A Path towards the New-Keynesian Model

John Kramer – University of Copenhagen

December 2022

Agenda

Second part of the course – Forget supply, Praise demand!

- Price differences – monopolistic competition (today)
- Rational expectations – Lucas model
- Different forms of price setting mechanisms
- Optimal monetary policy

Moving beyond the RBC model

The Real Business Cycle model is great!

- The model is widely used to study long-run issues
- It solves several problems of the earlier Keynesian analysis (Lucas critique)

BUT

- It is real model $\implies$ no prices
- It has nothing to say about inflation dynamics
- Central bankers have no purpose
- The world is governed by labor supply and productivity shocks

Inflation matters!

Price level in Germany during hyperinflation

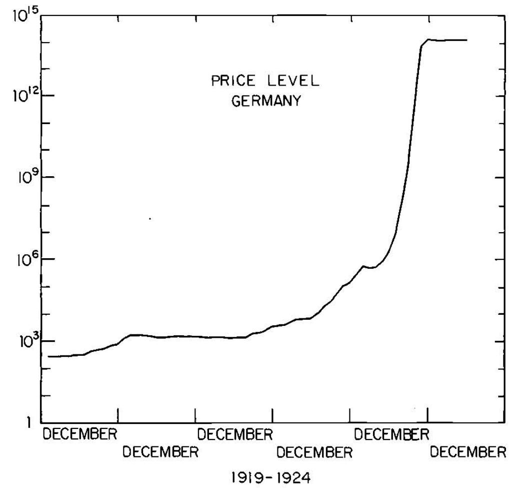


Fig. 2.4 Wholesale prices in Germany.

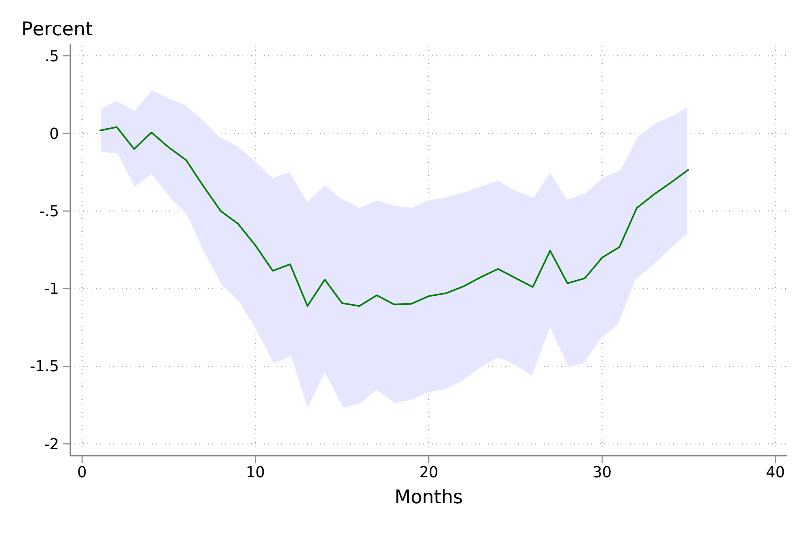
Prices are not perfectly flexible!

Data on US price changes from Nakamura & Steinsson (2008)

		Regular prices			
		Median			
Major group	Weight	Freq.	Impl. dur.	Mean freq.	Frac. up
Processed food	8.2	10.5	9.0	10.6	65.4
Unprocessed food	5.9	25.0	3.5	25.4	61.2
Household furnishing	5.0	6.0	16.1	6.5	62.9
Apparel	6.5	3.6	27.3	3.6	57.1
Transportation goods	8.3	31.3	2.7	21.3	45.9
Recreation goods	3.6	6.0	16.3	6.1	62.0
Other goods	5.4	15.0	6.1	13.9	73.7
Utilities	5.3	38.1	2.1	49.4	53.1
Vehicle fuel	5.1	87.6	0.5	87.4	53.5
Travel	5.5	41.7	1.9	43.7	52.8
Services (excl. travel)	38.5	6.1	15.8	8.8	79.0
All sectors	100.0	8.7	11.0	21.1	64.8

Monetary policy has real effects!

German output after an interst rate increase by the ECB



The New Keynesian model

Start from an RBC model

- Microfoundations (model relies on individual optimization)
- Rational expectations

Modifications

- Throw out capital
- Introduce rigidities (e.g., price/wage stickiness)
- Abandon perfect competition (today)

Outcome

- Monetary non-neutrality ($\implies$ money and interest rates have real effects)

The New Keynesian model

Workhorse model in modern economics

- Every central bank uses a version of the NK model to analyze policy
- The NK model is the starting point for most of modern business cycle research

Extensions

- Differences across consumers
- Labor markets
- Financial markets
- Supply chains
- Multi-country settings

Today

Monopolistic competition

- What happens if every producer is a monopolist?
- Very powerful framework
- Discuss output and welfare effects of money supply

Rigid prices

- What happens if prices don't change in response to changes in the economy?

Monopolistic Competition

Monopolistic competition

Market structure

- There are many small firms
- Each firm produces a differentiated good
- Consumers have an **inelastic** demand function across **all** goods

$\Longrightarrow$ small price increases don't drive demand to zero

- Firms are profit maximizers and price setters

$\Longrightarrow$ they take into account how price changes affect their demand

Monopolistic competition

Market structure

- There are many small firms
- Each firm produces a differentiated good
- Consumers have an **inelastic** demand function across **all** goods

$\Longrightarrow$ small price increases don't drive demand to zero

- Firms are profit maximizers and price setters

$\Longrightarrow$ they take into account how price changes affect their demand

What it does it get us

- Money is still neutral with flexible prices
- Welfare loss relative to perfect competition

BUT: If prices are rigid, changes in money supply affect output (money is not neutral anymore)

First steps

Starting point (DR 6.5-6.6)

- Static model, no dynamics (no price changes, inflation or growth)
- Money has no use, people hold it for fun (liquidity services)
- Representative household
- Household consumes many similar (but different) goods

Two step optimization

- Outer layer: How much to consume and work (RBC)
- Inner layer: What to consume, given prices

Representative Household

Household utility

$$U = C^{\gamma}\left(\frac{M}{P}\right)^{1-\gamma} - \frac{1}{\phi}N^{\phi} \quad \text{with } 0 < \gamma < 1, \phi > 1$$

(Nominal) budget constraint

$$\text{Nominal: } PC + M = \underbrace{M_0 + WN + \Pi}_{\text{Endowment } I}$$

- C: Consumption aggregator
- P: price index
- M: money holdings
- M_0: initial money holdings
- WN: labor income
- Π: profit income

Representative Household

Household utility

$$U = C^{\gamma}\left(\frac{M}{P}\right)^{1-\gamma} - \frac{1}{\phi}N^{\phi} \quad \text{with } 0 < \gamma < 1, \phi > 1$$

(Nominal) budget constraint

$$\text{Real: } C + \frac{M}{P} = \frac{M_0}{P} + \frac{W}{P}N + \frac{\Pi}{P}$$

- C: Consumption aggregator
- P: price index
- M: money holdings
- M_0: initial money holdings
- WN: labor income
- Π: profit income

Solve high level consumer problem

Household Problem: $\max_{C,N,\frac{M}{P}} U \implies$ Lagrangian

$$\mathcal{L} = C^{\gamma}\left(\frac{M}{P}\right)^{1-\gamma} - \frac{1}{\phi}N^{\phi} + \lambda\left[\frac{M_0}{P} + \frac{W}{P}N + \frac{\Pi}{P} - C - \frac{M}{P}\right]$$

$$\begin{aligned}
\frac{\partial L}{\partial C}: &\quad \gamma C^{\gamma-1}\left(\frac{M}{P}\right)^{1-\gamma} - \lambda = 0 &&\iff \gamma C^{\gamma-1}\left(\frac{M}{P}\right)^{1-\gamma} = \lambda \\
\frac{\partial L}{\partial M/P}: &\quad (1-\gamma)C^{\gamma}\left(\frac{M}{P}\right)^{-\gamma} - \lambda = 0 &&\iff (1-\gamma)C^{\gamma}\left(\frac{M}{P}\right)^{-\gamma} = \lambda \\
\frac{\partial L}{\partial N}: &\quad -N^{\phi-1} + \lambda\frac{W}{P} = 0 &&\iff N^{\phi-1} = \lambda\frac{W}{P}
\end{aligned}$$

First order conditions

$$\gamma C^{\gamma-1}\left(\frac{M}{P}\right)^{1-\gamma} = \lambda \tag{1}$$

$$(1-\gamma)C^{\gamma}\left(\frac{M}{P}\right)^{-\gamma} = \lambda \tag{2}$$

$$N^{\phi-1} = \lambda\frac{W}{P} \tag{3}$$

$$\underbrace{\frac{M_0}{P} + \frac{W}{P}N + \frac{\Pi}{P}}_{I} = C + \frac{M}{P} \tag{4}$$

Expenditure shares → if we know total income, we know how it's spent

$$(1) + (2) \implies \frac{M}{P} = \frac{(1-\gamma)}{\gamma}C$$

$$+(4) \implies C = \gamma I; \quad \frac{M}{P} = (1-\gamma)I$$

Remember: many different goods

Consumption aggregator

$$C = \left(\int_0^\infty c_i^{\frac{\theta-1}{\theta}} \quad di \right)^{\frac{\theta}{\theta-1}}$$

- θ is elasticity of substitution – interesting special case: $\theta \to \infty$

Remember: many different goods

Consumption aggregator

$$C = \left(\int_0^\infty c_i^{\frac{\theta-1}{\theta}} \quad di \right)^{\frac{\theta}{\theta-1}}$$

- θ is elasticity of substitution – interesting special case: $\theta \to \infty$

Solve for demand of each c_i → given some expenditure Z, maximize C?

$$\max_{c_i} \left(\int_0^\infty c_i^{\frac{\theta-1}{\theta}} \quad di \right)^{\frac{\theta}{\theta-1}} \text{ s.t. } \sum_{i=0}^{m} p_i c_i = Z$$

- Take some hypothetical Z as given
- Take p_i as given
- Find the optimal basket of goods to buy

Low level optimization

Lagrangian

$$\mathcal{L} = \left(\int_0^\infty c_i^{\frac{\theta-1}{\theta}} \quad di \right)^{\frac{\theta}{\theta-1}} + \xi \left(Z - \int_0^\infty p_i c_i \quad di \right)$$

First order condition

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_i}: \qquad & \frac{\theta}{\theta-1} \left(\int_0^\infty c_i^{\frac{\theta-1}{\theta}} \quad di \right)^{\frac{\theta}{\theta-1}-1} \frac{\theta-1}{\theta} c_i^{\frac{\theta-1}{\theta}-1} - \xi p_i = 0 \\
\implies \qquad & \left(\int_0^\infty c_i^{\frac{\theta-1}{\theta}} \quad di \right)^{\frac{1}{\theta-1}} c_i^{\frac{-1}{\theta}} = \xi p_i \\
\implies \qquad & C^{\frac{1}{\theta}} c_i^{\frac{-1}{\theta}} = \xi p_i
\end{aligned}$$

Low level fist order conditions

Optimal choices

$$\underbrace{Z = \int_0^\infty p_i c_i \quad di}_{\text{Budget constraint}} \quad \underbrace{c_i = C\left(\frac{1}{\xi}\right)^\theta p_i^{-\theta}}_{\text{First order cond.}}$$

Low level fist order conditions

Optimal choices

$$\underbrace{Z = \int_0^\infty p_i c_i \quad di}_{\text{Budget constraint}} \quad \underbrace{c_i = C\left(\frac{1}{\xi}\right)^\theta p_i^{-\theta}}_{\text{First order cond.}}$$

What is ξ?

- ξ measures increase in C when constraint is relaxed by 1 unit
- $1/\xi$ measures increase in constraint for increase in C by 1 unit

$\implies$ $1/\xi$ is the price of consumption **bundle**

$\implies$ $1/\xi = P$

Low level fist order conditions

Optimal choices

$$\underbrace{Z = \int_0^{\infty} p_i c_i \quad di}_{\text{Budget constraint}} \quad \underbrace{c_i = C \left(\frac{1}{\xi}\right)^{\theta} p_i^{-\theta}}_{\text{First order cond.}}$$

What is ξ?

- ξ measures increase in C when constraint is relaxed by 1 unit
- $1/\xi$ measures increase in constraint for increase in C by 1 unit

$\implies$ $1/\xi$ is the price of consumption **bundle**

$\implies$ $1/\xi = P$

One can (with some algebra) derive a **price index** Algebra

$$P = \left(\int_0^{\infty} p_i^{1-\theta} \quad di\right)^{\frac{1}{1-\theta}}$$

Consumers demand for goods

Demand function

$$\boxed{c_i = \left(\frac{p_i}{P}\right)^{-\theta} C} \tag{5}$$

- If P rises, demand for all goods rises
- If p_i rises, consumers substitute away with elasticity $\frac{\partial c_i}{\partial p_i}\frac{p_i}{c_i} = -\theta$
- $p_i \neq P$ does not drive demand to zero
- If $\theta \to \infty$, pricing power disappears

Firms take this demand function into account when pricing their goods

- They are monopolists!

Firm problem

Firms maximize profits by choosing their price, hours and output

$$\max_{p_i, y_i, n_i} = \frac{p_i}{P} y_i - \frac{W}{P} n_i$$

- y_i: firm specific output
- W/P: all firms pay the same real wage
- n_i: labor demanded by the firm
- If there are price-setting frictions, things get more complicated

Firm problem

Firms maximize profits by choosing their price, hours and output

$$\max_{p_i, y_i, n_i} = \frac{p_i}{P} y_i - \frac{W}{P} n_i$$

- y_i: firm specific output
- W/P: all firms pay the same real wage
- n_i: labor demanded by the firm
- If there are price-setting frictions, things get more complicated

Nothing goes to waste – market clearing at the firm level

$$y_i = c_i = \left(\frac{p_i}{P}\right)^{-\theta} C$$

Firm problem

Firms maximize profits by choosing their price, hours and output

$$\max_{p_i, y_i, n_i} = \frac{p_i}{P} y_i - \frac{W}{P} n_i$$

- y_i: firm specific output
- W/P: all firms pay the same real wage
- n_i: labor demanded by the firm
- If there are price-setting frictions, things get more complicated

Nothing goes to waste – market clearing at the firm level

$$y_i = c_i = \left(\frac{p_i}{P}\right)^{-\theta} C$$

Production function

$$y_i = n_i^{\alpha}$$

Firm optimization

Problem only depends on prices

$$\Pi = \frac{p_i}{P}\left(\frac{p_i}{P}\right)^{-\theta} C - \frac{W}{P}\left[\left(\frac{p_i}{P}\right)^{-\theta} C\right]^{1/\alpha}$$

First order conditions

$$\frac{\partial}{\partial p_i} : (1-\theta) p_i^{-\theta} P^{\theta-1} C + \frac{\theta}{\alpha} p_i^{-\theta/\alpha - 1} P^{\theta/\alpha} C^{1/\alpha} \frac{W}{P} = 0$$

$\vdots$ (tedious algebra)

Firm optimization

Problem only depends on prices

$$\Pi = \frac{p_i}{P}\left(\frac{p_i}{P}\right)^{-\theta} C - \frac{W}{P}\left[\left(\frac{p_i}{P}\right)^{-\theta} C\right]^{1/\alpha}$$

First order conditions

$$\frac{\partial}{\partial p_i} : (1-\theta) p_i^{-\theta} P^{\theta-1} C + \frac{\theta}{\alpha} p_i^{-\theta/\alpha - 1} P^{\theta/\alpha} C^{1/\alpha} \frac{W}{P} = 0$$

$\vdots$ (tedious algebra)

Optimal pricing Algebra

$$\frac{p_i}{P} = \left[\frac{1}{\alpha}\frac{\theta}{\theta-1}\frac{W}{P} C^{\frac{1-\alpha}{\alpha}}\right]^{\frac{\alpha}{\theta+\alpha(1-\theta)}}$$

- The optimal price pins down output and labor demand of the firm (through consumer demand)

Discussion of pricing policy

Pricing rule

$$\frac{p_i}{P} = \left[\frac{1}{\alpha} \frac{\theta}{\theta - 1} \frac{W}{P} C^{\frac{1-\alpha}{\alpha}} \right]^{\frac{\alpha}{\theta + \alpha(1-\theta)}}$$

- The firm's price setting policy depends on its marginal cost W/P and consumption C (due to decreasing returns)
- If $\alpha = 1$ (linear production) firms always set a constant markup above marginal cost

Discussion of pricing policy

Pricing rule

$$\frac{p_i}{P} = \left[\frac{1}{\alpha}\frac{\theta}{\theta-1}\frac{W}{P}C^{\frac{1-\alpha}{\alpha}}\right]^{\frac{\alpha}{\theta+\alpha(1-\theta)}}$$

- The firm's price setting policy depends on its marginal cost W/P and consumption C (due to decreasing returns)
- If $\alpha = 1$ (linear production) firms always set a constant markup above marginal cost

Markup

- Market power makes goods "too expensive", since $\theta/(\theta-1) > 1$
- With perfect competition, the markup disappears ($\theta \to \infty$)

General Equilibrium I

Firms and consumers

$$\frac{p_i}{P} = \left[\frac{1}{\alpha}\frac{\theta}{\theta-1}\frac{W}{P}C^{\frac{1-\alpha}{\alpha}}\right]^{\frac{\alpha}{\theta+\alpha(1-\theta)}}$$

$$y_i = c_i = \left(\frac{p_i}{P}\right)^{-\theta} C$$

- Without adjustment frictions, all firms set the same price
  $\implies p_i = P$

General Equilibrium I

Firms and consumers

$$\frac{p_i}{P} = \left[\frac{1}{\alpha}\frac{\theta}{\theta-1}\frac{W}{P}C^{\frac{1-\alpha}{\alpha}}\right]^{\frac{\alpha}{\theta+\alpha(1-\theta)}}$$

$$y_i = c_i = \left(\frac{p_i}{P}\right)^{-\theta} C$$

- Without adjustment frictions, all firms set the same price $\implies p_i = P$

Goods market equilibrium

$$\frac{W}{P} = \alpha\frac{\theta-1}{\theta}C^{\frac{\alpha-1}{\alpha}}$$

- As output (C) rises, wages fall (lower marginal product of labor)
- If $\alpha = 1$, firms charge a constant markup

General Equilibrium II

Labor market equilibrium

$$N_D = \int_0^\infty y_i^{\frac{1}{\alpha}} \quad di = Y^{\frac{1}{\alpha}} = C^{\frac{1}{\alpha}} \tag{6}$$

$$N_S = \left[(1-\gamma)^{1-\gamma} \gamma^\gamma \frac{W}{P} \right]^{\frac{1}{\phi-1}} \tag{7}$$

- Second equation comes from household optimization

Labor market clearing

$$N_D = N_S \iff \frac{W}{P} = \frac{1}{(1-\gamma)^{1-\gamma}\gamma^\gamma} C^{\frac{(\phi-1)}{\alpha}}$$

Equilibrium conditions

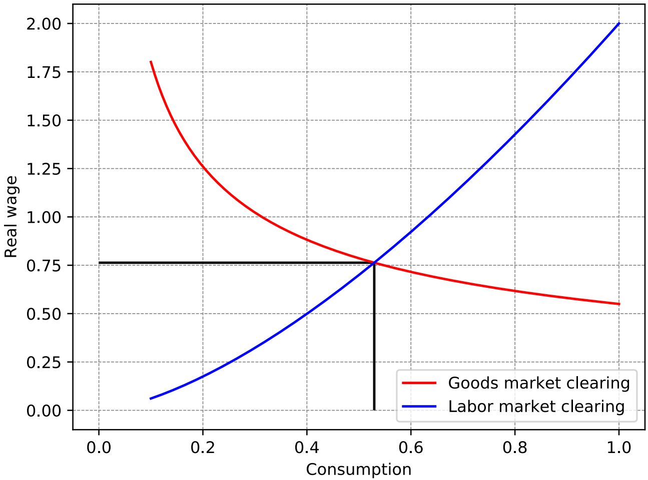
Goods market

$$\frac{W}{P} = \alpha \frac{\theta - 1}{\theta} C^{\frac{\alpha-1}{\alpha}}$$

Labor market

$$\frac{W}{P} = \frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} C^{\frac{(\phi-1)}{\alpha}}$$

Graphical representation



Takeaways

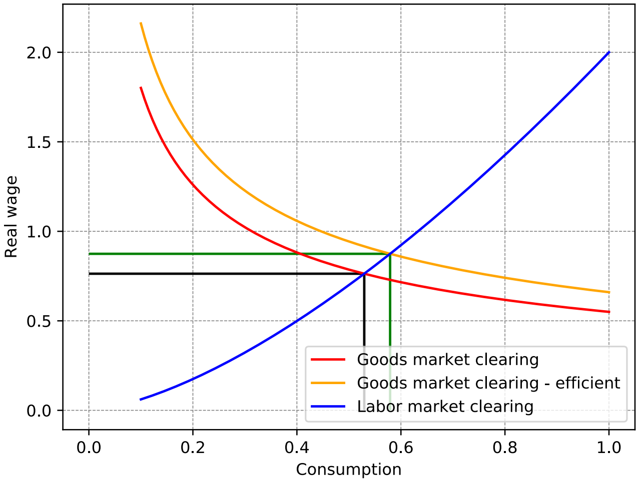
Surprising results

- Money does not matter, it's only in the background $\frac{M}{P} = \frac{(1-\gamma)}{\gamma}C$
- If we issue any amount of money, in equilibrium, the price level P adjusts
- Without adjustment frictions, this is **almost** a normal RBC model

Major innovation

- Firms **could** charge different prices (will be important later)
- More intuitive business cycles: more output = good (RBC model has symmetric costs)

Graphical representation – Efficient equilibrium



Deviations from efficiency

Why does monopolistic competition lead to lower output?

- Firms charge prices that are too high relative to worker productivity

$\Longrightarrow$ wages are too low (decreases incentives for workers)

$\Longrightarrow$ labor supply is too low

$\Longrightarrow$ output is too low

Deviations from efficiency

Why does monopolistic competition lead to lower output?

- Firms charge prices that are too high relative to worker productivity

$\implies$ wages are too low (decreases incentives for workers)

$\implies$ labor supply is too low

$\implies$ output is too low

Why are wages too low/prices too high?

- There is a coordination failure (externality) among firms
- If **one** firm lowers its price, demand at **all** firms rises
- Firms fail to account for this general equilibrium effect

$$\Pi_i = p_i \left(\frac{(p_i)}{P(p_i)} \right)^{-\theta} C - W \left[\left(\frac{(p_i)}{P(p_i)} \right)^{-\theta} C \right]^{1/\alpha};$$

Aside: Potential remedy for loss of efficiency

Inefficiency can be solved with labor subsidy (take $\alpha = 1$ for simplicity)

$$\frac{W}{P} = \frac{\theta - 1}{\theta}(1 + \tau) = 1$$

- If $\tau = \frac{1}{\theta-1}$, the economy is restored to its efficient state
- Only works if the subsidy is paid for by lump-sum taxes ($\implies$ no incentive effects on labor supply)

Summary

Assume $\alpha = 1$

- Real wage: $\frac{W}{P} = \frac{\theta-1}{\theta}$ (remember that $N^{\phi-1} = \frac{W}{P}$ and $C = N$)
- Real money balances: $\frac{M}{P} = \left(\frac{\theta-1}{\theta}\right)^{\frac{1}{\phi-1}} \frac{1}{\kappa}$ with $\kappa = \frac{1}{\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}}$
- Output: $C = \kappa \frac{M}{P} = \left(\frac{\theta-1}{\theta}\right)^{\frac{1}{1-\phi}}$
- Real profits: $\frac{\Pi}{P} = \underbrace{\left(1 - \frac{\theta-1}{\theta}\right)}_{\text{Profit share of output}} \left(\frac{\theta-1}{\theta}\right)^{\frac{1}{\phi-1}}$
- Labor earnings: $\frac{\Pi}{P} = \underbrace{\left(\frac{\theta-1}{\theta}\right)}_{\text{Labor share of output}} \left(\frac{\theta-1}{\theta}\right)^{\frac{1}{\phi-1}}$

Price Adjustment Costs

Demand effects in the model

Changes in M

- If firms can freely adjust prices, changes in M are immediately swalled up by changes in the price level
- Hence, the nominal money supply is irrelevant

Changes in M with fixed prices

- Imagine all firms' prices are fixed (for whatever reason) at the equilibrium level from above
- In this case, an increase in M can create demand
- This would bring the economy towards the efficient level of output

Price adjustment frictions

Types of frictions

- Menu costs $\implies$ every price change is costly
- Taylor contracts $\implies$ prices can only be changed after t periods
- Calvo fairy $\implies$ constant probability of price adjustment

Today: simple menu cost

- Changing price tags, reprinting menus, etc.
- Renegotiation
- Information-gathering

Price adjustment costs – a thought experiment

How high would adjustment costs need to be?

- Starting at the equilibrium from before, the government increases M
- Complete surprise to firms and consumers
- At what adjustment cost do prices remain constant?

Options for the firm

(1) Keep the price fixed

(2) Change the price (without assuming that anyone else will)

Plan of attack

(1) Derive profit under each option

(2) Quantify difference using numerical solution

Price adjustment frictions – thought experiment

Freal firm profits as a function of the money supply

$$\Pi = \left(\frac{p_i}{P} - \underbrace{\left(\kappa \frac{M}{P}\right)^{\phi-1}}_{\text{Real Wage}} \right) \underbrace{\left(\frac{p_i}{P}\right)^{-\theta} \underbrace{\left(\kappa \frac{M}{P}\right)}_{\text{Output}}}_{\text{Demand}}$$

Option 1 – Keep price fixed (assuming everyone else does)

$$p_i = P$$
$$\Pi_{fix} = \kappa \frac{M}{P} - \left(\kappa \frac{M}{P}\right)^{\phi}$$

Price adjustment frictions – thought experiment

Firm profits as a function of the money supply

$$\Pi = \left(\frac{p_i}{P} - \underbrace{\left(\kappa \frac{M}{P}\right)^{\phi - 1}}_{\text{Real Wage}} \right) \underbrace{\left(\frac{p_i}{P}\right)^{-\theta} \underbrace{\left(\kappa \frac{M}{P}\right)}_{\text{Output}}}_{\text{Demand}}$$

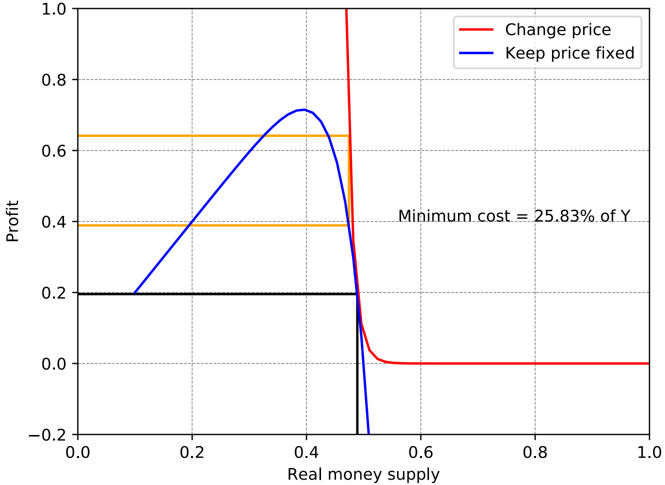
Option 2 – Change the price (assuming nobody else does)

$$\Pi_{change} = \frac{1}{\theta - 1} \left(\frac{\theta}{\theta - 1}\right)^{-\theta} \left(\kappa \frac{M}{P}\right)^{1-(1-\theta)(\phi-1)}$$

Note: Option 1 & 2 give the same result at equilibrium levels (check!)

