Open economy macroeconomics

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Second half so far

Pricing frictions

- Monopolistic competition leads to welfare losses
- Sticky/Rigid prices allow the central bank to influence output
- Taylor, Fischer, Calvo

Expectations matter

- Expected "shocks" do not affect output
- Prices adjust to keep output constant

The New Keynesian model

- Three equations to rule the world: PC, IS, TR
- Useful model of the world
- Heterogeneity matters for aggregate movements

Optimal monetary policy

- Central banks may want to neutralize demand, but not supply shocks

Today: change of topic!

Small open economy

- Back to **real**-ity: perfect competition

New ground

- What is a small open economy?
- Important concepts: trade balance & current account
- Gains from trade (heterogeneity)

Real exchange rate

- Multiple goods
- Short & long run dynamics

The small open economy

Setup

Open economy

- Markets don't clear internally anymore
- On the world level there is market clearing: $\mathbf{k}_{t+1} = \mathbf{a}_{t+1}$, but not in each country
- Goods, capital and assets can be exchanged

Small open economy

- Saving decisions do not affect the world interest rate
- Only focus on home country today, everything else is the rest of the world (ROTW)

Neoclassical open economy I

Representative consumer

$$\max_{c_t, a_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t. } a_{t+1} + c_t = a_t R_t + w_t$$

- Capital does not show up directly
- $R_t = 1 + r_t - \delta$
- Interest rate r_t is the world interest rate, capital can move freely across borders

Optimality

$$u'(c_t) = \beta R_t \mathbb{E}[u'(c_{t+1})]$$

Neoclassical open economy II

Representative firm

$$\max_{K_t, L_t} f(K_t, L_t) - r_t K_t - w_t L_t$$

- Firms are completely standard
- r_t is the world interest rate

Optimality

$$f_K(k_t, 1) = r_t$$
$$f_L(k_t, 1) = w_t$$

- Without technological progress and constant r_t, everything is constant

New concepts

Trade balance: Exports - Imports (flow)

$$tb_t = \underbrace{f(k_t, 1)}_{y_t} - c_t - \underbrace{(k_{t+1} - (1-\delta)k_t)}_{i_t}$$

- In a closed economy, production y equals consumption and investment, because markets clear internally
- Not true in the open economy $\implies$ not all output has to be consumed at home

Net foreign assets (stock)

$$N_t = a_t - k_t$$

- Not all assets need to be held at home
- Difference between demand a_t and supply k_t must be held in the rest of the world (ROTW)

Current account

Total goods received from/sent to ROTW

$$ca_t = \underbrace{tb_t}_{\text{trade}} + \underbrace{r_t N_t}_{\text{interest}} - \underbrace{\delta N_t}_{\text{depreciation}}$$

- The current account is a flow, just like the trade balance

Link between net foreign asset position and current account

- If the current account is positive, a country produced (y_t) than it used (c_t, i_t)
- This overproduction has to be stored somewhere $\implies$ more foreign asset savings

Net foreign asset dynamics – Algebra

$$
\begin{aligned}
ca_t &= tb_t + r_t N_t - \delta N_t \\
&= f(k_t, 1) - c_t - (k_{t+1} - (1-\delta)k_t) + (r_t - \delta)N_t \\
&= f(k_t, 1) - a_t R_t - w_t + a_{t+1} - (k_{t+1} - (1-\delta)k_t) + (r_t - \delta)N_t \\
&= f(k_t, 1) - w_t - a_t R_t + \underbrace{a_{t+1} - k_{t+1}}_{N_{t+1}} + (1-\delta)k_t + (r_t - \delta)N_t \\
&= f(k_t, 1) - w_t - a_t R_t + N_{t+1} + (1-\delta)k_t + \underbrace{(r_t - \delta)N_t}_{(R_t - 1)(a_t - k_t)} \\
&= f(k_t, 1) - w_t - a_t R_t + N_{t+1} + (1-\delta)k_t + R_t(a_t - k_t) - N_t \\
&= f(k_t, 1) - w_t + N_{t+1} - \underbrace{(R_t - 1 + \delta)}_{r_t} k_t - N_t \\
&= f(k_t, 1) - w_t - r_t k_t + N_{t+1} - N_t \\
&= N_{t+1} - N_t \\
&= \Delta N_{t+1}
\end{aligned}
$$

