Rational Expectations

John Kramer – University of Copenhagen

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Last time

The New Keynesian model

- Static RBC model

\+ Monopolistic competition

\+ Prices

Monetary non-neutrality

- In equilibrium with flexible prices, money is neutral
- Small price adjustment frictions may allow changes in money to have real effects

Agenda

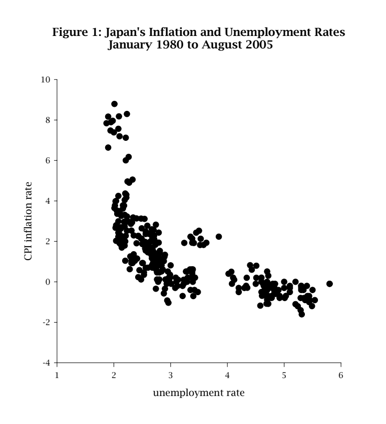
Economic Dynamics (no more static models)

- Allows the study of business cycles and economic policy

Rational expectations and the Phillips Curve

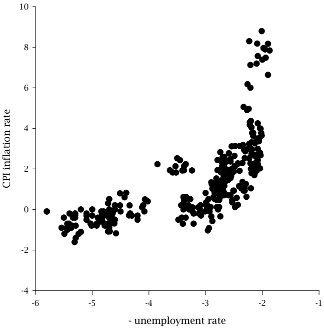
- Expectations of optimizing agents
- Law of iterated expectations
- The Lucas islands model

Japan's Phillips Curve



Japan's Phillips Curve

Figure 2: Japan's Inflation Rate and (Minus) Unemployment Rate
January 1980 to August 2005



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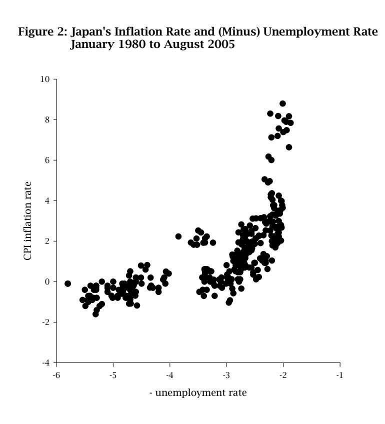


The Phillips Curve and economic policy

Robust relationship in the data

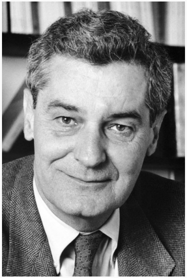
- When inflation is high, unemployment is low

Very attractive for policy makers

- Just drive up inflation and unemployment will fall!

Helmut Schmidt: "Rather 5% inflation than 5% unemployment"

Robert E. Lucas



- Nobel Laureate in 1995 "for having developed and applied the hypothesis of rational expectations, and thereby having transformed macroeconomic analysis and deepened our understanding of economic policy"
- Lucas critique: We cannot predict the effects of changes in policy based on historical data
- Printing money will not solve unemployment if people expect money to be printed!

or maybe Jack Muth

silly before Jack came along. Now, then came the macro stuff: When Tom and Neil and I started plugging the same principles that Jack had advised into Keynesian models . . . Jack didn't care about Keynesian economics, and it wouldn't have occurred to him to use that as an illustration, but it occurred to us. Neil and Tom took an IS–LM model and just changed the expectations and nothing else, and just showed how that seemingly modest change completely, radically alters the operating characteristics of the system. People noticed at that point. Now we were applying Jack's ideas to something that wasn't a straw man. It was something a lot of people had invested in, cared a lot about. It was helping to answer some real questions about macro policy, and his, Muth's, ideas start[ed] to really matter. There's no question that we got some undue credit for the basic concept, where what we had, I would say, was a more sexy implementation of an idea that Muth had offering a boring implementation of.

- Robert Lucas in 2011

Forecasting by optimizing agents

In the static environment, we assume agents optimize their choices.
What does that mean in the dynamic context?

- How should agents optimize in the presence of uncertainty?
- What information is known, which is used?

Agents form rational expectations

- They know the structure of the economy
- They use all available information

Agents do not make systematic forecast errors

Rational expectations

If some variable in our economy behaves stochastically, then agents form the expectation

$$\mathbb{E}_t[X_{t+1}] = \mathbb{E}_t[X_{t+1}|I_t]$$

where X is some economic variable, e.g., output, and I_t is the information set available to the agent.