Can price adjustment frictions rationalize sticky prices?

David Romer's calibration: $\phi = 11, \theta = 5$ – labor **very** inelastic



David Romer's take

Required frictions are too large

- Changing price tags does not cost 25% of GDP
- Firms should change prices all the time

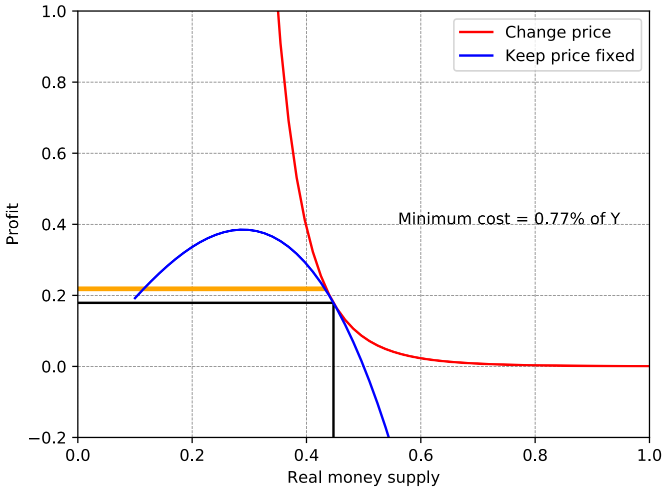
Important

- Inelastic labor supply (large ϕ) means changes in Y have big effects on W/P
- Recall $Y^{\phi-1} = \frac{W}{P}$ – For Y to fall, wages have to adjust
- $\frac{M}{P} \uparrow \implies Y \uparrow \implies \frac{W}{P} \uparrow\uparrow$
- Firms' marginal costs rise, which makes raising prices **very** attractive

What if the labor supply was a little more elastic?

Can price adjustment frictions rationalize sticky prices?

Slightly different calibration: $\phi = 3, \quad \theta = 5$



Menu costs can lead to welfare gains

If prices remain unchanged, more money can bring the economy closer to its efficient state

- Prices are "too high", more money counteracts this
- Real wages fall

Small adjustment frictions may be enough for firms to not change prices

But expectations will matter, too!

Summary

The New Keynesian model

- The New Keynesian model can rationalize the non-neutrality of money
- Today: first step towards the full framework

Monopolistic competition

- Under monopolistic competition, firms set prices that are too high
- Real wages are low, which leads to low labor supply and lower output
- Welfare is lower in this framework
- Increases in output bring the economy closer to its efficient state (booms are good)

Pricing frictions – Menu costs

- If prices are sticky, more money brings about higher output
- Small price adjustment costs may be enough

Appendix

Derivation of price index

$$C = \left(\int_0^\infty c_i^{\frac{\theta-1}{\theta}} \quad di\right)^{\frac{\theta}{\theta-1}} = \left(\int_0^\infty \left(C\left(\frac{1}{\xi}\right)^\theta p_i^{-\theta}\right)^{\frac{\theta-1}{\theta}} \quad di\right)^{\frac{\theta}{\theta-1}}$$

$$= \left(\int_0^\infty \left(CP^\theta p_i^{-\theta}\right)^{\frac{\theta-1}{\theta}} \quad di\right)^{\frac{\theta}{\theta-1}}$$

$$= C\left(\int_0^\infty P^{\theta-1}\left(p_i^{-\theta}\right)^{\frac{\theta-1}{\theta}} \quad di\right)^{\frac{\theta}{\theta-1}}$$

$$1 = P^\theta\left(\int_0^\infty p_i^{1-\theta} \quad di\right)^{\frac{\theta}{\theta-1}}$$

$$P^{-\theta} = \left(\int_0^\infty p_i^{1-\theta} \quad di\right)^{\frac{\theta}{\theta-1}}$$

$$P = \left(\int_0^\infty p_i^{1-\theta} \quad di\right)^{\frac{1}{1-\theta}}$$

Back!

Firm optimization – Algebra

$$
\begin{aligned}
(1-\theta)p_i^{-\theta}P^{\theta-1}C + \frac{\theta}{\alpha}p_i^{-\frac{\theta}{\alpha}-1}P^{\frac{\theta}{\alpha}}C^{\frac{1}{\alpha}}\frac{W}{P} &= 0\\
\frac{\theta}{\alpha}p_i^{-\frac{\theta}{\alpha}-1}P^{\frac{\theta}{\alpha}}C^{\frac{1}{\alpha}}\frac{W}{P} &= (\theta-1)p_i^{-\theta}P^{\theta-1}C\\
\frac{\theta}{\alpha}p_i^{-\frac{\theta}{\alpha}-1}P^{\frac{\theta}{\alpha}}\frac{W}{P} &= (\theta-1)p_i^{-\theta}P^{\theta-1}C^{\frac{\alpha-1}{\alpha}}\\
\frac{\theta}{\theta-1}\frac{1}{\alpha}p_i^{\theta-\frac{\theta}{\alpha}-1}P^{\frac{\theta}{\alpha}-\theta+1}\frac{W}{P} &= C^{\frac{\alpha-1}{\alpha}}\\
\frac{\theta}{\theta-1}\frac{1}{\alpha}\frac{W}{P}C^{\frac{1-\alpha}{\alpha}} &= \left(\frac{p_i}{P}\right)^{1-\theta+\frac{\theta}{\alpha}}\\
\frac{\theta}{\theta-1}\frac{1}{\alpha}\frac{W}{P}C^{\frac{1-\alpha}{\alpha}} &= \left(\frac{p_i}{P}\right)^{\frac{\theta+\alpha(1-\theta)}{\alpha}}\\
\left[\frac{\theta}{\theta-1}\frac{1}{\alpha}\frac{W}{P}C^{\frac{1-\alpha}{\alpha}}\right]^{\frac{\alpha}{\theta+\alpha(1-\theta)}} &= \left(\frac{p_i}{P}\right)
\end{aligned}
$$

Back!

Rational Expectations

John Kramer – University of Copenhagen

November 2022

Last time

The New Keynesian model

- Static RBC model

\+ Monopolistic competition

\+ Prices

Monetary non-neutrality

- In equilibrium with flexible prices, money is neutral
- Small price adjustment frictions may allow changes in money to have real effects

Agenda

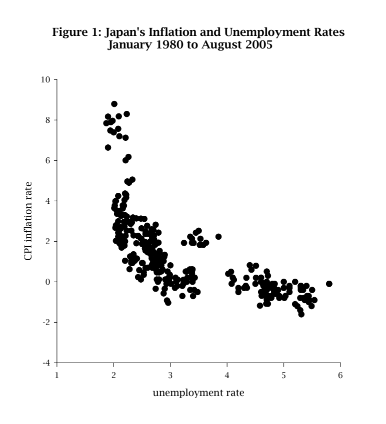
Economic Dynamics (no more static models)

- Allows the study of business cycles and economic policy

Rational expectations and the Phillips Curve

- Expectations of optimizing agents
- Law of iterated expectations
- The Lucas islands model

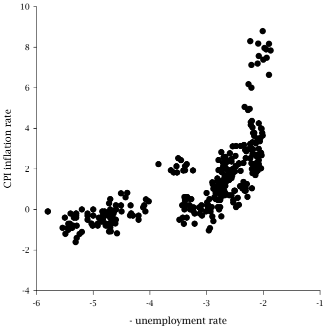
Japan's Phillips Curve



The Phillips Curve

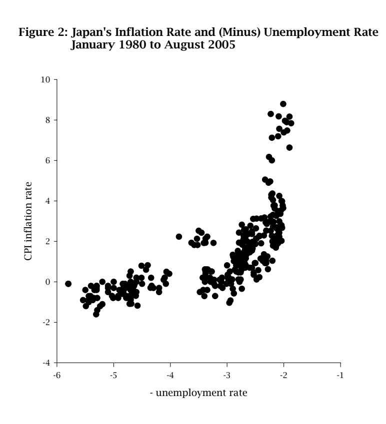
Japan's Phillips Curve

Figure 2: Japan's Inflation Rate and (Minus) Unemployment Rate
January 1980 to August 2005



The Phillips Curve

Japan's Phillips Curve



The Phillips Curve and economic policy

Robust relationship in the data

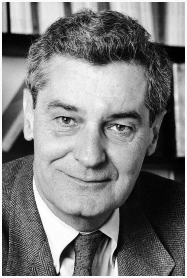
- When inflation is high, unemployment is low

Very attractive for policy makers

- Just drive up inflation and unemployment will fall!

Helmut Schmidt: "Rather 5% inflation than 5% unemployment"

Robert E. Lucas



- Nobel Laureate in 1995 "for having developed and applied the hypothesis of rational expectations, and thereby having transformed macroeconomic analysis and deepened our understanding of economic policy"
- Lucas critique: We cannot predict the effects of changes in policy based on historical data
- Printing money will not solve unemployment if people expect money to be printed!

or maybe Jack Muth

silly before Jack came along. Now, then came the macro stuff: When Tom and Neil and I started plugging the same principles that Jack had advised into Keynesian models . . . Jack didn't care about Keynesian economics, and it wouldn't have occurred to him to use that as an illustration, but it occurred to us. Neil and Tom took an IS–LM model and just changed the expectations and nothing else, and just showed how that seemingly modest change completely, radically alters the operating characteristics of the system. People noticed at that point. Now we were applying Jack's ideas to something that wasn't a straw man. It was something a lot of people had invested in, cared a lot about. It was helping to answer some real questions about macro policy, and his, Muth's, ideas start[ed] to really matter. There's no question that we got some undue credit for the basic concept, where what we had, I would say, was a more sexy implementation of an idea that Muth had offering a boring implementation of.

- Robert Lucas in 2011

Forecasting by optimizing agents

In the static environment, we assume agents optimize their choices.
What does that mean in the dynamic context?

- How should agents optimize in the presence of uncertainty?
- What information is known, which is used?

Agents form rational expectations

- They know the structure of the economy
- They use all available information

Agents do not make systematic forecast errors

Rational expectations

If some variable in our economy behaves stochastically, then agents form the expectation

$$\mathbb{E}_t[X_{t+1}] = \mathbb{E}_t[X_{t+1}|I_t]$$

where X is some economic variable, e.g., output, and I_t is the information set available to the agent.

Example: Efficient market hypothesis implies that all information I_t is priced into the stock price X_t

Note: If information sets differ, not everyone needs to form the same expectations.

Very controversial at the time. No more animal spirits (Keynes), only rational agents (Lucas, Sargeant).

The Law of Iterated Expectations

What do you think the rate of inflation will be in November 2023?

$$\mathbb{E}_t[\pi_{t+12}|I_t]?$$

The Law of Iterated Expectations

What do you think the rate of inflation will be in November 2023?

$$\mathbb{E}_t[\pi_{t+12}|I_t]?$$

What do you think **you will think** the rate of inflation will be in November 2023, **next month**?

$$\mathbb{E}_t\left[\mathbb{E}_{t+1}[\pi_{t+12}|I_{t+1}]|I_t\right]?$$

Expectational difference equations (EDEs)

Current economic conditions may depend on what we expect in the future

$$y_t = a\mathbb{E}_t[y_{t+1}|I_t] + cx_t$$

- The current endogenous variable y_t depends on exogenous variable x_t and it's own expected future value
- Rational expectations imply that agents know I_t
- Importantly, agents know all past values of y_t and x_t and the model itself

Expectational difference equations (EDEs)

Agents can solve the equation forward

$$\begin{aligned}
y_t &= a\mathbb{E}_t[y_{t+1}|I_t] + cx_t \\
&= a\mathbb{E}_t[a\mathbb{E}_{t+1}[y_{t+2}|I_{t+1}] + cx_{t+1}|I_t] + cx_t \\
&= a^2 \underbrace{\mathbb{E}_t[\mathbb{E}_{t+1}[y_{t+2}|I_{t+1}]|I_t]}_{\text{Apply law of iterated expectations!}} + ac\mathbb{E}_t[x_{t+1}|I_t] + cx_t \\
&= a^2\mathbb{E}_t[y_{t+2}|I_t] + ac\mathbb{E}_t[x_{t+1}|I_t] + cx_t \\
&= a^2\mathbb{E}_t[y_{t+2}] + ac\mathbb{E}_t[x_{t+1}] + cx_t \\
&= a^3\mathbb{E}_t[y_{t+3}] + a^2c\mathbb{E}_t[x_{t+2}] + ac\mathbb{E}_t[x_{t+1}] + cx_t
\end{aligned}$$

- A pattern emerges: y_t depends on exogenous variables and distant expectations of y

Expectational difference equations (EDEs)

Repeat this procedure T times:

$$y_t = a^T \mathbb{E}_t[y_{t+T}] + c \sum_{i+0}^{T} a^i \mathbb{E}_t[x_{t+i}]$$

- Usually assume that $a < 0$, or more generally $\lim_{T\to\infty} a^T \mathbb{E}_t[y_{t+T}] = 0$

$$y_t = c \sum_{i=0}^{\infty} a^i \mathbb{E}_t[x_{t+i}]$$

- y_t only depends on the expected value of exogenous shocks
- Example: Stock prices depend on the value of the dividends they are expected to pay

Example

Assume that x_t follows the AR(1) process

$$x_{t+1} = \rho\ x_t + \varepsilon_{t+1} \text{ with } \mathbb{E}[\varepsilon_{t+1}|I_t] = 0$$

Example

Assume that x_t follows the AR(1) process

$$x_{t+1} = \rho\ x_t + \varepsilon_{t+1} \text{ with } \mathbb{E}[\varepsilon_{t+1}|I_t] = 0$$

Rational expectation:

$$\mathbb{E}[x_{t+j}|I_t] = \rho^j x_t + \rho^{j-1} \sum_{i=0}^{j} \mathbb{E}[\varepsilon_{t+i}|I_t]$$

- Innovations ε_t are zero in expectation
- Agents **know** that further predictions have larger uncertainty, but the best guess is still given by $\rho^i x_t$.

Example

Assume that x_t follows the AR(1) process

$$x_{t+1} = \rho \; x_t + \varepsilon_{t+1} \text{ with } \mathbb{E}[\varepsilon_{t+1}|I_t] = 0$$

Rational expectation:

$$\mathbb{E}[x_{t+j}|I_t] = \rho^j x_t + \rho^{j-1} \sum_{i=0}^{j} \mathbb{E}[\varepsilon_{t+i}|I_t]$$

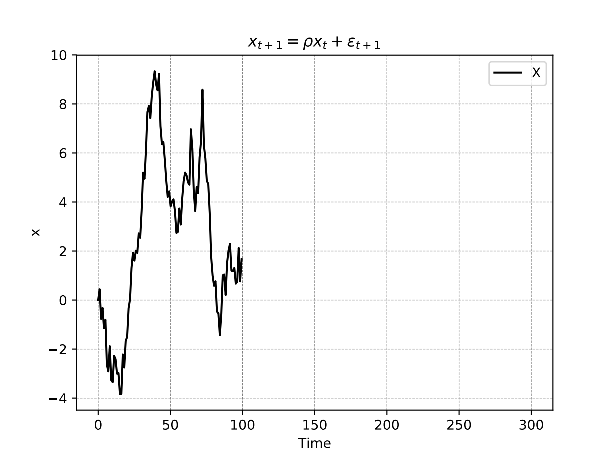
- Innovations ε_t are zero in expectation
- Agents **know** that further predictions have larger uncertainty, but the best guess is still given by $\rho^i x_t$.

Plug into equation on previous slide to obtain (assume $a\rho < 1$)

$$\begin{aligned} y_t &= c \sum_{i+0}^{\infty} a^i \mathbb{E}_t[x_{t+j}] = c \sum_{i+0}^{\infty} (a\rho)^i x_t \\ &= \frac{c}{1 - a\rho} x_t \end{aligned}$$

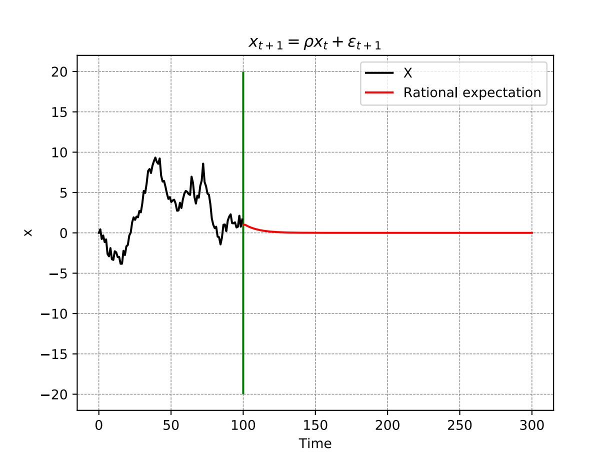
Example in pictures

Exogenous process x



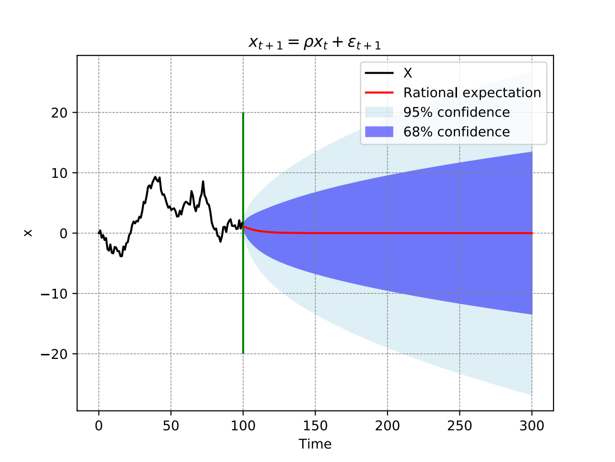
Example in pictures

Rational expectation starting from vertical line



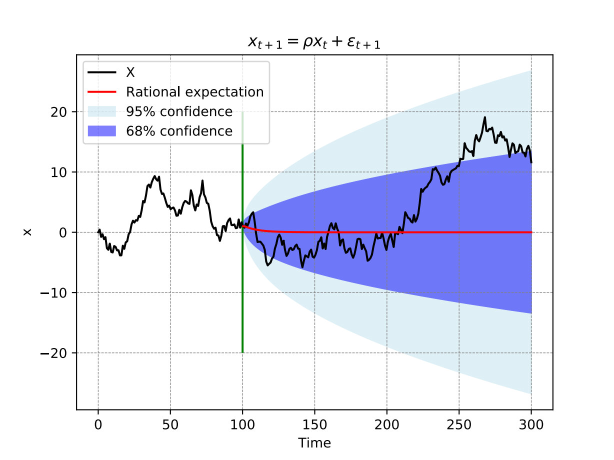
Example in pictures

Rational expectation including uncertainty



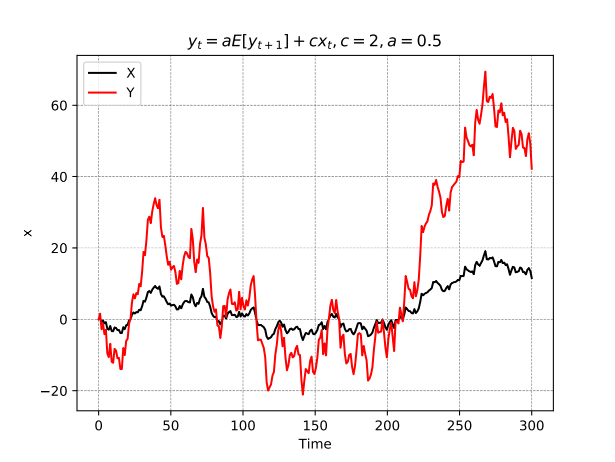
Example in pictures

Process realization



Example in pictures

Exogenous and endogenous variable



Lucas' island model (DR 6.9)

Lucas' island model I

Back to the Phillips Curve

- Lucas (1972) is a dynamic model
- Perfect competition: all firms are price takers
- All agents know the structure of the economy and are rational

Archipelago

- Each household lives on a small island
- They produce a differentiated good i
- **But they cannot see what anyone else is doing!**

$\Longrightarrow$ Informational frictions

[We will make some simplifying assumptions along the way]

Lucas' island model II

Economic decisions under uncertainty

- Producers see their own price p_i, but don't know the economy's price level P
- Only the relative price $\frac{P_i}{P}$ matters for production, but producers don't know it
- Higher P_i can mean demand for good i (should produce more) or more demand overall (produce as before)

The model delivers

- A Phillips Curve
- Strong predictions about the non-neutrality of money

Producers

Maximize utility

$$\max_{C_i, L_i} U_i = C_i - \frac{1}{\phi} L_i^{\phi}$$

- C_i is household consumption (basket of all goods in the economy)
- L_i is labor supply
- The production technology is linear, hence $L_i = Y_i$

Problem in terms of Y_i

- The households budget constraint is $PC_i = P_i Y_i$

$$\max_{Y_i} U_i = \frac{P_i}{P} Y_i - \frac{1}{\phi} Y_i^{\phi}$$

$$\implies Y_i = \left(\frac{P_i}{P} \right)^{\frac{1}{\phi - 1}}$$

Demand

Demand Function

$$Y_i = e^{z_i} \left(\frac{P_i}{P}\right)^{-\theta} Y = e^{z_i} \left(\frac{P_i}{P}\right)^{-\theta} \left(\frac{M}{P}\right)$$

- Similar last week, results from consumption aggregator
- $Y = M/P$ is a simplification
- e^{z_i} is a demand shock for good i

Limited information

- Each island (i.e., producer) can only observe its price P_i
- They don't know z_i or M - but both move demand

Linearize

Take logs of everything

$$Y_i = e^{z_i}\left(\frac{P_i}{P}\right)^{-\theta}\left(\frac{M}{P}\right) \qquad \Longrightarrow y_i = z_i - \theta(p_i - p) + m - p$$

$$Y_i = \left(\frac{P_i}{P}\right)^{\frac{1}{\phi-1}} \qquad \Longrightarrow y_i = \frac{1}{\phi - 1}(p_i - p)$$

- Producers observe p_i
- Crucial piece of information is $r_i = p_i - p$, not known
- With perfect information, $m \uparrow \rightarrow p, p_i \uparrow$, but (r_i) stays constant
- However, $z_i \uparrow \rightarrow p_i \uparrow$, hence $(r_i) \uparrow$

Rational expectations

Producers have to infer relative price from p_i

- Base production decision on $\mathbb{E}[r_i|p_i]$: $y_i = \frac{1}{\phi-1}\mathbb{E}[r_i|p_i]$
- They know the process of m and z_i, but not the realizations

Money and taste shocks

$$m \sim N(E(m), V_m)$$
$$z_i \sim N(0, V_z)$$

- Now: guess that p and r_i are independent and normally distributed variables (need to verify this later) with variances V_z and V_p

Intuition

What can be inferred about the price?

- $p_i = p_i - p + p = r_i + p$
- Fluctuations in the price are driven by fluctuations in r_i and p
- The variances V_r and V_p will depend on the underlying shocks (more on that later)

What if $V_r >> V_p$?

Intuition

What can be inferred about the price?

- $p_i = p_i - p + p = r_i + p$
- Fluctuations in the price are driven by fluctuations in r_i and p
- The variances V_r and V_p will depend on the underlying shocks (more on that later)

What if $V_r >> V_p$?

- Fluctuations in p_i most likely driven by fluctuations in r_i
- More likely to produce more output in response to observed changes in p_i

Model solution

Infer r_i from p_i's deviation from expected price level

$$\begin{aligned}\mathbb{E}[r_i|p_i] &= \mathbb{E}[r_i] + \frac{V_r}{V_r + V_p}(p_i - \mathbb{E}[p]) \\ &= \frac{V_r}{V_r + V_p}(p_i - \mathbb{E}[p])\end{aligned}$$

- $V_r/(V_r + V_p)$ is p_i's variance driven by r_i's variance
- If the *signal to noise ratio* is large, rely more on p_i to infer r_i

Individual producer's output

- For simplicity, assume that producers simply plug $\mathbb{E}[r_i|p_i]$ into their maximization problem (this is not true, they maximize $\mathbb{E}[U_i|P_i]$, but it simplifies things)

$$y_i = \frac{1}{\phi - 1}\frac{V_r}{V_r + V_p}(p_i - \mathbb{E}[p])$$

General Equilibrium I

Aggregating over all workers gives the Lucas supply function

$$\begin{aligned} y_i &= \frac{1}{\phi - 1} \frac{V_r}{V_r + V_p} (p - \mathbb{E}[p]) \\ &= b(p - \mathbb{E}[p]) \end{aligned}$$

- Further simplifying assumption: the price level p is simply the average of all prices (technically it's more complicated)
- This equation is almost a Phillips Curve (output ~ unemployment on the LHS and prices ~ inflation on the RHS)

Demand = Supply

$$m - p = b(p - \mathbb{E}[p]) \implies p = \frac{1}{1+b} m + \frac{b}{1+b} \mathbb{E}[p]$$

Money and prices

$$p = \frac{1}{1+b}m + \frac{b}{1+b}\mathbb{E}[p]$$
$$y = \frac{b}{1+b}m - \frac{b}{1+b}\mathbb{E}[p]$$

Now, find p in terms of primitives m by taking rational expectations

$$\begin{aligned}\mathbb{E}[p] &= \frac{1}{1+b}\mathbb{E}[m] + \frac{b}{1+b}\mathbb{E}[p] \\ &= \mathbb{E}[m]\end{aligned}$$

- As last week, prices (on average) adjust to equal the money supply
- Individual demand shocks wash out in the aggregate
- *In expectation*, money is neutral

Punchline

Use $m = \mathbb{E}[m] + (m - \mathbb{E}[m])$

$$p = \frac{1}{1+b}(m - \mathbb{E}[m]) + \mathbb{E}[m]$$
$$y = \frac{b}{1+b}(m - \mathbb{E}[m])$$

Only deviations from expectation matter

- Expected money growth will affect the price level, but not output
- **Un**expected money growth affects both

These conclusions are relevant for policy makers and particularly important for central banks

Unexpected money growth

Unexpected money growth affects output

$$p = \frac{1}{1+b}(m - \mathbb{E}[m]) + \mathbb{E}[m]$$
$$y = \frac{b}{1+b}(m - \mathbb{E}[m])$$

- Unexpected money growth raises everyone's prices
- Producers can't observe what's going on with other islands
- Prices are unexpectedly high $\implies$ raise output by b
- Aggregate output rises

Expected money growth

Expected money growth does not affect output

$$p = \frac{1}{1+b}(m - \mathbb{E}[m]) + \mathbb{E}[m]$$
$$y = \frac{b}{1+b}(m - \mathbb{E}[m])$$

- Expected money growth raises everyone's prices
- Producers can't observe what's going on with other islands, but expected prices to rise
- They don't raise output, because relative prices stay the same

Almost done: verify the guess for distributions of p and r

Start with V_p: simply take the variance of the aggregate price level

$$\begin{aligned} p &= \frac{1}{1+b}(m - \mathbb{E}[m]) + \mathbb{E}[m] \\ \implies \mathrm{Var}(p) &= \frac{1}{(1+b)^2}\mathrm{Var}(m) \end{aligned}$$

