Microeconomics III, Ex. Class 7: Problem Set 2[a]

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Outline

PS2, Ex. 1 (A): A Beautiful Mind

As a "Game of Chicken":

PS2, Ex. 1: A Beautiful Mind

As a "Game of Chicken":

Hansen

	Brunette	Blonde
Brunette	1, 1	1, 2
Blonde	2, 1	0, 0

John Nash

Nash Equilibria (NE):
{(*Brunette*, *Blonde*); (*Blonde*, *Brunette*)}

As a "Game of Chicken":

		Hansen	
		Brunette	Blonde
John Nash	Brunette	1, 1	1, 2
	Blonde	2, 1	0, 0

Nash Equilibria (NE):
$\{(Brunette, Blonde); (Blonde, Brunette)\}$
Pareto Optimal (PO):
$\{(Brunette, Blonde); (Blonde, Brunette)\}$

As a "Game of Chicken":

Hansen

		Brunette	Blonde
John Nash	Brunette	1, 1	1, 2
	Blonde	2, 1	0, 0

Nash Equilibria (NE):
$\{(Brunette, Blonde); (Blonde, Brunette)\}$
Pareto Optimal (PO):
$\{(Brunette, Blonde); (Blonde, Brunette)\}$

Can it be modelled to better reflect Hansen's view?

> "Nash, this is some way for you to get the blonde on your own... You can go to hell."
>
> ⸻
>
> *- Hansen*

Two ways of viewing the game depending on the degree of envy.

As a "Game of Chicken" (no envy):   As a "Prisoners' Dilemma" (high envy):

| | | Hansen | |
		Brunette	Blonde
John Nash	Brunette	1, 1	1, 2
	Blonde	2, 1	0, 0

Nash Equilibria (NE):
$\{(Brunette, Blonde); (Blonde, Brunette)\}$
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John Nash (row labels)

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$\{(Brunette, Blonde); (Blonde, Brunette)\}$
Pareto Optimal (PO):
$\{(Brunette, Blonde); (Blonde, Brunette)\}$

As a "Prisoners' Dilemma" (high envy):

	Hansen	
	Brunette	Blonde
Brunette	1, 1	-1, 2
Blonde	2, -1	0, 0

John Nash (row labels)

NE:
$\{(Blonde, Blonde)\}$

Two ways of viewing the game depending on the degree of envy.

As a "Game of Chicken" (no envy):

		Hansen	
		Brunette	Blonde
John Nash	Brunette	1, 1	1, 2
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Nash Equilibria (NE):
{(*Brunette*, *Blonde*); (*Blonde*, *Brunette*)}
Pareto Optimal (PO):
{(*Brunette*, *Blonde*); (*Blonde*, *Brunette*)}

As a "Prisoners' Dilemma" (high envy):

		Hansen	
		Brunette	Blonde
John Nash	Brunette	1, 1	-1, 2
John Nash	Blonde	2, -1	0, 0

NE:
{(*Blonde*, *Blonde*)}
PO:
Anything but {(*Blonde*, *Blonde*)}

Def. Pareto Optimal: No player can be better off without the other being worse off.

PS2, Ex. 2 (A): Nash Equilibria

a) $NE = \{(T, L); (B, R)\}$

	L	R
T	9, 9	0, 0
B	0, 0	8, 8

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	L	R
T	9, 9	0, 0
B	0, 0	8, 8

b) $NE = \{(T, L); (B, R)\}$

	L	R
T	9, 9	0, 7
B	7, 0	8, 8

a) $NE = \{(T, L); (B, R)\}$

	L	R
T	9, 9	0, 0
B	0, 0	8, 8

b) $NE = \{(T, L); (B, R)\}$

	L	R
T	9, 9	0, 7
B	7, 0	8, 8

c) In a) $\{T, L\}$ more reasonable as it's a pareto improvement of $\{B, R\}$.
   However, $\{B, R\}$ could be more likely in b) if both players are risk averse.

PS2, Ex. 3 (A): Elimination of weakly dominated strategies

a) As opposed to IESDS, the order of elimination is crucial for iterated elimination of *weakly* dominated pure strategies.

