ax^2 + bx + c$, where $a \neq 1$

PRIME: A polynomial that cannot be factored is called **prime.** (Just like a number that cannot be factored.)

Method 1 – Greatest Common Factor

Main Topics	Examples
Greatest Common Factor Largest number that will go into all terms Prime Can't be factored	**Example 1:** Identify the GCF of the following: $18, 30, 42$ Factors of 18: 1, 2, 3, 6, 9, 18 Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30 Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42 $\boldsymbol{GCF = 6}$ **Example 2:** Identify the GCF of the following: $10x^2y, 20x^2, 35x^2y^2$ Factors of 10: 1, 2, 5, 10 Factors of 20: 1, 2, 4, 5, 10, 20 Factors of 35: 1, 5, 7, 35 x^2 *is also common* $\boldsymbol{GCF = 5x^2}$ **Example 3:** These are prime polynomials. $2x + 7$ $4x - 19$ $9y^2 + 20$
Steps 1. Identify GCF. 2. Un-Distribute. a. Place GCF outside. b. Divide each term by GCF.	**Example 4:** Factor $3a - 6b$ GCF = 3 $3(a - 2b)$ **Example 5:** Factor $4x - 5xy$ GCF = x $x(4 - 5y)$ **Example 6:** Factor $15x^3 - 5x^2 + 30xy$ GCF = $5x$ $5x(3x^2 - x + 6y)$ **Example 7:** Factor $21x^2y + 35xy^2 - 7xy$ GCF = $7xy$ $7xy(3x + 5y - 1)$

Method 2 – Grouping – 4 terms

Main Topics	Examples
Steps 1. Chop the 4 terms in half Separate into two binomials. 2. Pull GCF out of each half. 3. Pull out identical pieces from both halves.	**Example 7:** Factor $x^3 + 4x^2 + 3x + 12$ $\underbrace{x^3 + 4x^2}\quad\underbrace{+3x + 12}$ $x^2(x+4) + 3(x+4)$ $(x+4)(x^2+3)$ **Example 8:** Factor $2x^3 + 10x^2 - 3x - 15$ $\underbrace{2x^3 + 10x^2}\quad\underbrace{-3x - 15}$ $2x^2(x+5) - 3(x+5)$ $(x+5)(2x^2-3)$ **Example 9:** Factor $7xy + 28x + y + 4$ $\underbrace{7xy + 28x}\quad\underbrace{+y + 4}$ $7x(y+4) + (y+4)$ $(y+4)(7x+1)$
Use Method 1 first if possible. Then use Grouping on remaining 4 terms.	**Example 10:** Factor $2x^3y + 16x^2y + 6xy + 48y$ GCF = $2y$ $2y(x^3 + 8x^2 + 3x + 24)$ $\underbrace{x^3 + 8x^2}\quad\underbrace{3x + 24}$ $x^2(x+8) + 3(x+8)$ $(x+8)(x^2+3)$ **Answer:** $\boldsymbol{2y(x+8)(x^2+3)}$

Section 5-1 Exercises

Perform the indicated operations.

4-3

1. $3a(4b^2 - a)$

2. $(x + 4)^2$

3. $(x + 1)(x^2 - 3x - 4)$

4. $(-x + 1)(3x - 7)$

4-4

5. $(-4m^3 - 16m^2 + 6m) \div (-2m)$

Identify which method(s) you should try in factoring the following polynomials (refer back to the beginning of section 5-1).

5-1

6. $3x^2 + x$

7. $x^2 - 4x + 4$

8. $5c^3 + 10c^2 - 2c - 6$

9. $14x^3 + 7x^2 - 21x$

10. $a^2b - bc + 3bc^2 - c^2$

11. $-4t^2 - 5tr$

Identify the greatest common factor between the terms.

12. $18, 24, 48$

13. $3x, 9x^2, 6$

14. $-14s^3, -7s$

15. $a^2b, -bc, 3bc^2$

16. $125x^3, 50x^2, 10x$

17. $y^4, 4y^3, 2y^2, 8y$

Factor the following by pulling out the greatest common factor if there is one. If not, the expression is prime.

18. $9k + 3$

19. $-18y^5 - 6y$

20. $2x^4 - 3y^2 + 7y$

21. $-z^2 - 7a - 2$

22. $-12a^3b + 8a^2b^2 - 16ab^2$

23. $19xy^2 - 38xy + 57y$

24. $2a^2x^4 + 6a^2x^3 - 10ax^3$

25. $-39s^5 - 18s^3 - 81s$

Factor the following by grouping if possible.

26. $3x^3 - 9x^2 + 4x - 12$

27. $-2x^3 - 2x^2 - 3x - 3$

28. $4x^3 - 20x^2 - 6x + 10$

29. $x^3 + 5x^2 - 2x - 10$

30. $a^2x + 5a^2 + bx + 5b$

31. $8x^3 + 18x^2 - 20xy - 45y$

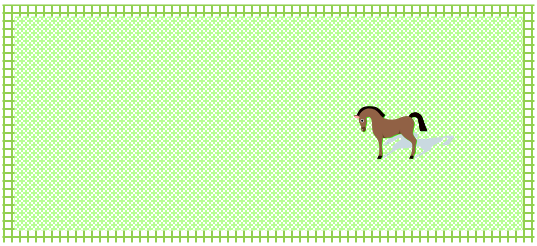
32. $x^3 + 3x^2 - 12x - 36$

33. $-4d^3 + 2d^2y - 6dy + 3y^2$

34. $x^3 + x^2 - x - 1$

Story Problem.

35. This pasture has an area described by the polynomial, $10x^3 + 4x^2 - 15x - 6$. Its length and width are described by binomials. Find a pair of binomials that will give the given area.



Preparation.

36. Multiply the following.

$\boldsymbol{a}.\ (\boldsymbol{x+7})(\boldsymbol{x+5})$ $\boldsymbol{b}.\ (\boldsymbol{x+2})(\boldsymbol{x+8})$ $\boldsymbol{c}.\ (\boldsymbol{x+9})(\boldsymbol{x+3})$ $\boldsymbol{d}.\ (\boldsymbol{x+5})(\boldsymbol{x+4})$

37. Given binomials like those in #36, notice that all the answers simplified to trinomials. Describe how to obtain the middle term of those trinomials.

Section 5-1 Answers

1. $12ab^2 - 3a^2$

2. $x^2 + 8x + 16$

3. $x^3 - 2x^2 - 7x - 4$

4. $-3x^2 + 10x - 7$

5. $2m^2 + 8m - 3$

6. Method 1: $Greatest\ Common\ Factors$
Method 5: $Special\ Cases$

7. Method 1: $Greatest\ Common\ Factors$
Method 3: $ax^2 + bx + c, where\ a = 1$

8. Method 1: $Greatest\ Common\ Factors$
Method 2: $Grouping$

9. Method 1: $Greatest\ Common\ Factors$
Method 4: $ax^2 + bx + c, where\ a \neq 1$

10. Method 1: $Greatest\ Common\ Factors$
Method 2: $Grouping$

11. Method 1: $Greatest\ Common\ Factors$
Method 5: $Special\ Cases$

12. 6

13. 3

14. $-7s$

15. b

16. $5x$

17. y

18. $3(3k + 1)$

19. $-6y(3y^4 + 1)\ \ or\ \ 6y(-3y^4 - 1)$

20. $prime$

21. $-1(z^2 + 7a + 2)$

22. $-4ab(3a^2 - 2ab + 4b)\ or\ \ 4ab(-3a^2 + 2ab - 4b)$

23. $19y(xy - 2x + 3)$

24. $2ax^3(ax + 3a - 5)$

25. $-3s(13s^4 + 6s^2 + 27)$

26. $(x - 3)(3x^2 + 4)$

27. $(x + 1)(-2x^2 - 3)$

28. $2(2x^3 - 10x^2 - 3x + 5)$
$Not\ factorable\ by\ grouping$

29. $(x^2 - 2)(x + 5)$

30. $(a^2 + b)(x + 5)$

31. $(2x^2 - 5y)(4x + 9)$

32. $(x^2 - 12)(x + 3)$

33. $(-2d + y)(2d^2 + 3y)$

34. $(x + 1)(x^2 - 1)$

35. $side\ 1: (2x^2 - 3)$
$side\ 2: (5x + 2)$

36. In class.

37. In class.

Section 5-2 Factoring: Trinomial Fast

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

THE 5 METHODS of FACTORING	
Method	**Type of Polynomial used on:**
1: Pull out the Greatest Common Factor (GCF)	All
2: Grouping	4 Terms
3: $ax^2 + bx + c$, where $a = 1$	Trinomials, where $a = 1$
4: $ax^2 + bx + c$, where $a \neq 1$ $ac - method$	Trinomials, where $a \neq 1$
5: Special Cases	Binomials & Perfect Squares

Method 3 - $x^2 + bx + c$ – Trinomial Fast

Main Topics	Examples
Steps 1. Write in descending order. 2. Find factors of c, 3. That add to b. 4. $(x \quad)(x \quad)$. Note that negatives can work.	**Example 1:** Factor $x^2 + 12x + 35$ $c = 35$: 1, 35; 5, 7 $(x + 5)(x + 7)$ **Example 2:** Factor $x^2 - 12x + 35$ $c = 35$: 1, 35; -5, -7 $(x - 5)(x - 7)$ **Example 3:** Factor $x^2 - 18x + 56$ $c = 56$: 1, 56; 2, 28; -4, -14; 7, 8 $(x - 4)(x - 14)$

Remember:
Step 1: Write in descending order.

