Book Two

The Short Run

Economic Fluctuations, Short-run Unemployment and Stabilization Policy

Chapter 13

Business cycles: Facts

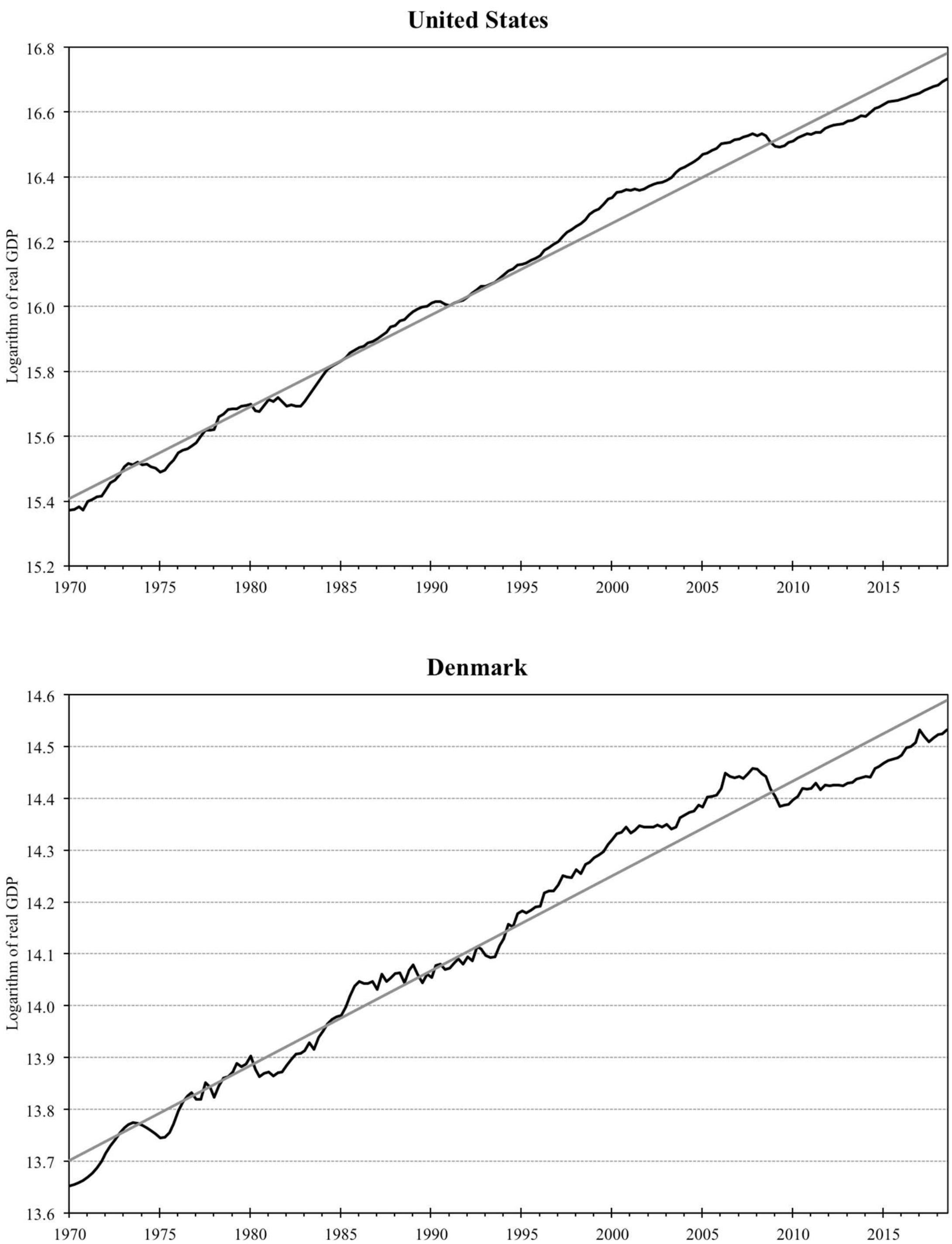
Introduction

Since the time of the Industrial Revolution the Western world has experienced a tremendous growth of total output. In Book One we focused on this long-run aspect of economic development. But history tells us that economic growth has been far from steady.

In the short and medium term the growth rate fluctuates considerably, as you can see from Figure 13.1, which plots quarterly data for the logarithm of real GDP in the large US economy and in the small open economy of Denmark. If the growth rate of the economy were constant, the log of GDP would follow the straight lines in Figure 13.1. The fact that the graphs for the log of actual GDP are sometimes steeper and sometimes flatter than the straight lines reflects that periods of rapid growth tend to alternate with intervals of slow growth. Indeed, the graphs have frequently had a negative slope, indicating that the growth rate sometimes becomes negative. Note that the data underlying Figure 13.1 are seasonally adjusted, so the fluctuations do not reflect the regular changes in business activity occurring with the changing seasons of the year, for example the seasonal swings in construction activity due to changing weather conditions. Thus, there are more fundamental forces causing an uneven pace of economic growth.

The rest of this book studies these short-term fluctuations in economic activity, commonly known as *business cycles*. How can we explain that the state of the economy repeatedly alternates between business cycle *expansions* characterized by rapid growth, and business cycle *contractions* or *recessions* characterized by slow growth or even declining economic activity?

Figure 13.1 The evolution of real GDP in the United States and Denmark



Note: Seasonally adjusted, quarterly data, 1970Q1 to 2018Q3
Source: OECD, National Accounts Statistics database.

The fact that economic growth is repeatedly interrupted by recessions is a major source of concern for economic policy makers and the general public. This is rooted in the perception that recessions bring considerable economic hardship to workers who lose their jobs, to entrepreneurs and homeowners who go bankrupt, and to ordinary consumers who suffer capital losses on their assets. Even for those who are not directly affected by layoffs and bankruptcies, recessions may cause a decline in well-being by generating fears of job losses and of future reductions in income and wealth.

Understanding business cycles is therefore not only of academic interest; it may also help the economist to offer advice to policy makers on the possibility of reducing business fluctuations through macroeconomic *stabilization policy*, that is, active monetary and fiscal policy. At the very least an insight into the workings of the business cycle may enable the economist to suggest how policy makers can avoid *amplifying* the business cycle through misguided macroeconomic policies.

The reason why it makes sense to theorize about business cycles is that, even though no two cycles are identical, they usually have some important features in common. Nobel laureate Robert Lucas of the University of Chicago made this point in the following way:

> Though there is absolutely no theoretical reason to anticipate it, one is led by the facts to conclude that, with respect to the qualitative behavior of comovements among series (economic variables), *business cycles are all alike*. To theoretically inclined economists, this conclusion should be attractive and challenging, for it suggests the possibility of a unified explanation of business cycles, grounded in the *general laws* governing market economies, rather than in political or institutional characteristics specific to particular countries or periods.[1]

In this chapter we will describe some of those co-movements of economic variables, which are characteristic of business cycles. In the chapter to follow we will discuss the nature of the social costs caused by business fluctuations and whether and how economic and social welfare could be improved if macroeconomic stabilization policy manages to dampen business fluctuations.

We begin in the next section by stating a definition of business cycles. We then move on to the question how we can measure business cycles in quantitative terms. That is, how can we separate short-term business cycle fluctuations in economic activity from the long-term economic growth trend? Following this, we will be ready to describe in quantitative terms the co-movements of important economic variables during a 'typical' business cycle.

[1] Robert E. Lucas, Jr., 'Understanding Business Cycles', in K. Brunner and A.H. Meltzer, eds, *Carnegie-Rochester Conference Series on Public Policy*, **5**, Autumn 1977, p. 10.

13.1 Defining business cycles

In a famous book, which became a milestone in empirical business cycle research, the American economists Arthur Burns and Wesley Mitchell gave the following classic definition of business cycles:

> Business cycles are a type of fluctuations found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent, but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.[2]

This definition of business cycles emphasizes several points:

- *Aggregate economic activity*: Business cycles are characterized by a co-movement of a large number of economic activities and not just by movements in a single variable like real GDP.
- *Organization in business enterprises*: Business cycles are a phenomenon occurring in decentralized market economies. Although they had several other economic problems, the former socialist economies of Eastern Europe did not go through business cycles of the type known in the Western world.
- *Expansions and contractions*: Business cycles are characterized by periods of expansion of economic activity followed by periods of contraction in which activity declines.
- *Duration of more than a year (persistence)*: A full business cycle lasts for more than a year. Fluctuations of shorter duration do not have the features characteristic of business cycles. This means that purely seasonal variations in activity within a year are not business cycles. We may also say that business cycle movements display *persistence*: once an expansion gets going, it usually lasts for some time during which the expansionary forces tend to be self-reinforcing, and once a contraction sets in, it tends to breed further contraction for a while.
- *Recurrent but not periodic*: Although business cycles repeat themselves, they are far from being strictly periodic. The duration of cycles has varied from slightly more than a year to 10–12 years, and the severity of recessions has also varied considerably, with recessions sometimes (but not always) turning into *depressions*.

[2] Arthur F. Burns and Wesley C. Mitchell, *Measuring Business Cycles*, National Bureau of Economic Research (NBER), New York, 1946, p. 1. Mitchell was a principal founder of the NBER, and Burns was a student of his who later became chairman of the US Federal Reserve Board from 1970 until 1978.

13.2 Dating business cycles

The contribution of Burns and Mitchell was to document the movements over time of a large number of economic variables. Through their work it became possible to identify the turning points in economic activity and hence to offer a dating of business cycles.

Since the movements of the different economic variables are not perfectly synchronized, it is a matter of judgement to identify the point in time at which the business cycle reaches its peak and moves from expansion into contraction, and to determine when it reaches its trough, moving from contraction (recession) to expansion.

Building on the tradition established by Burns and Mitchell, the US National Bureau of Economic Research (NBER) has for many years had a Business Cycle Dating Committee consisting of experts in empirical business cycle research. The NBER committee defines a recession (contraction) as a period of significant decline in total output, income, employment and trade, usually lasting from six months to a year, and marked by widespread contractions in many sectors of the economy.

On this basis the committee has arrived at a dating of US business cycles between 1854 and 2009. This dating is reproduced in Table 13.1.

The length of the business cycle is measured from trough to trough, and the last column of the table measures the duration of the expansion phase relative to the duration of the contraction phase. Table 13.1 illustrates the point stressed by Burns and Mitchell: business cycles are far from regular and periodic. The duration of the cycle varies greatly, and though the expansion phase usually lasts longer than the contraction phase – reflecting the economy's long-term potential for growth – there are also examples of cycles where the contraction has lasted longer than the previous expansion. At the bottom of the Great Depression in March 1933, the US economy had been contracting for 43 months. During this economic nightmare, real GDP fell by almost 30 per cent, and unemployment rose to 25 per cent. Notice, however, that the contraction of the 1870s lasted considerably longer than the Great Depression, although economic historians tell us that the decline in activity was less catastrophic.

The dates in Table 13.1 indicate that business cycle expansions have tended to last longer and that contractions have on average been shorter after the Second World War than before that time. The expansion from February 1961 to December 1969 (lasting 106 months) was at that time the longest economic boom in US history. That record was beaten by the ten-year-long expansion starting in March 1991 (lasting 120 months). Although the subsequent recession lasted only eight months, this cycle was the longest business cycle on record in the USA. However, since the through of June 2009 occurring under the Great Recession of the financial crisis, a new peak has not been identified by January 2019. This means that the expansion phase has lasted for 127 months at the beginning of 2019, and will be the longest ever recorded.

Table 13.1 US business cycle expansions and contractions

Business cycle reference dates			Duration in months			
Trough	Peak	Trough	1. Expansion	2. Contraction	Cycle[1]	1./2.
December 1854	June 1857	December 1858	30	18	48	1.7
December 1858	October 1860	June 1861	22	8	30	2.8
June 1861	April 1865	December 1867	46	32	78	1.4
December 1867	June 1869	December 1870	18	18	36	1
December 1870	October 1873	March 1879	34	65	99	0.5
March 1879	March 1882	May 1885	36	38	74	1
May 1885	March 1887	April 1888	22	13	35	1.7
April 1888	July 1890	May 1891	27	10	37	2.7
May 1891	January 1893	June 1894	20	17	37	1.2
June 1894	December 1895	June 1897	18	18	36	1
June 1897	June 1899	December 1990	24	18	42	1.3
December 1990	September 1902	August 1904	21	23	44	0.9
August 1904	May 1907	June 1908	33	13	46	2.5
June 1908	January 1910	January 1912	19	24	43	0.8
January 1912	January 1913	December 2014	12	23	35	0.5
December 2014	August 2018	March 1919	44	7	51	6.3
March 1919	January 1920	July 1921	10	18	28	0.6
July 1921	May 1923	July 1924	22	14	36	1.6
July 1924	October 1926	November 1927	27	13	40	2.1
November 1927	August 1929	March 1933	21	43	64	0.5
March 1933	May 1937	June 1938	50	13	63	3.9
June 1938	February 1945	October 1945	80	8	88	10
October 1945	November 1948	October 1949	37	11	48	3.4
October 1949	July 1953	May 1954	45	10	55	4.5
May 1954	August 1957	April 1958	39	8	47	4.9
April 1958	April 1960	February 1961	24	10	34	2.4
February 1961	December 1969	November 1970	106	11	117	9.6
November 1970	November 1973	March 1975	36	16	52	2.3
March 1975	January 1980	July 1980	58	6	64	9.7
July 1980	July 1981	November 1982	12	16	28	0.8
November 1982	July 1990	March 1991	92	8	100	12
March 1991	March 2001	November 2001	120	8	128	15
November 2001	December 2007	June 2009	73	18	91	4.1
June 2009	February 2020	April 2020	128	2	130	64
Average for pre-First World War period (15 cycles)[2]			25	23	48	1.1
Average for interwar period (5 cycles)[3]			26	20	46	1.3
Average for post-Second World War period (11 cycles)[4]			64	10	75	6.2

[1] The duration of the full business cycle is measured from trough to trough.

[2] December 1854 to December 1914.

[3] March 1919 to June 1938.

[4] October 1945 to June 2009.

Source: National Bureau of Economic Research.

The long US boom of the 1990s inspired many commentators to speculate about the arrival of a 'New Economy' in which the expansionary forces stemming from innovations in information technology were so strong that serious recessions would be a thing of the past, at least in the USA. Interestingly, the long boom of the 1960s gave rise to a similar unfounded optimism. In 1967, several leading US economists gathered for a conference asking: 'Is the Business Cycle Obsolete?'[3] But the recession in the USA and in many other countries at the beginning of this century and the dramatic international financial and economic crisis in 2008–09 – highly visible in Figure 13.1 – show that the business cycle is still with us.

Let us therefore move from the *dating* of business cycles to the problem of *quantitative measurement* of business fluctuations.

13.3 Measuring business cycles

Most economic time series fluctuate around a growing time trend. The growth trend reflects the forces described in the theory of economic growth, while the task of business cycle theory is to explain the fluctuations around that trend. For example, if Y_t is real GDP in period t, it is useful to think of Y_t as the product of a growth component $\overline{Y}_t$ indicating the trend value which Y_t would assume if the economy were always on its long-term growth path, and a cyclical component Y_t^c, which fluctuates around a long-run mean value of 1:

$$Y_t = \overline{Y}_t \cdot Y_t^c. \tag{1}$$

Our assumption on the mean value of Y_t^c implies that $Y_t = \overline{Y}_t$ on average. Equation (1) also implies that as long as the amplitude of the fluctuations in the cyclical component remains constant, the *absolute* amplitude of the business cycle fluctuations in real GDP will rise in proportion to the trend level of output so that the *percentage* deviations of actual output from trend output over the business cycle will tend to stay constant over time.

It will be convenient to work with the natural logarithms of the various variables rather than with the variables themselves, because changes in the log of some variable X approximate percentage changes in X. Taking logs on both sides of equation (1) and defining $y_t \equiv \ln Y_t$, $\overline{y}_t \equiv \ln \overline{Y}_t$, and $c_t \equiv \ln Y_t^c$, we get:

$$y_t = \overline{y}_t + c_t \tag{2}$$

[3] The conference participants tended to answer the question in the affirmative. The conference papers were published in Martin Bronfenbrenner (ed.), *Is the Business Cycle Obsolete?* Wiley, New York, 1969.

In this section we will discuss how we can estimate the trend component $\bar{y}_t$ and the cyclical component c_t separately, given that we only have observations of y_t. Let us start by going back to Figure 13.1. The straight lines in that figure are regression lines with a slope roughly equal to the average growth rate over the period of observation. Technically, the intercept and the slope of the regression line are chosen so as to minimize the sum of the squared deviations between the observed values of the log of GDP and the points on the line.

It might be tempting to let the straight regression line represent the trend value of output and to measure the cyclical component of GDP as the deviation from that hypothetical steady growth path. But a moment's reflection should convince you that this is not a satisfactory procedure. Recall that along the straight regression line the economy's real growth rate is constant. If we take the regression line to represent the trend growth path, we are therefore postulating that the economy would always be in a steady state equilibrium with a constant growth rate if it were not disturbed by business fluctuations.

However, the theory of economic growth gives no reason to believe that the economy is always in a steady state. Conventional growth theory tells us that the economy's growth rate will decline over time if it starts out with a capital to (effective) labour ratio below the steady state level, and vice versa. Moreover, the modern theory of endogenous growth suggests that the rate of technical progress may vary with the endogenous innovation activity of firms. Indeed, even if major technological innovations are exogenous to the economic system, they are unlikely to arrive at an equal pace over time, and this is sufficient reason to discard the assumption of an absolutely constant long-term growth rate. An inspection of Figure 13.1 also suggests that the longer-term movement of the economy does not follow a straight line, even if we abstract from the short-term ups and downs of the graphs for real GDP.

Hence we need a more sophisticated method for separating the growth trend from the cyclical component of a variable like GDP. Since we do not perfectly understand how the economy works, we cannot claim that there is a single objectively correct way of separating c_t from $\bar{y}_t$. Still, our reasoning above suggests that we need a method which allows for variation over time in the underlying growth trend, but which nevertheless ensures that the short-term fluctuations are categorized as temporary cyclical deviations from trend. One such method which has become widely used in recent decades is the so-called *Hodrick–Prescott filter*, named after American economists Robert Hodrick and Edward Prescott who popularized its use.[4] Under this method of 'filtering' (that is, detrending) an economic time series like $(y_t)_{t=1}^{T}$ for the log of GDP, the growth component $\bar{y}_t$ is determined by minimizing the magnitude:

[4] Robert J. Hodrick and Edward C. Prescott, 'Post-war U.S. Business Cycles: An Empirical Investigation', *Journal of Money, Credit, and Banking*, **29**, 1997, pp. 1–16 (originally published as a working paper in 1980).

$$HP = \sum_{t=1}^{T}(y_t - \bar{y}_t)^2 + \lambda \sum_{t=2}^{T-1}[(\bar{y}_{t+1} - \bar{y}_t) - (\bar{y}_t - \bar{y}_{t-1})]^2 \qquad (3)$$

with respect to all of the $\bar{y}_t$, where observations are available for the time periods $t = 1, 2, \ldots, T$, and where λ is a parameter which is chosen by the observer. Note that since y_t is measured in logarithms, the magnitudes $\bar{y}_{t+1} - \bar{y}_t$ and $\bar{y}_t - \bar{y}_{t-1}$ are approximately the *percentage growth rates* of the trend value of real GDP in periods $t + 1$ and t, respectively. The term in the square bracket in (3) thus measures the change in the estimated trend growth rate from one period to the next. Note also that, by definition, the term $y_t - \bar{y}_t$ in (3) measures the cyclical component c_t of log(GDP) in period t. Minimizing the expression in (3) therefore forces us to compromise between two objectives. On the one hand, we want to choose the $\bar{y}_t$'s so as to minimize changes in the estimated trend growth rate over time, since this will minimize the expression in the second term in (3). On the other hand, we want to bring $\bar{y}_t$ as close as possible to (the log of) actual output y_t so as to minimize the first sum in (3). The relative weight placed on each of these conflicting objectives depends on our choice of λ.

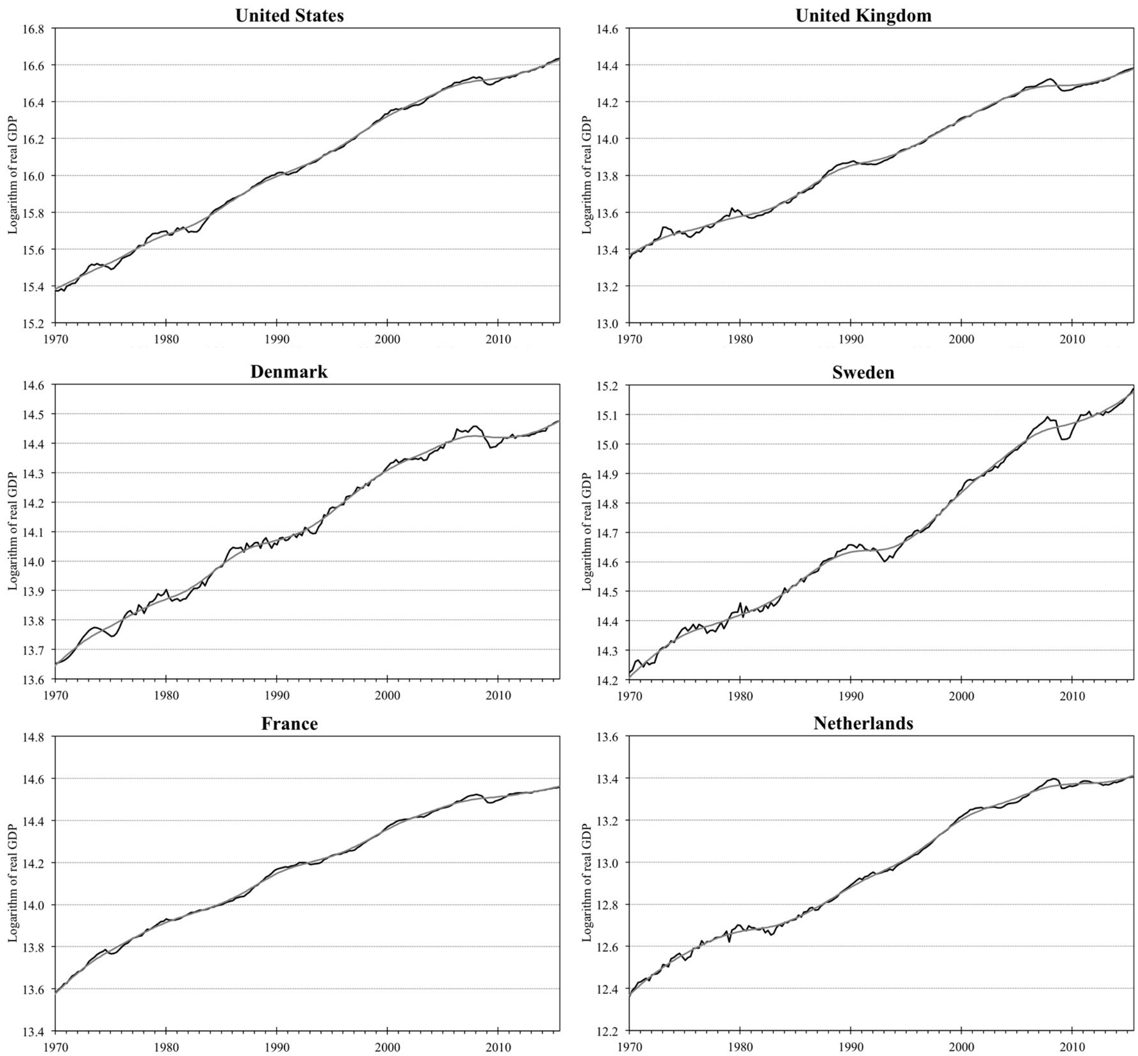
Consider first the extreme case where we set $\lambda = 0$. In this special case we will minimize *HP* by simply choosing $y_t = \bar{y}_t$ for all $t = 1, 2, \ldots, T$, since *HP* will then attain its lowest possible value of zero. But this amounts to postulating that all observed fluctuations in y_t reflect changes in the underlying growth trend! This clearly does not make sense, unless we want to deny the existence of business fluctuations.

Consider next the opposite extreme where we let λ tend to infinity. In that case the first sum in (3) does not carry any weight, and *HP* will then attain its lowest possible value of zero if we choose the $\bar{y}_t$'s to ensure that the estimated trend growth rate is *constant* throughout the period of observation, that is $\bar{y}_{t+1} - \bar{y}_t = \bar{y}_t - \bar{y}_{t-1}$ for all $t = 2, 3, \ldots, T - 1$. This would give us the straight lines in Figure 13.1, but we have already seen that it is unreasonable to assume that the trend growth rate is a constant.

Clearly, then, λ should be strictly positive but finite. The greater the value of λ, the more we will try to avoid variation over time in the estimated trend growth rate, that is, the smoother will be our estimated growth trend (the closer it will be to a straight line). On the other hand, the smaller the value of λ, the smaller will be the deviation between our estimated $\bar{y}_t$ and the actual value of output y_t, that is, the greater is that part of the movement in actual output which we ascribe to changes in the underlying growth trend.

Among business cycle researchers it is customary to set $\lambda = 1600$ when filtering *quarterly* data. This is basically a convention, based on a consensus that this value of λ produces a fitted growth trend that a ‘reasonable’ student of business cycles would draw through a time plot of quarterly data for (the log of) real GDP. Figure 13.2 shows the fitted growth trend for the log of GDP for six different countries, when the trend is estimated via the HP filter using $\lambda = 1600$. We see that the trend estimated by the HP filter (the grey curves) does indeed seem to capture the gradual changes in the growth trend which are apparent to the naked eye. However, researchers have found that the HP filter tends to give imprecise estimates of the trend at the end-points of a time series. For this reason, the analysis throughout this chapter excludes (at least) the first and the last 12 estimated cyclical components of all quarterly time series, in line with common practice.

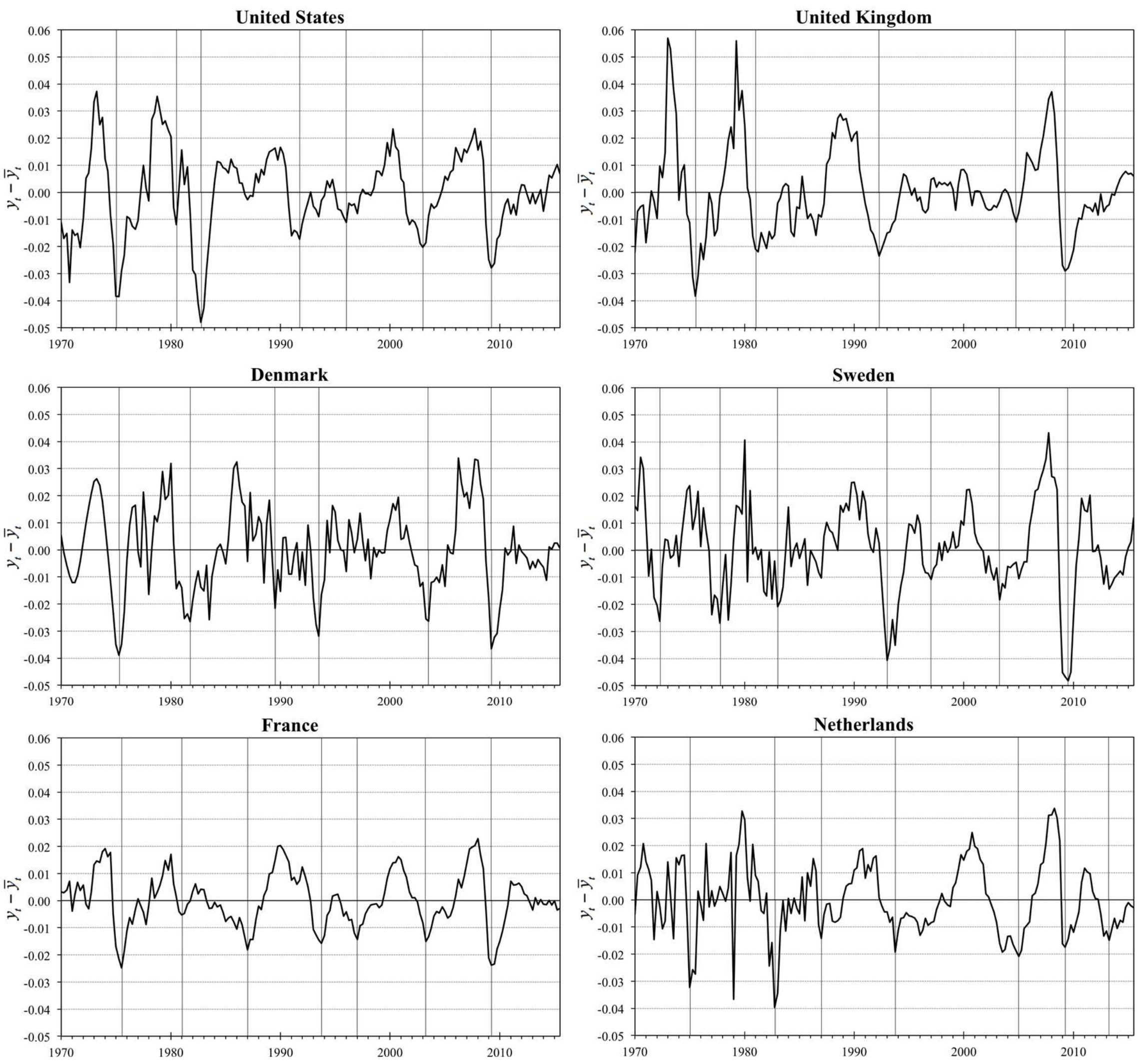
Figure 13.2 Real GDP and its trend, six countries, 1970Q1-2015Q3



Note: Quarterly data. The growth trend (grey curve) has been estimated by the Hodrick–Prescott filter with $\lambda = 1600$. In both ends, observations and trend values have been excluded for at least 12 quarters.
Source: OECD, National Accounts Statistics database and own calculations.

Once we have fitted a growth trend using the HP filter, we immediately obtain an estimate of the cyclical component of the (log of) quarterly real GDP by rearranging equation (2) to get $c_t = y_t - \bar{y}_t$. In Figure 13.3 we have plotted the resulting estimates of business cycles, that is, the values of the c_t's for our six countries. This cyclical component of GDP is usually termed "the output gap".

Figure 13.3 Cycles in real GDP, six countries, 1970Q1-2015Q3



Note: Based on quarterly data. In both ends, points for at least 12 quarters have been excluded.
Source: OECD, National Accounts Statistics database and own calculations.

The graphs in Figure 13.3 confirm what we have already emphasized: periods of expansion and periods of contraction tend to alternate, and there seems to be some regularity in the cycles although the business cycles are far from strictly periodic and regular, and even within each longer cycle there are erratic quarterly fluctuations in economic activity.

Figure 13.3 also gives some overall qualitative indication that business cycles are similar in different capitalist economies in line with the quote of Robert E. Lucas stated above: "business cycles are all alike".

It is tempting to offer a dating of recent business cycles based on Figure 13.3, for although business cycles involve the co-movement of many economic variables (as we shall see in the next section), GDP is, after all, the most common single indicator of aggregate economic activity. We have inserted vertical lines in Figure 13.3 to indicate business cycles measured from trough to trough. The identification of business cycle troughs and peaks is based on the following simple rules of thumb:

1. A trough must be followed by a peak, and a peak must be followed by a trough.
2. The expansion phase as well as the contraction phase must last for a minimum of two quarters.
3. A business cycle must span a minimum of five quarters.

If the first criterion is not met, it does not make sense to speak of a 'cycle'. The second criterion implies that we require a minimum degree of persistence in the movement of economic activity before we can speak of a systematic tendency for activity to expand or contract. The last criterion reflects the convention that fluctuations spanning only a year or less do not count as business cycles.[5]

We see from Figure 13.3 that more or less all the six countries experienced a serious contraction in the beginning- or mid-1970s and also fell into recession in the early 1980s, in the early 1990s, in the early 2000s, and in 2008-2009. In Exercise 1 we test your knowledge of recent economic history by asking you to mention some factors or 'shocks' which might help explain these downturns in economic activity.

13.4 What happens during business cycles?

The HP filter is a useful method of separating the growth trend from the cyclical component of an economic time series. In this section, we will study the statistical properties and interrelations of the estimated cyclical components of a number of time series. In order to keep the amount of material at a reasonable level, we will do so for just

[5] The computer algorithm used to date the business cycles was developed and kindly provided by Jesper Linaa from the secretariat of the Danish Economic Councils.

two of the countries considered above, USA and Denmark, thus covering a very large and relatively closed economy at the one end and a small and very open economy at the other. By our analysis, we will get more information on what happens during business cycles.

Our study is based on seasonally adjusted quarterly data going back to the first quarter of 1947 for USA and to the first quarter of 1971 for Denmark. The data sets include series for real GDP, as well as series for its real components, private consumption, private investment (with housing investment singled out), public consumption and investment, exports and imports, all measured in fixed prices. In addition, data for total employment and inflation, and at some places for nominal interest rates, are included. All variables have been detrended by means of the HP filter, using the λ-value of 1600, which is customary for business cycle analysis of quarterly data. Variables displaying a growing time (all other than inflation and interest rates) trend have been transformed into natural logarithms before detrending. Also, because the HP filter gives imprecise estimates of the trend at the end-points of a time series, we have excluded the first and the last 12 estimated cyclical components of each series from our statistical analysis, as mentioned earlier.

Volatility

The first question we will ask is: how large is the variability in different economic variables during a 'typical' business cycle? We may quantify this variability by calculating the *standard deviations* of the estimated cyclical components of the various time series. The empirical standard deviation s_x of a series of observations of variable x_t over the time interval $t = 1, 2, \ldots, T$ is defined as:

$$s_x = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(x_t - \overline{x})^2}, \qquad \overline{x} \equiv \frac{1}{T}\sum_{t=1}^{T} x_t, \tag{4}$$

where $\overline{x}$ is the empirical mean value of the x_t's. The empirical standard deviation measures the 'average' deviation of x_t from its mean over the period of observation. It is thus a natural indicator of the degree of *volatility* of the economic variable x_t.

In the middle columns of Table 13.2 we use the absolute standard deviation as a measure of the volatility of the cyclical components of our macroeconomic variables. Recall that when variables are measured in logarithms, the absolute standard deviation of some variable $x \equiv \ln X$ roughly indicates the average *percentage* deviation of X from its mean. For example, the first number in the middle column for the United States and Denmark indicate that in both countries, on average over the business cycle, the cyclical component of real quarterly GDP deviates about 1.6 per cent from its mean value.

Table 13.2 Macroeconomic volatility in the USA and Denmark

United States			
	Average share in GDP	**Absolute standard deviation (%)[1]**	**Relative standard deviation[2]**
GDP	100%	1.58	1.00
Private consumption	63%	1.29	0.82
Private investment	14%	7.07	4.48
Housing investment	6%	9.88	6.26
Public consumption & investment	26%	3.35	2.12
Exports	6%	5.23	3.32
Imports	8%	5.07	3.22
Employment		1.41	0.90
Inflation		0.52	0.33

Note: Based on quarterly data from 1947Q1 to 2017Q1. 24 end-point observations excluded. The inflation data starts in 1947Q2.

[1] Standard deviations of the quartertly cyclical components, for the real variables in percent, for inflation in percentage points.

[2] Standard deviation relative to standard deviation of GDP.

Sources: Bureau of Economic Analysis, Bureau of Labour Statistics and Federal Reserve Bank of St. Louis.

Denmark			
	Average share in GDP	**Absolute standard deviation (%)[1]**	**Relative standard deviation[2]**
GDP	100%	1.59	1.00
Private consumption	49%	2.06	1.30
Private investment	10%	6.94	4.35
Housing investment	5%	9.58	6.01
Public consumption	26%	1.17	0.73
Public investment	4%	6.42	4.03
Exports	35%	3.15	1.98
Imports	30%	4.50	2.82
Employment		1.08	0.68
Inflation		0.43	0.27

Note: Based on quarterly data from 1971Q1 to 2017Q1. 24 end-point observations excluded. The inflation data and the private investments data start in 1971Q2 and 1972Q1, respectively.

[1] Standard deviations of the quartertly cyclical components, for the real variables in percent, for inflation in percentage points.

[2] Standard deviation relative to standard deviation of GDP.

Sources: Danmarks Nationalbank and OECD, Economics Outlook Database Number 101.

In the last columns in Table 13.2 we have measured the standard deviations of the various variables relative to the standard deviation of GDP. A figure in excess of (smaller than) 1 means that the variable considered tends to be more (less) volatile than GDP.

One striking feature of Table 13.2 is that investment is around 4 to 5 times as volatile as GDP, and housing investment, in particular, is even more volatile, up to more than 6 times as volatile as GDP. The volumes of exports and imports also fluctuate a lot more than GDP, indicating that the foreign trade sector is relatively unstable. Employment is about as volatile as GDP in the USA but in Denmark somewhat less volatile than GDP. Denmark shares this feature other European countries (this is not documented here).

Inflation in Table 13.2 is the quarterly, not annualized rate of inflation and the table reports the standard deviation of the inflation itself, not the log of it, in percentage points. Measured that way, the *absolute* volatility of inflation seems to be around one third of the *relative* volatility of GDP.

The main messages from Table 13.2 and from Figure 13.3 may be summarized as three stylized facts regarding business cycles.

STYLIZED BUSINESS CYCLE FACT 1

Real GDP moves considerably up and down over the business cycle. In advanced economies like the USA and Denmark the average deviation of GDP from its trend value is more than 1.5 percent. This covers that the deviations sometimes go down to 4-5 percent below trend and sometimes up to 3-4 percent above, while at other times they are considerable smaller than the average deviation. A broad band of +/- 4-5 percent from trend seems to include almost all business cycle fluctuations in GDP over long periods.

STYLIZED BUSINESS CYCLE FACT 2

Investment is around 4-5 times more volatile over the business cycle than GDP and housing investment 6 times as volatile or more. Investment is the most unstable component of aggregate demand. The volatility of private consumption is in the vicinity of that of GDP, a bit below or above, and employment seems to be a bit less volatile than GDP. Foreign trade volumes are considerably more volatile than GDP, although not as volatile as investment.

STYLIZED BUSINESS CYCLE FACT 3

The average deviation of quarterly, non-annualized inflation from its trend value is around one half of a percentage point, and the absolute volatility of inflation measured this way is thus around one third of the relative volatility of GDP.

Correlation, leads and lags

Table 13.2 tells us how much different variables fluctuate relative to the fluctuations in GDP. But we are also interested in studying whether and to what extent the cyclical component, x_t, of some economic variable, private consumption e.g., moves in the same direction as or opposite to the cyclical component of real GDP, c_t. For this purpose we introduce the empirical *covariance* between x_t and c_t, defined as:

$$s_{xc} = \frac{1}{T-1}\sum_{t=1}^{T}(x_t - \bar{x})(c_t - \bar{c}), \qquad \bar{c} \equiv \frac{1}{T}\sum_{t=1}^{T} c_t, \tag{5}$$

where $\bar{c}$ is the mean value of the c_t's. The covariance measures the degree to which x and c move together, but its magnitude will depend on our choice of the units in which we measure x and c. To obtain an indicator that is independent of the choice of units, it is preferable to normalize the observations of $x_t - \bar{x}$ and $c_t - \bar{c}$ by the respective standard deviations s_x and s_c and to study the covariation of the normalized deviations, $(x_t - \bar{x})/s_x$ and $(c_t - \bar{c})/s_c$. Following this procedure, we obtain the *coefficient of correlation* between x and c, which we have already encountered in Chapter 10, and which is defined as:

$$\rho(x_t, c_t) = \frac{s_{xc}}{s_x s_c} = \frac{\sum_{t=1}^{T}(x_t - \bar{x})(c_t - \bar{c})}{\sqrt{\sum_{t=1}^{T}(x_t - \bar{x})^2} \cdot \sqrt{\sum_{t=1}^{T}(c_t - \bar{c})^2}}. \tag{6}$$

It can be shown that the coefficient of correlation will always assume a value somewhere in the interval from −1 to +1. If $\rho(x_t, c_t)$ is equal to 1 we say that x_t and c_t are perfectly positively correlated, and if $\rho(x_t, c_t)$ equals −1 we say that the two variables are perfectly negatively correlated. In both cases there is a strict linear relationship between the two variables. If $\rho(x_t, c_t)$ is positive but less than 1, x and c will tend to move in the same direction, with the co-movement being more systematic the smaller the deviation of $\rho(x_t, c_t)$ from 1. On the other hand, a negative value of $\rho(x_t, c_t)$ indicates that the two variables tend to move in opposite directions. Clearly, if $\rho(x_t, c_t)$ is (close to) 0, there is no (little) systematic relationship between x and c.

In the present context where c_t represents the cyclical component of real GDP, we say that x_t varies *procyclically* when $\rho(x_t, c_t)$ is substantially greater than 0, since the positive correlation indicates that x tends to rise and fall with GDP. By analogy, if $\rho(x_t, c_t)$ is substantially negative, we say that x_t moves in a *countercyclical* fashion because x tends to move in the opposite direction of GDP.

The co-movements of the different economic variables are not always synchronized over the business cycle: some variables may reach their turning point before others do. To investigate whether a variable x moves out of sync with real GDP, we may measure the coefficient of correlation $\rho(x_{t-n}, c_t)$ between c_t and the value of x observed n periods earlier (x_{t-n}), and the correlation coefficient $\rho(x_{t+n}, c_t)$ between c_t and the value of x observed n periods later (x_{t+n}). If $\rho(x_{t-n}, c_t)$ is significantly different from 0 and numerically greater than $\rho(x_t, c_t)$, we say that x_t is a *leading indicator*, because a change in x observed n periods earlier tends to be associated with a change in GDP in the current period. In other words, movements in x tend to *lead* movements in aggregate output, so a turnaround in x indicates a later turnaround in c. Similarly, we say that x is a *lagging variable* if $\rho(x_{t+n}, c_t)$ is significantly different from 0 and numerically greater than $\rho(x_t, c_t)$, since this is an indication that x tends to reach its peaks and troughs later than c.

In Table 13.3 we show coefficients of correlation between the logs of various variables and the log of GDP, including leads and lags. The middle columns headed '0' show the contemporaneous correlation coefficients, $\rho(x_t, c_t)$, where x_t is the relevant variable indicated in the left part of the table. The first two columns for each country show correlations between current GDP (c_t) and x_{t-2} and x_{t-1}, respectively, whereas the last two columns show correlations between c_t and x_{t+1} and x_{t+2}, respectively. Recall that the length of a time period is one quarter. From the second row for the USA we therefore infer that the coefficient of correlation between current GDP and private consumption two quarters earlier is 0.66, the correlation between current GDP and private consumption one quarter later is 0.78, while the contemporaneous correlation is 0.8 etc.

Table 13.3 shows that whereas public consumption and public investment are rather weakly correlated with GDP, private consumption, private investment (and in particular housing investment) and imports are all *procyclical*, displaying strong positive correlation with contemporaneous GDP.

Not surprisingly, we see that employment varies procyclically, since an increase in output requires an increase in labour input. The mirror image (not shown in the table) is that unemployment is countercyclical. We also see that employment is a *lagging variable*, since it is more strongly correlated with the GDP of the two previous quarters than with contemporaneous GDP. The finding that employment is a lagging variable is a general feature of OECD economies. It may be explained by the fact that hiring and firing of workers are costly. When the demand for a firm's product changes, it will therefore typically want to wait and see if the change in demand is of some duration before it adjusts its work force. In the meantime the firm may adapt to the change in demand by adjusting its inventory, its rate of utilization of its capital stock and the working hours of its existing employees.

All the variables discussed so far have been defined in real terms. The bottom rows in Table 13.3 show the correlation between real output and some important *nominal* variables.

Table 13.3 Macroeconomic correlations, leads and lags in the USA and Denmark

United States

	GDP and X_t				
	Quarterly leads and lags				
	-2	-1	0	1	2
X_t (Real variables)					
GDP	0.61	0.85	1.00	0.85	0.61
Private consumption	0.66	0.78	0.80	0.62	0.40
Private investment	0.59	0.76	0.85	0.68	0.43
Housing investment	0.69	0.68	0.57	0.33	0.09
Public consumption & investment	-0.08	0.00	0.12	0.17	0.22
Exports	0.02	0.24	0.43	0.50	0.47
Imports	0.50	0.65	0.72	0.64	0.42
Employment	0.29	0.58	0.81	0.89	0.84
X_t (Nominal variables)					
Inflation rate (CPI)	0.08	0.19	0.29	0.33	0.33
Short-term nominal interest rate	0.02	0.25	0.42	0.49	0.48
Long-term nominal interest rate	-0.23	-0.06	0.07	0.10	0.11

Note: Based on quarterly data from 1947Q1 to 2017Q1. 24 end-point observations excluded. The inflation data starts in 1947Q2.

Sources: See Table 13.2.

Denmark

	GDP and X_t				
	Quarterly leads and lags				
	-2	-1	0	1	2
X_t (Real variables)					
GDP	0.54	0.74	1.00	0.74	0.54
Private consumption	0.59	0.69	0.73	0.48	0.34
Private investment	0.32	0.47	0.65	0.60	0.59
Housing investment	0.52	0.65	0.69	0.65	0.53
Public consumption	-0.09	-0.02	0.07	0.07	0.06
Public investment	0.22	0.24	0.27	0.27	0.29
Exports	0.10	0.24	0.39	0.33	0.33
Imports	0.45	0.64	0.75	0.65	0.50
Employment	0.26	0.48	0.63	0.70	0.69
X_t (Nominal variables)					
Inflation rate (CPI)	-0.25	-0.16	-0.04	-0.02	-0.02
Short-term nominal interest rate	-0.24	-0.09	0.06	0.23	0.31
Long-term nominal interest rate	-0.31	-0.16	0.00	0.12	0.13

Note: Based on quarterly data from 1971Q1 to 2017Q1. 24 end-point observations excluded. The inflation data, the private investments data and the short-term nominal interest rate data start in 1971Q2, 1972Q1 and 1979Q2, respectively.

Sources: See Table 13.2.

In the US the rate of inflation tends to be positively correlated with GDP contemporaneously and subsequently, but the correlation is not strong. As our theoretical analysis in later chapters will make clear, a positive correlation between output and inflation indicates that business cycles are driven mainly by shocks to aggregate demand. For Denmark the correlation between inflation and contemporaneous and subsequent GDP is weak, while there may be a tendency for inflation to be a leading variable, so that a upturn of inflation can be an indication of a coming slowdown in GDP growth. The weak correlation between GDP and inflation in current and subsequent quarters may reflect nominal rigidities, the fact that most nominal wages and prices tend to be "sticky", adjusting only slowly to changes in demand and output as explained in Chapter 1.

Moreover, in the US (and less pronounced in Denmark) the short-term nominal interest rate tends to go up in the current quarter and in the two quarters following a rise in GDP, which can reflect that the central bank typically tightens monetary policy in reaction to higher economic activity.

Let us sum up the main lessons from Table 13.3.

STYLIZED BUSINESS CYCLE FACT 4
Private consumption, investment (particularly housing investment) and imports are strongly positively correlated with GDP.

STYLIZED BUSINESS CYCLE FACT 5
Employment (unemployment) is also strongly procyclical (countercyclical).

STYLIZED BUSINESS CYCLE FACT 6
Inflation tends to be positively correlated with GDP, although the correlation is not very strong.

STYLIZED BUSINESS CYCLE FACT 7
Employment is a lagging variable and so are nominal interest rates.

Persistence

Another interesting property of an economic variable is its degree of *persistence*. As you recall, one characteristic of business cycles is that, once the economy moves into an expansion or a contraction, it tends to stay there for a while. Persistence in some variable x means that the observed value of x in period t, x_t, is not independent of the value, x_{t-n}, of

Table 13.4 Macroeconomic persistence in the USA and Denmark

United States

	Coefficient of autocorrelation			
	1-quarter lag	2-quarter lag	3-quarter lag	4-quarter lag
X_t (Real variables)				
GDP	0.85	0.61	0.36	0.14
Private consumption	0.82	0.65	0.42	0.18
Private investment	0.82	0.59	0.33	0.10
Housing investment	0.91	0.73	0.51	0.30
Public consumption & investment	0.91	0.75	0.55	0.31
Exports	0.72	0.55	0.30	0.06
Imports	0.74	0.49	0.28	0.08
Employment	0.92	0.73	0.50	0.26
X_t (Nominal variables)				
Inflation rate (CPI)	0.40	0.15	0.21	0.02
Short-term nominal interest rate	0.83	0.59	0.45	0.27
Long-term nominal interest rate	0.77	0.50	0.28	0.04

Note: Based on quarterly data from 1947Q1 to 2017Q1. 24 end-point observations excluded. The inflation data starts in 1947Q2.
Sources: See Table 13.2.

Denmark

	Coefficient of autocorrelation			
	1-quarter lag	2-quarter lag	3-quarter lag	4-quarter lag
X_t (Real variables)				
GDP	0.74	0.54	0.32	0.15
Private consumption	0.69	0.53	0.39	0.21
Private investment	0.65	0.51	0.42	0.26
Housing investment	0.85	0.66	0.49	0.34
Public consumption	0.77	0.54	0.36	0.18
Public investment	0.50	0.37	0.16	0.14
Exports	0.69	0.56	0.31	0.13
Imports	0.75	0.48	0.18	-0.05
Employment	0.88	0.73	0.55	0.40
X_t (Nominal variables)				
Inflation rate (CPI)	0.43	0.11	0.03	-0.07
Short-term nominal interest rate	0.71	0.44	0.21	-0.09
Long-term nominal interest rate	0.79	0.45	0.13	-0.14

Note: Based on quarterly data from 1971Q1 to 2017Q1. 24 end-point observations excluded. The inflation data, the private investments data and the short-term nominal interest rate data start in 1971Q2, 1972Q1 and 1979Q2, respectively.
Sources: See Table 13.2.

x in some previous period $t - n$, where $n \geq 1$. In other words, if x assumed a high (low) value in previous period $t - n$, there is a greater chance that it will also assume a high (low) value in the current period t. We can measure such persistence in a time series $(x_t)_{t=1}^{T}$ by calculating the coefficient of correlation between x_t and its own lagged value x_{t-n}, for $n = 1, 2, \ldots$ This particular correlation coefficient $\rho(x_t, x_{t-n})$ is called the *coefficient of autocorrelation* and was already introduced in Chapter 10. If $\rho(x_t, x_{t-n})$ is significantly above zero for several positive values of n, there is a high degree of persistence: once x has been pushed above or below its mean value, it tends to continue to be above or below its mean for a long time.

Table 13.4 measures the persistence of business fluctuations by the coefficients of autocorrelation. The first figure in the first column for the USA (0.85) means that if real GDP goes up by one percentage point in the current quarter, then on average 0.85 percentage points of that increase will remain in the next quarter. We see from Table 13.4 that there is considerable persistence in the movements of GDP, private consumption and employment. The high persistence of employment is another reflection of the fact that firms are reluctant to hire and fire workers because hiring and firing is costly.

To sum up, we have:

STYLIZED BUSINESS CYCLE FACT 8
There is considerable persistence in GDP and about the same degree of persistence in private consumption and private investment.

STYLIZED BUSINESS CYCLE FACT 9
Employment tends to be even more persistent than GDP.

13.5 HP filtering: a final word of caution

In this chapter we have used the HP filter intensively in order to extract basic properties of the business cycle in advanced economies. We have done so by estimating the trend in (the log of) macroeconomic time series by the filter and then defined the cyclical components as the difference between the actual and the trend values. Our stylized business cycle facts were derived by studying the properties and interrelations between these cyclical components.

In particular, we obtained an estimate of the output gap by HP filtration of the log of GDP etc. There are more sophisticated ways of estimating the output gap and an exercise of this chapter will take you through one such method, the production function approach. However, this method also relies on the use of the HP filter, e.g., for the purpose of estimating the trend in labour input and TFP. The same is true for even more sophisticated versions of the production function method.

Although convenient, the method of HP filtering is not unproblematic. We have already mentioned that the HP trend tends to be imprecisely estimated at the endpoints of the time series. Hence, the serious researcher should disregard some end-point observations, as we have done above. However, this is unfortunate since one is often particularly interested in estimating the output gap for the most recent periods in order to evaluate the need for active macroeconomic policy to smooth the business cycle.

Another problem is that there is no objectively correct value of the parameter λ, which determines the estimated HP trend. An exercise will ask you to estimate annual output gaps based on the HP filter with two different values, $\lambda = 100$ and $\lambda = 1000$, respectively. You will probably see that the difference between the two measures is non-negligible in certain periods, so the basic arbitrariness in the choice of λ adds another element of uncertainty to measures of cyclical fluctuations based on the HP filter.

Despite of these critical remarks on the HP filter we think that the business cycle facts pointed to in the stylized facts listed above are so robust and well-established that one can have trust in them. We therefore feel safe to conclude this chapter by summarizing our main findings:

Summary

1. **1.** Business cycles are periods of expansion of aggregate economic activity followed by periods of contraction in which activity declines. These fluctuations are recurrent but not periodic; in duration business cycles have varied from a little more than a year to 10 –12 years. The severity of recessions has also varied considerably, with recessions sometimes turning into depressions where output and employment fall dramatically.
2. **2.** The expansion phase of the business cycle usually lasts considerably longer than the contraction phase, reflecting the economy's capacity for long-term growth. In the period since the Second World War business cycle expansions have tended to last longer, and recessions have tended to be shorter and milder than was the case before the war. For the post-Second World War period the average duration of the US business cycle expansions has been 58 months, while recessions have on average lasted 11 months. By contrast, the average recession lasted about 22 months before the war.
3. **3.** The business cycle fluctuations in a macroeconomic time series may be measured as the deviation of the actual time series from its long-term trend. The trend may be estimated by means of the HP filter, which allows for smooth changes in the underlying (growth) trend in a series.
4. **4.** The volatility of the cyclical component in a macroeconomic time series may be quantified by its standard deviation. By this measure, investment is a lot more volatile over the business cycle than GDP, whereas employment is somewhat less volatile.
5. **5.** The co-movements in different economic variables over the business cycle may be measured by their coefficients of correlation with GDP. Private investment, consumption and imports all have a strong positive correlation with GDP. Employment also displays a clear positive correlation with GDP, but it is a lagging variable, being more strongly correlated with the GDP of the two previous quarters than with current GDP. Inflation tends to vary positively with GDP, although the correlation is not very strong. The nominal short-term interest rate tends to go up in the two quarters following a rise in GDP, reflecting a tightening of monetary policy.
6. **6.** The degree of persistence in a macroeconomic variable may be measured by its coefficients of correlation with its own lagged values, the so-called coefficients of autocorrelation. There is considerable persistence in GDP and private consumption, and even more persistence in employment. This means that once these variables start to move in one direction, they will continue to move in the same direction for a while, unless they are significantly disturbed by new shocks.

Exercises

Exercise 1. Accounting for recent recessions

1. Almost all advanced economies experienced serious recessions in the mid-1970s, in the early 1980s and in the early 1990s. Try to give a brief account of the factors and exogenous 'shocks' which are likely to have generated these contractions in economic activity.
2. In 2008-09 almost all countries in the world fell into a serious recession. Mention some factors and 'shocks' which in your view contributed to this economic downturn.

Exercise 2. Measuring the output gap by means of the HP filter

At the web page for this book you will find a guide on how to perform HP filtering of economic time series in an Excel spread sheet on your computer. You will also find an Excel sheet containing annual data on real GDP for four different countries for a long period starting in 1950. [6] The data are taken from the Penn World Table. You may want to find annual data for real GDP for your own country, if this is not included, and include these. You can find such data in the Penn World Table, an easily accessible source. This exercise asks you to do HP filtering etc. of the GDP series following the instructions in the guide.

1. For each country in the data file: Create the transformed series, the natural log of GDP, and find the trend of this series by HP-filtering with λ=100. In a diagram plot the actual series for the log of GDP as well as its trend value against years for the entire period. Comment on an intuitive basis on how well you think that the HP filter finds the trend of the series. Comment particularly on this aspect for the end points, that is, the first and last 4-5 years.
2. For each country: Compute for each year the output gap in percent as ln(GDP) minus the trend value for ln(GDP) times 100 and plot the output gap against years. You may make four separate plots or one with all four output gap series. Comment on these aspects: Do the output fluctuations overall (qualitatively) look similar across countries? Do business cycles seem to become smaller or larger over time? And how strongly do the business cycles seem to be synchronized between the countries? For these questions you may want to disregard the first and last 4-5 years.
3. Perform the same exercise with $\lambda = 1000$. Are there significant differences in the estimated output gaps with the two alternative values of λ?
4. Use your estimated series for the output gap to offer a rough dating of recent business cycles in each country. Are the major peaks and troughs in your estimated output gap in line with your expectations, given your knowledge of recent economic history?

[6] For students in Macro II (or B) in Copenhagen the instruction on HP filtering and the data file are found on the Absalon page for the courrse under Files in the folder OPGAVER: The instruction file is "MAKRO_II_Instruction_HP Filtering_Jan22" and the data file is "Chap 13_Exc 2_GDP data_US_UK_SV_DK".

Exercise 3. Volatility of GDP, private consumption and private investment over the business cycle

At the web page for this book you will find a guide on how to perform HP filtering of economic time series in an Excel spread sheet on your computer. You will also find an Excel sheet containing quarterly data on real GDP, real private consumption and real investment for Denmark for a long period. [7] You may want to find corresponding quarterly data for your own country and use these instead. This exercise asks you to do HP filtering etc. of the time series following the instructions in the guide.

1. Apply the HP filter with λ=1600 to the series for the (natural) log of GDP, consumption, and investment to obtain the trends for each of these three series. Plot the actual data as well as the trends against time (quarters), inspect the graphs and comment on how well the HP filter seems to find the trend of the series. Comment on end point problems. Try changing λ to 100 and do the same. Intuitively, from inspection of the plots, does this substantially affect how well the HP filter captures the trend of quarterly data?
2. Compute the cyclical components (for λ=1600) for the log of each of the series, that is, the log of the actual value minus the trend of this. Also compute the cyclical components in percent (actual ln(variable) minus trend ln(variable) times 100); the latter are the "output gap", the "consumption gap" and the "investment gap" in percent, respectively.
3. Plot these gaps against time in the same diagram disregarding the first and last 16 quarters. Which of the variables can be seen to be more volatile, as measured by the size of the fluctuations around trend? Is this in line with the evidence presented in this chapter?

Let c_t be the cyclical component of ln(GDP) in period t, and x_t the cyclical component of the log of different series (GDP, consumption, investment).

4. Disregarding again the first and last 16 quarters, compute the contemporaneous coefficients of correlation, $\rho(x_t,c_t)$, for x_t being the cyclical component of the log of consumption and investment, respectively. Then compute the lagged and leaded coefficients of correlation, $\rho(x_{t-n}, c_t)$ for n = 2,1,-1,-2 and for x_t being the cyclical component of the log of GDP, consumption and investment, respectively (you may have to disregard a few more quarters at the ends). Use this to set up a table like Table 13.3 for Denmark for the involved variables. Compare to the findings for Denmark in Table 13.3. Is the overall picture confirmed? Is any of consumption or investment a

[7] For students in Macro II (or B) in Copenhagen the instruction and the data file are found on the Absalon page for the course under Files in the folder OPGAVER: The instruction file is "MAKRO_II_Instruction_HP Filtering_Jan22" and the data file is "Chap 13_Exc 3_data_Denmark".

leading or lagging variable to GDP?

Exercise 4. Measuring and decomposing the output gap: the production function approach

As we have seen, the output gap may be measured by detrending the time series for the log of real GDP by means of the HP filter. However, there is another and more elaborate way of estimating the output gap which allows an interesting decomposition of the gap. This exercise introduces you to this method which makes use of the concept of the aggregate production function known from the theory of economic growth.

Specifically, suppose that real GDP is given by the following Cobb–Douglas production function:

$$Y_t = B_t K_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{7}$$

where K_t is the aggregate capital stock, L_t is the aggregate number of hours worked, and B_t is the 'total factor productivity' measuring the combined productivity of capital and labour. By definition, total working hours are given as:

$$L_t = (1 - u_t)N_t H_t, \tag{8}$$

where u_t is the unemployment rate, N_t is the total labour force, and H_t is the average number of working hours per person employed. Hence we can specify GDP as:

$$Y_t = B_t K_t^{\alpha}[(1-u_t)N_t H_t]^{1-\alpha}. \tag{9}$$

Suppose now that Y_t, B_t, u_t, N_t and H_t all tend to fluctuate around some long-run trend levels denoted by $\bar{Y}_t, \bar{B}_t, \bar{u}_t, \bar{N}_t$ and $\bar{H}_t,$ respectively. By analogy to (54), we may then write trend output (also referred to as *potential output*) as:

$$\bar{Y}_t = \bar{B}_t K_t^{\alpha}[(1-\bar{u}_t)\bar{N}_t \bar{H}_t]^{1-\alpha}. \tag{10}$$

Note that this specification does not distinguish between the actual capital stock and its trend level, since we assume for simplicity that the capital stock is always fully utilized.

The output gap may be approximated by $y_t - \bar{y}_t,$ where $y_t \equiv \ln Y_t$ and $\bar{y}_t \equiv \ln \bar{Y}_t$. Taking logs on both sides of (9) and (10), subtracting the resulting expressions from one another, and using the approximation $\ln(1 - u) \approx -u$, we get:

$$y_t - \bar{y}_t \approx \ln B_t - \ln \bar{B}_t + (1-\alpha)[(\ln N_t - \ln \bar{N}_t) + (\ln H_t - \ln \bar{H}_t) - (u_t - \bar{u}_t)]. \tag{11}$$

Thus the output gap may be found as the cyclical component of total factor productivity, $\ln B_t - \ln \bar{B}_t,$ plus $1 - \alpha$ times the cyclical component of total labour input (the term within the square brackets). The latter may in turn be decomposed into the cyclical component of the labour force, $\ln N_t - \ln \overline{N}_t$, the cyclical component of average working hours, $\ln H_t - \ln \overline{H}_t$, and the amount of cyclical unemployment, $u_t - \overline{u}_t$.

Equation (11) may be used to estimate the output gap provided one has access to data on real GDP, the labour force, average working hours, unemployment, and the total capital stock. The cyclical components of labour supply and unemployment may be estimated by detrending the time series for $\ln N_t$, $\ln H_t$ and u_t by means of the HP filter. To estimate the cyclical component of (the log of) total factor productivity, one may proceed as follows. First, take logs on both sides of (7) and rearrange to find:

$$\ln B_t = \ln Y_t - \alpha \ln K_t - (1 - \alpha) \ln L_t, \tag{12}$$

where L_t may be calculated from (8) (or where data on L_t may be directly available). From the theory and empirics of economic growth we know that the magnitude $1 - \alpha$ should correspond roughly to the labour income share in total GDP which is close to $\frac{2}{3}$ in most countries. Hence, we may set $\alpha = \frac{1}{3}$. Plugging the data for Y_t, K_t and L_t into equation (12) then gives an estimate of total factor productivity. In a second step, one may detrend the estimated time series for $\ln B_t$ through HP filtering to obtain an estimate of the cyclical component of total factor productivity. The estimates for $\ln B_t - \ln \overline{B}_t$, $\ln N_t - \ln \overline{N}_t$, $\ln H_t - \ln \overline{H}_t$, and $u_t - \overline{u}_t$ may finally be inserted into (56) along with $\alpha = \frac{1}{3}$ to give an estimated time series for the output gap.

1. Try to find annual data on Y, K, N, H, L and u for your country, going back in time as far as the data allow (as a default, you may also use the data for the United States available at the web page for the book). Use Equations (11) and (12) and the HP filter to estimate the output gap and its components, setting $\lambda = 100$ in accordance with standard practice for annual data (you may get access to a computer facility performing HP filtering at the web page for the book).

2. Use your estimates from Question 1 to calculate the variables $(1-\alpha)(\ln L_t - \ln \overline{L}_t)$, $(1-\alpha)(\ln \overline{u}_t - \ln u_t)$, $(1-\alpha)(\ln H_t - \ln \overline{H}_t)$, and $(1-\alpha)(\ln N_t - \ln \overline{N}_t)$. Construct a diagram showing the evolution of the output gap $y_t - \overline{y}_t$ and the variable $(1-\alpha)(\ln L_t - \ln \overline{L}_t)$. This diagram will illustrate how much fluctuations in total working hours contribute to fluctuations in the output gap. Construct three similar diagrams where each diagram shows the evolution of the output gap along with one of the three gaps $(1-\alpha)(\ln \overline{u}_t - \ln u_t)$, $(1-\alpha)(\ln H_t - \ln \overline{H}_t)$, and $(1-\alpha)(\ln N_t - \ln \overline{N}_t)$. Describe which of the four gaps that contribute the most to the fluctuations in the total output gap. (Note: because of the imprecise end-point estimates produced by the HP filter, you should leave out your estimates for the first and last three years in each of your estimated time series).

3. Given the assumption that the capital stock is always fully utilized (so that there is no need to distinguish between K_t and $\bar{K}_t$), it follows from Equation (12) that the vertical distance between the curve for the output gap and the curve $(1-\alpha)(\ln L_t - \ln \bar{L}_t)$ measures the contribution of cyclical fluctuations in total factor productivity to the total gap. Describe how important these fluctuations in the productivity gap $\ln B_t - \ln \bar{B}_t$ are for the fluctuations in the output gap, compared to the contribution $(1-\alpha)(\ln L_t - \ln \bar{L}_t)$ from the fluctuations in the total hours gap. Discuss whether the estimated fluctuations in the productivity gap are likely to reflect "genuine" fluctuations in the level of technology or whether they could also be due to other factors.

4. Estimate a time series for the output gap for your country by running an HP filter through the time series for the natural log of real GDP with $\lambda = 100$ (you may also reuse your HP estimate from Exercise 2, assuming it is based on the same time series for real GDP as the one used in this exercise). Draw your estimate in a diagram and compare it to your estimate of the output gap based on the production function method. Are the two estimates reasonably close?

Chapter 14

Business cycles: Costs

Introduction

In the previous chapter we saw that there are considerable business cycle fluctuations in output, employment, consumption and inflation etc. The swings in GDP are quite persistent and on average a bit larger than 1.5 percent of the trend level, sometimes much larger, but almost always contained within a band of 4-5 percent on both sides of the trend. Aggregate consumption and employment fluctuate a bit less than, but highly correlated with GDP and with similar persistence. Inflation typically moves up and down by about 0.5 percentage points around trend and the movements are not very closely correlated to the swings in GDP.

Now we ask: why are business cycles harmful? In other words, why should fiscal and monetary stabilization policies aim at dampening business cycles?

To be more precise, the question is: Why and to what extent is 'society' worse off when variables such as GDP, consumption, employment and inflation fluctuate around their trend levels rather than just stay at those trend levels all the time?

The answer is *not* that the fluctuations are harmful because of the losses that occur when, say, consumption drops below its trend level, because the mirror image of these downswings are the consumption gains that occur when consumption rises above trend. The subtle issue is why the welfare effects of these fluctuations in opposite directions do not cancel out.

To answer this question we must study what business cycles do to the individuals in society, i.e., the people that public policy is concerned with. For example, we must explain why it would be better for the citizens in a country if itsGDP evolved over time as 100, 100, 100 … rather than as 98, 102, 98, 102 …? Presumably people are concerned with their own consumption and employment levels and the consumption and employment level of other people they care about, but not with business cycles per se. So why should the fluctuations in *aggregate* output, consumption, employment and inflation be harmful to the evolution of *individual* consumption and employment levels?

In the following we start by explaining the harmful effects of fluctuations in the *real* variables such as GDP, consumption and employment. We then turn to the costs of fluctuations in *nominal* variables such as inflation. The structure of our exposition is illustrated in Figure 14.1, and as we go along you may want to return to this overview to see where in the big picture our explanations belong.

Note that instability of output, consumption and employment could be costly for two kinds of reasons.

First, the instability could be harmful in itself. As we shall see, the instability of aggregate output does create fluctuations at the individual level in consumption and employment that could damage the welfare of individuals.

Second, the instability could have a negative impact on the average, trend values of aggregate output and consumption that the actual values fluctuate around, an effect that would translate into a negative impact on the average consumption level of individuals. This would mean that the higher the instability of GDP, consumption etc. at the aggregate level, the lower the structural (trend) levels of GDP, consumption etc. would be.[1]

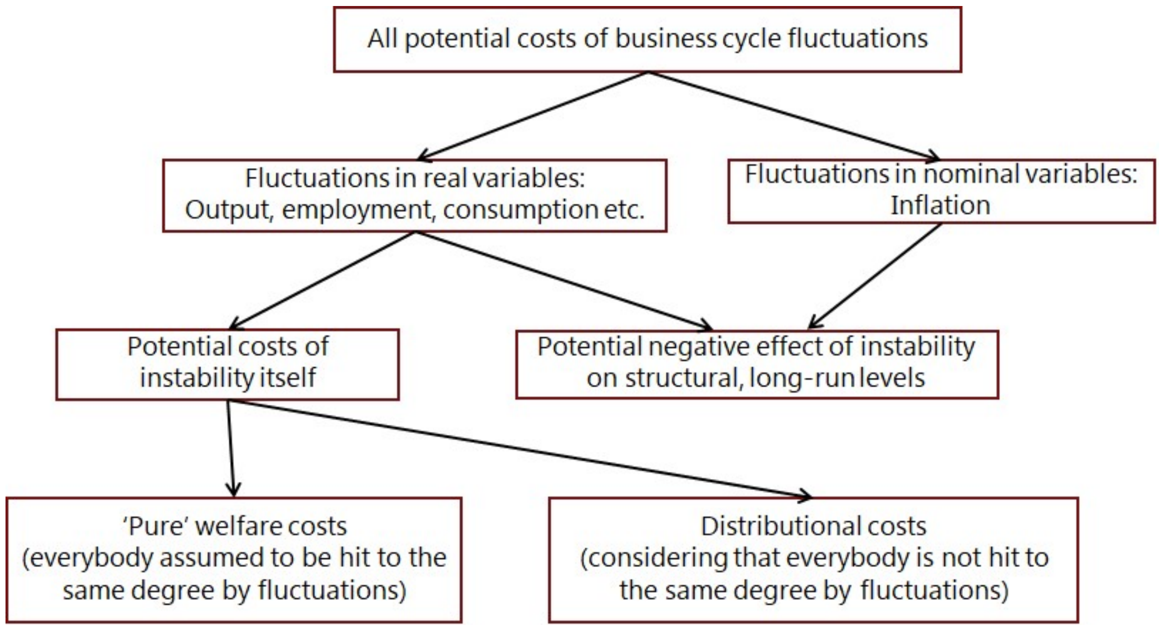
In terms of our example above, the first type of reason says that it is worse if GDP follows the time path 98, 102, 98, 102 … than if it stays constant at 100. The second type of reason says that if the fluctuation could be eliminated, then the stable outcome would not be the average of the unstable outcomes, the 100, all the time, but a better value, 101 say. In that case, an additional cost of the instability would be the one unit of average income that is lost permanently in all periods due to the instability, i.e., the extra average income that could be gained from stability.

In our discussion of the costs of swings in aggregate output, consumption and employment, we will first disregard the second type of reason, that is, we assume that the degree of aggregate instability does not affect the structural levels of important macroeconomic variables. Later on we will return to this assumption and see that it may be implausible.

We will divide the costs of instability itself into 'pure' welfare costs and distributional costs, as illustrated in Figure 14.1. We explain this division in more detail below. Our discussion of the costs of the swings in inflation will focus on the potential negative effects of such swings on structural levels, see again Figure 14.1.

[1] We already touched upon this possibility in Chapter 1 in our discussion of the correlation between the instability of GDP growth and the final level of GDP per worker shown in Figure 1.6.

Figure 14.1 Overview: A taxonomy of the potential social costs of business cycle fluctuations



14.1 The 'pure' welfare costs of output, employment and consumption fluctuations

Aggregate and individual fluctuations

Note first that the fluctuations in GDP and aggregate income would not *have* to result in fluctuations in aggregate or individual consumption. At an *aggregate level*, the national accounting identity is $Y = C + I + NX$, where Y is GDP, C is aggregate consumption (private plus public), I is aggregate investment and NX is net exports. Hence, a downswing in GDP would not have to imply a downswing in aggregate consumption if only it were offset by a downswing in the sum of aggregate investment and net exports, that is, a savings reduction of the same size. At an *individual level*, such consumption smoothing could be brought about through transactions on credit and insurance markets.

To some extent, individual consumers can use the private capital market to smooth consumption over time, borrowing or spending accumulated wealth to finance additional consumption when their income is low and postponing consumption through saving when their income is unusually high. In addition to borrowing in the capital market, people can insure themselves against temporary income losses by taking out insurance, say, insurance against unemployment. In this way, they can be at least partly compensated for income losses due to 'bad times'.

These considerations are correct, but the business cycle facts in the previous chapter tell us that the fluctuations in GDP are indeed accompanied by fluctuations in private consumption that are only a bit smaller than those of GDP and highly correlated with these. Despite the presence of credit and insurance markets, aggregate consumption is not smoothed over the business cycle.

There are good reasons why capital and insurance markets cannot do a perfect job of consumption smoothing. Capital markets are not perfect, so some consumers may be subject to credit constraints. In particular, during a serious recession the market values of many assets may fall considerably and destroy much of the collateral that consumers normally use to obtain credit. Furthermore, those who do not have much collateral but would borrow against their expected future incomes in a normal situation will typically lose their borrowing possibilities if they lose their jobs.

Likewise, unemployment insurance markets cannot be perfect either. A person who is fully insured against any income loss from unemployment has little incentive to avoid joblessness. If he or she values leisure, he or she may therefore choose to stay out of work for long periods. Because of this so-called *moral hazard problem* it is suboptimal for private insurance companies (as well as for the government) to offer full insurance against income losses from unemployment.

On top of this, the demand for unemployment insurance is likely to come mainly from those who face the highest risks of unemployment. If the premium for unemployment insurance is set to cover the *average* income loss from unemployment across a large group of insured workers, individuals in very risky jobs have a strong incentive to buy insurance, because the value of their expected unemployment benefits will exceed the value of the insurance premium. Workers facing a relatively low risk of losing their jobs may find that the premium is too high relative to their expected unemployment benefits. It is reasonable to assume that the insurance company cannot fully tell apart high-risk and low-risk people, which is why it cannot differentiate the premiums and must set the premium to cover average income losses. On this background, the workers with a low probability of unemployment may decide to drop out of the insurance scheme. It will then only cover the high-risk people and the premium will be accordingly high. Because of this so-called *adverse selection problem*, private insurance companies may rightfully fear that those who want to take out insurance represent only the highest risks, and companies must therefore either set a relatively high insurance premium preventing some participation or decide that offering insurance will not be profitable. In this way, the adverse selection problem will cause too little unemployment insurance supplied by the market.

Thus, the problems of moral hazard and adverse selection limit the scope for individual consumption smoothing through private credit and insurance markets. The moral hazard problem also makes it suboptimal for the government to offer full insurance against private income losses through the system of public transfer payments.[2] In

[2] A successful macroeconomic stabilization policy may substitute for what could have been obtained through credit and insurance markets if these had not been plagued by the market failures described. Stabilization policy can thus raise consumer welfare by evening out the time path of real income. We may

consequence, consumption is not perfectly smoothed over the business cycle.

Likewise, since capital is quite fixed in the short run being the result of capital accumulation over many years, the fluctuations in GDP have to be accompanied by fluctuations in aggregate employment. In a globalized world, one could imagine that the employment of domestic residents were smoothed over the cycle by inflow and outflow of foreign workers. Indeed, this happens to some extent in the modern world, but not to an extent close to smoothing the total employment of residents.

In the following, we will simply take for granted what the business cycle facts tell us: the fluctuations in aggregate output are accompanied by about equally sized fluctuations in aggregate consumption and employment that are highly correlated with the GDP fluctuations. Given the fact that there are fluctuations in consumption and employment *at the aggregate level*, so there must be *at the individual level*.

The individual fluctuations will typically not mimic the aggregate fluctuations completely. Constant aggregate consumption, for instance, could reflect that individual consumption goes up for some persons and down for others, so the individual fluctuations will be larger than the aggregated. Likewise, if aggregate consumption goes down by two percent, this could reflect that consumption is actually unchanged for a majority of persons, but goes down by much more than two percent for a minority.

To highlight the distinction between 'pure' welfare effects and distributional effects, we will first make the heroic assumption that all individuals are hit in exactly the same way by the aggregate fluctuations. Afterwards we relax this assumption, but initially we assume that if aggregate consumption swings by +/- 1.3 percent around trend on average, so does the consumption for all individuals. Likewise, if aggregate employment fluctuates by +/- 1.4 percent on average, so do work hours for each individual worker. We know that this assumption is far from being true. Particularly, when aggregate employment falls, the employment of most people will typically be unaffected while much fewer people will experience large drops in employment. Initially we abstract from this important fact for analytical reasons in order to disentangle the 'pure' welfare costs of instability (in aggregate output, consumption and employment), i.e., the costs if everybody were hit the same way, from the social costs associated with the distributional effects of the instability, i.e., the costs that society incurs when some people are more severely affected than others.

To recap, for now we operate under two idealized assumptions. First, we assume that instability does not have an impact on structural levels. This is to disentangle the business cycle costs arising from instability itself from costs arising from an impact on trend values. Second, we assume that everybody is hit by the business fluctuations in the same way. This is to distinguish the pure welfare costs of instability from the costs arising from the inequalities caused by business cycles. We will in due time relax both of these assumptions. Indeed, we will argue that a large part of the social costs of business cycles are the costs that appear when we do not impose these two assumptions.

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say that effective stabilization policy provides a type of *income insurance* that substitutes for the smoothing that cannot be delivered through the private market or the public transfer system.

Consumption fluctuations

What are the welfare costs of fluctuations in individual consumption levels? Suppose the consumption of a person swings symmetrically between 98 and 102. Why should this be worse for the person than consumption staying constantly at the average level 100? The answer lies in a standard assumption in microeconomics: *diminishing marginal utility*. If the extra utility from an additional unit of consumption decreases with the level of consumption already attained, then, compared to a consumption stream of 100 units all the time, the extra utility from the two additional units of consumption obtained in the 'good periods' is smaller than the utility loss resulting in the 'bad periods' when consumption is two units below average. Hence, on average over time the fluctuating consumption stream gives less total utility than the constant one.

The fluctuating consumption stream we consider does not have to be deterministic and go like 98, 102, 98, 102 … over the periods. It might take the form that in any given period the individual faces a 50 percent chance of a consumption level of 98 and a 50 percent chance of 102, not knowing before the period which will be the case, but knowing that consumption will be pulled from the 'lottery' described. We may use the notation $\frac{1}{2}\times 98 \oplus \frac{1}{2}\times 102$ for this lottery.[3]

In both cases, the fundamental and standard assumption of diminishing marginal utility says that it is better to have a safe constant consumption of 100 than having consumption fluctuating symmetrically between 98 and 102. In both cases we can express this preference by means of a concave utility function $u(C)$, that gives the single period utility generated by the consumption level C in the period. It is further assumed that the total utility over all periods of a consumption stream $C_1, C_2, C_1, C_2, \ldots$ where $C_1 \neq C_2$, is the average value of the instantaneous utilities, that is, $U(C_1, C_2, C_1, C_2, \ldots) = \frac{1}{2}u(C_1) + \frac{1}{2}u(C_2)$,[4] and likewise that the total utility of a consumption lottery $\frac{1}{2}\times C_1 \oplus \frac{1}{2}\times C_2$ is the expected instantaneous utility, that is, $U(\frac{1}{2}\times C_1 \oplus \frac{1}{2}\times C_2) = \frac{1}{2}u(C_1) + \frac{1}{2}u(C_2)$. We thus apply the 'expected utility hypothesis'.

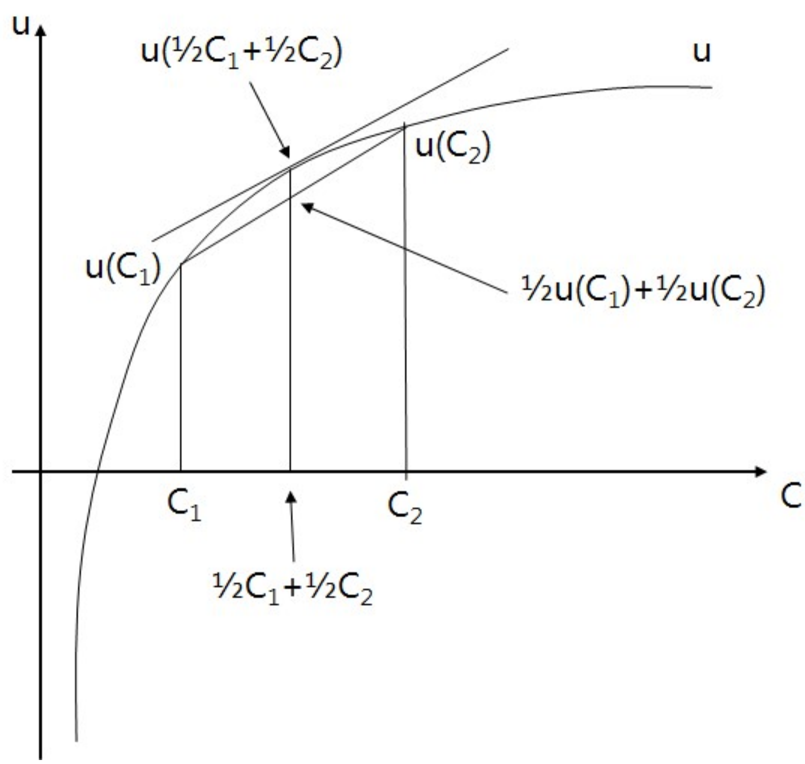
We assume that u is increasing, $u'(C) > 0$: more consumption gives more utility. If we also assume that u is concave, $u''(C) < 0$, exhibiting diminishing marginal utility, then the utility of the consumption stream $C_1, C_2, C_1, C_2, \ldots$ and of the consumption lottery $\frac{1}{2}\times C_1 \oplus \frac{1}{2}\times C_2$ will be smaller than the utility of obtaining the average consumption, $\frac{1}{2}C_1 + \frac{1}{2}C_2$ constantly or for sure as long as $C_1 \neq C_2$, that is, $u(\frac{1}{2}C_1 + \frac{1}{2}C_2) > \frac{1}{2}u(C_1) + \frac{1}{2}u(C_2)$. Figure 14.2 illustrates and in fact proves this statement.

Thus, the concavity of the utility function u captures the assumed aversion to

[3] These highly regular cycles disregard realistic features such as the irregularity and persistence of real world cycles. However, as we shall explain later on, our analysis carries over to more general fluctuations.
[4] For simplicity, discounting of future utilities is disregarded or, in other words, the discount factor is assumed to be one.

instability and riskiness of consumption, and from Figure 14.2 it is obvious that the more concave (curved) u is, the more aversion to instability and risk there is.

Figure 14.2 Illustration: concavity and aversion to instability and risk



The figure also illustrates that the loss due to fluctuating around the average rather than staying at the average all the time is of 'second order': it only arises because the utility function has a (concave) curvature. Had the utility function been linear over the considered range of consumption levels as indicated by the tangent in the figure, that is, had its first derivative been a constant, there would be no total loss (gains and losses would cancel). It is only because the first derivative is decreasing (the second derivative is negative) that there is a loss.

The reasoning above can help us to quantify how much worse the unstable or risky consumption stream is, compared to the smoothed and sure one. To do so we will use a famous measure of the degree of concavity of the utility function: $\rho(C) \equiv -u''(C)C/u'(C)$. As you can see, this is defined so that $\rho(C) > 0$, and it is the elasticity of the slope of the utility function, $u'(C)$, with respect to C, so it measures the relative decrease in this slope for a given relative increase in C. Hence, it is a measure of the degree of curvature or concavity of U. It is referred to as the 'Arrow-Pratt coefficient of *relative* aversion to instability or risk'.[5] It has a beautiful and intuitive interpretation:

Compare a stable/sure consumption stream of C in all periods to an unstable/risky one alternating between $(1-x)C$ and $(1+x)C$ with weight/frequency ½ for both of these consumption levels. Here $x>0$ is *the relative* deviation from average/trend in the fluctuating consumption stream. We already know that with a concave utility function the first stream is preferred to the second, but how much is it preferred? One measure of this is the degree by which we must increase the weight of the good outcome, $(1+x)C$, and consequently reduce the weight of the bad outcome, $(1-x)C$, in order for the unstable/risky stream to be just as good as the stable/safe one of C all the time. In other words, an intuitive measure of the degree of instability/risk aversion is the 'frequency or probability premium' $\pi(x)$ that exactly ensures that:

$$u(C)=\left[\frac{1}{2}+\pi(x)\right]u\big((1+x)C\big)+\left[\frac{1}{2}-\pi(x)\right]u\big((1-x)C\big) \tag{7}$$

It is obvious that $\pi(x)\to 0$ for $x\to 0$. We will be interested in $\pi'(0)\equiv \lim_{x\to 0}\pi'(x)$ as our intuitive measure of the degree of (relative) instability or risk aversion *just at the consumption level C*. It is the rate by which we must increase the weight of the good outcome just as we begin to spread the consumption levels on both sides of C if the consumer is to be indifferent between the unstable/risky situation and the stable/safe one. To find $\pi'(0)$ one can differentiate (7) twice with respect to x, and then in the resulting expression let x go to zero. It is left as an exercise for you to show that this (miraculously) gives: $4\pi'(0)=-u''(C)C/u'(C)$. The factor 4 here is just a scaling parameter, so we have justified that the coefficient of relative risk aversion, $\rho(C)=-u''(C)C/u'(C)$, is an appropriate and intuitive measure of the degree of aversion to instability and risk locally at the consumption level C.[6]

An often considered specific utility function is:

$$u(C)=\frac{C^{1-\theta}}{1-\theta}\ \text{ for } \theta>0,\neq 1 \quad\text{and}\quad u(C)=\ln C\ \text{ for } \theta=1. \tag{8}$$

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[5] An alternative measure of the degree of curvature is the 'Arrow-Pratt coefficient of *absolute* aversion to instability or risk' defined as $-u''(C)/u'(C)$, which is the semi-elasticity of u' with respect to C.

[6] If we had considered a deviation in absolute terms, so that consumption would fluctuate between $C - z$ and $C + z$, where $z > 0$, we could in a similar way define the premium $\pi(z)$ by $u(C)=[½+\pi(z)]u(C+z)+[½-\pi(z)]u(C-z)$. In that case, $4\lim_{z\to 0}\pi(z)=-u''(C)/u'(C)$, thus rationalizing and giving an interpretation of the Arrow-Pratt coefficient of absolute aversion. Show this as an exercise (this is Exercise 3 at the end of the chapter).

Note that u is an increasing function also for $\theta > 1,$ it just takes negative values in that case. You will easily verify that for this function $\rho(C) \equiv -u''(C)C / u''(C) = \theta$ independently of C, and this holds both for $\theta \neq 1$ and for $\theta = 1.$ Hence, for this function the degree of relative instability/risk aversion is θ everywhere, and the larger θ is, the more instability and risk averse the person in question is. The function is called a CRRA (Constant Relative Risk Aversion) utility function.[7]

Economists have tried through experimental and empirical studies to obtain plausible estimates of the size of θ in the utility function (8). They study behavior in different laboratory, survey or real world choice situations involving instability or risk and ask: if we assume that people act as if they were maximizing a utility function like (8), of what size should θ be in order for the observed choices to fit best with the behavior predicted from utility maximization? There is some controversy about the most plausible size of θ and estimates vary a great deal and depend on, e.g., whether it is consumption, income or wealth (or something else) that is inside the function u. We think it is fair to say that particularly for consumption, an estimate of θ around 2 can be considered a reasonable central (average) estimate with θ around 4-5 an upper estimate.

Equipped with the function u specified in (8), let us again compare the stable/sure consumption stream of 'always C' to the unstable/risky one of '$(1-x)C$ and $(1+x)C$ with fifty-fifty weights'. Even though real world business cycles are much less regular, in view of Figure 13.3 and the standard deviations reported in Table 13.2, a value of the relative deviation x of around 1.5 percent would reasonably mimic the average volatility of real world consumption cycles. A value of x of 4 percent would more or less 'envelope' most real world fluctuations.[8]

The total utility of the stable/safe stream is $\bar{U}(C) = C^{1-\theta} / 1-\theta,$ while the utility of the unstable/risky stream is:

$$\tilde{U}(C,x) = \frac{1}{2}\frac{\left[(1-x)C\right]^{1-\theta}}{1-\theta} + \frac{1}{2}\frac{\left[(1+x)C\right]^{1-\theta}}{1-\theta} = \frac{C^{1-\theta}}{1-\theta}\frac{1}{2}\left[(1-x)^{1-\theta} + (1+x)^{1-\theta}\right]$$
$$= F(x)\bar{U}(C), \quad \text{where } F(x) \equiv \frac{1}{2}\left[(1-x)^{1-\theta} + (1+x)^{1-\theta}\right]. \tag{9}$$

We know in advance that $\tilde{U}(C,x) < \bar{U}(C),$ and from (9) follows that $F(x)$ is the

[7] One can show that for the utility function $u(C) = -\exp(-\omega C)$, the coefficient of absolute aversion, $-u''(C)/u'(C),$ is ω independently of C (this is Exercise 4). This function is therefore called a CARA (Constant Absolute Risk Aversion) function. Exercise 4 also invites you to discuss what you find most convincing, a CRRA or a CARA utility function defined on consumption levels?

[8] One might wonder whether the evaluations of the cost of fluctuations we derive here depend on the assumed highly regular form of the consumption cycles considered. An exercise at the end of this chapter, where a more general type of fluctuation is considered, will show that this is not the case.

utility loss factor caused by the fluctuations. For $\theta > 1,$ utility is negative, so in that case a utility loss corresponds to $F(x) > 1.$ If we consider the plausible valuation $\theta = 2,$ we easily compute that for $x = 0.02,$ which corresponds to a consumption volatility on the upper side of the average volatility of real world business cycles, we get $F(0.02) = 1.0004,$ amounting to a utility loss of 0.04 percent. If we consider the upper envelope of volatility of consumption fluctuations, $x = 0.04,$ we get $F(0.04) = 1.0016,$ a utility loss of 0.16 percent. If, again, we let $x = 0.02,$ and now consider the upper evaluation of our instability and risk aversion parameter $\theta = 5,$ we get $F(0.04) = 1.004,$ corresponding to a utility loss of 0.4 percent. Even for these upper evaluations of volatility and instability/risk aversion, the utility loss from fluctuations seems small.

It may be argued that utility is really more of an ''ordinal' concept (that the only aspect of the utility function of importance is what it implies for *choices*, so that the exact *level* of utility is not important), and that a utility loss of, say, 0.4 percent does not say much.[9] Therefore, we may instead want to ask: by what fraction *e* should we reduce the stable/safe consumption level *C* in order for the stable and the fluctuating streams to become equally good? This *e* will then measure the consumption loss that the fluctuations give rise to, or in other words, the cost of the fluctuations measured as their consumption equivalent, a physical measure that only depends on the ordinal aspect of the utility function. The condition that *e* should fulfil is $\tilde{U}(C,x) = \bar{U}([1-e]C),$ where:

$$\bar{U}([1-e]C) = \frac{([1-e]C)^{1-\theta}}{1-\theta} = (1-e)^{1-\theta}\frac{C^{1-\theta}}{1-\theta} = (1-e)^{1-\theta}\bar{U}(C). \tag{10}$$

From (9) and (10) it is easy to see that $\tilde{U}(C,x) = \bar{U}([1-e]C)$ is equivalent to:

$$(1-e)^{1-\theta} = F(x) \quad \text{or} \quad e = 1 - F(x)^{\frac{1}{1-\theta}} \equiv e(x). \tag{11}$$

Now, for $\theta = 2,$ we have $e(x) = 1 - F^{-1} = (F-1)/F \approx F - 1,$ where the latter approximation holds for *F* not too far from one. So, $e(x) \approx F(x) - 1,$ and from our former calculations the cost of consumption fluctuations will, for an amplitude of 2 percent, amount to around 0.04 percent of consumption and for an amplitude of 4 percent to around 0.16 percent of consumption. For $x = 0.02$ and $\theta = 5,$ we get from (11) that $e(0.02) \approx 0.001,$ a consumption sacrifice of 0.1 percent. Whether we evaluate directly by the utility loss or by the consumption sacrifice, the individual costs of consumption

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[9] If the utility function *u* we consider is multiplied by a constant and has a constant added to it (an 'affine' transformation) it will not change what is most preferred (highest utility yielding consumption streams), but it can affect the relative difference between the utilities of different consumption streams.

fluctuations in isolation seem to be quite small under the assumption that everybody is hit to the same degree by the fluctuations.

Employment fluctuations

We have only considered consumption fluctuations so far. People are also concerned with work hours. Perhaps the instantaneous utility function should not be just $u(C)$, but rather $u(C)-v(L)$, where u could be given by (8), and $v(L)$ would be a utility subtraction, the disutility of working L hours, where the 'disutility' stems from the loss of leisure implied by spending time at work. A common formulation for the disutility function is:

$$v(L)=\beta\frac{L^{1+\mu}}{1+\mu},\quad \beta>0,\ \mu>0, \tag{12}$$

where β is a scaling factor. Maintaining the heroic assumption that everybody is affected by the business cycles to the same degree, we will now show that the disutility function (12) implies a larger relative welfare loss from employment fluctuations than the relative welfare cost of consumption fluctuations.

Comparing again a highly regular cycle where employment fluctuates between $(1-z)L$ and $(1+z)L$ with weights ½ and ½ around the constant trend level L, derivations parallel to those above imply that the expected disutility of the fluctuating stream of work hours is larger than the disutility of the stable one by a factor $G(z)\equiv \tfrac{1}{2}\left[(1-z)^{1+\mu}+(1+z)^{1+\mu}\right]$, where β does not appear. According to Table 13.2, employment fluctuations are on average 1.1 to 1.4 percent around trend, so a plausible (upper) evaluation of z could be $z=0.015$. What could a plausible value of μ be?

Assume that a person decides on consumption and labour supply in any period by maximizing $u(C)-v(L)$ subject to the budget constraint $C=wL+I$, where w is the real wage and I is other income, and v is given by (12). From the first order condition for maximizing $U(wL+I)-V(L)$ with respect to L, labour supply is (check this):

$$L=\left[\frac{1}{\beta}U'(wL+I)\right]^{\frac{1}{\mu}}w^{\frac{1}{\mu}}=\left[\frac{1}{\beta}U'(C)\right]^{\frac{1}{\mu}}w^{\frac{1}{\mu}}. \tag{13}$$

This shows that the *compensated* elasticity of labour supply is $1/\mu$, that is, the percentage change in L for a one percent increase in w under the hypothetical situation that I is adjusted so that the old C and L could still be obtained (they fit in the budget constraint), amounting to holding the C in (13) unchanged. Economists have put much effort into estimating realistic compensated labour (hours) supply elasticities and often found these to be quite small, e.g. around 0.1, which would correspond to a value of μ around 10. For $z = 0.015$ and $\mu = 10$, one has $G(0.015) \approx 1.0124$. Hence, for this value of μ, an evaluation of the welfare cost of employment fluctuations of a realistic size would be an increase in the disutility of working of around 1.25 percent. Again, we can convert this measure into an 'employment equivalent', which turns out to be $d(z) = G(z)^{\frac{1}{1+\mu}} - 1$.[10] A plausible value of this d calculated for the same values of z and μ is 0.0113 or 1.13 percent, implying that the representative worker would be willing to accept a permanent 1.13 percent increase in working hours relative to the current average number of hours worked if she could be sure that her employment would always stay constant at this higher level.

This is a non-negligible loss, but probably also exaggerated. Other estimates of the compensated elasticity of labour supply that attempt to account for short run frictions in working hours are around 0.3 corresponding to a value of μ around 3. For this value one gets $G(0.015) \approx 1.0014$. Now the welfare loss (disutility increase) is down to 0.14 percent, and the corresponding employment sacrifice is down to 0.003 percent (check this), so the loss from fluctuations is again quite small. The latter estimate is probably closer to the truth, since individual decisions on working hours are often restricted by collective bargaining agreements or by legislation, so probably the small estimated elasticities of hours of labour supply exaggerate the true value of the preference parameter μ.

Given the simple separable form of the period utility function assumed here, $u(C) - v(L)$, we can simply add the cost of consumption fluctuations and the cost of employment fluctuations to obtain the total cost of business cycles, but 'small plus small' remains small. For example, the estimated plausible 0.04 percent welfare loss from consumption fluctuations plus the plausible 0.14 percent loss from employment fluctuations is still below 0.2 percent.

Overall, from sober evaluations the conclusion seems unavoidably to be:

THE PURE WELFARE COSTS OF REAL BUSINES CYCLE FLUCTUATIONS

Under the idealized assumptions that 1) instability of output, consumption and employment does not affect the structural levels of these variables, and 2) all people are affected by business cycles in the same way and to the same extent, the welfare costs of realistically sized business cycle fluctuations in output, consumption and employment seem to be quite small.

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[10] An exercise at the end of the chapter will ask you to derive the expressions for both $G(z)$ and $d(z)$.

Stabilization policy motivated by the pure welfare costs of fluctuations in output, employment and consumption

One may therefore ask if it is really worth the effort to conduct fiscal and monetary stabilization policies attempting at smoothing business cycles. The risks of timing and dosing such policies badly (as we will return to in coming chapters on stabilization policy) only makes this question more relevant.

In fact, some economists do think and advocate that stabilization policies should be avoided altogether exactly because they consider business cycle costs to be small *and* the probability of policy error to be high. A more widespread view among economists and politicians is that policy makers should not attempt to 'fine tune' the economy to avoid even small fluctuations, but that large fluctuations should be mitigated by stabilization policies. According to this view events such as the overheating of many western economies in the period running up to the great financial crisis of 2008-09, and the large economic downturn that occurred during that crisis, should be counteracted by stabilization policy.[11]

An important point in this discussion is that business cycles and their welfare costs could be small exactly because governments and central banks do conduct successful stabilization policies, and that they might have been much larger in the absence of such policies. In that case, the smallness of the fluctuations is not an argument against stabilization policy, of course. It could even be that the general knowledge that governments and central banks do try to minimize fluctuations have a stabilizing effect. If households and firms expect that an economic downturn will trigger policy reactions that reduce the downturn, they will be less inclined to reduce consumption and investment when the economy seems to be on the verge of a recession, and this may in itself, by reducing the fall in demand, reduce the fall in activity. Without actually activating the stabilization policies too much, the fact that people know they will be activated if necessary may be partly responsible for the limited size of observed business cycles in the period since World War II when the rationale for stabilization policy became widely accepted.

Furthermore, our conclusion that the social costs of business fluctuations in output, consumption and employment are small rested on the idealized assumptions 1) and 2) of the box above. It is now time to relax these assumptions. We start with the second one.

14.2 The distributional costs of output, employment and consumption fluctuations

To illustrate that people are not all hit to the same degree by business cycles, Table 14.1

[11] Many economists argue that the financial crisis became so severe because governments and central banks had failed to conduct a sufficiently tight stabilization policy during the upswing before the crisis.

restates Table 10.2 of Chapter 10.

In a ‘good year’ (here 2006) with an overall rate of unemployment of 4-5 percent (in USA or Denmark), only 16-17 percent of workers are affected by unemployment at all during the year. And out of these a great majority, up to 70 percent, are unemployed for less than 14 weeks during the year, and less than 20 percent are unemployed for more than half the year, where the latter is often considered a criterion for long-term unemployment. In a ‘bad year’ with an overall unemployment rate of 12 percent (1993 in in Denmark), almost 30 percent are affected by unemployment during the year, so still a great majority of more than 70 percent is not affected at all, but now, out of the roughly 30 percent affected, more than 35 percent are long-term unemployed. This shows that in an economic downturn, most people are not directly affected (although they may suffer from the fear of losing job and income or from family and friends experiencing that), while a smaller group of people carry most of the drop in employment and income on their shoulders. If society is concerned with equality, this is bad in itself.

Table 14.1 Duration and incidence of unemployment

1993			2006		
Percentage of people experiencing unemployment during the year, who were unemployed for	**USA**	**Denmark**	**Percentage of people experiencing unemployment during the year, who were unemployed for**	**USA**	**Denmark**
Less than 5 weeks	37	21	Less than 5 weeks	37	38
5–14 weeks	29	23	5–14 weeks	32	27
15–26 weeks	14	19	15–26 weeks	15	16
27 or more weeks	20	36	27 or more weeks	16	19
	100	100		100	100
Persons having experienced unemployment as a percentage of the labour force	20	29	Persons having experienced unemployment as a percentage of the labour force	16	17
Unemployment rate, %	7	12	Unemployment rate %	5	4

Sources: Bureau of Labor Statistics (US) and Statistics Denmark (Statistisk Tiårsoversigt).

Above we saw that if individual household consumption levels mirror the aggregate consumption fluctuations, households would probably not be much better off if these fluctuations were smoothed out. In a world where unemployment hits unevenly and with imperfect credit markets, however, the consumption of specific individual households may be far more volatile than aggregate consumption, and hence these households would benefit more from the elimination of macroeconomic volatility. Economists Kjetil Storesletten, Chris Telmer and Amir Yaron have documented that the standard deviation of earnings across households in the USA more than doubles during a recession and that shocks to individual household earnings are highly persistent. When a specific household's earnings falls in the current year, its earnings are thus likely to be lower for quite a while.[12] The sharp rise in the dispersion of expected individual incomes during recessions shows that the costs of business cycle swings are much larger for some households than we found under the 'everybody-equally-affected' assumption.

Moreover, economic research has documented that the employment and income losses caused by recessions tend to be concentrated among the poorer members of society. Manual workers – and in particular low-paid unskilled workers and young workers – tend to suffer a much larger decline in consumption than other people do during times of recession.[13] For these groups, who already earn low incomes while employed, recurring recessions and the associated spells of unemployment can imply a serious welfare loss. Hence, a society concerned about inequality should also be concerned about the instability of output and employment.

To recap:

THE DISTRIBUTIONAL COSTS OF BUSINESS CYCLES

There are great differences in how much people are affected by business cycles and, in particular, the employment and income losses during economic downturns are concentrated on relatively few people. In a society averse to inequality, this means that business cycles generate distributional costs. These costs are reinforced by the fact that people who already have low income and education levels are disproportionately affected by the downturns.

In summary, a large part of the social costs of business cycles in real variables like output, consumption and employment seems to take the form of higher inequality in the distribution of income and welfare, so a main motivation for stabilization policies aiming at smoothing out these variables may lie in distributional concerns.

[12] See Kjetil Storesletten, Chris Telmer and Amir Yaron, 'The Welfare Cost of Business Cycles Revisited: Finite Lives and Cyclical Variation in Idiosyncratic Risk', *European Economic Review*, **45**, 2001, pp. 1311–1339.

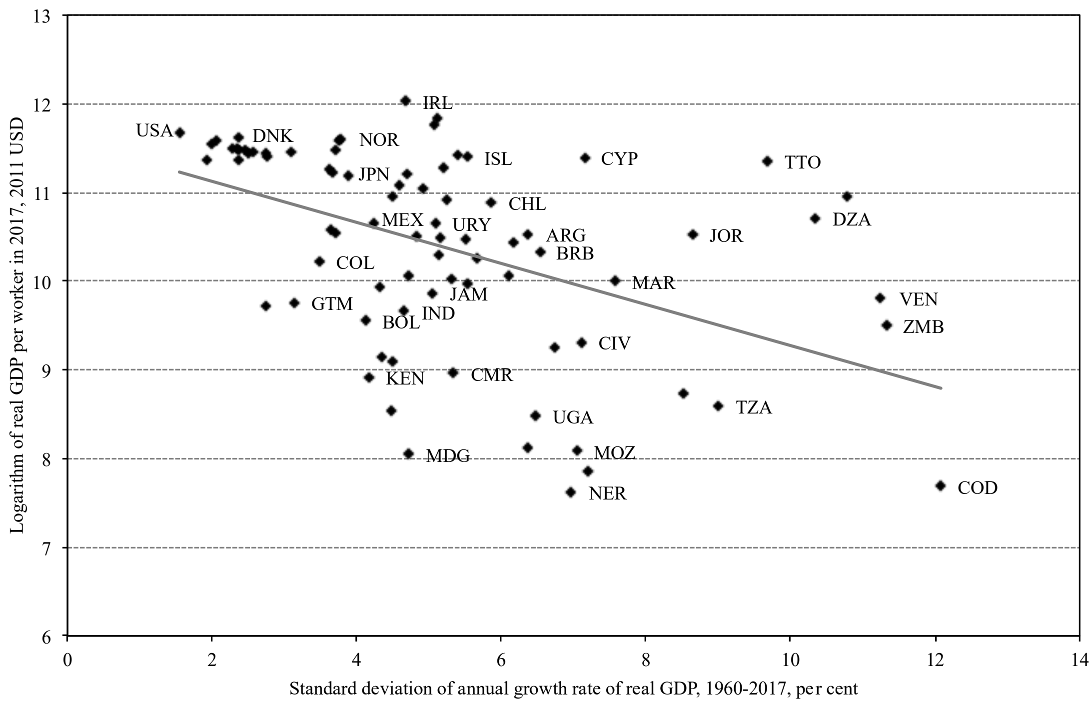
[13] See, e.g., Kenneth Clark, Derek Leslie and Elizabeth Symons, 'The Cost of Recessions', *Economic Journal*, **104**, 1994, pp. 20 – 36.

14.3 Does instability of GDP imply lower structural levels of activity?

If instability in aggregate output and income has a substantial negative impact on the structural (trend) levels of aggregate output and income, this constitutes a very direct cost of fluctuations: the income loss on average over all periods that would be avoided if output could be stabilized

Figure 1.6 of Chapter 1 is repeated here as Figure 14.3 for convenience. It shows a clear and substantial negative correlation between instability of GDP (measured by the standard deviation of its growth rate) and general prosperity in the 'end period' (measured by GDP per worker). The estimated slope of the grey tendency line is -0.23. If the correlation in the figure could be viewed as representing a causal effect from instability to prosperity, a country that could bring down the instability of its growth rate by one standard deviation, which from the figure would be around one tenth of the total variation in growth rates, would achieve an increase in its long-run, average income of around 23 percent. This is a very large effect.

Figure 14.3 Short-run economic volatility and long-run economic performance across 80 countries



Note: The grey line is estimated by OLS. The estimate of the slope is -0.23 with *t*-values -4.73. The *R*-square is 0.22. Country abbreviations can be seen in the appendix, Table A.
Source: Penn World Table 9.1.

However, as discussed in Chapter 1, the figure does not prove a causal effect from instability to prosperity (average income). It could cover all sorts of causality. If we control for other variables that may influence long-run income per worker (e.g., investment rates in physical and human capital and labour force growth rates), the negative correlation of the figure remains, though.[14] Still this does not does not prove a causal effect from instability to the level of income, and indeed such an effect is hard to prove. However, we can think of several reasons why instability should be bad for the kind of decisions that typically lead to prosperity.

Instability of aggregate output and income means that income from capital and labor also must be unstable and therefore uncertain at the individual level, and as we have seen more so for some individuals than for others. When people are risk averse, higher instability could therefore imply lower investment in physical and human capital. In Book One we found this to have a negative effect on income levels. A similar argument goes for innovation, another key factor explaining prosperity. The profit from successful innovation is a form of income, and the more uncertain the private gain from innovation is, the less innovation there may be.

A large downturn in economic activity creates losses for banks and other financial institutions that reduce their ratio of equity to loans. Typically, they will react to this by cutting back on their loans in an effort to restore their equity-to-loans ratios (an effect that may be reinforced by tightened bank regulation). This, in turn, can have a negative impact on investment and innovation for a while.

Furthermore, as Table 14.s above showed, the number of spells of long-term unemployment increases significantly during recessions. This can have longer lasting negative effects on incomes. A worker who is unemployed for a long time may lose important job skills and/or the motivation to search intensively for work. A long spell of unemployment may also stigmatize a worker in the eyes of employers, making it harder for the person to obtain a job offer even if he/she is perfectly qualified and motivated. For these reasons, a long and deep recession may cause some workers to drop out of the labour force and reduce the productivity of some of those remaining in the work force, effects referred to as 'hysteresis'.[15] Hysteresis effects can imply that a deep recession will have long lasting negative impact on the natural levels of employment and productivity, which may add significantly to the costs of business cycles.

The possibility of a longer-run (if not permanent) negative impact of a substantial economic downturn on the structural level of output and income has been much debated in the aftermath of the financial crisis in 2008-09, that lead to the Great Recession. Figure 14.4 shows some empirical evidence relevant for that debate.

The figure shows the evolution of the log of quarterly real GDP for the six different countries studied in Chapter 13 over a period from 19 years before the crisis to 9 years after. It also shows for each country the linear trends of log GDP for the two subperiods,

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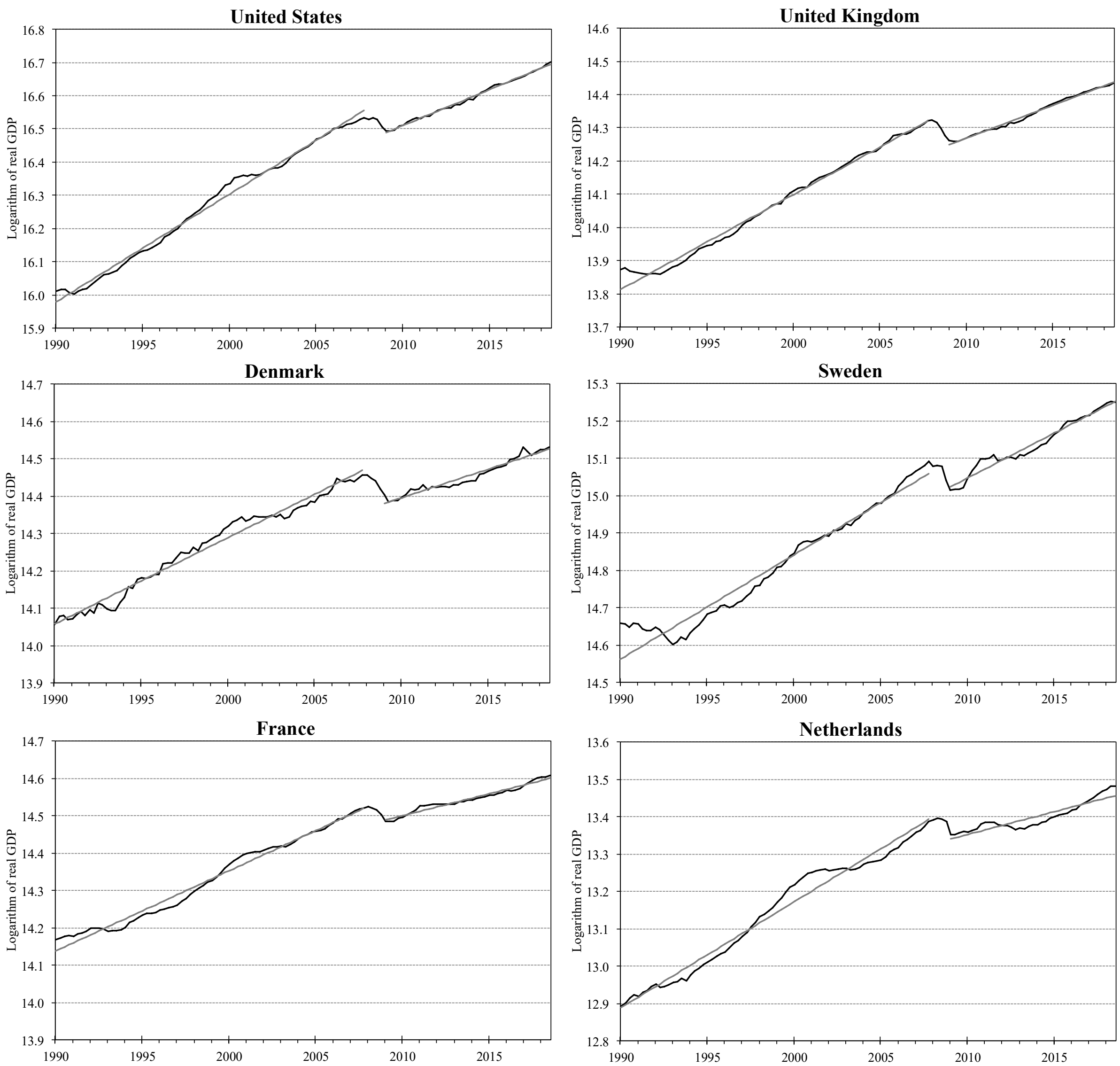
[14] This was a main point in the original contribution by Ramey and Ramey that gave inspiration to this figure: Valerie and Garey Ramey, 'Cross-Country Evidence on the Link Between Volatility and Growth', *American Economic Review*, **85**, 1995, pp. 1138–1151.

[15] The word 'hysteresis' is derived from an ancient Greek word meaning 'deficiency' or 'lagging behind'. In modern language hysteresis is the dependence of the state of a system on its history.

1999-2007 and 2009- 2018 (with the crisis year 2008 taken out of the trends). The figure clearly shows that all the countries experienced a serious drop in output during the crisis. It also shows that during the nine years after the crisis none of them returned to the levels of the growth trends from before the crisis. For most countries (all except Sweden, perhaps), even the growth rates along the post-crisis trends (the slopes of the grey trend lines) seem to be lower than the trend growth rates before the crisis.

Figure 14.4 GDP and its linear trend before and after the financial crisis of 2008-09, six countries, 1990Q1-2018Q3



Note: Based on quarterly data.

Source: National Accounts Statistics database, OECD.

The figure does not strictly *prove* a long-lasting negative impact of the Great Recession, since other growth dampening factors could have played a role at the same time. This could be, for instance, the ageing of the populations and perhaps that the growth potentials arising from IT technologies and digitization began to be exhausted at that time. Nevertheless, the coincidence of the time of crisis and the shift in the growth trends is more than conspicuous and not easily explained without reference to some negative and long-lasting effects of the crisis; this goes particularly for the *level* drop of the growth trends. Note that in so far as the overheating of the economies that took place up to the financial crisis was a main trigger of the crisis, the potential long-lasting negative effects of the crisis on structural levels do not only give a motivation for stimulating the economy in a slump, but also for dampening activity in a boom. [16]

In conclusion:

THE COSTS OF BUSINESS CYCLES ARISING FROM LONG-LASTING EFFECTS ON STRUCTURAL ACTIVITY LEVELS

Instability of aggregate output and income may have negative effects on the structural levels of output and income for a considerable period of time, perhaps even permanently. Although this 'hysteresis' effect is not easy to demonstrate empirically, it is highly plausible, particularly in connection with larger business cycle downturns. In the presence of hysteresis a main motivation for stabilization policy is that it may mitigate the long-lasting negative effect of instability on average incomes. If it is mainly the larger fluctuations that have considerable and longer-lasting negative effects, mainly stabilization policies that attempt to avoid the larger downturns and upswings are motivated, while fine tuning is not.

14.4 The welfare costs of unstable inflation

The fluctuations in consumption and employment are not the only sources of welfare losses from business cycles. The variations in the rate of inflation occurring during the cycle – documented in Table 13.2 and Table 13.3 – add further to the welfare costs of business fluctuations. We will now explain how inflation fluctuations can be socially costly and why society would therefore be better off if inflation could be kept constant around its average level over the business cycle. In particular, we will show that when

[16] A few studies concerned with long-lasting effects of the financial crisis are: Olivier Blanchard, Euginio Cerutti and Lawrence Summers, 'Inflation and Activity – Two Explorations and their Monetary Policy Implications', *IMF Working Paper* 15/230, 2015 and Valerie Cerra and Sweta Saxena, 'Booms, Crisis, and Recoveries: a New Paradigm of the Business Cycle and its Policy Implications', *IMF Working Paper* 17/250, 2018 and Maarten De Ridder, M. (2016) 'Investment in Productivity and the Long-Run Effect of Financial Crises on Output', *Centre for Macroeconomics and Cambridge-INET working paper*, 2016.

(some) nominal prices are rigid, a fluctuating and unpredictable inflation will distort the structure of relative prices which in turn will reduce consumer welfare.

