Recap lecture

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December 2022

The punchline from each lecture

Monopolistic competition

- Money can affect output (but there is a welfare loss)

Lucas model

- Expectations matter for the effects of monetary policy

Fischer and Taylor pricing

- Monetary policy can reduce the cost of rigid prices

Calvo pricing and the New Keynesian model

- Three equation model, fully forward looking

Optimal monetary policy

- Central banks may want to neutralize demand, but not supply shocks

Small open economy

- The interest rate is exogenously given and agents can save abroad

The secret ingredient is algebra!

Useful concepts

- Setting up first order conditions
- Lagrangian optimization
- Taking derivatives correctly
- Forward iterating of expectational difference equations
- Law of iterated expectations
- Taking infinite sums
- Logarithm rules (e.g., $\log(ab) = \log(a) + \log(b)$)

Advice

- Read the questions carefully
- Make sure to understand timing assumptions, etc.

Monopolistic competition

Monopolistic competition

Setup

- Large number of producers
- Each one has (a little bit of) market power
- Consumers have inelastic demand functions

Importance

- Firms are price setters (not price takers)
- Price changes don't make demand evaporate
- Allows the introduction of rigid prices

Derivation

Representative consumer

$$U = C^{\gamma}\left(\frac{M}{P}\right)^{1-\gamma} - \frac{1}{\phi}N^{\phi} \quad \text{with } 0 < \gamma < 1, \phi > 1$$

$$\text{s.t. } \sum_{i=1}^{m} P_i C_i + M = M_0 + WN + \sum_{i=1}^{m} \Pi_i$$

$$C = m^{\frac{1}{1-\theta}}\left(\sum_{i=1}^{m} C_i^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \text{ not } C = \left(\int_0^{\infty} C_i^{\frac{\theta-1}{\theta}} \quad di\right)^{\frac{\theta}{\theta-1}}$$

$$P = \left(\frac{1}{m}\sum_{i=1}^{m} P_i^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

- Assume there is a population of consumers of total size equal to one $\Longrightarrow$ representative consumer
- Special here: not an infinite number of firms, but m

Lagrangian

Solution methods

- Lecture 8: First solve for aggregate consumption, then solve for consumption of each variety i
- Today: Simple approach, just plug in
- More convoluted to solve, but straight forward to set up
- Both will give the same result

$$\max_{C_i, N, \frac{M}{P}} \mathcal{L} = \left[m^{\frac{1}{1-\theta}} \left(\sum_{i=1}^{m} C_i^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^{\gamma} \left(\frac{M}{P} \right)^{1-\gamma} - \frac{1}{\beta} N^{\beta}$$
$$- \lambda \left(\sum_{i=1}^{m} \frac{P_i}{P} C_i + \frac{M}{P} - \frac{M_0}{P} - \frac{W}{P} N - \sum_{i=1}^{m} \frac{\Pi_i}{P} \right)$$

Optimality

First order conditions

$$\frac{\partial \mathcal{L}}{\partial C_i} = 0 \Rightarrow \left(\frac{M}{P}\right)^{1-\gamma} \frac{\gamma\theta}{\theta - 1} m^{\frac{\gamma}{1-\theta}} \left(\sum_{i=1}^{m} C_i^{\frac{\theta-1}{\theta}}\right)^{\frac{\gamma\theta}{\theta-1}-1} \frac{\theta - 1}{\theta} C_i^{-\frac{1}{\theta}} = \lambda \frac{P_i}{P}$$

$$\frac{\partial \mathcal{L}}{\partial (M/P)} = 0 \Rightarrow (1-\gamma)\left(\frac{M}{P}\right)^{-\gamma} C^{\gamma} = \lambda$$

$$\frac{\partial \mathcal{L}}{\partial N} = 0 \Rightarrow N^{\beta-1} = \lambda \frac{W}{P}$$

Optimality (combine conditions 1 and 2)

$$\frac{M}{PC}\frac{\gamma}{1-\gamma}\left(\frac{C}{mC_i}\right)^{\frac{1}{\theta}} = \frac{P_i}{P}$$

