Advanced Microeconometrics - Project 3

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1   Introduction

The use of force by the American police is a continuous and highly relevant problem especially concerning racial biases. According to Fryer (2019) black and hispanic people are more likely to experience non-lethal use of force than white people. After his publications, Fryer's methodology was heavily criticized by Knox et al. (2020) and Durlauf and Heckman (2020). They both find limitations in the use of administrative records. In this paper, we estimate the change in probability of police using force on black and hispanic individuals using a Logit and Probit model with the some of the data used by Fryer. Our estimates are positive, but we do not report any statistical significant results.

2   Data

We use data from the Police Public Contact Survey (PPCS) which documents police–civilian interactions from the perspective of civilians. The 2011 sample contains 3,799 observations and 19 variables. The dataset includes background variables on the civilian (variables starting with s), incident information such as whether the officer used force and the race of the officer involved (starting with omaj).

Only 19 encounters involve police use of force, our outcome variable. This could raise concerns about rare events and the potential for quasi-separation issues, which we will discuss more in section 3.2. The raw data does indicate possible racial disparity wrt. use of force. While black people are 11 pct. of the group where there is no use of force, they are 16 pct. of the group who experience the use of force. Even starker are the differences in share of hispanics that make up 10 pct. of the no use of force group, but 32 pct. of the use of force group. This could indicate a racial disparity in police of force.

3   Econometric theory and method

3.1   Models

We model the relationship between police use of force and racial profiles as a binary response model, where use of force is a binary outcome, $y_i \in (0, 1)$. That is, $y_i = 1$ indicates that a police officer used force in encounter i, and $y_i = 0$ if not. The probability of using force is given as follows on the right hand side:

$$p(\mathbf{x}_i) := P(y_i = 1 | \mathbf{x}_i = x) = G(\mathbf{x}_i \boldsymbol{\beta_0}) \tag{1}$$

where $\mathbf{x}_i$ is a $(1 \times K)$ vector of regressors, $x_{i,j}$, for each encounter i and variable j. The vector includes our variables of interest, (sblack, shisp, sother), a set of controls, given in Table 1, and a constant. $\boldsymbol{\beta_0}$ is a $(K \times 1)$ vector of 'true parameter' coefficients. On the left hand side, the function $G(\mathbf{x}\boldsymbol{\beta_0}) \in (0, 1)$ is a CDF, indicating that we use a probability index model rather than a linear probability model. In a probability index model, each regressor $x_{i,j} \in \mathbf{x}_i$ affects the response probability through the scalar $\mathbf{x}\boldsymbol{\beta_0}$. We choose to look at two specifications for the CDF; a normal distribution and a logistic distribution.

Assuming a normal distribution results in a **probit model**, with the following CDF:

$$G(\mathbf{x}_i\boldsymbol{\beta_0}) = \Phi(\mathbf{x}_i\boldsymbol{\beta_0}) := \int_{-\infty}^{\mathbf{x}_i\boldsymbol{\beta_0}} \frac{1}{\sqrt{2\pi}} e^{-t^2} dt \qquad \mathbf{x}_i\boldsymbol{\beta_0}, \in \mathbb{R} \tag{2}$$

Assuming a logistic distribution yields a **logit model**, with the following CDF:

$$G(\mathbf{x}_i\boldsymbol{\beta_0}) = \Lambda(\mathbf{x}_i\boldsymbol{\beta_0}) := \frac{1}{1 + e^{-\mathbf{x}_i\boldsymbol{\beta_0}}}, \qquad \mathbf{x}_i\boldsymbol{\beta_0} \in \mathbb{R} \tag{3}$$

As mentioned above, we do not consider the linear probability model. This is because the prediction of that a model is not restricted to the unit interval, the variance can be negative, and it assumes constant partial effects.