At the heart of our explanation is the time-honoured idea in economics that the efficiency of the capitalist market economy lies in its ability to allocate the productive resources effectively so that the value of the marginal product of each resource is the same in all uses of the resource. If, for simplicity, we consider a world with just one resource, labour, and many outputs indexed by *i*, the condition for efficiency is that $P_i \cdot MP_i(L_i)$ takes the same value for all commodities *i*, where P_i is the price of output *i*, L_i is the amount of labour used in production of output *i*, and $MP_i(L_i)$ is the marginal productivity of labour in the production of output i, assumed to be decreasing in L_i. We can interpret the price P_i as the consumers' common marginal willingness to pay for commodity *i*, and hence $P_i \cdot MP_i(L_i)$ is the marginal benefit to the consumers of the last unit of input used in production of output *i*. If there are two commodities for which $P_i \cdot MP_i(L_i) < P_j \cdot MP_j(L_j),$ then more benefit for the consumers could be obtained by reallocating some labour from the production of i to the production of j. On the other hand, if $P_i \cdot MP_i(L_i) = P_j \cdot MP_j(L_j),$ then no additional benefit can be obtained from such a reallocation. A basic idea in economics is that a free market system will indeed tend to create a situation where the optimality condition, $P_i \cdot MP_i(L_i) = P_j \cdot MP_j(L_j)$ is met for all *i*,and *j*, for if, e.g., $P_i \cdot MP_i(L_i) < P_j \cdot MP_j(L_j),$ an extra profit could be gained by moving some labour from the production of commodity *i*, thus losing a revenue of $P_i \cdot MP_i(L_i),$ to the production of commodity *j* where it would generate a larger extra revenue $P_j \cdot MP_j(L_j).$

Since this story is important for our purposes, we will go into more detail with it. Assume that the various outputs are produced under 'monopolistic competition' (as described in Chapter 1, when we explained about nominal price rigidities, and studied again in Chapters 11 and 12), where there is one producer for each type of output with some market power to set the price. The firms buy a labour input at the common price *W*. The marginal cost of output type *i* produced by firm *i* is then $W / MP_i(L_i)$. As we saw in Chapters 11 and 12, under monopolistic competition each firm will set its price and hire labour up to a point where the price is a markup over marginal cost, that is, $P_i = mW / MP_i(L_i),$ where $m \geq 1$ is a markup factor that we assume to be constant and the same for all firms. Perfect competition (price equal to marginal cost) is the special case where $m = 1$.[17]

If all firms behave like this and all are faced with the same wage rate *W* and have the same degree of market power *m*, then for all *i* we will have $P_i \cdot MP_i(L_i) = mW,$ and hence $P_i \cdot MP_i(L_i)$ will be the same everywhere, the condition for optimality.

[17] When firms maximize profits, the mark-up factor will be $m = \sigma / (\sigma - 1)$ which will be constant when the price elasticity of demand, σ, is constant. Under perfect competition, $\sigma \to \infty$, implying $m \to 1$.

Further, when all consumers have optimized their consumption pattern and face the same output prices, then for each consumer the marginal rate of substitution between any pair of commodities *i* and *j* (the number of additional units of *j* it takes to compensate for one less unit of *i*) must equal the price ratio P_i / P_j, that is, $MRS_{ij} \equiv MU_i / MU_j = P_i / P_j$, where MU_i is the marginal utility from commodity *i* at the optimal consumption bundle etc. Otherwise, more utility could be obtained by changing the composition of consumption. And when all firms have maximized profit, the marginal rate of transformation between any two commodities *i* and *j* (the number of additional units of *j* that can be obtained if one unit less if *i* is produced) must also equal the price ratio that is, $MRT_{ij} \equiv MP_j / MP_i = P_i / P_j$. Hence, when all agents have optimized and all markets have cleared:

$$MRS_{ij} = \frac{MU_i}{MU_j} = \frac{P_i}{P_j} = \frac{MP_j}{MP_i} = MRT_{ij}. \tag{14}$$

This is exactly the condition ensuring that one cannot make (all) consumers better off by reallocating resources between different uses.

The traditional version of this story assumes perfect competition, $m = 1$, but as we have seen, imperfect competition does not *necessarily* ruin the story: with monopolistic competition the optimality condition can still be fulfilled if ‘only’ the degree of market power, *m*, is the same everywhere. In the following, we will assume monopolistic competition with $m > 1$, which is more realistic than perfect competition, and to have a point of departure without efficiency problems (in order to explain in a sharp way how unstable, unpredictable inflation can imply inefficiency) we maintain the assumption that all firms have the same degree of a market power (*m*).

For a clean case we simplify a bit further (this is in no way essential to our story): we assume that all the local monopolistic competitors have the same production function with a constant marginal and average product *A* (output equals *A* times labour input), so that marginal cost in every firm is *W*/*A*. Each firm will then want to set the price $P_i = mW / A$. We also assume that the different commodities enter the preferences (utility functions) of the consumers in a symmetric way so that the demand for each commodity *i* arising from utility maximization depends negatively on the price ratio P_i / P, where *P* is an appropriate price index (average price) over all commodities. Hence, demand for commodity *i* is assumed to be a decreasing function, $D(P_i / P)$.[18]

This means that when all prices are identical, as the firms want to set them, the demands for all commodities are also identical and the resource, labour power, is allocated evenly between all firms. We depart from this optimal situation.

Now recall the ‘vision’ of nominal price rigidities explained in Section 1.3 of Chapter

[18] In Chapters 11 and 12, we saw examples of a demand side of this kind and it was explained that so-called CES utility functions of the consumers will result in such a system.

1. The many price setters will not adjust their nominal prices simultaneously to changed circumstances. Due to costs of price adjustment, so-called menu costs that only need to be 'small', the firms choose to only adjust their nominal prices occasionally, and there is no reason that the dates for price adjustment should be the same for different firms. Hence, price adjustments take place at non-synchronized dates. Whenever a firm adjusts its nominal price, it keeps the new price unchanged for a period, so it must be set based on the firm's expectations about future circumstances over that period. As explained above, the price that a firm wants to prevail is $P_i = mW / A,$ so the price choice must be based on the firm's expectations concerning future levels of nominal wages, which will probably depend on future inflation. Thus, at the occasions of price adjustment each firm must set its price in view of the expected inflation over the period the firm expects its price to be fixed.

Assume that firm i has just changed its price, and in view of an expectation of a relatively low inflation in wages and prices it has decided on the price $P_i = mW^{\text{low}} / A,$ where W^{low} is the relatively low average wage expected over the period of a fixed price. Assume further that subsequently the firm is surprised by a higher than expected inflation, so that the cost level actually becomes $W^{\text{high}} > W^{\text{low}}$. The firm would then have liked the price to be $mW^{\text{high}} / A,$ but is stuck with the lower price, $P_i = mW^{\text{low}} / A$. However, since $m > 1$, the price it is stuck with may still be higher than the actual marginal cost, that is, $P_i > W^{\text{high}} / A.$ It will then pay for the firm to produce additional units of output during the period when its price is fixed if the extra output can be sold.

Consider another firm j that sets its price after the new and higher inflation has been observed by everybody. This firm will set a price reflecting the new expected cost level, $P_j = mW^{\text{high}} / A.$ In consequence, $P_j > P_i.$ Therefore the consumers will want to consume more of commodity i than of j, $D(P_i / P) > D(P_j / P),$ and because price is still higher than marginal cost for firm i, it will meet the higher demand for its product. Firm j will have to accept the (lower) demand for its product. The result is that production of commodity i becomes larger than production of commodity j, and therefore more of the labour resource is used in production of commodity i than in production of commodity j. This is sub-optimal, however. Obviously, the described phenomenon does not depend on the specific simple and symmetric framework considered. Essentially, the general lesson is that an unstable and therefore unpredictable inflation in combination with nominal price rigidities throw grit in the machinery of the market mechanism by making relative prices less precise signals of the relative cost of producing different economic goods and services.

We have illustrated this phenomenon in a framework of rigid nominal goods prices, but nominal wage rigidities in a framework of monopolistically competitive wage setters (trade unions) who set prices at unsynchronized dates for a period would create similar inefficiencies arising from unpredictable inflation. If the nominal wage of a certain type of labour is set just before an unexpected rise in inflation, then the real wage rate for this type of labour will be relatively low, compared to another type of labour for which the nominal wage is set after the higher inflation has been recognized. Therefore, employment of the first type of labour will be higher than employment of the second type. This twist in employment between different types of labour occurs despite the fact that both types are equally productive and have the same disutility of work at any given level of employment. This is obviously an inefficient allocation of labour which arises because real wages become imprecise market signals of the relative needs for different types of labour.

In fact, even a stable and fully anticipated rate of inflation will generate an inefficient allocation of resources in an economy with nominal rigidities where prices and wages in individual firms are adjusted at different points in time. At every point in time, some firms have set their prices a while ago whereas other firms have adjusted their prices more recently. Under a steady rate of inflation firms that have changed their prices more recently will charge a higher price than similar firms that have reset their prices at an earlier date. Thus, even a constant and fully anticipated inflation causes a distortion of relative prices that does not reflect a difference in the relative marginal costs of production. But this distortion tends to become worse when inflation changes in unexpected ways, because then the differences in relative prices across firms with similar production costs will not only reflect the difference in the timing of their price setting, it will also reflect that they had different information on the future rate of inflation when they set their prices.

Another inefficiency caused by unstable, unpredictable inflation arises in the financial markets. Most financial assets are specified in nominal terms (remaining debt is not regulated with inflation) and it is a nominal interest, *i* say, that is set in the contract between borrower and lender. The real interest rate that will unfold over the contract period ex-post is the nominal interest rate minus the inflation rate, $i-\pi$, say. The actual future inflation cannot be known when the asset is traded (the contract is made), so borrower and lender must base their actions on the expected rate of inflation, π^e, and thus on the ex-ante real interest rate, $i-\pi^e$. If inflation becomes higher than expected it will benefit the borrower, but hurt the lender, and vice versa. Hence, an unstable and unpredictable inflation will cause arbitrary and unintended ex-post redistributions between borrowers and lenders. This may itself be an undesirable distributional effect, but the uncertainty about the real interest rate of credit arrangements may also hamper financial transactions, leading to an inefficiently low degree of financial intermediation.

To recap:

NOMINAL RIGIDITIES AND THE WELFARE COSTS OF UNSTABLE INFLATION

Nominal price and wage stickiness, where (some) price and wage setters keep their nominal prices and wages constant for a certain period and reset their prices at different points in time, is widespread throughout the economy. In such circumstances, an unstable and hence unpredictable inflation will make relative prices, including real wages, less precise market signals of the relative usefulness of resources in different uses. This causes an inefficient allocation of resources, which in turn causes losses of consumer welfare. Unstable, unpredictable inflation also gives rise to arbitrary redistribution between borrowers and lenders trading in nominal financial assets, which may hamper financial transactions and lead to inefficiently low volumes of credit.

14.5 Summing up the economic costs of business cycles: the social loss function

The welfare and distributional costs of an unstable and fluctuating inflation may motivate stabilization policies aimed at keeping inflation stable just as the welfare and distributional costs of fluctuations in real variables such as output, consumption and employment can justify stabilization policies aimed at keeping these variables relatively constant. Intuitively, in both cases the costs will be relatively small when the fluctuations are relatively small, but can be substantial if the fluctuations are large. Because of the risks of bad timing and dosing, stabilization policies should probably not attempt at fine tuning, but only be used in case of substantial fluctuations.

We will formalize and bring together the costliness of fluctuations in real variables and inflation in a 'social loss function'. For each period *t*, this associates a social loss with the period's 'output gap', the relative discrepancy between actual and structural GDP, and the period's 'inflation gap', the difference between the actual rate of inflation and a 'target level' of inflation that we will soon return to:

$$SL_t = a_y\left(y_t - \overline{y}_t\right)^2 + a_\pi\left(\pi_t - \pi^*\right)^2, \quad a_y > 0, \;\; a_\pi > 0, \tag{15}$$

Here y_t and $\overline{y}_t$ are the natural logs of GDP and structural GDP in period *t*, respectively, π and π^* are the actual and the target rate of inflation, respectively, and a_y and a_π are two exogenous parameters that express the relative weights that society puts on output and inflation stability, respectively. Since the fluctuations in consumption and employment over the business cycle are highly correlated with the output fluctuations it seems reasonable in (15) to let the real fluctuations be represented just by the output gap.

Note that the loss function above captures the view that society should not be too concerned with small fluctuations, but only with substantial ones: the quadratic form implies that the social loss of small deviations is small, and generally, the social loss increases more than proportionally with the size of these deviations.

If we define the output gap and the inflation gap, $\hat{y}_t \equiv y_t - \overline{y}$ and $\hat{\pi}_t \equiv \pi_t - \pi^*$, respectively, we may write the single period loss as:

$$SL_t = a_y \hat{y}_t^2 + a_\pi \hat{\pi}_t^2 \tag{15'}$$

Equation (15) or (15') gives the social loss in one period, but fluctuations occur over many periods. We will assume that the total social loss to society from fluctuations occurring over many periods is a simple sum or average of the losses in the single periods. If the policy makers who conduct the stabilization policies have a time horizon of *T* periods, planning over the periods from the current one and *T* periods ahead, the total loss over time as seen from period *t* will then be:

$$\overline{L}_t \equiv \frac{1}{T}\sum_{\tau=0}^{T}\left[a_y \hat{y}_{t+\tau}^2 + a_\pi \hat{\pi}_{t+\tau}^2\right] = a_y \frac{1}{T}\sum_{\tau=0}^{T}\hat{y}_{t+\tau}^2 + a_\pi \frac{1}{T}\sum_{\tau=0}^{T}\hat{\pi}_{t+\tau}^2 \tag{16}$$

In Chapter 20 on stabilization policy we will develop the formula (16) further in two directions. We will account for the facts that policy makers cannot know the future output and inflation gaps for sure, but must work on the basis of their expected values, and that they may discount the future, so that distant periods do not count as much a current ones in the summation in (16) etc. For now (16) serves us well for illustration.

Note that if $\hat{y}_t$ and $\hat{\pi}_t$ fluctuate around mean values of zero, meaning that y_t and π_t fluctuate around $\overline{y}_t$ and π^*, respectively, then $\sum_{\tau=0}^{T}\hat{y}_{t+\tau}^2 / T$ is really the variance of $\hat{y}_t$ over time. Calling the variances of the output and inflation gaps σ_y^2 and σ_π^2, respectively, we may write (16) as:

$$\overline{L}_t = a_y \sigma_y^2 + a_\pi \sigma_\pi^2 \tag{16'}$$

which shows that it is really the variances of the output and inflation gaps that policy makers are assumed to be concerned about.

When discussing macroeconomic stabilization policy in later chapters, we will often assume that stabilization policy aims to minimize a social loss function like $\overline{L}_t$ in (16) or (16'). It is only the ratio between the two parameters a_y and a_π that matters for such decisions. In other words, what matters for optimal stabilization policies is the *relative* weight that society puts on inflation vis-à-vis output stability. Therefore we may as well write our loss functions as

$$SL_t = \hat{y}_t^2 + \kappa \hat{\pi}_t^2, \quad \text{and} \tag{17}$$

$$\bar{L}_t = \bar{L}_t = \sigma_y^2 + \kappa\sigma_\pi^2, \quad \kappa > 0 \tag{18}$$

where we can think of κ as $\kappa \equiv a_\pi / a_y$, the relative weight on inflation stability.

It is natural to think of the target level of output in the social loss function, $\bar{y}$, as the structural GDP that can be produced when resources are used up to their 'natural' rates of utilization. There seems to be no good reason for stabilizing output at any other level.[19] How should we think of the 'target' level of inflation, π^*, from which we measure the deviations in inflation of relevance in the social loss function? This is not in any natural way given from the economic structure itself. At different times, economies have experienced quite different average levels of inflation over long periods.

The only reasonable way to think of π^* is the level of inflation that the relevant policy makers would want to prevail under circumstances of a relatively stable inflation. Minimizing the $\bar{L}_t$ above involves an attempt to keep inflation as close as possible to π^*, at least as long as this does not conflict with keeping output close to $\bar{y}$. We will now discuss the welfare and distributional concerns that might guide the choice of the stable target rate of inflation.

14.6 The social costs of a high level of inflation and the choice of inflation target

So far we have discussed the social costs of unstable and therefore unpredictable inflation, but there are a number of traditional reasons for also being concerned with the level of a stable inflation and these reasons point to social costs of too high inflation, but also to potential costs of too low inflation.

A first observation is that high average inflation tends to go hand in hand with unstable inflation according to historical experience. Hence, all the reasons we have provided above for avoiding unstable inflation in practice also turns into arguments for avoiding too high levels of inflation.

Second, we have already seen in section 14.4 that even a stable and fully anticipated rate of inflation will generate consumer welfare losses due to relative price distortions when nominal prices and wages are temporarily rigid and firms reset their prices at different points in time.

Third, the classical case for relatively low inflation is the so-called 'shoe leather cost' of a high (but stable and predictable) inflation. The Fisher equation says $i = r + \pi^e$, where r is the real interest rate and the notation is otherwise as above. Given that the real

[19] As we discussed in Book One, structural policies are the most relevant instruments to raise the structural GDP. Nevertheless, in one of our later discussions of the limits of stabilization policy (in Chapter 22), a policy maker who insists on establishing a higher level of output than the structural level will play an important role.

interest rate stays relatively stable around its natural rate, a high and stable actual inflation rate will create a high nominal interest rate because it will work itself into the expected rate of inflation, π^e. The nominal interest rate is an opportunity cost of holding non-interest bearing money. Therefore, a relatively high (but stable) inflation will induce households and firms to economize on their money balances to be able to invest a larger share of their wealth in interest-bearing assets. When people hold a lower average stock of real money balances, they have to shift more frequently from interest-bearing assets to money balances when they need money for transactions purposes.[20] The more frequent such portfolio shifts are, the larger are the resulting transactions costs. More generally, when firms and households hold smaller cash balances, they will have to spend more time and effort on cash flow management to ensure that they can always meet their payment obligations. Such resource costs are part of the social costs of inflation since resources spent on cash management could alternatively have been used in productive activities.

Fourth, high inflation exacerbates the distortionary effects of taxation even if it is relatively stable. A positive rate of inflation drives a wedge between the nominal and the real rates of return on saving and investment, as we have seen. The income tax is typically levied on the entire nominal interest rate, including that part which the investor must set aside to preserve the real value of his or her nominal asset. As a result, inflation may cause the return to nominal assets to be overtaxed relative to the return on real assets such as land and buildings, thereby inducing too much investment in the latter type of assets. Further, the tax code typically only allows business assets to be written down on the basis of the historical purchase price of the asset. Hence, inflation tends to erode the real value of depreciation allowances, again implying a risk of over-taxation. In summary, the fact that the income tax system is not indexed for inflation means that inflation will tend to distort savings and investment decisions.

Adding all of these arguments, there would seem to be a formidable case for choosing a zero target inflation rate, $\pi^* = 0$. In a provocative essay, the late Nobel Prize winner Milton Friedman even argued that the target inflation rate ought to be *negative* and numerically equal to the long-run equilibrium (natural) real interest rate, $\overline{r} > 0$, so that the nominal interest rate $i = \overline{r} + \pi^*$ would drop to zero.[21] His argument was that the marginal social cost of supplying money to the public is roughly zero, since printing money is virtually costless. To induce people to hold the socially optimal amount of money balances, the marginal private opportunity cost of money holding – given by the nominal interest rate – should therefore also be zero. If the nominal interest rate is positive, households and firms will economize on their money balances to hold more of their wealth in the form of interest-bearing assets. Hence, as explained above, they will incur socially wasteful costs of cash flow management, which may be avoided by driving the nominal interest rate to zero. This in turn requires a steady rate of *deflation* to ensure

[20] They must go more often to the bank, which will wear out their shoes causing a 'shoe leather cost'.
[21] See Milton Friedman, 'The Optimum Quantity of Money', in *The Optimum Quantity of Money and Other Essays*, pp. 1–50, Chicago, Aldine Publishing, 1969.

a real interest rate equal to the economy's 'natural' rate of interest.

However, most economists believe that a policy of deflation could be very dangerous in practice, since it might trigger a destabilizing wave of bankruptcies if debtors have not fully anticipated the future fall in prices and the resulting increase in their real debt burdens.

Indeed, rather than aiming at a negative inflation target as proposed by Friedman, or a zero inflation target as our arguments could seem to point to, modern central banks typically seek to keep the rate of inflation stable around a moderately positive level. The reason is that a zero inflation rate may impair the central bank's ability to stabilize the economy. The Fisher equation shows that with zero or very low actual and anticipated inflation, the short-term nominal interest rate will typically be rather low. In that situation, the central bank will not be able to counteract a severe recession by a large cut in the interest rate, since this is already low and the nominal interest rate cannot fall much below zero.[22] If policy makers want interest rate policy to be an effective tool of stabilization policy, they may therefore have to accept a positive average rate of inflation to preserve room for substantial interest rate cuts in times of recession.

A further argument against a zero inflation target is that nominal wage rates, and probably also other nominal prices, tend to be particularly sticky in the downward direction.[23] If the economy is hit by a negative shock which calls for a fall in real wages, this adjustment is therefore easier to achieve through price inflation than through a fall in nominal wages. This is one additional reason why it may be easier to stabilize the real economy if there is a moderate positive rate of inflation. Generally, a modest positive inflation can be oil in the machinery of the market mechanism when there is downwards stickiness of some nominal prices and wages.

Finally, official price indices tend to overestimate the true rate of inflation since they cannot fully account for the fact that some price increases reflect improvements in product quality rather than genuine inflation.

For these reasons countries with explicit official inflation targets, such as Australia, Canada, New Zealand, Norway, Sweden, Switzerland, the UK and the USA, have typically adopted a positive target inflation rate of 2 – 2.5 percent per annum. The European Central Bank accepts an inflation rate of 2 percent as being consistent with its goal of 'price stability'.

In the wake of the financial crisis starting in 2008-09 many central banks have indeed hit the lower (close to zero) bound for their nominal policy interest rates making further

[22] If the interest rate were negative and there were no cost of holding money balances, it would be profitable to borrow without limit to acquire cash. Such behaviour would drive the nominal interest rate out of the negative territory. In practice, there are some costs of holding large cash balances (mainly costs of keeping them safe from theft), so the nominal interest rate can become somewhat negative without inducing endless borrowing, but the costs of money-holding impose a narrow limit on how far the nominal interest rate can drop below zero, the so-called lower bound often referred to as the zero lower bound (ZLB) although it may be somewhat smaller than zero.

[23] There is a lot of evidence of downward nominal wage rigidity. See, for example, George A. Akerlof, William T. Dickens, and George L. Perry, 'The Macroeconomics of Low Inflation', *Brookings Papers on Economic Activity*, no. 1, 1996, pp. 1–59.

reductions impossible even if a more expansionary monetary policy could have been wished for. This has led many economists, among them the former chief economist of the International Monetary Fund, IMF, Olivier Blanchard, and consultant for the IMF, Lawrence M. Ball, to advocate a higher target rate of inflation, e.g., 4 percent per year. This would imply that in normal times nominal interest rates would be higher, allowing more room for action in the downward direction when needed.[24] The arguments for higher inflations targets than hitherto has indeed had impact. As a part of its new monetary policy strategy announced in July 2021, the European central bank, the ECB, has changed its official target for inflation over the medium term from "below but close to 2%", to "a symmetric 2 per cent target". Whereas the ECB formerly considered positive deviations from the 2 per cent target as more undesirable than negative deviations, it now considers negative and positive deviations equally undesirable. Average inflation (in the harmonized index of consumer prices, HICP) over the medium term must therefore be expected to be somewhat, but not overwhelmingly much higher after 2021 than before. Likewise, the American central bank, the Fed, announced in August 2020 a change of strategy that explicitly states a *symmetric* 2 percent inflation target. There is no doubt that part of the motivation of the ECB and the Fed for these changes is to make more room for demand stimulating monetary policy in situations of severe economic downturns.

14.7 Political costs of business cycles

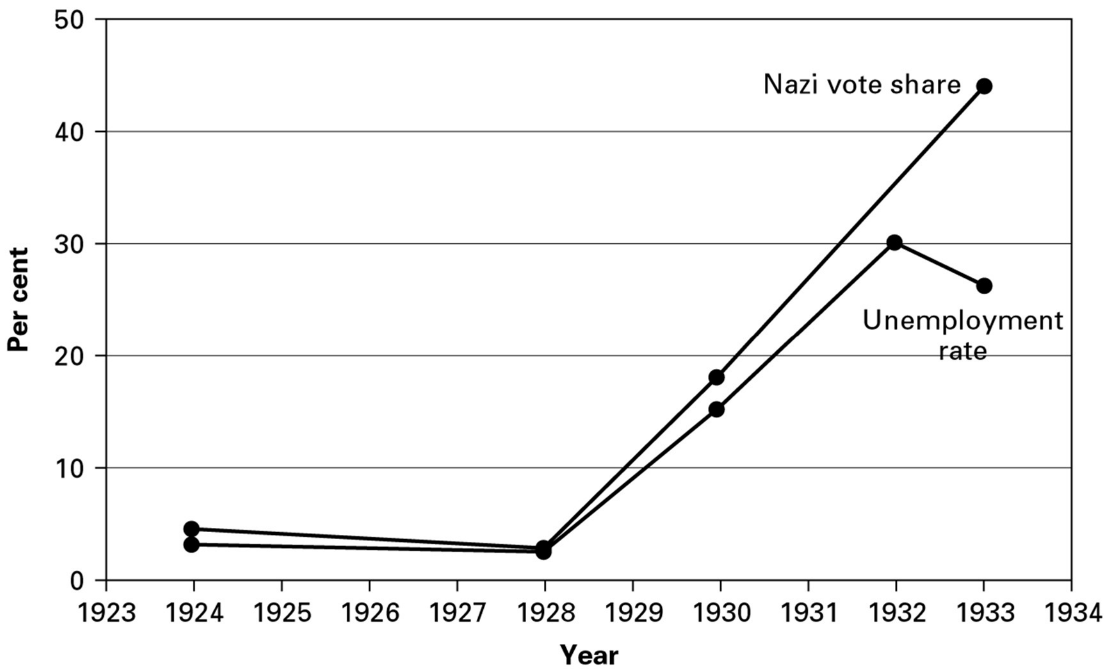
We have so far focused on the economic costs of business cycles and on the unequal distribution of those costs. We should also note that, on several occasions in history, recessions have developed into severe economic *depressions* paving the way for social and political disaster.

A glance at Fig. 14.5 should convince you why it is important to understand the causes of depressions and the means to avoid them. The figure shows a striking correlation between the unemployment rate in interwar Germany and the share of total votes in Reichstag elections going to Adolf Hitler's Nazi Party. In the 1928 election, when unemployment stood at the low level of 2.8 per cent, the Nazi Party captured only 2.6 per cent of the votes and was not considered a serious political force. As the democratic system of the Weimar Republic proved unable to prevent the mass unemployment and human suffering caused by the Great Depression, a rapidly growing number of Germans became receptive to Hitler's radical critique of the parliamentary system. By 1933, with unemployment close to 30 per cent of the labour force (in 1932), Hitler obtained a vote share of almost 44 per cent in the last free election before he established his Nazi dictatorship and steered Germany towards the Second World War. Although there were several, other factors explaining Hitler's rise to power, there is no doubt that the

[24] For a clear statement, the reader is referred to Lawrence M. Ball, 'The Case for a Long-Run Inflation Target of Four Percent', *IMF Working Paper*, WP/14/92, June 2014.

economic depression made it easier for him to gather support.[25]

Figure 14.5 Unemployment and political extremism in Germany, 1924 –1933



Note: Nazi vote share in 1933 is an average across two elections.
Sources: Unemployment rate from B.R. Mitchell, *International Historical Statistics, Europe 1750–1988*, Macmillan, New York, 1993. Nazi vote share from Richard F. Hamilton, *Who Voted for Hitler?*, Princeton University Press, 1982.

The Great Depression of the 1930s was exceptional in its severity and in its social and political consequences. Table 14.2 shows that voter support for anti-system political parties increased greatly in many European countries during the depression and that democracy was put to an end in several countries. There are also several, more recent examples of economic downturns that have caused social upheaval, including the South-East Asian crisis of 1997-98, which brought down the Indonesian government and led to serious civil unrest, and the economic crisis in Argentina in 2001, which forced the

[25] The Nazi leaders themselves were fully aware of the opportunities that economic depression played into their hands. Already in 1926 the later Nazi propaganda minister Joseph Goebbels wrote the following sentence in a pamphlet on *Die Zweite Revolution*: 'We can achieve anything if we can make hunger, despair and deprivation march in our ranks …' (quoted from Joachim C. Fest: *Das Gesicht des Dritten Reiches. Profile einer Totalitären Herrschaft*, Piper Verlag, München, 1980; quote translated by the present authors).

government to resign after riots in the streets. Further, the mass unemployment experienced by several European countries in the wake of the European sovereign debt crisis that started in 2010 has generated widespread voter dissatisfaction with established political parties and has weakened popular support for cooperation among the EU countries. By studying business cycles we will not only learn more about the workings of a market economy; we will also improve our understanding of the general course of social and political events.

Table 14.2 Election results for anti-system political parties before and after the onset of the Great Depression

	Last election result before 1929		Peak election result after 1929		Coup/end of democracy
	% of votes	% of seats	% of votes	% of seats	
Austria	8.5	7.2	9.8	10.8	Yes (1933)
Belgium	8.2	6.4	24.7	22.8	No
Bulgaria	2.5	0	13.0	11.4	Yes (1934)
Denmark	0.3	0	4.2	2.7	No
France	n.a.	8.9	n.a.	19.8	No
Germany	13.2	13.4	58.3	59.6	Yes (1933)
Greece	6.7	0.4	9.7	7.3	Yes (1936)
Hungary	3.8	0.8	22.8	17.4	No
Netherlands	2.0	2.0	7.6	7.0	No
Romania	1.2	0	25.1	27.2	Yes (1938)
Spain	n.a.	n.a.	n.a.	16.2	Yes (1936)
Sweden	6.4	3.5	8.9	3.5	No

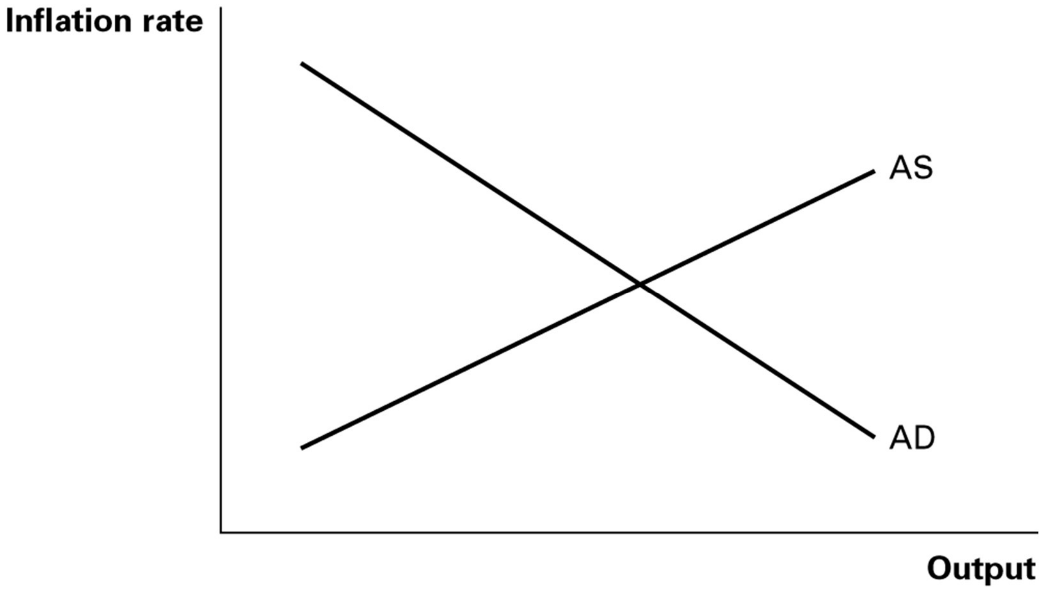
Note: Anti-system parties are defined as parties that would change, if they could, not the government, but the system of government. They include fascist, monarchist and secessionist parties on the right and communist parties on the left. The peak election results are the strongest results obtained by anti-system parties during the 1930s.
Source: Table 1 in Alan de Bromhead, Barry Eichengreen and Kevin H. O'Rourke, *Right-Wing Political Extremism in the Great Depression*, National Bureau of Economic Research Working Paper 17871, February 2012.

14.8 A look ahead

We have now described facts about business cycles and elaborated on their social costs. In the rest of this book, our main goal will be to construct economic models that may explain short-run fluctuations in aggregate output, consumption, employment and inflation and how such fluctuations may be dampened through macroeconomic stabilization policy. Specifically, we will gradually build up a model which may be summarized in the manner depicted in Fig. 14.6, where real output and the rate of inflation are determined by the intersection of an upward-sloping aggregate supply curve and a downward-sloping aggregate demand curve. We will then use this model to study how the economy reacts over time to various shocks to the aggregate supply curve and the aggregate demand curve.

Our goal is to construct a model in which the effects of aggregate demand and supply shocks tend to *persist* over time due to so-called *propagation mechanisms* arising from the links between central macroeconomic variables. In this view of the world, business cycles are *initiated* by random shocks to the economy such as an unanticipated change in the oil price or a change in business confidence due, say, to unexpected political events. However, the *cumulative and systematic* character of business fluctuations documented in the former chapter we will seek to explain endogenously by the properties of our macroeconomic model.

Figure 14.6 Aggregate supply and aggregate demand



We hasten to add that we cannot promise to explain all the features of business cycles – economists are still far from having a perfect understanding of this complex phenomenon – but our model economy will at least have the property that random shocks tend to generate irregular cycles displaying a certain persistence.

As indicated in Fig. 14.6, the aggregate demand curve and the aggregate supply curve are the central building blocks of our model. To construct the aggregate demand curve we must build a theory of private investment and private consumption. This is our agenda for Chapters 15 and 16. In Chapter 17, we combine the insights from Chapters 15 and 16 with a study of monetary policy and the financial sector to derive the aggregate demand curve. Chapter 18 then analyses the relation between inflation and unemployment as a basis for constructing the aggregate supply curve. This enables us to set up our basic AS–AD model in Chapter 19, where we use the model to reproduce some of the stylized facts about business cycles. The subsequent Chapters 20 through 22 apply and extend the basic model in order to analyse the problems of macroeconomic stabilization policy. In Chapters 23 through 25 we extend the AS–AD model to the open economy, studying how monetary and fiscal policy works in regimes with fixed as well as flexible exchange rates. Chapter 25 also contains a discussion of the pros and cons of fixed vs. flexible exchange rate regimes. Our explorations end in Chapter 26 with a discussion of optimal currency areas and an overview of the special challenges faced by a group of countries forming a currency union like the European Monetary Union. This naturally leads us also to a discussion of the European sovereign debt crisis in 2010-12.

Summary

1. **1.** The instability of output and the associated fluctuations in real income impose a welfare loss on instability and risk averse consumers who prefer a smooth to an uneven time path of consumption. Problems of moral hazard and adverse selection prevent consumers from insuring themselves fully against the unexpected temporary income losses caused by business cycles, and credit constraints often prevent unemployed consumers from borrowing against their expected future labour income. Hence, the unanticipated income losses generated by recessions force some consumers into cutting their consumption, resulting in welfare losses, which are not fully offset by the welfare gains from higher consumption during economic booms, due to diminishing marginal utility of consumption.
2. **2.** The employment fluctuations resulting from business cycles also create welfare losses for (some) workers.
3. **3.** Theory and evidence suggest that the welfare losses from the observed business cycle fluctuations in real economic variables would be small if instability had no impact on structural levels and if all individuals were exposed to fluctuations of the same relative size as the aggregate fluctuations. Both of the assumptions are, however, doubtful.
4. **4.** Theoretical considerations as well as suggestive empirical material suggest that, ceteris paribus, the more unstable aggregate output and income are, the lower the structural levels of output and income, and hence of consumption, are likely to be. This may well constitute a serious cost of business cycle instability.
5. **5.** Furthermore, empirical studies clearly document that the bulk of the losses of income and consumption caused by recessions is borne by relatively few people and in particular by relatively low-paid and unskilled workers. Hence, business cycles tend to exacerbate the unequal distribution of income. Distributional concerns constitute yet another reason why society may wish to stabilize output and employment.
6. **6.** Nominal price rigidities mean that (some) firms keep their prices constant for a certain period and reset their prices at different points in time. An unstable inflation will then distort relative prices by causing them to deviate from the relative marginal costs of production. The resulting economic inefficiency causes a loss of consumer welfare. Generally, unanticipated inflation arising from a volatile inflation rate is a social problem since it causes relative prices (including real wages and real interest rates) to deviate from the expected levels that households and businesses base their plans on.
7. **7.** A stable and high inflation also creates welfare costs, for instance stemming from an inefficiently low demand for cash balances and a distorted measurement of taxable

income due to an unindexed tax system.

8. Apart from stabilizing output and employment, another goal for stabilization policy therefore is to minimize the volatility of the rate of inflation around a low target rate. Even though a constant and anticipated inflation generates welfare costs, it is widely agreed that the target inflation rate should be kept above zero to enable the central bank to cut the nominal interest rate significantly in a serious recession without hitting the (close to) zero bound for nominal interest. A further reason for aiming at a positive (stable) inflation of some size is to facilitate the downward adjustment of real wages (that can be needed from time to time in reaction to negative shocks) when nominal wages are rigid.

Exercises

Exercise 1. The goals of stabilization policy

In this exercise you are invited to restate and discuss some of the main results and arguments presented in this chapter. The case for macroeconomic stabilization policy rests on the assumption that it is desirable to stabilize the rates of output and inflation around some target values. This raises a number of questions.

1. Discuss why it is socially desirable to stabilize the rate of inflation around some constant target value. What are the costs of a fluctuating rate of inflation?
2. What are the arguments for avoiding fluctuations in output and employment? What factors determine the magnitude of the welfare costs of output and employment instability?
3. Explain why even a constant rate of inflation generates welfare costs. Do you consider these costs to be large or small? Discuss the factors which policy makers should take into account when they choose the target inflation rate. Is zero inflation an optimal inflation target?

Exercise 2. The instability cost of more general consumption fluctuations

In this chapter we considered the cost to an individual consumer of consumption fluctuations of a particularly simple form: swings between the two levels $(1-x)\bar{C}$ and $(1+x)\bar{C}$ with probability ½ for each (independently). We derived the cost as the difference in 'value' (expected utility or consumption equivalent) for the consumer between being exposed to this consumption lottery and having $\bar{C}$ for sure all the time. In the real world consumption fluctuations are not that simple and regular. Therefore, we will consider a more general situation where consumption can take different, strictly positive values $C_1,...,C_N$ with probabilities $\pi_1,...,\pi_N$, where $0<\pi_i<1$, and $\sum_{i=1}^{N}\pi_i=1$. The mean value (or mathematical expectation) of consumption is $\bar{C}=\sum_{i=1}^{N}\pi_i C_t \equiv EC$, where we have used E for mean value, so for any function $f(C)$ of consumption, $Ef(C)\equiv\sum_{i=1}^{N}\pi_i f(C_i)$. We may write the consumption lottery in relatively short form as $F\equiv\left[(C_1,\pi_1);...;(C_N,\pi_N)\right]$. According to this, consumption swings stochastically around the mean value $\bar{C}$.

In line with the chapter it is assumed, that there is a von Neumann-Morgenstern utility function defined on consumption, $u(C), u'>0,\ u''<0,$ such that the utility $U(F)$ of the lottery F is the expected value of $u(C)$:

$$U(F)=\sum_{i=1}^{N}\pi_i u(C_i)\equiv Eu(C)$$

To be specific, the consumer considers consumption lottery $F \equiv \left[(C_1, \pi_1); ...; (C_N, \pi_N)\right]$ to be at least as good as $F' \equiv \left[(C_1', \pi_1'); ...; (C_N', \pi_N')\right]$, if and only if $U(F) \geq U(F') \Leftrightarrow \sum_{i=1}^{N} \pi_i u(C_i) \geq \sum_{i=1}^{N} \pi_i' u(C_i')$. In other words, the consumer orders the lotteries in preference according to their expected utilities, $Eu(C)$.

Obviously, the expected utility of the 'lottery' that gives $\bar{C}$ for sure is $Eu(\bar{C}) = u(\bar{C})$. We will consider truly stochastic lotteries $F \equiv \left[(C_1, \pi_1); ...; (C_N, \pi_N)\right]$, (meaning that for some *i* and *j*, $\pi_i > 0$ and $\pi_j > 0$, and $C_i \neq C_j$), with $EC = \bar{C}$, and be interested in the difference $U(F) - Eu(\bar{C}) = Eu(C) - u(\bar{C})$, that is, the utility difference between swinging around $\bar{C}$ and obtaining $\bar{C}$ for sure. We know in advance that $Eu(C) - u(\bar{C})$ is negative; this follows from the concavity of *u*.

1. Show first that $Eu(C) - u(\bar{C}) = E\left[u(C) - u(\bar{C})\right]$. Begin by making clear for yourself what the difference between the left and the right hand side is.

2. By using a second order Taylor approximation of $u(C) - u(\bar{C})$ around $\bar{C}$, show that:[26]

$$Eu(C) - u(\bar{C}) \approx \frac{1}{2} u''(\bar{C}) E(C - \bar{C})^2 = \frac{1}{2} u''(\bar{C}) \bar{C}^2 E\left(\frac{C - \bar{C}}{\bar{C}}\right)^2 \quad (20)$$

3. Now show that:

$$\frac{Eu(C) - u(\bar{C})}{u'(\bar{C})\bar{C}} \approx -\frac{1}{2}\left(-\frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\right) E\left(\ln C - \ln \bar{C}\right)^2 \quad (21)$$

4. Explain why the left hand side in (21) can be interpreted as the relative deduction in consumption from the sure level $\bar{C}$ needed for the stochastic consumption lottery to be just as good as the sure consumption level reduced by this deduction. (In other words, the left hand side is the relative decrease in consumption that the consumer is ready to pay for avoiding the consumption swings and, as such, a real measure of the cost of the instability). Describe the ingredients on the right hand side of (21).

5. By assuming some realistic values as well as some upper evaluations of the degree of instability of consumption over the business cycle and of the Arrow-Pratt measure of relative risk aversion, make some evaluations of how costly business cycle swings in consumption seem to be (at most) under the assumption that everybody is hit by the swings to the same degree. (For the Arrow-Pratt measure of relative risk aversion, you

[26] For a function *f*(*x*) and a specific value, $\bar{x}$, of the independent variable, this approximation is:

$$f(x) - f(\bar{x}) \approx f'(\bar{x})(x - \bar{x}) + \tfrac{1}{2} f''(\bar{x})(x - \bar{x})^2$$

can consider 2 as a reasonable value and 5 as an upper evaluation).

6. I view of your results, discuss if stabilization policy intended to mitigate aggregate fluctuations in consumption seems could be worth the effort.

Exercise 3. The Arrow-Pratt measure of absolute risk (or instability) aversion

This exercise asks you to show the claim in Footnote 6 of this chapter. Define the premium $\pi(z)$ by $u(C)=[½+\pi(z)]u(C+z)+[½-\pi(z)]u(C-z)$, and show that $4\lim_{z\to 0}\pi'(z)=-u''(C)/u'(C)$, thus rationalizing and giving an interpretation of the Arrow-Pratt coefficient of absolute risk (or instability) aversion.

Exercise 4. The constant absolute risk (or instability) aversion, CARA, utility function

Show that for the utility function $u(C)=-\exp(-\omega C)$, the coefficient of absolute risk (or instability) aversion, $-u''(C)/u'(C)$, equals ω independently of *C*. What do you find most convincing, a CRRA or a CARA utility function defined on consumption levels?

Exercise 5. The welfare loss of employment fluctuations

This exercise asks you to verify some claims from the chapter (Section 14.1, the subsection on employment fluctuations). Let the disutility of work function be (12) from the chapter, that is, $v(L)=\beta\frac{L^{1+\mu}}{1+\mu}$, $\beta>0$, $\mu>0$. Compare a cycle where employment fluctuates between $(1-z)L$ and $(1+z)L$ with weights ½ and ½ to the stable stream of *L* all the time. Show by steps similar to those leading to (9) in the chapter, that the (discounted/expected) disutility of the fluctuating stream is larger than the disutility of the stable one by a factor $G(z)\equiv ½\left[(1-z)^{1+\mu}+(1+z)^{1+\mu}\right]$. Now show by derivations similar to those leading to (11) in the chapter that this measure of the loss corresponds to an 'employment equivalent' in relative terms of $d(z)=G(z)^{\frac{1}{1+\mu}}-1$. Make some evaluations of $d(z)$ based on plausible values of *z* and μ.

PART 5

The Building Blocks for Short-run Macroeconomics

Chapter 15

Investment and asset prices

Introduction

Our previous chapter on business cycle facts showed that private investment is the most volatile component of aggregate demand, that it is highly correlated with total output and of about the same persistence as output. Hence, economic up- and downturns are typically associated with large decreases and increases in the firms' investments. A commonly held view is that the turning points of the business cycle are often triggered by shifts in private investment caused by shifts in the moods and confidence of investors, often referred to as their 'animal spirits'.

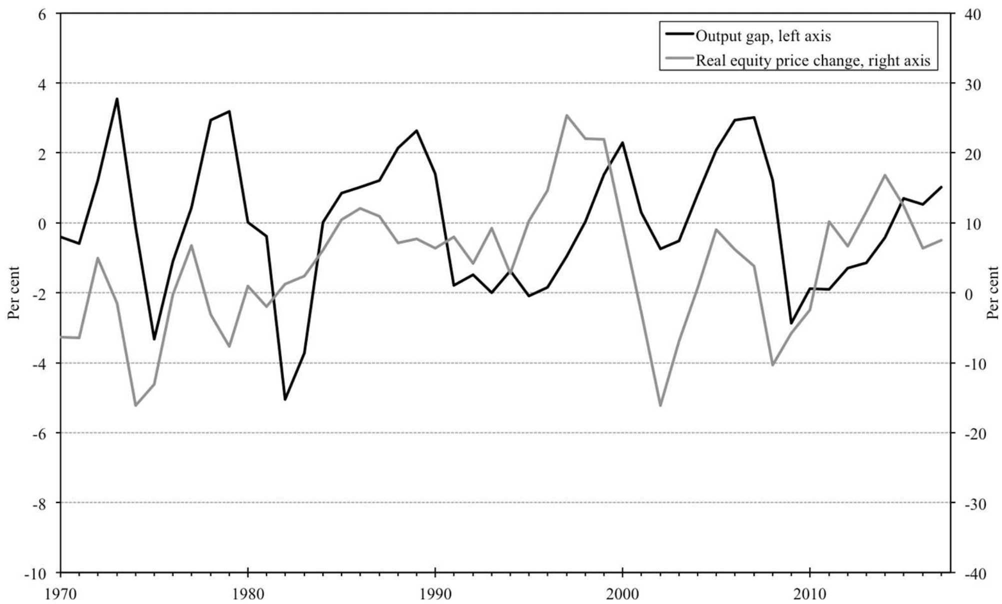
Understanding the forces driving private investment is therefore crucial for understanding business cycles. In this chapter we present a theory of business and housing investment, the so-called 'Tobin's q-theory'. This theory emphasizes the importance of the values of firms and houses for investment.

Presenting the q-theory therefore gives us an opportunity to study two of the most important asset markets in the economy: the stock market and the market for owner-occupied housing. As we shall see, there is a systematic link between stock prices and business investment, and a similar link between housing prices on housing investment.[1] To understand investment, we must therefore study how asset prices are formed.

[1] Nobel Laureate James Tobin was the first economist to give a systematic formal account of the link between stock prices and business investment. The classic statement of the theory was James Tobin, 'A General Equilibrium Approach to Monetary Theory', *Journal of Money, Credit, and Banking*, **1**, 1969, pp. 15 – 29. The theory was later refined and extended by Fumio Hayashi, 'Tobin's Marginal q and Average q: A Neoclassical Interpretation', *Econometrica*, **50**, 1982, pp. 213 – 224.

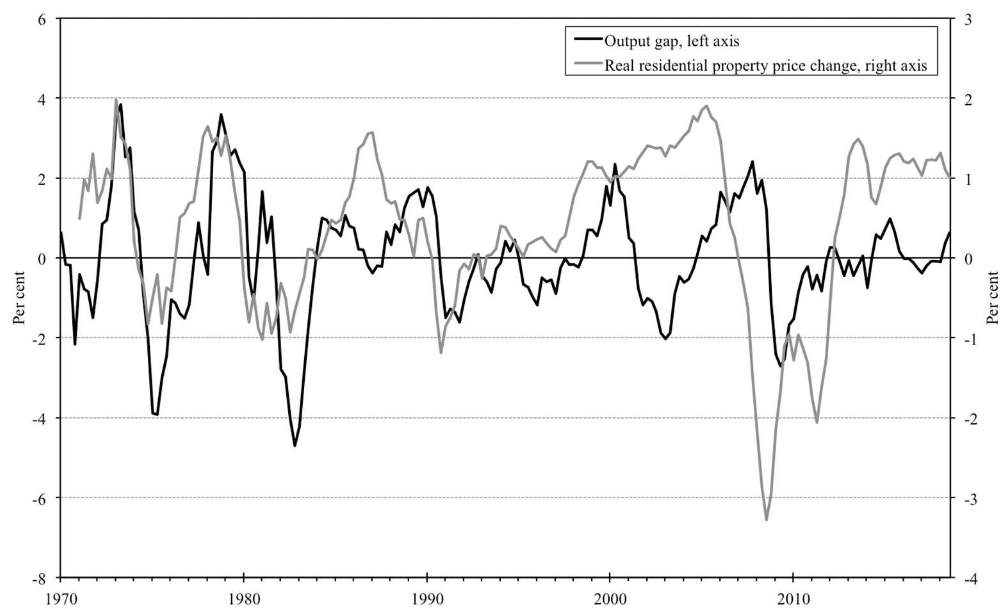
A glance at Figure 15.1 should make clear why we are interested in asset prices. Figure 15.1a shows the link between the rate of change in real equity prices and the output gap, and Figure 15.1b shows the link between the rate of change of real residential prices and the output gap, both for the USA. *There is a close relationship between asset price fluctuations and output fluctuations and a clear tendency for asset price movements to lead movements in output.* Data for most other OECD countries show a similar picture.

Figure 15.1a Real stock prices and the output gap in the USA



Note: Based on annual data. Real equity price change is a three-year moving average; calculation based on Standard and Poor's Composite Stock Price index. Output gap is calculated from a HP-filter with $\lambda = 100$. No endpoints removed at upper end.

Figure 15.1b Real property prices and the output gap in the USA



Note: Based on quarterly data. The real residential price change is a four-quarter moving average. Output gap is calculated from HP-filter with $\lambda = 1600$. No endpoints removed at upper end.
Sources: Output gap from own calculations; real equity price index from Robert J. Shiller's web page: http://www.econ.yale.edu/~shiller/data.htm; real residential property price index from the Analytical House Prices Indicators database, OECD.

Thus, the evidence suggests that a larger-than-normal increase in stock prices or in housing prices will trigger an increase in economic activity, whereas a significant drop in asset prices may be a signal of a future economic downturn. As we will show in this chapter and the next one, Figure 15.1 reflects that higher asset prices tend to stimulate private consumption and investment. In particular, the present chapter will explain why higher stock prices tend to lead to higher business investment, and why higher housing prices provide a boost to housing investment.

We can most easily illustrate the basic idea underlying our theory of investment by looking at the housing market. At any point in time there is a certain market price for houses of a given size and quality. This price may well exceed the cost of constructing a new house of similar size and quality (the replacement cost). The more the market price exceeds the replacement cost, the more profitable it will be for construction firms to build and sell new houses. Hence, we will observe a higher level of housing investment the greater the discrepancy between the market price and the replacement cost of housing. More construction should tend to bring down house prices again, but the market price can deviate from the replacement cost for a long time. The reason is that it takes time for new construction to produce a significant increase in the existing housing stock, because houses last for long time and because it is costly to shift economic resources into the construction industry if construction activity becomes more profitable due to a rise in the market price of housing.

For business investment, a similar basic principle applies. The perceived value of the business assets owned by a corporation is reflected in the market price of the shares in the firm. The replacement cost of the firm's assets is given by the price at which it can acquire machinery, etc., from its suppliers of capital goods. If the stock market value of the firm's assets is higher than their replacement cost, the firm can increase the wealth of its shareholders by purchasing additional capital goods, that is, by investing. The higher the stock price relative to the replacement cost, the greater is the incentive to invest, so the higher the level of investment will be. One might think that the firm would instantaneously adjust its capital stock to eliminate immediately any discrepancy between the stock market value and the replacement cost of its assets. However, this is not realistic since in practice, the firm will incur various costs of adjusting the capital stock, and these costs are likely to increase more than proportionally to the level of investment. Hence, it will be more profitable to allow a gradual adjustment of the capital stock, and during this (potentially long) adjustment period the stock market value of the firm will deviate from the replacement cost of its assets.

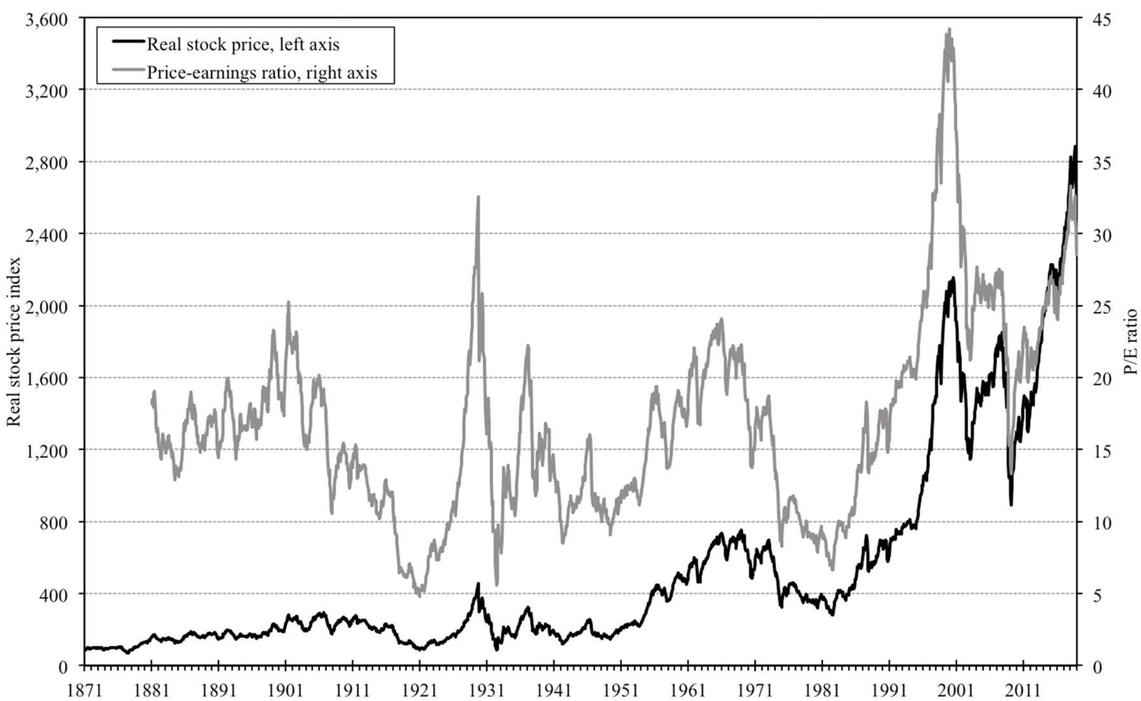
In the following sections we will explain this theory of the link between asset prices and investment in more detail, starting with a theory of stock prices and business investment.

15.1 The stock market

A few facts about the stock market

It is well known that stock prices are highly volatile and sometimes experience dramatic swings. For example, on 19 October 1987 the US Dow Jones index fell by 22.6 per cent in a single day. This was even more apocalyptic than the notorious crash in the 'Black October' of 1929 when the Wall Street stock market dropped by 23 per cent in the course of two days (however, the recovery of stock prices after October 1987 was much faster, so the macroeconomic effects of the crash of 1929 were much more serious).

Figure 15.2 Real stock price index and the price-earnings ratio in the United States**,** 1871M1-2018M12



Note: Based on monthly data. Time period 1871M1-2018M12

Sources: Calculated by Robert J Shiller based on Standard and Poor's Composite Stock Price index, the US Consumer Price index, and a 10-year moving average of the earnings of the companies included in the Standard and Poor's stock index. Data downloadable from: http://www.econ.yale.edu/~shiller/data.htm.

Figure 15.2 illustrates some long-term tendencies in the US stock market, documenting the evolution of real (inflation-adjusted) stock prices and the so-called price-earnings ratio, defined as the market value of shares relative to the profits of the companies which have issued the shares. If stock prices always just reflected recent earnings of companies, then the grey curve in the figure should be quite flat, but it is not.

Rather it seems to go up and down when the stock prices themselves go up and down, thus demonstrating that stock prices are much more volatile than they would be if their movements were simple and immediate reflections of the movements up and down in earnings.

The curve for the real stock price index highlights, for instance, the enormous stock market boom of the 1990s, which was followed by a sharp downturn after the turn of the new century. The subsequent strong recovery was interrupted by a new dramatic drop in stock prices during the financial and economic crisis of 2008, which was in turn followed by a new recovery.

Mainly because of rising stock prices over time, the market value of outstanding shares (the 'stock market capitalization') as a percentage of GDP rose sharply during the 1990s, as shown in Figure 15.3. Overall, stocks have become a much more important component of total financial wealth in USA and Denmark and many other countries.

Figure 15.3 Stock market capitalization as a percentage of GDP, 1980-2017

Note: Based on annual data. After 2004 the World Bank stops providing market capitalization for Denmark, hence from 2006 and onwards CEIC Data is used instead (only for Denmark).
Sources: World Bank, World Development Indicators and CEIC Data.

Although many persons choose not to hold a substantial part of their wealth *directly* in the form of stock, the evolutions on the stock market and in stock prices are nevertheless of great importance for a majority of households since these often hold substantial amounts of stock *indirectly*. Households channel a large part of their savings through pension funds, life insurance companies and other financial intermediaries, which in turn invest a substantial part of their funds in shares. Hence, the performance of the stock market directly or indirectly determines the value of and the return to a large fraction of household wealth. In this way, stock market developments may determine when people feel they can afford to retire from the labour market or when they can afford to buy new consumer durables. Moreover, the evolution of stock prices may have an important impact on the level of output and employment, because of its influence on consumption and business investment. In short, the stock market is important both for individual consumers and for the macro economy, so it is worthwhile investing some effort in understanding how it works, and how it affects investment decisions.

15.2 The price of stock

The value of a firm and the fundamental stock price

The starting point for our analysis is the assumption that business investment is guided by a desire to *maximize the wealth of the owners of the firm*. In modern Western economies, the bulk of business activity is carried out by firms organized as joint stock companies (corporations). For these, maximization of the wealth of the firm's owners is equivalent to maximization of the market value of the outstanding shares in the corporation. This is the reason for our focus on the stock market. However, our theory of investment will also be relevant for unincorporated firms or for corporations, which are not quoted in an official stock exchange. As we shall see, our theory of the stock market implies that *the market value of shares equals the discounted value of the expected future cash flow from the firm to its owners*. This is exactly how a rational outside investor would also value an unquoted or unincorporated firm if he or she were contemplating buying or investing in such a firm. If the owner of an unincorporated firm wants to maximize the market value of his or her business assets, appropriate investment behaviour will therefore be similar to the investment behaviour of a corporation whose shares are traded in a public stock exchange. We will explain this in more detail later.

You may wonder why we assume that the objective of the firm is to maximize the *wealth* of its owners? The answer is that maximization of the current market value of the firm will also maximize the *consumption possibilities* of its owners. This will become clear in the next chapter where we show that a person's potential present and future consumption is constrained by the sum of the person's financial wealth and current and discounted future labour income. Therefore, if a firm can change its operations so as to increase the market value of its assets, it will increase the financial wealth of its owners and enable them to increase their consumption either now or in the future. In both cases, the owners will obviously be better off. In your basic microeconomics course, you may have learned that firms maximize their profits rather than the market value of their assets. Fortunately, there is no contradiction between these two goals. A firm that maximizes its discounted stream of profits over time will also maximize its market value, since ultimately the source of the cash flows from a firm to its owners is its profits.

If a corporation plans its investment with the purpose of maximizing the market value of its shares, we must base our theory of investment on a theory of the value of the firm. Our starting point for such a theory is an *arbitrage condition*, which says that the market value of the shares in the firm must adjust to ensure that the investment of a certain amount of wealth in shares is equally attractive as investing a similar amount in bonds.

We will consider the choice between buying stock and investing the same amount in risk free bonds, e.g., government bonds of a safe country. Our notation will be as follows: V_t is the actual and known real market price of shares in the firm (the stock price measured in units of consumption gooods) at the beginning of period t, V_{t+1}^e is the real stock price expected to prevail at the start of period $t + 1$, D_t^e is the real dividend expected by shareholders to be paid out per share at the end of the period t, and r is the safe and known real interest rate on bonds assumed to be constant. If an investor buys one share at the beginning of period t at price V_t, the real value of the investor's holdings at the beginning of the next period will be $D_t^e + V_{t+1}^e$, so that the increase in wealth or the return on the placement is $D_t^e + (V_{t+1}^e - V_t)$, composed of the expected dividend plus the expected capital gain. If the investor instead places the amount V_t in bonds, the real value of the holdings at the beginning of period t+1 will be $V_t + rV_t$ and the increase or return will be rV_t.

It may be tempting to think that the condition for the two placements to be considered equally good is $V_t + rV_t = D_t^e + V_{t+1}^e$, or $rV_t = D_t^e + (V_{t+1}^e - V_t)$, but this would not take into consideration that the stock is more risky than the bond. For the bond, the return is known to be rV_t, whereas for the stock it is uncertain and only *expected* to be $D_t^e + (V_{t+1}^e - V_t)$, which we may interpret as the mean value of a stochastic variable taking different values with different probabilities. Since stock prices and dividends are generally more volatile than bond prices and interest payments, shares are a riskier investment than bonds. Because investors are risk averse, the expected rate of return on shares must therefore include a risk premium $\varepsilon > 0$ on top of the market interest rate to ensure that shareholding is just as attractive as the holding of bonds. Therefore, the relevant arbitrage condition is really $V_t + rV_t + \varepsilon V_t = D_t^e + V_{t+1}^e$, where ε is the required risk premium on stock, or:

$$\overbrace{(r+\varepsilon)V_t}^{\substack{\text{required}\\ \text{return}}} = \underbrace{D_t^e + \overbrace{V_{t+1}^e - V_t}^{\substack{\text{expected}\\ \text{capital gain}}}}_{\substack{\text{total expected}\\ \text{return on shares}}}, \tag{1}$$

We must expect market forces to ensure that (1) is fulfilled. If, for instance, the left hand side in (1) were larger than the right hand side, bonds would be considered a better placement than stock. This would increase the demand for bonds and decrease the demand for stock, thus driving down the price V_t of stock (and drive down the interest rate on bonds) until equality would be established as in (1).

We may rearrange (1) to get:

$$V_t = \frac{D_t^e + V_{t+1}^e}{1 + r + \varepsilon} \tag{2}$$

This is a very important relationship in our analysis below. It says:

THE VALUE OF A FIRM AND THE SHARE PRICE

The value of a firm (the price of its stock if it is a stock company) at the beginning of any period equals the present value of the sum of the period's expected cash flow to the owners (the dividend if it is a stock company) and the expected market value of the firm at the end of the period, where the rate at which future values are discounted is the market interest rate, *r*, plus the required risk premium, ε.

A further rewriting of (2) will be of use and lead us to the so-called 'fundamental share price'. Since an arbitrage condition similar to (2) must hold for each subsequent period, rational financial investors will expect that future stock prices will satisfy the relationships:

$$V_{t+1}^e = \frac{D_{t+1}^e + V_{t+2}^e}{1+r+\varepsilon},\ V_{t+2}^e = \frac{D_{t+2}^e + V_{t+3}^e}{1+r+\varepsilon}, ...,\ V_{t+n}^e = \frac{D_{t+n}^e + V_{t+n+1}^e}{1+r+\varepsilon},\ \text{etc.} \tag{3}$$

Now, if we start out from (2) and for the V_{t+1}^e on the right hand side of this insert the expression from the first equation of (3) we get:

$$\begin{aligned} V_t &= \frac{D_t^e}{1+r+\varepsilon} + \frac{V_{t+1}^e}{1+r+\varepsilon} = \frac{D_t^e}{1+r+\varepsilon} + \frac{1}{1+r+\varepsilon}\frac{D_{t+1}^e + V_{t+2}^e}{1+r+\varepsilon} \\ &= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{V_{t+2}^e}{(1+r+\varepsilon)^2} \end{aligned}$$

Here, for the V_{t+2}^e on the right hand side we can insert the expression from the second equation of (3) etc. and thus by successive substitutions find that

$$\begin{aligned} V_t &= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + \frac{V_{t+3}^e}{(1+r+\varepsilon)^3} \\ &= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + \ldots + \frac{V_{t+n}^e}{(1+r+\varepsilon)^n} \end{aligned} \tag{4}$$

The term V_{t+n}^e in (4) is the expected stock price *n* periods ahead. If we call the expected average rate of increase in the stock price g^e, then $V_{t+n}^e = V_t(1+g^e)^n$. The last term in Equation (4) is then:

$$\frac{V_{t+n}^e}{(1+r+\varepsilon)^n} = \left(\frac{1+g^e}{1+r+\varepsilon}\right)^n V_t$$

It is reasonable to assume that investors do not expect that the growth rate g^e of future real stock prices can be larger than the growth rate G of GDP indefinitely. For if the opposite were the case, the expected stock price relative to GDP would at last become infinitely large, which is highly implausible since all income, including that from stock, must ultimately be a part of GDP. Hence, it is reasonable to assume $g^e < G$. Over long periods, the growth rate of GDP has typically been smaller than the real interest, $G < r$, and therefore even more so should $G < r + \varepsilon$ hold. This means that $g^e < r + \varepsilon$, which implies that

$$\lim_{n\to\infty} \frac{V_{t+n}^e}{(1+r+\varepsilon)^n} = \lim_{n\to\infty} \left(\frac{1+g^e}{1+r+\varepsilon} \right)^n V = 0 \tag{5}$$

We will assume this to hold. If we continue the successive substitutions indicated in (4) indefinitely and use our assumption (5), we end up with:

$$\begin{aligned} V_t &= \frac{D_t^e}{(1+r+\varepsilon)} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + \ldots + 0 \\ &= \sum_{n=0}^{\infty} \frac{D_{t+n}^e}{(1+r+\varepsilon)^{n+1}} \end{aligned} \tag{6}$$

Equation (6) is our second important relationship for the stock price, the so-called fundamental share price:

THE FUNDAMENTAL SHARE PRICE

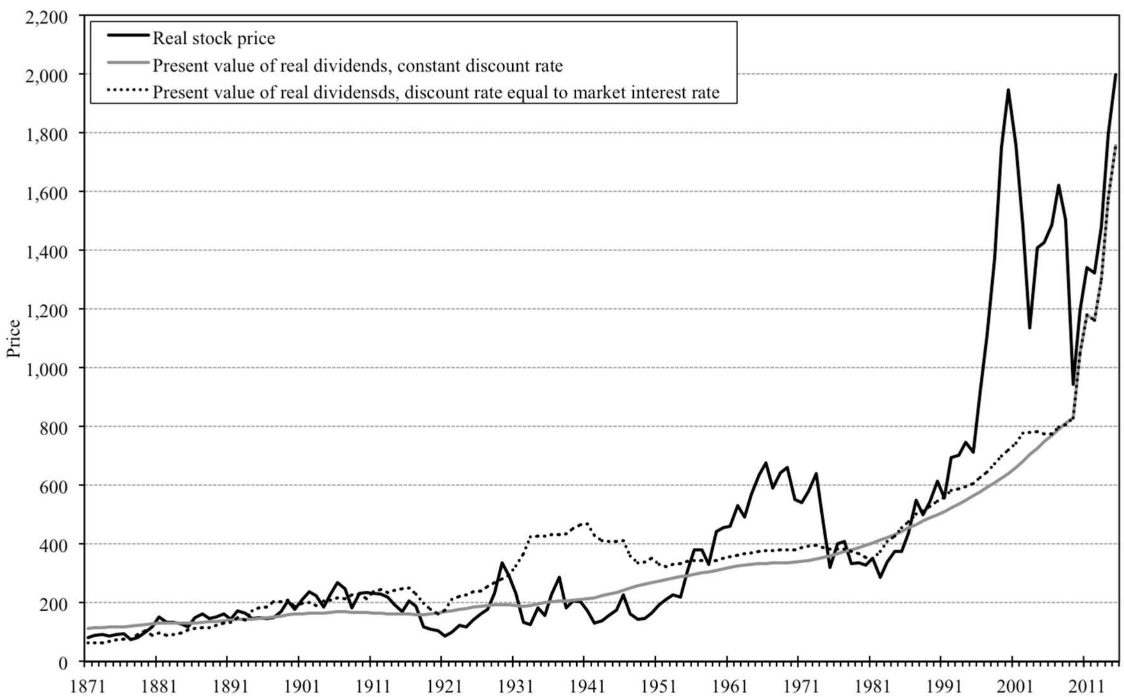
The market value of the shares in a firm equals the present discounted value of the expected future dividends paid out by the firm, where the discount rate is the market interest rate, *r*, plus the required risk premium, ε.

Note that there must be a close correlation between a firm's dividends and its profits, since the latter are the source of the former. This observation is the basis for our earlier claim that maximization of (the present value of) profits is roughly equivalent to maximization of market value.

How well does the theory of the fundamental share price explain stock prices?

If investor expectations of future dividends, interest rates and risk levels are reasonably accurate, Equation (6) predicts a fairly close link between the stock price observed at time *t* and the present discounted value (PDV) of the actual dividends paid out after that time. Figure 15.4 shows the evolution since 1871 of the real value of the Standard & Poor 500 stock price index for the USA and two alternative measures of the real present value of future dividends. (Real dividends after the end year were estimated by assuming that future dividends will grow at the average historical growth rate for dividends).The smooth grey curve shows the PDV when future dividends are discounted at a constant rate equal to the observed average return on shares between 1871 and the end year. The dotted curve ('discount rate = market interest rate') indicates the PDV when dividends are discounted by actual one-year interest rates plus a constant risk premium equal to the average actual risk premium on shares observed over the entire period. We see that at least up until the late 1990s, there is indeed a long-run link between the (trends of) stock prices and the PDV of dividends, as Equation (6) would lead us to expect.

Figure 15.4 Real stock prices and present values of subsequent real dividends in the USA, 1871-2015



Note: Based on monthly data.
Source: Update of Figure 1 in Robert J. Shiller: 'From Efficient Markets Theory to Behavioural Finance', *Journal of Economics Perspectives*, 17, Winter 2003, pp. 83-104. Data downloadable from: http://www.econ.yale.edu/~shiller/data.htm

However, we also see from Figure 15.4 that there are substantial and systematic deviations between stock prices and the PDV of dividends. First, stock prices are consistently much more volatile than our two measures of the PDV of dividends. Second, there is a high degree of persistence in the deviations between the stock price and the PDV of dividends, such that if the stock price is above the PDV at a certain time there is a high probability that it will also be above for a considerable amount of time before and after. These systematic deviations have lead many observers to ask two interrelated questions: Is the theory of the fundamental share price simply wrong in the sense that the pricing in the stock markets does not, after all, work in the 'efficient' manner expressed by the formula (6) (and hence (1) that leads to (6))? Why are stock prices so volatile meaning so much more volatile than the PDV of dividends as computed above?

The systematic discrepancies shown in figure 15.4 do not themselves imply that the formula (6) is wrong. The computation of the PDVs in the figure were ex-post, based on observed dividends. This can be done for a past point in time, in 1950 say. We can simply observe the dividends paid out by the companies over a long subsequent period and use these in the computation of the PDV for 1950. However, the pricing in the stock market at any point in time cannot be contingent on the *actual* dividends paid out in future periods since these are not known. It therefore has to depend on *expected* dividends as the formula (6) expresses by having expected dividends on the right hand side. Although we have here for simplicity assumed constant interest rates and risk premia, something similar holds for these. At any point in time the investors do not know the actual levels of future interest rates and risk evaluations (of stock vs. bonds) involved in the computation of the PDV, but must base themselves on expected interest rates and risk levels. Hence, the reason for the systematic discrepancies evidenced in Figure 15.4 may simply be that the expectations of dividends, interest rates and risk levels are often systematically different from the actual levels as these later unfold. This means that the formula (6) may be correct on its own premises as depending on *expected* values.

Accepting this view, Equation (6) suggests three possible explanations of the excess volatility of stock prices and of the persistence in the deviations between actual stock prices and the PDV of dividends:

1. Misperceptions of and fluctuations in expected future real dividends.
2. Misperceptions of and fluctuations in expected future real interest rates.
3. Misperceptions of and fluctuations in future risk levels leading to fluctuations in required risk premia.

Consider as an example the great stock market boom of the 1990s. This was probably driven by optimistic and, as it later turned out, exaggerated expectations regarding future real dividends and risk levels. Financial investors came to believe that the rapid innovations in information technology and the tendencies to more globalization would create a 'New Economy' characterized by significantly higher real growth rates in output and business profits and considerably lower risk levels as business cycle fluctuations would more or less be a phenomenon of the past. Presumably, also expectations of too low future interest rates played a role. A similar story could be told about the great boom in stock prices in the 1950s and 1960s, which was probably also to some extent built on overly optimistic expectations about future profitability, risks and interest rates.

In general, the theory summarized in Equation (6) is compatible with both the larger and longer-term stock market fluctuations observed over decades and the more frequent, short-term fluctuations in stock prices. This is the case if investors sometimes have overly optimistic or pessimistic expectations for longer periods and if they frequently revise their forecasts of future dividends, their evaluations of future risks and their interest rate expectations (the latter, for instance, because they are often faced with unanticipated changes in real interest rates).

Even though we can reconcile Equation (6) with the observed behavior of stock prices, many observers of the stock market believe that stock prices can sometimes deviate from the fundamental share price. During such periods of ‘speculative bubbles’, stocks become objects of pure short-term speculation (price increases or decreases being created by *expectations* of coming price increases or decreases), and their prices cease to be pinned down by the discounted value of expected future dividends. We do not deny that speculative bubbles may sometimes occur, but in this chapter, we will assume that stock prices do reflect expectations of future dividends, even if those expectations may not always be rational.

Summing up, Equation (6) makes no specific assumptions regarding the formation of expectations and risk premia and it does *not* exclude the possibility that financial investors may at times hold unduly optimistic or pessimistic expectations about future dividends, or that they may sometimes require ‘unreasonably’ high or low risk premia due to an inability to make realistic forecasts of the riskiness of business investment. It only assumes for given dividend expectations and risk evaluations that the expected return on shares is systematically related to the return obtainable on bonds. This is the feature needed for the theory of business investment that we now turn to.

15.3 A Tobin's *q*-theory of business investment

Stock prices and investment

Our working hypothesis is that the firm chooses its level of investment with the purpose of maximizing its market value V_t. From Equation (2), maximization of V_t is equivalent to maximizing the sum of its owners’ expected dividends and expected end-of-period wealth, $D_t^e + V_{t+1}^e$, since the individual firm has no influence on r and ε. The question is: what level of investment will maximize $D_t^e + V_{t+1}^e$?

We will denote by K_t the real capital stock of the firm and normalize the acquisition price of a unit of capital to the value 1. This means that the replacement value of the capital stock is simply K_t. The real value of the stock in the firm is V_t.

At any specific point in time the stock of the firm will have a certain actual (real) value V_t^a, and the capital stock will have a given size, K_t^a, where a stands for 'actual'. As a pure definition we will let the variable q_t indicate the ratio between the market value of the (stock of the) firm and the replacement value of the firm’s capital stock, that is, $q_t \equiv V_t^a / K_t^a$. Taking the view that the firm really consists of its capital (buildings, machines, know-how etc.), we may write the definition:

$$q_t \equiv \frac{V_t^a}{K_t^a} = \frac{\text{Market value of the firm's capital stock}}{\text{Re lacement value of the firm's capital stock}}$$

This is the famous Tobin's q. Note the direct link between stock prices and q: if the market price of shares in the firm goes up, the value of q increases correspondingly. A great advantage of specifying our theory of investment in terms of q is that this variable can be measured empirically since stock market values as well as replacement values of business assets can in principle be observed. This makes our theory of investment empirically testable.

A value of q larger than one means that one unit of capital is worth more if placed inside the firm than outside, i.e., than the price it can be bought for in the market for equipment. Intuitively this is exactly what should motivate investment (addition to the capital stock) if the firm wants to increase its market value.

A said, the formula for q above is a pure definition, and a definition does not make any theory; for that assumptions are needed. We can, of course, rewrite $q_t = V_t^a / K_t^a$ as $V_t^a = q_t K_t^a$, thus spreading out the firm's total actual value over its units of capital associating to each unit the average value q_t. This still does not make a theory. If, however, we assumed that also for other potential sizes K_t of the capital stock than the actual one K_t^a (but possibly no too far from this), the average value per unit of capital would be the same q_t, then we would have a theory saying what the value of the firm would be for alternative sizes of the capital stock. This theory would be $V_t = q_t K_t$ for K_t in the appropriate neighbourhood where q_t was assumed constant.

An assumption of a constant q over a certain range of capital stocks is an assumption of 'average q = marginal q'. If the general relationship between the firm's capital stock and its value is $V_t = Q(K_t)$, where of course $V_t^a = Q(K_t^a)$ and $q_t = Q(K_t^a) / K_t^a$, the assumption says that at least in some neighbourhood around K_t^a, this function is linear with slope q_t, that is, $V_t = q_t K_t$. It is not given by nature that the function Q should look like this, so this is really an assumption.

Now, the firm cannot really decide on the size of the capital stock K_t in period t, since at that time, K_t is predetermined from investment (and depreciation) in the past. It can, however, determine the size of next period's capital stock by its investment I_t in the current period t through the identity:

$$K_{t+1} = K_t + I_t \tag{7}$$

For simplicity, this abstracts from depreciation of the existing capital stock, but as you will learn from an exercise, our theory of investment generalizes to allow for depreciation.

We will assume that the firm and its owners perceive that over the relevant range of capital stocks next period (the stocks that can be reached by the levels of investment under consideration by the firm), the value of the firm will be proportional to the size of the capital stock by a proportionality factor q_{t+1}^e, the next period's expected *q*. We can state this assumption as follows: There is a $q_{t+1}^e > 0,$ such that for all levels of capital K_{t+1} reachable by the investment levels I_t under consideration in period *t*, the expected value of the firm in the next period is $V_{t+1}^e = q_{t+1}^e K_{t+1}.$

For the expected *q* we make the simplest possible assumption of static expectations: The expectation of next period's *q* equals this period's observed *q,* that is, $q_{t+1}^e = q_t.$

Combining $V_{t+1}^e = q_{t+1}^e K_{t+1}$ and $q_{t+1}^e = q_t$ gives $V_{t+1}^e = q_t K_{t+1}$ and then by using also the identity (7):

$$V_{t+1}^e = q_t(K_t + I_t) \tag{8}$$

We have now expressed V_{t+1}^e in terms of $I_t,$ which will be useful for finding the optimal level of investment. Let us next consider the other determinant of current market value $V_t = (D_t^e + V_{t+1}^e)/(1 + r + \varepsilon),$ that is, the expected dividend D_t^e for period *t*, and see how this relates to investment.

Suppose for the moment that the firm finances all of its current investment spending I_t via retained profits (we shall consider external financing later on) and pays out as dividends exactly its profit minus its investment costs. Furthermore, suppose realistically that changes in the firm's capital stock, $\Delta K_t = K_{t+1} - K_t = I_t,$ imply *adjustment costs*, including costs of installing new machinery, costs of training workers to use the new equipment, and possibly costs of changing the firm's organization. For convenience, all such costs will be called 'installation costs' and will be denoted by $c(I_t)$ indicating that they are a function of investment spending.

We assume that installation costs are increasing and convex, so that $c(0) = c'(0) = 0,$ and $c'' > 0.$ It is reasonable that installation costs will rise more than proportionately with investment spending. If investment is low, the changes in the capital stock are small and can be accommodated without significant changes in the firm's organization. But when investment is high, the firm may have to undertake significant organizational changes and extensive training of employees, and the attention of managers will be diverted from the firm's day-to-day business. Such an organizational overhaul is typically very expensive. Note that $c(I_t)$ is a cost in excess of the direct investment cost $I_t.$ A simple installation cost function capturing our assumptions is:

$$c(I_t) = \frac{a}{2} I_t^2, \quad a > 0 \tag{9}$$

This implies that $c'(I_t) = aI_t,$ that is, the *marginal* installation cost increases in proportion to the level of investment, reflecting that large changes in the capital stock are disproportionately more costly than small changes

With our assumptions, the expected dividend for period *t* will be equal to the expected profit in period *t*, denoted by Π_t^e (and measured before deduction of installation costs) minus that part of profit which is retained in order to finance the expenditure $I_t + c(I_t)$ associated with new investment:

$$D_t^e = \Pi_t^e - I_t - c(I_t) \tag{10}$$

Starting from (2), and inserting the expression in (8) for V_{t+1}^e and the expression in (10) for D_t^e, we find the following expression for the firm's market value at the start of period *t* as linked to the current investment level I_t:

$$V_t = \frac{D_t^e + V_{t+1}^e}{1+r+\varepsilon} = \frac{\overbrace{\Pi_t^e - I_t - c(I_t)}^{D_t^e} + \overbrace{q_t(K_t + I_t)}^{V_{t+1}^e}}{1+r+\varepsilon}. \tag{11}$$

The firm chooses its level of investment I_t so as to maximize the initial wealth of its owners, V_t, *taking the stock market's valuation of a unit of capital,* q_t *, as given and assuming this valuation also to prevail in the next period for varying levels of the capital stock*. Since the profit in period *t* cannot be expected to depend on the investment level in period *t* (this influences the profit of period *t*+1 the earliest), the first-order condition for the solution to this maximization problem is:

$$\begin{aligned} &\frac{\partial V_t}{\partial I_t} = \frac{-1 - c'(I_t) + q_t}{1+r+\varepsilon} = 0 \quad \Leftrightarrow \\ &q_t = 1 + c'(I_t) \end{aligned} \tag{12}$$

Note that the second order condition for a maximum, $-c''(I_t) < 0,$ is fulfilled under our assumptions, and since (12) can only have one solution $I_t > 0$, such a solution really gives a maximum.

Assuming the specific form (9) of the installation cost function, where $c'(I_t) = aI_t$, we get for the optimal level of investment:

$$\begin{aligned} &q_t = 1 + aI_t \quad \Leftrightarrow \\ &I_t = \frac{q_t - 1}{a} \end{aligned} \tag{12'}$$

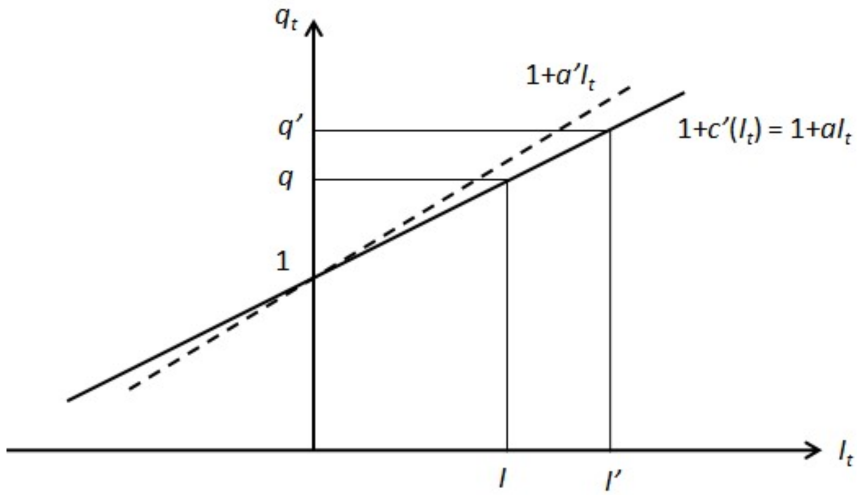
The investment rule $q_t = 1 + c'(I_t)$ of (12) may be explained as follows: to finance the acquisition and installation of an extra unit of capital in period *t*, the firm must reduce its dividend payout in period *t* by an amount equal to the acquisition cost of a unit of capital – which we have set equal to 1 – *plus* the marginal installation cost $c'(I_t)$. This forgone dividend $1+c'(I_t)$ is the shareholders' marginal opportunity cost of allowing the firm to undertake an extra unit of investment. The shareholder's marginal benefit from investment is the gain q_t in the value of shares resulting from the installation of an extra unit of capital. At the optimal level of investment, the marginal dividend forgone is just compensated by the extra capital gain on shares. Clearly, the higher the market valuation q_t of an extra unit of capital, the further the firm can push its level of investment before the marginal installation cost reaches the threshold where the shareholder's additional capital gain is offset by the extra dividend forgone.

With the simple installation cost function (9), we obtain the simple investment schedule in (12'), which also says that investment will be higher the higher the level of the stock price q_t. Particularly (12') says that $q_t > 1$ will induce investment $I_t > 0,$ while $q_t < 0$ will induce disinvestment $I_t < 0.$ The latter would be perfectly possible in our model and correspond to capital being uninstalled and sold in the market at the going price (normalized to one) for capital. In the real world, such selling off of capital only takes place rarely and mostly in connection with shutdowns of firms. Usually an adjustment of capital downwards will occur through depreciation, that is, through zero gross investment implying negative net investment. Equation (12') further shows that higher marginal installation costs (reflected in a higher value of the cost parameter a) will reduce the optimal level of investment, as one would expect.

These findings are also clear from the graphical illustration of the determination of investment in Figure 15.5: a higher value of q_t implies higher investment, as illustrated by the shift from q to the larger q', and a higher value of a increases the slope of the curve $1+c'(I_t)=1+aI_t$ as illustrated by the dashed line and thereby other things being equal reduces the value-maximizing level of investment where $1+c'(I_t)=q_t.$

The investment schedule (12) or (12') also holds when investment outlays are financed by issuing new debt or new shares rather than by retaining profits. Regardless of the mode of finance, the installation of an extra unit of capital will increase the expected market value of the firm's assets by the amount q_t, still assuming that the current stock price gives an indication of the expected value. If the cost of buying and installing the extra unit of capital, $1+c'(I_t),$ is financed by an increase in the firm's outstanding debt or by the issue of new shares, the expected increase in the market value of the shares owned by the firm's *existing* shareholders will be equal to the rise in total market value q_t *minus* the value of the newly issued debt or equity, $1 + dc/dI_t$. Of course, it is optimal for existing shareholders to let the firm expand its investment until the expected marginal gain in the value of their shares is driven down to zero, that is, until $q_t = 1+c'(I_t).$ But this is exactly the investment rule leading to the investment schedule in (12)! Hence we obtain the important result that the investment function (12) or (12') is valid *regardless of the method of investment finance*.

Figure 15.5 The optimal level of investment



In summary, we have:

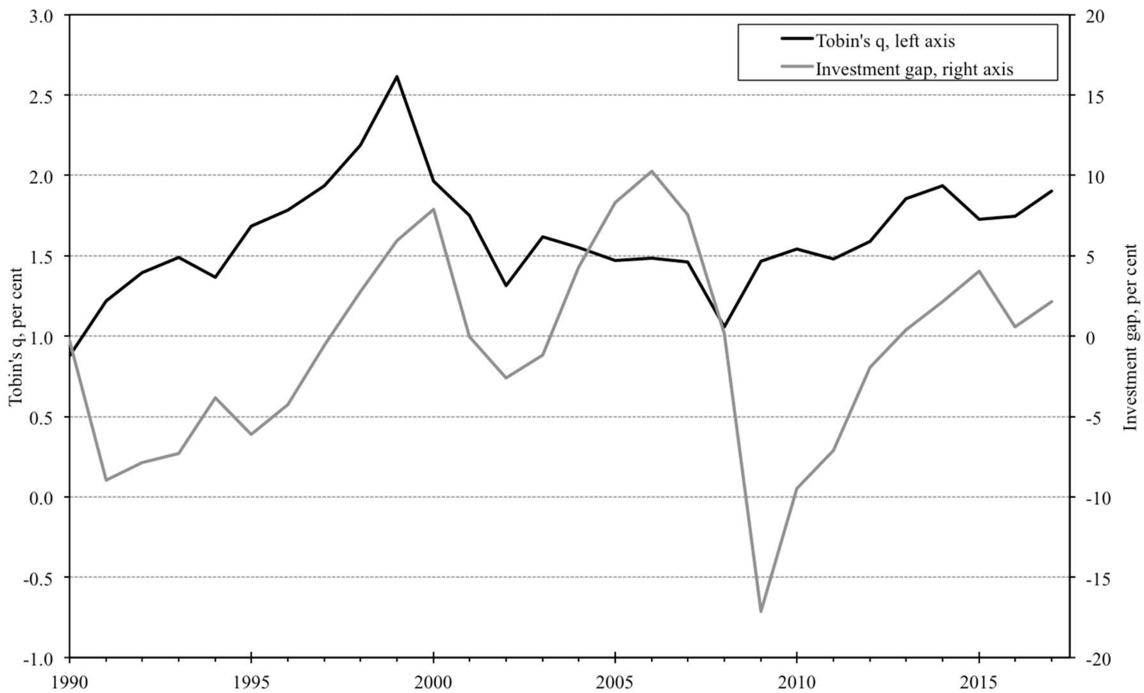
THE q-THEORY OF BUSINESS INVESTMENT

A value-maximizing firm will invest up to the point where the rise in its market value induced by an extra unit of investment is just equal to the sum of the acquisition and installation cost of buying and installing an additional unit of capital. Because the marginal installation cost increases with the volume of investment, the optimal investment level is higher, the higher an increase in the firm's capital stock is valued by the stock market. An increase in the ratio of stock prices to the acquisition cost of the firm's assets (that is, a rise in q) will therefore stimulate its investment.

Figure 15.6 plots the investment gap in the USA (relative deviation in gross domestic investment from its HP trend) against an estimate of Tobin's q, defined as the ratio of the stock market value of firms to the replacement value of their capital stock, as above.

We see that total investment (its deviation from trend) and the q-ratio do tend to move together to a decent degree, and even with some tendency for turning points in q to lead turning points in investment, but the relationship is not very tight. Part of the problem may be that, in practice, capital and stock prices reflect many 'intangible' business assets besides physical capital, for example the value of know-how or goodwill, and the amount and price of know-how or goodwill are not easy to measure and incorporate into the replacement value of a firm's capital stock as required for measuring Tobin's q. Another part of the explanation for the sometimes weak relationship between investment and Tobin's q may be the earlier mentioned problem of average vs. marginal q. The estimated value of q reflects the *average* ratio between the *total* market value and the *total* replacement value of the capital stock, whereas investment decisions should depend on the *marginal* value of q, that is, on the *increase* in market value relative to the acquisition price of an *additional* unit of capital. In our analysis above, the marginal and average values of q were, as mentioned, *assumed* to be identical, in consequence of our simplifying assumption of perceived proportionality between the firm's future capital stock and the future value of the firm. If this proportionality breaks down, the marginal q will no longer coincide with the average q. One could easily build a theory similar to the above one on the *marginal* q different from the *average* one, but this would suffer from not being empirically testable since only the latter can be measured empirically.

Figure 15.6 Investment gap and Tobin's q in the USA, 1990-2017



Note: Investment is gross domestic investments in chained 2009 USD. Investment gap calculated by HP-filter with $\lambda = 100$. No end points have been removed.
Sources: Tobin's q from Smither & Co. Ltd. Investment data from the Bureau of Economic Analysis.

The role of interest rates, profits and sales

How does the q-theory of investment square with the conventional assumption that investment depends negatively on the real interest rate? The claim that investment varies positively with stock prices is fully consistent with the hypothesis that it varies negatively with the real interest rate.

To see this, let us go back to Equation (6) and let us assume for simplicity (since this will not affect our qualitative conclusion) that real dividends are expected to stay constant at the level D^e from period t and onwards. Equation (6) then becomes:

$$V_t = D^e\left[\frac{1}{1+r+\varepsilon}+\frac{1}{(1+r+\varepsilon)^2}+\frac{1}{(1+r+\varepsilon)^3}+\dots\right] \tag{13}$$

If we multiply both sides of (13) by $1 + r + \varepsilon$ and subtract (13) from the resulting equation, we get:

$$V_t = \frac{D^e}{r+\varepsilon} \tag{14}$$

Equation (14) is just a special version of the general formula stating that the value of the firm equals the present discounted value of expected future dividends. Now recall that by definition, $V_t = q_tK_t$. From this relationship and (14) it follows that:

$$q_t = \frac{D^e/K_t}{r+\varepsilon} \tag{15}$$

If we insert this expression for q in our optimal investment rule (12') we get:

$$I_t = \frac{q_t - 1}{a} = \frac{1}{a}\left(\frac{D^e / K_t}{r+\varepsilon} - 1\right) \tag{16}$$

According to (15) the market value of a unit of the firm's capital stock, q_t, equals the discounted value of the expected future dividends per unit of capital. Hence, a rise in the real interest rate r will, *ceteris paribus*, reduce the stock price and hence q_t, and this will reduce investment as shown by (16).

Equation (15) and (16) show that q_t and I_t depend not only on the interest rate r, but also on the risk premium ε and the expected dividend ratio, D^e/K. An increase in the interest rate as well as an increase in the risk premium will imply that expected future dividends are discounted down by a larger rate implying a smaller V_t and q_t and hence lower investment. A higher expected dividend ratio on the other hand implies a larger q_t and larger investment.

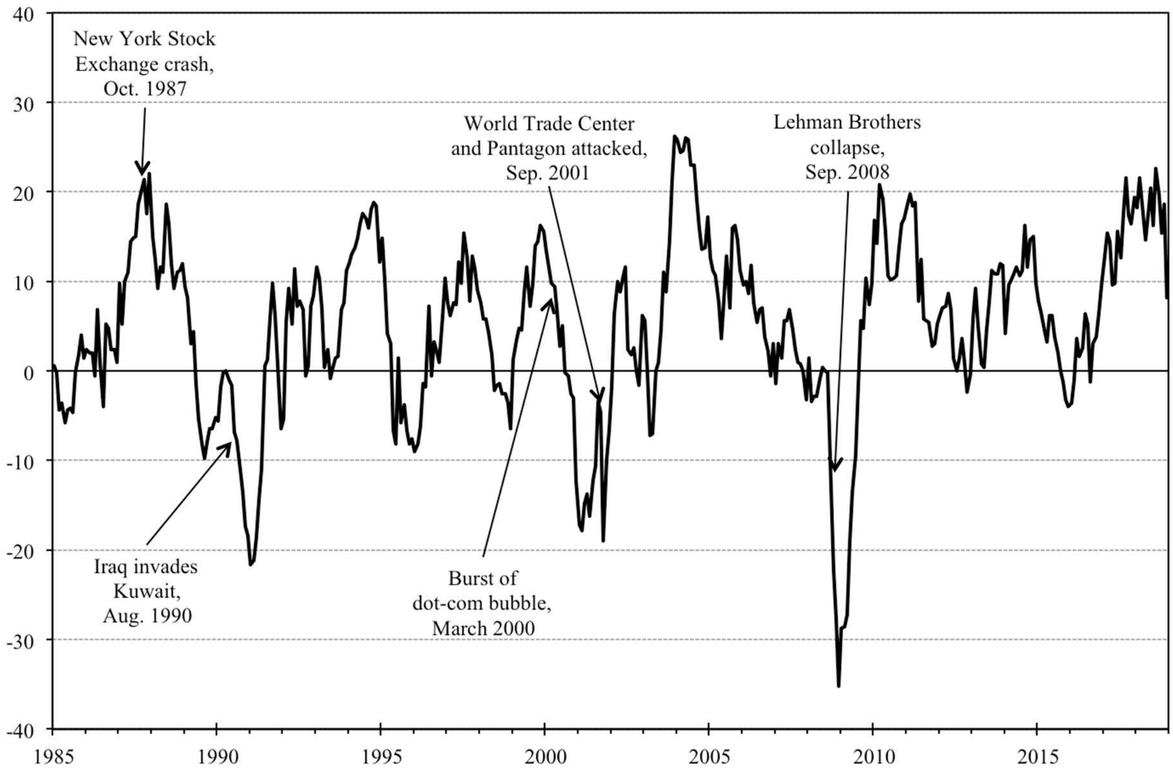
Let us take a closer look at the dividend ratio and its likely determinants. Since dividends ultimately come from profits, expected dividends must be tightly related to expected future profits Π^e, that is, $D^e \approx \Pi^e$. It seems reasonable to assume that current actual profits Π_t have a positive influence on expected profits, but at the same time that actual profits do not completely tie down expected profits. Expected profits probably also depend on subjective elements of investors' expectations such as the expected future level of demand, activity and income, expectations about future productivity, input prices (e.g., oil price shocks), policy interventions and geopolitical events such as crisis, riots, revolution and war in various countries. We will denote these subjective expectations by E. We can think of the combination of ε and E as business 'confidence' or investors' 'animal spirits', where ε covers expected risk (more precisely the required premium implied by expected risk) and E covers the expected (average) values of future demand, productivity, input prices etc.

It may thus be reasonable to assume about expected profits that $\Pi^e = \Pi_t \cdot E$, where E is a confidence variable that corrects current profits up or down to arrive at expected profits according to expectations being optimistic or pessimistic (E relatively large or small). All in all these considerations would suggest that $D^e \approx \Pi_t \cdot E$, and inserting this into (16) gives:

$$I_t = \frac{1}{a}\left(E \frac{\Pi_t / K_t}{r + \varepsilon} - 1 \right) \tag{17}$$

This theory emphasizes the importance for business investment of investors' subjective levels of confidence (animal spirits): although a higher current profit is likely to boost expected future profits and dividends, it is too primitive to expect a mechanical one-to-one impact of the former on the latter variable. Firms and investors may sometimes have good reason to expect that future profitability will deviate from realized current profits. Indeed, the fact that the economy moves up and down in cycles suggests that intelligent investors will not mechanically extrapolate current earnings into the future. Instead, they will revise their expectations regarding future sales and profits as they receive new information on relevant economic and political events. Most advanced countries publish indices of 'business confidence' to measure business expectations regarding the near future. Usually these indices build on survey data where a sample of business managers report current and expected movements in their future output, sales, employment, investment etc. They are typically in some way asked if they expect the situation to be better or worse some time ahead than today and the confidence index is simply computed as the percentage answering better minus the percentage answering worse. Figure 15.7 shows the evolution of one such index of business confidence in the USA. We see that business confidence can fluctuate quite a lot and that sometimes unanticipated political and economic events affect the index significantly. Although confidence is probably very important for investment demand (and for consumption demand as well) it is not easy to incorporate into a theory as other than an (important) exogenous variable, as will also be the case for our theory.

Figure 15.7 Business confidence in the USA



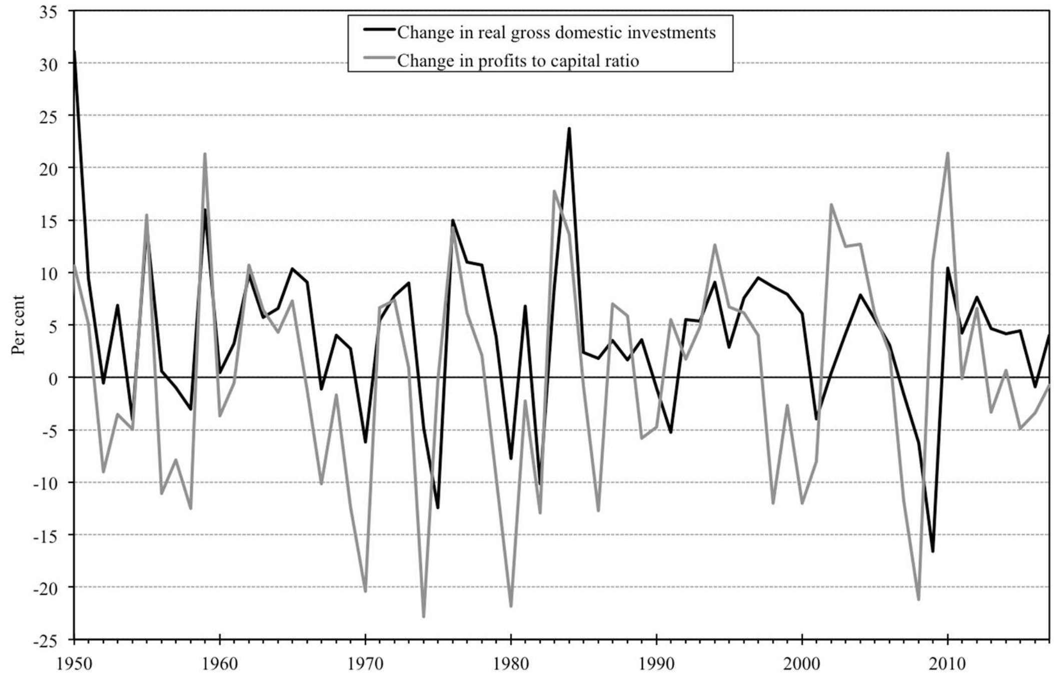
Note: Note: Based on monthly data. The figure shows the balance of the per cent of positive responses over the per cent of negative responses. Hence, a value of 20 implies that the survey contains 20 percentage points more positive responses than negative responses.
Source: Federal Reserve Bank of St. Louis**.**

Equation (17) says that for a given level of 'animal spirits', investment should be positively related to the firm's current *rate* of profit (the profit to capital ratio). Hence, we would expect overall to observe a positive correlation between changes in the current profit rate and changes in current investment, although not a very tight one (as investment to a large degree depends a lot on the shifting moods of investors). Figure 15.8 below suggests that such a positive and not too tight relationship does in fact exist.[2]

Finally, we would also expect a positive relationship between the profit rate and the output-capital ratio. In 'good times' where output is high, either because of high demand or high productivity, it will typically also be good times for businesses so that firms earn relatively high profits. Figure 15.9 suggests that a positive relationship between the rate of profit rate and the output-capital ratio is not farfetched.

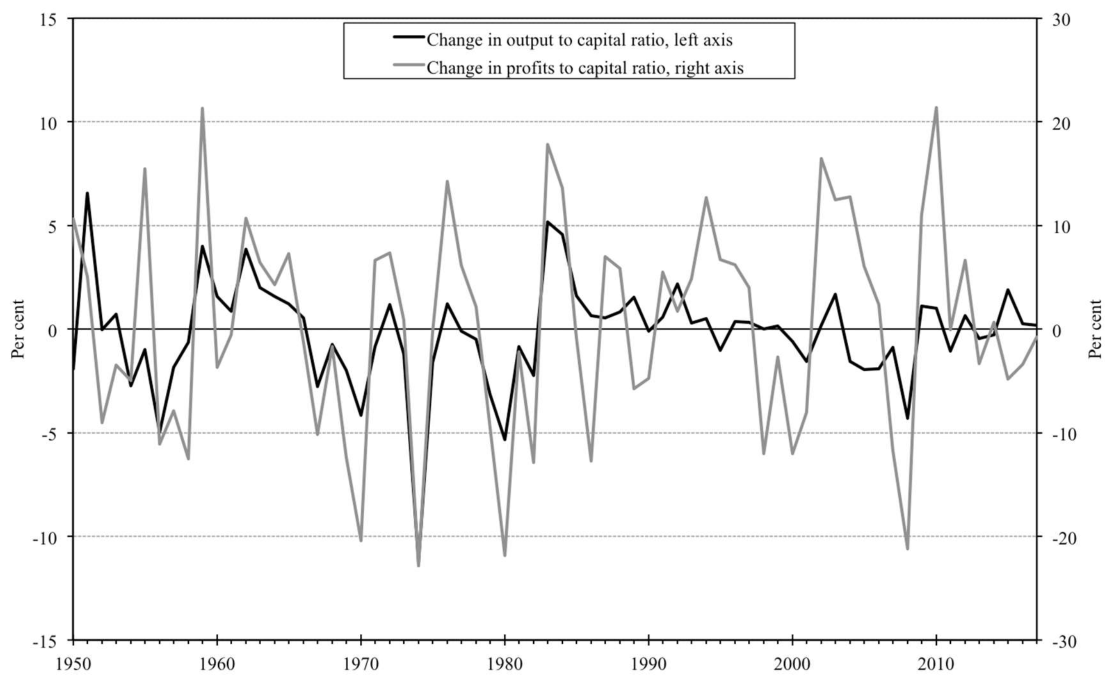
[2] By looking at changes rather than levels we take out the underlying trend increase in investment over time that would disturb a levels comparison.

Figure 15.8 Relative changes in investment and profit to capital, USA, 1950–2017



Note: Based on annual data. Investments are gross domestic investments in 2009 USD.
Source: Bureau of Economic Analysis

Figure 15.9 Relative changes in ratio of output to capital and profit to capital, USA, 1950-2017



Note: Based on annual data.
Source: Bureau of Economic Analysis

If we approximate Π_t / K_t by $= \alpha Y_t / K_t$ we can rewrite (17) as:

$$I_t = \frac{1}{a}\left(\alpha E \frac{Y_t / K_t}{r+\varepsilon} - 1\right) \tag{18}$$

This shows that the q-theory of investment plausibly supports the hypotheses that:

- business investment depends negatively on the (expected) real interest rate, r
- business investment depends negatively and the expected level of risk, ε
- business investment depends positively on output, Y
- business investment depends negatively on the existing level of capital, K
- business investment depends positively on confidence, E

The expression (18) contains specific functional forms. In a more general form, and dropping time subscripts for convenience, we may write our investment function as $I = I(Y, r, \varepsilon, E, K)$. Here both ε and E represent subjective expectations, where ε contains the expected riskiness of (variability in) future return on investment and E contains expectations of relevance for future average returns. It will be convenient to collect these subjective expectations in one variable that we will call ε, which thus includes all aspects of business confidence or investors' animal spirits. This gives rise to the general investment function:

$$I = I\Big(\underset{+}{Y}, \underset{-}{r}, \underset{+}{\varepsilon}, \underset{-}{K}\Big). \tag{19}$$

The signs below the variables indicate the signs of the corresponding partial derivatives of the I-function. In terms of the q-theory, an increase in Y or ε will stimulate investment by raising q through an increase in the expected dividend ratio D^e/K and possibly through a fall in the risk premium ε. An increase in the current capital stock K reduces investment by driving down D^e/K, and an increase in the real interest rate r likewise discourages investment via a negative impact on q.

In the case of firms whose market value is not directly observable because they are not quoted on the stock exchange, it is inappropriate to interpret Equation (19) literally in terms of the q-theory. Nevertheless, as Exercise 3 will make clear, the investment behaviour of such firms may still be described by an equation like (19) if they invest with the purpose of maximizing the present value of the net cash flow to their owner (thereby maximizing his or her wealth). Equation (19) therefore summarizes our general theory of business investment. In words, we can sum up our insights as follows:

THE BUSINESS INVESTMENT FUNCTION

The value of Tobin's q reflects expected future dividends per unit of capital, which react positively to a rise in the current profit rate. The profit rate in turn varies positively with the output–capital ratio. Hence, investment is an increasing function of current output and a decreasing function of the current capital stock. A higher real interest rate or a higher risk premium increases the rate at which expected future dividends are discounted, thus depressing investment by reducing the value of Tobin's q.

Econometric research has confirmed that changes in Y, K and r do influence investment in the manner indicated in (19). However, researchers have also found that it is quite difficult to explain just close to fully all of the observed movements in investment. This is likely because investment is very much influenced by the animal spirits of investors (represented by E in equation 19)) a variable that it is hard to measure in a reliable way. As it has turned out, business investment is just very hard to predict.

15.4 A Tobin's q-theory of housing investment

Housing investment is an important component of total private investment. As it appears from Table 13.2 and 13.3 of Chapter 13, housing investment is typically around one third of total private investment and, if anything, it is even more volatile than total private investment and about as correlated with GDP. Hence, fluctuations in residential investment often play an important role during business cycles and, as we shall see below, there are long and large swings in housing prices and investment over the cycle.

A basic factor contributing to the volatility of housing investment is the fact that housing capital is highly durable. In any year, the construction of new housing is only a very small fraction of the existing housing stock. To accommodate even small percentage changes in consumer *demand* for housing capital, construction activity may therefore have to undergo large relative changes.

In this section, we will show that housing investment may be explained along lines, which are similar to the q-theory of business investment.[3] The present section may therefore be seen as an illustrative special version of the q-theory, adapted to fit the housing market. As a by-product of our theory of housing investment, we will develop a theory of the formation of housing prices and identify the factors, which may cause fluctuations in the market value of the housing stock. Since the stock of housing capital is an important component in total household wealth, and since the next chapter will show

[3] The theory of the housing market we are about to present is inspired by the following influential article by James A. Poterba: 'Tax Subsidies to Owner-Occupied Housing: An Asset-Market Approach', *Quarterly Journal of Economics*, **99**, 1984, pp. 729–752. However, while Poterba assumed that homeowners rationally anticipate future capital gains or losses on their property, we simplify by treating the expected rate of capital gains as an exogenous variable.

that private consumption depends on private wealth, the theory of the housing market developed below will also help us to understand fluctuations in private consumption.

Construction activity and Tobin's *q*-for housing investment

In any given period, the housing stock is predetermined at a specific level $H > 0$ by the accumulated historical levels of housing investment and depreciation. The current level of gross housing investment or construction activity, I^H, adds to the housing stock at the start of the next period while depreciation suctrtacts. From one period to the next, the housing stock will change by an amount ΔH according to the equation $\Delta H = I^H - \delta H$, where δ is the rate of depreciation of housing capital. In our model, we will for simplicity disregard economic growth (growth in the labour force or in technology), so in a long-run equilibrium where all variables are at their structural levels, the stock of houses will be constant, $\Delta H = 0$, implying that $I^H = \delta H$. For a given housing stock *H*, the 'normal' level of gross housing investment will thus be δH, but in the short run actual housing investment I^H can deviate from δH in response to various shocks.

Housing investment or construction activity, I^H, is the same as the output of the economy's construction sector. We will assume there is one representative firm in this sector, and we make the following assumptions on the production possibilities of this:

In the long run, the production function of the construction sector exhibits constant returns to scale. To build one unit of house requires specific amounts of labour power, land, building materials, construction equipment, organization etc., when these inputs are used in cost minimizing combination, and to build I^H houses requires I^H times as many of these inputs. Hence, if the real price (cost) of the inputs needed to construct one unit of house is $P > 0$, measured in units of consumption goods, then in the long run, where all inputs in the construction sector can be adjusted, the cost of building I^H houses is $P \cdot I^H$. In the short run, however, some inputs are fixed, e.g., the equipment and organizational aspects of the construction sector. In any given period, the inputs that are fixed in the short run will be geared to a specific level of investment that we will call $\bar{I}^H > 0$. Therefore, in the short run there will be an additional adjustment cost that is larger the further away from $\bar{I}^H$ that actual investment I^H is. We assume that this adjustment cost is convex and equal to $½\alpha\left(I^H - \bar{I}^H\right)^2 P$. Hence the total cost in the short run of the construction firm is:

$$TC(I^H) = \left[I^H + \frac{1}{2}\alpha\left(I^H - \bar{I}^H\right)^2\right]P \tag{20}$$

By differentiation, the marginal cost of an additional unit of housing investment is:

$$MC(I^H) = \left[1 + \alpha\left(I^H - \bar{I}^H\right)\right]P \tag{21}$$

Our assumption of a convex adjustment cost plays a role similar to the effect of convex installation costs in our theory of business investment above: an increase in construction within a period takes place at increasing marginal cost. This speaks for, other things being equal, creating a given increase in the stock of housing in smaller portions over some periods rather as one big construction activity in one period.

If p^H is the market price of a unit of housing, again measured in units of consumption goods, the real sales revenue of the representative construction firm will be: $TR(I^H) = P^H I^H$. Marginal revenue will then be $MR(I^H) = P^H$, and real profits will be:

$$\Pi(I^H) = TR(I^H) - TC(I^H) = p^H I^H - \left[I^H + \frac{1}{2}\alpha\left(I^H - \bar{I}^H\right)^2 \right] P \tag{22}$$

Taking the housing price p^H and the composite input price P as given, the construction firm chooses its level of activity I^H with the purpose of maximizing its profit. According to (22), the first-order condition for profit maximization, $\Pi'(I^H) = 0$, implies:

$$\overbrace{p^H}^{MR(I^H)} = \overbrace{\left[1 + \alpha\left(I^H - \bar{I}^H\right)\right]P}^{MC(I^H)} \quad \Leftrightarrow \quad I^H = \frac{1}{\alpha}\left(\frac{p^H}{P} - 1\right) + \bar{I}^H$$

The profit-maximizing construction firm will push construction activity to the point where the marginal construction cost equals the marginal revenue, in turn equal to the market price of a housing unit.

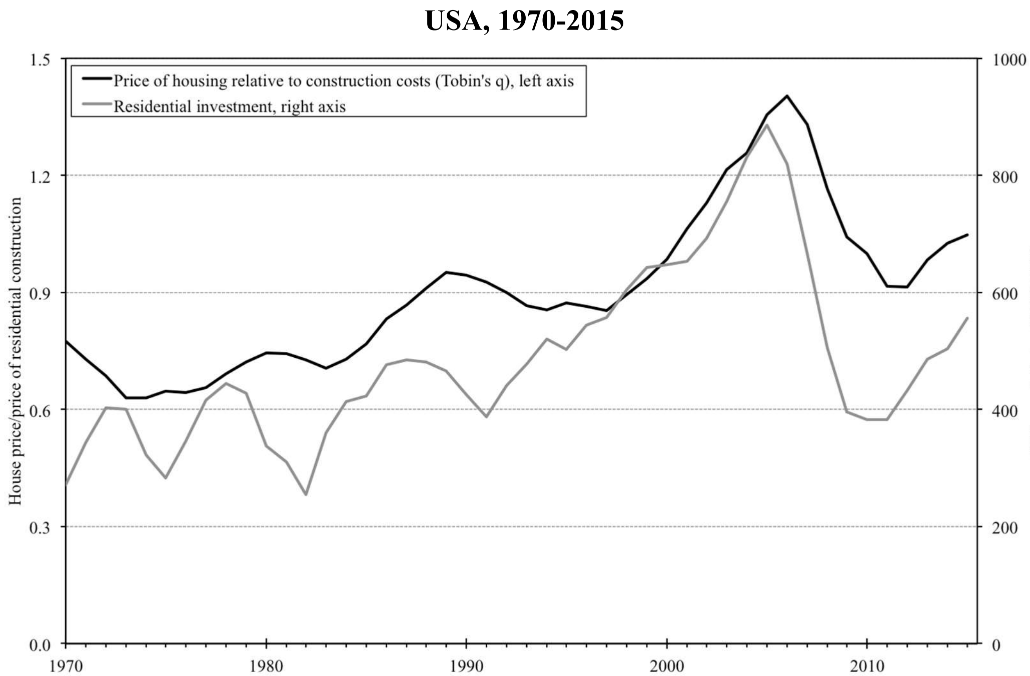
Very naturally we will assume that the level of investment that the equipment etc. of the construction sector is geared to is the normal level of gross housing investment of a long-run equilibrium, that is, $\bar{I}^H = \delta H$. Hence, the supply curve for the construction sector, or gross housing investment as a function of the house price, is:

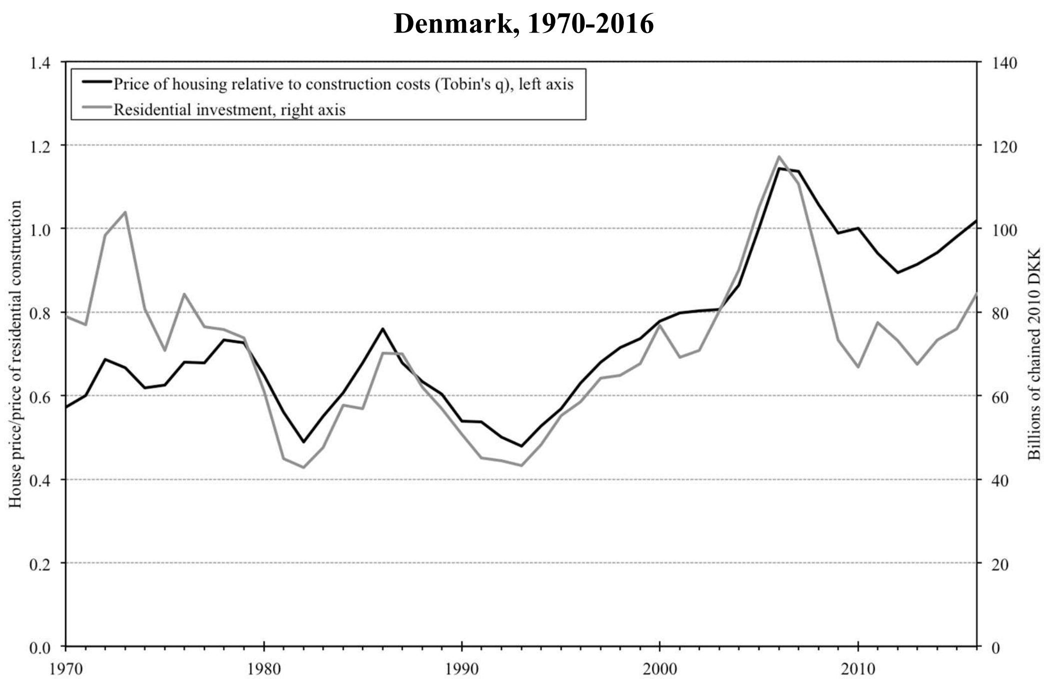
$$I^H = k\left(\frac{p^H}{P} - 1\right) + \delta H, \quad k \equiv \frac{1}{\alpha} \tag{23}$$

According to this investment function, total gross housing investment consists of the normal investment level δH required to preserve the existing housing stock, H, plus an addition that depends on the ratio between the house price and construction costs and is negative when $p^H < P$. The ratio p^H/P is an analogue of Tobin's q here. Housing investment I^H will be larger the higher the q-ratio of the housing price to the construction cost is, but, because marginal cost of construction is increasing from zero, a $q > 1$ will only imply a finite level of investment, not an infinite one. And a $q < 1$ will also give some gross investment as long as $k\left(p^H / P - 1\right) > -\delta H$, which we will assume.