- Plugging this into the equation for the price index, we get $P = \frac{\gamma}{1-\gamma}\frac{M}{C}$

$\Longrightarrow$ cancel to get demand as function of price

Consumer demand

Consumer optimality

$$C_i = \left(\frac{P_i}{P}\right)^{-\theta} \frac{C}{m}$$

Very similar expression to infinite-variety case (Lecture 8)

$$C_i = \left(\frac{P_i}{P}\right)^{-\theta} C$$

Firm optimality (same as before)

$$\max_{\frac{P_i}{P}, N_i, Y_i} \Pi_i = P_i Y_i - W N_i$$

$$\text{s.t. } Y_i = C_i = \left(\frac{P_i}{P}\right)^{-\theta} \frac{C}{m},$$

$$Y_i = N_i^{\alpha}, \quad 0 < \alpha < 1$$

Firm FOCs, optimality and market clearing

Firm optimality

$$\frac{P_i}{P} = \left[\frac{\theta}{\theta - 1} \frac{1}{\alpha} \frac{W}{P} \left(\frac{C}{m} \right)^{\frac{1-\alpha}{\alpha}} \right]^{\frac{\alpha}{\alpha + \theta(1-\alpha)}}$$

Imposing homogeneity gives goods market equilibrium ($P_i = P$)

$$\frac{W}{P} = \frac{\theta - 1}{\theta} \alpha \left(\frac{C}{m} \right)^{\frac{\alpha - 1}{\alpha}}$$

- Real wage is less than marginal cost
- θ, the degree of demand elasticity, governs how much
- $\theta \to \infty$ implies perfect competition

Implications of monopolistic competition

Welfare loss

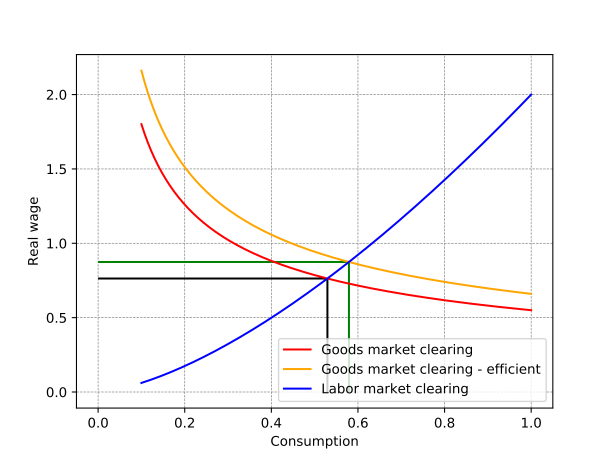
- Wages are too low $\implies$ output is too low
- Firms don't price in the demand effect they have on other producers
  - If one firm lowers its P_i, consumers become richer and demand more of everything
  - All firms make slightly more profit $\implies$ welfare could be improved
- A labor subsidy can eliminate the distortion

Rigid prices

- $C = \frac{\gamma}{1-\gamma}\frac{M}{P}$ implies that M can affect C if P is rigid
- If ϕ is high (el. of lab. supply is low), price adjustment costs need to be very high
- Welfare loss is larger if ϕ is low

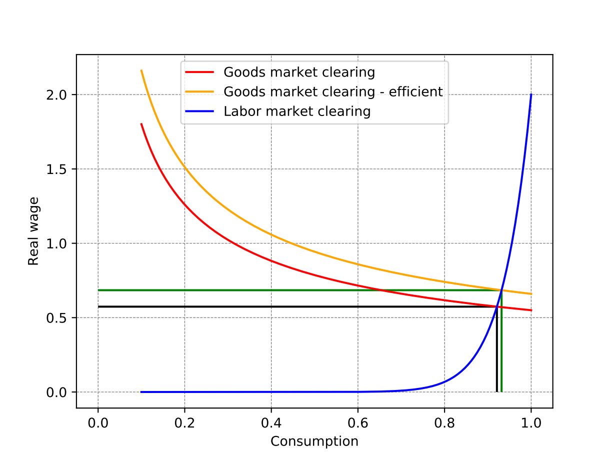
Welfare loss graphic

Low ϕ (high elasticity of labor supply)



