Sand H04

1) Formlen for tætheden i en standard normalfordeling er $\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

Vi ved de er uafhængige. Altså må de simultane være lig de marginale

Ergo kan tætheden omskrives som ℓ^2

$$P(X,Y) = \frac{1}{2\pi} e^{-\frac{1}{2}(x^2+y^2)}$$

2) Formlen lyder $g(Z) = \int_{-\infty}^{\infty} p_1(x)\, p_2(Z-x)\,dx$

Vi kender de marginale, men da de er uafhængige slåes de sammen som produkt

$$g(Z) = \int_{-\infty}^{\infty} \frac{1}{2\pi} e^{-\frac{1}{2}((Z-Y)^2+Y^2)} dy$$ hvor Y repræsenterer w

3)

$$g(z) = \frac{1}{\sqrt{4\pi}} e^{-\frac{1}{4}z^2} \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} e^{\frac{1}{4}z^2 - \frac{1}{2}(z^2 - 2zY + 2Y^2)} dY$$

Z er en konstant, som flyttes ind i integralet.

$$g(Z) \int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi}} e^{\cancel{-\frac{1}{4}z^2}} \frac{1}{\sqrt{\pi}} e^{\cancel{\frac{1}{4}z^2}} \frac{1}{\sqrt{\pi}} e^{-\frac{1}{2}(z^2 - 2zY + 2Y^2)} dy$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{4}\cdot\sqrt{\pi}} \frac{1}{\sqrt{\pi}} e^{-\frac{1}{2}((Z-Y)^2+Y^2)}$$

$$\int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi}} \cdot \frac{1}{\sqrt{\pi}} e^{-\frac{1}{2}((Z-Y)^2+Y^2)} = \int_{-\infty}^{\infty} \frac{1}{2\pi} e^{-\frac{1}{2}((Z-Y^2)+Y^2)} dy$$

4) Bruger formlen $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$

$$\frac{1}{\sqrt{2}\sqrt{\pi}\sqrt{2}} e^{-\frac{1}{2}\frac{(z-0)^2}{2}} = \frac{1}{\sqrt{4\pi}} e^{-\frac{1}{4}z^2}$$

5)

$$m = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} \exp\left(-\frac{1}{2}\left(\sqrt{2}\,w - \frac{1}{\sqrt{2}}z\right)^2\right)$$

$$— \quad '' \quad -\left(-\frac{1}{2}\left(2w^2 - \frac{1}{\sqrt{2}}z^2 - \frac{\sqrt{2}\,wz}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}}wz\right.\right.$$

$$— \quad '' \quad -\left(-\frac{1}{2}\left(2w^2 - \frac{1}{2}z^2 - 2wz\right)\right.$$

$$m = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} \exp\left(\frac{1}{4}z^2 - \frac{1}{2}\left(z^2 - 2zw + 2w^2\right)\right)$$

Ved substitution kan dette yderligere omskrives. $v = \sqrt{2}\,w - \frac{1}{2}z$.

Derfor

$$\frac{dv}{dw} = \sqrt{2} \iff dw = \frac{1}{\sqrt{2}}dv$$

Ergo

$$m = \int \frac{1}{\sqrt{\pi}} \exp\left(-\frac{1}{2}(v)^2\right) \frac{1}{\sqrt{2}} dv$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(v)^2\right) dv$$

Det må stadig gælde at $m=1$, da vi stadig beskæftiger os med en normalfordelt variabel

Ho 4 Sand Fortsæt

$$\frac{Z}{M} = \frac{1}{M} \sum_{i=1}^{M} X_i \sim N\left(0, \frac{1}{M}\right)$$

Bruger reglen

$$Var(bX) = b^2 Var(X)$$

$$Var\left(\frac{Z}{M}\right) = \left(\frac{1}{M}\right)^2 M$$

$$Var\left(\frac{Z}{M}\right) = M^{-2} M$$

$$= M^{-1}$$

$$= \frac{1}{M}$$

Dette kan skrives som $\frac{Z}{M} = \sum_{i=1}^{M} X_i \sim N\left(\frac{1}{M}\right)$