Figure 15.10 shows that this theory of housing investment fits the facts quite well. There is a relatively tight relationship between the ups and downs of the 'housing q' and residential investment. This is particularly true in the longer run. The figure also demonstrates the long swings of housing prices and investment.

Figure 15.10 Residential investment and the price of housing relative to construction





Note: Based on annual data.
Sources: Price of housing and construction costs for USA are from Robert J Shiller, http://www.econ.yale.edu/~shiller/data.htm. Data on residential investments for USA are from Bureau of Economics Analysis. Data for Denmark are from SMEC database, the Danish Economics Councils.

The tight correlation that the figure shows does not *prove* that the housing q causes housing investment. It could be a simple reflection of good times pulling upwards in both house prices and housing investment, and vice versa. However, the idea that a higher housing q should *cause* higher housing investment is theoretically convincing and *not* contradicted by the data.

Summarizing:

THE q-THEORY OF HOUSING INVESTMENT

When the market price of existing residential property rises relative to the cost of constructing a new unit of housing, it becomes profitable for firms in the construction industry to increase the supply of new housing. Hence, housing investment goes up. This q-theory of housing investment is in accordance with the data.

The demand for housing and the short-run house price

Like the q-theory of business investment, our theory of housing investment is consistent with the hypothesis that investment varies negatively with interest rates and positively with total income. To demonstrate this, we will now first develop a theory of housing demand in order to explain, in the next step, the housing price p^H, appearing in Tobin's q for housing investment, $q = p^H / P$.

Consider a representative household that, in a given period, must decide on how many units of (owner-occupied) housing, H^d, and how many units of ordinary, non-durable consumption goods, C, it will consume. The trade-off between housing and ordinary consumption takes place by maximizing utility U, which we assume to be given by the Cobb–Douglas function:

$$U(H^d, C) = \left(H^d\right)^{\eta} C^{1-\eta}, \quad 0 < \eta < 1 \tag{24}$$

In practice, the consumer will derive utility from the housing *service* flowing from the housing stock H^d, and not from the housing stock as such. The specification in (24) just assumes that the housing service is proportional to the housing stock.

The price of ordinary consumption is normalized to one, so C units cost C. What is the price of a unit of the durable good, housing, used for one period? The answer is the so-called 'user-cost', in general connected to a unit of capital good, here specializing to a unit of house.

Consider a household that has borrowed to acquire a housing stock H^d at the going market price p^H per unit of housing. Suppose the consumer has to spend an amount δp^H per unit of house on repair and maintenance each period in order to maintain the quality of the house and thereby to maintain the value p^H per unit of house in the absence of general increases or decreases in house prices (relative to consumption goods). Suppose further that the real interest rate on mortgage debt is r. The consumer's total cost of housing consumption if the real house price remains constant will then be $(r + \delta)p^H H^d$, or $(r + \delta)p^H$ per unit. However, if the consumer expects the market price of houses to increase over the period considered relative to the price of ordinary consumption goods, this is really a deduction in the cost of holding the house (it will then take a smaller spending on repairs etc. to maintain the value of the house). If we denote by g^e the expected relative increase in house prices (which can be negative),

$$g^e = \frac{\left(p_{+1}^H\right)^e - p^H}{p^H}, \tag{25}$$

then one unit of house is expected to increase in price by $g^e p^H = (p_{+1}^H)^e - p^H$ consumption units. The total cost of holding one unit of house for one period will then be,

$$uc = (r + \delta - g^e)p^H = v \cdot p^H, \quad v \equiv r + \delta - g^e \tag{26}$$

the user-cost of 'housing capital', reflecting the financial cost, r, as well as the costs of maintenance, δ, and the expected capital gains or losses, captured by g^e. Note that if the household has financed the acquisition of its house by own past savings (and not borrowed), the appropriate user-cost is still $(r + \delta - g^e)p^H$. This should include the interest rate r as an opportunity cost, since this is the income the consumer forgoes by investing in a house rather than in interest-bearing assets.[4]

In the analysis below, we will treat δ and g^e as exogenous constants, but it is important to keep in mind that changes in expected capital gains or losses on houses (changes in g^e) will cause shifts in the user-cost of housing.

If the consumer sets aside a real budget Y to spend on housing and ordinary consumption, an amount that we will assume to be closely related to the consumer's income, the consumer's budget constraint in any given period will be:

$$C + v \cdot p^H H^d = Y, \tag{27}$$

where we have used that the price of non-durables is equal to 1.

The consumer wishes to allocate total spending between housing and non-durables so as to maximize his utility U. Using the budget constraint (27) to eliminate C from (24), we get:

$$U = \left(H^d\right)^\eta [Y - v \cdot p^H H^d]^{1-\eta} \tag{28}$$

The consumer's optimal level of housing demand is found by maximizing the utility U of (28) with respect to H^d. The first-order condition, $\partial U / \partial H^d = 0$, for a solution is:

[4] In practice, the tax system also affects the user-cost of housing. This is a theme of Exercise 4.

$$\overbrace{\eta\left(H^d\right)^{\eta-1}[Y-v\cdot p^H H^d]^{1-\eta}}^{\partial U/\partial H^d}-v\cdot p^H\,\overbrace{(1-\eta)\left(H^d\right)^{\eta}[Y-v\cdot p^H H^d]^{-\eta}}^{\partial U/\partial C}=0, \tag{29}$$

or

$$\frac{\partial U/\partial H^d}{\partial U/\partial C}=v\cdot p^H=(r+\delta-g^e)p^H. \tag{30}$$

Equation (30) says that in the consumer's optimum situation, the marginal rate of substitution between housing and non-durables (the left-hand side) must equal the relative price of housing, $(r+\delta-g^e)p^H$. If we solve (29) for H^d, we get the *demand for housing*:

$$H^d=\frac{\eta Y}{v\cdot p^H}=\frac{\eta Y}{(r+\delta-g^e)p^H} \tag{31}$$

This has the familiar form that follows from Cobb-Douglas utility: the parameter η is the budget share gong to housing, and total housing demand is the part of the budget going to housing, ηY, divided by the price of one unit of housing, which is the user-cost, $v\cdot p^H=(r+\delta-g^e)p^H$. We have derived (31) as resulting from the optimization of one representative household, but we will assume that the market demand function for houses is of the same form.

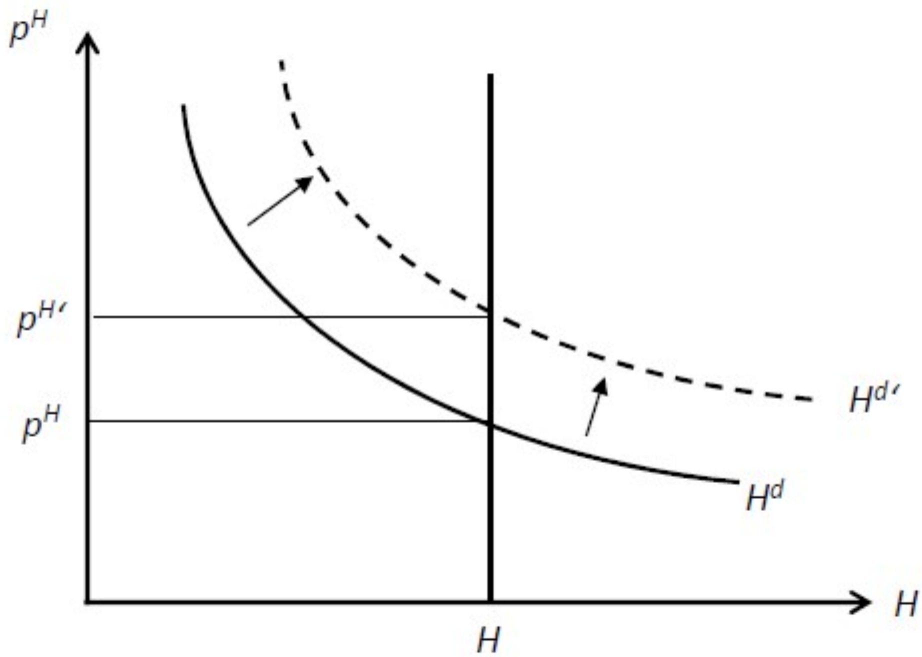
While (31) thus gives the *demand* for housing, the aggregate *supply* of housing is fixed (predetermined) in the short run as explained above. In the short run the market price of houses, p^H, must therefore adjust to bring the demand for housing, H^d, in line with the existing supply, H. Inserting the expression for H^d from (31) into the equilibrium condition $H^d=H$ and solving for p^H, we get the market-clearing price of houses:

$$p^H=\frac{\eta Y}{v\cdot H}=\frac{\eta Y}{(r+\delta-g^e)H}. \tag{32}$$

Figure 15.11 illustrates how the equilibrium price of houses is determined in the short run where the supply of housing is fixed at the level H. *Ceteris paribus*, a higher income Y or a lower real interest rate r will shift the demand curve to the right and upwards as illustrated by the dashed demand curve implying a higher housing price. A higher housing stock would imply a lower current housing price.

Figure 15.1b showed that residential property prices can fluctuate quite a lot over time. From (32) one would expect fluctuations in incomes and real interest rates to be important for an explanation of the shorter-term house price fluctuations, since the housing stock evolves only gradually over time, as current construction activity typically makes up only a very small proportion of the existing housing stock. However, expectations may also contribute to instability in housing prices. In the expected user-cost of housing, $r+\delta-g^e$, the expected rate of real capital gain on a house, g^e, appears. Thus, if for some reason households come to expect that house prices will rise less rapidly or even fall, the resulting drop in g^e will immediately reduce demand and current property prices by driving up the user cost of housing. On the other hand, when expectations of large future capital gains on housing arise, there is an immediate boost to current demand for housing and property prices.

Figure 15.11 Short-run equilibrium in the housing market



Housing investment, interest rates and income

We can now combine (23) and (32) to get the housing investment function:

$$I^H = k\left(\frac{p^H}{P}-1\right)+\delta H = k\left(\frac{\eta Y}{(r+\delta-g^e)HP}-1\right)+\delta H. \tag{33}$$

This shows that housing investment is increasing with income and decreasing with user-cost $r+\delta-g^e$, and construction cost, P. The existing stock of houses, H, has two opposing effects on housing investment: a larger stock gives, *ceteris paribus*, a lower house price and through that channel lower housing investment, but at the same time a large stock calls, *ceteris paribus*, for larger gross investment to preserve the stock. Since the rate of depreciation on houses is relatively small, one would usually assume that the first effect dominates so that housing investment is smaller the larger the existing stock is.

In particular, we have found that housing investment is decreasing in the real interest rate other things being equal. We may write our housing investment function more generally as

$$I^H = I^H(\underset{+}{Y}, \underset{-}{r+\delta-g^e}, \underset{-}{H}), \tag{34}$$

or, letting a separate confidence variable ε capture expected capital gains on houses, and disregarding the depreciation rate δ, which is really a technical parameter:

$$I^H = I^H(\underset{+}{Y}, \underset{-}{r}, \underset{+}{\varepsilon}, \underset{-}{H}), \tag{35}$$

This is of the exact same form as the business investment function (19).

HOUSING PRICES AND HOUSING INVESTMENT

In the short run where the housing stock is predetermined, property prices must adjust to keep housing demand equal to the existing housing stock. A larger housing stock puts downward pressure on property prices, thereby discouraging housing investment. A rise in income or a fall in the real interest rate will stimulate housing demand, thus raising property prices and the construction of new homes. By reducing the user cost of housing, expectations of future capital gains on residential property will also boost property prices and housing investment.

Housing market dynamics

At the aggregate level, the housing stocks in period t and in period $t + 1$ are linked by the identity $H_{t+1} = (1-\delta)H_t + I_t^H$. We may combine this with Equation (23) and Equation (32) to get, altogether, a simple dynamic model of the housing market now including time subscripts:

$$I_t^H = k\left(\frac{p_t^H}{P} - 1\right) + \delta H_t \tag{23}$$

$$p_t^H = \frac{\eta Y}{(r+\delta-g^e)H_t} \tag{32}$$

$$H_{t+1} = (1-\delta)H_t + I_t^H \tag{36}$$

For given values of Y, r, and g^e, the predetermined housing stock, H_t, determines the housing price for period t via (32). Given the value of P, Equation (23) then determines the current level of housing investment I_t^H, which subsequently determines the next period's housing stock H_{t+1} via (36). We then get a new housing price p_{t+1}^H via (32), which enables us to determine I_{t+1}^H by use of (23), giving a new housing stock H_{t+2} via (36), and so on. This dynamic process will continue until the housing price has reached a level where construction activity is just sufficient to compensate for the depreciation of the existing housing stock so that the stock of housing remains constant. Thus, whereas an upward shift in housing demand is fully absorbed by a rise in house prices in the short run, over the longer run it will cause an increase in the housing stock which will dampen the initial price increase.[5]

If we allow for endogenous expectations formation, our housing market model (23), (32) and (36) may help to explain why housing markets tend to go through long cycles of boom and bust. For example, suppose the interest rate is cut, thereby shifting the housing demand curve in Figure 15.11 outwards and driving up the short-run equilibrium housing price. Suppose further that this initial price increase generates expectations of further increases in house prices so that $\delta - g^e$ decreases. According to (32) this will cause a further outward shift of the housing demand curve that will tend to initiate yet another round of price increases, thus confirming the expectations that drove up prices and helping to sustain expectations of further capital gains. Even though the rising prices of existing houses will induce construction of new houses (cf. (23)), which will help to keep the price increases in check over the longer run, it will take a while for the increased housing supply to catch up with the rising demand. The reasons are that construction is time-consuming and the existing housing stock is so large relative to the number of newly constructed housing units even in 'boom' years. Hence, observed increases in actual house prices that generate expectations of further capital gains may keep a housing boom going for quite some time. However, as house prices continue to rise above construction costs, construction activity will continue to increase, and at some point the resulting increase in housing supply will bring the rise in housing prices to a halt and force a decline in the prices of existing homes. Once consumers observe a decline in prices, they are likely to revise the expected rate of capital gains g^e downwards, leading to a fall in housing demand as a result of a rise in the expected user cost $r + \delta - g^e$. This may cause prices to fall even further, causing yet another reduction in housing demand via a fall in g^e, and so on.

Note that such a self-reinforcing slump in the housing market may be quite prolonged. For even in an extreme case, where the price decline completely eliminates all new construction activity, the annual decrease in total housing supply can never exceed the total physical depreciation δH (cf. (36)), which is small relative to the existing

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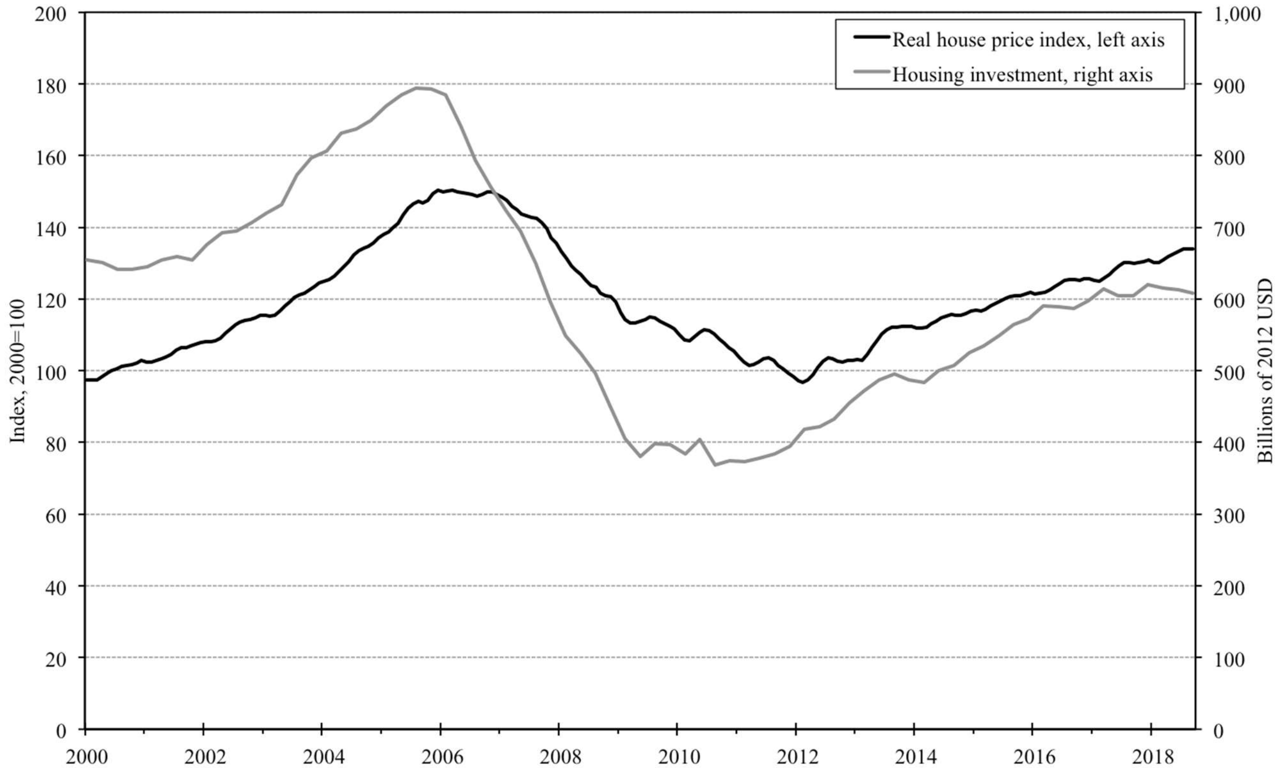
[5] You should try to show using (23), (32) and (36) that the long-run values of the housing stock, the house price and housing investment (for given P, Y, r and g^e) are $H^* = \eta Y/[(r+\delta+g^e)P]$, $p^{H*} = P$ and $I^{H*} = \delta H^*$. (Remember that in long-run equilibrium the housing stock is constant, which you can use in (36) to find H^* etc.). Exercise 4 and 5 will ask you to explore the dynamics of the housing market in an extended model allowing for property taxes and capital income taxes.

housing stock for any realistic value of the depreciation rate δ (often estimated to be in the neighbourhood of 1-2 per cent).

Figure 15.12 Boom and bust in the US housing market



Note: The House price index is based on monthly data and housing investment is based on quarterly data. The house price index is the Case-Shiller composite index for 20 metropolitan regions. Housing investment comes from a decomposition of private fixed investment.

Sources: Shiller data downloadable from: http://www.econ.yale.edu/~shiller/data.htm. Data on housing investment are from Bureau of Economics Analysis.

Figure 15.12 illustrates the quite dramatic boom–bust cycle in the US housing market during the past two decades. Supported by lax lending standards and a marked decline in interest rates between 2001 and 2004, American home prices rose sharply up until 2006, inducing a strong increase in residential construction. With the gradual increase in housing supply, the record high housing prices became unsustainable, and prices started to fall from 2006. As many so-called subprime (i.e. less creditworthy) borrowers in the US housing market relied on continuous price increases to be able to refinance their mortgage debts, the downturn in home prices left many new homeowners insolvent and caused a steep increase in the number of foreclosures and in defaults on many complex derivative financial assets backed by mortgage loans. This in turn triggered one of the worst financial crises in history. As the financial crisis weakened the real economy, causing income and employment to fall, the downturn in the housing market was exacerbated, and prices and construction took a further strong dive lasting until 2010-12, thus contributing to the steepest economic downturn since the 1930s. Since 2012 house prices and housing investment have again taken a joint swing upwards, but not as steeply as in the period leading up to the financial crisis. As illustrated by this case study, instability in the housing market and the long swings in house prices and housing investment can be a major source of business fluctuations.

Summary

1. Empirically, changes in stock prices and in housing prices tend to be followed by changes in output in the same direction. In part, this reflects that higher asset prices lead to higher investment. This chapter explains the links between asset prices and investment.
2. A firm seeking to maximize the wealth of its owners will choose an investment plan, which maximizes the market value of the firm's assets. The value of the firm, referred to as the fundamental stock value, is the present discounted value of the expected future dividends paid out by the firm. This follows from the shareholder's arbitrage condition, which says that the expected return to shareholding, consisting of dividends and capital gains on shares, must equal the return to bondholding plus an appropriate risk premium.
3. When share prices reflect the fundamental value of firms, there are three possible reasons for the observed volatility of stock prices: (i) fluctuations in (the growth rate of) expected future dividends, (ii) fluctuations in the real interest rate, and (iii) fluctuations in the required risk premium on shares. There is indirect evidence that the required risk premium fluctuates quite a lot.
4. Many observers believe that stock prices can sometimes deviate from the fundamentals as captured by the fundamental stock price. The analysis in this chapter abstracts from such 'bubbles' in stock prices.
5. Increases in the firm's capital stock imply adjustment costs (installation costs), including costs of installing new machinery, costs of training workers to use the new equipment, and perhaps costs of adapting the firm's organization. These installation costs will typically increase more than proportionally to the firm's level of investment.
6. The value-maximizing firm will push its investment to the point where the shareholder's capital gain from a unit increase in the firm's capital stock is just offset by the dividend that must be forgone to finance the purchase and instalment of an extra unit of capital. Because the marginal installation cost is increasing in the volume of investment, this investment rule implies that the firm's optimal level of investment will be higher, the higher a unit increase in the firm's capital stock is valued by the stock market. An increase in the ratio of stock prices to the replacement cost of the firm's assets will therefore stimulate its investment.
7. The market value of stocks relative to the replacement value of the underlying business assets is referred to as Tobin's q. Our theory of investment may be summarized by saying that business investment is an increasing function of this q.
8. Stock prices reflect expected future dividends, which tend to be positively affected by a rise in current profits. The value of Tobin's q therefore tends to vary positively with the current profit rate, which in turn varies positively with the output–capital ratio. Hence, investment is an increasing function of current output and a decreasing

function of the existing capital stock.

9. *Ceteris paribus*, a rise in the real interest rate implies that expected future dividends are discounted more heavily, leading to a fall in Tobin's q via lower stock prices. Thus, a higher real interest rate tends to depress investment. A rise in the required risk premium on shares, generated by more uncertainty about the future, will have a similar negative impact on investment.

10. A version of the q-theory can explain investment in owner-occupied housing. When the market price of residential property increases relative to the cost of housing construction, it becomes profitable for firms in the construction sector to increase the supply of new housing units. As a consequence, housing investment (construction activity) goes up. There is empirical evidence in favour of this hypothesis.

11. In the short run, the market price of housing varies positively with current income and with the expected rate of capital gain on housing and negatively with the real interest rate and with the existing housing stock. Since construction increases with the market price of housing, it follows that housing investment is an increasing function of income and a decreasing function of the real interest rate and the current housing stock.

Exercises

Exercise 1. Stock market valuation and 'fundamentals'

The purpose of this exercise is to illustrate how our equation (6) for the fundamental stock price may be used to evaluate whether stock prices are unrealistically high or low, that is, whether the stock market is 'overvalued' or 'undervalued'. Suppose that real dividends are expected to grow at the constant rate g^e. If the actual real dividend for period t is D_t, the expected real dividend for future period n will then be given by:

$$D_n^e = D_t(1+g^e)^{n-t} \qquad \text{for } n = t+1, t+2, \ldots \tag{37}$$

1. Use Equation (6) in the text to demonstrate that, when expected future dividends are given by (37), the value of shares will be given by:

$$V_t = \frac{D_t}{r+\varepsilon-g^e}. \tag{38}$$

Stock market analysts often focus on the so-called 'trailing dividend yield', D_t/V_t, defined as the current dividend relative to the current market value of shares. We will simply refer to this ratio as the 'dividend yield'.

2. Suppose you have information on the current dividend yield, D_t/V_t, the real interest rate, r, and the required risk premium on shares, ε. Use (38) to solve for the value of the expected real growth rate g^e, which is necessary to justify current stock prices.
3. Suppose alternatively that, in addition to information on the current dividend yield and the current real interest rate, you have somehow obtained information on the expected future growth of real dividends whereas you do not know the required risk premium on shares. Use (38) to derive the value of the risk premium, which will justify current stock prices.
4. In the USA in 1999, the average dividend yield was 1.2 per cent and the real interest rate on (approximately) risk-free, ten-year government bonds was 3.4 per cent. Moreover, the average historical risk premium on shares in the period 1980 – 99 was 2.8 per cent per annum (all of these figures are taken from the IMF's World Economic Outlook, May 2000). What was the annual growth rate of real dividends which US financial investors expected in 1999, if they required a risk premium equal to the historical average? On the basis of your result, would you say that the US stock market was overvalued or undervalued in 1999? Justify your answer.
5. Over the period 1980 – 99, the average growth rate of US real GDP was 3.0 per cent. Suppose now that US investors in 1999 expected future real dividends to grow in line with historical GDP growth so that $g^e = 0.03$ (discuss whether this might be a reasonable assumption). Given the other pieces of information in the previous

question, what was the risk premium on shares required by US investors in 1999? Would you say that this risk premium was 'reasonable'? Can you imagine any reasons why US investors in 1999 should require a lower or a higher risk premium than the average historical premium?

Exercise 2. A generalized q-theory of investment

Suppose that instead of Equation (9) in the main text of this chapter, the installation cost function takes the more general form:

$$c(I_t) = \frac{a}{1+\eta} I_t^{1+\eta}, \qquad \eta > 0. \tag{39}$$

1. Derive an expression for investment as a function of q_t (a generalized version of Equation (12')). Give a brief verbal explanation of the idea and economic mechanisms underlying the q-theory of investment. Explain why the q-theory is consistent with the well-documented fact that investment tends to vary negatively with the real interest rate and positively with economic activity.

Suppose now that during each period, a fraction δ of the capital stock has to be scrapped because of wear, tear and technical obsolescence, where $0 < \delta < 1$. So, the change in the capital stock is given by:

$$K_{t+1} - K_t = I_t - \delta K_t, \tag{40}$$

where I_t indicates *gross* investment, including the replacement investment serving to compensate for depreciation. Suppose that only *net* additions to the capital stock generate adjustment costs. In that case installation costs will be determined by *net* investment $I_t - \delta K_t$ so that (39) must be replaced by:

$$c(I_t) = \frac{a}{\eta+1}(I_t - \delta K_t)^{\eta+1}. \tag{41}$$

2. Discuss whether it is reasonable to assume that only net investment (but not replacement investment) generates adjustment costs. Derive a revised expression for gross investment I_t, assuming that installation costs are given by (41).

In a stationary economy with no long-run growth, a long-run equilibrium requires the capital stock to be constant over time. In such a situation where net investment is zero there is no need for firms to retain any part of their net profit. Hence, all net profits will be paid out as dividends. According to Equation (15) in the main text, this implies $q_t = (\Pi_t / K_t)/(r + \varepsilon)$.

3. What is the ratio of the market value to the replacement value of the firm's capital stock in a stationary long-run equilibrium? Furthermore, what is the relationship between the profit rate Π_t / K_t and the required return on shares in such a long-run equilibrium? Try to provide some economic intuition for your results.

Exercise 3. Tax policy and investment

This exercise serves two purposes. First, you are asked to demonstrate that the investment behaviour of unincorporated firms is similar to the investment behaviour of corporations, which are quoted on the stock market. Second, you are invited to study how various forms of capital income taxation will influence investment.

We consider an entrepreneur who owns a private unincorporated business firm. We divide the entrepreneur's time horizon into two periods which may be thought of as 'the present' (period 1) and 'the future' (period 2). At the beginning of period 1, the entrepreneur has accumulated a predetermined capital stock, K_1, which is invested in his firm. During period 1, the entrepreneur incurs gross investment expenditure, I, with the purpose of maintaining and increasing the capital stock. In each of the two periods, the capital stock depreciates at the rate δ, so at the beginning of period 2, the entrepreneur's capital stock will be given by:

$$K_2 = K_1(1 - \delta) + I, \qquad 0 < \delta < 1. \tag{42}$$

The entrepreneur undertakes investment and employs labour with the purpose of maximizing the present value V of the net cash flows Π_1 and Π_2 withdrawn from the firm during periods 1 and 2, respectively. In other words, the entrepreneur wants to maximize:

$$V = \frac{\Pi_1}{1+r} + \frac{\Pi_2}{(1+r)^2}. \tag{43}$$

At the end of period 2, the entrepreneur plans to liquidate the firm and sell its remaining assets at their replacement value $(1 - \delta)K_2$. For simplicity, we assume that there are no adjustment costs associated with changing the firm's capital stock. If Y_t is the firm's output, L_t is labour input, and w_t is the real wage rate, the cash flows from the firm to the owner during the two periods will be:

$$\Pi_1 = Y_1 - w_1L_1 - I, \tag{44}$$

$$\Pi_2 = Y_2 - w_2L_2 + (1 - \delta)K_2. \tag{45}$$

where (45) includes the revenue from the sale of the firm's remaining capital stock at the end of period 2. Output in the two periods is given by the Cobb–Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{\beta}, \quad 0 < \alpha < 1,\ 0 < \beta < 1,\ 0 < \alpha + \beta \leq 1, t = 1, 2. \tag{46}$$

1. Demonstrate that the entrepreneur's optimal gross investment during period 1 can be written as:

$$I = \frac{\alpha Y_2}{r + \delta} - (1 - \delta)K_1. \tag{47}$$

(Hint: use (42) to eliminate K_2 before you derive your first-order condition.) Does the investment function (47) have the same qualitative properties as the business investment function derived in the main text of the chapter? (Hint: note that the variable Y_2 must be interpreted as *expected* output in period 2).

We now invite you to study the effects of a profits tax. In accordance with existing tax rules, suppose that the firm is allowed to deduct its labour costs and the depreciation on its capital stock from taxable profits. If the profits tax rate is τ, the firm's tax bill T then becomes:

$$T_t = \tau(Y_t - w_t L_t - \delta K_t), \qquad 0 < \tau < 1, \qquad t = 1, 2, \tag{48}$$

and the after-tax cash flows from the firm to the entrepreneur become equal to:

$$\Pi_1 = Y_1 - w_1 L_1 - I - T_1, \tag{49}$$

$$\Pi_2 = Y_2 - w_2 L_2 + (1 - \delta)K_2 - T_2. \tag{50}$$

2. Derive the analogue of the firm's investment function (47) in the presence of the profits tax. Explain how the profits tax affects investment.

We have so far assumed that investment is financed by retained profits. Consider now the alternative case where only replacement investment δK is financed by retained earnings whereas net investment expenditure $I - \delta K$ is financed by debt. In that case the firm's stock of debt B will always be equal to its capital stock, that is:

$$B_t = K_t \qquad t = 1, 2, \tag{51}$$

and the firm's revenue ΔB_1 from new borrowing during period 1 will be equal to its net investment during that period:

$$\Delta B_1 = I - \delta K_1. \tag{52}$$

Using (51) and (52) and noting that the firm's expenses on interest payments are $rB = rK$, we may then write the cash flows to the entrepreneur as:

$$\begin{aligned}\Pi_1 &= Y_1 - w_1 L_1 - rB_1 - T_1 - I + \Delta B_1 \\ &= Y_1 - w_1 L_1 - (r+\delta)K_1 - T_1,\end{aligned} \tag{53}$$

$$\begin{aligned}\Pi_2 &= Y_2 - w_2 L_2 + (1-\delta)K_2 - T_2 - (1+r)B_2 \\ &= Y_2 - w_2 L_2 - (r+\delta)K_2 - T_2,\end{aligned} \tag{54}$$

assuming that the entrepreneur must repay all of his debt with interest at the end of the second period. Since the tax code allows interest payments as well as depreciation to be deducted from taxable profits, the tax bills for the two periods will be

$$T_t = \tau(Y_t - w_t L_t - (r+\delta)K_t), \qquad t = 1, 2. \tag{55}$$

3. Derive the firm's investment function on the assumption that net investment is fully financed by debt. (Hint: remember to use (42) and (46).) Does the profits tax affect investment? Does it yield any revenue? Is the tax system neutral towards the firm's choice of financing method? Explain your results.

Exercise 4. Tax policy and the housing market

In this exercise we will extend our model of the housing market to allow for taxes on capital income and property. We then ask you to study how the housing market reacts to tax policy in the short run and the long run.

We assume that the government levies a proportional property tax at the rate s on the current value $p^H H$ of the consumer's housing stock H, so the property tax paid is $sp^H H$. We also assume that the government imposes a proportional tax on positive as well as negative capital income at the rate τ. If the tax payer has real financial wealth V, if negative a debt, and the real interest rate is r, then the tax payer pays a real capital income tax of $\tau r V$. If V is negative, this is a deduction in the tax liability. As in the main text, the consumer has to spend an amount $\delta p^H H$ on repair and maintenance during each period to maintain the value of his house, where δ is thus be the depreciation rate for housing capital. Expenses on repair and maintenance are assumed not to be deductible from taxable income and for simplicity we ignore expected capital gains or losses on houses in this exercise.

This implies that the user-cost of one unit of housing capital is changed from (26) in the main text to:

$$uc = [r(1-\tau) + \delta + s]p^H = v \cdot p^H, \quad v \equiv r(1-\tau) + \delta + s \tag{56}$$

In all other respects, our model is as the model of Section 15. 4. Housing investment is still given by (23), housing demand is still given by (31), and hence the short-run housing price is still given by (32), but in (31) and (32) the variable v now has the new content given by (26). All in all, the system describing the dynamics of housing investment, the housing price and the stock of houses is:

$$I_t^H = k\left(\frac{p_t^H}{P} - 1\right) + \delta H_t \tag{23}$$

$$p_t^H = \frac{\eta Y}{v H_t} \tag{32}$$

$$H_{t+1} = (1-\delta)H_t + I_t^H \tag{36}$$

where now $v \equiv r(1-\tau) + \delta + s$. The variables without subscript t are exogenous parameters and here constant over time.

1. Consider the modified expression (56) for the user-cost of housing capital. Explain each of its elements and explain why a *lower* tax on capital income implies a *higher* user-cost, ceteris paribus. Your explanation must cover both the case where the house owner has borrowed to finance the house and the case where the house is bought out of accumulated wealth.

At the beginning of each period the existing housing stock H_t is predetermined. Given H_t, Equation (32) determines the short-run equilibrium housing price P_t^H, which inserted into (23) gives the current level of housing investment, I_t^H. This I_t^H inserted into (36) along with H_t gives the housing stock H_{t+1} at the beginning of the next period, which then determines P_{t+1}^H via (32), and so on.

2. Show that that the three equations of our housing market model, (23), (32) and (36), can be condensed into the following single non-linear first-order difference equation in the housing stock H_t:

$$H_{t+1} = H_t + k\left(\frac{\eta Y}{vPH_t} - 1\right) \tag{57}$$

and show further that the stationary point (the long-run equilibrium point) of this is at the housing stock:

$$H^* = \frac{\eta Y}{vP} \tag{58}$$

Then show that the long-run housing price and investment levels are:

$$p^{H*} = P \quad \text{and} \quad I^{H*} = \delta H^* \tag{59}$$

3. This question is concerned with the stability of the long-run equilibrium. Show that the partial derivative of H_{t+1} as function of H_t according to (57) is:

$$\frac{\partial H_{t+1}}{\partial H_t} = 1 - k\frac{\eta Y}{vP}\frac{1}{H_t^2}, \tag{60}$$

and then show that measured at the long-run equilibrium this derivative is:

$$\left(\frac{\partial H_{t+1}}{\partial H_t}\right)^* = 1 - \frac{k}{H^*}. \tag{61}$$

It can safely be assumed that $k < H^*$. Show that the derivative in (61) is then strictly between zero and one. In a diagram with H_t along the horizontal axis and H_{t+1} along the vertical, where you also draw the 45-degree line ($H_{t+1} = H_t$), sketch the 'transition curce' showing H_{t+1} as a function of H_t (note that from (60) the graph must be U-formed). Show by iterations in the figure that the long-run equilibrium must be (locally) stable.

In the following we consider a tax change, an increase in s and/or a decrease in τ, that implies an increase in v.

4. Describe how this affects the housing stock, price and investment in the long run.

5. Describe the transition from the short to the long run by applying a diagram like Figure 15.11 illustrating short-run equilibria of the housing market: Assume that the market is first in long-run equilibrium and illustrate this in the figure. Then the tax change occurs. Explain how this affects the housing demand curve, H^d, according to (31), and the short-run housing price. Then explain and illustrate the implication of the change in the house price for the short-run housing supply curve and the housing price in the next period. Illustrate the transition towards the new long-run equilibrium over the following periods.

6. Now explain how the tax change affects the transition diagram and curve you derived in Question 3 above and illustrate the transition from the old long-run equilibrium to the new by 'staircase iteration' in the diagram.

Consider the following initial parameter values, which can be considered reasonable, some of them as normalizations: $\delta = 0.025,\ k = 1,\ \eta = 0.2,\ v = 0.05,\ Y = 100,\ P = 2$. For v, the underlying initial parameters implying a value of 0.05 could reasonably be: $r = 0.03,\ \tau = 1/3,\ \delta = 0.025$ (again) and $s = 0.005$.

7. For these parameter values, derive the long-run equilibrium values H^*, I^{H*} and p^{H*}. Do ratios such as H^*/Y and I^{H*}/Y seem reasonable?

From the initial parameter values, consider a tax change that increases v from 0.05 to 0.06, all other things being equal. This could be a raise in the property tax from 0.5 percent (0.005) to 1.5 percent (0.015).

8. Fist describe how this affects the long-run equilibrium values H^*, I^{H*} and p^{H*}. Now, starting from the initial long-run equilibrium, simulate the housing market model consisting of Eqs. (23), (32) and (36), on the computer (e.g., in Excel), using the new parameter values (the new $v' = 0.06$) over 100 periods. This is most easily done by simulating over periods 1-100 the transition equation (57) starting in period zero with the old long-run equilibrium housing stock ($H_0 = 200$, as you should have found above), but using in the simulation the new value $v' = 0.06$, thus constructing a sequence of H_t over $t = 1, \ldots, 100$. You can then compute the associated p_t^H and I_t^H using (32) and (23). Draw diagrams that show the time lapses of the housing stock, the housing price and housing investment. Comment on the sluggishness in the transition.

Chapter 16

Consumption, income, interest rates and wealth

Introduction

Private consumption is by far the largest component of aggregate demand for goods and services and typically makes up at least one-half of GDP. Chapter 13 showed that GDP and private consumption are highly positively correlated and about equally volatile. Although private consumption is considerably less volatile than investment, changes in consumption are still a dominant source of changes in total demand: private investment may typically be 4-5 times as volatile as consumption, but consumption is typically 4-5 times larger than private investment. While most economists probably consider swings in investment to be more important drivers of business cycle turning points than changes in consumption, the movements of consumption will reinforce the effects of fluctuations in investment, and the reactions of private consumption are very important for the effects of stabilization policies. For instance, to what degree will an interest rate cut brought about by monetary policy, or temporarily lower taxes or higher government expenditure brought about by fiscal stabilization policy, stimulate private consumption? A theory of private consumption is therefore an essential building block in any theory of aggregate demand.

Studying private consumption will not only help us to build a short-run model of the business cycle. Since consumption is a basic determinant of economic welfare, our theory of the link between consumption and other macroeconomic variables will also help us understand how business cycles and economic policies affect consumer welfare. Moreover, our theory of consumption will imply a theory of saving, because saving is equal to income minus consumption. Since saving is the basis for capital accumulation, our analysis of consumption is also relevant for the theory of economic growth presented in Book One.

In the section on housing demand in the previous chapter we studied how the consumer allocates total consumption between housing consumption and consumption of nondurables *within a given time period*. This chapter complements the previous one by analysing how the consumer will wish to allocate consumption *over time*. Since we now wish to explain *aggregate* consumption, this chapter will not elaborate on the previous chapter's analysis of the *composition* of consumption. Thus, we will treat consumption as a single aggregate which, of course, must be thought of as a bundle of commodities, including housing services.

We will start this chapter by briefly restating and discussing the simple Keynesian theory of private consumption. We will then introduce a richer model of consumption to illustrate how consumption relates to income, wealth and interest rates. In the final part of the chapter, we will show how our theory of consumption can be used to analyse the effects of the government's tax and debt policies on aggregate demand.[1]

16.1 The simple Keynesian consumption function

In his famous *General Theory of Employment, Interest and Money*, published in 1936, John Maynard Keynes wrote that '… the propensity to consume is a fairly stable function so that, as a rule, the amount of aggregate consumption mainly depends on aggregate income …'. In other words, Keynes argued that real private consumption during period t, denoted by C_t, is mainly determined by real disposable income Y_t^d during that period. In formal terms, $C_t = C(Y_t^d)$. Keynes went on to argue that:

> The fundamental psychological law, upon which we are entitled to depend with great confidence both *a priori* from our knowledge of human nature and from the detailed facts of experience, is that men are disposed, as a rule and on the average, to increase their consumption as income increases, but not by as much as the increase in their income.

The claim made by Keynes in this passage is that the *marginal* propensity to consume, $C' \equiv dC_t/dY_t^d$, is positive but less than 1. In a subsequent passage he asserted that

> … it is also obvious that a higher absolute level of income will tend, as a rule, to widen the gap between income and consumption. For the satisfaction of the immediate primary needs of a man and his family is usually a stronger motive than the motives towards accumulation, which only acquire effective sway when a margin of comfort has been attained. These reasons will lead, as a rule, to a *greater proportion* of income

[1] Parts of this chapter borrow from the teaching note by Henrik Jensen, 'Mikrofundament for konventionelle makroadfærdsrelationer', Københavns Universitets, Økonomiske Institut, Marts 1996. We are grateful to Henrik for the inspiration, but of course he should not be held responsible for any shortcomings in our exposition.

being saved as real income increases.[2]

Thus, Keynes believed that the *average* propensity to consume, C_t/Y_t^d, will decrease with the level of income. In other words, the rich are assumed to have a higher average savings rate than the poor.

A consumption function with these Keynesian properties is the simple linear one:

$$C_t = a + bY_t^d, \quad a > 0, \quad 0 < b < 1. \tag{1}$$

In this consumption function, the marginal propensity to consume is the constant *b*, which is less than 1, and the average propensity to consume is $C_t/Y_t^d = b + a/Y_t^d$ which is obviously decreasing with income.

The consumption function (1) has the virtue of being simple, but there are at least two problems with it. The first problem is theoretical: although it seems plausible that consumption is positively related to current income, it is not clear why the current consumption of an optimizing consumer should depend *only* on current income and not also on, say, expected future income, wealth and the real rate of interest. Thus one must doubt whether a consumption function like (1) is consistent with optimizing behaviour.

The second problem is empirical. Although microeconomic *cross-section* data on the relationship between consumption and income for different families within a given period do indicate that the rich save a larger fraction of their current income than the poor, macroeconomic *time series* data for most countries indicate that the ratio of aggregate consumption to aggregate income is roughly constant over the long run. These apparently contradictory stylized facts are illustrated in Figure 16.1, where Figure 16.1a indicates the relationship between income and consumption for *different households within a given period* such as a year, while Figure 16.1b indicates the relation between *aggregate* household income and *aggregate* consumption *over time*.

The rough long-run constancy of the average propensity to consume is illustrated for the USA Figure 16.2. Apart from a temporary drop due to consumption rationing during the Second World War, the average propensity to consume in the USA has been remarkably stable over the long run, despite the tremendous growth in income since 1930. Figure 16.2 clearly contradicts Equation (1) which implies that the ratio of consumption to income should *decline over time* as income grows. Instead of Equation (1), we therefore need a theory of consumption which has a solid theoretical foundation and which is able to explain why we observe different relationships between consumption and income in microeconomic cross-section data and in macroeconomic time series data. In the rest of this chapter we shall try to build such a theory.

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[2] This quotation and the two previous ones can be found on pp. 96 – 97 of John Maynard Keynes, *The General Theory of Employment, Interest and Money*, London and Basingstoke, Macmillan Press, 1936. These brief quotations do not do full justice to Keynes' theory of consumption. He did in fact discuss a host of other factors likely to influence private consumption. Still, it is fair to say that as a first approximation, he believed that current consumption depends mainly on current income.

Figure 16.1a Stylized relationship between income and consumption in microeconomic cross-section data

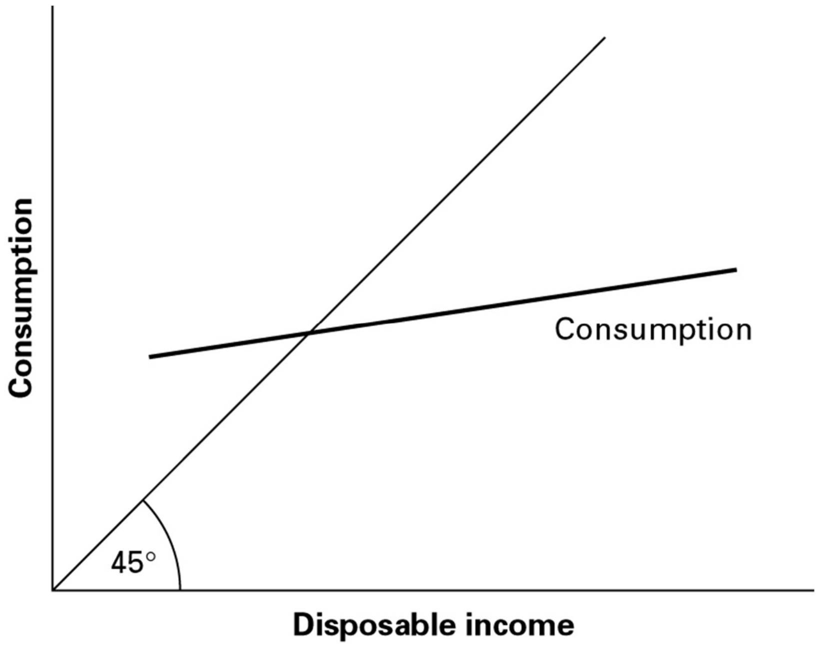


Figure 16.1b Stylized relationship between income and consumption in macroeconomic time series data

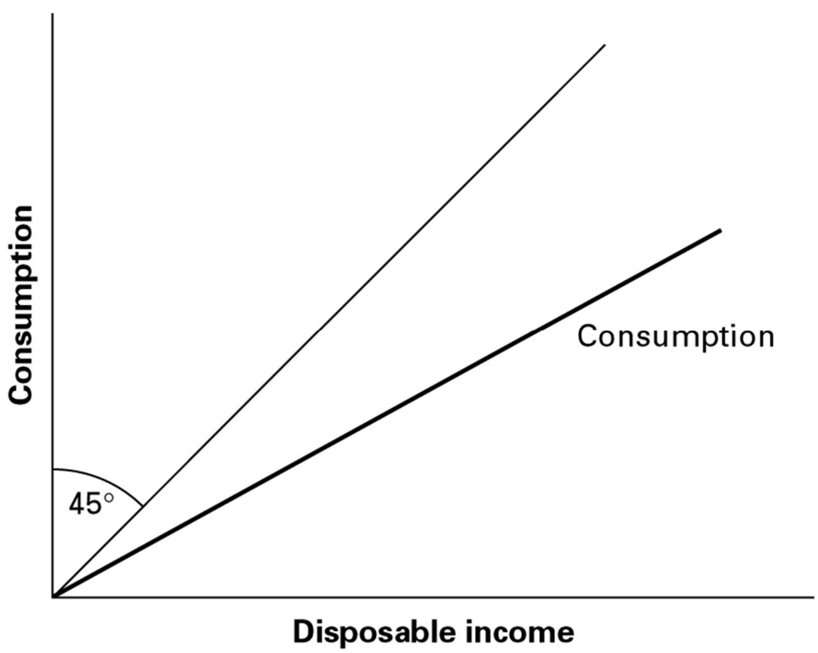
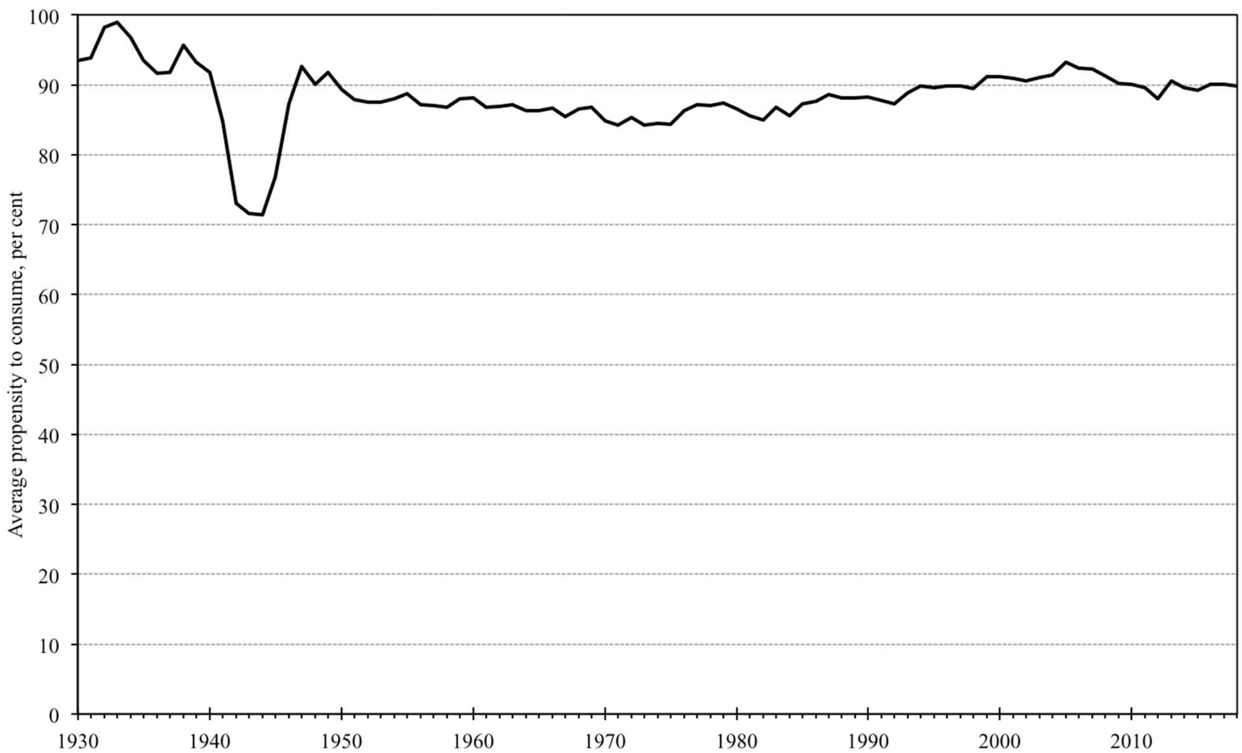


Figure 16.2 The average propensity to consume in the USA, 1930-2018



Note: Based on annual data. The average propensity to consume is the ratio of private consumption to disposable income.
Source: Bureau of Economic Analysis.

16.2 A consumption theory based on intertemporal optimization

Consumer preferences

The starting point for a micro-based theory of consumption is a specification of consumer preferences. Consider a consumer who plans for a certain finite time horizon. We will divide this time interval into two periods, which may be thought of as 'the present' (the current period 1) and 'the future' (period 2). The limitation to only two periods is just a simplification; one can show that all our qualitative conclusions will continue to hold in a setting with many periods.

In each period t (where $t = 1, 2$), the consumer derives utility $u(C_t)$ from consumption.

However, because the consumer is 'impatient', he or she prefers a unit of utility today to a unit of utility tomorrow. When evaluated at the beginning of period 1, the consumer's *lifetime* utility U is therefore given by:

$$U = u(C_1) + \frac{u(C_2)}{1+\phi}, \quad u' > 0, \quad u'' < 0, \quad \phi > 0. \tag{2}$$

The consumer's impatience is captured by the parameter ϕ which is referred to as *the rate of time preference*. The positive rate of time preference means that if $C_1 = C_2$, a given additional amount of consumption today is valued more highly than a similar extra amount of consumption tomorrow. On the other hand, as long as ϕ is not infinitely high, the consumer is not indifferent about the future, and he or she will then have to decide how to allocate his or her consumption optimally over time. The assumptions $u' > 0$ and $u'' < 0$ reflect that the marginal utility of consumption in any period is positive but decreasing. An increase in consumption in any period will thus reduce the marginal utility gain from a further consumption increase in that period.

We derive our theory of consumption from the assumption that the consumer trades off present against future consumption to maximize the lifetime utility function (2). The terms of this trade-off will depend on the consumer's intertemporal budget constraint to which we now turn.

The intertemporal budget constraint

When specifying the consumer's budget constraints for the two periods, we will first assume that *capital markets are perfect*. This means that the consumer can freely lend and borrow as much as he or she likes at the going market rate of interest. In practice, some consumers may face *credit constraints* preventing them from borrowing as much as they would have preferred at the going interest rate. We will discuss credit constraints later in the chapter, and exercises will ask you to consider their implications in detail, but for the moment we will assume perfect capital markets.

We specify the consumer's budget constraints in real terms. At the beginning of period 1, the consumer is endowed with a predetermined stock of real financial wealth V_1 (which could be negative if the consumer is indebted). During period 1, he or she earns real labour income Y_1^L, pays the real amount of taxes T_1, and spends the real amount C_1 on consumption. For convenience, we assume that all payments are made at the beginning of the period.[3] After having received this income and incurred this expenditure

[3] If some or all payments were made at the end of the period instead, we would obtain a consumption function with the same qualitative properties as those described below. However, the assumption that payments take place at the start of each period leads to slightly more elegant analytical expressions. Whenever we divide the time axis into discrete finite intervals, there is no objectively 'correct' assumption on the timing of payments (beginning-of-period versus end-of-period). Hence, we are free to choose the assumption on timing that is most convenient for analytical purposes.

on taxes and consumption, the consumer has an amount $V_1 + Y_1^L - T_1 - C_1$ left over for investment in interest-bearing financial assets. If the real interest rate is *r*, the consumer will therefore end up with a real stock of financial wealth $V_2 = (1+r)(V_1 + Y_1^L - T_1 - C_1)$ at the beginning of period 2. Hence the budget constraint for period 1 is:

$$V_2 = (1+r)(V_1 + Y_1^L - T_1 - C_1). \tag{3}$$

Note that V_2 may well be negative corresponding to $C_1 > V_1 + Y_1^L - T_1$. In that case, the consumer is a net borrower during period 1. Since he or she does not plan any consumption beyond period 2, the consumer will simply spend all his or her resources during that period, including the financial wealth accumulated during period 1. Using the same notation as before, we may therefore write the budget constraint for period 2 as:

$$C_2 = V_2 + Y_2^L - T_2. \tag{4}$$

Equation (4) states that the consumer will spend his or her initial financial wealth plus the after-tax labour income on consumption during period 2. Although borrowing in period 1, $V_2 < 0$, is possible, we assume that the consumer must be able to pay back all debt in period 2 at the end of his or her planning horizon, that is, we impose the restriction $V_2 \geq -(Y_2^L - T_2)$, from (4) corresponding to $C_2 \geq 0$. Hence, if the consumer borrowed during period 1, he or she must reserve part of the period 2 labour income for debt repayment.

It will turn out to be convenient to consolidate (3) and (4) into a single constraint. We therefore use (3) to eliminate V_2 from (4), divide through by $(1 + r)$ on both sides of the resulting equation and rearrange to get:

$$C_1 + \frac{C_2}{1+r} = V_1 + Y_1^L - T_1 + \frac{Y_2^L - T_2}{1+r}. \tag{5}$$

Equation (5) is the consumer's *intertemporal budget constraint*. It states that the present value of the consumer's lifetime consumption (the left-hand side) must equal the present value of his or her after-tax labour income plus the initial financial wealth (the right-hand side).[4] In other words, with a perfect capital market current consumption does not have to equal current income, but over the life cycle the consumer cannot spend any more than his or her total resources. These resources consist of labour income and the initial financial wealth.

We can write (5) in a simpler form by introducing

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[4] Of course, the consumer could choose to consume less than his or her total lifetime resources, but since an increase in consumption today or tomorrow will always increase lifetime utility, he or she will always choose to consume as much as the budget constraint permits. This is why (5) is written with an equality sign rather than an inequality sign.

$$H_1 \equiv Y_1^L - T_1 + \frac{Y_2^L - T_2}{1+r}. \tag{6}$$

The variable H_1 is the present value of the consumer's disposable lifetime labour income, and it is referred to as *human wealth* or *human capital* because it measures his or her capitalized earnings potential in the labour market. Note that H_1 carries the time subscript 1 because it includes labour income from period 1 and onwards. Inserting (6) into (5), we get

$$C_1 + \frac{C_2}{1+r} = V_1 + H_1, \tag{7}$$

where we assume throughout that $V_1 + H_1 > 0$. Hence, the consumer's intertemporal budget constraint simply states that the present value of real lifetime consumption is constrained by *total real initial wealth*, consisting of the sum of financial wealth and human wealth.

We shall now study how consumers will wish to allocate consumption over time.

The allocation of consumption over time

The consumer chooses his or her time path of consumption to maximize the lifetime utility function (2) subject to the intertemporal budget constraint (7). The consumer's wage rate is taken as given, and we assume that his or her working hours are institutionally determined, say, by collective bargaining agreements or by law. This means that the labour incomes Y_1^L and Y_2^L and thereby H_1 are exogenously given to the consumer.[5] Since the market value of financial assets is also beyond the control of the individual consumer, it follows that the consumer's total initial wealth, $V_1 + H_1$, can be taken as given when optimizing consumption. Because we are mainly interested in studying the determinants of current consumption C_1, we will use the intertemporal budget constraint (7) to eliminate C_2 from the lifetime utility function (2). Doing that, we obtain:

$$U = u(C_1) + \frac{u((1+r)(V_1 + H_1 - C_1))}{1+\phi}. \tag{8}$$

The consumer's problem then boils down to choosing C_1 so as to maximize (8). The first-order condition for a maximum is:

[5] In Exercise 2, you will be asked to study the determination of consumption in the more complicated case where the consumer can choose his or her working hours and hence labour income.

$$\partial U / \partial C_1 = u'(C_1) - \frac{1+r}{1+\phi} u'((1+r)(V_1 + H_1 - C_1)) = 0.$$ [6]

Can we be sure that the solution is 'interior', meaning $C_1 > 0$ and $C_2 > 0$, and hence indeed given by the first order condition? Yes, if the consumer has some resources to spend at all, $V_1 + H_1 > 0$, and marginal instantaneous utility becomes very large for a very small level of consumption, $U'(C) \to \infty$ for $C \to 0$, the consumer will always plan to have strictly positive consumption in both periods. We can write the first-order condition as:

$$u'(C_1) = \frac{1+r}{1+\phi} u'(C_2), \tag{9}$$

or as:

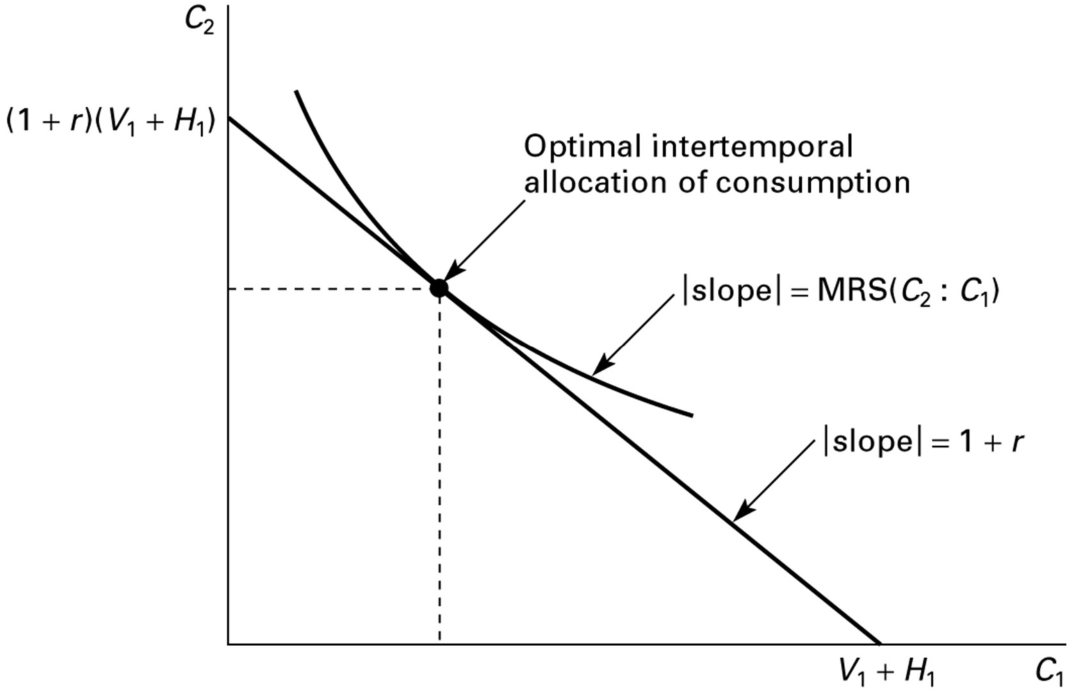
$$\frac{u'(C_1)}{\frac{u'(C_2)}{1+\phi}} \equiv MRS(C_2 : C_1) = 1 + r. \tag{10}$$

Equations (9) and (10) are just two alternative ways of writing the consumer's optimum condition. Consider first the interpretation of (9). If the consumer increases consumption by one unit in period 1, his or her lifetime utility will increase by $u'(C_1)$. If he or she chooses instead to save an extra unit and invest the funds in the capital market to earn the real interest rate r, the consumer will be able to increase consumption in period 2 by $1 + r$ units. This will generate an increase in lifetime utility equal to $((1 + r)/(1 + \phi))u'(C_2)$. According to (9) these two alternatives must be equally attractive. In other words, in optimum the consumer must be indifferent between consuming an extra unit today and saving an extra unit today.

Equation (10) is a version of the usual optimum condition that the consumer's marginal rate of substitution, MRS, between any two goods must equal the price ratio between the two goods. In this case, the two goods are 'present consumption', C_1, and 'future consumption', C_2, so the MRS measures the additional units of future consumption needed to compensate the consumer for a unit reduction in present consumption. The optimum condition (10) is illustrated in Figure 16.3.

[6] Check that the second order condition, $\partial^2 U / \partial C_1^2 < 0$, is fulfilled.

Figure 16.3 The consumer's optimal intertemporal allocation of consumption



To interpret the diagram, note that along any consumer indifference curve lifetime utility is constant. According to the lifetime utility function (2) a constant utility level implies:

$$dU = u'(C_1)dC_1 + \frac{u'(C_2)}{1+\phi}dC_2 = 0,$$

or :

$$-\frac{dC_2}{dC_1} = \frac{u'(C_1)}{\frac{u'(C_2)}{1+\phi}}. \tag{11}$$

Equation (11) shows that the numerical slope of the indifference curve is equal to the marginal rate of substitution between present and future consumption, $MRS(C_2 : C_1)$, defined as the ratio of the marginal lifetime utility of present consumption $u'(C_1)$ to the marginal lifetime utility of future consumption $u'(C_2)/(1 + \phi)$. Figure 16.3 also shows that the numerical slope of the consumer's lifetime budget constraint equals $1 + r$: if the consumer gives up one unit of present consumption, $1 + r$ additional units of future consumption will result because his or her saving earns the real interest rate r. Hence we can say that $1 + r$ is the *relative price of present consumption*, since it measures the amount of future consumption which must be given up to enable the consumer to increase present consumption by one unit. When the consumer attains the highest possible level of lifetime utility consistent with his or her lifetime budget constraint, we see from Figure 16.3 that the slope of the indifference curve must equal the slope of the budget constraint, given by the relative price of present consumption. This is exactly what Equation (10) says.

The optimum consumption and saving rule (9) or (10) is called the *Keynes–Ramsey rule* after its discoverers, and it may be restated as follows:

THE KEYNES–RAMSEY RULE OF OPTIMAL SAVING

When consumers are impatient, the net utility gain from postponing a unit of consumption from the current to the next period is $-u'(C_1) + \gamma u'(C_2)$, where $\gamma = (1 + r)/(1 + \phi)$, r is the real interest rate and ϕ is the rate of time preference. In a perfect capital market, a consumer who optimizes the allocation of consumption over time will carry saving out of current income to the point where this net gain is zero.

Although formally derived by Cambridge economist Frank Ramsey in 1928,[7] this rule had been verbally anticipated by the very same John Maynard Keynes who later (in his *General Theory* of 1936) came to believe in a consumption function like (1)! As we will discuss later in this chapter (and in some exercises), for some households current consumption is indeed likely to depend only on current income, as postulated in the consumption function (1). However, we shall also see that the consumption function implied by the Keynes–Ramsey rule (10) is consistent with several empirical observations on consumption, which cannot be reconciled with the simple Keynesian consumption function.

One important implication of (10) is that consumers will typically want to use the capital market to smooth their consumption over time. This is seen most clearly in the benchmark case where the real interest rate equals the rate of time preference, $r = \phi$, so that the consumer's impatience ϕ is exactly offset by the capital market reward r for postponing consumption. When $r = \phi$, condition (9) or (10) can only be met if $C_1 = C_2$, that is, if consumption is constant over the consumer's life cycle. Suppose for simplicity that the consumer starts out in period 1 with zero financial wealth, $V_1 = 0$. Unless the consumer's labour income happens to be the same in periods 1 and 2, he or she will then have to engage in financial saving in one period and financial dissaving in the other period to keep consumption constant over time.

Figure 16.4a shows the case (still for $r = \phi$ and $V_1 = 0$), where labour income in

[7] Frank Ramsey, 'A Mathematical Theory of Saving', *Economic Journal*, **38**, 1928, pp. 543 – 559. This was one of two path-breaking articles published by the unusually gifted Frank Ramsey before his premature death at the age of 25.

period 1 is lower than labour income in period 2 (perhaps because the consumer spends a part of period 1 going through education rather than working in the labour market). The consumer will then want to borrow in period 1 to smooth consumption over the life cycle even though this means having to reserve part of the higher future labour income for payment of interest.

Figure 16.4b shows the alternative case where future labour income is lower than current labour income (perhaps because the consumer plans to retire some time in period 2). In this case, the consumer will want to save part of his or her current labour income and will partly finance period 2 consumption out of previous savings.

Figure 16.4a Consumption smoothing for a consumer with relatively low income during period 1 ($r = \phi$ and $V_1 = 0$)

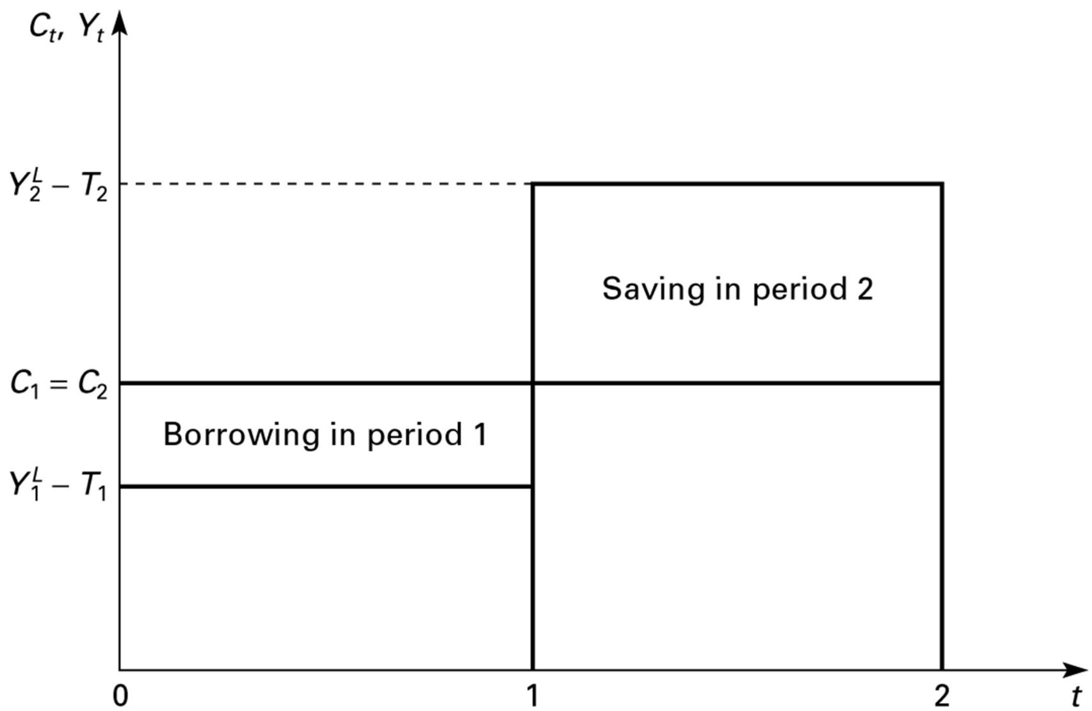
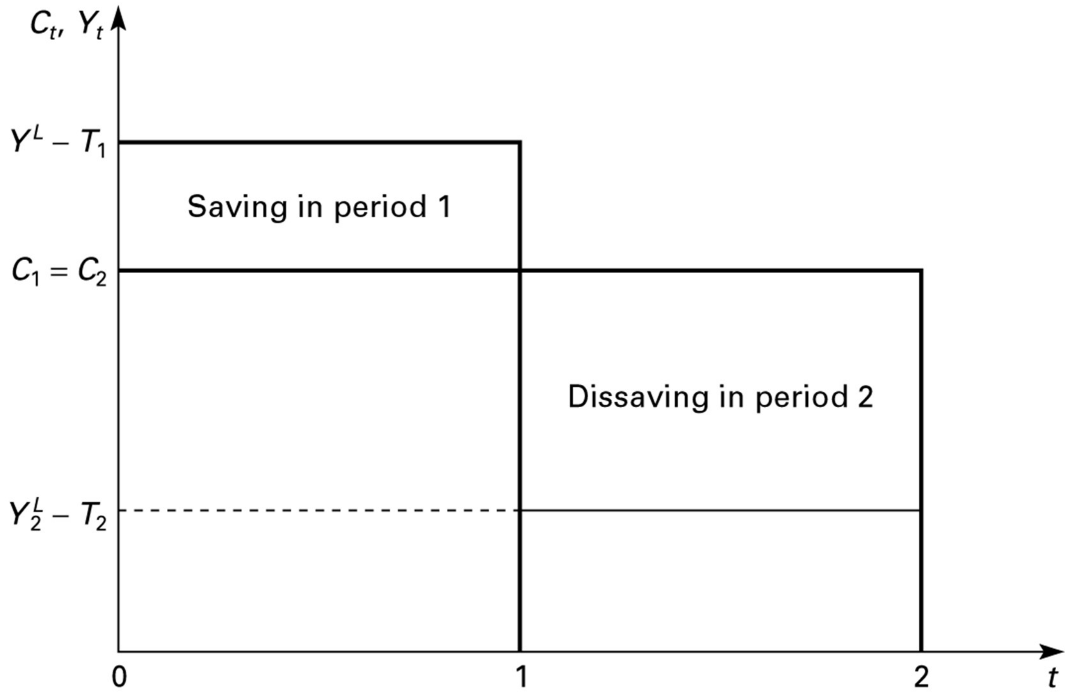


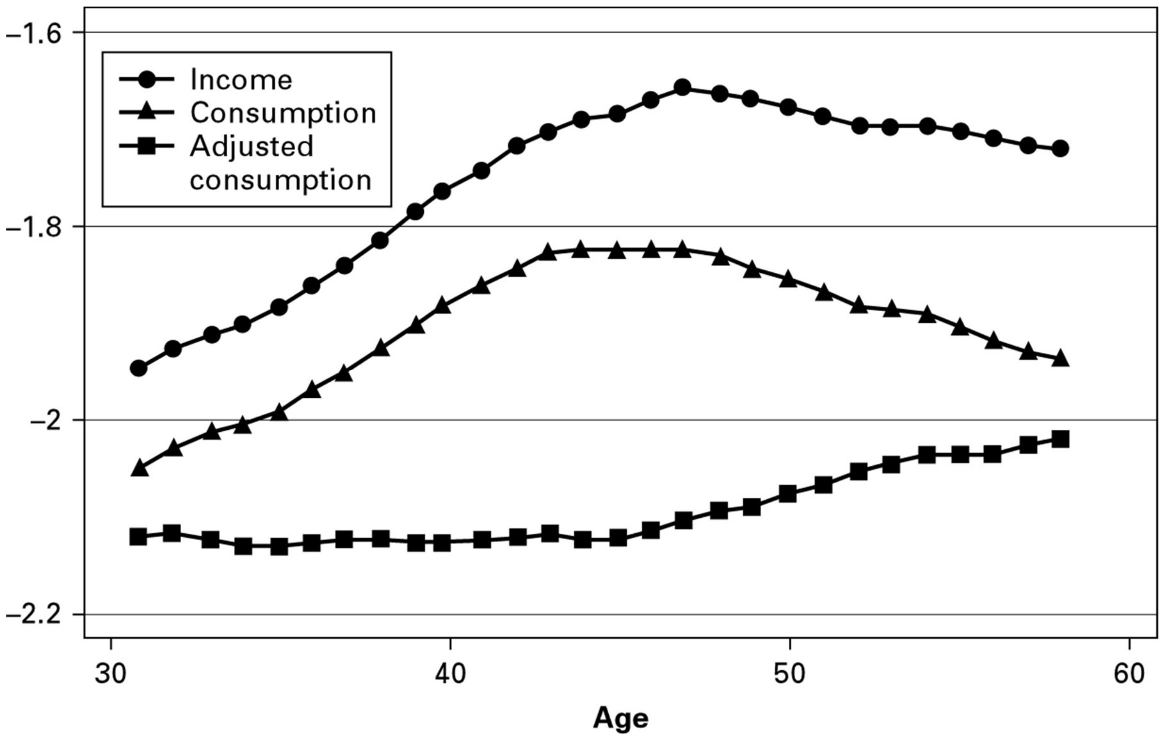
Figure 16.4b Consumption smoothing for a consumer with relatively high income during period 1 ($r = \phi$ and $V_1 = 0$)



If there were no capital market allowing saving and dissaving, consumption would have to equal income within each period. If income is low in one period and high in another, the marginal utility of consumption would then differ across periods, because marginal utility decreases with increasing consumption. The consumer will therefore enjoy a welfare gain to the extent that he or she is able to smooth consumption by borrowing or saving through the capital market. Capital markets enable consumers to decouple current consumption from current income, and utility-maximizing consumers will typically want to take advantage of that in order to spread their consumption more evenly over time. Although total lifetime consumption is constrained by total lifetime resources (wealth), we should not necessarily expect to observe a close link between current consumption and current income.

Do consumers actually smooth their consumption over their working life? A glance at the upper two curves in Figure 16.5 might suggest that the answer is 'No!'. The figure tracks data for the average consumption-age profile and the average income-age profile for a cohort of British married couples. The figures for income and consumption have been deflated by the consumer price index and transformed into logarithms. The age of the household is defined by the age of the female household member. We see that both consumption and income follow a hump-shaped pattern over the life cycle, peaking a little before the age of 50. In particular, current consumption seems to follow current income fairly closely, in apparent contrast to the hypothesis of consumption-smoothing.

Figure 16.5 Consumption and income profiles for households in the UK (born 1935 – 39)



Source: Background material for Martin Browning and Mette Ejrnæs: 'Consumption and Children'. *Review of Economics and Statistics*, **91**, 2009, pp. 93 –111, provided by Mette Ejrnæs.

However, using data from the UK Family Expenditure Survey, economists Martin Browning and Mette Ejrnæs have demonstrated that if one corrects the consumption figures for the systematic impact of differences in the number of children across households, consumers *do* actually tend to smooth their consumption over time. This is illustrated by the bottom curve in Figure 16.5, which shows 'adjusted consumption', that is, the level of consumption adjusted for the estimated impact of the numbers and ages of children on the consumption needs of the household. We see that the adjusted consumption-age profile is much flatter than the income profile, suggesting that consumers do indeed prefer to smooth consumption per household member over time, in accordance with our theory.

The determinants of current consumption

To derive an explicit analytical solution for current consumption C_1, we need to specify the form of the consumer's utility function. One specification, which has often been used in economic research, is the following:

$$u(C_t) = \frac{C_t^{\frac{\sigma-1}{\sigma}} - 1}{\frac{\sigma-1}{\sigma}} \quad \text{for } \sigma > 0, \neq 1, \tag{12}$$

$$u(C_t) = \ln C_t \qquad \text{for } \sigma = 1. \tag{13}$$

This specification satisfies the assumptions in (2) that $u' > 0$ and $u'' < 0$, and the utility function in (12) converges on the logarithmic utility function (13) when σ approaches 1.[8] The 'minus one' in the numerator of (12) is important for this latter property, but is not important for the choices following from maximizing utility. Without this 'minus one', we could write the utility function in (12) as $u(C) = C^{1-\rho}/(1-\rho)$, where $\rho \equiv 1/\sigma$. You have seen this utility function before in Chapter 13, where it represented preferences concerning unstable or uncertain consumption and the interpretation of ρ was the Arrow-Pratt measure of relative risk or instability aversion. We now prefer to write the utility function as $u(C) = C^{1-1/\sigma}/(1-1/\sigma)$, because the parameter $\sigma \equiv 1/\rho$ then has a nice interpretation for our present purposes as ρ had for our purposes in Chapter 13.

The parameter σ is called the *intertemporal elasticity of substitution* because it measures the degree to which consumers are willing to substitute between present and future consumption. To see this, note from (12) that $u'(C_t) = C_t^{-1/\sigma}$. Inserting this into the consumer's optimum condition (9), we get (check!):

$$\frac{C_2}{C_1} = \left(\frac{1+r}{1+\phi}\right)^{\sigma}. \tag{14}$$

This shows that σ is the elasticity of the consumption ratio C_2/C_1 with respect to the relative (real) price of period 1 consumption, $1+r$.[9] In our case:

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[8] To see this, note that the expression on the right-hand side of (12) may be written as $f(\sigma)/g(\sigma)$, where $f(\sigma) = C^{1-1/\sigma} - 1$ and $g(\sigma) = 1 - 1/\sigma$. For $\sigma \to 1$ we see that $f(\sigma) \to 0$ and $g(\sigma) \to 0$. Using L'Hopital's rule, we then get $\lim_{\sigma\to1}(f(\sigma)/g(\sigma)) = \lim_{\sigma\to1}(f'(\sigma)/g'(\sigma)) = \ln C$.

[9] For a function $y = kx^{\varepsilon}$, where k and ε are constants, ε is the elasticity of y with respect to x: Taking logs on both sides gives $\ln y = \ln k + \varepsilon \ln x$, and then differentiating with respect to x gives $(1/y)(\partial y/\partial x) = \varepsilon/x$ or $(\partial y/y)/(\partial x/x) = \varepsilon$.

$$\frac{\partial \frac{C_2}{C_1} / \frac{C_2}{C_1}}{\partial(1+r)/(1+r)} = \sigma. \tag{15}$$

Hence, σ is the percentage increase in C_2/C_1 for a one percent increase in $1+r$. In other words, σ indicates how willing consumers are to reallocate consumption over time when the relative price of present consumption changes.

Since $MRS(C_2 : C_1) = 1 + r$ in the consumer's optimum, we may as well say that the intertemporal elasticity of consumption, *IES*, is the percentage change in the ratio of future to present consumption (C_2/C_1) implied by a 1 per cent change in the consumer's marginal rate of substitution:

$$IES = \frac{d\ln(C_2/C_1)}{d\ln MRS(C_2 : C_1)}. \tag{16}$$

The utility function (12) thus has the property that the intertemporal elasticity of substitution is constant and equal to σ. By varying σ we may therefore study the effects of variations in the consumer's willingness to substitute consumption over time. As σ tends to zero, the consumer becomes quite unwilling to trade present for future consumption, and his or her indifference curves become rectangular, as shown in Figure 16.6. In this case the consumer will choose a fixed consumption ratio (C_2/C_1) regardless of the slope of the intertemporal budget constraint. On the other hand, as σ tends to infinity, substitution possibilities also become infinite, and the indifference curves converge to straight lines.

We are now ready to derive the consumption function. According to (14) we have $C_2 = C_1\,(1 + r)^{\sigma}/(1 + \phi)^{\sigma}$. This can be inserted for C_2 in the lifetime budget constraint, $C_1 + C_2/(1 + r) = V_1 + H_1$, to give $C_1 + C_1\,(1 + r)^{\sigma-1}(1 + \phi)^{-\sigma} = V_1 + H_1$, from which we can find C_1 and then also C_2 by inserting C_1 into $C_2 = C_1\,(1 + r)^{\sigma}/(1 + \phi)^{\sigma}$:

$$C_1 = \theta(V_1 + H_1), \quad \theta \equiv \frac{1}{1+(1+r)^{\sigma-1}(1+\phi)^{-\sigma}} = \frac{(1+\phi)^{\sigma}}{(1+\phi)^{\sigma}+(1+r)^{\sigma-1}}, \quad 0 < \theta < 1. \tag{17}$$

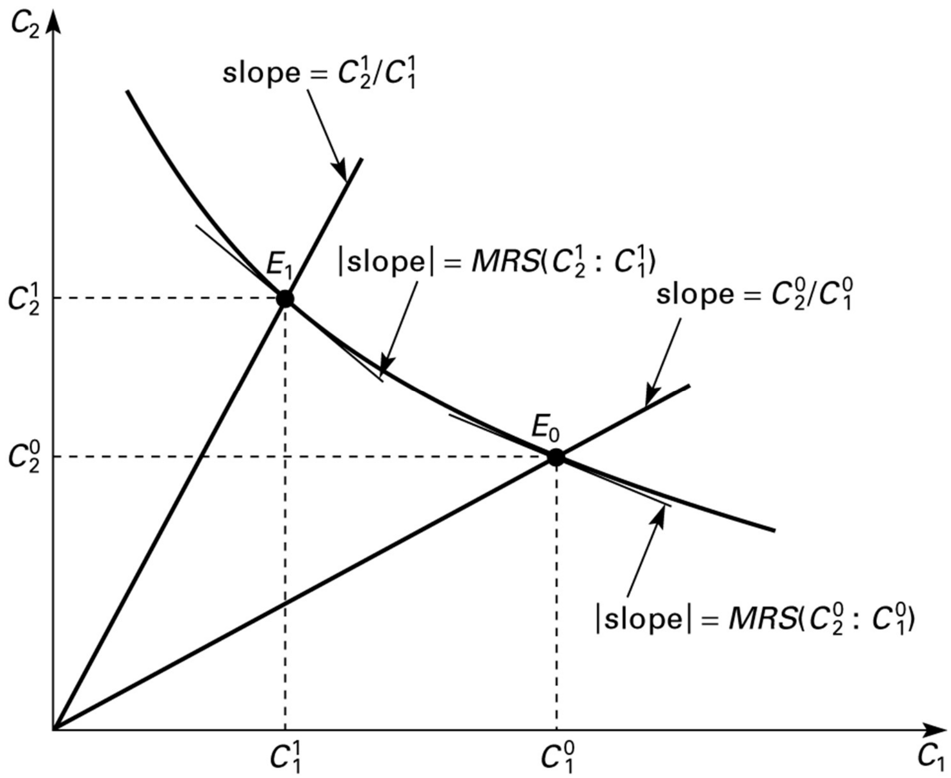
$$C_2 = \frac{(1+r)^{\sigma}}{(1+\phi)^{\sigma}+(1+r)^{\sigma-1}}(V_1 + H_1). \tag{17'}$$

Equation (17) is rather different from the simple Keynesian consumption function (1), and it contains the following important message:

CONSUMPTION AND WEALTH

With a constant intertemporal elasticity of substitution in consumption and perfect capital markets, current consumption is proportional to total current wealth, financial plus human. The propensity to consume out of current wealth, θ, is positive and less than one. Its magnitude generally depends on the real rate of interest.

Figure 16.6 The relation between the shape of the indifference curve and the intertemporal substitution elasticity



16.3 The properties of the consumption function

Consumption and income

Let us now explore the implications of (17) for the relationship between current consumption C_1 and current disposable labour income, $Y_1^d \equiv Y_1^L - T_1$. If we insert our definition of human wealth (6) into (17), we get:

$$C_1 = \theta\left(Y_1^d + \frac{Y_2^d}{1+r} + V_1\right), \qquad Y_t^d \equiv Y_t^L - T_t, \qquad t = 1, 2. \tag{18}$$

This may be rewritten as:

$$C_1 = \hat{\theta} Y_1^d, \tag{19}$$

$$\hat{\theta} \equiv \theta\left(1 + \frac{R}{1+r} + v_1\right), \qquad R \equiv \frac{Y_2^d}{Y_1^d}, \qquad v_1 \equiv \frac{V_1}{Y_1^d}, \tag{20}$$

where $\hat{\theta}$ measures the propensity to consume (out of) current income, R is the ratio of future to current income, and v_1 is the current wealth–income ratio. Based on (19) and (20) we may now explain the empirical puzzle that the average propensity to consume current income seems to decrease with income when we consider microeconomic cross-section data whereas it seems to be roughly constant when we consider macroeconomic time series data.

Consider first the observed cross-section relationship between individual household income and individual consumption suggesting that the rich have a higher average savings rate than the poor. Within any given time period, some of those individuals who record a high current income level cannot expect to earn similar high incomes in the future. One important reason is that workers must some day retire from the labour market in which case they will no longer earn income from labour. Moreover, for self-employed individuals a high current income will sometimes reflect that business has been unusually good during that year. Thus, if a high current income level is expected to be *temporary*, the ratio of future to current income (R) will be relatively low, and according to (20) this will imply a low average propensity to consume current income. In a similar way, some consumers with low current incomes may have good reason to expect that they will earn more in the future. This will typically be the case for university students and other young people who have not yet realized their earnings potential in the labour market, and for entrepreneurs who have just started up a business or who experience unusually bad business during the current year. For such individuals the ratio R in (20) will be high, and hence they will have a high propensity to consume out of their low current income. If some of the observed variation in income levels in a cross-section of consumers reflects such temporary factors, Equation (20) may thus explain why the average propensity to consume appears to fall as income goes up.

In a celebrated article, Franco Modigliani and Richard Brumberg provided the first statement of the so-called *life cycle theory* of consumption according to which consumers in different stages of the life cycle will have different propensities to consume current income because of their desire to smooth consumption over time.[10] In a similarly famous contribution, Milton Friedman developed the so-called *permanent income hypothesis* which says that transitory changes in income will mainly lead to temporary changes in savings, whereas current consumption will depend on the consumer's *permanent* income, that is, his expected long-run average income, which is proportional to his total wealth.[11] Note that the consumer's stock of total wealth equals the discounted value of his or her expected future income. Permanent income is that hypothetical constant level of income, which has the same present value as the consumer's expected future income stream.

Thus these writers pointed out that some of the high incomes observed during a given year are only temporarily high and hence will not induce high levels of current consumption, and some of the low incomes recorded in a given period are just temporarily low and thus will not cause similarly low levels of consumption. Because of this, a cross-section analysis of the relationship between current consumption and current income in any given year will give the impression that high-income people have a lower average propensity to consume than low-income people.

Let us next see how Equations (19) and (20) may explain the empirical observation that the average propensity to consume is roughly constant in the long run when we look at aggregate time series data. If we denote the growth rate of real income by *g*, we have $Y_2^d \equiv (1+g)Y_1^d$ which may by inserted into (20) to give:

$$\hat{\theta} = \theta\left(1 + \frac{1+g}{1+r} + v_1\right). \tag{21}$$

While the growth rate *g* fluctuates in the short run, on average it has been fairly constant over the long run, as we saw in Figure 2.8 of Chapter 2. Moreover, the real interest rate *r* shows no systematic long-run trend as we also argued in Chapter 2, and over the long run we also found a tendency that the capital stock grows at the same rate as income so that the long-run wealth–income ratio $v_1 \equiv V_1/Y_1^d$ is roughly constant. It then follows from (21) that the long-run average propensity to consume will also tend to be constant, as we do indeed observe.

To sum up, our reconciliation of the microeconomic cross-section data and the macroeconomic time series data runs as follows:

[10] See Franco Modigliani and Richard Brumberg, 'Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data', in Kenneth K. Kurihara, ed., *Post-Keynesian Economics*, pp. 388 – 436, New Brunswick, NJ, Rutgers University Press, 1954.

[11] Milton Friedman, *A Theory of the Consumption Function*, Princeton, NJ, Princeton University Press, 1957.

CONSUMPTION AND INCOME: EXPLAINING THE CONSUMPTION PUZZLES

A temporary increase in income for an individual consumer will reduce his or her expected future income relative to current income and reduce his or her short-run wealth–income ratio. For these reasons, the average propensity to consume will tend to fall with rising income levels in a cross-section of consumers. Over the long run, the average growth rate of income across all consumers is roughly constant, and wealth moves roughly in line with income. From the consumption function (21), this implies a constant long-run average propensity to consume at the macro level.

Going back to (17), we see that the propensity to consume out of current wealth, θ, is unambiguously increasing in the time preference rate, ϕ. From our discussion of Equation (21) it follows that the long-run propensity to consume current income will then also be higher the higher the rate of time preference. This is intuitive, since a higher value of ϕ indicates a higher degree of "impatience", i.e., a stronger preference for current over future consumption.

Consumption and wealth

While the average propensity to consume current income tends to be constant in the long run, it fluctuates quite a lot in the short run, as indicated in Figure 16.7. Equation (21) suggests three possible reasons for this:

1. short-run changes in the expected growth rate of income (g),
2. short-run changes in the wealth–income ratio (v_1), and
3. short-run changes in the real rate of interest.

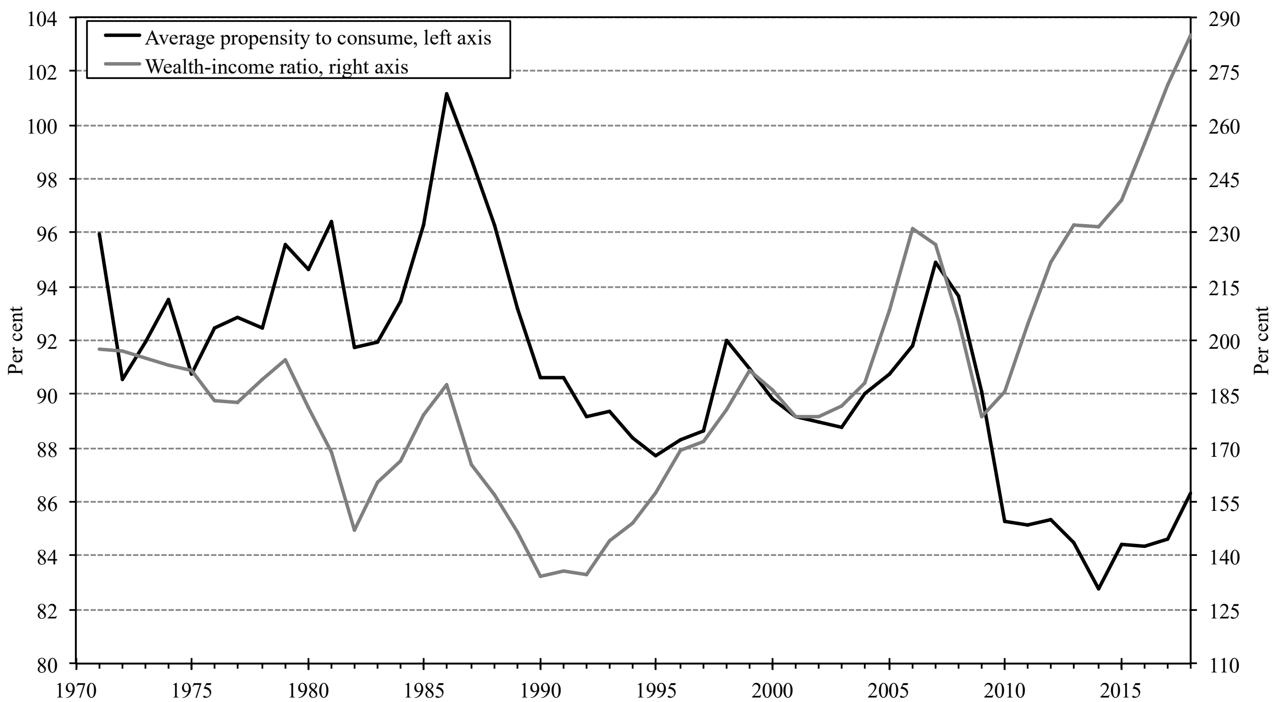
According to (21) a higher expected income growth or a rise in the market value of financial wealth relative to current income will increase the propensity to consume. Note that changes in g and v_1 will often go hand in hand, since a higher expected income growth is likely to drive up the market prices of stocks and owner-occupied housing, thereby increasing V_1, because a higher expected growth rate will tend to raise expected future corporate dividends and to drive up the demand for housing.

Figure 16.7 shows that the average propensity to consume does indeed tend to rise when the wealth–income ratio goes up, and vice versa, as our theory of consumption would lead us to expect.[12] The two curves move up and down synchronously most of the

[12] It is worth noting that even though he did not include financial wealth as an explanatory variable in his basic consumption function, Keynes was fully aware of the potential influence of wealth on private consumption. On pp. 92 – 93 of his *General Theory* he wrote: 'Windfall changes in capital-values . . . are

time, but during the severe crises that Denmark experienced from 1986/87 and again – as part of the great financial crisis – from 2008/09, first consumption and wealth drop together, but then the wealth-income ratio begins to recover considerably before the consumption-wealth ratio does. This is probably due to pessimistic expectations concerning future incomes in the aftermath of larger crises, which is consistent with our consumption theory based on intertemporal optimization.

Figure 16.7 Propensity to consume and wealth-income ratio in Denmark, 1971-2018



Note: Based on annual data. The average propensity to consume is the ratio of private consumption to disposable income, and the wealth-income ratio measures the ratio of private wealth to disposable income. *Source*: SMEC database, Danish Economic Councils.

Consumption and interest rates

The third determinant of the propensity to consume current income is the real interest rate r. The interest rate affects consumption through three different channels. First, it

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of much more importance in modifying the propensity to consume, since they will bear no stable or regular relationship to the amount of income. The consumption of the wealth-owning class may be extremely susceptible to unforeseen changes in the money-value of its wealth. This should be classified amongst the major factors capable of causing short-period changes in the propensity to consume.’

influences the propensity θ to consume out of wealth. Second, it affects the market value of financial wealth. Third, the real interest rate also affects the value of human wealth. We will consider each of these channels in turn.

For convenience we restate (17) as (22):

$$C_1 = \theta(V_1 + H_1), \quad \theta \equiv \frac{1}{1+(1+r)^{\sigma-1}(1+\phi)^{-\sigma}} = \frac{(1+\phi)^{\sigma}}{(1+\phi)^{\sigma}+(1+r)^{\sigma-1}}. \tag{22}$$

Clearly, the propensity to consume wealth, θ, increases with r if $\sigma < 1$, and decreases if $\sigma > 1$, and we cannot say *a priori* which direction is correct as σ can be either smaller or larger than 1. The indeterminacy in how r affects the propensity to consume wealth is due to offsetting substitution and income effects. On the one hand, an increase in the real interest rate raises the relative price $1 + r$ of current consumption. *Ceteris paribus*, this will induce the consumer to substitute future for present consumption through an increase in current saving. Obviously, this *substitution effect* will tend to reduce the propensity to consume in the current period. On the other hand, the higher interest rate also increases the amount of future consumption generated by a given amount of current saving. Hence, the consumer can afford a higher level of current consumption without having to reduce future consumption. Other things equal, this *income effect* of the rise in the interest rate works in favour of a rise in current as well as future consumption and thus tends to increase θ (as can be seen from (17) and (17'), present and future consumption are normal goods with the preferences assumed here).

Since σ measures the strength of the substitution effect, it is not surprising that the direction in the dependence of θ on r depends on the magnitude of σ, such that a larger σ pulls in the direction of a negative relationship. We see that, for an intertemporal substitution elasticity equal to 1, the income and substitution effects will exactly offset each other, and the propensity to consume will be unaffected. Over the years many researchers have tried to estimate the empirical magnitude of the intertemporal substitution elasticity, σ. Most authors have found values of σ well below 1.[13] This suggests that the propensity to consume wealth will *rise* when the real interest rate goes up.

However, a higher interest rate will also influence the level of wealth itself. As already mentioned, the stock of financial wealth, V_1, includes the value of stock and of housing capital. In Chapter 15, we saw that both of these important wealth components will be negatively affected by a rise in the real interest rate. Thus, a rise in r means that expected future dividends are discounted more heavily, leading to a fall in share prices. Moreover, by raising the user cost of owner-occupied housing, an increase in r reduces housing demand, which in turn drives down the market value of the existing housing stock. For these reasons, a rise in the real interest rate will reduce the wealth–income ratio, v_1.

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[13] Note that this accords with the Arrow-Pratt measure of instability ρ being well above 1, as we said in Chapter 13. Since $\rho = 1/\sigma$, a ρ larger than 1 corresponds to a σ smaller than 1.

In addition, the higher interest rate means that expected future labour income is discounted more heavily. As a consequence, the value of human wealth specified in (6) goes down. Intuitively, a higher real interest rate makes future labour income less valuable by making it easier to attain a given level of future consumption through saving out of current income.

The fall in human and financial wealth induced by a rise in the real interest rate clearly tends to reduce the propensity to consume current income. Since we have also seen that there may be an offsetting increase in the propensity to consume wealth, θ, the net effect of a rise in r on the ratio of current consumption to current income, $\hat{\theta}$, is ambiguous. Against this background, it is not surprising that empirical research has found it difficult to document a strong effect of real interest rates on consumption, although the dominant view is that a rise in r tends to reduce consumption because of the negative impact on wealth.

CONSUMPTION AND THE REAL INTEREST RATE

When combined with empirical evidence on the intertemporal elasticity of substitution in consumption, the micro-based theory of consumption smoothing in perfect capital markets predicts that the propensity to consume *out of total current wealth* is most probably increasing in the interest rate. In other words, ignoring how the real interest rate affects current financial and human wealth, consumption is likely to increase when the interest rate goes up. However, current financial and human wealth depend negatively on the real interest rate and consumption depends positively on wealth. Taking this into account, consumption most probably depends negatively on the real interest rate.

16.4 Consumption, taxation and debt: Ricardian quivalence

We will now show the implications of our optimization-based theory of consumption for the effects of government's tax and debt policies on private consumption demand, still first ignoring credit or liquidity constraints. This will give us an opportunity to highlight the importance of the way in which consumers form their expectations about the future. It will also enable us to illustrate the important distinction between temporary and permanent changes in tax policy.

Consumption and taxes: temporary versus permanent tax cuts

To focus on tax policy, we start by rewriting the consumption function (18) as:

$$C_1 = \theta\left(Y_1^L - T_1 + \frac{Y_2^L - T_2}{1+r} + V_1\right). \tag{23}$$

Suppose now that the government wishes to stimulate consumption demand by cutting net taxes T_1 during period 1, say, because the economy is in recession. Let the change in net taxes be $dT_1 < 0$. For the moment, we assume that consumers expect the tax cut to be *temporary*, perhaps because the government has announced that it may have to raise taxes again once the recession is over. In that case, consumers will expect T_2 to be unchanged, and according to (23) the immediate impact dC_1 of the temporary tax cut will then be:

$$dC_1 = \frac{\partial C_1}{\partial T_1} dT_1 = -\theta dT_1 > 0. \tag{24}$$

In other words, if the government cuts taxes temporarily by one unit, current consumption will go up by θ units.[14] Thus, the temporary tax cut will indeed succeed in stimulating current consumption, but recalling from (17) that $\theta < 1$, we also see that part of the increase in disposable income will be saved for future consumption. Of course, this saving reflects the consumer's desire to smooth consumption over time.

If the government changes the net taxes in period 1 by dT_1 and announces taxes in period 2 to be changed by dT_2, and the consumers believe this, the implied change in a period 1 consumption is from (23):

$$dC_1 = \frac{\partial C_1}{\partial T_1} dT_1 + \frac{\partial C_1}{\partial T_2} dT_2 = -\theta\left(dT_1 + \frac{dT_2}{1+r}\right). \tag{25}$$

For comparison, suppose that consumers expect the tax cut in period 1 to be *permanent* so that T_2 goes down by the same amount as T_1, that is, $dT_1 = dT_2 < 0$. It then follows that:

$$dC_1 = -\theta\left(1 + \frac{1}{1+r}\right) dT_1 = -\theta \frac{2+r}{1+r} dT_1 > -\theta dT_1 > 0. \tag{26}$$

Not surprisingly, we see from (24) and (26) that *a permanent tax cut will have a stronger impact on current consumption than a temporary tax cut*. In the benchmark case where the real interest rate r equals the rate of time preference ϕ so that the consumer wants to consume equal amounts in the two periods, we saw earlier that $\theta = (1 + r)/(2 + r)$. In that case it follows from (26) that $dC_1 = -dT$. In other words, when the consumer wants to

[14] Note that this is a *partial equilibrium* analysis. If the tax cut succeeds in stimulating economic activity, it will indirectly cause a further rise in consumption by raising aggregate labour income Y_1^L. However, our focus here is on the immediate direct impact on consumption, for a given level of activity.

smooth consumption perfectly over time, he or she will consume all of the period 1 tax cut during that period. There is no need to save any part of the increase in current disposable income to smooth consumption, since his or her expected future disposable income has gone up by a similar amount as his or her current net income.

The government budget constraint

In the analysis above we have not specified whether the tax cuts are associated with changes in public spending. As we shall argue below, the effects of a tax cut on private consumption may depend on whether or not it reflects a cut in public expenditure. To see this, we must make a slight detour to study the government's budget constraint. Let us assume for the moment that the government plans for the same time horizon as the household sector (later we will discuss the implications of relaxing this assumption). As before, we will divide this time horizon into two periods representing the 'present' (period 1) and the 'future' (period 2). At the beginning of period 1, the government starts out with a given real stock of public debt D_1 which is predetermined by the accumulated historical government budget deficits. In period 1 (at the beginning) the government spends a real amount G_1 on public consumption and collects a real net tax revenue T_1, where T_1 measures taxes net of government transfer payments (transfers are simply treated as negative taxes). Interest payment rD_1 on the existing debt is not included in G_1 or T_1. If the government runs a primary deficit, $G_1 - T_1 > 0$,[15] in period 1 this will itself add to the debt and, of course, a primary surplus will subtract from it. The interest payment on existing debt, rD_1, as well as the interest payment on the new debt created in period 1, $r(G_1 - T_1)$, will also add the debt.[16] Recall here that transactions are assumed to take place at the beginning of each period, so the deficit $G_1 - T_1$ will therefore imply an interest payment in period 1. Hence, the debt at the beginning of period 2 will be $D_2 = D_1 + rD_1 + (G_1 - T_1) + r(G_1 - T_1)$, which can also be written:

$$D_2 = (1 + r)(D_1 + G_1 - T_1). \tag{27}$$

This is the government budget constraint for period 1. Likewise, at the beginning of period 3 the government debt would be $D_3 = (1 + r)(D_2 + G_2 - T_2)$. However, since the planning horizon is only two periods for consumers and government, the government cannot carry debt into period 3, so it must plan so that $D_3 = 0$ implying $D_2 + G_2 - T_2 = 0$, or:

$$T_2 = D_2 + G_2, \tag{28}$$

This is the government budget constraint for period 2. In period 2, the government must

[15] The primary public balance is the surplus or deficit excluding interest payments.
[16] We abstract from money printing (seigniorage) as a way of financing government consumption, since this plays very little role in developed countries.

collect taxes, which are sufficient to enable it to repay the public debt accumulated during the previous period and to cover expenditure in period 2.

In our simple two-period framework the relevant criterion for *sustainability of fiscal policy* is that the government pays back all debt at the end of period two: no one would lend to the government if the government did not pay back within the relevant time horizon. In a more general setting with many periods, long-run sustainability of fiscal policy still requires a balance between current and future tax payments on the one hand and initial debt plus current and future government spending on the other. The ratio of government debt to GDP must not 'explode', that is, it must not rise to such levels that lenders to the government no longer trust that it can pay back its debt by raising the necessary tax revenue. Since the future is not known with certainty, it may be difficult to judge at a given point in time whether fiscal policy is sustainable or not, but the government has to aim at sustainability to be able to finance its deficits. That is, it must convince the public that it has taken or will take actions that will ensure sustainability.

Inserting (27) into (28) and dividing through by 1 + r, we obtain *the intertemporal government budget constraint:*

$$D_1 + G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r} \tag{29}$$

Equation (29) makes a basic, but very important point:

THE INTERTEMPORAL GOVERNMENT BUDGET CONSTRAINT AND SUSTAINABILITY OF FISCAL POLICY

Fiscal policy is said to be **sustainable** if and only if the present value of current and future tax revenues cover the present value of current and future government spending plus the initial government debt. A government must aim at sustainability to finance its deficits.

As we shall now see, the intertemporal government budget constraint may have profound implications for the effects of tax policy.

Tax finance versus debt finance of government spending: the Ricardian Equivalence Theorem

It follows directly from (29) that if the government does not reduce its current or planned future spending – that is, if G_1 and G_2 are unchanged – a tax cut $dT_1 < 0$ in the current period must be followed by a future increase $dT_2 > 0$ in taxes so that:

$$dT_1 + \frac{dT_2}{1+r} = 0 \quad \text{or} \quad dT_2 = -(1+r)\,dT_1. \tag{30}$$

To understand this relationship between current and future taxes, consider (27) and (28) once again. According to (27), if the government cuts current taxes by an amount $|dT_1|$ without reducing current public spending, G_1, the stock of real public debt will have risen by the amount $dD_2 = (1 + r)\,|dT_1|$ at the beginning of period 2. If the government does not cut back on its period 2 consumption, G_2, it follows from (28) that it will have to raise taxes in period 2 by the amount $dT_2 = (1 + r)\,|dT_1|$ to pay for the principal and interest on the extra debt created by the tax cut in period 1.

Suppose now that consumers have *rational expectations* in the sense that they look forward and understand the implications of the intertemporal government budget constraint. In that case they will realize that if the government reduces current taxes *without* reducing current or planned future public spending, the present value of future taxes will have to increase by as much as the cut in current taxes. It then follows from (26) and (30) that:

$$dC_1 = -\theta\left(dT_1 + \frac{dT_2}{1+r}\right) = -\theta\left(dT_1 - \frac{(1+r)\,dT_1}{1+r}\right) = 0 \tag{31}$$

The implication of (31) is striking: a cut in current taxes which is *not* accompanied by a cut in present or planned future public spending will have *no* effect on private consumption! In other words, a switch from tax finance to debt finance of current public spending leaves private consumption unaffected, since consumers realize that the tax cut today will be offset by the future tax increase needed to service the additional government debt. Indeed, because the present value of their lifetime tax burden is unchanged, consumers know that their net human wealth is also unchanged, and hence they feel unable to afford an increase in current consumption. Instead, they will save the entire current tax cut and invest it in the capital market. This increase in current private saving will raise consumer cash receipts in period 2 by the amount $(1 + r)\,|dT_1|$ which is just sufficient to pay for the higher future tax bill. Hence, future private consumption is also unchanged. The increased supply of government bonds in period 1 is thus matched by a similar increase in private demand for bonds, and the higher period 2 taxes are matched by an increase in private sector income from bond holdings.

The observation that tax finance and debt finance of government spending are in principle equivalent was first made as early as 1820 by the classical British economist David Ricardo. In a treatise on public debt, Ricardo discussed three alternative ways of financing a war. For concreteness, he assumed that the war would generate military expenditure of £20 million per year. One financing option would be to impose additional taxes amounting to £20 million per year until the end of the war. Alternatively, the government could borrow £20 million every year during the war and increase tax collections by just £1 million each year to cover the interest payments on a £20 million loan, assuming an interest rate of 5 per cent. In this case, the public debt would continue to rise until the war was over and would never be repaid, and taxes would be permanently higher. The third possibility considered by Ricardo was one where the war was mainly debt-financed, but where taxes would be raised by £1.2 million per year for every £20 million borrowed, enabling the government to pay off its debt in 45 years. Under the assumptions made, the present value of tax payments would be the same under the three modes of finance. As Ricardo put it: ‘In point of economy, there is no real difference in either of the modes; for twenty millions in one payment, one million per annum for ever, or 1,200,000 pounds for 45 years, are precisely of the same value’.[17]

Because of this statement by Ricardo, the claim that taxes and debt are equivalent methods of public finance is referred to as the *Ricardian Equivalence Theorem.* Before you dismiss the theorem as being utterly unrealistic, take a look at Figure 16.8.

Figure 16.8 Private and public saving in Denmark, percent of GDP, 1970-2018

Note: Based on annual data. Private saving is gross saving of companies and households relative to GDP. Public saving is gross public saving relative to GDP.
Source: SMEC database, Danish Economic Councils.

In that figure we have plotted private and public saving in Denmark as a percentage

[17] From Ricardo’s paper on the ‘Funding System’, *The Works and Correspondence of David Ricardo*, vol. IV, Piero Sraffa, ed., Cambridge University Press, 1951, p. 186. The subtle phrase: ‘In point of economy …’ has been interpreted to mean something like: ‘From a rational economic perspective …’.

of GDP against time. As we have seen above, the Ricardian Equivalence Theorem implies that whenever the public sector reduces its saving, thereby increasing new issues of public debt (or reducing the rate at which public debt is retired), we should observe an offsetting increase in private saving, and vice versa. Figure 16.8 shows that there is indeed a clear tendency for private and public saving to move in opposite directions almost as mirror images, suggesting that Ricardian equivalence may not be that unrealistic after all.

However, while Figure 16.8 does not seem inconsistent with Ricardian equivalence, it does not 'prove' that the theorem is correct, since there may be other explanations for the observed relationship between private and public saving. One potential Keynesian explanation is that Figure 16.8 simply reflects the so-called automatic stabilizers built into the public budget. If the private propensity to save goes up for some reason, the resulting fall in consumption demand will tend to generate a fall in economic activity which in turn will increase the public budget deficit by automatically reducing government revenue from taxes on income and consumption and by automatically increasing public expenses on unemployment benefits. Alternatively, if the economy is hit by some other contractionary shock, say, a drop in investment which reduces output and employment, thereby increasing the budget deficit via the automatic stabilizers, consumers may become more pessimistic and uncertain about their future income and may react by increasing their propensity to save.

Why Ricardian equivalence is likely to fail

Although Figure 16.8 suggests that the Ricardian Equivalence Theorem cannot be dismissed so easily, most economists remain skeptical of the theorem in its strong form. Indeed, Ricardo himself did not believe in the practical relevance of his theorem. Right after having explained that his three alternative methods of war finance would imply the same present value of taxes (cf. the quotation above), he proceeded to write:

> … but the people who pay the taxes never so estimate them, and therefore do not manage their private affairs accordingly. We are too apt to think that the war is burdensome only in proportion to what we are at the moment called to pay for it in taxes, without reflecting on the probable duration of such taxes.[18]

What Ricardo says here is that ordinary taxpayers simply do not have the foresight or sophistication to calculate the present value of their expected future taxes. Hence, they do not realize that lower taxes today (more government debt) must mean higher taxes tomorrow as long as the time path of public consumption is unchanged. Casual observation suggests that many people may indeed be myopic in this sense. However, even if consumers are not irrational or myopic, there may still be several reasons why taxes and debt are not equivalent modes of public finance. Let us briefly consider these.

[18] Ricardo, *op. cit.*, pp. 186 –187.

Finite horizons and intergenerational distribution effects

Our analysis above assumed that the private and public sectors have identical planning horizons. In practice, the state will continue to exist beyond the finite lifetime of the individual consumer. Many current taxpayers (especially the elderly) may therefore rationally expect that some of the future taxes needed to service the existing public debt will be levied on future generations and not on those currently alive. In that case a shift from tax finance to debt finance is a way of shifting (part of) the burden of paying for current government spending onto future generations. A debt-financed tax cut will therefore increase the human wealth and consumption of the currently living generations.

This argument against Ricardian equivalence may sound plausible, but it was met by an ingenious objection from the American economist Robert Barro. He pointed out that parents care about their children. If the government tries to shift part of the tax burden from current to future generations through debt finance, parents may use their tax savings to increase their bequests to compensate their children for the higher future tax burden. When parents internalize the welfare of their children who in turn care about the welfare of *their* children and so on, the current generation will effectively behave as if it has an infinite time horizon. Any attempt by the government to redistribute resources across generations will then be neutralized by offsetting private intergenerational transfers, according to Barro.[19] However, this assumes that all parents actually plan to leave bequests to their children. In reality, some parents may not do so, for example if they believe that their children will be much richer than themselves. Moreover, the Barro argument abstracts from population growth. If population is growing, the future tax burden will be spread out across a larger number of taxpayers, and parents will not have to pass on all of their current tax savings to their children to compensate the latter for the higher future taxes needed to service the higher public debt. Still, Barro's analysis is an important reminder that human mortality as such is not a sufficient reason to dismiss Ricardian equivalence.

Intragenerational redistribution

The macro analysis of the preceding section hides the fact that most taxes are redistributive in nature. For the individual taxpayer or the individual family dynasty, a tax cut today may not be matched by an equivalent present-value tax increase tomorrow, even if the government obeys its aggregate intertemporal budget constraint. If a shift in the tax burden over time also involves a shift in the lifetime tax burdens across different

[19] Robert J. Barro, 'Are Government Bonds Net Wealth?', *Journal of Political Economy*, **82**, 1974, pp. 1095 –1117. This seminal article is the authoritative modern statement of the Ricardian Equivalence Theorem. Interestingly, even Barro's sophisticated reasoning was anticipated by Ricardo, for our previous quotation from Ricardo's text continues: 'It would be difficult to convince a man possessed of 20,000 pounds, or any other sum, that a perpetual payment of 50 pound per annum was equally burdensome with a single tax of 1000 pounds. He would have some vague notion that the 50 pounds per annum would be paid by posterity, and would not be paid by him; but if he leaves his fortune to his son, and leaves it charged with this perpetual tax, where is the difference whether he leaves him 20,000 pounds with the tax, or 19,000 pounds without it?'.

individuals or families, the resulting *intragenerational* redistribution across agents belonging to the same generation may have macroeconomic effects. The reason is that different individuals or families have different characteristics and hence different propensities to consume so that redistribution of income does not necessarily leave aggregate consumption unchanged. However, intragenerational redistribution does not mean that a switch from tax finance to debt finance will necessarily raise current consumption. As a counter-example, if a lot of people fear that they will have to bear a disproportionate share of the higher future tax burden implied by an increase in public debt, a debt-financed cut in current taxes may actually *reduce* current private consumption by reducing expected aggregate human wealth.

Distortionary taxes
The Ricardian Equivalence Theorem assumes that taxes take the form of lump sum payments, which are unrelated to individual economic behaviour. In practice, a person's tax bill is typically linked to his or her income or consumption. Taxes on income and consumption may discourage labour supply and savings and induce consumers to change their pattern of consumption. In particular, if a debt-financed cut in current income taxes generates expectations of higher future tax rates to service the higher public debt, consumers may want to increase their current labour supply and reduce their future labour supply to take advantage of the fact that marginal tax rates are lower today than they will be tomorrow. By inducing such *intertemporal substitution in labour supply*, the government may be able to stimulate current economic activity through a switch from tax finance to debt finance of current public spending.

Credit constraints
Our derivation of the Ricardian Equivalence Theorem relied on our assumption that capital markets are perfect so that no consumers are credit-constrained. In reality, many consumers (such as university students!) will be unable to borrow as much as they would like at the going market interest rate. For example, a person may expect that he will earn much more in the future than today, but if his bank does not have the information necessary to estimate his future earnings potential, it may be reluctant to grant him a credit against his expected future labour income if he cannot produce any collateral. Such a credit-constrained consumer will wish to spend all of his current disposable income here and now, since he would prefer to increase his current consumption at the expense of future consumption if he could only obtain more credit. If the government implements a debt-financed tax cut, credit-constrained consumers will therefore increase their current consumption even if they realize that they will face higher taxes in the future. By means of the debt-financed cut in present taxes, the government is using its access to the capital market to shift the consumption possibilities of credit-constrained consumers from the future to the present, thereby offsetting the imperfections in the private capital market. In this scenario Ricardian equivalence breaks down, as you are invited to demonstrate in Exercise 4.

Empirical research has found that current consumption tends to react more strongly to changes in current income than one would expect if consumption were governed only by expected lifetime income (of which current annual or quarterly income is usually only a small fraction). This so-called 'excess sensitivity' of current consumption to current income may reflect that many consumers are indeed credit-constrained and will hence wish to consume all of an increase in their current income, or it may reflect that they are simply short-sighted.

The debate on Ricardian Equivalence has been an important controversy in macroeconomics, so let us restate what we have learned:

THE RICARDIAN EQUIVALENCE THEOREM

The theorem says that a temporary switch from (lump-sum) tax finance to debt finance of public spending will not affect private consumption, since the intertemporal government budget constraint implies that the present value of future taxes will rise by the same amount as current taxes go down. Realizing this, rational forward-looking consumers will save all of the current tax cut and use the savings and the interest thereon to pay for their higher future tax bill. However, in practice the equivalence between tax finance and debt finance is likely to break down, as consumers may be short-sighted or may expect part of the higher future tax burden to fall on future generations, and as credit-constrained consumers will wish to spend their tax cut immediately even if they realize that future taxes will go up.

16.5 Towards a plausible consumption function

We end this chapter by summarizing our theory of private consumption.

