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1

1.1

As a movie I have chosen The Dark Knight. At the pinnacle of the film, The Joker has let two boats leave, one full of prisoners and one full of regular citizens. The Joker reveals, that he has equipped both boats with heavy explosives and placed a lever for the explosives at in the opposite boat. Therefore, each boat has the option of blowing up the other boat. If one boat decides to blow up the other, then they will live and vice versa. If no boat blows up, The Joker will blow up both boats.

1.2

I would define this a static game with complete information as The Joker has defined both boats strategies and the possible outcomes. I can classify this as a static game, because I assume, that when one boat plays pull, the other instantly plays do not pull. If both pulls at the same time, both boats sink.

1.3

I will now denote the game in normal form:

- Players: Boat 1(Citizens) and boat 2(Prisoners)
- Strategies: $S_i \in \{s_1, s_2\} = \{Pull, Don't\}$
- Payoff: $u_i(s_i, s_j) = \begin{cases} 1 & \text{if } \{s_1, s_2\} = \{Pull, Dont\} \\ 0 & \text{if } \{s_1, s_2\} = \{Dont, Pull\} \\ 0 & \text{if } \{s_1, s_2\} = \{Dont, Dont\} \end{cases}$

We write the Bimatrix as:

$$\begin{array}{ccc} & Pull & Don't \\ Pull & \underline{0}, 0 & \underline{1}, \underline{0} \\ Don't & \underline{0}, \underline{1} & 0, \underline{0} \end{array}$$

1.4

We use the underline technique in order to find the Nash Equilibrium. This leaves us with two NE's which is where one of the boats decide to pull the lever. Both of these are pure.

2

2.1

We suppose = 2, which gives the following matrix.

	A	B	C	D
a	$8,8$	$0,10$	$2,2$	$3,4$
b	$10,1$	$6,2$	$0,0$	$6,0$
c	$5,3$	$3,0$	$2,2$	$5,4$
d	$3,2$	$2,1$	$1,5$	$3,4$

We use IESDS eliminate strategies.

1. For player 1 c strictly dominates d
2. For player 2 A strictly dominates C
3. For player 1 b strictly dominates a
4. For player 1 b strictly dominates c
5. For player 2 B strictly dominates A
6. For player 2 B strictly dominates D

This leaves the possible strategy left:

$$\{(b, B)\}$$

2.2

For $a \leq 1$ there will be no strictly dominating strategies, because then d will not be dominated by c, and we will not be able to eliminate other strategies.

2.3

From the above we know, that there will not be any unique equilibriums, if $a \leq 1$. Furthermore, we will only have an unique solution if strictly dominates C, which is only possible if $a < 3$. We combine these to conditions and we obtain, that we only have a unique solution if $1 < a < 3$.

3

3.1

The inverse demand function is given as:

$$P(q_1, q_2) = 8 - (q_1 + q_2)$$

The firms has the following cost-function:

$$C_i(q_i) = \frac{5}{2}(q_i)^2$$

We are now able to derive the profit functions. As the firms are identical, we know their functions are symmetrical.

$$\pi_1 = (8 - q_1 - q_2) \cdot q_1 - \frac{5}{2}(q_1)^2$$

FOC yields the best-response functions:

$$\frac{\partial \pi_1}{\partial q_1} = 8 - 2q_1 - q_2 - 5q_1 = 0$$

Solving for q_1:

$$7q_1 = 8 - q_2 \Leftrightarrow$$
$$BR_1 = q_1 = \frac{8 - q_2}{7}$$

And symmetrical for q_2, following the argument that the firms are identical:

$$BR_2 = q_2 = \frac{8 - q_1}{7}$$

In the NE we know that $q_1^\star = q_2^\star$, since the firms are identical and faces same demand. With this knowledge, we insert into the best-response function, and solve for their respective quantities:

$$BR_1 = q_1 = \frac{8 - q_2}{7} \Leftrightarrow$$
$$7q_1 = 8 - q_1 \Leftrightarrow$$
$$8q_1 = 8$$
$$q_1 = \frac{8}{8}$$
$$q_1^\star = q_2^\star = 1$$

