Calvo pricing and the New Keynesian Model

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Last time

Price frictions

- Prices remain fixed **exogenously**, they cannot be adjusted at will
- Different forms of price frictions lead the same shock to have longer/shorter effects on output

Fischer & Taylor contracts

- Fischer contracts: set a price path
- Taylor contracts: constant prices

Effects

- Anticipated **shocks** matter
- Output can be manipulated using a money printing machine

Today

The missing piece: Calvo pricing

- Preeminent assumption to obtain the New Keynesian Phillips Curve (NKPC)
- Very elegant solution to a complicated price setting problem

The New Keynesian Model

- Three equations to rule the world:
    - Phillips Curve
    - IS Curve
    - Taylor rule
- Output responses to different shocks

Calvo pricing

Calvo pricing

Fischer and Taylor contracts get intractable for long durations

- More periods to keep track of
- With risk aversion (not present in our case) things get complicated quickly

Calvo contracts

- Price setting is stochastic: Prices are reset with a constant **probability**
- That implies that **on average**, prices stay constant for a number of periods, agents don't know if they will be able to change prices next period $\implies$ they form expectations
- "Calvo fairy" visits with some probability $\alpha \in [0, 1]$
- Elegant solution to a complicated problem

The price level in a Calvo world

Prices depend on past

$$p_t = \underbrace{\alpha x_t}_{\text{Resetters}} + \underbrace{(1-\alpha)p_{t-1}}_{\text{Cannot reset}} \implies \pi_t = \alpha(x_t - p_{t-1})$$

- x_t is the optimal price level for resetters in period t
- Only a fraction of α can reset their price
- All others have to keep their old price p_{t-1}
- Note that this is the aggregate price level. Some firms have been stuck with their price for potentially many periods

Model modifications

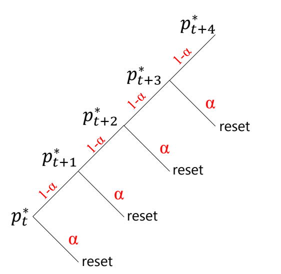
- Because agents have to look far into the future now, we have to introduce discounting into the utility function:

$$U_i = \sum_0^{\infty} \beta^t \mathbf{C}_{i,t} - \sum_0^{\infty} \beta^t \frac{1}{\phi} L_{i,t}^{\phi}$$

Optimal reset price I

Optimal reset price is an average of expected optimal future prices

- Under Taylor pricing: weighted average of two periods
- Under Calvo: weighted average of periods that will carry the price



$$\text{Taylor: } x_t = \frac{p^*_t + p^*_{t+1}}{2}$$

$$\text{Calvo: } x_t = \frac{p^*_t + (1-\alpha)p^*_{t+1} + (1-\alpha)^2 p^*_{t+2} + \cdots}{1 + (1-\alpha) + (1-\alpha)^2 + \cdots}$$

Optimal reset price II

Simplification (introducing discounting of the future)

$$
\begin{aligned}
x_t &= \frac{p_t^* + \beta(1-\alpha)p_{t+1}^* + \beta^2(1-\alpha)^2 p_{t+2}^* + \cdots}{1 + \beta(1-\alpha) + \beta^2(1-\alpha)^2 + \cdots} \\
&= \frac{\sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \mathbb{E}[p_{t+j}^*]}{\sum_{j=0}^{\infty} \beta^j (1-\alpha)^j} \\
&= (1-\beta(1-\alpha)) \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \mathbb{E}[p_{t+j}^*]
\end{aligned}
$$

- Last line uses the fact that $\sum_{j=0}^{\infty} x^j = \frac{1}{1-x}$
- If $\alpha = 1$, we're back to flexible prices: $x_t = p_t^*$
- We want to write this equation recursively (meaning: on the right side, we want future values of the left side)