For V_r, start from island market clearing

$$\begin{aligned} y_i &= z_i - \theta r_i + m - p && \text{(Demand for good i)} \\ y_i &= b(p_i - p + p - \mathbb{E}[p]) && \text{(Supply for good i)} \\ br_i + \underbrace{b(p - \mathbb{E}[p])}_{\text{Agg. Supply}} &= z_i - \theta r_i + \underbrace{m - p}_{\text{Agg. Demand}} \\ \mathrm{Var}(r_i) &= \frac{1}{(b+\theta)^2}\mathrm{Var}(z_i) && \text{(p \& r are normal and indep.)} \end{aligned}$$

Solve for b

Recall

$$y_i = b(p_i - \mathbb{E}[p])$$

$\implies$ b governs how strongly producers react to price signals

$$\begin{aligned} b &= \frac{1}{\phi - 1} \frac{V_r}{V_r + V_p} \\ &= \frac{1}{\phi - 1} \frac{V_z}{V_z + \frac{(b+\theta)^2}{(1+b)^2} V_m} \end{aligned}$$

- Equation gives b as an *implicit function* of V_z and V_m
- $\frac{\partial b}{\partial V_z} > 0$: If $V_z \uparrow$ producers lean on p_i as signal
- $\frac{\partial b}{\partial V_m} < 0$: If $V_m \uparrow$ producers don't trust p_i, too much noise

The Economy

Money supply and demand shifters

$$m_t = c + m_{t-1} + u_t \text{ where } u_t \sim N(0, V_m) \quad ; z_t \sim N(0, V_z)$$

Equilibrium equations

$$y_t = m_t - p_t \qquad \text{(Aggregate Demand)}$$

$$y_t = b(p_t - \mathbb{E}[p_t]) \qquad \text{(Aggregate Supply)}$$

Useful equations

$$p_t = \mathbb{E}[m_t] + \frac{1}{1+b}(m_t - \mathbb{E}[m_t]) = c + m_{t-1} + \frac{1}{1+b}u_t$$

$$\pi_t = p_t - p_{t-1} = c + \frac{1}{1+b}u_t - \frac{b}{1+b}u_{t-1}$$

$$y_t = \frac{b}{1+b}(m_t - \mathbb{E}[m_t]) = \frac{b}{1+b}u_t$$

Finally: The Phillips Curve Alternative derivation

$$m_t = c + m_{t-1} + u_t \text{ where } u_t \sim N(0, V_m) \quad ; z_t \sim N(0, V_z)$$

Inflation: $\pi_t = p_t - p_{t-1}$

$$y_t - y_{t-1} = m_t - m_{t-1} - (\pi_t) \qquad \text{(Aggregate Demand)}$$

$$y_t - y_{t-1} = b(\pi_t - (\mathbb{E}[m_t] - \mathbb{E}[m_{t-1}]) \qquad \text{(Aggregate Supply)}$$

$$m_t = c + m_{t-1} + u_t \text{ where } u_t \sim N(0, V_m) \quad ; z_t \sim N(0, V_z)$$

Inflation: $\pi_t = p_t - p_{t-1}$

$$y_t - y_{t-1} = m_t - m_{t-1} - (\pi_t) \qquad \text{(Aggregate Demand)}$$
$$y_t - y_{t-1} = b(\pi_t - (\mathbb{E}[m_t] - \mathbb{E}[m_{t-1}]) \qquad \text{(Aggregate Supply)}$$

Plug in using the process above

$$y_t - y_{t-1} = c + u_t - \pi_t$$
$$y_t - y_{t-1} = b(\pi_t - (m_{t-1} - m_{t-2})) = b(\pi_t - c - u_{t-1})$$

Phillips Curve

$$\rightarrow \quad \pi_t = \mathbb{E}_{t-1}[\pi_t] + \frac{1}{b} y_t \leftarrow$$

Unexpected rise in c

The central bank sneakily raises c to c'

- under the old regime , inflation would have been

$$\pi = \underbrace{c + \frac{b}{1+b} u_{t-1}}_{\mathbb{E}_{t-1}[\pi_t]} + \underbrace{\frac{1}{1+b} u_t}_{\frac{1}{b} y_t}$$

- but instead, it is

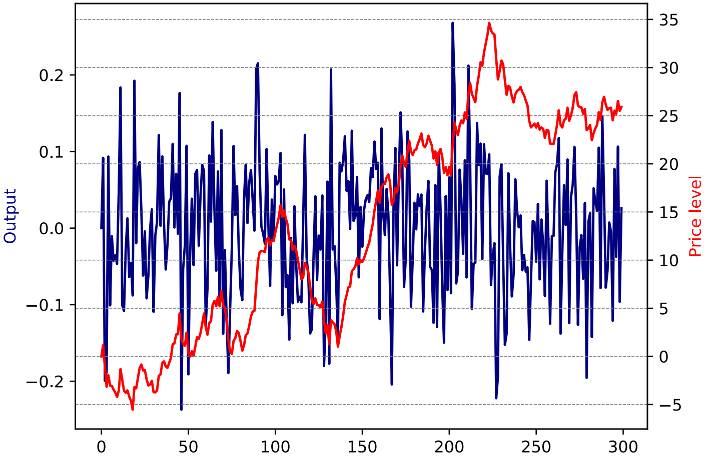
$$\pi = \underbrace{c' + \frac{b}{1+b} u_{t-1}}_{?} + \underbrace{\frac{1}{1+b} u_t + \frac{1}{1+b}(c' - c)}_{\frac{1}{b} y_t \text{ (output rises)}}$$

$$= c' - c + \underbrace{c + \frac{b}{1+b} u_{t-1}}_{\mathbb{E}_{t-1}[\pi_t]} + \frac{1}{b} y_t = c' - c + \mathbb{E}_{t-1}[\pi_t] + \frac{1}{b} y_t$$

- The Phillips Curve shifts up

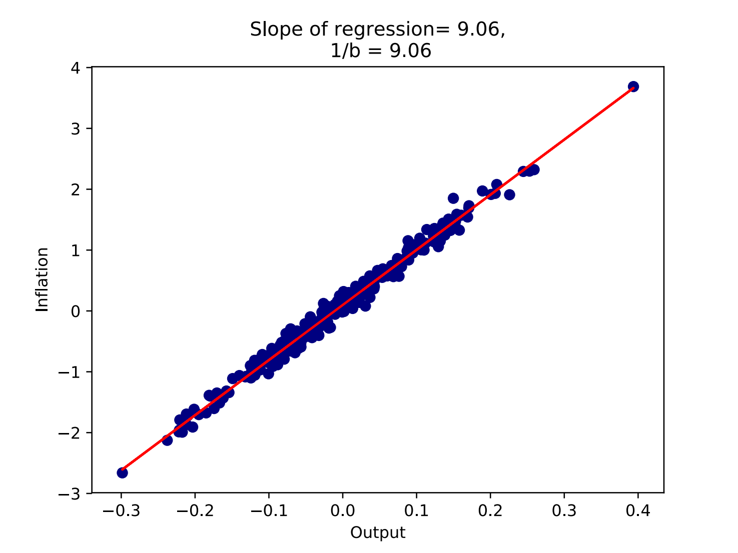
Example – Output and Prices

Set parameters and simulate: $c = 0.1, V_z = 6, V_m = 1, \theta = 5, \phi = 3$



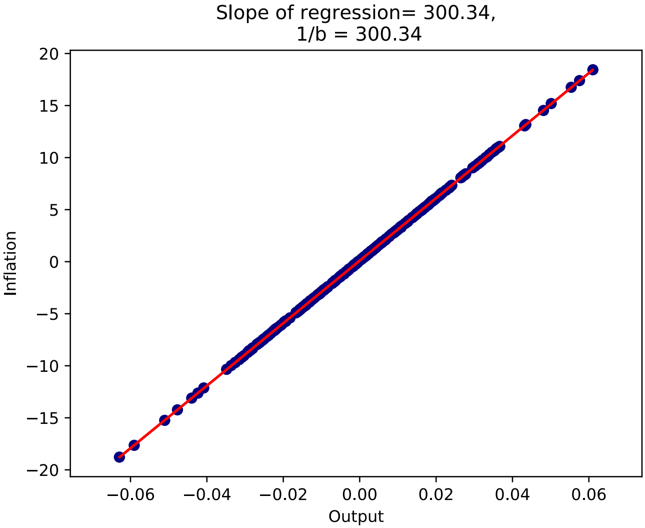
Example – Phillips Curve

Set parameters and simulate: $c = 0.1, V_z = 6, V_m = 1, \theta = 5, \phi = 3$

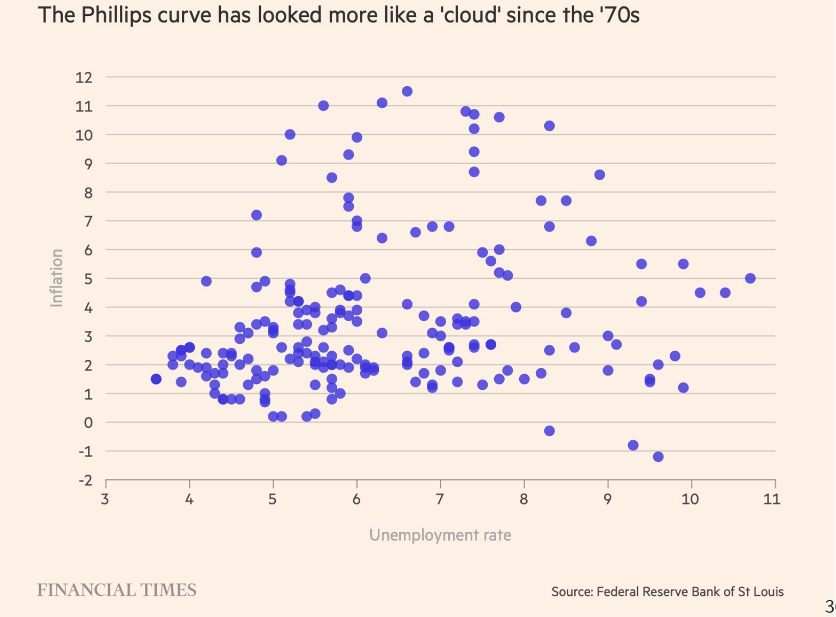


Example – Phillips Curve

Set parameters and simulate: $c = 0.1, V_z = 1, V_m = 6, \theta = 5, \phi = 3$



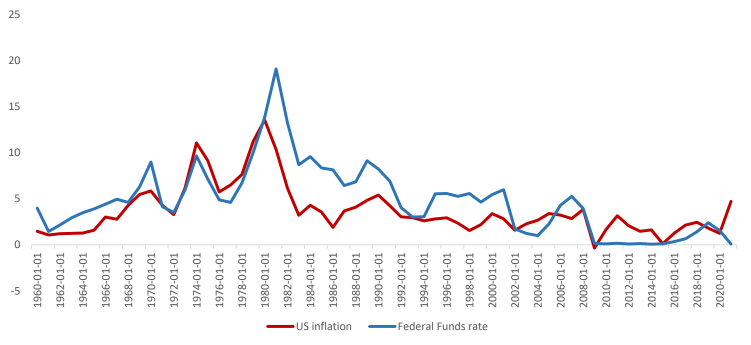
The modern Phillips Curve



What explains this breakdown

A new monetary policy regime

- US inflation was very high in the 70s
- Paul Volcker took over as chairman of the Federal Reserve
- He started aggressively hiking interest rates
- Monetary policy has become more predictable and conservative

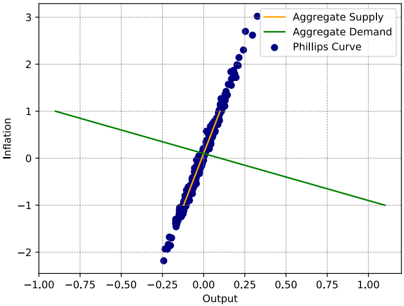


The Phillips Curve is a general equilibrium object

Identification is difficult

- The Phillips curve traces the aggregate supply curve
- Can only be identified through *shocks* to aggregate demand
- If central banks work **against** demand shocks, that's difficult

$\Longrightarrow$ No obvious slope anymore



Discussion

- The Lucas model produces a positive relationship between output and inflation
- But policy makers cannot exploit it (unless they surprise everyone)
- If central bankers raise money growth c unexpectedly, it will only affect output in the first period

Implications for policy

- Unless the CB knows more than the agents in the model, no role for stabilitzation policy
- If CB knows u_t before everyone else does, it can adjust c accordingly
- Unlikely in today's world: everything is online anyway

Lucas critique

- Don't take an empirical relationship for granted! Agents might reoptimize when we try to exploit it
- Expectations affect the equilibrium
- Physicists do not teach atoms how to behave - "Luigi Zingales"

Next time

Second few New Keynesian steps

- Lucas model with monopolistic competition

Exogenous pricing frictions

- Some empirical results about price changes
- Theoretical predictions of different models
  - Fischer pricing
  - Taylor contracts
- Inflation persistence

Monetary policy

- Demand stabilization

The Phillips Curve—alternative derivation Back

Equilibrium equations

$$p_t = \mathbb{E}[m_t] + \frac{1}{1+b}(m_t - \mathbb{E}[m_t]) = c + m_{t-1} + \frac{1}{1+b}u_t$$

$$y_t = \frac{b}{1+b}(m_t - \mathbb{E}[m_t]) = \frac{b}{1+b}u_t$$

Inflation

$$p_t - p_{t-1} = c + m_{t-1} + \frac{1}{1+b}u_t - \left(c + m_{t-2} + \frac{1}{1+b}u_{t-1}\right)$$

$$\pi_t = \underbrace{c + \frac{b}{1+b}u_{t-1}}_{\mathbb{E}_{t-1}[\pi_t]} + \underbrace{\frac{1}{1+b}u_t}_{\frac{1}{b}y_t}$$

Phillips Curve

$$\rightarrow \quad \pi_t = \mathbb{E}_{t-1}[\pi_t] + \frac{1}{b}y_t \leftarrow$$

Rigid prices

John Kramer – University of Copenhagen

November 2022

Last time

Lucas' island model

- Rational expectations
- The model produces a positive relationship between output and inflation
- But policy makers cannot exploit it (unless they surprise everyone)

Lucas critique

- Don't take an empirical relationship for granted! Agents might reoptimize when we try to exploit it
- Expectations affect the equilibrium
- Physicists do not teach atoms how to behave - "Luigi Zingales"

Today

Some Empirics

New Keynesian model – light

- Lucas' model with monopolistic competition
- Main message remains intact: only unexpected shocks have effects

Price rigidities

- Fischer contracts
- Taylor contracts

Optimal policy

- What should central banks do against demand shocks?

Empirics about price changes

Important research

Jon Steinsson & Emi Nakamura

- Five Facts about Prices: A Reevaluation of Menu Cost Models (2008)
- Price Rigidity: Microeconomic Evidence and Macroeconomic Implications (2013)

Typical price path

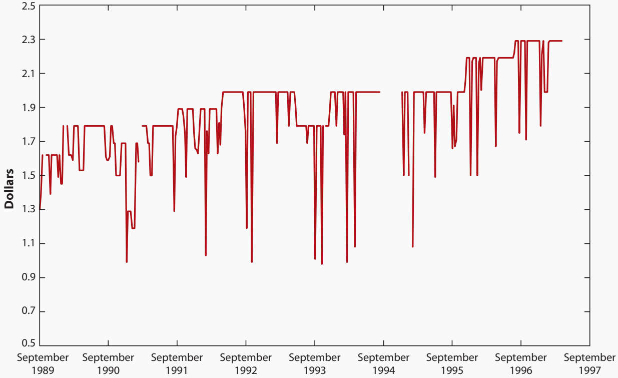


Figure 2

Price series of Nabisco Premium Saltines (16 oz) at a Dominick's Finer Foods store in Chicago.

- Is a sale a price change?

Sales

Table 2 Transience of temporary sales

	Fraction return after one-period sales	Frequency of regular price change	Frequency of price change during one-period sales	Average duration of sales
Processed food	78.5	10.5	11.4	2.0
Unprocessed food	60.0	25.0	22.5	1.8
Household furnishings	78.2	6.0	11.6	2.3
Apparel	86.3	3.6	7.1	2.1

The sample period is 1998–2005. The first data column gives the median fraction of prices that return to their original level after one-period sales. The second is the median frequency of price changes excluding sales. The third lists the median monthly frequency of regular price change during sales that past one month. The monthly frequency is calculated as $1 - (1 - f)^{0.5}$, where f is the fraction of prices that return to their original levels after one-period sales. The fourth data column gives the weighted average duration of sale periods in months. Data taken from Nakamura & Steinsson (2008).

- Most prices return to original level
- Still an important question → more work needs to be done

State dependent pricing? – Gagnon (2009)

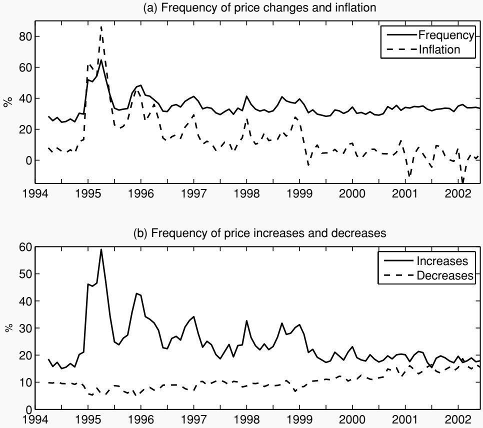


FIGURE III
Monthly Frequency of Price Changes (Nonregulated Goods)
All statistics in the figure, including inflation, are computed using the sample of nonregulated goods.

- In Mexico, high inflation means more adjustment
- Frequency is flat for low inflation period

Size of price changes – Klenow & Krystov (2008)

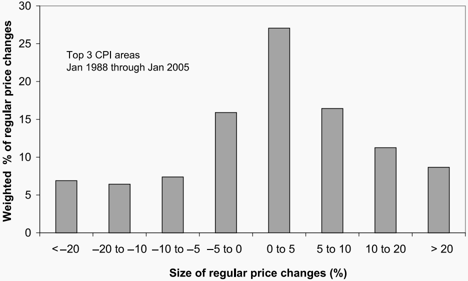


FIGURE II
Weighted Distribution of Regular Price Changes

- Average price change is positive and big $> 5\%$
- Many price changes are very small

Adjustment size vs duration – Klenow & Krystov (2008)

FIGURE VIII
Size of Regular Price Changes by Age vs. Decile Fixed Effects

- Price change doesn't seem to depend on when price was last set

Empirical summary

Measuring price changes is complicated

- Sales
- New goods
- Better quality products

Price changes

- Higher inflation leads to higher and more frequent price changes
- In low inflation periods, most price changes are small
- Price change doesn't depend on how old the price is

NK Model – light

The New Keynesian model – light

The model today is not the actual standard New Keynesian model

- Romer discusses a version of the full DSGE model in chapter 7, but it is slightly more difficult to derive (it features firms)
- Instead, we use Lucas' assumption that households are consumers and producers at the same time – gets around firm behavior and wage-setting
- Most important insights are in our model

Setup – same as last lecture

Representative household

$$U_i = \mathbf{C}_i - \frac{1}{\phi} L_i^{\phi}$$

- $\mathbf{C}$ is a consumption basket, as in previous lectures, composed of C_i
- Households use their labor L to produce output according to $Y_i = L_i$
- P is the aggregate price level, P_i is the price of the household's variety i

Budget constraint

$$P\mathbf{C}_i = P_i Y_i \implies \mathbf{C}_i = \frac{P_i}{P} Y_i$$

Demand function

$$Y_i = \left(\frac{P_i}{P}\right)^{-\theta} Y \iff \frac{P_i}{P} = \left(\frac{Y_i}{Y}\right)^{-1/\theta}$$

Optimality condition

Program

$$\max_{Y_i, P_i, L_i} \quad \frac{P_i}{P} Y_i - \frac{1}{\phi} L_i^{\phi}$$

$$\max_{Y_i} \quad \left(\frac{Y_i}{Y}\right)^{-1/\theta} Y_i - \frac{1}{\phi} Y_i^{\phi}$$

- Households take the demand function into account when making production decisions

First order condition

$$-\frac{1}{\theta}\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}-1} \frac{Y_i}{Y} + \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}} = Y_i^{\phi-1}$$

Rearrange

Optimal output

$$
\begin{aligned}
-\frac{1}{\theta}\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}-1}\frac{Y_i}{Y}+\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}} &= Y_i^{\phi-1} \\
-\frac{1}{\theta}\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}}+\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}} &= Y_i^{\phi-1} \\
\left(1-\frac{1}{\theta}\right)\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\theta}} &= Y_i^{\phi-1} \\
\underbrace{\left(\frac{\theta-1}{\theta}\right)}_{\text{Inverse markup}}\left(\frac{P_i}{P}\right) &= Y_i^{\phi-1} \leftarrow \text{ p/P = markup } * \text{ "marginal cost"}
\end{aligned}
$$

Take logs

$$
\begin{aligned}
\log\left(\frac{\theta-1}{\theta}\right)+p_i-p &= (\phi-1)y_i \\
p_i-p &= (\phi-1)y_i+\mathcal{M}
\end{aligned}
$$

Price setting

Optimal price setting

$$p_i^* - p = (\phi - 1)y_i + \mathcal{M}$$

- Optimal price depends on elasticity of labor supply $\frac{1}{\phi}$ and markup $\mathcal{M}$

Aggregate up (all firms are symmetric $\implies$ make the same choices)

$$p^* - p = (\phi - 1)\underbrace{(m - p)}_{y} + \mathcal{M}$$

$$p^* = (\phi - 1)m + (2 - \phi)p + \mathcal{M}$$

- Note that $p = m$ is not the outcome if $p^* = p$, due to monopolistic competition. Prices are higher in this model, compared to Lucas'
- As ϕ rises (labor becomes less elastic) m's effect on p^* increases
- $m \uparrow$ raises demand, which means y_i must rise. If L_i is inelastic, this requires large price movements
- Careful! Romer's $\phi^R = \phi^J - 1$. His notation is slightly different!

Introducing expectations

Money supply follows some stochastic process

- Households need to form expectations about future m to set prices p
- Hence, they must form expectations about how other price setters will behave

$$p^* = (\phi - 1)\mathbb{E}[m|I] + (2 - \phi)\mathbb{E}[p|I] + \mathcal{M}$$

- Since everyone behaves the same, take expectations of the whole expression

$$\mathbb{E}[p|I] = \mathbb{E}\left[(\phi - 1)\mathbb{E}[m|I] + (2 - \phi)\mathbb{E}[p|I] + \mathcal{M}|I\right]$$
$$\mathbb{E}[p|I] = \mathbb{E}[m|I] + \frac{1}{\phi - 1}\mathcal{M}$$

Equilibrium

Prices and output

$$
\begin{aligned}
p &= (\phi - 1)\mathbb{E}[m|I] + (2-\phi)\left(\mathbb{E}[m|I] + \frac{1}{\phi - 1}\mathcal{M}\right) + \mathcal{M} \\
&= \mathbb{E}[m|I] + \frac{1}{\phi - 1}\mathcal{M} \\
y &= m - \mathbb{E}[m|I] - \frac{1}{\phi - 1}\mathcal{M}
\end{aligned}
$$

- If prices are flexible, the equilibrium is almost the same as in the Lucas model (with $b \to \infty$)
- Only unanticipated movements in aggregate demand ($\mathbb{E}[m] \neq m$) have real effects
- Monopolistic competition still leads to lower output and, therefore, a welfare loss
- Next: Pricing frictions

Two different approaches today



- Fischer contracts: Set price **schedule** in advance
- Taylor pricing: Fix prices for a certain time
- Calvo fairy: Fixed probability of adjusting

Not here, but still interesting

- Menu costs: pay fixed price to change a price

Fischer contracts (DR 7.2)

Environment

- Firms set price schedules in advance and stick to them
- Only some fraction of firms renews their schedules each period
- Everyone has to fulfil demand $\Longrightarrow$ work more if prices too low

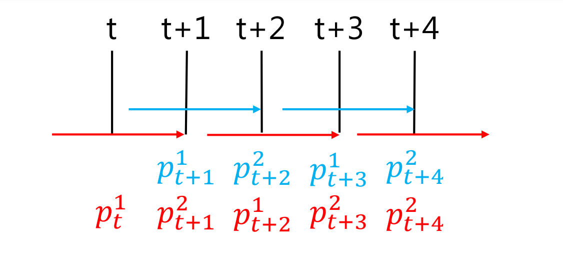
Operationalization

- Each price-setter sets prices for two periods – potentially different ones
- Assume that half of all producers set prices in even periods, the rest in odd
- Rational expectations: everyone knows the environment

Timing

- Those who reset do so right before the end of period $t-1$, setting prices for t and $t+1$
- They **do not** know the shocks in t

Timing and notation



- Price schedules are set with the information set of the previous period
- Prices can change over time
- Subscript: period for which prices were set
- Superscript: how many periods ago prices were set

Price level with Fischer contracts

Price level

$$p_t = \frac{1}{2}\left(p_t^1 + p_t^2\right)$$
$$p_t^* = (\phi - 1)m + (2 - \phi)p + \mathcal{M}$$

Optimal price setting in expectation

$$p_t^1 = \mathbb{E}_{t-1}[p_t^*] = (\phi - 1)\mathbb{E}_{t-1}[m_t] + (2 - \phi)\frac{1}{2}\left(p_t^1 + p_t^2\right) + \mathcal{M}$$
$$p_t^2 = \mathbb{E}_{t-2}[p_t^*] = (\phi - 1)\mathbb{E}_{t-2}[m_t] + (2 - \phi)\frac{1}{2}\left(\mathbb{E}_{t-2}[p_t^1] + p_t^2\right) + \mathcal{M}$$

- p_t^2 is observed in period $t-1$, so no expectation needed
- Price setters don't need expectations about their own prices

Rearrange and solve

$$p_t^1 = \frac{2(\phi-1)}{\phi}\mathbb{E}_{t-1}[m_t] + \frac{(2-\phi)}{\phi}p_t^2 + \frac{2}{\phi}\mathcal{M}$$
$$p_t^2 = \frac{2(\phi-1)}{\phi}\mathbb{E}_{t-2}[m_t] + \frac{(2-\phi)}{\phi}\mathbb{E}_{t-2}[p_t^1] + \frac{2}{\phi}\mathcal{M}$$

- Rational expectations imply that resetters 2 periods ago knew the other's policy function $\implies$ plug in + some tedious algebra

$$p_t^2 = \mathbb{E}_{t-2}[m_t] + \frac{1}{(\phi-1)}\mathcal{M}$$

- Exactly the same as before: base prices on expected demand
- Use this expression to solve for p_t^2

$$p_t^1 = \mathbb{E}_{t-1}[m_t] + \frac{\phi-2}{\phi}\underbrace{(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t])}_{\text{Updated information set}} + \frac{1}{(\phi-1)}\mathcal{M}$$

Equilibrium with Fischer contracts

$$
\begin{aligned}
p_t &= \frac{1}{2}\left(p_t^1 + p_t^2\right) \\
&= \frac{1}{2}\left(\left[\mathbb{E}_{t-1}[m_t] + \frac{\phi-2}{\phi}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right)\right] + \left[\mathbb{E}_{t-2}[m_t]\right]\right) + \frac{1}{(\phi-1)}\mathcal{M} \\
&= E_{t-1}[m_t] - \frac{1}{\phi}\left(E_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right) + \frac{1}{(\phi-1)}\mathcal{M} \\
y_t &= m_t - E_{t-1}[m_t] + \frac{1}{\phi}\left(E_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right) - \frac{1}{(\phi-1)}\mathcal{M} \\
&= \varepsilon_t + \frac{1}{\phi}\varepsilon_{t-1} - \frac{1}{(\phi-1)}\mathcal{M}
\end{aligned}
$$

Consider a surprising increase in m_t, announced after p_t^2 was set

- Prices are **lower** compared to flexprice: p^2s were locked in in $t-1$ at too low level
- Output is **higher** compared to flexprice
- If labor is inelastic the costs of fixing prices are lower

Monetary policy

Thinking of m as completely random seems strange

- Central banks are not trying to surprise anybody
- They try to "lean against the wind" and work against demand shocks

New definition of aggregate demand

- Postulate the following relationship:

$$y_t = m_t - p_t + v_t$$

- New definition: v_t represents shocks to aggregate demand
- Think of m_t as (potentially active) monetary policy
- Important: monetary policy does not know more than the market

New equilibrium

Note: m_t and v_t always enter as a sum $\implies$ substitute

$$p_t = E_{t-1}[m_t + v_t] - \frac{1}{\phi}\left(E_{t-1}[m_t + v_t] - \mathbb{E}_{t-2}[m_t + v_t]\right) + \frac{1}{(\phi - 1)}\mathcal{M}$$
$$y_t = m_t + v_t - E_{t-1}[m_t + v_t] + \frac{1}{\phi}\left(E_{t-1}[m_t + v_t] - \mathbb{E}_{t-2}[m_t + v_t]\right) - \frac{1}{(\phi - 1)}\mathcal{M}$$

Question

- If v_t is random, but m_t can be controlled, what should policy makers do?
- What's the **optimal** monetary policy?