We have just argued that in the real world many consumers are constrained in borrowing and that the consumption of such credit- or liquidity-constrained consumers will depend only on current disposable income. The framework of intertemporal optimization may still be relevant for such consumers, but they are subject to an additional constraint. When introducing the theory, we said that we would *first* consider perfect credit markets where the only constraint on the borrowing of the consumer was that he or she should be able to pay back all debt at the end of period 2, amounting to $V_2 \geq -(Y_2^L - T_2) = -Y_2^d$. However, in Chapter 14 we described several reasons why credit markets may not be perfect, among these moral hazard and adverse selection problems relevant for credit markets. A person who cannot borrow at all against future income will be subject to the tougher constraint $V_2 \geq 0$, and if he or she can only borrow up to some limit $-\overline{V}_2 > 0$, where $\overline{V}_2$ is a borrowing constraint, the constraint for period 2 is $V_2 \geq \overline{V}_2$ which is still tougher than the constraint in a perfect capital market if $\overline{V}_2 > -Y_2^d$.

A consumer subject to a credit constraint is not necessarily affected by it. If the consumption determined by intertemporal optimization in the absence of the credit constrained implies a degree of borrowing, possibly no borrowing, that does not violate the constraint, then this is not binding and has no influence on the consumer's behaviour. However, if the consumer's current income is sufficiently small relative to expected future income, and the consumer's time preference rate is sufficiently large, then intertemporal optimization will make the consumer want to front load his or her consumption relative to income. If also the consumer's financial wealth is sufficiently small, this may well imply borrowing to a degree that violates the borrowing constraint. Such a consumer will probably already have run down financial assets to zero, and if he or she cannot borrow at all, it is optimal for the consumer simply to consume all of his or her disposable income, that is, $C_1 = Y_1^d$.[20] Young families who expect higher future than current incomes or persons who just temporarily experience a relatively low income could be among such actively liquidity-constrained people. Other people may not be optimizers at all, but live 'from hand to mouth' or according to simple 'rules of thumb' (e.g., 'always spend 90 percent of current income on consumption'). For such myopic consumers current consumption will also be determined by and narrowly linked to current disposable income.

However, it is also realistic to assume that many consumers will not be myopic or credit-constrained in any particular year. For example, people with positive net financial assets are unlikely to be liquidity-constrained, since they can always sell off some of their assets if they want to increase their present consumption relative to planned future consumption. For these individuals we have seen that consumption depends on expected future income as well as on current income. In other words, aggregate consumption depends positively on current disposable labour income, Y_1^d, and on expected future disposable labour income $(Y_2^d)^e$ for those consumers who are not credit-constrained. We have also seen that for such consumers consumption is affected by the real rate of interest, r, and by the market value of initial private wealth, V_1.

How important are credit constraints for consumption? A policy experiment in Denmark can shed some light on this. In 1998 Denmark introduced the mandatory so-called 'Special Pension (SP) Saving' system mainly to dampen the activity level in the economy which was threatening to overheat. Every income earner would have to pay 1 percent of annual income per year into a new individual pension account (separate from other pension plans of the person). The accumulated funds were to be paid out at the age of 65. In 2004, further contributions to the SP accounts were suspended, but the existing accounts lived on. In March 2009, as part of the stimulus policy during the financial crisis, the government gave people the opportunity to have their SP savings paid out during the second half of 2009 with a considerable tax rebate (roughly a tax of 35 percent rather than the 40 percent to be paid at pension age).

Many people decided to have their savings paid out and the amounts were large. The average balance of accounts in the SP system was around 15,000 DKK. For those who

[20] Exercises 3 and 4 ask you to dig deeper into the issue of credit-constrained consumers.

chose to withdraw their savings, the average payout was around 15,500 DKK corresponding to around 9,500 DKK after tax (100 DKK ≈ 13 Euro ≈ 20 USD at the time). The total payout after tax was around 23 billion DKK corresponding to 1.4 percent of GDP.

For several reasons one should expect the consumption reactions to the potential and actual payouts to be very small if consumers were unconstrained intertemporal optimizers. First, people had their own money paid out. It was thus very transparent that more money now would mean less money later on, a transparency that was further supported by a letter sent from the SP administrator to all SP savers detailing the possibilities and consequences of keeping or liquidating one's account. Thus, for people who were not liquidity-constrained, Ricardian equivalence should hold even at an individual level and not depend on complicated anticipated tax reactions. Second, even people who mistakenly considered their payouts as true income would smooth out the resulting additional consumption over many periods if they were unconstrained optimizers.

In an article, the Danish economists Claus Thustrup Kreiner, David Dreyer Lassen and Søren Leth-Petersen have studied the effects of the SP payouts.[21] They combined survey data on the spending reactions of people to the payouts (asking people directly about this) with register data on the marginal interest rates on consumption loans of people. The idea is that larger interest rates correspond to tighter credit rationing.

One main finding of the authors was that around one third of people reported not to have changed their consumption at all in consequence of their payouts (but used this to repay debt or accumulate other assets including pension savings), while around two thirds reported to have increased their consumptions with the full amount of the payout. Very few people (4 percent) reported to have increased their consumption with in-between amounts. The average propensity to consume the payouts were 65 percent. This suggests that in the real world there are both true, unconstrained intertemporal optimizers and other consumers who behave much more 'from hand to mouth' and the latter constitute a sufficiently large fraction of all consumers to ensure that the total consumption response to a once and for all payout (of own money) is quite considerable.

A second finding in the article suggests that a large part of the consumption reaction is due to consumers being credit-constrained and not just myopic. Figure 16.9, which is reproduced from the discussion paper version of the article, shows a very clear and positive correlation between the marginal interest rate of people and their propensity to consume their SP payouts. This strongly supports the idea that the more credit-constrained consumers are, the more their consumption will be determined by directly available means, which in the absence of extraordinary payouts will typically be the current disposable income.

If we envisage that in the real world there is always a mix of credit-constrained (or rule-of-thumb or hand-to-mouth) consumers and true, unconstrained intertemporal

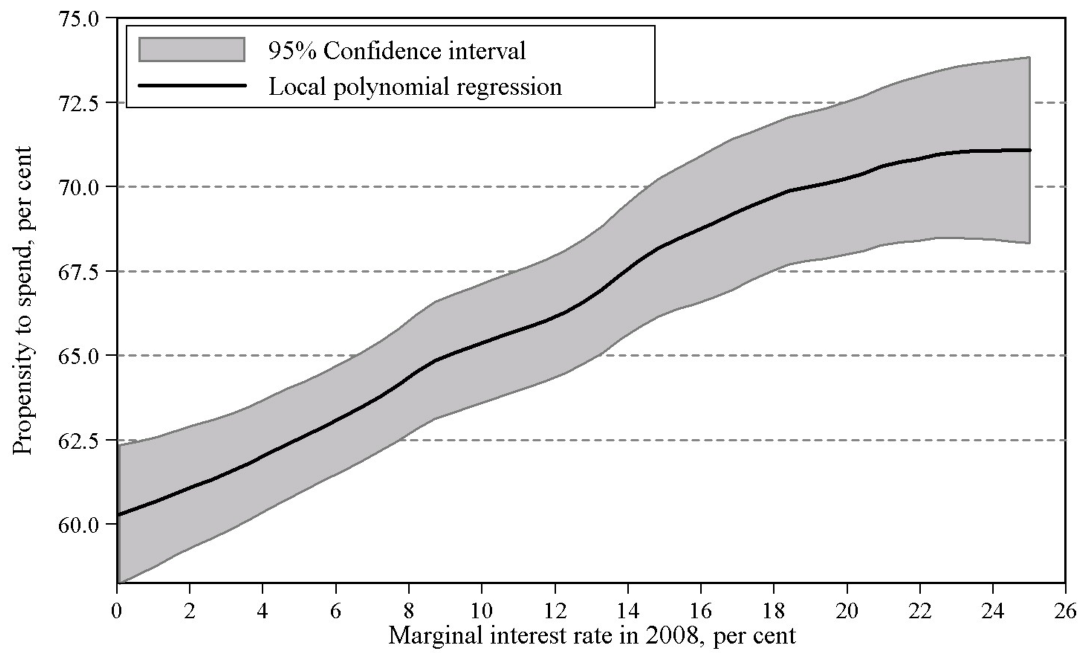
[21] Claus Thustrup Kreiner, David Dreyer Lassen and Søren Leth-Petersen, 'Liquidity Constraint Tightness and Consumer Responses to Fiscal Stimulus Policy', American Economic Journal: Economic Policy, 11(1) 2019, pp. 351–379.

optimizers, we may sum up our theory of consumption in the following generalized consumption function $C(\cdot)$, where $\left(Y_2^d\right)^e$ is expected future disposable income:

$$C_1 = C\left(\underset{+}{Y_1^d}, \underset{+}{(Y_2^d)^e}, \underset{(-)}{r}, \underset{+}{V_1}\right). \tag{32}$$

The signs below the variables in (32) indicate the signs of the partial derivatives implied by our theory. The brackets around the sign of the derivative with respect to the interest rate indicates that the effect is in principle uncertain, but likely to be negative. Due to credit-constrained or myopic consumers, we expect the marginal propensity consume current income, $\partial C / \partial Y_1^d$, to be much larger than it would be according to unconstrained intertemporal optimization and also much larger than $\partial C / \partial (Y_2^d)^e$.

Figure 16.9 Household propensity to spend and the marginal interest rate in Denmark



Note: The figure shows the result from a local polynomial regression of the spending share of the SP payout on the marginal interest rate. The spending share is obtained from survey questions. The marginal interest rate denotes the interest on marginal liquidity and is computed from a register data on all loan and deposit accounts of the household obtained from the Danish Tax Agency. Observations: 5038.
Source: Claus Thustrup Kreiner, David Dreyer Lassen and Søren Leth-Petersen: 'Consumer Responses to Fiscal Stimulus Policy and Households' Cost of Liquidity', CEPR Discussion Paper Series No. 9161, 2012.

Summing up:

THE CONSUMPTION FUNCTION

Aggregate private consumption varies positively with current disposable income and with expected future disposable income as well as with the current level of private wealth. Because of offsetting income and substitution effects, we cannot say for sure whether a rise in the real interest rate will raise or lower consumption, but once we recognize that the value of financial and human wealth varies negatively with the interest rate, it becomes likely that a higher real interest rate will cause private consumption to fall.

Notice how expectations feed into consumption. When consumers are optimistic about the future, they will expect a high rate of income growth g^e and hence a relatively high level of future disposable income, $(Y_2^d)^e = (1+g^e)Y_1^d$. This will have a direct positive effect on current consumption. To stress the dependence of current consumption on the expected income growth (or decline) in the economy, we may write our generalized consumption function in the following way, now dropping explicit reference to the time period:

$$C = C\left(\underset{+}{Y^d}, \underset{(-)}{r}, \underset{+}{g^e}, \underset{+}{V}\right). \qquad (32')$$

In the next chapter, we shall see how our consumption function (32) or (32') combined with our investment function from Chapter 15 leads to a theory of aggregate demand for goods and services.

Summary

1. **1.** Private consumption is by far the largest component of the aggregate demand for goods and services. A satisfactory theory of private consumption must explain the paradoxical stylized facts that the average propensity to consume is a decreasing function of disposable income in microeconomic cross-section data, whereas it is roughly constant in long-run macroeconomic time series data.
2. **2.** The properties of the aggregate consumption function may be derived by studying the behaviour of a representative consumer who must allocate consumption optimally over time, subject to an intertemporal budget constraint. With perfect capital markets, this constraint implies that the present value of lifetime consumption cannot exceed the sum of the consumer's initial financial and human wealth. Human wealth is the present value of current and future disposable labour income.
3. **3.** In the consumer's optimum, the marginal rate of substitution between present and future consumption equals the relative price of future consumption, given by one plus the real rate of interest. When disposable income varies over time, the optimizing consumer will want to smooth the time path of consumption relative to the time path of income. There is evidence that such consumption smoothing does indeed take place.
4. **4.** Given the assumption of perfect capital markets, the optimal intertemporal allocation of consumption implies that current consumption is proportional to total current wealth (the sum of financial and human wealth). The propensity to consume current wealth depends on the real interest rate, the consumer's rate of time preference, and on the consumer's intertemporal elasticity of substitution.
5. **5.** A rise in the real interest rate will have offsetting income and substitution effects on the propensity to consume current wealth. If the intertemporal substitution elasticity is greater than 1, reflecting a strong willingness of consumers to substitute future for present consumption, the substitution effect will dominate. A rise in the real interest rate will then reduce the propensity to consume current wealth. The opposite will happen if the intertemporal substitution elasticity is smaller than 1. Even if a rise in the interest rate does not significantly affect the propensity to consume a given amount of wealth, it may reduce current consumption by reducing the present value of future labour income, that is, by reducing human wealth, and by reducing the market value of the consumer's stockholdings and housing wealth.
6. **6.** For an optimizing consumer, the average propensity to consume current *income* will vary positively with the ratio of current financial wealth to current income. There is strong empirical evidence that such a positive relationship exists. The rough long-run constancy of the average propensity to consume observed in macroeconomic time series data may be explained by the fact that the wealth–income ratio, the

growth rate of real income, and the real rate of interest tend to be roughly constant over the long run.

7. The negative correlation between income and the average propensity to consume observed in microeconomic cross-section data may be explained by the fact that, in any given period, many consumers will have a relatively low current income relative to their average income over the life cycle. Such consumers will therefore have a high level of current consumption relative to their current income, because they expect higher future incomes, or because they have accumulated wealth by saving out of higher past incomes.

8. A tax cut which is expected to be permanent will have a stronger positive impact on current consumption than a tax cut which is believed to be temporary. When the real interest rate equals the rate of time preference, a permanent tax cut will induce a corresponding rise in current consumption.

9. The government's intertemporal budget constraint implies that the present value of current and future taxes must be sufficient to cover the present value of current and future government spending plus the initial stock of government debt. For given levels of current and future government spending, a tax cut today must therefore be offset by a future tax increase of equal present value.

10. If consumers have sophisticated (rational) expectations, they will realize the implications of the intertemporal government budget constraint. This means that a cut in current (lump sum) taxes, which is not accompanied by a cut in present or future public spending, will have no effect on private consumption: consumers will save all of the current tax cut to be able to finance the higher future taxes without having to reduce future consumption. This equivalence between tax finance and debt finance of current public spending is referred to as Ricardian equivalence.

11. In practice, consumers are unlikely to save the full amount of a current tax cut, even if they realize that lower taxes today must imply higher taxes in the future. First, consumers may believe that some of the future taxes will be levied on future generations. Second, some consumers may be credit-constrained. A switch from current to future taxes will help these individuals to achieve a desired rise in current consumption at the expense of future consumption. The use of redistributive and distortionary taxes also means that a switch from tax finance to debt finance of current public spending is likely to have real effects on current consumption and labour supply.

12. The theory of private consumption is summarized in the generalized consumption function, which states that aggregate consumption is an increasing function of current disposable income, of expected future disposable income, and of current financial wealth. A rise in the real interest rate has a theoretically ambiguous effect, although it is likely to reduce current consumption due to its negative impact on human and financial wealth.

Exercises

Exercise 1. Important concepts and results in the theory of private consumption

1. The theory of consumption presented in this chapter says that private consumption is proportional to private wealth. Explain the definition of wealth, including the concept of 'human wealth'. Explain the factors which determine the propensity to consume current wealth.
2. Explain the assumptions underlying our theory of consumption. Which assumptions do you find most important and problematic?
3. Empirical evidence suggests that many consumers tend to spend all of their current disposable income immediately. Is this irrational? Discuss.
4. Explain why we cannot say whether an increase in the real (after-tax) interest rate will raise or lower the propensity to consume. Explain the concept of the intertemporal elasticity of substitution in consumption and its role in determining the effect of a change in the interest rate on consumption. On balance, what do you consider the most likely effect of a rise in the real interest rate on consumption (positive or negative?). Justify your answer.
5. Explain the conflicting evidence on the relationship between the average propensity to consume and disposable income found in microeconomic cross-section data and in macroeconomic time series data. Explain how the theory of consumption presented in this chapter helps to resolve the apparent inconsistency between the two types of evidence.
6. Explain the consumer's intertemporal budget constraint and the government's intertemporal budget constraint and how the two constraints are linked, assuming that the representative consumer and the government have the same two-period planning horizon. (Hint: try to eliminate the consumer's lump sum tax payments from his budget constraint by using the government's intertemporal budget constraint.) What does your finding imply for the relationship between private and public spending? Explain and discuss.

Exercise 2. The consumption function with endogenous labour supply

In the main text of this chapter we made the simplifying assumption that the consumer's labour income is exogenously given to him, say, because wage rates as well as working hours are regulated by collective bargaining agreements. This exercise invites you to derive the consumption function when the representative consumer may freely choose his preferred number of working hours, h, while still taking the real wage rate w as given by the market.

To simplify, we will assume that the consumer is retired from the labour market during the second period of his life, earning labour income only during the first period. In real terms, his budget constraints for the two periods of life may then be written as:

$$V_2 = (1 + r)[V_1 + w(1 - \tau)h - C_1] \tag{33}$$

$$C_2 = V_2 \tag{34}$$

where τ is a proportional labour income tax so that $w(1 - \tau)h$ is disposable labour income. For concreteness, we assume that the consumer's lifetime utility is given by the function:

$$U = \ln C_1 + \beta \ln(1 - h) + \frac{\ln C_2}{1 + \phi} \tag{35}$$

where ϕ is the rate of time preference. In (35) we have assumed that the total time available to the consumer in the first period is equal to 1. Hence the magnitude $1 - h$ is the amount of leisure enjoyed during period 1, and β is a parameter indicating the strength of the consumer's preference for leisure. Furthermore, the after-tax real wage rate $w(1 - \tau)$ may be called the consumer's *potential* labour income (or the market value of his time endowment), since it measures the amount of wage income he could earn if he worked all the time. Note that $w(1 - \tau)$ may also be seen as the 'price' of leisure, since it measures the net income forgone by the consumer if he chooses to consume one more unit of leisure.

1. Derive and interpret the consumer's intertemporal budget constraint. Does lifetime consumption depend on actual or on potential labour income? (Hint: for later purposes it may be useful for you to rewrite the lifetime budget constraint so that the value of the consumption of leisure, $w(1 - \tau)(1 - h)$, appears on the left-hand side as part of total lifetime consumption.)

2. Rewrite the utility function (35) and the intertemporal budget constraint by introducing the variable $F \equiv 1 - h$, where F is leisure, and use the intertemporal budget constraint to eliminate C_2 from the utility function. Then show that the utility-maximizing choices of C_1 and F will be given by:

$$C_1 = \tilde{\theta}[V_1 + w(1 - \tau)], \qquad \tilde{\theta} \equiv \frac{1 + \phi}{1 + (1 + \phi)(1 + \beta)}, \tag{36}$$

$$F = \frac{\beta C_1}{w(1 - \tau)}. \tag{37}$$

Compare your consumption function to the consumption function (17) derived in the main text of the chapter and comment on the similarities and differences.

3. Derive an expression for the consumer's optimal labour supply h during period 1 (recall that $h = 1 - F$). How does labour supply depend on the labour income tax rate τ? Give an intuitive explanation.

4. Suppose now that the consumer's desired working hours h exceed the amount of hours $\bar{h}$ that he is actually allowed to work as a member of his trade union. In that case he will work the maximum hours allowed, and his disposable labour income in period 1 will be $w(1-\tau)\bar{h}$. Derive his optimal period 1 consumption level in this case and compare with the consumption function derived in Question 2. Comment on the difference.

Exercise 3. Consumption demand and credit constraints

The consumption function is important for macroeconomic theory about the activity level in the short run and in particular, the propensity to consume out of current income is important for the Keynesian multiplier and the effects of stabilization policy that we come to in later chapters. This exercise asks you to analyse what the optimization-based consumption model considered in this chapter may say about these issues potentially with credit constraints.

We consider the model of Section 16.2. The consumptions of period 1 and 2 are $C_1 \geq 0$ and $C_2 \geq 0$, respectively. For notational ease, we call the consumer's disposable income in the two periods Y_1 and Y_2, respectively, both assumed to be strictly positive. For simplicity we assume that $V_1 = 0$. The consumer's saving in period 1 is $S = Y_1 - C_1$. The budget constraints of period 1 and 2, respectively, are thus: $C_1 + S = Y_1$ and $C_2 = (1+r)S + Y_2$, respectively, where $r > 0$ is the real interest rate. Saving in period 1 may be negative, $S < 0$, but the consumer must plan so that he or she can pay back all debt at the end of period 2, that is, such that $C_2 \geq 0$. Intertemporal utility is $U = u(C_1) + u(C_2)/(1+\phi)$, where $\phi > 0$, just as in the chapter. It is assumed that $u' > 0$, $u'' < 0$, $u'(C) \to \infty$ for $C \to 0$, and $u'(C) \to 0$ for $C \to \infty$.

1. Explain what kind of behavior on credit markets that $S < 0$ corresponds to. Show that the constraint $C_2 \geq 0$ is equivalent to $S \geq -Y_2/(1+r)$ and give an intuitive interpretation of this.
2. Show that the consolidated budget constraint is

$$C_1 + \frac{C_2}{1+r} = H, \quad H \equiv Y_1 + \frac{Y_2}{1+r},$$

describe its content and give an interpretation of H.

3. Explain verbally and intuitively that an optimal consumption plan (C_1, C_2), that is, one that maximizes U given the consolidated budget constraint, must fulfil $C_1 > 0$

and $C_2 > 0$ and the Keynes-Ramsey rule:

$$u'(C_1) = \frac{1+r}{1+\phi} u'(C_2).$$

4. In a diagram with C_1 and C_2 along the axes: Illustrate the consolidated budget constraint given H, sketch some indifference curves for U and indicate the optimal consumption plan. For given H, illustrate by indicating alternative Y_1 on the C_1 axis a situation where $S > 0$ as well as a situation where $S < 0$ in the optimum. Explain and illustrate, still for a given H, how the budget constraint is affected by an increase in the real interest rate.

In all of the following assume that $u(C) = \ln C$.

5. Show that the optimal consumption plan is:

$$C_1^* = \frac{1+\phi}{2+\phi} H, \quad C_2^* = \frac{1+r}{2+\phi} H.$$

Describe this with respect to consumption smoothing. Describe how C_1^* depends on the interest rate, first for a given H, and then for a situation where only Y_1 and Y_2 are given. Comment.

6. Show that for the optimal consumption plan saving is:

$$S^* = \frac{1}{2+\phi} Y_1 - \frac{1+\phi}{2+\phi} \frac{Y_2}{1+r}.$$

Show that this fulfils the requirement $S \geq -Y_2/(1+r)$ of Question 1 above. Show further that $S^* \geq 0$ if and only if:

$$\frac{1+\phi}{1+r} \leq \frac{Y_1}{Y_2}. \qquad (*)$$

Describe and explain intuitively what creates positive and negative optimal saving, respectively.

From now we assume that the consumer is also subject to the credit constraint $S \geq 0$.

7. In a figure like that of Question 4, illustrate (again) a situation where the (not credit-constrained) optimal saving S^* is strictly negative, $S^* < 0$. Illustrate also the credit constraint $S \geq 0$ in the figure as a modification to the budget set. Show on basis of the figure and explain intuitively that for the consumer considered here, the optimal consumption plan is:

$$C_1^c = Y_1, \quad C_2^c = Y_2.$$

8. State the propensity to consume out of current income, $\partial C_1^* / \partial Y_1$ for a consumer who is not credit-constrained (either because he or she is not subject to the constraint $S \geq 0$ or because $S^* > 0$). Evaluate the size of this on basis of a period length of one year for which a ϕ around 0.03 can be reasonable. What would a realistic size of the current propensity to consume be in a framework with many periods do you think? What is the current propensity to consume for a consumer who is credit constrained (who is subject to the constraint $S \geq 0$ *and* for whom (*) is not fulfilled). Comment on the difference between these propensities to consume current income.

Envisage that the economy has two types of consumers. All have the same disposable incomes Y_1 and Y_2, all are confronted with the same real interest rate r, and all are subject to the credit constraint $S \geq 0$. The fraction λ of consumers (the patient) have a time preference rate ϕ^{low}, such that (*) is fulfilled with strict inequality, while the fraction $1-\lambda$ has a time preference rate ϕ^{high}, such that (*) is not fulfilled.

9. Express the average consumption in period 1 over all consumers, $\overline{C}_1$, as function of the average current income, the average expected future income and the real interest rate. What is the average marginal propensity to consume, $\partial \overline{C}_1 / \partial Y_1$? What could be a plausible value of this if one half or two thirds of consumers are credit-constrained?

Exercise 4. Fiscal policy and consumption with credit-constrained consumers

In Section 16.4 we saw that the Ricardian Equivalence Theorem rests on the assumption of perfect capital markets. In this exercise you are asked to analyse the effects of a switch from tax finance to debt finance of public consumption when some consumers are credit-constrained.

For concreteness, suppose that all consumers in the economy earn the same labour income and pay the same amount of taxes in each period, but that one group of consumers ('the poor') enters the economy with a zero level of initial wealth at the beginning of period 1, whereas the remaining consumers ('the rich') start out with a level of initial wealth equal to V_1. Moreover, suppose that disposable labour income in period 1 is so low that the poor would like to borrow during that period, but that the banks are afraid of lending them money because they cannot provide any collateral. In that case the poor will be credit-constrained during period 1, and the consumption of a poor person during that period, C_1^p, will then be given by the budget constraint:

$$C_1^p = Y_1^L - T_1 \tag{38}$$

A rich consumer does not face any borrowing constraint, and his optimal consumption in period 1, C_1^r, will therefore be given by the consumption function (16) from the main text, that is:

$$C_1^r = \theta\left(V_1 + Y_1^L - T_1 + \frac{Y_2^L - T_2}{1+r}\right), \quad 0 < \theta < 1 \tag{39}$$

Suppose that the total population size is equal to 1 (we can always normalize population size in this way by appropriate choice of our units of measurement). Suppose further that a fraction μ of the total population is 'poor' in the sense of having no initial wealth.

1. Derive the economy's aggregate consumption function for period 1, that is, derive an expression for total consumption $C_1 = \mu C_1^p + (1-\mu)C_1^r$. Derive an expression for the economy's marginal propensity to consume current disposable income, $\partial C_1 / \partial(Y_1^L - T_1)$. Compare this expression with the value of the marginal propensity to consume in an economy with no credit-constrained consumers. Explain the difference.

Suppose now that the government enacts a debt-financed reduction of current taxes T_1 by one unit, without cutting current or planned future public consumption. Assume further that the government and the private sector have the same planning horizons, and that the private sector understands that the tax cut today will have to be matched by a tax hike tomorrow of the same present value, due to the intertemporal government budget constraint. In other words, suppose that all consumers realize that:

$$dT_1 + \frac{dT_2}{1+r} = 0 \tag{40}$$

2. Derive the effect of the switch from tax finance to debt finance on aggregate private consumption in period 1. Compare with the situation with no credit-constrained consumers and explain the difference. Discuss whether the increase in public debt in period 1 will improve the lifetime welfare of consumers.

Instead of financing the tax cut in period 1 by debt, the government may finance the tax reduction by cutting public consumption. In the question below we will distinguish between two scenarios. In the first scenario the fall in public consumption is expected to be *temporary*, that is, $dG_1 = dT_1$ and $dG_2 = dT_2 = 0$. In the second scenario the cut in public consumption is expected to be *permanent* so that $dG_2 = dG_1 = dT_1 = dT_2$.

3. Derive the effect on current private consumption C_1 of a temporary tax cut financed by a temporary cut in public consumption. Compare this to the effect on C_1 of a permanent tax cut financed by a permanent cut in public consumption. (In the latter case you may assume that $r = \phi$.) Explain the difference between your expressions. Does it make any difference for the effects of temporary and permanent tax cuts whether or not consumers are credit-constrained?

Exercise 5. Ricardian Equivalence?

1. Explain the content of the Ricardian Equivalence Theorem and its assumptions. Discuss whether the theorem is likely to hold in practice.

2. Try to find data for private saving and public sector saving as ratios to GDP in your country and plot the two series against each other. (If you are a Danish student, try to update Figure 16.8 in the text.) Do the movements in the two time series seem consistent with the Ricardian Equivalence Theorem? Explain.

3. In many official statistics public sector saving is simply measured by the balance on the public sector budget (the negative of the budget deficit), defined as $T - C^g - I^g - rD$, where T is net taxes (taxes net of transfers), C^g is public consmption, I^g is public investment, and rD is the net interest payment on government debt. Discuss whether the ideal measure of public sector saving for the purpose of testing Ricardian Equivalence should include public investment, I^g. (Hint: if the budget deficit increases due to an increase in I^g, and if public investment yields a positive return, how will this affect the private sector's future tax liability?) Try to check if your empirical data for public sector savings include or exclude the public sector's net investment.

Exercise 6. Wage taxes versus consumption taxes versus wealth taxes

In the main text of the chapter we assumed for simplicity that taxes took the form of lump sum payments which were unrelated to the consumer's behaviour. Now we assume instead that the consumer must pay a proportional tax, τ^w, on his wage income, a proportional value-added tax, τ^c, on all his consumption expenditure, and potentially also a one-time tax, τ^v, on his initial wealth. This exercise asks you to consider some similarities and differences between these taxes.

We assume that the consumer's working hours are exogenously given to him and equal to 1 so that his total pre-tax labour income in period 1 is equal to the real wage rate w. We also assume that he is retired from the labour market during period 2. We begin with a situation without any taxes on wealth. The consumer budget constraints for the two periods of life are then given by:

$$V_2 = (1+r)[V_1 + (1-\tau^w)w - (1+\tau^c)C_1], \qquad 0 < \tau^w < 1, \qquad \tau^c \geq 0 \tag{41}$$

$$(1+\tau^c)C_2 = V_2 \tag{42}$$

1. Derive the consumer's intertemporal budget constraint and comment on your expression. From the consumer's perspective, what is the similarity and the difference between the wage tax and the consumption tax?

The consumer has the lifetime utility function:

$$U = \ln C_1 + \frac{\ln C_2}{1+\phi}, \qquad \phi > 0 \tag{43}$$

2. Show that the consumer's optimal consumption is given by:

$$C_1 = \left(\frac{1+\phi}{2+\phi}\right)\left[\frac{V_1 + (1-\tau^w)w}{1+\tau^c}\right], \tag{44}$$

$$C_2 = \left(\frac{1+r}{2+\phi}\right)\left[\frac{V_1 + (1-\tau^w)w}{1+\tau^c}\right]. \tag{45}$$

Comment on these expressions.

Suppose now that the government needs to raise more tax revenue to finance additional public spending. The government presents two alternative proposals in parliament. The first proposal implies that the consumption tax rate (the VAT) will be raised from 20 per cent to 25 per cent while the wage tax rate will be kept unchanged at 50 per cent. The second proposal implies that the consumption tax rate is maintained at 20 per cent, while the wage tax rate is raised from 50 per cent to 52 per cent. In addition, the second proposal includes a one-time proportional tax of 4 per cent on existing initial wealth V_1. The government stresses that this is a once-and-for-all wealth tax which will not be imposed on future wealth V_2.

3. Would consumers prefer one of the government's proposals to the other one? (Hint: how do the two alternative tax plans affect the consumer's total wealth, including his human wealth?) What difference does it make to your answer whether the wealth tax is a one-time levy or a permanent tax?

Chapter 17

Monetary policy and aggregate demand

Introduction

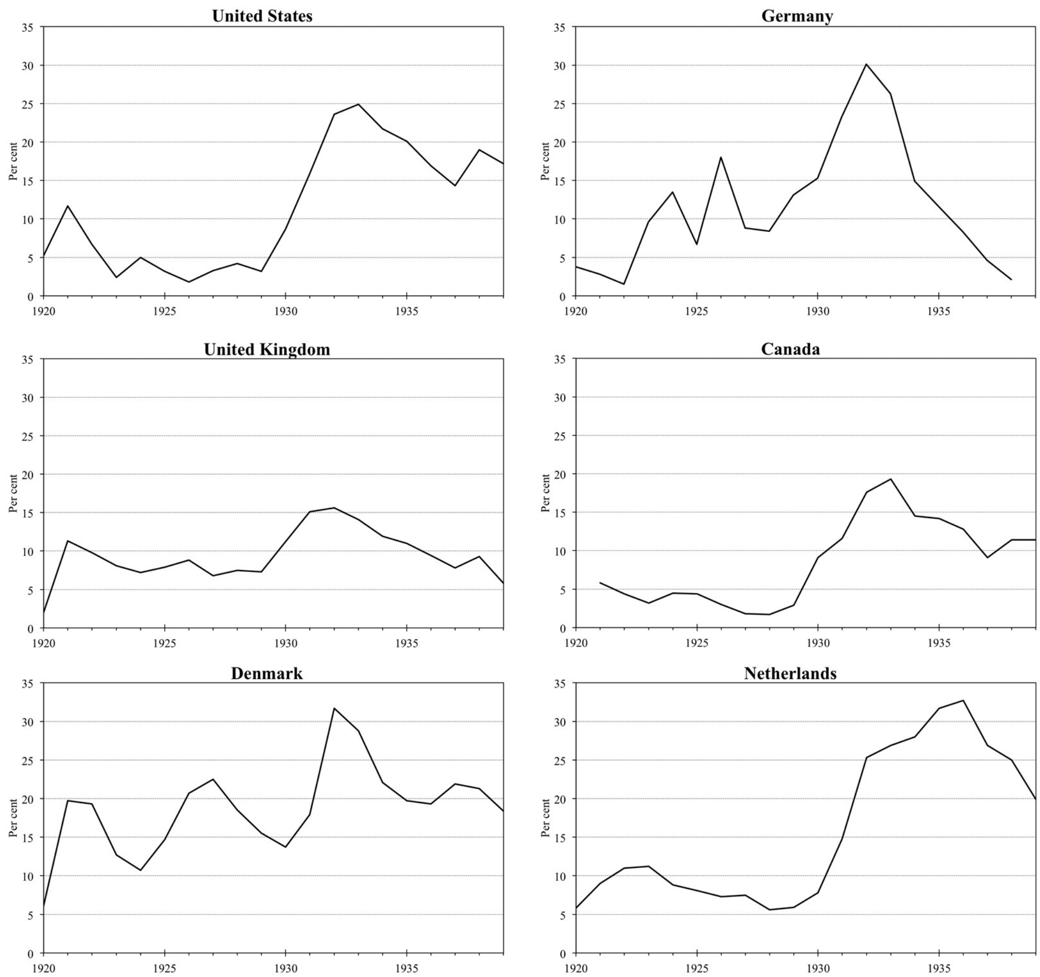
According to the classical macroeconomic theory which dominated economic thought before the Great Depression of the 1930s, the levels of total output and employment are predominantly determined from the economy's supply side and deviations between the actual and structural levels of output, employment etc. are small and short-lived. In the classical world, wages and prices adjust to ensure that the available supplies of labour and capital are utilized at their 'natural' rates determined by the structure of labour and product markets. In Chapter 1 we argued that this is a useful working assumption when we analyse the long-run economic phenomena which are the subject matter of the theory of economic growth, but in the short and medium term economic activity often deviates from its long-run growth trend. To understand these short-run macroeconomic fluctuations, we must explain why the aggregate demand for goods and services does not necessarily correspond to the aggregate supply of goods produced when all resources are utilized at their natural rates. Building on the previous two chapters, the present chapter therefore develops a theory of aggregate demand.

17.1 Keynes, the classics and the Great Depression

The classical economists did not literally claim that a capitalist market economy could never deviate from its natural rate of employment and output, but they did believe that if only market forces were allowed to work, such disturbances would be temporary and quite short-lived. The classical economists therefore saw no need for the government to engage in macroeconomic stabilization policy. In their view, the only role of monetary policy was to ensure price stability, and the task of fiscal policy was to avoid budget deficits which would crowd out private capital formation and thereby hamper economic growth. Winston Churchill, who was Secretary of the Treasury in Britain for several years during the 1920s, was in agreement with this classical view when he explained his approach to fiscal policy as follows: 'It is the orthodox Treasury dogma, steadfastly held, that whatever might be the political or social advantages, very little employment can, in fact, as a general rule, be created by state borrowing and state expenditure.'[1]

When the Great Depression of the 1930s struck the Western world, this classical laissez-faire position came under heavy attack. The Great Depression was an economic earthquake. Due to a catastrophic combination of negative shocks and macroeconomic policy failures, output in several countries fell by 25 – 30 per cent between 1929 and 1932 – 33, with disastrous consequences for employment as illustrated by Figure 17.1. For USA it took almost a decade for output to return to its pre-1929 peak!

Figure 17.1 Unemployment rates 1920 – 39



Note: The source for this figure is not the same as for Figure 10.1, which explains the differences.
Source: R. B. Mitchell, *International Historical Statistics: Europe 1750-1993*, Macmillan Press, 1998, and R. B. Mitchell, *International Historical Statistics: The Americas 1750-1988*, Macmillan Press, 1993.

Against this background the British economist John Maynard Keynes and several

[1] Quoted in Richard T. Froyen, *Macroeconomics – Theories and Policies*, 6th edition, Prentice-Hall, Inc., 1999, p. 68.

others attacked the classical view that resource utilization at natural rates is the normal state of affairs. Indeed, Keynes challenged the time-honoured definition of economics as 'the study of the allocation of scarce resources to satisfy competing ends'. Keynes' point was that resources are *not* always scarce; on the contrary they are often underutilized due to a lack of demand. In such circumstances the government will be able to raise total employment and output through a fiscal or monetary policy which stimulates aggregate demand. These ideas were laid out in 1936 in Keynes' famous book, *The General Theory of Employment, Interest and Money*. That book is often considered to mark the birth of modern macroeconomics, because it revolutionized the way economists thought about the problem of business cycles.

Today, most macroeconomists believe that economic activity in the short and medium run is determined by the interaction of aggregate demand and aggregate supply. In the long run the forces of aggregate supply stressed by the classical economists carry the day, but in the short run aggregate demand plays a key role in the determination of output and employment. As a step on the way to constructing a model of short-run macroeconomic fluctuations, we must therefore develop a theory of aggregate demand.

We have already seen that private investment as well as private consumption are influenced by the real rate of interest. In the long run the equilibrium real interest rate – the so-called natural rate of interest – is determined by the forces of productivity and thrift, as we explained in Chapter 3. However, in the short run monetary policy can have a significant impact on the real interest rate. Hence, much of this chapter will focus on the conduct of monetary policy and on how monetary policy affects aggregate demand.

We start our analysis by specifying the equilibrium condition for the goods market, drawing on the theory of consumption and investment developed in Chapters 15 and 16. We then move on to a study of the monetary sector and the conduct of monetary policy. Incorporating our specification of monetary policy into the equilibrium condition for the goods market, we then end up deriving a systematic link between the level of output and the rate of inflation, which must hold whenever the goods market clears. This link is called the aggregate demand curve, and it will be one of the two central building blocks of the short-run macroeconomic model we set up in Chapter 19.

17.2 The goods market and the IS curve

Goods market equilibrium

For the product market to clear, the aggregate demand for goods must be equal to total output, Y. In this chapter we will focus on a closed economy (we will consider the open economy in later chapters). Aggregate demand for goods then consists of the sum of real private consumption, C, real private investment, I, and real government demand for goods and services, G. Hence goods market equilibrium requires:

$$Y = C + I + G. \tag{1}$$

In Chapter 15 we saw that private investment behaviour can be summarized in an investment function of the form $I = I(Y, r, \varepsilon, K)$, where Y is current aggregate income and activity, r is the real interest rate, ε is an indicator of 'business confidence' capturing expected future activity, profitability and riskiness, and K is the predetermined capital stock existing at the beginning of the current period. For the purpose of short-run analysis, we may treat the predetermined capital stock as a constant and leave it out of our behavioural equations.[2] We may then write private investment demand as:

$$I = I(Y, r, \varepsilon), \quad I_Y \equiv \frac{\partial I}{\partial Y} > 0, \quad I_r \equiv \frac{\partial I}{\partial r} < 0, \quad I_\varepsilon \equiv \frac{\partial I}{\partial \varepsilon} > 0, \tag{2}$$

where the signs of the partial derivatives of the investment function follow from the theory developed in Chapter 15. Thus, investment increases with current output and with the state of confidence, whereas it decreases with the real interest rate. Note that the partial derivatives in (2) generally depend on Y, r and ε.

Our theory of private consumption presented in Chapter 16 ended with a consumption function $C = C(Y^d, r, g^e, V)$. We write this in the form $C = C(Y - T,\ r, g^e, V)$, where T denotes total net tax payments so that $Y - T$ is current disposable income, g^e is expected income growth and V is non-human wealth. Our analysis in Chapter 15 showed that the market value of non-human wealth is the present discounted value of expected future returns on the relevant assets. Hence, V should depend positively on the state of consumer expectations and negatively on the real interest rate r. We assume that consumer confidence equals the state of confidence of business firms, ε. It follows that V is determined by r and ε. We will use this dependence to eliminate V from the consumption function and let it be replaced by r and ε. Finally, we will assume that expected income growth g^e is well captured by the consumer (and business) confidence variable, ε. Our consumption function thus becomes:

$$C = C(Y - T, r, \varepsilon), \qquad 0 < C_Y \equiv \frac{\partial C}{\partial (Y - T)} < 1, \qquad C_\varepsilon \equiv \frac{\partial C}{\partial \varepsilon} > 0. \tag{3}$$

The signs of the partial derivatives were explained in Chapter 16. From that chapter we recall that the real interest rate has an ambiguous effect on consumption, due to offsetting income and substitution effects, although the negative impact of a higher interest rate on private wealth suggests that the net effect on consumption is likely to be negative. The analysis in Chapter 16 also implied that the marginal propensity to consume current income is generally less than 1, as we assume above.

Our variable T measures 'net taxes', defined as total tax payments minus total transfers from the government to private households. In practice, net tax payments vary positively with economic activity, so we will assume that

$$T = \tau Y, \qquad 0 < \tau < 1. \tag{4}$$

We shall refer to τ as 'the net tax rate'. Note that this parameter captures the fact that

[2] The dynamics of capital accumulation were dealt with in Book One. To include it here as well would complicate the formal analysis considerably, given that we also want to study the dynamics of output and inflation. We therefore leave the inclusion of capital stock adjustment in the economy's short-run dynamics for a more advanced macroeconomics course.

unemployment benefits and social assistance to the unemployed vary negatively with output and employment as well as the fact that the revenue from taxes on income and consumption varies positively with GDP. In an extended modern welfare state, the value of τ can therefore be quite high. For example, in Denmark it is estimated that τ is in the neighbourhood of 0.85. We assume that the values of τ and G are set such that, on average over the business cycle, the government balances its budget so that the government does not systematically accumulate or decumulate debt over time. This allows us to ignore complications arising from the dynamics of government debt accumulation. At the same time, the specification in (4) allows for the fact that public revenues may deviate from public spending in the short run when actual GDP deviates from its trend value.

Let us denote total private demand by $D \equiv C + I$. It then follows from (2) through (4) that the goods market equilibrium condition (1) may be stated in the form:

$$Y = D(Y, \tau, r, \varepsilon) + G. \tag{5}$$

Properties of the private demand function

We will now consider the signs and magnitudes of the partial derivatives of the private demand function $D(Y, \tau, r, \varepsilon)$.

Since $Y - T = Y(1 - \tau)$ and hence $D \equiv C + I = C(Y(1-\tau), r, \varepsilon) + I(Y, r, \varepsilon)$, it follows that: $D_Y \equiv \partial D / \partial Y = (1-\tau)C_Y + I_Y > 0$ and $D_\tau \equiv \partial D / \partial \tau = -C_Y \cdot Y < 0$. The derivative D_Y is the marginal private propensity to spend out of current income, defined as the increase in total private demand induced by a unit increase in (pre-tax) income. We will assume that this marginal spending propensity is less than 1. Thus:

$$0 < D_Y \equiv \frac{\partial D}{\partial Y} = (1-\tau)C_Y + I_Y < 1, \quad D_\tau \equiv \frac{\partial D}{\partial \tau} = -C_Y \cdot Y < 0. \tag{6}$$

The assumption that $D_Y < 1$ guarantees that the *Keynesian multiplier*, $\tilde{m} \equiv 1/(1 - D_y)$, is positive. Recall from your basic macroeconomics course that the Keynesian multiplier measures the total increase in aggregate demand for goods generated by a unit increase in some exogenous demand component. The Keynesian multiplier captures the phenomenon that as economic activity goes up, the resulting rise in output and income induces a further increase in private consumption and investment, which generates an additional rise in output and income that in turn causes a new round of private spending increase, and so on. Below we shall return to the role played by the Keynesian multiplier in our theory of aggregate demand.

The effect of a rise in the real interest rate on private demand is $D_r \equiv \partial D/\partial r \equiv C_r + I_r$. The derivative D_r measures the effect of a rise in the real interest rate on the private sector savings surplus. The private sector savings surplus is defined as $SS \equiv S - I$, where private saving is given by $S \equiv Y - T - C$. Hence we have $\partial SS/\partial r = -C_r - I_r \equiv -D_r$. There is strong empirical evidence that a higher real interest rate raises the private sector savings surplus. Even though economic theory does not unambiguously determine the sign of the derivative C_r, we may therefore safely assume that:

$$D_r \equiv \frac{\partial D}{\partial r} \equiv C_r + I_r < 0, \qquad C_r \equiv \partial C / \partial r. \tag{7}$$

Finally, we see from (2) and (3) that the effect on private demand of higher consumer and business confidence is:

$$D_\varepsilon \equiv \frac{\partial D}{\partial \varepsilon} \equiv C_\varepsilon + I_\varepsilon > 0. \tag{8}$$

Restating the condition for goods market equilibrium: the IS curve

In business cycle analysis we study the deviations in the economic variables from their structural levels that will prevail in a long-run equilibrium. We will denote structural levels by a bar over the variable in question, so that $\overline{Y}$ is structural output, and $\overline{\tau}, \overline{r}, \overline{\varepsilon}$ and $\overline{G}$ are the trend values of τ, r, ε and G, respectively. We can think of $\overline{Y}$ as the GDP level along a growth path as studied in Book One, and we can think of $\overline{\varepsilon}$ as the state of confidence in a 'neutral' business cycle situation where the economy is on its long-run trend and expectations are neither optimistic nor pessimistic. Furthermore, we can interpret $\overline{\tau}$ and $\overline{G}$ as the net tax rate and government spending observed when the economy is on its structural growth path where there is no reason for stabilization policy, while $\tau - \overline{\tau} \neq 0$ and $G - \overline{G} \neq 0$ will represent impulses to demand from fiscal stabilization policy. Finally, we can think of the long-run equilibrium real interest rate $\overline{r}$ as the Wicksellian 'natural' rate of interest discussed in Section 3.3 of Chapter 3.

It will be convenient to rewrite the goods market equilibrium condition directly in terms of the deviations between actual and structural levels that interest us. From (5) we have for actual values: $Y = D\left(Y, \tau, r, \varepsilon\right) + G$. Equivalently, for the structural values along the economy's long-run growth trend we must have: $\overline{Y} = D(\overline{Y}, \overline{\tau}, \overline{r}, \overline{\varepsilon}) + \overline{G}$. A first-order linear approximation of the deviation between actual and structural current output is then:

$$Y - \overline{Y} = D_Y \cdot (Y - \overline{Y}) + D_\tau \cdot (\tau - \overline{\tau}) + D_r \cdot (r - \overline{r}) + D_\varepsilon \cdot (\varepsilon - \overline{\varepsilon}) + G - \overline{G} \iff$$

$$Y - \overline{Y} = \tilde{m} \cdot [D_r(r - \overline{r}) + D_\varepsilon(\varepsilon - \overline{\varepsilon}) + G - \overline{G} + D_\tau(\tau - \overline{\tau})], \tag{9}$$

$$\tilde{m} \equiv \frac{1}{1 - D_y} \equiv \frac{1}{1 - (1 - \tau)C_Y - I_Y}.$$

Our next step is to rewrite (9) in terms of relative changes in Y, G and ε:

$$\frac{Y - \overline{Y}}{\overline{Y}} = \tilde{m} \cdot \left[\frac{D_r}{\overline{Y}}(r - \overline{r}) + \frac{\overline{\varepsilon} D_\varepsilon}{\overline{Y}} \frac{\varepsilon - \overline{\varepsilon}}{\overline{\varepsilon}} + \frac{\overline{G}}{\overline{Y}} \frac{G - \overline{G}}{\overline{G}} + \frac{D_\tau}{\overline{Y}}(\tau - \overline{\tau}) \right]. \tag{10}$$

We will consider the coefficients in the square bracket, $D_r / \overline{Y}$, $\overline{\varepsilon} D_\varepsilon / \overline{Y}$, $\overline{G} / \overline{Y}$

and $D_\tau / \overline{Y}$ as constants, which is natural for an underlying long run growth path.[3]

In the final step, we use the fact that the change in the log of some variable is approximately equal to the relative change in that variable. Defining

$$y \equiv \ln Y, \quad \overline{y} \equiv \ln \overline{Y}, \quad g \equiv \ln G, \quad \overline{g} \equiv \ln \overline{G}, \tag{11}$$

we may then write (10) in the form:

$$y - \overline{y} = \alpha_1(g - \overline{g}) - \alpha_2(r - \overline{r}) - \alpha_3(\tau - \overline{\tau}) + v, \tag{12}$$

where

$$\alpha_1 \equiv \tilde{m}\frac{\overline{G}}{\overline{Y}} > 0, \quad \alpha_2 \equiv -\tilde{m}\frac{D_r}{\overline{Y}} > 0, \quad \alpha_3 \equiv -\tilde{m}\frac{D_\tau}{\overline{Y}} > 0, \quad v \equiv \tilde{m}\left(\frac{\overline{\varepsilon} D_\varepsilon}{\overline{Y}}\right)(\ln \varepsilon - \ln \overline{\varepsilon}). \tag{13}$$

Equation (12) says that the percentage deviation of output from trend (the output gap) can be approximated by a linear function of the percentage deviations of G and ε from their trend values and of the absolute deviations of r and τ from their trend values. Of course, (12) is just a particular way of stating that the aggregate demand for goods varies negatively with the real interest rate, positively with government spending, negatively with the net tax rate and positively with confidence in the future. The interpretation of v is that this is a 'confidence shock' to demand.

Keeping g, τ and v fixed, Equation (12) implies a negative relationship between the real interest rate and the (demand for) GDP, or between the interest rate gap, $r - \overline{r}$, and the output gap, $y - \overline{y}$. This is the famous 'IS curve'.

Notice also the role played by the *Keynesian multiplier* $\tilde{m} \equiv 1/(1 - D_Y)$ in the definitions of the coefficients α_1 and α_2 given in (13). For example, if government consumption rises by one unit, the immediate impact is a net increase in aggregate demand equal to 1, but when the Keynesian multiplier effect is accounted for, the total increase in demand adds up to $\tilde{m} > 1$. Therefore, if public consumption increases by 1 per cent, the resulting percentage increase in total demand will be $\tilde{m}(\overline{G}/\overline{Y})$, given that the initial ratio of public consumption to total output is $\overline{G}/\overline{Y}$. This explains the coefficient α_1 on the percentage increase in government consumption, $g - \overline{g}$, in (12). Similarly, if the real interest rate goes up by one percentage point, the resulting immediate *percentage* change in total demand is $D_r/\overline{Y}$, which is negative since $D_r < 0$. The immediate percentage fall is thus $-D_r/\overline{Y}$. When this initial fall in demand is magnified by the Keynesian multiplier, the total percentage fall in demand adds up to $-\tilde{m}(D_r/\overline{Y})$, in

[3] For instance, to justify the assumption that $D_r/\overline{Y}$ remains roughly constant despite the fact that $\overline{Y}$ is growing over time, we must assume that the derivative D_r takes the form $D_r = f(r)\overline{Y}, f'(r) < 0$. This is equivalent to assuming that the investment *ratio*, I/Y, will stay constant as long as the real interest rate stays constant. This assumption is warranted by our analysis of economic growth in Book One where we found that once the real interest rate has settled down to its long-run equilibrium value, the level of investment tends to grow at the same rate as output.

accordance with the definition of α_2 in (13). Thus the familiar Keynesian multiplier theory is built into our theory of aggregate demand. Note from the definition of $\tilde{m}$ in (9) that the presence of the net tax rate τ reduces the size of the multiplier, thereby reducing the repercussion effects of shocks to aggregate demand such as changes in ε or G. Hence the net tax rate is a so-called *automatic stabilizer* which helps to reduce the fluctuations in aggregate demand even when the government does not actively change its fiscal policy. In a modern welfare state with high marginal tax rates and generous unemployment benefits (resulting in a high value of τ), the automatic responses of taxes and transfers to changes in economic activity may therefore play an important role in reducing the amplitude of business fluctuations.

Summing up our main findings so far, we have:

AGGREGATE DEMAND, GOODS MARKET EQUILIBRIUM AND THE IS CURVE

The aggregate demand for goods and services varies positively with current income, with public consumption and investment and with consumer and business confidence, whereas it varies negatively with the real rate of interest and the net tax rate. In goods market equilibrium, any shock to aggregate demand is magnified by the Keynesian multiplier process, but the presence of automatic stabilizers such as income taxes and unemployment insurance reduces the size of the multiplier. For given fiscal policy and confidence in the future, the goods market equilibrium implies a negative impact of the real interest rate on the total demand for output, a relationship known as the IS curve.

Equation (12) is our preliminary description of the economy's aggregate demand side. Below we will show that (12) implies a systematic link between output and inflation, once one allows for the way that monetary authorities typically conduct monetary policy. To understand this link, we must study the relationship between inflation and the real interest rate, and that requires taking a closer look at the money market and the behaviour of central banks.

17.3 The money market and monetary policy

The money market and the IS-LM model revisited

From your basic macroeconomics course you will recall that nominal money demand is often formulated as a function, $M^d = P \cdot L(Y, i)$, $L_Y \equiv \partial L / \partial Y > 0$, $L_i \equiv \partial L / \partial i < 0$, where $L(Y, i)$ is the demand for real money balances, i is the nominal interest rate, and P is the price level. Real money demand varies positively with income, since a rise in income leads to more transactions, which in turn requires more liquidity. At the same time money demand varies negatively with the nominal interest rate, because a higher interest rate raises the opportunity cost of holding money rather than interest-bearing assets, inducing agents to economize on their money balances to be able to invest a larger share of their wealth in interest-bearing financial instruments.

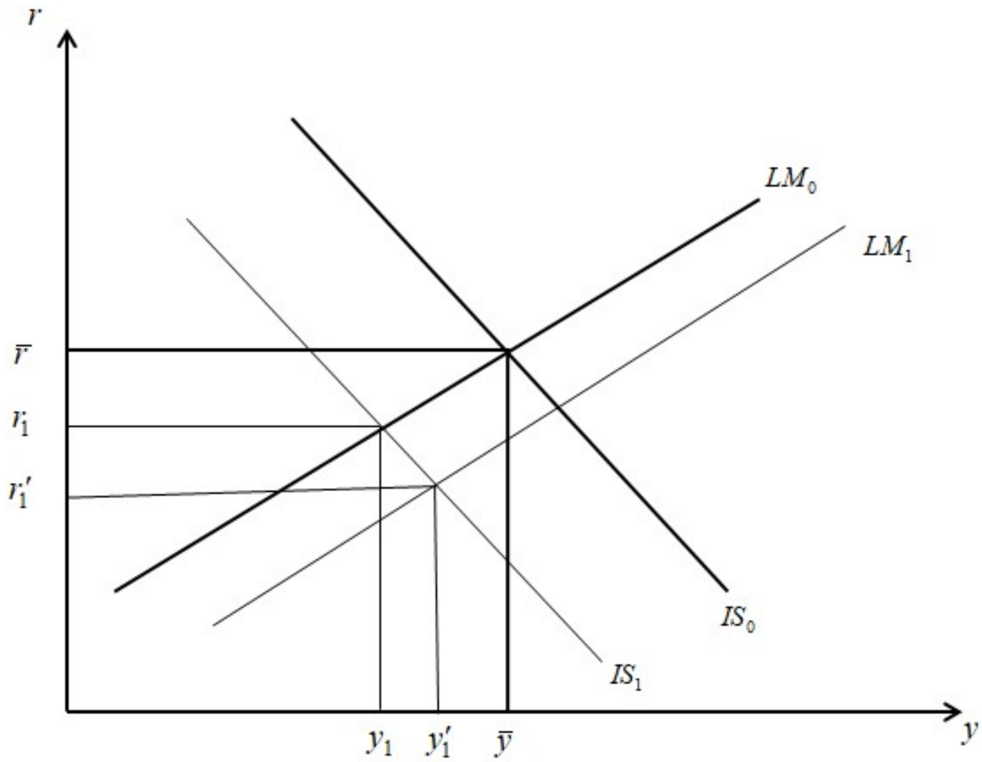
If we denote the nominal money supply by M, the equilibrium condition for the money market is:

$$\frac{M}{P} = L(Y, i), \tag{14}$$

where the left-hand side is the supply of real money balances, which must be equal to real money demand in equilibrium.

Equation (14) lies behind the LM curve of the famous IS-LM model. The IS curve is essentially (12) above. In Figure 17.2 the curve labelled IS_0 illustrates (12) for a situation where the fiscal policy impulses and the confidence shock are zero, $g - \overline{g} = \tau - \overline{\tau} = v = 0$, so the decreasing curve passes through $(\overline{y}, \overline{r})$. With an appropriate rescaling of the function L, we may write (14) as $M / P = L(y, r + \pi^e)$, where π^e is the expected inflation rate so that $i = r + \pi^e$. Considering the expected inflation rate, the money supply and the price level as given, this equation gives an upward sloping relationship between y and r, the LM curve: a higher y implies higher money demand, which pulls the interest rate upwards for a given real money supply and expected inflation. In Figure 17.2, LM_0 is the LM curve in a situation where M, P and π^e are such that this curve passes through $(\overline{y}, \overline{r})$. When we take π^e, M and P (and hence the real money supply M/P) as given, the full IS-LM model determines the real variables Y and r. Equilibrium according to the IS-LM model is at the intersection between the IS curve and the LM curve. For IS_0 and LM_0 this happens at $(\overline{y}, \overline{r})$, so here the economy is in long run equilibrium. Now suppose that a negative confidence shock $v < 0$ occurs, shifting the IS curve to the left by the amount $|v|$ as illustrated by IS_1 in the figure. If the real interest rate did not change, log output would thus fall by $|v|$, but this is not what happens in the IS-LM-model. In this model, the (real) money supply is controlled by the monetary authority (the central bank). If the central bank keeps the money supply unchanged, the LM curve will also be unchanged and the new equilibrium will be at (y_1, r_1) in the figure, so that output has fallen, but not by the full amount $|v|$, and the real interest has fallen in consequence of the reduced transactions demand for money. If the central bank still thinks that the fall in output is too large, it can choose to increase the money supply, thereby, for given P, shifting the LM curve to the right as illustrated by LM_1 in the figure. The increase in the money supply could result from market operations where the central bank buys bonds in the financial markets. It could also result from the central bank lowering the short-term interest rates at which private banks can borrow in the central bank, thus stimulating private banks to provide more liquidity to the rest of the economy. In any case, the IS-LM model assumes that the central bank can control the money supply. In the situation considered, we see that the shift in the LM curve will move the equilibrium to (y_1', r_1'), where the fall in output is reduced compared to (y_1, r_1).

Figure 17.2 The IS-LM model



We emphasize two features of this IS-LM analysis. First, even if the central bank does not change its policy variable, the money supply, the interest rate falls, implying that output falls by less than the size of the demand shock. Second, in the real world central banks are not much preoccupied with the money supply. Their policy instruments are most often various interest rates as we will explain below. However, in our example, what the central bank obtains by changing the money supply is really the interest rate, or more precisely the point on the relevant IS curve, that it thereby creates. We could turn things upside down and consider the interest rate an exogenous variable and the money supply as endogenous (the money supply required to achieve the interest rate desired given other circumstances).

We will follow up on both of these themes in the remainder of this chapter, but for the moment we will maintain the assumption that the central bank seeks to control the money supply since this will give us a nice transition to a more realistic situation where the central bank is directly preoccupied with interest rates. For concreteness, we will assume that the demand for real money balances can be approximated by a function of the form:

$$L(Y,i) = kY^{\eta}e^{-\beta i}, \quad k>0, \quad \eta>0, \quad \beta>0, \tag{15}$$

where e is the exponential function, η is the income elasticity of money demand, and β is the semi-elasticity of money demand with respect to the interest rate.[4] Notice that the

[4] The semi-elasticity β measures the *percentage* drop in real money demand induced by a one-percentage *point* increase in the interest rate.

interest rate i appearing in the money demand function should be interpreted as a *short-term* interest rate, since the closest substitutes for money are the most liquid interest-bearing assets with a relatively short term to maturity.

The constant money growth rule

To find the link between output and inflation on the economy's demand side, we need to know how the real interest rate r appearing in (12) is related to these two variables. This depends on the way monetary policy is conducted. Monetary policy regimes vary across time and space. Here we shall focus on two benchmark monetary policy rules, which have received widespread attention in the literature and in practical policy making. A monetary policy rule is a rule or principle prescribing how the monetary policy *instrument* of the central bank should be chosen. In practice, the main monetary policy instrument of the central bank is its set of short-term interest rates charged or offered vis-à-vis the commercial banking sector. Through their control of the central bank interest rates, monetary policy makers can normally control the level of short-term interest rates prevailing in the interbank market.[5] The interbank market is the market for short-term credit where commercial banks with a temporary surplus of liquidity meet other commercial banks with a temporary liquidity shortage. The interbank interest rate in turn heavily influences the level of market interest rates on all types of short-term credit.

Under the *constant money growth rule* for the conduct of monetary policy the central bank adjusts its short-term interest rates and performs market operations to ensure that *the nominal monetary base grows at a constant rate*. Assuming a constant money multiplier (that is, a constant ratio between the broader money supply and the monetary base), this will also ensure a constant growth rate of the broader money supply, which includes bank deposits as well as base money. In an influential book published in 1960, the American economist Milton Friedman argued that a constant money supply growth rate would in practice ensure the highest degree of macroeconomic stability which could realistically be achieved, since it would imply a stable increase in aggregate nominal income.[6] This argument was based on Friedman's belief in a stable money demand function with a low interest rate elasticity.

To see his point most clearly, suppose for a moment that our parameter β in (15) is close to zero, and that the income elasticity of money demand η is close to 1, as Friedman assumed Money market equilibrium then roughly requires $M = kPY$, where k is a constant. Hence aggregate nominal income PY must grow roughly in proportion to the nominal money supply M. Securing a stable growth rate of M will then secure a stable growth rate of nominal income. Furthermore, if the money market equilibrium in the current period is (close to) $M = kPY$, then for the preceding period it is (close to) $M_{-1} = kP_{-1}Y_{-1}$. By taking natural logs and subtracting the latter equation from the former, it follows that $\ln M - \ln M_{-1} = \ln P - \ln P_{-1} + \ln Y - \ln Y_{-1}$. If we define $\mu \equiv \ln M - \ln M_{-1}$, the growth rate of money supply, $\pi \equiv \ln P - \ln P_{-1}$, the inflation rate, and $g \equiv \ln Y - \ln Y_{-1}$, the growth rate of real output GDP, we thus have $\mu = \pi + g$. This is just a restatement of

[5] In times of financial crisis, the ability of central banks to control interest rates in the interbank market may be weakened, though. We return to this issue below.

[6] See Milton Friedman, *A Program for Monetary Stability*, New York, Fordham University Press, 1960.

the claim that if the central bank secures a constant money growth rate, it will also secure a constant growth rate of nominal GDP. If we also adopt Friedman's assumption that GDP adjusts quickly to its structural, long run level $\overline{Y}$, then g will normally not be too far from the economy's structural growth rate $\overline{g} \equiv \ln \overline{Y} - \ln \overline{Y}_{-1}$ which is quite stable. By choosing a constant money growth rate, the central bank can then ensure a fairly stable inflation rate $\pi = \mu - \overline{g}$, and by choosing a low μ it can ensure a relatively stable *and low* rate of inflation.

In Chapter 14, we mentioned several reasons for wanting a stable and relatively low inflation rate (although not too low). The desire to avoid the long run social cost of high and unstable inflation was one reason why Friedman recommended the constant money growth rule, but he also claimed that in the short run this rule would help to stabilize output and employment, thereby reducing the social costs of fluctuations in these variables.

Suppose now that the interest sensitivity of money demand (β) is in fact positive and that the income elasticity of money demand η is not necessarily close to 1. What does the constant money growth rule then imply for short run interest rate reactions? To investigate this, we still suppose that the central bank can implement its desired constant growth rate μ of the nominal money supply by applying its various policy instruments. We want to study how the market interest rate *i* in the money demand function reacts to deviations in the economy's inflation rate and output from their long run levels when the money supply grows at a constant rate. We assume that up to and including the former period called period -1, the economy has been in long run equilibrium. This means that up to period -1 we have had $Y = \overline{Y}$, $r = \overline{r}$ and $\pi = \mu - \overline{g}$. For simplicity we now ignore underlying growth and now assume $\overline{g} = 0$. Since inflation has been constant for some time, it has also been predictable, so expected inflation has been $\pi^e = \pi = \mu$, and hence the nominal interest rate has been constant at $i = \overline{r} + \pi^e = \overline{r} + \mu$. The money market equilibrium condition for period -1 is then:

$$\frac{M_{-1}}{P_{-1}} = k\overline{Y}^{\eta} e^{-\beta(\overline{r}+\mu)}, \tag{16}$$

where M_{-1} and P_{-1} are the nominal money supply and the price level prevailing in the previous period, respectively.

In the current period, one can have $Y \neq \overline{Y}$ and $\pi \neq \mu$ as result of shocks to the economy. The nominal interest rate will then have to adjust to maintain a money market equilibrium where the money supply grows at a constant growth rate, so in the current period *i* will deviate from its long run equilibrium value $\overline{r} + \mu$, and the money market equilibrium condition will just be:

$$\frac{M}{P} = kY^{\eta} e^{-\beta i}. \tag{17}$$

Taking natural logs on both sides of (16) and (17), subtracting the former from the latter equation, and again using our definitions $\mu \equiv \ln M - \ln M_{-1}$, $\pi \equiv \ln P - \ln P_{-1}$, and

$y \equiv \ln Y$, we get:

$$\mu - \pi = \eta\left(y - \overline{y}\right) - \beta i + \beta\overline{r} + \beta\mu, \tag{18}$$

which can be rewritten as:

$$i = \overline{r} + \pi + \left(\frac{1-\beta}{\beta}\right)(\pi - \mu) + \left(\frac{\eta}{\beta}\right)(y - \overline{y}). \tag{19}$$

Equation (19) shows how the short-term nominal interest rate i will adjust in response to changes in inflation and output if monetary policy aims at establishing a constant growth rate μ of the nominal money supply. It is thus an implicit monetary policy rule for the nominal interest rate. Since η and β are both positive, we see that the interest rate varies positively with the output gap, $y - \overline{y}$. If the numerical semi-elasticity β of money demand with respect to the interest rate is not too high ($\beta < 1$), as Friedman assumed, we also see that the nominal interest rate will increase more than one-to-one with the rate of inflation, implying an increase in the *real* interest rate. These reactions to variations in output and inflation will often be stabilizing, leading the economy back towards its long run equilibrium. For instance, if a demand shock drives output and inflation above their long run levels, the nominal interest rate will rise by more than inflation has increased, thereby raising the real interest rate which will dampen demand. These stabilizing short run interest rate reactions are thus implicit in the constant money growth rule and, according to Friedman, the best stabilization that one could hope for from monetary policy. Note that since the long-term equilibrium inflation rate equals the monetary growth rate, our parameter μ may be interpreted as the central bank's *target inflation rate*.

Friedman emphasized that we have only limited knowledge of the way the economy works. His studies of American monetary history also suggested that monetary policy tends to affect the real economy with long and variable lags.[7] Friedman therefore argued that the central bank may often end up *destabilizing* the economy if it attempts to manage aggregate demand through activist monetary policy by constantly varying the growth rate of money supply in response to changing economic conditions. Moreover, according to Friedman, the self-regulating market forces are sufficiently strong to ensure that real output and employment will be pulled fairly quickly towards their 'natural' rates following an economic disturbance. Given that activist monetary policy may fail to stabilize the economy, and that the need for stabilization is limited anyway, Friedman concluded that his constant money supply growth rule would be the best way to conduct monetary policy and actually achieve some stabilization.

Friedman's arguments did not go unchallenged, but they had a substantial impact on many central banks. In particular, the German Bundesbank adopted stable target growth rates for the money supply from the 1970s, and after the formation of the European Monetary Union, the European Central Bank maintained a target for the evolution of the money supply to support its target for (low) inflation.

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[7] Milton Friedman and Anna Schwartz, *A Monetary History of the United States, 1867 –1960*, Princeton, NJ, Princeton University Press, 1963. In Chapter 20 we shall discuss the lags in monetary policy in more detail.

The Taylor rule

However, one problem with a constant monetary growth rate is that it may not succeed in stabilizing the evolution of nominal aggregate demand if the parameters of the money demand function are changing over time in an unpredictable fashion. Such unanticipated shifts in the money demand function may occur when new financial instruments and methods of payment emerge because of financial innovation, or when the appetite for risk in financial markets changes. An extreme example of instability of money demand was seen during the financial crisis of 2007 – 08 when the demand for liquid assets including bank reserves soared and central banks throughout the world allowed a drastic increase in bank reserves to keep interest rates down.

If the purpose of the constant money growth rule is the pattern of interest rate reactions in the markets given by (19), it seems an obvious idea to let monetary policy focus directly on these interest rate reactions. In an influential article, American economist John Taylor argued that rather than worrying too much about the evolution of the money supply as such, the central bank might simply adjust its short-term interest rates in accordance with a rule like (19).[8] Assuming that policy makers wish to stabilize output around its trend level, and denoting the inflation target by π^*, we may specify the monetary policy rule proposed by Taylor as:

$$i = \bar{r} + \pi + h(\pi - \pi^*) + b(y - \bar{y}), \quad h > 0, \quad b > 0. \tag{20}$$

Equation (20) is the famous *Taylor rule*. Recalling that the monetary growth rate μ under the constant money growth rule could be interpreted as an inflation target, we see from (19) and (20) that the nominal interest rate follows an equation of the same form under the constant money growth rule and under the Taylor rule. Yet there is an important difference. Under the constant money growth rule, the coefficients in the equation for the interest rate depend on the parameters η and β of the money demand function, corresponding to $h = (1-\beta)/\beta$ and $b = \eta/\beta$ in (20). In contrast, under the Taylor rule these parameters have been 'set free' such that h and b in (20) are chosen directly by policy makers, depending, e.g., on their preferences concerning inflation and output instability. This makes an important difference since if β is small (as believed by Friedman), Equation (19) may imply drastic reactions in the interest rate to deviations in π and y from μ and $\bar{y}$, whereas this is not implied when h and b are chosen freely. According to Taylor, it is important that the value of h is positive so that the real interest rate goes up when inflation increases. If $1 + h$ is less than 1, a rise in inflation will drive down the real interest rate $i - \pi$, and this in turn will further feed inflation by stimulating aggregate demand for goods, leading to economic instability. The recommendation of a positive reaction parameter h in the Taylor rule, that is, a stronger than one-to-one reaction in the nominal interest rate to inflation, is called the *Taylor principle*.

Based on the US macroeconomic experience in the 1980s and early 1990s, Taylor suggested that the parameter values $\pi^* = \bar{r} = 0.02$ (2 percent) on annual basis and $h = b =$

[8] See John B. Taylor, 'Discretion versus Policy Rules in Practice', *Carnegie-Rochester Conference Series on Public Policy*, **39**, 1993, pp. 195 – 214.

0.5 would lead to good economic performance for the US economy. He also argued that a Taylor rule with these parameter values was descriptive of how the US central bank, the Fed, had set its main policy interest rate during the period, as illustrated by Figure 17.3.

Figure 17.3 The actual federal funds target rate and the rate according to the Taylor rule with $\pi^* = \overline{r} = 2$ percent per year and $h = b = 0.5$

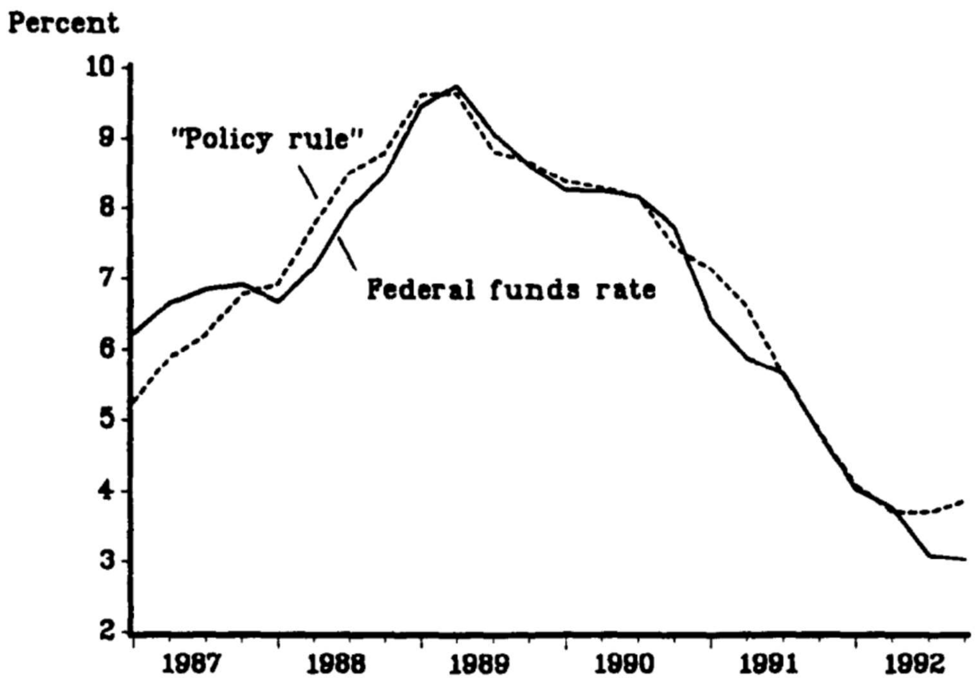
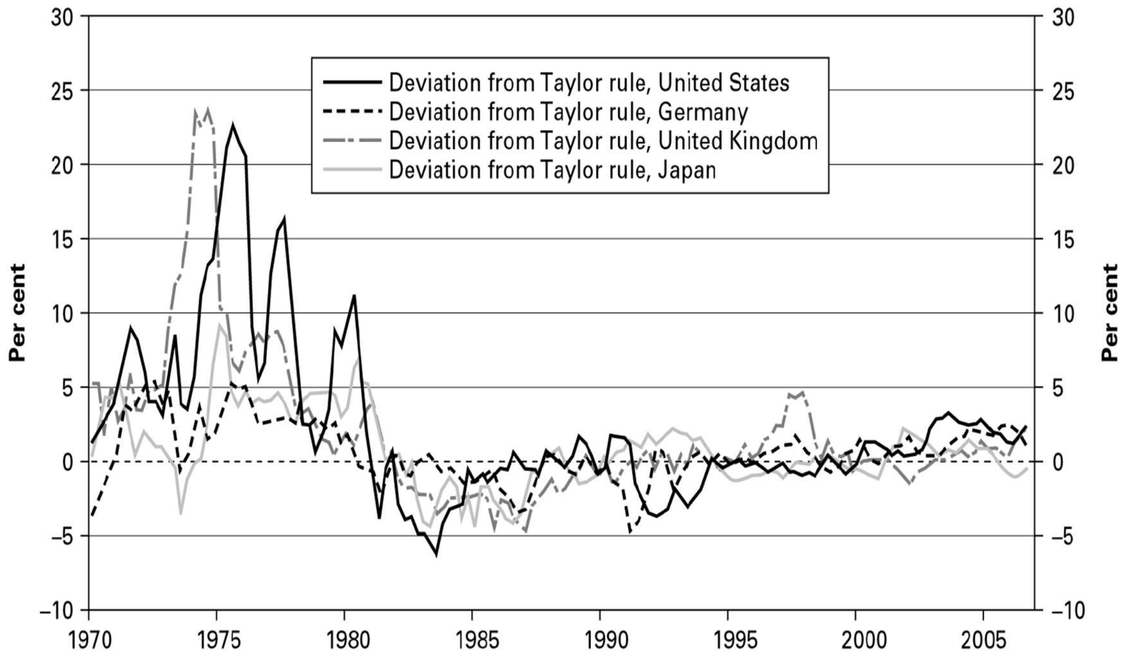


Figure 1. Federal funds rate and example policy rule.

Source: The figure is reproduced from John B. Taylor: `Discretion versus policy rules in practice'. *Carnegie-Rochester Conference Series on Public Policy*, **39**, 1993, pp. 195-214

Figure 17.4 shows the deviation between a 'Taylor interest rate' and the actual policy interest rate set by the central bank in four important countries over a longer period.

Figure 17.4 Monetary policy deviations from the Taylor rule



Source: Figure reproduced from p. 40 in Stephen G. Cecchetti, Peter Hooper, Bruce C. Kasman, Kermit L. Schoenholtz and Mark W. Watson: `Understanding the Evolving Inflation Process'. Report to the U.S. Monetary Policy Forum 2007, July 2007.

The 'Taylor rates' in the figure are given by the following monetary policy rules:[9]

Germany:	$i = 2.88 + \pi + 0.25(\pi - 2.0) + 0.25(y - \bar{y})$
Japan:	$i = 1.91 + \pi + 0.5(\pi - 1.0) + 0.25(y - \bar{y})$
United Kingdom:	$i = 3.63 + \pi + 0.25(\pi - 2.0) + 0.5(y - \bar{y})$
United States:	$i = 2 + \pi + 0.5(\pi - 2.0) + 0.5(y - \bar{y})$

From Figure 17.4 we see that, during the 1970s, the actual central bank interest rates were much lower than prescribed by the Taylor rule whereas they were somewhat higher than the 'Taylor rates' during the early 1980s. However, from the mid-1980s the deviations of the actual interest rates from those predicted by the above Taylor rules have been relatively small. Note that all of these rules involve a positive value of the parameter *h* in (21), so in recent years all of the four central banks have indeed followed Taylor's principle that *h* should be substantially above zero to ensure a rise in the real interest rate in response to a rise in inflation.[10] Overall, Figure 17.4 gives a clear indication that actual

[9] In the case of Germany, the actual interest rate is the one set by the German central bank (Bundesbank) prior to 1999 and the rate set by the European Central Bank after the formation of the European Monetary Union.

[10] Figure 17.4 only goes up to 2007 and thus does not include the period during and after the great financial crisis. In that period the Taylor rate may well have gone way below zero and thus have bumped into the

monetary policy interest rates since the mid-1980s seem to have become much more anchored by simple Taylor rules than they were before in major developed economies.

In Chapter 20 we shall discuss whether the Taylor rule is in fact also an *optimal* monetary policy. In the meantime, let us sum up the important monetary policy principles advocated by John Taylor:

THE TAYLOR RULE

The Taylor rule says that the nominal interest rate that the central bank targets, possibly its own policy interest rate, should vary positively with the output gap and with the deviation of inflation from the central bank's target inflation rate. In particular, the nominal interest rate should rise by more than one-to-one with the inflation rate to ensure that the real interest rate goes up when inflation increases, and vice versa. When output is at its trend level and inflation is at its target level, the central bank should aim at a nominal interest rate that yields a real interest rate equal to the economy's estimated 'natural' interest rate. While this Taylor rule was originally advanced as a normative prescription for monetary policy, it has turned out to provide a rather good empirical description of the actual interest rate policies followed by central banks.

Monetary policy and long-term interest rates: the yield curve

The policy interest rates directly set (or induced) by central banks typically concern quite short-term lending and borrowing of private banks in the central banks with maturities down to one day, one week or two weeks ahead and in normal times no longer than three months. Through its control over such short-term policy interest rates a central bank will normally be able to control the interest rates of other short term credits from banks to other banks or to customers. If a private bank can lend to and borrow from the central bank for two weeks at an annual interest rate of 2 percent, say, it will not lend to another customer for two weeks for less than 2 percent. And if it asks for considerably more than 2 percent on a two week credit, 3 percent say, another bank can increase its profit by underbidding the first one, borrowing in the central bank at 2 percent and lending out at 2½ percent, say. In this way, the short term policy rate set by the central bank, call it i^p, is transmitted into the short-term market interest rate of loans in the credit markets, i^s say, where *s* stands for 'short': $i^s \approx i^p$. However, the incentive to invest in a real asset depends on the expected cost of capital over the entire useful life of the asset. This lifetime may be many years if the asset is, say, a building, a truck, or a piece of machinery, so the interest rate relevant for investment decisions (and for decisions concerning the purchase of durable consumption goods, which are much like investment decisions) will typically concern debt instruments with longer maturity. The crucial question is: to what extent can monetary policy affect the incentive to acquire long-lived assets which make up the bulk of investment? In other words: what influence does the central bank have on the relevant longer term interest rates on loans with, e.g., 10-30

lower bound on nominal interest rates that we discussed in Chapter 14 and return to below. In that case, the Taylor rule cannot be descriptive of actual policy interest rates.

years to maturity through its indirect control over the short-term interest rates on loans with up to, say, 3 months to maturity?

Consider a firm which is contemplating investment in a real asset with an expected lifetime of *n* periods to be thought of as a considerable amount of time, e.g., 10 years. Suppose first that the firm plans to finance the investment with short-term debt which is 'rolled over' in each period, e.g., quarter, so that the interest rate varies with the movements in the short-term interest rate. For simplicity, suppose further that the firm does not need to pay any interest until the end of the *n* periods when the entire loan is paid back with compound interest. For each euro of debt incurred in the current period, the *expected* amount A^s to be repaid at the end of the *n* periods will be:

$$A^s = (1 + i^s) \times (1 + i_1^e) \times (1 + i_2^e) \times \ldots \times (1 + i_{n-1}^e), \tag{22}$$

where i^s is the current short-term (one-period) interest rate which is known at the time the debt is incurred, and i_j^e is the future short-term interest rate *expected* to prevail in the period j periods ahead.

As an alternative, the firm may finance the investment by a long-term loan with *n* terms to maturity (i.e., a loan lasting for *n* periods), where the long-term interest rate per period, i_n^ℓ, is fixed and known at the time when the debt is incurred. Assuming again that no interest is paid until the loan expires at time $t + n$, the amount A^l to be repaid at that time will then be:

$$A^l = (1 + i_n^l)^n. \tag{23}$$

Since the future short-term interest rates are not known at the time the loan is made, for the borrower the short-term financing is relatively risky compared to the long-term one, since in the first case the borrower needs to refinance at unknown interest rates. For the lender, the long-term placement is relatively risky because if he or she needs to realize the asset on the way this will be at an unkown price of the bond, say. Hence, if the agents are risk averse, the borrower will be ready to pay, and the lender will require, a risk premium on the long-term financing for the long-term and the short-term financing/placement to be equally attractive.

However, for simplicity and illustration we will first assume that the agents are risk neutral, meaning that they are only concerned with the mean value i_j^e of the expected distribution over the short-term interest rate *j* periods ahead. In this case, short-term and long-term credit are perfect substitutes in the sense that only the expected payments, A^s and A^l, are of importance for the choice between them. The *arbitrage condition* stating that the two forms of credit should be equally attrractive for both borrower and lender is then $A^s = A^l$, or from (22) and (23):

$$(1 + i_n^l)^n = (1 + i^s) \times (1 + i_1^e) \times (1 + i_2^e) \times \ldots (1 + i_{n-1}^e). \tag{24}$$

Indeed, market forces will tend to enforce this arbitrage condition. If, for instance, one had > instead of = in Equation (24), amounting to $A^s < A^l$, then borrowers would tend to demand more short-term credit and lenders would tend to supply more long-term credit, pushing the short-term interest rate i^s up (and possibly the expected ones, i_j^e, as well) and the long-term interest rate i_n^l down. These movements would continue until (24) is fulfilled.

The arbitrage behaviour of financial investors as expressed by (24) creates a link between the long-term interest rate on the one hand and the current and expected short-term interest rates on the other. This link is of great importance for monetary policy since it implies that the long-term interest rate relevant for investments is influenced by the short-term interest rate controlled by the central bank. We will therefore explore this link further.

The dependence of the current long-term interest rate on the current and the expected future short-term interest rates is referred to as the *expectations hypothesis*. If the length of our period is, say, a year, a quarter, or a month, the interest rates appearing in (24) will not be far above 0, and the approximation $\ln(1 + i) \approx i$ will be fairly accurate. Taking logs on both sides of (24) and dividing through by n, we then get:

$$i_n^\ell \approx \frac{1}{n}(i^s + i_1^e + i_2^e + \ldots + i_{n-1}^e). \tag{25}$$

Equation (25) says that *the current long-term interest rate is a simple average of the current and the expected future short-term interest rates*. This relationship assumes that agents are risk neutral. If we assume, more realistically, that they are risk averse, one must add a risk premium to the right-hand side of (25), as argued above. Hence, the more realistic version of the expectations hypothesis is:

$$i_n^\ell \approx \frac{1}{n}(i^s + i_1^e + i_2^e + \ldots + i_{n-1}^e) + \varepsilon_n, \tag{26}$$

where $\varepsilon_n > 0$ is a risk premium on long-term credit which depends positively on *n,* since the more terms the loan covers, the more risky is the long-term placement for the lender, and the more risky is the short-term financing for the borrower.

We have so far considered only two different debt instruments. In reality, a large number of securities with many different terms to maturity are traded in financial markets. But the reasoning which led to equations (25) and (26) is valid for any $n \geq 2$, so for given current and expected short-term interest rates, i^s, i_1^e, i_2^e ..., and a given function ε_n for the dependence of the risk premium on the investment horizon, (25') determines longer-term interest rates i_n^ℓ for all possible n. This means that it determines the entire *term structure of interest rates*, the relationship between interest rates on securities, i_n^l, and their durations, n.

This also leads to the so-called *yield curve*, which shows the effective interest rates on instruments of different terms to maturity at a given point in time. From (26) we see that the yield curve has the following properties:

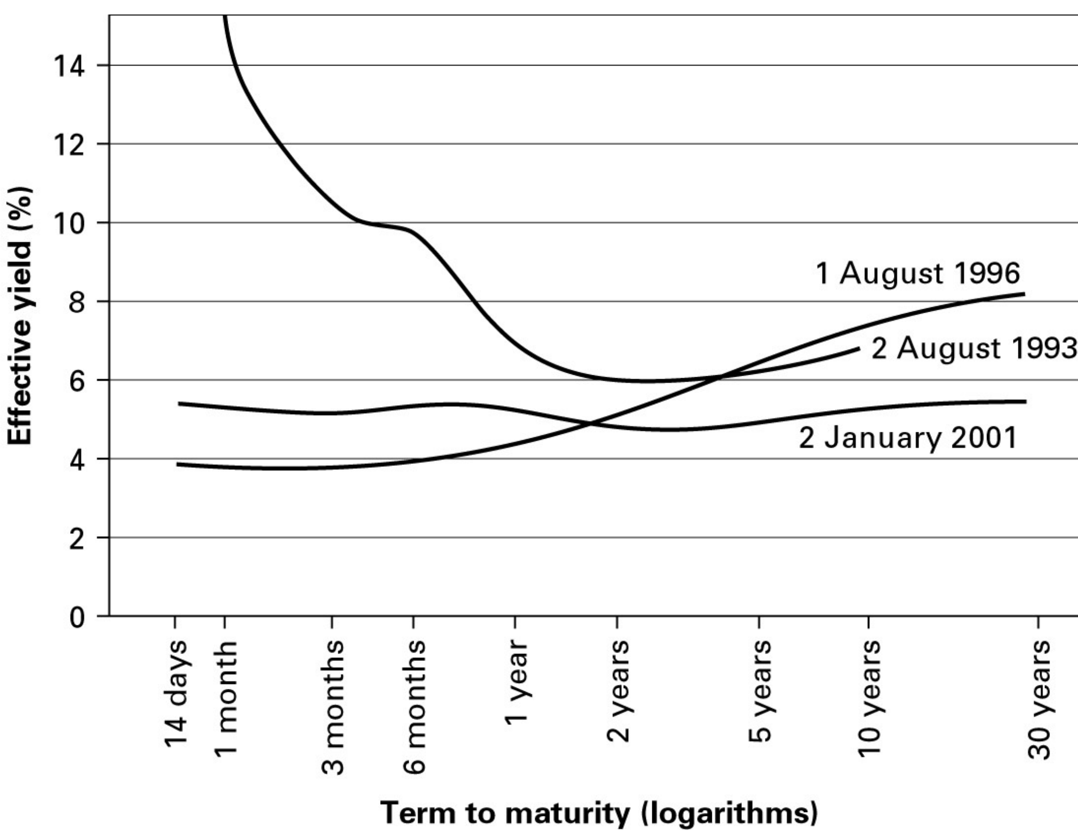
- Since ε_n is typically increasing in n, *ceteris paribus* securities with longer periods to maturity will have higher interest rates

- Expectations of unchanged short-term interest rates give *ceteris paribus* a flat yield curve, that is, if $i_j^e = i^s$ for all $j = 1, 2, \ldots, n-1$, then $i_n^\ell = i^s + \varepsilon_n$ for all n
- Expectations of declining (increasing) short-term interest rates give *ceteris paribus* a decreasing (increasing) yield curve

In other words, if financial investors happen to expect no changes in future short-term interest rates compared to the current rate, the interest rates on loans of all durations will be relatively close to the current short-term interest rate, and the yield curve will be relatively flat. Figure 17.5 shows that the yield curve in Denmark did in fact look this way in the beginning of January 2001. Likewise, if investors expect future short-term interest rates to increase gradually, then the interest rate on securities will be increasing in the number of terms to maturity etc.

As we move from left to right on the horizontal axis in Figure 17.5, we consider instruments with increasing terms to maturity. The first point on the yield curve shows the market interest rate on interbank credit with 14 days until maturity. This interest rate is almost perfectly controlled by the interest rate policy of the Danish central bank (Danmarks Nationalbank). The last point on the yield curve plots the effective market interest rate on 30-year Danish government bonds. The flatness of the yield curve suggests that investors in Denmark roughly expected constant short-term interest rates at the beginning of 2001.

Figure 17.5 The term structure of interest rates in Denmark



Source: Danmarks Nationalbank.

A rather flat yield curve, that is, one that is only moderately increasing due to the risk premium ε_n being increasing in n, is often considered to represent a 'normal' situation where investors have no particular reason to believe that tomorrow will be much different from today. Of course, the situation is not always normal. Figure 17.5 shows that short-term interest rates were far above long-term rates on 2 August 1993. Around that date, Denmark and many other European countries suffered from a speculative attack on the European Monetary System, the fixed exchange rate system that existed before the formation of the European Monetary Union. To stem the capital outflow generated by fears of a devaluation of the Danish krone, Danmarks Nationalbank drove up the 14-day interbank interest rate to the exorbitant height of 45 per cent per annum! The fact that long-term interest rates remained much lower indicates that investors did not expect the extreme situation at the short end of the market to last long.

In contrast, the yield curve had an unusually steep upward slope on 1 August 1996, as illustrated in Figure 17.5. At that time it was widely expected that the pace of growth in the European economy was about to increase significantly. Market participants therefore expected future monetary policy to be tightened to counteract inflationary pressures, and the expectation of higher future short-term interest rates drove current long-term rates significantly above the current short rate.

The relationship between short-term and long-term interest rates is very important for the effectiveness of monetary policy. We therefore restate the theory of

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THE TERM STRUCTURE OF INTEREST RATES

According to the expectations hypothesis, the current long-term interest rate is an average of the current and the expected future short-term interest rates plus a risk premium that typically increases with time to maturity. This arbitrage condition ensures that lenders obtain the same risk-adjusted return on long-term and short-term instruments and borrowers pay the same. The expectations hypothesis implies that the yield curve (showing the interest rate on instruments with different terms to maturity) will be relatively flat when investors expect the short-term interest rate to remain constant over time (it will only be slightly increasing reflecting that the risk premium increases with time to maturity). When future short-term interest rates are expected to increase (fall), the yield curve will be relatively steeply increasing (relatively little increasing or decreasing). In practice, risk-averse investors will require a risk premium on longer-term assets, so even if the short-term interest rate is expected to remain constant, the yield curve will tend to slope upwards.

Implications for monetary policy

What does the expectations hypothesis imply for monetary policy? Recall that the left-hand side of (26) reflects the cost of financing investment by long-term debt while the right-hand side represents the cost of financing investment through a sequence of short-term loans (including a risk premium). Moreover, also if a real investment is financed by equity, the cost of finance is still represented by either of the two sides of (26), since the opportunity cost of equity finance is the rate of interest, which the owners of the firm could have earned if they had chosen instead to invest their wealth in the capital market. Regardless of the mode of finance, (26) implies that *monetary policy can only have a significant impact on the incentive to invest in long-lived real assets if it affects expectations about future short-term interest rates*. For example, if the central bank engineers a unit increase in the current short rate i^s, which the market considers to be purely temporary, the expected future interest rates appearing on the right-hand side of (26) will be unaffected, and the interest rate on long-term debt with *n* periods to maturity will only increase by $1/n$ (for given ε_n). If the short-term rate applies to an instrument with a term of one month, and the long-term rate relates to a 10-year bond, *n* will be equal to $12 \times 10 = 120$. In that case a one-percentage point increase in the short term interest rate will only raise the long-term bond rate by a negligible 0.0083 percentage points, i.e., less than one basis point. Thus, there is very little impact on the incentive to invest in long-lived real assets, regardless of whether the investment is financed by long-term bond issues or by a sequence of short-term loans. At the other end of the spectrum is the situation where a change in the current short-term interest rate i^s is *expected* to be permanent. According to (26) the long-term interest rate i_n^ℓ will then rise by the full amount of the increase in the short rate (given ε_n).

The difficulties of controlling the cost of long-term investment finance through central bank interest rate policy are illustrated in Figures 17.6a and 17.6b.

For example, despite the many successive cuts in the target short-term interest rate of the US Federal Reserve Bank (the Federal funds target rate) undertaken during 2001 in reaction to economic recession, the interest rate on long-term government bonds was much more slow to come down, as shown by Figure 17.6a. This suggests that market participants expected a quick economic recovery, which would induce the Federal Reserve Bank to raise its interest rate again.

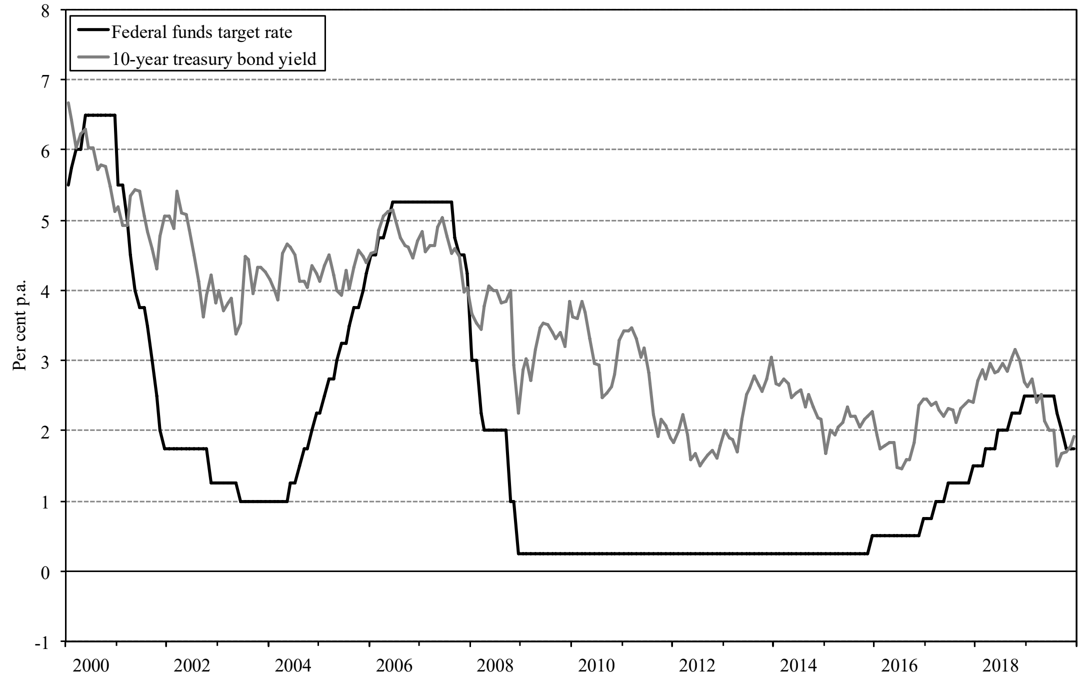
As another example, after the financial crisis and great recession of 2008-9, the central banks on both sides of the Atlantic lowered their policy interest rates dramatically, but the longer-term interest rates decreased with a considerable delay and for a long time by much less than the policy rates. Apparently, at first the financial agents did not believe that the lower short-term interest rates would last for a sufficient amount of time to pull down the long-term interest rates to the degree intended by the central banks.

The fact that monetary policy works to a large extent through its impact on market expectations explains why central banks care so much about their communication strategies, and why market analysts scrutinize every statement by central bankers to find hints about future monetary policy. As a former American central banker has put it, '*talking* about (monetary) policy is part of *making* it'.[11] In any given situation, the transmission from a change in the central bank interest rate to the change in long-term market interest rates will depend on market expectations. These in turn will depend on context and historical circumstances.

Overall, Figure 17.6 gives the impression that there is indeed a certain, positive relationship between the short-term policy (or target or signaling) interest rates of central banks and the longer-term market interest rates that they intend to influence, but the connection is not very tight and involves considerable time lags.

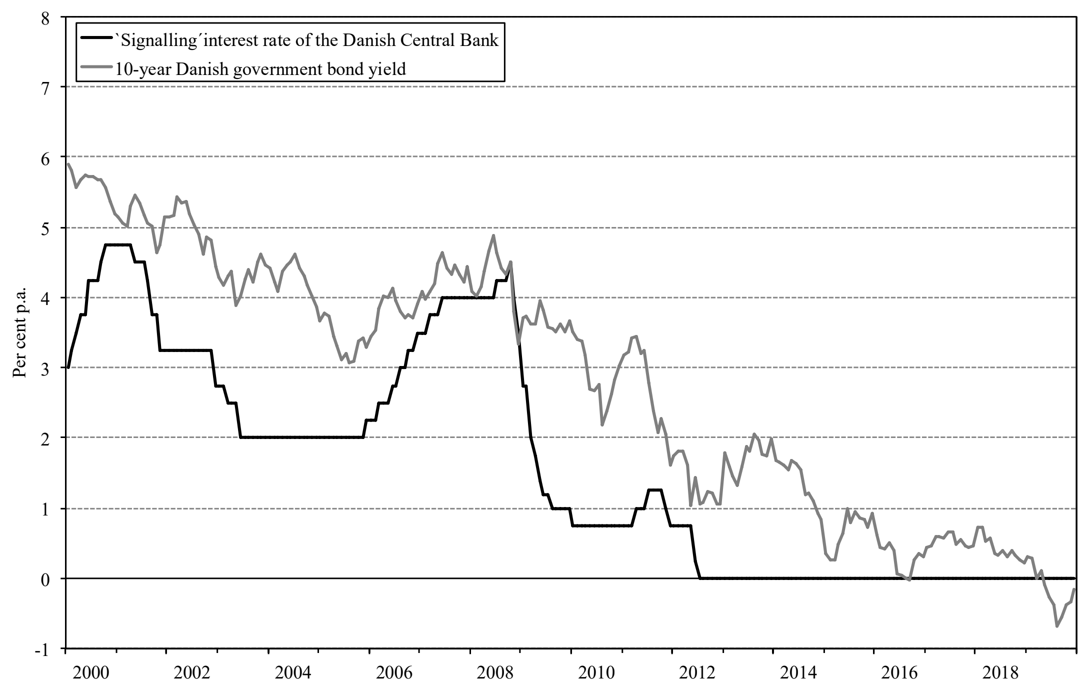
[11] This saying is due to Alan Blinder, Professor of Economics and former member of the Board of Governors of the US Federal Reserve Bank System.

Figure 17.6a The central bank signaling interest rate and the long-term market interest rate in the USA, 2000M1-2019M12



Note: End of month data. From December 2008 the FED stopped providing a target rate and instead reported a target range. The figure shows the upper limit of the range after December 2008.
Source: Federal Reserve Bank of St. Louis.

Figure 17.6b The central bank signaling interest rate and the long-term market interest rate in Denmark, 2000M1-2019M12



Note: End of month data. The signalling interest rate is the Danish Central Bank's so-called Discount Rate.
Source: Danmarks Statistik and The Wall Street Journal.

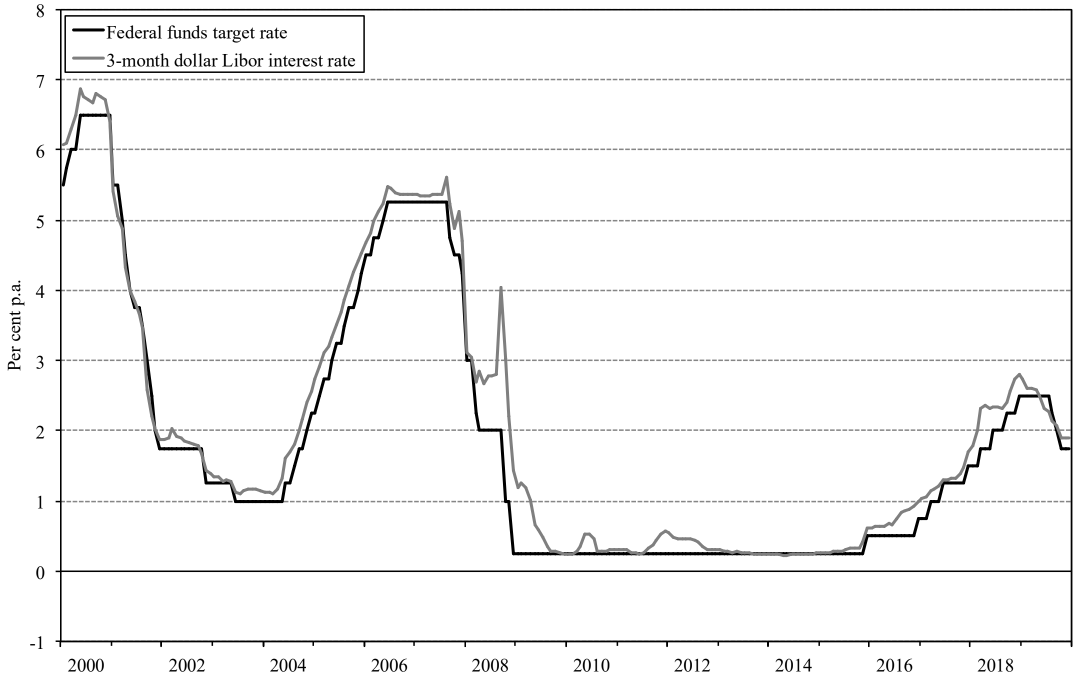
Figure 17.6 studies directly the influence of the central bank's policy interest rate, which we have called i^p, on the longer-term interest rate that the central bank aims at controlling, i_n^ℓ, with a reasonably large *n* in our notation, that is, $i^p \to i_n^\ell$. In our explanations, this influence ran through the short-term market interest rates that we called i^s, that is, $i^p \to i^s \to i_n^\ell$. We argued that there ought to be a rather tight connection at least from i^p to i^s. Does this hold true?

Actually, another complication for monetary policy is that, in times of financial crisis, it may be difficult for central banks even to control tightly the short-term market rates of interest. This is illustrated by Figures 17.7a and 17.7b. We see that normally there is a very tight link between the central bank interest rates and the three-month interest rates formed in the interbank markets. However, in 2007, financial turmoil was triggered by growing defaults on mortgage loans to 'subprime' (that is, less creditworthy) borrowers in the US housing market and a bit later the turmoil led to a full-blown financial crisis. This began to drive a highly unusual wedge between short-term market interest rates and the rates at which commercial banks could borrow from the central banks, as Figure 17.7a shows. Via complex securities backed by US mortgage loans, through sophisticated financial derivatives, and via (often implicit) guarantees from one financial institution to another, the risks associated with lending to subprime debtors had been dispersed throughout the international banking system in a way that was hard to dissect. As a result, it became unclear which banks were most exposed to the expected losses created by the crisis, so the risk premium on short-term interbank credit – usually quite close to zero – suddenly rose to a very high and volatile level. In the fall of 2008, the size and volatility of the risk premium rose even further to historically unprecedented levels, following the bankruptcy of the large US investment bank Lehman Brothers in mid-September. In other words, the link between the central bank interest rates and the short-term market interest rates became looser for an extended period with the short-term interbank rates lying considerably above the central bank interest rate for a period around 2008. Something similar happened in the euro zone during the financial crisis as Figure 17.7b shows.

This figure also shows that the three-month interbank rate in the euro area has been considerably lower than the European Central Bank's (the ECB's) main policy interest rate, the interest rate on its so-called 'main refinancing operations' (MRO), on several occasions since 2009. Most remarkably this has been the case for a long period after 2015 where the interbank interest rate has been negative.

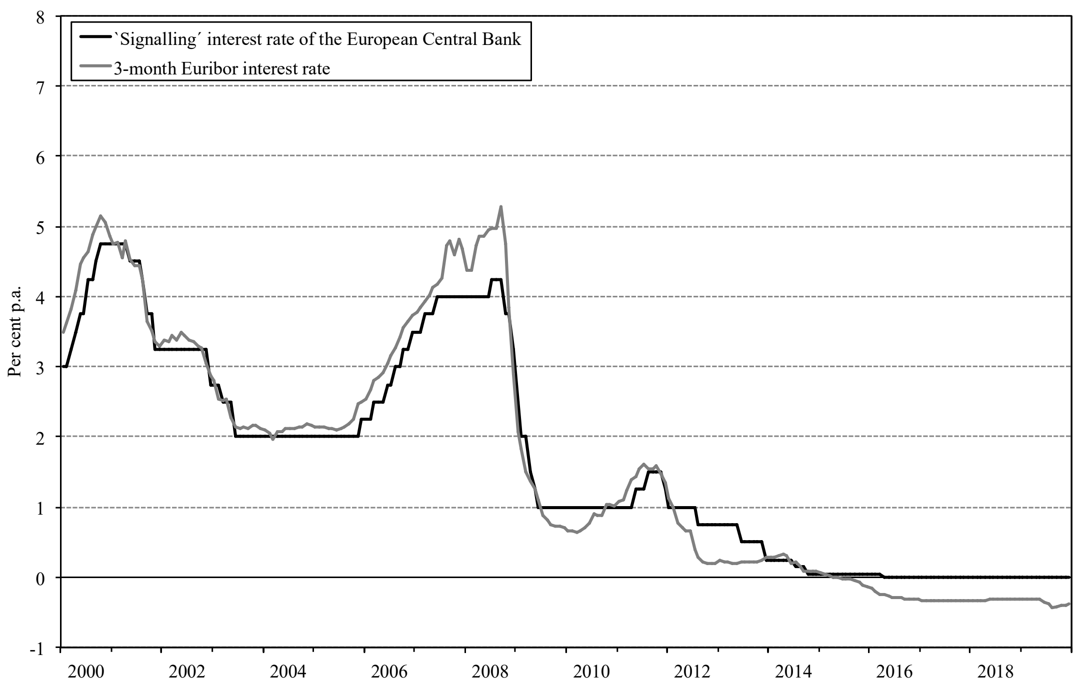
This is not another indication of a loose connection between the ECB's policy interest rates and short-term market interest rates, however, since the low and negative inter-bank interest rate has actually been very close to another important policy interest rate of the ECB and thereby has evolved as intended by the ECB. The ECB has several policy interest rates. Two of these relate to the bank's 'standing facilities' which are very short-term lending and borrowing arrangements between the ECB and the commercial banks. The 'marginal lending facility' allows private banks to obtain one-day loans (overnight liquidity) in the ECB against collateral in eligible assets at an interest rate set by the ECB. The 'deposit facility' offers private banks the opportunity to make overnight deposits in the ECB at an interest rate chosen by the ECB. Both of these interest rates were drastically cut after the financial crisis and, in particular, from June 2014 the interest rate on the deposit facility became negative with an annual rate of -0.1 percent and was from there gradually lowered in steps of 0.1 point to reach -0.5 percent from September 2019.

Figure 17.7a The central bank signaling interest rate and the interbank market interest rate in the USA, 2000M1-2019M12



Note: As figure 16.6a
Source: Federal Reserve Bank of St. Louis

Figure 17.7b The central bank signaling interest rate and the interbank market interest rate in the euro zone, 2000M1-2019M12



Note: Based on end of month data. The signalling interest rate is the European Central Bank's interest rate on the main refinancing operations (MROs).
Source: European Central Bank, Statistical Data Warehouse and Quandl.

The interest rate on the marginal lending facility was not lowered to a similar degree and stayed positive, but during the period considered it was more of a concern for private banks to have liquidity placed than to obtain liquidity by borrowing in the central bank and therefore it was the interest rate on the deposit facility that became binding. The negative interest rate on the ECB‘s deposit facility therefore explains the negative short-term inter-bank interest rates. A private bank with (temporary) excess liquidity will have to pay a negative interest rate if it places this in the central bank. Another bank can therefore also require payment to hold the money. The motivation for the ECB's negative interest rate on the deposit facility was exactly to force liquidity out into the credit markets to create there lower interest rates and more lending to stimulate activity. Hence, the negative interest rates in the interbank market followed the ECB interest rate on the deposit facility closely as intended by the ECB.

Another liquidity-providing service of the ECB are the above-mentioned 'main refinancing operations' (MROs) where private banks can obtain loans with a maturity of one week against collateral. The interest rate on the MRO is considered the ECB's main policy and signaling interest rate and it is the policy rate shown in Figure 17.7b. It reached zero in March 2016. Figure 17.7a shows that also the American Federal funds target interest rate was close to zero for a longer period between 2009 and 2015. As we explained in Chapter 14, nominal interest rates have a lower bound close to zero. The situation where the policy interest rates reach their (close to) zero lower bound is referred to as the 'liquidity trap' and will be an important theme in our analyses of stabilization policy in Chapter 20.

The overall picture from Figure 17.76 and 17.7 is that normally there is a rather tight connection between the central banks' main policy interest rates and short-term market interest rates. There is also a further connection to longer-term interest rates although this connection is at times weak and delayed. If a central bank wants the longer-term interest rates to decrease in order to stimulate the economy but cannot achieve this just by lowering its own short-term policy interest rates, it still has options; it can apply 'unconventional monetary policy'.

Unconventional monetary policy

The potential lack of influence of the central banks' short-term policy interest rates on the longer-term interest rates comes from a lack of influence on the expectations of future short-term interest rates. Unconventional monetary policy instruments aim at either creating such an influence on expectations or at influencing more directly the longer-term nominal interest rates.

Longer-term liquidity provision

There is no law saying that a central bank can only provide very short-term liquidity services to private banks. This is just the main service needed by private banks in normal times. If the central bank wants to, it can offer loans with longer maturities, thereby trying to obtain an influence further out on the yield curve, that is, on longer-term market interest rates.

The ECB, for instance, also has 'longer-term refinancing operations' (LTROs) where it regularly offers loans to banks with three months to maturity. This is not itself considered unconventional or non-standard monetary policy, but the ECB has sometimes chosen to offer loans with considerably longer maturities. For instance, in the aftermath of the financial crisis (since 2014), the ECB has offered low interest rate loans to private banks with maturities of up to four years under the heading ‘targeted longer-term refinancing operations’ (TLTROs). The intention of this unconventional monetary policy is to influence more directly longer-term market interest rates.

Forward guidance

As we noted above, central banks are generally eager to explain their actions in order to influence the longer-term market interest rates by affecting the expected future short-term interest rates. In the aftermath of the financial crisis, the central banks in both Europe and America took this kind of communication to a new level. Not only did they explain carefully why they had set their policy interest rates very low, they also promised to keep them at least as low for a specified and considerable amount of time, e.g., one year or two years. The intention was to create expectations of low short-term interest rates also in the future, which should work to establish lower longer-term interest rates here and now through the arbitrage mechanisms underlying the expectations hypothesis.

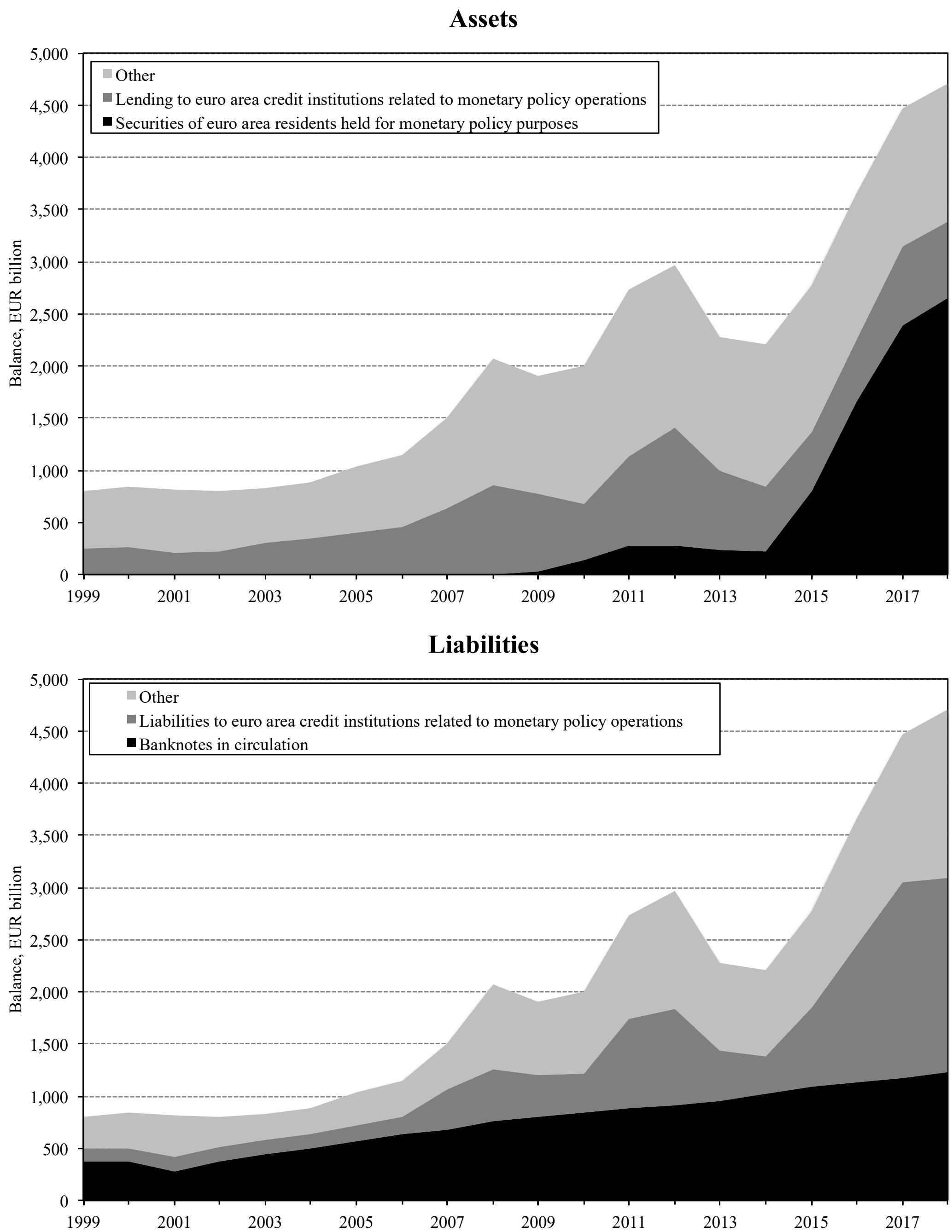
It is debated whether such ‘forward guidance’ has any effect. For forward guidance to work, people must believe that the central bank will stick to its promise of keeping the future interest rate low even in a potential future situation where the bank would normally want to increase its policy interest rate because of higher inflation and activity. In other words, people must believe that the central bank will keep the interest rate low even if the goal of creating higher economic activity is attained. If people do not believe that the central bank will stick to its promise no matter what, forward guidance will not have any impact.

Asset purchase programs - quantitative easing

A third way a central bank can try to influence longer-term interest rates over and above what can be obtained by setting (and explaining) its short-term policy interest rates is by directly buying up securities with longer maturities like government bonds, corporate bonds and real estate bonds in the markets. Using its position as a monopoly supplier of legal tender, the bank can print its own money to buy bonds, thus creating a demand for bonds that will drive up their prices and thereby lower their interest rates.

During the low-inflation period of economic crisis after the great financial crisis, all the major central banks engaged in variants of this type of policy. For example, the ECB has performed such policy under the heading of ‘asset purchase programs’ (APPs). Starting in March 2015, the ECB began to buy bonds in the markets at a pace of 60-80 billion euros per month until December 2017, then lowering the pace to 15-30 billion euros per month. From January 2019 new purchases stopped, but the ECB reinvested payments from maturing securities. From November 2019, the ECB restarted its asset purchase program at a pace of 20 billion euros per month. In March 2020, in view of the corona crisis, the ECB decided to add temporary additional net asset purchases of 120 billion euros in total until the end of 2020. From January 2021, net purchases are back to 20 billion euros per month and the Governing Council of the ECB has expressed that it ‘expects them to run for as long as necessary … and to end shortly before it starts raising the key ECB interest rates’. The declared purpose of these operations was to ‘support the monetary transmission mechanism'. In other words, since the ECB had not achieved as low longer-term, nominal interest rates as it wanted (to increase inflation and activity) by its more conventional instruments (its short-term policy interest rates), it resorted to directly buying securities with longer maturities to force down longer-term interest rates. Figure 17.8 illustrates the strong impact of the APPs on the balance sheet of the ECB.

