The Value of Early Exercise in Option Prices: An Empirical Investigation

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Abstract

Previous studies in the valuation of American options apparently undervalue the right of early exercise. This study uses actual prices from the CBOE's S&P 100 option instead of model-generated values. Deviations from the theoretical put-call parity relationship are caused by the possibility of early exercise. These deviations are used to infer the value of early exercise. The actual value of early exercise is both statistically and economically significant. As expected from theoretical considerations, the value of early exercise for put options is greater than for call options.

I. Introduction

Merton (1973a) has shown that an American call option on a non-dividend-paying stock should not be exercised prematurely. Rational early exercise can occur for call options on dividend-paying stocks if the annualized dividend yield received over the remaining life of the option exceeds the risk-free rate of interest. In this case, the opportunity cost of not receiving the dividends now outweighs the benefit of paying a lower present value for the stock later. Since such high dividend yields are rare for stock options, however, the value of early exercise for call options is often neglected.[1]

Merton (1973b) has shown that American put options, on the other hand, can be rationally exercised prior to expiration. Thus, the value of early exercise is often a concern of both traders and researchers in evaluating put options. The true value of early exercise is ultimately an empirical question.

This study uses actual prices from the CBOE's S&P 100 index option to determine the market value of early exercise. The value of early exercise is defined here to mean the difference in price between an American option and an otherwise identical European option. Several methods of analysis are possible. First, if American and European options exist on the same asset, the early exercise premium can be computed directly. However, simultaneous liquid markets

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[1] Stoll and Whaley (1986) show that American call options on stocks paying continuous dividends may be rationally exercised when the option is deep-in-the-money.

for otherwise identical American and European options do not exist. Alternately, the difference between the market price of the American option and the value of a European option generated by an option pricing model can be used as the value of early exercise. Unfortunately, the observed measure is a function of the particular option pricing model employed. For example, the option pricing model may not exactly capture the marketplace's assessment of the impact of dividends, changing volatility, changing interest rates, jump probability, or process misspecification as well as the early exercise probability.

Another means of measuring the value of early exercise would be to examine deviations from European put-call parity. Put-call parity does not rely upon any particular option pricing model for validity, but rather upon the idea of a duplicating portfolio. Because the duplicating portfolio consists entirely of observable market prices, deviations from parity permit computation of the early exercise premium without requiring the joint testing of a particular option pricing model. In particular, the approach used in this paper is robust to the problems faced by option pricing models itemized in the preceding paragraph.[2]

Section II summarizes the previous research into the value of early exercise, which has been dependent upon the assumed accuracy of theoretical option pricing models. The third section describes the option pricing model-free methodology used to estimate the early exercise premium. Section IV presents the results, which show that the value of early exercise is both statistically and economically significant. As expected from theoretical considerations, the value of early exercise for put options is greater than for call options. Furthermore, the value of early exercise increases with time to expiration and with the degree the option is in-the-money. The final section summarizes the findings and explains how previous empirical findings of biases in the Black-Scholes model are consistent with a premium for the right of early exercise.

II. Previous Research

Table 1 summarizes the prior research on the valuation of American options. As Table 1 indicates, two phases of research have occurred. The first research phase observed that European option pricing models misvalued actual American options. Theoretical models were introduced that reduced, but did not eliminate, the observed pricing errors. Whaley (1982), Sterk (1983), and Geske and Roll (1984) each observed that option pricing models that reflect a positive probability of early exercise produce better fits with market data.

The second research phase into pricing American options has focused on finding faster methods of computing the value of an American option. Geske and Shastri (1985), Barone-Adesi and Whaley (1987), Johnson (1983), Blomeyer (1986), and Geske and Johnson (1984) all provide mathematical approximation techniques that give very similar predicted values for American options.

[2] Since the put-call parity relationships hold exactly for European options, even with the complications (excepting early exercise, of course), enumerated above, deviations from put-call parity with American options are necessarily due to the presence of the probability of early exercise. The source of mispricing is of concern for theoretical purposes in creating a ''better'' American option pricing model. The purpose of this paper is to illustrate a simple, robust method of determining the amount of mispricing.