Optimal reset price III

Recursive notation

$$
\begin{aligned}
x_t &= (1-\beta(1-\alpha))\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j\mathbb{E}_t[p^*_{t+j}] \\
&= (1-\beta(1-\alpha))p^*_t + (1-\beta(1-\alpha))\sum_{j=1}^{\infty}\beta^j(1-\alpha)^j\mathbb{E}_t[p^*_{t+j}] \\
&= (1-\beta(1-\alpha))p^*_t + (1-\beta(1-\alpha))\sum_{j=0}^{\infty}\beta^{j+1}(1-\alpha)^{j+1}\mathbb{E}_t[p^*_{t+1+j}] \\
&= (1-\beta(1-\alpha))p^*_t + \beta(1-\alpha)\underbrace{(1-\beta(1-\alpha))\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j\mathbb{E}_t[p^*_{t+1+j}]}_{\mathbb{E}_t[x_{t+1}]} \\
&= (1-\beta(1-\alpha))p^*_t + \beta(1-\alpha)\mathbb{E}_t[x_{t+1}]
\end{aligned}
$$

- The optimal reset price depends on expectations of future optimal prices

Inflation under Calvo

Tease out inflation

$$x_t - p_t = (1-\beta(1-\alpha))(p_t^* - p_t) + \beta(1-\alpha)\mathbb{E}_t[\underbrace{x_{t+1} - p_t}_{\pi_{t+1}/\alpha}]$$

$$\underbrace{x_t - p_{t-1}}_{\pi_t/\alpha} - \underbrace{(p_t - p_{t-1})}_{\pi_t} = (1-\beta(1-\alpha))(p_t^* - p_t) + \beta(1-\alpha)\mathbb{E}_t\left[\frac{\pi_{t+1}}{\alpha}\right]$$

$$\frac{1-\alpha}{\alpha}\pi_t = (1-\beta(1-\alpha))(p_t^* - p_t) + \beta\frac{1-\alpha}{\alpha}\mathbb{E}_t\left[\pi_{t+1}\right]$$

$$\pi_t = \frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}(p_t^* - p_t) + \beta\mathbb{E}_t\left[\pi_{t+1}\right]$$

$$\pi_t = \frac{\alpha(1-\beta(1-\alpha))(\phi-1)}{1-\alpha}y_t + \beta\mathbb{E}_t\left[\pi_{t+1}\right]$$

- Remember from last lecture:
  $p^* = (\phi-1)m_t + (2-\phi)p_t \implies p_t^* - p_t = (\phi-1)y_t$

The New Keynesian Phillips Curve

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \left[\pi_{t+1} \right] \text{ with } \kappa = \frac{\alpha(1-\beta(1-\alpha))(\phi-1)}{1-\alpha}$$

- Inflation and output are positively related: high output means high inflation
- The slope of the Curve depends on three parameters:
  - Discounting parameter β
  - Price adjustment parameter α
  - Inelasticity of labor supply ϕ
- This is the economy's supply curve: higher prices → more output

A tale of two Phillips Curves

New Keynesian

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \left[\pi_{t+1} \right] \text{ with } \kappa > 0$$

- Inflation depends on expectations of future inflation
- The New Keynesian model is entirely **forward looking**

Lucas

$$\pi_t = \frac{1}{b} y_t + \mathbb{E}_{t-1} \left[\pi_t \right] \text{ with } b > 0$$

- Inflation depends on expectations of contemporary inflation

Notes on derivation

Assumptions on consumers as producers

- In the derivations so far, consumers are assumed to produce the goods themselves
- They choose production Y_i and price P_i
- There were no firms, the optimization problem was not dynamic (i.e., without taking the future into account)

Towards dynamic optimization

- This is not essential. It's possible to derive the same PC with consumers who work at profit maximizing firms

The IS curve

The Euler equation I

Intertemporal optimality

- The representative household's optimality condition for consumption can be derived from a standard dynamic optimization problem (see DR 7.1)

$$u'(C_t) = \beta\mathbb{E}\left[\frac{1+i_t}{1+\pi_{t+1}}u'(C_{t+1})\right]$$
$$u'(C_t) = \beta\mathbb{E}\left[(1+r_t)u'(C_{t+1})\right]$$

The Euler equation – Intuition

- There is a trade-off between consumption today tomorrow
- Cutting consumption today lowers utility by $u'(C)dC$
- The saved consumption can be invested into a nominal bond at gross nominal interest rate $1+i_t$
- Inflation between t and $t+1$ decreases the worth of the investment
- Saving dC today yields $u'(C)dC\beta(1+r_t)$ tomorrow

