Mikro II - HO6

Jeppe Vanderhaegen

March 31, 2022

Exercise 1

We are giving the following utility function:

$$u(w) = w^{\frac{1}{2}}$$

An employee has an reservation utility of $u = 10$ and the starting wage is $w = 100$. This starting wage makes an employer meet the reservation utility from the beginning. We can compute the following:

$$u(100) = \sqrt{100} \Leftrightarrow$$

$$u(100) = 10$$

Therefore, the employer is indifferent between working and the reservation utility. We know the condition must be binding, thus we have the following equation:

$$u(w) \geq 10$$

This equation states, that the utility of working must be greater or equal to the best alternative for the employee.

a) We now introduce the probability of things going better or worse than planned. Both have an equal probability of 50%. When things go better than planned, the employee will obtain an bonus. We denote this as a. When things go worse than planned, the employee will we subtracted an amount. We denote this as b. Therefore, we can compute the following equation:

$$\frac{1}{2}(u(w+a)) + \frac{1}{2}(u(w-b)) \geq 10$$

When things go worse than planned, the managing director will set b to 10. We insert this:

$$\frac{1}{2}(u(w)+a) + \frac{1}{2}(u(w)-10) \geq 10$$

$$\frac{1}{2}(100+a)^{\frac{1}{2}} + \frac{1}{2}(90)^{\frac{1}{2}} \geq 10$$

$$
\begin{aligned}
\frac{1}{2}(100+a)^{\frac{1}{2}} + \frac{1}{2} * 9,49 &\geq 10 \\
(100+a)^{\frac{1}{2}} + 9,49 &\geq 20 \\
100+a &\geq 10,51^2 \\
100+a &\geq 110,46 \\
a &\geq 10,46
\end{aligned}
$$

Therefore, the bonus, when things go better than expected, has to be $a = 10,46$.

This shows $b = 10$ the bonus must be greater or equal to 10,46 for the employee to keep working. We can conclude, that the quantity needed to keep the employee happy is greater than 10. This is because the employee is risk avers, and has to be compensated extra because of the variance of the lottery. We, can solve for different levels of b

$$
\begin{aligned}
&\frac{1}{2}(u(w+a)) + \frac{1}{2}(u(w-b)) \geq 10 \Leftrightarrow \\
&(u(w+a)) + (u(w-b)) \geq 20 \Leftrightarrow \\
&(u(w+a)) \geq 20 - (u(w-b)) \Leftrightarrow \\
&(100+a)^{\frac{1}{2}} \geq 20 - (u(w-b)) \Leftrightarrow \\
&100+a \geq (20 - (100-b)^{\frac{1}{2}})^2 \Leftrightarrow \\
&a \geq (20 - (100-b)^{\frac{1}{2}})^2 - 100 \Leftrightarrow
\end{aligned}
$$

We can now insert different values for b: 20,....,100

$$
\begin{aligned}
a &\geq (20 - (100-20)^{\frac{1}{2}})^2 - 100 = 22 \\
a &\geq (20 - (100-30)^{\frac{1}{2}})^2 - 100 = 35 \\
a &\geq (20 - (100-40)^{\frac{1}{2}})^2 - 100 = 50 \\
a &\geq (20 - (100-50)^{\frac{1}{2}})^2 - 100 = 67
\end{aligned}
$$

These results indicate the compensation needed is exponentially growing when b increases. The reason is, that the variance gets bigger, with increasing values of b. For the IR to be satisfied the compensation has to greater than the loss. We see, that the curve is constant increasing,

$$
\frac{\partial^2 a}{\partial^2 b} > 0
$$

b) From **a)** we see that the employee has quasi-linear utility, which implies the employee is risk avers. Assuming the firm is risk-neutral, an efficient allocation for in this PA-problem would be for the firm to take on all risk. Securing insurance through their wage policy for the employee to keep a constant wage and therefore level of utility, would keep the IR condition satisfied.