Assignment #2

Econometric II

Approx. 11500 characters

October 25, 2022

Introduction

¨ In this assignment we are examining if Tobin's Q drive changes in residential investment or visa versa in Denmark. We have a time series sample from the early 70'ties to just before the corona-lockdown. We used a VAR model to analyze the dynamic relationship between between Tobin's Q and changes in residential investments.
We found that Tobin's Q does in fact Granger cause residential investments in the Danish market.

Econometric Theory

VAR model

The VAR (Vector Autoregressive) model describes the dynamic evolution of a number of variables from their common history. Our model consists of two variables, **Inv** and **Q**, thus our model is a two-dimensional vector $Z_t = (\mathbf{\Delta_4 Inv_t}, \mathbf{\Delta_4 Q_t})' \in \mathbb{R}^2$.

$$\begin{pmatrix} \Delta_4 Inv_t \\ \Delta_4 Q_t \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \Pi_{11}^1 & \Pi_{12}^1 \\ \Pi_{21}^1 & \Pi_{22}^1 \end{pmatrix} \begin{pmatrix} \Delta_4 Inv_{t-1} \\ \Delta_4 Q_{t-1} \end{pmatrix} + \begin{pmatrix} \Pi_{11}^2 & \Pi_{12}^2 \\ \Pi_{21}^2 & \Pi_{22}^2 \end{pmatrix} \begin{pmatrix} \Delta_4 Inv_{t-2} \\ \Delta_4 Q_{t-2} \end{pmatrix}$$
$$+ \ldots + \begin{pmatrix} \Pi_{11}^k & \Pi_{12}^k \\ \Pi_{21}^k & \Pi_{22}^k \end{pmatrix} \begin{pmatrix} \Delta_4 Inv_{t-k} \\ \Delta_4 Q_{t-k} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}, \quad t = 1, 2, \ldots T \qquad (1)$$

From above we see that all explanatory variables are predetermined(endogenous), thus there is no prior assumptions of the contemporaneous effects between the endogenous variables. .

Contemporanous effects

The contemporaneous effect are measured through the correlation of the error terms. More specifically it is given by the covariance matrix of the error terms:

$$E_{t-1}\left(\begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} (\epsilon_{1t}, \epsilon_{2t})\right) = \begin{pmatrix} E_{t-1}(\epsilon_{1t}\epsilon_{1t}) & E_{t-1}(\epsilon_{1t}\epsilon_{2t}) \\ E_{t-1}(\epsilon_{2t}\epsilon_{1t}) & E_{t-1}(\epsilon_{2t}\epsilon_{2t}) \end{pmatrix} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} = \Omega, \quad (2)$$

Where by symmetry $\Omega_{12} = \Omega_{21}$. Which is the covariance between ϵ_{1t} and ϵ_{2t}. Whilst the diagonal is the covariance between a given error term and itself, thus it is the variance of each of the error terms.

Stationarity condition

From **Theorem 6.2** from Nielsen (2020a). We have that the var(k)-model is stable and that Z_t is stationary and weakly dependent if the eigenvalues of the *companion matrix*:

$$\begin{pmatrix} \Pi_1 & \Pi_2 & \Pi_3 & \cdots & \Pi_k \\ I_p & 0 & 0 & \cdots & 0 \\ 0 & I_p & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & I_p & 0 \end{pmatrix} \tag{3}$$

All are inside the unit circle. Thus $\max\{|\boldsymbol{\lambda}|\} < 1$ where $\boldsymbol{\lambda}$ is a vector of all eigenvalues.

We have that if the model is stationary and weekly dependent then the MLEs are consistent and normally distributed. Furthermore the test statistics will have standard normal and χ^2 distributions.