Welfare loss graphic

High ϕ (low elasticity of labor supply)



Lucas model

Rational expectations

Usefulness

- Allows for the analysis of dynamic models
- Can think about feedback loop between policy and expectations

Formulation

$$\mathbb{E}_t[X_{t+1}] = \mathbb{E}_t[X_{t+1}|I_t]$$

- Agents base their expectations on an information set I_t
- They know the processes underlying the economy
- They don't make systematic forecast errors

  Expectational difference equations can be solved fowards

What does this mean for the Phillips Curve?

Lucas' island model

- Perfect competition, everyone is a price taker
- Dynamic model
- Households cannot see what is happening on other islands (info frictions)
- Producers need to disentangle movements in P_i from movements in P

Movements in P_i vs P

- If P_i rises but P remains constant, want to produce more
- If P_i and P rise, produce as before
- Decision links output to inflation (Phillips Curve)

Equilibrium

General equilibrium in the Lucas model (for derivation see lecture 9)

$$p = \frac{1}{1+b}(m - \mathbb{E}[m]) + \mathbb{E}[m]$$
$$y = \frac{b}{1+b}(m - \mathbb{E}[m])$$

Implications

- Only unexpected money growth matters for output
- Expected money growth $\mathbb{E}[m]$ raises prices, but not output
- Central banks (CB) cannot generate demand by raising the money supply
- CB can only affect output if they have an informational advantage or can make decisions after expectations are locked in

Phillips Curve

Feedback loop between inflation and output

$$\pi_t = \mathbb{E}_{t-1}[\pi_t] + \frac{1}{b} y_t$$

Intuition

- Expected inflation drives current inflation
- Higher output leads to higher prices (depending on b)
- High variance of firm demand shock z increases b
- Variance of money growth m (aggregate demand shock) lowers b

Conclusion

- An empirical relationship cannot be taken for granted if agents' decisions change when new information arrives

Pricing frictions

Overview

Non-microfounded pricing frictions

- Fischer pricing: set unchangeable price schedule
- Taylor pricing: set unchangeable prices
- Calvo pricing: Fixed reset probability

Implications

- No need for elaborate justification (like informational frictions)
- Reasonable approximation of the real world
- Rigid prices imply that demand shocks affect output

Set up the problem (Exam Feb 2022)

Representative household

- Assume a continuum of identical households, whose total number is normalized to one

$$U_i = \mathbf{C}_i - \frac{1}{\phi} L_i^{\phi}$$

Budget constraint and demand function

$$\mathbf{C}_i = \frac{P_i}{P} Y_i \quad \& \quad Y_i = \left(\frac{P_i}{P} \right)^{-\theta} Y$$

- **C** is a consumption basket
- Households use their labor L to produce output according to $Y_i = L_i^{\alpha}$
- P is the aggregate price level, P_i is the price of the household's variety i

a) Utility maximization + F.O.C.

Problem

$$\max_{P_i} U_i = \frac{P_i}{P}\left(\frac{P_i}{P}\right)^{-\theta} Y - \frac{1}{\phi}\left[\left(\left(\frac{P_i}{P}\right)^{-\theta} Y\right)^{\frac{1}{\alpha}}\right]^{\phi}$$

First order condition (after some tedious algebra)

$$\frac{P_i}{P} = \frac{\theta}{\theta - 1}\frac{1}{\alpha} Y_i^{\frac{\phi}{\alpha}-1}$$