Figure 17.8 Balance of European Central Bank, 1999-2018

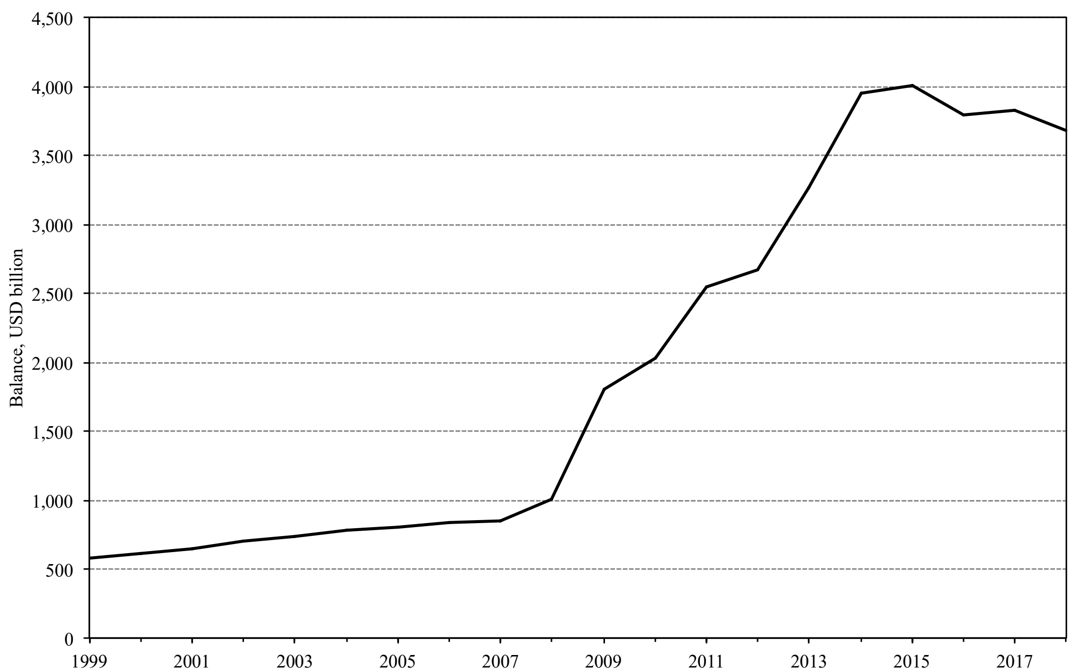


Note: Based on annual data.
Source: European Central Bank.

The overall balance has increased from a 'normal' less than 1,000 billion euros up to around 2005 to above 4,500 billion euros in 2018. On the asset side, the bulk of the increase comes from increased holdings of 'securities of euro area residents held for monetary policy purposes', mostly bonds. The counterpart of this on the liability side is the huge increase in 'banknotes in circulation' and 'liabilities to euro area credit institutions related to monetary policy operations', which are bank reserves. The sum of these two is more or less the monetary base, which is seen to have increased from around 500 billion euros to more than 3,000 billion euros since 1999.

In the US, the Fed has conducted similar asset purchasing programs under the heading of quantitative easing (QE) with a similar impact on the American monetary base, as Figure 17.9 illustrates. The Bank of England and the Bank of Japan have also engaged in large scale quantitative easing programs, and in September 2016 the Bank of Japan (BoJ) supplemented its QE policy by so-called Yield Curve Control. Under this framework, the BoJ sets two key interest rates – the short-term policy rate and an operating target for the interest rate on 10-year Japanese government bonds. To achieve the latter target, the BoJ conducts purchases of government bonds.

Figure 17.9 Federal Reserve Adjusted Monetary Base, 1999-2018



Note: Based on annual data.
Source: Federal Reserve Bank of St. Louis.

Many researchers have tried to assess how much influence the unconventional monetary policies and most notably the APP and QE programs have had on the longer-term interest rates they intend to affect. Whereas there can be little doubt about the direction of this influence – the programs have implied lower longer-term interest rates than there would have been in the absence of the programs – it is much more difficult to obtain a reliable estimate of the size of this effect, because it is hard to evaluate how longer-term interest rates would have evolved without the programs.

Summing up on monetary policy

In the remainder of this chapter and in coming chapters, where we consider a central bank with a rule-based, independent monetary policy, we will describe its influence on the relevant interest rates as follows.

We will assume that the central bank's short-term policy interest rate i^p is set according to a Taylor rule similar to Equation (20):

$$i^p = \bar{r} + \pi^e_{+1} + h(\pi - \pi^*) + b(y - \bar{y}), \quad h > 0, \quad b > 0, \tag{27}$$

where $\bar{r}$, π^* and $\bar{y}$ have the same meanings as in (20). There are two differences to the Taylor rule of Equation (20). First, here we explicitly state that the interest rate that the central bank controls is the short-term policy interest rate i^p, which therefore appears on the left hand side. Second, on the right hand side we have substituted the forward *expected* inflation rate from the current period to the next, π^e_{+1}, for the actual inflation rate, π, appearing first on the right hand side of (20). This π^e_{+1} should be thought of as the inflation expectation of the private economic agents. We thus assume that the central bank knows this expectation.

Since expected inflation enters into the real interest rate, as we will soon explain in more detail, it is important for the central bank to have a good estimate of inflation expectations. In many countries consumer and/or business surveys provide an estimate of the rate of inflation expected by the private sector. Several countries also have markets for indexed bonds whose principal and coupon payments are automatically adjusted in accordance with the change in some index of the general price level. For such bonds, the interest rate does not have to include an inflation premium to compensate the creditor for the erosion of his or her real wealth caused by inflation. By comparing the interest rate on indexed bonds to that on conventional non-indexed bonds of similar maturity, one may therefore obtain an estimate of the expected rate of inflation.

In one of these ways, the central bank will usually be able to measure the private sector's expected rate of inflation. Alternatively, we might assume that the central bank forms its own estimate of the future inflation rate and uses this as a proxy for the private sector's expected inflation rate. If the central bank and the private sector are using the same information, they should arrive at roughly the same value of π^e_{+1}.

To have expected inflation in the Taylor rule (27) is logical. The central bank wants to affect the *real* interest rate, and the real interest rate on some asset is the excess of the nominal interest rate on the asset over forward expected inflation, as we explain below. According to (27), if the current inflation rate π increases by *x* percentage point, the central bank will raise its nominal policy interest rate i^p by *hx* points *above the expected inflation rate* π^e_{+1}, thereby increasing what we could call the *policy real interest rate*, $i^p - \pi^e_{+1}$, by *hx* points.

However, the ultimate goal of the central bank is the interest rate on longer-term debt instruments such as a longer-term bank credit or a corporate or real estate bond which is of importance for decisions on investment and consumption. We call the relevant longer-term nominal interest rate that the central bank aims at determining *i*, and our assumption will be that the central bank can, through its control over i^p and possibly through its use of other, less conventional instruments, ensure that *i* is linked to i^p as follows:

$$i = i^p + \hat{\rho}, \tag{28}$$

where $\hat{\rho}$ is an 'error term' typically distributed around zero.

Note that we can interpret the policy interest rate i^p so as to include any normal risk premium included in the relevant longer-term nominal interest rate over the short-term nominal interest rate that the central bank decides on. Figure 17.6 indicates that on average over long periods the ten year bond yield lies somewhat above the short-term target or signaling interest rate of the central bank, perhaps by 1-2 percentage points. It is only natural that there is a systematic risk premium in the longer-term over the short-term interest rate, since buying a ten year bond, say, involves an uncertainty with respect to the future price of the bond that a short credit of one week or one month does not involve. If the longer-term nominal interest rate of relevance normally lies, say, 2 percentage points above the central bank's target interest rate (ceteris paribus), then we view these 2 percentage points as included in our i^p, so if the actually chosen policy interest rate of the central bank is 1 percent, then i^p is 3 percent. This is just an assumption that the central bank takes into account the systematic discrepancy between the longer-term market interest rates that it wants to influence and its own target interest rate when it decides on the latter. With this interpretation of i^p it is natural to assume that the longer-term interest rate *i* is distributed around i^p, as done in (28).

Combining the Taylor rule (27) with the link between the policy rate and the long-term nominal market interest rate *i* stated in (28), we get that the central bank controls the relevant nominal interest *i* up to:

$$i = \overline{r} + \pi_{+1}^e + h(\pi - \pi^*) + b(y - \overline{y}) + \hat{\rho}, \quad h > 0, \quad b > 0. \tag{29}$$

Through the error term $\hat{\rho}$, our analysis allows for incomplete central bank control over the relevant, longer-term nominal interest rate prevailing in private markets. Thus, we include the possibility of a 'backlash' in the central bank's control and assume that $\hat{\rho}$ is a random term with mean zero. The size of the variance of $\hat{\rho}$ then describes the degree of imprecision in the central bank's control over the relevant market interest rate through its control of the nominal policy rate i^p and its other less conventional policy instruments. For example, during a financial crisis $\hat{\rho}$ may become large and highly variable as the risk premia in financial markets become high and volatile. In contexts where the possibility of imprecision in the central bank's control over the relevant nominal interest rate is not important for our analysis, we will simply assume $\hat{\rho} = 0$.

Note that in the realistic case where the central bank does not always slavishly follow the Taylor rule, we should include a residual additive 'error term' on the right-hand side of (28) and (29). This term would then become absorbed in $\hat{\rho}$ in (29). In a broader interpretation of our model, the variable $\hat{\rho}$ thus reflects not only imprecision in the central bank's control over the relevant nominal interest rate, but also deliberate monetary policy deviations from the Taylor rule.

17.4 The aggregate demand curve

The real interest rate: *ex ante* versus *ex post*

We are now ready to derive the relationship between the inflation rate and the aggregate demand for goods and services. This relationship, called the aggregate demand (AD) curve, will be one of the two cornerstones of our model of the macro economy.

The first step in our derivation of the AD curve is the specification of the relationship between the nominal interest rate, the real interest rate and inflation. We have previously used the popular definition according to which the real interest rate is given by $r = i - \pi$, or $r = i - \pi_{+1}^e$, but will now be more precise on this. For a saver or a borrower the actual real interest rate r^a earned or paid between the current period and the next one is:

$$1 + r^a \equiv \frac{1+i}{1+\pi_{+1}}. \tag{30}$$

The reasoning behind (30) is this: if the current price level is P, giving up one unit of consumption today will enable you to invest the amount P in the capital market. Your nominal wealth one year from now will then be $P(1 + i)$. With an inflation rate π_{+1} between the current and the next period, a unit of consumption tomorrow will cost you $P(1 + \pi_{+1})$, so the purchasing power of your wealth one year from now will be $P(1+i)/P(1+\pi_{+1}) = (1+i)/(1+\pi_{+1})$. Thus your *real* rate of return is $r^a = (1+i)/(1+\pi_{+1}) - 1$, which is just another way of writing (30).

The variable r^a is called the *ex post* real interest rate, because it measures the real interest rate implied by the *actual* rate of inflation, measured *after* the relevant time period has passed ('*ex post*'). However, since saving and investment decisions must be made *'ex ante', before* the future price level is known with certainty, the real interest rate affecting aggregate demand for goods is the so-called *ex ante* real interest rate (r) which is based on the rate of inflation π_{+1}^e *expected* to prevail over the next period:

$$1 + r \equiv \frac{1+i}{1+\pi_{+1}^e}.$$

You may easily verify that it follows that:

$$r = \frac{i - \pi_{+1}^e}{1+\pi_{+1}^e} \approx i - \pi_{+1}^e, \tag{31}$$

where the latter approximation holds as long as π_{+1}^{e} does not deviate too much from zero. In the special case of static inflation expectations where agents assume that the rate of price increase over the next period will correspond to the rate of inflation experienced between the previous and the current period, we have $\pi_{+1}^{e} = \pi$. It then follows from (31) that the *ex ante* real interest rate may be proxied by $r = i - \pi$, corresponding to the popular definition of the real interest rate. Still, you should keep in mind that the more correct specification of the real interest rate influencing saving and investment decisions is given by (31).

Deriving the aggregate demand curve

Summing up, we assume that the central bank has an influence on the relevant longer-term nominal interest rate as given by (29), $i = \overline{r} + \pi_{+1}^{e} + h(\pi - \pi^{*}) + b(y - \overline{y}) + \hat{\rho}$, and, as we have just seen in (31), the real interest rate is $r = i - \pi_{+1}^{e}$. Combining these two gives the implied policy rule for the real interest rate:

$$r = \overline{r} + h(\pi - \pi^*) + b(y - \overline{y}) + \hat{\rho}. \tag{32}$$

This has a natural property. If output is at the structural level, $y = \overline{y}$, and inflation is right on target, $\pi = \pi^{*}$, a situation that the central bank should be perfectly happy with, the bank will impose a real interest rate equal to the structural or natural real interest rate, $\overline{r}$, except for the imprecision in the central bank's control over the relevant nominal interest rate as expressed by $\hat{\rho}$. According to the IS curve (12), repeated here for convenience

$$y - \overline{y} = \alpha_1(g - \overline{g}) - \alpha_2(r - \overline{r}) - \alpha_3(\tau - \overline{\tau}) + v, \tag{12}$$

a real interest rate equal to the structural one, $r = \overline{r}$, will exactly create demand for the full structural output, that is $y = \overline{y}$, in a situation with no fiscal policy impulses and no confidence shock. This should be exactly what the central bank wants since there is no reason to stimulate or dampen activity when both inflation and output are 'on target' in the absence of shocks.

The demand side of the economy is described by the IS curve (12) in combination with the monetary policy rule (32). We may express the demand side in just one equation. Inserting (32) into (12) gives:

$$y - \overline{y} = \alpha_1(g - \overline{g}) - \alpha_2[\overbrace{h(\pi - \pi^{*}) + b(y - \overline{y}) + \hat{\rho}}^{=r-\overline{r}}] - \alpha_3(\tau - \overline{\tau}) + v,$$

which is equivalent to:

$$y - \overline{y} = -\frac{\alpha_2 h}{1+\alpha_2 b}(\pi - \pi^{*}) + \frac{\alpha_1}{1+\alpha_2 b}(g - \overline{g}) - \frac{\alpha_3}{1+\alpha_2 b}(\tau - \overline{\tau}) + \frac{v}{1+\alpha_2 b} - \frac{\alpha_2 \hat{\rho}}{1+\alpha_2 b}. \tag{33}$$

This is one way of writing the economy's *aggregate demand* or *AD curve*. For some purposes, for instance if one does not want to analyse fiscal stabilization policy explicitly, it will be convenient to write the AD curve in compact form as:

$$y-\bar{y}=z-a(\pi-\pi^{*}),$$
$$a\equiv\frac{\alpha_2 h}{1+a_2 b}>0,\qquad z\equiv\frac{\alpha_1(g-\bar{g})-\alpha_3(\tau-\bar{\tau})+v-\alpha_2\hat{\rho}}{1+\alpha_2 b}. \tag{34}$$

The shock variable z captures *aggregate demand shocks* arising from the fiscal policy impulses $g-\bar{g}$ and $\tau-\bar{\tau}$, the confidence shock v and from the financial market shock $\hat{\rho}$ reflecting the central bank's incomplete control of market interest rates. The shock variable $\hat{\rho}$ also covers changes in monetary policy (deliberate deviations from the Taylor rule), while a captures the strength by which inflation affects the demand for output.

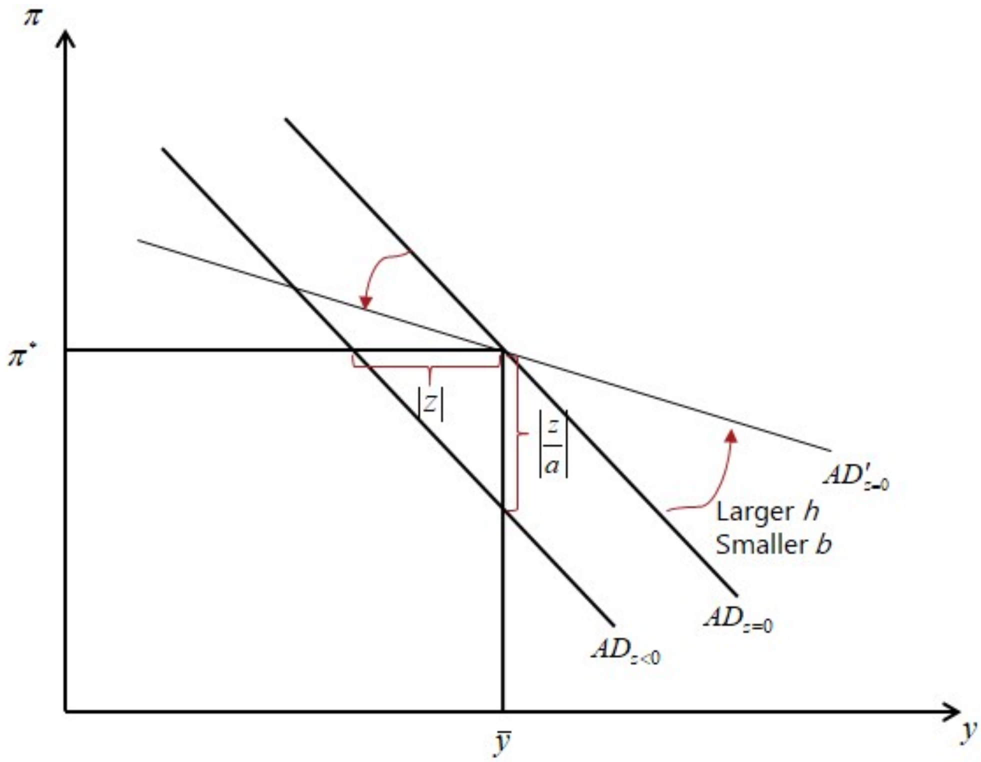
We see from (33) or (34) that *the aggregate demand curve is downward-sloping* in the y,π-space: a higher rate of inflation is associated with lower aggregate demand for output, as illustrated in Figure 17.10, showing AD curves for alternative values of z, h and b. The reason for the negative slope is that higher inflation induces monetary policy makers to raise the real interest rate, given that the parameter h in the central bank's reaction function (30) is positive. The higher real interest rate in turn dampens aggregate private demand for goods and services.

Note that for $z = 0$, obtained, e.g., when $g-\bar{g}=\tau-\bar{\tau}=v=\hat{\rho}=0$, the AD curve passes through $(\bar{y},\pi^{*})$, as illustrated by the AD curve labelled $AD_{z=0}$ in Figure 17.10. To identify the determinants of the position and the slope of the AD curve in the y,π-plane, it may be convenient to rearrange (34) as:

$$\pi=\pi^{*}-\frac{1}{a}(y-\bar{y})+\frac{z}{a}. \tag{34'}$$

A negative demand shock, $z < 0$, arising, e.g., from a more contractionary fiscal policy (a fall in g or an increase in τ), more pessimistic growth expectations in the private sector (a fall in ε and thereby v) or a financial shock driving up the risk premia in the financial sector (an increase in $\hat{\rho}$), will shift the aggregate demand curve downwards and to the left in the y,π-plane as illustrated by $AD_{z<0}$ in Figure 17.10. Given our definitions of v and z in (12) and (34), the value of z will be zero under 'normal' conditions where fiscal policy and private sector growth expectations are at their trend levels and monetary policy is able to control market interest rates in accordance with the Taylor rule.

Figure 17.10 The aggregate demand curve



According to (34) or (34'), the position of the aggregate demand curve is also affected by the central bank's inflation target π^*. If the central bank becomes more 'hawkish' in fighting inflation (if π^* falls), the aggregate demand curve will shift downwards, to the left.

Monetary policy influences the *slope* of the aggregate demand curve (-1/a) as well as its position. If the central bank puts relatively strong emphasis on fighting inflation and relatively little on stabilizing output, the parameter h in the Taylor rule will be relatively high, and b will be relatively low. According to the definition of a in (34), this means that the aggregate demand curve will be relatively flat (a will be relatively high), meaning that a change in the inflation rate will have a relatively large impact on the demand for output. On the other hand, if monetary policy reacts strongly to the output gap and only weakly to inflation, we have a low value of h and a high value of b, generating a low value of a and a steep aggregate demand curve, meaning that output demand will be relatively insensitive to inflation. These properties are also illustrated in Figure 17.10 by the anticlockwise rotation of the AD curve labelled $AD_{z=0}$ to the one labelled $AD'_{z=0}$.

Summing up with respect to the AD curve:

THE AGGREGATE DEMAND CURVE

The AD curve shows the relationship between the inflation rate and the aggregate demand for goods and services. The AD curve is derived from the goods market equilibrium condition and the Taylor rule for monetary policy. A rise in the rate of inflation reduces aggregate demand because it induces the central bank to raise the real interest rate, thereby reducing private investment demand and possibly also private consumption. The AD curve shifts upwards / to the right in case of a rise in public spending, a cut in the net tax rate, an increase in the private sector's expected future income growth or a drop in market risk premia.

The aggregate demand curve is one of the two key relationships in our short-run model of the macro economy. To identify the point on the AD curve where the economy will settle down, we need to bring in the aggregate supply side. This is the topic of the next chapter.

Summary

1. **1.** The IS curve is derived by combining the aggregate consumption and investment functions with the goods market equilibrium condition that total output must equal the total demand for output consisting of private consumption, private investment and government demand for goods and services. Goods market equilibrium implies that aggregate saving equals aggregate investment. The IS curve assumes that the private sector savings surplus (savings minus investment) is an increasing function of the real rate of interest. The evidence supports this assumption.
2. **2.** According to the IS curve, aggregate demand depends on the real rate of interest. It is therefore crucially influenced by the interest rate policy of the central bank. Historically some central banks have followed Milton Friedman's suggested constant money growth rule, choosing its policy instruments with the purpose of attaining a steady growth rate of the nominal money supply. A constant money growth rate generates systematic responses of the nominal interest rate to output and inflation where the interest rate increases when output and inflation rise above their targets, and vice versa. More recently, many important central banks have tended to follow a more direct rule suggested by John Taylor with similar properties for the bank's policy interest rates. According to the Taylor rule, the central bank should raise the short-term real interest rate when faced with a rise in the rate of inflation or in output above their targets, and vice versa.
3. **3.** The central bank can normally control the short-term interest rate, but not the long-term interest rate. The *expectations hypothesis* states that the long-term interest rate is a simple average of the current and expected future short-term interest rates. If a change in the short-term interest rate has little effect on expected future short-term rates, it will also have little effect on the long-term interest rate. The ability of the central bank to influence the long-term interest rate therefore depends very much on its ability to affect market expectations.
4. **4.** The incentive to invest in a real asset depends on the expected cost of finance over the lifetime of the asset. Under debt finance, a long-lived asset may be financed by a long-term loan or by a sequence of short-term loans. Risk neutral investors will choose the mode of finance that has the lowest expected cost. When the expectations hypothesis holds, the expected cost of finance is the same whether real investment is financed by equity, by long-term debt, or by a sequence of short-term loans. Therefore, the ability of the central bank to influence incentives for long-term real investment depends on its ability to influence the long-term interest rate which in turn hinges on its ability to affect market expectations of future short-term rates.
5. **5.** When market agents are risk averse, the long-term interest rate will equal the average of the current and expected future short-term interest rates plus a risk premium which will vary with the appetite for risk-taking. Fluctuating risk premia are another reason why the central bank cannot fully control the long-term market interest rate.
6. **6.** In times of financial crisis, the ability of the central bank to lower the real interest rate may be limited by the lower bound on the nominal interest rate. The central bank may

then resort to unconventional monetary policy such as 'forward guidance', where the bank seeks to increase the expected inflation rate by promising to keep its policy interest rate very low for a long time, or various forms of 'quantitative easing', where the central banks engages in direct purchases of financial assets, including long-term assets, thereby driving down the long-term real interest rate.

7. Because of its empirical relevance, our theory of aggregate demand assumes that monetary policy follows the Taylor rule, which implies that the central bank raises the real interest rate when the rate of inflation or output goes up. Combining the Taylor rule with the IS curve gives the AD curve. Because a higher rate of inflation will be accompanied by a higher real interest rate from the Taylor rule, it will also lead to a fall in aggregate demand from the IS curve, so the AD curve will be downward sloping in y, π -space. The AD curve will shift down if the central bank lowers its target rate of inflation or if the economy is hit by a negative demand shock, due to a tightening of fiscal policy or a fall in private sector confidence or a rise in the non-controllable part of the interest rate.

Exercises

Exercise 1. Topics in the theory of aggregate demand

1. Explain the concepts of the *ex post* real interest rate and the *ex ante* real interest rate. Which of these measures is most relevant as an indicator of the incentive to save and invest? Which of the two measures is most relevant for judging how inflation affects the distribution of income between borrowers and lenders? How are the two measures of the real interest rate related to the popular measure $i_t - \pi_t$?
2. Why does the AD curve slope downwards? Which factors can cause the AD curve to shift? Which factors determine the slope of the AD curve? In particular, explain in economic terms why a higher value of the parameter b in the Taylor rule makes the AD curve steeper.

Exercise 2. Interest rate setting under a constant money growth rule

When we derived the interest rate reaction function (19) under the central bank's constant money growth rule, we assumed for simplicity that there was no growth in trend output. We will now assume instead that trend output grows at the constant rate x so that:

$$\overline{Y} = (1+x)\overline{Y}_{-1} \quad \Rightarrow \quad \overline{y} \approx \overline{y}_{-1} + x, \quad \overline{y} \equiv \ln \overline{Y}, \quad \overline{y}_{-1} \equiv \ln \overline{Y}_{-1}, \tag{35}$$

where we have used the approximation $\ln(1 + x) \approx x$. Following Section 17.3 and the notation used there, we specify the demand for real money balances as:

$$L = kY^{\eta} e^{-\beta i}. \tag{36}$$

1. Show by means of (35) and (36) that the growth rate of the demand for real money balances in a long-run equilibrium is approximately equal to ηx. (Hint: approximate the growth rate in real money demand by $\ln L - \ln L_{-1}$). Show that in long-run equilibrium, the rate of inflation will be given by $p = \mu - hx$, where μ is the constant growth rate of the nominal money supply, defined by $M = (1+\mu)M_{-1}$. (Hint: you may approximate the growth rate of the real money supply by $\ln(M/P) - \ln(M_{-1}/P_{-1})$ using the fact that $P = (1 + \pi)P_{-1}$).

We now invite you to derive an equation showing how the nominal interest rate will react if the central bank maintains a constant growth rate μ of the nominal money supply, in accordance with Milton Friedman's recommendation. You may assume that the economy is in long-run equilibrium in the previous period, with a nominal interest rate equal to $i_{-1} = \overline{r} + \mu - \eta X$. Given that the money market clears in every period, we then have:

$$\frac{M/P}{M_{-1}/P_{-1}} \equiv \frac{1+\mu}{1+\pi} = \frac{kY^{\eta}e^{-\beta i}}{k\overline{Y}_{-1}^{\eta}e^{-\beta(\overline{r}+\mu-\eta x)}}. \tag{37}$$

2. Use (35) and (37) to show that Friedman's constant money growth rule requires the central bank to set the nominal interest rate (approximately) in accordance with the rule:

$$i = \overline{r} + \pi + \left(\frac{\eta}{\beta}\right)(y - \overline{y}) + \left(\frac{1-\beta}{\beta}\right)[\pi - (\mu - \eta x)]. \tag{38}$$

(Hint: take logs on both sides of (37) and use the approximation $(1 + \mu)/(1 + \pi) = \mu - \pi$). Comment on the similarities and differences between (38) and Equation (20) in the main text.

3. Friedman has argued that the interest sensitivity of money demand (β) is very low (although positive). What does this imply for the evolution over time in the nominal and real interest rate if monetary policy makers follow the constant money growth rule? Do you see any problem in this?

Exercise 3. Nominal GDP targeting

In the main text of this chapter we discussed the constant money growth rule which is intended to ensure a stable evolution of aggregate nominal income. Given this goal, some economists have proposed that the central bank should not focus on the evolution of the nominal money supply as such but rather adopt a target growth rate for nominal GDP. Such a rule would allow real GDP to grow faster when inflation falls and would require real growth to be dampened when inflation rises. In formal terms, if the target growth rate of nominal GDP is μ, the central bank would adjust its short-term interest rates and other policy measures to ensure that:

$$\overbrace{y - y_{-1} + \pi}^{\text{nominal GDP growth}} = \mu, \tag{39}$$

where y is the log of GDP so that $y - y_{-1}$ is the growth rate of real GDP. Ignoring fluctuations in confidence and government spending ($v = 0$), and assuming static inflation expectations so that the *ex ante* real interest rate becomes equal to $i - \pi$, we may write the goods market equilibrium condition or IS curve (12) as:

$$y - \overline{y} = -\alpha_2(i - \pi - \overline{r}). \tag{40}$$

Finally, suppose that trend output grows at the constant rate x so that:

$$\overline{y} \approx \overline{y}_{-1} + x. \tag{41}$$

1. Derive the implied interest rate reaction function under nominal GDP targeting. How

does the interest rate react to inflation? How does it react to the lagged output gap $y_{-1} - \bar{y}_{-1}$? Explain in economic terms why and how the parameter α_2 affects the central bank's interest rate response to changes in the rate of inflation and in the lagged output gap.

2. Compare interest rate setting under nominal GDP targeting to interest rate setting under the Taylor rule and under the constant money growth rule. Explain similarities and differences. Do you see any advantages of nominal GDP targeting compared to Friedman's constant money growth rule? Give reasons for your answer.

Exercise 4. Topics in monetary policy

1. Explain and discuss the arguments underlying the constant money growth rule for monetary policy. Explain the similarities and differences between the constant money growth rule and the Taylor rule. What could be the argument for choosing a Taylor rule rather than a constant money growth rule? Is it possible to determine by empirical analysis whether a central bank follows a constant money growth rule or a Taylor rule?
2. Explain the expectations hypothesis of the link between short-term interest rates and long-term interest rates. What is the crucial assumption underlying the expectations hypothesis? Is this assumption reasonable? Discuss the central bank's possibility of controlling long-term interest rates through its control of the short-term interest rate.
3. Discuss why financial market analysts study the official statements of central bankers so carefully and why central bankers seem to be so careful about what they say. Most observers argue that central banks should be as open and transparent about their analysis of the economic situation and their policy intentions as possible, but some argue that complete openness may not be the optimal policy. Try to think of arguments for and against maximum transparency of central banks. (As a source of inspiration, you may want to consult the article 'It's not always good to talk', *The Economist*, 24 – 30 July 2004, p. 65.)

Chapter 18

Inflation, unemployment and aggregate supply

Introduction

Inflation and unemployment are two of the most important macroeconomic problems. Indeed, the main goals of macroeconomic stabilization policy are to fight cyclical unemployment and to avoid high and unstable inflation. In this chapter we explore the relationship between inflation and unemployment. As we shall see, understanding the link between these two variables is crucial for understanding how the supply side of the economy works in the short run and for how the economy reacts to shocks. Therefore, studying the relationship between inflation and unemployment is fundamental for understanding business fluctuations.

18.1 Background: a brief history of the Phillips curve

The wage inflation Phillips curve of A.W. Phillips

For many years after the Second World War, most economists and policy makers believed that there was an inescapable trade-off between inflation and unemployment: if you want lasting less inflation, you have to live with permanently higher unemployment, and vice versa. Figures 18.1a and 18.1b, taken from a famous article published in 1958 by the New Zealand-born economist A.W. Phillips, suggest why most observers came to believe in a permanent unemployment-inflation trade-off.[1]

Figure 18.1a reproduces a figure from Phillips' article. It shows data points and a curve fitted to them, which Phillips used to describe the relationship between unemployment and the rate of money wage inflation in the United Kingdom in the period 1861–1913. We see that he found a tight (although non-linear) negative correlation between the two variables. Phillips then showed that the curve fitted to the 1861–1913 data was able to explain the relationship between UK

[1] A.W. Phillips, 'The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957', *Economica*, New Series, **25** (100), Blackwell Publishing (Nov. 1958), pp. 283 – 299.

unemployment and wage inflation in the much later period 1948 –57, as shown in Figure 18.1b. Apparently, Phillips had discovered a very stable and fundamental trade-off between the rate of unemployment and the rate of wage inflation.

Figure 18.1a The Phillips curve in the UK, 1861–1913

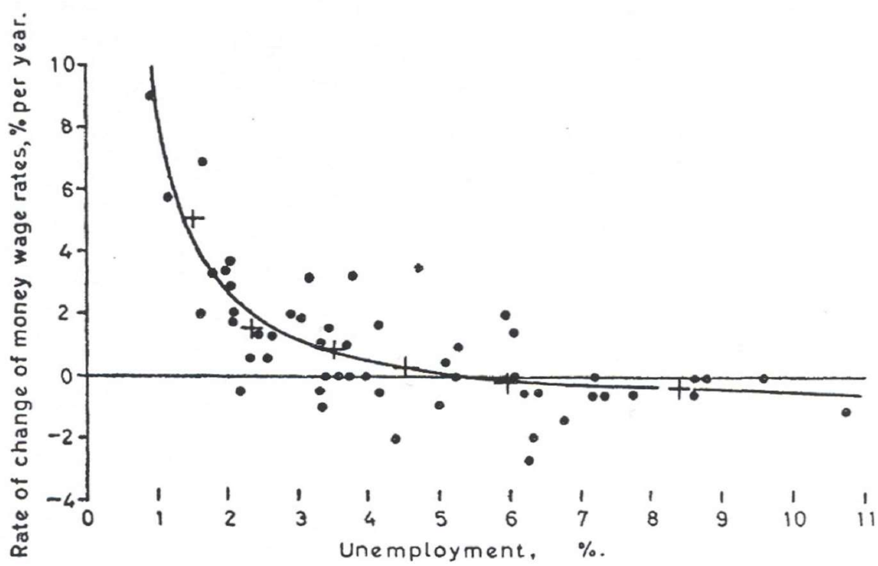
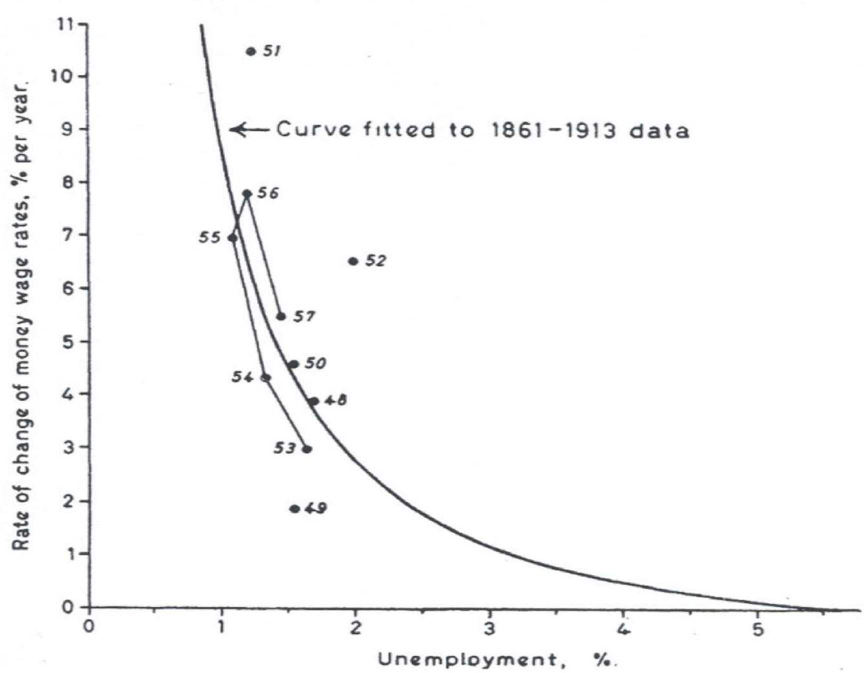


Figure 18.1b The Phillips curve in the UK, 1948 –1957



Source: Figure 1 and 10, respectively, of A.W. Phillips, 'The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957', *Economica*, New Series, **25** (100), Blackwell Publishing (Nov. 1958), pp. 283 – 299.

A.W. Phillips himself studied the relationship between *wage* inflation and unemployment and suggested a 'labour market pressure', excess demand/supply explanation, by Phillips called a 'demand pull' effect. In his own words:

> When the demand for labour is high and there are few unemployed we should expect employers to bid wage rates up quite rapidly, each firm and each industry continually tempted to offer a little above the prevailing rates to attract the most suitable labour from other firms and industries.[2]

Appealing to a close relationship between wage and price levels, and hence between wage and price inflation (caused by the fact that labour costs constitute the main part of production costs), it was soon suggested that there should be an equally stable relationship between *price* inflation and unemployment.

The simple price inflation Philips curve

Figure 18.2a seems to confirm such an idea. It is based on US data on unemployment and the rate of consumer price inflation up through the 1960s and it suggests a rather stable Phillips curve trade-off between inflation and unemployment in USA for that period. In a general form the relationship could be written as $\pi = f(\bar{u} - u)$, where f is an increasing function with $f(0) = 0$, π is the rate of inflation, u is the actual rate of unemployment, and $\bar{u}$ is the natural unemployment rate, here defined as the unemployment rate that would give zero inflation. We will, however, consider a simplified linear form involving a parameter, α:

$$\pi = \alpha(\bar{u} - u), \quad \alpha > 0. \tag{1}$$

The finding of an apparently stable Phillips curve trade-off was received with enthusiasm by macroeconomists who saw it as a 'missing link' between the real and nominal sides of the macro economy. It was therefore incorporated into macroeconomic models in the 1960s under the name of the Phillips curve. One can think of real output and the real interest rate as being determined for given levels of government demand, taxation and real money supply in a *real* IS-LM type of model (as briefly reviewed in Chapter 17) that had become the canonical Keynesian macro model at the time.[3] According to this, it should be possible to increase output and reduce unemployment, or vice versa, by appropriate fiscal and/or monetary policy. The incorporation of the Phillips curve into the model would then give an explanation of the rate of inflation, a nominal variable, and thus connect the real and nominal sides of the economy: The cost of increasing output and reducing unemployment by means of fiscal or monetary policy would be the increased level of inflation going with it.

However, in the 1970s the relationship broke down completely as Figure 18.2b shows. Many times during the 1970s the USA experienced a simultaneous rise in inflation and unemployment, a phenomenon much to the perplexity and frustration of economic policy makers referred to as

[2] Quoted from page 283 of Phillips' article mentioned in Footnote 1.

[3] Money demand should, of course, depend on a nominal interest rate, but considering inflation expectations as given and stable, the LM curve could be seen as a relationship between output and the real interest rate, so the model would determine real output and the real interest rate.

‘stagflation’. The same thing happened in practically all OECD countries during that decade. What was going on?

Figure 18.2a The Phillips curve in the USA of the 1960s

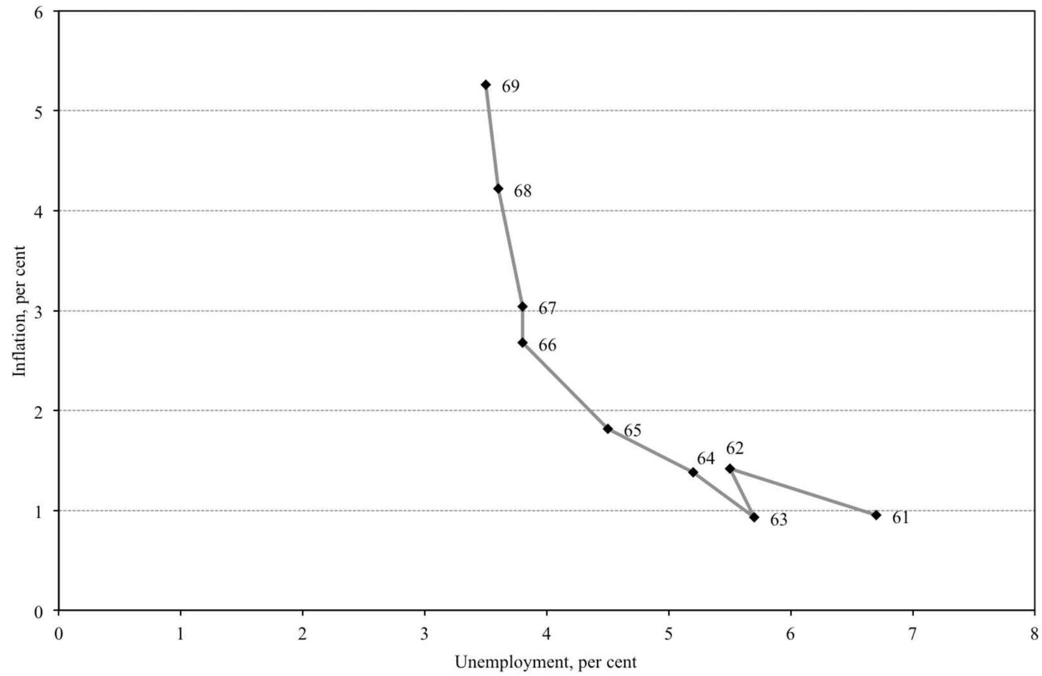
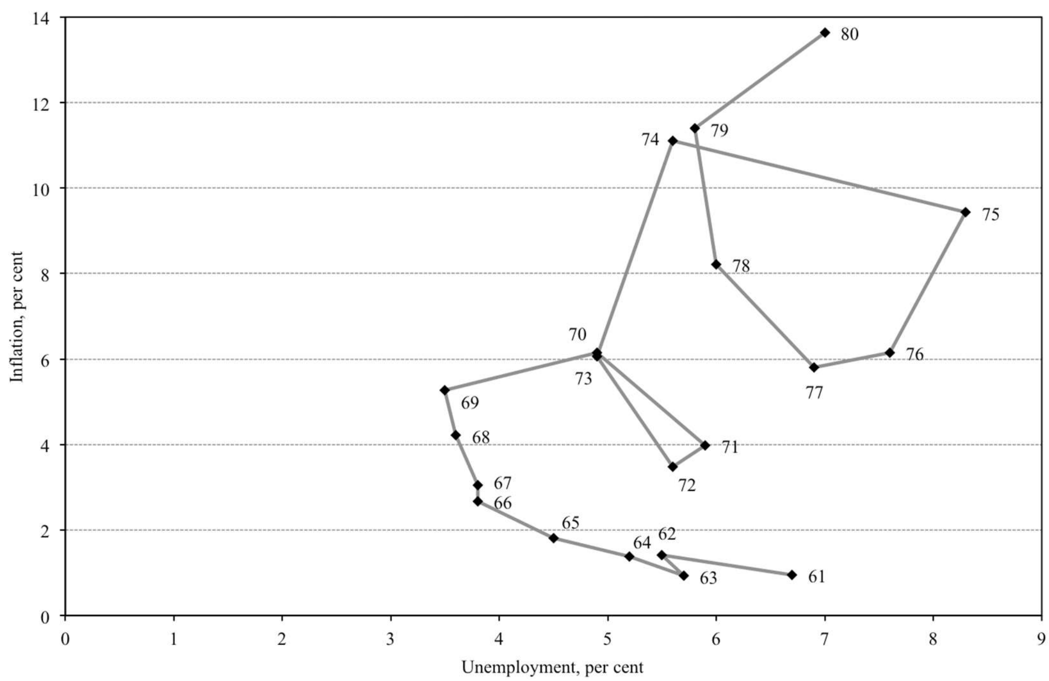


Figure 18.2b The breakdown of the simple Phillips curve in the USA



Source: R. B. Mitchell, *International Historical Statistics The Americas 1750-1988*, Macmillan Press, 1993.

The expectations-augmented Phillips curve

Even before the simple Phillips curve broke down, the two US economists Milton Friedman and Edmund Phelps had in the late sixties more or less anticipated the breakdown and suggested a modification of the Phillips curve that came to be known as the expectations-augmented Phillips curve.[4] Friedman and Phelps were probably both inspired by the rapid increases in inflation occurring in the second half of the sixties for an only slightly reduced rate of unemployment as shown in Figure 18.2a. They suggested two related ideas.

The first one that leads to the expectations-augmented Phillips curve was that *an expected inflation will have a tendency to work itself into actual inflation on a one-for-one basis*, so that the 'correct' relationship (in simplified linear form) would really be:

$$\pi = \pi^e + \alpha(\bar{u} - u), \tag{2}$$

where π^e is the expected inflation rate. The reason lies in a logic very similar to the idea behind the 'Fisher effect' saying that expected inflation will work itself one-for-one into nominal interest rates.

Assume that everybody expects a rate of inflation of 10 percent. It then takes a nominal wage increase of 10 percent just to keep the real wage rate intact, so workers will see wage negotiations, or their labour supply behaviour, as 'starting', so to say, from a nominal wage increase of 10 percent. Only a deviation from that can create a change in the real wage. Employers will tend to accept the 10 percent wage increase as a starting point since on average they expect price increases on their products of 10 percent and thus will have their cost increases covered. If productivity increases by 2 percent, say, making room for a real wage increase of 2 percent, and the degree of pressure in the labour market is 'neutral' corresponding to $\alpha(\bar{u} - u) \approx 0$ above, then supply and demand behaviour in the labour market, or behaviour in wage negotiations, will tend to create a 12 percent increase in nominal wages. As a result, the employers' costs per unit will increase by 10 percent creating a rate of inflation of 10 percent. In this way, expected inflation builds itself into actual inflation. A positive labour market pressure $\alpha(\bar{u} - u) > 0$ would tend to create a wage increase larger than 12 percent and thereby a rate of inflation higher than π^e, but again this effects starts from the 10 percent of expected inflation.

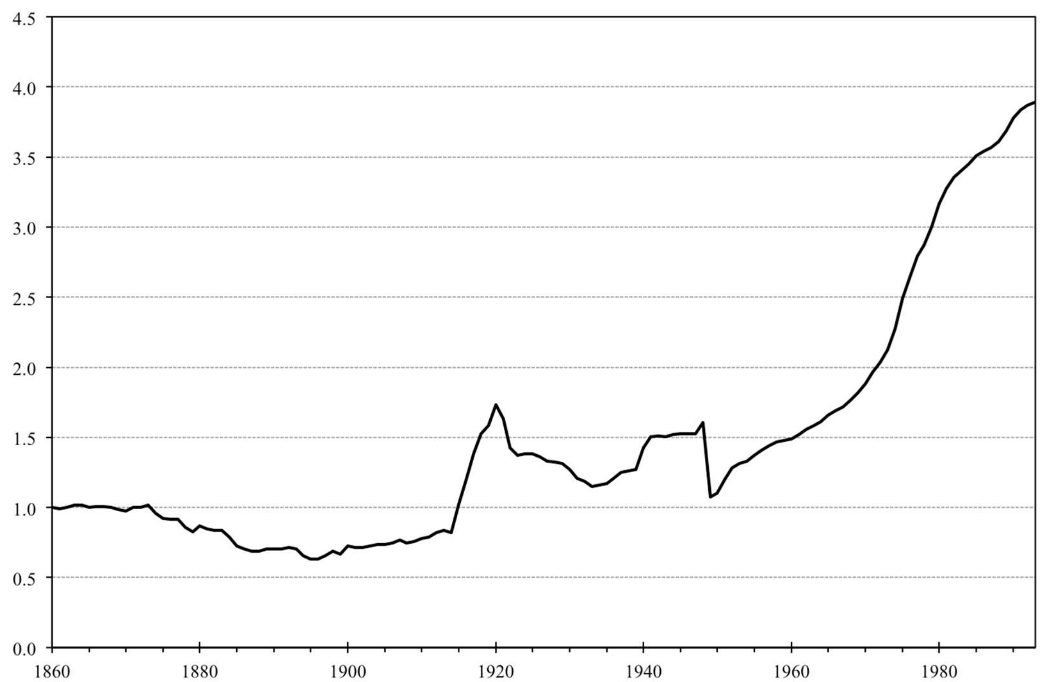
The second idea was that *an observed and longer lasting change in actual inflation will eventually work itself into expected inflation on a one-for-one basis*. If the actual and expected rate of inflation are first both 5 percent, and the actual rate of inflation increases permanently to 10 percent (and stays there), then at least after some time economic agents will realize what has happened and revise their expected rate of inflation from 5 to 10 percent.

Equipped with these two ideas we may now offer an explanation why the simple unemployment–inflation trade-off broke down in the USA and elsewhere in the OECD from around 1970. Over the long historical period considered by Phillips – from around 1860 to the 1950s – there was no systematic tendency for prices to rise by much for extended periods of time, as you can see from Figure 18.3. Except for war inflations and post war deflations at WWI

[4] Theories of the expectations-augmented Phillips curve were developed almost simultaneously by Friedman and Phelps: Milton Friedman, 'The Role of Monetary Policy', *American Economic Review*, **58**, 1968, pp. 1–17, and Edmund S. Phelps, 'Money-Wage Dynamics and Labor Market Equilibrium', *Journal of Political Economy*, **76**, 1968, pp. 678 – 711.

and WWII, the nominal consumer price level remains rather constant and is very close to unchanged from 1860 to 1950, from where an era of significant positive inflation begins. However, for a very long period up to the point in time where Phillips wrote his original article, the overall picture was very close to price stability.[5] Because of this long experience of approximate price stability, it was natural for economic agents to expect prices to be roughly constant. In such circumstances where $\pi^e \approx 0$, Equation (2) does indeed predict that a lower unemployment rate will always be associated with a higher inflation rate, and vice versa.

Figure 18.3 The natural log of the consumer price index in the UK, 1860 –1993, 1860 = 1



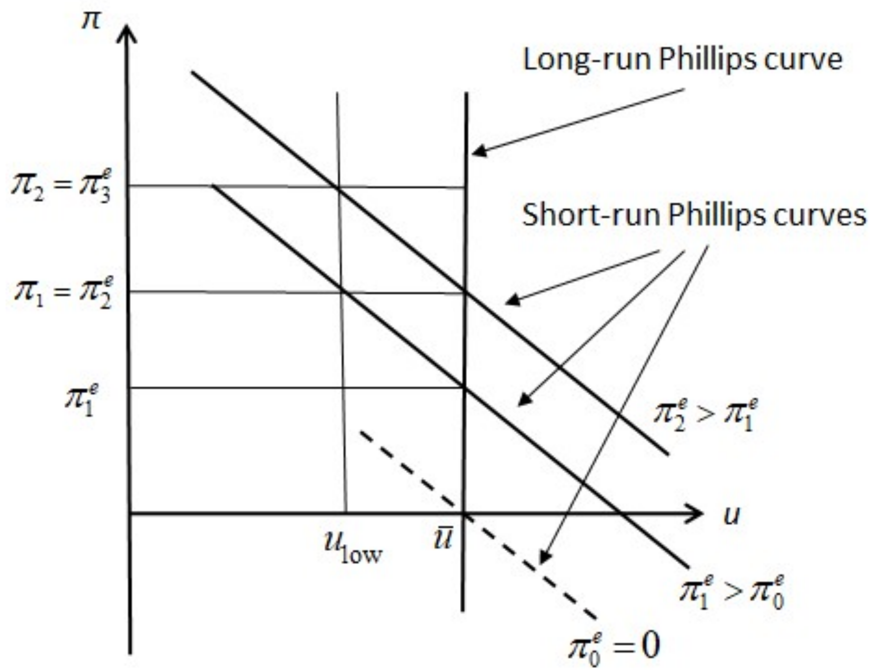
Note: The figure shows a normalization of the natural logarithm of the British consumer price index where year 1860 equals one.
Source: B.R. Mitchell, *International Historical Statistics: Europe 1750 –1993*, Macmillan Press, 1998.

However, towards the end of the 1960s inflation had been systematically positive and gradually rising for several years, so people started to consider a positive inflation rate as a normal state of affairs. Therefore, the expected inflation rate started to increase. According to (2) this tended to drive up the actual rate of inflation associated with any given level of unemployment, just as portrayed in Figure 18.2b which showed that many years during the 1970s were characterized by simultaneous increases in inflation and unemployment. There were also other reasons for these developments, such as dramatic increases in the price of oil due to turmoil in the Middle East, but rising inflation expectations probably played an important role in the breakdown of the simple Phillips curve from the end of the 1960s.

[5] The stability was supported by the UK being on some type of gold standard up to the early 1930s.

The implication of all this is that the simple negative Phillips curve relationship between inflation and unemployment is a *short-run* trade-off, which will hold only as long as the expected rate of inflation stays constant. For this reason, the simple downward-sloping Phillips curve (defined for a given expected rate of inflation) may also be called the *short-run* Phillips curve. Whenever the expected inflation rate π^e increases, the short-run Phillips curve will shift upwards, as illustrated in Figure 18.4, which shows three different short-run Phillips curves, each corresponding to different levels of expected inflation.

Figure 18.4 The expectations-augmented Phillips curve



Note from (2) that the short-run Phillips curve passes through the point $(\bar{u}, \pi^e)$, as illustrated in Figure 18.4, so if unemployment is at its natural rate actual inflation becomes equal to expected inflation. A lower rate of unemployment, as u_{low} in the figure, will create a higher than expected rate of inflation, as also illustrated, e.g., by the inflation rate $\pi_1 > \pi_1^e$. This means that *the short run trade-off between inflation and unemployment exists, but only as a result of erroneous expectations*.

Combining the two ideas of Friedman and Phelps implies that in the longer run there is no trade-of between inflation and unemployment. Assume that the expected inflation rate is the $\pi_1^e > 0$ shown in Figure 18.4, so the appropriate short-run Phillips cure is the one marked $\pi_1^e > \pi_0^e$. Assume further that the rate of unemployment is kept at the low level $u_{\text{low}} < \bar{u}$ (e.g., by expansionary monetary or fiscal policies) for a long time. Inflation will then be above the expected level π_1^e, as illustrated by $\pi_1 > \pi_1^e$. However, according to the second idea of Friedman this will imply that sooner or later the expected rate of inflation will shift up to the constant actual inflation rate π_1, so that the new expected rate of inflation will be $\pi_2^e = \pi_1 > \pi_1^e$. The short-run Phillips curve will then shift up to the one passing through $(\bar{u}, \pi_2^e)$, as illustrated, and, if unemployment is still at the low level u_{low}, actual inflation will be pushed up to the new higher level π_2. After some time this new and higher inflation will (again) work itself into expectations, the short-run Phillips curve will shift up once more and inflation will be pushed to an even higher level etc. As long as unemployment stays at the level below the natural rate, inflation will not only be high but *accelerating*, going eventually to infinity. This means that a lower rate of unemployment cannot really be 'bought' by accepting a higher level of inflation, only by accepting accelerating inflation.

Only one level of unemployment is compatible with a non-accelerating or non-decelerating rate of inflation in the long run, the structural level $\bar{u}$ which for this reason is often referred to as the NAIRU, the Non-Accelerating-Inflation-Rate-of-Unemployment. One may say that *in the long run the Phillips curve is vertical at the NAIRU*. Hence, there is no *permanent* trade-off between inflation and unemployment.

Here are some famous quotations of Milton Friedman summing up the ideas we have explained:

> ... there is always a temporary trade-off between inflation and unemployment; there is no permanent trade-off. The temporary trade-off comes not from inflation per se, but from unanticipated inflation, which generally means, from a rising rate of inflation … A rising rate of inflation may reduce unemployment, a high rate will not.
>
> But how long, you will say, is "temporary"? … the intial effects of a higher and unanticipated rate of inflation last for something like two to five years ...[6]

Another important implication of the expectations-augmented Phillips curve (2) is that there is *nominal inertia*: if unemployment is at its natural rate, the inflation prevailing today will automatically continue tomorrow because it is built into expectations. To bring down inflation, it is necessary to push the actual unemployment rate above the NAIRU for a while. To recap:

THE SHORT-RUN PHILLIPS CURVE, THE LONG-RUN PHILLIPS CURVE AND THE NAIRU

In the short run where the expected rate of inflation is roughly predetermined, there is a trade-off between inflation and unemployment, described by the short-run Phillips curve. However, when the actual inflation rate deviates from the expected inflation rate, the latter will adjust over time, causing the short-run Phillips curve to shift. If the expected inflation rate varies positively with the actual inflation rates observed in the past, inflation will accelerate (decelerate) whenever actual unemployment is lower (higher) than natural unemployment. Hence a long-run equilibrium with constant inflation can only be established when unemployment is at its natural rate, also referred to as the Non-Accelerating-Inflation-Rate-of-Unemployment (NAIRU). Thus the long-run Phillips curve is vertical.

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[6] Quoted from the article by Friedman mentioned in Footnote 4.

Our next step in this chapter is to develop a micro-founded theory of inflation and unemployment, which will indeed lead to, and thus give a foundation of, the expectations-augmented Phillips curve. Many roads lead to the expectations-augmented Phillips curve. Above we have seen one such road: the combination of Phillips' original idea of demand pull effects in the labour market and Friedman's idea that expected inflation tends to work itself into actual inflation. Now we will present a micro-founded theory consistent with the theories of structural unemployment presented in Part 4. This will make the assumptions underlying the theory very clear and will help us understand the determinants of the parameters α and $\overline{u}$ of the Philips curve.

In Section 18.2 we offer a theory of the expectations-augmented Phillips curve in line with the theory of trade union behaviour introduced in Chapter 12. However, we stress that the same qualitative results can be obtained from the theory of efficiency wages presented in Chapter 11, as demonstrated in an appendix to this chapter. Hence, a micro-founded expectations-augmented Phillips curve does not presume that structural unemployment (the $\overline{u}$ of the curve) is rooted in trade union behaviour, it may as well be rooted in efficiency wage effects.[7]

18.2 A micro-foundation of the expectations-augmented Phillips curve with basis in trade union theory

Inflation is a continuous rise in the general price level. A theory of inflation therefore requires a theory of price formation. Since prices depend on the cost of inputs, and since labour is the most important input, our theory of price formation will build on a theory of wage formation. The theory will allow for imperfect competition in the markets for goods as well as labour. Introducing imperfect competition in output markets complicates the analysis, but in return it enables us to illustrate how structural changes in product markets affect inflation and the natural rate of unemployment.

In Book One, where we focused on the long run, we assumed that agents had correct expectations about the general level of wages and prices, as must be the case in any long-run equilibrium. By contrast, in the present short-run context we assume that people do not have perfect information about the current general price level. As we shall see, this means that employment and output may deviate from their long-run equilibrium levels.

This section assumes that nominal wages are 'sticky' in the short run, being pre-set by trade unions. Thus, a description of trade union behaviour and wage formation will be central to our analysis.

The overall framework

Just as in Chapter 11 and 12, we will consider a macro framework of monopolistic competition where each firm produces a differentiated product that is not quite the same as the product of any other firm. Each firm therefore is a monopolist in its own production sector. Its market power

[7] The exposition in this chapter does not assume that you have studied Chapter 11 or 12, so you should still be able to understand all parts of the present chapter even if you have not had the opportunity to go through Book One.

may nevertheless be limited because there is competition between the different products arising from the substitution possibilities of the consumers. If one firm raises its price, it will lose some demand because the consumers will substitute towards other and now relatively cheaper products. Although each firm may be large in its own market (having there some market power), it is assumed to be small in the entire economy taking all aggregates, e.g., total production and the general price level, as given.

A number of workers are connected to each production sector and are organized in the trade union of the sector. The trade union determines the nominal wage rate of the sector, whereas the firm determines how many workers to hire at that rate. Thus each trade union is large in its own sector but small relative to the economy as a whole, so it takes all aggregates as given. Unlike our assumptions in Chapter 11 and 12, where we studied the long run, but in line with Chapter 1 and 14 about the short run, we will here assume nominal rigidities in wages and sometimes prices, so that these are preset before a period and remain fixed within the period.

The details of our framework are as follows. There are n production sectors each containing one firm. The representative firm in sector i uses a technology described by the production function:

$$Y_i = BL_i^{1-\alpha}, \quad B > 0, \quad 0 < \alpha < 1, \tag{3}$$

where α and B are parameters, Y_i is the volume of real output produced and sold, and L_i is labour input in sector i. For simplicity, we assume that the number of working hours for the individual worker is fixed, so total labour input is proportional to the number of workers employed. Since we are concentrating on the short run where the capital stock is fixed, we have not included capital explicitly in the production function.[8] According to (3), the marginal product of labour is:

$$MPL_i \equiv \frac{\partial Y_i}{\partial L_i} = (1-\alpha)BL_i^{-\alpha}, \tag{4}$$

which is seen to diminish as labour input increases, due to the fixity of the capital stock. The marginal cost in sector i is W_i / MPL_i, where W_i is the nominal wage rate in sector i:

$$MC_i = \frac{W_i}{MPL_i} = \frac{W_i}{(1-\alpha)B} L_i^{\alpha}, \tag{5}$$

which increases with labour input and production in sector i.

We assume that the firm of sector i faces a demand curve of the form:

$$Y_i = \left(\frac{P_i}{P}\right)^{-\sigma} \frac{Y}{n}, \quad \sigma > 1, \tag{6}$$

[8] In Book One we worked with the Cobb–Douglas production function $Y = BK^{\alpha}L^{1-\alpha}$. Equation (3) is just a version of this production function where we have fixed the capital stock K at unity.

where σ is a parameter, P_i is the nominal price per unit of output set by the firm in sector i, and P is the general price level, that is, an appropriate average of the nominal prices of all sectors.[9] This demand curve has a constant numerical price elasticity of demand equal to $\sigma = -(dY_i/dP_i)(P_i/Y_i)$. The variable Y is total GDP, and n is the number of different sectors in the economy. Aggregate output Y is a measure of the total size of the national market, and Y/n is the market share captured by each industry if they all charge the same prices (so that $P_i = P$).

In each production sector there is the same number of workers, a number we will normalize to one. The trade union of sector i sets the nominal wage rate W_i which is obtained by the workers who find employment in the sector. Workers in sector i are educated and trained to work in that particular sector, so they cannot move to another sector to look for a job. A worker who fails to find a job in his or her sector therefore becomes unemployed and will then obtain a real income equal to the real rate of unemployment benefit b, which is assumed to be fixed in real terms through indexation to the price level P.[10] An employed worker in sector i earns the real wage $w_i \equiv W_i/P$, so his or her *net* income gain from being employed is $w_i - b$.

The trade union for sector i cares about this real income gain for its employed members, but it also cares about the total number of jobs L_i secured for the membership. We formalize this by assuming that the union sets the nominal wage rate with the purpose of maximizing a utility function Ω of the form:

$$\Omega = (w_i - b)L_i^{\eta}, \quad \eta > 0 \tag{7}$$

The parameter η reflects the weight that the union attaches to high employment relative to the goal of a high real wage for employed union members. The more the union is concerned about employment relative to wages, the higher is the value of η. In the benchmark case where $\eta = 1$ (corresponding to the case analysed in Chapter 12), the union is simply interested in the aggregate net income gain obtained by employed members or, what amounts to the same, the ex-ante expected income by a representative union member.

When setting the wage rate, the union must account for the fact that a higher real wage will lower the employer's demand for labour. Our next step is to derive this constraint on the union's optimization problem, that is, the labour demand curve for each sector. To derive this we must first consider the optimal price determination of each firm.

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[9] The demand curves of form (6) are here simply taken as given, but can, as also explained in Chapter 11, be derived from optimization of a consumer side with a constant elasticity of substitution (CES) utility function. This was the subject of an exercise for Chapter 11 which showed that the parameter σ has the interpretation of the elasticity of substitution between any two products. The exercise also derived the appropriate average formula for the price index P. The exact formula is not of importance for our purposes here, however.

[10] In Chapters 11 and 12, where we focused on the long run, we assumed that workers who fail to find a job in their initial sector have time to retrain so that they can move into other sectors to look for alternative employment opportunities. In that case the expected real income obtainable by a sector i worker who loses his initial job would be $v = (1-u)w + ub$, where u is the general unemployment rate, w is the average real wage in the economy, and where the employment rate $1-u$ represents the probability of finding a job outside sector i. In Exercise 1 you are asked to consider such a case with intersectoral labour mobility and to show that this case leads to the expectations-augmented Phillips curve as well.

Price setting and the labour demand curve

The total revenue of firm i is $TR_i \equiv P_iY_i$, so, using (6), its marginal revenue (the increase in total revenue from selling an extra unit of output) will be:

$$MR_i \equiv \frac{dTR_i}{dY_i} = P_i + \frac{dP_i}{dY_i}Y_i = P_i\left(1+\frac{dP_i}{dY_i}\frac{Y_i}{P_i}\right) = P_i\left(1-\frac{1}{\sigma}\right). \tag{8}$$

From microeconomic theory we know that a profit-maximizing firm will expand output to the point where marginal revenue equals marginal cost, $MR_i = MC_i$. Hence from (8), the necessary condition for maximization of profits, $MR_i = MC_i$, becomes:

$$P_i\frac{\sigma-1}{\sigma} = MC_i \quad \text{or} \quad P_i = \frac{\sigma}{\sigma-1}MC_i \tag{9}$$

Equation (9) shows that the profit-maximizing representative firm in sector i will set its price as a mark-up over its marginal cost. Our previous assumption $\sigma > 1$ guarantees that the mark-up factor $m^p = \sigma/(\sigma-1)$ is positive and greater than one. The price elasticity, σ, is a measure of the strength of product market competition. The higher the elasticity, the greater is the fall in demand induced by a higher price (the flatter is the demand curve), and the lower is the mark-up of price over marginal cost. In the limiting case where the price elasticity tends to infinity, the price is driven down to the level of marginal cost ($\sigma \to \infty$ implies that $m^p \to 1$), corresponding to perfect competition.

Inserting here the expression for MC_i of (5) into (9) gives:

$$P_i = m^p\frac{W_i}{(1-\alpha)B}L_i^{\alpha} \quad \Rightarrow \quad \frac{P_i}{P} = m^p\frac{L_i^{\alpha}}{(1-\alpha)B}\frac{W_i}{P} \tag{10}$$

We can now derive the labour demand curve of sector i, showing the relationship between the real wage W_i/P claimed by the union in sector i and the level of employment in that sector. Inserting the expression for P_i/P of (10) into the demand curve (6) gives sector i production as:

$$Y_i = \left(m^p\frac{L_i^{\alpha}}{(1-\alpha)B}\frac{W_i}{P}\right)^{-\sigma}\frac{Y}{n} \tag{11}$$

Finally, we can compute how much employment, L_i, is needed to produce this level of output by equating the Y_i of (3) with the Y_i of (11),

$$BL_i^{1-\alpha} = \left(m^p\frac{L_i^{\alpha}}{(1-\alpha)B}\frac{W_i}{P}\right)^{-\sigma}\frac{Y}{n}, \tag{12}$$

and solve for L_i. This gives:

$$L_i = \left(\frac{Y}{nB}\right)^{\varepsilon/\sigma} \left(\frac{(1-\alpha)B}{m^p}\right)^{\varepsilon} \left(\frac{W_i}{P}\right)^{-\varepsilon} \equiv L_i\left(\frac{W_i}{P}\right), \quad \varepsilon \equiv \frac{\sigma}{1+\alpha(\sigma-1)} > 1 \tag{13}$$

This is the labour demand curve for sector *i*, giving labour demand as a function of the real wage rate, W_i / P. The numerical real wage elasticity of labour demand at the sectoral level, defined as $-(dL_i/d(W_i/P))((W_i/P)/L_i)$, is equal to the constant ε. From the expression for ε it follows that $\varepsilon > 1$ as claimed in (13) and that a higher numerical price elasticity of product demand (tougher competition in product markets) increases the wage elasticity of sectoral labour demand and thereby diminishes the degree of market power of the trade union in sector i.[11] This is intuitive: a rise in the wage rate will drive up the output price by raising the firm's marginal cost. The higher the price elasticity of output demand, the greater is the fall in sales and output, so the greater is the resulting fall in labour demand.

Wage setting

The labour demand curve (13) implies that employment in sector *i* is a declining function, $L_i(w_i)$, of the real wage in that sector, $w_i \equiv W_i/P$. The union's utility function (7) may therefore be written as:

$$\Omega(w_i) = (w_i - b)[L_i(w_i)]^{\eta} \tag{14}$$

The union will want to set a nominal wage W_i so as to obtain a real wage $w_i = W_i / P$ that maximizes $\Omega(w_i)$. In Chapter 12, when we studied the long run, we assumed that in doing so, the union knew the actual price level *P*. This is a natural long-run assumption. Now when we are studying the short run we will *at first* make this assumption again, but afterwards relax it. So, suppose for the moment that the union has perfect information about the current price level and perfectly controls the real wage $w_i \equiv W_i/P$ via its control of the money wage W_i. The union will choose w_i so as to maximize $\Omega(w_i)$. The necessary condition for maximizing $\Omega(w_i)$ with respect to w_i, $\partial\Omega(w_i)/\partial w_i = 0$, is $L_i^{\eta} + (w_i - b)\eta L_i^{\eta-1}(dL_i/dw_i) = 0$, which is equivalent to:

$$1 + \frac{\eta(w_i - b)}{w_i}\left(\frac{dL_i}{dw_i}\frac{w_i}{L_i}\right) = 0.$$

Using the fact that $(dL_i/dw_i)(w_i/L_i) = -\varepsilon$, we may rewrite this expression as:

$$w_i = m^w \cdot b, \qquad m^w \equiv \frac{\varepsilon\eta}{\varepsilon\eta - 1} \tag{15}$$

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[11] Dividing through by σ in the expression for ε, we get $\varepsilon = 1/\left[(1-\alpha)\sigma^{-1} + \alpha\right]$. The denominator in this expression is less than 1, since $0 < \alpha < 1$ and $\sigma^{-1} < 1$. Hence ε will exceed 1 and will be larger, the higher the value of σ.

According to (15) the union's target real wage is a mark-up over the opportunity cost of employment. The opportunity cost of employment is the rate of unemployment benefit b, since this is the income a worker forgoes by being employed rather than unemployed. To secure that (15) actually implies a positive real wage, we assume that $\varepsilon\eta > 1$ (note that since $\varepsilon > 1$, an assumption of $\eta \geq 1$ will indeed imply that $\varepsilon\eta > 1$). The wage mark-up factor, m^w, is then greater than 1. The target money wage rate for the union is, of course, $W_i = m^w b \cdot P$.

Equation (15) implies that the union's real wage claim will be lower the greater the weight it attaches to the goal of high employment, i.e., the higher the value of η. It also follows from (15) that the target real wage will be lower the higher the elasticity of labour demand, ε. The reason is that a higher labour demand elasticity increases the loss of jobs resulting from any given increase in the real wage. Finally, we see from (15) that a higher rate of unemployment benefit drives up the target real wage because it reduces the income loss incurred by those union members who lose their jobs when the union charges a higher wage rate.

We have so far assumed that the union has perfect information about the current price level and therefore perfectly controls the real wage W_i/P through its control of the money wage rate, W_i. However, in practice, nominal wage rates are almost always *pre-set* for a certain period of time, that is, in the short run the nominal wage rate is *rigid*. Moreover, at the start of the period when wages are set, trade union leaders cannot perfectly foresee the price level P which will prevail over the period during which the nominal wage rate will be fixed by the wage contract. A trade union setting the wage rate for the current period must therefore base its money wage claim on its *expectation* of the price level, P^e, that will prevail in the current period, where the expectation is formed in the period before the current one. Given that the union strives to obtain the real wage specified in (15), it will then set the *money* wage rate to achieve an *expected* real wage equal to the target real wage $m^w b$.[12] The nominal wage rate set by the union at the start of the period will thus be:

$$W_i = m^w b \cdot P^e \tag{16}$$

Having developed a theory of wage and price setting as well as a theory of labour demand, we are now ready to derive the link between inflation and unemployment.

The expectations-augmented Phillips curve

Equation (16) implies that the *actual* real wage may be written as:

$$\frac{W_i}{P} = m^w b \cdot \frac{P^e}{P} \tag{17}$$

[12] We assume for simplicity that the union has a correct estimate of the level of b. For example, we may assume that the nominal rate of unemployment benefit is automatically indexed to the current price level to protect its real value. The union will then be able to forecast the level of the real rate of unemployment benefit even if it cannot perfectly foresee the price level. In Exercise 1 you are asked to consider the alternative case where the union does not have perfect information about the real value of the nominal rate of unemployment benefit.

This means that if the price level turns out to be higher than expected, $P > P^e$, then the actual real wage will be lower than the union's target, $W_i / P < m^w b$, because the union and the workers are surprised by a higher than expected price level eroding the purchasing power of the nominal wages. Inserting the expression for the real wage rate of (17) into the labour demand curve (13), we obtain the level of employment in sector *i*:

$$L_i = \left(\frac{Y}{nB}\right)^{\varepsilon/\sigma} \left(\frac{(1-\alpha)B}{m^p}\right)^{\varepsilon} \left(m^w b \cdot \frac{P^e}{P}\right)^{-\varepsilon} = \left(\frac{Y}{nB}\right)^{\varepsilon/\sigma} \left(\frac{(1-\alpha)B}{m^p m^w b}\frac{P}{P^e}\right)^{\varepsilon} \tag{18}$$

The higher the actual price level relative to the expected price level, P/P^e, that is, the more the trade union underestimates the price level, the lower is its nominal wage claim relative to the actual price level, so the lower is the real wage and the higher is the level of sectoral employment.

We will now show that a similar qualitative relationship between employment and the ratio of actual to expected prices will prevail at the aggregate level. Recall that we have assumed that all sectors in the economy are *symmetric* so that output and employment in each sector are given by Equations (3) and (18), respectively, where all the parameters as well as the ratio P/P^e are the same across sectors. This implies from (18) that L_i must be the same in all sectors and hence total employment, L, will be $L = nL_i$, and total GDP will be $Y = nY_i = nBL_i^{1-\alpha}$. Substituting the latter expression into (18) gives:

$$\begin{aligned} &L_i = \left(L_i^{1-\alpha}\right)^{\varepsilon/\sigma} \left(\frac{(1-\alpha)B}{m^p m^w b}\frac{P}{P^e}\right)^{\varepsilon} \quad \Leftrightarrow \quad L_i^{1-(1-\alpha)\frac{\varepsilon}{\sigma}} = \left(\frac{(1-\alpha)B}{m^p m^w b}\frac{P}{P^e}\right)^{\varepsilon} \quad \Leftrightarrow \\ &L_i = \left(\frac{(1-\alpha)B}{m^p m^w b}\frac{P}{P^e}\right)^{\frac{\varepsilon}{1-(1-\alpha)\frac{\varepsilon}{\sigma}}} = \left(\frac{(1-\alpha)B}{m^p m^w b}\frac{P}{P^e}\right)^{\frac{1}{\frac{1}{\varepsilon}-\frac{1-\alpha}{\sigma}}} \end{aligned} \tag{19}$$

For the latter exponent on the right hand side we get (miraculously) by using the expression for ε of (13):

$$\frac{1}{\frac{1}{\varepsilon} - \frac{1-\alpha}{\sigma}} = \frac{1}{\frac{1+\alpha(\sigma-1)}{\sigma} - \frac{1-\alpha}{\sigma}} = \frac{1}{\alpha} \tag{20}$$

Inserting into (19) and computing $L = nL_i$ then gives for total employment:

$$L = nL_i = n \cdot \left(\frac{(1-\alpha)B}{m^p m^w b} \cdot \frac{P}{P^e}\right)^{\frac{1}{\alpha}}, \tag{21}$$

Note that since according to (17), the real wage is the same in all sectors and equal to $m^w b(P^e/P)$, we may define the general real wage as $w \equiv W/P = m^w b(P^e/P)$ and write (21) as:

$$L = n\left(\frac{(1-\alpha)B}{m^p}\right)^{1/\alpha}\left(\frac{W}{P}\right)^{-\frac{1}{\alpha}}. \tag{22}$$

This expression shows that at the *aggregate* level the numerical real wage elasticity of labour demand is $1/\alpha$, whereas at the level of the individual sector we found it to be equal to $\varepsilon = \sigma/[1+\alpha(\sigma-1)]$, which is smaller than $1/\alpha$. In Exercise 2 we ask you to provide an intuitive explanation for this difference between the labour demand elasticities at the macro and the micro levels.

Structural employment is the level of employment in a long-run equilibrium where expectations are fulfilled. Inserting $P^e = P$ into (21), we therefore obtain the long-run equilibrium level of aggregate employment, $\bar{L}$, also called the 'natural' level of employment:

$$\bar{L} = n\left(\frac{(1-\alpha)B}{m^p m^w b}\right)^{\frac{1}{\alpha}} \tag{23}$$

Equation (23) gives the level of employment which will prevail when price expectations are correct so that trade unions actually obtain their target real wage. Dividing (21) by (23), we get a simple relationship between the actual and the natural level of employment:

$$\frac{L}{\bar{L}} = \left(\frac{P}{P^e}\right)^{\frac{1}{\alpha}} \tag{24}$$

A price level above the expected one will drive employment above the natural level. You should try to explain why for yourself. The aggregate labour force has been normalized to n (remember that we chose to measure labour in units of the amount of labour in one sector, so the number of workers in each sector is 1, and then the total amount of labour equals the number of sectors). If we denote the unemployment rate by u, it follows by definition that $L = (1-u)n$. Similarly, the 'natural' unemployment rate, $\bar{u}$, is defined by the relationship $\bar{L} \equiv (1-\bar{u})n$.[13] Substitution of these identities into (24) gives:

$$\frac{1-u}{1-\bar{u}} = \left(\frac{P}{P^e}\right)^{\frac{1}{\alpha}} \tag{25}$$

Taking natural logarithms on both sides and using the approximation $\ln(1-u) \approx -u$ etc. we get:

$$\bar{u} - u = \frac{1}{\alpha}\left(\ln P - \ln P^e\right) \quad \Leftrightarrow \quad (\ln P - \ln P_{-1}) - (\ln P^e - \ln P_{-1}) = \alpha(\bar{u} - u), \tag{26}$$

where the subscript '-1' indicates that the variable in question refers to the previous time period. Defining the actual and expected rates of inflation from the previous to the current period,

[13] For simplicity, we do not distinguish between the current labour force and its trend level.

$\pi \equiv \ln P - \ln P_{-1}$ and $\pi^e \equiv \ln P^e - \ln P_{-1}$, respectively, finally gives:

$$\pi = \pi^e + \alpha(\bar{u} - u). \tag{27}$$

Equation (27) has exactly the form of an *expectations-augmented Phillips curve* like (2) providing a link between inflation and unemployment. We can read the expectations-augmented Philips curve ‘both ways’. One way is that increased inflation for given inflation expectations implies lower real wages (because money wages are pre-set in view of expected inflation and fixed) leading firms with decreasing marginal productivity of labour to demand more labour and thus increase production and employment, which lowers unemployment. The other way is that a shock to the economy that brings unemployment down and output and employment up drives the firms further out on their increasing marginal cost curves leading them (as expressed buy (10)) to increase their prices thereby, all in all, increasing inflation from the previous period.

The expectations-augmented Philips curve (27) shows, as we also discussed above, that *unanticipated* inflation, that is, $\pi > \pi^e$, and *only unanticipated inflation* (not an anticipated higher level of inflation), can drive unemployment below its natural rate. The reason is that an unexpected rise in the rate of inflation causes the real value of the pre-set money wage rate to fall below the target real wage of trade unions, thereby inducing firms to expand employment beyond the natural level.

Since we already had the expectations-augmented Phillips curve in hand, you may think that the derivations of this section have been a long journey, but note that now we have established a link between the α and the $\bar{u}$ of the Phillips curve and the fundamentals of the underlying economic model. The slope parameter α should be 1 minus the output elasticity with respect to labour input in the production function (often considered to be around 1/3), and we will soon return to the determinants of $\bar{u}$, but before that let us take stock of what we have learned so far:

THE EXPECTATIONS-AUGMENTED PHILLIPS CURVE AND THE NATURAL RATE HYPOTHESIS

According to the expectations-augmented Phillips curve, the actual rate of inflation varies positively and one-to-one with the expected rate of inflation and negatively with the excess of the actual rate of unemployment over the natural rate of unemployment. The natural rate of unemployment is the long-run equilibrium level of unemployment ensuring that actual and expected inflation coincide so that workers obtain their target real wages.

What determines the natural rate of unemployment?

It is obvious that the natural rate of unemployment plays an important role in our theory of inflation, given that inflation will tend to rise if unemployment is pushed below that level. But what determines the natural rate? Our expression (23) for the natural level of employment provides the key to an answer. Recall that by definition, $\bar{L} \equiv (1-\bar{u})n.$ Inserting this into (23) and rearranging, we get:

$$\bar{u} = 1 - \left(\frac{B(1-\alpha)}{m^p m^w b} \right)^{1/\alpha} \tag{28}$$

Equation (28) shows that the natural unemployment rate depends on the level of the real unemployment benefit, b, among other things.

It is reasonable to assume that the government allows the rate of unemployment benefit to grow in line with real income per capita, at least over the longer run. We will therefore assume that the level of unemployment benefits is tied to the level of GDP per worker for a given (the natural) level of employment so that $b = cB$, where $c > 0$ is a parameter reflecting the generosity of the system of unemployment compensation. Substituting cB for b in (28), we get the following expression for the natural rate of unemployment, where we assume that the combination of parameter values ensures a positive value of $\bar{u}$:

$$\bar{u} = 1 - \left(\frac{(1-\alpha)}{m^p m^w c} \right)^{1/\alpha} \tag{29}$$

According to (29) the natural unemployment rate is higher the higher the mark-ups in wage and price setting, and the more generous the level of unemployment benefits (the higher the value of c). A rise in $m^p = \sigma/(\sigma - 1)$ reflects a fall in the representative firm's numerical price elasticity of demand (σ) which means that it takes a larger cut in the firm's relative price P_i/P to obtain a given increase in sales. To sell the extra output produced by an extra worker, the firm must therefore accept a larger price cut the lower the value of σ. For any given wage level, the profit-maximizing level of employment will thus be lower the lower the value of σ. This is why the natural unemployment rate will be higher the higher the mark-up factor m^p.

A fall in σ will also increase the wage mark-up, since the sectoral labour demand elasticity $\varepsilon \equiv \sigma/[1 + \alpha(\sigma - 1)]$ is increasing in σ, and since $m^w = \eta\varepsilon/(\eta\varepsilon - 1)$ is decreasing in ε. The intuition for this rise in the wage mark-up is that a lower price elasticity of demand for the output of the representative firm reduces the drop in sales and employment occurring when a higher union wage claim drives up the firm's marginal cost and price. Hence it becomes less costly (in terms of jobs lost) for the union to push up the wage rate, and this invites more aggressive wage claims.

The representative firm's price elasticity of demand reflects the degree of competition in product markets. The greater the substitutability of the products of different firms, the tougher competition will be, and the greater will be the price elasticity of demand faced by the individual firm or industry. Thus, our analysis shows that a lower degree of competition in product markets (a lower σ) will spill over to the labour market and raise the natural rate of unemployment, partly because it lowers labour demand, and partly because it induces more aggressive wage claims. This is an interesting example of how imperfections in some markets may exacerbate imperfections in other markets.

It is worth noting two more points from (29). First, a greater union concern about employment, reflected in a higher value of the parameter η, will reduce the natural rate of unemployment by lowering the wage mark-up $m^w = \eta\varepsilon/(\eta\varepsilon - 1)$. Second, the level of productivity B does not affect the natural rate of unemployment. This prediction is in line with empirical observations. As illustrated by Figure 10.1, the unemployment rate tends to fluctuate around a constant level over the very long run despite the fact that productivity is steadily growing over time. However, as we shall see later in this section, short-run fluctuations in productivity growth do affect the short-run unemployment–inflation trade-off.

Let us restate these important points:

DETERMINANTS OF THE NATURAL UNEMPLOYMENT RATE

The natural rate of unemployment increases with the mark-ups in wage and price setting and with the replacement rate in the system of unemployment compensation. A greater union concern about employment relative to real wages reduces natural unemployment by reducing the wage mark-up. Stronger competition in product markets, reflected in a higher price elasticity of output demand in each production sector, lowers the natural unemployment rate in two ways. First, it reduces the optimal mark-up of price over marginal cost in the representative firm. Second, it reduces wage mark-ups because a higher elasticity of output demand increases the wage elasticity of labour demand, inducing unions to moderate their wage claims.

Nominal price rigidity

For simplicity, we have so far assumed that while nominal wages are rigid in the short run, all output prices adjust immediately, although imperfectly competitively, to changes in demand and marginal costs. This is a way of capturing the stylized fact that nominal wages tend to be fixed for longer periods than most goods prices. However, in reality many output prices are also held constant for considerable periods, as we noted in Chapter 1. In that chapter we also saw that small 'menu costs' of price adjustment – such as the costs of printing new price catalogues and communicating new prices to customers – may make it suboptimal for firms to adjust prices too frequently. In this subsection, we sketch how our model of wage and price formation may be extended to allow for nominal price stickiness. As we shall see, this extension does not necessarily alter the *qualitative* properties of the expectations-augmented Phillips curve, but it does change its *quantitative* properties in a potentially important way.

We will rely on an informal exposition. Nominal price rigidity is assumed to resemble nominal wage rigidity with pre-set nominal prices one period ahead: a price setter subject to the nominal rigidity has to decide the price, P_i^{fix} say, for the current period at the end of the previous period and the price stays fixed during the current period. To capture the fact that different products are characterized by different degrees of price rigidity, we assume that a fraction ω of firms are 'flex-price' firms and not subject to nominal price rigidity while a fraction $1-\omega$ are 'sticky-price' firms and have to pre-set their prices one period ahead, $0 \leq \omega < 1$.

Now consider a positive demand shock to the economy that increases the aggregate demand for goods and services. Both fix-price and flex-price firms will then hire more labour and increase their production to meet the increase in demand. Along with the increase in their output, the flex-price firms will raise their nominal prices as their marginal costs go up. The fix-price firms will not change their nominal price, of course, but since they have already set their (fixed) nominal price above their expected marginal cost MC_i^e at their expected level of output, that is, $P_i^{\text{fix}} = m^P MC_i^e > MC_i^e$, it will still be profitable for them to expand production as long as $P_i^{\text{fix}} > MC_i$, that is, as long as their actual marginal cost MC_i does not rise too much above their expected marginal cost.

Hence, both types of firms will expand output (although to different degrees), but only the fraction ω of flex-price firms will also increase their price. Intuitively we will therefore expect the price response to be the fraction ω of what it would be if all firms were of the flex-price type. Accordingly the expectations-augmented Phillips curve should be:

$$\pi = \pi^e + \omega \cdot \alpha(\bar{u} - u). \qquad (30)$$

We see that this equation has the same form as our previous expectations-augmented Phillips curve (27) except that the introduction of nominal price rigidity lowers the slope coefficient on the unemployment gap from α to $\omega\alpha$. Obviously, the size of ω has an impact on the sensitivity of inflation to the unemployment gap and the slope of the Phillips curve is thus no longer completely determined by (one minus) the elasticity of output with respect to labour input in the production function, but can be smaller than that. We may conclude:

NOMINAL PRICE RIGIDITY AND THE PHILLIPS CURVE

Because of 'menu costs', many firms do not change their prices instantaneously in response to cost changes; instead they pre-set their prices for some period ahead. Allowing for such nominal price rigidity does not change the qualitative properties of the expectations-augmented Phillips curve, but reduces the slope of the short-run Phillips curve.

We will thus proceed in this chapter and in the book with Phillips curves like (27) or (30), where the expected inflation rate π^e concerns inflation from the preceding to the current period. This reflects the type of nominal rigidity we take as our basis, where wages and prices are only pre-set one period ahead.

However, in practice wage and price setters may actually adjust their wages and prices at unsynchronized points in time and keep them fixed for different amounts of time that could exceed just one period. For agents with large transactions and menu costs of wage or price adjustment, it may be optimal to keep nominal wages or prices fixed for several periods before adjusting them to changes in the general price level. These agents must form expectations of inflation several periods ahead and set their wages and prices accordingly in order not to be stuck with a fixed nominal wage or price that is much too low or much too high relative to the future cost of living or the future cost of production. Hence, also the expected future rate of inflation, π^e_{+1}, may matter for current wage and price setting and thus for the current actual rate of inflation. If a fraction θ of price setters keep prices fixed for just one period (group 1 firms), and a fraction $1-\theta$ maintain fixed prices for more than one period (group 2 firms), a plausible Phillips curve might look like $\pi = \theta\pi^e + (1-\theta)\pi^e_{+1} + \alpha(\bar{u} - u)$, where π^e is the expected rate of inflation between the preceding and the current period, and π^e_{+1} is the inflation rate expected to prevail over the average number of future periods during which group 2 firms plan to keep their prices fixed.

Furthermore, if wages and prices on average stay fixed for a considerable period after having just been set, then probably the future expected inflation rate is much more important than the current one. In this case where the parameter θ is close to one, the relevant Phillips curve would be (approximately) like:

$$\pi = \pi^e_{+1} + \alpha(\bar{u} - u). \tag{31}$$

This is a (slightly modified) version of the so-called New Keynesian Phillips Curve (NKPC) that is often a part of modern short-run New Keynesian macroeconomic models.[14] However, including forward expected inflation π^e_{+1} in the Phillips curve complicates the models considerably and is therefore saved for your later courses in more advanced macroeconomics. Here and in the coming chapters where we study full macroeconomic models, we make the simplification that only current expected inflation enters the Phillips curve as in (27) or (30).

Supply shocks

Our expectations-augmented Phillips curve as derived so far, $\pi = \pi^e + \alpha(\bar{u} - u),$ postulates a strict deterministic relation between the unemployment rate u and the amount of unanticipated inflation $\pi - \pi^e$. In this section we shall see that the link between these two variables is not really that tight. The reason is that the labour market is frequently hit by shocks which generate 'noise' in the relationship between unemployment and inflation. These so-called supply shocks take the form that the productivity variable B can deviate from its long-run trend level $\bar{B}$, and likewise the mark-up factors m^p and m^w can fluctuate around their long-run trend levels, $\bar{m}^p$ and $\bar{m}^w$, due to possible fluctuations in the underlying parameters α, σ or η or just 'errors' in price and wage setting. We now extend our model of unemployment and inflation to account for supply shocks.

We will assume that the rate of unemployment benefit, b, is itself structural (always at its structural level), which follows if it is tied to the underlying *trend* level of productivity and not to the actual B, for instance through the indexation rule $b = c\bar{B},\ c > 0$. This is indeed more natural and realistic than a rule $b = cB$ which would imply that short-run fluctuations in productivity would give rise to similar fluctuations in the benefit rate.

Equation (21), repeated here for convenience,

$$L = nL_i = n \cdot \left(\frac{(1-\alpha)B}{m^p m^w b} \cdot \frac{P}{P^e} \right)^{\frac{1}{\alpha}}, \tag{21}$$

[14] According to the sophisticated microeconomic theory underlying the NKPC, the coefficient on the expected future inflation rate π^e_{+1} in (31) should actually be slightly less than one, reflecting that firms discount their future profits. However, this detail is not important here. The theoretical foundations of the NKPC were laid by the Argentine-American economist Guillermo Calvo in Calvo, G. (1983): "Staggered prices in a utility-maximizing framework", *Journal of Monetary Economics*, **12** (3), 383-398.

gives actual employment, L, as a function of the surprise factor, P/P^e, and the actual values of the productivity variable and the mark-up factors, B, m^p and m^w (and other parameters). We now define natural employment as the level of employment, which will prevail when expectations are fulfilled *and* when productivity as well as the price and wage mark-ups are all at their ‘normal’ long-run trend levels. It follows from (31) that the natural employment level previously stated in (23) now modifies to:

$$\bar{L} = n \cdot \left(\frac{(1-\alpha)\bar{B}}{\bar{m}^p \bar{m}^w b} \right)^{\frac{1}{\alpha}} \tag{32}$$

Dividing (31) by (32) and using again $L \equiv (1-u)n$ and $\bar{L} \equiv (1-\bar{u})n$, we get:

$$\frac{L}{\bar{L}} = \frac{1-u}{1-\bar{u}} = \left(\frac{B}{\bar{B}} \cdot \frac{\bar{m}^p}{m^p} \cdot \frac{\bar{m}^w}{m^w} \cdot \frac{P}{P^e} \right)^{\frac{1}{\alpha}} \tag{33}$$

Taking logs on both sides of the second equality in (33) and using again $\ln(1 - u) \approx -u$ etc. plus the definitions $\pi \equiv \ln P - \ln P_{-1}$ and $\pi^e \equiv \ln P^e - \ln P_{-1}$, we end up with:

$$\pi = \pi^e + \alpha(\bar{u} - u) + \tilde{s}, \quad \tilde{s} \equiv \ln\left(\frac{m^p}{\bar{m}^p} \right) + \ln\left(\frac{m^w}{\bar{m}^w} \right) - \ln\left(\frac{B}{\bar{B}} \right). \tag{34}$$

Equation (34) is an expectations-augmented Phillips curve, extended to allow for supply shocks. The specification of the supply shock variable, $\tilde{s}$, shows that a positive shock to inflation occurs if the wage mark-up or the price mark-up rises above its normal level, whereas a negative shock to inflation occurs if productivity rises above its trend level. By construction, $\tilde{s}$ will fluctuate around a mean value of 0, since $\bar{m}^p$ and $\bar{m}^w$ are the average values of m^p and m^w, respectively, and since B is on average on its trend growth path $\bar{B}$.

We are now ready to confront our theory of inflation and unemployment with some data.

18.3 Testing the Phillips curve theory

If we assume the simplest possible ‘backward-looking’ type of inflation expectations, so-called static expectations saying that the expected rate of inflation will equal the latest observed actual rate of inflation, $\pi_t^e = \pi_{t-1}$, then from (34), we obtain an equation of the form

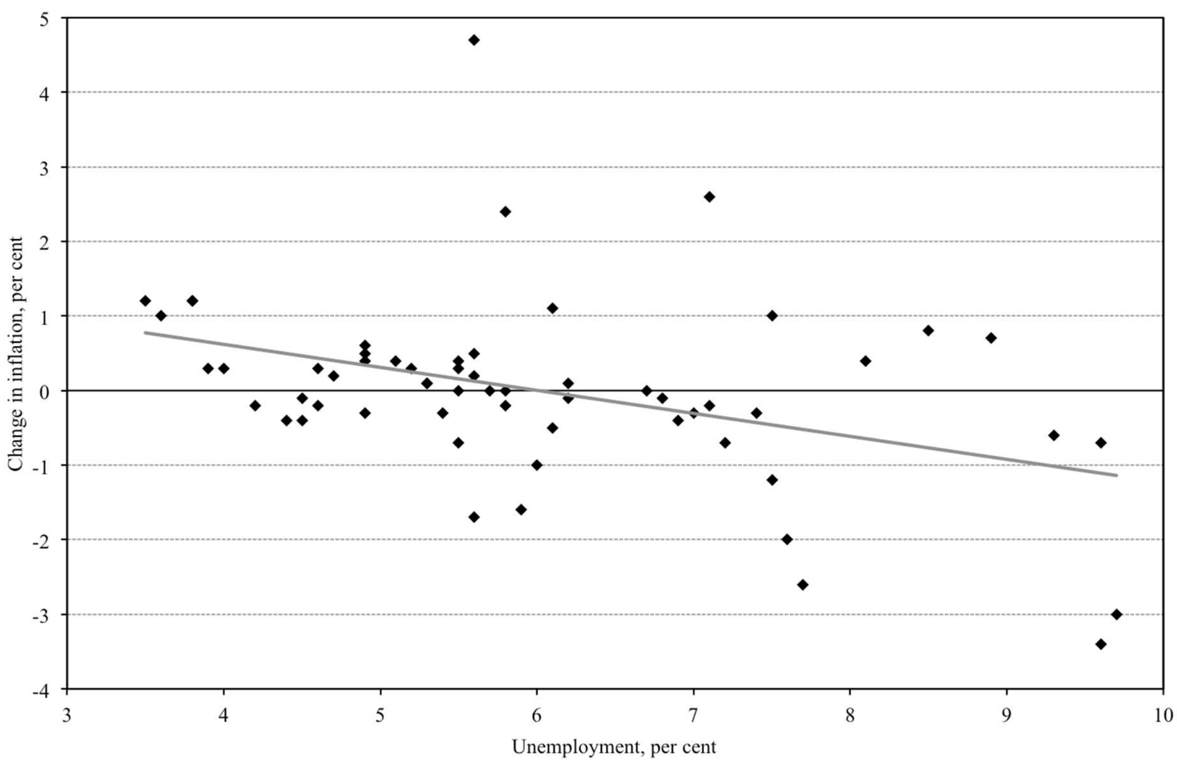
$$\Delta\pi_t \equiv \pi_t - \pi_{t-1} = \alpha_0 + \alpha_1 u_t + \tilde{s}, \quad E[\tilde{s}] = 0, \tag{35}$$

where $\alpha_0 = \alpha\bar{u}$ and $\alpha_1 = -\alpha$ are constants, and $E[\cdot]$ is the expectations operator. Thus, our theory implies that the *change* in the rate of inflation should be negatively related to the rate of unemployment. With time series data for inflation and unemployment we may test this relationship by plotting the change in the rate of inflation against the rate of unemployment in a figure and by using the econometric techniques described in the Appendix to this book, as we have done many times in other chapters. In this way we can check whether the estimated parameter values seem to have the 'correct' signs (α_0 positive, α_1 negative), and we can test whether they are significantly different from zero in a statistical sense.

Figure 18.5 plots observations of the annual change in the rate of consumer price inflation, $\pi_t - \pi_{t-1}$, in the USA against the unemployment rate, u_t, for years *t* between 1960 and 2018. Indeed, there seems to be a negative relationship. The downward-sloping straight line in the figure is a regression line estimated by OLS (Ordinary Least squares) indicating the 'average' relationship between unemployment and the change in inflation. The regression line has the following quantitative properties, where the figures in brackets indicate the standard errors of the estimated coefficients, and where R^2 is the so-called coefficient of determination measuring the share of the variation in $\Delta\pi$, which is explained by our estimated regression line:

$$\Delta\pi_t = \underset{(0.59)}{1.85} - \underset{(0.09)}{0.31} \cdot u_t, \qquad R^2 = 0.16 \tag{36}$$

Figure 18.5 Relation between unemployment and the change in inflation in the USA, 1960 – 2018



Source: Data from US Bureau of Labour Statistics and calculations based on these.

The coefficients in (36) do indeed have the signs we would expect from theory. They are also significantly of the right sign. As a general rule, if one has a number of observations that exceeds the number of explanatory variables just moderately as we do here, an estimated coefficient is significantly of the relevant sign at a one per cent significance level if it lies more than 2.8 standard errors away from zero, it is significant at a five per cent level if it lies more than 1.8 (in practice often rounded up to 2) standard errors away from zero, and at a ten percent level if it lies more than 1.4 standard errors away. The condition for the slope parameter above to be significantly negative is easily met at a one percent significance level. The data material of Figure 18.5 thus shows a clear tendency for the USA that inflation increases less or falls more the higher the rate of unemployment is.

Quantitatively the estimated relation points to a one percentage point higher rate of unemployment implying an increase in annual inflation of around 0.3 percentage points. Recall that, according to our theory of the Phillips curve as expressed by Equation (27), the value of $-\alpha_1$ should equal the production function parameter α, which is typically believed to be around one third. This is strikingly close to the estimate in Equation (36). Taking uncertainty of the estimate of α_1 into account one could consider the 95 percent confidence interval that goes roughly from two standard errors below the estimate to two standard errors above. This gives here (roughly) the interval from 0.1 to 0.5. Interestingly, a standard rule of thumb for the US economy says that for each percentage point that the unemployment rate exceeds the natural rate, inflation falls by around one-half percentage point per year. Although, as we shall see below, this rule is perhaps better supported by estimations based on earlier than later time periods, it is not significantly contradicted by our estimation in (36).

The estimated coefficients in (36) enable us to offer an estimate of the natural rate of unemployment in the USA. Since $\alpha_0 = \alpha\bar{u}$ and $\alpha_1 = -\alpha$, we have $\bar{u} = \alpha_0 / \alpha = -\alpha_0 / \alpha_1$. Inserting the estimated parameter values from (36), we find that $\bar{u} = 1.85/0.31 \approx 6.0$ percent. This implies that the natural unemployment rate in the USA viewed over the entire period should on average have been around 6 percent. This is actually a very sensible estimate in complete accordance with standard estimates of structural unemployment in the USA.

How about other countries? Table 18.1 reports estimations of Philips curve relations of the form $\Delta\pi = \alpha_0 + \alpha_1 u + \tilde{s}$ as in (35) above for eight different countries (including USA) for periods as long as data availability allowed.

Table 18.1 Phillips curves for eight countries

Dependent variable: Change in annual inflation rate, percentage points					
Country	Constant, α_0	Coefficient on unemployment rate, α_1	Implied structural rate of unemployment, $-\alpha_0/\alpha_1$	Period	R^2
United States	1.85 (0.59)	-0.31 (0.09)	6.01	1960-2018	16%
Australia	1.56 (0.70)	-0.27 (0.11)	5.79	1966-2017	11%
France	0.47 (0.44)	-0.08 (0.06)	5.65	1960-2017	3%
Japan	0.56 (0.88)	-0.20 (0.28)	2.75	1960-2017	1%
Netherlands	0.81 (0.42)	-0.16 (0.07)	5.00	1971-2017	10%
New Zealand	0.82 (0.58)	-0.22 (0.12)	3.76	1960-2017	6%
Sweden	0.54 (0.57)	-0.13 (0.10)	4.30	1963-2017	3%
Switzerland	0.35 (0.28)	-0.19 (0.12)	1.85	1960-2018	4%

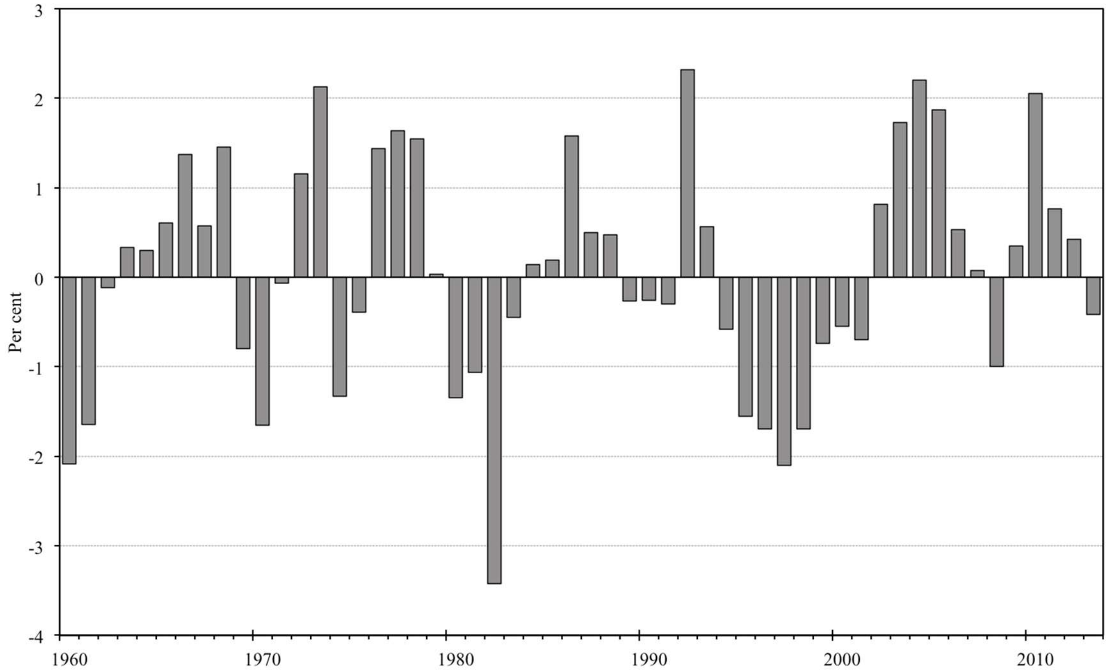
Note: Standard errors in parenthesis.
Source: Estimations based on data from US Bureau of Labour Statistics and Main Economic Indicators, OECD.

For all the countries considered the slope estimate of the Phillips curve is negative, and in most cases significantly so at least at a ten percent significance level,[15] but in most cases at much higher significance levels. Furthermore, the implied levels of structural unemployment seem very sensible for most countries.

In summary, the theory of the expectations-augmented Phillips curve seems roughly consistent with the data for the US and other developed economies over long periods. At the same time we also see both from the figure and the estimations that the observed change in inflation has often deviated quite a lot from the estimated regression line (reflected in the low values of R^2 in the estimations). However, it is not surprising that our regression equation leaves a lot of the variation in inflation changes unexplained since a lot of variation can be due to considerable variations in the supply shock $\tilde{s}$ that is really part of our equation. This is not a problem for the estimation of the parameters α_0 and α_1 as long as $\tilde{s}$ is nicely and independently behaved and, e.g., not correlated with the difference between the rates of actual and structural unemployment. The shock $\tilde{s}$ contains productivity, so to give an impression of how it may behave, Figure 18.6 shows the deviations from trend in annual labour productivity in USA computed by HP filtering.

[15] The only exception from this is Japan that in many years in the considered period has been in a particular, low inflation state, where the reactions in inflation to changes in unemployment have probably been low because inflation was already very low and there can be a certain downwards inertia. This contradicts the linear form of the Phillips curve imposed here for all countries.

Figure 18.6 Deviations from trend in labour productivity in the USA, 1960-2013



Note: As a measure of labour productivity, we use output per hour in non-farm business sector. We calculate the deviations from the structural, trend level of labour productivity. The trend level is calculated by HP-filtering with $\lambda = 100$. End-point observations excluded.
Source: US Bureau of Labour Statistics.

The deviations are of considerable size and they are quite strongly serially correlated. This gives the idea to incorporate them explicitly in an estimation. We therefore consider an equation of the form:

$$\Delta\pi_t = \alpha_0 + \alpha_1 u_t + \alpha_2(\ln B_{t-1} - \ln \overline{B}_{t-1}) + \alpha_3(\ln B_t - \ln \overline{B}_t) + \hat{s}_t, \tag{37}$$

where $\hat{s}_t$ is a new error term after productivity has been taken out. The idea is that a relatively high level of productivity in period *t* (a level much above trend) should hold down prices in period *t* and thus *ceteris paribus* imply a relatively low inflation from period *t*-1 to period *t* (pointing to a negative value of α_3) and at the same time, a relatively high level of productivity in period t-1 should imply relatively low prices in period *t*-1 and therefore *ceteris paribus* a relatively high inflation from period *t*-1 to period *t* if productivity moves towards a normal level (pointing to a positive value of α_2).

An estimation by OLS for the USA, where we have used the productivity deviations calculated for Figure 18.6 for $\ln B_t - \ln \overline{B}_t$ (measured in per cent) for the period 1960-2013 and with standard errors in parentheses, looks as follows:

$$\Delta\pi_t = 1.40 - \underset{(0.09)}{0.23} \cdot u_t + \underset{(0.13)}{0.61} \cdot (\ln B_{t-1} - \ln \overline{B}_{t-1}) - \underset{(0.13)}{0.30} \cdot (\ln B_t - \ln \overline{B}_t) \quad \text{adj. } R^2 = 0.38 \tag{38}$$

The estimated coefficient on the rate of unemployment is still significantly negative at a high significance level (although now somewhat smaller than before), the coefficients on the productivity variables significantly have the expected signs and the R-square is now considerably larger.

This seems again to support the idea of a fundamental expectations augmented Phillips curve relationship. There is, however, some discussion about the size of the slope parameter (our α_1) and whether the relationship has changed over time.

Table 18.2 reports some Phillips curve estimations for the USA. In the upper part we have returned to the simple equation (35) and estimated this for 1960-2018 (repeating the estimation shown above) and for the two sub-periods 1960-1990 and 1990-2018.

Table 18.2 Phillips curves for the USA for different periods

Dependent variable: Change in annual inflation rate, percentage points				
Period	**Constant**	**Coefficient on unemployment rate**	**R^2**	
1960-2018	1.85	-0.31 (0.09)	16%	
1960-1990	3.28	-0.52 (0.17)	25%	
1990-2018	0.42	-0.08 (0.05)	8%	
Dependent variable: Annual inflation rate				
Period	**Constant**	**Coefficient on unemployment rate**	**Coefficient on lagged inflation**	**R^2**
1960-2018	1.84	-0.28 (0.11)	0.96 (0.07)	80%
1960-1990	3.33	-0.55 (0.23)	1.02 (0.12)	76%
1990-2018	0.85	-0.08 (0.05)	0.82 (0.08)	80%

Note: Standard errors in parenthesis.
Source: US Bureau of Labour Statistics; Federal Reserve Bank of St. Louis;

In the early sub-period 1960-1990, the slope estimate is numerically considerably larger than in the later sub-period, 1990-2018, and in the full period (and so is the R-square). In the early sub-period the slope estimate is close to one half which is just bull's eye with respect to the above-mentioned rule of thumb for the US economy that a one percentage point lower unemployment rate should give a one half of a percentage point higher rate of annual inflation. In more recent times this sensitivity seems to have dropped to a lower level, but the slope estimate is still significantly negative for the period 1990-2018 at a ten per cent level (and indeed almost at a five per cent level). The Phillips curve relationship thus seems to have survived but apparently with a lower sensitivity of inflation to unemployment.[16]

In our estimations so far we assumed that expected inflation is approximately given by observed past inflation, $\pi_t^e = \pi_{t-1}$. This hypothesis is obviously highly simplified and could be discussed. Perhaps it would be more plausible to assume that expected inflation to some extent (for some agents) is determined by short run observations, where the most recently observed inflation rate π_{t-1} could be of importance, and to some extent (for other agents) is tied down by longer-run inflation expectations $\overline{\pi}_t$. The latter could, e.g., be affected by and anchored in the inflation target of the central bank, π^*, so that $\overline{\pi}_t \approx \pi^*$, in which case the longer-run inflation expectations would be relatively constant. Inflation expectations could then on average be an average, $\pi_t^e = v\pi_{t-1} + (1-v)\pi^*$, $0 < v < 1$. Inserting this into (34) gives:

$$\pi_t = \alpha_0 + \alpha_1 u_t + \alpha_2 \pi_{t-1} + \tilde{s}, \tag{39}$$

where now $\alpha_0 = \alpha\overline{u} + (1-v)\pi^*$, $\alpha_1 = -\alpha$ and $\alpha_2 = v.$ In the special case $v = 1,$ we are back at (35), but one idea suggested in the debate on the Phillips curve has been that the parameter v has decreased over time, so inflation expectations have come to put more weight on the long run anticipations and less on recent observations. In Chapter 17 we saw in connection with Figure 17.4 that since the late 1980s many central banks have begun to stick more closely to an inflation targeting Taylor rule, which may have led economic agents to put relatively more weight on the central banks' inflation targets and less on recently observed inflation in their expectation formation. In the lower part of Table 18.2, we take this to a test by estimating an equation of form (39) for different periods for the USA. In the early period, 1960-1990, the estimate of the coefficient on lagged inflation is very close to, and certainly not significantly different from, one, which accords well with inflation expectations being to a high degree determined by recent observations and with static inflation expectations. In the later period, the estimate of the coefficient on lagged inflation is considerably and significantly smaller than one, in accordance with the idea that inflation expectations may have become more long-run in nature and perhaps more anchored in the central banks' inflation targets. Still the estimate of the coefficient on the unemployment rate is significantly negative at the ten percent level and almost at the five percent level.[17]

In the paper by Olivier Blanchard, Eugenio Cerutti, and Lawrence Summers mentioned in Footnote 15 above, the authors find support for both a flatter Phillips curve (a smaller coefficient on the unemployment term) and more anchored inflation expectations in recent times for the US economy. However, it is debated whether the Philips curve has really flattened. Its apparent flattening could be an expression exactly of the more anchored inflation expectations and not an indication of a lower impact of unemployment on inflation. To see why, we will return for a

[16] The possibility of a 'flatter' Phillips curve is discussed in, e.g., the IMF working paper, Olivier Blanchard, Eugenio Cerutti, and Lawrence Summers (2015): "Inflation and Activity – Two Explorations and their Monetary Policy Implications", *IMF WP/15/230*.

[17] The much larger R-squared in the lower than in the upper part of Table 18.2 is just an artefact of the dependent variable being the *change* in the rate of inflation in the upper part and the *level* of inflation in the lower part. It is therefore not an indication of the estimations in the lower part having really much more explanatory power.

moment to the Phillips curve (the NKPC) in (31), $\pi = \pi_{+1}^e + \alpha(\bar{u} - u)$, where the forward inflation expectation π_{+1}^e appears. Compare two different periods, an early one where inflation expectations π_{+1}^e are only weakly anchored by the inflation target of the central bank and therefore highly sensitive to observed inflation π, and a later one where inflation expectations are highly anchored and therefore little sensitive to π, but where the impact of unemployment on current inflation, α, is the same. Consider a positive demand shock to the economy that drives u down, thereby increasing current inflation π by the amount α (per unit of change in unemployment) according to the Phillips curve. Now, in the early period the increase in current inflation π will make inflation expectations π_{+1}^e increase more than in the later period. This implies from $\pi = \pi_{+1}^e + \alpha(\bar{u} - u)$ that current inflation will increase more in the early period and therefore a stronger negative correlation between current unemployment and current inflation is observed. In estimations based on this correlation, one will therefore easily find that unemployment influences inflation more in the early than in the late period, without this being a true indication of the effect of unemployment itself. Of course, if one measures expected inflation correctly in one's estimations, one will not make such an error, but expectations are not easily observed. It can therefore be questioned whether the often found flattening of the Phillips curve is really a result of more anchored (forward) inflation expectations and not of a weaker impact of unemployment on inflation.[18]

All in all, the idea that for given inflation expectations there is a (short-run) trade-off between unemployment and inflation of considerable magnitude seems to stand up well against the data, although there may be considerable uncertainty about the size of the slope in the relationship.

18.4 From the Phillips curve to the aggregate supply curve

Our theory of aggregate demand presented in Chapter 17 implied a systematic link between the output gap (the percentage deviation of output from trend) and the rate of inflation. We shall now show that our theory of inflation and unemployment implies another systematic link between these two variables.

Recall that in a symmetric general equilibrium of the model considered in this chapter, total GDP is $Y = nY_i$, and total employment is $L = nL_i$. Using also that with our normalization, $L = (1-u)n$, we get from (3):

$$Y = nB\left(\frac{L}{n}\right)^{1-\alpha} = nB\left(\frac{(1-u)n}{n}\right)^{1-\alpha} = nB\left(1-u\right)^{1-\alpha} \qquad (40)$$

[18] In the paper Peter Lihn Jorgensen and Kevin J. Lansing (2019), "Anchored Inflation Expectations and the Flatter Phillips Curve", Federal Reserve Bank of San Francisco Working Paper Series 2019-27, the authors exactly find that with a specific and careful handling of forward inflation expectations, one need not find any flattening of the Phillips curve over time. It follows from their analysis that the most reliable estimates of the slope of the Phillips curve in other investigations are the typically smaller ones (numerically) found for later periods where inflation expectations are more anchored. Their paper is inspired by a speech given by then FED governor Ben Bernanke in 2007 (https://www.federalreserve.gov/newsevents/speech/bernanke20070710a.htm).

Let us now define 'natural' or structural output, $\overline{Y}$, as the volume of output produced when employment and unemployment are at their natural levels *and* productivity is at its trend level. In view of (40) we must then have:

$$\overline{Y} = n\overline{B}\left(1-\overline{u}\right)^{1-\alpha}, \tag{41}$$

and then, dividing (40) by (41):

$$\frac{Y}{\overline{Y}} = \frac{B}{\overline{B}}\left(\frac{1-u}{1-\overline{u}}\right)^{1-\alpha} \tag{42}$$

Taking natural log on both sides of (42), letting $y \equiv \ln Y$ etc. and using $\ln(1-u) \approx -u$ etc. we get:

$$y - \overline{y} = \ln\frac{B}{\overline{B}} + (1-\alpha)(\overline{u}-u) \quad \Leftrightarrow \quad \overline{u} - u = \frac{y-\overline{y}}{1-\alpha} - \frac{\ln\frac{B}{\overline{B}}}{1-\alpha} \tag{43}$$

Inserting this expression into our Phillips curve (34) gives:

$$\pi = \pi^e + \alpha\left(\frac{y-\overline{y}}{1-\alpha} - \frac{\ln\frac{B}{\overline{B}}}{1-\alpha}\right) + \tilde{s} = \pi^e + \frac{\alpha}{1-\alpha}(y-\overline{y}) - \frac{\alpha}{1-\alpha}\ln\frac{B}{\overline{B}} + \tilde{s} \tag{44}$$

Recalling the definition of $\tilde{s}$ from (34), this can be written as:

$$\begin{gathered} \pi = \pi^e + \gamma(y-\overline{y}) + s, \\ \gamma \equiv \frac{\alpha}{1-\alpha} \quad \text{and} \quad s \equiv \tilde{s} - \frac{\alpha}{1-\alpha}\ln\frac{B}{\overline{B}} = \ln\left(\frac{m^p}{\overline{m}^p}\right) + \ln\left(\frac{m^w}{\overline{m}^w}\right) - \frac{1}{1-\alpha}\ln\frac{B}{\overline{B}} \end{gathered} \tag{45}$$

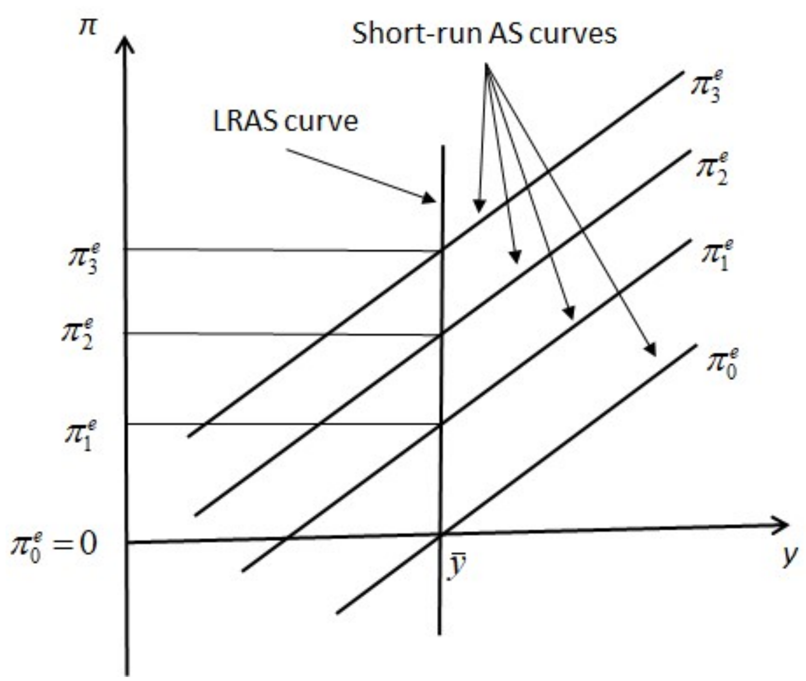
Equation (45) is our final short-run aggregate supply curve. We refer to it as the AS curve or, if we want to emphasize its short run nature, as the SRAS curve. Had we also had nominal price rigidity, the coefficient γ would have been $\gamma = \omega\alpha/(1-\alpha)$.