General equilibrium

Asset accumulation

- Asset accumulation is governed by the representative household's Euler equation and their budget constraint

$$u'(c_t) = \beta R_t \mathbb{E}[u'(c_{t+1})]$$
$$a_{t+1} + c_t = a_t R_t + w_t$$

Capital

- The amount of capital is governed by the firm's investment choice

$$f_K(k_t, 1) = r_t$$

- There is no resource constraint $(k_{t+1} = k_t(1-\delta) + f(k_t, 1) - c_t)$ within the economy
- The interest rate is not endogenous

Consumption in equilibrium

Solve the budget constraint forward

$$\begin{aligned} c_t &= a_t R_t + w_t - a_{t+1} \\ &= a_t R_t + w_t - \left(\frac{a_{t+2} + c_{t+1} - w_{t+1}}{R_{t+1}} \right) \\ &= a_t R_t + \sum_{s=0}^{\infty} \frac{w_{t+s}}{\Pi_{j=0}^{s} R_{t+j}} - \sum_{s=1}^{\infty} \frac{c_{t+s}}{\Pi_{j=0}^{s} R_{t+j}} \end{aligned}$$

- Assume a transversality condition: $\lim_{s\to\infty} \frac{1}{\Pi_{j=0}^{s} R_{t+j}} a_{t+s} = 0$
- Assume $R_t = R = \frac{1}{\beta} \implies c$ is constant

$$c = \rho a_t R_t + \rho \sum_{s=0}^{\infty} \frac{w_{t+s}}{R^s} \quad \text{where} \quad \left(\rho = \sum_{s=0}^{\infty} \frac{1}{R^s} \right)^{-1} = \frac{R-1}{R}$$

- Consumption depends on lifetime wealth (perfect smoothing)

An open endowment economy

Setup

- Assume the home economy cannot accumulate capital: $k_t = 0$
- Assume that workers receive an (time varying) endowment ω_t

Implications

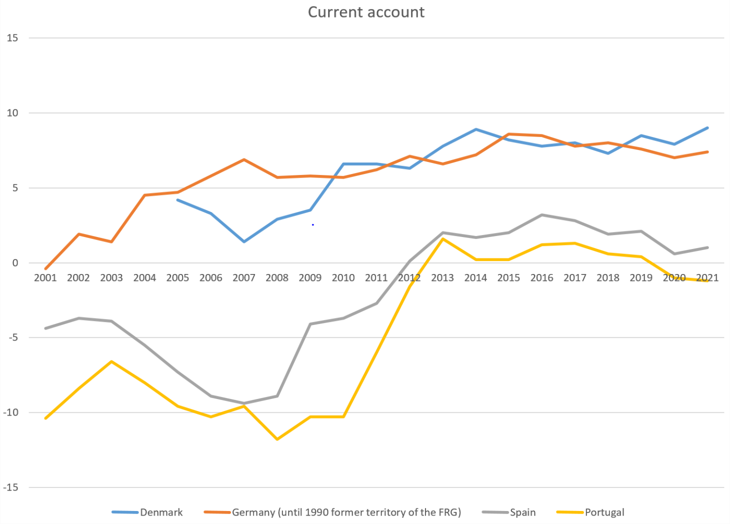
- The net foreign asset position is $N_t = a_t$
- Let permanent income be $\widetilde{\omega}_t = \rho \sum_{t=0}^{\infty} \frac{\omega_t}{R^t}$
- The period budget constraint is $c_t = N_t R - N_{t+1} + \omega_t$

$$
\begin{aligned}
N_t R - N_{t+1} + \omega_t &= \rho R N_t + \widetilde{\omega}_t \\
N_t R(1-\rho) - N_{t+1} &= \widetilde{\omega}_t - \omega_t \\
ca_t = N_{t+1} - N_t &= \omega_t - \widetilde{\omega}_t
\end{aligned}
$$