3.2 Maximum likelihood estimation

We use maximum likelihood estimation as both probit and logit are non-linear models. We assume that the observations are i.i.d. and Bernoulli distributed when we condition on $\mathbf{x}_i$ with success probability $G(\mathbf{x}_i\boldsymbol{\beta_0})$. The conditional density is therefore as shown in eq. 4.

$$f(y_i|\mathbf{x}_i, \boldsymbol{\beta}) = G(\mathbf{x}_i\boldsymbol{\beta_0})_i^y [1 - G(\mathbf{x}_i\boldsymbol{\beta_0})]^{1-y_i}, \quad \boldsymbol{\beta}_0 \in \mathcal{B} \subset \mathbb{R}^K \tag{4}$$

The maximum likelihood estimator is obtained by solving

$$\hat{\boldsymbol{\beta}}^{MLE} \in \arg\max \left\{ \frac{1}{N} \sum_{i=1}^{N} \ln f(y_i|\mathbf{x_i}, \boldsymbol{\beta}) \right\} \tag{5}$$

Because no closed-form solution exists, we find the estimator numerically. We rely on the BFGS quasi-Newton method, which uses numerical gradients and an iterative approximation of the Hessian matrix to locate the maximum of the sample log-likelihood.

Consistency of the MLE in both models relies solely on identification, as the log-likelihood functions for probit and logit are globally log-concave and $\mathcal{B} \subset \mathbb{R}^K$ is

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convex. Identification ensures that the estimator converges to the unique maximizer of the population log-likelihood. Identification requires a normalization of scale for the error term, as the latent model is unchanged by proportional rescaling, preventing point identification. In the probit model, we assume a standard normal distribution eq. 6, while we assume a standard logistic distribution in the logit model eq. 7.

$$e_i \sim N(0, \sigma_o^2), \quad \sigma_o := 1 \tag{6}$$

$$e_i \sim \text{logistic}(0, s), \quad s := 1, \text{std}(e_i) = \frac{\pi}{\sqrt{3}} \approx 1.8 \tag{7}$$

Additionally, covariate patterns must not exhibit (quasi-)separation, meaning that no linear combination of regressors may perfectly classify the outcome. If separation occurs, the likelihood becomes unbounded and finite-valued parameter estimates fail to exist. As we only have 19 instances of police use of force in our data ($y_i = 1$), this could be a potential issue in our model.

Assuming the standard set of regularity conditions for maximum likelihood estimation: i) the true parameter is an interior solution, ii) that the estimator is consistent, and iii) β is twice differentiable on the log-likelihood function, the MLE is asymptotically normally distributed:

$$\sqrt{N} \, (\hat{\beta} - \beta_0) \xrightarrow{d} N\left(0, \mathbf{V}\right), \tag{8}$$

where $\mathbf{V} = A^{-1}BA^{-1}$ denotes the robust (sandwich) variance matrix, with A representing the negative average Hessian of the log-likelihood and B denoting the outer product of the score. Under conditions for maximum likelihood, there are three candidates for A where the information matrix equality should hold for all if the model is well-specified. We use the sandwich formula $\hat{\mathbf{V}} = \hat{A}^{-1}\hat{B}\hat{A}^{-1}$ as we view this as the most robust of the three candidates.

3.3   Partial effects

In non-linear binary response models such as the probit and logit, the coefficient vector β_0 only determine the sign of the effect. It does not say anything about the the magnitude of changes in the response probability. Those coefficients therefore does not have any meaningful interpretations, and we therefore rely on partial effects, which quantify how the predicted probability of police use of force changes when one regressor is varied.

For a binary response model, the partial effect of regressor $x_{i,j}$ equals the change in $p(x)$ when only $x_{i,j}$ varies, holding all other elements of vector $\mathbf{x_i}$ fixed. For a continuous regressor, it equals the derivative $\frac{\partial p(x)}{\partial x_j} = g(x\beta_0)\frac{\beta_{0,j}}{\sigma}$ and for a binary regressor, it equals a discrete difference, as given in eq. 9 below.

$$
\text{PE}_j(\mathbf{x}_i) = \begin{cases} g(\mathbf{x}_i\boldsymbol{\beta}_0)\beta_{0,j}, & \text{if} \quad x_{i,j} \quad \text{is continuous} \\ G(\mathbf{x}_{i,-j}\boldsymbol{\beta}_{0,-j} + \beta_{0,j}) - G(\mathbf{x}_{i,-j}\boldsymbol{\beta}_{0,-j}), & \text{if} \quad x_{i,j} \quad \text{is binary} \end{cases} \tag{9}
$$

where $g(\cdot) = G'(\cdot)$ and $\mathbf{x}_{-j}$ is the regressor vector excluding variable j.

We look at the average partial effect, $APE_j = \mathbb{E}[PE_j(\mathbf{x}_i)]$, This will provide the average effect for each variable j on the response probability, e.g. how the probability of police use of force changes if the encounter was with a black individual on average, holding all other variables fixed.