The magnitude $y - \overline{y}$ is the percentage deviation of output from trend, referred to as the *output gap*. From (45) we see that, *ceteris paribus*, the rate of inflation varies positively with the output gap. A rise in output requires a rise in employment, and because of diminishing marginal productivity of labour, higher employment generates an increase in marginal cost, which translates into an increase in prices and hence inflation via the mark-up pricing behaviour of firms. Or the other way around, an unanticipated rise in inflation will imply lower real wages because nominal wages were set in expectation of a lower inflation, and lower real wages induce firms to supply more output. Equation (45) also implies that the actual rate of inflation varies positively with the expected rate of inflation and with the supply shock variable, s, capturing shocks to mark-ups and to productivity.

The short-run aggregate supply curve in (45) summarizes the supply side of the economy. Because the expected inflation rate is here taken as given, the curve is a *short-run* relationship. Over time, the expected inflation rate will gradually adjust in reaction to previous inflation forecast errors. When π^e changes, it follows from (45) that the short-run aggregate supply curve will shift upwards or downwards. This is illustrated in Figure 18.7, which shows four different AS curves (all for $s = 0$) corresponding to four different levels of the expected inflation rate. In long-run equilibrium, when expected inflation equals actual inflation and there are no shocks ($s = 0$), we see from (45) that output must be equal to its 'natural' level, $\bar{y}$.

The natural rate of output is independent of the rate of inflation, since the natural unemployment rate $\bar{u}$ is independent of π. The *long-run aggregate supply (LRAS) curve* is therefore vertical, as shown in Figure 18.7.

Figure 18.7 Aggregate supply in the short run (AS) and in the long run (LRAS)



Apart from depending on the expected rate of inflation, the position of the short-run aggregate supply curve also depends on the supply shock variable, s. When $s = 0$, it follows from (45) that the AS curve passes through $(\bar{y}, \pi^e)$ as depicted in Figure 18.7. From (45) we see that the AS curve will shift upwards in case of a positive shock to one of the mark-ups m^w or m^p, or in case of a negative shock to productivity, $B < \bar{B}$. Note that several types of supply shocks may be modelled as productivity shocks. For example, a loss of output due to industrial conflict may be interpreted as a temporary fall in labour productivity. A politically decided lock-down of the economy in order to protect people against a pandemic like the covid-19 can also be modelled as a negative supply shock. Likewise, an unusually bad harvest due to bad weather conditions may be seen as a temporary drop in productivity. An exogenous increase in the real price of imported raw materials such as oil will likewise work very much like a negative productivity shock. If the price of oil increases relative to the general price level, an economy dependent on imported oil will have to reserve a greater fraction of domestic output for exports to maintain a given volume of oil imports. Thus, for given inputs of domestic labour and capital, a lower amount of domestic output will be available for domestic consumption, just as if factor productivity had declined. More generally, any exogenous change in the economy's international terms of trade (a shift in import prices relative to export prices) may be modelled as a productivity shock in our AS–AD model.

The aggregate supply curve is a cornerstone in our models of the macroeconomy, so let us recapitulate its properties:

THE AGGREGATE SUPPLY CURVE

The AS curve describes a relationship between total output and the rate of inflation. In the short run where the expected rate of inflation is predetermined, the actual rate of inflation varies positively with the output gap, as higher output and employment generates higher marginal production costs which are passed on to prices. This positive link between output and inflation defines the short-run aggregate supply (AS) curve. The AS curve shifts upwards (downwards) one-to-one with a rise (fall) in the expected rate of inflation. The AS curve also shifts up in case of a negative (unfavourable) supply shock. A negative supply shock may take the form of a rise in the price or wage mark-ups, a temporary drop in productivity or a fall in the country's international terms of trade. In long-run equilibrium, inflation expectations must be fulfilled, and supply shocks must be zero. Output will then be at its 'natural' (trend) level, so the long-run aggregate supply (LRAS) curve is vertical.

This completes our theory of the aggregate supply side. In the next chapter we shall see how aggregate supply interacts with aggregate demand to determine total GDP as well as the rate of inflation.

Summary

1. The link between inflation and unemployment determines how the supply side of the economy works. For some decades after WWII most economists and policy makers believed in the simple Phillips curve which postulates a permanent trade-off between inflation and unemployment: a permanent reduction in the rate of unemployment can be achieved by accepting a permanent increase in the rate of inflation, and vice versa.
2. Empirically, the simple Phillips curve broke down in the stagflation of the 1970s. This led to the theory of the expectations-augmented Phillips curve, which says that the simple Phillips curve is just a short-run trade-off between inflation and unemployment, existing only as long as the expected rate of inflation is constant. When the expected inflation rate goes up, the actual inflation rate increases by a corresponding amount, other things equal.
3. The expectations-augmented Phillips curve implies the existence of a 'natural' rate of unemployment, defined as the level of unemployment, which will prevail in a long-run equilibrium where the expected inflation rate equals the actual inflation rate. Since any fully anticipated rate of inflation is compatible with long-run equilibrium, the long-run Phillips curve is vertical. When the actual unemployment rate falls below the natural unemployment rate, the actual inflation rate exceeds the expected inflation rate, and vice versa.
4. Several different theories of wage and price formation lead to the expectations-augmented Phillips curve. One such theory is the 'sticky-wage model' in which nominal wage rates are pre-set at the start of each period. In the sticky wage model presented in this chapter, money wages are dictated by trade unions who seek to achieve a certain target real wage, given their expectations of the price level that will prevail over the next period. Given the wage rate set by unions, profit-maximizing monopolistically competitive firms set their prices as a mark-up over marginal costs and choose a level of employment which is declining in the actual real wage. According to this model, employment increases above its natural rate when the actual price level exceeds the expected price level, and vice versa. The model also implies that, in general, there is some amount of involuntary unemployment. The appendix to this chaper shows that an efficiency wage framework leads to an expectations augmented Phillips curve with essentially the same properties as the one derived from the sticky wage model.
5. In the sticky-wage model, the target real wage is a mark-up over the opportunity cost of employment, which is given by the rate of unemployment benefit. The wage mark-up factor – and hence the target real wage – is higher the lower the wage elasticity of labour demand, and the lower the weight the union attaches to the goal of high employment relative to the goal of a high real wage. The mark-up of prices over marginal costs is higher the lower the price elasticity of demand for the output of the representative firm. The natural rate of unemployment is higher the higher the wage and price mark-ups and the more generous the level of unemployment benefits.
6. The evidence strongly suggests that, just as money wages are sticky, many nominal product prices are pre-set and held constant for a while, presumably because of menu costs of frequent price changes. The sticky-wage model can be extended to include nominal price rigidity as well as nominal wage rigidity. This extension of the model reduces the short-run sensitivity of inflation to the unemployment gap.

7. According to the hypothesis of static expectations, the expected inflation rate for the current period equals the actual inflation rate observed during the previous period. Combined with the expectations-augmented Phillips curve, the assumption of static expectations implies that the rate of inflation will keep on accelerating (decelerating) when actual unemployment is below (above) the natural unemployment rate.

9. Supply shocks in the form of fluctuations in productivity and in the wage and price mark-ups create 'noise' in the relationship between inflation and unemployment. An unfavourable supply shock implies an increase in the actual rate of inflation for any given levels of unemployment and expected inflation. In the presence of supply shocks, the natural unemployment rate is defined as the rate of unemployment prevailing when inflation expectations are fulfilled and productivity as well as the wage and price markups are at their trend levels.

10. An expectations-augmented Phillips curve with static inflation expectations is basically consistent with data from US and many other developed 'western' economies on inflation and unemployment in the period from the 1960s to recent time. There may be some uncertainty about the slope of the Phillips curve and about how expectations should be modelled. It is possible that the sensitivity of inflation to unemployment has fallen after (around) 1990 and that inflations expectations have come to put less weight on recently experienced inflation and more on inflation targets, but overall the expectations-augmented Phillips curve seems to be alive.

11. The economy's short-run aggregate supply (AS or SRAS) curve implies a positive link between the output gap and the actual rate of inflation, given the expected rate of inflation. The AS curve may be derived from the expectations-augmented Phillips curve, using the production function which links the unemployment rate to the level of output. The AS curve shifts upwards when the expected inflation rate goes up, or when the economy is hit by an unfavourable supply shock. When there are no supply shocks and expected inflation equals actual inflation, the economy is on its long-run aggregate supply curve (the LRAS curve) which is vertical at the natural level of output corresponding to the natural rate of unemployment.

Exercises

Exercise 1. Intersectoral labour mobility and the expectations-augmented Phillips curve

In Section 18.2 we abstracted from intersectoral labour mobility by assuming that individual workers are educated and trained to work in a particular sector. In that case a worker's outside option (the income he could expect to earn if he were not employed by his current employer) is simply equal to the rate of unemployment benefit. In this exercise we ask you to show that one can still derive the expectations-augmented Phillips curve from a trade union model of wage setting even if unemployed workers can move between sectors in their search for jobs, as we assumed in our analysis of the labour market in Chapters 1, 11 and 12.

To simplify matters a bit, we now set our productivity parameter $B = 1$ and work with a linear production function with constant marginal productivity of labour, corresponding to $\alpha = 0$ in Equation (3) in the main text. For $\alpha = 0$ and $B = 1$, it follows from (7) in Section 18.2 that the representative firm will set its product price as:

$$P_i = m^p W_i, \qquad m^p > 1. \tag{46}$$

As in Section 18.2, an optimizing monopoly union for workers in firm (sector) i will set the nominal wage rate W_i to attain an expected real wage which is a mark-up over the expected real value of its members' outside option, denoted by v^e. The union expects the current price level to be P^e. Hence, it will set the wage rate:

$$\frac{W_i}{P^e} = m^w v^e, \qquad m^w > 1. \tag{47}$$

This equation is a parallel to Equation (15) in Section 18.2 where we assumed that v^e is simply equal to the real rate of unemployment benefit, b. Here we assume instead that workers who are initially members of the trade union for sector i are qualified to apply for a job in other sectors if they fail to find one in sector i. For an average job seeker, the probability of finding work is equal to the rate of employment, $1 - u$, where u is the unemployment rate which gives the probability that an average job seeker will remain unemployed. If the ratio of unemployment benefits to the average wage level is equal to the constant c, we thus have:

$$v^e = (1-u)w^e + ub^e = (1-u+uc)w^e, \qquad 0 < c < 1, \tag{48}$$

where w^e is the expected average level of real wages, and $b^e = cw^e$ is the expected real rate of unemployment benefit.

Since the outside option v^e is the same across all sectors and all unions are assumed to hold the same price expectations, it follows from (46) and (47) that all unions will charge the same nominal wage rate, W, and that all firms will charge the same output price, P:

$$W_i = W, \qquad P_i = P. \tag{49}$$

According to (46) the average real wage will then be $W/P = 1/m^p$. Let us suppose that union wage setters have a realistic estimate of the average real wage so that:

$$w^e = 1/m^P. \tag{50}$$

1. Show that the relationship between the actual and the expected price level may be written as:

$$P = m^w(1 - \gamma u)P^e, \qquad \gamma \equiv 1 - c > 0. \tag{51}$$

2. Use (51) to derive an expectations-augmented Phillips curve of the same form as equation (27) in Section 2. (Hint: use the approximation $\ln(1 - \gamma u) \approx -\gamma u$ and define $\bar{u} \equiv \ln m^w/\gamma$.) Explain intuitively what determines the natural unemployment rate, u. Explain intuitively what determines the slope (γ) of the expectations-augmented Phillips curve.
3. Does the theory embodied in (46)–(51) assume short-run nominal wage rigidity?
4. The specification of the outside option (48) assumes that unions know the current aggregate unemployment rate when they set the wage rate. Suppose instead that unemployment statistics are published with a lag so that unions base their estimate of the outside option on last period's recorded unemployment rate, u_{-1}:

$$v^e = (1-u_{-1})w^e + u_{-1}b^e = (1-u_{-1}+u_{-1}c)w^e. \tag{52}$$

Derive the expectations-augmented Phillips curve on the assumption that the perceived outside option is given by (52) rather than (48). Do we now have nominal wage rigidity in the short run?

Exercise 2. Wage setting, labour demand and unemployment

1. Explain why union wage claims are moderated by a higher price elasticity in the representative firm's product demand curve.
2. In the text, we found that the wage elasticity of labour demand is lower at the sectoral level than at the aggregate level. Explain why this is so.
3. 'Tougher product market competition will reduce structural unemployment.' Explain this statement. Discuss what the government could do in practice to promote fiercer product market competition.

Exercise 3. The Phillips curve with endogenous price mark-ups

In the main text we assumed for simplicity that the representative firm's mark-up m^p of price over marginal cost was an exogenous constant. However, empirical research for the USA suggests that price mark-up factors in that country tend to move in a countercyclical fashion. In other words, the mark-up tends to fall during business cycle expansions and to rise during recessions. There are several potential reasons for this countercyclical behaviour of mark-ups, including the possibility that during booms when the demand pressure is high, more new firms find it profitable to enter the market, thereby increasing the degree of competition and forcing existing firms to reduce their profit margins.

Since the rate of unemployment moves countercyclically, the countercyclical variation of the price mark-up means that m^p will tend to move in the same direction as the unemployment rate. For concreteness, suppose this relationship takes the form:

$$m^p = \tilde{m} \cdot e^{\varphi u}, \quad \tilde{m} > 1, \quad \varphi \geq 0, \tag{53}$$

where e is the exponential function, and φ is a parameter measuring the sensitivity of the mark-up to changes in the unemployment rate, u. In the main text of the chapter we focused on the case of $\varphi = 0$, corresponding to a constant mark-up. In this exercise we ask you to study the implications of assuming $\varphi > 0$ which is more in line with the empirical evidence for the USA.

As in Exercise 1, we consider an economy with intersectoral labour mobility, union wage setting and mark-up price setting. Hence, we have (see Exercise 1 in case you need further explanation):

$$P_i = m^p W_i, \tag{54}$$

$$\frac{W_i}{P^e} = m^w \cdot v^e, \quad m^w > 1. \tag{55}$$

$$v^e = (1-u)w^e + ub^e = (1-u+uc)w^e, \qquad 0 < c < 1, \tag{56}$$

where w^e is the expected average level of real wages, $b^e = cw^e$ is the expected real rate of unemployment benefit, and υ^e is the individual worker's expected outside option. The outside option is the same across all sectors, so all unions charge the same nominal wage rate and all firms charge the same price:

$$W_i = W, \qquad P_i = P. \tag{57}$$

Union wage setters assume that the average real wage is:

$$w^e = 1/\bar{m}, \quad \bar{m} > 1, \tag{58}$$

where the constant $\bar{m}$ is the expected 'normal' price mark-up.

1. Show that the model consisting of (53)–(58) implies an expectations-augmented Phillips curve of the form:

$$\pi = \pi^e + (1-c-\varphi)(\bar{u}-u), \quad \bar{u} \equiv \frac{\ln \tilde{m} + \ln m^w - \ln \bar{m}}{1-c-\varphi}, \tag{59}$$

where $\pi \equiv p - p_{-1}$, $\pi^e \equiv p^e - p^e_{-1}$, and $p \equiv \ln P$, and where you may assume that $1 - c - \varphi >$ 0 to ensure a positive solution for the natural unemployment rate, $\bar{u}$. Explain intuitively how the parameter φ affects the sensitivity of inflation to unemployment. Explain and discuss the various factors determining the natural unemployment rate.

2. Suppose that union wage setters expect the 'normal' price mark-up $\bar{m}$ appearing in (58) to be

$$\bar{m} = \tilde{m} \cdot e^{\varphi \bar{u}}. \tag{60}$$

Discuss whether the assumption made in (60) is reasonable. Derive a new expression for the natural unemployment rate $\bar{u}$ on the assumption that (60) holds. Compare the new expression to the expression for $\bar{u}$ given in (59) and comment on the differences.

Exercise 4. The Phillips curve and active labour market policy

The model with intersectoral labour mobility presented in Exercise 1 assumes that all workers are competing on equal terms and with equal intensity for the available jobs. In that case it seems reasonable that any individual worker's probability of finding a job is simply equal to the overall rate of employment, $1 - u$, as we assumed when specifying a worker's outside option.

In the present exercise we assume instead that some of the workers recorded in the unemployment statistics are not fully 'effective' in competing for jobs, perhaps because their skills do not fully match the qualifications demanded by employers, or perhaps because they are not actively searching for a job all the time. We may model this in a simple way by assuming that only a fraction, s, of the registered unemployed workers contribute fully to the available labour supply in the sense of being immediately ready and able to accept the jobs available. Thus we may specify the 'effective' labour supply from the pool of unemployed workers as su. In the following, we will refer to s as the Job-Search-and-Matching-Efficiency parameter (the JSME parameter). For the moment, we will assume that s is an exogenous constant.

If we normalize the total labour force to equal 1, there are thus $(1 - s)u$ unemployed workers who are not really competing with their fellow workers for a job. Hence we may measure the 'effective' labour force as $1 - (1 - s)u$. For an average member of the effective labour force (a qualified person who is ready to accept the available jobs), the probability p^u of ending up in the unemployment pool is the ratio of the 'effective' number of unemployed to the 'effective' labour force, $p^u = su/[1 - (1 - s)u]$. By implication, a qualified person's probability of finding a job in the labour market is $1 - p^u$. If the expected average real wage is w^e, and if the ratio of the unemployment benefit to the average wage level is c, we may therefore specify the real value of a qualified worker's expected outside option as:

$$v^e = \left(1 - \frac{su}{1-(1-s)u}\right) w^e + \left(\frac{su}{1-(1-s)u}\right) \overbrace{cw^e}^{\substack{\text{unemployment} \\ \text{benefit}}} = \left(\frac{1-(1-cs)u}{1-(1-s)u}\right) w^e, \quad 0 < c < 1. \tag{61}$$

The trade union for sector i expects the current price level to be P^e and sets its nominal wage rate W_i to attain an expected real wage W_i/P^e which is a mark-up over the outside option of the employable union members:

$$\frac{W_i}{P^e} = m^w v^e, \quad m^w > 1. \tag{62}$$

The representative firm uses a linear production function with $\alpha = 0$ and $B = 1$, so according to Equation (7) in the main text it sets its price P_i as:

$$P_i = m^p W_i, \qquad m^p > 1. \tag{63}$$

Since υ^e is the same across all sectors, it follows from (61) and (62) that all unions will set the same wage rate and that all firms will charge the same price:

$$W_i = W, \qquad P_i = P. \tag{64}$$

In accordance with (63) and (64), the representative union expects the average real wage to be:

$$w^e = 1/m^p \tag{65}$$

1. Demonstrate through a logarithmic approximation that the model (61)–(65) leads to an expectations-augmented Phillips curve of the form

$$\pi = \pi^e + s\gamma(\bar{u} - u), \quad \gamma \equiv 1 - c > 0, \quad \bar{u} \equiv \frac{\ln m^w}{s\gamma}, \tag{66}$$

where $\pi \equiv p = p_{-1}$, $\pi^e \equiv p^e = p_{-1}$, and $p \equiv \ln P$. (Hint: use the approximations $\ln[1 - (1 - cs)u] \approx -(1 - cs)u$ and $\ln[1 - (1 - s)u] \approx -(1 - s)u$.) Explain in economic terms how the JSME parameter s affects the sensitivity of inflation to unemployment.

Let us now analyse the effects of active labour market policy. Suppose that a fraction l of the unemployed workers is enrolled in public education and training programmes aimed at improving their qualifications for the available jobs. Such programmes may increase the JSME parameter s partly by improving the match between the qualifications demanded by employers and the skills possessed by the unemployed, and partly by increasing workers' motivation to look for jobs (say, because it makes more attractive jobs available to them). Hence we assume that s is an increasing function of l:

$$s = \bar{s} \cdot l^{\eta}, \quad 0 < \bar{s} < 1, \quad \eta > 0, \quad 0 < l < 1. \tag{67}$$

The elasticity η is a parameter measuring the degree to which the labour market programmes succeed in actually upgrading the skills and motivation of the unemployed. However, (78) does not capture all of the effect of active labour market programmes. When people are enrolled in such a programme, they will often not be immediately available for a job should they receive a job offer, or they may not have the time to look for a job. For simplicity, let us assume that only that fraction $1 - l$ of the unemployed who are not currently engaged in education and training are able to take a job. Moreover, let us assume that these job seekers have benefited from previous training so that their Job-Search-and-Matching-Efficiency corresponds to the value of s specified in (67). The 'effective' labour supply coming from the pool of unemployed workers is then given by:

$$\text{'Effective' unemployment rate} = s(1-l)u \tag{68}$$

where s is determined by (67). The effective labour force consists of those unemployed workers who are effectively available for work, $s(1-l)u$, plus those who are already employed, $1-u$. Thus a qualified worker's probability of being unemployed is:

$$p^u = \frac{s(1-l)u}{1-u+s(1-l)u} = \frac{s(1-l)u}{1-[1-s(1-l)]u}. \tag{69}$$

In the questions below we will assume that l is a policy instrument controlled by the makers of labour market policy. Furthermore, we assume that workers enrolled in active labour market programmes receive the same unemployment benefits and enjoy the same utility as unemployed workers who are not enrolled in programmes. This implies that active labour market policy has no effect on the outside option other than the effect working through the impact of l on p^u.

2. Following the same procedure as in Question 1, demonstrate through a logarithmic approximation that when effective unemployment is given by (79) and the JSME parameter is given by (78), we obtain an expectations-augmented Phillips curve of the form:

$$\pi = \pi^e + \hat{\gamma}(\overline{u} - u), \qquad \hat{\gamma} \equiv (1-c)\overline{s}l^{\eta}(1-l), \qquad \overline{u} \equiv \frac{\ln m^w}{\gamma}. \tag{70}$$

   (Hints: start by using (69) to respecify v^e. Later on, when you take logs, use the approximations $\ln\{1-[1-cs(1-l)]u\} \approx -[1-cs(1-l)]u$ and $\ln\{1-[1-s(1-l)]u\} \approx -[1-s(1-l)]u$.)
3. How does the natural unemployment rate react to an increase in the proportion of the unemployed enrolled in active labour market programmes? (Hint: derive $\partial\overline{u}/\partial l$.) Explain the offsetting effects of an increase in l.
4. Suppose that the government wishes to minimize the natural rate of unemployment through its active labour market programmes. Derive the value of l which will achieve this goal. (Hint: remember that a necessary condition for minimization of is $\overline{u}$ is $\partial\overline{u}/\partial l = 0$.) Give an intuitive interpretation of your result. Discuss briefly whether the government should necessarily push active labour market policy to the point implied by your formula. (Hint: are there any costs and benefits of active labour market policy which we have not included in our analysis?)
5. Suppose that unemployed workers prefer not to be enrolled in active labour market programmes, say, because they consider enrolment to be stigmatizing, or because it reduces their leisure time. Discuss whether this would make active labour market policy more or less effective as a means of reducing structural unemployment.

Exercise 5. The Phillips curve with a time-varying NAIRU

Empirical estimates of the natural unemployment rate (the NAIRU) typically find that the evolution of the NAIRU tends to track the evolution of the actual rate of unemployment, at least in the short and medium run. This exercise extends our theory of the Phillips curve in order to explain why the NAIRU tends to move in the same direction as the actual unemployment rate in the shorter run.

We consider the following model with intersectoral labour mobility, where we apply the same notation as in Exercise 1, and where p^u is an average worker's probability of remaining out of work if he fails to find a job in his original sector:

Price formation: $P_i = m^p W_i, \qquad m^p > 1,$ (71)

Wage claim of the representative union: $\dfrac{W_i}{P^e} = m^w \upsilon^e, \qquad m^w > 1,$ (72)

The 'outside option' of union members: $v^e = (1 - p^u)w^e + p^u b^e,$ (73)

Expected average real wage: $w^e = 1/m^p,$ (74)

Expected real rate of unemployment benefit: $b^e = cw^e, \qquad 0 < c < 1,$ (75)

Probability of remaining unemployed in case of job loss:

$p^u = u + \theta(u - u_{-1}), \qquad \theta \geq 0, \qquad p^u \leq 1$ (76)

The variables u and u_{-1} are the unemployment rates in the current and in the previous period, respectively. The new feature of the model above is Equation (76) which says that, *ceteris paribus*, an unemployed worker has a smaller chance of finding a job if unemployment is rising than if unemployment is falling.

1. Discuss briefly whether the specification in (76) is plausible.
2. Use the model (71)–(76) to derive an expectations-augmented Phillips curve of the form:

$$\pi = \pi^e + \ln m^w - \gamma u - \gamma\theta(u - u_{-1}) \qquad \gamma \equiv 1 - c > 0, \tag{77}$$

where $\pi \equiv p - p_{-1}, \pi^e \equiv p^e - p^e_{-1}$ and $p \equiv \ln P$. (You may use the usual approximation $\ln(1 - x) \approx -x$ which is valid as long as x is not too far from zero.) Comment on the expression in (88) and compare with the expectations-augmented Phillips curve derived in the main text of the chapter.

In the following we assume that inflation expectations are static so that $\pi^e = \pi_{-1}$.

3. Define the *long-run NAIRU* as the rate of unemployment u which will be realized when the rate of inflation as well as the rate of unemployment are constant over time, that is when $\pi = \pi_{-1}$ and $u = u_{-1}$. Derive an equation for the long-run NAIRU and use this expression to

explain the factors which determine the equilibrium rate of unemployment in the long run.

4. Define the *short-run NAIRU* as the rate of unemployment $\bar{u}_s$ which will be compatible with a constant inflation rate in the *short* run where we do not necessarily have $u = u_{-1}$. (At the short-run NAIRU we thus have $\pi = \pi_{-1}$ but not necessarily $u = u_{-1}$). Derive an expression for the short-run NAIRU and show that $\bar{u}_s$ may be written as a weighted average of the long-run NAIRU and last period's actual rate of unemployment u_{-1}. Which parameter determines how much the short-run NAIRU is affected by last period's actual unemployment rate?
5. Assume that in period 0 unemployment increases from the long-run NAIRU $(\bar{u})$ to the level $\bar{u} + \Delta u_0$. Will it be possible to return to the unemployment rate $\bar{u}$ in period 1 without creating higher inflation? Give reasons for your answer.

Exercise 6. Estimating the time-varying NAIRU

From Table 18.2 in Section 18.3 one can compute estimates of the natural unemployment rate in the USA for different periods. First, do that. They seem to have varied over time. In this exercise, we invite you to estimate the level and variation in the NAIRU and to investigate whether changes in the underlying rate of productivity growth may help to explain the evolution of the NAIRU.

If inflation expectations are static, the expectations-augmented Phillips curve allowing for supply shocks $(\tilde{s})$ takes the form:

$$\pi - \pi_{-1} = \alpha(\bar{u} - u) + \tilde{s}$$

which may be rearranged to give:

$$\bar{u} + \tilde{s}/\alpha = u + \Delta\pi/\alpha, \quad \Delta\pi \equiv \pi - \pi_{-1}. \tag{78}$$

Thus, we may see the movements in the magnitude $u + \Delta\pi / \alpha$ on the right-hand side of (78) to be a result of gradual movements in the NAIRU, $\bar{u}$, as well as a result of the shorter-term and more erratic supply shocks captured by $\tilde{s}$. If we have somehow obtained an estimate of the parameter α, we may construct an estimate of $\bar{u} + \tilde{s}/\alpha$ by calculating $u + \Delta\pi / \alpha$, using available data on unemployment and inflation. We may then use the HP filter introduced in Chapter 13 to split our estimate of $\bar{u} + \tilde{s}/\alpha$ into a smooth underlying trend, which we interpret as an estimate of $\bar{u}$, and a residual term which we take to reflect $\tilde{s}/\alpha$. This is the methodology we ask you to follow below.

1. The first step is to obtain an estimate of our parameter α. At the web page for this book, you will find annual data on the unemployment rate and the rate of inflation in the USA for the period since 1960. Using these data, estimate a standard expectations-augmented Phillips curve of the form:

$$\Delta\pi = a_0 + a_1 u + \tilde{s}, \tag{79}$$

by performing an OLS regression analysis for the period. (You may assume that $\tilde{s}$ is normally distributed with a zero mean and a constant variance). Is your estimate of α_1 statistically significant and does it have the expected sign? Assuming you can answer in the affirmative, you may use your estimate of the numerical value of α_1 as an estimate of α in your further analysis.

2. Armed with your estimate of α and your data set, you can now calculate a time series for the magnitude $u + \Delta\pi / \alpha$. In this way you obtain an estimated time series for $\bar{u} + \tilde{s}/\alpha$ for the period after 1960. By the method also applied in Chapter 13 (and used in Exercise 2 of that chapter, which also refers to an instruction in HP filtering in Excel to be found at the web page for this book), estimate an HP trend for your time series for $\bar{u} + \tilde{s}/\gamma$, setting the λ parameter equal to 1000 to obtain a quite smooth trend. Interpret your HP trend as an estimate of $\bar{u}$ and construct a diagram in which you plot your estimated time series for the NAIRU. What is the range within which the NAIRU has varied?

It is sometimes argued that the trade-off between the level of unemployment and the change in the rate of inflation will tend to improve in periods of accelerating productivity growth, and vice versa. According to (78), a longer-lasting improvement in the trade-off between u and $\Delta\pi$ (that is, a fall in $u + \Delta\pi / \alpha$) must reflect a fall in the NAIRU, $\bar{u}$. Thus the NAIRU should tend to fall when the underlying rate of productivity growth accelerates, whereas $\bar{u}$ should tend to rise when underlying productivity growth slows down. The next question asks you to explore this relationship.

3. At the web page for this book, you will also find annual data on the growth rate in output per hour worked in the USA for the period since 1960. Use the HP filter to estimate an underlying trend in this productivity growth rate, setting $\lambda = 100$. Plot the resulting estimate of trend productivity growth against your estimate of the NAIRU. Do the two time series tend to move in opposite directions, as our theory predicts? Explain the theoretical reasons why accelerating productivity growth may be expected to reduce the NAIRU, at least temporarily.

(Postscript: if you would like to know more about the likely reasons for the variations in the US NAIRU, you may want to consult the following readable article from which the idea for this exercise was taken: Laurence Ball and N. Gregory Mankiw, ‘The NAIRU in Theory and Practice’, *Journal of Economic Perspectives*, **16** (4), Fall 2002, pp. 115 –136).

Appendix: A micro-foundation of the expectations-augmented Phillips curve and the aggregate supply curve with basis in efficiency wage theory

We will now illustrate how a simplified version of the so-called 'shirking' model of efficiency wages introduced in Chapter 11 may provide an alternative micro-theoretic foundation for the expectations-augmented Phillips curve.[19] The basic idea underlying this model is that work effort generates disutility, so workers have an incentive to 'shirk' on the job (taking coffee breaks, chatting with colleagues, using the office computer to surf on the internet, etc.) instead of working hard all the time. To keep the incentive for shirking in check, firm managers therefore have to monitor the work effort of the employees and impose some sanction such as firing a worker who is caught shirking. However, since perfect and constant monitoring is practically impossible or prohibitively expensive, there is always some chance that a worker can get away with shirking without being caught.

We consider again a framework of monopolistic competition as in Section 18.2 with many different firms indexed by i,each producing its own product variety from labour input. Each firm sets its own nominal output price P_i and the nominal wage rate W_i paid to the workers employed in the firm.

At this nominal wage, each worker employed in firm i is formally obliged to contribute a certain level of effort. Shirking means providing less than this required effort. To simplify matters, we assume that a worker can either fully provide the required effort or shirk all the time. If the worker provides full effort, the expected utility level U_i^w of the worker will be:

$$U_i^W = \left(\frac{W_i}{P^e}\right)(1-\mu), \qquad 0 < \mu < 1, \tag{A1}$$

where P^e is the expected general price level, that is, the expectation of an appropriate average of the nominal prices of the commodities (coming from different production sectors) that the worker needs for consumption. Hence, W_i/P^e is the expected purchasing power of the money wage offered by firm i.

Note that we assume in (A1) that workers do not know the exact prices of all consumption goods at the time when they must decide on work effort. Whereas the worker may be perfectly aware of the wage rate (and perhaps the price) of the worker's own firm, he or she cannot know all prices of all other goods simply because there so many of them and. The worker must therefore rely on an expected price level. We also assume that the individual firm does not know the exact prices and wages of all other firms, and hence the general price and wage levels, at the time when it decides on its own wage and price. This is due to nominal wage and price rigidities: each firm sets its nominal wage and price for the current period before that period begins and maintains the chosen wage and price levels throughout the period. At the time of wage and price setting the firm cannot know the general wage and price levels prevailing in the coming period, so it must rely on their expected levels.

Equation (A1) assumes that the worker's utility from consumption is W_i / P^e, but it also

[19] Even if you have not studied Chapter 11, you should still be able to follow the exposition below.

assumes that there is a disutility of providing the required effort equal to $\mu\,(W_i/P^e)$, where μ is a parameter reflecting the worker's preference for 'leisure on the job'. The higher the worker's income, the more highly he thus values leisure on the job, so the more income he or she is willing to sacrifice in return for the benefits from coffee breaks, internet surfing, etc.[20]

A worker who chooses instead to shirk all the time faces a probability θ of being caught by the manager and fired for poor work performance. In that case, the worker must look for a job elsewhere in the labour market, but since there is unemployment, he or she cannot be sure to find one. We assume that the worker expects to be able to obtain the utility level v if he or she is fired due to shirking. The variable v is referred to as the worker's 'outside option', i.e., his or her fall-back position if sacked (we will discuss the determinants of v in a moment). The probability that the worker is not caught shirking is $1 - \theta$. In that situation the worker's utility will simply be his or her expected real consumer wage W/P^e, since full shirking does not involve any disutility from effort. Overall, a shirking worker's expected utility is therefore given by:

$$U_i^s = (1-\theta)\left(\frac{W_i}{P^e}\right) + \theta v, \qquad 0 < \theta < 1, \qquad \theta > \mu, \tag{A2}$$

where the assumption $\theta > \mu$ is made for a meaningful model (see Equation (A.3) below). A utility-maximizing worker will choose to shirk if $U_i^s > U_i^w$ and will choose to work when $U_i^s < U_i^w$. In the borderline case where $U_i^s = U_i^w$, the worker is in principle indifferent between working and shirking, but we assume that in this case social norms induce him or her to work.

When workers choose to provide the required effort, their output per person is determined exogenously by the firm's level of technology which we assume to be sufficient to generate a weakly positive profit in the firm. We can now derive the cost-minimizing wage rate that the employer will want to offer. If the wage is so low that $U_i^s > U_i^w$, workers will produce nothing and the employer will make a loss as long as the wage rate is positive. Firing the shirking workers will not solve the problem since newly hired workers will have an incentive to shirk as well, assuming they have the same preferences as those who were fired. On the other hand, if the wage is so high that $U_i^s < U_i^w$, the employer is paying more than necessary to induce work effort without obtaining a higher labour productivity in return (since the productivity of a non-shirking worker is exogenously determined by technology). Hence, the cost-minimizing wage rate is found where the *non-shirking condition* $U_i^w \geq U_i^s$ is met with equality. Setting $U_i^w = U_i^s$ and using (A1) and (A2), we thus obtain the cost-minimizing wage rate offered by firm i:

$$\overbrace{\left(\frac{W_i}{P^e}\right)(1-\mu)}^{U_i^w} = \overbrace{(1-\theta)\left(\frac{W_i}{P^e}\right) + \theta v}^{U_i^s} \quad \Leftrightarrow \quad \frac{W_i}{P^e} = \frac{\theta}{\theta - \mu} v. \tag{A3}$$

We see that the expected consumer real wage is set as a mark-up over the representative

[20] If we did not allow the valuation of leisure to rise with the worker's income level, the incentive to shirk would tend towards zero as the real wage grows over time due to increasing productivity. Assuming that the demand for leisure varies positively with income is equivalent to assuming that leisure is a 'normal' good.

worker's outside option v. The mark-up factor $\theta/(\theta - \mu)$ is positive and greater than 1, given our assumption in (A2) that $\theta > \mu$. A higher value of θ reduces the firm's optimal wage rate, because a higher risk of being fired for shirking strengthens the incentive to work, thereby lowering the wage rate necessary to induce effort. On the other hand, the stronger the preference μ for 'leisure on the job', the higher is the wage rate which must be paid to secure effort.

Let us now specify the representative worker's outside option, i.e., the utility he or she expects to obtain if fired for shirking. For an average worker who is sacked, the probability of finding a job elsewhere in the labour market is given by the employment rate $1 - u$, where u is the rate of unemployment. If the worker succeeds in finding a job, it follows from the analysis above that he or she may expect to obtain a utility level $U^s = U^w = (W^e/P^e)(1 - \mu)$, where W^e is the expected average money wage level in the labour market. On the other hand, the worker faces a probability u of remaining unemployed, in which case he or she expects to be able to receive a nominal unemployment benefit B^e with an expected real purchasing power B^e/P^e. We may therefore specify the representative worker's outside option as follows:

$$v = (1-u)(1-\mu)\frac{W^e}{P^e} + u\frac{B^e}{P^e}. \tag{A4}$$

The rate of unemployment benefit is assumed to be indexed to the general wage level, with a constant replacement ratio c known to all agents. The expected benefit rate is then given by $B^e = cW^e$. We impose the restriction that $0 < c < 1 - \mu$, since otherwise the expected real wage net of the disutility of work, $(W^e/P^e)(1 - \mu)$, would be lower than the expected real rate of unemployment benefit, cW^e/P^e, so that unemployed workers would have no incentive to look for a job.

Inserting $B^e = cW^e$ into (A4) gives:

$$v = (1-u)(1-\mu)\frac{W^e}{P^e} + u\frac{cW^e}{P^e} = \left(1-\mu-\gamma u\right)\frac{W^e}{P^e}, \quad \gamma \equiv 1-\mu-c > 0, \tag{A5}$$

and then inserting this expression for v into (A3) and multiplying through by P^e, we find the optimal nominal wage rate offered by firm i:

$$W_i = M^w\left(1-\mu-\gamma u\right)W^e, \quad M^w = \frac{\theta}{\theta-\mu} > 1. \tag{A6}$$

We see that the expected price level P^e does not affect wage formation.[21] As (A6) makes clear, the individual firms seeks to maintain a certain relationship between its own nominal wage rate and the expected general nominal wage level. A change in nominal prices does not disturb this relationship, since it affects workers inside and outside each firm in the same manner. Thus our efficiency wage model implies 'relative wage resistance' in the sense that individual firms resist a change in the ratio of their own wage to the general wage level. However, the model does not

[21] This result relies on our assumption that the nominal rate of unemployment benefit is indexed to the nominal wage level. If benefits were instead indexed to the price level, (expected) changes in P would affect the workers' outside option and thereby influence wage setting. In reality, most countries index unemployment benefits to wages rather than to consumer prices.

imply 'real wage resistance', since a rise in nominal prices does not automatically lead to a rise in nominal wages.

If we make the *symmetry assumption* that expectations as well as the parameters θ and μ are the same across all firms, (A6) implies that all employers will set the same wage rate. Hence we have $W_i = W$, where W is the general level of nominal wages. From (A6) we then get:

$$W = M^w(1-\mu-\gamma u)W^e. \tag{A7}$$

To establish the link between wages and prices, we now consider the representative firm's price-setting behaviour. For simplicity, suppose the production function is linear and that one unit of labour produces A units of output, where A is the exogenous level of average and marginal labour productivity. The representative firm's average and marginal unit costs will then be $W_i/A = W/A$. According to the theory of monopolistically competitive price formation developed in this chapter, a monopolistically competitive firm will set its price as a mark-up, M^p, over its marginal labour cost. Since all firms pay the same wages, use the same technology and charge the same mark-up, the output price charged by the representative firm will then be:

$$P_i = P = M^p \cdot \frac{W}{A}, \qquad M^p > 1. \tag{A8}$$

Let us assume that M^p is constant over time whereas labour productivity fluctuates around some trend level $\overline{A}$. Equation (A8) then implies that, on average, the relationship between the general wage level and the domestic price level is:

$$W = \frac{\overline{A}}{M^p} P. \tag{A9}$$

Suppose, reasonably, that by experience firms (the wage and price setters) are aware of this average relationship between the general wage and price levels. It follows that reasonably there must be the following relationship between each individual firm's expected wage level and price level at the time when the firm sets its own wage and price for the next period:

$$W^e = \frac{\overline{A}}{M^p} P^e. \tag{A10}$$

Using (A9) and (A10) to eliminate W and W^e from (A7), taking natural logarithms on both sides of the resulting equation, writing $p \equiv \ln P$, $p^e \equiv \ln P^e$, $m^w \equiv \ln M^w$, $a \equiv \ln A$ and $\overline{a} \equiv \ln \overline{A}$, and exploiting the approximation $\ln(1-\mu-\gamma u) \approx -\mu-\gamma u$, we get:

$$\overbrace{\frac{A}{M^p}\cdot P}^{W} = M^w(1-\mu-\gamma u)\overbrace{\frac{\overline{A}}{M^p}\cdot P^e}^{W^e} \quad \Rightarrow \quad p \approx p^e + m^w - \mu - \gamma u + \overline{a} - a. \tag{A11}$$

If we subtract p_{-1} on both sides and use $\pi \equiv p - p_{-1}$ and $\pi^e \equiv p^e - p_{-1}$ etc., we get:

$$\pi = \pi^e + m^w - \mu - \gamma u + \bar{a} - a. \tag{A12}$$

Defining the natural unemployment rate $\bar{u}$ as the rate of unemployment u that gives $\pi = \pi^e$ for $a = \bar{a}$, it follows from (A12) that $m^w - \mu = \gamma \bar{u}$. Now, writing $\gamma \bar{u}$ for $m^w - \mu$ in (A12) gives:

$$\pi = \pi^e + \gamma(\bar{u} - u) + \bar{a} - a, \quad \bar{u} \equiv \frac{m^w - \mu}{\gamma}. \tag{A13}$$

We see that (A13) is an expectations-augmented Phillips curve. From this we may derive an aggregate supply curve. Denoting the total labour force by N and the trend level of output by $\bar{Y}$, we introduce the following definitions and approximations as in this chapter:

$$Y \equiv A(1-u)N \quad \Rightarrow \quad y \equiv \ln Y \approx a - u + \ln N, \tag{A14}$$

$$\bar{Y} \equiv \bar{A}(1-\bar{u})N \quad \Rightarrow \quad \bar{y} \equiv \ln \bar{Y} \approx \bar{a} - \bar{u} + \ln N. \tag{A15}$$

It follows that $\bar{u} - u = y - \bar{y} + \bar{a} - a$. Inserting this into (A13), we end up with a short-run aggregate supply curve of the form:

$$\text{AS:} \qquad \pi = \pi^e + \gamma(y - \bar{y}) + s, \quad s \equiv (1+\gamma)(\bar{a} - a), \tag{A16}$$

where s is a supply shock variable capturing productivity shocks.[22] Equation (A16) shows that our theory of efficiency wage setting leads to an aggregate supply curve of exactly the same form as the AS curve based on union wage setting considered in the main text of this chapter.

[22] For simplicity, we have assumed that the wage and price mark-ups m^w and m^p are constant. If they are fluctuating over time, the supply shock variable s would also include the cyclical components of m^w and m^p.

PART 6

Short-run Macroeconomics for the Closed Economy

Chapter 19

Explaining business cycles: Aggregate supply and aggregate demand in action

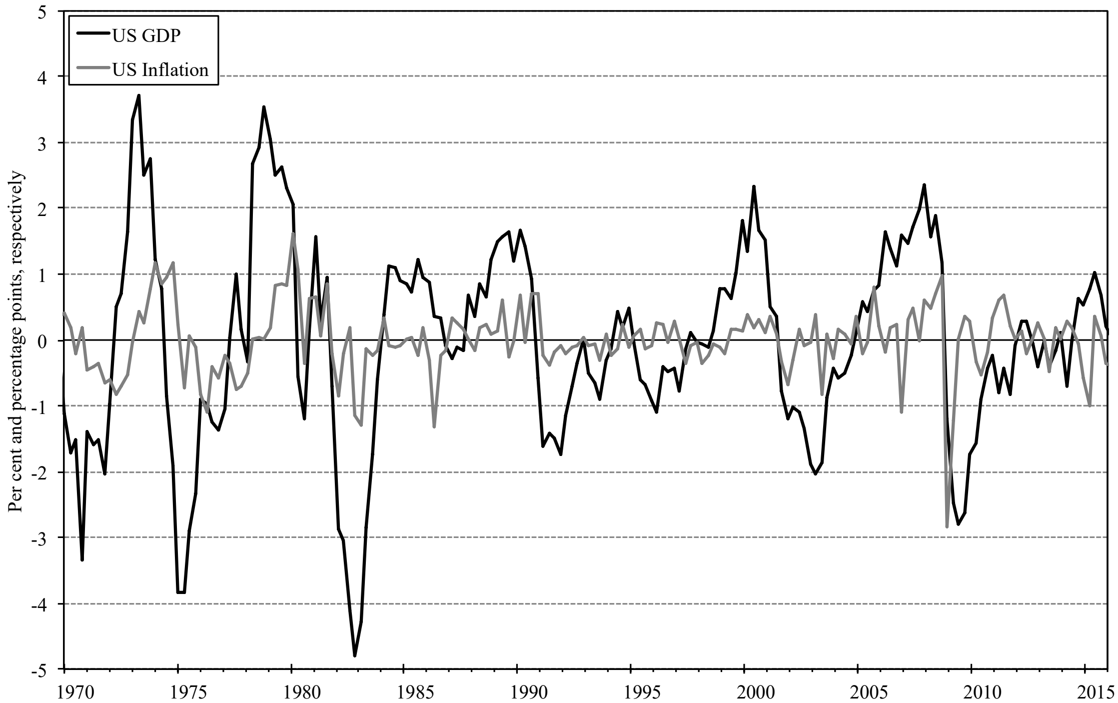
Introduction

Throughout economic history, the capitalist market economies of the world have gone through recurrent periods of boom and bust. This is the fascinating phenomenon of business cycles described in Chapter 13. Although long periods of high economic growth have sometimes led people to believe that the business cycle was dead, Figure 19.1 and Figure 19.2 show that it is still very much alive. Economic activity and inflation continue to fluctuate in an irregular cyclical manner around their long-run trend levels, and in the steepest economic downturn since the 1930s, output per capita in all the major developed economies fell sharply between 2008 and 2009. A fundamental challenge for macroeconomic theory is to explain why the economy goes through these cyclical movements rather than evolving smoothly over time.

The two previous chapters derived the economy's aggregate supply curve and its aggregate demand curve. In this chapter, we bring the two curves together in a complete macro model which enables us to determine the levels of total output and inflation in the short run. This model allows us to investigate the causes of the fluctuations in economic activity which we observe in the real world. We will illustrate how business fluctuations may be seen as the economy's reaction to various shocks that tend to shift the aggregate supply and demand curves. We will also investigate whether our model is able to reproduce the picture of Figure 19.1, that is, the degrees of volatility in GDP and inflation, the contemporaneous correlation between the fluctuations in the those two variables, and the degree of persistence in each of them in USA.[1] Our analysis will show you a simple example of a *stochastic, dynamic, general equilibrium model* (DSGE) model. This will introduce you to an important subfield in modern macroeconomic theory and empirics: constructing DSGE models and confronting them with the empirical facts of the business cycle.

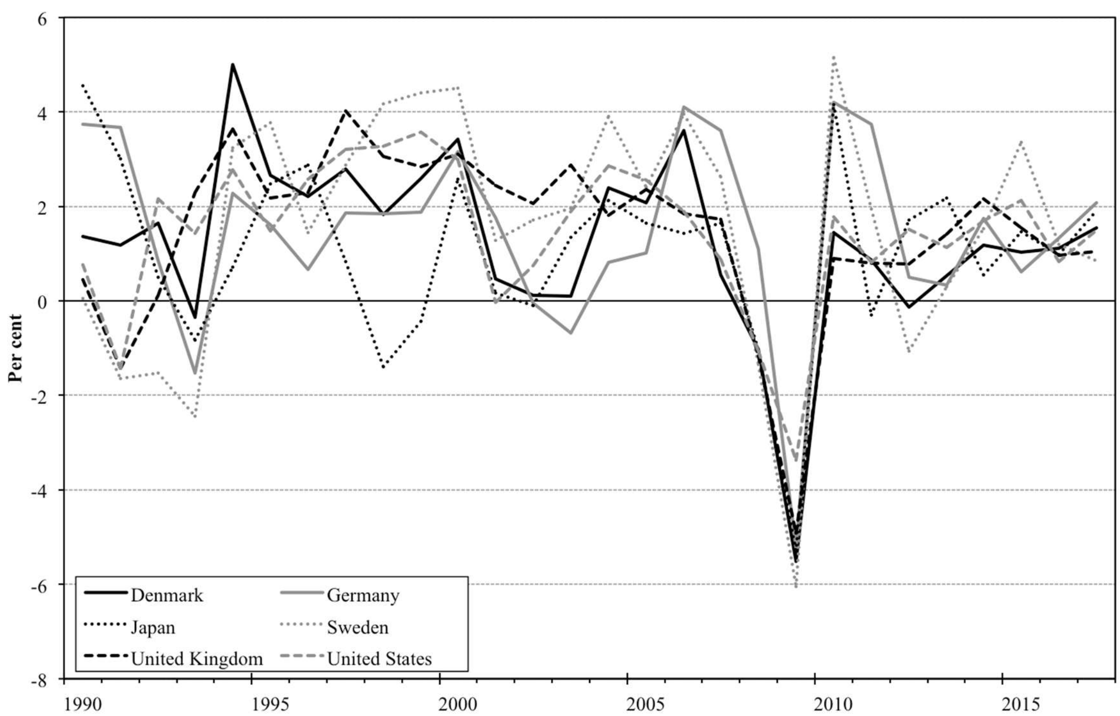
[1] The inflation rate used in Figure 19.1 is quarterly and *not* annualized.

Figure 19.1 Cyclical components of real GDP and inflation in the USA, 1970Q1-2015Q4



Note: Quarterly data, 1970Q1-2015Q4. Inflation rate not annualized, cyclical component for inflation in percentage points. Structural levels based on HP-filter with $\lambda = 1600$. End-point observations excluded. *Source*: Bureau of Economic Analysis and Bureau of Labor Statistics and own computations.

Figure 19.2 Growth in real per capita GDP in selected countries, 1990-2017



Source: World Economic Outlook, October 2018, IMF.

The perspective on business cycles adopted here is sometimes referred to as the *Frisch–Slutsky paradigm*, named after the Norwegian economist and Nobel Prize winner Ragnar Frisch and the Russian economist and statistician Eugen Slutsky who first introduced this way of interpreting business cycles.[2] The Frisch–Slutsky paradigm distinguishes between the *impulse* which *initiates* a movement in economic activity, and the *propagation mechanism* which subsequently *transmits* the shock through the economic system *over time*. In our model framework, the impulse is a sudden exogenous change in one of the 'shock' variables determining the position of the aggregate supply and demand curves. The propagation mechanism is the endogenous economic mechanism which converts the impulse into *persistent* business fluctuations. The propagation mechanism reflects the structure of the economy and determines the manner in which it reacts to shocks and how long it takes for it to adjust to a shock. Ragnar Frisch stressed that even though shocks to the economy may follow an unsystematic pattern, the structure of the economy may imply that it reacts to disturbances in a systematic way which is very different from the pattern of the shocks themselves. Frisch was inspired by the famous Swedish economist Knut Wicksell who used the following metaphor to explain the difference between the unsystematic impulse to the economy and the systematic business cycle response implied by the propagation mechanism: 'If you hit a wooden rocking chair with a club, the movement of the chair will be more or less regular because of its form, even if the hits are quite irregular'.[3]

In short, this chapter raises two basic questions:

1. Why do movements in economic activity display persistence?

2. Why do these movements tend to follow a cyclical pattern?

We start out in Section 19.1 by setting up our model of aggregate supply and aggregate demand, termed the AS–AD model. In Section 19.2 we then use the model to illustrate how the economy reacts to demand and supply shocks in a so-called deterministic world. In this *deterministic* version of our AS–AD model, the demand and supply shocks are non-random, occurring either within a limited time span, or representing a permanent level shift in some exogenous variable. Following a qualitative graphical analysis, we will set up a quantitative version of the deterministic AS–AD model to study the *impulse-response functions* showing how the economy responds to various shocks over time. We find that the deterministic AS–AD model is capable of explaining the observed *persistence* of the movements in economic activity following a shock, but it cannot really explain why business fluctuations tend to follow a *cyclical* pattern. To deal with this problem, Section 19.3 sets up a *stochastic* version of the AS–AD model in which the exogenous demand and supply shock variables are *random variables*. As we shall see, this model is able to reproduce the most important stylized business cycle facts reasonably well.

Expectations matter a great deal for explaining business cycle regularities by DSGE models. In this chapter, we only study *backward-looking expectations*, where the

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[2] See Ragnar Frisch, 'Propagation Problems and Impulse Problems in Dynamic Economics', in *Economic Essays in Honour of Gustav Cassel*, London, Allen and Unwin, 1933; and Eugen Slutsky, 'The Summation of Random Causes as the Source of Cyclic Processes', *Econometrica*, **5**, 1937, pp. 105 –146 (the latter published in Russian ten years earlier).

[3] This statement by Wicksell was made in a discussion at a meeting of the Swedish Economic Association in Uppsala in 1924. See 'Nationalekonomiska Föreningens Förhandlingar 1924', Uppsala, 1925.

expectation of some future variable is rooted in past observations of the same variable. In Chapter 21 we will study business fluctuations in a DSGE model with *forward-looking expectations*, namely so-called *rational expectations*.

19.1 The model of aggregate supply and aggregate demand

Assembling the AS–AD model

To prepare the ground for our analysis of business cycles, we will now assemble the building blocks from the two previous chapters to form a model of aggregate supply and aggregate demand – the AS–AD model – which allows us to determine the short-run levels of output and inflation.

In extensive form, our AS–AD model may be stated as follows, where the subscript t indicates that we consider period t:

$$y_t - \bar{y} = \alpha_1(g_t - \bar{g}) - \alpha_2(r_t - \bar{r}) - \alpha_3(\tau_t - \bar{\tau}) + v_t \tag{1}$$

$$r_t \equiv i_t - \pi_{t+1}^e \tag{2}$$

$$i_t = \bar{r} + \pi_{t+1}^e + h(\pi_t - \pi^*) + b(y_t - \bar{y}) + \hat{\rho}_t \tag{3}$$

$$\pi_t = \pi_t^e + \gamma(y_t - \bar{y}) + s_t \tag{4}$$

$$\pi_t^e = \pi_{t-1} \tag{5}$$

As you recall from Chapter 17, Equation (1) is the IS curve derived from the condition for goods market equilibrium, where v_t captures shifts in private sector confidence, $g_t - \bar{g}$ and $\tau_t - \bar{\tau}$ are fiscal policy impulses, and the long-run equilibrium real market interest rate is $\bar{r}$. Multiplier effects are included in the parameters α_1 through α_3 and in the shock v_t. Equation (2) is the definition of the *ex ante* market real interest rate. Equation (3) is the Taylor rule describing the longer-term, nominal interest rate that a central bank pursuing an inflation target π^* and the natural output target $\bar{y}$ will try to obtain. The stochastic error term $\hat{\rho}_t$ reflects fluctuations in risk premia which mean that changes in the central bank's short term policy interest rate are not always transmitted one-to-one into changes in the relevant longer-term market interest rate. All of these equations were explained in Chapter 17. Equation (4) is the short-run aggregate supply curve derived in Chapter 18. The variable s_t captures supply shocks such as fluctuations in wage and price mark-ups and shifts in productivity. The final equation (5) restates the assumption of static expectations according to which the expected current inflation rate equals last period's observed inflation rate.

Chapter 17 showed how (2) and (3) may be inserted into (1) to give the economy's aggregate demand curve. Repeating this operation, we get the AD curve:

$$\text{AD:}\quad y_t - \bar{y} = -a(\pi_t - \pi^*) + z_t,$$

$$a \equiv \frac{\alpha_2 h}{1+\alpha_2 b}, \qquad z_t \equiv \frac{\alpha_1(g_t - \bar{g}) - \alpha_3(\tau_t - \bar{\tau}) + v_t - \alpha_2 \hat{\rho}_t}{1+\alpha_2 b}. \tag{6}$$

The AD curve may be rearranged as:

$$\text{AD:}\quad \pi_t = \pi^* - \frac{1}{a}(y_t - \bar{y}) - \frac{z_t}{a}. \tag{7}$$

Furthermore, we may write the short-run aggregate supply curve in the following form by merging (4) and (5):

$$\text{AS:}\quad \pi_t = \pi_{t-1} + \gamma(y_t - \bar{y}) + s_t \tag{8}$$

Equations (6) (or (7)) and (8) summarize the AS–AD model with static expectations in compact form. It is worth recalling the mechanisms underlying the AD and SRAS curves:

PROPERTIES OF THE AS–AD MODEL

The AD curve is downward-sloping in y_t, π_t -space because a fall in the rate of inflation induces the central bank to cut the real interest rate, thereby stimulating aggregate demand. The AS curve slopes upwards in (y_t, π_t) space because a rise in output requires a rise in employment which drives down the marginal productivity of labour, thereby increasing marginal costs and inducing firms to raise their prices at a faster rate.

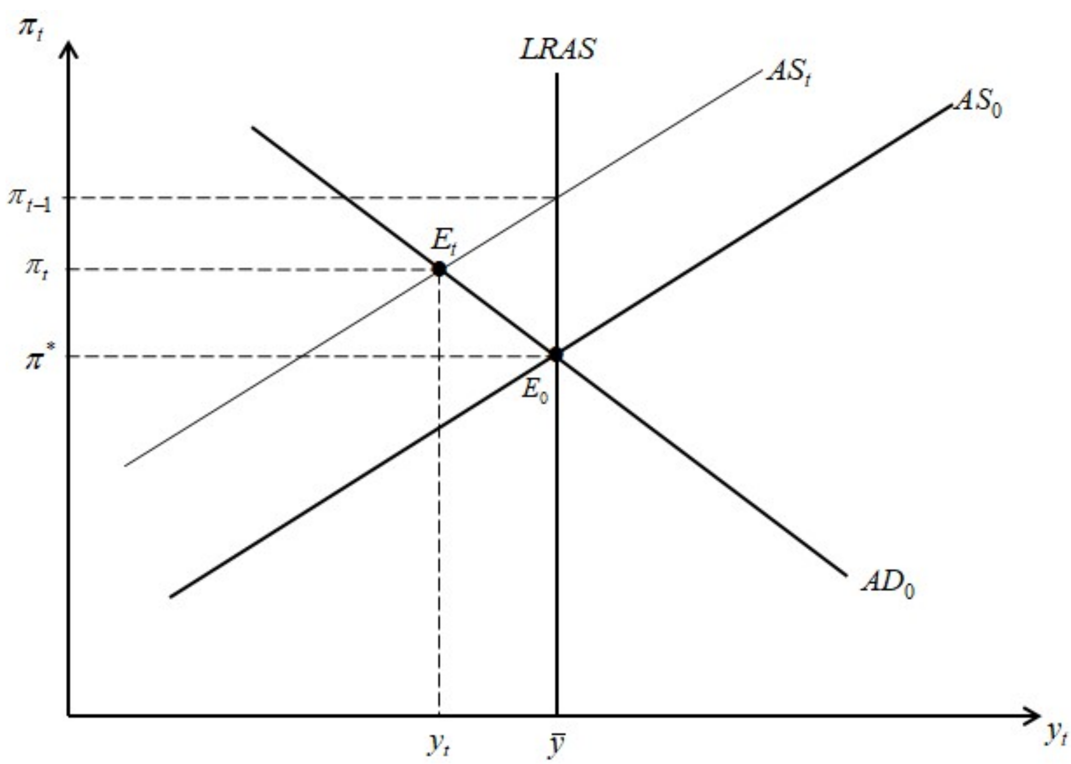
Long-run and short-run macroeconomic equilibrium

As explained in Chapter 17 and evident from (6) or (7), in a period where $z_t = 0$, the AD curve passes through the point $(\bar{y}, \pi^*)$ in the y_t, π_t -plane. In Figure 19.3 the curve labelled AD_0 illustrates such an AD curve. And as explained in Chapter 18 and evident from (4) above, in a period where $s_t = 0$, the AS curve passes through $(\bar{y}, \pi_t^e)$ and since we now assume $\pi_t^e = \pi_{t-1}$, it passes through $(\bar{y}, \pi_{t-1})$. If it happens that $\pi_{t-1} = \pi^*$, the AS curve passes through $(\bar{y}, \pi^*)$ as illustrated by AS_0 in Figure 19.3. The figure also contains another AS curve labelled AS_t relevant for a period *t* for which $s_t = 0$ and the rate of inflation the period before (period *t*-1) was the π_{t-1} indicated in the figure. Note that AS_t passes through $(\bar{y}, \pi_{t-1})$.

In any period, the relevant short-run equilibrium is where the AS curve for the period intersects the AD curve for the period.

Consider a period that we will call period 0, where up to and including the period no shocks have occurred for some time, that is, $z_t = s_t = 0$ for some $t \le 0$, and where for some time inflation has been stable, that is, $\pi_t = \pi_{t-1} = \pi_t^e$ for some $t \le 0$. The AD curve is then AD_0 in Figure 19.3 passing through $(\bar{y}, \pi^*)$. The stable (constant) inflation must have been equal to the inflation target, $\pi_t = \pi_{t-1} = \pi_t^e = \pi^*$. As Figure 19.3 shows, if for instance $\pi_{t-1} > \pi^*$, then the rate of inflation in the short-run equilibrium of period *t* will fulfill $\pi_t < \pi_{t-1}$ and inflation is then not stable. Since then $\pi_{t-1} = \pi^*$, the AS curve of period 0 will be like AS_0 in the figure passing through $(\bar{y}, \pi^*)$. The two curves AS_0 and AD_0 will then intersect each other exactly in the point $(\bar{y}, \pi^*)$ denoted by $\bar{E}$ in the figure, establishing this point as the appropriate short-run equilibrium in period 0. Such a situation, where no shocks have occurred for some time, and where inflation and inflation expectations have come to rest at π^*, so that the relevant AS and AD curve intersect at $(\bar{y}, \pi^*)$, we will refer to as a long-run equilibrium, since in the absence of any new shocks no forces will lead away from the situation. For the same reason we refer to the vertical line at $\bar{y}$ as the long-run aggregate supply curve, *LRAS*. The position of the LRAS curve is determined by the economy's natural rate of output, as we explained in Chapter 18.

Figure 19.3 Short-run macroeconomic equilibrium with cyclical unemployment



In any given period *t*, the short-run equilibrium need not coincide with the long-run equilibrium. A deviation can occur either because of shocks occurring in the period,

$z_t \neq 0$ or , $s_t \neq 0$, or because the rate of inflation in period $t-1$ did not happen to be exactly π^* , perhaps because of earlier shocks. Hence, in a period *t*, the AS curve can deviate from AS_0, and the AD curve can deviate from AD_0 in Figure 19.3, creating an intersection between the two in another point than $(\bar{y}, \pi^*)$. This other point will then be the relevant short-run equilibrium in the period. Figure 19.3 illustrates a situation where for some unspecified reason (due to shocks occurring before period *t*), the rate of inflation π_{t-1} in period *t*-1, and thereby the expected rate of inflation π_t^e for period *t*, are different from and both larger than π^* , that is, $\pi_t^e = \pi_{t-1} > \pi^*$, but otherwise there are no disturbances in period *t*, so $z_t = s_t = 0$. The AS curve of period *t* will then be like AS_t in the figure, while the AD curve will be like AD_0. The two intersect at E_t , the short-run equilibrium of period *t*, where $y_t < \bar{y}$ and $\pi_t > \pi^*$ and $\pi_t < \pi_{t-1}$.

The short-run equilibrium E_t illustrated in Figure 19.3 is characterized by cyclical unemployment, since actual output y_t falls short of natural output. The question is, are there forces in the economy whereby the economy is able to work itself out of this recession and back towards the long-run equilibrium? [4]

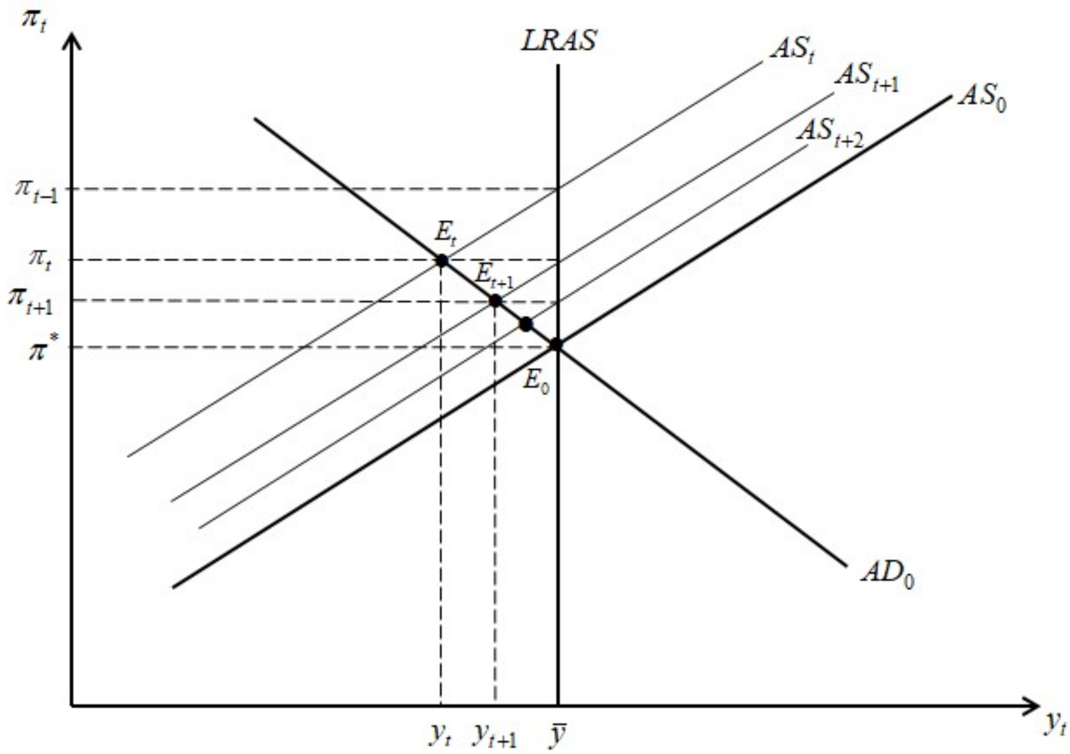
The adjustment to long-run equilibrium

To focus on the economy's adjustment mechanisms, suppose there are no further demand and supply shocks following the initial shocks, which created the recession. Furthermore, remember that the position of the AS curve depends on the expected rate of inflation π_t^e , which is equal to last period's actual inflation rate π_{t-1} , such that in any period where $s_t = 0$, the AS curve passes through $(\bar{y}, \pi_{t-1})$.

Using this insight, Figure 19.4 traces the economy's adjustment over time. In the initial period *t*, where the economy is in recession at point E_t, the inflation rate is π_t . Hence, in the next period *t*+1, people expect the previous period's level of inflation to continue, so the AS curve for period *t*+1 cuts the LRAS curve at the inflation rate π_t . Hence, the SRAS curve shifts down from AS_t to AS_{t+1} between period *t* and period *t*+1, generating a new short-run equilibrium at point E_{t+1}, where inflation is lower and output is higher than in period *t*. When the economy enters period 2, agents expect an inflation rate $\pi_{t+1} < \pi_t$, so the AS curve shifts down again, cutting the LRAS curve at the inflation rate π_{t+1}. This creates a new short-run equilibrium at E_{t+2}, where inflation is even lower and output has risen once more. As illustrated in Figure 19.4, this process of successive downward shifts in the SRAS curve will continue as long as $y_t < \bar{y}$. Thus, the economy will gradually move down along the AD curve until it finally settles in the long-run equilibrium $\bar{E}$ where output is at its natural rate $\bar{y}$. In this long-run equilibrium, inflation is at its target level, since it follows from (7) that $\pi_t = \pi^*$, when $y_t = \bar{y}$ and $z_t = 0$.

[4] Below we analyse more carefully how the economy may be pushed into a recession in the first place. Here we just take the economy's starting point as given, since we are currently focusing on its dynamic adjustment mechanisms.

Figure 19.4 The adjustment to long-run macroeconomic equilibrium



The economics underlying this macroeconomic adjustment mechanism may be explained as follows. When the economy is in recession, wage setters tend to overestimate the rate of inflation. This will motivate them to reduce their inflation forecasts over time, and as a consequence they will lower their required rate of increase in money wages. Hence, firms will experience a lower rate of increase in marginal costs, and via mark-up price setting this will translate into a lower rate of price inflation. As inflation goes down, the central bank cuts the real interest rate, thereby ensuring that weaker inflationary pressure increases aggregate demand and output. As long as output and employment remain below their natural rates, wage setters will continue to overestimate the rate of inflation (although to a falling degree) and will therefore revise their inflation forecasts downwards, resulting in a lower rate of wage and price inflation, which triggers another interest rate cut and generates yet another fall in expected inflation, and so on. In this way, the gradual reduction in actual and expected inflation paves the way for successive reductions in the rate of interest that stimulate aggregate demand and pull the economy out of the recession.

Note the crucial importance of central bank behaviour for this macroeconomic adjustment process. To make sure that falling inflation actually increases aggregate demand, the central bank must cut the nominal interest rate by *more* than one percentage point for each percentage point drop in inflation (and hence in expected inflation up to the next period), that is, our parameter h in the Taylor rule must be positive.[5] Otherwise,

[5] Inserting $\pi^e_{t+1} = \pi_t$ into (3), we get $i_t = \overline{r} + \pi_t + h(\pi_t - \pi^*) + b(y_t - \overline{y}) + \hat{\rho}_t$ which shows that a one

falling inflation will cause the real interest rate to rise, exacerbating the initial recession. The stronger monetary policy reacts to falling inflation (the greater the value of h), the flatter is the aggregate demand curve, and the faster is the convergence of output to its natural rate.

The need to keep the monetary policy parameter h positive is the 'Taylor Principle' also mentioned in Chapter 17. While adherence to this principle is crucial for the stability of the macro economy, our specific assumption of static expectations $(\pi_t^e = \pi_{t-1})$ is less important. The only thing needed to ensure convergence towards long-run equilibrium is that the expected inflation rate will fall over time when people learn that they have overestimated the rate of inflation, and that expected inflation will increase when people find out that they have underestimated the inflation rate. In that case, the AS curve will gradually shift down when the economy is in recession, whereas it will shift up over time when the economy is in a boom (where $y_t > \overline{y}$).

Calibrating the model: how fast is convergence - how long is the long run?

According to the analysis above, the changes in the inflation rate generated by market forces will automatically pull the economy out of a recession as long as the central bank follows the Taylor Principle. In the long run, the rates of output and employment will thus converge on their natural rates. However, for workers who cannot find a job and for entrepreneurs struggling to avoid bankruptcy it may not be very interesting to know that the economy will eventually recover from a recession if the process of recovery is very slow. As John Maynard Keynes once said: 'In the long run we are all dead'. When deciding whether political action such as a tax cut, an increase in public spending or larger reaction parameters h and b in the Taylor rule are needed to fight a recession, it is obviously very important to know whether the economy's convergence to the natural rate is relatively slow or relatively fast as policy is presently conducted. In the former case (additional) policy intervention may be needed; in the latter case it may not.
To study the question 'How long is the long run?', we may consider a *quantitative* version of our AS–AD model. By assigning plausible values to the parameters of the model, we can investigate how fast the economy is likely to move from its current short-run equilibrium towards its long-run equilibrium after having been hit by a shock. For this purpose we maintain the assumption that there are no further supply or demand shocks following the initial shock that created the recession. In formal terms this means that $s_t = z_t = 0$ for $t \geq 0$. Let $\hat{y}_t \equiv y_t - \overline{y}$ denote the relative deviation of output from trend (the output gap), and let $\hat{\pi}_t \equiv \pi_1 - \pi^*$ indicate the deviation of inflation from the target inflation rate (the inflation gap). Setting $s_t = z_t = 0$, we may restate our AS–AD model (6) and (8) as follows:

$$\text{AD:} \quad \hat{y}_t = -a\hat{\pi}_t, \qquad a \equiv \frac{\alpha_2 h}{1+\alpha_2 b}. \tag{9}$$

---

percentage point decrease in the current inflation rate will trigger a more than one percentage point decrease in the nominal policy interest rate when $h > 0$.

$$\text{AS:} \quad \hat{\pi}_t = \hat{\pi}_{t-1} + \gamma \hat{y}_t. \tag{10}$$

Inserting the expression in (10) for $\hat{\pi}_t$ into (9) and then using that from (9), $\hat{\pi}_t = -a / \hat{y}_t$, we get:

$$\hat{y}_t = -a\left(\hat{\pi}_{t-1} + \gamma \hat{y}_t\right) = -a\hat{\pi}_{t-1} - a\,\gamma \hat{y}_t = -a\left(-\hat{y}_{t-1} / a\right) - a\,\gamma \hat{y}_t = \hat{y}_{t-1} - a\,\gamma \hat{y}_t,$$

which we can write as:

$$\hat{y}_t = \beta \hat{y}_{t-1}, \quad \beta \equiv \frac{1}{1 + a\gamma}. \tag{11}$$

Inserting the expression in (9) for $\hat{y}_t$ into (10) gives $\hat{\pi}_t = \hat{\pi}_{t-1} + \gamma\left(-a\hat{\pi}_t\right)$, from which:

$$\hat{\pi}_t = \beta \hat{\pi}_{t-1}. \tag{12}$$

Essentially, (11) and (12) contain the same linear, first-order difference equation. By iteration, ($\hat{y}_1 = \beta \hat{y}_0$ and then $\hat{y}_2 = \beta \hat{y}_1 = \beta^2 \hat{y}_0$ and then $\hat{y}_3 = \beta \hat{y}_2 = \beta^3 \hat{y}_0$ etc.), the solutions are:

$$\hat{y}_t = \hat{y}_0 \beta^t, \quad t = 0, 1, 2, \ldots \tag{13}$$

$$\hat{\pi}_t = \hat{\pi}_0 \beta^t, \quad t = 0, 1, 2, \ldots \tag{14}$$

where $\hat{y}_0$ and $\hat{\pi}_0$ are the initial values of $\hat{y}_t$ and $\hat{\pi}_t$, respectively. According to the definition given in (11), $\beta \equiv 1/(1 + a\gamma)$, we have $0 < \beta < 1$, so the term β^t on the right-hand sides of (13) and (14) will tend to 0 as time t tends to infinity. In other words, y_t will converge on $\bar{y}$ and π_t will converge on π^*. This proves that the economy is *stable* in the sense that it tends towards its long-run equilibrium. Obviously, the smaller β is, the faster is convergence.

Literally speaking, it will take infinitely long for the economy to reach the long-run equilibrium, but we may ask how long it will take before, say, half the adjustment to equilibrium has been completed. Let t_h denote the number of time periods that must elapse before half of the initial gap $\hat{y}_0$ between actual output and long-run equilibrium output has been closed (in the absence of new shocks). According to (13), the value of t_h may be found from the equation:

$$\hat{y}_t = \hat{y}_0 \beta^{t_h} \equiv \frac{1}{2}\hat{y}_0 \quad \Leftrightarrow \quad \beta^{t_h} = \frac{1}{2} \quad \Leftrightarrow \quad t_h \ln \beta = -\ln 2 \quad \Leftrightarrow$$

$$t_h = -\frac{\ln 2}{\ln \beta}, \quad \beta \equiv \frac{1}{1 + a\gamma}. \tag{15}$$

Hence, the half-life t_h is uniquely determined by the value of the parameter β which depends on the values of γ and a. We will therefore take a closer look at each of these in turn in order to find some plausible values for them from which we can estimate a plausible speed of adjustment to long-run equilibrium.

According to the AS curve (4) (or (8) or (10)), a 1 percentage increase in output should go hand in hand with an absolute increase in inflation (or in the inflation gap) of γ percentage points; or the other way round, a 1 percentage point increase in inflation should be accompanied by a relative increase in output of $1/\gamma$ percent. The rule of thumb relating to the Phillips curve for USA that we mentioned in Chapter 18 was that a one percentage point decrease in unemployment generates an increase in the annual rate of inflation of ½ percentage point. Furthermore, from Chapter 10 you will recall that Okun's Law for the US economy says that a decrease in unemployment of one percentage point goes hand in hand with approximately a 2 percent increase in output. Hence, a one percent increase in output should give a decrease in unemployment of around ½ of a percentage point, which should induce an increase in inflation of around $½ \cdot ½ = 0.25$ percentage points. All in all this suggests a plausible value of our γ on annual basis of around 0.25, which we will round off to 0.3.

With respect to a we first recall from the AD curve (6) (or (9)) that $a \equiv a_2 h/(1+\alpha_2 b)$, where from the IS curve (1), α_2 is the percentage increase in aggregate demand (including multiplier effects) resulting from a 1 percentage point decrease in the real interest rate. A plausible value, again based on an approximate rule of thumb and nothing more, could be α_2 around 1, perhaps a bit larger. If we use this value of α_2 and set the monetary policy parameters h and b equal to 0.5, which was proposed by John Taylor and fits the US economy reasonably well, we get a slightly above $0.5/(1+0.5)=1/3$, which we will round up to 0.4

With $\gamma \approx 0.3$ (on annual basis) and $a \approx 0.4$, we get $\beta \approx 1/(1+0.4 \cdot 0.3) \approx 0.89$ on an annual basis, and hence a half-life of $t_h \approx -\ln 2 / \ln 0.89 \approx 6$ years.

In other words, for reasonable parameter values our model implies that it will take roughly six years for the economy to complete half of the adjustment to its new long-run equilibrium after it has been hit by a shock. In a similar way one can show that it will take almost 20 years according to our model before the economy has completed 90 percent of the total adjustment to long-run equilibrium. Thus, our model implies that it will take quite a long time for the output gap to be closed if the economy is exposed to a shock.[6] This is just another way of saying that there is a strong *persistence* in the deviations of output from trend. The reason for this persistence is that actual and expected inflation adjust only slowly over time, so in the short and medium run output and employment have to bear a large part of the burden of adjusting to a shock.

Note from the definitions of a and β in (9) and (15) that a higher value of the monetary policy parameter h will push β downwards, thereby speeding up the economy's convergence to long-run equilibrium. This confirms our earlier conclusion that the

[6] One may think that our estimates of both γ and a above were a bit to the large side. This may be true, but note that smaller values of these variables would only make convergence slower. The conclusion of a rather slow convergence is robust to this alternative specification of the parameters.

stronger the interest rate response to a change in the rate of inflation, the faster is the adjustment of output to its natural rate. It also follows from (9) and (15) that a lower value of the policy parameter b will tend to raise the speed of convergence. This might seem to suggest that the central bank should choose the highest possible value of h and the lowest possible value of b to minimize deviations from long-run equilibrium. However, through their influence on the slope of the AD curve, the magnitude of h and b also determine how far output and inflation are pushed away initially from their long-run equilibrium levels when the economy is hit by demand or supply shocks. Choosing the optimal values of h and b is therefore a complicated matter to which we return in Chapter 20.

For the moment, let us sum up the main insights from the above analysis:

THE MACROECONOMIC ADJUSTMENT MECHANISM IN THE AS–AD MODEL

When the economy is in recession, wage setters tend to overestimate the rate of inflation and will therefore reduce their inflation forecasts and their required rate of increase in money wages. This will reduce the rate of inflation and will induce a central bank adhering to the Taylor Principle to cut the real interest rate. In this way the weaker inflationary pressure increases aggregate demand and output. In the absence of new shocks to the economy, the gradual fall in actual and expected inflation and the concomitant fall in the real interest rate will continue until the negative output gap is closed. If the economy starts out with a positive output gap, the reverse process will occur, with actual and expected inflation and the real interest rate gradually increasing until a long-run equilibrium is reached. For realistic parameter values and according to our AS-AD model, this macroeconomic adjustment process is likely to take a long time, since actual and expected inflation tend to adjust rather slowly. The stronger monetary policy reacts to a change in inflation, the flatter is the aggregate demand curve, and the faster is the convergence of output to its natural rate.

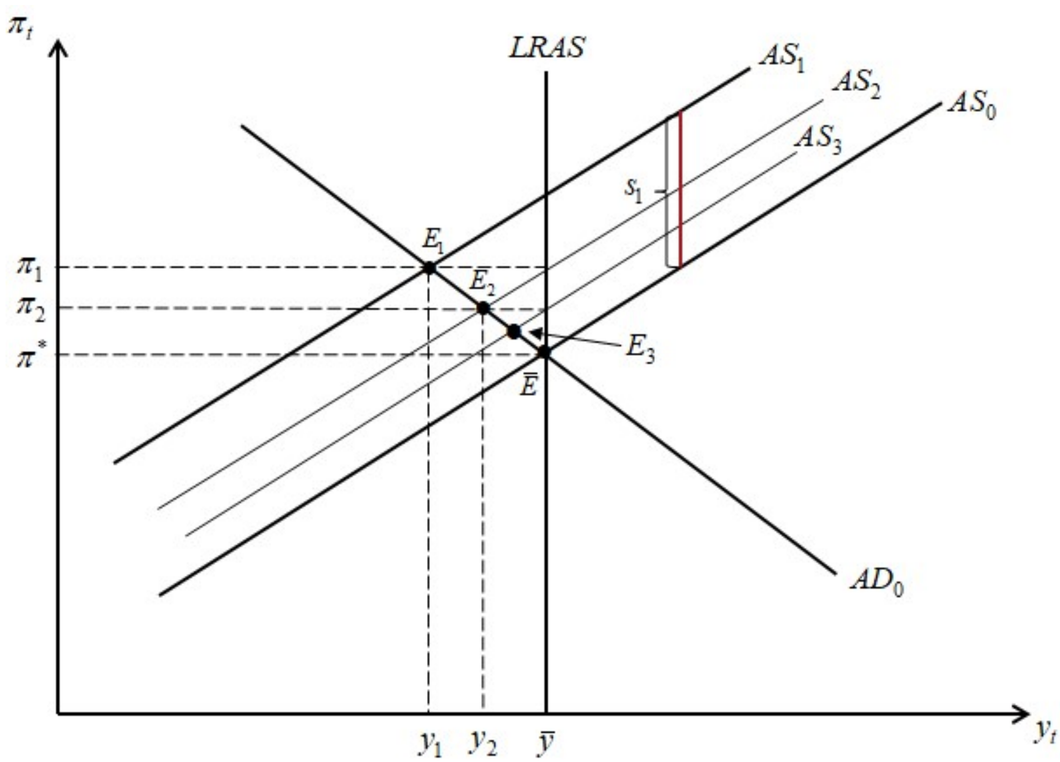
19.2 Business fluctuations in a deterministic world

The analysis above studied the economy's convergence on long-run equilibrium, but without explaining how output and inflation came to deviate from long-run equilibrium in the first place. In this section, we will illustrate how the long-run equilibrium may be disturbed by shocks to aggregate supply and aggregate demand, and how the economy will react on impact and over time to such shocks. Although in practice new shocks occur all the time, we can learn about the workings of the economy by tracing the isolated effects of a single shock, so this is what we will do in the present section. Specifically, we will treat a shock as a one-time disturbance, which hits the economy in a single period. In this way, we are able to highlight how the economy's reaction to the shock will generate persistent (long-lasting) deviations of output and inflation from trend even if the shock itself is purely temporary. We assume that, after the initial shock, there are no further subsequent shocks. The economy will then evolve in a deterministic manner over time, that is, we can use our AS–AD model to calculate the exact values of the output gap and the inflation gap in each period following the shock. In Section 19.3 we will extend the model by assuming that the economy is repeatedly hit by random shocks which turn our AS–AD model into a simple dynamic, stochastic, general equilibrium (DSGE) model.

A temporary negative supply shock

We start by studying the economy's reaction to a temporary negative supply shock such as an industrial conflict or a bad harvest. Let us assume that in period 0, before the shock, the economy is in the long-run equilibrium illustrated by point $\bar{E}$ where AS_0 and AD_0 intersect in Figure 19.5. Suppose then that the economy is hit by a temporary negative supply shock in period 1, causing the value of our shock variable s_t to rise from 0 to some positive number s_1 in period 1. Because of the temporary nature of the shock, it will not affect the long-run aggregate supply curve, but according to (8) the short-run aggregate supply curve will move up by the vertical distance s_1 from AS_0 to AS_1 in the figure. The new short-run equilibrium for period 1 is therefore given by point E_1.

Figure 19.5 Effects of a temporary negative supply shock



In moving from $\bar{E}$ to E_1 we see that the economy goes through a period of *stagflation*, defined as the simultaneous occurrence of rising inflation and falling output. The fact that inflation goes up is not surprising, since a negative supply shock is essentially an exogenous increase in production costs. The reason why output falls is that the central bank reacts to the rise in inflation by raising the interest rate, thereby depressing aggregate demand.

As mentioned, we assume that the source of the supply shock disappears from period 2 onwards so that s_t returns to its original value of 0. One might then think that the SRAS curve would shift down to its original position AS_0, thereby restoring long-run equilibrium already in period 2. But since the inflation rate has risen to the level π_1 in period 1, $\pi_1 > \pi^*$, our assumption of static expectations implies that the expected inflation rate for period 2 will be π_1, which is higher than the expected inflation rate π^* corresponding to the original AS curve, AS_0. Because of this rise in expected inflation, the short-run aggregate supply curve only shifts down to the level AS_2 passing through $(\bar{y}, \pi_1)$ in period 2, even though s_t drops back down to 0. The result is the new short-run equilibrium E_2 illustrated in Figure 19.5, where inflation is still above the target rate and output is still below its natural rate. Yet, since the actual inflation rate falls between periods 1 and 2, the expected inflation rate for period 3 also falls, generating a further downward shift in the AS curve in period 3 to AS_3 in the figure. This in turn leads to the new short-run equilibrium E_3 with lower inflation, causing another downward shift in the AS curve, and so on. Thus the continued downward revision of the expected inflation rate implies that the economy will move gradually down the AD curve back to the original long-run equilibrium $\bar{E}$. However, the important point is that because of the dynamic adjustment of expectations, even a short-lived supply shock has a long-lasting effect on output and inflation. In summary,

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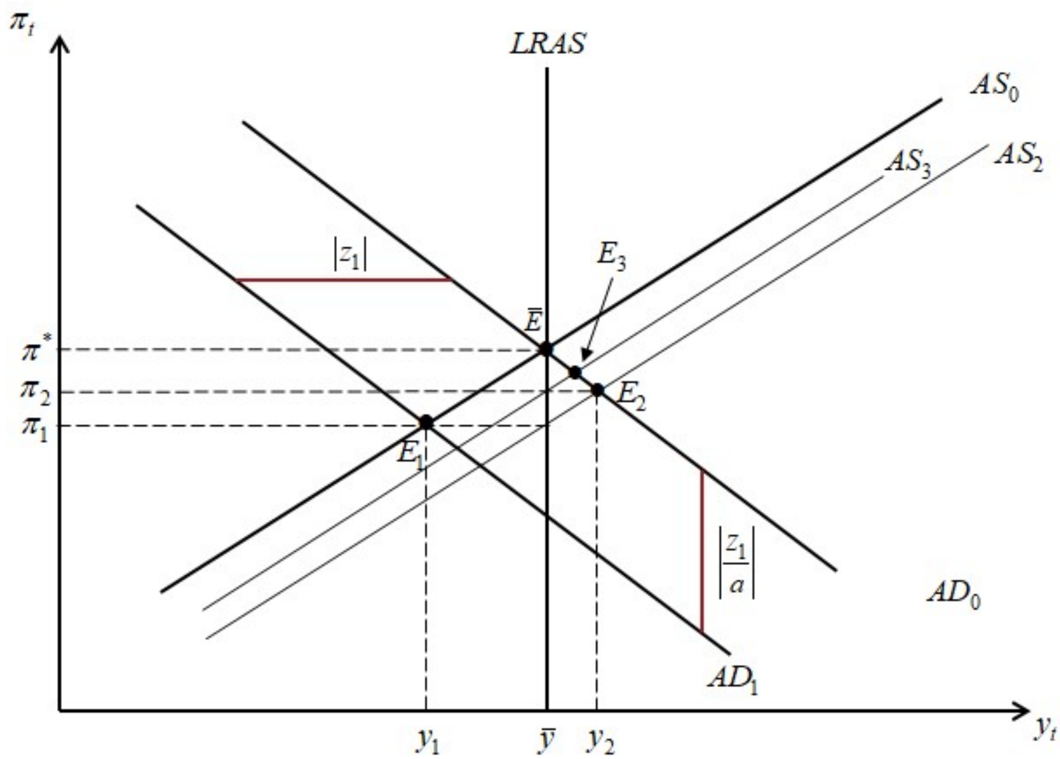
A TEMPORARY NEGATIVE SUPPLY SHOCK

A temporary negative supply shock causes stagflation (a simultaneous increase in unemployment and inflation) in the short run. Even when the shock is temporary, economic activity will stay below its natural rate for a while, because the central bank will keep the actual real interest rate above the 'natural' real interest rate as long as inflation remains above the target rate.

A temporary negative demand shock

Let us now consider the effects of a shock to aggregate demand. Suppose that, after having been in long-run equilibrium in period 0, the economy is hit by a temporary negative demand shock in period 1, say, because private agents temporarily become more pessimistic about the prospects for income growth. Figure 19.6 illustrates how the economy will react to the temporary weakening of private sector confidence. Between period 0 and period 1 the demand shock variable z changes from 0 to some negative number z_1. According to (6) and (7), the AD curve therefore shifts to the left by the distance $|z_1|$ and down by the distance $|z_1 / a|$ as illustrated by the shift from AD_0 to AD_1 in Figure 19.6. The AS curve is unaffected in period 1, because there has been no supply shock. This drives the economy from the initial long-run equilibrium to the new short-run equilibrium E_1 where output as well as inflation are lower.

Figure 19.6 Effects of a temporary negative demand shock



However, in period 2 private sector confidence is restored (we assume), pushing the aggregate demand curve back to its original position AD_0 as the variable z_t in (7) returns to its initial value of 0. You might think that this would immediately pull the economy back to its initial equilibrium $\bar{E}$. Yet this is not what happens, since the observed fall in inflation during period 1 causes a fall in expected inflation from π^* to the lower level π_1 as the economy moves from period 1 to period 2. Hence the short-run aggregate supply curve shifts down to AS_2 in period 2 generating a new short-run equilibrium at point E_2. Remarkably, we see that output in period 2 *overshoots* its long-run equilibrium value $\bar{y}$. Real GDP will only gradually return to its normal trend level as the above-normal level of activity drives up actual and expected inflation. As expected inflation goes up, the AS curve will gradually shift back towards its original position AS_0, and the economy will move back along the AD curve AD_0 to the initial long-run equilibrium $\bar{E}$. The interesting point is that the initial recession generated by the temporary demand shock is followed by an extended economic boom. This shows how the economy's propagation mechanism may generate a pattern of adjustment, which is rather different from the time pattern of the driving shock itself.

By analogy, in case of a one-shot, temporary *positive* demand shock, $z_1 > 0$, there would be an initial rise in output and inflation to levels above $\bar{y}$ and π^* in period 1, but in period 2 a fall in output below $\bar{y}$, and from then a gradual adjustment back towards the long-run equilibrium by successive increases in output and decreases in inflation. You may find this 'opposite reaction' in output (first above and then below the long-run level) after a one-shot demand shock unconvincing since we have learned from Chapter 13 that there is considerable persistence in output: once it goes up, there is a high probability that it will stay high for some time.[7] However, the opposite reaction is perfectly logical when the demand shock that occurs is literally a single-period shock. It will first pull activity and inflation up, but when the shock has vanished after just one period, only the contractionary effect of higher inflation (through the central bank reaction) will remain. What happens in the real world is that shocks are typically not of a one-shot nature but correlated over time. If consumer and business confidence go up in one quarter, say, giving rise to a positive demand shock in that quarter, confidence will most probably also be above its initial level the quarter after and for several more quarters, which will counteract the contractionary effect from higher inflation for some time.

In sum,

A TEMPORARY NEGATIVE DEMAND SHOCK

A temporary negative demand shock initially causes a recession where output and inflation are below their long-run equilibrium levels. However, when the aggregate demand curve returns to its original position, the economy enters a boom that will last for several periods, because the fall in actual and expected inflation caused by the initial recession allows the central bank to keep the real interest rate below the natural rate until actual and expected inflation catch up with the target inflation rate.

Impulse-response functions

In Section 19.1 we set up a quantified version of our model to study how long it will take the economy to converge towards long-run equilibrium. In a similar way, we may use a quantitative version of our AS–AD model to investigate how strongly the economy is likely to react to demand and supply shocks, assuming plausible values of the key parameters. For this purpose we must of course allow our shock variables z_t and s_t to deviate from 0. Using our previous notation for the output gap, $\hat{y}_t \equiv y_t - \bar{y}$, and our definition of the inflation gap, $\hat{\pi} \equiv \pi_t - \pi^*$, we may write the AD curve (6) and the AS curve (8) in the form:

[7] Specific empirical investigations of reactions to demand shocks based, e.g., on so-called SVAR (Structural Vector Auto Regressive) techniques also typically find that the reaction to a positive demand shock, e.g., a sudden increase in government demand, is that output increases and stays at a higher level for some time.

AD: $\hat{y}_t = -a\hat{\pi}_t + z_t,$ (16)

AS: $\hat{\pi}_t = \hat{\pi}_{t-1} + \gamma\hat{y}_t + s_t.$ (17)

If we insert the expression (17) for $\hat{\pi}_t$ into (16) we get: $\hat{y}_t = -a\left(\hat{\pi}_{t-1} + \gamma\hat{y}_t + s_t\right) + z_t$.
If we insert here the expression for $\hat{\pi}_{t-1}$ obtained by lagging (16) by one period, $\hat{\pi}_{t-1} = \left(z_{t-1} - \hat{y}_{t-1}\right)/a$, we get:

$$\hat{y}_t = -a\left(\frac{z_{t-1} - \hat{y}_{t-1}}{a} + \gamma\hat{y}_t + s_t\right) + z_t = \hat{y}_{t-1} - a\,\gamma\hat{y}_t + z_t - z_{t-1} - as_t, \tag{18}$$

which can easily be solved for $\hat{y}_t$ to find:

$$\hat{y}_t = \beta\hat{y}_{t-1} + \beta(z_t - z_{t-1}) - a\beta s_t, \tag{19}$$

where β has the same definition as above: $\beta \equiv 1/(1 + a\gamma)$ and $0 < \beta < 1$. Likewise, if we insert the expression (16) for $\hat{y}_t$ into (17) and afterwards solve for $\hat{\pi}_t$ we get:

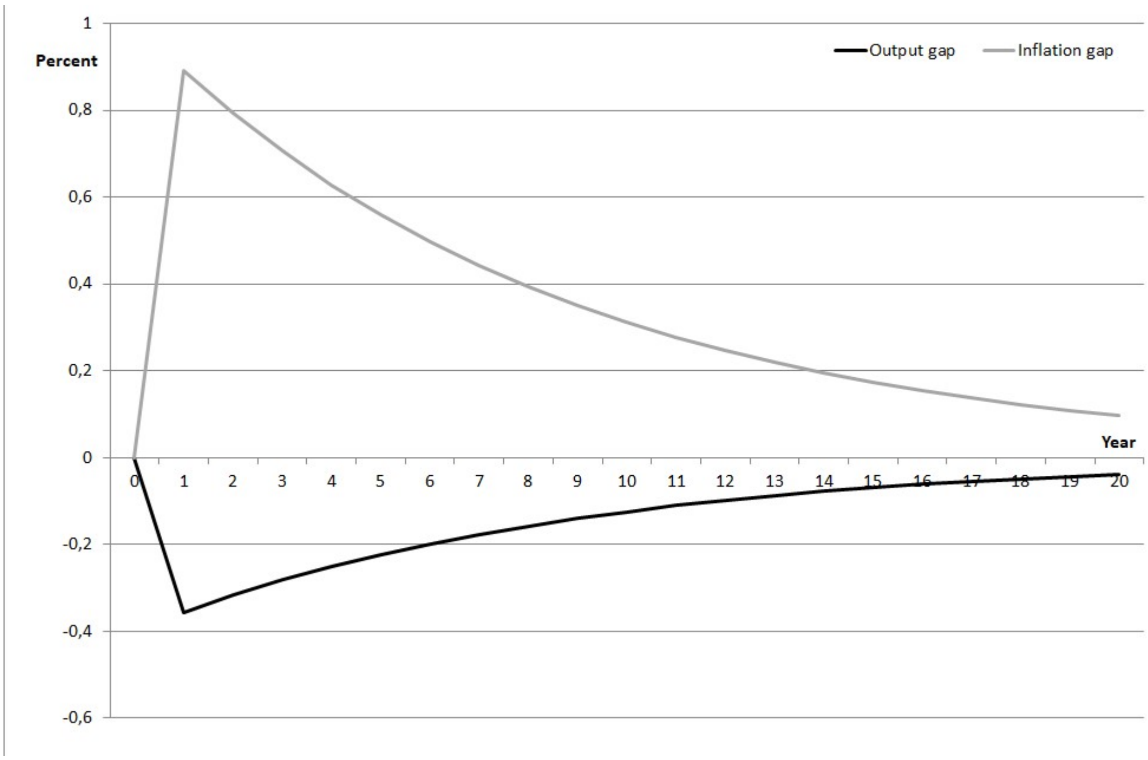
$$\hat{\pi}_t = \beta\hat{\pi}_{t-1} + \gamma\beta z_t + \beta s_t. \tag{20}$$

Equations (19) and (20) are stochastic, first-order, linear difference equations in $\hat{y}_t$ and $\hat{\pi}_t$, respectively. Each equation states how the current value of the variable in question depends on its own lagged value and on past and current shocks. Note that for given shock realizations z_t and s_t and for given parameter values for a , γ and β , one can easily simulate with (19) and (20) to create time paths for $\hat{y}_t$ and $\hat{\pi}_t$.

Using plausible parameter values, we may therefore simulate (19) and (20) from period 0 and onwards to obtain so-called *impulse-response functions* showing how output and inflation react over time to various shocks. Since output y_t is measured in logarithms, a 0.01 increase in our demand shock variable z_t reflects an exogenous increase in demand corresponding to 1 per cent of initial GDP. Moreover, a 0.01 increase in our supply shock variable s_t corresponds to an exogenous 1 percentage point increase in the rate of inflation.

In Figure 19.7 we show the impulse-response functions for the output gap and for the inflation gap generated by a temporary increase in our supply shock variable s_t from 0 to 0.01 occurring in period 1 and only in that period ($s_t = 0$ for $t \neq 1$ and $z_t = 0$ for all t). The figure is called an impulse-response diagram. We use the same parameter values as before, $\gamma = 0.3$, $a = 0.4$ (the latter based on $\alpha_2 \approx 1$ and $h = b \approx 0.5$), giving $\beta \approx 0.89$, where the appropriate length of the period is one year. Not surprisingly, Figure 19.7 confirms the qualitative analysis in Figure 19.5: in the short and medium run, the temporary negative supply shock generates a positive inflation gap and a negative output gap. In period 1, the inflation rate does not quite rise by 1 percentage point, because the drop in output and employment reduces inflationary pressure a bit by driving down the marginal costs of production. The short-run decrease in output is quite substantial, amounting to almost 0.4 percent.

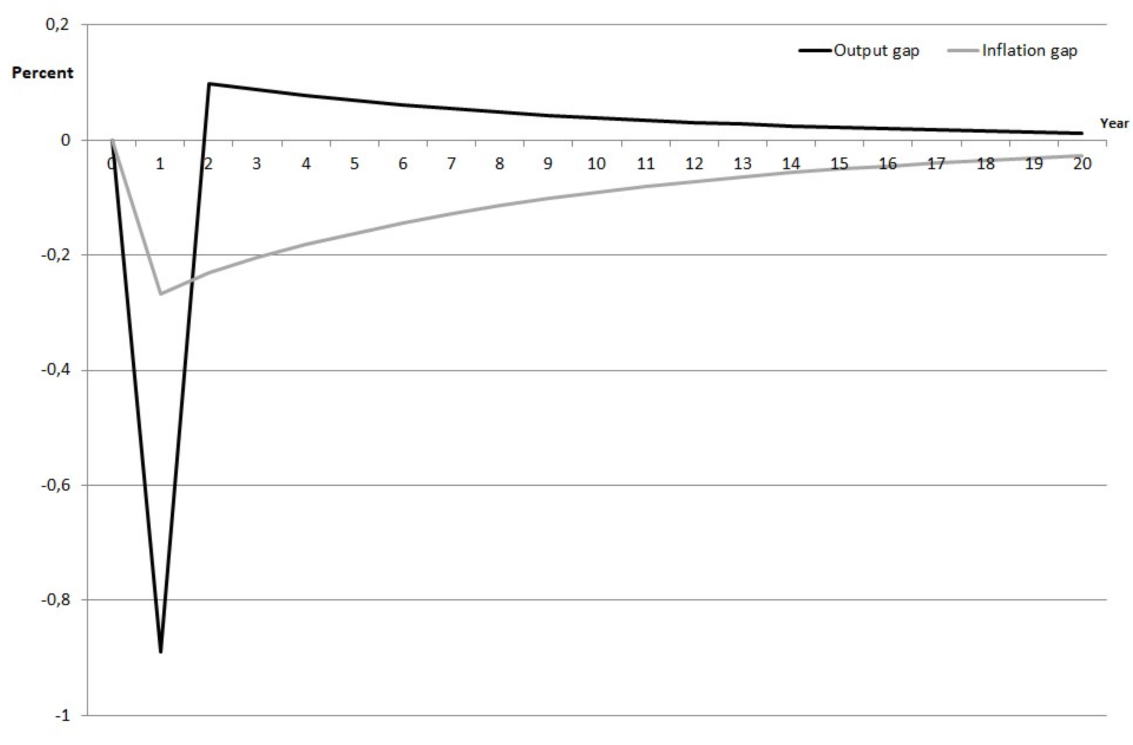
Figure 19.7 The adjustment to a temporary negative supply shock, $s_1 = 0.01 = 1\%$



Note: Parameter values: $\beta = 0.89$ based on $\gamma = 0.3$ and $a = 0.4$ (the latter based on $\alpha_2 \approx 1,\ h = b \approx 0.5$).

Figure 19.8 is the impulse-response diagram for a demand shock where z_t temporarily falls from 0 to –0.01 in period 1 ($z_t = 0$ for all $t \neq 1$ and $s_t = 0$ for all t). In the short run output falls by a little less than 1 per cent, since the drop in inflation and economic activity induces the central bank to cut the real interest rate to limit the fall in demand. When z_t returns to its normal level of 0 from period 2 onwards, we see that the output gap changes from negative to positive and then gradually falls back towards 0. This is quite in accordance with the qualitative analysis of a temporary demand shock in Figure 19.6.

Figure 19.8 The adjustment to a temporary negative demand shock, $z_1 = -0.01 = -1\%$



Note: Parameter values: $\beta = 0.89$ based on $\gamma = 0.3$ and $a = 0.4$ (the latter based on $\alpha_2 \approx 1,\ h = b \approx 0.5$).

Permanent shocks

We have so far considered deterministic temporary shocks that last for one period, and in the next subsection we will consider stochastic shocks distributed around zero. However, sometimes a shock may be permanent. One example could be a tax reform that permanently changes the propensities to consume and invest, and thereby has permanent effects on both supply and demand. Another could be the adoption of a new, revolutionizing technology that gives a once-and-for-all, permanent lift in productivity. A third could be a liberalization of markets that permanently affect the markups of prices over marginal costs by changing the degree of product market competition.

Permanent changes like these can have permanent effects on the structural/natural rate of output and on the structural equilibrium real interest rate. Once these changes in $\bar{y}$ and $\bar{r}$ have been taken into account, our macroeconomic model consisting of equations (1) through (5) above, or in the shorthand AS-AD form of equations (6) and (8), remains valid, it is only understood that now the short run deviations occur around the new long-run equilibrium involving the new levels of $\bar{y}$ and $\bar{r}$.

However, we may also use our model to analyze explicitly how the long-run equilibrium is affected by a permanent change in, e.g., our model's v_t or s_t. As we shall now see, a permanent *demand* shock will change the long-run equilibrium, structural real interest rate and thereby the central bank's monetary policy rule into which the structural real interest rate enters. A permanent *supply* shock will change the economy's natural rate of output and thereby the structural real interest rate and hence again the monetary policy rule.

In order to keep track of these changes we have to consider our model in a more extensive form than the AS-AD version (6) and (8), where the real interest rate has been eliminated. The following version of our model comes from restating the IS curve (1), collecting (2) and (3) into a monetary policy rule for the real interest rate (MP), and combining (4) and (5) into an AS curve as also done above. This way of stating our model is sometimes referred to as an IS-MP-AS model:

$$y_t - \bar{y} = \alpha_1(g_t - \bar{g}) - \alpha_2(r_t - \bar{r}) - \alpha_3(\tau_t - \bar{\tau}) + v_t \tag{IS}$$

$$r_t = \bar{r} + h(\pi_t - \pi^*) + b(y_t - \bar{y}) + \hat{\rho}_t \tag{MP}$$

$$\pi_t = \pi_{t-1} + \gamma(y_t - \bar{y}) + s_t \tag{AS}$$

We also have to distinguish between the structural levels before and after the permanent shock considered. Let $\bar{r}_0$ and $\bar{y}_0$ denote, respectively, the natural real interest rate and the natural rate of output prevailing in the *initial* long-run equilibrium before the economy is hit by a permanent shock, and let $\bar{r}$ and $\bar{y}$ be the structural levels after the shock.

Hence, before any permanent shock has occurred, the IS curve must have been:

$$y_t - \bar{y}_0 = \tilde{v}_t - \alpha_2(r_t - \bar{r}_0), \qquad \tilde{v}_t \equiv v_t + \alpha_1(g_t - \bar{g}) - \alpha_3(\tau - \bar{\tau}), \tag{21}$$

where we have collected the confidence shock and the fiscal policy impulses into one demand shock $\tilde{v}_t$. Likewise, the monetary policy (MP) relation must have been:

$$r_t = \bar{r}_0 + h(\pi_t - \pi^*) + b(y_t - \bar{y}_0) + \hat{\rho}_t, \tag{22}$$

and the AS curve must have been:

$$\pi_t = \pi_{t-1} + \gamma(y_t - \bar{y}_0) + s_t. \tag{23}$$

In the initial equilibrium we have $\tilde{v}_t = s_t = 0$. Suppose now that the economy experiences a supply shock, which permanently changes s_t from 0 to some constant $s \neq 0$. The *new* long-run equilibrium level of output, $\bar{y}$, may then be found from (23) by inserting $\pi_t = \pi_{t-1}$, $y_t = \bar{y}$, and $s_t = s$, and solving for $\bar{y}$ to get:

$$\bar{y} = \bar{y}_0 - \frac{s}{\gamma}. \tag{24}$$

That is, a permanent supply shock $s > 0$ reduces structural output by the amount s/γ.

Recall that the long-run equilibrium real interest rate (the 'natural' interest rate) is the real interest rate ensuring that the goods market clears at the natural rate of output. The *new* structural real interest rate, $\bar{r}$, may therefore be found from (21) by setting actual output y_t equal to the new natural rate of output $\bar{y} = \bar{y}_0 - (s/\gamma)$, and by setting $r_t = \bar{r}$. Assuming that no permanent demand shocks have occurred so that $\tilde{v}_t = 0$, it then follows from (21) that:

$$\bar{r} = \bar{r}_0 + \frac{s}{\gamma\alpha_2}. \tag{25}$$

The intuition for (25) is straightforward: a negative supply shock ($s > 0$) which reduces the natural rate of output as given by (24) requires a fall in aggregate demand to maintain long-run equilibrium in the goods market. To curb aggregate demand, the structural real interest rate must go up, as stated in (25).

To steer the economy towards the new long-run equilibrium characterized by (24) and (25) and in this equilibrium achieving its target inflation rate π^*, the central bank must therefore revise its estimates of both the long-run, structural rate of output and the long-run structural equilibrium real interest rate entering the monetary policy rule (MP). That is, it must revise its policy rule to include its estimates of the new values of the structural levels as given by (24) and (25).

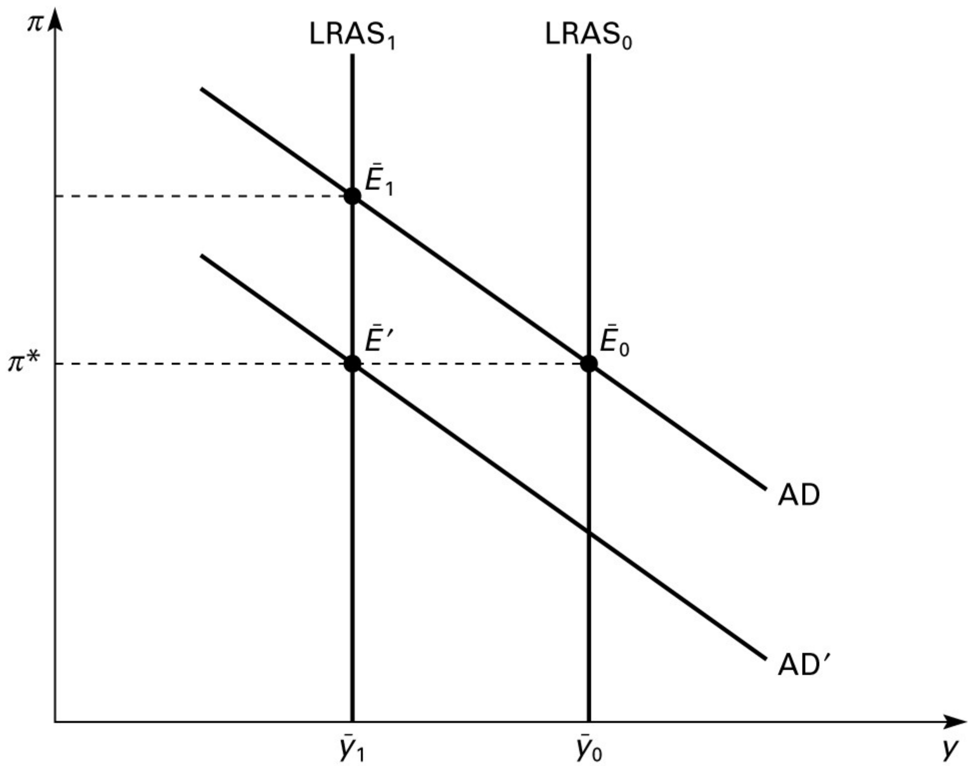
Suppose now that the economy is instead hit by a permanent *demand* shock changing $\tilde{v}_t$ from 0 to some constant $\tilde{v} \neq 0$. We see from (23) that there is no effect on the natural rate of output (given that $s = 0$), but the long-run equilibrium real interest rate changes. We can find the new level $\bar{r}$ by setting $\tilde{v}_t = \tilde{v}$, $\bar{y} = \bar{y}_0$ and $r_t = \bar{r}$ in (21) and solving for $\bar{r}$ to obtain:

$$\bar{r} = \bar{r}_0 + \frac{\tilde{v}}{\alpha_2}. \tag{26}$$

This result is also intuitive: since natural output is unchanged, an autonomous permanent increase in aggregate demand $(\tilde{v} > 0)$ must be offset by a rise in the real interest rate which reduces the interest-sensitive components of demand so that total demand does not exceed the constant long-run equilibrium level of output. Again, to steer the economy towards long-run equilibrium with an inflation rate of π^*, the central bank must revise its policy rule to include its estimate of the new structural real interest rate $\bar{r}$ rather than the old one $\bar{r}_0$.

Figure 19.9 illustrates why the central bank needs to revise its estimate of the variables $\bar{r}$ and $\bar{y}$ entering the monetary policy rule (MP) when the economy is hit by permanent shocks. The figure assumes that a permanent negative supply shock has permanently shifted the long-run aggregate supply curve from the position $LRAS_0$ to the position $LRAS_1$. If the central bank does not account for the facts that the structural real interest rate has increased and the natural rate of output has decreased, then the AD-curve will be unchanged despite of the shock as illustrated by the curve labelled AD in the figure. This would lead the economy towards the long-run equilibrium $\bar{E}_1$, where the inflation rate permanently exceeds the inflation target π^*. To prevent this systematic deviation of inflation from target, the central bank must revise its estimate of $\bar{y}$ downwards in accordance with (24) and its estimate of $\bar{r}$ upwards in accordance with (25). When this revision occurs, the AD curve will shift down to the level AD' in Figure 19.9, so that the new long run equilibrium $\bar{E}'$ becomes consistent with the inflation target.

Figure 19.9 The long-run effects of a permanent negative supply shock



Of course, requiring that the central bank should update its estimates of the 'natural' rates of output and real interest to account for permanent shocks is easier said than done, since it may take quite a while before it is possible to judge whether some shock is temporary or permanent in nature. When a permanent shock occurs, the economy's adjustment over time will depend on how long it takes for the central bank to discover the permanent character of the shock. In Exercise 2 we invite you to study the effects of permanent demand and supply shocks in more detail.

We may sum up these insights as follows:

PERMANENT SHOCKS

When the economy is hit by a permanent demand or supply shock which changes the natural real interest rate and possibly the natural rate of output, the central bank must adjust its estimate of the equilibrium real interest rate accordingly to ensure that inflation will stay on target in the long run. A permanent negative (positive) supply shock reduces (increases) natural output and increases (reduces) the natural real interest rate. A permanent negative (positive) demand shock does not affect natural output but reduces (increases) the natural real interest rate. In practice, it may take a long time before the central bank is able to discover that a shock is permanent. In the meantime, inflation may deviate substantially from its target.

19.3 Business cycles in a stochastic world

As we have seen, our deterministic AS–AD model does quite a good job in accounting for the observed *persistence* in the movement of output over time. However, the deterministic model does not really explain the crucial feature of business cycles that economic booms tend repeatedly to be followed by recessions, and vice versa. A satisfactory model of the business cycle must be able to replicate at least to some extent the *recurrent fluctuations* in output and inflation, like those in Figure 19.1 which illustrated the evolution of the cyclical components of real GDP and the rate of inflation in the USA in recent decades.