Example 4: Factor

$$x - 56 + x^2$$

$$x^2 + x - 56$$

$c = -56$	
1	56
2	28
4	14
-7	+8

$$(x - 7)(x + 8)$$

In Ex 5 and Ex 6, we see that the numbers must multiply to c exactly (including the sign) and add to b exactly (including the sign.

Example 5: Factor

$$x^2 + 13x - 30$$

$c = -30$	
1	30
-2	+15
3	10
5	6

$$(x - 2)(x + 15)$$

Example 6: Factor

$$x^2 - 13x + 30$$

$c = 30$	
1	30
2	15
-3	-10
5	6

$$(x - 3)(x - 10)$$

Example 7: Factor

$$x^2 - 10x - 24$$

$c = -24$	
1	24
+2	-12
3	8
4	6

$$(x + 2)(x - 12)$$

Watch Out

Main Topics	Examples
Don't forget Method 1 – GCF, first.	**Example 8:** Factor $5x^3 - 10x^2 - 120x$ GCF = $5x$ $\mathbf{5x(x^2 - 2x - 24)}$ $c = -24$: 1 & 24; 2 & 12; 3 & 8; +4 & -6 $(x - 6)(x + 4)$ *Final Answer*: $5x(x - 6)(x + 4)$
A polynomial that looks like $x^4 + bx^2 + c$ can use this method as well.	**Example 9:** Factor $x^4 + 25x^2 + 24$ $c = 24$: 1 & 24; 2 & 12; 3 & 8; 4 & 6 $(x^2 + 1)(x^2 + 24)$

Section 5-2 Exercises

Identify the greatest common factor between the terms.

5-1

1. $24,\ 96,\ 336$ **2.** $120,\ 480,\ 960$ **3.** $77, 154, 968$

4. $-13b^2, -12ab, -25b^2$ **5.** $12a^2, 16a^2, 96a^2$ **6.** $14m^2n, 28mn^2, 77m^2$

Factor the following by pulling out the greatest common factor if possible.

7. $-3x^2 + 6x^2y - 27x^3$ **8.** $7xy^2 - 2xy + 4x^2y + 2x^2y^2$

9. $12p^{24} + 56p^2 - 48s$ **10.** $3r^2 + 27r^3 - 33r^4$

Factor the following by grouping if possible.

11. $28x^3 - 12x^2 + 7x - 3$ **12.** $6x^3 + 3x^2 + 2x + 1$

13. $x^3 - 3x^2 + x - 3$ **14.** $16x^3 + 52x^2 - 52x - 169$

15. $2a^2x - 2a^2y + 3bx - 3by$ **16.** $2x^3 + 2x^2y - 3x - 3y$

17. $x^3 - 3x^2 - 9x + 27$ **18.** $x^4 + 4x^3 + 2x + 8$

Factor the following using the $ax^2 + bx + c$, where $a = 1$ method.

5-2

19. $x^2 + 2x - 24$ **20.** $x^2 + 11x + 18$

21. $x^2 - 8x + 15$ **22.** $x^2 - x - 20$

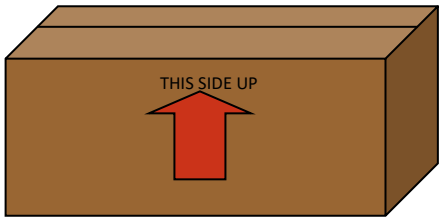
23. $x^2 + 16x + 63$ **24.** $x^2 - 4x - 60$

25. $3x^2 + 27x + 24$ **26.** $x^2 - 7x - 60$

27. $x^2 + 6x - 27$ **28.** $7x^2 - 7x - 14$

Story problem.

29. This box's volume is described by the polynomial $4x^3 - 12x^2 - 72x$. Its height is described by a monomial and its length and width are described by binomials. Find a solution set using prime factors.



Preparation: The following are trinomials in the form $ax^2 + bx + c$ where $a \neq 1$.

30. Multiply the following polynomials together.

a. $(3x + 5)(2x + 7)$ ***b.*** $(5x - 1)(4x + 3)$ ***c.*** $(8x + 1)(2x + 1)$

Section 5-2 Answers

1. 24
2. 120
3. 11
4. $-b \quad or \quad b$
5. $4a^2$
6. $7m$
7. $-3x^2(1-2y+9x) \quad or \quad 3x^2(-1+2y-9x)$
8. $xy(7y-2+4x+2xy)$
9. $4(3p^{24}+14p^2-12s)$
10. $3r^2(1+9r-11r^2)$
11. $(7x-3)(4x^2+1)$
12. $(3x^2+1)(2x+1)$
13. $(x^2+1)(x-3)$
14. $(4x^2-13)(4x+13)$
15. $(2a^2+3b)(x-y)$
16. $(2x^2-3)(x+y)$
17. $(x^2-9)(x-3)$
18. $(x^3+2)(x+4)$
19. $(x+6)(x-4)$
20. $(x+9)(x+2)$
21. $(x-3)(x-5)$
22. $(x-5)(x+4)$
23. $(x+9)(x+7)$
24. $(x-10)(x+6)$
25. $3(x+8)(x+1)$
26. $(x-12)(x+5)$
27. $(x+9)(x-3)$
28. $7(x-2)(x+1)$
29. $height: 4x$
    $length\ and\ width: (x+3) \quad and \quad (x-6)$
30. In class

Section 5-3 Factoring: ac-method

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

THE 5 METHODS of FACTORING	
Method	**Type of Polynomial used on:**
1: Pull Out the Greatest Common Factor (GCF)	All
2: Grouping	4 Terms
3: $ax^2 + bx + c$, where $a = 1$	Trinomials, where $a = 1$
4: $ax^2 + bx + c$, where $a \neq 1$ $ac - method$	Trinomials, where $a \neq 1$
5: Special Cases	Binomials & Perfect Squares

Method 4 - $ax^2 + bx + c$ - *ac method*

Main Topics	Examples

Steps

1. Write in descending order.
2. Find factors of ac,
3. That split b.
4. $4 - terms \ldots Grouping!$

Example 1: Factor $6x^2 + 11x - 10$

$ac = -60$	
1	60
2	30
3	20
-4	+15
5	12
6	10

$$6x^2 - 4x + 15x - 10$$

$$\underbrace{6x^2 - 4x} \qquad \underbrace{+15x - 10}$$

$$2x(3x - 2) + 5(3x - 2)$$

$$(2x + 5)(3x - 2)$$

Example 2: Factor $2x^2 - 19x + 9$

$ac = 18$	
-1	-18
2	9
3	6

$$2x^2 - x - 18x + 9$$

$$\underbrace{2x^2 - x} \qquad \underbrace{-18x + 9}$$

$$x(2x - 1) - 9(2x - 1)$$

$$(x - 9)(2x - 1)$$

Example 3: Factor $6y^2 + 25y + 25$

$ac = 150$	
1	150
2	75
3	50
5	30
6	25
+10	+15

$6y^2 + 10y + 15y + 25$

$\underbrace{6y^2 + 10y}\quad\underbrace{+ 15y + 25}$

$2y(3y + 5) + 5(3y + 5)$

$(2y + 5)(3y + 5)$

Note: If there are two variables, write in descending order with respect to one of them.