Example: Efficient market hypothesis implies that all information I_t is priced into the stock price X_t

Note: If information sets differ, not everyone needs to form the same expectations.

Very controversial at the time. No more animal spirits (Keynes), only rational agents (Lucas, Sargeant).

The Law of Iterated Expectations

What do you think the rate of inflation will be in November 2023?

$$\mathbb{E}_t[\pi_{t+12}|I_t]?$$

The Law of Iterated Expectations

What do you think the rate of inflation will be in November 2023?

$$\mathbb{E}_t[\pi_{t+12}|I_t]?$$

What do you think **you will think** the rate of inflation will be in November 2023, **next month**?

$$\mathbb{E}_t\left[\mathbb{E}_{t+1}[\pi_{t+12}|I_{t+1}]|I_t\right]?$$

Expectational difference equations (EDEs)

Current economic conditions may depend on what we expect in the future

$$y_t = a\mathbb{E}_t[y_{t+1}|I_t] + cx_t$$

- The current endogenous variable y_t depends on exogenous variable x_t and it's own expected future value
- Rational expectations imply that agents know I_t
- Importantly, agents know all past values of y_t and x_t and the model itself

Expectational difference equations (EDEs)

Agents can solve the equation forward

$$
\begin{aligned}
y_t &= a\mathbb{E}_t[y_{t+1}|I_t] + cx_t \\
&= a\mathbb{E}_t[a\mathbb{E}_{t+1}[y_{t+2}|I_{t+1}] + cx_{t+1}|I_t] + cx_t \\
&= a^2 \underbrace{\mathbb{E}_t[\mathbb{E}_{t+1}[y_{t+2}|I_{t+1}]|I_t]}_{\text{Apply law of iterated expectations!}} + ac\mathbb{E}_t[x_{t+1}|I_t] + cx_t \\
&= a^2\mathbb{E}_t[y_{t+2}|I_t] + ac\mathbb{E}_t[x_{t+1}|I_t] + cx_t \\
&= a^2\mathbb{E}_t[y_{t+2}] + ac\mathbb{E}_t[x_{t+1}] + cx_t \\
&= a^3\mathbb{E}_t[y_{t+3}] + a^2c\mathbb{E}_t[x_{t+2}] + ac\mathbb{E}_t[x_{t+1}] + cx_t
\end{aligned}
$$

- A pattern emerges: y_t depends on exogenous variables and distant expectations of y

Expectational difference equations (EDEs)

Repeat this procedure T times:

$$y_t = a^T \mathbb{E}_t[y_{t+T}] + c \sum_{i+0}^{T} a^i \mathbb{E}_t[x_{t+i}]$$

- Usually assume that $a < 0$, or more generally $\lim_{T\to\infty} a^T \mathbb{E}_t[y_{t+T}] = 0$

$$y_t = c \sum_{i=0}^{\infty} a^i \mathbb{E}_t[x_{t+i}]$$

- y_t only depends on the expected value of exogenous shocks
- Example: Stock prices depend on the value of the dividends they are expected to pay

Example

Assume that x_t follows the AR(1) process

$$x_{t+1} = \rho\ x_t + \varepsilon_{t+1} \text{ with } \mathbb{E}[\varepsilon_{t+1}|I_t] = 0$$

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Rational expectation:

$$\mathbb{E}[x_{t+j}|I_t] = \rho^j x_t + \rho^{j-1} \sum_{i=0}^{j} \mathbb{E}[\varepsilon_{t+i}|I_t]$$

- Innovations ε_t are zero in expectation
- Agents **know** that further predictions have larger uncertainty, but the best guess is still given by $\rho^i x_t$.

Example

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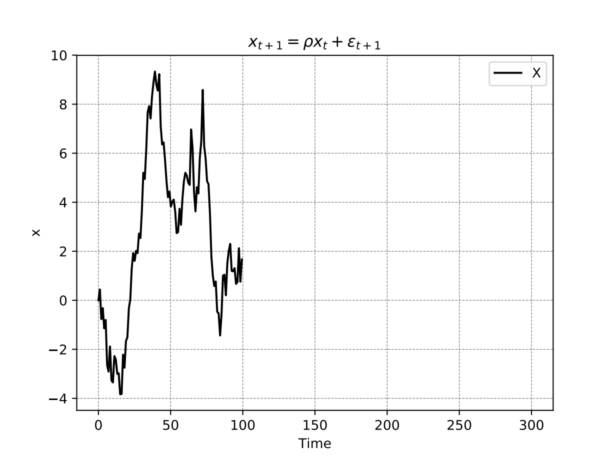
Plug into equation on previous slide to obtain (assume $a\rho < 1$)

$$\begin{aligned} y_t &= c \sum_{i+0}^{\infty} a^i \mathbb{E}_t[x_{t+j}] = c \sum_{i+0}^{\infty} (a\rho)^i x_t \\ &= \frac{c}{1 - a\rho} x_t \end{aligned}$$