We also look at the partial effect at the average, $PEA_j = PE_j(\bar{\mathbf{x}})$, which according to Wooldridge (2010), is often similar to the APE. For the probit and logit model, partial effects are identified through the identification of β_0 to scale.

To estimate the partial effects, we apply the analogy principle by replacing the theoretical β coefficients from eq. 9 with the estimates $\hat{\beta}^{MLE}$ to find $\widehat{APE_j} = \frac{1}{N}\sum_{i=1}^{N}\widehat{PE}_j(\mathbf{x}_i)$. PEA is estimated through the sample mean $\bar{\mathbf{x}} = \frac{1}{N}\sum_i \mathbf{x}_i$.

For inference of the partial effects, we rely on the delta method. For each encounter

i, we can write the partial effect as $PE_j(\mathbf{x}_i) = h_j(\beta_0, \mathbf{x}_i)$, where $h_j(\cdot)$ follows directly from the expressions in equation 9. The delta method quantifies how the uncertainty of our estimator $\hat{\beta}^{MLE}$ is transferred through such a nonlinear function like the PE's. Since $h_j(\beta, \mathbf{x}_i)$ is a nonlinear function of β, the variation in $\hat{\beta}$ does not translate linearly into variation of $\widehat{PE}_j(\mathbf{x}_i)$. We estimate the variance through a first-order Taylor expansion.

The estimated partial effect, as mentioned above, is found by plugging in the MLE for β: $\widehat{PE}_j(\mathbf{x}_i) = h_j(\hat{\beta}, \mathbf{x}_i)$. Following the inference of the MLE, we have: $\sqrt{N}(\hat{\beta} - \beta_0) \xrightarrow{d} N(0, V)$, as given in equation 8. A first-order Taylor expansion of $h_j(\hat{\beta}, \mathbf{x}_i)$ around β_0 gives us the following expression for the asymptotic variance of the partial effects:

$$\sqrt{N}\big(h_j(\hat{\beta}, \mathbf{x}_i) - h_j(\beta_0, \mathbf{x}_i)\big) \xrightarrow{d} N\big(0, \nabla h_j(\beta_0, \mathbf{x}_i)\mathbf{V}\nabla h_j(\beta_0, \mathbf{x}_i)'\big) \tag{10}$$

where $\nabla h_j(\beta_0, \mathbf{x}_i)$ is the gradient of h_j with respect to β.

Applying the analogy principle, the delta method yields the following estimated variance for the average partial effect:

$$AVar[\widehat{APE}_j(\mathbf{x}_i)] = \left(\frac{1}{N}\sum_{i=1}^{N}\nabla h_j(\hat{\beta}, \mathbf{x}_i)\right)' \widehat{\mathbf{V}} \left(\frac{1}{N}\sum_{i=1}^{N}\nabla h_j(\hat{\beta}, \mathbf{x}_i)\right) \tag{11}$$

The delta method is also applied to find the asymptotic variance of the PEA. Where the APE averages over all $\mathbf{x}_i$, the function $\widehat{PEA}_j = h_j(\hat{\beta}, \bar{\mathbf{x}})$ is used as the 'bread part' of the sandwich variance matrix.

4   Empirical Analysis

We estimate the partial effects APE and PEA in both of our models using MLE. We test the null-hypotheses $H_0 : PE_j = 0$ against the alternative hypothesis $H_A : PE_j \neq 0$. We use the asymptotic variance to test the statistical significance of our results. We estimate the effects both with and without using the controls from table

1. We report all results in Table 2.

In Table 2, we see that there are only neglible differences in the standard errors of the estimates. For the estimates of the APE that excludes control variables, we find the probability of force increases 0.5% if the civillian is black, but the estimate is not statistically significant. The probability increases to 0.13% for Hispanic individuals, but only at a 10% significance level. It shall be noted that including controls renders all coefficients insignificant even at a 10% significance level. For the PEA estimates, we see slight changes in the coefficients, but the overall results and conclusions are the same as for the APE.

5   Discussion and conclusion

We do not find any significant effects of racial disparities in experiencing police force. This conclusion supports the critique of Fryer, yet there are several limitations of the PPCS data affect the interpretation of our findings.