TABLE 1
Representative Prior Research on Value of Early Exercise

Paper	Principal Finding
Panel A. Comparison of Values Generated by Option Pricing Models with Market Prices	
Whaley (1982)	American option pricing models fit data better than Black-Scholes European option pricing model.
Sterk (1983)	American option pricing models fit data better than Black-Scholes European option pricing model.
Geske and Roll (1984)	Conclude biases reported in Black-Scholes model reduced or eliminated using American option models.
Panel B. Accuracy and Computational Efficiency of Alternative Models	
Johnson (1983)	Analytic approximation to American put prices.
Blomeyer (1986)	Modifies Johnson (1983) result to account for dividends.
Geske and Johnson (1984)	Polynomial approximation formula for American put options using a compound option approach.
Geske and Shastri (1985)	Numerous techniques give same answer; only computing cost varies.
Barone-Adesi and Whaley (1987)	A simple approximation gives essentially identical results to more complicated formulas.

This second phase of improving computational efficiency makes the implicit assumption that the American option pricing models generate correct prices. Blomeyer and Johnson (1988) have recently published a study that marks the beginning of a third stage of research into the value of early exercise. They find that actual market prices differ substantially from theoretical model prices, even after including a theoretical value for early exercise. Furthermore, the difference between model prices is slight compared to the difference between any model price and the market price of a put, except for those cases in which it is optimal to exercise the put prematurely. A careful reading of their study suggests that the existing theoretical models for pricing American put options do not adequately capture the market value of early exercise.

Overdahl (1988) similarly finds that the analytic expressions for predicting the price at which an option on a Treasury bond futures contract would be exercised provide biased estimates when compared with the marketplace. Again, the indication is that the existing analytic models for American options do not completely capture the market value of early exercise.

Unlike the previous studies, which use option pricing models to determine the value of early exercise, this paper will use a simple arbitrage relationship to impute the early exercise premium in put and call prices. Stoll (1969) has shown that the prices of puts and calls are deterministically related by the put-call parity relationship. Merton (1973b) points out that the relationship strictly holds only for European options, since the possibility of early exercise will destroy the risk-free hedge that is the foundation of put-call parity. Klemkosky and Resnick (1979) have tested the put-call parity relationship using a small sample of listed puts and calls. They report that a large portion of the hedges violated parity in the absence of transactions costs. In particular, they find that calls were overpriced relative to puts. Nonetheless, they conclude that the results were generally consistent with put-call parity theory.

Evnine and Rudd (1985) find, for a small sample of days in 1984, that S&P 100 index options violate the European put-call parity relationship, with calls tending to be relatively underpriced. When the weaker bounds of American put-call parity are used, the observed parity relationship is stronger. In their novel approach, Evnine and Rudd assume the at-the-money call is properly priced. They then use a binomial model with the implied volatility from this call to value the other options. They find that puts are selling for more than the prices predicted by the model, whereas calls sell for approximately the model prices. This paper explores an alternative explanation for their finding: a nonzero value for the right to early exercise.

III. Methodology

A. Concept

The basic put-call parity relationship as described in Stoll (1969) contains six variables: the price of the call, C; the price of the put, P; the price of the underlying asset, S; the exercise or striking price, X; the risk-free rate of interest, r; and the time to expiration, T. The European form of put-call parity assuming no dividends are paid is

(1) $$C - P = S - Xe^{-rT}.$$

Equation (1) can be generalized to accommodate varying interest rates over the life of the option resulting from a sloping term structure or interest rates following a stochastic process. The generalization replaces the factor e^{-rT} with $B(t,T-t)$, the price of a default-free one dollar discount bond at time t, maturing at time T.

The basic parity relationship also can be adjusted to allow for the payment of dividends on the underlying asset,

(2) $$C - P = S - D_T - Xe^{-rT},$$

where D_T is the present value of dividends paid between the present time and expiration time T.

Given observable market prices, C, P, S, the known dividend payment schedule,[3] and the fixed contract variables X and T, one can readily solve for the implied interest rate, r',

(3) $$r' = -\left\{\mathrm{Ln}\left[\left(S - D_T - C + P\right)/X\right]\right\}/T.$$

Whenever put-call parity holds, then r' equals r, the risk-free rate of interest. Brenner and Galai (1986) use this put-call parity relationship to estimate an implied risk-free rate from American options' prices. Brenner and Galai find the

[3] Although the dividends paid are not entirely certain, the payment pattern is highly predictable for the S&P 100 index used in this study.

implied risk-free rate in the expected range, but the rate is dependent upon the relationship between the stock price and the striking price. They argue that the implied rate varies because the possibility, and hence the value, of early exercise varies. Brenner and Galai also contend that the bias in the estimated implied interest rate is small for at-the-money options.