The Euler equation II

Functional form

- Assume that utility is given by $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$
- Special case: $\sigma = 1 \implies \log(c)$

$$C_t^{-\sigma} = \beta \mathbb{E}\left[(1 + r_t) C_{t+1}^{-\sigma}\right]$$

Interpretation

- Patient households (high β) consume less today
- Higher real interest rates lead to more consumption tomorrow
- Higher risk aversion σ leads to less volatile C over time

The Dynamic IS curve

Market clearing

- Everything that is produced will be consumed ($Y_t = C_t$)

$$Y_t^{-\sigma} = \beta \mathbb{E}\left[(1 + r_t) Y_{t+1}^{-\sigma}\right]$$

Linearize by applying logs

$$\begin{aligned} -\sigma y_t &= \log(\beta) + \mathbb{E}\left[\log(1 + r_t) - \sigma y_{t+1}\right] \\ \implies y_t &= \mathbb{E}[y_{t+1}] - \frac{1}{\sigma}\left(\log(\beta) + \log(1 + r_t)\right) \\ y_t &\approx \mathbb{E}[y_{t+1}] - \frac{1}{\sigma}\left(r_t - \rho\right) \end{aligned}$$

Important simplification

- $\log(1 + x) \approx x$ and $\log(1 - x) \approx -x$

The three-equation model

IS curve – Consumer optimality

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}\left(E_t[r_t] - \rho\right)$$

Phillips curve – Firm optimality

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right]$$

Taylor rule – Monetary policy

$$r_t = \rho + \phi_y \mathbb{E}_t[y_{t+1}] + \phi_\pi \mathbb{E}_t[\pi_{t+1}]$$

Discussion

Is this a useful model?

- The three equation model is at the heart of every DSGE model currently in use
- It is a grotesque simplification of reality, but
  - It is based on micro foundations, meaning that it survives the Lucas critique
  - Most of its predictions make intuitive sense
  - Monetary policy has a place in the world after all
- Most models are much more complicated and account for all kinds of things in the economy

Forward iteration

IS curve – Consumer optimality

$$
\begin{aligned}
y_t &= \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}\left(E_t[r_t] - \rho\right) \\
&= \mathbb{E}_t[y_{t+T}] - \frac{1}{\sigma}\left(\sum_{j=0}^{T} r_{t+j} - \rho\right) \qquad = -\frac{1}{\sigma}\left(\sum_{j=0}^{T} r_{t+j} - \rho\right)
\end{aligned}
$$

- If consumers expect the real interest rate to be high, they consume less today

Phillips curve – Firm optimality

$$
\begin{aligned}
\pi_t &= \kappa y_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right] \\
&= \kappa \sum_{j=0}^{T} \beta^j y_{t+j}
\end{aligned}
$$

- The rate of inflation is pinned down by the expectations of future economic performance

Shocks to the economy

Demand shock

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}\left(E_t[r_t] - \rho\right) + u_{IS}$$

Cost-push shock

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right] + u_{CP}$$

Monetary policy shock

$$r_t = \rho + \phi_y \mathbb{E}_t[y_{t+1}] + \phi_\pi \mathbb{E}_t[\pi_{t+1}] + u_{MP}$$

- To find how shocks affect output and inflation, plug r_t into the IS curve and the IS curve into the PC

The effect of shocks

Output

$$y_t = \left(1 - \frac{\phi_y}{\sigma}\right)\mathbb{E}_t[y_{t+1}] - \frac{\phi_\pi}{\sigma}E[\pi_{t+1}] - \frac{1}{\sigma}u_{MP} + u_{IS}$$

Inflation

$$\pi_t = \left(1 - \frac{\phi_y}{\sigma}\right)\kappa\mathbb{E}_t[y_{t+1}] + \left(\beta - \frac{\phi_\pi}{\sigma}\right)E[\pi_{t+1}] - \kappa\left(\frac{1}{\sigma}u_{MP} - u_{IS}\right) + u_\pi$$