$\Longrightarrow$ Perfect insurance **against temporary shocks** but without domestic capital to save in

Empirical evidence

The current account



The trade balance

Permanent income consumers

$$\sum_{s=0}^{\infty} \frac{c}{R^s} = N_t R + \sum_{s=0}^{\infty} \frac{\omega_{t+s}}{R^s}$$

Trade balance in the endowment economy

$$tb_t = \omega_t - c$$

Can a country run a trade-deficit ($tb_t < 0$) forever?

The trade balance

Permanent income consumers

$$\sum_{s=0}^{\infty} \frac{c}{R^s} = N_t R + \sum_{s=0}^{\infty} \frac{\omega_{t+s}}{R^s}$$

Trade balance in the endowment economy

$$tb_t = \omega_t - c$$

Can a country run a trade-deficit ($tb_t < 0$) forever?

$$\sum_{s=0}^{\infty} \frac{c}{\Pi_{j=0}^{s} R_{t+j}} - \sum_{s=0}^{\infty} \frac{\omega_{t+s}}{\Pi_{j=0}^{s} R_{t+j}} = a_t R_t$$

$$\sum_{s=0}^{\infty} \frac{tb_{t+s}}{\Pi_{j=0}^{s} R_{t+j}} = -a_t R_t$$

- Only rich countries can run deficits for long periods of time
- If $a_t < 0$, then the country must eventually run trade surpluses

The trade balance over time

Consumption

$$c = \rho a_t R_t + \rho \sum_{s=0}^{\infty} \frac{w_{t+s}}{R^s}$$

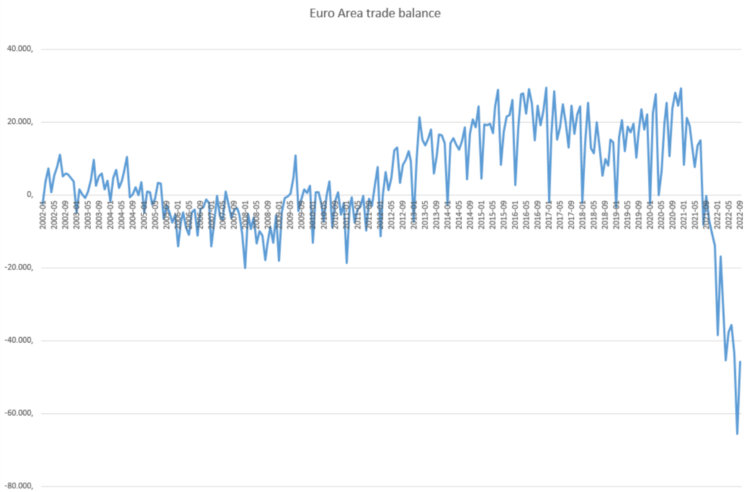
Trade balance in the endowment economy

$$tb_t = \omega_t - \rho a_t R_t - \underbrace{\rho \sum_{s=0}^{\infty} \frac{w_{t+s}}{R^s}}_{\text{perm. inc.}}$$

- The trade balance is **procyclical**
- When current income ω_t is higher than permanent income, the economy exports more

Empirical evidence

Trade balance of the euro area



The real exchange rate

Extending the model

Exchange rate determination

- So far, there is only one consumption good that all countries consume
- Hence, the real exchange rate between the home country and the ROTW is one

$\Longrightarrow$ To talk about exchange rates, we need at least two goods

Extending the model

Exchange rate determination

- So far, there is only one consumption good that all countries consume
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$\Longrightarrow$ To talk about exchange rates, we need at least two goods