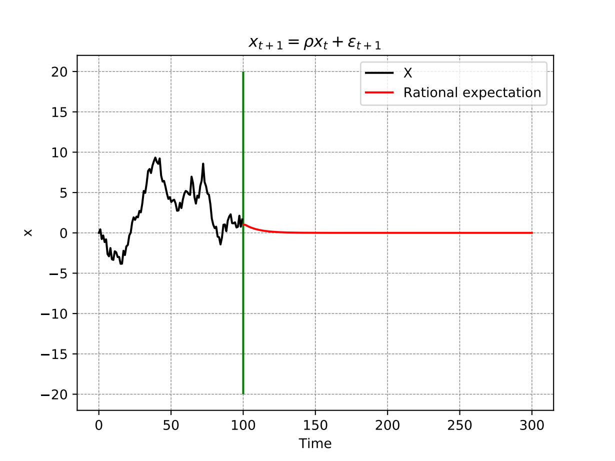
Example in pictures

Exogenous process x



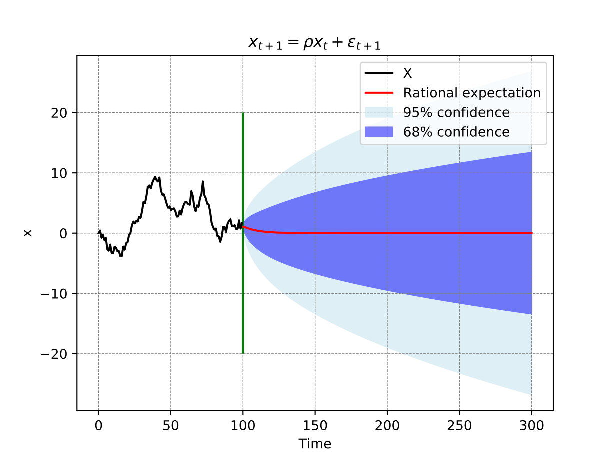
Example in pictures

Rational expectation starting from vertical line



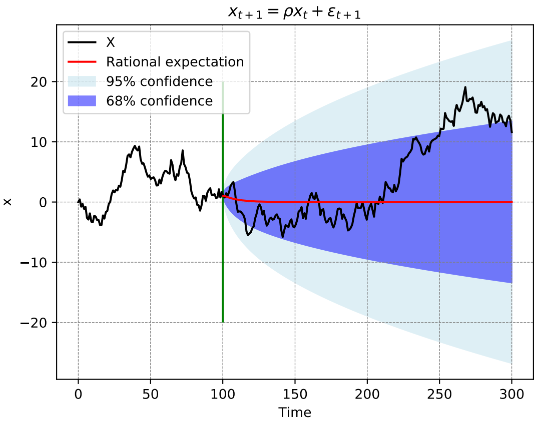
Example in pictures

Rational expectation including uncertainty



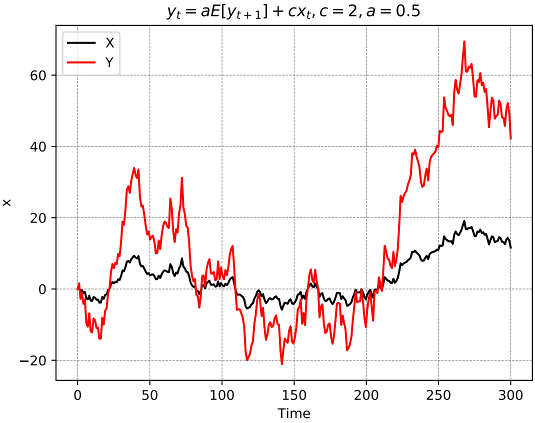
Example in pictures

Process realization



Example in pictures

Exogenous and endogenous variable



Lucas' island model (DR 6.9)