Fischer contracts with demand stabilization

Environment

- Assume v_t follows a random walk: $v_t = v_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$
- Assume monetary policy is given by

$$m_t = a_1 \varepsilon_{t-1} + a_2 \varepsilon_{t-2} + \cdots$$

- Policy makers know all past information
- The policy rule only contains linear terms – this turns out to be enough, given certain assumptions on society's preferences. We shall return to this issue.
- Question: What should be the weight on each a_x term?

Fischer contracts with demand stabilization II

Rewriting

- Plugging in the information on the previous slide gives

$$m_t + v_t = \underbrace{a_1\varepsilon_{t-1} + a_2\varepsilon_{t-2} + \cdots}_{m_t} + \underbrace{v_{t-1} + \varepsilon_t}_{v_t}$$

- This is a model with rational expectations, hence the policy rule enters expectations

$$y_t = m_t + v_t - E_{t-1}[m_t + v_t] + \frac{1}{\phi}\left(E_{t-1}[m_t + v_t] - \mathbb{E}_{t-2}[m_t + v_t]\right) - \frac{1}{(\phi - 1)}\mathcal{M}$$

Fischer contracts with demand stabilization III

$$y_t = m_t + v_t - E_{t-1}[m_t + v_t] + \frac{1}{\phi}\left(E_{t-1}[m_t + v_t] - \mathbb{E}_{t-2}[m_t + v_t]\right) - \frac{1}{(\phi - 1)}\mathcal{M}$$

Future can be expressed in terms of the past

$$\begin{aligned} E_{t-1}[m_t + v_t] &= E_{t-1}[a_1\varepsilon_{t-1} + a_2\varepsilon_{t-2} + a_3\varepsilon_{t-3} + \cdots + v_{t-1} + \varepsilon_t] \\ &= m_t + v_{t-1} + E_{t-1}[\varepsilon_t] \\ &= m_t + v_{t-1} \end{aligned}$$

$$\begin{aligned} E_{t-2}[m_t + v_t] &= E_{t-2}[a_1\varepsilon_{t-1} + a_2\varepsilon_{t-2} + a_3\varepsilon_{t-3} + \cdots + v_{t-2} + \varepsilon_{t-1} + \varepsilon_t] \\ &= E_{t-2}[a_1\varepsilon_{t-1} + a_2\varepsilon_{t-2} + a_3\varepsilon_{t-3} + \cdots + \varepsilon_{t-1} + \varepsilon_t] + v_{t-2} \\ &= E_{t-2}[a_1\varepsilon_{t-1} + \varepsilon_{t-1} + \varepsilon_t] + a_2\varepsilon_{t-2} + a_3\varepsilon_{t-3} + \cdots + v_{t-2} \\ &= a_2\varepsilon_{t-2} + a_3\varepsilon_{t-3} + \cdots + v_{t-2} \end{aligned}$$

Fischer contracts with demand stabilization IV

$$y_t = m_t + v_t - E_{t-1}[m_t + v_t] + \frac{1}{\phi}\left(E_{t-1}[m_t + v_t] - \mathbb{E}_{t-2}[m_t + v_t]\right) - \frac{1}{(\phi - 1)}\mathcal{M}$$

$$E_{t-1}[m_t + v_t] = m_t + v_{t-1}$$

$$E_{t-2}[m_t + v_t] = a_2\varepsilon_{t-2} + a_3\varepsilon_{t-3} + \cdots + v_{t-2}$$

Deriving output with monetary policy

$$\begin{aligned}
E_{t-1}[m_t + v_t] - E_{t-2}[m_t + v_t] &= m_t - a_2\varepsilon_{t-2} - a_3\varepsilon_{t-3} - \cdots + v_{t-1} - v_{t-2} \\
&= a_1\varepsilon_{t-1} + \varepsilon_{t-1} = 1 + a_1\varepsilon_{t-1} \\
m_t + v_t - E_{t-1}[m_t + v_t] &= m_t + v_t - m_t - v_{t-1} \\
&= v_t - v_{t-1} = \varepsilon_t
\end{aligned}$$

$$y_t = \varepsilon_t + \frac{1 + a_1}{\phi}\varepsilon_{t-1} - \frac{1}{(\phi - 1)}\mathcal{M}$$

Optimal monetary policy

The central bank can affect output with a_1

$$y_t = \varepsilon_t + \frac{1 + a_1}{\phi}\varepsilon_{t-1} - \frac{1}{(\phi - 1)}\mathcal{M}$$

Other terms are irrelevant

- Contracts are only set for two periods
- The central bank (CB) cannot see what will happen tomorrow

$\Longrightarrow$ CB can eliminate effects of anticipated shocks

Minimize variance of output

$$\begin{aligned}\mathrm{Var}(y_t) &= \mathrm{Var}(\varepsilon_t) + \frac{(1 + a_1)^2}{\phi^2}\mathrm{Var}(\varepsilon_{t-1}) \\ &= \sigma_\varepsilon^2 + \frac{(1 + a_1)^2}{\phi^2}\sigma_\varepsilon^2\end{aligned}$$

Optimal monetary policy

The central bank can affect output with a_1

$$y_t = \varepsilon_t + \frac{1 + a_1}{\phi}\varepsilon_{t-1} - \frac{1}{(\phi - 1)}\mathcal{M}$$

Other terms are irrelevant

- Contracts are only set for two periods
- The central bank (CB) cannot see what will happen tomorrow

$\Longrightarrow$ CB can eliminate effects of anticipated shocks

Minimize variance of output

$$\begin{aligned}
\mathrm{Var}(y_t) &= \mathrm{Var}(\varepsilon_t) + \frac{(1 + a_1)^2}{\phi^2}\mathrm{Var}(\varepsilon_{t-1}) \\
&= \sigma_\varepsilon^2 + \frac{(1 + a_1)^2}{\phi^2}\sigma_\varepsilon^2 \\
&= \sigma_\varepsilon^2 \text{ with } a_1 = -1
\end{aligned}$$

Central bankers save firms from frictions

The central bank can return the economy to the frictionless equilibrium

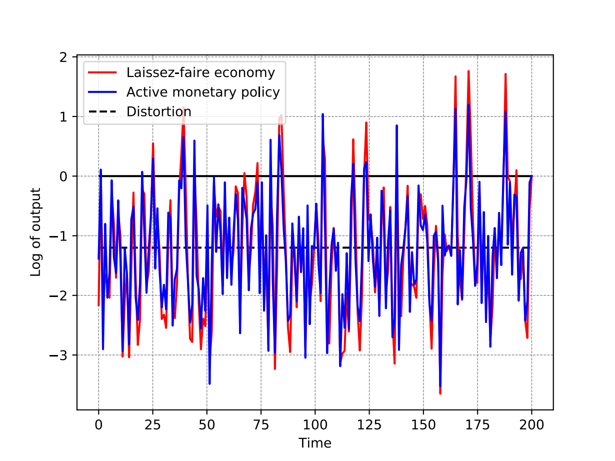
- It neutralizes all anticipated **demand** shocks it observes
- Firms anticipate the actions of the CB, which returns us to the frictionless world
- Unanticipated demand shocks still have effects, but only for one period

Optimal policy

- If the policy goal is to minimize output volatility, a linear rule is enough
- "Lean against the wind": If aggregate demand is high, decrease money supply

Optimal monetary policy in a picture

Output volatility is reduced but only little, because of $1/\phi$



Taylor contracts

Taylor contracts (DR 7.3)

Environment

- Firms set prices in advance and stick to them
- Only some fraction of firms resets every period

Operationalization

- Each price-setter sets prices for two periods (same price this time)
- Assume that half of all producers set prices in even periods, the rest in odd
- Rational expectations: everyone knows the environment

Timing

- Those who reset do so at the beginning of period t, setting the price for t and $t+1$
- They **know** the shocks in t (change! – makes algebra easier)

Taylor contracts – Firm decisions

Let x_t be the optimal price for firms firms who set in t

$$
\begin{aligned}
x_t &= \frac{1}{2}(p^*_{i,t} + \mathbb{E}_t[p^*_{i,t+1}]) \\
&= \frac{1}{2}\left\{(\phi - 1)m_t + (2 - \phi)p_t + (\phi - 1)\mathbb{E}_t[m_{t+1}] + (2 - \phi)\mathbb{E}_t[p_{t+1}]\right\} \\
&= \frac{1}{2}\left\{(\phi - 1)m_t + (2 - \phi)p_t + (\phi - 1)m_t + (2 - \phi)\mathbb{E}_t[p_{t+1}]\right\}
\end{aligned}
$$

The realized price level each period is

$$p_t = \frac{1}{2}(x_t + x_{t-1})$$

Combining the two:

$$x_t = \frac{(2 - \phi)}{4}\left\{(x_t + x_{t-1}) + \mathbb{E}_t[x_{t+1} + x_t]\right\} + (\phi - 1)m_t$$

Taylor contracts – Optimal prices

Solve for the optimal reset price

$$
\begin{aligned}
x_t &= \frac{(2-\phi)}{4}\left\{(x_t + x_{t-1}) + \mathbb{E}_t[x_{t+1} + x_t]\right\} + (\phi - 1)m_t \\
&= 2\left(\frac{\phi-1}{\phi}\right)m_t + \underbrace{\frac{1}{2}\left(\frac{2-\phi}{\phi}\right)}_{A}\left[x_{t-1} + \mathbb{E}_t[x_{t+1}]\right] \\
&= (1-2A)m_t + A\left[x_{t-1} + \mathbb{E}_t[x_{t+1}]\right]
\end{aligned}
$$

- Both past and future matter for reset price
- Difficult to solve (don't just plug in, it's futile!)

Guess that x_t follows some process

$$x_t = \mu + \lambda x_{t-1} + v m_t$$

Method of undetermined coefficients I

Guess that x_t follows some process

$$x_t = \mu + \lambda x_{t-1} + v m_t$$

- Could just plug in, but if it is true, this equation must always hold
- Even if there are no shocks, i.e., $x_t = x_{t-1} = m_t \implies \mu = 0; v = 1 - \lambda$
- Then $x_t = \lambda x_{t-1} + (1-\lambda) m_t$. Now plug in!

$$\begin{aligned}
\mathbb{E}_t[x_{t+1}] &= \mathbb{E}_t[\lambda x_t + (1-\lambda) m_{t+1}] \\
&= \lambda\left(\lambda x_{t-1} + (1-\lambda) m_t\right) + (1-\lambda) m_t \\
&= \lambda^2 x_{t-1} + (1-\lambda^2) m_t
\end{aligned}$$

From before:

$$\begin{aligned}
x_t &= (1-2A) m_t + A\left[x_{t-1} + \mathbb{E}_t[x_{t+1}]\right] \\
&= (1-2A) m_t + A\left[x_{t-1} + \lambda^2 x_{t-1} + (1-\lambda^2) m_t\right] \\
&= (1-2A + A(1-\lambda^2)) m_t + A(1+\lambda^2) x_{t-1}
\end{aligned}$$

Method of undetermined coefficients II

Guess that x_t follows some process

$$x_t = \lambda x_{t-1} + (1-\lambda) m_t$$

$$x_t = (1 - 2A + A(1-\lambda^2)) m_t + A(1+\lambda^2) x_{t-1}$$

$$\implies \lambda = A(1+\lambda^2)$$

$$\implies \lambda = \frac{\phi \pm 2\sqrt{\phi - 1}}{2 - \phi} \quad \text{only one } \lambda : -1 < \lambda < 0! \text{ (-)}$$

Take aways

- $|\lambda_+| > 1$, which means it would never converge back to steady state, even after only a single shock
- This slide pins down the evolution of the optimal price, output is next

Output dynamics under Taylor Contracts

Using this result for the process of x_t, we can solve for output

$$\begin{aligned}
y_t &= m_t - \frac{1}{2}\left(x_{t-1} + x_t\right) \\
&= m_t - \frac{1}{2}\left(\lambda x_{t-2} + (1-\lambda)m_{t-1} + \lambda x_{t-1} + (1-\lambda)m_t\right) \\
&= m_t - \frac{1}{2}(1-\lambda)(m_t + m_{t-1}) - \lambda \underbrace{\frac{1}{2}(x_{t-2} + x_{t-1})}_{p_{t-1}} \\
&= \lambda y_{t-1} + \frac{1+\lambda}{2}\varepsilon_t
\end{aligned}$$

- As $\phi \to \infty$, $\lambda \to -1$. A lower labor supply elasticity means that output oscillates around the steady state. For $\phi < 2$, output converges slowly back to the steady state.

Persistence of inflation I

Fischer Contracts (ignoring markup) – m_t is random walk

$$
\begin{aligned}
p_t &= \mathbb{E}_{t-1}[m_t] - \frac{1}{\phi}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right) \\
&= m_{t-1} - \frac{1}{\phi}\varepsilon_{t-1} \\
\implies p_t - p_{t-1} &= m_{t-1} - \frac{1}{\phi}\left(\varepsilon_{t-1}\right) - m_{t-2} + \frac{1}{\phi}\left(\varepsilon_{t-2}\right) \\
&= \varepsilon_{t-1} - \frac{1}{\phi}\varepsilon_{t-1} + \frac{1}{\phi}\varepsilon_{t-2} \\
&= \frac{\phi - 1}{\phi}\varepsilon_{t-1} + \frac{1}{\phi}\varepsilon_{t-2}
\end{aligned}
$$

- Remember: prices are reset in $t-1$ for t and $t+1$ $\implies$ shock lasts two periods
- Inflation depends on an anticipated component and an unanticipated component
- As labor becomes inelastic ($\phi \uparrow$), anticipated component less important

Persistence of inflation II

Taylor Contracts (ignoring markup) – m_t is random walk

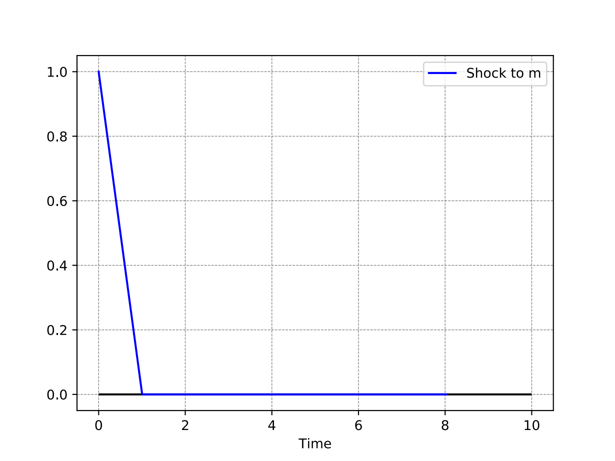
$$
\begin{aligned}
p_t &= \frac{1}{2}\left(x_t + x_{t-1}\right) \\
&= \frac{1}{2}(1-\lambda)(m_t + m_{t-1}) + \lambda \underbrace{\frac{1}{2}(x_{t-2} + x_{t-1})}_{p_{t-1}} \\
&= \frac{1}{2}(1-\lambda)(2m_{t-1} + \varepsilon_t) + \lambda p_{t-1} \\
&= (1-\lambda)m_{t-1} + \frac{1-\lambda}{2}\varepsilon_t + \lambda p_{t-1} \\
p_t - p_{t-1} &= \frac{1-\lambda}{2}\varepsilon_t + (1-\lambda)m_{t-1} + (\lambda - 1)p_{t-1} \\
&= \frac{1-\lambda}{2}\varepsilon_t + (1-\lambda)y_{t-1}
\end{aligned}
$$

- Inflation depends on **past output** $\implies$ shocks last longer
- Resetters know that the competition is locked in → change prices just enough to capture some market share → sluggishness

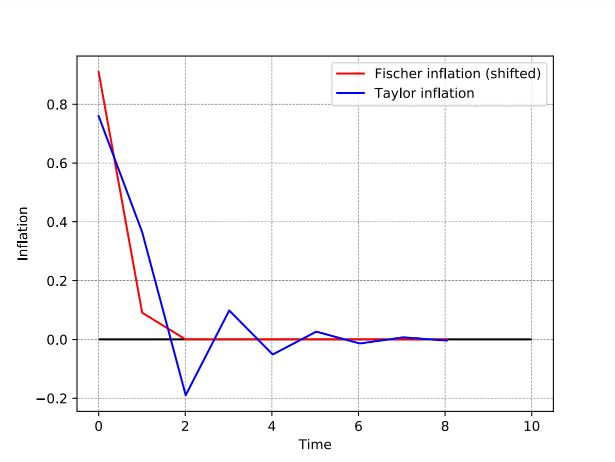
Inflation path comparison in pictures

Impulse response function of inflation – unanticipated shock

- What does inflation look like after a **single shock**? – $\varepsilon_0 = 1$
- Inflation persistence: for how long does the shock affect inflation?



Inflation impulse responses



- Note: we changed the timing assumptions, so the Fischer graph is shifted by one period
- Taylor contracts lead to much more persistence than Fischer prices

Punchline

Pricing frictions

- Romer's conclusion carries through: without frictions, anticipated shocks have no effect on output
- If agents cannot adjust prices freely, however, demand shocks have real effects
- Monetary policy can help stabilize output

Different forms of rigidity

- Fischer contracts: set price schedule and stick to it
- Taylor contracts: set prices for fixed number of periods
- Taylor has longer lasting effects

Next time

- Third (and most influential) form of price rigidity: the Calvo fairy

Calvo pricing and the New Keynesian Model

John Kramer – University of Copenhagen

November 2022

Last time

Price frictions

- Prices remain fixed **exogenously**, they cannot be adjusted at will
- Different forms of price frictions lead the same shock to have longer/shorter effects on output

Fischer & Taylor contracts

- Fischer contracts: set a price path
- Taylor contracts: constant prices

Effects

- Anticipated **shocks** matter
- Output can be manipulated using a money printing machine

Today

The missing piece: Calvo pricing

- Preeminent assumption to obtain the New Keynesian Phillips Curve (NKPC)
- Very elegant solution to a complicated price setting problem

The New Keynesian Model

- Three equations to rule the world:
  - Phillips Curve
  - IS Curve
  - Taylor rule
- Output responses to different shocks

Calvo pricing

Calvo pricing

Fischer and Taylor contracts get intractable for long durations

- More periods to keep track of
- With risk aversion (not present in our case) things get complicated quickly

Calvo contracts

- Price setting is stochastic: Prices are reset with a constant **probability**
- That implies that **on average**, prices stay constant for a number of periods, agents don't know if they will be able to change prices next period $\implies$ they form expectations
- "Calvo fairy" visits with some probability $\alpha \in [0, 1]$
- Elegant solution to a complicated problem

The price level in a Calvo world

Prices depend on past

$$p_t = \underbrace{\alpha x_t}_{\text{Resetters}} + \underbrace{(1-\alpha)p_{t-1}}_{\text{Cannot reset}} \implies \pi_t = \alpha(x_t - p_{t-1})$$

- x_t is the optimal price level for resetters in period t
- Only a fraction of α can reset their price
- All others have to keep their old price p_{t-1}
- Note that this is the aggregate price level. Some firms have been stuck with their price for potentially many periods

Model modifications

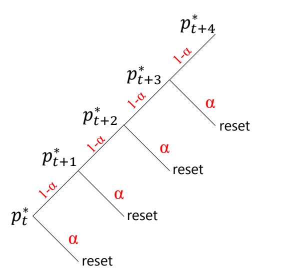
- Because agents have to look far into the future now, we have to introduce discounting into the utility function:

$$U_i = \sum_0^{\infty} \beta^t \mathbf{C}_{i,t} - \sum_0^{\infty} \beta^t \frac{1}{\phi} L_{i,t}^{\phi}$$

Optimal reset price I

Optimal reset price is an average of expected optimal future prices

- Under Taylor pricing: weighted average of two periods
- Under Calvo: weighted average of periods that will carry the price



$$\text{Taylor: } x_t = \frac{p^*_t + p^*_{t+1}}{2}$$

$$\text{Calvo: } x_t = \frac{p^*_t + (1-\alpha)p^*_{t+1} + (1-\alpha)^2 p^*_{t+2} + \cdots}{1 + (1-\alpha) + (1-\alpha)^2 + \cdots}$$

Optimal reset price II

Simplification (introducing discounting of the future)

$$
\begin{aligned}
x_t &= \frac{p_t^* + \beta(1-\alpha)p_{t+1}^* + \beta^2(1-\alpha)^2 p_{t+2}^* + \cdots}{1 + \beta(1-\alpha) + \beta^2(1-\alpha)^2 + \cdots} \\
&= \frac{\sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \mathbb{E}[p_{t+j}^*]}{\sum_{j=0}^{\infty} \beta^j (1-\alpha)^j} \\
&= (1-\beta(1-\alpha)) \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \mathbb{E}[p_{t+j}^*]
\end{aligned}
$$

- Last line uses the fact that $\sum_{j=0}^{\infty} x^j = \frac{1}{1-x}$
- If $\alpha = 1$, we're back to flexible prices: $x_t = p_t^*$
- We want to write this equation recursively (meaning: on the right side, we want future values of the left side)

Optimal reset price III

Recursive notation

$$
\begin{aligned}
x_t &= (1-\beta(1-\alpha))\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j\mathbb{E}_t[p^*_{t+j}] \\
&= (1-\beta(1-\alpha))p^*_t + (1-\beta(1-\alpha))\sum_{j=1}^{\infty}\beta^j(1-\alpha)^j\mathbb{E}_t[p^*_{t+j}] \\
&= (1-\beta(1-\alpha))p^*_t + (1-\beta(1-\alpha))\sum_{j=0}^{\infty}\beta^{j+1}(1-\alpha)^{j+1}\mathbb{E}_t[p^*_{t+1+j}] \\
&= (1-\beta(1-\alpha))p^*_t + \beta(1-\alpha)\underbrace{(1-\beta(1-\alpha))\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j\mathbb{E}_t[p^*_{t+1+j}]}_{\mathbb{E}_t[x_{t+1}]} \\
&= (1-\beta(1-\alpha))p^*_t + \beta(1-\alpha)\mathbb{E}_t[x_{t+1}]
\end{aligned}
$$

- The optimal reset price depends on expectations of future optimal prices

Inflation under Calvo

Tease out inflation

$$x_t - p_t = (1-\beta(1-\alpha))(p_t^* - p_t) + \beta(1-\alpha)\mathbb{E}_t[\underbrace{x_{t+1} - p_t}_{\pi_{t+1}/\alpha}]$$

$$\underbrace{x_t - p_{t-1}}_{\pi_t/\alpha} - \underbrace{(p_t - p_{t-1})}_{\pi_t} = (1-\beta(1-\alpha))(p_t^* - p_t) + \beta(1-\alpha)\mathbb{E}_t\left[\frac{\pi_{t+1}}{\alpha}\right]$$

$$\frac{1-\alpha}{\alpha}\pi_t = (1-\beta(1-\alpha))(p_t^* - p_t) + \beta\frac{1-\alpha}{\alpha}\mathbb{E}_t\left[\pi_{t+1}\right]$$

$$\pi_t = \frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}(p_t^* - p_t) + \beta\mathbb{E}_t\left[\pi_{t+1}\right]$$

$$\pi_t = \frac{\alpha(1-\beta(1-\alpha))(\phi-1)}{1-\alpha}y_t + \beta\mathbb{E}_t\left[\pi_{t+1}\right]$$

- Remember from last lecture:
  $p^* = (\phi-1)m_t + (2-\phi)p_t \implies p_t^* - p_t = (\phi-1)y_t$

The New Keynesian Phillips Curve

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right] \text{ with } \kappa = \frac{\alpha(1-\beta(1-\alpha))(\phi-1)}{1-\alpha}$$

- Inflation and output are positively related: high output means high inflation
- The slope of the Curve depends on three parameters:
    - Discounting parameter β
    - Price adjustment parameter α
    - Inelasticity of labor supply ϕ
- This is the economy's supply curve: higher prices → more output

A tale of two Phillips Curves

New Keynesian

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \left[\pi_{t+1} \right] \text{ with } \kappa > 0$$

- Inflation depends on expectations of future inflation
- The New Keynesian model is entirely **forward looking**

Lucas

$$\pi_t = \frac{1}{b} y_t + \mathbb{E}_{t-1} \left[\pi_t \right] \text{ with } b > 0$$

- Inflation depends on expectations of contemporary inflation

Notes on derivation

Assumptions on consumers as producers

- In the derivations so far, consumers are assumed to produce the goods themselves
- They choose production Y_i and price P_i
- There were no firms, the optimization problem was not dynamic (i.e., without taking the future into account)

Towards dynamic optimization

- This is not essential. It's possible to derive the same PC with consumers who work at profit maximizing firms

The IS curve

The Euler equation I

Intertemporal optimality

- The representative household's optimality condition for consumption can be derived from a standard dynamic optimization problem (see DR 7.1)

$$u'(C_t) = \beta \mathbb{E}\left[\frac{1+i_t}{1+\pi_{t+1}} u'(C_{t+1})\right]$$
$$u'(C_t) = \beta \mathbb{E}\left[(1+r_t) u'(C_{t+1})\right]$$

The Euler equation – Intuition

- There is a trade-off between consumption today tomorrow
- Cutting consumption today lowers utility by $u'(C)dC$
- The saved consumption can be invested into a nominal bond at gross nominal interest rate $1+i_t$
- Inflation between t and $t+1$ decreases the worth of the investment
- Saving dC today yields $u'(C)dC\beta(1+r_t)$ tomorrow

The Euler equation II

Functional form

- Assume that utility is given by $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$
- Special case: $\sigma = 1 \implies \log(c)$

$$C_t^{-\sigma} = \beta \mathbb{E}\left[(1 + r_t) C_{t+1}^{-\sigma}\right]$$

Interpretation

- Patient households (high β) consume less today
- Higher real interest rates lead to more consumption tomorrow
- Higher risk aversion σ leads to less volatile C over time

The Dynamic IS curve

Market clearing

- Everything that is produced will be consumed ($Y_t = C_t$)

$$Y_t^{-\sigma} = \beta \mathbb{E}\left[(1 + r_t) Y_{t+1}^{-\sigma}\right]$$

Linearize by applying logs

$$\begin{aligned} -\sigma y_t &= \log(\beta) + \mathbb{E}\left[\log(1 + r_t) - \sigma y_{t+1}\right] \\ \Longrightarrow \; y_t &= \mathbb{E}[y_{t+1}] - \frac{1}{\sigma}\left(\log(\beta) + \log(1 + r_t)\right) \\ y_t &\approx \mathbb{E}[y_{t+1}] - \frac{1}{\sigma}\left(r_t - \rho\right) \end{aligned}$$

Important simplification

- $\log(1 + x) \approx x$ and $\log(1 - x) \approx -x$

The three-equation model

IS curve – Consumer optimality

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}\left(E_t[r_t] - \rho\right)$$

Phillips curve – Firm optimality

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right]$$

Taylor rule – Monetary policy

$$r_t = \rho + \phi_y \mathbb{E}_t[y_{t+1}] + \phi_\pi \mathbb{E}_t[\pi_{t+1}]$$

Discussion

Is this a useful model?