Example 4: Factor $12x^2 - 13xy - 4y^2$

$ac = -48$	
1	48
2	24
+3	-16
4	12
6	8

$12x^2 + 3xy - 16xy - 4y^2$

$\underbrace{12x^2 + 3xy}\quad\underbrace{- 16xy - 4y^2}$

$3x(4x + y) - 4y(4x + y)$

$(3x - 4y)(4x + y)$

Example 5: Factor $-28x - 60 + 8x^2$

Rewrite in descending order.

$8x^2 - 28x - 60$

GCF = 4

$4(2x^2 - 7x - 15)$

Don't forget to check Method 1 – GCF on all polynomials.

$ac = -30$	
1	30
2	15
+3	-10
5	6

$2x^2 + 3x - 10x - 15$

$\underbrace{2x^2 + 3x}\quad\underbrace{- 10x - 15}$

$x(2x + 3) - 5(2x + 3)$

Answer: $\mathbf{4(x - 5)(2x + 3)}$

Section 5-3 Exercises

Identify the greatest common factor between the terms.

5-1

1. $14, 49, 112$ **2.** $15x, 39xy, 52x$ **3.** $45j^2k, -80j^2k^3, 105k^2$

Factor the following by pulling out the greatest common factor.

4. $49x^5 + 21x^3 - 14x^2$ **5.** $24x^3y^3 + 96x^2y^3 - 72x^3y^2$

6. $15x^2y^3z^2 - 12x^2y^3z + 9x^2y^3$ **7.** $18p^3 - 6p^2 + 14p^4 + 2rs$

Factor the following by grouping if possible.

8. $8x^3 + 2x^2 - 12x - 15$ **9.** $3x^3 - 15x^2 + 5x - 25$

10. $2x^3 + 3x^2 + 2x + 3$ **11.** $x^3 + x^2 - x - 1$

Factor the following using the $ax^2 + bx + c$, where $a = 1$ method.

5-2

12. $x^2 - 5x - 84$ **13.** $x^2 - x - 6$

14. $x^2 - 2x - 35$ **15.** $x^2 - 15x + 54$

16. $x^2 - 18x + 81$ **17.** $x^2 - 8x - 33$

Factor the following using the $ax^2 + bx + c$, where $a \neq 1$ method.

5-3

18. $10x^2 - 7x - 6$ **19.** $8x^2 + 2x - 3$

20. $3 + 16x + 5x^2$ **21.** $2m^2 + 11m + 12$

22. $2x^2 - 5x - 25$ **23.** $6x^2 + 25xy + 14y^2$

24. $2s^2 - 21s + 40$ **25.** $-6 + 16x^2 + 20x$

26. $5x^2 - 14xy - 3y^2$ **27.** $7y^2 - 30y + 8$

Preparation: Multiply.

4-3

28. $(x - 5)(x + 5)$ **29.** $(a + b)^2$

Section 5-3 Answers

1. 7
2. x
3. $5k$
4. $7x^2(7x^3 + 3x - 2)$
5. $24x^2y^2(xy + 4y - 3x)$
6. $3x^2y^3(5z^2 - 4z + 3)$
7. $2(9p^3 - 3p^2 + 7p^4 + rs)$
8. $Not\ factorable\ by\ Grouping$
9. $(3x^2 + 5)(x - 5)$
10. $(x^2 + 1)(2x + 3)$
11. $(x^2 - 1)(x + 1)$
12. $(x - 12)(x + 7)$
13. $(x - 3)(x + 2)$
14. $(x - 7)(x + 5)$
15. $(x - 9)(x - 6)$
16. $(x - 9)(x - 9)\ \ or\ \ (x - 9)^2$
17. $(x - 11)(x + 3)$
18. $(5x - 6)(2x + 1)$
19. $(2x - 1)(4x + 3)$
20. $(5x + 1)(x + 3)$
21. $(2m + 3)(m + 4)$
22. $(2x + 5)(x - 5)$
23. $(3x + 2y)(2x + 7y)$
24. $(2s - 5)(s - 8)$
25. $2(4x - 1)(2x + 3)$
26. $(5x + y)(x - 3y)$
27. $(y - 4)(7y - 2)$
28. In class.
29. In class.

Section 5-4 Factoring: Special Cases

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

THE 5 METHODS of FACTORING	
Method	**Type of Polynomial used on:**
1: Pull Out the Greatest Common Factor (C.F.)	All
2: Grouping	4 Terms
3: $ax^2 + bx + c$, where $a = 1$	Trinomials, where $a = 1$
4: $ax^2 + bx + c$, where $a \neq 1$ $ac - method$	Trinomials, where $a \neq 1$
5: Special Cases	Binomials & Perfect Squares

2-Terms, Difference of Squares

Squares

Squares of numbers to recognize:

$1^2 = 1$ $\quad$ $8^2 = 64$
$2^2 = 4$ $\quad$ $9^2 = 81$
$3^2 = 9$ $\quad$ $10^2 = 100$
$4^2 = 16$ $\quad$ $11^2 = 121$
$5^2 = 25$ $\quad$ $12^2 = 144$
$6^2 = 36$ $\quad$ $13^2 = 169$
$7^2 = 49$ $\quad$ $14^2 = 196$

Difference means subtraction.

$$A^2 - B^2 = (A + B)(A - B)$$

Example 1: Factor

$$\mathbf{25x^2 - 16}$$

square $\quad$ difference $\quad$ square

$25x^2 = (5x)^2 \qquad 16 = 4^2$

$(5x + 4)(5x - 4)$

Example 2: Factor

$$\mathbf{49x^2 - 9}$$

$49x^2 = (7x)^2 \qquad 9 = 3^2$

$(7x + 3)(7x - 3)$

Difference of squares: $A^2 - B^2 =$ $(A + B)(A - B)$	**Example 3:** Factor $\mathbf{121x^2 - 36y^2}$ $121x^2 = (11x)^2 \qquad 36y^2 = (6y)^2$ $(11x + 6y)(11x - 6y)$ **Example 4:** Factor $\mathbf{4x^2 - 81y^2}$ $4x^2 = (2x)^2 \qquad 81y^2 = (9y)^2$ $(2x + 9y)(2x - 9y)$ **Example 5:** Factor $\mathbf{100t^2 - 1}$ $100t^2 = (10t)^2 \qquad 1 = (1)^2$ $(10t + 1)(10t - 1)$ **Example 6:** Factor $\mathbf{y^4 - 1}$ $y^4 = (y^2)^2 \qquad 1 = (1)^2$ $(y^2 + 1)(y^2 - 1)$ another difference of squares **Answer:** $\mathbf{(y^2 + 1)(y + 1)(y - 1)}$

Sum of Squares is Prime

Sum means addition. $A^2 + B^2$ *is prime.* (cannot be factored) Why? Look at the possibilities. • $(A + B)(A - B) = A^2 - B^2$ • $(A + B)(A + B) = A^2 + 2AB + B^2$ • $(A - B)(A - B) = A^2 - 2AB + B^2$ None work.	**Example 7:** Factor $\mathbf{x^2 + 9}$ Prime. **Example 8:** Factor $\mathbf{25x^2 + 64}$ Prime. **Example 9:** Factor $\mathbf{36x^2 + 49}$ Prime.

Perfect square trinomial

Identify a perfect square

POLYNOMIAL	Checklist to recognize a **Perfect Square Trinomial.**				**Perfect Square Trinomials**
	Tri-nomial	Ends = Perfect Squares	Middle term = ***2AB***	Last term is positive	
$16y^2 + 24y + 9$	✔	✔	✔	✔	YES
$16y^2 + 25y + 9$	✔	✔	NO	✔	NO
$4x^2 + 12x + 9$	✔	✔	✔	✔	YES
$4x^2 + 12x - 9$	✔	✔	✔	NO	NO
$x^2 - 20x + 100$	✔	✔	✔	✔	YES
$x^2 + 20x - 100$	✔	✔	✔	NO	NO
$x^2 + 25x + 100$	✔	✔	NO	✔	NO
$x^2 + 20x + 96$	✔	NO	NO	✔	NO

Factor a perfect square

$$A^2 + 2AB + B^2 = (A + B)(A + B) = (A + B)^2$$

Note: If you find a trinomial isn't a perfect square, try Method 3 or 4 to see if it will factor another way.