Multiple goods

- Assume that there is a tradable good c^T and a non-tradable good c^N
- For c^N, the home market clears, c^T is traded internationally
- Households consume both goods such that $c_t = g(c^T, c^N)$
- Let p_t be the price of c^N, while the price of $c^T = 1$.
- $\mathcal{P}_t$ is the price index for a unit of the consumption aggregate c_t

Updated setup

Household problem

$$\max_{c_t^T, c_t^N, a_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c(c_t^N, c_t^T))$$

$$a_{t+1} + \underbrace{\mathcal{P}_t c_t}_{c_t^T + p_t c_N^T} = a_t R_t + \omega_t^T + p_t \omega_t^N$$

- $\mathcal{P}_t$ is the real exchange rate (price of c_t at home rel. to ROTW)
- Households choose how much to save (as before)
- Pick how much of each good to buy
- R_t is exogenous (as before)

Optimization

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c(c_t^N, c_t^T)) + \sum_{t=0}^{\infty} \lambda_t \left(a_t R_t + \omega_t^T + p_t \omega_t^N - c_t^T - p_t c_t^N - a_{t+1}\right)$$

Household optimality

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c(c_t^N, c_t^T)) + \sum_{t=0}^{\infty} \lambda_t \left(a_t R_t + \omega_t^T + p_t \omega_t^N - \underbrace{c_t^T - p_t c_t^N}_{\mathcal{P}_t c_t} - a_{t+1} \right)$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} : \lambda_t = \lambda_{t+1} R_t$$

$$\frac{\partial \mathcal{L}}{\partial c_t^T} : \beta^t u'(c_t) c_T(c_t^N, c_t^T) = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial c_t^N} : \beta^t u'(c_t) c_N(c_t^N, c_t^T) = \lambda_t p_t$$

Optimality conditions

$$u'(c_t) c_T(c_t^N, c_t^T) = \beta R_t u'(c_{t+1}) c_T(c_{t+1}^N, c_{t+1}^T)$$

$$p_t = \frac{c_N(c_t^N, c_t^T)}{c_T(c_t^N, c_t^T)}$$

$$u'(c_t) = \beta R_t \frac{\mathcal{P}_{t+1}}{\mathcal{P}_t} u'(c_{t+1})$$

Intuition

Optimality conditions

$$u'(c_t)c_T(c_t^N, c_t^T) = \beta R_t u'(c_{t+1})c_T(c_{t+1}^N, c_{t+1}^T)$$

$$p_t = \frac{c_N(c_t^N, c_t^T)}{c_T(c_t^N, c_t^T)}$$

$$u'(c_t) = \beta R_t \frac{\mathcal{P}_{t+1}}{\mathcal{P}_t} u'(c_{t+1})$$

- Consumption c_t follows an IS-curve $\implies$ marginal utility is related across periods
- $\frac{\mathcal{P}_{t+1}}{\mathcal{P}_t}$ represents changes in the real exchange rate
- $\mathcal{P}_t(p_t) \uparrow \implies$ appreciation, home goods become more expensive
- p_t (price of c^N in terms of c_T) is dictated by the marginal rate of substitution