The extremely small number of force incidents (19 cases) raises the risk of quasi-separation and limits the variation within key regressors, particularly racial subgroups, making the probit and logit estimates potentially unstable. The PPCS also relies on civilian self-reports and does not distinguish between levels of force, introducing measurement error and likely reducing the estimated effects. There is also a risk of omitted variable bias, as important determinants of use of force - such as officer experience, neighborhood risk, and policing intensity - are unobserved, limiting any causal interpretation of the racial coefficients. In addition, the PPCS excludes incarcerated individuals, which restricts external validity.

Finally, although the PPCS avoids the non-random selection issues documented in administrative police records used by Fryer, Knox et al. (2020), inference remains fragile in rare-event settings even when using robust (sandwich) standard errors. Alternative approaches such as penalized likelihood or rare-events corrections could be

more appropriate. We could also have used the White's information matrix misspecification test, to test whether either probit, logit or both are well-specified, as this could help us in deciding whether to trust our results and which model provides the best estimate. Overall, while our results indicate nonsignificant racial disparities in the likelihood of experiencing force, stronger conclusions would require richer and more comprehensive data.

6   Appendix

Table 1: Summary statistics for variables based on use of force

Variable	Count (no force)	Share (no force)	Count (force)	Share (force)
swhite	2799	0.74	9	0.47
sblack	417	0.11	3	0.16
shisp	380	0.10	6	0.32
sother	184	0.05	1	0.05
smale	1997	0.53	15	0.79
sempl	2633	0.70	9	0.47
sincome_low	1099	0.29	6	0.32
sincome_medium	957	0.25	6	0.32
sincome_high	1724	0.46	7	0.37
spop_small	2913	0.77	8	0.42
spop_medium	545	0.14	7	0.37
spop_large	151	0.04	1	0.05
spop_huge	171	0.05	3	0.16
inctype_traffic	3628	0.96	13	0.68
daytime	2523	0.67	9	0.47
sbehavior	237	0.06	10	0.53
omajwhite	3415	0.90	18	0.95
omajblack	231	0.06	0	0.00
omajhisp	90	0.02	1	0.05
omajother	44	0.01	0	0.00

Source: Police Public Contact Survey

Table 2: Estimation results

Variables	Average Partial Effects				Partial Effects at the Average			
	Probit NC	Logit NC	Probit C	Logit C	Probit NC	Logit NC	Probit C	Logit C
sblack	0.005	0.005	0.002	0.001	0.004	0.004	0.001	0.000
	(0.005)	(0.006)	(0.005)	(0.004)	(0.005)	(0.005)	(0.002)	(0.001)
shisp	0.013*	0.013*	0.005	0.004	0.013*	0.013*	0.002	0.001
	(0.007)	(0.007)	(0.005)	(0.004)	(0.007)	(0.007)	(0.003)	(0.002)
sother	0.003	0.003	0.001	-0.001	0.003	0.003	0.000	-0.000
	(0.007)	(0.008)	(0.008)	(0.006)	(0.007)	(0.007)	(0.002)	(0.002)
smale			0.002**	0.002*			0.001	0.001
			(0.001)	(0.001)			(0.001)	(0.001)
sempl			-0.002	-0.002			-0.000	-0.000
			(0.001)	(0.001)			(0.000)	(0.000)
sincome_medium			0.001	0.001			0.000	0.000
			(0.004)	(0.003)			(0.001)	(0.001)
sincome_high			0.002	0.001			0.000	0.000
			(0.003)	(0.003)			(0.001)	(0.001)
spop_medium			0.007*	0.007*			0.003	0.003
			(0.004)	(0.004)			(0.002)	(0.002)
spop_large			0.007	0.009			0.002	0.003
			(0.016)	(0.018)			(0.007)	(0.007)
spop_huge			0.007	0.009			0.002	0.003
			(0.011)	(0.012)			(0.005)	(0.005)
inctype_traffic			-0.001	-0.001			-0.000	-0.000
			(0.001)	(0.001)			(0.000)	(0.000)
daytime			-0.001	-0.001			-0.000	-0.000
			(0.001)	(0.001)			(0.000)	(0.000)
sbehavior			0.025	0.025*			0.015	0.013
			(0.017)	(0.014)			(0.017)	(0.013)
sage_scaled			-0.000	-0.000			-0.000	-0.000
			(0.004)	(0.004)			(0.001)	(0.002)

Note: Standard errors in parentheses. (* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$)

References

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