Take-aways

- For alternative derivation see lecture 10 (same result, obviously)
- Monopolistic competition makes prices too high, relative to marginal cost
- $\frac{1}{\alpha} Y_i^{\frac{\phi}{\alpha}-1}$ is marginal cost of output
- More competition ($\theta \uparrow$) lowers the markup
- Lower α raises prices (decreasing returns)

b) Compute the desired price under flexible prices

Taking logs

$$p_i^* - p = \log\left(\frac{\theta}{(\theta-1)\alpha}\right) + \frac{\phi-\alpha}{\alpha} y_i$$

Aggregate

$$p^* - p = \log\left(\frac{\theta}{(\theta-1)\alpha}\right) + \frac{\phi-\alpha}{\alpha} y$$

- Use the market clearing condition in logs $m - p = y$

$$\begin{aligned} p^* - p &= \log\left(\frac{\theta}{(\theta-1)\alpha}\right) + \frac{\phi-\alpha}{\alpha}(m-p) \\ \implies p^* &= \log\left(\frac{\theta}{(\theta-1)\alpha}\right) + \frac{\phi-\alpha}{\alpha} m + \left(1 - \frac{\phi-\alpha}{\alpha}\right) p \end{aligned}$$

b) Compute the desired price under flexible prices (cont.)

$$p^* = \underbrace{\log\left(\frac{\theta}{(\theta-1)\alpha}\right)}_{\mathcal{M}} + \underbrace{\frac{\phi-\alpha}{\alpha}}_{v} m + \left(1 - \frac{\phi-\alpha}{\alpha}\right) p$$

Interpretation

- This equation pins down what prices will be set to
- The first term is the markup
- Under flexible prices, $p^* = p$, under rigid prices, the two may differ

Pricing frictions

- Fundamentally, all these models result in this optimality condition
- The next step is to introduce some form of price rigidity

c) (modified) Prices under Fischer contracts

- Half of agents set prices each period
- p_t^{-1} represents the price for period t set in period $t-1$

Price level (suppress the markup)

$$p_t = \frac{1}{2}\left(p_t^{-1} + p_t^{-2}\right)$$
$$p_t^* = vm_t + (1-v)p_t$$

Optimal price setting in expectation

$$p_t^{-1} = \mathbb{E}_{t-1}[p_t^*] = v\mathbb{E}_{t-1}[m_t] + (1-v)\frac{1}{2}\left(p_t^{-1} + p_t^{-2}\right)$$
$$p_t^{-2} = \mathbb{E}_{t-2}[p_t^*] = v\mathbb{E}_{t-2}[m_t] + (1-v)\frac{1}{2}\left(\mathbb{E}_{t-2}[p_t^{-1}] + p_t^{-2}\right)$$

Equilibrium under Fischer contracts

Optimal price setting in expectation

$$p_t^{-2} = \frac{2v}{1+v}\mathbb{E}_{t-2}[m_t] + \frac{1-v}{1+v}\mathbb{E}_{t-2}[p_t^{-1}]$$
$$p_t^{-1} = \mathbb{E}_{t-2}[m_t] + \frac{2v}{1+v}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right)$$

- Use law of iterated expectations: $\mathbb{E}_{t-1}[x_t] = \mathbb{E}_{t-2}[\mathbb{E}_{t-1}[x_t]]$

Once price setting is solved, get price level and output

$$\begin{aligned} p_t &= \frac{1}{2}\left(p_t^{-1} + p_t^{-2}\right) \\ &= \mathbb{E}_{t-1}[m_t] + \frac{1}{1+v}\left(\mathbb{E}_{t-2}[m_t] - \mathbb{E}_{t-1}[m_t]\right) \\ y_t &= m_t - \mathbb{E}_{t-1}[m_t] - \frac{1}{1+v}\left(\mathbb{E}_{t-2}[m_t] - \mathbb{E}_{t-1}[m_t]\right) \end{aligned}$$

Rigid prices – Interpretation

General equilibrium (not a typo, just rewritten)

$$p_t = \mathbb{E}_{t-2}[m_t] + \frac{v}{1+v}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right)$$
$$y_t = m_t - \mathbb{E}_{t-1}[m_t] - \frac{1}{1+v}\left(\mathbb{E}_{t-2}[m_t] - \mathbb{E}_{t-1}[m_t]\right)$$