- The three equation model is at the heart of every DSGE model currently in use
- It is a grotesque simplification of reality, but
    - It is based on micro foundations, meaning that it survives the Lucas critique
    - Most of its predictions make intuitive sense
    - Monetary policy has a place in the world after all
- Most models are much more complicated and account for all kinds of things in the economy

Forward iteration

IS curve – Consumer optimality

$$
\begin{aligned}
y_t &= \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}\left(E_t[r_t] - \rho\right) \\
&= \mathbb{E}_t[y_{t+T}] - \frac{1}{\sigma}\left(\sum_{j=0}^{T} r_{t+j} - \rho\right) \qquad = -\frac{1}{\sigma}\left(\sum_{j=0}^{T} r_{t+j} - \rho\right)
\end{aligned}
$$

- If consumers expect the real interest rate to be high, they consume less today

Phillips curve – Firm optimality

$$
\begin{aligned}
\pi_t &= \kappa y_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right] \\
&= \kappa \sum_{j=0}^{T} \beta^j y_{t+j}
\end{aligned}
$$

- The rate of inflation is pinned down by the expectations of future economic performance

Shocks to the economy

Demand shock

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}\left(E_t[r_t] - \rho\right) + u_{IS}$$

Cost-push shock

$$\pi_t = \kappa y_t + \beta\mathbb{E}_t\left[\pi_{t+1}\right] + u_{CP}$$

Monetary policy shock

$$r_t = \rho + \phi_y\mathbb{E}_t[y_{t+1}] + \phi_\pi\mathbb{E}_t[\pi_{t+1}] + u_{MP}$$

- To find how shocks affect output and inflation, plug r_t into the IS curve and the IS curve into the PC

The effect of shocks

Output

$$y_t = \left(1 - \frac{\phi_y}{\sigma}\right)\mathbb{E}_t[y_{t+1}] - \frac{\phi_\pi}{\sigma}E[\pi_{t+1}] - \frac{1}{\sigma}u_{MP} + u_{IS}$$

Inflation

$$\pi_t = \left(1 - \frac{\phi_y}{\sigma}\right)\kappa\mathbb{E}_t[y_{t+1}] + \left(\beta - \frac{\phi_\pi}{\sigma}\right)E[\pi_{t+1}] - \kappa\left(\frac{1}{\sigma}u_{MP} - u_{IS}\right) + u_\pi$$

Singular, unexpected shocks

- If we interpret y_t and π_t as deviations from a steady state, then without shocks, $y_t = \pi_t = 0$
- That makes it very easy to quantify the effect of single unexpected shocks, since $E[y_{t+1}] = \mathbb{E}[\pi_{t+1}] = 0$
- The model has no way to generate endogenous persistence

A monetary policy shock

Effects on output and inflation

$$y_t = -\frac{1}{\sigma} u_{MP}$$
$$\pi_t = -\kappa \frac{1}{\sigma} u_{MP}$$

- If monetary policy raises the interest rate **surprisingly** output falls
- Consumption tomorrow is relatively attractive $\implies$ delay consumption
- Because agents want to buy less today, prices have to fall

Elasticity of substitution $\frac{1}{\sigma}$

- If σ is high, consumers are very unwilling to substitute across periods
- Monetary policy will be less effective

This is how we think the world works!

Economists warn of deeper US downturn as Fed keeps up inflation fight

Central bank expected to implement fourth 0.75 point rate rise despite calls for slower pace

Fed chair Jay Powell has refused to rule out the possibility of a recession in the world's largest economy © FT Montage/Reuters /Dreamstime

A demand shock

Effects on output and inflation

$$y_t = u_{IS}$$

$$\pi_t = \kappa u_{IS}$$

- This shock is **in addition** to normal demand (i.e., outside the consumer's problem)
- Example: Government spending
- Because of more goods demand, market clearing requires that prices rise

Opposite of monetary policy shock

- Conventional wisdom: If demand is too high, raise interest rates to cool the economy

A cost-push shock

Effects on output and inflation

$$y_t = 0$$

$$\pi_t = u_\pi$$

- Over night, everything becomes more expensive
- Inflation rises, but output is unaffected
- A cost-push shock does not have real consequences in this New-Keynesian model

Assumptions are key

- In the model, there is nobody who cannot afford to eat when prices rise
- Nominal wages adjust immediately, such that real wages stay constant

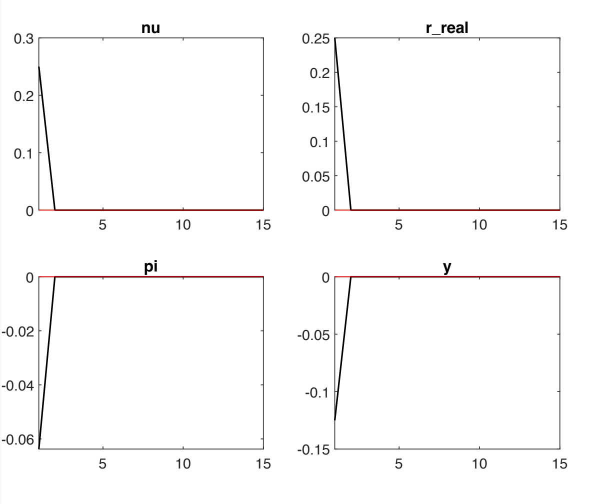
Persistent shocks

Effects on output and inflation

- If the shock processes are persistent, meaning $u_{t,MP} = \rho_{MP} u_{t-1,MP} + \epsilon_{t,MP}$, things get complicated
- The model can still be solved by hand, but it is cumbersome – we can just force a computer to solve it
- Rational expectations: agents in the model know ρ_{MP}! Upon the shock ϵ realization, they know the path of u

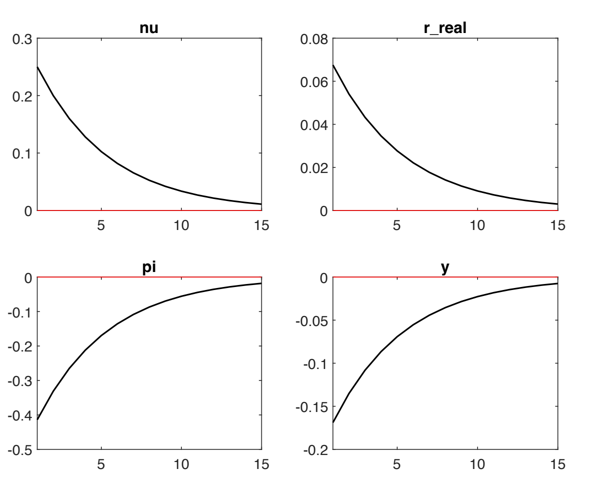
Computer output

Start with single-period shock



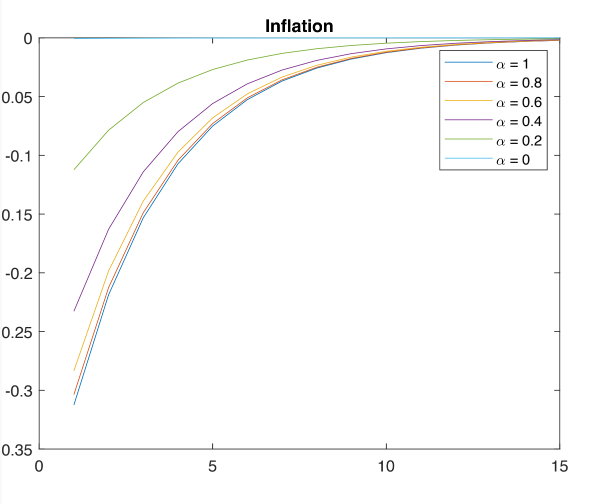
Computer output

Persistent shock



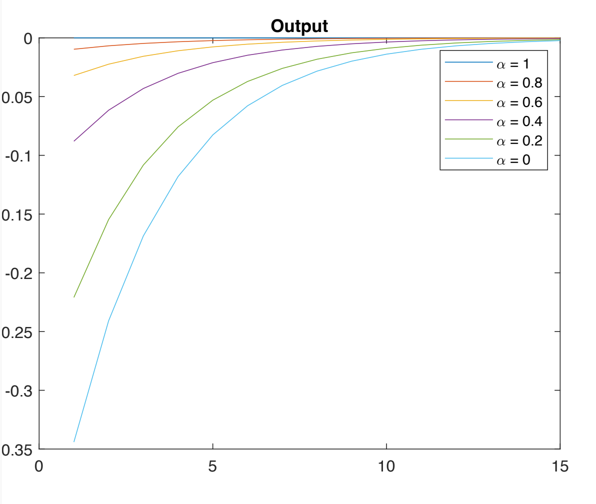
Computer output

Effect of price stickiness on inflation



Computer output

Effect of price stickiness on output



The role of monetary policy

The Taylor rule

- In the New Keynesian model, monetary policy is described by a rule
- If ϕ_π is high, the interest rates respond strongly to keep inflation in line
- If ϕ_y is high, the central bank tries to close the output gap quickly
- Is there a trade-off? Not allowing prices to change might mean that the effect of shocks on output will be larger

Expectations

- Agents in the model know the central banks response function
- If agents don't expect inflation, there will be no inflation

Example

Extreme case: $\phi_\pi = \infty$

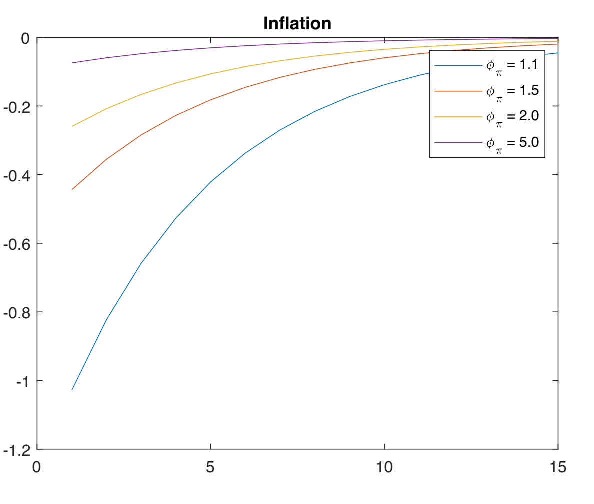
- The central bank will not allow any inflation in periods it has influence over
- The anticipation of potentially very large interest rate movements keeps output close to the baseline
- Through the Phillips curve, smaller output movements keep inflation low

Commitment and Credibility

- If the central bank can convince agents that it will not allow inflation to move, output will not move
- This is called the "divine coincidence": keeping inflation low will keep output low
- Fun fact: if ϕ_π is not high enough (usually > 1), the model cannot be solved

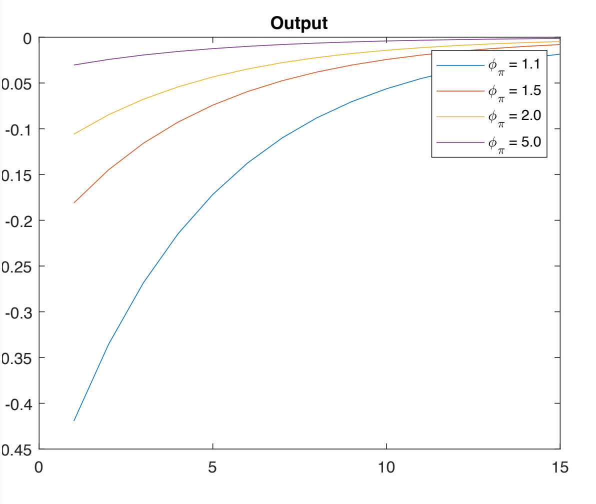
Aggregate responses as a function of ϕ_π

Effect of inflation aversion on inflation



Aggregate responses as a function of ϕ_π

Effect of inflation aversion on output



Shortcomings of the model

No labor market

- One can extend the model with Calvo adjustment frictions on wages. This way, researchers can think about questions regarding **involuntary** unemployment

Inflation persistence

- Inflation and output do not move instantaneously in response to changes in monetary policy. The baseline model can be extended with behavioral, backward-looking terms that lead to more hump-shaped responses

The model does not contain inequality

- There is only a representative agent in the model, but recently, questions of inequality have received a lot of attention.

Heterogeneity

Relevance of heterogeneity

Monetary policy

- Some people hold variable interest rate mortgages which are very vulnerable to interest rate changes
- People consume different goods, meaning their inflation rates differ

Government spending

- High earners pay higher income taxes, wealthy people pay most capital gains taxes
- The unemployed receive benefits, parents receive transfers

Labor market

- Some people work for the government, in relatively save jobs, others carry more risk

Demographics

- Women face different labor market trends than men
- Younger people face different labor market risks than the old

Measure of heterogeneous consumption (Almgren et al, 2022)

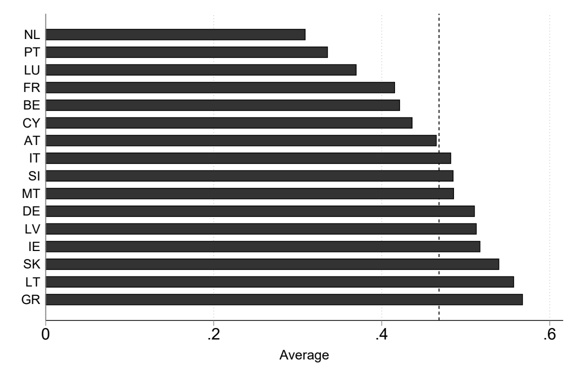
Propensity to spend unexpected income

Imagine you unexpectedly receive money from a lottery, equal to the amount of income your household receives in a month. What percent would you spend over the next 12 months on goods and services, as opposed to any amount you would save for later or use to repay loans?

Measure of heterogeneous consumption (Almgren et al, 2022)

Propensity to spend unexpected income

Imagine you unexpectedly receive money from a lottery, equal to the amount of income your household receives in a month. What percent would you spend over the next 12 months on goods and services, as opposed to any amount you would save for later or use to repay loans?



Vulnerability to shocks

Propensity to spend unexpected income


CNBC
Search quotes, news & videos
WATCHLIST | SIGN IN
CREATE FREE ACCOUNT
MARKETS BUSINESS INVESTING TECH POLITICS CNBC TV INVESTING CLUB PRO
MAKE IT
USA • INTL

PERSONAL FINANCE

63% of Americans are living paycheck to paycheck — including nearly half of six-figure earners

PUBLISHED MON, OCT 24 2022•10:29 AM EDT

Jessica Dickler
@JDICKLER

SHARE

KEY POINTS

- With persistent inflation eroding wage gains, the number of Americans living paycheck to paycheck is near a historic high, according to a recent report.
- Almost half of those earning more than $100,000 say they are just getting by.

Marginal propensity to consume

Representative agent model

$$u'(C_t) = \beta(1 + r_t)u'(C_{t+1})$$
$$C_t + A_t = A_{t-1}(1 + r_{t-1}) + I + \tau$$

- In the steady state, the representative agent always consumes the same: $C_t = C_{t+1}$, since $\beta(1 + r) = 1$
- What if we give him a bit more money for a single period?

Still consume the same in each period $\implies$ only consume such that it can be recouped by saving

$$(1 - \alpha)\tau(1 + r) = \tau$$
$$\frac{dC}{dI} = \alpha = 1 - \frac{1}{1 + r} \approx 2\%$$

Step forward

Representative agent cannot capture heterogeneity

- A marginal propensity to consume of 2% is unrealistically low
- Even the average MPC of an economy is likely much higher

Suggested solution:

- Some people always consumer their whole income
- Hand-to-mouth agents:

$$
\begin{aligned}
C_t &= I_t \\
\implies \frac{dC}{dI} &= 1
\end{aligned}
$$

Model modifications

Re-derive the model

- Include a fraction of λ of hand-to-mouth households
- Firm part of the model stays exactly the same

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \left[\pi_{t+1} \right]$$

- The IS curve changes to

$$y_t = \mathbb{E}[y_{t+1}] - \frac{1}{\sigma} \frac{1-\lambda}{1-\lambda\chi} (\mathbb{E}_t r_t - \rho)$$

The new IS-curve

$$y_t = \mathbb{E}[y_{t+1}] - \frac{1}{\sigma}\frac{1-\lambda}{1-\lambda\chi}(\mathbb{E}_t r_t - \rho)$$

New insights

- A demand shock (i.e., surprising change in $r_t - \rho$) has bigger or smaller effects depending on whether $\frac{1-\lambda}{1-\lambda\chi}$ is smaller or bigger than one
- If there are no HtM consumers ($\lambda = 0$), the fraction collapses to 1 and we are back to the original model
- If $\lambda > 0$, the effect of a demand shock depends on the parameter χ
- χ governs how strongly the income of the HtM agents moves with the aggregate

The elasticity of income to output

The new IS-curve

$$y_t = \mathbb{E}[y_{t+1}] - \frac{1}{\sigma}\frac{1-\lambda}{1-\lambda\chi}(\mathbb{E}_t r_t - \rho)$$

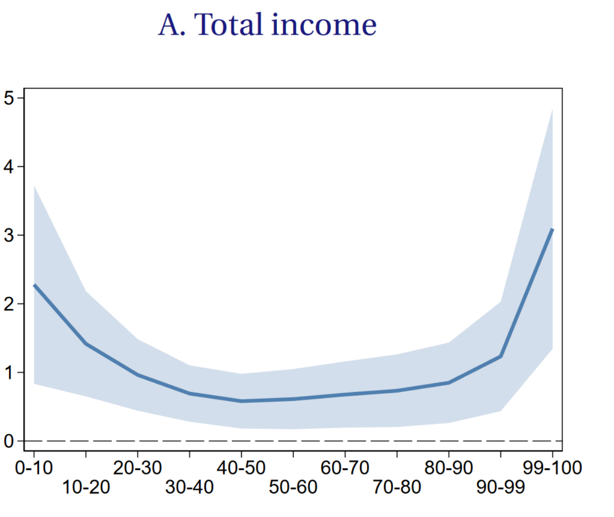
The magical χ parameter

$$y_t^{htm} = \chi y_t$$
$$y_t^n = \frac{1-\lambda\chi}{1-\lambda} y_t$$
$$\text{(note that } y_t = \lambda y_t^{htm} + (1-\lambda) y_t^n\text{)}$$

- Remember: small letter y means logs, hence $\chi = \frac{d\log(Y_t^{htm})}{d\log(Y_t)}$ is an elasticity
- $\chi > 1$ implies that the income of the HtM consumers is more volatile than aggregate income
- $\chi > 1$ monetary policy has a bigger effect on output

Empirical estimates

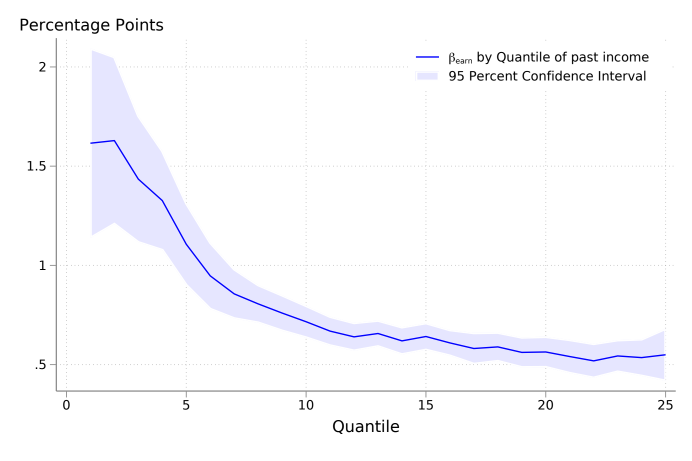
Sweden – Amberg et al (2022)



- χ seems to lie between 1.5 and 2 $\Longrightarrow$ plug into the NK model

Empirical estimates

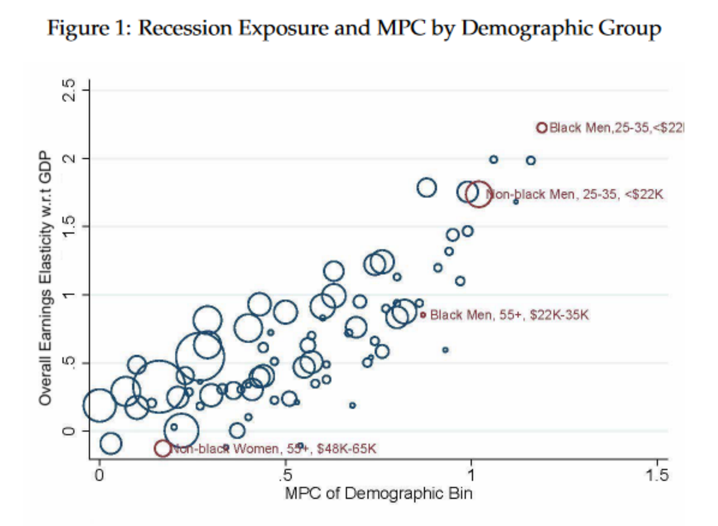
Germany – Broer et al (2022)



- χ seems to lie between 1.5 and 2 $\Longrightarrow$ plug into the NK model

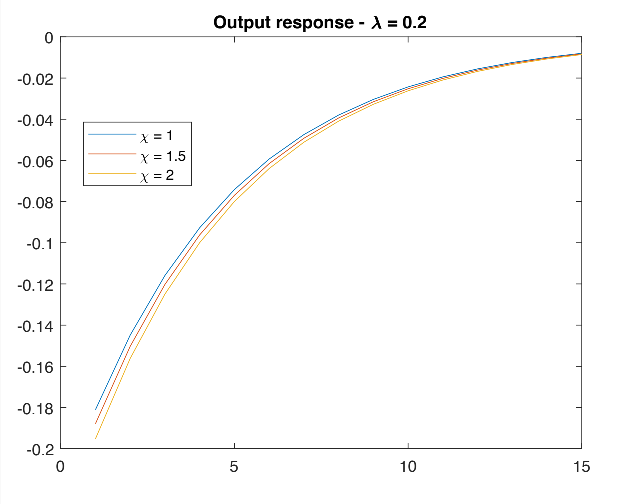
Empirical estimates

US – Patterson (2022)



- χ seems to lie between 1.5 and 2 $\Longrightarrow$ plug into the NK model

Influence of χ for output



Importance of heterogeneity

Intuition

- Some individuals in the economy cannot/do not insure against shocks to the economy (e.g., monetary policy). If their income falls, their consumption falls 1:1.
- The interest rate has no effect on their consumption decision
- The strong fall in consumption, in equilibrium, leads to less demand, which feeds back to the incomes of the unconstrained agents $\Longrightarrow$ even less consumption
- Heterogeneity amplifies the effects of aggregate shocks

  Just as Keynes always predicted!

Conclusions have implications beyond monetary policy

- Government spending: cutting taxes vs giving transfers

Conclusion

Calvo price setting

- Elegant way to introduce price stickiness into a model

The New Keynesian model

- The equations: PC, IS, TR
- Entirely forward looking: if we don't expect inflation, there will be no inflation
- Allows economists to speak intelligently about the effects of monetary policy

Heterogeneity

- Heterogeneity matters for the effect of shocks
- The representative agent model likely underestimates the effects of monetary policy

Optimal monetary policy

John Kramer – University of Copenhagen

December 2022

Last time

Calvo pricing

- Sellers adjust prices with a given probability
- Very elegant solution to a complicated price setting problem

The New Keynesian Model

- Three equations to rule the world: PC, IS, TR
- Output responses to different shocks

Heterogeneity

- The representative agent model must be extended to allow for better policy analysis
- Different marginal propensities to consume are a good starting point

Today

Central bankers are (rational) people, too

- Policy makers have goal functions, but where do they come from?
- Can central bankers be expected to adhere to rules?

Policy and politics

- Assuming some exogenous rule for monetary policy is too simple
- The policy itself is an outcome of its environment: it is endogenous
- In this context, credibility and reputation will play key roles

Optimal Monetary policy

- Agents and the central bank play a non-cooperative "game" ($\implies$ Nash)
- Outcome depends on the ability of the central bank to **commit** to a plan

Monetary policy makers care about their credibility

Isabel Schnabel

- "Instead, for monetary policy to remain credible in the current environment, it must not be an inflationary source itself."

Christine Lagarde

- "What became evident [during 2012] is that the perceived commitment of policymakers was a crucial variable in effective policymaking."

Janet Yellen

- "My remarks today will focus on the issue of credibility—in particular on the Federal Reserve's credibility regarding its announced commitment to maintaining price stability."

Mario Draghi

- "Within our mandate, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough."