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Example:

		Player 2	
		a	b
Player 1	A	0, 1	0, 0
	B	0, 0	1, 0

1st step: Initially Player 1 can eliminate A or Player 2 can eliminate b.

a) As opposed to IESDS, the order of elimination is crucial for iterated elimination
of *weakly* dominated pure strategies.

Example:

Player 2

		a	b
Player 1	A	0, 1	0, 0
	B	0, 0	1, 0

1st step: Initially Player 1 can eliminate A or Player 2 can eliminate b.

2nd step: In the reduced form game the other player can eliminate any strategy.

b) $NE = \{(A, a); (B, a); (B, b)\}$

	Player 2	
Player 1	a	b
A	0, 1	0, 0
B	0, 0	1, 0

b) $NE = \{(A, a); (B, a); (B, b)\}$

		Player 2	
		a	b
Player 1	A	0, 1	0, 0
	B	0, 0	1, 0

(B, a) is a NE as there is no incentive to deviate as either player is indifferent given the strategy of the other:

$$u_1(B, a) \geq u_1(A, a) \Leftrightarrow 0 \geq 0$$
$$u_2(B, a) \geq u_2(B, b) \Leftrightarrow 0 \geq 0$$

The (non-strict) inequalities can be turned around to explain the other equilibria:

$$u_1(B, a) \leq u_1(A, a) \Leftrightarrow 0 \leq 0$$
$$u_2(B, a) \leq u_2(B, b) \Leftrightarrow 0 \leq 0$$

b) $NE = \{(A, a); (B, a); (B, b)\}$

	Player 2	
	a	b
A	0, 1	0, 0
B	0, 0	1, 0

Player 1

(B, a) is a NE as there is no incentive to deviate as either player is indifferent given the strategy of the other:

$$u_1(B, a) \geq u_1(A, a) \Leftrightarrow 0 \geq 0$$
$$u_2(B, a) \geq u_2(B, b) \Leftrightarrow 0 \geq 0$$

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$$u_2(B, a) \leq u_2(B, b) \Leftrightarrow 0 \leq 0$$

As opposed to IESDS, iterated elimination of *weakly* dominated pure strategies can eliminate NE. E.g. Player 1 can eliminate A or Player 2 can eliminate b, eliminating one NE and giving us one of the reduced form games:

b) $NE = \{(A, a); (B, a); (B, b)\}$

	Player 2	
Player 1	a	b
A	0, 1	0, 0
B	0, 0	1, 0

(B, a) is a NE as there is no incentive to deviate as either player is indifferent given the strategy of the other:

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	a	b
B	0, 0	1, 0

	a
A	0, 1
B	0, 0

And another NE can be eliminated by further reducing to just one of the NE:

b) $NE = \{(A, a); (B, a); (B, b)\}$

		Player 2	
		a	b
Player 1	A	0, 1	0, 0
	B	0, 0	1, 0

(B, a) is a NE as there is no incentive to deviate as either player is indifferent given the strategy of the other:

$$u_1(B, a) \geq u_1(A, a) \Leftrightarrow 0 \geq 0$$
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	a	b
B	0, 0	1, 0

	a
A	0, 1
B	0, 0

And another NE can be eliminated by further reducing to just one of the NE:

	a
B	0, 0

	b
B	1, 0

	a
A	0, 1

PS2, Ex. 4-5: More Nash Equilibria

4. Find all Nash equilibria in the following game:

Player 2

		a	b	c
Player 1	A	7, 7	3, 0	1, 6
	B	2, 8	5, 4	9, 3
	C	3, 0	5, 4	2, 1

5. Find the Nash equilibrium in the following game:

Player 2

		A	B
Player 1	A	100, 100	1, 99
	B	99, 1	0, 0

Find the NE after adding 2 to the payoff of strategy B for each player:

Player 2

		A	B
Player 1	A	100, 100	1, 101
	B	101, 1	2, 2

Comment. What has changed?

4. Find all Nash equilibria in the following game:

<div style="text-align:center">Player 2</div>

	a	b	c
A	7, 7	3, 0	1, 6
B	2, 8	5, 4	9, 3
C	3, 0	5, 4	2, 1

Player 1

4. Find all Nash equilibria in the following game:

<div align="center">

Player 2

		a	b	c
Player 1	A	7, 7	3, 0	1, 6
	B	2, 8	5, 4	9, 3
	C	3, 0	5, 4	2, 1

</div>

$NE = \{(A, a); (C, b)\}$ as there is no incentive to deviate for any player, e.g.

$$u_1(C, b) \geq u_1(B, b) \Leftrightarrow 5 \geq 5$$

Player 1 is indifferent between (A, b) and (B, b), but Player 1 knows that Player 2's best response to B would be a.