Singular, unexpected shocks

- If we interpret y_t and π_t as deviations from a steady state, then without shocks, $y_t = \pi_t = 0$
- That makes it very easy to quantify the effect of single unexpected shocks, since $E[y_{t+1}] = \mathbb{E}[\pi_{t+1}] = 0$
- The model has no way to generate endogenous persistence

A monetary policy shock

Effects on output and inflation

$$y_t = -\frac{1}{\sigma} u_{MP}$$
$$\pi_t = -\kappa \frac{1}{\sigma} u_{MP}$$

- If monetary policy raises the interest rate **surprisingly** output falls
- Consumption tomorrow is relatively attractive $\implies$ delay consumption
- Because agents want to buy less today, prices have to fall

Elasticity of substitution $\frac{1}{\sigma}$

- If σ is high, consumers are very unwilling to substitute across periods
- Monetary policy will be less effective

This is how we think the world works!

Economists warn of deeper US downturn as Fed keeps up inflation fight

Central bank expected to implement fourth 0.75 point rate rise despite calls for slower pace

Fed chair Jay Powell has refused to rule out the possibility of a recession in the world's largest economy © FT Montage/Reuters /Dreamstime

A demand shock

Effects on output and inflation

$$y_t = u_{IS}$$

$$\pi_t = \kappa u_{IS}$$

- This shock is **in addition** to normal demand (i.e., outside the consumer's problem)
- Example: Government spending
- Because of more goods demand, market clearing requires that prices rise

Opposite of monetary policy shock

- Conventional wisdom: If demand is too high, raise interest rates to cool the economy

A cost-push shock

Effects on output and inflation

$$y_t = 0$$

$$\pi_t = u_\pi$$

- Over night, everything becomes more expensive
- Inflation rises, but output is unaffected
- A cost-push shock does not have real consequences in this New-Keynesian model

Assumptions are key

- In the model, there is nobody who cannot afford to eat when prices rise
- Nominal wages adjust immediately, such that real wages stay constant

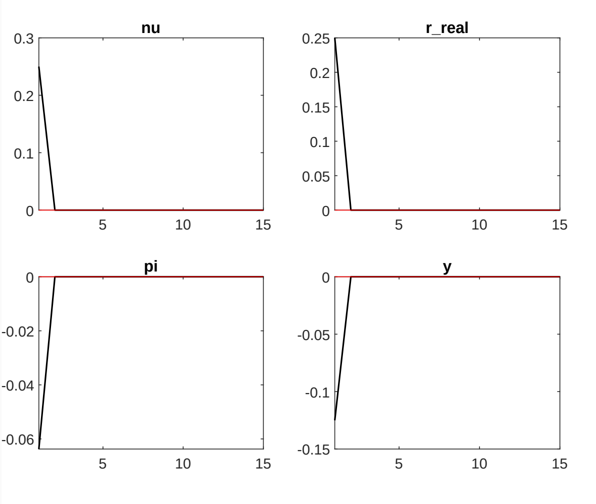
Persistent shocks

Effects on output and inflation

- If the shock processes are persistent, meaning $u_{t,MP} = \rho_{MP} u_{t-1,MP} + \epsilon_{t,MP}$, things get complicated
- The model can still be solved by hand, but it is cumbersome – we can just force a computer to solve it
- Rational expectations: agents in the model know ρ_{MP}! Upon the shock ϵ realization, they know the path of u