Equilibrium

- Steady state: set $R = 1/\beta$ and $\omega_t^N = \omega^N$

Market clearing

- Non-tradables have to clear within the country $\implies$ $\omega^N = c^N$

$$u'(c_t)c_T(\omega^N, c_t^T) = \beta R_t u'(c_{t+1})c_T(\omega^N, c_{t+1}^T)$$

$$p_t = \frac{c_N(\omega^N, c_t^T)}{c_T(\omega^N, c_t^T)}$$

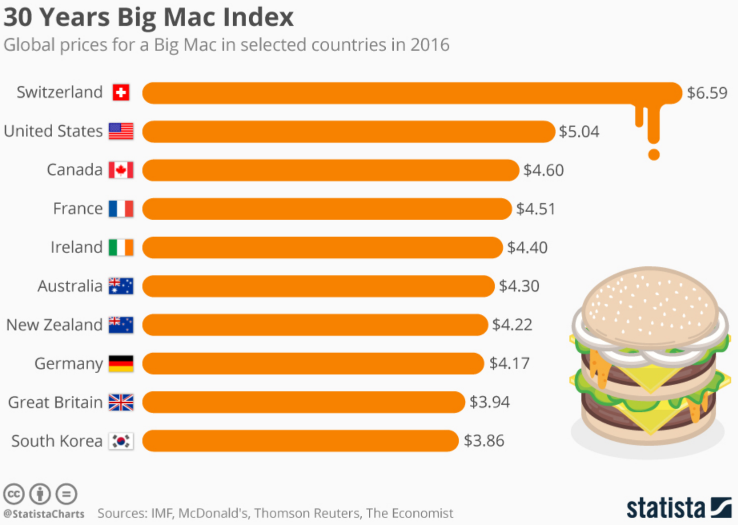
$$c_t^T = \rho N_t R_t + \rho \sum_{s=0}^{\infty} \frac{\omega_{t+s}^T}{R^s}$$

$\implies$ Tradable consumption is higher for richer countries (more net foreign assets or higher future endowment)

$\implies$ Richer countries want to consume more, but c^N is fixed in the short run $\implies$ $\mathcal{P}_t(p_t)$ higher

Empirical real exchange rate

The Big Mac index



Short run vs long run

Long run adjustments

- In the short run, non-tradable production may be fixed
- Over time, factors of production will realign to exploit price differences

$\Longrightarrow$ Move away from endowment assumption

Short run vs long run

Long run adjustments

- In the short run, non-tradable production may be fixed
- Over time, factors of production will realign to exploit price differences

$\Longrightarrow$ Move away from endowment assumption

Two goods

$$Y^T = A_t^T F(K_t^T, L_t^T)$$
$$Y^N = A_t^N F(K_t^N, L_t^N)$$

- Capital is perfectly mobile across the world (returns R_t)
- Labor is mobile within the home country, with $L_t^T + L_t^N = 1$
- As before, relative price of tradable good it p_t

Competitive Equilibrium

First order conditions

$$r_t = A_t^T f_K'(k_t^T)$$
$$r_t = p_t A_t^N f_K'(k_t^N)$$
$$w_t = A_t^T f_L'(k_t^T)$$
$$w_t = p_t A_t^N f_L'(k_t^N)$$

- The tradable sector is the numeraire, non-tradable goods have to be transformed at price p_t
- Competitive firms make sure that cost of capital R_t is equal to the marginal benefit
- Wages (measured in tradable goods) equate to the MPL

Determinants of the RER in the long-run

First order conditions

$$
\begin{aligned}
r_t &= A_t^T f_K'(k_t^T) \\
r_t &= p_t A_t^N f_K'(k_t^N) \\
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w_t &= p_t A_t^N f_L'(k_t^N)
\end{aligned}
$$

Intuition – an increase in A_t^T

Determinants of the RER in the long-run

First order conditions

$$
\begin{aligned}
r_t &= A_t^T f_K'(k_t^T) \\
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w_t &= A_t^T f_L'(k_t^T) \\
w_t &= p_t A_t^N f_L'(k_t^N)
\end{aligned}
$$

Intuition – an increase in A_t^T

- An increase in A^T drives factors of production towards tradables
- This raises wages in both sectors
- p_t or $f_L'(k_t^N)$ have to rise, but they cannot move in opposite directions because r_t is constant
- p_t rises $\implies$ RER rises