Example 10: Factor

$$16x^2 + 88x + 121$$

$$16x^2 = (4x)^2 \qquad 121 = (11)^2 \qquad 88 = 2(4)(11)$$

$$(4x + 11)^2$$

Example 11: Factor

$$25x^2 - 90x + 81$$

$$25x^2 = (5x)^2 \qquad 81 = (-9)^2 \qquad -90 = 2(5)(-9)$$

$$(5x - 9)^2$$

Example 12: Factor

$$9x^2 - 42x + 49$$

$$9x^2 = (3x)^2 \qquad 49 = (-7)^2 \qquad -42 = 2(3)(-7)$$

$$(3x - 7)^2$$

Section 5-4 Exercises

Factor the following by pulling out the greatest common factor if possible.

5-1

1. $8x^6 - 48x^3 + 64x^4$

2. $3j^2kb^3 - 2j^3k^5b^2 + 5j^3k^2b - 7j^3k^3b^3$

Factor the following by grouping if possible.

5-1

3. $12k^3b + 15kb + 8k^2 + 10$

4. $84m^3n^2 + 35m^2n^2 - 12m - 5$

Factor the following using the $ax^2 + bx + c$, where $a = 1$ method.

5-2

5. $x^2 - 2x - 63$

6. $x^2 + 20x + 75$

Factor the following using the $ax^2 + bx + c$, where $a \neq 1$ method.

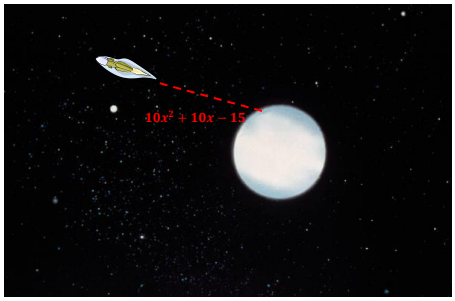
5-3

7. $12x^2 + 7x - 10$

8. $-14x^2 + 17x + 6$

Story problem.

9. An alien spaceship has traveled $10x^2 + 19x - 15$ miles from their home planet. Their speed and time can both be represented by binomials. Find two suitable binomials that will represent them. (Recall that $d = rt$, and you have been given distance.)



Determine if the following are differences of squares, then factor. If unfactorable, explain why.

5-4

10. $x^2 - 36$

11. $16y^4 + 9$

12. $4x^2 - 9$

13. $54x^2 - 24$

14. $-25 + 4y^2$

15. $25g^8 - 81$

16. $27 - 3m^2$

17. $y^5 + 4y$

18. $2x^2 - 1$

19. $16x^2 - 49$

Determine if the following are perfect square trinomials. If they are, factor using method 5. If they are not or you are unsure, use method 3 or 4.

20. $x^2 + 10x + 25$

21. $4x^2 - 12x + 9$

22. $3x^2 + 5x - 2$

23. $6x^2 - 84x + 294$

24. $9y^4 - 66y^2 + 121$

25. $2m^2 + 16m + 32$

26. $4x^2 - 16x + 16$

27. $4x^2 + 30x - 100$

Preparation: The following polynomials have been factored already. Determine if they are completely factored. If not, finish factoring the polynomials.

28. $4(x^2 - 6x + 9)$

29. $(x^2 + 4)(x^2 - 4)$

30. $3(m - 8)^2$

31. $7(100y^4 - 16)$

Section 5-4 Answers

1. $8x^3(x^3 + 8x - 6)$

2. $j^2kb(3b^2 - 2jk^4b + 5jk - 7jk^2b^2)$

3. $(3kb + 2)(4k^2 + 5)$

4. $(7m^2n^2 - 1)(12m + 5)$

5. $(x - 9)(x + 7)$

6. $(x + 5)(x + 15)$

7. $(3x - 2)(4x + 5)$

8. $(-2x + 3)(7x + 2)$ or $(-7x - 2)(2x - 3)$ or $-(2x - 3)(7x + 2)$

9. rate: $(2x + 5)$ time: $(5x - 3)$ *or*
rate: $(5x - 3)$ time: $(2x + 5)$

10. $(x + 6)(x - 6)$

11. Prime. (It is prime because a *sum* of squares is not factorable.)

12. $(2x + 3)(2x - 3)$

13. $6(3x + 2)(3x - 2)$

14. $(2y + 5)(2y - 5)$

15. $(5g^4 + 9)(5g^4 - 9)$

16. $3(3 + m)(3 - m)$

17. $y(y^4 + 4)$

18. Prime. (It is prime because 2 is not a perfect square.)

19. $(4x + 7)(4x - 7)$

20. $(x + 5)^2$

21. $(2x - 3)^2$

22. $(x + 2)(3x - 1)$

23. $6(x - 7)^2$

24. $(3y^2 - 11)^2$

25. $2(m + 4)^2$

26. $4(x - 2)^2$

27. $2(2x - 5)(x + 10)$

28. In class.

29. In class.

30. In class.

31. In class.

Section 5-5 Factoring (All together)

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

THE 5 METHODS of FACTORING	
Method	**Type of Polynomial used on:**
1: Pull out the Greatest Common Factor (GCF)	All
2: Grouping	4 Terms
3: $ax^2 + bx + c$, where $a = 1$	Trinomials, where $a = 1$
4: $ax^2 + bx + c$, where $a \neq 1$	Trinomials, where $a \neq 1$
5: Special Cases	Binomials & Perfect Squares

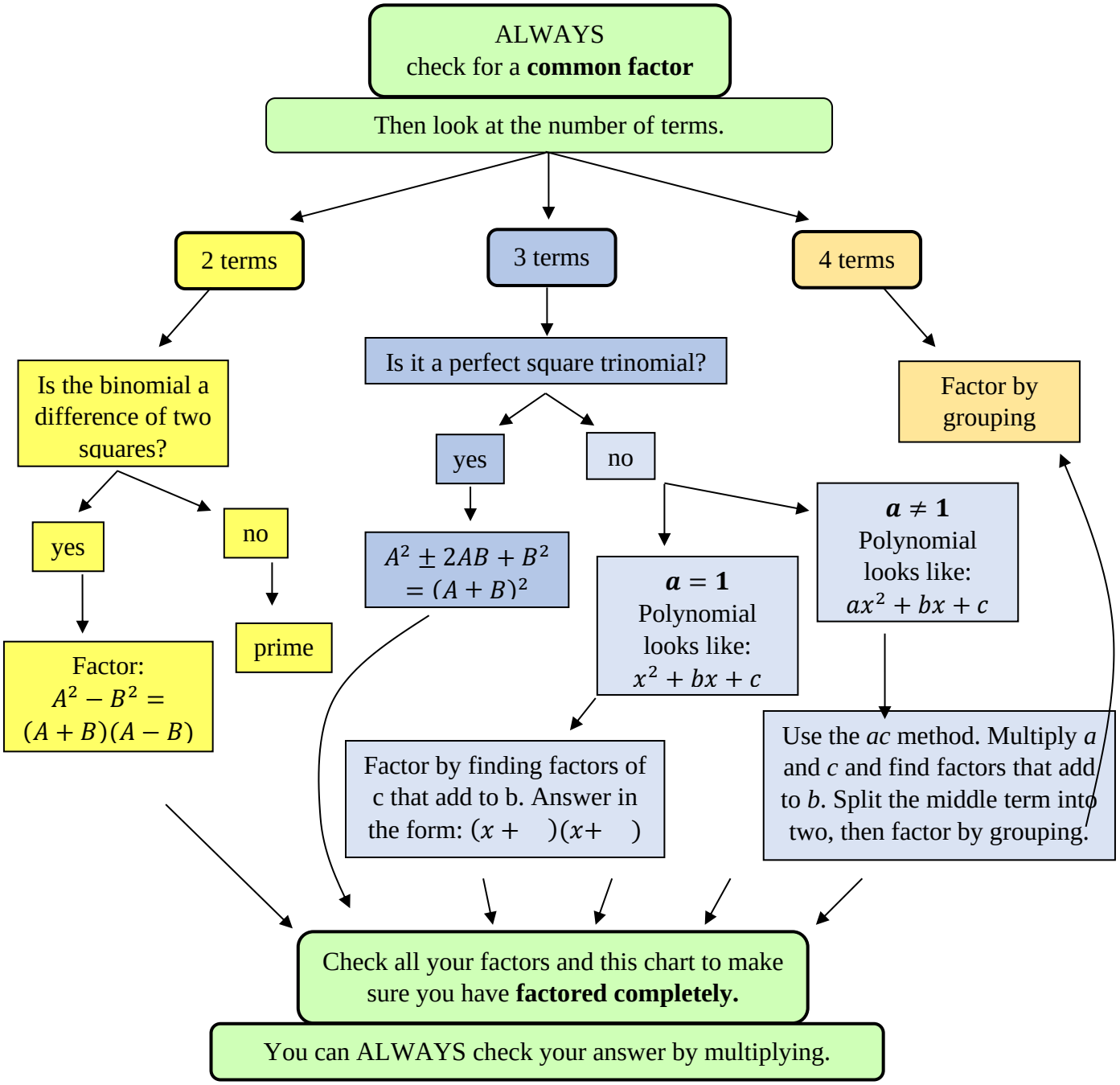
REVIEW FROM SECTION 5-1: WHEN TO TRY EACH METHOD:

Polynomial	Factoring Methods Possible
$y^3 + 4y^2 + 2y + 8$	Method 1: Pull out Greatest Common Factor Method 2: Grouping
$x^2 + 6x + 9$	Method 1: Pull out Greatest Common Factor Method 3: $ax^2 + bx + c$, where $a = 1$
$16x^2 - 16x + 4$	Method 1: Pull out Greatest Common Factor Method 4: $ax^2 + bx + c$, where $a \neq 1$
$54x^3 - 6x$	Method 1: Pull out Greatest Common Factor Method 5: Special Cases (because it's a binomial)
$x^2 - 5x + 6$	Method 1: Pull out Greatest Common Factor Method 3: $ax^2 + bx + c$, where $a = 1$
$2c^3 + 8c^2 - 6c - 12$	Method 1: Pull out Greatest Common Factor Method 2: Grouping
$36a^2 - 25$	Method 1: Pull out Greatest Common Factor Method 5: Special Cases (because it's a binomial)
$35x^3 + 42x^2 - 14x - 77xy - 14y + 7$	Method 1: Pull out Greatest Common Factor (only method because it has 6 terms)
$6y^2 + 25y + 25$	Method 1: Pull out Greatest Common Factor Method 4: $ax^2 + bx + c$, where $a \neq 1$

Notice that you may be using a couple of different methods on each problem.

We have learned several methods of factoring, and each method is used in different circumstances. If you are unsure what to do when factoring a polynomial, this chart will be helpful.

Steps to Factoring Polynomials



Factoring All Together

Example 1: Factor: $2x^2 + 36x + 154$

Check for a common factor.	• $2(x^2 + 18x + 77)$
How many terms?	• Three terms
Is it a perfect square trinomial?	• Not a perfect square trinomial
What is the first coefficient? ($a = 1$ or $a \neq 1$)	• $a = 1$, polynomial looks like $x^2 + bx + c$.
Factor – find factors of c that add to b.	• $c = 77 = \mathbf{11} \times \mathbf{7}$; $b = 18 = \mathbf{11} + \mathbf{7}$ Factored form: $(x + 11)(x + 7)$
Check – can any of the factors be factored?	• No, all factors are now prime.

Answer: $\mathbf{2(x + 11)(x + 7)}$

Example 2: Factor: $36y^2 - 36y + 9$

Check for a common factor.	• $9(4y^2 - 4y + 1)$
How many terms?	• Three terms
Is it a perfect square trinomial?	• Yes, it follows the form $A^2 + 2AB + B^2$ where $A = 2y$ and $B = -1$
Factor knowing that $A^2 \pm 2AB + B^2 = (A + B)^2$.	• $9(2y - 1)^2$
Check – can any of the factors be factored?	• The factors are 9 and $(2y - 1)$. • $(2y - 1)$ is prime.

Answer: $\mathbf{9(2y - 1)^2}$

Example 3: Factor: $\mathbf{36x^3 + 72x^2 - x - 2}$

Check for a common factor.	• Nope
How many terms?	• Four terms – Grouping $36x^3 + 72x^2 \quad -x - 2$ $36x^2(x + 2) - 1(x + 2)$ $(36x^2 - 1)(x + 2)$
Look harder. Is there more that can be done? Yes. The first binomial is a difference of squares	• $(36x^2 - 1)(x + 2)$
Factor knowing that $A^2 - B^2 = (A + B)(A - B)$.	• $(36x^2 - 1)(x + 2)$ • $(6x + 1)(6x - 1)(x + 2)$
Check – can any of the factors be factored?	• Nope. We are now done

Answer: $\mathbf{(6x + 1)(6x - 1)(x + 2)}$

Section 5-5 Exercises

Factor the following. If non-factorable label as prime.

5-5

1. $1 - y^2$

2. $2x^2 - 8x + 8$

3. $13x + 2 + 18x^2$

4. $81 + x^4$

5. $-2s^2 + 21s - 40$

6. $6x^2 + 25xy + 14y^2$

7. $-5x + 2x^4 + 2x^2 - 5x^3$

8. $x^2 - y^2$

9. $16b^2 - 9b$

10. $4x^2 + 30x - 100$

11. $28x^2 + 65x + 28$

12. $4a^5 + 16$

13. $x^3 + 3x^2 - 4x - 12$

14. $r^3 + r^2 - 4r - 4$

15. $2x^3y + 4x^2y - 30xy$

16. $16z^3 + 48z^2 + 36z + 108$

17. $4x^5 + 12x^4 - 4x^3 + 12x^2$

18. $3x^2 + 5x - 17$

19. $x^8 - 81$

20. $12x^3y - 27xy^3$

21. $x^2 - 3x - 18$

22. $y^8 - k^{12}$

23. $16a^2 + 40a + 25$

24. $z^4 + 8z^2 + 16$

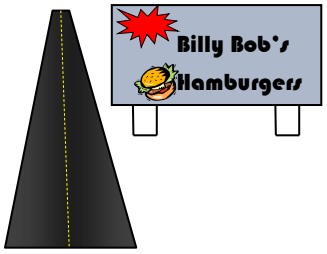
25. $2\pi r^2 - 2\pi$

26. $9x^2 + 3x$

27. $4b^2 + 256$

28. $18x^3 + 54x^2 + 6x + 18$

Story Problems.



29. A billboard along the side of I-15 has an area represented by the polynomial $9y^8 - 100$. Find 2 binomials that represent the length and width of billboard. Recall that $Area = (length) \times (width)$.

30. A telephone booth with a square bottom has a volume of $175x^3 + 140x^2 + 28x$. Its height is represented by a monomial and its length and width by binomials. Find a monomial and two binomials that will represent these three dimensions.

Preparation:

31. Solve the following equations for the variable:

a. $x^2 = 4$ **b.** $25 - y^2 = 0$

Section 5-5 Answers

1. $(1+y)(1-y)$
2. $2(x-2)^2$
3. $(2x+1)(9x+2)$
4. Prime
5. $(-2s+5)(s-8)\ or\ (2s-5)(-s+8)$
6. $(3x+2y)(2x+7y)$
7. $x(x^2+1)(2x-5)$
8. $(x+y)(x-y)$
9. $b(16b-9)$
10. $2(2x-5)(x+10)$
11. $(4x+7)(7x+4)$
12. $4(a^5+4)$
13. $(x+2)(x-2)(x+3)$
14. $(r-2)(r+2)(r+1)$
15. $2xy(x-3)(x+5)$
16. $4(4z^2+9)(z+3)$
17. $4x^2(x^3+3x^2-x+3)$
18. Prime
19. $(x^4+9)(x^2+3)(x^2-3)$
20. $3xy(2x-3y)(2x+3y)$
21. $(x+3)(x-6)$
22. $(y^4+k^6)(y^2+k^3)(y^2-k^3)$
23. $(4a+5)^2$
24. $(z^2+4)^2$
25. $2\pi(r+1)(r-1)$
26. $3x(3x+1)$
27. $4(b^2+64)$
28. $6(3x^2+1)(x+3)$
29. $side\ 1\colon\ 3y^4+10$
    $side\ 2\colon\ 3y^4-10$
30. $height\colon\ 7x$
    $base\ length\colon\ 5x+2$
    $base\ width\colon\ 5x+2$
31. In class.