The Frisch-Slutsky paradigm

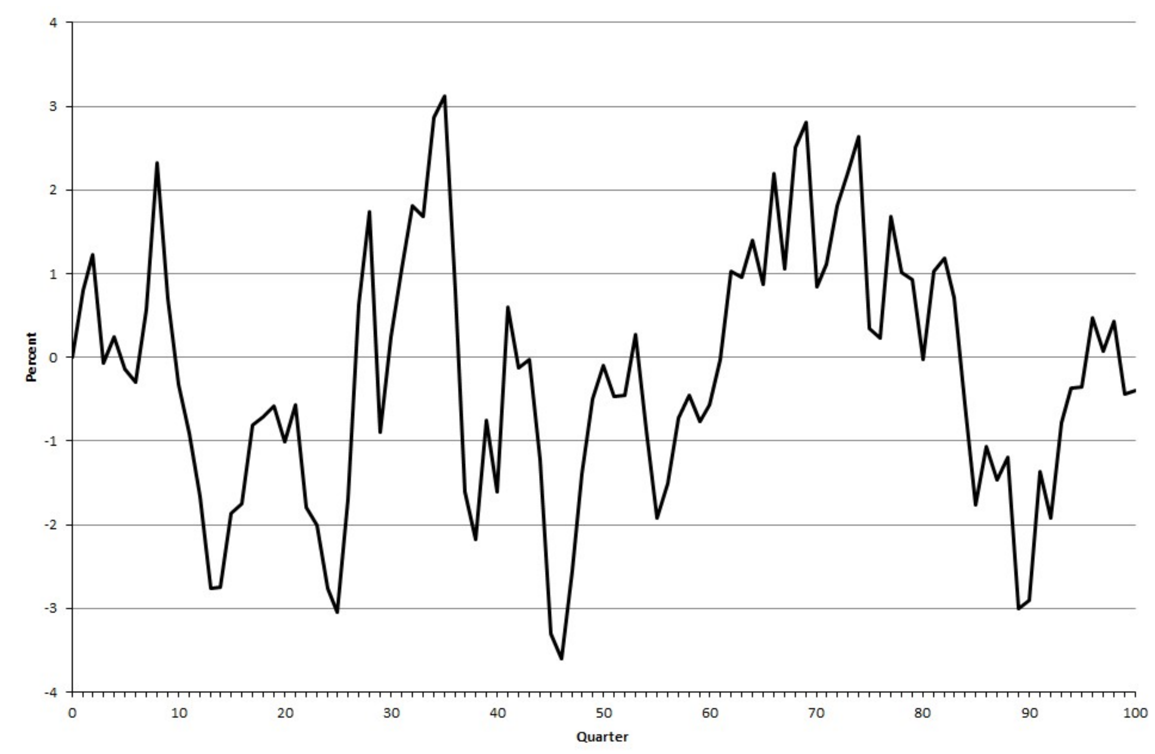
To explain the cyclical pattern of output and inflation, we will now set up a *stochastic* version of our AS–AD model in which our demand and supply shock variables z and s are assumed to be *random variables*. In taking this step, we are building on a remarkable discovery made in 1937 by the Russian economist-statistician Eugen Slutsky (see the reference in Footnote 2). Slutsky discovered that if one adds a stochastic term with a zero mean and a constant variance to a first-order linear difference equation, and if the coefficient on the lagged endogenous variable is not too far below unity, the resulting stochastic difference equation will generate a time series which looks very much like the irregular cyclical pattern of output displayed in Figure 19.1!

To illustrate Slutsky's fundamental insight, suppose the variable q_t evolves according to the difference equation:

$$q_t = 0.9q_{t-1} + e_t, \tag{26}$$

where e_t in each period is drawn independently from a normal distribution with a zero mean. Note that (26) has the same general form as our previous equation (19) for the output gap if we treat the additive 'shock' terms $\beta(z_t - z_{t-1}) - \alpha\beta s_t$ in (19) as a single stochastic variable. Drawing a sample from the standard normal distribution with a zero mean and a unit variance to obtain a sequence of e_t, assuming e_t to be independently distributed over time and setting the initial value of q_t equal to zero, we simulated Equation (26) over 100 periods, here thought of as quarters. In this way we obtained the time path for q_t displayed in Figure 19.10.[8] Comparing this to the graph for the evolution of the US output gap in Figure 19.1, we see that the simple difference equation (26) is able to generate a time series which qualitatively looks remarkably like the recurrent business cycles observed in the real world.

Figure 19.10 Illustrating Slutsky's discovery: an example of simulation of Equation (26) when e_t follows the standard normal distribution



Slutsky's discovery suggests that it may be fruitful to treat our shock variables z_t and s_t as stochastic processes. By nature, the shocks to demand and supply, which we have been discussing, are very hard to predict. Recall that supply side shocks include phenomena such as industrial conflicts, fluctuations in agricultural output due to changing weather conditions, oil price shocks due perhaps to military conflict or political unrest in oil-producing countries, changes in productivity stemming from irregularly

[8] It is easy to do a simulation and create a Slutsky diagram like Figure 19.10 in Microsoft Excel, if one knows how to get a random draw from the normal distribution with mean μ and standard deviation σ in Excel. This can be obtained by the order: '=NORM.INV(RAND();μ,σ)', where $\mu = 0$ and $\sigma = 1$ is one particular possibility. RAND() gives a random number between zero and one drawn from the uniform distribution and NORM.INV(;μ,σ) is the inverse of the cumulative normal distribution with mean μ and standard deviation σ. Hence, NORM.INV(z;μ,σ) is the number for which the probability of ending below that number according to the normal distribution with mean μ and standard deviation σ is z.

arriving technological breakthroughs, etc. On the demand side, shocks may occur due to sudden shifts in market psychology, or due to political regime shifts involving significant changes in fiscal policies, among other things. Whether events such as these occur with deterministic necessity – that is, whether they were unavoidable, given the way things had developed – or whether they are fundamentally unpredictable, just like the outcome of the toss of a coin, is a deep scientific question. However, as long as our understanding of the causes of such events – and hence our ability to predict them – is limited, it seems to make sense to treat the supply and demand shocks in macroeconomic models as random variables. In doing so, we admit that we can only predict what demand and supply will be on average, while acknowledging that the actual levels of demand and supply may deviate from their average positions in a way we cannot anticipate.

As mentioned earlier, this approach to analysing business cycles is sometimes referred to as the Frisch–Slutsky paradigm and it may be summarized as follows:

THE FRISCH–SLUTSKY PARADIGM FOR BUSINESS CYCLE ANALYSIS

The paradigm sees business cycles as being initiated by stochastic demand and supply shocks. Even though these shocks (impulses) occur at irregular intervals in an unpredictable manner, they interact with the economy's endogenous persistence (propagation) mechanisms in a way that generates recurrent cycles in economic activity, provided the persistence mechanisms are strong enough.

Let us now investigate how far a stochastic version of our AS–AD model can take us towards explaining the stylized facts of business cycles, taking in this project a humble attitude: given the simplicity of our model, we should at most hope for an overall, qualitative similarity between model predictions and facts.

The stylized facts

The stylized business cycle facts, which we would like our model to explain, are basically expressed by Figure 19.1 and summarized in Table 19.1. The figures in the table based on quarterly data for the USA from 1947 to 2017 are taken from Tables 13.2-13.4 of Chapter 13 (and rounded off to one decimal). We have chosen to focus on the relatively closed US economy because we still have not extended our AS–AD model to allow for international trade in goods and capital (we will do so in Chapter 23). Table 19.1 shows in the first two columns the degree of volatility in output and inflation, measured by the standard deviations of their cyclical components (note throughout that quarterly inflation has not been annualized). In the third column, it shows the degree to which output and inflation move together, measured by the contemporaneous coefficient of correlation. In the remaining columns, it shows for each of output and inflation the degree of persistence measured by the coefficients of autocorrelation between the current and the up to four quarters lagged values of (the cyclical components of) the variable in question. Ideally, simulations of our stochastic AS–AD model should be able to reproduce these statistical measures of the US business cycle.

Table 19.1 Volatilities and correlations in the US business cycle, 1947-2017

Standard deviation		Correlation	Auto correlation							
GDP (Percent)	Inflation (Points)	GDP/inflation	GDP				Inflation			
			t-1	t-2	t-3	t-4	t-1	t-2	t-3	t-4
1.6	0.5	0.3	0.9	0.6	0.4	0.1	0.4	0.2	0.2	0.0

Note: Based on quarterly data. The quarterly rate of inflation has not been annualized.
Source: Tables 13.2, 13.3 and 13.4 of Chapter13.

The stochastic AS–AD model

Our AS-AD model will be based on the following two equations:

$$y_t - \bar{y} = -a(\pi_t - \pi^*) + z_t, \qquad \text{(AD)}$$

$$\pi_t = \pi_t^e + \gamma(y_t - \bar{y}) + s_t, \qquad \text{(AS)}$$

where notation and definitions are as above. To complete the model we need to specify an expectations hypothesis for the π_t^e appearing in (AS) and specify the random processes for the shocks z_t and s_t. With respect to expectations, we will at first, as above, assume static expectations, $\pi_t^e = \pi_{t-1}$, but later employ a more general expectations hypothesis, so-called adaptive expectations, in order to improve the predictive power of our model.

To perform simulations with our model, we will also need specific parameter values for a and γ. As above we will use $a = 0.4$, but since we now consider a period length of one quarter, we will bring down the value of γ to around one quarter of the value we found reasonable on annual basis, that is, $0.3/4 \approx 0.1$, so we assume $\gamma = 0.1$. Over one quarter the price level should increase by around one fourth of its annual increase. Since we measure quarterly inflation in non-annualized form, γ on quarterly basis should therefore be around one fourth of γ on annual basis. With $a = 0.4$ and $\gamma = 0.1$, we get $\beta \equiv 1/(1+a\gamma) \approx 0.96$.

With these specifications, we can simulate with our stochastic AS-AD model over many periods and thereby compute long series for output and inflation gaps – we explain the details below. The series found can then be confronted with the empirical facts presented in Figure 19.1 and Table 19.1.

Economists have often debated whether demand shocks or supply shocks are the most important type of disturbances driving the business cycle. One way of resolving this issue is to investigate whether a model driven by demand shocks is better at replicating the stylized business cycle facts than a model driven by supply shocks, or vice versa. We will therefore first perform two pre-experiments, one where we only include demand shocks and assume static expectations, and one where we only include supply shocks and also assume static expectations. As we will see, none of these experiments generates time series for output and inflation gaps that come close to resembling the empirical series.

We therefore perform a third and more elaborate experiment where both demand and supply shocks occur and where we employ the more general expectations hypothesis of adaptive expectations (to be explained).

We have already seen above that with static expectations our AS-AD model implies that the output and inflation gaps in any period t depend on their own lagged values and on shock realizations as given by (19) and (20), repeated here for convenience:

$$\hat{y}_t = \beta \hat{y}_{t-1} + \beta(z_t - z_{t-1}) - a\beta s_t, \tag{19}$$

$$\hat{\pi}_t = \beta \hat{\pi}_{t-1} + \gamma\beta z_t + \beta s_t. \tag{20}$$

Experiment 1: only demand shocks and static expectations

So, first we consider our stochastic AS–AD model with static expectations and include only demand shocks. Thus, we set s_t equal to 0 in all time periods. The demand shocks are assumed to evolve according to the following first-order autoregressive stochastic process:

$$z_t = \delta z_{t-1} + x_t, \quad 0 \leq \delta < 1, \quad x_t \sim N(0, \sigma_x^2), \quad x_t \text{ i.i.d.} \tag{27}$$

The notation $x_t \sim N(0, \sigma_x^2)$, x_t i.i.d. means that x_t is assumed to follow a normal distribution with a zero mean value and a constant finite variance σ_x^2, and that it is identically and independently distributed over time (i.i.d.). Hence, the probability distribution of x_t is the same in all periods, and the realized value of x_t in any period t is independent of the realized value of x_j in any other time period j.[9] A stochastic process x_t with these 'i.i.d.' properties is called 'white noise'. Note from (27) that, by allowing the parameter δ to be positive, we allow for the possibility that a demand shock occurring in a given quarter may not die out entirely within that same quarter, but may be felt partly in subsequent quarters. At the same time the restriction $\delta < 1$ implies that demand shocks do not last 'forever'. As we argued above in connection with our discussion of temporary, deterministic shocks of a one-shot nature, this is a reasonable assumption.

To simulate our model we now proceed as follows. We choose the value of the autocorrelation coefficient δ in (27) to be 0.8, and we choose to begin with the standard deviation in the distribution of the i.i.d. shock x_t to be $\sigma_x = 1$. We then go through the following steps:

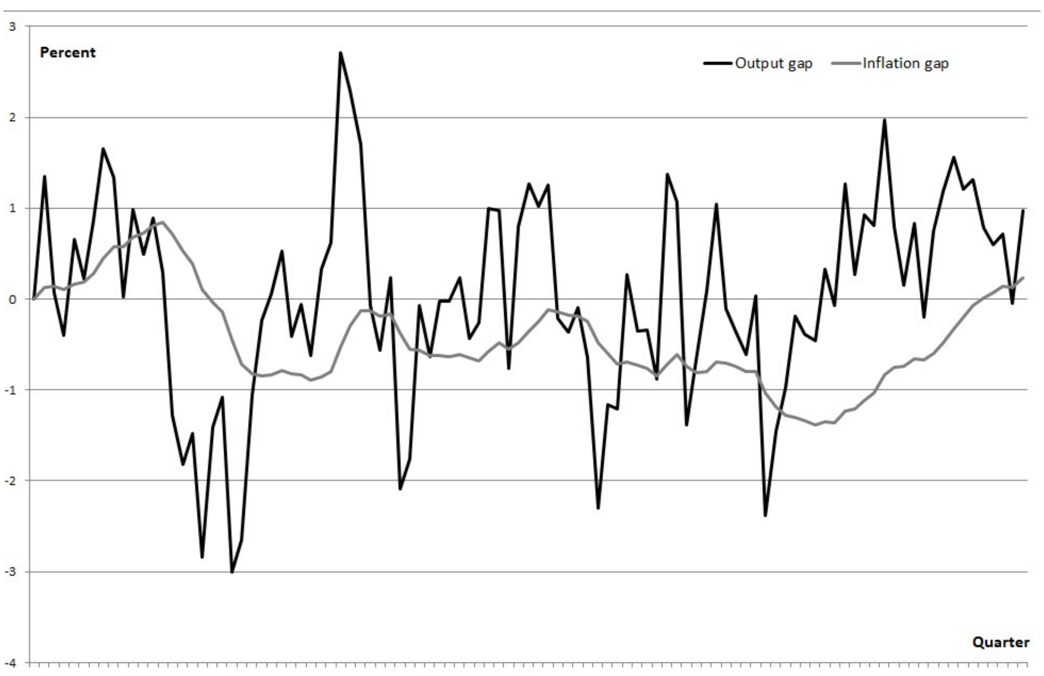
1. We start from a long-run equilibrium in period 0, that is, $\hat{y}_0 = \hat{\pi}_0 = z_0 = s_0 = 0$.
2. For all t we let $s_t = 0$.
3. For each t = 1, 2, 3, … up to a final period, which could be period 100 or 1000, we let the computer determine x_t by a draw from the normal distribution, $x_t \sim N(0, \sigma_x^2)$, where to begin with, $\sigma_x = 1$. Footnote 8 above explained how this is done in Excel.

[9] Formally, $E\left[x_t x_{t'}\right] = 0$ for all $t \neq t'$, where $E[\cdot]$ is the expectations operator.

4. Given these x_t for $t = 1, 2, 3, \ldots$, we construct the demand shock z_t for $t = 1, 2, 3, \ldots$ according to (27), that is, we repeatedly compute z_t as $z_t = \delta z_{t-1} + x_t$ with $z_0 = 0$, where $\delta = 0.8$. This is easily done in Excel.
5. Given these z_t (and $s_t = 0$ for all t), one can compute $\hat{y}_t$ and $\hat{\pi}_t$ from Equations (19) and (20) over the periods $t = 1, 2, 3, \ldots$, starting from $\hat{y}_0 = \hat{\pi}_0 = 0$ for $t = 0$. This is again easily done in Excel.
6. We then compute the standard deviation of $\hat{y}_t$ for the computed series, which should be long, e.g., 1000 quarters, and if this does not fit reasonably well with the empirical value of 1.6 for the US economy, we adjust σ_x upwards or downwards and repeat the simulation until a better fit is obtained. In this case, actually $\sigma_x = 1$ gives a decent fit, so we stop here.
7. For the finally resulting simulated values of $\hat{y}_t$ and $\hat{\pi}_t$, we investigate whether these fit reasonably well with the empirical series. This can be done graphically or by calculating the standard deviations, the cross-correlation, and the coefficients of autocorrelation for the two endogenous variables over the simulated period.

Figure 19.11 shows a typical simulation output from this procedure over 100 quarters.

Figure 19.11 Simulation of the stochastic AS–AD model with static expectations and no supply shocks



We do not need to compute standard deviations or coefficients of correlation to see that this picture does not resemble Figure 19.1 much. Our simulation without supply shocks reproduces the cyclicality and persistence in the output gap reasonably well, but this is not so for the inflation gap, which according to the simulation drifts too much away from zero and can stay negative (or positive) for around 80 quarters in a row. The model simulation thus substantially underestimates the positive correlation between output and inflation, and wildly exaggerates the volatility (standard deviation) of inflation as well as the degree of persistence (autocorrelation) in inflation. This suggests that an AS–AD model driven solely by demand shocks cannot give a nearly satisfactory account of the business cycle.

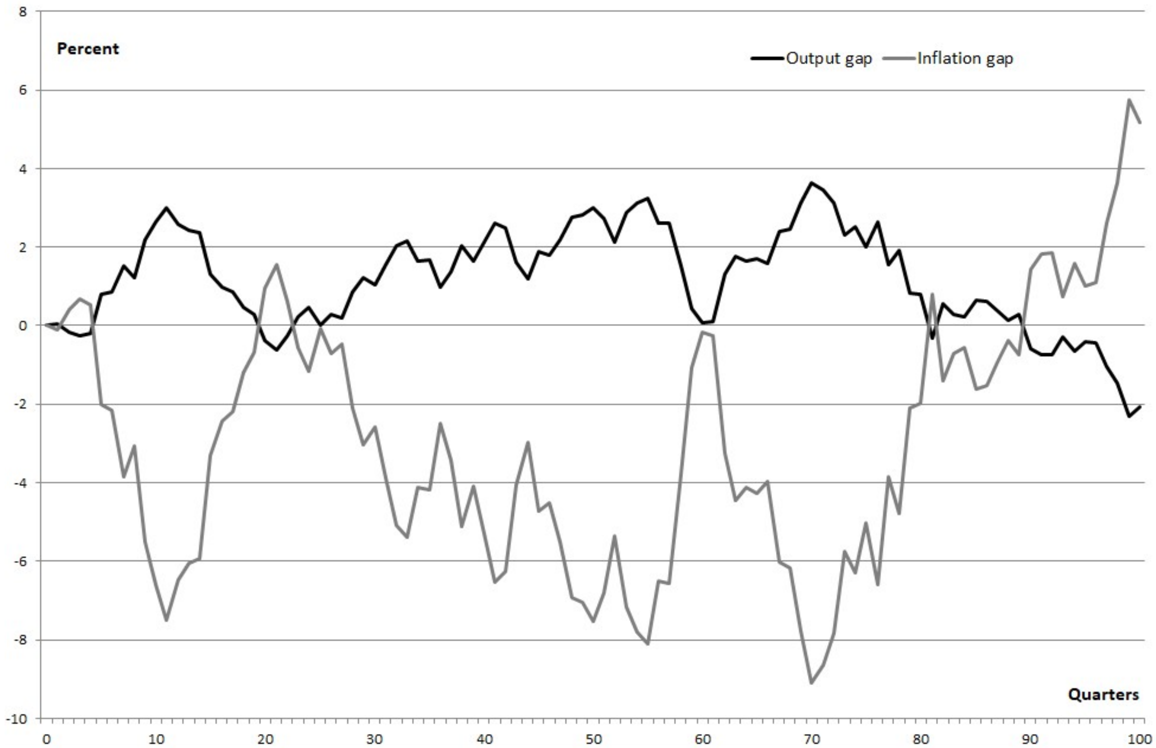
Experiment 2: only supply shocks and static expectations

Figure 19.12 below shows a typical simulation output for the opposite benchmark case where the stochastic disturbances occur only on the economy's supply side. By analogy to (27), we assume that the supply shocks follow a stochastic process of the form:

$$s_{t+1} = \omega s_t + c_{t+1}, \quad 0 \leq \omega < 1, \quad c_t \sim N(0, \sigma_c^2), \quad c_t \text{ i.i.d.} \tag{28}$$

To create Figure 19.12, we have followed a procedure similar that explained above, starting from long-run equilibrium in period 0 etc., except that we now set the demand shock variable $z_t = 0$ for all t. The 'persistence' parameter ω in (28) was set to 0, since positive values of ω generate an even poorer fit to the data than that displayed in Figure 19.12. The standard deviation σ_c of the white noise variable c_t used, resulting from Step 6 of the procedure, is 1.2, which thus ensures a simulated standard deviation of the output gap roughly in line with the empirical standard deviation. Even so, we see clearly from comparing Figure 19.12 to Figure 19.1 that the purely supply shock-driven AS–AD model is inconsistent with the stylized business cycle facts. The model generates far too much persistence in output and in inflation, far too much volatility of inflation, and a counterfactual perfect negative correlation between output and inflation. Of course, this negative correlation is not surprising, since we have previously seen that supply shocks, which shift the AS curve, will drive inflation and output in opposite directions along the fixed AD curve, creating a contemporaneous coefficient of correlation between output and inflation of exactly minus 1. By contrast, in the US economy the correlation between output and inflation has been positive in recent decades, indicating that supply shocks cannot have been the only driver of the business cycle.

Figure 19.12 Simulation of the stochastic AS–AD model with static expectations and no demand shocks



It should be stressed that the simulation results reported in Table 19.1 are *sample specific*, relying on particular samples from the normal distribution. If we feed different samples of x_t or c_t into the model, we get somewhat different sample statistics, but the general picture remains that neither a purely demand-driven nor a purely supply-driven AS–AD model can account fully for the stylized facts of the business cycle. To sum up:

THE STOCHASTIC AS–AD MODEL WITH STATIC EXPECTATIONS AND THE STYLIZED BUSINESS CYCLE FACTS

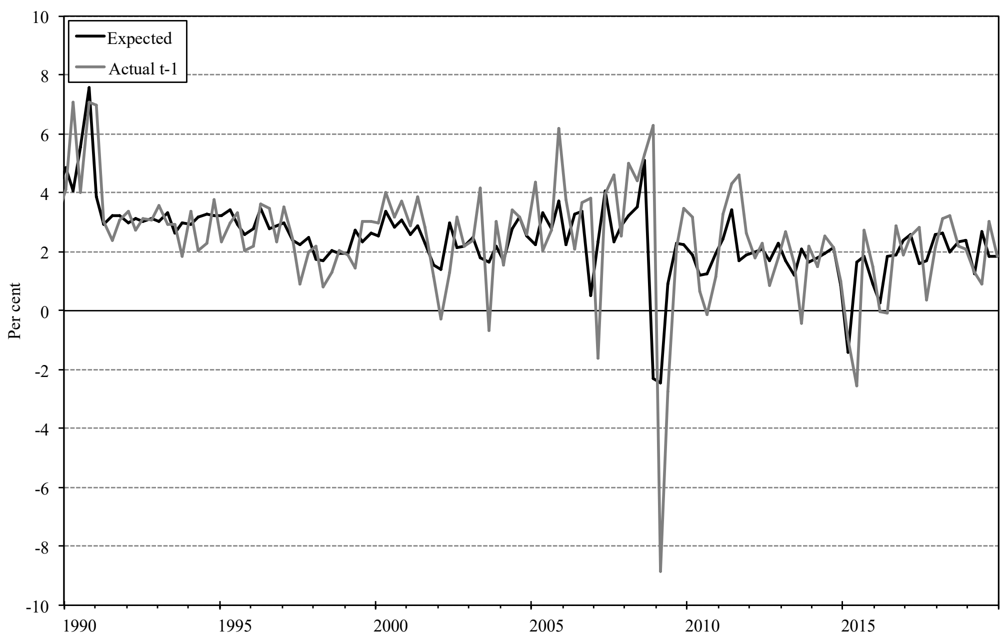
A stochastic AS–AD model with static inflation expectations where business fluctuations are driven solely by auto-correlated demand shocks can reproduce the observed volatility of output and the observed degree of output persistence reasonably well, but it generates an unrealistically high volatility and persistence of inflation. A stochastic AS–AD model with static expectations where fluctuations are driven only by supply shocks produces a counterfactual perfect negative correlation between output and inflation and far too much persistence of inflation and output as well as far too much volatility of inflation.

These findings suggest that, overall, demand shocks are more important for the business cycle than supply shocks, but at the same time a satisfactory business cycle model will have to account for both shocks to demand and supply, without assigning too great a role to supply shocks. In the next section we will therefore consider an extended model allowing both types of shocks to occur at the same time.

The stochastic AS–AD model with adaptive expectations

Apart from allowing for simultaneous demand and supply shocks, we will also generalize our description of the formation of expectations, since this will improve the ability of our AS–AD model to reproduce the empirical business cycle. We have so far assumed that expectations of inflation are *static*, meaning that this period's expected inflation rate is equal to last period's observed inflation rate, $\pi_t^e = \pi_{t-1}$. This assumption may well be too simplistic. The bolded curve in Figure 19.13 shows the *expected* inflation rate for the current quarter, π_t^e, calculated as an average of the inflation forecasts made by a number of professional economic forecasters in the USA. The forecasts were reported before the middle of the current quarter, at a time when the forecasters knew the *actual* inflation rate in the previous quarter, π_{t-1}. The latter variable is drawn as the grey line in Figure 19.13. If expectations were entirely static $(\pi_t^e = \pi_{t-1})$, the two curves in the diagram should more or less coincide. In reality, this is not the case. Instead, we see from the figure that expected inflation tends to systematically fluctuate less than lagged actual inflation.

Figure 19.13 Expected current inflation and lagged actual inflation in the USA



Note: The figure covers 1990Q1-2019Q4
Sources: Federal Reserve Bank of Philadelphia, Survey of Professional Forecasters and Bureau of Labor Statistics. Inflation is measured by the consumer price index.

This suggests that our AS–AD model will become more realistic if we replace the assumption of static expectations by a hypothesis, which implies that the expected inflation rate fluctuates *less* than last period's actual inflation rate. We will therefore assume that expectations are *adaptive*, adjusting in accordance with the formula:

$$\overbrace{\pi_t^e - \pi_{t-1}^e}^{\text{revision of expected inflation rate}} = (1-\phi)\cdot\overbrace{(\pi_{t-1} - \pi_{t-1}^e)}^{\text{last period's inflation forecast error}}, \quad 0 \le \phi < 1. \tag{29}$$

Equation (29) says that the expected inflation rate is adjusted upwards (downwards) over time if last period's actual inflation rate exceeded (fell short of) its expected level. We also see that a change in last period's actual inflation rate is not fully translated into a corresponding change in the expected inflation rate, provided $\phi > 0$. Isolating π_t^e in (29) gives:

$$\pi_t^e = \phi\pi_{t-1}^e + (1-\phi)\pi_{t-1} \tag{30}$$

From (30) we see that this period's expected inflation rate is a weighted average of last period's expected and actual inflation rates, so the inflation expectation is 'adapted' in the direction of observed inflation. We also see that static expectations $(\pi_t^e = \pi_{t-1})$ is that special case of adaptive expectations where the parameter ϕ is equal to 0. If we include the adaptive expectations hypothesis (29) in our AS–AD model, we can therefore easily reproduce all our previous results by simply setting $\phi = 0$, but perhaps we can obtain better fits by allowing $\phi > 0$. Note also that ϕ is a measure of the 'stickiness' of expectations: a relatively high value of ϕ means that people tend to be conservative in their expectations formation, being reluctant to revise their expected inflation rate in response to previous inflation forecast errors.

By lagging (30) repeatedly, we get:

$$\pi_{t-1}^e = \phi\pi_{t-2}^e + (1-\phi)\pi_{t-2} \tag{31}$$

$$\pi_{t-2}^e = \phi\pi_{t-3}^e + (1-\phi)\pi_{t-3} \tag{32}$$

...

and so on. We can gain further insight into the implications of adaptive expectations if we use the expressions for π_{t-1}^e, π_{t-2}^e etc., to eliminate π_{t-1}^e from the right-hand side of (30). Using such a series of successive substitutions, we obtain:

$$\begin{aligned}
\pi_t^e &= \phi\pi_{t-1}^e + (1-\phi)\pi_{t-1} \\
&= \phi^2\pi_{t-2}^e + (1-\phi)\pi_{t-1} + \phi(1-\phi)\pi_{t-2} \\
&= \phi^3\pi_{t-3}^e + (1-\phi)\pi_{t-1} + \phi(1-\phi)\pi_{t-2} + \phi^2(1-\phi)\pi_{t-3} \\
&\;\;\cdots \\
&= \phi^n\pi_{t-n}^e + (1-\phi)\pi_{t-1} + \phi(1-\phi)\pi_{t-2} + \phi^2(1-\phi)\pi_{t-3} + \cdots + \phi^{n-1}(1-\phi)\pi_{t-n}.
\end{aligned}$$

Since $\phi < 1$, the term $\phi^n\pi_{t-n}^e$ will vanish as we let n tend to infinity. Hence, we get:

$$\pi_t^e = \sum_{n=1}^{\infty} \phi^{n-1}(1-\phi)\pi_{t-n}, \quad 0 \le \phi < 1. \tag{33}$$

Equation (33) shows that the expected inflation rate for the current period is a weighted average of all inflation rates observed in the past, with geometrically declining weights as we move further back into history. Thus, adaptive expectations put more weight on the experience of the recent past than on the more distant past. However, unlike the special case of static expectations, adaptive expectations imply that people do not base their expectations on only the experience of the most recent period. The higher the value of ϕ, the longer are people's memories, that is, the greater is the impact of the more distant inflation history on current expectations.

Our AS–AD model with adaptive expectations may now be summarized as follows:[10]

AD: $$y_t - \bar{y} = -a(\pi_t - \pi^*) + z_t, \tag{34}$$

AS $$\pi_t = \pi_t^e + \gamma(y_t - \bar{y}) + s_t, \tag{35}$$

Expectations: $$\pi_t^e = \phi\pi_{t-1}^e + (1-\phi)\pi_{t-1}. \tag{36}$$

As for our earlier AS-AD model we will derive dynamic equations for the output gap, $\hat{y}_t \equiv y_t - \bar{y}$, and the inflation gap, $\hat{\pi}_t \equiv \pi_t - \pi^*$, expressing each of these in terms of its own lagged value and shock realizations.

Moving the AS curve (35) one period back in time and rearranging, we get:

$$\pi_{t-1}^e = \pi_{t-1} - \gamma(y_{t-1} - \bar{y}) - s_{t-1},$$

which may be inserted into (36) to give:

$$\begin{aligned} \pi_t^e &= \phi\left[\pi_{t-1} - \gamma(y_{t-1} - \bar{y}) - s_{t-1}\right] + (1-\phi)\pi_{t-1} \\ &= \pi_{t-1} - \phi\gamma(y_{t-1} - \bar{y}) - \phi s_{t-1}. \end{aligned} \tag{37}$$

Substituting (37) into (35) gives:

$$\pi_t = \pi_{t-1} + \gamma(y_t - \bar{y}) - \phi\gamma(y_{t-1} - \bar{y}) + s_t - \phi s_{t-1}, \tag{38}$$

Now, using our definitions of the output and inflation gaps, we may restate (34) and (38),

[10] You may wonder if the AD curve is unaffected by the switch from static to adaptive expectations. The answer is 'Yes', provided the central bank has a good estimate of the expected inflation rate, as we assume (such an estimate may be obtained through consumer and business surveys, or by comparing the market interest rates on indexed and non-indexed bonds). In that case, the central bank's control over the relevant, longer-term nominal interest rate i_t (however tight or loose that control is) translates into a similar control over the real interest rate $i_t - \pi_{t+1}^e$ regardless of the way inflation expectations are formed.

and thus our entire AS–AD model with adaptive expectations, in the compact form:

$$\hat{y}_t = -a\hat{\pi}_t z_t \ , \tag{39}$$

$$\hat{\pi}_t = \hat{\pi}_{t-1} + \gamma\hat{y}_t - \phi\gamma\hat{y}_{t-1} + s_t - \phi s_{t-1}, \tag{40}$$

where (39) is the AD curve and (40) is a restatement of the AS curve. In the same way as we derived our stochastic, linear difference equations above for the model with static expectations (the way we derived (19) and (20) from (16) and (17)), equations (39) and (40) can be shown to imply:

$$\hat{y}_t = d\hat{y}_{t-1} + \beta(z_t - z_{t-1}) - \alpha\beta s_t + \alpha\beta\phi s_{t-1}, \tag{41}$$

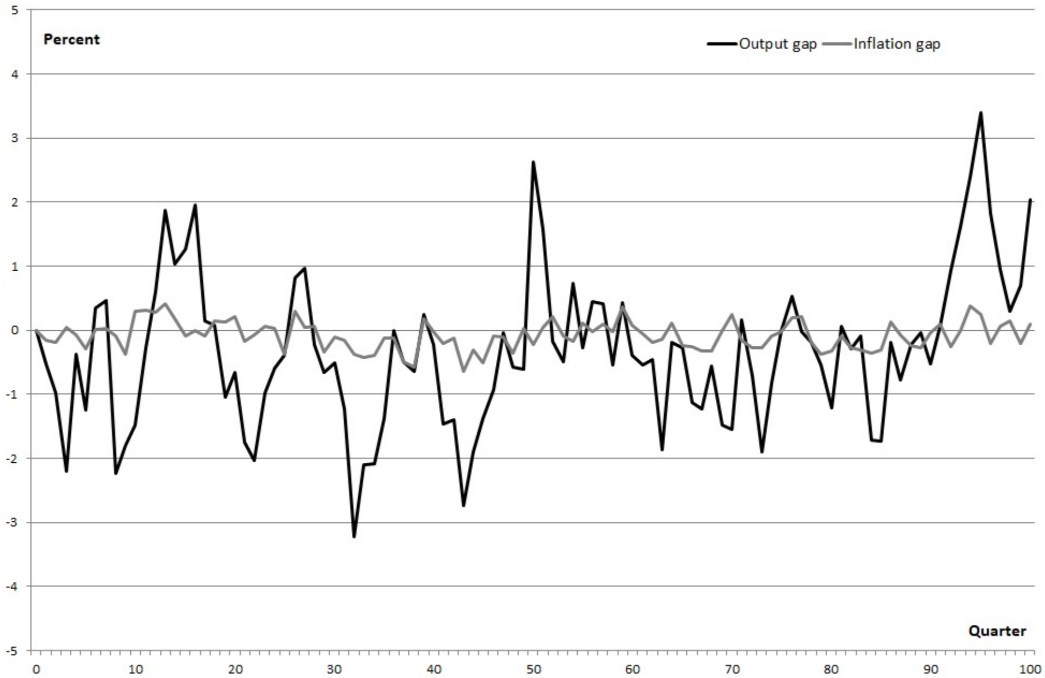
$$\hat{\pi}_t = d\hat{\pi}_{t-1} + \beta\gamma z_t - \beta\gamma\phi z_{t-1} + \beta s_t - \beta\phi s_{t-1}, \tag{42}$$

$$d \equiv \frac{1 + a\gamma\phi}{1 + a\gamma} < 1, \qquad \beta \equiv \frac{1}{1 + a\gamma} < 1. \tag{43}$$

These are our stochastic, linear, first order difference equations in $\hat{y}_t$ and $\hat{\pi}_t$, respectively, for the AS-AD model with adaptive expectations. We can simulate equations (41) and (42) in the same way as we did above for equations (19) and (20), only now we must also specify the expectations stickiness parameter ϕ. In our simulations we set $\phi = 0.9$, while the other parameter values remain at $a = 0.4$ and $\gamma = 0.1$. These parameter values imply $\beta \approx 0.96$ and $d = 0.996$. We assume that z_t and s_t are uncorrelated with each other and that they follow the stochastic processes (27) and (28) with auto-regressive coefficients, $\delta = 0.8$ and $\omega = 0.15$, and standard deviations, $\sigma_x = 1$ and $\sigma_c = 0.2$. Given these parameter values, we simulate the model according to the procedure described above, starting from long-run equilibrium, $\hat{y}_0 = \hat{\pi}_0 = z_0 = s_0 = 0$, in period 0.

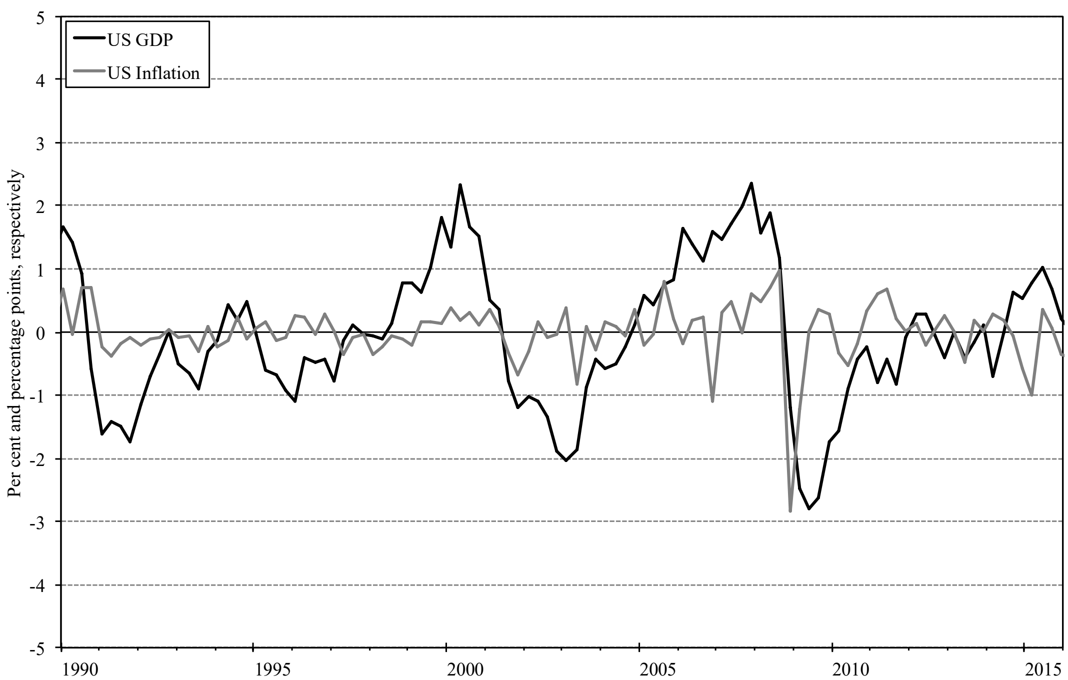
The model-generated time series for $\hat{y}_t$ and $\hat{\pi}_t$ are plotted in Figure 19.14a for 100 periods. For convenience, we have also reproduced the latest around 100 quarters from Figure 19.1 as Figure19.14b. This shows the actual US business cycle for a recent time interval covering about the same number of periods as the simulation output in Figure 19.14a, allowing us to compare the model-generated data with reality. Of course, since the timing of the random shocks hitting our model economy does not coincide with the timing of the historical shocks to the US economy, our model cannot reproduce the historical turning points of the American business cycle, but it may reproduce the overall qualitative features. A glance at the figures suggests that this is indeed so. The variances of output and inflation and the correlation between the two according to our calibrated AS–AD model with adaptive expectations and auto-correlated shocks seem fairly realistic. The persistence of output and of inflation also seem somewhat OK, but perhaps eye-balling reveals a bit too high persistence according to the model.

Figure 19.14a Simulation of output and inflation gap from the stochastic AS–AD model with adaptive expectations and auto-correlated demand and supply shocks



Parameter values: See notes to Table 19.1.

Figure 19.14b Cyclical components of real GDP and inflation in the USA, 1990-2015



Source: As for Figure 19.1.

Looking at the details of the matter, Table 19.2 reports the business cycle statistics for the US economy (reproduced from Table19.1) and the statistics generated by our simulation of the model (41) and (42) over 1,000 periods.

Table 19.2 Volatilities and correlations of the US business cycle, 1947-2017, and according to the AS-AD model with adaptive expectations and auto-correlated shocks

	Standard deviation		Correlation	Auto correlation							
	GDP (Percent)	Inflation (Points)	GDP/inflation	GDP				Inflation			
				t-1	t-2	t-3	t-4	t-1	t-2	t-3	t-4
US economy	1.6	0.5	0.3	0.9	0.6	0.4	0.1	0.4	0.2	0.2	0.0
AS-AD model	1.6	0.5	0.3	0.8	0.6	0.5	0.4	0.8	0.7	0.7	0.7

Note: Empirical part as for Table 19.1 Model part based on simulation over 1,000 periods with parameters: $a = 0.4$, $\gamma = 0.1$, $\phi = 0.9$, $\delta = 0.8$, $\omega = 0.15$, $\sigma_x = 1$, $\sigma_c = 0.2$.
Source: As for Table 19.1 and own simulations.

Comparing the two rows for the US economy and for our AS-AD model of Table 19.2 confirms that the extended AS–AD model with adaptive expectations and simultaneous, auto-correlated demand and supply shocks fits the empirical business cycle facts fairly well, given the parameter values we have chosen. The simulated volatility of output and inflation and the correlation between the two variables are realistic, but the model tends to overestimate the degree of persistence in output and particularly of inflation, due to its simplified dynamic structure.

We have tried to show that a simple stochastic AS–AD model allowing for shocks to supply as well as demand can provide a reasonably good account of the cyclical movements in output and inflation. In so doing, we have also tried to illustrate the basic methodology of modern business cycle analysis, showing how macroeconomists build dynamic stochastic general equilibrium (DSGE) models and calibrate these models to reproduce the stylized statistical facts of the business cycle.

We must emphasize once again that the simulation results reported in Figure 19.14a and Table 19.2 are sample specific. Even though the simulation reported on in the table runs over 1,000 periods (amounting to 250 years if the period length is one quarter), the computed statistics are not completely stable and will differ a bit from simulation to simulation. They do stay relatively stable, however. The overall impression is that the model fits the data reasonably well, once we consider how simple the model really is compared to the staggering complexity of the real-world economy.

Yet we must keep in mind that although our AS–AD model with adaptive expectations seems roughly consistent with the data on output and inflation, this does not imply that we have found *the* explanation for business cycles. It is possible to construct other types of models that match the data on output and inflation equally well. Hence, we cannot claim to have found the only correct theory of the business cycle. All we can say is that our theory does not seem to be clearly rejected by the data.

Let us sum up our latest findings:

THE STOCHASTIC AS–AD MODEL WITH ADAPTIVE EXPECTATIONS

The hypothesis of adaptive inflation expectations says that the expected rate of inflation is a weighted average of all inflation rates observed in the past, with a greater weight attached to more recent observations. A stochastic AS–AD model with adaptive expectations and auto-correlated shocks, where business fluctuations are mainly driven by demand shocks, but where supply shocks also play some role, can reproduce the stylized business cycle facts reasonably well.

Summary

1. The AS–AD model determines the short-run equilibrium values of output and inflation as the point of intersection between the upward-sloping short-run aggregate supply curve (the AS curve) and the downward-sloping aggregate demand curve (the AD curve). The model also determines the long-run equilibrium levels of output and inflation as the point of intersection between the vertical long-run aggregate supply curve (the LRAS curve) and the AD curve.
2. When expectations are static, the expected inflation rate for the current period equals the actual inflation rate observed during the previous period. Under this assumption, the AS–AD model is globally stable, converging gradually towards the long-run equilibrium where output is at its natural rate and inflation is at its target rate. The adjustment to long-run equilibrium takes place through successive shifts in the AS curve, as economic agents gradually revise their inflation expectations in reaction to observed changes in the actual inflation rate. During the adjustment process, the economy moves along the AD curve, as the central bank gradually adjusts the real interest rate in reaction to the changes in the inflation rate.
3. With plausible parameter values, the AS–AD model suggests that it will take around six years for the economy to complete half of the adjustment towards the steady state and almost 20 years to complete 90 per cent of the adjustment. Hence, output and inflation may deviate from their trend values for quite a long time, once the long-run equilibrium has been disturbed by a shock.
4. The AS–AD model may be specified in deterministic terms or in stochastic terms. In the deterministic version of the model, the demand and supply shock variables are non-stochastic. In the stochastic AS–AD model, the shocks to demand and supply are treated as random variables.
5. The deterministic model may be used to study the isolated effects of a single temporary or permanent shock to supply or demand. The stochastic AS–AD model may be used to generate simulated time series for output and inflation, allowing a calculation of the variance, covariance and autocorrelation in these variables. These statistics may then be compared to the statistical properties of empirical time series to investigate how well the stochastic AS–AD model is able to reproduce the stylized facts of the business cycle.

6. In the short run, a temporary negative supply shock generates stagflation, defined as an increase in inflation combined with a fall in output. Even after the negative supply shock has disappeared, inflation will remain above the target rate and output will remain below the natural rate for many successive periods, because it takes time for inflation expectations to adjust back to the target inflation rate.
7. A temporary (one-shot) negative demand shock generates a bust–boom adjustment pattern in output. In the period when the negative demand shock occurs, there is a drop in output as well as inflation. When the negative demand impulse disappears, output rises above its natural rate because the AS curve shifts down, due to a fall in expected inflation. In the subsequent periods, output gradually falls back to its trend level, and inflation gradually rises towards the target rate.
8. By quantifying the parameter values, the deterministic AS–AD model may be used to generate impulse-response functions showing how output and inflation will evolve over time in response to an impulse such as a temporary or permanent demand or supply shock.
9. The deterministic AS–AD model can explain the observed persistence (autocorrelation) in economic time series, that is, it can explain why even a temporary shock generates protracted, long-lasting deviations of output and inflation from their trend levels. However, the deterministic model cannot explain the observed recurrent cyclical fluctuations in macroeconomic variables. If the demand and supply shock variables are treated as stochastic processes, then the AS–AD model is able to generate irregular, cyclical fluctuations in output and inflation.
10. A calibrated stochastic AS–AD model with static expectations where all shocks take the form of auto-correlated demand shocks is reasonably good at reproducing the statistical properties of the empirical time series for output, but such a model generates an unrealistically high degree of persistence in the rate of inflation. A stochastic AS–AD model with static expectations where all shocks originate from the supply side is incapable of reproducing the statistical properties of the time series for output as well as inflation. This suggests that a model intended to explain the business cycle must allow for demand shocks as well as supply shocks and that the assumption of static inflation expectations may be too simple.
11. The hypothesis of adaptive expectations says that the expected inflation rate for the current period is a weighted average of all inflation rates observed in the past, with more weight being put on the experience of the recent past than on the more distant past. Static expectations are a special case of adaptive expectations where the weight given to last period's observed inflation rate is 100 per cent.
12. A calibrated stochastic AS–AD model with adaptive expectations allowing for auto-correlated demand shocks as well as supply shocks is able to reproduce the statistical properties of the time series for US output and inflation reasonably well. The stochastic AS–AD model offers an explanation for the business cycle in line with the Frisch–Slutsky paradigm. In this paradigm, business fluctuations are initiated by random demand or supply shocks, which are then propagated through the economic system in a way that generates persistence in macroeconomic variables. In our AS–AD model, the persistence in the macroeconomic time series stems from the fact that the expected and actual inflation rates adjust sluggishly over time. However, one can construct other macroeconomic models with other

persistence mechanisms, so our AS–AD model is not the only possible explanation for business cycles.

Exercises

Exercise 1. Temporary shocks in the deterministic AS–AD model

1. Use the AS–AD model with static expectations to undertake a graphical analysis of the effects of a positive supply shock (a fall in s) which lasts for one period. You may assume that the economy starts out in long-run equilibrium in period 0 and that the temporary shock occurs in period 1. Explain the short-run effects as well as the economy's adjustment over time.

2. Suppose now that the positive supply shock emerging in period 1 also persists throughout period 2 before it goes away from period 3 onwards. Illustrate the effects of this two-period shock graphically and explain the difference compared to the one-period shock analysed in Question 1. Be careful to explain exactly how the AS curves shift in the two scenarios. (Hint: indicate precisely where the AS curves cut the LRAS curve in the various periods).

3. Now use the AS–AD model with static expectations to perform a graphical analysis of the effects of a positive demand shock occurring in period 1 and dying out from period 2 onwards. Explain the mechanisms underlying the evolution of output and inflation over time.

4. Assume instead that the positive demand shock emerging in period 1 lasts for *two* periods before it dies out. Illustrate the effects graphically. Will the fluctuations in output and inflation be larger or smaller than those occurring when the shock lasts for only one period? Explain.

Exercise 2. Permanent shocks in the deterministic AS–AD model

1. Suppose that the economy is hit by a *permanent* negative demand shock, say, because a tax reform permanently raises the private sector's propensity to save. Illustrate by a graphical analysis that such a shock will lead to a permanent violation of the inflation target if the central bank does not adjust its estimate of the structural (natural) real interest rate, $\bar{r}$.

2. Now suppose that the economy is in long-run equilibrium in period 0 and that the permanent negative demand shock hits the economy from period 1 onwards. Suppose further that it takes the central bank only one period to realize that the shock is permanent. From period 2 onwards, the bank thus revises its estimate of the structural real interest rate from the original level to the new level given by Equation (26) in the text. Analyse mathematically and graphically how these developments will shift the AD curve in periods 1 and 2. Illustrate and explain the economy's adjustment over time. Compare your graphical analysis with the analysis of a purely *temporary* negative demand shock in Figure 19.6. Is there any difference?

3. Suppose alternatively that it takes until period 3 before the central bank realizes the permanency of the negative demand shock and revises its estimate of the equilibrium real interest rate. Perform a graphical analysis of the economy's adjustment from period 0 onwards and compare with the analysis in Question 2. (Be precise in your indication of the shifts in the AD and AS curves). How does the delayed adjustment of the estimate for $\overline{r}$ affect the magnitude of the fluctuations in output and inflation? Explain.
4. Suppose again that the economy starts out in long-run equilibrium in period 0 and assume that a *permanent negative supply shock* (a rise in s) occurs from period 1 onwards, reducing natural output from $\overline{y}_0$ to the new lower level $\overline{y} = \overline{y}_0 - (s/\gamma)$. Suppose further that the central bank realizes the permanency of the shock already in period 2 and hence revises its estimate of the equilibrium real interest rate in accordance with Equation (25) from period 2 onwards, so that the current real market interest rate is given by the following equation (where we abstract from shocks to market risk premia):

$$r_t = \overline{r}_0 + \frac{s}{\gamma \alpha_2} + h(\pi_t - \pi^*) + b\left[y_t - \left(\overline{y}_0 - \frac{s}{\gamma} \right) \right] \quad \text{for } t \geq 2.$$

Derive the equation for the new AD curve which will hold for $t \geq 2$ (where will the new AD curve cut the new LRAS curve?). Illustrate the new short-run equilibria in periods 1 and 2. (Hint: note that from period 2 onwards, the AS curves will cut the *new* LRAS curve at the level of the previous period's inflation rate). Illustrate by arrows along the new AD curve how the economy will evolve after period 2 and explain the economic adjustment mechanisms. Compare your results with the analysis of a purely temporary negative supply shock in Fig. 19.5.

Exercise 3. Interest rate smoothing in the AS–AD model

We have so far assumed that the interest rate adjusts immediately to the target level given by the Taylor rule. However, empirical research has found that central banks tend to adjust their interest rates only gradually towards the target level because they do not like to change the interest rate too abruptly. We may model such 'interest rate smoothing' by assuming that:

$$r_t = (1-\lambda) r_t^* + \lambda r_{t-1} \Leftrightarrow r_t - r_{t-1} = (1-\lambda)(r_t^* - r_{t-1}), \quad 0 \leq \lambda < 1, \tag{44}$$

where λ is a parameter indicating the sluggishness of interest rate adjustment, and where r_t^* is the *target* real interest rate given by the Taylor:

$$r_t^* = \overline{r} + h(\pi_t - \pi^*) + b(y_t - \overline{y}). \tag{45}$$

In the main text we have analysed the special case where $\lambda = 0$, but here we will focus on the general case where $\lambda > 0$. The IS curve or goods market equilibrium condition may still be written as:

$$y_t - \bar{y} = \alpha_1(g_t - \bar{g}) - \alpha_2(r_t - \bar{r}) + v_t, \tag{46}$$

and the economy's supply side is still given by:

$$\pi_t = \pi_t^e + \gamma(y_t - \bar{y}) + s_t. \tag{47}$$

As a new element, we will assume that economic agents are sufficiently sophisticated to realize that on average over the long run, the inflation rate must equal the central bank's inflation target. Hence, we assume that the expected inflation rate is:

$$\pi_t^e = \pi^* \quad \text{for all } t. \tag{48}$$

This completes the description of our revised AS–AD model.

1. Discuss whether the assumption made in (48) is reasonable.
2. Define $\hat{y}_t \equiv y_t - \hat{y}$ and $\hat{\pi}_t \equiv \pi_t - \pi^*$ and show that the AD curve is given by the equation:

$$\hat{y}_t = \rho\hat{y}_{t-1} - \alpha\hat{\pi}_t + z_t - \lambda z_{t-1}, \tag{49}$$

$$\rho \equiv \frac{\lambda}{1+\alpha_2 b(1-\lambda)}, \qquad \alpha \equiv \frac{\alpha_2 h(1-\lambda)}{1+\alpha_2 b(1-\lambda)}, \qquad z_t \equiv \frac{\alpha_1(g_t - \bar{g}) + v_t}{1+\alpha_2 b(1-\lambda)},$$

3. Show that the model can be reduced to the difference equation:

$$\hat{y}_t = \beta\rho\hat{y}_{t-1} + \beta(z_t - \lambda z_{t-1}) - \alpha\beta s_t, \quad \beta \equiv \frac{1}{1+\gamma\alpha}, \tag{50}$$

and prove that, in the absence of shocks, the economy will converge on a long-run equilibrium where $\hat{y}_t = \hat{\pi}_t = 0$. How does the parameter λ affect the economy's speed of adjustment? Explain.

4. Suppose that the economy starts out in a short-run equilibrium in period 0 where $y_0 < \bar{y}$. Give a graphical illustration of the initial short-run equilibrium and of the economy's adjustment to long-run equilibrium, assuming there are no further shocks. (Hint: note that the adjustment now takes place through successive *shifts in the AD curve* rather than through shifts in the SRAS curve). Explain the economic mechanism, which ensures convergence on long-run equilibrium.
5. Now assume that the economy is in long-run equilibrium in period 0, but is hit by a *temporary negative supply shock* (a rise in s_t) in period 1. The shock vanishes again from period 2 onwards. Give a graphical illustration of the new short-run equilibria in

periods 1 and 2. Could y_2 be lower than y_1? (Hint: use (67) to justify your answer). Indicate by arrows (along the AS curve) how the economy will evolve after period 2. Explain your findings.

6. Suppose again that the economy is in long-run equilibrium in period 0, but assume now that it is hit by a *temporary negative demand shock* which occurs only in period 1. Illustrate graphically how this will affect the economy in periods 1 and 2 and use arrows to indicate the economy's adjustment after period 2. Explain your findings. (Hint: you may want to use Equations (66) and (67) to show that the AD curve for period 2 will lie *above* the AD curve for period 0).

Exercise 4. Making your own Slutsky diagram

In the chapter, we explained rather carefully how to do simulations with a dynamic equation like (26). In particular, Footnote 8 explained how to make a random draw from the normal distribution in Microsoft Excel. Use this to do own simulations with the stochastic dynamic equation $q_t = bq_{t-1} + e_t,\ 0 < b < 1$, and create your own Slutsky diagrams like Figure 19.6, varying the stickiness parameter b and the standard deviation of the normal distribution og e_t.

Exercise 5. Implementing the stochastic AS–AD model with adaptive expectations and auto-correlated shocks yourself

In the chapter, we explained rather carefully how to do simulations with the AS-AD models considered in Microsoft Excel. You are now challenged to do such simulations yourself (in Excel) based on the dynamic equations (41) and (42). Your job is to see if you can make the model perform a bit better with respect to auto-correlation in the output and inflations gaps than we obtained in the chapter, where the model's persistence was found to be somewhat to the high side.

In an Excel sheet, set aside some cells for entering parameter values for a, γ, and ϕ, for the auto-regressive coefficients δ and ω, and for the standard deviations in the demand and supply shocks, σ_x and σ_c. Start with the same values as considered in the chapter. Let Excel compute the resulting β and d, so that these turn up in two cells.

Then in two columns each containing at least 500 cells, make two times 500 separate draws from the normal distribution, both with mean zero, and with standard deviation σ_x in the first column and σ_c in the second. From these draws, covering periods $t = 1, 2, \ldots, 500$, let Excel compute two associated series for z_t and s_t for periods $t = 0, 1, 2, \ldots, 500$, letting $z_0 = s_0 = 0$, and using Equations (27) and (28) and your chosen δ and ω. From your computed shocks z_t and s_t, let Excel compute a series for the output gap $\hat{y}_t$ and the inflation gap $\hat{\pi}_t$, using Equations (40) and (41) and your computed β and d for $t = 1, 2, \ldots, 500$, starting from long-run equilibrium in period 0. From your time series, compute the standard deviations and coefficients of correlation also appearing in Table 19.2.

Now, from Equations (41) and (42) it seems an obvious idea that a smaller value of

ϕ (than the 0.9 used in the simulations in the chapter) may help to obtain less persistence in the output gap and the inflation gap. Therefore, try a lower value for ϕ and alternative values for other parameters of your own choice and do simulations to investigate, whether you can obtain a (lower) degree of persistence in better accordance with the statistics for the US economy and at the same time preserve a god fit in the other respects found in the chapter.

Exercise 6. Developing and implementing a stochastic AS–AD model with a more general AS curve

In this exercise you are asked to set up a stochastic AS–AD model, implement it on the computer, and undertake some simulations to illustrate the effects of monetary policy. In this way you will become even more familiar with the modern methodology for business cycle analysis which was described in Section 19.3 than from the former exercise.

Our starting point is a generalized version of the short-run aggregate supply curve. Many econometricians studying the labour market have found that wage inflation is moderated not only by the *level* of unemployment (u_t), but also by the *increase* in the unemployment rate between the previous and the current period ($u_t - u_{t-1}$). The reason is that, *ceteris paribus*, it is more difficult for a dismissed worker to find an alternative job when unemployment is rising than when it is falling. When labour is mobile across sectors, a rising unemployment rate thus reduces the value of the representative worker's outside option. Assuming static inflation expectations $(\pi_t^e = \pi_{t-1})$, we therefore get the following generalized version of the expectations-augmented Phillips curve:

$$\pi_t = \pi_{t-1} + \gamma(\bar{u} - u_t) - \gamma\theta(u_t - u_{t-1}), \quad \gamma > 0, \quad \theta > 0, \tag{51}$$

where $\bar{u}$ is the constant natural rate of unemployment, and the parameter θ indicates the degree to which the wage claims of workers are moderated by rising unemployment. We assume that current output is given by a simple linear production function of the form $Y_t = a_t L_t$, where a_t is the exogenous current average labour productivity, and L_t is current employment. If the constant labour force is N, we have $L_t \equiv (1 - u_t)N$. Using the approximation $\ln(1 - u_t) \approx u_t$ and the definition $y_t \equiv \ln Y_t$, we thus have:

$$y_t = \ln a_t + \ln N - u_t. \tag{52}$$

Trend output (natural output) is $\bar{Y} = \bar{a}\bar{L} = \bar{a}(1-\bar{u})N$, where $\bar{L}$ is natural employment and $\bar{a}$ is the 'normal' (trend) level of productivity. Hence we have:

$$\bar{y}_t = \ln \bar{a} + \ln N - \bar{u}. \tag{53}$$

Let us define the supply shock variable:

$$s_t \equiv \ln \bar{a} - \ln a_t. \tag{54}$$

Note that s_t is *positive* when productivity is unusually low, that is, when unit labour costs are above their normal level.

1. Use (51)–(54) to demonstrate that the economy's short-run aggregate supply curve may be written as:

$$\hat{\pi}_t = \hat{\pi}_{t-1} + \gamma(1+\theta)\hat{y}_t - \gamma\theta\hat{y}_{t-1} + \gamma(1+\theta)s_t - \gamma\theta s_{t-1}, \tag{55}$$

$$\hat{y}_t \equiv y_t - \bar{y}, \quad \hat{\pi}_t \equiv \pi_t - \pi^*.$$

How does Equation (55) deviate from the AS curve in the model in the main text? Explain briefly why the lagged output gap $\hat{y}_{t-1}$ appears with a negative coefficient on the right-hand side of (72). Explain in economic terms how the parameter θ affects the sensitivity of inflation to the current output gap.

As usual, the economy's aggregate demand curve is given by:

$$\hat{y}_t = z_t - a\hat{\pi}_t, \quad \hat{\pi}_t \equiv \pi_t - \pi^*, \tag{56}$$

where π^* is the central bank's inflation target.

2. Show that the solutions for the output gap and the inflation gap take the form:

$$\hat{y}_{t+1} = a_1\hat{y}_t + \beta(z_{t+1} - z_t) - a_2 s_{t+1} + a_3 s_t, \tag{57}$$

$$\hat{\pi}_{t+1} = a_1\hat{\pi}_t + c_1(z_{t+1} + s_{t+1}) - c_2(z_t + s_t), \tag{58}$$

where

$$\beta \equiv \frac{1}{1+\gamma\alpha(1+\theta)}, \quad a_1 \equiv \beta(1+\gamma\alpha\theta), \quad a_2 \equiv \beta\gamma\alpha(1+\theta), \tag{59}$$

$$a_3 \equiv \beta\gamma\alpha\theta, \qquad c_1 \equiv \beta\gamma(1+\theta), \qquad c_2 \equiv \beta\gamma\theta. \tag{60}$$

(Hints: write Equation (55) for period $t+1$, insert (56) into the resulting expression and collect terms to get (57). Then use (56) to write (57) in terms of $\hat{\pi}$ rather than $\hat{y}$ and collect terms to get (58)).

The demand and supply shock variables are assumed to follow stochastic processes of the form:

$$z_{t+1} = \delta z_t + x_{t+1}, \quad 0 \le \delta < 1, \quad x_t \sim N(0, \sigma_x^2), \quad x_t \text{ i.i.d.} \tag{61}$$

$$s_{t+1} = \omega s_t + c_{t+1}, \quad 0 \leq \omega < 1, \quad c_t \sim N(0, \sigma_c^2), \quad c_t \text{ i.i.d.} \tag{62}$$

We also remember that:

$$\alpha_2 = \frac{\eta(1-\tau)}{1-D_\gamma}, \quad a = \frac{\alpha_2 h}{1+\alpha_2 b}, \tag{63}$$

where the parameters are as defined and explained in the main text of this chapter.

3. *Implementing the model*. In order to undertake simulation exercises, you are now asked to program the model consisting of (57)–(63) on the computer using Microsoft Excel. Use the procedure described in footnote 8 of this chapter to draw two different 500-period samples taken from the standardized normal distribution with mean zero and standard deviation 1. You may take the first sample to represent the stochastic shock variable x_t, and the second sample to represent the shock variable c_t. In your first Excel spreadsheet you should list the parameters of the model as well as the standard deviations of the output and inflation gaps, the coefficient of correlation between the two gaps, and the coefficients of autocorrelation emerging from your simulations. It will also be useful to include diagrams illustrating the simulated values of the output gap and the inflation gap. We suggest that you use the parameter values:

$\gamma = 0.075$, $\tau = 0.2$, $\eta = 3.6$, $D_\gamma = 0.5$, $h = b = 0.5$, $\theta = 0.5$.

(All of these values except the one for θ were used in the main text.) To calibrate the magnitude of the supply and demand shocks, x_t and c_t, you must choose their respective standard deviations σ_x and σ_c and multiply the samples taken from the standardized normal distribution by these standard deviations. As a starting point, you may simply choose:

$\sigma_x = 1$, $\sigma_c = 1$.

You also have to choose the value of the parameters δ and ω. For a start, just set:

$\delta = 1$, $\omega = 1$.

Finally, you must choose the initial values of the endogenous variables in period 0. We assume that the economy is in a long-run equilibrium in period 0 so that:

$$\hat{y}_0 = \hat{\pi}_0 = z_0 = s_0 = 0 \quad \text{for } t = 0.$$

Now program this model in Excel and undertake a simulation over 500 periods. Compare the model-generated statistics on the standard deviations and the coefficients of correlation and autocorrelation of output and inflation with the corresponding statistics for the USA given in the bottom row of Table 19.1 in the main text. Comment on the differences.

4. Experiment with alternative constellations of the parameters σ_x, σ_c, δ and ω until you find a constellation which enables your AS–AD model to reproduce the US business cycle statistics for output and for the correlation between output and inflation reasonably well. (You should not bother too much about the behaviour of inflation implied by the model, since we know from the text that a model with static inflation expectations is not very good at reproducing the statistical behaviour of inflation). State a set of values for σ_x, σ_c, δ and ω which in your view gives a reasonable account of the behaviour of output and of the correlation between output and inflation. What does your choice of parameter values imply for the relative importance of demand and supply shocks? Comment.
5. The simulations above use the monetary policy parameter values suggested by John Taylor, that is, $h = b = 0.5$. Now suppose that the central bank decides to react more aggressively to changes in the rate of inflation by raising the value of h from 0.5 to 1.0. Simulate your model to investigate the effects of such a policy change. Try to explain the effects. Discuss whether the policy change is desirable.
6. Suppose again that the central bank decides to raise h from 0.5 to 1.0, but suppose also that supply shocks are very important so that $\sigma_c = 5$. Is the policy change now desirable? Discuss.

Chapter 20

Monetary and Fiscal Stabilization policy

Introduction

We have previously explained the multiple reasons why fluctuations in output and inflation and the associated fluctuations of consumption and employment cause net losses of economic welfare. The social costs of business cycles mean that social welfare increases if policy makers succeed in reducing the volatility of output and inflation through active monetary and fiscal policy. But whether the government can indeed dampen the business cycle through active stabilization policy is a long-standing controversy in macroeconomics. For example, in Chapter 17 we reported the view of Milton Friedman that since monetary policy works with long and variable lags that are hard to predict, the best contribution monetary policy can make to economic stability is to ensure a constant growth rate of the nominal money supply. Essentially Friedman argued that because the economy is such a complex mechanism that we do not understand very well, attempts by governments to manipulate aggregate demand will often fail as monetary or fiscal interventions may occur at the wrong time or in the wrong dosage. In this way, active demand management policies may end up amplifying rather than dampening the business cycle. Some other economists have argued that if monetary or fiscal policy follow a systematic pattern over the business cycle, rational private sector agents will come to anticipate this pattern and will adapt their behaviour in a manner that will tend to nullify the intended effects of stabilization policy.[1]

The debate on what macroeconomic stabilization policy can and should try to achieve continues and was recently stimulated by the severe financial and economic crisis starting in 2008 – 09 where monetary and fiscal policy makers intervened on a massive scale in their efforts to stabilize the economy. In this chapter, we take some first steps in the analysis of the optimal design of stabilization policy, building on the findings of the

[1] Chapter 21 will explain this argument in detail.

previous chapters. We start, in Part A, with some general remarks on stabilization policy that are not specific to monetary or fiscal policy. First we revisit the social loss function that expresses the costs of business cycles. This will lead us on to a discussion whether stabilization policy should be based on a fixed rule such as the Taylor rule or whether it should be 'discretionary', reacting to the particular circumstances of the day without following a predetermined pattern. We conclude that there can be advantages of following policy rules, although discretionary deviations from a strict policy rule are sometimes warranted.

We go on by studying monetary stabilization policy in Part B and fiscal stabilization policy in Part C. Taking as granted that that rules *can* be desirable, in part B we first study how monetary policy can dampen the business cycle, *given that it follows a Taylor rule*. Then we go on to characterize *the optimal monetary policy rule* when it is not taken as given that monetary policy should follow a Taylor rule. Subsequently we will gradually relax some underlying strong assumptions by analysing the optimal monetary stabilization rule when there are time lags in the economy's reaction to policy changes. Having gone through these issues for monetary stabilization policy, we do the same for fiscal stabilization policy in Part C. Here we also consider some important issues related to the interactions between monetary and fiscal stabilization policies. In this connection we will study the so-called 'liquidity trap' which gained renewed relevance during and after the great financial crisis starting in 2008.

Most of the analysis in this chapter maintains our previous assumption that expectations are *backward looking*, i.e., that the expected future rate of inflation is determined solely by the actual rates of inflation observed in the past. In Chapters 21 and 22 we shall analyse the scope for stabilization policy when expectations are *forward looking* in the sense that agents try to form the best possible estimate of future variables, utilizing all the relevant information available to them at the time expectations are formed. Chapter 22 will also consider the challenges for stabilization policy arising when there is uncertainty in the measurement of the output and inflation gaps, and when the authorities have difficulties establishing the credibility of their commitment to a low and stable inflation rate.

In the present and the next two chapters we will thus gradually introduce many of the issues raised by those who are sceptical with regard to the ability of governments and central banks to actively stabilize the economy. In this chapter, however, our point of departure will be that there is indeed an important role for macroeconomic stabilization policy, at least in mitigating the more substantial business cycles. The crucial question in this chapter is *how* such policy should be designed.

In the first part of this chapter, Sections 201.1 and 20.2, we discuss some issues concerning monetary as well as fiscal stabilization policy in general: what should stabilization policy try to accomplish, and should it be conducted according to rules or discretion? We then turn to monetary stabilization policy in Sections 20.3-20.5 and finally to fiscal stabilization policy in Sections 20.6-20.8.

20.1 The social loss function revisited and a look ahead

Looking back: the social loss function

We concluded our discussion of the social costs of business cycles in Chapter 14 by summarizing them in a social loss function of the form

$$SL_t = \left(y_t - \overline{y}\right)^2 + \kappa\left(\pi_t - \pi^*\right)^2 = \hat{y}_t^2 + \kappa\hat{\pi}_t^2, \quad \kappa > 0, \tag{1}$$

which attaches a social loss in period *t* to the output gap, $\hat{y}_t \equiv y_t - \overline{y}$, and the inflation gap, $\hat{\pi}_t \equiv \pi_t - \pi^*$, where the parameter κ expresses the relative weight society puts on inflation stability vis-à-vis output stability, and π^* is the target rate of inflation.

Looking many periods ahead from period *t*, it is natural to assume that the total social cost of instability to society is the expected sum of the single period losses with the future discounted by a discount factor δ:

$$\overline{L}_t \equiv E_t\left(\sum_{\tau=0}^{\infty}\delta^\tau L_{t+\tau}\right) = \sum_{\tau=0}^{\infty}\delta^\tau E_t(SL_{t+\tau}) = \sum_{\tau=0}^{\infty}\delta^\tau\left[E_t\left(\hat{y}_{t+\tau}^2\right) + \kappa E_t\left(\hat{\pi}_{t+\tau}^2\right)\right], \quad 0 < \delta < 1, \tag{2}$$

where E_t means the expected value as seen from period *t*, that is, given the information available in period *t*.[2]

Considering just the single period *t*, the marginal losses from a bit larger output gap and inflation gap, respectively, are from (1):

$$MSL_{y,t} \equiv \frac{\partial SL_t}{\partial \hat{y}_t} = 2\hat{y}_t \quad \text{and} \quad MSL_{\pi,t} \equiv \frac{\partial SL_t}{\partial \hat{\pi}_t} = 2\kappa\hat{\pi}_t \tag{3}$$

We note that the marginal loss from a higher gap is negative if the gap is already negative, so if, for instance, inflation is too low compared to the target level, then a higher inflation implies a smaller loss etc. We also note that the marginal losses are strictly increasing in the gaps themselves, so the further away from target a gap is, the worse it is if the gap moves even further away from target.

Indifference curves for the single-period social loss function are combinations of

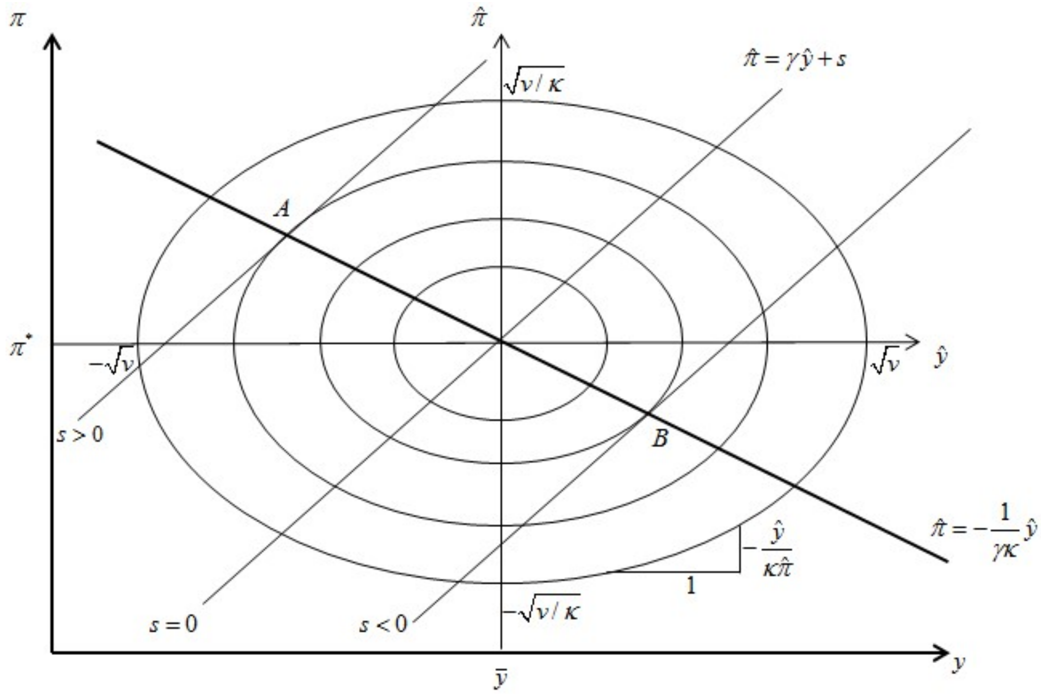
[2] Since the economy is exposed to shocks in future periods, which are not known in period *t*, the future output and inflation gaps will be stochastic as seen from period *t*, taking different values with different probabilities. A calculation of the total social loss over the current as well as the future periods must then be based on the expected values (each possible value times the probability of this value summed up over all possible values) of the future squared output and inflation gaps, given the knowledge about their distributions available at time *t*.

$\hat{y}_t$ and $\hat{\pi}_t$ for alternative values of the social loss $v \geq 0$, such that:

$$\hat{y}_t^2 + \kappa \hat{\pi}_t^2 = \left(y_t - \overline{y}\right)^2 + \kappa\left(\pi_t - \pi^*\right)^2 = v. \tag{4}$$

In a diagram with $\hat{y}_t$ along the horizontal axis and $\hat{\pi}_t$ along the vertical axis, the indifference curves are ellipses around the 'centre', $(0,0)$. In a diagram with y_t and π_t along the axes, they are ellipses around $(\overline{y}, \pi^*)$, as illustrated in Figure 20.1. It is easy to see that the ellipses intersect the $\hat{y}_t$- axis at $\hat{y}_t = \pm\sqrt{v}$, and the $\hat{\pi}_t$- axis at $\hat{\pi}_t = \pm\sqrt{v/\kappa}$, as shown in Figure 20.1. Hence, the closer to the centre the indifference curve is, the smaller is the associated social loss, that is, the better is the situation for 'society' or the stabilization authority. In any point $(\hat{y}_t, \hat{\pi}_t)$, the slope of the indifference curve is $-MSL_{y,t} / MSL_{\pi,t} = -\hat{y}_t/(\kappa\hat{\pi}_t)$, which is also illustrated in Figure 20.1.

Figure 20.1 The trade-off for the stabilization authority



If the authority responsible for macroeconomic stabilization can freely choose a point in the $\hat{y}_t, \hat{\pi}_t$- plane it will, of course, choose $(\hat{y}_t, \hat{\pi}_t) = (0,0)$. However, it may be that the economic structure imposes some restrictions on what can be obtained. Just for illustration, assume that the economic structure implies that only $\hat{y}_t, \hat{\pi}_t$- combinations along positively sloped straight 'restriction lines' of the form:

$$\hat{\pi}_t = s + \gamma\hat{y}_t, \quad \gamma > 0, \quad s \text{ given,} \tag{5}$$

can be realized and that the authority has access to instruments that can move the $\hat{y}_t, \hat{\pi}_t$- combination (in the directions north-east and south-west) along such lines. You may think that Equation (5) looks pretty much like an AS-curve (though not completely so) and that the policy instruments we consider (that can increase both output and inflation, or decrease both, but not increase one and decrease the other) resembles monetary and fiscal policy. This is by no means accidental, but for the moment we will consider the limitations described in (5) 'in the abstract'.

Now, if $s = 0$, the authority can move the economy along the line labelled $'s = 0'$ in the figure and will, of course, choose the point $(\hat{y}_t, \hat{\pi}_t) = (0,0)$. But if $s \neq 0$, as illustrated by the lines labelled $'s > 0'$ and $'s < 0'$ in the figure, the authority cannot just obtain the best of all possible situations and is in a true dilemma. It will have to balance its goals against each other. The best possible point on the line is where it just hits the indifference curve closest to the centre, as indicated by the tangent points *A* and *B* in the figure. At these points the slope of the relevant restriction line, γ, equals the slope of the indifference curve, $-\hat{y}_t/(\kappa\hat{\pi}_t)$, so $\gamma = -\hat{y}_t/(\kappa\hat{\pi}_t)$, or:

$$\hat{y}_t = -\gamma\kappa\hat{\pi}_t \quad \text{or} \quad \hat{\pi}_t = -\frac{1}{\gamma\kappa}\hat{y}_t \tag{6}$$

Hence, for all possible values of *s*, the authority will choose points along the negatively sloped 'policy line' given by (6) and indicated in Figure 20.1 as the line through the points *A* and *B*. This line is flatter the larger *κ* and *γ* are. As a useful exercise, try to provide the intuition for why this must be the case, given the ways these two parameters enter into (1) and (5). You may think that the policy line looks much like an AD-curve and again this is by no means accidental and it will turn up as a theme in this chapter. For now, we just want you to pick up the following:

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If the stabilization policy authority in the different periods is limited by 'restriction lines' of form (5), but can (by appropriate stabilization policy) freely choose a point on the relevant restriction line, and if the authority seeks to minimize the value of a social loss function as (1) in each period, it will choose combinations of the output gap $\hat{y}_t$ and the inflation gap $\hat{\pi}_t$ situated on a negatively sloped 'policy line' as given by (6). This policy line will be flatter in the $\hat{y}_t, \hat{\pi}_t$- diagram the larger the values of *κ* and *γ*, and vice versa.

Looking forward: the perspective of this chapter

The main question in this and the coming chapter will be: what is the best possible stabilization policy that the central bank or the government can pursue to minimize the total losses from business cycles stated in (2), given that they must respect the limitations implied by the structure of the economy? The 'limitations' on stabilization policy consist of the following three elements studied in earlier chapters:

1) An IS-curve stating how the 'demand output gap' in a period depends on the deviations in government demand, the net tax rate and the real interest rate from their structural levels and on a confidence shock. In the notation known from previous chapters, the IS curve is:

$$y_t - \bar{y} = \alpha_1(g_t - \bar{g}) - \alpha_2(r_t - \bar{r}) - \alpha_3(\tau_t - \bar{\tau}) + v_t \tag{7}$$

2) An AS-curve stating the 'supply output gap' determined by the difference between inflation and expected inflation and a productivity/mark-up supply shock. The AS curve relates inflation to expected inflation, the output gap and the supply shock as follows:

$$\pi_t = \pi_t^e + \gamma(y_t - \overline{y}) + s_t \tag{8}$$

3) An expectations hypothesis specifying how the expected rate of inflation for the current period (the π_t^e above) is formed. In this chapter expectations will (mostly) be 'backward looking' and take the simplest possible form of 'static expectations'. Thus the expected rate of inflation will simply equal the latest observed actual rate of inflation (so $\pi_t^e = \pi_{t-1}$ and $\pi_{t+1}^e = \pi_t$ etc.), but in the next two chapters we will consider 'forward looking' expectations.

Assuming that the stabilization authority *cannot* influence the elements 1) – 3), the question will be: how should the monetary or fiscal authority determine the instruments that it *can* decide in order to minimize the social loss (2), given the knowledge it has about the economic structure and given its information about economic variables? For a central bank, the policy instruments will typically include a short-term nominal policy interest rate and possibly less conventional instruments as discussed in Chapter 17. For a government they will typically be government demand for goods and services and the net tax rate.

Particularly, if the authority in question is a central bank we will *not* a priori assume that it follows a Taylor rule, but investigate *how* it should try to affect real interest rates (indirectly through its instruments) in order to minimize the social costs of business cycles.[3] Afterwards we can relate the optimal policy rule for the interest rate so derived to the Taylor rule to see if this is an optimal way of conducting monetary stabilization policy.

We are thus looking for an endogenous optimal policy *rule*, that is, the best possible systematic way for the stabilization authority to relate the instruments it controls to the variables it observes. This investigation presupposes that the authority wants to follow a rule at all. Is this a good idea? Should stabilization policy be guided by somewhat strict and perhaps publicly announced rules that give a predictable pattern in the policy makers' reactions to economic events, or should policy makers rather make up their minds as economic events unfold?

[3] On the way, however, we will first, in Section 20.3, take the Taylor rule as given and investigate what it implies for stabilization.

20.2 Rules versus discretion

It is indeed a basic issue in the debate on macroeconomic stabilization policy whether policy makers should follow a fixed *policy rule*, or be left with *discretion* in their policy choices.

Under the rules approach, stabilization policy is essentially automatic, since the policy rule prescribes how the policy instruments should be set in any given situation. The Taylor rule considered in Chapter 17 is an example of a fixed monetary policy rule specifying how the central bank interest rate should be set, given the observed state of the economy. The Friedman rule prescribing a constant growth rate of the nominal money supply is another example of a fixed monetary policy rule. In an open economy, a fixed exchange rate regime can also be seen as a monetary policy rule, which requires the central bank to adjust its interest rate and possibly other instruments in reaction to any departure of the foreign exchange rate from its target (parity) value or interval.

By contrast, under discretion, policy makers are free to conduct monetary and fiscal policy in any way that they believe will help advance the objectives of stabilization policy in any particular situation. The idea is that policy makers should use all the available information and take account of any special circumstances without necessarily following the same pattern of reaction from one situation to the next.

It might seem that discretion should be preferred to rules, since reliance on a simple fixed policy rule reduces the ability of policy makers to react to all relevant information. A policy maker can always by discretion make exactly the choice that some policy rule would prescribe, but could also deviate from the rule if circumstances make it desirable. So, conducting policy by discretion simply allows more choice flexibility. Doesn't this mean that conducting policy by discretion always dominates conducting it by rules?

The flexibility argument certainly has some appeal, but many economists nevertheless believe that there are advantages to a stabilization policy based on simple rules. In Chapter 22 we discuss the issue of rules vs. discretion in detail, but for now the argument in favour of rules is essentially that they can help policy makers to establish *credibility* which will help them realize the goals of stabilization policy. If a specific rule is followed consistently, it can build up a general trust that this is the way policy will also be conducted in the future, and this can affect people's expectations in a way that helps the authority to achieve its goals.

For example, consistent inflation targeting in monetary policy can influence expectations such that if, say, inflation rises above its target level due to a boom in economic activity, a tightening of monetary policy with a negative impact on activity will be anticipated. This may help to prevent a rise in the expected rate of inflation, and it will probably make households and firms more cautious in their spending decisions than if no such dampening of economic activity were expected. The resulting negative impact on demand and on expected inflation will in itself dampen activity and inflation and thus help the policy makers to achieve their objectives with less actual intervention. Thus, a consistent inflation targeting policy rule may create an anchor for inflation expectations that helps realizing the inflation target. Similarly, if policy makers announce a rule which implies that policy is automatically tightened when output rises above its structural level and is automatically eased if it falls below, this may help to stabilize the growth expectations of the private sector, which in turn can help to stabilize aggregate demand.[4]

Of course, policy makers can only 'buy' credibility by announcing a fixed policy rule if they can convince the public that they are really bound by the rule. This will be easier if the rule is written into the law or into the mandate or statutes of the central bank, and if policy makers face some kind of sanction if they break the rule. For example, the law regulating the central bank could specify that the central bank governor will be fired if he or she consistently misses the bank's inflation target. For a rule to be credible, it must also be reasonably simple so the public can easily understand the rule and check that policy makers actually stick to it.

The advocates of discretion argue that fixed policy rules can only establish credibility if they are overly simple and rigidly adhered to. Hence, *credibility* can only be bought at the price of lost *flexibility*: by sticking to a simple policy rule no matter what the situation is, policy makers lose the ability to account for whatever special circumstances might prevail. For instance, suppose that because of some unexpected event the stock market suddenly takes an exceptional plunge which provides good reason to believe that the economy is headed for a deep recession, but that inflation has not yet reacted. In that situation, should the central bank really keep the interest rate unchanged just because only the rate of inflation, but not stock prices enter into the policy reaction function it has announced?

Most adherents of rules acknowledge that complete loss of flexibility would be a problem. For example, John Taylor has not argued that central banks should slavishly follow his rule. Rather, they should use it as a *guideline* to be followed under normal circumstances, but deviations from the rule would be acceptable when exceptional circumstances prevail.

In practice, central banks do not announce a fixed quantified interest rate reaction function, presumably because they wish to preserve some amount of policy flexibility. Nevertheless, as we saw in Chapter 17, the interest rate policies of the most important central banks seem to be fairly well described by the Taylor rule. In the following, we will therefore start by assuming that monetary policy follows a Taylor rule and study what this rule does for stabilization under different assumptions on the basic parameters of the rule. We will then analyse how an *optimal* policy rule should be designed, starting with monetary policy and moving on to fiscal policy.

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[4] These arguments presuppose that expectations are not just backward looking, but possibly affected by knowledge of economic policy reactions and of the responses of the economy to policy interventions, in contrast to how expectations are otherwise described in this chapter. The more forward looking nature of expectations implicit in the arguments anticipates the concept of 'rational expectations' to be studied in subsequent chapters. It should cause no confusion, we hope, to appeal already in the present discussion of rules vs. discretion to the possibility of expectations being more sophisticated than simply backward looking.

20.3 Monetary stabilization policy by the Taylor rule

We focus (first) on just one period *t,* taking the expected inflation rate from the previous to that period, π_t^e, as given. We will analyse what monetary policy conducted by a Taylor rule implies for the economy's reactions to demand and supply shocks.

The economic model considered

Repeating from earlier chapters, the central bank is assumed to set its nominal policy interest rate i_t^p according to the Taylor rule: $i_t^p = \bar{r} + \pi_{t+1}^e + h(\pi_t - \pi^*) + b(y_t - \bar{y})$, where the notation is well-known and we remember that $\bar{r}$ is the structural or natural real interest rate. Recall that we assume i_t^p to include any systematic risk premium in the relevant longer-term nominal interest over the nominal policy interest rate. We also assume that the central bank knows the expected inflation rate of the private agents and that it can react to current inflation and output when setting the current nominal policy interest rate. Furthermore, the link between the relevant longer-term nominal interest rate i_t and the nominal policy rate (achieved through all the central bank's instruments) is assumed to be $i_t = i_t^p + \hat{\rho}_t$, where $\hat{\rho}_t$ is an unsystematic and unknown shock to the risk premium fluctuating around zero (or a deliberate deviation from the Tylor rule). Combining the two above equations gives a Taylor rule directly for the relevant longer-term nominal interest rate:

$$i_t = \bar{r} + \pi_{t+1}^e + h(\pi_t - \pi^*) + b(y_t - \bar{y}) + \hat{\rho}_t, \quad h > 0,\ b > 0. \tag{9}$$

Since the real interest rate is $r_t = i_t - \pi_{t+1}^e$, we get the implied Taylor rule for the *real* interest rate (as we have also seen before):

$$r_t = \bar{r} + h(\pi_t - \pi^*) + b(y_t - \bar{y}) + \hat{\rho}_t. \tag{10}$$

In a situation where inflation is on target, $\pi_t = \pi^*$, output is at the structural level, $y_t = \bar{y}$, and there is no imprecision in the central bank's influence on the long-term market interest rate, $\hat{\rho}_t = 0$, the central bank will want the real interest rate to equal the natural interest rate, $\bar{r}$, which is exactly what (10) dictates.

The remaining parts of our model are the IS-curve (7) and the AS-curve (8) above. Since our focus is now on *monetary* stabilization policy, we will disregard fiscal stabilization policy and set $g_t = \bar{g}$ and $\tau_t = \bar{\tau}$ in the IS-curveor alternatively we let the fiscal impulses $\alpha_1(g_t - \bar{g}) - \alpha_3(\tau - \bar{\tau})$ be defined into the demand shock v_t. Our model thus consists of an IS-curve, a monetary policy rule MP, and an AS-curve:

$$y_t - \bar{y} = v_t - \alpha_2(r_t - \bar{r}) \tag{IS}$$

$$r_t = \bar{r} + h(\pi_t - \pi^*) + b(y_t - \bar{y}) + \hat{\rho}_t \tag{MP}$$

$$\pi_t = \pi_t^e + \gamma(y_t - \bar{y}) + s_t \tag{AS}$$

Until further notice the expected rate of inflation, π_t^e, is considered an exogenous variable (which could be $\pi_t^e = \pi_{t-1}$). The endogenous variables are y_t, r_t and π_t.

As we have also seen earlier, the IS- and the MP-relations above can be combined into the AD-curve:

$$y_t - \bar{y} = z_t - a(\pi_t - \pi^*) \tag{AD}$$

where:

$$a \equiv \frac{\alpha_2 h}{1+\alpha_2 b}, \quad z_t \equiv \frac{v_t - \alpha_2 \hat{\rho}_t}{1+\alpha_2 b} \tag{11}$$

The IS-MP-AS model can thus be reduced to the AS-AD model consisting of the equations (AS) and (AD) with endogenous variables y_t and π_t.

From (11) we make the following important observations on the AD-curve: First, the parameter *b* of the Taylor rule directly works to dampen the effect of the demand shock v_t and the monetary policy shock $\hat{\rho}_t$ in the sense that for $b > 0$ one has $|z_t| < |v_t - \alpha_2 \hat{\rho}_t|$ whenever $v_t - \alpha_2 \hat{\rho}_t \neq 0$. The parameter *h* does not have such an effect. The reason is that for $b > 0$, the central bank reacts to the direct effect of the demand shock on output by changing the interest rate, and *b* measures the strength of this reaction. Second, the slope a of the AD-curve depends on both *h* and *b*, and a larger *h* and a smaller *b* tend to give a larger value of a, that is, a flatter AD-curve in a diagram with y_t along the horizontal axis and π_t along the vertical axis. If inflation falls, the central bank will react by lowering the interest rate, which works to raise output. The increase in output itself, however, will *ceteris paribus* make the central bank raise the interest rate, which dampens the increase in output. The larger *h* is, the larger the first effect will be, and the smaller *b* is, the smaller is the second, counteracting effect. Third, irrespective of the value of *b*, any positive value of the slope variable a of the AD-curve can be obtained by appropriate choice of *h*, whereas the opposite is not true. In particular, for $h = 0$, one will have $a = 0$, irrespective of *b*.

In the following we will study the first-period effects of demand and supply shocks, starting out from a long-run equilibrium where $v_t = \hat{\rho}_t = z_t = s_t = 0,$ and $y_t = \bar{y},$ and $\pi_t = \pi^* = \pi_t^e$. Our aim is to investigate the implications for stabilization of different values of the parameters *h* and *b* of the Taylor rule.

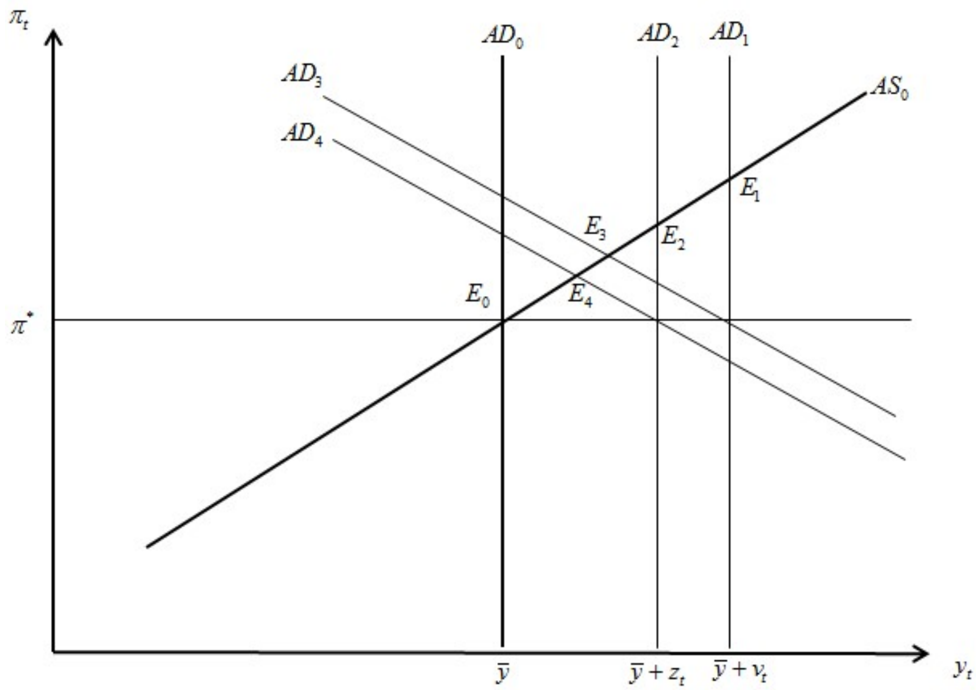
Demand shocks

As a point of comparison we will first consider the case where the central bank is 'passive' and does not react to changes in output or inflation by altering the interest rate, that is, we assume $h = b = 0$. This means that the AD-curve becomes vertical, $y_t = \bar{y} + z_t$. Figure 20.2 illustrates the long-run equilibrium for this special case by the intersection between the upward sloping AS-curve labelled AS_0 passing through $(\bar{y}, \pi_t^e) = (\bar{y}, \pi^*)$ and the vertical AD-curve labelled AD_0. The point of intersection is $(\bar{y}, \pi^*)$, which is labelled E_0 in the figure. We will compare this long-run equilibrium to alternative equilibria in the period arising after a demand shock has occurred for alternative constellations of the monetary policy parameters, *h* and *b*.

So, assume that a positive demand shock, $v_t > 0,$ occurs, while $s_t = \hat{\rho}_t = 0,$ and consider first the case where $h = b = 0$. When $b = 0$ we see from (AD) that $z_t = v_t,$ so the demand shock will shift the AD-curve to the right by the full amount of $v_t,$ as illustrated by the vertical AD-curve AD_1 in Figure 20.2. The new equilibrium will be in the point E_1 where output and inflation are higher than their target levels.

Now, consider a case where $b > 0,$ but $h = 0.$ Then the AD-curve is still vertical, but Equation (11) shows that for the given demand shock $v_t > 0,$ the shift to the right of the AD-curve will be smaller than when $b = 0$, that is, $z_t < v_t,$ as indicated by the AD-curve labelled AD_2 in the figure (all the AD-curves in Figure 20.1 pertain to the same period *t*). The equilibrium will then be in the point E_2, where, as compared to $E_1,$ the increases of both output and inflation are smaller than with passive monetary policy. Even though monetary policy only reacts to the increase in output, the result is that both output and inflation are stabilized. It also follows from Equation (AD) above that by choosing *b* appropriately, any value of z_t between zero and v_t can be obtained, so any point between E_0 and E_1 on the AS-curve can result. In principle, the shift in the AD-curve can be made very small and thus an almost complete stabilization can be obtained by the choice of a very large *b*.

Figure 20.2 Stabilization of a demand shock



Next assume that $h > 0,$ but $b = 0.$ The AD-curve will now be downward sloping rather than vertical, $a = \alpha_2 h > 0,$ but the horizontal shift in it will be the same as for AD_1, since now again $z_t = v_t$. The new AD-curve is labelled AD_3 in the figure and the new equilibrium is $E_3,$ in which both output and inflation have (again) been stabilized to some extent. By choosing *h* appropriately any positive value of a can be achieved meaning that any negative slope of the AD-curve can result, so, again, any point between E_0 and E_1 on the AS-curve can be obtained by appropriate choice of *h*. In particular, the AD-curve can in principle be made arbitrarily flat for very large *h*, wherebycomplete stabilization of both output and inflation is obtained.

Finally, assume that both *b* and *h* are strictly positive and, for the sake of illustration, assume further that *b* is of the same size as considered above for the case *h* = 0 and *b* > 0, while *h* is set so that the slope parameter of the AD-curve, $a = \alpha_2 h / (1 + \alpha_2 b),$ becomes the same as in the just considered case where we had *h* > 0 and *b* = 0. The horizontal shift in the AD-curve will then be the same as for $AD_2,$ but the slope will be the same as for AD_3. The relevant new AD-curve is labelled AD_4 in the figure and the new equilibrium will be at E_4, where further stabilization of both output and inflation has been obtained as compared to both E_2 and E_3.

All in all, demand shocks can be stabilized to any degree wanted by appropriate choice of just one of the parameters h or b. In particular, they can be completely stabilized by choosing either h or b or both large enough. Furthermore, in connection with demand shocks there is no conflict between output stabilization and inflation stabilization. This should not surprise you. Looking at the IS-curve (IS), one sees that the demand shock, v_t, and the real interest rate term, $-\alpha_2(r_t - \bar{r})$, affect the demand for output and the output gap in 'the same way', and looking at the monetary policy relation (MP) one sees that monetary policy works through affecting $r_t - \bar{r}$. Hence, it is no wonder that monetary policy can neutralize demand shocks.

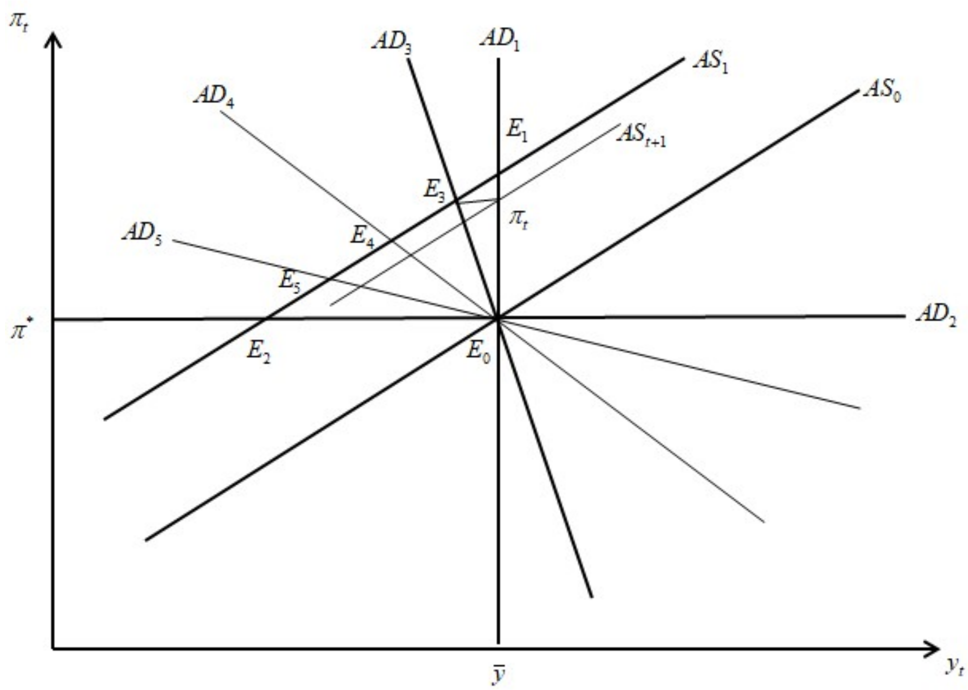
A particular observation above is that any degree of stabilization of demand shocks can be obtained by a monetary rule that only reacts to the output gap ($h = 0$, $b > 0$) *and* by a rule that only reacts to inflation ($h > 0$, $b = 0$). The latter fact is of some importance since data on inflation are more quickly available and are perhaps more reliable than initial data on the overall activity level (GDP). Furthermore, there is no uncertainty about the inflation target, π^*, since this is chosen by the central bank, whereas structural output, $\bar{y}$, is a hypothetical estimated magnitude and therefore a variable of great uncertainty. This could speak for a policy rule of so-called 'pure (or strict) inflation targeting' that only reacts to the inflation gap and not to the output gap ($h > 0$, $b = 0$). As far as stabilization of demand shocks (in isolation) is concerned and strictly on the model's assumptions, nothing is lost by limiting monetary policy to react only to inflation.

Supply shocks

Now, comparing again to a long-run equilibrium like E_0 in Figure 20.3, consider a so-called 'negative' (unfavourable) supply shock $s_t > 0$, and now assume $v_t = \hat{\rho}_t = 0$. The supply shock will shift the AS-curve upwards by s_t from AS_0 to AS_1, as illustrated. Depending on the slope of the AD-curve, and only on this slope, an equilibrium somewhere on the line between E_1 and E_2 will be obtained. If $h = 0$, then $a = 0$, and the AD-curve will be vertical like AD_1 in the figure, leading to the equilibrium E_1. If the central bank does not react to inflation gaps ($h = 0$), then a supply shock will come through as only a deviation in inflation and no deviation in output.

If, on the other hand, h is very large and b is not, then a will be large (in the limit, $a = \infty$), the AD-curve will be horizontal like AD_2 in the figure and the new equilibrium will be E_2. If the central bank reacts much more strongly to inflation gaps than to output gaps, then supply shocks will only cause a deviation in output and (almost) no deviation in inflation. By appropriate choices of h and b, any slope of the AD-curve can be obtained and hence any point along the line from E_1 to E_2 can be obtained as the new equilibrium after the supply shock, e.g., the equilibria E_3, E_4 and E_5 illustrated.

Figure 20.3 Stabilization of a supply shock



Hence, in connection with supply shocks there *is* a conflict between the goals of output stabilization and inflation stabilization, so monetary policy makers must trade-off these objectives against each other. The result of this trade-off will manifest itself in the relative sizes of the parameters h and b in the Taylor rule or, to be specific, in the value of the 'slope parameter' a implied by the constellation of h and b. Note that any value of $a \geq 0$ can be obtained by appropriate choice of h alone given that $b = 0$, so also in connection with supply shocks (in isolation) any stabilization that can be obtained by a Taylor rule can also be obtained by a pure inflation targeting rule with $b = 0$. *How* the trade-off should be resolved is the subject of the next section, but before turning to this issue we will discus monetary policy by the Taylor rule a bit further.[5]

[5] Exercise 1 at the end of the chapter asks you to establish algebraically (from the two equations (AS) and (AD) etc.) all the results about how the effects of demand and supply shocks depend on the policy parameters h and b that were derived graphically above.

Discussing monetary stabilization by the Taylor rule

You may have got the impression from the analysis above that a central bank may as well rely on pure inflation targeting since all that can be obtained by a Taylor rule with general parameters h and b can also be obtained by a rule with $b = 0$. Although this is true for both demand shocks in isolation and supply shocks in isolation, it is not true when both types of shocks can occur. If one fixes $b = 0$, obtaining a desired *degree* of stabilization in connection with demand shocks will dictate one value of h, while obtaining a desired *mix* of output and inflation stabilization in connection with supply shocks will dictate another value of h, and these two values would only coincide by pure accident. Hence, probably an appropriate Taylor rule will have both $h > 0$ and $b > 0$.

On the other hand, by allowing both h and b to be strictly positive, we have seen that one can balance these two parameters relatively to obtain any mix of output and inflation stabilization desired in connection with supply shocks, and one can then scale h and b up to very large levels and thus obtain full stabilization of demand shocks. This suggests that an appropriate Taylor rule would always involve 'large' values of the parameters h and b.

This is true strictly on the model's premises that the central bank knows and can react instantaneously to the true values of the current inflation level and the output gap. However, in the real world there is a lot of uncertainty involved in evaluating the stance of the business cycle. Data on both inflation and output come with a delay and when they appear, they may suffer from measurement problems. Furthermore, there is particular uncertainty regarding the structural level of output. Therefore, the central bank will always be in some doubt about the true values of the current inflation and output gaps. Furthermore, the central bank does not have complete knowledge about how the economy will react to its interest rate interventions, another real world fact from which we have abstracted. All this uncertainty speaks for cautiousness, meaning that the central bank should not choose very high values of h and b.

Our analysis nevertheless suggests that an appropriate Taylor rule should include values of h and b that are both strictly positive and balanced to obtain the desired mix of output and inflation stabilization in the presence of supply shocks.

Let us sum up:

MONETARY STABILIZATION POLICY BY THE TAYLOR RULE

When monetary stabilization policy follows a Taylor rule, it would seem desirable to choose a strong positive response of the policy interest rate to the inflation gap as well as to the output gap to neutralize demand shocks and then balance these policy parameters (*h* and *b*) to obtain the preferred mix of inflation and output when the economy is hit by supply shocks. However, because of the uncertainties relating to the data and the economic mechanisms involved, a central bank should be reluctant to react very strongly to changes in its estimates of the inflation and output gaps. Still, for appropriate stabilization both of the policy parameters *h* and *b* should probably be strictly positive.

By focusing on only one period, we have in all of the above neglected the dynamics of expectations formation and thereby the influence that stabilization in one period can have on stabilization possibilities in the future. This is, however, a highly relevant issue. Consider for instance a case where the central bank has chosen to put relatively much emphasis on output stabilization, so it has chosen a relatively large value of *b* compared to *h* and thus has obtained a rather steep AD-curve such as AD_3 in Figure 20.3. After a supply shock $s_t > 0,$ output will only fall a little short of the structural level, but inflation will be considerably above target as illustrated by E_3 in the figure. If inflation expectations are backward looking, e.g. static, $\pi_{t+1}^e = \pi_t,$ then the relatively high level of inflation in period *t* will imply a relatively high level of expected inflation in period *t*+1. Since the AS-curve in any period passes through the point $(\bar{y}, \pi_t^e)$ for $s_t = 0,$ this means that even if there is no supply shock in period *t*+1, that is, $s_{t+1} = 0$, the AS-curve in period *t*+1 will be located at a relatively high position as indicated by the AS-curve AS_{t+1} in Figure 20.3. This is an undesirable situation: it is as if a relatively large, unfavourable supply shock (of size $\pi_t - \pi^*$) had occurred in period *t*+1. If the central bank had put relatively less emphasis on stabilizing output and more on stabilizing inflation, thus creating a flatter AD-curve like AD_5 in Figure 20.3, then the inflation rate in period *t* would have been smaller as illustrated by $E_5,$ and therefore the upward shift $\pi_t - \pi^*$ in the AS-curve from period *t* to *t*+1 would have been smaller. In that case stabilization would be easier to handle in period *t*+1 and onwards (but at the cost of a relatively large negative output gap in period *t*). Incorporating these dynamic links will be a theme in the investigation of optimal monetary stabilization that we now turn to.

20.4 Optimal monetary stabilization

The economic model considered

We now assume that the central bank does not necessarily follow a Taylor rule. Therefore we eliminate the relation (MP) from our model structure, so that only (IS) and (AS) remain. We still assume that the central bank adjusts its short-term nominal policy interest rate, i_t^p, in each period and that the link between this and the relevant longer-term nominal interest is $i_t = i_t^p + \hat{\rho}_t$ in the usual notation. The real interest rate is then $r_t = i_t - \pi_{t+1}^e = i_t^p - \pi_{t+1}^e + \hat{\rho}_t$. Since we also assume that the central bank knows the expected inflation rate, π_{t+1}^e, it can control $r_t^p \equiv i_t^p - \pi_{t+1}^e$ which we will refer to as the 'policy real interest rate'. Hence, the bank's imprecise control over the real interest rate implies $r_t = r_t^p + \hat{\rho}_t$, where the bank chooses r_t^p.

We will assume static expectations, so the expected rate of inflation in the AS-curve will be $\pi_t^e = \pi_{t-1}$. Our model structure is thus:

$$y_t - \bar{y} = v_t - \alpha_2 (r_t - \bar{r}) \tag{IS}$$

$$r_t = r_t^p + \hat{\rho}_t \tag{RP}$$

$$\pi_t = \pi_{t-1} + \gamma (y_t - \bar{y}) + s_t \tag{AS}$$

The economy is considered from some period *t* and onwards where the rate of inflation in period *t*-1 is a given constant.

For simplicity, the shocks v_t, s_t and $\hat{\rho}_t$ are assumed to be 'white noise' processes. This means that each of them in each period has a mean of zero and some variance and each is uncorrelated with the other shocks and with itself across time, so the realization of the shock in one period has no implication for what its realizations should be expected to be in future periods. This means that as viewed from period *t*, there will be specific shock realizations v_t and s_t and $\hat{\rho}_t$ in the period, which are probably different from zero, but independently of these realizations the best expectations for future periods $t+\tau$, $\tau \geq 1$, are: $E_t(v_{t+\tau}) = E_t(s_{t+\tau}) = E_t(\hat{\rho}_{t+\tau}) = 0$.

The model has four endogenous variables, y_t, r_t, r_r^p and π_t. In order to close it, we add a description of the objective of the central bank. In line with our discussion in Section 20.1, and as expressed by Equation (2), we assume its aim is to minimize the present value of the current and the expected future social loss from instability of output and inflation:

$$\overline{L}_t \equiv \sum_{\tau=0}^{\infty} \delta^{\tau} E_t(SL_{t+\tau}) = \sum_{\tau=0}^{\infty} \delta^{\tau} \left[E_t\left(\hat{y}_{t+\tau}^2\right) + \kappa E_t\left(\hat{\pi}_{t+\tau}^2\right) \right] \tag{SL}$$

where it is remembered that δ is a discount factor, $0<\delta<1$, and $\hat{y}_t \equiv y_t - \overline{y}$ and $\hat{\pi}_t \equiv \pi_t - \pi^*$, are the output gap and the inflation gap, respectively. Hence, the central bank will in each period *t* choose its policy real interest rate r_t^p (indirectly through i_t^p etc.) so that, given the way that r_t^p affects the evolution of the economic system in period *t* and over all future periods according to the equations (IS), (RP) and (AS), the loss function $\overline{L}_t$ is minimized. Thus the central bank takes into account how its stabilization effort in one period will affect its stabilization possibilities in future periods.

In deriving the central bank's optimal monetary policy, it is important to specify the information available to it.. We assume that the bank cannot directly observe the shock realizations in the current period *t*, but we maintain the assumption also built into the Taylor rule that the bank can observe and react to the current levels of output and inflation within the period. Furthermore, we assume that its choice of interest rate will have an impact on output and inflation within the period.[6]

Figure 20.4 illustrates the constraints and possibilities of the central bank in period *t*. In the figure it is assumed that $\pi_{t-1} > \pi^*$, or $\hat{\pi}_{t-1} \equiv \pi_{t-1} - \pi^* > 0$, and furthermore that an unfavourable supply shock $s_t > 0$ has occurred in period *t*. The AS-curve of period *t*, labelled AS_t in the figure, passes through the point $(\overline{y}, \pi_{t-1} + s_t) = (\overline{y}, \pi^* + \hat{\pi}_{t-1} + s_t)$, which is situated strictly above $(\overline{y}, \pi^*)$. As the equations (IS) and (RP) show, the bank can by its choice of r_t^p and the way r_t^p affects demand establish any value of y_t that it wishes irrespective of the shock realizations v_t and $\hat{\rho}_t$. Here our assumption that the bank can observe y_t and react to it without any delay is crucial. Irrespective of the shocks, the bank can observe the resulting y_t and, if not satisfied with it, change r_t^p to establish the y_t it wants. In the figure this means that the bank can pick any point on the AS-curve it wants for period *t*, but it has to respect the AS-curve itself. Of course, instead of saying that the bank picks a y_t and lets π_t be determined by the relevant AS-curve, we may as well say that the bank picks π_t and lets the AS-curve determine y_t.

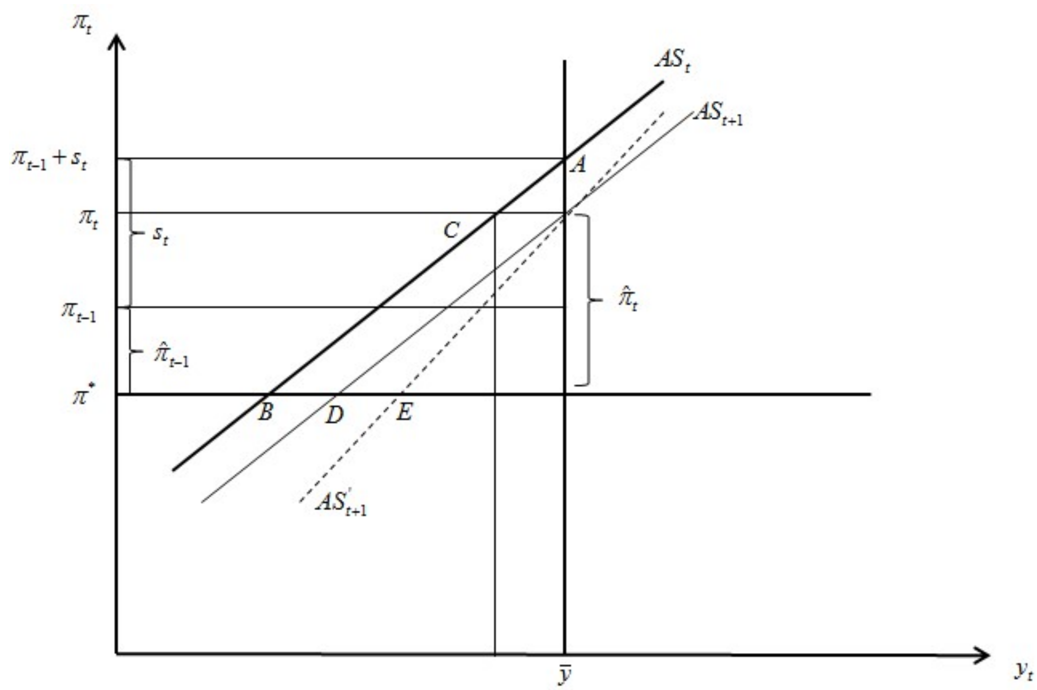
Only the points on the segment $[A, B]$ of the AS-curve in Figure 20.4 are relevant for the bank. From any point to the north-east of *A*, e.g., where both $\hat{y}_t > 0$ and $\hat{\pi}_t > 0$, the bank could unambiguously improve the situation in period *t* by lowering both gaps, which it is capable of. However, along $[A, B]$ there is a true trade-off between establishing a higher output gap, which is desirable because $\hat{y}_t < 0$, and establishing a

[6] Later in this section, we will depart from these ideal circumstances and consider a case where there is a time lag between when interest rates are set and they take effect on demand.

lower inflation gap, which is desirable because $\hat{\pi}_t > 0$. The bank must therefore choose according to it preferences for output and inflation stability as these are expressed by (SL).

Say that this leads the bank to choose the point C in the figure, which involves the indicated rate of inflation π_t and hence the inflation gap $\hat{\pi}_t = \pi_t - \pi^*$ also indicated. This means that in period t+1, the expected rate of inflation will be π_t and the AS-curve for period t+1 will pass through $(\bar{y}, \pi_t) = (\bar{y}, \pi^* + \hat{\pi}_t)$ for $s_{t+1} = 0$. So, even if $s_{t+1} = 0$, as the bank indeed expects, it will be as if a supply shock of size $\hat{\pi}_t$ hits the economy in period t+1, and the closer to zero this 'artificial supply shock' $\hat{\pi}_t$ is, the more favourable the stabilization situation in period t+1 will be. The central bank should, of course, take this 'intertemporal spill-over effect' of its action in period t to its stabilization possibilities in period t+1 into account when deciding which point on $[A, B]$ to choose in period t.

Figure 20.4 The possibilities of the central bank



If we insert the expression for r_t of (RP) into (IS) and use the definition of the output gap, (IS) becomes:

$$\hat{y}_t = v_t - \alpha_2(r_t^p - \bar{r} + \hat{\rho}_t) \tag{12}$$

If we subtract π^* on both sides of (AS) and use the definitions of the inflation gap and the output gap we get:

$$\hat{\pi}_t = \hat{\pi}_{t-1} + \gamma\hat{y}_t + s_t \tag{13}$$

Here we can insert the expression in (12) for the output gap and obtain:

$$\hat{\pi}_t = \hat{\pi}_{t-1} + \gamma\left[v_t - \alpha_2(r_t^p - \bar{r} + \hat{\rho}_t)\right] + s_t \tag{13'}$$

Given $\hat{\pi}_{t-1}$ the two equations (12) and (13) (or (13')) imply a specific evolution of the inflation and output gaps in period *t* and all future periods as this evolution depends on the central bank's choice of the real policy interest rate r_t^p in each period and the realizations of the shocks v_t, s_t and $\hat{\rho}_t$ in each period.

The central bank is free to choose its real policy interest rate in each period independently and can re-optimize over the new remaining future every time a new period is reached. We can therefore focus on how the bank will determine its real policy interest rate in the specific period *t*, since this could be any period. In its optimization, the bank should take into account how its choice of r_t^p affects the expected future values of the (squared) output and inflation gaps. However, we will first consider a simple case where the central bank does not take into account the implications of current stabilization for future stabilization, which will provide a good point of comparison for the more general case.

The optimal policy rule when the central bank disregards the future

In this case the objective of the central bank is to minimize the single period social loss in period *t*, $SL_t = \hat{y}_t^2 + \kappa\hat{\pi}_t^2$, which appears as the limit of $\bar{L}_t$ defined in (SL) for $\delta \to 0$. Note here that in period *t*, the shocks v_t, s_t and $\hat{\rho}_t$ have been realized and are no longer stochastic, but we have assumed that the central bank does not observe the shock realizations directly.