Determinants of the RER in the long-run – Algebra

Start from zero profit conditions

$$A_t^T f(k_t^T) = w_t + k_t^T r_t$$
$$p_t A_t^N f(k_t^N) = w_t + k_t^N r_t$$

Total derivatives $f(k_t^T) + A_t^T \frac{df(k_t^T)}{dk_t^T} \frac{dk_t^T}{dA_t^T} = \frac{dw_t}{dA_t^T} + \frac{dk_t^T}{dA_t^T} r_t$

$$A_t^N f(k_t^N) \frac{dp_t}{dA_t^T} + A_t^N \frac{df(k_t^N)}{dk_t^N} \frac{dk_t^N}{dA_t^T} p_t = \frac{dw_t}{dA_t^T} + \frac{dk_t^N}{dA_t^T} r_t$$
$$\implies \frac{dp_t}{dA_t^T} = \frac{f(k_t^T)}{A_t^N f(k_t^N)} \qquad \implies \frac{A_t^T}{p_t} \frac{dp_t}{dA_t^T} = \frac{A_t^T f(k_t^T)}{p_t A_t^N f(k_t^N)}$$

- Tradable productivity increases the RER (through a rise in the price of non-tradable goods p_t)
- Countries with higher tradable productivity should have higher RERs (Harrod-Balassa-Samuelson)

Gains from trade

Trade is good, right?

The benefits of trade

- Ricardo says: trade is always good!
- Trade in assets can allow for more risk sharing
- Capital can flow to its most productive uses
- Consumption increases $\implies$ higher welfare

Complications

- Heterogeneity
- Unequal gains from trade

Two period model

Budget constraints

$$c_0 = f(k_0) - \underbrace{(k_0 - a_0)r_0}_{N_0} + a_0(1-\delta) - a_1$$

$$c_1 = f(k_1) - (k_1 - a_1)r_1 + a_1(1-\delta)$$

Households

$$\max_{c_0, c_1, a_1} u(c_0) + \beta u(c_1)$$

- If the economy is closed, $N_t = 0$
- Production functions are the same across the world

Optimality conditions

$$\max_{c_0, c_1, k_1} u(c_0) + \beta u(c_1)$$

First order condition in the closed economy

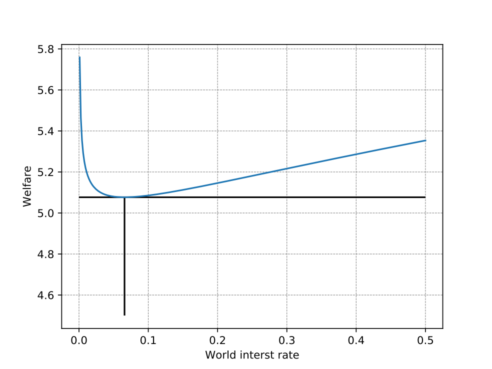
$$u'(c_0) = \beta(1 + f'(k_1) - \delta)u'(c_1)$$

First order condition in the small open economy

$$u'(c_0) = \beta(1 + r_t - \delta)u'(c_1)$$

- For the closed economy, the country-specific interest rate equals the country's marginal product of capital
- If the economy opens up, its savings pay the world interest rate (because capital is perfectly mobile)

Welfare



- Opening up always increases welfare
- If $r < f'(k_t)$, cheap capital flows into the economy, raising output
- If $r > f'(k_t)$, domestic capital moves abroad and earns higher return (home output falls)

Capitalists and workers

- Some agents own the firms and the capital
- The rest just work and collect wages (no saving)

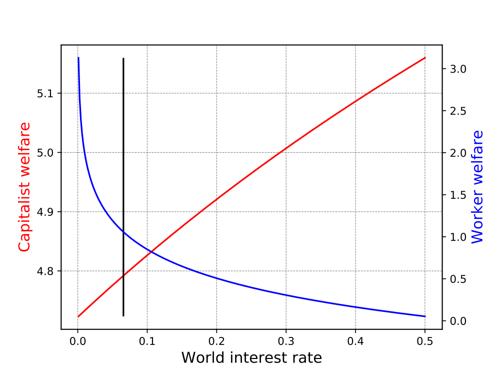
Capitalists (can save $\implies$ on their Euler equation)

$$c_0^K = f(k_0) - \underbrace{(k_0 - a_0)r_0}_{N_0} + a_0(1-\delta) - w_0 - a_1$$
$$c_1^K = f(k_1) - (k_1 - a_1)r_1 + a_1(1-\delta) - w_1$$