Maximum Likelihood Estimation

For $Z_t = (z_{1t}, \ldots, z_{pt})' \in \mathbb{R}^p$ and the parameters $\theta = \{\mu, \Pi^i, \Omega\}$ Then the maximum likelihood estimator for the var(k)-model is given by:

$$\hat{\theta} = \arg\max_{\theta} \log L\left(\theta \mid Z_1, \ldots Z_T\right) \tag{4}$$

Where the log likelihood function is given by:

$$\log L(\theta) = -\frac{Tp}{2}\log(2\pi) - \frac{T}{2}\log|\Omega| - \frac{1}{2}\sum_{t=1}^{T}\epsilon_t(\theta)'\Omega^{-1}\epsilon_t(\theta) \tag{5}$$

For the MLEs to be asymptotically gaussian distributed and to allow for standard inference we assume that $\epsilon_t|\mathcal{I}_{t-1} \sim i.i.d.N(0,\Omega)$. Thus we do not assume normality of errors.

If it is the case, through testing, that we find no normality of error or we find the presence of heteroskedacity or both. Then we opt to use Quasi-Maximum Likelihood (QMLE). This is done by using "robust" test statistics. The QMLE does not ensure efficient estimators as the MLE does, but under the staionarity condition is still produces consistent estimates.

Granger Causality

A series x is said to be Granger causal for y, if the forecast error obtained by forecasting y_{T+h} on $\mathcal{I}_T = \{y_T, x_T, y_{T-1}, x_{T-1}, ..\}$ is less than the forecast error obtained only forecasting on $\tilde{\mathcal{I}}_T = \{y_T, y_{T-1}\}$.

Testing Granger-causality

We can test the hypothesis of no-Granger causality by applying a restriction for the given parameter we want to test for. Given equation **1**, and if we want to test if $\Delta_4 Q_t$ Granger causes $\Delta_4 Inv_t$ we can test the null hypotheses against the alternative hypthethis:

$$H_0: \;\; \Delta_4 Q_t \nrightarrow \Delta_4 \text{Inv}_t: \;\; \Pi_{12}^i = 0 \text{ for } i = 1, 2, \ldots, k \tag{6}$$

$$H_A: \;\; \Delta_4 Q_t \rightarrow \Delta_4 \text{Inv}_t: \;\; \Pi_{12}^i \neq 0 \text{ for } i = 1, 2, \ldots, k \tag{7}$$

The null hypothesis can then be tested using either likelihood ratio tests or Wald test. For which both statistics will be a $\chi^2(k)$. If we then reject the null hypothesis we accept the alternative hypothesis instead and say that $\Delta_4 Q_t$ does Granger causes $\Delta_4 Inv_t$.

Misspecification tests

Below is a overview of our different test:

Test for misspecification				
	Autocorrelation (LM)	Vector Hetero	Hetero-X	Normality test
H_0	no autocorrelation	Homoscedasticity	Homoscedasticity	Normality
H_A	Autocorrelation in error term	Heteroscedasticity	Heteroscedasticity	Non-normal dist. of error term
Asymptotic distribution	F(20,322)	F(162,375)	F(54,480)	$\text{Chi}^2(4)$

Lag determination

In this given paper we utilize **general to specific**. Thus we need to determine if we can remove certain lags from our model. To determine the number of lags we calculate the likelihood ratio statistics

$$LR(k = i - \frac{r}{p} | k = i) \overset{d}{=} \chi^2(r) \quad \text{where r=number of restrictions \& p=dimensions} \tag{8}$$

as twice the difference in log-likelihoods. For our case we have p=2.

Trough *Oxmetrics* this is done using *Exclusion restrictions* from the test menu.