- Unanticipated shocks to m_t have real effects (as always)
- Anticipated shocks to m_t affect those who cannot reset their prices in time
- Importance of rigidity is governed by $v = \frac{\phi - \alpha}{\alpha}$

Interpretation

- $\phi \uparrow$ implies less elastic labor supply $\implies$ price changes matter less
- Similarly, $\alpha \downarrow$ increases marginal costs $\implies$ changing output (when prices are wrong) is more expensive

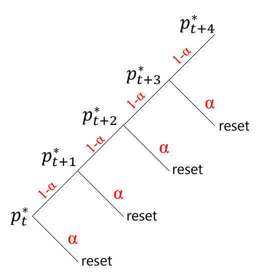
Calvo pricing and the New Keynesian model

Calvo pricing I

Constant reset probability

- Every firm resets its price with constant probability
- Price level is $p_t = \gamma x_t + (1 - \gamma) p_{t-1}$
- Optimal price (no markup) under flexible prices is $p_t^* = v m_t + (1 - v) p_t$

Optimal reset price



Calvo pricing II

Optimal reset price

$$x_t = \sum_{j=0}^{\infty} \frac{\beta^j (1-\gamma)^j \mathbb{E}[p^*_{t+j}]}{\sum_{j=0}^{\infty} \beta^j (1-\gamma)^j}$$

$$= (1 - \beta(1-\gamma)) \sum_{j=0}^{\infty} \beta^j (1-\gamma)^j \mathbb{E}[p^*_{t+j}]$$

$$= (1 - \beta(1-\gamma)) p^*_t + \beta(1-\gamma) \mathbb{E}_t[x_{t+1}]$$

Intuition

- The current optimal reset price depends on the expected ideal price over the foreseeable future
- A higher γ gives less weight to the distant future
- A higher β gives more weight to the distant future

Inflation and Phillips Curve

Inflation

$$\pi_t = \underbrace{\frac{\gamma(1-\beta(1-\gamma))v}{1-\alpha}}_{\kappa} y_t + \beta\mathbb{E}_t\left[\pi_{t+1}\right]$$

Interpretation

- The Phillips curve is steeper when v is large and/or γ is close to 1
- Inflation if pinned down by future inflation (i.e., future output)

Inflation as a function of expected output (Exam June 2021)

$$\pi_t = \kappa\sum_{s=0}^{\infty}\beta^s y_{t+s}$$

$$\mathrm{Var}(\pi_t) = \kappa^2\sum_{s=0}^{\infty}\beta^{2s}\mathrm{Var}(y_{t+s}) = \kappa^2\frac{\sigma_y}{1-\beta^2}$$

The New Keynesian model

Equations

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \left[\pi_{t+1}\right]$$
$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}\left(E_t[r_t] - \rho\right)$$
$$r_t = \rho + \phi_y \mathbb{E}_t[y_{t+1}] + \phi_\pi \mathbb{E}_t[\pi_{t+1}]$$

Important takeaways

- Inflation is pinned down by expectations about future output
- Output is dictated by expected changes in the real interest rate

$\Longrightarrow$ Expectations determine the state of the economy

- Effect of single period shocks can be solved for by hand
- Heterogeneity alters the equations slightly

Shocks

Output

$$y_t = \left(1 - \frac{\phi_y}{\sigma}\right)\mathbb{E}_t[y_{t+1}] - \frac{\phi_\pi}{\sigma}E[\pi_{t+1}] - \frac{1}{\sigma}u_{MP} + u_{IS}$$

Inflation

$$\pi_t = \left(1 - \frac{\phi_y}{\sigma}\right)\kappa\mathbb{E}_t[y_{t+1}] + \left(\beta - \frac{\phi_\pi}{\sigma}\right)E[\pi_{t+1}] - \kappa\left(\frac{1}{\sigma}u_{MP} - u_{IS}\right) + u_\pi$$