4. Find all Nash equilibria in the following game:

Player 2

		a	b	c
Player 1	A	7, 7	3, 0	1, 6
	B	2, 8	5, 4	9, 3
	C	3, 0	5, 4	2, 1

$NE = \{(A, a); (C, b)\}$ as there is no incentive to deviate for any player, e.g.

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5. Find the Nash equilibrium in the following game:

Player 2

		A	B
Player 1	A	100, 100	1, 99
	B	99, 1	0, 0

Find the NE after adding 2 to the payoff of strategy B for each player:

Player 2

		A	B
Player 1	A	100, 100	1, 101
	B	101, 1	2, 2

Comment. What has changed?

4. Find all Nash equilibria in the following game:

Player 2

		a	b	c
	A	7, 7	3, 0	1, 6
Player 1	B	2, 8	5, 4	9, 3
	C	3, 0	5, 4	2, 1

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Player 1 is indifferent between (A, b) and (B, b), but Player 1 knows that Player 2's best response to B would be a.

5. Find the Nash equilibrium in the following game:

Player 2

		A	B
	A	100, 100	1, 99
Player 1	B	99, 1	0, 0

$NE = \{(A, A)\}$

4. Find all Nash equilibria in the following game:

<div align="center">Player 2</div>

	a	b	c
A	7, 7	3, 0	1, 6
B	2, 8	5, 4	9, 3
C	3, 0	5, 4	2, 1

Player 1 (rows A, B, C)

$NE = \{(A, a); (C, b)\}$ as there is no incentive to deviate for any player, e.g.

$$u_1(C, b) \geq u_1(B, b) \Leftrightarrow 5 \geq 5$$

Player 1 is indifferent between (A, b) and (B, b), but Player 1 knows that Player 2's best response to B would be a.

5. Find the Nash equilibrium in the following game:

<div align="center">Player 2</div>

	A	B
A	100, 100	1, 99
B	99, 1	0, 0

Player 1 (rows A, B)

$NE = \{(A, A)\}$

<div align="center">Player 2</div>

	A	B
A	100, 100	1, 101
B	101, 1	2, 2

Player 1 (rows A, B)

Comment. What has changed?

4. Find all Nash equilibria in the following game:

Player 2

		a	b	c
Player 1	A	7, 7	3, 0	1, 6
	B	2, 8	5, 4	9, 3
	C	3, 0	5, 4	2, 1

$NE = \{(A, a); (C, b)\}$ as there is no incentive to deviate for any player, e.g.

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5. Find the Nash equilibrium in the following game:

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$NE = \{(A, A)\}$

Find the NE after adding 2 to the payoff of strategy B for each player:

Player 2

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Player 1	A	100, 100	1, 101
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$NE = \{(B, B)\}$

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5. Find the Nash equilibrium in the following game:

Player 2

		A	B
Player 1	A	100, 100	1, 99
	B	99, 1	0, 0

$NE = \{(A, A)\}$
Find the NE after adding 2 to the payoff of strategy B for each player:

Player 2

		A	B
Player 1	A	100, 100	1, 101
	B	101, 1	2, 2

$NE = \{(B, B)\}$
Now B is strictly dominated by A instead.
$NE = \{B, B\}$ as coordination on $\{A, A\}$ isn't credible due to incentive to deviate.

PS2, Ex. 6: Cournot equilibrium

There are two bakeries in the same village. Every morning, they simultaneously decide how many breads to produce. Denote the quantities they produce by q_1 and q_2. The price for which they can sell the bread is a function of the overall quantity, such that $p = 11 - (q_1 + q_2)$. The cost of producing one bread is 2.

a) Compute the quantities in the Cournot equilibrium, i.e., the Nash Equilibrium of the game where the firms simultaneously choose quantities.

b) Draw a diagram of the best response functions, and check whether they really intersect in the Nash Equilibrium.