The implied interest rate r' is the effective continuously compounded rate for a risk-free discount bond maturing at the expiration date of the option pair. This is not necessarily the same as the locally riskless market-clearing interest rate discussed in Cox, Ingersoll, and Ross (1985), who note that some otherwise reasonable interest rate processes imply arbitrage opportunities over finite holding periods. Rather, the implied interest rate r' used here is the rate that prevents arbitrage opportunities over the life of the European options because European put-call parity is a no-arbitrage relationship.

In general, the European put-call parity relation will not hold if there is a potential for early exercise in one of the options. As an arbitrage relationship, however, put-call parity is robust to problems such as those explored by Cox and Ross (1976), Johnson and Shanno (1987), and Merville and Pieptea (1985). In any given pair of puts and calls with identical striking prices and time to expiration, one contract will be in-the-money and the other out-of-the-money. Ceteris paribus, we expect the in-the-money option is more likely to be exercised and, therefore, to have a larger value for early exercise built into its market price. The out-of-the-money option would tend to have a very small value of early exercise. Thus, $C-P$ will be ''too high'' if the call is in-the-money and ''too negative'' if the put is in-the-money. Other researchers into the value of early exercise have used mathematical model values for C and P to judge if the prices are too high or low. The methodology used in this paper avoids any known problems with mathematical option pricing models.

Instead of using a model such as the Black-Scholes model to evaluate the theoretical prices for C and P and comparing them with observed prices, deviations of the American options prices from European put-call parity are used to directly impute a value of early exercise. The put-call parity relationship for American options with known dividends is never an equality solely because of the possibility of early exercise. The implied interest rate r_o' from the nearest-the-money American put-call combination is used to determine the theoretical values of the put-call differential, $(C-P)'$, for otherwise equivalent European options,

$$(4) \qquad (C-P)' = S - D_T - Xe^{-r_o'T}.$$

The observed difference between American call and put prices, $(C-P)$, includes the difference between European call and put prices as well as the net difference in the value of early exercise for the American options. By subtracting the theoretical value of European put-call parity, $(C-P)'$, from the net observed prices of the American options, $(C-P)$, we can estimate the net value of early exercise for the pair of options,

$$(5) \qquad A = (C-P) - (C-P)'.$$

This estimated value of early exercise consists of a relatively large value (in absolute terms) for the option in-the-money and a relatively small value for the option out-of-the-money. Thus, A is expected to be positive for in-the-money calls, and negative for in-the-money puts. The small at-the-money values for early exercise approximately cancel each other when determining the implicit interest rate r_o'. Since the two early exercise premia have opposite signs, $|A|$ is a conservative estimate of the value of early exercise for the option in-the-money.

B. Data

The S&P 100 (OEX) index option family was selected for empirical examination of the early exercise premium. This family has several advantages over other contracts. First, the OEX is the most widely traded contract in the world, with daily volumes of over 100,000 contracts for both calls and puts. This high frequency of trading minimizes the problems of unsynchronized prices. Prices of the call, the put, and the underlying index should be simultaneous, since we are investigating their relationship. Second, the nearly continuous dividend stream of the index option makes ''optimal'' early exercise unlikely for the call option. Third, the large volume of trading in the OEX ensures that observations are available for several striking prices and maturities each trading day. This leads to a much larger number of observations than would be possible with most stock options.

Closing prices for each striking price and the expiration date for each trading day in 1985 were obtained from the CBOE, as were the values for the S&P 100 index. The option and index prices were checked for internal consistency using the put-call relationship (2) to identify obvious outliers that were then removed from the data base. After the screening, 3635 usable pairs of options remained.

C. Estimation of Early Exercise Premium

Each trading day, Equation (3) was used to compute an implied interest rate r_o' for the pair of put and call options nearest-the-money. A different implied rate was used for each contract month to allow for term structure effects as well as different dividend patterns.[4] For each pair of put and call prices, the value of early exercise was estimated using Equation (5).