Lucas' island model I

Back to the Phillips Curve

- Lucas (1972) is a dynamic model
- Perfect competition: all firms are price takers
- All agents know the structure of the economy and are rational

Archipelago

- Each household lives on a small island
- They produce a differentiated good i
- **But they cannot see what anyone else is doing!**

$\Longrightarrow$ Informational frictions

[We will make some simplifying assumptions along the way]

Lucas' island model II

Economic decisions under uncertainty

- Producers see their own price p_i, but don't know the economy's price level P
- Only the relative price $\frac{P_i}{P}$ matters for production, but producers don't know it
- Higher P_i can mean demand for good i (should produce more) or more demand overall (produce as before)

The model delivers

- A Phillips Curve
- Strong predictions about the non-neutrality of money

Producers

Maximize utility

$$\max_{C_i, L_i} U_i = C_i - \frac{1}{\phi} L_i^{\phi}$$

- C_i is household consumption (basket of all goods in the economy)
- L_i is labor supply
- The production technology is linear, hence $L_i = Y_i$

Problem in terms of Y_i

- The households budget constraint is $PC_i = P_i Y_i$

$$\max_{Y_i} U_i = \frac{P_i}{P} Y_i - \frac{1}{\phi} Y_i^{\phi}$$

$$\implies Y_i = \left(\frac{P_i}{P} \right)^{\frac{1}{\phi - 1}}$$

Demand

Demand Function

$$Y_i = e^{z_i} \left(\frac{P_i}{P} \right)^{-\theta} Y = e^{z_i} \left(\frac{P_i}{P} \right)^{-\theta} \left(\frac{M}{P} \right)$$

- Similar last week, results from consumption aggregator
- $Y = M/P$ is a simplification
- e^{z_i} is a demand shock for good i

Limited information

- Each island (i.e., producer) can only observe its price P_i
- They don't know z_i or M - but both move demand

Linearize

Take logs of everything

$$Y_i = e^{z_i}\left(\frac{P_i}{P}\right)^{-\theta}\left(\frac{M}{P}\right) \qquad \Longrightarrow y_i = z_i - \theta(p_i - p) + m - p$$

$$Y_i = \left(\frac{P_i}{P}\right)^{\frac{1}{\phi-1}} \qquad \Longrightarrow y_i = \frac{1}{\phi - 1}(p_i - p)$$

- Producers observe p_i
- Crucial piece of information is $r_i = p_i - p$, not known
- With perfect information, $m \uparrow \rightarrow p, p_i \uparrow$, but (r_i) stays constant
- However, $z_i \uparrow \rightarrow p_i \uparrow$, hence $(r_i) \uparrow$

Rational expectations

Producers have to infer relative price from p_i

- Base production decision on $\mathbb{E}[r_i|p_i]$: $y_i = \frac{1}{\phi-1}\mathbb{E}[r_i|p_i]$
- They know the process of m and z_i, but not the realizations

Money and taste shocks

$$m \sim N(E(m), V_m)$$
$$z_i \sim N(0, V_z)$$

- Now: guess that p and r_i are independent and normally distributed variables (need to verify this later) with variances V_z and V_p

Intuition

What can be inferred about the price?

- $p_i = p_i - p + p = r_i + p$
- Fluctuations in the price are driven by fluctuations in r_i and p
- The variances V_r and V_p will depend on the underlying shocks (more on that later)

What if $V_r >> V_p$?

Intuition

What can be inferred about the price?

- $p_i = p_i - p + p = r_i + p$
- Fluctuations in the price are driven by fluctuations in r_i and p
- The variances V_r and V_p will depend on the underlying shocks (more on that later)

What if $V_r >> V_p$?