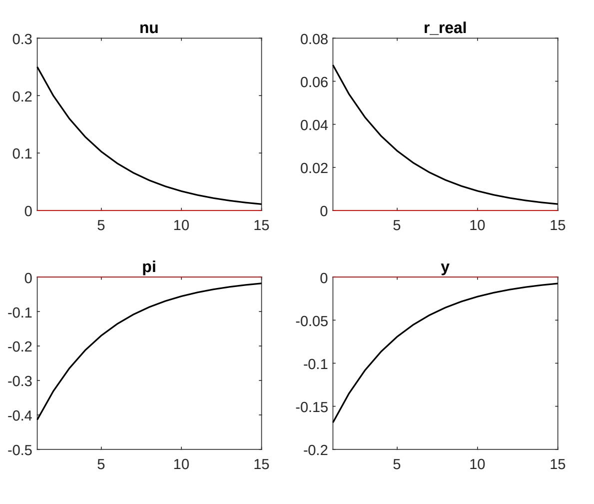
Computer output

Start with single-period shock



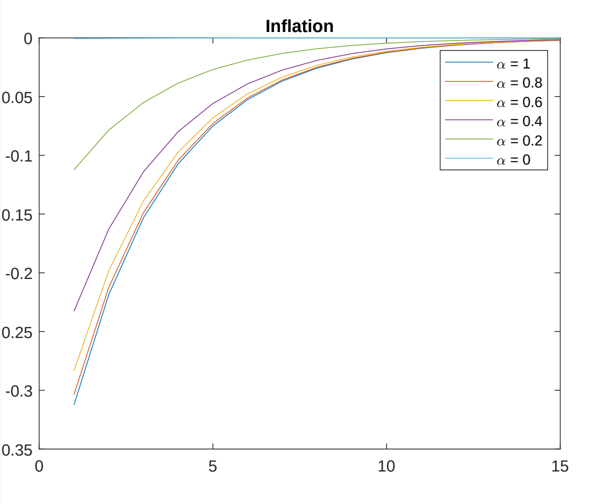
Computer output

Persistent shock



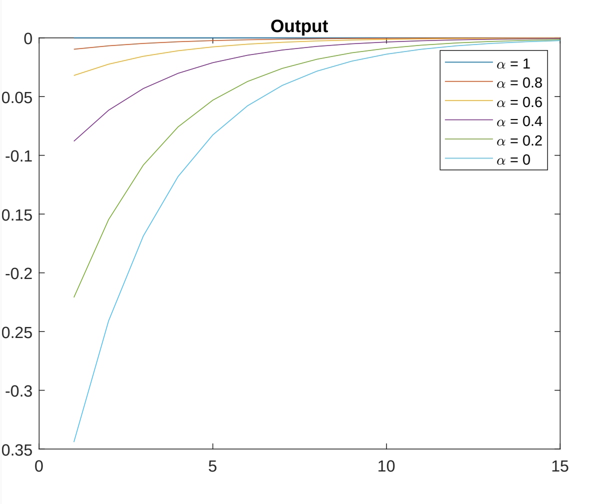
Computer output

Effect of price stickiness on inflation



Computer output

Effect of price stickiness on output



The role of monetary policy

The Taylor rule

- In the New Keynesian model, monetary policy is described by a rule
- If ϕ_π is high, the interest rates respond strongly to keep inflation in line
- If ϕ_y is high, the central bank tries to close the output gap quickly
- Is there a trade-off? Not allowing prices to change might mean that the effect of shocks on output will be larger

Expectations

- Agents in the model know the central banks response function
- If agents don't expect inflation, there will be no inflation

Example

Extreme case: $\phi_\pi = \infty$

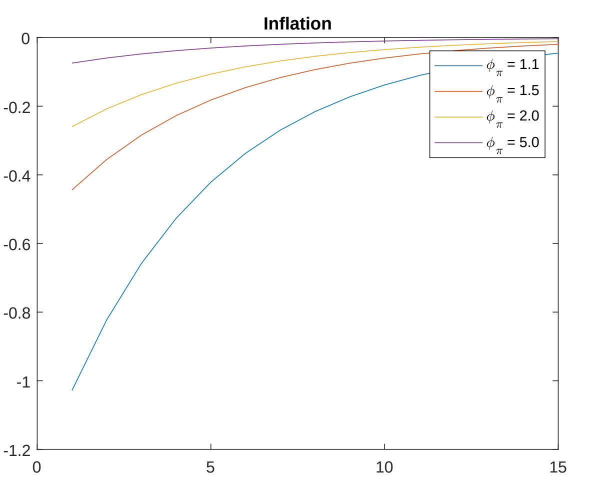
- The central bank will not allow any inflation in periods it has influence over
- The anticipation of potentially very large interest rate movements keeps output close to the baseline
- Through the Phillips curve, smaller output movements keep inflation low

Commitment and Credibility

- If the central bank can convince agents that it will not allow inflation to move, output will not move
- This is called the "divine coincidence": keeping inflation low will keep output low
- Fun fact: if ϕ_π is not high enough (usually > 1), the model cannot be solved