The resulting estimates of the net value of early exercise, A, were then analyzed. The early exercise premia were separately examined for the cases where calls were in-the-money and the cases where puts were in-the-money.

[4] The implied interest rate, r', was found to vary with the maturity of the option,

$$\underset{}{r'(x1000)} = \underset{(19.34)}{0.378} - \underset{(-6.70)}{0.00195}\text{ Days} + \underset{(4.56)}{0.00487}(S-X).$$

This is consistent with the findings of Brenner and Galai (1986), who find a statistically significant maturity effect in the implied interest rate.

Three testable hypotheses are summarized by the relationship in Equation (6),

$$(6) \qquad A = a + \overset{+}{\underset{+}{b_1}} f((S-X)) + \overset{+}{\underset{-}{b_2}} g(T) + \overset{+}{\underset{-}{b_3}} h\left(r_o'\right),$$

where $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ are functions of the moneyness, $(S-X)$, the time to expiration, T, and the implied interest rate, r_o', respectively. The signs above the coefficients are the expected signs of the coefficients for calls, and the signs below the coefficients are the expected signs for puts.

The first testable hypothesis is that the value of early exercise increases with the amount the option is in-the-money. For calls, this means that A increases as $(S-X)$ increases, whereas for puts, A increases (decreases in absolute terms) as $(S-X)$ increases (the put is less in-the-money). The second testable hypothesis is that the value of the option to early exercise increases with time to expiration. Early exercise itself is an option. Since the sign of A differs for calls and puts, the coefficient of $g(T)$ should change sign and be nonzero. Finally, the value of early exercise will depend upon the risk-free rate of interest. A higher opportunity cost would increase the desire to exercise the option early. The coefficient of the interest rate should change sign and be nonzero.

IV. Results

The data were divided into two parts for estimation of Equation (6), depending upon which option (call or put) was in-the-money. Only those observations that had positive (negative) estimates of the early exercise premium for calls (puts) were used. Although Equation (6) is consistent with many functional forms, a linear regression equation was used to test the hypotheses. A stepwise regression involving the dependent variables and their squares and square roots (to attempt to catch any persistent nonlinearities) on the first half and then the second half of the year's data showed that the linear terms were the only ones consistently significant.[5] Furthermore, the adjusted-R^2s increased by only 0.01 when all the nonlinear terms were forced into the regression with the entire year's data. Since the linear regression provides insight into the three testable hypotheses in Equation (6), only these results are summarized in Table 2.

In both cases, the coefficients were of the hypothesized sign. The coefficients of the "moneyness" variable, $(S-X)$, and the time to expiration, T, were highly significant. As expected, the coefficients were larger in magnitude for puts than for calls. The large positive intercept for the in-the-money puts is consistent with early studies of put-call parity, which found that puts tended to be undervalued relative to calls (see, e.g., Klemkosky and Resnick (1979)).

As hypothesized in the preceding section, the value of the early exercise premium varies like a well-behaved option in that it (1) increases with increasing time to expiration, (2) increases with increases in the risk-free rate of interest, (3)

[5] The nonlinear terms are highly collinear with their related linear terms, while the three basic explanatory variables ($S-X$, T, and r') are reasonably independent.

TABLE 2
Regression Estimates of Value of Early Exercise
$A = a + b_1(S-X) + b_2T + b_3r'_o$

$\overline{A}$	$\overline{(C-P)}$	a	b_1	b_2	b_3	R^2
Case 1: Calls in-the-Money, Positive A Only, 2117 Observations						
0.372	10.37	−0.1630	+0.0411	+0.00311	+26.94	0.3298
(37.98)	(74.33)	(−6.67)	(31.34)	(11.82)	(1.47)	
Case 2: Puts in-the-Money, Negative A Only, 1518 Observations						
−0.521	−4.10	+0.4304	+0.1119	−0.00441	−57.92	0.5118
(−31.67)	(−36.68)	(11.97)	(38.66)	(−11.58)	(−2.18)	

$(S-X)$ is the difference between the S&P 100 index value and the common exercise price of the put and call options. T is the number of days to expiration of the options, and r'_o is the implied daily interest rate from put-call parity for a near-the-money pair of options with the same time to expiration. A, the net value of early exercise, is computed as the difference between the actual difference in call and put prices, $(C-P)$, and the difference implied by put-call parity and the implied interest rate r'_o. $\overline{A}$ is the average early exercise premium. Numbers in parentheses are t-statistics.

increases with increases in the stock price, and (4) decreases with increases in the exercise price. Because of the conversion process from calls to puts, the time and interest rate sensitivity are reversed in sign for put options. Also, the greater value of early exercise for put options relative to similar call options is reflected in the increased intercept in the second casc in Table 2.