Section 5-6 Solving Polynomial Equations

CHAPTER OVERVIEW (Video Instruction and Solutions Link)

THE 5 METHODS of FACTORING	
Method	**Type of Polynomial used on:**
1: Pull out the Greatest Common Factor (GCF)	All
2: Grouping	4 Terms
3: $ax^2 + bx + c$, where $a = 1$	Trinomials, where $a = 1$
4: $ax^2 + bx + c$, where $a \neq 1$	Trinomials, where $a \neq 1$
5: Special Cases	Binomials & Perfect Squares

Solve Polynomial Equations

Polynomial Equations

Main Topics	Examples
Multiplication with 0 • Easiest multiplication ever	$5 \cdot 0 = 0$ $73 \cdot 0 = 0$ $0 \cdot (-371) = 0$ $0 \cdot x = 0$ $3x = 0$ $57y = 0$ $97(m-7) = 0$ $x = 0$ $y = 0$ $m - 7 = 0$, $m = 7$
• If the answer is 0, then one of the factors has to be 0.	$(x-3)(2x+5)(x+7)(3x-11) = 0$ $x - 3 = 0$ $2x + 5 = 0$ $x + 7 = 0$ $3x - 11 = 0$ $x = 3$ $x = -\frac{5}{2}$ $x = -7$ $x = \frac{11}{3}$
Factor to Solve 1. Get = 0. 2. Factor. 3. Set each factor = 0.	**Example 1:** $2x^2 - 10x = 0$ $2x(x-5) = 0$ $2x = 0$ $x - 5 = 0$ $x = 0$ $x = 5$ **Example 2:** -18: 1 & 18; **2** & **-9**; 3 & 6 $x^2 - 7x = 18$ $x^2 - 7x - 18 = 0$ $(x-9)(x+2) = 0$

	$x = 9, -2$
Factor to Solve 1. Get = 0. 2. Factor. 3. Set each factor = 0.	**Example 3:** $6x^2 - 10 = -11x$ $6x^2 + 11x - 10 = 0$ $ac = -60$: 1 & 60; 2 & 30; 3 & 20; **-4** & **+15**; 5 & 12; 6 & 10 $6x^2 - 4x + 15x - 10 = 0$ $2x(3x - 2) + 5(3x - 2) = 0$ $(2x + 5)(3x - 2) = 0$ $x = -\frac{5}{2}, \frac{2}{3}$ **Example 4:** $49x^2 = 56x - 16$ $49x^2 - 56x + 16 = 0$ $(7x - 4)^2 = 0$ $x = \frac{4}{7}$ *happens twice, a "double root"* **Example 5:** $25x^2 - 49 = 0$ $(5x - 7)(5x + 7) = 0$ $x = \frac{7}{5}, -\frac{7}{5}$ $= \pm\frac{7}{5}$

Word Problems

Main Topics	Examples
Formulas Plug in the given data. 1. Get = 0. 2. Factor. 3. Set each factor = 0.	**Example 6:** The profit of a small company when they make x thingamabobs is $P = 2x^2 - 17x$. How many will they need to make to get a profit of \$30? $30 = 2x^2 - 17x$ $0 = 2x^2 - 17x - 30$ $ac = -60$: 1 & 60; 2 & 30; **3** & **-20**; 4 & 15; 5 & 12; 6 & 10 $0 = 2x^2 - 20x + 3x - 30$ $0 = 2x(x - 10) + 3(x - 10)$ $0 = (2x + 3)(x - 10)$ $x = -\frac{3}{2}$ $x = 10$

Answer: They must make 10 to have a profit of $30.

Shapes – Use the formulas from section R-4 or in the appendix.

Example 7: A box has a height of 3 in. Its length is two inches longer than its width. If it has a total surface area of $222\ in^2$, what are the dimensions?

$$h = 3 \quad l = w + 2$$

$$SA = 2hl + 2lw + 2hw$$

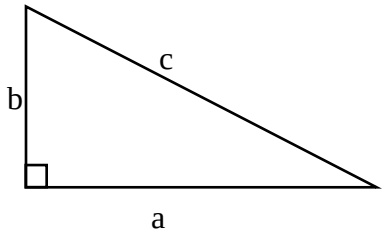
$$222 = 2 \cdot 3(w + 2) + 2(w + 2)w + 2 \cdot 3w$$

$$222 = 6w + 12 + 2w^2 + 4w + 6w$$
$$0 = 2w^2 + 16w - 210$$

$$0 = 2(w^2 + 8w - 105)$$
$$0 = 2(w + 15)(w - 7)$$
$$w = -15 \quad w = 7$$

Answer: The dimensions are 3 in, 7 in, and 9 in.

Pythagorean Theorem:



$$a^2 + b^2 = c^2$$

Example 8: Jessie is mowing her back yard that is in the shape of a right triangle. The shortest side is 7 meters shorter than the second side, and the hypotenuse is 13 meters long. What are the lengths of the sides?

$$b = a - 7 \qquad c = 13$$

$$a^2 + (a - 7)^2 = 13^2$$

$$a^2 + a^2 - 14a + 49 = 169$$

$$2a^2 - 14a - 120 = 0$$
$$2(a^2 - 7a - 60) = 0$$
$$2(a - 12)(a + 5) = 0$$
$$a - 12 = 0 \qquad a + 5 = 0$$

$$a = 12 \quad a = -5$$

$$b = a - 7 = 5$$

−60	
1	60
2	30
3	20
4	15
5	**-12**
6	10

Answer: The sides are 12 m, 5 m, and 13 m.

Section 5-6 Exercises

Factor the following.

5-5

1. $4x^2 + 36x$

2. $r^2 - 64$

3. $x^2 + 4x + 4$

4. $5x^2 - 4x - 1$

5. $x^2 - 25$

6. $x^3 + 5x^2 - 9x - 45$

7. $-6 + 16a^2 + 20a$

8. $x^3 - 3x^2 + 2x$

9. $2x^3 + x^2 - 98x - 49$

10. $x^2 + 16x + 63$

11. $25x^2 - 100x + 100$

12. $x^2 + 8x - 48$

The following are identical to the previous twelve problems, except that they are now equations. Now that you have factored them, solve for the variable.

5-6

13. $4x^2 + 36x = 0$

14. $r^2 - 64 = 0$

15. $x^2 + 4x + 4 = 0$

16. $5x^2 - 4x - 1 = 0$

17. $x^2 - 25 = 0$

18. $x^3 + 5x^2 - 9x - 45 = 0$

19. $-6 + 16x^2 + 20x = 0$

20. $x^3 - 3x^2 + 2x = 0$

21. $2x^3 + x^2 - 98x - 49 = 0$

22. $x^2 + 16x + 63 = 0$

23. $25x^2 - 100x + 100 = 0$

24. $x^2 + 8x - 48 = 0$

Factor the following and solve for the variable.

25. $4x^2 + 16x + 16 = 0$

26. $x^2 = 9$

27. $x^2 - 14x + 14 = -35$

28. $y^2 + 16 = 0$

29. $3x^2 - 36 = 3x$

30. $x^2 - 169 = 0$

31. $4x^2 + 36x - 15 = 25$

32. $x^3 + 3x^2 - 4x - 12 = 0$

33. $100x^2 + 80x + 16 = 0$

34. $x^2 - 4 = 0$

35. $6x^2 = -36x - 54$

36. $x^2 - x - 20 = 0$

Story Problems.

37. The energy of an object is dependent on its mass and can be described by the following equation: $E = 2m^2 - 12m$, where E stands for energy and m stands for mass. If the energy of the object is 14 units, what is the mass of the object?

38. The area of a window is 192 in^2. The width of the window is four inches more than half the length of the window. What are the dimensions of the window?

39. A cone has a surface area of 36π cm^2 and a slant height of 9 cm. What is the radius of the cone? (See section 2.2 for formulas)

40. Jefferson's back yard is in the shape of a right triangle. One leg of the triangle is seven feet longer than the other, with a hypotenuse of 17 feet. What are the lengths of the two legs?

41. Carl is building a right triangle hot tub that has a leg ten feet more than twice the other leg. The hypotenuse is 25 feet. What are the lengths of the two legs?