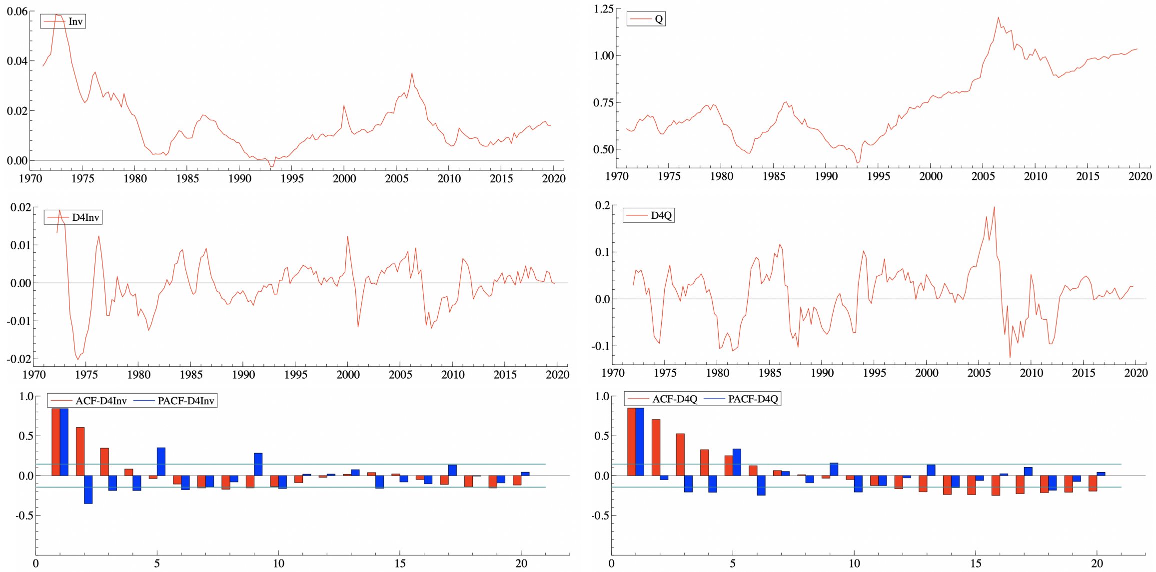

Figure 1: Graphs of data

Description of Data

In this assignment we examine quarterly data fro real residential investment and Tobin's Q. Where *Inv* is the real residential investment divided with housing stock the previous quarter and Q is the house prices divided by residential investment deflator$(2010 = 1)$. The data has been taken from the model *Mona*, which is maintained by the Danish Central Bank.

We have the following defined transformed variables:

- $\Delta_4 \text{Inv}_t = \text{Inv}_t - \text{Inv}_{t-4}$
- $\Delta_4 \text{Q}_t = \text{Q}_t - \text{Q}_{t-4}$

Below are plots of the data and the ACF and PACF of the variables D4Inv and D4Q.

Empirical Analysis

Model Selection

We using a GETS (general to specific) method and we have selected to use a VAR model with 9 lags, that is two years with addition of a lag. The first lag might be affected by the 4 as we have quarterly data and then the present lag might be previous lag, which is why we add an extra lag.

We then remove lags that are not significant in regards to their F-tests for both $\Delta_4 Inv$ and $\Delta_4 Q$. Thereafter we test with the option in Oxmetrics '*Exclusion Restriction*' under the test menu to exclude individual lags for the respective variables. We arrive at a VAR model where we keep lag 1,4,5,8 and 9 for the variable $\Delta_4 Inv$ and lag 1,4,5 and 6 for the variable $\Delta_4 Q$.

Model Estimation results

From the previous section we described our model selection and we can now write the model, as in equation (1) under the econometric theory section, with our outputs from Oxmetrics:

$$
\begin{aligned}
\begin{pmatrix} \Delta_4\text{Inv}_t \\ \Delta_4 Q_t \end{pmatrix} &= \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \underset{(0.0475)}{0.9143} & \underset{(0.0040)}{0.0120} \\ \underset{(0.5753)}{-0.019} & \underset{0.0491}{0.9641} \end{pmatrix} \begin{pmatrix} \Delta_4\text{Inv}_{t-1} \\ \Delta_4 Q_{t-1} \end{pmatrix} + \\
& \begin{pmatrix} \underset{(0.0777)}{-0.6742} & \underset{(0.0064)}{0.0023} \\ \underset{(0.9411)}{-0.0059} & \underset{(0.0780)}{-0.4577} \end{pmatrix} \begin{pmatrix} \Delta_4\text{Inv}_{t-4} \\ \Delta_4 Q_{t-4} \end{pmatrix} + \begin{pmatrix} \underset{(0.0825)}{0.6474} & \underset{(0.0075)}{0.0060} \\ \underset{(0.9994)}{0.8325} & \underset{(0.0909)}{0.5159} \end{pmatrix} \begin{pmatrix} \Delta_4\text{Inv}_{t-5} \\ \Delta_4 Q_{t-5} \end{pmatrix} + \\
& \begin{pmatrix} \underset{(0)}{0} & \underset{(0.0056)}{-0.0176} \\ \underset{(0)}{0} & \underset{(0.0688)}{-0.2877} \end{pmatrix} \begin{pmatrix} \Delta_4\text{Inv}_{t-6} \\ \Delta_4 Q_{t-6} \end{pmatrix} + \begin{pmatrix} \underset{(0.0728)}{-0.3820} & \underset{(0)}{0} \\ \underset{(0.8811)}{0.6136} & \underset{(0)}{0} \end{pmatrix} \begin{pmatrix} \Delta_4\text{Inv}_{t-8} \\ \Delta_4 Q_{t-8} \end{pmatrix} + \\
& \begin{pmatrix} \underset{(0.064)}{0.2606} & \underset{(0)}{0} \\ \underset{(0.7837)}{-0.2091} & \underset{(0)}{0} \end{pmatrix} \begin{pmatrix} \Delta_4\text{Inv}_{t-9} \\ \Delta_4 Q_{t-9} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}, \quad t = 1, 2, \ldots T
\end{aligned}
$$