The results in this study cast a different light on previous research. MacBeth and Merville (1979) find that the implied variance from the Black-Scholes model is a function of the contractual time to expiration and the degree of moneyness. MacBeth and Merville are actually reporting evidence that there is a value to early exercise. They report that out-of-the-money calls with shorter times to expiration have smaller implied variances than those with longer times to expiration. Furthermore, out-of-the-money calls have smaller implied variances than in-the-money calls. Both of these observations are consistent with the market price of the call including an early exercise premium of the form described in Equation (5). Since the model used by them for estimating implied variance is for European calls, the market price of the American call used as an input is too large by the value of early exercise. Because the value of early exercise increases with time to expiration and the degree of in-the-money, the implied variance computed also will increase with time to expiration and the degree of moneyness, as found by MacBeth and Merville.

The results of this study also stand in contrast to those of Blomeyer and Johnson (1988). Their European put prices are generated by the Black-Scholes model and assume that the put-call parity (conversion) process is operational. The difference between the prices generated by their American put model, which is developed by Geske and Johnson (1984), and their European model is defined as the value of early exercise. Their Table 1 shows a typical value for early exercise of about 4 percent of the market put price for puts in-the-money. This value is perhaps overstated in that it includes the difference for puts so far in-the-money that early exercise would certainly be optimal (see Blomeyer and Johnson's Figure 2). For these puts, the true value of the put is $X-S$, which is greater than the value generated by the Black-Scholes model. No puts or calls were observed in our sample that were as extreme as these puts in the Blomeyer-Johnson sample (greater than 10 percent in-the-money). Thus, the figure computed by Blomeyer and Johnson for the value of early exercise, 4 percent, is much smaller than the approximately 10 percent value found for puts in this study. The 10 percent value found here is conservative in that it includes the offsetting value of early exercise for the corresponding call option.

Given the well-behaved nature of the early exercise premium expressed in Table 2, a modification of Hull and White's (1988) control variate approach to valuing American options seems appropriate until better American option pricing models become available. This approach would add an estimate of the early exercise premium derived from empirical observations of the deviations from put-call parity to the Black-Scholes European price. This varies from the technique used in Hull and White, which would add the *theoretical* value of early exercise to the *observed* European price. Instead, the *observed* value of early exercise from Equation (5) would be added to the *theoretical* value of a European option. This is similar to the matrix pricing approach used to price bonds described in Nunn, Hill, and Schneeweis (1986). In fact, many traders do trade off charts that comprise a matrix pricing system. It is possible that the widespread use of such charts induces a ''self-fulfilling prophecy'' effect. If so, the ''best'' model (in terms of predicting the actual market price) may *not* be theoretically derivable using the usual techniques.

V. Summary and Conclusions

Contrary to several recent papers that indicate that the value of early exercise is probably quite small, a substantial early exercise premium is found in both S&P 100 calls and puts. The average value of early exercise is found to be about 3.5 percent for call options and 10 percent for put options on the S&P 100 index. These are much greater than the theoretical values shown in Blomeyer and Johnson (1988) for put options. Yet, the value of early exercise should be smaller for index options than for options on individual stocks. The findings of this paper suggest that the existing theoretical American option pricing models fail to capture all the nuances of the early exercise decision.

The value of the early exercise premium varies like a well-behaved option in that it (1) increases with increasing time to expiration, (2) increases with increases in the risk-free rate of interest, (3) increases with increases in the stock price, and (4) decreases with increases in the exercise price. Also, the value of early exercise for put options is greater than for similar call options.

Given the well-behaved nature of the early exercise premium, a modified Hull and White (1988) control variate approach to valuing American options seems appropriate until better American option pricing models become available. This approach would add an estimate of the early exercise premium derived from empirical observations of the deviations from put-call parity to the Black-Scholes European price. In fact, many traders do trade off charts that comprise a matrix pricing system. It is possible that the widespread use of such charts induces a ''self-fulfilling prophecy'' effect. If so, the ''best'' model (in terms of predicting the actual market price) may *not* be theoretically derivable using the usual techniques.