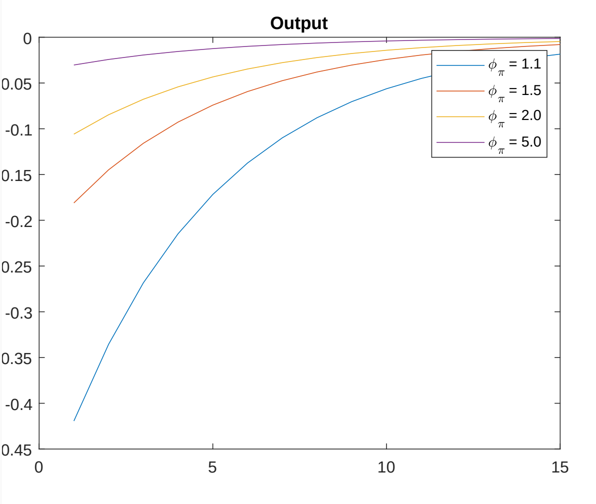
Aggregate responses as a function of ϕ_π

Effect of inflation aversion on inflation



Aggregate responses as a function of ϕ_π

Effect of inflation aversion on output



Shortcomings of the model

No labor market

- One can extend the model with Calvo adjustment frictions on wages. This way, researchers can think about questions regarding **involuntary** unemployment

Inflation persistence

- Inflation and output do not move instantaneously in response to changes in monetary policy. The baseline model can be extended with behavioral, backward-looking terms that lead to more hump-shaped responses

The model does not contain inequality

- There is only a representative agent in the model, but recently, questions of inequality have received a lot of attention.

Heterogeneity

Relevance of heterogeneity

Monetary policy

- Some people hold variable interest rate mortgages which are very vulnerable to interest rate changes
- People consume different goods, meaning their inflation rates differ

Government spending

- High earners pay higher income taxes, wealthy people pay most capital gains taxes
- The unemployed receive benefits, parents receive transfers

Labor market

- Some people work for the government, in relatively save jobs, others carry more risk

Demographics

- Women face different labor market trends than men
- Younger people face different labor market risks than the old

Measure of heterogeneous consumption (Almgren et al, 2022)

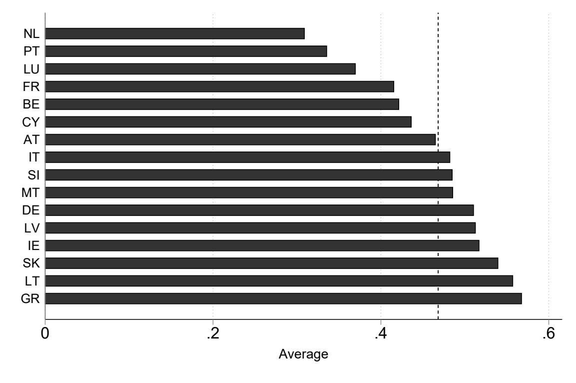
Propensity to spend unexpected income

Imagine you unexpectedly receive money from a lottery, equal to the amount of income your household receives in a month. What percent would you spend over the next 12 months on goods and services, as opposed to any amount you would save for later or use to repay loans?

Measure of heterogeneous consumption (Almgren et al, 2022)

Propensity to spend unexpected income

Imagine you unexpectedly receive money from a lottery, equal to the amount of income your household receives in a month. What percent would you spend over the next 12 months on goods and services, as opposed to any amount you would save for later or use to repay loans?



Vulnerability to shocks

Propensity to spend unexpected income

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63% of Americans are living paycheck to paycheck — including nearly half of six-figure earners

PUBLISHED MON, OCT 24 2022•10:29 AM EDT

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KEY POINTS

- With persistent inflation eroding wage gains, the number of Americans living paycheck to paycheck is near a historic high, according to a recent report.
- Almost half of those earning more than $100,000 say they are just getting by.

Marginal propensity to consume

Representative agent model

$$u'(C_t) = \beta(1 + r_t)u'(C_{t+1})$$
$$C_t + A_t = A_{t-1}(1 + r_{t-1}) + I + \tau$$

- In the steady state, the representative agent always consumes the same: $C_t = C_{t+1}$, since $\beta(1 + r) = 1$
- What if we give him a bit more money for a single period?