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a) Compute the quantities in the Cournot equilibrium, i.e., the Nash Equilibrium of the game where the firms simultaneously choose quantities.

b) Draw a diagram of the best response functions, and check whether they really intersect in the Nash Equilibrium.

Cookbook (general, not just for bread):

1. Write up the payoff (profit) functions and take the derivative with respect to one's quantity.

2. From the first order conditions (FOCs) find the Best-Response (BR) functions given the quantity chosen by the other player.

3. Substitute in the BR quantity for the other player to solve for the equilibrium quantities.

a) Quantities in the Cournot equilibrium

Inverse market demand is given as:

$$p = 11 - (q_1 + q_2) \qquad (1)$$

The payoff (profit) for firm i with constant marginal cost $c = 2$ is:

$$\begin{aligned}
\pi_i &= (p - c)q_i \\
&= (11 - q_i - q_j - 2)q_i \\
&= (9 - q_i - q_j)q_i \text{ for } i \in \{1, 2\} \qquad (2)
\end{aligned}$$

FOC of the payoff function (2) for firm i:

$$\frac{\delta \pi_i}{\delta q_i} = 9 - 2q_i - q_j = 0$$

$$\Rightarrow q_i = \frac{9 - q_j}{2} \qquad (3)$$

This is firm i's Best-Response (BR) function given the quantity chosen by the other firm j.

Due to the symmetry of the payoff functions the Best-Response quantity $q_i^* = q_j^*$ which we insert in (3) to find the equilibrium quantities:

$$q_i^* = \frac{9 - q_i^*}{2}$$

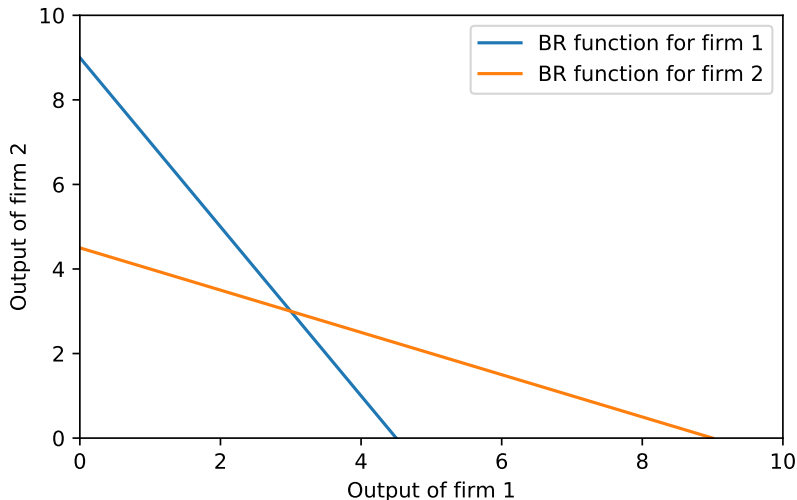$$\frac{3}{2}q_i^* = \frac{9}{2}$$

$$q_i^* = \frac{2}{3} \cdot \frac{9}{2}$$

$$q_i^* = 3 \qquad (4)$$

I.e. the Cournot/Nash equilibrium is:

$$q_i^* = q_j^* = 3 \equiv q^{NE}$$

b) To plot the intersection we need:

- The inverse of the BR function for firm 1: $q_1(q_2) = \frac{9-q_2}{2} \Rightarrow q_2 = 9 - 2q_1$
- And the BR function for firm 2: $q_2 = \frac{9-q_1}{2}$

PS2, Ex. 7: Letter from an American foundation

Heidi and Jakup have both received a letter from an American foundation:

We are willing to give the two of you one million US dollars in total. There are, however, some conditions involved. Take a pen and a paper, write down how much you would like to get and send it to us in a sealed envelope. If the sum of your two claims is equal to or below one million, you get exactly what you have asked for. Otherwise you get punished for being greedy - none of you gets a single cent.