References

Barone-Adesi, G., and R. E. Whaley. "Efficient Analytic Approximation of American Option Values." *Journal of Finance*, 42 (June 1987), 301–320.

Black, F., and M. S. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy*, 81 (May–June 1973), 637–654.

Blomeyer, E. C. "An Analytical Approximation for the American Put Price for Options on Stocks with Dividends." *Journal of Financial and Quantitative Analysis*, 21 (June 1986), 229–233.

Blomeyer, E. C., and H. Johnson. "An Empirical Examination of the Pricing of American Put Options." *Journal of Financial and Quantitative Analysis*, 23 (March 1988), 13–22.

Brenner, M., and D. Galai. "Implied Interest Rates." *Journal of Business*, 59 (July 1986), 493–507.

Cox, J. C.; J. E. Ingersoll; and S. A. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*, 53 (March 1985), 385–407.

Cox, J. C., and S. A. Ross. "The Valuation of Options for Alternative Stochastic Processes." *Journal of Financial Economics*, 3 (Jan./March 1976), 145–166.

Evnine, J., and A. Rudd. "Index Options: The Early Evidence." *Journal of Finance*, 40 (July 1985), 743–756.

Geske, R., and H. E. Johnson. "The American Put Valued Analytically." *Journal of Finance*, 39 (Dec. 1984), 1511–1524.

Geske, R., and R. Roll. "On Valuing American Call Options with the Black-Scholes European Formula." *Journal of Finance*, 39 (June 1984), 443–455.

Geske, R., and K. Shastri. "Valuation by Approximation: A Comparison of Alternative Valuation Techniques." *Journal of Financial and Quantitative Analysis*, 20 (March 1985), 45–71.

Hull, J., and A. White. "The Use of the Control Variate Technique in Option Pricing." *Journal of Financial and Quantitative Analysis*, 23 (Sept. 1988), 237–251.

Johnson, H. E. "An Analytic Approximation for the American Put Option." *Journal of Financial and Quantitative Analysis*, 18 (March 1983), 141–148.

Johnson, H., and D. Shanno. "Option Pricing when the Variance is Changing." *Journal of Financial and Quantitative Analysis*, 22 (June 1987), 143–151.

Klemkosky, R. C., and B. G. Resnick. "Put-Call Parity and Market Efficiency." *Journal of Finance*, 34 (Dec. 1979), 1141–1155.

MacBeth, J. D., and L. J. Merville. "An Empirical Examination of the Black-Scholes Call Option Pricing Model." *Journal of Finance*, 34 (Dec. 1979), 1173–1186.

Merton, R. C. "The Theory of Rational Option Pricing." *Bell Journal of Economics and Management Science*, 4 (Spring 1973a), 141–183.

________________. "The Relationship between Put and Call Option Prices: Comment." *Journal of Finance*, 28 (March 1973b), 183–184.

Merville, L. J., and D. R. Pieptea. "On the Stochastic Nature of the Stock Price Variance Rate and Strike Price Bias in Option Pricing." Working Paper, Univ. of Texas at Dallas (Dec. 1985).

Nunn, K. P., Jr.; J. Hill; and T. Schneeweis. "Corporate Bond Price Data Sources and Return/Risk Measurement." *Journal of Financial and Quantitative Analysis*, 21 (June 1986), 197–208.

Overdahl, J. A. "The Early Exercise of Options on Treasury Bond Futures." *Journal of Financial and Quantitative Analysis*, 23 (Dec. 1988), 437–449.

Sterk, W. E. "Comparative Performance of the Black-Scholes and Roll-Geske-Whaley Option Pricing Models." *Journal of Financial and Quantitative Analysis*, 18 (Sept. 1983), 345–354.

Stoll, H. R. "The Relationship between Put and Call Prices." *Journal of Finance*, 24 (Dec. 1969), 810–822.

Stoll, H. R., and R. E. Whaley. "New Option Instruments: Arbitrageable Linkages and Valuation." *Advances in Futures and Options Research*, 1 (1986), 25–62.

Whaley, R. E. "Valuation of American Call Options on Dividend-Paying Stocks: Empirical Tests." *Journal of Financial Economics*, 10 (March 1982), 29–58.