- Fluctuations in p_i most likely driven by fluctuations in r_i
- More likely to produce more output in response to observed changes in p_i

Model solution

Infer r_i from p_i's deviation from expected price level

$$\begin{aligned}\mathbb{E}[r_i|p_i] &= \mathbb{E}[r_i] + \frac{V_r}{V_r + V_p}(p_i - \mathbb{E}[p]) \\ &= \frac{V_r}{V_r + V_p}(p_i - \mathbb{E}[p])\end{aligned}$$

- $V_r/(V_r + V_p)$ is p_i's variance driven by r_i's variance
- If the *signal to noise ratio* is large, rely more on p_i to infer r_i

Individual producer's output

- For simplicity, assume that producers simply plug $\mathbb{E}[r_i|p_i]$ into their maximization problem (this is not true, they maximize $\mathbb{E}[U_i|P_i]$, but it simplifies things)

$$y_i = \frac{1}{\phi - 1}\frac{V_r}{V_r + V_p}(p_i - \mathbb{E}[p])$$

General Equilibrium I

Aggregating over all workers gives the Lucas supply function

$$\begin{aligned} y_i &= \frac{1}{\phi - 1} \frac{V_r}{V_r + V_p} (p - \mathbb{E}[p]) \\ &= b(p - \mathbb{E}[p]) \end{aligned}$$

- Further simplifying assumption: the price level p is simply the average of all prices (technically it's more complicated)
- This equation is almost a Phillips Curve (output ~ unemployment on the LHS and prices ~ inflation on the RHS)

Demand = Supply

$$m - p = b(p - \mathbb{E}[p]) \implies p = \frac{1}{1+b} m + \frac{b}{1+b} \mathbb{E}[p]$$

General Equilibrium II

Money and prices

$$p = \frac{1}{1+b}m + \frac{b}{1+b}\mathbb{E}[p]$$
$$y = \frac{b}{1+b}m - \frac{b}{1+b}\mathbb{E}[p]$$

Now, find p in terms of primitives m by taking rational expectations

$$\begin{aligned}\mathbb{E}[p] &= \frac{1}{1+b}\mathbb{E}[m] + \frac{b}{1+b}\mathbb{E}[p] \\ &= \mathbb{E}[m]\end{aligned}$$

- As last week, prices (on average) adjust to equal the money supply
- Individual demand shocks wash out in the aggregate
- *In expectation*, money is neutral

Punchline

Use $m = \mathbb{E}[m] + (m - \mathbb{E}[m])$

$$
\begin{aligned}
p &= \frac{1}{1+b}(m - \mathbb{E}[m]) + \mathbb{E}[m] \\
y &= \frac{b}{1+b}(m - \mathbb{E}[m])
\end{aligned}
$$

Only deviations from expectation matter

- Expected money growth will affect the price level, but not output
- **Un**expected money growth affects both

These conclusions are relevant for policy makers and particularly important for central banks

Unexpected money growth

Unexpected money growth affects output

$$p = \frac{1}{1+b}(m - \mathbb{E}[m]) + \mathbb{E}[m]$$
$$y = \frac{b}{1+b}(m - \mathbb{E}[m])$$

- Unexpected money growth raises everyone's prices
- Producers can't observe what's going on with other islands
- Prices are unexpectedly high $\implies$ raise output by b
- Aggregate output rises

Expected money growth

Expected money growth does not affect output

$$p = \frac{1}{1+b}(m - \mathbb{E}[m]) + \mathbb{E}[m]$$
$$y = \frac{b}{1+b}(m - \mathbb{E}[m])$$

- Expected money growth raises everyone's prices
- Producers can't observe what's going on with other islands, but expected prices to rise
- They don't raise output, because relative prices stay the same

Almost done: verify the guess for distributions of p and r

Start with V_p: simply take the variance of the aggregate price level

$$p = \frac{1}{1+b}(m - \mathbb{E}[m]) + \mathbb{E}[m]$$
$$\implies \operatorname{Var}(p) = \frac{1}{(1+b)^2}\operatorname{Var}(m)$$

For V_r, start from island market clearing

$$y_i = z_i - \theta r_i + m - p \qquad \text{(Demand for good i)}$$
$$y_i = b(p_i - p + p - \mathbb{E}[p]) \qquad \text{(Supply for good i)}$$
$$br_i + \underbrace{b(p - \mathbb{E}[p])}_{\text{Agg. Supply}} = z_i - \theta r_i + \underbrace{m - p}_{\text{Agg. Demand}}$$
$$\operatorname{Var}(r_i) = \frac{1}{(b+\theta)^2}\operatorname{Var}(z_i) \qquad \text{(p \& r are normal and indep.)}$$

Solve for b

Recall

$$y_i = b(p_i - \mathbb{E}[p])$$

$\implies$ b governs how strongly producers react to price signals

$$\begin{aligned} b &= \frac{1}{\phi - 1} \frac{V_r}{V_r + V_p} \\ &= \frac{1}{\phi - 1} \frac{V_z}{V_z + \frac{(b+\theta)^2}{(1+b)^2} V_m} \end{aligned}$$