Misspecification tests

The results of our misspecification test are given by the table below:

Results of misspecifications tests			
AR	Hetero	Hetero-X	Normality
1.4856 (0.0838)	2.1103 (0.000**)	1,6361 (0.0001**)	39.803 (0.000**)

As we can see from the result we cannot reject the H_0 for the AR test but we reject

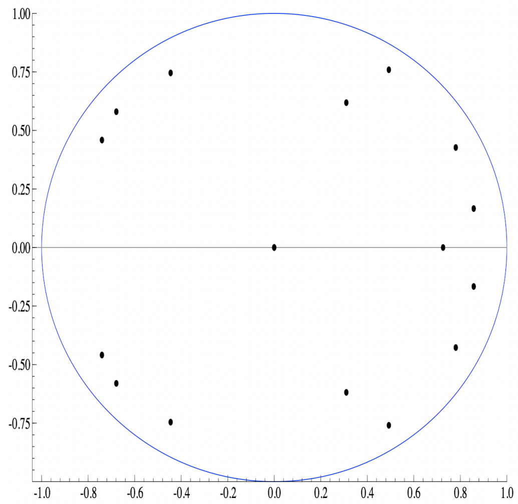

Figure 2: Plotted roots of companion matrix

all other tests. The model suffers from homoscadasticity and we don't have normality. The most important aspect is that the test for no autocorrelation is not rejected thus we can be fairly certain that the estimates are consistent. As we have heteroscadasticity we are using robust errors.

Stationarity

To test for stationarity we follow the procedure in section **Stationarity condition**. From figure **2** we see that all the eigenvalues of the companion matrix are within the unit circle. Thus the model is staionary and weakly dependent. To see the actual eigenvalues of the companion matrix please refer to table **1**.

Test for Granger Causality

We want to test tyhe Granger-causality between Tobin's Q and residential investment. More specifically we want to test if $\Delta_4 Q_t$ Granger causes $\Delta_4 Inv_t$. We follow the procedure of section **Testing Granger-causality** where we use a LR-test with a $\chi^2(4)$-distribution. Since we have 4 lags of $\Delta_4 Q_t$. With a 5%-level the critical value is 9.488. To test if $\Delta_4 Inv_t$ Granger causes $\Delta_4 Q_t$. We use a LR-test with a $\chi^2(5)$-distribution(5 lags of $\Delta_4 Inv_t$). Which with a 5%-level has a critical value of 11.070.