Still consume the same in each period $\implies$ only consume such that it can be recouped by saving

$$(1 - \alpha)\tau(1 + r) = \tau$$
$$\frac{dC}{dI} = \alpha = 1 - \frac{1}{1 + r} \approx 2\%$$

Step forward

Representative agent cannot capture heterogeneity

- A marginal propensity to consume of 2% is unrealistically low
- Even the average MPC of an economy is likely much higher

Suggested solution:

- Some people always consumer their whole income
- Hand-to-mouth agents:

$$
\begin{aligned}
C_t &= I_t \\
\implies \frac{dC}{dI} &= 1
\end{aligned}
$$

Model modifications

Re-derive the model

- Include a fraction of λ of hand-to-mouth households
- Firm part of the model stays exactly the same

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \left[\pi_{t+1} \right]$$

- The IS curve changes to

$$y_t = \mathbb{E}[y_{t+1}] - \frac{1}{\sigma} \frac{1 - \lambda}{1 - \lambda \chi} (\mathbb{E}_t r_t - \rho)$$

The new IS-curve

$$y_t = \mathbb{E}[y_{t+1}] - \frac{1}{\sigma}\frac{1-\lambda}{1-\lambda\chi}(\mathbb{E}_t r_t - \rho)$$

New insights

- A demand shock (i.e., surprising change in $r_t - \rho$) has bigger or smaller effects depending on whether $\frac{1-\lambda}{1-\lambda\chi}$ is smaller or bigger than one
- If there are no HtM consumers ($\lambda = 0$), the fraction collapses to 1 and we are back to the original model
- If $\lambda > 0$, the effect of a demand shock depends on the parameter χ
- χ governs how strongly the income of the HtM agents moves with the aggregate

The elasticity of income to output

The new IS-curve

$$y_t = \mathbb{E}[y_{t+1}] - \frac{1}{\sigma}\frac{1-\lambda}{1-\lambda\chi}(\mathbb{E}_t r_t - \rho)$$

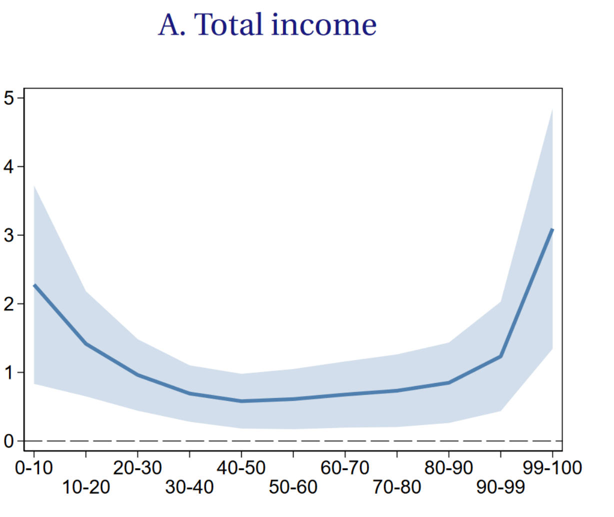
The magical χ parameter

$$y_t^{htm} = \chi y_t$$
$$y_t^n = \frac{1-\lambda\chi}{1-\lambda} y_t$$
$$\text{(note that } y_t = \lambda y_t^{htm} + (1-\lambda) y_t^n\text{)}$$

- Remember: small letter y means logs, hence $\chi = \frac{d\log(Y_t^{htm})}{d\log(Y_t)}$ is an elasticity
- $\chi > 1$ implies that the income of the HtM consumers is more volatile than aggregate income
- $\chi > 1$ monetary policy has a bigger effect on output