- Equation gives b as an *implicit function* of V_z and V_m
- $\frac{\partial b}{\partial V_z} > 0$: If $V_z \uparrow$ producers lean on p_i as signal
- $\frac{\partial b}{\partial V_m} < 0$: If $V_m \uparrow$ producers don't trust p_i, too much noise

The Economy

Money supply and demand shifters

$$m_t = c + m_{t-1} + u_t \text{ where } u_t \sim N(0, V_m) \quad ; z_t \sim N(0, V_z)$$

Equilibrium equations

$$y_t = m_t - p_t \qquad \text{(Aggregate Demand)}$$
$$y_t = b(p_t - \mathbb{E}[p_t]) \qquad \text{(Aggregate Supply)}$$

Useful equations

$$p_t = \mathbb{E}[m_t] + \frac{1}{1+b}(m_t - \mathbb{E}[m_t]) = c + m_{t-1} + \frac{1}{1+b}u_t$$
$$\pi_t = p_t - p_{t-1} = c + \frac{1}{1+b}u_t - \frac{b}{1+b}u_{t-1}$$
$$y_t = \frac{b}{1+b}(m_t - \mathbb{E}[m_t]) = \frac{b}{1+b}u_t$$

Finally: The Phillips Curve

Alternative derivation

$$m_t = c + m_{t-1} + u_t \text{ where } u_t \sim N(0, V_m) \quad ; z_t \sim N(0, V_z)$$

Inflation: $\pi_t = p_t - p_{t-1}$

$$y_t - y_{t-1} = m_t - m_{t-1} - (\pi_t) \qquad \text{(Aggregate Demand)}$$

$$y_t - y_{t-1} = b(\pi_t - (\mathbb{E}[m_t] - \mathbb{E}[m_{t-1}]) \qquad \text{(Aggregate Supply)}$$

Finally: The Phillips Curve Alternative derivation

$$m_t = c + m_{t-1} + u_t \text{ where } u_t \sim N(0, V_m) \quad ; z_t \sim N(0, V_z)$$

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$$y_t - y_{t-1} = b(\pi_t - (\mathbb{E}[m_t] - \mathbb{E}[m_{t-1}]) \qquad \text{(Aggregate Supply)}$$

Plug in using the process above

$$y_t - y_{t-1} = c + u_t - \pi_t$$

$$y_t - y_{t-1} = b(\pi_t - (m_{t-1} - m_{t-2})) = b(\pi_t - c - u_{t-1})$$

Phillips Curve

$$\rightarrow \quad \pi_t = \mathbb{E}_{t-1}[\pi_t] + \frac{1}{b} y_t \leftarrow$$

Unexpected rise in c

The central bank sneakily raises c to c'

- under the old regime , inflation would have been

$$\pi = \underbrace{c + \frac{b}{1+b}u_{t-1}}_{\mathbb{E}_{t-1}[\pi_t]} + \underbrace{\frac{1}{1+b}u_t}_{\frac{1}{b}y_t}$$

- but instead, it is

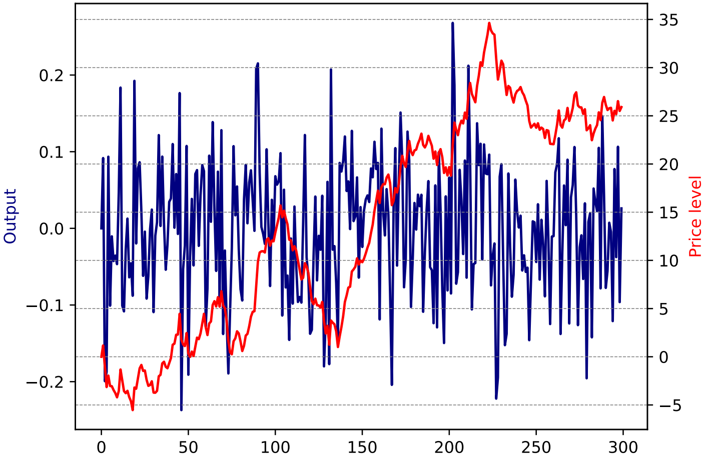
$$\pi = \underbrace{c' + \frac{b}{1+b}u_{t-1}}_{?} + \underbrace{\frac{1}{1+b}u_t + \frac{1}{1+b}(c' - c)}_{\frac{1}{b}y_t \text{ (output rises)}}$$

$$= c' - c + \underbrace{c + \frac{b}{1+b}u_{t-1}}_{\mathbb{E}_{t-1}[\pi_t]} + \frac{1}{b}y_t = c' - c + \mathbb{E}_{t-1}[\pi_t] + \frac{1}{b}y_t$$