Does Tobin's Q Granger cause residential investments

$$H_0: \ \Delta_4 Q_t \nrightarrow \Delta_4 \text{Inv}_t: \ \Pi_{12}^1 = \Pi_{12}^4 = \Pi_{12}^5 = \Pi_{12}^6 = 0 \tag{9}$$

$$H_A: \ \Delta_4 Q_t \rightarrow \Delta_4 \text{Inv}_t: \ \Pi_{12}^1 = \Pi_{12}^4 = \Pi_{12}^5 = \Pi_{12}^6 \neq 0 \tag{10}$$

$$LR(\Delta_4 Q_t \nrightarrow \Delta_4 \text{Inv}_t) = 23.146 > 9.488 \tag{11}$$

From above we can, with 5% confidence level reject the null hypothesis that $\Delta_4 Q_t \nrightarrow \Delta_4 \text{Inv}_t$ and accept the alternative hypothesis. Therefore we can conclude that Tobin's Q does Granger cause residential investments.

Does residential investments Granger cause Tobin's Q

$$H_0: \ \Delta_4 \text{Inv}_t \nrightarrow \Delta_4 Q_t: \ \Pi_{21}^1 = \Pi_{21}^4 = \Pi_{21}^5 = \Pi_{21}^8 = \Pi_{21}^9 = 0 \tag{12}$$

$$H_A: \ \Delta_4 \text{Inv}_t \rightarrow \Delta_4 Q_t: \ \Pi_{21}^1 = \Pi_{21}^4 = \Pi_{21}^5 = \Pi_{21}^8 = \Pi_{21}^9 \neq 0 \tag{13}$$

$$LR(\Delta_4 \text{Inv}_t \nrightarrow \Delta_4 Q_t) = 6.4460 \ngtr 11.070 \tag{14}$$

From above we can not, with 5% confidence level, reject the null hypothesis that $\Delta_4 \text{Inv}_t \nrightarrow \Delta_4 Q_t$. Thus we can't conclude that residential investments Granger causes Tobin'Q.

Contemporaneous Effects

From our empirical results we get that the contemporaneous effect between $\Delta_4 \text{Inv}_t$ and $\Delta_4 Q_t$ is $\hat{\rho} = Corr(\epsilon_{1t}, \epsilon_{2t}) = 0.34611$. To test if this is significant different from zero, the test statistic is given by:

$$t = \frac{\hat{\rho}}{se(\hat{\rho})} = \frac{\hat{\rho}}{\frac{1}{\sqrt{T}}} = \frac{0.34611}{\frac{1}{\sqrt{182}}} = 4.6693 \tag{15}$$

If we assume that $r\hat{h}o$ is assymptotically normal distributed. Then the t-statistic becomes standard Gaussian. For which with a 5% confidence level, we get that the critical value is 1.645. Our calcualted test-statistic is 4.6693 thus larger than 1.645. Which means that the contemporanoeus effect between $\Delta_4 \text{Inv}_t$ and $\Delta_4 Q_t$ is significantly different from 0.

Discussion

From the misspecification test we concluded that we our model suffers from no normality of the error terms. To overcome the issue with normality we used recursive estimation to investigate the behaviour of the residuals. We identified several outliers which we tried to dummy out to correct the issue with normality but without success. We therefore accept that the model doesn't not satisfy the assumptions of normality. Since the model is stationary, we have chosen to use quasi maximum log-likelihood estimator (QMLE). Under this it is still not efficient, but it is consistent. Thus we find that it is an acceptable model to use.

Conclusion

Through our analysis of the given data. We have found that there is significant contemporaneous effect between Tobin's Q and residential investments in the danish housing market from the first quarter of 1971 until last quarter of 2019, and that Tobin's Q does Granger cause residential investments. While the opposite can not be stated.

Appendix

Eigenvalues of companion matrix:

real	imag	modulus
0.4929	-0.7593	0.9053
0.4929	0.7593	0.9053
-0.6789	-0.5802	0.8930
-0.6789	0.5802	0.8930
0.7806	-0.4272	0.8898
0.7806	0.4272	0.8898
0.8578	-0.1663	0.8737
0.8578	0.1663	0.8737
-0.7407	0.4590	0.8714
-0.7407	-0.4590	0.8714
-0.4453	-0.7458	0.8686
-0.4453	0.7458	0.8686
0.7261	0.000	0.7261
0.3099	-0.6187	0.6920
0.3099	0.6187	0.6920
0.000	0.000	0.000
0.000	0.000	0.000
0.000	0.000	0.000

Table 1: Eigenvalues of the companion matrix