Value judgements

Optimal monetary policy

- So far, all of our analyses have been positive
- Given the initial assumptions, they contained no value judgements, only descriptive conclusions
- Optimal monetary policy (i.e., what the central bank **should** do) requires us to do normative analysis

Rational expectations

- How trustworth/predictable the central bank's actions are is important
- Policy outcomes differ based on commitment/discretion

The model

Setup (Persson & Tabellini 15)

- The model is as simple as possible to isolate the channels we care about: the influence of central bank policy on output and inflation
- There is a relationship between output and inflation (PC) and an IS curve (reduced form)

The goal is to

- identify a rule for monetary policy that optimizes a loss function
- analyze how the economy's aggregates change under different assumption on credibility

Background

Unions

- In the Persson-Tabellini model, labor unions negotiate for some wage growth w such that

$$w = \omega + \pi^e$$

- Output growth, in turn, depends on the negotiated real wage:

$$x = \gamma - (w - \pi) - \varepsilon$$

- γ is a parameter, if wages are too high, output is too low
- supply shocks ε lower domestic output

Resulting output

$$x = \underbrace{(\gamma - \omega)}_{\theta} + (\pi - \pi^e) - \varepsilon$$

Equations

Phillips Curve

$$\pi_t = m_t + \underbrace{v}_{\text{Demand shock}} + \underbrace{\mu}_{\text{MP shock}}$$

Demand equation

$$x = \underbrace{\theta}_{\text{natural rate of output}} + (\pi_t - \pi^e) - \underbrace{\varepsilon}_{\text{Supply shock}}$$

- Inflation depends on money growth, unexpected demand and monetary policy mistakes
- Output depends on unexpected inflation, supply and its natural rate
- Expected inflation is $\mathbb{E}_t[\pi_t] \equiv \pi^e$
- All shocks are independent and 0 in expectation

Timing assumptions

Perfect commitment

1. Announcement of monetary rule
2. Everyone observes the natural level of output θ
3. Expectations π^e are formed, given the information about θ
4. Everyone observes v and ε
5. The central bank decides the money supply m
6. μ is realized, pinning down output x and inflation π

Consequences

- The central bank has an informational advantage (expectations are pinned down before the money supply is set)

Intuition

Monetary policy can move after expectations are formed

- This is a reduced form way to make monetary policy powerful
  $\Longrightarrow$ it can stabilize output against shocks
  $\Longrightarrow$ it can save the agents from themselves
- Monetary policy is decided every six weeks, wages are only renegotiated at longer intervals

Lucas again

- After θ realizes, only unexpected changes in monetary policy have an effect
- Moves in m can stabilize shocks to v and μ

$$\mathbb{E}_t[\pi_t|\theta] = \mathbb{E}_t[m_t|\theta]$$

$$x = \theta + (\underbrace{m_t + v_t + \mu_t}_{\pi_t} - \mathbb{E}_t[m_t|\theta]) + \varepsilon$$

Policy

Quadratic loss function

$$\mathcal{L} = \frac{1}{2}\left[a(\pi - \overline{\pi})^2 + \lambda(x - \overline{x})^2\right]$$

- Loss function implies that the central bank dislikes deviations from some inflation benchmark $\overline{\pi}$, and deviations from some output target $\overline{x}$
- The degree of "pain" such deviations cause the banker are governed by the parameters a and λ
- The parameters a and λ are known to all agents in the economy

Policy rule

Linear policy rule

- With a quadratic objective and linear shock processes, it can be shown that a policy rule which is linear in the shocks is optimal
- It can achieve the minimization of the loss function given the realizations of the shocks

Assumes the rule

$$m = \varphi + \varphi_\theta \theta + \varphi_v v + \varphi_\varepsilon \varepsilon$$

- The central bank reacts to shocks to natural output θ, demand shocks v and productivity shocks ε
- By definition, it cannot do anything about μ
- Recall: θ is observed before expectations are formed, v and ε realize after

Perfect credibility

Credible policy rule

- If agents know the rule and it is perfectly credible, they will include it into their expectations
- Strong assumption: central bankers may have an incentive to deviate (more on that later)

Expectations

$$\mathbb{E}_t[m_t|\theta] = \varphi + \varphi_\theta \mathbb{E}[\theta|\theta] + \varphi_v \mathbb{E}[v|\theta] + \varphi_\varepsilon \mathbb{E}[\varepsilon|\theta]$$
$$\mathbb{E}_t[\pi_t|\theta] = \mathbb{E}[m_t|\theta] = \varphi + \varphi_\theta \theta$$

- The shocks are independent $\implies$ conditional expectations don't help
- Expected inflation only depends on the realization of θ
- Other shocks are 0 in expectation

Perfect credibility

$$\mathbb{E}_t[\pi_t] = \varphi + \varphi_\theta \theta$$

Realized inflation

$$\begin{aligned}\pi_t &= \underbrace{\varphi + \varphi_\theta \theta + \varphi_v v + \varphi_\varepsilon \varepsilon}_{m_t} + v + \mu \\ &= \varphi + \varphi_\theta \theta + (1 + \varphi_v) v + \varphi_\varepsilon \varepsilon + \mu\end{aligned}$$

Realized output

$$\begin{aligned}x &= \theta + (\underbrace{\varphi + \varphi_\theta \theta + (1 + \varphi_v) v + \varphi_\varepsilon \varepsilon + \mu}_{\pi_t} - \underbrace{\varphi - \varphi_\theta \theta}_{\pi_t^e}) - \varepsilon \\ &= \theta + (1 + \varphi_v) v + (\varphi_\varepsilon - 1)\varepsilon + \mu\end{aligned}$$

What is the optimal policy?

Ex-ante optimality

- What parameters should be set for the policy rule **ex-ante**?
- Crucially: Implies optimal policy **in expectation**

Minimize the loss function

- If the rule is credible, then output and inflation will behave as on the previous slide
- Plug into the loss function
- Minimize **the expectation**

Expected loss

$$\begin{aligned}
\mathbb{E}[\mathcal{L}] &= \frac{1}{2}\mathbb{E}\left[a(\pi - \overline{\pi})^2 + \lambda(x - \overline{x})^2\right] \\
&= \frac{1}{2}\mathbb{E}\Bigg[a(\underbrace{\varphi + \varphi_\theta\theta + (1+\varphi_v)v + \varphi_\varepsilon\varepsilon + \mu}_{\pi} - \overline{\pi})^2 \\
&\quad + \lambda(\underbrace{\theta + (1+\varphi_v)v + (\varphi_\varepsilon - 1)\varepsilon + \mu}_{x} - \overline{x})^2 \Bigg] \\
&= \frac{1}{2}\mathbb{E}\left[a(A) + \lambda(B)\right]
\end{aligned}$$

- Want to minimize $\implies$ take derivatives
- But: Expectation of square term is complicated, better multiply out first (next slide)

Algebra

$$
\begin{aligned}
A &= (\varphi + \varphi_\theta\theta + (1+\varphi_v)v + \varphi_\varepsilon\varepsilon + \mu - \overline{\pi})^2 \\
&= \varphi^2 + \varphi\varphi_\theta\theta + \varphi(1+\varphi_v)v + \varphi\varphi_\varepsilon\varepsilon + \varphi\mu - \varphi\overline{\pi} + \varphi\varphi_\theta\theta + \varphi_\theta^2\theta^2 + (1+\varphi_v)v\varphi_\theta\theta \\
&+ \varphi_\varepsilon\varepsilon\varphi_\theta\theta + \mu\varphi_\theta\theta - \overline{\pi}\varphi_\theta\theta + \varphi(1+\varphi_v)v + \varphi_\theta\theta(1+\varphi_v)v + (1+\varphi_v)^2v^2 \\
&+ \varphi_\varepsilon\varepsilon(1+\varphi_v)v + \mu(1+\varphi_v)v - \overline{\pi}(1+\varphi_v)v + \varphi_\varepsilon\varepsilon\varphi + \varphi_\theta\varphi_\varepsilon\varepsilon\theta + (1+\varphi_v)\varphi_\varepsilon\varepsilon v \\
&+ \varphi_\varepsilon^2\varepsilon^2 + \varphi_\varepsilon\varepsilon\mu - \varphi_\varepsilon\varepsilon\overline{\pi} + \varphi\mu + \varphi_\theta\mu\theta + (1+\varphi_v)\mu v + \varphi_\varepsilon\mu\varepsilon + \mu^2 - \mu\overline{\pi} \\
&+ \varphi\overline{\pi} + \varphi_\theta\overline{\pi}\theta + (1+\varphi_v)\overline{\pi}v + \varphi_\varepsilon\overline{\pi}\varepsilon + \overline{\pi}\mu - \overline{\pi}\overline{\pi}
\end{aligned}
$$

All shocks are independent!

- In expectation, shock terms multiplied by constants are zero, e.g. $\mathbb{E}[\varphi\varphi_\theta\theta] = \varphi\varphi_\theta\mathbb{E}[\theta] = 0$
- In expectation, cross-terms are zero: $\mathbb{E}[\varphi_\varepsilon\varepsilon\varphi_\theta\theta] = \varphi_\varepsilon\varphi_\theta\mathbb{E}[\varepsilon\theta] = \varphi_\varepsilon\varphi_\theta\mathbb{E}[\varepsilon]\mathbb{E}[\theta] = 0$

Light at the end of the tunnel

$$
\begin{aligned}
\mathbb{E}[A] &= \mathbb{E}[\varphi + \varphi_\theta \theta + (1+\varphi_v)v + \varphi_\varepsilon \varepsilon + \mu - \overline{\pi})^2] \\
&= \varphi^2 - \varphi\overline{\pi} + \varphi_\theta^2 \mathbb{E}[\theta^2] + (1+\varphi_v)^2 \mathbb{E}[v^2] + \varphi_\varepsilon^2 \mathbb{E}[\varepsilon^2] + \mathbb{E}[\mu^2] - \varphi\overline{\pi} + \overline{\pi}^2 \\
&= \varphi^2 - \varphi\overline{\pi} + \varphi_\theta^2 \sigma_\theta^2 + (1+\varphi_v)^2 \sigma_v^2 + \varphi_\varepsilon^2 \sigma_\varepsilon^2 + \sigma_\mu^2 - \varphi\overline{\pi} + \overline{\pi}^2
\end{aligned}
$$

Expectations depend on variances of shocks

- $\sigma_q^2 = \mathrm{Var}(q) = \mathbb{E}[(q - \overline{q})^2]$ – If mean of random variable is 0, the expectation of its square is the variance
- The zero-mean and independence assumptions are doing **a lot** of heavy lifting for us

Second square term

Apply the same principle to the square variable B

$$\begin{aligned}\mathbb{E}[B] &= \mathbb{E}[(\theta + (1+\varphi_v)v + (\varphi_\varepsilon - 1)\varepsilon + \mu - \overline{x})^2] \\ &= \mathbb{E}[\theta^2] + (\varphi_v + 1)^2\mathbb{E}[v^2] + (1-\varphi_\varepsilon)^2\mathbb{E}[\varepsilon^2] + \mathbb{E}[\mu^2] + \overline{x}^2 \\ &= \sigma_\theta^2 + (\varphi_v + 1)^2\sigma_v^2 + (1-\varphi_\varepsilon)^2\sigma_\varepsilon^2 + \sigma_\mu^2 + \overline{x}^2\end{aligned}$$

- Now we have all the ingredients to fill in the expectation of the loss function

Minimize expected loss

Plugging in from the previous slides:

$$\min_{\varphi,\varphi_\theta,\varphi_v,\varphi_\varepsilon} \frac{1}{2}a\left(\varphi^2 - \varphi\overline{\pi} + \varphi_\theta^2\sigma_\theta^2 + (1+\varphi_v)^2\sigma_v^2 + \varphi_\varepsilon^2\sigma_\varepsilon^2 + \sigma_\mu^2 - \varphi\overline{\pi} + \overline{\pi}^2\right)$$
$$+ \frac{1}{2}\lambda\left(\sigma_\theta^2 + (\varphi_v+1)^2\sigma_v^2 + (1-\varphi_\varepsilon)^2\sigma_\varepsilon^2 + \sigma_\mu^2 + \overline{x}^2\right)$$

- A hypothetical social planner wants to set the rule (i.e., the parameters in the central bank's response function) to minimize this loss
- The rule is in place forever $\implies$ minimizing single period expectation of loss is the same as discounted infinite sum of all future periods' losses
- Take the derivatives w.r.t. $\varphi, \varphi_\theta, \varphi_v, \varphi_\varepsilon$

Minimum expected loss

$$\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial \varphi} : a(\varphi - \overline{\pi}) = 0 \implies \varphi = \overline{\pi}$$
$$\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial \varphi_\theta} : a\vartheta_\theta \sigma_\theta^2 = 0 \implies \varphi_\theta = 0$$
$$\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial \varphi_v} : \sigma_v^2(a + \lambda)(1 + \varphi_v) = 0 \implies \varphi_v = -1$$
$$\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial \varphi_\varepsilon} : \sigma_\varepsilon^2(a\varphi_\varepsilon - \lambda(1 - \varphi_\varepsilon)) = 0 \implies \varphi_\varepsilon = \frac{\lambda}{a + \lambda}$$

Implications

- Anchor inflation where society wants it
- Shocks that are priced into expectations need no reaction (θ)
- Neutralize demand shocks
- Supply shocks: it depends. Countering supply shocks causes less deviations from $\overline{x}$, but at the cost of deviations from $\overline{\pi} \implies$ tradeoff

Equilibrium under commitment

Optimal rule

$$m_t = \overline{\pi} - v_t + \frac{\lambda}{a + \lambda}\varepsilon$$

Equilibrium inflation

$$\pi^C = \overline{\pi} + \frac{\lambda}{a + \lambda}\varepsilon + \mu$$

Equilibrium output

$$x^C = \theta - \frac{a}{a + \lambda}\varepsilon + \mu$$

- Output may fluctuate due to changes in the natural rate, supply shocks or policy errors
- Inflation only changes due to supply shocks and policy errors
- Depending on preferences, supply shocks will feed more into output, or more into inflation

Commitment conclusions

Benefits

- If the policy maker can commit to a rule, inflation and output are stable around their natural levels
- As we will see, this is the best possible outcome

Simplify the problem

- Demand shocks are neutralized $\implies$ we can ignore them
- Policy errors μ are not interesting to study because there is little we can do about them $\implies$ ignore for now
- **The only important shocks left are θ and ε**

Credibility

Credibility

Problems with rules

- Central bankers are not computers. They may want to exploit their informational advantage
- Once expectations are locked in, it's possible to decrease societal losses even further
- The rule may not be credible if bankers have **discretion** (i.e., ability) to deviate

Discretion

- It's more realistic to assume policy makers don't stick to a rule
- This feeds back into agents (rational) expectations
- Equilibrium outcomes are different without commitment

New timing

Discretion/Non-credible rule

1. ~~Announcement of monetary rule~~
2. Everyone observes the natural level of output θ
3. Expectations π^e are formed, given the information about θ
4. Everyone observes ε
5. The central bank decides the money supply m
6. Output x and inflation π are pinned down

Implications

- Without a (credible) rule, the central bank is free to do what it wants each period

New optimality

Ex-post optimality

- When the CB could commit to a credible rule, that rule was **ex-ante** optimal: $\mathbb{E}[\frac{\mathcal{L}}{\partial m}] = 0$
- Without a rule, policy will be ex-post optimal: $\frac{\partial \mathcal{L}}{\partial m} = 0$
- What seems like a small difference has big consequences

Nash-equilibrium

- Central bank and consumers play a game. In equilibrium nobody wants to deviate from decision
- Solve by backwards induction

Central bank optimum under discretion (second stage)

$$\mathcal{L} = \frac{1}{2}\mathbb{E}\left[a(\pi_t - \overline{\pi})^2 + \lambda(x_t - \overline{x})^2\right]$$

$$\pi_t = m_t \quad \text{remember: } v \text{ and } \mu \text{ set to } 0$$

$$x_t = \theta + (\pi_t - \pi_t^e) - \varepsilon_t$$

- Since $\pi_t = m_t$, just assume that the CB sets π_t directly
- Recall that the CB takes π_t^e as given

$$\frac{\mathcal{L}}{\pi_t} : a(\pi_t - \overline{\pi}) + \lambda(\theta + (\pi_t - \pi_t^e) - \varepsilon_t - \overline{x}) = 0$$

$$\implies \pi = \frac{a}{a+\lambda}\overline{\pi} + \frac{\lambda}{a+\lambda}(\pi_t^e - \theta + \varepsilon + \overline{x})$$

- Note: If we plug in the result from the commitment equilibrium $\pi = \pi^e = \overline{\pi}$, $\frac{\mathcal{L}}{\pi_t} > 0 \implies$ CB can do better!

Consumer expectation under discretion (first stage)

Take expectation of central banks decision function

$$\begin{aligned}\mathbb{E}[\pi|\theta] &= \frac{a}{a+\lambda}\overline{\pi} + \frac{\lambda}{a+\lambda}\mathbb{E}[(\mathbb{E}[\pi|\theta] - \theta + \varepsilon + \overline{x})|\theta] \\ &= \overline{\pi} + \underbrace{\frac{\lambda}{a}(\overline{x} - \theta)}_{\text{Inflation bias}}\end{aligned}$$

- Expected inflation is higher than in the commitment case
- Because θ is known to consumers, they know what the CB will do. If $\theta < \overline{x}$: increase m, if $\theta > \overline{x}$: decrease m
- However, these actions are pointless, because prices adjust. As always, if higher m is expected, p (and therefore π) adjusts, and x stays constant

Realized values of inflation and output

Output

$$x^D = \theta - \frac{a}{a + \lambda}\varepsilon$$

- Output is the same as under commitment!

Inflation

$$\pi^D = \overline{\pi} + \frac{\lambda}{a}(\overline{x} - \theta) + \frac{\lambda}{a + \lambda}\varepsilon$$

- Inflation is higher and more volatile

Giving central banks discretion leaves output constant, but $\mathcal{L}$ is actually lower than it could be

Reputation

Longer time horizon

Single period

- The commitment and discretion cases before are single-period games
- In reality, central banks make decisions all the time

$\Longrightarrow$ Current decisions affect future reputation

Multi-period game

- The central bank makes decisions every period, proclaiming a rule
- Consumers decide whether they trust the bank or not
- Trust can never be rebuilt

Longer run optimality

Infinite loss function

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^j \mathcal{L}(\pi_{t+j}, x_{t+j})$$

- The central bank now cares about all future periods

Simplifying assumption

$$\mathcal{L}(\pi_t, x_t) = \frac{\pi_t^2}{2} - \lambda x$$
$$\implies \pi^C = 0, \quad \pi^D = \lambda, \quad x^C = x^D = \theta - \varepsilon$$

- As always: this contains the most important intuition
- Using the double square loss function is much more messy
- Inflation volatility is costly $\implies$ CB let's ε only affect output

Betraying trust

Inflation expectations

$$\pi_t^e = 0 \text{ if } \pi_{t-1} = \pi_{t-1}^e$$
$$\pi_t^e = \lambda \text{ otherwise}$$

- If realized inflation was in line with the agents expectations yesterday, the bank has not deviated from its rule
- In this case: keep trusting the central bank
- In any other case the bank has deviated $\implies$ don't trust the CB ever again

The central bank's problem

Adhere to rule or break trust?

- Each period, the CB faces a choice
- If it deviates, it can decrease its loss function today
- But at the cost of never being able to do so ever again

Determinants of decision

- Because the CB is a rational agent, it computes the one-time benefits of deviating and compares the to the future costs
- Whichever is more attractive is the equilibrium outcome

Contemporary benefit of deviating

Loss in case of exploitation

$$\pi_t = \lambda$$

$$x_t = \theta + \lambda - \varepsilon$$

$$\mathcal{L}(\lambda, \theta + \lambda - \varepsilon) = +\frac{1}{2}\lambda^2 - \lambda(\theta + \lambda - \varepsilon)$$

Loss in case of continuous commitment

$$\pi_t = 0$$

$$x_t = \theta - \varepsilon$$

$$\mathcal{L}(0, \theta - \varepsilon) = -\lambda(\theta - \varepsilon)$$

One-time loss from deviating

$$\mathcal{L}(\lambda, \theta + \lambda - \varepsilon) - \mathcal{L}(0, \theta - \varepsilon) = -\frac{1}{2}\lambda^2 \text{ (loss is lower)}$$

Long-run cost of deviating

Loss in case of deviating (starting at period $t = s + 1$—tomorrow)

$$\pi_s = \lambda, \quad x_s = \theta - \varepsilon$$

$$\mathbb{E}\left[\sum_{t=s+1}^{s+T} \beta^{t-s} \mathcal{L}(\lambda, \theta - \varepsilon)\right] = \sum_{t=s+1}^{s+T} \beta^{t-s} \left(\frac{1}{2}\lambda^2 - \lambda \mathbb{E}[(\theta - \varepsilon)]\right)$$

Loss in case of continuous commitment

$$\pi_s = 0, \quad x_s = \theta - \varepsilon$$

$$\mathbb{E}\left[\sum_{t=s+1}^{s+T} \beta^{t-s} \mathcal{L}(0, \theta - \varepsilon)\right] = -\sum_{t=s+1}^{s+T} \beta^{t-s} \lambda \mathbb{E}\left[(\theta - \varepsilon)\right]$$

Long-run loss from deviating

$$\mathbb{E}\left[\sum_{t=s+1}^{s+T} \beta^{t-s} \mathcal{L}(\lambda, \theta - \varepsilon)\right] - \mathbb{E}\left[\sum_{t=s+1}^{s+T} \beta^{t-s} \mathcal{L}(0, \theta - \varepsilon)\right] = \beta \frac{1}{2} \frac{(1 - \beta^{T-1})}{1 - \beta} \lambda^2$$

Overall cost-benefit analysis

Add up single-period and long-run losses from deviating

$$Q = \beta \frac{1}{2} \frac{(1-\beta^{T-1})}{1-\beta} \lambda^2 - \frac{1}{2} \lambda^2$$

- If $Q < 0$, the effect of deviating on the loss function (contemporaneous + long-run) is negative $\implies$ desirable! Smaller loss means gain
- If $Q > 0$, the loss is positive (that's bad) and the CB does not want to deviate
- The central bank will deviate if:

$$\frac{1}{2} \lambda^2 \left(\beta \frac{(1-\beta^{T-1})}{1-\beta} - 1 \right) < 0 \iff \beta \frac{(1-\beta^{T-1})}{1-\beta} < 1$$

Intuition

$$\beta \frac{(1 - \beta^{T-1})}{1 - \beta} < 1$$

Special cases

- If the world end tomorrow ($T = 1$), $0 < 1$ implies that the central bank will deviate with certainty
- If the world never ends, we need $\beta < 0.5$ for the CB to find deviating attractive
- If the discount factor is low (0.5 is very low), the CB doesn't care about the future and will deviate

Implications

- The repeated game nature of this example, together with the threat of higher inflation forever, keep the central bank honest
- Once the CB has deviated, the economy can never go back

Institutions

Policy mandates

European Central Bank

- The primary objective of the European System of Central Banks (hereinafter referred to as 'the ESCB') shall be to maintain price stability. (Article 127, TFEU)
- In pursuing price stability, the ECB seeks to hold inflation below but close to 2 percent over a medium-term horizon.
- (...) support the general economic policies in the Union with a view to contributing to the achievement of the objectives of the Union as laid down in Article 3 of the Treaty on European Union.

Federal Reserve

- (...) so as to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates.

Central bank appointments

Guidance

- Both ECB and Fed have been given clear guidelines on what to focus their policies on
- ECB: Price stability – everything else is secondary
- Fed: Dual mandate – more in line with the formulas above

Doves or Hawks

- However, it is impossible for a central bank to credibly commit to a rule
- Still, governments can at least appoint the right person to head the central bank
- Who are they?

Finding the right central banker

$$x_B^D = \theta - \frac{a_B}{a_B + \lambda_B}\varepsilon$$

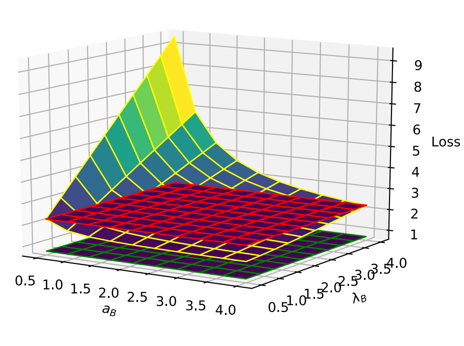
$$\pi_B^D = \overline{\pi} + \frac{\lambda_B}{a_B}(\overline{x} - \theta) + \frac{\lambda_B}{a_B + \lambda_B}\varepsilon$$

- Each central banker has their own a_B and λ_B
- Which one should be chosen to make decisions?

$\implies$ minimize conditional loss function

$$\mathbb{E}\left[\mathcal{L}(x_B^D, \pi_B^D)\right] = \mathbb{E}\left[\frac{1}{2}a\left(\frac{\lambda_B}{a_B}(\overline{x} - \theta) + \frac{\lambda_B}{a_B + \lambda_B}\varepsilon\right)^2 + \frac{1}{2}\lambda\left(\theta - \frac{a_B}{a_B + \lambda_B}\varepsilon - \overline{x}\right)^2\right]$$

Finding the right central banker



- red: discretion; green: commitment; yellow: central banker
- Large values of a_B and small values of λ_B approach the optimum
- Inflation hawks minimize the loss function

Conclusion

Commitment and Discretion

- Commitment to a rule leads to lowest inflation
- Discretion creates an inflationary bias—output is unchanged

$\Longrightarrow$ Credibility is important

Repeated game

- Interaction across many periods can keep the central bank in check
- Future costs of deviating make optimum more attractive

The ideal central banker

- Inflation hawks lead to a lower loss function
- Can approach commitment optimum

Algebra

$$
\begin{aligned}
&\sum_{t=s+1}^{s+T}\beta^{t-s}\left(\frac{1}{2}\lambda^2-\lambda\mathbb{E}[(\theta-\varepsilon)]\right)+\sum_{t=s+1}^{s+T}\beta^{t-s}\lambda\mathbb{E}\left[(\theta-\varepsilon)\right]\\
&=\sum_{t=s+1}^{s+T}\beta^{t-s}\frac{1}{2}\lambda^2=\frac{1}{2}\lambda^2\sum_{t=s+1}^{s+T}\beta^{t-s}\\
&=\frac{1}{2}\lambda^2\sum_{t=s+1}^{s+T}\beta^{t-s}=\frac{1}{2}\lambda^2\beta\sum_{t=s}^{s+T-1}\beta^{t-s}=\frac{1}{2}\lambda^2\beta\sum_{j=0}^{T-1}\beta^{j}\\
&=\frac{1}{2}\lambda^2\beta\left(\sum_{j=0}^{\infty}\beta^j-\sum_{j=T-1}^{\infty}\beta^j\right)=\frac{1}{2}\lambda^2\beta\left(\frac{1}{1-\beta}-\sum_{j=T-1}^{\infty}\beta^j\right)\\
&=\frac{1}{2}\lambda^2\beta\left(\frac{1}{1-\beta}-\beta^{T-1}\sum_{j=0}^{\infty}\beta^j\right)\\
&=\frac{1}{2}\lambda^2\beta\frac{1-\beta^{T-1}}{1-\beta}
\end{aligned}
$$

Open economy macroeconomics

John Kramer – University of Copenhagen

December 2022

Second half so far

Pricing frictions

- Monopolistic competition leads to welfare losses
- Sticky/Rigid prices allow the central bank to influence output
- Taylor, Fischer, Calvo

Expectations matter

- Expected "shocks" do not affect output
- Prices adjust to keep output constant

The New Keynesian model

- Three equations to rule the world: PC, IS, TR
- Useful model of the world
- Heterogeneity matters for aggregate movements

Optimal monetary policy

- Central banks may want to neutralize demand, but not supply shocks

Today: change of topic!