Unfortunately, they do not know how to get hold of each other before answering, so there is no chance for them to communicate before sending the envelope.

a) What would you write, if you were one of them?

b) Formalize the game. Find all the pure-strategy Nash equilibria.

c) What do you think, which of the equilibria is much more likely to be played in reality? Does it correspond to your answer to a)?

d) If they had time to communicate with each other, what do you think would happen? What would you do, if you were one of them?

e) Now assume that Heidi could send a message to Jakup, but Jakup would not be able to answer. What advice would you give to Heidi?

b) The normal form game:

Players:

$$I = \textit{Heidi}, \textit{Jakup}$$

Strategy set:

$$S_i = \{0; 0.01; ...; 99999.99; 1000000\}$$

Payoffs for player $i \neq j$:

$$u_i(s_i, s_j) = \left\{ \begin{array}{ll} s_i & \text{if} \quad s_i + s_j \leq 1000000 \\ 0 & \text{if} \quad s_i + s_j > 1000000 \end{array} \right.$$

Pure Strategy Nash Equilibria (PSNE):

$$\{(s_1; s_2) \in S_1 \times S_2 : s_1 + s_2 = 1000000\}$$

c) What do you think, which of the equilibria is much more likely to be played in reality? Does it correspond to your answer to a)?

d) If they had time to communicate with each other, what do you think would happen? What would you do, if you were one of them?

e) Now assume that Heidi could send a message to Jakup, but Jakup would not be able to answer. What advice would you give to Heidi?

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Players:

$$I = Heidi, Jakup$$

Strategy set:

$$S_i = \{0; 0.01; ...; 99999.99; 1000000\}$$

Payoffs for player $i \neq j$:

$$u_i(s_i, s_j) = \begin{cases} s_i & \text{if} \quad s_i + s_j \leq 1000000 \\ 0 & \text{if} \quad s_i + s_j > 1000000 \end{cases}$$

Pure Strategy Nash Equilibria (PSNE):

$$\{(s_1; s_2) \in S_1 \times S_2 : s_1 + s_2 = 1000000\}$$

e) Now assume that Heidi could send a message to Jakup, but Jakup would not be able to answer. What advice would you give to Heidi?

Heidi can either play it safe by sending the message that she will write 500000.

Or try to pressure Jakup to accept an unequal equilibrium, e.g. send the message "I will write 800000, you can write 200000 or get nothing."

PS2, Ex. 8: Stopping the bike thief

There are *n* people observing someone trying to steal a parked bike. Each of the witnesses would like the thief to be stopped, but prefers not to do it him/herself (because it is unpleasant and perhaps even dangerous). More precisely, if the thief is stopped by someone else, each of the witnesses gets a utility of $v > 0$. Every person who stops the thief gets a utility of $v - c > 0$, where c is the cost of interaction with the thief. Finally, if nobody stops the thief and the bike gets stolen, every witness gets a utility of 0. The witnesses decide whether or not to stop the thief simultaneously and independently.

a) Write the game in normal form.

b) Describe all pure-strategy Nash equilibria of the game (an informal description is sufficient). It may be a good idea to start with the case $n = 2$.

a) Write the game in normal form:

Players: $I = 1, 2, ..., n$
Strategy set: $S_i = \{$Do nothing; Stop the thief$\}$
Payoffs for player $i \neq j$:

$$u_i(s_i, s_j) = \begin{cases} v > 0 & \text{if I do nothing and someone else stops the thief} \\ v - c > 0 & \text{if I stop the thief} \\ 0 & \text{if nobody stops the thief} \end{cases}$$

b) Describe all pure-strategy Nash equilibria of the game (an informal description is sufficient). It may be a good idea to start with the case $n = 2$.

PS2, Ex. 8: Stopping the bike thief

a) Write the game in normal form:

Players: $I = 1, 2, ..., n$
Strategy set: $S_i = \{$Stop the thief; Do nothing$\}$
Payoffs for player $i \neq j$:

$$u_i(s_i, s_j) = \begin{cases} v > 0 & \text{if I do nothing and someone else stops the thief} \\ v - c > 0 & \text{if I stop the thief} \\ 0 & \text{if nobody stops the thief} \end{cases}$$

b) Describe all pure-strategy Nash equilibria of the game (an informal description is sufficient). It may be a good idea to start with the case $n = 2$.

If $c > 0$, there exist n equilibria where exactly one person stops the thief, e.g.:

		Player 2	
		Stop the thief	Do nothing
Player 1	Stop the thief	$v - c$; $v - c$	$v - c$; v
	Do nothing	v ; $v - c$	0 ; 0