- The Phillips Curve shifts up

Example – Output and Prices

Set parameters and simulate: $c = 0.1, V_z = 6, V_m = 1, \theta = 5, \phi = 3$



Example – Phillips Curve

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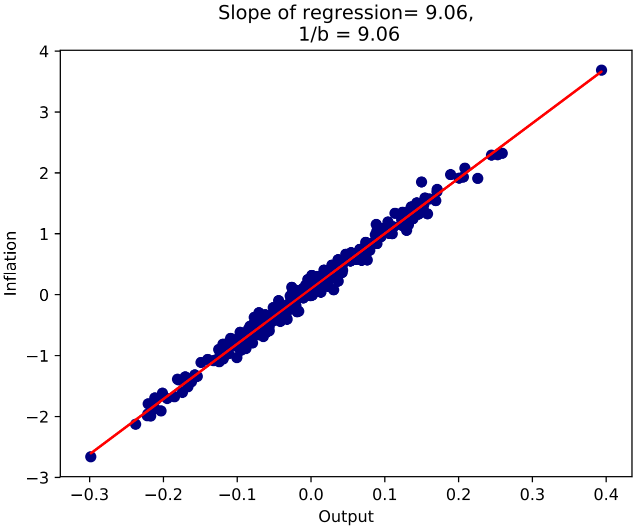
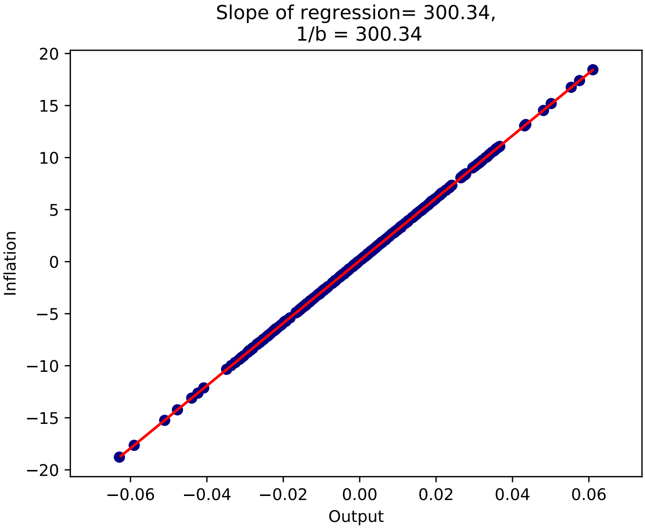
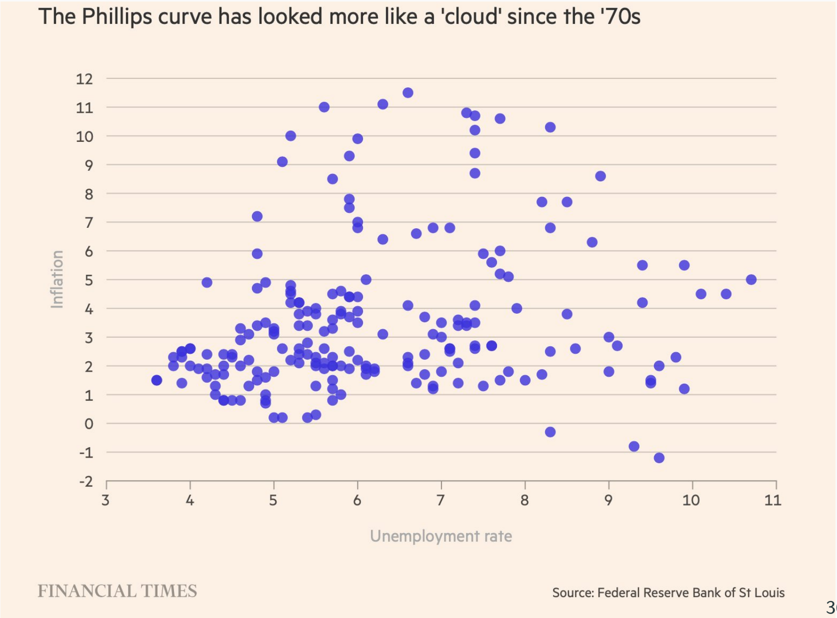