Small open economy

- Back to **real**-ity: perfect competition

New ground

- What is a small open economy?
- Important concepts: trade balance & current account
- Gains from trade (heterogeneity)

Real exchange rate

- Multiple goods
- Short & long run dynamics

The small open economy

Setup

Open economy

- Markets don't clear internally anymore
- On the world level there is market clearing: $\mathbf{k}_{t+1} = \mathbf{a}_{t+1}$, but not in each country
- Goods, capital and assets can be exchanged

Small open economy

- Saving decisions do not affect the world interest rate
- Only focus on home country today, everything else is the rest of the world (ROTW)

Neoclassical open economy I

Representative consumer

$$\max_{c_t, a_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t. } a_{t+1} + c_t = a_t R_t + w_t$$

- Capital does not show up directly
- $R_t = 1 + r_t - \delta$
- Interest rate r_t is the world interest rate, capital can move freely across borders

Optimality

$$u'(c_t) = \beta R_t \mathbb{E}[u'(c_{t+1})]$$

Neoclassical open economy II

Representative firm

$$\max_{K_t, L_t} f(K_t, L_t) - r_t K_t - w_t L_t$$

- Firms are completely standard
- r_t is the world interest rate

Optimality

$$f_K(k_t, 1) = r_t$$
$$f_L(k_t, 1) = w_t$$

- Without technological progress and constant r_t, everything is constant

New concepts

Trade balance: Exports - Imports (flow)

$$tb_t = \underbrace{f(k_t, 1)}_{y_t} - c_t - \underbrace{(k_{t+1} - (1-\delta)k_t)}_{i_t}$$

- In a closed economy, production y equals consumption and investment, because markets clear internally
- Not true in the open economy $\implies$ not all output has to be consumed at home

Net foreign assets (stock)

$$N_t = a_t - k_t$$

- Not all assets need to be held at home
- Difference between demand a_t and supply k_t must be held in the rest of the world (ROTW)

Current account

Total goods received from/sent to ROTW

$$ca_t = \underbrace{tb_t}_{\text{trade}} + \underbrace{r_t N_t}_{\text{interest}} - \underbrace{\delta N_t}_{\text{depreciation}}$$

- The current account is a flow, just like the trade balance

Link between net foreign asset position and current account

- If the current account is positive, a country produced (y_t) than it used (c_t, i_t)
- This overproduction has to be stored somewhere $\implies$ more foreign asset savings

Net foreign asset dynamics – Algebra

$$
\begin{aligned}
ca_t &= tb_t + r_t N_t - \delta N_t \\
&= f(k_t, 1) - c_t - (k_{t+1} - (1-\delta)k_t) + (r_t - \delta)N_t \\
&= f(k_t, 1) - a_t R_t - w_t + a_{t+1} - (k_{t+1} - (1-\delta)k_t) + (r_t - \delta)N_t \\
&= f(k_t, 1) - w_t - a_t R_t + \underbrace{a_{t+1} - k_{t+1}}_{N_{t+1}} + (1-\delta)k_t + (r_t - \delta)N_t \\
&= f(k_t, 1) - w_t - a_t R_t + N_{t+1} + (1-\delta)k_t + \underbrace{(r_t - \delta)N_t}_{(R_t - 1)(a_t - k_t)} \\
&= f(k_t, 1) - w_t - a_t R_t + N_{t+1} + (1-\delta)k_t + R_t(a_t - k_t) - N_t \\
&= f(k_t, 1) - w_t + N_{t+1} - \underbrace{(R_t - 1 + \delta)}_{r_t} k_t - N_t \\
&= f(k_t, 1) - w_t - r_t k_t + N_{t+1} - N_t \\
&= N_{t+1} - N_t \\
&= \Delta N_{t+1}
\end{aligned}
$$

General equilibrium

Asset accumulation

- Asset accumulation is governed by the representative household's Euler equation and their budget constraint

$$u'(c_t) = \beta R_t \mathbb{E}[u'(c_{t+1})]$$
$$a_{t+1} + c_t = a_t R_t + w_t$$

Capital

- The amount of capital is governed by the firm's investment choice

$$f_K(k_t, 1) = r_t$$

- There is no resource constraint $(k_{t+1} = k_t(1-\delta) + f(k_t, 1) - c_t)$ within the economy
- The interest rate is not endogenous

Consumption in equilibrium

Solve the budget constraint forward

$$\begin{aligned} c_t &= a_t R_t + w_t - a_{t+1} \\ &= a_t R_t + w_t - \left(\frac{a_{t+2} + c_{t+1} - w_{t+1}}{R_{t+1}} \right) \\ &= a_t R_t + \sum_{s=0}^{\infty} \frac{w_{t+s}}{\Pi_{j=0}^{s} R_{t+j}} - \sum_{s=1}^{\infty} \frac{c_{t+s}}{\Pi_{j=0}^{s} R_{t+j}} \end{aligned}$$

- Assume a transversality condition: $\lim_{s\to\infty} \frac{1}{\Pi_{j=0}^{s} R_{t+j}} a_{t+s} = 0$
- Assume $R_t = R = \frac{1}{\beta} \implies c$ is constant

$$c = \rho a_t R_t + \rho \sum_{s=0}^{\infty} \frac{w_{t+s}}{R^s} \quad \text{where} \left(\rho = \sum_{s=0}^{\infty} \frac{1}{R^s} \right)^{-1} = \frac{R-1}{R}$$

- Consumption depends on lifetime wealth (perfect smoothing)

An open endowment economy

Setup

- Assume the home economy cannot accumulate capital: $k_t = 0$
- Assume that workers receive an (time varying) endowment ω_t

Implications

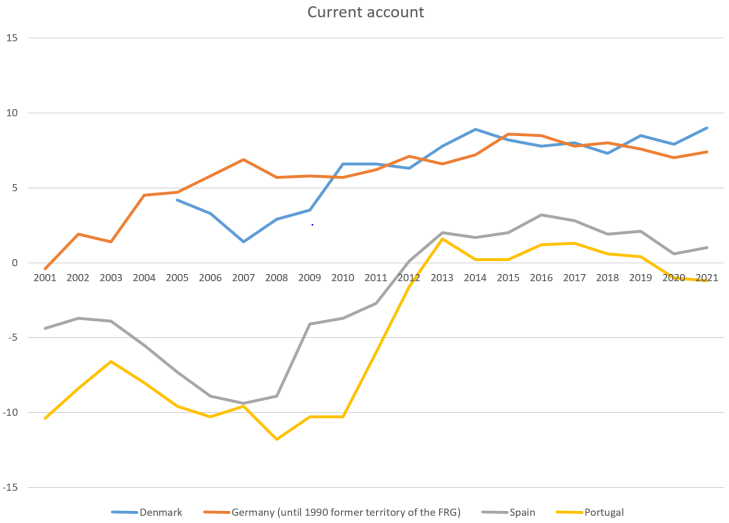
- The net foreign asset position is $N_t = a_t$
- Let permanent income be $\widetilde{\omega}_t = \rho \sum_{t=0}^{\infty} \frac{\omega_t}{R^t}$
- The period budget constraint is $c_t = N_t R - N_{t+1} + \omega_t$

$$
\begin{aligned}
N_t R - N_{t+1} + \omega_t &= \rho R N_t + \widetilde{\omega}_t \\
N_t R(1-\rho) - N_{t+1} &= \widetilde{\omega}_t - \omega_t \\
ca_t = N_{t+1} - N_t &= \omega_t - \widetilde{\omega}_t
\end{aligned}
$$

$\Longrightarrow$ Perfect insurance **against temporary shocks** but without domestic capital to save in

Empirical evidence

The current account



The trade balance

Permanent income consumers

$$\sum_{s=0}^{\infty} \frac{c}{R^s} = N_t R + \sum_{s=0}^{\infty} \frac{\omega_{t+s}}{R^s}$$

Trade balance in the endowment economy

$$tb_t = \omega_t - c$$

Can a country run a trade-deficit ($tb_t < 0$) forever?

The trade balance

Permanent income consumers

$$\sum_{s=0}^{\infty} \frac{c}{R^s} = N_t R + \sum_{s=0}^{\infty} \frac{\omega_{t+s}}{R^s}$$

Trade balance in the endowment economy

$$tb_t = \omega_t - c$$

Can a country run a trade-deficit ($tb_t < 0$) forever?

$$\begin{aligned} \sum_{s=0}^{\infty} \frac{c}{\Pi_{j=0}^{s} R_{t+j}} - \sum_{s=0}^{\infty} \frac{\omega_{t+s}}{\Pi_{j=0}^{s} R_{t+j}} &= a_t R_t \\ \sum_{s=0}^{\infty} \frac{tb_{t+s}}{\Pi_{j=0}^{s} R_{t+j}} &= -a_t R_t \end{aligned}$$

- Only rich countries can run deficits for long periods of time
- If $a_t < 0$, then the country must eventually run trade surpluses

The trade balance over time

Consumption

$$c = \rho a_t R_t + \rho \sum_{s=0}^{\infty} \frac{w_{t+s}}{R^s}$$

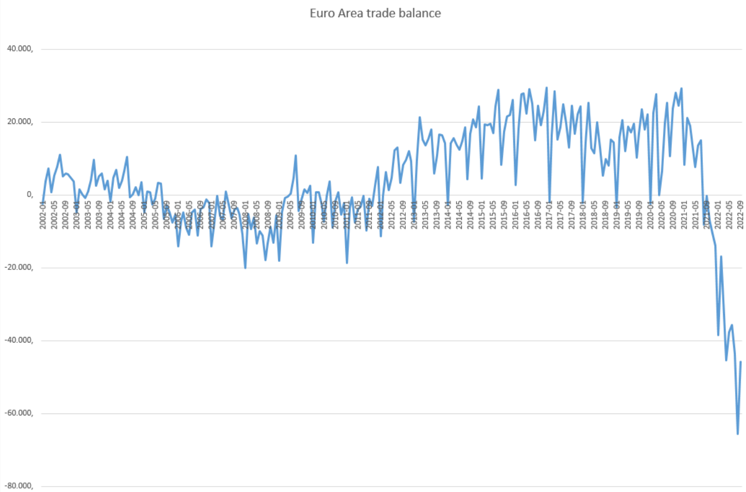
Trade balance in the endowment economy

$$tb_t = \omega_t - \rho a_t R_t - \underbrace{\rho \sum_{s=0}^{\infty} \frac{w_{t+s}}{R^s}}_{\text{perm. inc.}}$$

- The trade balance is **procyclical**
- When current income ω_t is higher than permanent income, the economy exports more

Empirical evidence

Trade balance of the euro area



The real exchange rate

Extending the model

Exchange rate determination

- So far, there is only one consumption good that all countries consume
- Hence, the real exchange rate between the home country and the ROTW is one

$\Longrightarrow$ To talk about exchange rates, we need at least two goods

Extending the model

Exchange rate determination

- So far, there is only one consumption good that all countries consume
- Hence, the real exchange rate between the home country and the ROTW is one

$\Longrightarrow$ To talk about exchange rates, we need at least two goods

Multiple goods

- Assume that there is a tradable good c^T and a non-tradable good c^N
- For c^N, the home market clears, c^T is traded internationally
- Households consume both goods such that $c_t = g(c^T, c^N)$
- Let p_t be the price of c^N, while the price of $c^T = 1$.
- $\mathcal{P}_t$ is the price index for a unit of the consumption aggregate c_t

Updated setup

Household problem

$$\max_{c_t^T, c_t^N, a_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c(c_t^N, c_t^T))$$

$$a_{t+1} + \underbrace{\mathcal{P}_t c_t}_{c_t^T + p_t c_N^T} = a_t R_t + \omega_t^T + p_t \omega_t^N$$

- $\mathcal{P}_t$ is the real exchange rate (price of c_t at home rel. to ROTW)
- Households choose how much to save (as before)
- Pick how much of each good to buy
- R_t is exogenous (as before)

Optimization

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c(c_t^N, c_t^T)) + \sum_{t=0}^{\infty} \lambda_t \left(a_t R_t + \omega_t^T + p_t \omega_t^N - c_t^T - p_t c_t^N - a_{t+1} \right)$$

Household optimality

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c(c_t^N, c_t^T)) + \sum_{t=0}^{\infty} \lambda_t \left(a_t R_t + \omega_t^T + p_t \omega_t^N - \underbrace{c_t^T - p_t c_t^N}_{\mathcal{P}_t c_t} - a_{t+1} \right)$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} : \lambda_t = \lambda_{t+1} R_t$$

$$\frac{\partial \mathcal{L}}{\partial c_t^T} : \beta^t u'(c_t) c_T(c_t^N, c_t^T) = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial c_t^N} : \beta^t u'(c_t) c_N(c_t^N, c_t^T) = \lambda_t p_t$$

Optimality conditions

$$u'(c_t) c_T(c_t^N, c_t^T) = \beta R_t u'(c_{t+1}) c_T(c_{t+1}^N, c_{t+1}^T)$$

$$p_t = \frac{c_N(c_t^N, c_t^T)}{c_T(c_t^N, c_t^T)}$$

$$u'(c_t) = \beta R_t \frac{\mathcal{P}_{t+1}}{\mathcal{P}_t} u'(c_{t+1})$$

Intuition

Optimality conditions

$$u'(c_t)c_T(c_t^N, c_t^T) = \beta R_t u'(c_{t+1})c_T(c_{t+1}^N, c_{t+1}^T)$$

$$p_t = \frac{c_N(c_t^N, c_t^T)}{c_T(c_t^N, c_t^T)}$$

$$u'(c_t) = \beta R_t \frac{\mathcal{P}_{t+1}}{\mathcal{P}_t} u'(c_{t+1})$$

- Consumption c_t follows an IS-curve $\implies$ marginal utility is related across periods
- $\frac{\mathcal{P}_{t+1}}{\mathcal{P}_t}$ represents changes in the real exchange rate
- $\mathcal{P}_t(p_t) \uparrow \implies$ appreciation, home goods become more expensive
- p_t (price of c^N in terms of c_T) is dictated by the marginal rate of substitution

Equilibrium

- Steady state: set $R = 1/\beta$ and $\omega_t^N = \omega^N$

Market clearing

- Non-tradables have to clear within the country $\implies \omega^N = c^N$

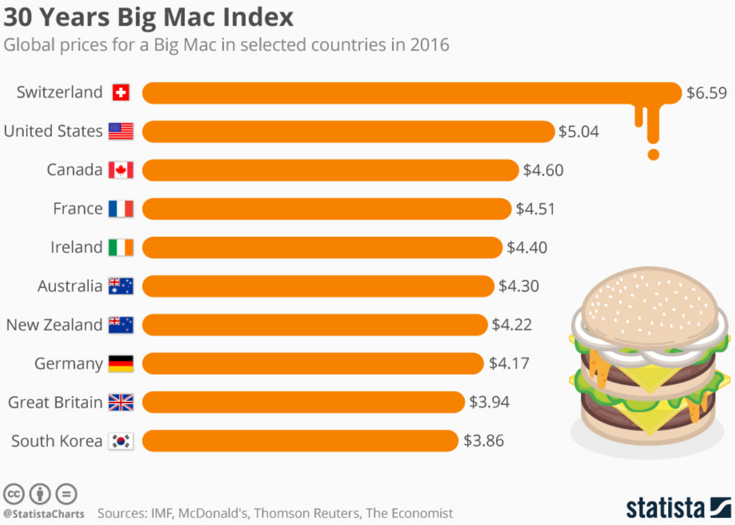
$$u'(c_t)c_T(\omega^N, c_t^T) = \beta R_t u'(c_{t+1})c_T(\omega^N, c_{t+1}^T)$$
$$p_t = \frac{c_N(\omega^N, c_t^T)}{c_T(\omega^N, c_t^T)}$$
$$c_t^T = \rho N_t R_t + \rho \sum_{s=0}^{\infty} \frac{\omega_{t+s}^T}{R^s}$$

$\implies$ Tradable consumption is higher for richer countries (more net foreign assets or higher future endowment)

$\implies$ Richer countries want to consume more, but c^N is fixed in the short run $\implies \mathcal{P}_t(p_t)$ higher

Empirical real exchange rate

The Big Mac index



Short run vs long run

Long run adjustments

- In the short run, non-tradable production may be fixed
- Over time, factors of production will realign to exploit price differences

$\Longrightarrow$ Move away from endowment assumption

Short run vs long run

Long run adjustments

- In the short run, non-tradable production may be fixed
- Over time, factors of production will realign to exploit price differences

$\Longrightarrow$ Move away from endowment assumption

Two goods

$$Y^T = A_t^T F(K_t^T, L_t^T)$$
$$Y^N = A_t^N F(K_t^N, L_t^N)$$

- Capital is perfectly mobile across the world (returns R_t)
- Labor is mobile within the home country, with $L_t^T + L_t^N = 1$
- As before, relative price of tradable good it p_t

Competitive Equilibrium

First order conditions

$$
\begin{aligned}
r_t &= A_t^T f_K'(k_t^T) \\
r_t &= p_t A_t^N f_K'(k_t^N) \\
w_t &= A_t^T f_L'(k_t^T) \\
w_t &= p_t A_t^N f_L'(k_t^N)
\end{aligned}
$$

- The tradable sector is the numeraire, non-tradable goods have to be transformed at price p_t
- Competitive firms make sure that cost of capital R_t is equal to the marginal benefit
- Wages (measured in tradable goods) equate to the MPL

Determinants of the RER in the long-run

First order conditions

$$
\begin{aligned}
r_t &= A_t^T f_K'(k_t^T) \\
r_t &= p_t A_t^N f_K'(k_t^N) \\
w_t &= A_t^T f_L'(k_t^T) \\
w_t &= p_t A_t^N f_L'(k_t^N)
\end{aligned}
$$

Intuition – an increase in A_t^T

Determinants of the RER in the long-run

First order conditions

$$
\begin{aligned}
r_t &= A_t^T f_K'(k_t^T) \\
r_t &= p_t A_t^N f_K'(k_t^N) \\
w_t &= A_t^T f_L'(k_t^T) \\
w_t &= p_t A_t^N f_L'(k_t^N)
\end{aligned}
$$

Intuition – an increase in A_t^T

- An increase in A^T drives factors of production towards tradables
- This raises wages in both sectors
- p_t or $f_L'(k_t^N)$ have to rise, but they cannot move in opposite directions because r_t is constant
- p_t rises $\implies$ RER rises

Determinants of the RER in the long-run – Algebra

Start from zero profit conditions

$$A_t^T f(k_t^T) = w_t + k_t^T r_t$$
$$p_t A_t^N f(k_t^N) = w_t + k_t^N r_t$$

Total derivatives $f(k_t^T) + A_t^T \dfrac{df(k_t^T)}{dk_t^T}\dfrac{dk_t^T}{dA_t^T} = \dfrac{dw_t}{dA_t^T} + \dfrac{dk_t^T}{dA_t^T} r_t$

$$A_t^N f(k_t^N)\frac{dp_t}{dA_t^T} + A_t^N \frac{df(k_t^N)}{dk_t^N}\frac{dk_t^N}{dA_t^T} p_t = \frac{dw_t}{dA_t^T} + \frac{dk_t^N}{dA_t^T} r_t$$

$$\implies \frac{dp_t}{dA_t^T} = \frac{f(k_t^T)}{A_t^N f(k_t^N)} \qquad \implies \frac{A_t^T}{p_t}\frac{dp_t}{dA_t^T} = \frac{A_t^T f(k_t^T)}{p_t A_t^N f(k_t^N)}$$

- Tradable productivity increases the RER (through a rise in the price of non-tradable goods p_t)
- Countries with higher tradable productivity should have higher RERs (Harrod-Balassa-Samuelson)

Gains from trade

Trade is good, right?

The benefits of trade

- Ricardo says: trade is always good!
- Trade in assets can allow for more risk sharing
- Capital can flow to its most productive uses
- Consumption increases $\implies$ higher welfare

Complications

- Heterogeneity
- Unequal gains from trade

Two period model

Budget constraints

$$c_0 = f(k_0) - \underbrace{(k_0 - a_0)r_0}_{N_0} + a_0(1-\delta) - a_1$$

$$c_1 = f(k_1) - (k_1 - a_1)r_1 + a_1(1-\delta)$$

Households

$$\max_{c_0, c_1, a_1} u(c_0) + \beta u(c_1)$$

- If the economy is closed, $N_t = 0$
- Production functions are the same across the world

Optimality conditions

$$\max_{c_0, c_1, k_1} u(c_0) + \beta u(c_1)$$

First order condition in the closed economy

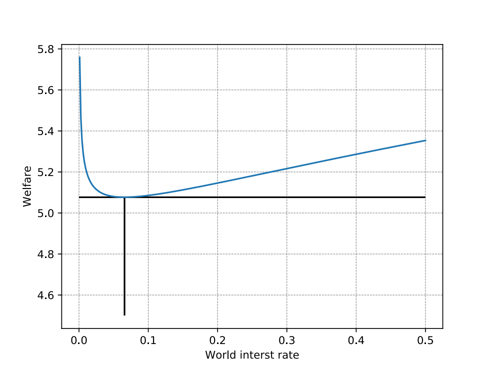
$$u'(c_0) = \beta(1 + f'(k_1) - \delta)u'(c_1)$$

First order condition in the small open economy

$$u'(c_0) = \beta(1 + r_t - \delta)u'(c_1)$$

- For the closed economy, the country-specific interest rate equals the country's marginal product of capital
- If the economy opens up, its savings pay the world interest rate (because capital is perfectly mobile)

Welfare



- Opening up always increases welfare
- If $r < f'(k_t)$, cheap capital flows into the economy, raising output
- If $r > f'(k_t)$, domestic capital moves abroad and earns higher return (home output falls)

Heterogeneity/Inequality

Capitalists and workers

- Some agents own the firms and the capital
- The rest just work and collect wages (no saving)

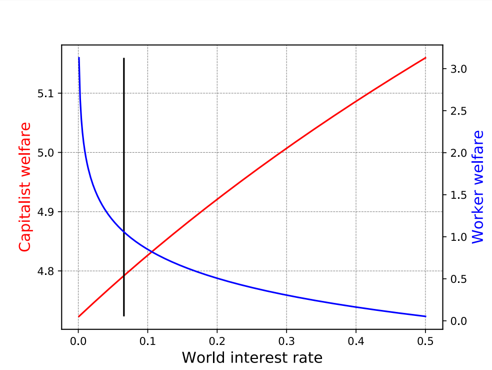
Capitalists (can save $\implies$ on their Euler equation)

$$
\begin{aligned}
c_0^K &= f(k_0) - \underbrace{(k_0 - a_0)r_0}_{N_0} + a_0(1-\delta) - w_0 - a_1 \\
c_1^K &= f(k_1) - (k_1 - a_1)r_1 + a_1(1-\delta) - w_1
\end{aligned}
$$

Workers (live hand-to-mouth)

$$
\begin{aligned}
c_0^K &= w_0 \\
c_1^K &= w_1
\end{aligned}
$$

Gains from trade with heterogeneity



- If $r < f'(k_t)$, cheap capital flows into the economy, raising labor productivity $\implies$ higher wages
- If $r > f'(k_t)$, domestic capital moves abroad $\implies$ home wages fall

$\implies$ Distributional aspects matter!

Heterogeneity matters for outcomes

Gains from trade

- Even small changes in the model lead to different conclusions
- Depending on the shares of workers/capitalists, opening an economy to the ROTW can have positive or negative effects on welfare
- Capital controls have the opposite effect
- **World interest rate shocks** also have heterogeneous effects

Beyond economics

- A social planner would redistribute resources such that trade is always beneficial
- How realistic this scenario is depends on the political environment (very much beyond the scope of this lecture)

World real interest rate

US 10-year interest rate

Two large open economies

2countries 2periods

Two countries

- Each country is large $\implies$ affects the interest rate
- Equivalent except for productivity in period 2
- Both enter the first period with the same capital stock k_0

Budget constaints

$$c_0^l = f(k_0^l) + k_0^l(1-\delta) - a_1^l$$
$$c_1^l = A^l f(k_1^l) - (k_1^l - a_1^l)r_1 + a_1^l(1-\delta)$$

World resource constraint

$$a_1^1 + a_1^2 = k_1^1 + k_1^2$$

Euler equations

$$u'(c_0^l) = \beta(1 + r_1 - \delta)u'(c_1^l)$$

How can this be solved?

Need to find:

- r_1 such that the world resource constaint is satisfied
- **given** r_1, a_1^l such that the Euler equations hold

Problem (similar for lots of problems in modern macro)

- There is no pencil-and-paper solution to this problem
- Once we know r_1, we know both k_1^l, a_0^l and k_0^l are given
- Even if we know r_1 (and log utility), a_1 is not easy to find:

$$f(k_1) = A_1^l k_t^\alpha \implies f'(k_t) = A_1^l \alpha k_1^{\alpha-1} = r_1 \implies k_1 = \left(\frac{r_1}{A_1^l \alpha}\right)^{\frac{1}{1-\alpha}}$$

$$\frac{1}{f(k_0^l) + k_0^l(1-\delta) - a_1^l} = \frac{\beta(1 + r_1 - \delta)}{A^l f(k_1^l) - (k_1^l - a_1^l) r_1 + a_1^l(1-\delta)}$$

Solution method

Turn to the computer

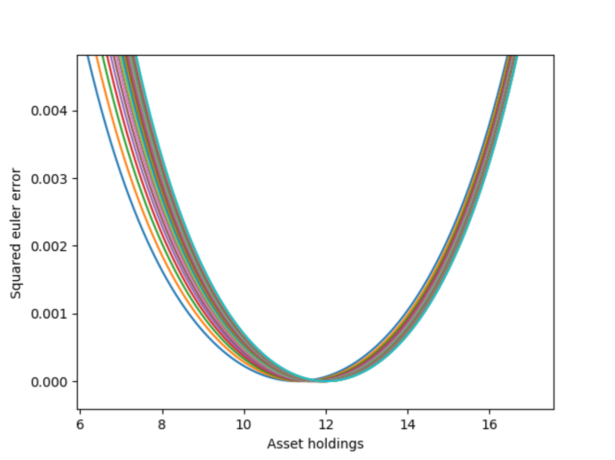
- On a computer, this is relatively easy to solve

Algorithm

- Guess a value of r_1
- Given r_1, guess values for a_1^1 and a_1^2
- Check if the Euler equations hold (if not, update the guess)
- Once the Euler equations hold, we have found a_1^1 and a_1^2
- Now check if the market clearing condition holds (if not, update the guess for r_1
- Once the market clearing condition holds, the problem is solved

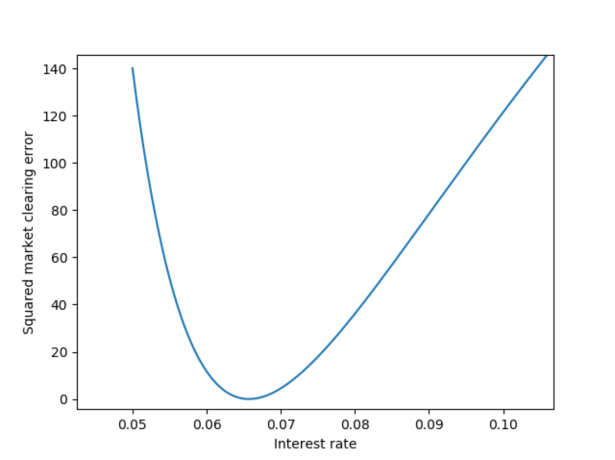
Solution method – pictures

Find asset holdings (for each country, given r_1)



Solution method – pictures

Find the correct interest rate



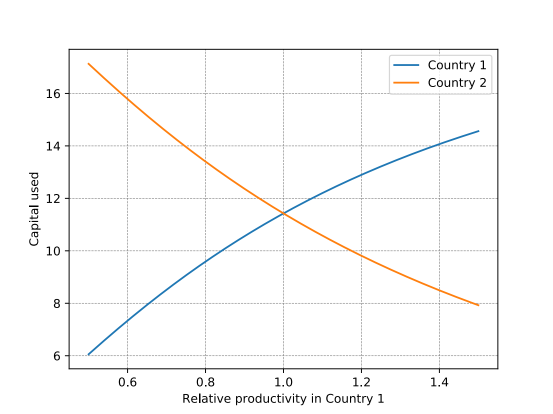
Different productivities

- If one country experiences a negative productivity shock, where does capital flow?

Different productivities

- If one country experiences a negative productivity shock, where does capital flow?

Capital

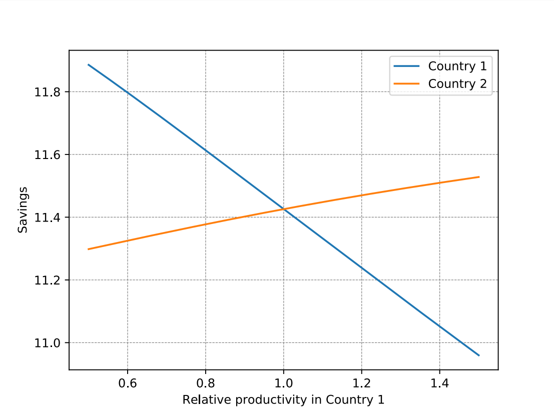


Global business cycles

Different productivities

- If one country experiences a negative productivity shock, where does capital flow?