Workers (live hand-to-mouth)

$$c_0^K = w_0$$
$$c_1^K = w_1$$

Gains from trade with heterogeneity



- If $r < f'(k_t)$, cheap capital flows into the economy, raising labor productivity $\implies$ higher wages
- If $r > f'(k_t)$, domestic capital moves abroad $\implies$ home wages fall

$\implies$ Distributional aspects matter!

Heterogeneity matters for outcomes

Gains from trade

- Even small changes in the model lead to different conclusions
- Depending on the shares of workers/capitalists, opening an economy to the ROTW can have positive or negative effects on welfare
- Capital controls have the opposite effect
- **World interest rate shocks** also have heterogeneous effects

Beyond economics

- A social planner would redistribute resources such that trade is always beneficial
- How realistic this scenario is depends on the political environment (very much beyond the scope of this lecture)

World real interest rate

US 10-year interest rate

Two large open economies

2countries 2periods

Two countries

- Each country is large $\implies$ affects the interest rate
- Equivalent except for productivity in period 2
- Both enter the first period with the same capital stock k_0

Budget constaints

$$c_0^l = f(k_0^l) + k_0^l(1-\delta) - a_1^l$$
$$c_1^l = A^l f(k_1^l) - (k_1^l - a_1^l)r_1 + a_1^l(1-\delta)$$

World resource constraint

$$a_1^1 + a_1^2 = k_1^1 + k_1^2$$

Euler equations

$$u'(c_0^l) = \beta(1 + r_1 - \delta)u'(c_1^l)$$

How can this be solved?

Need to find:

- r_1 such that the world resource constaint is satisfied
- **given** r_1, a_1^l such that the Euler equations hold

Problem (similar for lots of problems in modern macro)

- There is no pencil-and-paper solution to this problem
- Once we know r_1, we know both k_1^l, a_0^l and k_0^l are given
- Even if we know r_1 (and log utility), a_1 is not easy to find:

$$f(k_1) = A_1^l k_t^\alpha \implies f'(k_t) = A_1^l \alpha k_1^{\alpha-1} = r_1 \implies k_1 = \left(\frac{r_1}{A_1^l \alpha}\right)^{\frac{1}{1-\alpha}}$$

$$\frac{1}{f(k_0^l) + k_0^l(1-\delta) - a_1^l} = \frac{\beta(1 + r_1 - \delta)}{A^l f(k_1^l) - (k_1^l - a_1^l) r_1 + a_1^l(1-\delta)}$$

Solution method

Turn to the computer

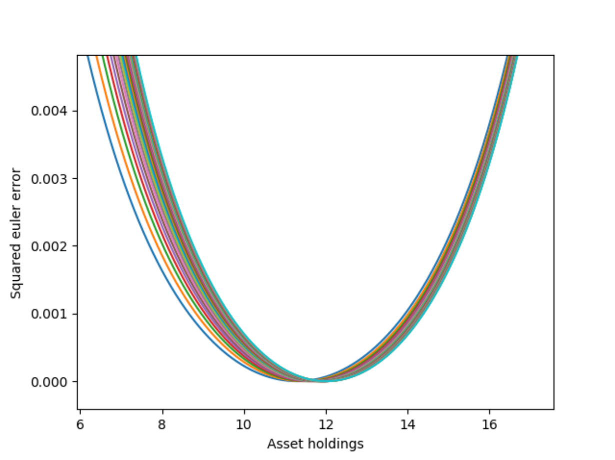
- On a computer, this is relatively easy to solve

Algorithm

- Guess a value of r_1
- Given r_1, guess values for a_1^1 and a_1^2
- Check if the Euler equations hold (if not, update the guess)
- Once the Euler equations hold, we have found a_1^1 and a_1^2
- Now check if the market clearing condition holds (if not, update the guess for r_1
- Once the market clearing condition holds, the problem is solved