Example – Phillips Curve

Set parameters and simulate: $c = 0.1, V_z = 1, V_m = 6, \theta = 5, \phi = 3$



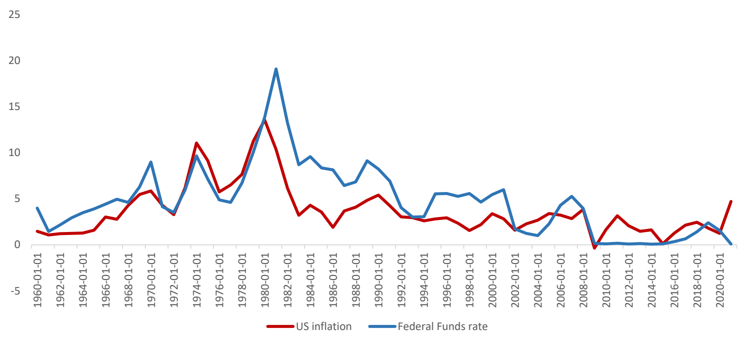
The modern Phillips Curve



What explains this breakdown

A new monetary policy regime

- US inflation was very high in the 70s
- Paul Volcker took over as chairman of the Federal Reserve
- He started aggressively hiking interest rates
- Monetary policy has become more predictable and conservative

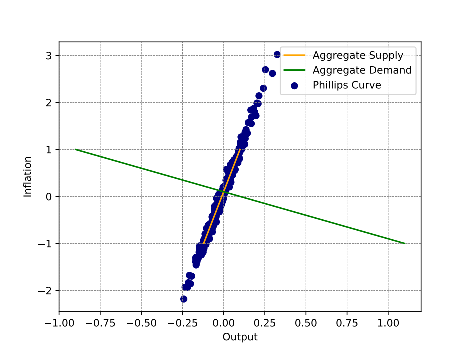


The Phillips Curve is a general equilibrium object

Identification is difficult

- The Phillips curve traces the aggregate supply curve
- Can only be identified through *shocks* to aggregate demand
- If central banks work **against** demand shocks, that's difficult

$\implies$ No obvious slope anymore



Discussion

- The Lucas model produces a positive relationship between output and inflation
- But policy makers cannot exploit it (unless they surprise everyone)
- If central bankers raise money growth c unexpectedly, it will only affect output in the first period

Implications for policy

- Unless the CB knows more than the agents in the model, no role for stabilitzation policy
- If CB knows u_t before everyone else does, it can adjust c accordingly
- Unlikely in today's world: everything is online anyway

Lucas critique

- Don't take an empirical relationship for granted! Agents might reoptimize when we try to exploit it
- Expectations affect the equilibrium
- Physicists do not teach atoms how to behave - "Luigi Zingales"

Next time

Second few New Keynesian steps

- Lucas model with monopolistic competition

Exogenous pricing frictions

- Some empirical results about price changes
- Theoretical predictions of different models
    - Fischer pricing
    - Taylor contracts
- Inflation persistence

Monetary policy

- Demand stabilization

The Phillips Curve—alternative derivation Back

Equilibrium equations

$$p_t = \mathbb{E}[m_t] + \frac{1}{1+b}(m_t - \mathbb{E}[m_t]) = c + m_{t-1} + \frac{1}{1+b}u_t$$

$$y_t = \frac{b}{1+b}(m_t - \mathbb{E}[m_t]) = \frac{b}{1+b}u_t$$

Inflation

$$p_t - p_{t-1} = c + m_{t-1} + \frac{1}{1+b}u_t - \left(c + m_{t-2} + \frac{1}{1+b}u_{t-1}\right)$$

$$\pi_t = \underbrace{c + \frac{b}{1+b}u_{t-1}}_{\mathbb{E}_{t-1}[\pi_t]} + \underbrace{\frac{1}{1+b}u_t}_{\frac{1}{b}y_t}$$

Phillips Curve

$$\rightarrow \quad \pi_t = \mathbb{E}_{t-1}[\pi_t] + \frac{1}{b}y_t \leftarrow$$