- Demand and monetary policy shocks can cancel each other
    - Monetary policy makes agents substitute intertemporally

$\implies$ if demand shock today, raise rates to push some of the demand to tomorrow

- Cost push shocks only raise prices
    - Nominal wages rise immediately, hence real wages stay constant

Optimal monetary policy

Model

Phillips Curve

$$\pi_t = m_t + \underbrace{v_t}_{\text{Demand shock}} + \underbrace{\mu_t}_{\text{MP shock}}$$

Demand equation

$$x_t = \underbrace{\theta_t}_{\text{natural rate of output}} + (\pi_t - \pi_t^e) - \underbrace{\varepsilon_t}_{\text{Supply shock}}$$

Timing assumptions

1. Announcement of monetary rule (credible or not)
2. Everyone observes the natural level of output θ
3. Expectations π^e are formed, given the information about θ
4. Everyone observes v and ε
5. The central bank decides the money supply m (advantage)
6. μ is realized, pinning down output x and inflation π

Further ingredients

Society's loss function

$$\mathcal{L} = \frac{1}{2}\left[a(\pi - \overline{\pi})^2 + \lambda(x - \overline{x})^2\right]$$

Monetary policy rule

$$m = \varphi + \varphi_\theta \theta + \varphi_v v + \varphi_\varepsilon \varepsilon$$

Implications

- The central bank has an informational advantage, it can move after agents have committed to their expectations
- Agents and the central bank play a game, with agents at first mover disadvantage
- Society's loss depends on the ability of the CB to commit or on the ability of the agents to punish deviations

Solution – Backwards induction

Commitment

- Society wants to set the parameters in the monetary policy rule to minimize the **expected** loss $\implies$ society moves first
- Solve the agents' problem given the loss function
- Optimize each parameter in the loss function

Discretion

- The central bank can do whatever it wants $\implies$ agents move first
- Solve the central bank's problem given the agents' expectations
- Then solve backwards (plug CB's actions into the agents' policy functions)

Reputation

- The central bank can do whatever it wants **but** the parties play a repeated game
- Single period decision exactly like discretion, but have to take into account the future costs

Result – Commitment

Optimal rule

$$m_t = \overline{\pi} - v_t + \frac{\lambda}{a + \lambda}\varepsilon$$

Equilibrium inflation

$$\pi^C = \overline{\pi} + \frac{\lambda}{a + \lambda}\varepsilon + \mu$$

Equilibrium output

$$x^C = \theta - \frac{a}{a + \lambda}\varepsilon + \mu$$

- Inflation is anchored at the desired level
- Output is anchored at its natural level
- The CB does not react to θ since it is priced into agents' expectations
- Responsiveness to supply shocks depends on society's preferences

Result – Discretion

Output

$$x^D = \theta - \frac{a}{a+\lambda}\varepsilon$$

Inflation

$$\pi^D = \overline{\pi} + \frac{\lambda}{a}(\overline{x} - \theta) + \frac{\lambda}{a+\lambda}\varepsilon$$

- Output is the same as under commitment: agents know the CB's objective function and price it into their expectations
- Inflation is higher and more volatile
- This is the optimal solution for the CB. If θ is too low, agents know the CB will increase m, so they raise their prices.
- The CB now has no choice but to actually print the money, otherwise prices are too high and output is too low

Result – Reputation

Long-run loss function

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^j \mathcal{L}(\pi_{t+j}, x_{t+j})$$

Punishment

- Some rule by which expectations change when the CB deviates
- Possible examples: tit-for-tat, assured destruction, limited-time punishment

Optimization

- Attractiveness of deviation depends on the **present value** of cost
- If cost is in every future period, or discount factor is high, deviation is painful

Open economy macroeconomics

Important ingredients

Neoclassical economy

- No money
- Prices are perfectly flexible
- Perfect competition

Openness

- Markets for goods and savings do not need to clear within the country

$\Longrightarrow$ Ship (invest) excess production (assets) abroad

Interest rates

- In a small open economy, interest rates are given from abroad (not pinned down by domestic savings behavior)
- Savings and investment decisions do not affect the level of the interest rate (everyone is a price taker)