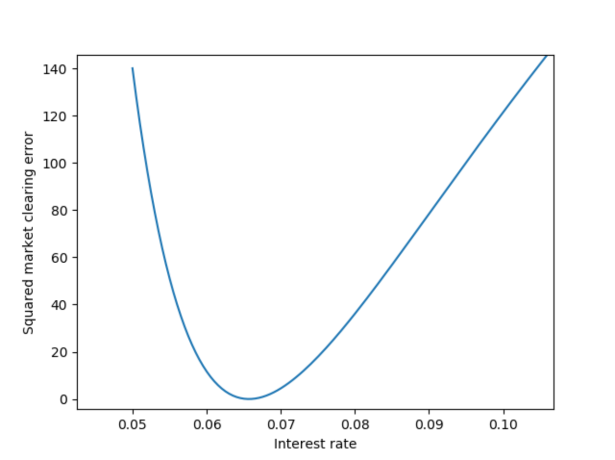
Solution method – pictures

Find asset holdings (for each country, given r_1)



Solution method – pictures

Find the correct interest rate



Global business cycles

Different productivities

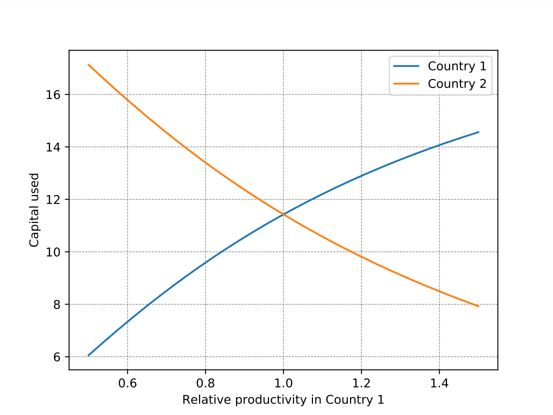
- If one country experiences a negative productivity shock, where does capital flow?

Global business cycles

Different productivities

- If one country experiences a negative productivity shock, where does capital flow?

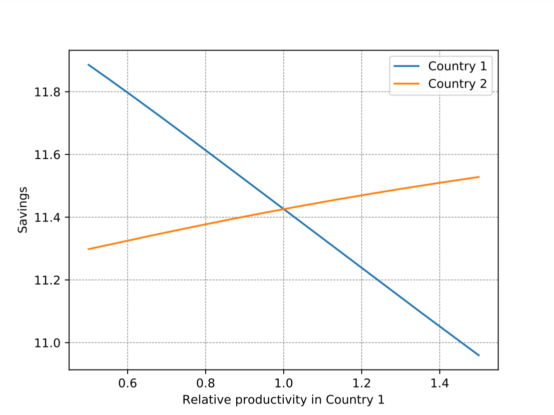
Capital



Different productivities

- If one country experiences a negative productivity shock, where does capital flow?

Savings



Global business cycles

Intuition

- If $A^1 \downarrow$, country 1 demands less capital
- Hence, world savings need to fall $\implies r_1 \downarrow$
- Total savings decrease, but only little
- Strong reallocation of capital

$\implies$ Capital responds strongly to interest rates, consumption does not

- Effect of savings stronger in country 1, since it is less productive in period 2

Total productivity increase (all countries)

- World interest rate rises due to more demand for capital

Conclusion

The small open economy

- Same setup as the closed economy but without internal market clearing
- Interest rates are given from abroad $\implies$ MPK externally determined
- Allows discussion of current account and trade balance

Implications

- External assets allow an economy access to insurance
- Current account and trade balance are procyclical
- The real exchange rate is determined by future permanent income in the short run
- In the long run, productivity differences matter

Gains from trade

- For the representative agent, trade is always good
- Inequality makes the conclusion more difficult