Section 5-6 Answers

1. $4x(x+9)$

2. $(r+8)(r-8)$

3. $(x+2)^2$

4. $(5x+1)(x-1)$

5. $(x+5)(x-5)$

6. $(x+5)(x-3)(x+3)$

7. $2(4a-1)(2a+3)$

8. $x(x-2)(x-1)$

9. $(2x+1)(x-7)(x+7)$

10. $(x+9)(x+7)$

11. $25(x-2)^2$

12. $(x+12)(x-4)$

13. $x=-9,0$

14. $r=-8,8$

15. $x=-2$

16. $x=-\frac{1}{5},1$

17. $x=-5,5$

18. $x=-5,-3,3$

19. $a=-\frac{3}{2},\frac{1}{4}$

20. $x=0,1,2$

21. $x=-7,-\frac{1}{2},7$

22. $x=-9,-7$

23. $x=2$

24. $x=-12,4$

25. $x=-2$

26. $x=-3,3$

27. $x=7$

28. $No\ solution$

29. $x=-3,4$

30. $x=-13,13$

31. $x=-10,1$

32. $x=-3,-2,2$

33. $x=-2/5$

34. $x=-2,2$

35. $x=-3$

36. $x=-4,5$

37. $m=7$

38. $l=16\text{ in}, w=12\text{ in}$

39. $r=3\text{ cm}$

40. 8 feet, 15 feet

41. 7 feet, 24 feet

Chapter 5 Review Exercises

1. Create a visual chart of all of the methods, formulas, and examples from studying how to factor polynomials.

Identify the greatest common factor between the terms.

5-1

2. $112,\ 148,\ 246$ **3.** $3j^2, 5j^2aj^3, 4j^3y$ **4.** $12m^4n^2p^4, 24m^3n^3p^4, 30m^3n^2p^5$

Factor the following by grouping.

5. $6x^3 + 10x^2 + 3x + 5$

6. $21x^3 - 14x^2 - 12x + 8$

7. $5ax^3 + 20a^2x^2 + 3x + 12a$

8. $28x + 7x^3 + 4 + x^2$

Factor the following using method 3 or method 4.

5-2,3

9. $2x^2 + 3x + 1$

10. $3x^2 - 12x - 63$

11. $2x^2 + 32x + 96$

12. $14x^2 + 29x - 15$

13. $-4x^2 - 24x + 108$

14. $7x^2 - 35x + 42$

Factor the following using method 5.

5-4

15. $-x^2 + 64$

16. $x^2 + 2x + 1$

17. $49x^2 - 28x + 4$

18. $x^4y^2 - 9z^2$

19. $-16x^2 - 4$

20. $8y^2 + 72y + 162$

Factor the following.

5-5

21. $18x^2 + 24x + 8$

22. $x^2 + 7x - 18$

23. $-18x^3 + 15x^2 + 24x - 20$

24. $-20x^2 - 19x - 3$

25. $-3a^2xy + 3b^2xy$

26. $x^2 - 15x + 56$

27. $x^2 - 23x + 132$

28. $x^2 - 4x - 45$

29. $14x^3 + 16x^2 + 35x + 40$

30. $-4x^2 + 52x - 168$

31. $24m^3 - 6m^2n - 63mn^2$

32. $49x^2 - 121$

33. $35x^4y^2 + 42x^4y + 15x^3y^2 + 18x^3y$

34. $4x^2 - 36xy + 81y^2$

Solve for the variable by factoring.

5-6

35. $x^2 - 13x + 22 = -20$

36. $-6x^2 - 11x + 10 = 0$

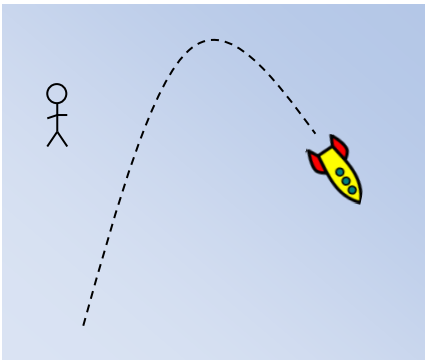
37. $-5x^3 - 10x^2 = 5x$

38. $4x^3 + 16x^2 - x - 4 = 0$

39. $100 - x^2 = -21$

40. $4x^3 + 4x^2 - 25x = 25$

Story Problems.



41. Henry stands on a platform and shoots a rocket in his 10th grade science class. He found that the motion of the rocket can be described by the equation $y = 60 + 28x - x^2$, where y is the vertical distance and x is the horizontal distance traveled. If this trajectory equation were to be graphed, find **the x-intercepts** of the equation. (Fun fact: The positive x-intercept is how far away from the platform the rocket hits the ground!)

42. A triangle has an area of 14 square inches. The height of the triangle is three inches more than the base. What are the base and height of the triangle?

43. Jill has a small treasure box that is 6 inches long. It can hold a volume of 72 inches cubed, and the width of the box is 5 inches less than twice the height of the box. What are the dimensions of the box?

44. Jessie is mowing her back yard that is in the shape of a right triangle. The shortest side is 7 meters shorter than the second side, and the hypotenuse is 13 meters long. What are the lengths of the two sides?

Chapter 5 Review Answers

1. This is your last one; make it good
2. 2
3. j^2
4. $6m^3n^2p^4$
5. $(2x^2+1)(3x+5)$
6. $(7x^2-4)(3x-2)$
7. $(5ax^2+3)(x+4a)$
8. $(x^2+4)(7x+1)$
9. $(x+1)(2x+1)$
10. $3(x-7)(x+3)$
11. $2(x+4)(x+12)$
12. $(2x+5)(7x-3)$
13. $-4(x-3)(x+9)$ or equivalent
14. $7(x-3)(x-2)$
15. $(8+x)(8-x)$
16. $(x+1)^2$
17. $(7x-2)^2$
18. $(x^2y+3z)(x^2y-3z)$
19. $-4(4x^2+1)$, not a special case
20. $2(2y+9)^2$
21. $2(3x+2)^2$
22. $(x+9)(x-2)$
23. $(-3x^2+4)(6x-5)$
24. $-1(5x+1)(4x+3)$ or equivalent
25. $-3xy(a+b)(a-b)$ or equivalent
26. $(x-7)(x-8)$
27. $(x-11)(x-12)$
28. $(x-9)(x+5)$
29. $(2x^2+5)(7x+8)$
30. $-4(x-7)(x-6)$
31. $3m(4m-7n)(2m+3n)$
32. $(7x-11)(7x+11)$
33. $x^3y(7x+3)(5y+6)$
34. $(2x-9y)^2$
35. $x=6,7$
36. $x=-\frac{5}{2},\frac{2}{3}$
37. $x=-1,0$
38. $x=-4,\ -\frac{1}{2},\ \frac{1}{2}$
39. $x=-11,11$
40. $x=-\frac{5}{2},-1,\frac{5}{2}$
41. $(-2,0),(30,0)$
42. base $=4$ inches, height $=7$ inches
43. height $=4$ inches, width $=3$ inches
44. 5 meters, 12 meters

Beginning Algebra Final Review Exercises

Find the prime factorization for each number.

Ch. R/1

1. 27 **2.** 360

Find the lowest common multiple for each set of numbers.

3. $72, 16$ **4.** $12,\ 15, 30$

Evaluate.

5. $V = \frac{1}{3}\pi r^2 h$ when $r = 4\ in.$, $h = 3\ in.$ **6.** $3x^2 - 12x - 6$, when $x = -2$

7. $\left|-\frac{12}{4}\right|$ **8.** $-\frac{4}{18} - \left(-\frac{2}{9}\right)$

Simplify.

9. $2[3(y + 2) - 2(4y - 5)] - 14$ **10.** $\frac{20(8-3)-4|3-10|}{-10(-2)^2-2(5+2)}$

Solve.

Ch. 2

11. $2 - 5(x + 5) = 3(x - 2) - 1$ **12.** $\frac{5}{3} + \frac{2}{3}\text{x} = 6$

13. $-5x + 7 = 5x - 10(x + 1)$ **14.** $6\text{x} - 10 \leq 7\text{x} + 5$

15. 2 is what percent of 40? **16.** Solve for c in $x = \frac{a+b+c}{3}$

17. Selene wants to buy a paddleboard for $468. The tax will be 8%. How much will she pay total?

18. The interior angles of a certain triangle have the following relationship: the second angle is three times the first angle and the third angle is 15° less than the first angle. Find the measure of all three angles.

Graph the following linear equations.