Optimality

Consumers

$$u'(c_t) = \beta(1 + r_{t+1} - \delta)\mathbb{E}[u'(c_{t+1})]$$

Firms

$$f_K(k_t, 1) = r_t$$
$$f_L(k_t, 1) = w_t$$

Interaction with the ROTW (all 0 in closed economy)

$$tb_t = \underbrace{f(k_t, 1)}_{y_t} - c_t - \underbrace{(k_{t+1} - (1-\delta)k_t)}_{i_t}$$
$$N_t = a_t - k_t$$
$$ca_t = \underbrace{tb_t}_{\text{trade}} + \underbrace{r_t N_t}_{\text{interest}} - \underbrace{\delta N_t}_{\text{depreciation}}$$

Equilibrium in an endowment economy

Setup

- Home country has no capital but can save abroad

$$c = \rho a_t R_t + \underbrace{\rho \sum_{s=0}^{\infty} \frac{w_{t+s}}{R^s}}_{\text{perm. inc.}} \quad (\text{ in closed economy: } c_t = \omega_t)$$

Use foreign assets to stabilize consumption

$$ca_t = N_{t+1} - N_t = \omega_t - \widetilde{\omega}_t$$
$$tb_t = \omega_t - \rho a_t R_t - \rho \sum_{s=0}^{\infty} \frac{w_{t+s}}{R^s}$$

- Opening up the economy allows agents to access savings instruments
- Consumption can be smoothed without the need for own productive capital

Real exchange rates (RER)

Setup

- Real exchange rate is one if there is only one good

$\Longrightarrow$ Need at least two goods

- Here: tradable & non-tradable

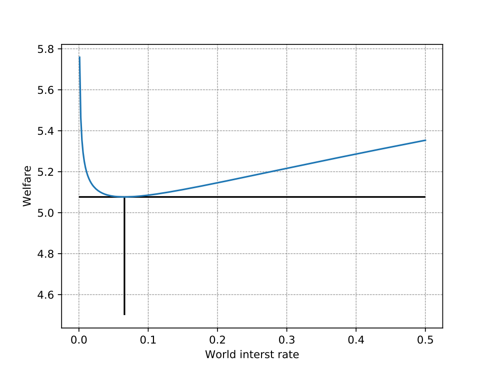
Equilibrium in an endowment economy

- Non-tradable consumption is more expensive in countries that are rich in tradables or foreign assets (higher RER)
- Non-tradable good is the limiting factor $\Longrightarrow$ more overall consumption demand makes it more expensive

Equilibrium in a production economy

- In the long-run, factors of production adjust to most productive use
- Intuition similar: countries that are productive at making tradables have higher RERs
- Higher productivity drives up wages in all sectors $\Longrightarrow$ all goods more expensive

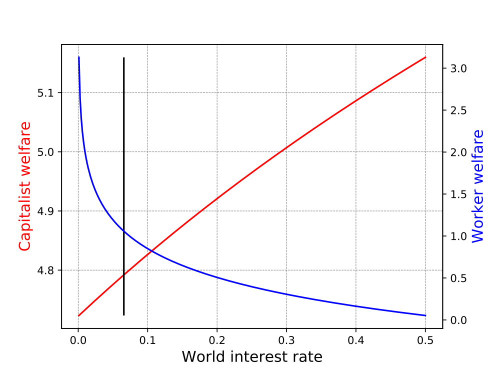
Gains from trade – representative agent



Opening up the economy is always beneficial

- If $r < f'(k_t)$, cheap capital flows into the economy, raising output
- If $r > f'(k_t)$, domestic capital moves abroad and earns higher return (home output falls)

Unequal gains from trade



Gains depend on distribution

- If $r < f'(k_t)$, cheap capital flows into the economy, wages rise
- If $r > f'(k_t)$, domestic capital moves abroad and earns higher return, capitalists gain, workers lose