This is the allocation in the Nash-equilibrium. We now derive the price as:

$$P(q_1^\star, q_2^\star) = 8 - (1 + 1) = 6$$

Which leads to the following NE-profits.

$$\pi_1 = \pi_2 = (8 - 1 - 1) \cdot 1 - \frac{5}{2}(1)^2 = 6 - \frac{5}{2} = \frac{7}{2}$$

3.2

The cost-functions of the merged firms is given as:

$$C_m(q_m) = (1 - s)\frac{5}{2}(q_m)^2$$

Calculating the derivative of the cost-function with respect to s represents the relation between the cost-function and s:

$$\frac{\partial C_m(q_m)}{\partial s} = -\frac{5}{2}(q_m)^2 < 0$$

When s increases, the cost-function will decrease. Naturally, higher costs will result in lower produced amount, since the firm is rational and profit-maximizing. Assuming that $s_1 > s_2$, the firm will produce more goods with s_1 than with s_2. In the NE, the produced quantity was 2. We want to find a value for s, that satisfies this. We derive the profit function as:

$$\pi_m = (8 - q_m)q_m - (1-s)\frac{5}{2}(q_m)^2$$

FOC yields:

$$\frac{\partial \pi_m}{\partial q_m} = 8 - 2q_m - (1-s)5q_m = 0 \Leftrightarrow$$

$$2q_m + (1-s)5q_m = 8 \Leftrightarrow$$

$$7q_m - 5sq_m = 8 \Leftrightarrow$$

$$q_m = \frac{8}{7-5s}$$

Assuming that $q_m = Q = 2$, and solving for s:

$$2 = \frac{8}{7-5s} \Leftrightarrow$$

$$s = \frac{3}{5}$$

3.3

We now observe three identical firms. The course of action is the same as before. The difference in this scenario is that $Q = q_1 + q_2 + q_3$. Therefore, we expand the profit function:

$$\pi_2 = (8 - q_1 - q_2 - q_3) * q_1 - \frac{5}{2}(q_1)^2$$

The FOC then yields:

$$\frac{\partial \pi}{\partial q_1} = 8 - 2q_1 - q_2 - q_3 - 5q_1$$

Therefore, the Best Responses for q_i are:

$$BR_1 = q_1 = \frac{8 - q_2 - q_3}{7}$$

$$BR_2 = q_2 = \frac{8 - q_1 - q_3}{7}$$

$$BR_3 = q_3 = \frac{8 - q_1 - q_2}{7}$$

As before, we use that $q_1^\star = q_2^\star = q_3^\star$. From BR_1 we get:

$$q_1 = \frac{8 - 2q_1}{7} \Leftrightarrow$$

$$7q_1 = 8 - 2q_1 \Leftrightarrow$$

$$9q_1 = 8$$

$$q_1^\star = q_2^\star = q_3^\star = \frac{8}{9}$$

It it shown here, that the quantity produced by each firm in the NE with three firms has fallen by $\frac{1}{9}$ units, compared to the NE with two firms. The total supply is now $\frac{24}{9}$, which earlier was $\frac{27}{9}$. We now find the price:

$$P(q_1, q_2, q_3) = 8 - (q_1 + q_2 + q_3) \Leftrightarrow$$

$$P = 8 - 3\frac{8}{9} \Leftrightarrow$$

$$P = \frac{72}{9} - \frac{24}{9} \Leftrightarrow$$

$$P = \frac{48}{9} = \frac{16}{3} \approx 5,333$$

This is true with the intuition as more firms on the market for a homogeneous product means lower prices for the consumers and vice versa. The individual firm produces less, but the overall production with more firms is higher, which in turn leads to lower prices.