Ch. 3

19. $y = 2x - 4$ **20.** $6\text{x} - 2\text{y} = 18$

21. $2x + 6 = 14$ **22.** $y = -2.5$

Write the equations of the lines with the following characteristics.

23. $\text{m} = -1$, goes through $(-2,5)$ **24.** Goes through $(2,1)$ and $(4,0)$

Write the following numbers in Scientific Notation.

Ch. 4

25. 678,100,000

26. 0.000036

Simplify.

27. $3^{-8} \cdot 3^{7}$

28. $\left(\frac{2a^2}{3b^4}\right)^{-2}$

29. $\left(\frac{4x^4y}{8x^{-3}yz^2}\right)^{3}$

30. $\frac{g^4m^3}{a^{-2}gm^7}$

Evaluate.

31. Evaluate $3x^3 - x^2 - x + 9$ when $x = -2$.

Multiply.

32. $(6x - 2y)(5x^2 - 3y)$

33. $(5x^3 - 7x)^2$

34. $(t + 3)(t^2 - 3t - 4)$

35. $(2h + 3)(2h - 3)$

Divide.

36. $(18x^6 - 27x^5 - 3x^3) \div (9x^3)$

Factor Completely.

Ch. 5

37. $x^6 - x^5 - 30x^4$

38. $m^2 + 5m + mt + 5t$

39. $16x^8 - 81$

40. $12a^2 + 84ab + 147b^2$

Solve.

41. $3x^2 + 8x = 9 + 2x$

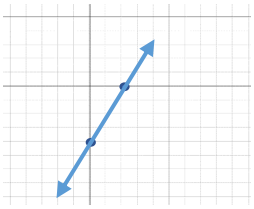
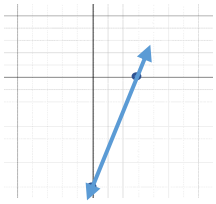
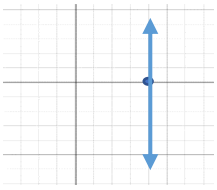
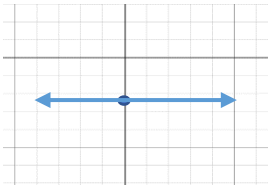
42. The height of a sail on a sailboat is 3 feet greater than the length of its "foot" (the base of the sail). The hypotenuse of the triangle formed by the sail is 15 feet long. Find the height of the sail and the length of its foot.

43. A rectangular picture is three times as long as it is wide. The area of the picture is 588 in.^{2} Find the dimensions of the picture.

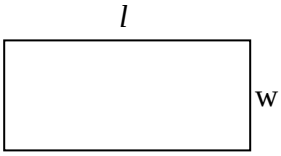
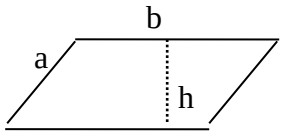
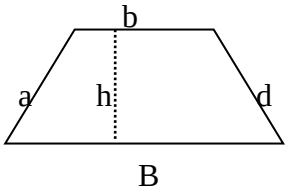
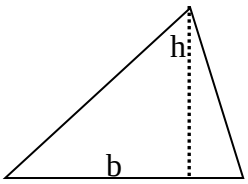
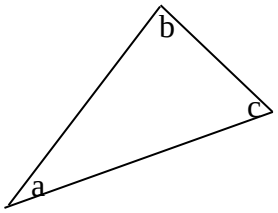
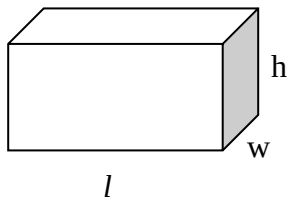
44. The product of two consecutive integers is 55 more than their sum. Find the integers. (Hint: There will be two sets of integers in the answer – negative integers and positive integers.)

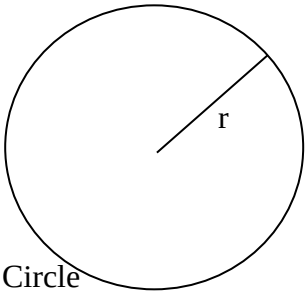
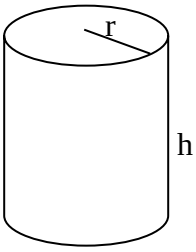
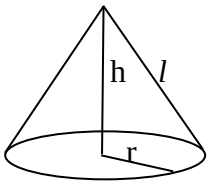
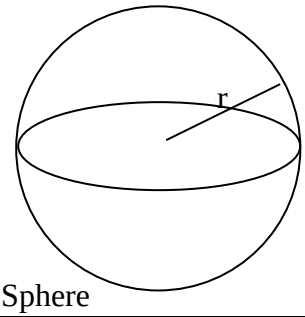
45. What are the x-intercepts of the graph of $y = x^2 + x - 12$?

Final Review Answers

1. $27=3^3$
2. $360 = 2^3 \cdot 3^2 \cdot 5$
3. 144
4. 60
5. $50.27\ in^3$.
6. 30
7. 3
8. 0
9. $-10y + 18$
10. $-\frac{4}{3}$
11. $x = -2$
12. $x = \frac{13}{2}\ or\ 6.5$
13. No Solution
14. $x \geq -15$
15. 5%
16. $c = 3x - a - b$
17. $\$505.44$
18. $1st: 39, 2nd: 117, 3rd: 24$
19. 
20. 
21. 
22. 
23. $y = -x + 3$
24. $y = -\frac{1}{2}x + 2$
25. 6.781×10^8
26. 3.6×10^{-5}
27. $\frac{1}{3}$
28. $\frac{9b^8}{4a^4}$
29. $\frac{x^{21}}{8z^6}$
30. $\frac{g^3a^2}{m^4}$
31. -17
32. $30x^3 - 18xy - 10x^2y + 6y^2$
33. $25x^6 - 70x^4 + 49x^2$
34. $t^3 - 13t - 12$
35. $4h^2 - 9$
36. $2x^3 - 3x^2 - \frac{1}{3}$
37. $x^4(x - 6)(x + 5)$
38. $(m + 5)(m + t)$
39. $(2x^2 - 3)(2x^2 + 3)(4x^4 + 9)$
40. $3(2a + 7b)^2$
41. $x = -3, 1$
42. foot $= 9$ ft., height $= 12$ ft.
43. width $= 14$ in., length $= 42$ in.
44. Negative integers: -7, -6
    Positive integers: 8, 9
45. (-4, 0) and (3, 0)

Shape Formulas

 Rectangle	$P = 2l + 2w$ $A = lw$	**P** is the Perimeter ***l*** is the length **w** is the width **A** is the Area
 Parallelogram	$P = 2a + 2b$ $A = bh$	**P** is the Perimeter **a** is a side length **b** is the other side length **A** is the Area
 Trapezoid	$P = b + a + B + d$ $A = \frac{1}{2}h(B + b)$	**P** is Perimeter **b** is the little base **B** is the Big base **a** is a leg **d** is a leg **A** is the Area
 Triangle	$P = s_1 + s_2 + s_3$ $A = \frac{1}{2}bh$	**P** is the Perimeter $\boldsymbol{s_1}$ is the 1st side, etc **A** is the Area
 Triangle	$a + b + c = 180$	**a** is one angle **b** is another angle **c** is another angle
 Rectangular Solid	$SA = 2lw + 2wh + 2lh$ $V = lwh$	***l*** is the length **h** is the height **w** is the width **SA** is the Surface Area **V** is Volume

 Circle	$C = 2\pi r$ $A = \pi r^2$	**C** is the Circumference or Perimeter **π** is a number, about 3.14159… it has a button on your calculator **r** is the radius of the circle **A** is the area inside the circle
 Cylinder	$LSA = 2\pi rh$ $SA = 2\pi rh + 2\pi r^2$ $V = \pi r^2 h$	**LSA** is Lateral Surface Area = Area just on the sides **h** is the height **SA** is total Surface Area **π** is a number, about 3.14159… it has a button on your calculator **r** is the radius of the circle **V** is Volume
 Cone	$LSA = \pi rl$ $SA = \pi r^2 + \pi rl$ $V = \frac{1}{3}\pi r^2 h$	**h** is the height **r** is the radius of the circle ***l*** is the slant height **π** is a number, about 3.14159… it has a button on your calculator **SA** is total Surface Area **LSA** is Lateral Surface Area = Area just on the sides **V** is the Volume
 Sphere	$SA = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$	**r** is the radius **SA** is the Surface Area **V** is the Volume