Savings



Intuition

- If $A^1 \downarrow$, country 1 demands less capital
- Hence, world savings need to fall $\implies r_1 \downarrow$
- Total savings decrease, but only little
- Strong reallocation of capital

$\implies$ Capital responds strongly to interest rates, consumption does not

- Effect of savings stronger in country 1, since it is less productive in period 2

Total productivity increase (all countries)

- World interest rate rises due to more demand for capital

Conclusion

The small open economy

- Same setup as the closed economy but without internal market clearing
- Interest rates are given from abroad $\implies$ MPK externally determined
- Allows discussion of current account and trade balance

Implications

- External assets allow an economy access to insurance
- Current account and trade balance are procyclical
- The real exchange rate is determined by future permanent income in the short run
- In the long run, productivity differences matter

Gains from trade

- For the representative agent, trade is always good
- Inequality makes the conclusion more difficult

Recap lecture

John Kramer – University of Copenhagen

December 2022

The punchline from each lecture

Monopolistic competition

- Money can affect output (but there is a welfare loss)

Lucas model

- Expectations matter for the effects of monetary policy

Fischer and Taylor pricing

- Monetary policy can reduce the cost of rigid prices

Calvo pricing and the New Keynesian model

- Three equation model, fully forward looking

Optimal monetary policy

- Central banks may want to neutralize demand, but not supply shocks

Small open economy

- The interest rate is exogenously given and agents can save abroad

The secret ingredient is algebra!

Useful concepts

- Setting up first order conditions
- Lagrangian optimization
- Taking derivatives correctly
- Forward iterating of expectational difference equations
- Law of iterated expectations
- Taking infinite sums
- Logarithm rules (e.g., $\log(ab) = \log(a) + \log(b)$)

Advice

- Read the questions carefully
- Make sure to understand timing assumptions, etc.

Monopolistic competition

Monopolistic competition

Setup

- Large number of producers
- Each one has (a little bit of) market power
- Consumers have inelastic demand functions

Importance

- Firms are price setters (not price takers)
- Price changes don't make demand evaporate
- Allows the introduction of rigid prices

Derivation

Representative consumer

$$U = C^{\gamma}\left(\frac{M}{P}\right)^{1-\gamma} - \frac{1}{\phi}N^{\phi} \quad \text{with } 0 < \gamma < 1, \phi > 1$$

$$\text{s.t. } \sum_{i=1}^{m} P_i C_i + M = M_0 + WN + \sum_{i=1}^{m} \Pi_i$$

$$C = m^{\frac{1}{1-\theta}}\left(\sum_{i=1}^{m} C_i^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \text{ not } C = \left(\int_0^{\infty} C_i^{\frac{\theta-1}{\theta}} \quad di\right)^{\frac{\theta}{\theta-1}}$$

$$P = \left(\frac{1}{m}\sum_{i=1}^{m} P_i^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

- Assume there is a population of consumers of total size equal to one $\Longrightarrow$ representative consumer
- Special here: not an infinite number of firms, but m

Lagrangian

Solution methods

- Lecture 8: First solve for aggregate consumption, then solve for consumption of each variety i
- Today: Simple approach, just plug in
- More convoluted to solve, but straight forward to set up
- Both will give the same result

$$\max_{C_i, N, \frac{M}{P}} \mathcal{L} = \left[m^{\frac{1}{1-\theta}} \left(\sum_{i=1}^{m} C_i^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^{\gamma} \left(\frac{M}{P} \right)^{1-\gamma} - \frac{1}{\beta} N^{\beta} - \lambda \left(\sum_{i=1}^{m} \frac{P_i}{P} C_i + \frac{M}{P} - \frac{M_0}{P} - \frac{W}{P} N - \sum_{i=1}^{m} \frac{\Pi_i}{P} \right)$$

Optimality

First order conditions

$$\frac{\partial \mathcal{L}}{\partial C_i} = 0 \Rightarrow \left(\frac{M}{P}\right)^{1-\gamma} \frac{\gamma\theta}{\theta-1} m^{\frac{\gamma}{1-\theta}} \left(\sum_{i=1}^{m} C_i^{\frac{\theta-1}{\theta}}\right)^{\frac{\gamma\theta}{\theta-1}-1} \frac{\theta-1}{\theta} C_i^{-\frac{1}{\theta}} = \lambda \frac{P_i}{P}$$

$$\frac{\partial \mathcal{L}}{\partial (M/P)} = 0 \Rightarrow (1-\gamma)\left(\frac{M}{P}\right)^{-\gamma} C^{\gamma} = \lambda$$

$$\frac{\partial \mathcal{L}}{\partial N} = 0 \Rightarrow N^{\beta-1} = \lambda \frac{W}{P}$$

Optimality (combine conditions 1 and 2)

$$\frac{M}{PC}\frac{\gamma}{1-\gamma}\left(\frac{C}{mC_i}\right)^{\frac{1}{\theta}} = \frac{P_i}{P}$$

- Plugging this into the equation for the price index, we get $P = \frac{\gamma}{1-\gamma}\frac{M}{C}$

$\Longrightarrow$ cancel to get demand as function of price

Consumer demand

Consumer optimality

$$C_i = \left(\frac{P_i}{P}\right)^{-\theta} \frac{C}{m}$$

Very similar expression to infinite-variety case (Lecture 8)

$$C_i = \left(\frac{P_i}{P}\right)^{-\theta} C$$

Firm optimality (same as before)

$$\max_{\frac{P_i}{P}, N_i, Y_i} \Pi_i = P_i Y_i - W N_i$$

$$\text{s.t. } Y_i = C_i = \left(\frac{P_i}{P}\right)^{-\theta} \frac{C}{m},$$

$$Y_i = N_i^{\alpha}, \quad 0 < \alpha < 1$$

Firm FOCs, optimality and market clearing

Firm optimality

$$\frac{P_i}{P} = \left[\frac{\theta}{\theta - 1} \frac{1}{\alpha} \frac{W}{P} \left(\frac{C}{m} \right)^{\frac{1-\alpha}{\alpha}} \right]^{\frac{\alpha}{\alpha + \theta(1-\alpha)}}$$

Imposing homogeneity gives goods market equilibrium $(P_i = P)$

$$\frac{W}{P} = \frac{\theta - 1}{\theta} \alpha \left(\frac{C}{m} \right)^{\frac{\alpha - 1}{\alpha}}$$

- Real wage is less than marginal cost
- θ, the degree of demand elasticity, governs how much
- $\theta \to \infty$ implies perfect competition

Implications of monopolistic competition

Welfare loss

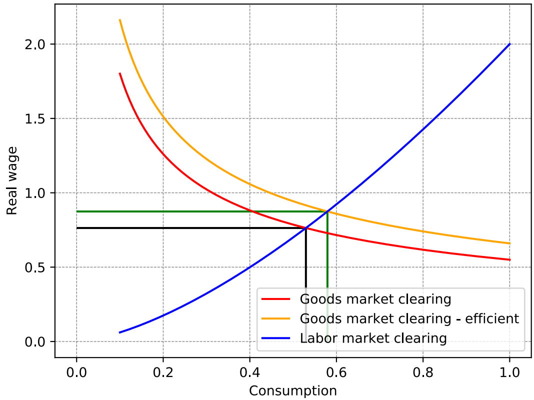
- Wages are too low $\implies$ output is too low
- Firms don't price in the demand effect they have on other producers
  - If one firm lowers its P_i, consumers become richer and demand more of everything
  - All firms make slightly more profit $\implies$ welfare could be improved
- A labor subsidy can eliminate the distortion

Rigid prices

- $C = \frac{\gamma}{1-\gamma}\frac{M}{P}$ implies that M can affect C if P is rigid
- If ϕ is high (el. of lab. supply is low), price adjustment costs need to be very high
- Welfare loss is larger if ϕ is low

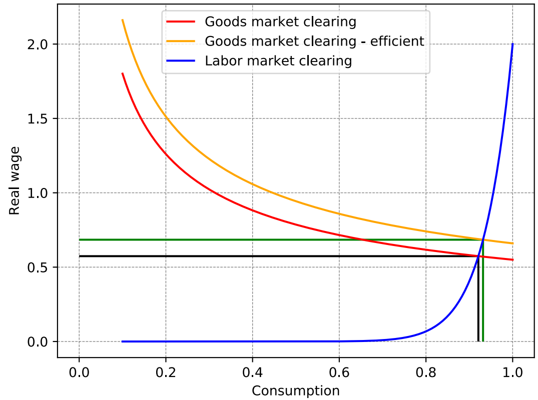
Welfare loss graphic

Low ϕ (high elasticity of labor supply)



Welfare loss graphic

High ϕ (low elasticity of labor supply)



Lucas model

Rational expectations

Usefulness

- Allows for the analysis of dynamic models
- Can think about feedback loop between policy and expectations

Formulation

$$\mathbb{E}_t[X_{t+1}] = \mathbb{E}_t[X_{t+1}|I_t]$$

- Agents base their expectations on an information set I_t
- They know the processes underlying the economy
- They don't make systematic forecast errors

  Expectational difference equations can be solved fowards

What does this mean for the Phillips Curve?

Lucas' island model

- Perfect competition, everyone is a price taker
- Dynamic model
- Households cannot see what is happening on other islands (info frictions)
- Producers need to disentangle movements in P_i from movements in P

Movements in P_i vs P

- If P_i rises but P remains constant, want to produce more
- If P_i and P rise, produce as before
- Decision links output to inflation (Phillips Curve)

Equilibrium

General equilibrium in the Lucas model (for derivation see lecture 9)

$$p = \frac{1}{1+b}(m - \mathbb{E}[m]) + \mathbb{E}[m]$$
$$y = \frac{b}{1+b}(m - \mathbb{E}[m])$$

Implications

- Only unexpected money growth matters for output
- Expected money growth $\mathbb{E}[m]$ raises prices, but not output
- Central banks (CB) cannot generate demand by raising the money supply
- CB can only affect output if they have an informational advantage or can make decisions after expectations are locked in

Phillips Curve

Feedback loop between inflation and output

$$\pi_t = \mathbb{E}_{t-1}[\pi_t] + \frac{1}{b} y_t$$

Intuition

- Expected inflation drives current inflation
- Higher output leads to higher prices (depending on b)
- High variance of firm demand shock z increases b
- Variance of money growth m (aggregate demand shock) lowers b

Conclusion

- An empirical relationship cannot be taken for granted if agents' decisions change when new information arrives

Pricing frictions

Overview

Non-microfounded pricing frictions

- Fischer pricing: set unchangeable price schedule
- Taylor pricing: set unchangeable prices
- Calvo pricing: Fixed reset probability

Implications

- No need for elaborate justification (like informational frictions)
- Reasonable approximation of the real world
- Rigid prices imply that demand shocks affect output

Set up the problem (Exam Feb 2022)

Representative household

- Assume a continuum of identical households, whose total number is normalized to one

$$U_i = \mathbf{C}_i - \frac{1}{\phi} L_i^{\phi}$$

Budget constraint and demand function

$$\mathbf{C}_i = \frac{P_i}{P} Y_i \quad \& \quad Y_i = \left(\frac{P_i}{P}\right)^{-\theta} Y$$

- **C** is a consumption basket
- Households use their labor L to produce output according to $Y_i = L_i^{\alpha}$
- P is the aggregate price level, P_i is the price of the household's variety i

a) Utility maximization + F.O.C.

Problem

$$\max_{P_i} U_i = \frac{P_i}{P}\left(\frac{P_i}{P}\right)^{-\theta} Y - \frac{1}{\phi}\left[\left(\left(\frac{P_i}{P}\right)^{-\theta} Y\right)^{\frac{1}{\alpha}}\right]^{\phi}$$

First order condition (after some tedious algebra)

$$\frac{P_i}{P} = \frac{\theta}{\theta - 1}\frac{1}{\alpha} Y_i^{\frac{\phi}{\alpha}-1}$$

Take-aways

- For alternative derivation see lecture 10 (same result, obviously)
- Monopolistic competition makes prices too high, relative to marginal cost
- $\frac{1}{\alpha} Y_i^{\frac{\phi}{\alpha}-1}$ is marginal cost of output
- More competition ($\theta \uparrow$) lowers the markup
- Lower α raises prices (decreasing returns)

b) Compute the desired price under flexible prices

Taking logs

$$p_i^* - p = \log\left(\frac{\theta}{(\theta-1)\alpha}\right) + \frac{\phi-\alpha}{\alpha} y_i$$

Aggregate

$$p^* - p = \log\left(\frac{\theta}{(\theta-1)\alpha}\right) + \frac{\phi-\alpha}{\alpha} y$$

- Use the market clearing condition in logs $m - p = y$

$$\begin{aligned} p^* - p &= \log\left(\frac{\theta}{(\theta-1)\alpha}\right) + \frac{\phi-\alpha}{\alpha}(m-p) \\ \implies p^* &= \log\left(\frac{\theta}{(\theta-1)\alpha}\right) + \frac{\phi-\alpha}{\alpha} m + \left(1 - \frac{\phi-\alpha}{\alpha}\right) p \end{aligned}$$

b) Compute the desired price under flexible prices (cont.)

$$p^* = \underbrace{\log\left(\frac{\theta}{(\theta-1)\alpha}\right)}_{\mathcal{M}} + \underbrace{\frac{\phi-\alpha}{\alpha}}_{v} m + \left(1 - \frac{\phi-\alpha}{\alpha}\right) p$$

Interpretation

- This equation pins down what prices will be set to
- The first term is the markup
- Under flexible prices, $p^* = p$, under rigid prices, the two may differ

Pricing frictions

- Fundamentally, all these models result in this optimality condition
- The next step is to introduce some form of price rigidity

c) (modified) Prices under Fischer contracts

- Half of agents set prices each period
- p_t^{-1} represents the price for period t set in period $t-1$

Price level (suppress the markup)

$$p_t = \frac{1}{2}\left(p_t^{-1} + p_t^{-2}\right)$$
$$p_t^* = v m_t + (1-v) p_t$$

Optimal price setting in expectation

$$p_t^{-1} = \mathbb{E}_{t-1}[p_t^*] = v\mathbb{E}_{t-1}[m_t] + (1-v)\frac{1}{2}\left(p_t^{-1} + p_t^{-2}\right)$$
$$p_t^{-2} = \mathbb{E}_{t-2}[p_t^*] = v\mathbb{E}_{t-2}[m_t] + (1-v)\frac{1}{2}\left(\mathbb{E}_{t-2}[p_t^{-1}] + p_t^{-2}\right)$$

Equilibrium under Fischer contracts

Optimal price setting in expectation

$$p_t^{-2} = \frac{2v}{1+v}\mathbb{E}_{t-2}[m_t] + \frac{1-v}{1+v}\mathbb{E}_{t-2}[p_t^{-1}]$$
$$p_t^{-1} = \mathbb{E}_{t-2}[m_t] + \frac{2v}{1+v}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right)$$

- Use law of iterated expectations: $\mathbb{E}_{t-1}[x_t] = \mathbb{E}_{t-2}[\mathbb{E}_{t-1}[x_t]]$

Once price setting is solved, get price level and output

$$\begin{aligned} p_t &= \frac{1}{2}\left(p_t^{-1} + p_t^{-2}\right) \\ &= \mathbb{E}_{t-1}[m_t] + \frac{1}{1+v}\left(\mathbb{E}_{t-2}[m_t] - \mathbb{E}_{t-1}[m_t]\right) \\ y_t &= m_t - \mathbb{E}_{t-1}[m_t] - \frac{1}{1+v}\left(\mathbb{E}_{t-2}[m_t] - \mathbb{E}_{t-1}[m_t]\right) \end{aligned}$$

Rigid prices – Interpretation

General equilibrium (not a typo, just rewritten)

$$p_t = \mathbb{E}_{t-2}[m_t] + \frac{v}{1+v}\left(\mathbb{E}_{t-1}[m_t] - \mathbb{E}_{t-2}[m_t]\right)$$
$$y_t = m_t - \mathbb{E}_{t-1}[m_t] - \frac{1}{1+v}\left(\mathbb{E}_{t-2}[m_t] - \mathbb{E}_{t-1}[m_t]\right)$$

- Unanticipated shocks to m_t have real effects (as always)
- Anticipated shocks to m_t affect those who cannot reset their prices in time
- Importance of rigidity is governed by $v = \frac{\phi-\alpha}{\alpha}$

Interpretation

- $\phi \uparrow$ implies less elastic labor supply $\implies$ price changes matter less
- Similarly, $\alpha \downarrow$ increases marginal costs $\implies$ changing output (when prices are wrong) is more expensive

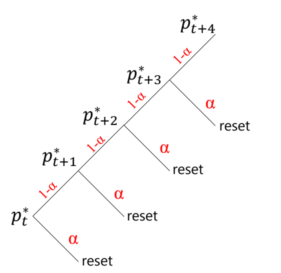
Calvo pricing and the New Keynesian model

Calvo pricing I

Constant reset probability

- Every firm resets its price with constant probability
- Price level is $p_t = \gamma x_t + (1-\gamma)p_{t-1}$
- Optimal price (no markup) under flexible prices is $p_t^* = v m_t + (1-v)p_t$

Optimal reset price



Calvo pricing II

Optimal reset price

$$
\begin{aligned}
x_t &= \sum_{j=0}^{\infty} \frac{\beta^j (1-\gamma)^j \mathbb{E}[p^*_{t+j}]}{\sum_{j=0}^{\infty} \beta^j (1-\gamma)^j} \\
&= (1-\beta(1-\gamma)) \sum_{j=0}^{\infty} \beta^j (1-\gamma)^j \mathbb{E}[p^*_{t+j}] \\
&= (1-\beta(1-\gamma)) p^*_t + \beta(1-\gamma) \mathbb{E}_t[x_{t+1}]
\end{aligned}
$$

Intuition

- The current optimal reset price depends on the expected ideal price over the foreseeable future
- A higher γ gives less weight to the distant future
- A higher β gives more weight to the distant future

Inflation and Phillips Curve

Inflation

$$\pi_t = \underbrace{\frac{\gamma(1-\beta(1-\gamma))v}{1-\alpha}}_{\kappa} y_t + \beta \mathbb{E}_t \left[\pi_{t+1}\right]$$

Interpretation

- The Phillips curve is steeper when v is large and/or γ is close to 1
- Inflation if pinned down by future inflation (i.e., future output)

Inflation as a function of expected output (Exam June 2021)

$$\pi_t = \kappa \sum_{s=0}^{\infty} \beta^s y_{t+s}$$

$$\mathrm{Var}(\pi_t) = \kappa^2 \sum_{s=0}^{\infty} \beta^{2s} \mathrm{Var}(y_{t+s}) = \kappa^2 \frac{\sigma_y}{1-\beta^2}$$

The New Keynesian model

Equations

$$
\begin{aligned}
\pi_t &= \kappa y_t + \beta \mathbb{E}_t \left[\pi_{t+1} \right] \\
y_t &= \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma} \left(E_t[r_t] - \rho \right) \\
r_t &= \rho + \phi_y \mathbb{E}_t[y_{t+1}] + \phi_\pi \mathbb{E}_t[\pi_{t+1}]
\end{aligned}
$$

Important takeaways

- Inflation is pinned down by expectations about future output
- Output is dictated by expected changes in the real interest rate

$\Longrightarrow$ Expectations determine the state of the economy

- Effect of single period shocks can be solved for by hand
- Heterogeneity alters the equations slightly

Shocks

Output

$$y_t = \left(1 - \frac{\phi_y}{\sigma}\right)\mathbb{E}_t[y_{t+1}] - \frac{\phi_\pi}{\sigma}E[\pi_{t+1}] - \frac{1}{\sigma}u_{MP} + u_{IS}$$

Inflation

$$\pi_t = \left(1 - \frac{\phi_y}{\sigma}\right)\kappa\mathbb{E}_t[y_{t+1}] + \left(\beta - \frac{\phi_\pi}{\sigma}\right)E[\pi_{t+1}] - \kappa\left(\frac{1}{\sigma}u_{MP} - u_{IS}\right) + u_\pi$$

- Demand and monetary policy shocks can cancel each other
    - Monetary policy makes agents substitute intertemporally

$\implies$ if demand shock today, raise rates to push some of the demand to tomorrow

- Cost push shocks only raise prices
    - Nominal wages rise immediately, hence real wages stay constant

Optimal monetary policy

Model

Phillips Curve

$$\pi_t = m_t + \underbrace{v_t}_{\text{Demand shock}} + \underbrace{\mu_t}_{\text{MP shock}}$$

Demand equation

$$x_t = \underbrace{\theta_t}_{\text{natural rate of output}} + (\pi_t - \pi_t^e) - \underbrace{\varepsilon_t}_{\text{Supply shock}}$$

Timing assumptions

1. Announcement of monetary rule (credible or not)
2. Everyone observes the natural level of output θ
3. Expectations π^e are formed, given the information about θ
4. Everyone observes v and ε
5. The central bank decides the money supply m (advantage)
6. μ is realized, pinning down output x and inflation π

Further ingredients

Society's loss function

$$\mathcal{L} = \frac{1}{2}\left[a(\pi - \overline{\pi})^2 + \lambda(x - \overline{x})^2\right]$$

Monetary policy rule

$$m = \varphi + \varphi_\theta \theta + \varphi_v v + \varphi_\varepsilon \varepsilon$$

Implications

- The central bank has an informational advantage, it can move after agents have committed to their expectations
- Agents and the central bank play a game, with agents at first mover disadvantage
- Society's loss depends on the ability of the CB to commit or on the ability of the agents to punish deviations

Solution – Backwards induction

Commitment

- Society wants to set the parameters in the monetary policy rule to minimize the **expected** loss $\implies$ society moves first
- Solve the agents' problem given the loss function
- Optimize each parameter in the loss function

Discretion

- The central bank can do whatever it wants $\implies$ agents move first
- Solve the central bank's problem given the agents' expectations
- Then solve backwards (plug CB's actions into the agents' policy functions)

Reputation

- The central bank can do whatever it wants **but** the parties play a repeated game
- Single period decision exactly like discretion, but have to take into account the future costs

Result – Commitment

Optimal rule

$$m_t = \overline{\pi} - v_t + \frac{\lambda}{a + \lambda}\varepsilon$$

Equilibrium inflation

$$\pi^C = \overline{\pi} + \frac{\lambda}{a + \lambda}\varepsilon + \mu$$

Equilibrium output

$$x^C = \theta - \frac{a}{a + \lambda}\varepsilon + \mu$$

- Inflation is anchored at the desired level
- Output is anchored at its natural level
- The CB does not react to θ since it is priced into agents' expectations
- Responsiveness to supply shocks depends on society's preferences

Result – Discretion

Output

$$x^D = \theta - \frac{a}{a+\lambda}\varepsilon$$

Inflation

$$\pi^D = \overline{\pi} + \frac{\lambda}{a}(\overline{x} - \theta) + \frac{\lambda}{a+\lambda}\varepsilon$$

- Output is the same as under commitment: agents know the CB's objective function and price it into their expectations
- Inflation is higher and more volatile
- This is the optimal solution for the CB. If θ is too low, agents know the CB will increase m, so they raise their prices.
- The CB now has no choice but to actually print the money, otherwise prices are too high and output is too low

Result – Reputation

Long-run loss function

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^j \mathcal{L}(\pi_{t+j}, x_{t+j})$$

Punishment

- Some rule by which expectations change when the CB deviates
- Possible examples: tit-for-tat, assured destruction, limited-time punishment

Optimization

- Attractiveness of deviation depends on the **present value** of cost
- If cost is in every future period, or discount factor is high, deviation is painful

Open economy macroeconomics

Important ingredients

Neoclassical economy

- No money
- Prices are perfectly flexible
- Perfect competition

Openness

- Markets for goods and savings do not need to clear within the country

$\Longrightarrow$ Ship (invest) excess production (assets) abroad

Interest rates

- In a small open economy, interest rates are given from abroad (not pinned down by domestic savings behavior)
- Savings and investment decisions do not affect the level of the interest rate (everyone is a price taker)

Optimality

Consumers

$$u'(c_t) = \beta(1 + r_{t+1} - \delta)\mathbb{E}[u'(c_{t+1})]$$

Firms

$$f_K(k_t, 1) = r_t$$
$$f_L(k_t, 1) = w_t$$

Interaction with the ROTW (all 0 in closed economy)

$$tb_t = \underbrace{f(k_t, 1)}_{y_t} - c_t - \underbrace{(k_{t+1} - (1 - \delta)k_t)}_{i_t}$$
$$N_t = a_t - k_t$$
$$ca_t = \underbrace{tb_t}_{\text{trade}} + \underbrace{r_t N_t}_{\text{interest}} - \underbrace{\delta N_t}_{\text{depreciation}}$$

Equilibrium in an endowment economy

Setup

- Home country has no capital but can save abroad

$$c = \rho a_t R_t + \underbrace{\rho \sum_{s=0}^{\infty} \frac{w_{t+s}}{R^s}}_{\text{perm. inc.}} \quad (\text{ in closed economy: } c_t = \omega_t)$$

Use foreign assets to stabilize consumption

$$ca_t = N_{t+1} - N_t = \omega_t - \widetilde{\omega}_t$$

$$tb_t = \omega_t - \rho a_t R_t - \rho \sum_{s=0}^{\infty} \frac{w_{t+s}}{R^s}$$

- Opening up the economy allows agents to access savings instruments
- Consumption can be smoothed without the need for own productive capital

Real exchange rates (RER)

Setup

- Real exchange rate is one if there is only one good

$\implies$ Need at least two goods

- Here: tradable & non-tradable

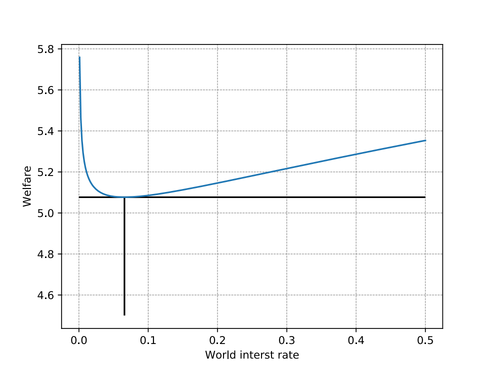
Equilibrium in an endowment economy

- Non-tradable consumption is more expensive in countries that are rich in tradables or foreign assets (higher RER)
- Non-tradable good is the limiting factor $\implies$ more overall consumption demand makes it more expensive

Equilibrium in a production economy

- In the long-run, factors of production adjust to most productive use
- Intuition similar: countries that are productive at making tradables have higher RERs
- Higher productivity drives up wages in all sectors $\implies$ all goods more expensive

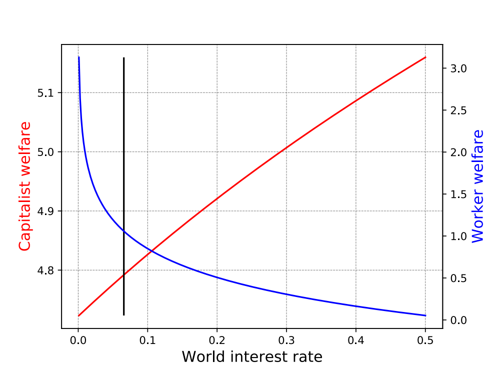
Gains from trade – representative agent



Opening up the economy is always beneficial

- If $r < f'(k_t)$, cheap capital flows into the economy, raising output
- If $r > f'(k_t)$, domestic capital moves abroad and earns higher return (home output falls)

Unequal gains from trade



Gains depend on distribution

- If $r < f'(k_t)$, cheap capital flows into the economy, wages rise
- If $r > f'(k_t)$, domestic capital moves abroad and earns higher return, capitalists gain, workers lose