Empirical estimates

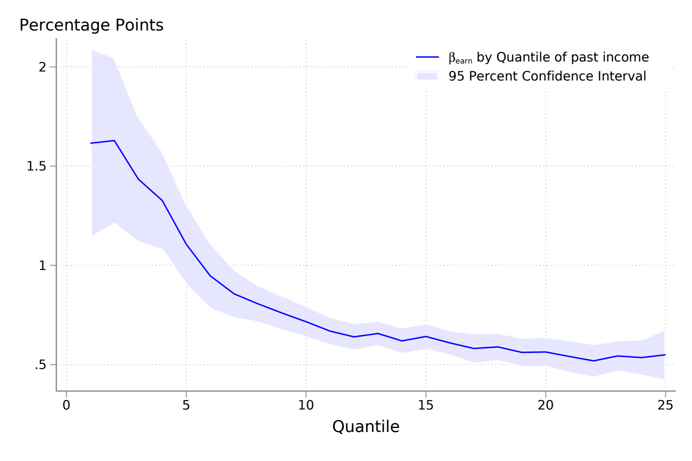
Sweden – Amberg et al (2022)



- χ seems to lie between 1.5 and 2 $\implies$ plug into the NK model

Empirical estimates

Germany – Broer et al (2022)

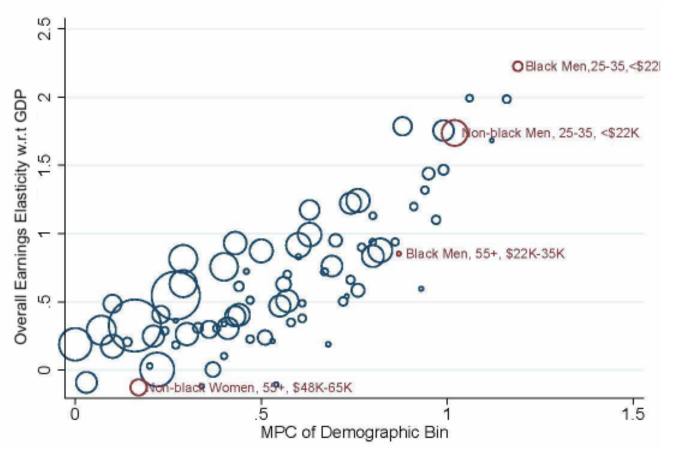


- χ seems to lie between 1.5 and 2 $\implies$ plug into the NK model

Empirical estimates

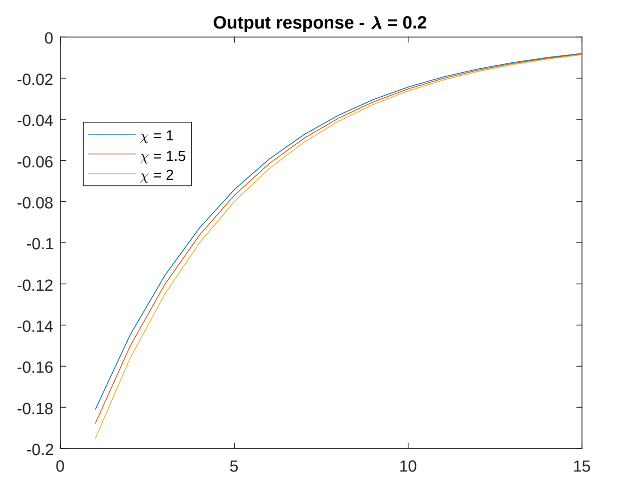
US – Patterson (2022)

Figure 1: Recession Exposure and MPC by Demographic Group



- χ seems to lie between 1.5 and 2 $\implies$ plug into the NK model

Influence of χ for output



Importance of heterogeneity

Intuition

- Some individuals in the economy cannot/do not insure against shocks to the economy (e.g., monetary policy). If their income falls, their consumption falls 1:1.
- The interest rate has no effect on their consumption decision
- The strong fall in consumption, in equilibrium, leads to less demand, which feeds back to the incomes of the unconstrained agents $\Longrightarrow$ even less consumption
- Heterogeneity amplifies the effects of aggregate shocks

  Just as Keynes always predicted!

Conclusions have implications beyond monetary policy

- Government spending: cutting taxes vs giving transfers

Conclusion

Calvo price setting

- Elegant way to introduce price stickiness into a model

The New Keynesian model

- The equations: PC, IS, TR
- Entirely forward looking: if we don't expect inflation, there will be no inflation
- Allows economists to speak intelligently about the effects of monetary policy

Heterogeneity

- Heterogeneity matters for the effect of shocks
- The representative agent model likely underestimates the effects of monetary policy