Using (12) and (13') we can write the single period social loss in period *t* as:

$$SL_t = [v_t - \alpha_2(r_t^p - \bar{r} + \hat{\rho}_t)]^2 + \kappa[\hat{\pi}_{t-1} + \gamma\left[v_t - \alpha_2(r_t^p - \bar{r} + \hat{\rho}_t)\right] + s_t]^2 \tag{14}$$

Note that the central bank does not know the specific shock realizations, but irrespective of what they are, it will want to choose r_t^p to minimize the SL_t of (14). For any given constellation of v_t, s_t and $\hat{\rho}_t$, the first order condition for this minimization is:

$$\frac{\partial SL_t}{\partial r_t^p} = -2\alpha_2[v_t - \alpha_2(r_t^p - \bar{r} + \hat{\rho}_t)] - 2\alpha_2\gamma\kappa[\hat{\pi}_{t-1} + \gamma\left[v_t - \alpha_2(r_t^p - \bar{r} + \hat{\rho}_t)\right] + s_t] = 0$$

(where you should verify that the second order derivative is strictly positive such that the second order condition for a minimum is fulfilled).

Reinserting the output gap and the inflation gap (writing $\hat{y}_t$ for $[v_t - \alpha_2(r_t^p - \bar{r} + \hat{\rho}_t)]$ and $\hat{\pi}_t$ for $[\hat{\pi}_{t-1} + \gamma\left[v_t - \alpha_2(r_t^p - \bar{r} + \hat{\rho}_t)\right] + s_t]$ etc.) this becomes:

$$\hat{y}_t = -\gamma\kappa\hat{\pi}_t \quad \text{or} \quad \hat{\pi}_t = -\frac{1}{\gamma\kappa}\hat{y}_t \tag{15}$$

Hence, in the conduct of its optimal interest rate policy the central bank should, if possible, seek to establish a tight relationship between the output and inflation gaps, such that irrespective of the shocks in period *t*, the combination of the output gap and the inflation gap is situated on the straight 'policy line' given by (15). Note that you have seen this policy line before: it is identical to Equation (6) of Section 20.1. This should be no surprise and it gives the appropriate interpretation of (15) in terms of marginal social losses (the same interpretation as for (6)). It is equivalent to the condition $MSL_{y,t} + \gamma MSL_{\pi,t} = 0$ saying that in an optimum it must not be possible to exchange one gap for the other in a way that is possible according to the AS-curve (e.g., $\Delta\hat{y}$ higher output gap and $\gamma\Delta\hat{y}$ higher inflation gap) and gives a smaller social loss.

The next question is whether and how the central bank can implement the policy line (15) through an appropriate monetary policy. We already more or less know the answer from Section 20.3. If the central bank follows the Taylor rule (9), thereby setting the real interest rate in accordance with (10), this will result in the AD-curve of Equation (AD) which we restate here using the definitions of the output and the inflation gap:

$$\hat{y}_t = z_t - a\hat{\pi}_t, \quad a \equiv \frac{\alpha_2 h}{1+\alpha_2 b}, \quad z_t \equiv \frac{v_t - \alpha_2\hat{\rho}_t}{1+\alpha_2 b} \tag{AD'}$$

By choosing a large value of *b*, the bank can ensure $z_t \approx 0,$ and then by choosing *h* such that

$$a \equiv \frac{\alpha_2 h}{1+\alpha_2 b} = \gamma\kappa, \tag{16}$$

the policy line of (15) will be (approximately) established. Note (again) that our assumptions that the central bank can observe and react to the current output and inflation gaps, and that the policy interest rate affects output and inflation already in the current period are fundamental for this conclusion. Also note that (16) requires $h > 0$.

We see that the larger γ and κ are, the flatter is the policy line (15) in the $\hat{y}, \hat{\pi}$-space, that is, the more the central bank will insist on stabilizing inflation rather than output. In terms of the Taylor rule (16), this calls for a larger responsiveness to inflation (h) relative to the responsiveness to output (b).

The effect of κ on the optimal Taylor rule is quite intuitive since measures the relative cost of inflation instability in the social loss function. To understand the impact of γ on the slope of the policy line (15), suppose the economy has been hit by an unfavourable supply shock that has driven output below the natural level and inflation above the target level. The central bank would then like to boost output, but with a high value of γ a rise in output will create a large further increase in the inflation gap, implying a high inflation cost of moving output in the desired direction. Because a higher value of γ makes it more costly (in terms of inflation instability) to stabilize output, the central bank will put less emphasis on output stability relative to inflation stability, and this is exactly what a flatter policy line means.

Our conclusion so far is that the Taylor rule is not just one reasonable way to conduct monetary policy; in fact it is a way to implement the best possible monetary policy. There are important reservations to mention in relation to this result, but before turning to these we will investigate how the optimal policy is modified if the central bank does not disregard the future.

The optimal policy rule when the central bank takes the future into consideration

As we explained in connection with figure 20.4 above, under our assumptions the central bank is capable of choosing any point on the current AS-curve through its control over the policy real interest rate, r_t^p, so the bank can choose the inflation gap and let the AS-curve decide the output gap. It turns out to be convenient to consider the inflation gap, $\hat{\pi}_t$, directly as the choice variable of the bank (although what we have in mind is that the bank chooses a policy real interest rate that results in this gap).

The central bank's view of the uncertainties involved in its choices is as follows: In period *t* the bank does not know the shock realizations for that period, but it can observe the y_t and π_t that result. Since the shocks are white noise, the central bank's best guess about the future shocks is that they will be zero, so in period *t* the bank's expectations about the shocks in the future period $t+\tau$ are $E_t(v_{t+\tau}) = E_t(s_{t+\tau}) = E_t(\hat{\rho}_{t+\tau}) = 0$. In period *t*, the bank only has to decide about $\hat{\pi}_t$, but since it also cares about the future, it will account for the impact of its choice of the current inflation gap $\hat{\pi}_t$ on the future course of the economy and make a plan for the optimal values of all the future inflation gaps $\hat{\pi}_{t+\tau}$, $\tau \geq 0$, given its expectations about the future involving $E_t(v_{t+\tau}) = E_t(s_{t+\tau}) = E_t(\hat{\rho}_{t+\tau}) = 0$ for $\tau \geq 1$. The bank knows, of course, that as the future unfolds, the shock realizations will generally be different from zero, but in period *t*, where only $\hat{\pi}_t$ has to be chosen, the bank can do no better than planning according to expectations as described. Hence, in its attempt to find the optimal $\hat{\pi}_t$, the bank calculates the planned sequence of the current and all future inflation gaps $\hat{\pi}_{t+\tau}$, $\tau \geq 0$, but it only implements $\hat{\pi}_t$, and so to say throws the rest away and re-optimizes in period *t*+1 when the shock realizations for that period have materialized. In period *t*, the different planned values of $\hat{\pi}_{t+\tau}$ for $\tau \geq 1$ are not stochastic as they are simply deterministic (but non-binding) decisions by the bank. The associated future output gaps will be stochastic, however.

From (13) we have:

$$\hat{y}_t = \frac{\hat{\pi}_t - \hat{\pi}_{t-1}}{\gamma} - \frac{s_t}{\gamma}, \tag{17}$$

expressing the current output gap in terms of the current inflation gap decided by the bank, the predetermined past inflation gap and the realized current supply shock. Since s_t is the realized in period *t*, this current output gap is not stochastic as seen from period *t*. For future periods we can roll (17) forwards, so:

$$\hat{y}_{t+\tau} = \frac{\hat{\pi}_{t+\tau} - \hat{\pi}_{t+\tau-1}}{\gamma} - \frac{s_{t+\tau}}{\gamma} \quad \text{for all } \tau \geq 1, \tag{18}$$

which shows that even when the inflation gaps are chosen deterministically, the future output gaps will be stochastic as they depend on the shocks $s_{t+\tau}$.

Now we write down the terms in $\overline{L}_t$ of (SL) from the beginning for $\tau = 0,1,2,\ldots$, taking into account that the inflation gaps are deterministic decisions while the output gaps for $\tau \geq 1$ are stochastic:

$$\begin{aligned}\overline{L}_t &= \hat{y}_t^2 + \kappa\hat{\pi}_t^2 + \delta E_t(\hat{y}_{t+1}^2) + \delta\kappa\hat{\pi}_{t+1}^2 + \delta^2 E_t(\hat{y}_{t+2}^2) + \delta^2\kappa\hat{\pi}_{t+2}^2 + \cdots \\ &= \left(\frac{\hat{\pi}_t - \hat{\pi}_{t-1}}{\gamma} - \frac{s_t}{\gamma}\right)^2 + \kappa\hat{\pi}_t^2 + \delta E_t\left[\left(\frac{\hat{\pi}_{t+1} - \hat{\pi}_t}{\gamma} - \frac{s_{t+1}}{\gamma}\right)^2\right] + \delta\kappa\hat{\pi}_{t+1}^2 \\ &\quad + \delta^2 E_t\left[\left(\frac{\hat{\pi}_{t+2} - \hat{\pi}_{t+1}}{\gamma} - \frac{s_{t+2}}{\gamma}\right)^2\right] + \delta^2\kappa\hat{\pi}_{t+2}^2 + \cdots\end{aligned} \tag{19}$$

The bank decides on the sequence $\hat{\pi}_{t+\tau}$, $\tau \geq 0$, by minimizing this $\overline{L}_t$ with respect to all the $\hat{\pi}_{t+\tau}$, $\tau \geq 0$. It does not know the current shock realization s_t, but irrespective of this it wants to minimize $\overline{L}_t$. The first order conditions for this maximization problem are

$\partial \overline{L}_t / \partial \hat{\pi}_{t+\tau} = 0$ for $\tau = 0,1,2,...$. We will focus on the first order condition particularly for $\hat{\pi}_t$, the only gap to which the central bank has to commit (i.e., the only irreversible decision it has to make) in period *t*. Note that $\hat{\pi}_t$ only appears in the first three terms in (19). Differentiating the third term involving the expectations operator may be a bit subtle. If we denote the different possible values of the shock s_{t+1} by s_j ,each having the probability p_j, then by definition:

$$E_t\left(\frac{\hat{\pi}_{t+1}-\hat{\pi}_t}{\gamma}-\frac{s_{t+1}}{\gamma}\right)^2 = \sum_j p_j \left(\frac{\hat{\pi}_{t+1}-\hat{\pi}_t}{\gamma}-\frac{s_j}{\gamma}\right)^2$$

Differentiation then gives:

$$\frac{\partial\left[E_t\left(\frac{\hat{\pi}_{t+1}-\hat{\pi}_t}{\gamma}-\frac{s_{t+1}}{\gamma}\right)^2\right]}{\partial\hat{\pi}_t} = -\frac{2}{\gamma}\sum_j p_j\left(\frac{\hat{\pi}_{t+1}-\hat{\pi}_t}{\gamma}-\frac{s_j}{\gamma}\right) = -\frac{2}{\gamma}E_t\left(\frac{\hat{\pi}_{t+1}-\hat{\pi}_t}{\gamma}-\frac{s_j}{\gamma}\right) \quad (20)$$

Now, differentiating the expression for $\overline{L}_t$ in (19) with respect to $\hat{\pi}_t$ and using (20) for the third term gives the first order condition:

$$\frac{\partial\overline{L}_t}{\partial\hat{\pi}_t} = \frac{2}{\gamma}\left(\frac{\hat{\pi}_t-\hat{\pi}_{t-1}}{\gamma}-\frac{s_t}{\gamma}\right)+2\kappa\hat{\pi}_t-\delta\frac{2}{\gamma}E_t\left(\frac{\hat{\pi}_{t+1}-\hat{\pi}_t}{\gamma}-\frac{s_{t+1}}{\gamma}\right)=0 \quad (21)$$

It is easy to see that the second order condition for a minimum, $\partial^2\overline{L}_t/\partial^2\hat{\pi}_t > 0$, is fulfilled.[7] Multiplying both sides of (21) by γ and reinserting the gaps etc., (21) becomes $2\hat{y}_t + 2\gamma\kappa\hat{\pi}_t - 2\delta E_t(\hat{y}_{t+1}) = 0$, or:

$$\hat{y}_t = -\gamma\kappa\hat{\pi}_t + \delta E_t(\hat{y}_{t+1}) \quad \text{or} \quad \hat{\pi}_t = -\frac{1}{\gamma\kappa}\hat{y}_t + \frac{\delta}{\gamma\kappa}E_t(\hat{y}_{t+1}) \quad (22)$$

This has an intuitive interpretation. Using the marginal social losses defined in (3), the expression $2\hat{y}_t + 2\gamma\kappa\hat{\pi}_t - 2\delta E_t(\hat{y}_{t+1}) = 0$ can be rewritten as:

$$MSL_{y,t} + \gamma MSL_{\pi,t} = \delta E_t(MSL_{y,t+1})$$

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[7] Indeed, $\dfrac{\partial\overline{L}_t^2}{\partial\hat{\pi}_t^2} = \dfrac{2}{\gamma^2} + 2\kappa + \delta\dfrac{2}{\gamma^2} > 0$.

Assume the bank reduces the real interest rate so as to increase the output gap by one unit in period *t*. The marginal social loss from this action is $MSL_{y,t} = 2\hat{y}_t$, which is negative if $\hat{y}_t < 0$. According to the AS-curve, the unit increase in the output gap will raise inflation in period *t* by γ points, resulting in a social loss of $\gamma MSL_{\pi,t} = 2\gamma\kappa\hat{\pi}_t$. Hence the left hand side above is the total social loss (if negative a gain) in period *t* from increasing the output gap in that period by one unit. This is not the end of story, however. The γ points higher inflation in period *t* means that expected inflation in period *t*+1 will also be γ points higher. To prevent this increase in expected inflation from driving the inflation gap for period *t*+1 above the optimal level planned by the central bank, it follows from the AS-curve for period *t*+1 that the output gap in that period must fall by one unit, since this will reduce inflation by γ points, ceteris paribus. The social loss from a one unit lower output gap in period *t*+1 is in expected terms $-E_t(MSL_{y,t+1}) = -2E_t(\hat{y}_{t+1})$. Discounted back to period *t* and expressed as a gain this is $\delta E_t(MSL_{y,t+1}) = 2\delta E_t(\hat{y}_{t+1})$. All in all, the optimality condition says that for policy to be optimal, the marginal loss (gain) from changing the output gap in period *t*, including the effect on inflation in period *t*, must equal the marginal gain (loss) from the implied change in next period's output gap, discounted back to period *t*. In other words, it must not be possible to reap a marginal net gain from a shift in the output and inflation gaps in period *t* that the bank could achieve.

Note that if the future does not matter for the central bank, corresponding to $\delta = 0$, then (22) reduces to (15) above, as should be the case.

For the optimality condition (15) we quite easily found a monetary policy rule that would secure its fulfilment. Can we find a policy rule that will satisfy the more complicated optimality condition (22)? Building on our previous analysis, we want to investigate whether fulfilment of (22) can be achieved by forcing each period's combination of output gap and inflation gap to be situated on a negatively sloped 'policy line' that optimizes the output-inflation trade-off,

$$\hat{\pi}_t = -\varphi\hat{y}_t \quad \text{or} \quad \hat{y}_t = -\frac{1}{\varphi}\hat{\pi}_t, \quad \varphi > 0 \tag{23}$$

where φ is to be specified. We know that we can enforce such a policy line by a specific form of the Taylor rule. So, is there a φ such that enforcement of (23) will imply that (22) is fulfilled in any period *t*?

If the bank enforces (23) in all periods, it will know that the output gap in period *t*+1 will be determined as the intersection between the policy line $\hat{\pi}_{t+1} = -\varphi\hat{y}_{t+1}$, and the AS-curve, $\hat{\pi}_{t+1} = \hat{\pi}_t + \gamma\hat{y}_{t+1} + s_{t+1}$. By setting the two right hand sides equal to each other one easily finds that this implies: $\hat{y}_{t+1} = -\hat{\pi}_t/(\gamma+\varphi) - s_{t+1}/(\gamma+\varphi)$, and then:

$$E_t(\hat{y}_{t+1}) = -\frac{\hat{\pi}_t}{\gamma + \varphi}$$

where we used $E_t(s_{t+1}) = 0.$ Inserting this into (22) gives:

$$\hat{\pi}_t = -\frac{1}{\gamma\kappa}\hat{y}_t - \frac{\delta}{\gamma\kappa}\frac{1}{\gamma + \varphi}\hat{\pi}_t$$

in which we can isolate the inflation gap:

$$\hat{\pi}_t = -\frac{1}{\gamma\kappa + \dfrac{\delta}{\gamma + \varphi}}\hat{y}_t \tag{24}$$

Comparing (23) and (24) we can see that enforcing (23) will indeed imply (22) provided that φ is set so that:

$$\varphi = \frac{1}{\gamma\kappa + \dfrac{\delta}{\gamma + \varphi}} \tag{25}$$

We will not try to solve (25) for φ, but simply note that there is a unique $\varphi > 0$ that solves it: the left hand side as a function of φ is the 45°-line, while the right hand side is a strictly positive function of φ starting at a strictly positive number (and intersection with the second axis) $\gamma / (\gamma^2\kappa + \delta)$ for $\varphi = 0$, and for $\delta > 0$ it increases monotonically up towards $1/(\gamma\kappa)$, which ensures a unique intersection with the 45°-line. It follows that at the intersection where $\varphi > 0$, we have $\varphi < 1/(\gamma\kappa)$. For $\delta = 0$, obviously $\varphi = 1/(\gamma\kappa)$ as it should be.

Hence, disregarding the future the central bank would seek to enforce the policy line $\hat{\pi}_{t+1} = -(1/\gamma\kappa)\hat{y}_{t+1}$, but taking the future into account the bank will try to enforce the policy line $\hat{\pi}_{t+1} = -\varphi\hat{y}_{t+1}$, where $0 < \varphi < 1/(\gamma\kappa)$, that is, a flatter policy line in the $\hat{y}, \hat{\pi}$-space. This reflects that when the future counts, it is more costly to create a strictly positive or negative inflation gap in a given period than when the future doesn't count, because the current inflation gap works itself into the AS-curve for the next period, thereby worsening next period's output-inflation trade-off the more the current inflation gap deviates from zero. It follows that the more the future counts (the higher the value of δ) and the greater the social cost of an inflation gap (the higher the value of κ), the lower is the optimal value of φ, as shown by (25).

The dependence of φ on γ is ambiguous. The first term in the denominator of (25) captures the effect we explained in our comments on the policy rule (15) applying when the central bank neglects the future: the larger γ is the greater is the worsening of the inflation gap required *in period t* to improve the output gap by one unit in period *t*. This effect of a higher γ tends to lower the optimal value of φ (flattening the policy line) by making it more costly to stabilize output. However, according to the second term in the denominator of (25) a higher γ calls for a higher φ. The reason is that a larger γ reduces the harmful effects *in period t+1* of an inflation gap occurring in period *t*, so by reducing the future cost of today's inflation gap, a larger γ makes stabilization of the inflation gap less important, inducing the central bank to put more emphasis on stabilization of output. This point is illustrated in Figure 20.4 which shows that for a given inflation gap $\hat{\pi}_t \neq 0$ in period *t*, the AS-curve in period *t*+1 will pass through $(\bar{y}, \pi_t) = (\bar{y}, \pi^* + \hat{\pi}_t)$ for $s_{t+1} = 0,$ causing a stabilization problem in period *t*+1 as if a supply shock of size $\hat{\pi}_t$ had occurred. The figure also shows that this stabilization problem in period *t*+1 will be smaller (for given $\hat{\pi}_t$) the steeper the AS-curve is. For the steeper AS-curve in period *t*+1, the dashed AS'_{t+1}, the worsening of the output gap it takes to improve the inflation gap by one unit is smaller than for the flatter AS-curve AS_{t+1} illustrated by the solid line. In particular, the negative output gap one must live with in order to bring the inflation gap down to zero in period *t*+1 is numerically smaller for the steeper AS-curve in period *t*+1 than for the flatter one, as the points *D* and *E* show. We cannot in general determine which of the two effects from γ is strongest. As (25) shows, if κ is sufficiently small relative to $\delta,$ the second effect will dominate and vice versa.

The central bank can (again) enforce the policy line $\hat{\pi}_t = -\varphi \hat{y}_t$ by an appropriate Taylor rule. The Taylor rule will, as before, create the AD-curve (AD') of the former subsection and then by setting *b* large (ensuring $z_t \approx 0$) and choosing *h* and *b* such that

$$a \equiv \frac{\alpha_2 h}{1 + \alpha_2 b} = \frac{1}{\varphi}, \tag{26}$$

the bank will all in all ensure that the economy is kept close to $\hat{\pi}_t = -\varphi \hat{y}_t$ in all periods.

This implies in particular that $h > 0$ – the famous 'Taylor principle'. From the Taylor rule (9) we see that if $h > 0,$ an increase in inflation π_t of sufficient duration to build itself into the expected inflation rate (so $\pi^e_{t+1} = \pi_t$) will induce a rise in the nominal policy interest rate i^p_t that is larger than the increase in inflation itself. Thus the Taylor principle says that the nominal policy interest rate should respond to longer-lasting changes in inflation in the same direction and *more than one-to-one*.

Optimal monetary stabilization policy can be implemented via a Taylor rule for the central bank's policy interest rate. If the central bank can estimate the output and inflation gaps with great certainty and react to them immediately, the Taylor rule's responsiveness to the inflation gap, h, and to the output gap, b, should both be very large and the two parameters should be balanced to obtain society's preferred mix of output and inflation reactions to supply shocks. In the real world there is uncertainty regarding the size of the output gap, and the monetary policy process may be subject to time lags, so the central bank should never choose extreme reaction parameters h and b. Nevertheless, our analysis suggests the following guidelines for monetary stabilization policy: 1) Both reaction parameters in the Taylor rule should be strictly positive and of considerable size, and 2) h should be larger relative to b the more emphasis society puts on inflation stability vs. output stability and the more weight society puts on future economic performance.

Lags in stabilization policy

Our analysis of monetary stabilization policy above assumed that the central bank could perfectly observe the output and inflation gaps and react to them immediately. We also assumed that changes in the policy interest rate affect economic activity already in the current period. These are highly idealized assumptions. In the real world, information is not perfect and is only available with delays, and it may take time before a change in the policy interest rate has a significant effect on the economy.

Imperfect information

In practice inflation, output and other economic variables cannot be perfectly observed and certainly not immediately. The GDP, for instance, is associated with all kinds of difficult measurement problems, and the structural GDP is simply not a measured magnitude, but a hypothetical estimated variable. Inflation may be easier to measure, but often it will be difficult to evaluate whether a change in inflation is purely temporary (and hence does not call for policy action) or longer-lasting. These uncertainties mean that the economic statistics will not give a perfect picture of the stance of the business cycle, so policy makers will typically not want to react strongly to the (preliminary) information they get for fear of overreacting. A further problem is that economic statistics come with substantial delays and are often revised several times, as we now turn to.

Time lags

Even the non-perfect economic statistics take time to produce. Data must be collected from primary sources and then be processed to generate understandable statistics, and all of this is time consuming.

This creates the problem of the *recognition lag*, the difficulty that since the sufficient amount of information only becomes available with a time lag, it takes time before policy makers recognize that the state of the economy has changed. In addition to the recognition lag, there may be a *decision lag*, because it may take some time to decide what change of policy is warranted by the change in the state of the economy. For instance, a tax change must typically be decided in parliament, which takes time. On top of that there may be an *implementation lag* in so far as the change in policy cannot be implemented immediately. Even when a tax change has been decided, it has to be written into law texts etc. The sum of the three lags just mentioned is sometimes referred to as the *inside lag*, covering the period from the time an economic disturbance arises until the time when a change in the economic policy instrument has been implemented. In the case of monetary policy the inside lag is usually considered to consist mainly of the recognition lag, since an independent central bank can typically decide and implement a change in the policy interest rate or other instruments quite quickly once the need for a change has been recognized. In our analysis above we assumed a very short inside lag.

But stabilization policy faces an additional complication in the form of the so-called *outside lag*. This is the period from the time the policy instrument has changed until this change achieves its (maximum) impact on the economy. The analysis above implicitly assumed that the outside lag is shorter than the length of the time period considered, so that a change in the interest rate attains its full impact on aggregate demand within that period. In the real world it may take a considerable amount of time before a change in the policy interest rate attains its full impact on aggregate demand and inflation.

Since the inside lag is usually considered relatively short and the outside lag relatively long in connection with monetary policy (compared to fiscal policy), we will perform an analysis where there is no inside lag (the central bank can still react very fast to changed circumstances), but there is a considerable outside lag (it takes a substantial amount of time before the interest rate decisions of the bank affects demand etc.).

The outside lag, inflation forecast targeting, and the Taylor rule

The experience from many Western countries is that it can take about two years for a change in the interest rate to attain its maximum impact on the rate of inflation because of delayed behavioural responses in the private sector. While it may be possible to reduce the inside lag by investing more resources in the collection and improvement of economic data and by reforming the procedures for policy decision making, it is much harder for policy makers to do anything about the outside lag, since this lag is rooted in the frictions and delays in the private sector's reactions to changes in the economic environment. Yet, if policy makers know the approximate length of the outside lag, they may be able to account for it when designing the rules for stabilization policy. As an example, we will now demonstrate how a specific version of the *Taylor rule* for monetary policy may be an optimal way of coping with the outside lag.

We will illustrate this by the following modified version of the IS-RP-AS model set up at the beginning of Section 20.4, where we now incorporate the outside lag:[8]

$$y_{t+1} - \bar{y} = v_{t+1} - \alpha_2 (r_t - \bar{r}) \qquad \text{(IS')}$$

$$r_t = r_t^p + \hat{\rho}_t \qquad \text{(RP)}$$

$$\pi_{t+1} = \pi_t + \gamma (y_t - \bar{y}) + s_{t+1} \qquad \text{(AS')}$$

The central bank still determines the real interest rate in period t with some imprecision as expressed by (RP), but now, as (IS') shows, it takes one period before a change in the real interest rate affects demand, and, as (AS') shows, it takes one period before a change in the output gap affects inflation.

The period length of the model may be thought of as one year. Thus Equation (IS') assumes that it takes one year before a change in the real interest rate attains its (full) impact on aggregate demand. This is not altogether unrealistic, since it takes time for firms to make new investment plans in response to a change in the cost of capital, and since it is time-consuming for firms in the investment goods sector to produce new equipment and (in particular) new business structures. Equation (AS') further assumes that it takes a year for a change in economic activity to affect the rate of inflation. Again, a certain delay in the impact of a change in activity on inflation can be realistic, since nominal wages and prices tend to be sticky in the short run because of the menu costs of wage and price changes, or because workers and firms are temporarily locked into nominal contracts. With these time lags in aggregate demand and aggregate supply, the model captures the stylized empirical fact mentioned above that it typically takes about two years for monetary policy to achieve its full effect on the rate of inflation.

Suppose now that the government has delegated monetary policy to an independent central bank with the prime mandate of ensuring a low and stable rate of inflation at the target level π^*. As we shall explain in detail in Chapter 22, it may be quite rational for a government to give its central bank such a mandate even if the government also cares about output and employment. In practice, many central banks have as their main goal to ensure 'price stability' (defined as a low and stable inflation rate). For example, Article 2 of the statutes for the European Central Bank says that the primary objective of the ECB shall be to maintain price stability. Given this one-dimensional goal, we may specify the central bank's loss function for period t as:

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[8] Our analysis is a simplified version of the one found in Lars E. O. Svensson, 'Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets', *European Economic Review*, **41**, 1997, pp. 1111-1146.

$$SL_t = (\pi_t - \pi^*)^2 \tag{27}$$

This expresses that the central bank dislikes negative as well as positive deviations of inflation from target. For a future period $t+\tau$, where the central bank cannot know the inflation rate but must act on its expectation $E_t(\pi_{t+\tau})$, we assume that the social loss of the bank is:

$$SL^e_{t+\tau} \equiv (E_t(\pi_{t+\tau}) - \pi^*)^2 \tag{27'}$$

Note that the $SL^e_{t+\tau}$ of (27') is not exactly the same as $E_t(SL_{t+\tau})$, but given the aims of the central bank it is a reasonable simplification that it will want to bring expected inflation in a future period as close as possible to the target π^*. In doing so, the central bank must account for the fact that a change in the interest rate in year *t* will not affect the inflation rate until year *t* + 2. In year *t*, the bank must therefore choose the policy real interest rate r^p_t to minimize SL^e_{t+2}, the 'expected loss' two years in the future. For simplicity, we assume here that in each period *t*, the central bank does not take into account periods after period *t*+2; it only worries about the effect of r^p_t on π_{t+2}.

To see how the current policy real interest rate affects the rate of inflation two years later, we first roll (AS') one period ahead

$$\pi_{t+2} = \pi_{t+1} + \gamma(y_{t+1} - \bar{y}) + s_{t+2} \tag{28}$$

and for the π_{t+1} on the we right hand side of this we insert the expression from (AS'), and for the $y_{t+1} - \bar{y}$, we insert the expression from (IS'), and for r_t we insert from (RP):

$$\pi_{t+2} = \pi_t + \gamma(y_t - \bar{y}) + s_{t+1} + \gamma[v_{t+1} - \alpha_2(r^p_t + \hat{\rho}_t - \bar{r})] + s_{t+2}$$

Since the shock processes are assumed to be white noise the best possible expectations of the central bank formed in period *t* are $E_t(v_{t+1}) = E_t(s_{t+1}) = E_t(s_{t+2}) = 0$, and since the central bank does not know the realization of the disturbance term $\hat{\rho}_t$ in period *t* either (otherwise it would not be a disturbance), it also expects $E_t(\hat{\rho}_t) = 0$. Therefore, the best possible expectation of the inflation rate in period *t*+2 that can be formed in period *t* based on the information available there is:

$$E_t(\pi_{t+2}) = \pi_t + \gamma(y_t - \bar{y}) - \alpha_2\gamma(r^p_t - \bar{r}) \tag{29}$$

The best the central bank can do to minimize $SL^e_{t+2} \equiv (E_t(\pi_{t+2}) - \pi^*)^2$ by its decisions in period t is to try to obtain $E_t(\pi_{t+2}) = \pi^*$. Thus, the central bank should follow a policy of *inflation forecast targeting*: if the forecast for the inflation rate two years ahead is higher than the inflation target, the interest rate should be raised until $E_t(\pi_{t+2}) = \pi^*$. If the opposite is the case, the interest rate should be lowered until the inflation forecast is on target. To find the interest rate which will ensure that the inflation forecast corresponds to the target inflation, we insert from (29) into $E_t(\pi_{t+2}) = \pi^*$:

$$\pi_t + \gamma(y_t - \bar{y}) - \alpha_2\gamma(r_t^p - \bar{r}) = \pi^*$$

which can be rewritten as:

$$r_t^p = \bar{r} + \frac{1}{\alpha_2\gamma}(\pi_t - \pi^*) + \frac{1}{\alpha_2}(y_t - \bar{y}) \tag{30}$$

This is indeed a rule for the policy real interest rate. Using (RP) it can also be written:

$$r_t = \bar{r} + \frac{1}{\alpha_2\gamma}(\pi_t - \pi^*) + \frac{1}{\alpha_2}(y_t - \bar{y}) + \hat{\rho}_t \tag{31}$$

This is a particular version of the monetary policy rule for the real interest rate (MP) stated in Section 20.3, $r_t = \bar{r} + h(\pi_t - \pi^*) + b(y_t - \bar{y}) + \hat{\rho}_t$, corresponding to:

$$h = \frac{1}{\alpha_2\gamma} \quad \text{and} \quad b = \frac{1}{\alpha_2} \tag{32}$$

In turn, (MP) was derived from the Taylor rule (9) for the nominal policy interest rate i_t^p. Our analysis based on the outside lag thus supports conducting policy by a Taylor rule with the coefficients h and b given as in (32).

Hence, our IS-RP-AS model with an outside lag implies that *the Taylor rule is an optimal monetary policy* provided the coefficients on the inflation and output gaps reflect the way these variables affect the future inflation rate (through the parameters α_2 and γ).

It is interesting to note that monetary policy should react to the output gap even though this variable does not enter the social loss function (27). The reason is that the current output gap is a *predictor* of future inflation. For example, if $y_t - \bar{y}$ rises by one unit, we see from (AS') and (28) that this will, *ceteris paribus*, raise π_{t+1} and π_{t+2} by γ units. But if the central bank in its reaction to this manages to raise the real interest rate r_t by $1/\alpha_2$ units, it follows from (IS) that this would reduce aggregate demand and output in the next period by one unit which in turn would reduce π_{t+2} by γ units. Thus, if the two-year-ahead inflation forecast is initially on target, the central bank can keep it there by raising the interest rate by an appropriate amount in response to a rise in the current output gap.

Likewise, if current inflation increases by one unit this will, *ceteris paribus*, as (28) shows raise π_{t+1} and π_{t+2} by one unit. Raising the interest rate by $1/(\alpha_2\gamma)$ units will according to (AS') decrease y_{t+1} by $1/\gamma$ units, which in turn, according to (28) will decrease π_{t+2} by one unit.

Let us draw some lessons from the above analysis:

THE OUTSIDE LAG, INFLATION FORECAST TARGETING AND THE TAYLOR RULE

The outside lag in monetary policy is the period from the time the central bank changes its policy rate until the (maximum) impact on the economy is felt. With a considerable outside lag a central bank charged with the responsibility to maintain a low and stable inflation rate should target a *forecast* for the future inflation rate within a time horizon corresponding to the outside lag, adjusting the policy rate until the inflation forecast corresponds to the target inflation rate. This insight is reflected in the contemporary practice of many central banks who often justify their interest rate decisions by reference to inflation forecasts for the next couple of years, published in so-called inflation reports. A policy of inflation forecast targeting can lead to a version of the Taylor rule for monetary policy.

All in all, we do not have to view the Taylor rule 'only' as a plausible description of actual central bank behaviour. It can also under various and plausible circumstances be rationalized as optimal monetary stabilization policy. In particular, the Taylor principle of a more than one-to-one reaction of the nominal policy interest rate to lasting changes in inflation is well founded, and macroeconomic theory also supports the idea that the nominal policy interest rate should react positively to both increases in inflation and in output.

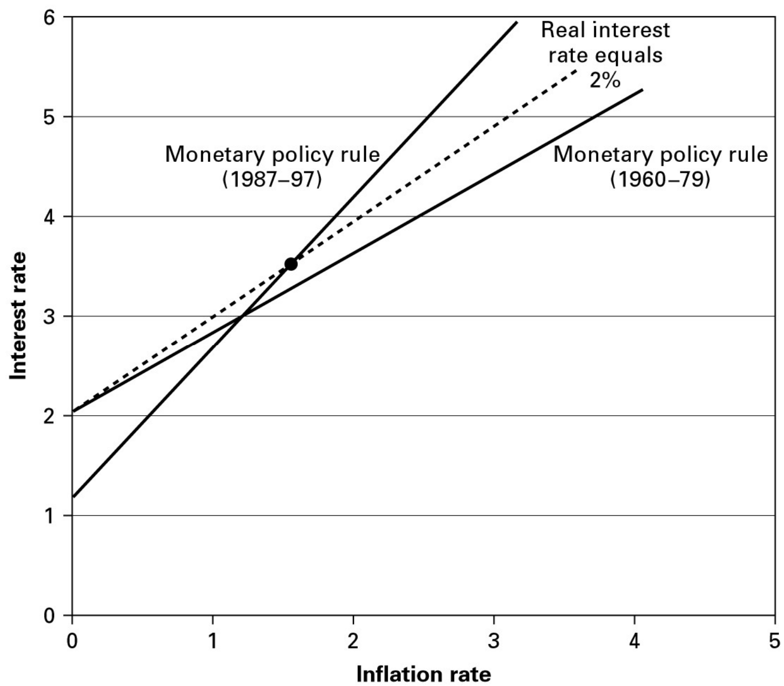
20.5 Monetary stabilization policy in practice: Alan Greenspan, John Taylor and the financial crisis of 2008 – 09

The monetary policy experience of the USA since 1960 provides a good illustration of some of the policy principles discussed above. One main guideline is the Taylor principle that the policy interest rate should vary more than one-to-one with the rate of inflation, that is, the coefficient *h* on the inflation gap in the Taylor rule (9) above should be strictly positive.

The danger of choosing a negative *h* is illustrated by the two diagrams below. Figure

20.5, created by John B. Taylor himself, shows the estimated reaction of the nominal short-term policy interest rate to the rate of inflation in the USA for the two periods 1960 – 79 and 1987 – 97, respectively. The slopes of the two solid straight lines reflect the estimates of the coefficient 1 + h in the Taylor rule $i_t^p = \overline{r} + \pi_t + h(\pi_t - \pi^*) + b(y_t - \overline{y})$ in the two periods.

Figure 20.5 Estimated monetary policy rules in the USA: 1960 – 79 versus 1987–97

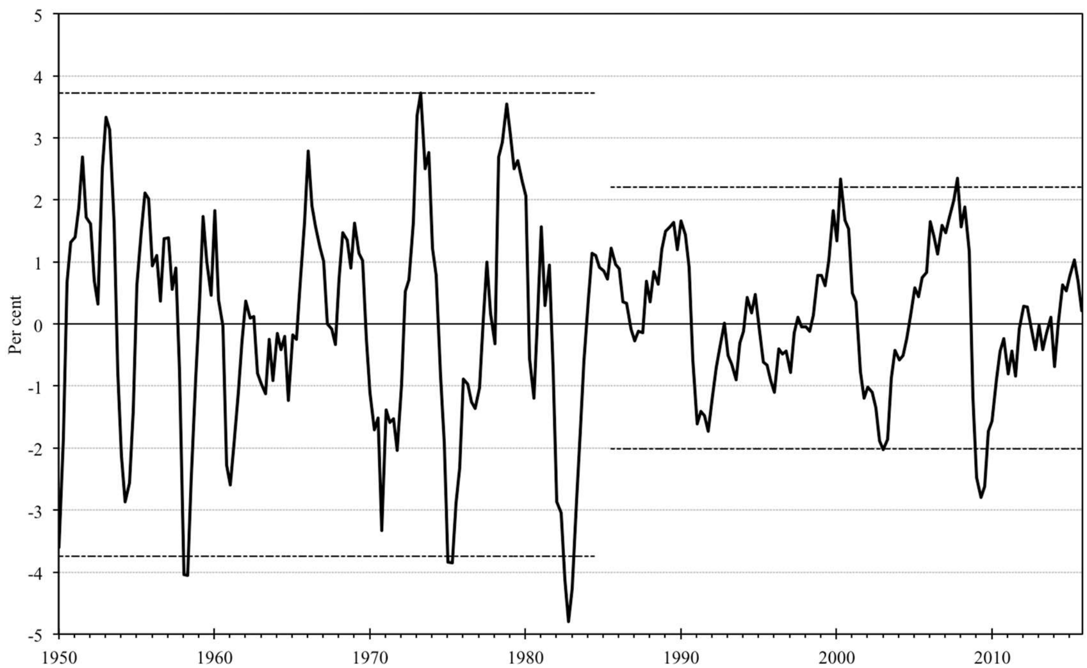


Source : Figure 7.3, p. 331 in John B. Taylor, 'A Historical Analysis of Monetary Policy Rules', Chapter 7 in John B. Taylor (ed.), *Monetary Policy Rules*, NBER Business Cycles Series, Volume 31, The University of Chicago Press © 1999 by the National Bureau of Economic Research.

We see that 1 + h < 1 in 1960 – 79, implying a negative value of h. In other words, in this period a rise in inflation was typically allowed to reduce the *real* interest rate, and vice versa. Thus, monetary policy did not obey the Taylor principle and did not succeed in curbing aggregate demand when inflation was rising during a boom period; nor did it systematically support aggregate demand during recessions when inflation tended to fall. By contrast, during the later period 1987 – 97 we see from Figure 20.5 that h became positive, so in this period the short-term real interest rate did in fact rise and fall with the rate of inflation. The response of the policy interest rate to the output gap also increased between the two periods considered. A study by John Taylor found that the coefficient b on the output gap in the interest rate reaction function of the US Federal Reserve Bank doubled from about 0.25 in the first period to about 0.5 during the second period. At the same time the coefficient h on the inflation gap rose from around -0.25 to 0.5.[9]

Apparently, a major shift in the conduct of US monetary policy from a more accommodating to a more active counter-cyclical modus operandi occurred between the period up to around 1980 and the period after the mid 1980's. Other things equal, one could therefore expect the business cycle to be less volatile in the later than in the earlier period. Figure 20.6 by and large confirms this expectation.

Figure 20.6 The volatility of real GDP (cyclical component) in the USA, 1950Q1-2015Q4



Note: Based on quarterly data. Trend GDP estimated by HP-filter with $\lambda = 1600$, 24 end-point observations excluded. Horizontal lines indicate 1.96 standard deviations for 1950-1983 and 1984-2015. *Source*: Bureau of Economic Analysis.

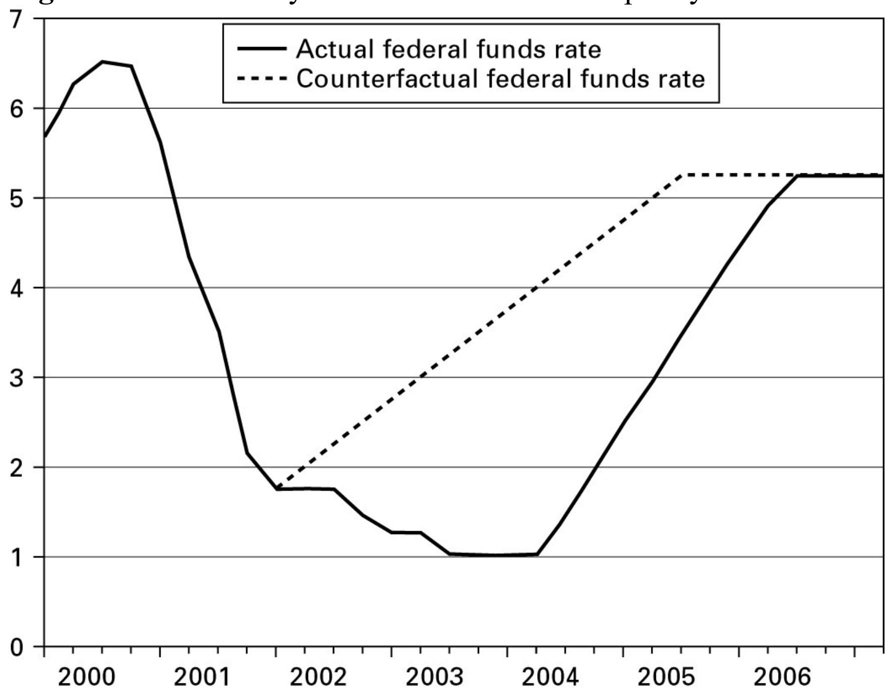
In the intervening period 1980–86 not shown in Figure 20.5, US monetary policy moved from being 'dovish' to being very 'hawkish' on inflation, reflecting the strong determination of the new Federal Reserve Board Chairman Paul Volcker to reduce inflation from the very high levels experienced during the 1970s. During this 'Volcker disinflation' period the short-term interest rate rose sharply above the rate prescribed by a Taylor rule with coefficients $h = b = 0.5$. This reversal of monetary policy was a main factor behind the deep US recession of 1981– 82 highly visible in Figure 20.6, but Paul Volcker did succeed in permanently lowering the actual and expected inflation rate.

The transition to a less volatile business cycle after the mid-1980s is often referred to

[9] John B. Taylor: 'The Explanatory Power of Monetary Policy Rules'. The Adam Smith Lecture. Annual Meeting of the National Association of Business Economics, September 10, San Francisco. Available at: www.stanford.edu/~johntayl/

as 'The Great Moderation'. A lot of the credit for this improvement in macroeconomic stability has been ascribed to Alan Greenspan who became Chairman of the Federal Reserve Board from the start of 1987 when the Fed began to react more strongly and systematically to the output and inflation gaps, in accordance with the Taylor rule. When Alan Greenspan retired from the Federal Reserve in the beginning of 2006, he was hailed as one of the greatest central bankers the world had ever seen, mainly because the US economy experienced only two very short and mild recessions during his long term as chairman of the Fed. However, more recently John Taylor and several other economists have criticized the Fed's interest rate policy from 2003 to 2005 for having contributed to the severe financial and economic crisis of 2008-09.

Figure 20.7 The Taylor rate versus the actual policy interest rate in the USA

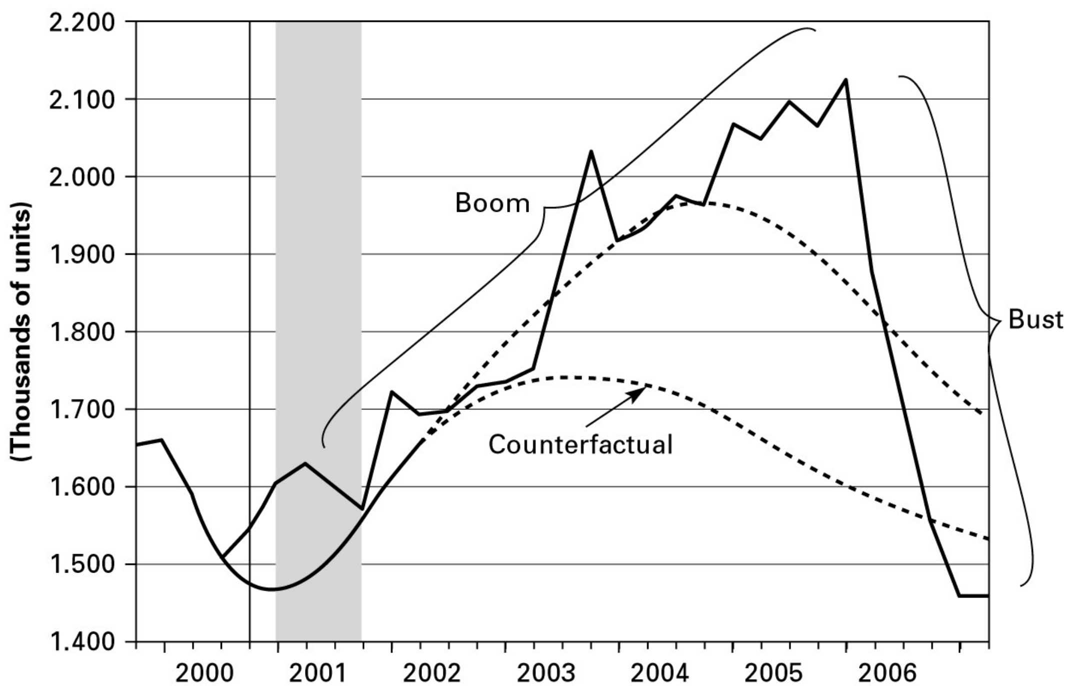


Source: Figure 1 in John B. Taylor: 'Housing and Monetary Policy', presented at the Policy Panel at the Symposium on Housing, Housing Finance, and Monetary Policy sponsored by the Federal Reserve Bank of Kansas City in Jackson Hole, Wyoming, September 2007. Available at: www.stanford.edu/~johntayl/

To understand this criticism, first take a look at Figure 20.7. The figure shows the actual US policy interest rate (the Federal Funds rate) compared to the (smoothed) time path for the interest rate prescribed by the standard Taylor rule (with $h = b = 0.5$) that had worked so well during most of the Great Moderation.

We see that the actual interest rate from late 2002 to 2006 was considerably below the 'Taylor rate'. The motivation for the low-interest-rate policy of this period was that the Fed worried about the danger of deflation, as inflation fell to very low levels in the aftermath of the bursting of the 'dot-com' stock market bubble and the 2001 recession. Apparently, the Fed was anxious to avoid falling into a Japanese-style deflation trap. The low-interest-rate policy contributed significantly to the record increase in US house prices between 2003 and 2006 and the resulting construction boom. The solid line in Figure 20.8 illustrates the boom–bust cycle in US construction activity, measured by the number of new housing starts. The upper dotted line shows the number of housing starts predicted by a simple regression equation with the Federal Funds rate as the explanatory variable. We see that the Fed's interest rate policy can explain a large part of the boom–bust pattern of residential investment. The lower dotted line in Figure 20.8 (labelled 'Counterfactual') shows the number of housing starts predicted by the regression equation if the Federal Funds rate had followed the Taylor rule illustrated in Figure 20.7.

Figure 20.8 Actual and estimated housing starts in the USA



Source: Figure 2 in John B. Taylor: 'The Financial Crisis and the Policy Responses: An Empirical Analysis of What Went Wrong'. Working Paper, Stanford University, November 2008. Available at: www.stanford.edu/~johntayl/

We see that in such a counterfactual scenario the housing cycle would most likely have been significantly dampened. Since the strong increase in the supply of new housing units was a major factor behind the drop in housing prices from 2006 (see Figure 15.17) – which in turn contributed to triggering the severe financial crisis starting in mid-2007 – many economists argue that the Fed's deviation from the Taylor rule unintendedly helped to pave the way for the crisis.

Many other countries also followed a loose monetary policy in the run-up to the financial crisis. Although many complex factors contributed to the financial and economic crisis in 2008 – 09, an inappropriate and too loose monetary policy in many countries before the crisis can help to explain why it became so severe.

20.6 Optimal fiscal stabilization policy

We now turn to fiscal stabilization policy. Like monetary policy, fiscal policy works by affecting aggregate demand through the IS curve of our macro model, so our analysis of optimal *fiscal* policy under 'normal circumstances' will be a simple translation of the analysis of optimal *monetary* stabilization policy. However, we will also point to real world circumstances that imply that monetary and fiscal policy are not just equivalent even under 'normal circumstances', and we will be much preoccupied with 'special circumstances' under which monetary policy loses its potency so that only fiscal stabilization can be relied on, the so-called liquidity trap.

A model framework for studying fiscal stabilization policy

Since we will now study *fiscal* policy, we will no longer disregard the fiscal impulses in the IS-curve (7), which is repeated here for convenience:

$$y_t - \bar{y} = \alpha_1(g_t - \bar{g}) - \alpha_2(r_t - \bar{r}) - \alpha_3(\tau_t - \bar{\tau}) + v_t \tag{IS''}$$

Fiscal stabilization policy consists of the government temporarily letting (the log of) government demand g_t and/or the net tax rate τ_t deviate from their structural levels with the purpose of stabilizing the business cycle. With respect to the contents of the parameters and the confidence shock in (IS''), we recall the following results from the derivation of the IS-curve in Chapter 17:

$$\begin{aligned}&\alpha_1 \equiv \frac{\tilde{m}\bar{G}}{\bar{Y}}, \quad \alpha_2 \equiv -\frac{\tilde{m}D_r}{\bar{Y}}, \quad \alpha_3 \equiv -\frac{\tilde{m}D_\tau}{\bar{Y}} = \tilde{m}C_Y, \quad v_t \equiv \frac{\tilde{m}\bar{\varepsilon}D_\varepsilon}{\bar{Y}}(\ln\varepsilon_t - \ln\bar{\varepsilon}), \quad \text{and} \\ &\tilde{m} \equiv \frac{1}{1-D_Y} = \frac{1}{1-(1-\bar{\tau})C_Y - I_Y} > 0,\end{aligned} \tag{33}$$

The other element of the economic structure will be the AS-curve unchanged from Equation (AS) above and repeated here for convenience:

$$\pi_t = \pi_{t-1} + \gamma(y_t - \bar{y}) + s_t \tag{AS}$$

Some observations on the IS-curve are of importance for our discussion of fiscal stabilization policy:

First, the coefficients α_1, α_2 and α_3 have obvious interpretations, e.g., for α_3: Starting from a long-run equilibrium, a unit rise in the income tax rate reduces disposable income by the amount $\overline{Y}$. On impact, private consumption therefore falls by the amount $C_Y\overline{Y}$, where C_Y is the marginal propensity to consume out of disposable income, but when the multiplier effect of the initial drop in demand is taken into account, the total fall in aggregate demand induced by the tax increase will be $\tilde{m}C_Y\overline{Y}$, where $\tilde{m}$ is the Keynesian multiplier also stated in (33). Measured relative to the initial level of GDP, a tax increase of magnitude $\tau - \overline{\tau}$ will thus reduce aggregate demand by the amount $\tilde{m}C_Y \cdot (\tau - \overline{\tau})$.

Second (and related), the Keynesian multiplier $\tilde{m}$ appears in all the coefficients and in the confidence shock v_t, and this multiplier itself depends on the structural net tax rate $\overline{\tau}$, such that the larger $\overline{\tau}$ is, the smaller is the multiplier. This means that the effects of the so-called 'automatic stabilizers' are included everywhere. If demand is boosted for instance by high confidence, $\varepsilon_t > \overline{\varepsilon}$, the total effect on demand will be smaller the larger $\overline{\tau}$ is (the smaller $\tilde{m}$ is). After the confidence shock has first increased activity by its direct effect, the multiplier effect will be smaller the higher the net tax rate is, because increases in income and activity will lead to higher net taxes that dampen the indirect consumption effects, and more so the larger the net tax rate is. This dampening of the effect of confidence shocks (and of other demand shocks) occurs without any active policy decisions having to be taken – it follows from the way the systems of taxes and transfers have *already* been constructed. The automatic stabilization effects are incorporated in the coefficients and in the specification of the demand shock v_t in (33). Automatic stabilization is an important part of fiscal stabilization policy, which is why we have bothered to repeat here how the coefficients α_1, α_2 and α_3 and the shock v_t depend on underlying parameters etc.

Third, we observe from the IS-curve (IS'') that the fiscal impulses $\alpha_2(g_t - \overline{g})$ and $-\alpha_3(\tau_t - \overline{\tau})$ appear in a parallel way as the impulse from monetary policy, $-\alpha_2(r_t - \overline{r})$, and all of these impulses only appear in the IS-curve, that is, they only affect the economic system through their effects on aggregate demand. Hence, seemingly, strictly on the model's premises one cannot obtain anything by fiscal stabilization that could not have been obtained by monetary stabilization, and vice versa. We are going to study an exception from this, the so-called 'liquidity trap', but outside this trap fiscal and monetary policy work in very similar ways in our model.

Optimal fiscal stabilization policy

In our analysis of *monetary* stabilization policy we started by disregarding the fiscal impulses from the IS-curve (IS'') (or rather putting them into the shock v_t), and then we asked in Section 20.4 how the central bank should control the interest rate for best possible stabilization. We concluded that it would be optimal in each period to keep the economy tight and close to a 'policy line' $\hat{y}_t = -(1/\varphi)\hat{\pi}_t$ with φ given by Equation (25), and indeed this could be obtained by letting monetary policy be governed by a Taylor rule with appropriate choices of the parameters h and b of the rule. We could now disregard monetary policy and set $r_t - \overline{r} = 0$ (or rather set $r_t - \overline{r} = \hat{\rho}_t$ and let $-\alpha_2\hat{\rho}_t$ be included in the shock v_t) and ask a similar question for the fiscal impulses.

However, we do not need to conduct any explicit analysis to obtain an answer because monetary and fiscal policy essentially work the same way according to our model. From (IS'') we see that a 1 per cent rise in government spending on goods and services has the same impact on aggregate demand as an interest rate cut of α_1/α_2 percentage points, and an increase in the net tax rate by one percentage point has the same effect on demand as in increase in the interest rate by α_3/α_2 points. It follows that if fiscal policy makers have the same information on the state of the economy as the central bank and react with the same time lag to this information, a fiscal policy using the policy instruments g_t and τ_t can achieve exactly the same stabilizing effect as the one that can be achieved via monetary policy.

To be specific and for illustration, assume that indeed monetary policy is kept passive, $r_t - \overline{r} = \hat{\rho}_t$, and assume also $g_t - \overline{g} = 0$, and that the net tax rate is set according to the rule:

$$\tau_t = \overline{\tau} + h'(\pi_t - \pi^*) + b'(y_t - \overline{y}), \quad h' > 0, \quad b' > 0 \tag{35}$$

Combining (35) with the IS-curve (IS''), it is easy to see that this implies the AD-curve:

$$y_t - \overline{y} = -\frac{\alpha_3 h'}{1+\alpha_3 b'}(\pi_t - \pi^*) + \frac{v_t - \alpha_2\hat{\rho}_t}{1+\alpha_3 b'} \tag{36}$$

Hence, by choosing both of h' and b' large and letting $\alpha_3 h'/(1+\alpha_3 b') = 1/\varphi$, the optimal policy line, $\hat{y}_t = -(1/\varphi)\hat{\pi}_t$, would be implemented. Of course, all the reservations we stated for optimal monetary policy concerning cautiousness to avoid overreactions to business cycle fluctuations are also valid for fiscal policy.

On the assumptions of our model, it makes no difference whether stabilization is conducted by monetary or fiscal policy. There are, however, many reasons why the two types of policy work differently in practice. Some of these reasons originate from the basic characteristics of the instruments and the information requirements for using them in stabilization and are discussed in the next subsection. Another reason originates from the fact that monetary interest rate policy can become completely ineffective if the nominal policy interest rate hits its lower bound around zero. This phenomenon, the so-called 'liquidity trap', has indeed been important in the aftermath of the great financial crisis starting in 2008-09 and will be considered in the section 20.8.

20.7 Fiscal versus monetary stabilization policy

The most fundamental debate on stabilization policy is whether there should be *active* stabilization at all, or whether the authorities should just let the *automatic* stabilizers do their job and leave it there. Economists are somewhat divided on this issue, but it is widely agreed that 'fine tuning' should be avoided, that is, policy makers should not attempt to mitigate even very small fluctuations by active stabilization efforts. Such attempts will probably do more harm than good because informational and time lag problems may imply bad dosage and timing of the policy interventions. On the other hand most economists agree that larger and more long lasting booms and busts should be counteracted by active stabilization attempts.

Another broad discussion reviewed in Section 20.2 is whether active stabilization policy should be conducted by *rules* or *discretion*. The general consensus here seems to be to that rules certainly have advantages mainly by strengthening the credibility of policy makers and serving as an anchor for expectations,, but active stabilization cannot rely entirely on pre-specified rules: if the rules should account for all relevant contingencies, they would become so complicated that they would be impossible to explain and understand whereby their credibility advantages would disappear.

Thus, accepting that there should be active stabilization policy, partly by rules and partly by discretion, should stabilization policy mainly involve measures of monetary policy or fiscal policy? We will discuss that below, but before turning to *active* stabilization policy we first note that almost everybody agrees that the *automatic* stabilizers are good and should be allowed to work themselves out. Societies arrange themselves differently with respect to the level and progressivity of direct and indirect taxation, the character of property taxes, the generosity of transfers, the size of the public sector etc. The Scandinavian countries and some continental European countries have in international comparison quite high taxes and transfers. In those countries the marginal net tax rate could be up to perhaps 80 percent, whereas some of the Anglo-American countries have lower taxes and transfers and marginal net tax rates below 50 percent. Given the level of taxation that society has decided, the fact that it implies at least some degree of automatic stabilization will almost always be considered desirable.

The effectiveness of the instruments

Although in our model all the instruments of monetary and fiscal policy work through the IS-curve, they nevertheless work through different channels. Government expenditure has a direct effect on demand (and on top of that a multiplier effect through disposable income, consumption and investment, but this is true for all instruments). Taxes affect disposable incomes directly, but they only affect demand (consumption) indirectly through disposable income. Interest rates affect demand indirectly through the sensitivity of consumption and investment to the real interest rate. Hence, government expenditure has the most direct effect and perhaps taxes the second-most direct effect.

Up through the 1950's and 1960's after the Keynesian views had been incorporated in macroeconomics, many Keynesian economists believed that fiscal policy would be much more effective in affecting demand than monetary policy (perhaps in line with what Keynes himself believed). They pointed to the fact that fiscal instruments affect demand more directly and tended to believe that demand was only little sensitive to the real interest rate. They also tended to believe that money demand was very sensitive to interest rates so that changes in the money supply would have little impact on interest rates, leaving monetary policy quite ineffective. On the other hand, 'Monetarist' economists, with Milton Friedman as a main exponent, argued that money demand was little sensitive to interest rates and highly sensitive to income whereas goods demand was substantially sensitive to real interest rates, leaving monetary policy more potent than fiscal policy.

Subsequent empirical research has shown that aggregate demand is normally quite sensitive to real interest rates perhaps corresponding to our model's α_2 being around 1-1.4. This means that under normal circumstances a monetary policy that affects interest rates can be quite effective in regulating demand. There are some further theoretical arguments why monetary policy may be more effective than fiscal policy. First, a tax cut that is only temporary has little impact on permanent income and may therefore have little impact on current consumption. Second, a debt-financed government spending programme may be counter-acted by the Ricardian equivalence effect described in Chapter 16 on consumption. However, as that chapter explained, a substantial number of households may be liquidity-constrained or live from 'hand-to-mouth', and in that case debt-financed government spending and tax cuts, even if temporary, may have significant effects on aggregate demand.

Today it is widely agreed that both monetary and fiscal policy instruments have the potential to affect demand and activity. However, there is also some consensus that a fundamental condition for fiscal policy being substantially effective is that government finances are seen as sustainable at the outset. If a government plagued by high debt and deficits and perhaps in danger of defaulting on its debt attempts to stimulate demand by tax cuts and expenditure increases, it may experience counteracting cuts in private demand as consumers and firms in fear of the future fiscal tightening bound to come begin to reduce consumption and investment. This cause for ineffectiveness of fiscal stimulating policy became sadly relevant after the financial crisis of 2008-09 that led on to an economic crisis from 2009 and a severe sovereign debt crisis in parts of the

European Union from around 2010. Some countries that had conducted loose fiscal policy in the good times up to the crisis and had seen their already large deficits severely increased because of the economic downturn simply did not have the fiscal capacity to stimulate demand when it was most needed. This provides a reason for keeping sound and sustainable government finances during normal times.

The suitability of the instruments

Most of our model's government demand G, and hence g, is government consumption, that is, government expenses for public services such as education, health care, care for children and the elderly, national defence, cultural activities etc. These expenses are not motivated at all by their effects on the business cycle. One may ask whether it makes sense to have more public education, health care and defence in a downturn and less in an upswing. The answer is no. For most of government consumption, it simply does not make sense to use it for regulating the business cycle. There may be smaller parts of public consumption where it does make sense, e.g., the public services provided for those who encounter social problems during recessions, but these are minor exceptions.

Can a similar objection be raised against using the interest rate for regulation? The interest rate changes of monetary stabilization policy work through affecting demand mainly for investment and durable consumption goods. Since investment is accumulated into capital which lasts for longer periods, it can in fact make sense for society to distribute investment over time such that more investment is undertaken when there is a lack of demand and less when demand is too high. This is exactly what monetary stabilization aims at.

Returning to government demand, what was just said about private investment holds as well for public investment. Therefore fiscal stabilization policy from the expenditure (not the tax) side will often consist of attempts to increase and bring forward public investment in the bust and decrease it in the boom. This may involve other problems that we return to below, but public investment is, contrary to much of public consumption, not by its character ill-suited for stabilization of the business cycle. In connection with the great financial crisis arising from 2008-09 those governments that had the fiscal space to stimulate demand partly did that by bringing public investment forward in time.

For the tax instrument, a first fact of importance is that taxes are from the outset collected to finance public expenditure and to redistribute income, not for affecting the business cycle. However, financing expenditure only requires an appropriate balance between net taxes and expenditure over the business cycle, not in every single year. It may therefore make sense to adjust taxation such that relatively more tax is collected when activity is considerably above the structural level and relatively less when it is below. If people anticipate this, some may react by smoothing consumption over time thus counteracting the intended effects on demand, but the credit-constrained (or 'rule-of-thumb' or 'hand-to-mouth') households will react as wanted. There are, however, also other limits to the suitability of taxes for regulating the business cycle: unstable and unpredictable taxation in general may well involve efficiency losses from distortions of private decisions on labour supply, saving etc. In connection with the financial crisis, several countries used various forms of temporary tax cuts in order to boost demand.

The main stream consensus on these issues seems to be that fiscal stabilization policy – in so far as there is a fiscal space for it – should be conducted by accelerating or postponing public investment and to some extent by adjusting taxes. Using government consumption for stabilization to a larger degree seems to conflict with the deeper purpose of government consumption. For monetary stabilization policy, use of a reaction pattern like the Taylor rule for the interest rate does not seem to substantially contradict other purposes and, furthermore, keeping inflation at a stable, moderate level serves its own efficiency purposes.

Information and time lags involved in using the instruments

The assumption that the fiscal and monetary authorities share the same information on the state of the macro economy is plausible, so fiscal and monetary policy is probably subject to roughly the same recognition lag. However, both the decision lag and the implementation lag is generally assumed to be shorter for monetary than for fiscal policy, so that the *inside lag* is shorter in monetary than in fiscal policy. The reason is that changes in fiscal policy most often first require time-consuming political negotiations for a decision and, once a decision has been made, it may have to be written into laws and regulation for implementation. On top of that, to change, e.g., the level of public investment may take a considerable amount of time for purely practical reasons; just think of a decision to bring forward in time the construction of a bridge. To the contrary, a central bank can change its policy interest rate very quickly after having recognized the need for a change in monetary policy. On the other hand, once a change in government consumption or investment has been implemented, it has an immediate and direct impact on aggregate demand, whereas a change in interest rates only achieves its maximum impact on demand with a considerable time lag. Thus the *outside lag* is typically longer in monetary than in fiscal policy.

An overall consensus on all the aspects we have discussed seems to be:

MONETARY VERSUS FISCAL STABILIZATION POLICY

In normal times where the business cycle swings are large enough to call for policy reaction but still of moderate size, demand management should predominantly be handled by monetary policy working through interest rates. In connection with a larger over-heating or downturn, monetary stabilization policy should be supplemented by active fiscal stabilization mainly consisting of adjusting public investment and taxation. An important reservation is that the government may only be able to stimulate demand by fiscal measures if it has sufficient fiscal strength, that is, if government finances are seen as sustainable.

The conclusion that monetary stabilization policy should sometimes be supplemented by fiscal stabilization is of particular importance in situations where the nominal interest rate after a serious negative demand shock reaches its lower bound.

20.8 Discretionary fiscal policy outside and inside a liquidity trap

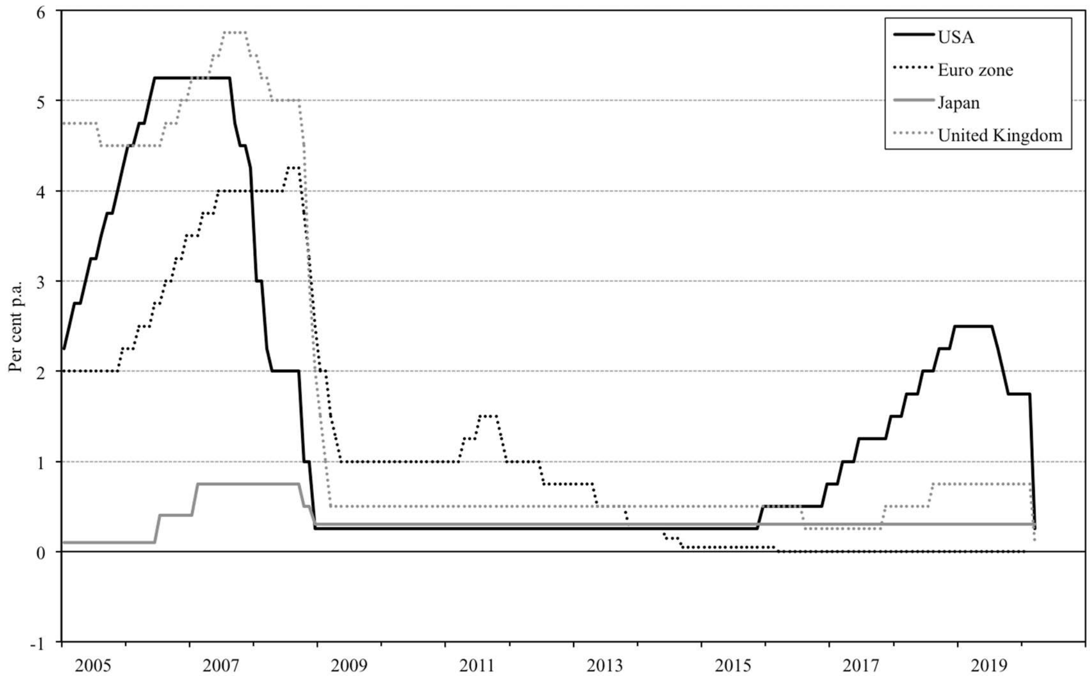
In all our analyses above on monetary policy and on the relation between monetary and fiscal policy we assumed that the central bank could always adjust its nominal policy interest rate to any level wanted, for instance adjust it downwards in reaction to a decrease in the output and inflation gaps. However, this is not possible if the policy interest rate is already equal or close to zero, because a nominal interest rate can never become negative, can it? A negative nominal interest rate means that the lender pays the borrower for borrowing, which really does not make sense since the lender could just choose to 'keep the money' and avoid paying anything. In some cases, though, a person or institution holding liquidity (money) can have a basic need to have its money stored under safe conditions and be ready to pay for that. In the wake of the financial crisis, for instance, some central banks have paid negative interest rates on the deposits of private banks in the central bank. The interest rate on the 'deposit facility' of the European Central Bank, for instance, went down to zero from July 2012 and has been negative since June 2014 and still was by March 2020. The central banks charge negative interest rates on deposits to encourage the private banks to send their liquidity out in the wider economy (lend it out to other economic agents) in order to decrease interest rates and stimulate demand. However, private banks often decide nevertheless to hold parts of their liquidity as deposits in the central bank despite of the negative interest rates. The alternative for the private banks not wanting to lend out their liquidity to firms or other banks would be to hold it as physical cash in their own vaults, so to say. Considering the situation of negative interest rates to be temporary, they chose not to build up the apparatus needed for that. Hence, nominal interest rates can in special situations be slightly negative, but they do have a lower bound not too far from zero. We will now investigate situations where monetary policy cannot stimulate aggregate demand to the extent wanted because the relevant nominal interest rate is already at its lower bound.

Since the days of John Maynard Keynes such a situation is described as a 'liquidity trap', meant to describe circumstances where the public's demand for money and other liquid assets becomes so elastic with respect to interest rates that these could not be pushed downwards by monetary policy. It is important to note that even though the central bank loses its ability to boost demand via its policy nominal interest rate, this does not mean that monetary policy becomes unimportant in a liquidity trap. On the contrary, the deep recessions that can create a liquidity trap often coincide with financial crises where the central bank plays a crucial role as a lender of last resort, providing financial institutions and the public with the liquidity and credit that private markets fail to supply.

This vital policy of the central bank may help to prevent a complete meltdown of the economy. Furthermore, when the policy interest rates can no longer be used to drive down the relevant longer term nominal interest rates, central banks can turn to other measures such as 'asset purchase programs' or 'quantitative easing', as they indeed have in the wake of the great financial crisis, but even such measures may not suffice to raise aggregate demand to levels needed to bring the economy out of deep recession. In such a case, fiscal policy may come to the rescue.

During the Great Depression of the 1930s, many countries fell into the liquidity trap and almost all of them failed to adopt the appropriate countercyclical fiscal policy, because the prevailing fiscal orthodoxy of those days prescribed a balanced government budget as the only responsible policy. The Keynesian Revolution in macroeconomics following the publication of Keynes' *General Theory* in 1936 made countercyclical fiscal policy intellectually respectable. Thus, one could imagine it would be used in a liquidity trap, but the generally good economic evolution without any depression-like larger downturns after the Second World War generated a widespread belief that liquidity traps were a thing of the past. At the same time as deficit spending during recessions became the accepted norm, most economists gradually came to see monetary policy as the main tool of stabilization policy, for the reasons explained in the previous section. In recent years, however, the liquidity trap has come back with a vengeance! Figure 20.9 shows how central banks in many countries cut their policy rates to near-zero levels after the financial crisis starting in 2008-09.

Figure 20.9 Policy interest rates before, during and after the financial crisis



Note: Based on monthly data. From Dec. 16, 2008 the FED stopped providing a target rate and instead reported a target range. The USA line shows the upper limit of the range after Dec. 16, 2008. The signalling interest rates of the Euro zone, Japan and the United Kingdom are the European Central Bank's interest rate on its Main Refinancing Operations (MRO), Bank of England's official Bank Rate and Bank of Japan's Basic Discount Rate, respectively.
Sources: Federal Reserve Bank of St. Louis; European Central Bank, Statistical Data Warehouse; Bank of Japan and Bank of England.

For our analysis of the liquidity trap, it will be of importance that the relevant, longer-term *nominal* interest rate i_t appears directly, so we will write the real interest rate r_t as $i_t - \pi^e_{t+1}$ and hence we write the IS-curve (7) as:

$$y_t - \bar{y} = \alpha_1(g_t - \bar{g}) - \alpha_2(i_t - \pi^e_{t+1} - \bar{r}) - \alpha_3(\tau_t - \bar{\tau}) + v_t \qquad \text{(IS''')}$$

The monetary policy rule (for the relevant longer-term nominal interest rate i_t) will be (9) above, but since the 'imprecision' $\hat{\rho}_t$ in (9) is not key to our present purposes, we will simplify and assume $\hat{\rho}_t = 0$. Hence the monetary policy rule will be:

$$i_t = \bar{r} + \pi^e_{t+1} + h(\pi_t - \pi^*) + b(y_t - \bar{y}) \qquad \text{(MP')}$$

We will assume that as long as it is possible, that is, as long as it does not conflict with the lower bound on the nominal interest rate, the central bank lets the nominal interest rate i_t be governed by the Taylor rule (MP').

Thus, outside the liquidity trap our model will consist of (IS'''), (MP'), the usual AS-curve:

$$\pi_t = \pi_t^e + \gamma(y_t - \bar{y}) + s_t \tag{AS}$$

and some expectations hypothesis. For the future expected rate of inflation π_{t+1}^e present in the IS-curve and in the monetary policy relation (MP') we assume that this is sensitive to the rate of inflation established in period *t*. For simplicity we assume static expectations, $\pi_{t+1}^e = \pi_t$. With static expectations the expected rate of inflation π_t^e up to period *t* present in the AS-curve and embedded into the current nominal wage contracts (negotiated in period *t*-1) is $\pi_t^e = \pi_{t-1}$. Until further notice we take π_{t-1} as exogenous, that is, until further notice we consider a period during which wage contracts are given and not re-negotiated.

Our first question will be under what circumstances the Taylor rule (MP') could imply that the nominal interest rate i_t would hit its lower bound, which we will now assume to be zero. A reasonable value for the natural real interest rate appearing in (MP') could perhaps be $\bar{r} = 0.02$, and in normal times where expected inflation is around the inflation target, $\pi_{t+1}^e = 0.02$ could be reasonable so that in 'peace time' $\bar{r} + \pi_{t+1}^e = 0.04$ seems plausible. With $h = b = ½$, which could also be reasonable, we see from (MP') that if, e.g., the inflation gap is -0.02 and the output gap is -0.06, the nominal interest will indeed become zero according to the Taylor rule. These are not unthinkably large negative values of the gaps in connection with a substantial negative demand shock as, e.g., in connection with the financial crisis, so the liquidity trap is a possibility one should take seriously.[10]

The combinations of output and inflation that imply a zero policy interest rate are found by letting $i_t = 0$ and $\pi_{t+1}^e = \pi_t$ in (MP'): $0 = \bar{r} + (1+h)\pi_t - h\pi^* + b(y_t - \bar{y})$, or:

$$y_t - \bar{y} = \frac{h}{b}\pi^* - \frac{\bar{r}}{b} - \frac{1+h}{b}\pi_t \quad \text{or} \quad \pi_t = \frac{h}{1+h}\pi^* - \frac{\bar{r}}{1+h} - \frac{b}{1+h}(y_t - \bar{y}) \tag{37}$$

[10] In assessing the plausibility of hitting the zero lower bound, one should bear in mind that we interpret our i_t^p to include any systematic risk premium in the relevant, longer-term nominal interest rate i_t over the true short-term, nominal policy interest rate. If this systematic risk premium is typically 2 percentage points, then already when the longer-term, nominal interest rate i_t equals 2 percent, the true nominal policy interest rate will hit zero. Hence, if there is a lower bound not too far below zero directly on the short-term, nominal policy interest rate, this bound may well be hit while the relevant longer-term nominal interest rate i_t is still in positive territory. As mentioned in Chapter 14, the fact that the zero lower bound can be reached relatively easily could be a reason for setting the annual target inflation rate larger than 2 percent.

In the y-π- diagram this is a line with slope $-b/(1+h)$ that runs below and to the left of the long-run equilibrium, $(\bar{y}, \pi^*)$. Indeed, for $y_t = \bar{y}$, Equation (37) could plausibly imply π_t close to (slightly below) zero, and for $\pi_t = \pi^*$ a value of $y_t - \bar{y}$ around -0.08 (follows from $\bar{r} = \pi^* = 0.02$ and $h = b = ½$). Above the line given by (37), the nominal interest rate implied by monetary policy will be governed by (MP'), but below it (MP') will be 'decoupled' and the nominal interest rate will be $i_t = 0$. We will refer to this line as the 'liquidity trap line' and it is illustrated in Figure 20.10 below.

For the area above the liquidity trap line we can insert (MP') into (IS''') to derive an AD-curve in the usual way (and thus exploiting that the central bank knows π_{t+1}^e no matter how this expectation is formed including if $\pi_{t+1}^e = \pi_t$) :

$$y_t - \bar{y} = \frac{\alpha_1(g_t - \bar{g}) - \alpha_3(\tau_t - \bar{\tau})}{1+\alpha_2 b} - \frac{\alpha_2 h}{1+\alpha_2 b}(\pi_t - \pi^*) + \frac{v_t}{1+\alpha_2 b} \tag{AD''}$$

Note that for all positive values of the parameters, $(1+h)/b > \alpha_2 h/(1+\alpha_2 b),$ so in the y-π- diagram the liquidity trap line (37) will be flatter than the line given by (AD'') as illustrated in Figure 20.10 by the upper parts of the AD-curves.

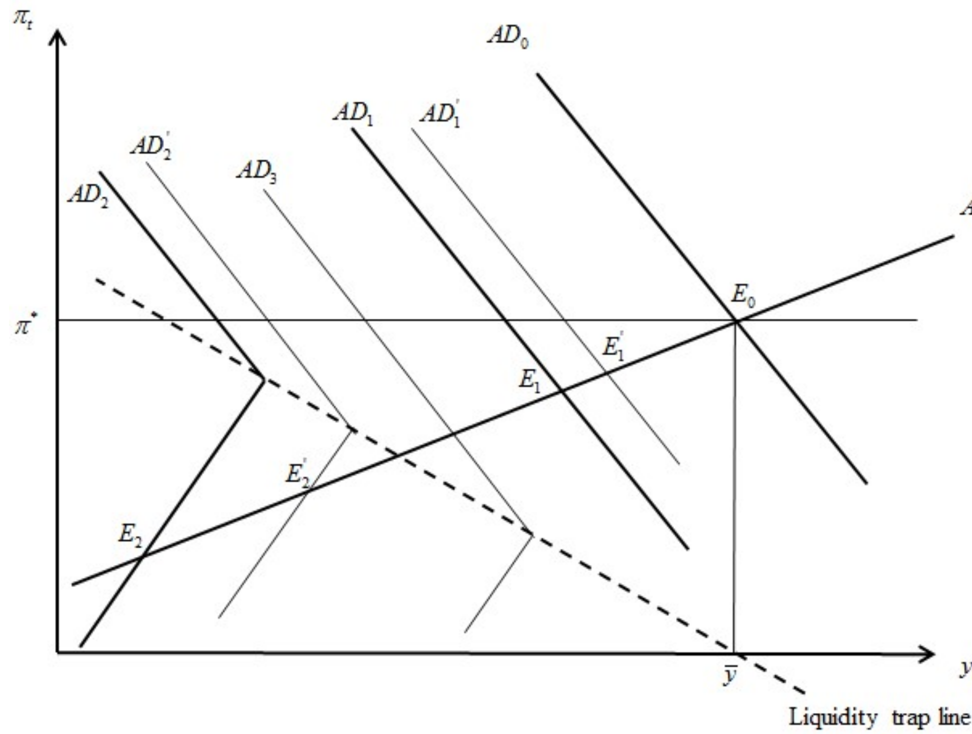
Below the liquidity trap line the Taylor rule (MP') is decoupled and replaced by $i_t = 0$. Inserting this as well as $\pi_{t+1}^e = \pi_t$ into (IS''') gives the appropriate AD-curve 'in the liquidity trap':

$$y_t - \bar{y} = \alpha_1(g_t - \bar{g}) + \alpha_2(\pi_t + \bar{r}) - \alpha_3(\tau_t - \bar{\tau}) + v_t \tag{AD'''}$$

This is *upward sloping* in the y-π- diagram with a slope of $1/\alpha_2$. The reason is that when the nominal interest rate i_t is locked at zero, an increase in current inflation which builds itself into higher expected inflation will imply a lower real interest rate, which stimulates demand with strength α_2. When the central bank does not respond to higher inflation with a higher interest rate, then the normal channel through which higher inflation gives a higher real interest rate and thereby lower demand is not present. Only the effect that higher inflation gives higher expected inflation remains, which unambiguously stimulates demand through a lower real interest rate.[11]

[11] As an exercise you should verify that along the liquidity trap line given by (37) the two AD-curves given by (AD'') and (AD''') meet, that is, the intersection between (37) and (AD'') is the same as the intersection between (37) and (AD''').

Figure 20.10 Active fiscal stimulus policy outside and inside a liquidity trap



Note that for reasonable parameter values and period lengths one will have $1/\alpha_2 > \gamma$, which implies that the upward sloping AD-curve given by (AD''') is steeper in the y-π- diagram than the AS-curve given by (AS). We have argued that on an annual basis α_2 in the range 1-1.4 and γ around 0.3 could be reasonable implying that $1/\alpha_2$ is way above γ. Figure 20.10 illustrates the AS-curve for $\pi_t^e = \pi^*$ and $s_t = 0$, labelled AS_0, and the AD-curve for $g_t - \bar{g} = \tau_t - \bar{\tau} = v_t = 0$, labelled AD_0. The figure also shows alternative AD-curves, AD_1, AD_2 and AD_3 for alternative sizes of a negative confidence shock, $v_t \leq 0$. Note that the AD-curves are kinked along the liquidity trap line (37). The figure illustrates that starting from the long-run equilibrium E_0 with $y_t = \bar{y}$, and $\pi_t^e = \pi_t = \pi^*$, a moderately sized negative confidence shock will imply a new equilibrium like E_1 in the figure given as the intersection between the upward sloping AS_0 and a downward sloping AD-curve like AD_1 in the figure. However, a very large negative demand shock will imply an equilibrium like E_2 in the figure occurring in the 'liquidity trap area' and given by the intersection between the upward-sloping AS-curve and the upward-sloping part of an AD-curve like AD_2 in the figure.

We will consider a discretionary fiscal shock, $\Delta \equiv \alpha_1(g_t - \bar{g}) - \alpha_3(\tau_t - \bar{\tau}) > 0,$ meant to boost activity after a negative demand shock starting from the two types of recession equilibria represented by E_1 (a moderate recession) and E_2 (a deep recession that has led the economy into the liquidity trap).

We first consider E_1 and note that in this equilibrium the stabilizing effects of monetary policy have already worked as explained in Section 20.3. The AD-curve has shifted to the left from AD_0 to AD_1 by $|v_t / (1+\alpha_2 b)|,$ as Equation (AD'') shows, which is less than the size of the shock $|v_t|,$ and furthermore in E_1 output has fallen by less than the horizontal shift $|v_t / (1+\alpha_2 b)|$ because of the central bank's interest rate reaction to lower inflation. The fiscal impulse will shift the AD-curve to the right from AD_1 to AD_1' by the amount $\Delta / (1+\alpha_2 b),$ which is smaller than the impulse itself, as Equation (AD'') shows. This will lead to the new equilibrium $E_1',$ where output has been pushed up compared to the equilibrium $E_1,$ but by less than the horizontal shift $\Delta / (1+\alpha_2 b).$ Fiscal policy thus has an activity boosting effect, but the effect is mitigated by counteracting monetary policy.

Now consider E_2 following the very large negative demand shock. In this equilibrium, monetary stabilization has again done its job, but it ran out of power when the nominal interest rate reached its lower bound. The fiscal shock Δ will again shift the AD-curve to the right from AD_2 to AD_2' in the figure. The size of the horizontal shift is again $\Delta / (1+\alpha_2 b)$ for the upper part of the AD-curve where this is decreasing as Equation (AD'') shows. For the lower part the shift to the right is the full size of $\Delta,$ as (AD''') shows. Hence, the shift to the right is larger than before for the relevant part of the AD-curve. Furthermore, the increase in output from E_2 to E_2' is this time not smaller than the horizontal shift of the AD-curve but larger. The reason is first that this time there is no counteracting monetary policy and second that the increase in inflation created by the fiscal expansion reduces the real interest rate through its effect on expected inflation, which further boosts demand.

Hence, in the liquidity trap not only can fiscal policy be effective in stimulating demand, which monetary interest rate policy cannot, but also in the liquidity trap, fiscal policy becomes highly effective. This is a further reason why it is undesirable and short-sighted for a country to accept weak government finances in 'peace time' since this may eliminate the country's fiscal space to ease fiscal policy if a huge negative demand shock should occur.

Just as fiscal policy is more effective in the upwards direction in a liquidity trap, contractionary fiscal policy will have a stronger-than-normal negative effect on output in a liquidity trap, since it cannot be counteracted by a lower nominal interest rate and since the decrease in inflation caused by it will increase the real interest rate and thereby further reduce demand. This fact became unpleasantly relevant for many European countries after the financial crisis in 2007-09, which lead on to the European sovereign debt crisis unfolding from 2010. The financial crisis had a huge negative impact on the budgets and debts of governments in most countries, since tax revenues fell and activity-dependent government expenditure increased. Some European countries had large deficits and debt already up to the crisis, and other countries experienced rapidly rising ratios of government debt to GDP as they had to bail out large private banks to prevent them from failing. As the crisis deepened, several European governments found it impossible to refinance their debt via the private capital market and had to rely on loans from other EU countries and the IMF as well as liquidity support from the ECB. As a condition for receiving these loans, the debtor countries were required to tighten their fiscal policies substantially in a situation where the ECB gradually lost its ability to support economic activity through lower nominal interest rates as it got caught in a liquidity trap. This was the much-debated European policy of *austerity*, which contributed further to the economic downturn in the debtor countries. In Chapter 26, we will consider the effects of the European austerity policies in more detail.

Deep recessions are normally long-lived. Assume now that the downturn illustrated in Figure 20.10 lasts so long that nominal wage contracts get renegotiated. The expected rate of inflation entering the AS-curve will then no longer be given, but will come to reflect the actually realized inflation rate in the recession. In our example of Figure 20.10 the expected rate of inflation will fall from π^* to the lower level of the relevant short run equilibrium, which could be E_1 or E_2, or, if stimulating fiscal policy has been attempted, E_1' or E_2'. This will shift the AS-curve downwards. The new AS-curve has not been drawn in the figure, but it is easy to see that in the mild recession corresponding to E_1 or to E_1', the downward shift in the AS-curve will cause output to increase. In this case the lower inflation generated by the downward shift of the AS-curve makes the central bank cut its interest rate, thus stimulating demand. In the deep and liquidity-trapped recession corresponding to E_2 or E_2', the downward shift in the AS-curve will cause output to *fall*. This time the lower inflation caused by the shift in the AS-curve does not trigger any interest rate reaction from monetary policy since the nominal interest rate is already at its lower bound, but the lower inflation makes expected inflation fall and thereby the real interest rate increase, which lowers demand further. Hence, in the liquidity trap the downward pressure on inflation caused by the recession can be disastrous, digging the recession even deeper. There is no doubt that this phenomenon has been of relevance in the wake of the financial crisis particularly in several Southern-European countries. The morale is that after a negative demand shock that is so big that it pushes the economy into a liquidity trap, the government should pursue an expansionary fiscal policy to bring the economy out of the liquidity trap area and do that so fast that the harmful inflation rate dynamics described do not begin to work.

The undesirable consequences of a downward shifting AS-curve in a liquidity trap materialize whatever the reason for the downward shift may be. If the government implements structural policies intended to increase the structural level of GDP, e.g., by measures that increase labour supply, this will increase our model's $\overline{y}$ and thus shift the AS-curve to the right and downwards. If this takes effect while the economy is still in a liquidity trap, it will work to depress the economy further in the short run, because the resulting fall in actual and expected inflation will drive up the real interest rate. This suggests that structural labour market reforms that increase labour supply, which can indeed be relevant even in a deep recession, should be designed such that they are decided *during* the recession as credibly and firmly as possible, but only take effect later and hopefully *after* the recession. In that case, the immediate effects on supply should be avoided and at the same time, the expectation of higher incomes and improved government finances in the future, that the decided reforms are intended to generate, may actually stimulate demand already in the recession

A central bank that has brought its policy interest rates down to the lower bound will not necessarily have lost all its possibilities of stimulating demand further. The intention behind lowering the policy interest rate is to affect some longer-term interest rates of relevance for investment and consumption decisions and this influence goes through the yield curve as we saw in Chapter 17. If it works, it works through affecting the expectations of future short-term interest rates. Even if the very short-term policy interest rate is already at its lower bound, a further influence on longer-term interest rates may be obtained if the central bank announces that it will keep its policy rate at the very low level for a specified future period, e.g., promises that it will not raise the policy rate for another one or two years, and manages to make people believe that. This kind of policy also discussed in Chapter 17 is referred to as 'forward guidance' and has been a part of the monetary policies of the major central banks after the financial crisis. The credibility of the central bank is, of course, key to the possibility of forward guidance having a separate effect on the expectations of future short-term interest rates. Therefore, the promises of the central banks to keep their interest rates low for a period were typically backed up by statements of the central banks about their expectations of the future state of the economy. If the bank can convince the public that there is no reason to expect an increase in inflation or a normalization of activity within the next two years then it may make the public believe that it is not going to raise its short-term interest rates for another two years.

The central bank may also attempt to influence the longer-term interest rates directly by systematically buying up bonds of the relevant maturities in the financial markets, thus seeking to press bond prices upwards. As also mentioned in Chapter 17, in the aftermath of the financial crisis all the major central banks, e.g., the FED in USA, the ECB of the euro area, the Bank of Japan, and the Bank of England, have under the names of 'Quantitative Easing', QE, or 'Asset Purchase Programs', APP, engaged in massive buying of bonds in the markets in order to press longer-term interest rates down. They began with this when they realized that they could no longer hope for such an effect from reducing their short-term policy interest rates. Probably the great QE programs of the central banks have kept the relevant interest rates lower than they would otherwise have been, but on the other hand, the programmes have not been able to bring interest rates as much down as the central banks wanted. This means that even taking QE programmes into consideration, the liquidity trap has been painfully relevant and fiscal policy therefore equally relevant.

Summarizing:

FISCAL POLICY IN A LIQUIDITY TRAP

When the economy is hit by a large negative demand shock, it may become stuck in a liquidity trap where the nominal policy interest rate reaches its zero bound so that monetary interest rate policy cannot further stimulate aggregate demand. In this situation a fiscal expansion can help to pull the economy out of recession, in part via its direct positive impact on aggregate demand, and partly by increasing the actual and hence the expected rate of inflation and thereby lowering the real rate of interest. The positive effect on aggregate demand is likely to be greater when the initial budget deficit and public debt are not too high.

Summary

1. The goal of macroeconomic stabilization policy is to minimize the business cycle fluctuations in the output and inflation gaps. A forward-looking policy authority will want to minize the present value of the social losses from the current and expected future output and inflation gaps.
2. When the economy is hit by a demand shock, starting from a long-run equilibrium, the output and inflation gaps move in the same direction away from their desired levels. Monetary and fiscal stabilization policy works by influencing aggregate demand and can therefore be used to push output as well as inflation back towards their desired levels when a demand shock occurs. Hence demand shocks do not represent a dilemma for stabilization policy.
3. When the economy is hit by a supply shock, starting from a long-run equilibrium, the output and inflation gaps move in opposite directions away from their desired levels. Because stabilization policy works by influencing aggregate demand, it will always move the output and inflation gaps in the same direction. Following a supply shock, a stabilization policy that drives output back towards its desired 'natural' level will always drive inflation further away from its target level, and vice versa. Hence supply shocks represent a true dilemma for stabilization policy. The optimal policy places the economy on a 'policy line' in the output-inflation space representing the least-bad combination of the output and inflation gaps.
4. A long-standing debate in macroeconomics is whether stabilization policy should follow a fixed rule or be decided on a discretionary basis. A monetary or fiscal policy rule prescribes how the instruments of stabilization policy should be set, given the observed state of the economy. Following a fixed policy rule may help policy makers to establish credibility and to increase the predictability of economic policy so as to minimize welfare-reducing expectational errors in the private sector. The alternative to fixed policy rules is discretionary policy. Under discretion, policy makers may conduct monetary and fiscal policy in any way they believe will help advance the goals of stabilization policy, taking account of any special circumstances which might prevail. Discretion allows more flexibility than rules, but typically at the cost of reduced credibility and predictability. The evidence suggests that the monetary policy of the most important central banks is fairly well described by some form of Taylor rule, although policy makers never mechanically follow a fixed rule.
5. A benchmark scenario for analysing monetary stabilization policy is that the central bank can observe the current output and inflation gaps and react to them instantaneously and that changes in the bank's policy interest rate affect output and inflation already in the current period. If the central bank focuses exclusively on

minimizing the social loss from the output and inflation gaps in the current period, the optimal monetary policy rule takes the form of a Taylor rule where the responsiveness of the policy interest rate to the output gap should be very large in order to neutralize the effects of demand shocks, and where the policy interest rate should respond more than one-to-one to a change in the inflation gap, with a responsiveness ensuring that the economy is placed on the policy line representing the least-bad combination of the output and inflation gaps when the economy is hit by supply shocks. The larger the social cost of inflation instability, and the larger the response of inflation to the output gap, the flatter is the policy line, that is, the more emphasis is put on avoiding large inflation gaps.

6. A forward-looking central bank will realize that its policy in the current period will affect next period's economic performance by influencing the position of next period's aggregate supply curve: a higher current inflation rate will shift the AS curve for the next period upwards by raising the expected inflation rate for the next period, thereby worsening the future output-inflation trade-off. Accounting for this effect, the central bank will be keener to avoid large current inflation gaps, so the optimal monetary policy will involve a flatter policy line in the output-inflation space. Again, the optimal policy can be implemented via a Taylor rule with appropriate coefficients on the output and inflation gaps. The optimal response of the policy interest rate to the inflation gap is stronger when the central bank accounts for the future effects of its current policy than when it does not.

7. In practice monetary and fiscal policy makers do not have perfect information on the output gap, and stabilization policy is also hampered by various time lags. Because economic statistics are only available with some delay and are often revised, there is a *recognition lag* before policy makers recognize that the state of the economy has changed. In addition there may be a *decision lag* before it is decided what policy is warranted by that change. This may be followed by an *implementation lag* before the policy decided can be implemented. The sum of the recognition lag, the decision lag and the implementation lag is the *inside lag* in stabilization policy. On top of this comes the outside lag which is the time from the policy instrument has changed until it achieves its (maximum) impact on the economy. Monetary policy will typically involve a shorter inside lag but a longer outside lag than fiscal policy. Imperfect information and time lags call for a less activist stabilization policy than the optimal policy under perfect information and no time lags.

8. The outside lag in monetary policy is the period from the time the central bank changes its policy rate until the (maximum) impact on the economy is felt. A central bank charged with the responsibility to maintain a low and stable inflation rate should target a forecast for the future inflation rate within a time horizon corresponding to the outside lag, adjusting the policy rate until the inflation forecast corresponds to the target inflation rate. An optimal policy of inflation forecast targeting can be implemented through a Taylor rule in which the coefficients on the current output and inflation gaps depend on the way these gaps affect the future inflation rate.

9. Government spending on goods and services may have important supply side effects in the long run, but in the short run they work mainly by affecting aggregate demand. If the policy interest rate is not constrained, public spending policies therefore cannot achieve any short-term stabilization effect that could not have been attained through monetary policy. But since the implementation lag in fiscal policy tends to be long, many economists argue that in normal times the task of short-term stabilization policy is best left to monetary policy makers whereas fiscal policy should focus on the government's longer-term goals regarding resource allocation and income distribution. Moreover, even if fiscal policy is not actively changed in response to changes in economic activity, the automatic stabilizers (income and consumption tax rates, unemployment benefits etc.) built into the public budget will dampen the activity effects of shocks by reducing the size of the Keynesian multiplier.

10. However, when the economy is hit by a large negative demand shock, it may become stuck in a liquidity trap where the nominal policy interest rate hits its lower bound so that monetary policy cannot further stimulate aggregate demand. In this situation a fiscal expansion can help to pull the economy out of recession, in part via its direct effect on aggregate demand, and partly by increasing the rate of inflation and thereby lowering the real rate of interest. The positive effect on aggregate demand may be dampened if the budget deficit and/or the level of public debt is already high and rapidly increasing, since in this case a further deterioration of the public finances may create uncertainty and fears of large future tax increases.

Exercises

Exercise 1. Monetary stabilization policy by the Taylor rule, algebraically

In Section 20.3, we gave a graphical analysis of monetary stabilization policy by the Taylor rule. This exercise asks you to do the same algebraically. Therefore, consider the two equations (AS) and (AD) in Section 20.3.

1. Show that these imply the following solutions for output and inflation in period *t* expressed in terms of parameters, $\bar{y}$ and π^*, shocks and the expected rate og inflation in period *t*:

$$y_t = \bar{y} + \frac{z_t - a s_t}{1 + a\gamma} - \frac{a}{1 + a\gamma}\left(\pi_t^e - \pi^*\right)$$

$$\pi_t = \pi^* + \frac{\gamma z_t + s_t}{1 + a\gamma} + \frac{1}{1 + a\gamma}\left(\pi_t^e - \pi^*\right)$$

2. Starting from long-run equilibrium in the previous period, and thus maintaining from our graphical analysis the assumption that the expected inflation rate in (AS) equals the target inflation rate, show that the output and inflation gaps in period *t* are:

$$\hat{y}_t = \frac{z_t - as_t}{1 + a\gamma}, \quad \hat{\pi}_t = \frac{\gamma z_t + s_t}{1 + a\gamma}$$

3. From these expressions, taking into account how a and z_t depend on more basic parameters and shocks as given by (11),

$$a \equiv \frac{\alpha_2 h}{1 + \alpha_2 b}, \quad \text{and} \quad z_t \equiv \frac{v_t - \alpha_2 \hat{\rho}_t}{1 + \alpha_2 b},$$

verify all the results derived graphically in Section 20.3 with respect to how the effects of demand and supply shocks, respectively, in the current period depend on the policy parameters *h* and *b*. [Hint: To verify some results in connection with supply shocks ($z_t = 0,\ s_t \neq 0$), you may want to compute the ratio $\hat{y}_t / \hat{\pi}_t$]. For each result also give an intuitive, verbal explanation.

Exercise 2. The outside lag and inflation forecast targeting

In most of chapter 20, we assumed that monetary policy affects aggregate economic activity and inflation immediately, i.e., within the same period. A vast empirical literature has shown, however, that monetary policy works with lags. Central bankers should ideally consider this when setting monetary policy. This was the subject of the latter part of Section 20.4, but we will now dig a bit deeper on this point.

Consider a central bank which has been charged with the main task of securing a low and stable inflation rate at the target level π^*. In accordance with the discussion in the main text, the bank thus seeks to minimize the following loss function for any future period $t+\tau$:

$$SL_{t+\tau}^e \equiv \frac{1}{2}(E_t(\pi_{t+\tau}) - \pi^*)^2 \tag{E1}$$

As also assumed in the main text, the central bank influences the real interest rate through its *real* policy real interest rate r_t^p as follows:

$$r_t = r_t^p + \hat{\rho}_t \tag{E2}$$

where:

$$r_t^p \equiv i_t^p - \pi_{t+1}^e \tag{E2'}$$

and i_t^p is the *nominal* policy interest rate. Finally, we assume that inflation expectations are static and (first) that the IS-curve and the AS-curve are as considered in the chapter involving the same delays:

$$y_{t+1} - \bar{y} = v_{t+1} - \alpha_2 (r_t - \bar{r}) \tag{E3}$$
$$\pi_{t+1} = \pi_t + \gamma (y_t - \bar{y}) + s_{t+1} \tag{E4}$$

1. Show that if the central bank in any period chooses its policy interest rate (r_t^p or i_t^p) so as to minimize $SL_{t+\tau}^e$ for the τ for which the policy interest has an influence on $SL_{t+\tau}^e$, it follows that it will set its nominal policy interest rate according to following version of the Taylor rule:

$$i_t^p = \bar{r} + \pi_t + h(\pi_t - \pi^*) + b(y_t - \bar{y}), \qquad h \equiv \frac{1}{\alpha_2 \gamma}, \qquad b \equiv \frac{1}{\alpha_2} \tag{E5}$$

[Hint: this is almost just the derivation done in the chapter, but remember here that from static expectations, $\pi_{t+1}^e = \pi_t$]. Explain (E5) intuitively, in particular why the central bank should react to the output gap even though the loss function (E1) only includes the inflation gap.

As explained in the main text, it is realistic to assume that a change in the interest rate only affects aggregate demand with a certain time lag as assumed in (E3). Moreover, empirical research has found that it also takes a while before a change in income attains its full impact on aggregate demand. Suppose therefore that, instead of being given by (E3), the equilibrium condition for the goods market is:

$$y_{t+1} - \bar{y} = v_{t+1} + \alpha_1 (y_t - \bar{y}) - \alpha_2 (r_t - \bar{r}), \qquad \alpha_1 > 0 \tag{E6}$$

Thus, a change in current income has some positive impact on demand in the next period, say, because it takes time before consumers adapt their consumption habits to a change in income.

2. Show that the optimal monetary policy is now given by the following version of the Taylor rule.:

$$i_t^p = \bar{r} + \pi_t + h(\pi_t - \pi^*) + b(y_t - \bar{y}), \qquad h \equiv \frac{1}{\alpha_2 \gamma}, \qquad b \equiv \frac{1 + \alpha_1}{\alpha_2}. \tag{E7}$$

[Hint: use the same procedure as the one used to derive (E5)]. Compare (E7) to (E5) and give an economic explanation for the difference. Explain why the various parameters enter the coefficients *h* and *b* in the manner indicated.

The above analysis assumed static inflation expectations. Suppose instead that the central bank has full credibility so that the public's expected inflation rate is firmly anchored by the official inflation target, i.e. $\pi_t^e = \pi^*$. Given the time lags assumed in the previous question, the goods market equilibrium condition will still be (E6), while the aggregate supply curve modifies to:

$$\pi_{t+1} = \pi^* + \gamma(y_t - \bar{y}) + s_{t+1}. \tag{E8}$$

3. Show that when the structure of the economy is given by (E6) and (E8), the optimal interest rate rule is:

$$i_t^p = \bar{r} + \pi^* + b(y_t - \bar{y}), \qquad b \equiv \frac{\alpha_1}{\alpha_2} \tag{E9}$$

Try to explain why the current inflation gap does not enter in (E9). [Hint: does the current inflation gap affect the inflation rate two periods later?] Explain also why the policy interest rate should react to the current output gap and give an economic explanation for the determinants of the coefficient *b* in (48).

Exercise 3. Creating disinflation by monetary policy: the role of expectations

The equations (IS) through (AS) below are part of a macroeconomic model for a closed economy. The notation is the same as in the chapter, e.g., v_t is a demand shock and s_t is a supply shock. The description of expectations formation remain.

$$y_t - \bar{y} = v_t - \alpha_2\left(r_t - \bar{r}\right) \tag{IS}$$

$$r_t = i_t - \pi_{t+1}^e \tag{RR}$$

$$i_t = \bar{r} + \pi_{t+1}^e + h(\pi_t - \pi^*), \qquad h > 0 \tag{MP}$$

$$\pi_t = \pi_t^e + \gamma(y_t - \bar{y}), \qquad \gamma > 0 \tag{AS}$$

We define the output and inflations gaps: $\hat{y}_t \equiv y_t - \bar{y}$ and $\hat{\pi}_t \equiv \pi_t - \pi^*$.

1. Explain Equation (MP), describe what type of monetary policy it expresses, and giver an interpretation of each of the parameters α_2, h and γ. Show that (IS), (RR) and (MP) imply:

$$y_t - \bar{y} = v_t - \alpha_2 h\left(\pi_t - \pi^*\right), \tag{AD}$$

and give an interpretation of this equation.

We will consider a situation where the rate of inflation for some time up to and including period 0 (zero) has been constant and relatively high at the level π^{high}. This has been accommodated by the central bank by the target inflation rate π^* having been equal to π^{high} during this period. After a new central bank governor has been appointed, the central bank decreases the inflation target π^* to π^{low}, where $0 < \pi^{\text{low}} < \pi^{\text{high}}$. The new target inflation rate is announced in period 0 and takes effects from period 1 and onwards. It is assumed that during the periods up to and including period 0, that is, for $t \leq 0$, the economy has been in a long-run equilibrium with $v_t = s_t = 0$, $y_t = \bar{y}$ and $\pi_t = \pi_t^e = \pi^* = \pi^{\text{high}}$. We also assume until further notice that for the periods $t \geq 1$, $v_t = s_t = 0$. And until further notice we assume static inflation expectations:

$$\pi_t^e = \pi_{t-1} \tag{SE}$$

The macroeconomic model then consists of (AS), (AD) and (SE).

2. What is the expected inflation rate π_1^e for period 1 created in period 0? In a $y-\pi-$ diagram (with y_t along the horizontal axis etc.), illustrate the (short-run) AS and AD curves relevant for period 0 and 1, respectively, and indicate the long-run equilibrium for period 0 and the short-run equilibrium for period 1. Explain why $y_1 < \bar{y}$ and $\pi^{\text{low}} < \pi_1 < \pi^{\text{high}}$.

3. Show that output and inflation in period 1 according to the model are: :

$$y_1 = \bar{y} - \frac{\alpha_2 h}{1+\alpha_2 h\gamma}\left(\pi^{\text{high}} - \pi^{\text{low}}\right) \tag{1}$$

$$\pi_1 = \pi^{\text{low}} + \frac{1}{1+\alpha_2 h\gamma}\left(\pi^{\text{high}} - \pi^{\text{low}}\right) \tag{2}$$

Explain, graphically and on basis of (1) and (2), how output (gap) and inflation (gap) in period 1 depend on $\alpha_2 h$ and γ, respectively, and explain the result. Based on the calibration $\alpha_2 = 1$, $h = \frac{1}{2}$ and $\gamma = \frac{1}{3}$, which is reasonable on an annual basis, assess the level og inflation and the percentage output loss in period 1 (the first period after the lowering of the inflation target) assuming $\pi^{\text{high}} = 0.12$ (12 percent) and $\pi^{\text{low}} = 0.02$ (2 percent), both on annual basis.

4. Illustrate (in the $y-\pi-$diagram) and explain the adjustments of the economy over the periods coming after period 1. What do y_t and π_t, respectively, converge to in the long run? Is something fishy about expectations formation here? In your answer to this, involve the relation between expected and actual inflation over the adjustment periods.

5. Show that the model consisting of (AS), (AD) and (SE) implies the following dynamic equation for the output gap:

$$\hat{y}_t = \beta \hat{y}_{t-1}, \quad \beta \equiv \frac{1}{1+\alpha_2 h\gamma}, \quad 0 < \beta < 1 \tag{3}$$

Define the cumulated output gap, $\hat{y} = \hat{y}_1 + \hat{y}_2 + \hat{y}_3 + \cdots$. Then $\hat{y}$ is the cumulated output loss relative to one period's structural GDP in consequence of the lowering of the inflation target.

6. Show that: :

$$\hat{y} = -\frac{1}{\gamma}\left(\pi^{\text{high}} - \pi^{\text{low}}\right) \tag{4}$$

[Hint: $1+\beta+\beta^2+\cdots = 1/(1-\beta)$]. Explain (4) intuitively. The cumulated percentage output loss per percentage point that the inflation rate is attempted lowered is referred to as 'sacrifice ratio'. What is the sacrifice ratio here? Asses the sacrifice ratio for the reasonable calibration above. Still with this calibration, what is the relative cumulated output loss if the central bank wishes to bring down inflation by 10 percentage points? Comment on the magnitude: does it in your eyes have a high cost to bring down inflation by that much?

[This exercise is continued in Chapter 21, where the results obtained here will be contrasted to the results obtained by assuming so-called rational expectations. The full exercise over the two chapters will illustrate to you both the motivation for and the striking consequences of rational expectations].

Exercise 4. Discretionary fiscal policy outside and in inside a liquidity trap

Consider a slightly simplified version of the modelling framework used for studying fiscal stabilization policy outside and in a liquidity trap in Section 20.6. The equilibrium condition for the output market (the IS curve) is:

$$y_t - \bar{y} = \Delta_t - \alpha_2 (i_t - \pi^e_{t+1} - \bar{r}) + v_t \tag{IS}$$

where the fiscal policy impulse, $\Delta_t \equiv \alpha_1 (g_t - \bar{g}) - \alpha_3 (\tau_t - \bar{\tau})$, includes Keynesian multiplier effects. The AS curve is:

$$\pi_t = \pi^e_t + \gamma (y_t - \bar{y}) \tag{AS}$$

where we have assumed no supply shocks ($s_t = 0$). As in the chapter, we (first) consider the π^e_t in the AS-curve as an exogenous variable. The background is that the current nominal wage contracts negotiated in period *t*-1 and based on the expected inflation rate π^e_t formed there are until further notice taken as given, that is, until further notice we consider a period during which wage contracts are not re-negotiated. However, when they are, we assume static expectations, $\pi^e_t = \pi_{t-1}$.

The monetary policy of the central bank implies the following Taylor rule for the relevant, longer-term nominal interest rate:

$$i_t = \bar{r} + \pi^e_{t+1} + h(\pi_t - \pi^*) + b(y_t - \bar{y}) \tag{MP}$$

as long as this rule does not imply a negative value of i_t. If it does, that is, in the liquidity trap, the central bank will have to choose $i_t = 0$.

Whenever we need an expectations hypothesis for the π^e_{t+1} in (IS) and in (MP), we assume again static expectations, $\pi^e_{t+1} = \pi_t$.

1. First show that if the economy is not in the liquidity trap, so that (MP) applies, then no matter how the expectation π^e_{t+1} is formed, as long as it is the same in (IS) and (MP), these two equations imply the AD-curve:

$$y_t - \bar{y} = \frac{\Delta_t}{1+\alpha_2 b} - a(\pi_t - \pi^*) + \frac{v_t}{1+\alpha_2 b}, \quad a \equiv \frac{\alpha_2 h}{1+\alpha_2 b} \tag{AD^{out}}$$

2. Then show that (AS) and (AD^{out}) imply the following solution for output and inflation in period *t outside the liquidity trap*:

$$y_t = \bar{y} + \frac{1}{1+a\gamma}\frac{\Delta_t}{1+\alpha_2 b} + \frac{1}{1+a\gamma}\frac{v_t}{1+\alpha_2 b} - \frac{a}{1+a\gamma}\left(\pi_t^e - \pi^*\right) \qquad \text{(E10)}$$

$$\pi_t = \pi^* + \frac{\gamma}{1+a\gamma}\frac{\Delta_t}{1+\alpha_2 b} + \frac{\gamma}{1+a\gamma}\frac{v_t}{1+\alpha_2 b} + \frac{1}{1+a\gamma}\left(\pi_t^e - \pi^*\right) \qquad \text{(E11)}$$

The fiscal multiplier is defined as $\partial y_t / \partial \Delta_t$.

3. Show that *outside the liquidity trap*, the fiscal multiplier is:

$$\frac{\partial y_t}{\partial \Delta_t} = \frac{1}{1+a\gamma}\frac{1}{1+\alpha_2 b} \qquad \text{(E12)}$$

Explain intuitively the elements in this. Comment on the interaction between fiscal and monetary policy: how does it affect the fiscal multiplier that $h > 0$ and $b > 0$, compared to a situation where $h = b = 0$?

In the following we assume that the economy is hit by a large negative demand shock $v_t < 0$, so that (according to (E10) and (E11)), the output and inflation gaps become negative and so small that the Taylor rule would imply a strictly negative value of i_t. In that case, monetary policy will imply $i_t = 0$.

4. Show that (IS) combined with $i_t = 0$ and assuming static expectations with respect to π_{t+1}^e gives the following AD-curve in the liquidity trap:

$$y_t - \bar{y} = \Delta_t + \alpha_2(\pi_t + \bar{r}) + v_t \qquad \text{(AD}^{\text{in}}\text{)}$$

Explain intuitively why higher inflation gives higher output demand according to (AD$^{\text{in}}$).

5. Now show that (AS) and (AD$^{\text{out}}$) imply the following solution for output and inflation in period *t in the liquidity trap* (where it is assumed that $\alpha_2\gamma < 1$, which, as argued in the chapter, is reasonable):

$$y_t = \bar{y} + \frac{1}{1-\alpha_2\gamma}\Delta_t + \frac{\alpha_2}{1-\alpha_2\gamma}\pi^e + \frac{\alpha_2}{1-\alpha_2\gamma}\bar{r} + \frac{1}{1-\alpha_2\gamma}v_t \qquad \text{(E13)}$$

$$\pi_t = \frac{\gamma}{1-\alpha_2\gamma}\Delta_t + \frac{1}{1-\alpha_2\gamma}\pi^e + \frac{\alpha_2\gamma}{1-\alpha_2\gamma}\bar{r} + \frac{\gamma}{1-\alpha_2\gamma}v_t \qquad \text{(E14)}$$

6. Show that *inside the liquidity trap* the fiscal multiplier is:

$$\frac{\partial y_t}{\partial \Delta_t} = \frac{1}{1-\alpha_2 \gamma} \qquad \text{(E15)}$$

Compare to the fiscal multiplier in (E12) and explain the difference intuitively.

7. Assume that the economy remains in the liquidity trap for so long that the wage contracts get renegotiated and new wages set in view of the new and lower inflation rate observed *after* the negative shock that first sent the economy into the liquidity trap. How will the lower expected inflation affect output and activity in the liquidity trap? How would lower expected inflation affect activity outside the liquidity trap? Explain the difference.

Exercise 4. Explaining time lags and their implications

Explain why there may be time lags in macroeconomic stabilization policy and why such lags reduce the ability of policy makers to stabilize the economy. Explain how monetary policy makers may try to cope with time lags through a policy of inflation forecast targeting. Discuss whether time lags are a more serious problem for fiscal than for monetary policy.

Chapter 21

Stabilization policy with rational expectations

Introduction

Economic activity today depends crucially on expected economic conditions tomorrow. A drop in the economy's expected future growth rate will tend to reduce the propensities to consume and invest by reducing the expected future earnings of households and firms. Hence, the aggregate demand curve will shift down, causing an immediate fall in current output. As another example, a change in the expected rate of inflation will shift the aggregate supply curve by feeding into the nominal wages negotiated by workers and firms. It may also move the aggregate demand curve through its impact on the expected real rate of interest. The expected inflation rate is thus an important determinant of current economic activity.

Conventional macroeconomic models often assume that the expected future values of economic variables depend only on the past history of those variables. Indeed, in the previous chapters we typically postulated that the expected inflation rate for the current period is a simple function of past inflation rates and often simply equal to the actual inflation rate experienced in the previous period. This assumption of *backward-looking expectations* may be plausible in 'quiet' times when the macroeconomy is not subject to significant shocks. When people have no particular reason to believe that the tightness of labour and product markets next year will be much different from what it is today, it seems reasonable for them to assume that next year's inflation rate will be more or less the same as this year's. However, if the economy is hit by an obvious and visible shock such as a dramatic change in the price of imported oil, or if there is a clear change in the economic policy regime, say, due to a change of government or central bank governor, it does not seem rational for people to assume that next year's economic environment will be the same as this year's. Instead of just mechanically extrapolating the past into the future, rational households and firms will seek to utilize all the relevant information available to them when they form expectations about the future state of the economy.

In the early 1970s, some macroeconomists often referred to as the new-classical economists took this idea of *forward-looking expectations* to its logical limit by advancing the *rational expectations hypothesis* (REH).[1] According to the REH, people use all the available information to make the best possible forecasts of the economic variables which are relevant to them. Moreover, *the available information includes information about the structure of the economy*. The idea is that, even though in practice the layman may not know much about the way the economy works, the economic forecasts produced by professional economists are available to the public through the media, so in this way people have access to the most competent forecasts of, say, next year's rate of inflation. Economists should therefore model the formation of expectations *as if* people use the relevant economic model to predict inflation and other economic variables which are important for their economic decisions. In other words, rational expectations are *model-consistent* expectations: they are identical to the forecasts one would make by using the available knowledge of the structure of the economy, as embodied in the relevant economic models. Another way of putting it is to say that economic analysts should not assume that they are smarter than the economic agents whose behaviour they are trying to predict. Instead, they should assume that agents form their expectations in accordance with the analysts' own description of the economy. If they did not, and if the analysts' model is correct, then agents would be making systematic expectational errors, and presumably this would induce them to change the rules of thumb by which they form their expectations until there is no discernible systematic pattern in their forecast errors. We can therefore also describe rational expectations as 'expectations without *systematic* expectational errors', but there can still be actual (unsystematic) expectational errors due to the influence of economic shocks that the agents could not possibly have foreseen.

This idea of rational expectations essentially revolutionized macroeconomic theory. The REH is obviously a very strong assumption, and as we shall see, it can be criticized on theoretical as well as empirical grounds. But before addressing these criticisms, this chapter will explain the case for the REH in more detail and derive some of its striking implications. Our main purpose is to illustrate the importance of the way expectations are formed. In particular, we will show how the effects of macroeconomic stabilization policy may differ significantly depending on whether expectations are rational or backward-looking. We will also show how the effects of stabilization policy under rational expectations depend on the amount of information economic agents and policy makers have when they form their expectations and take their actions. The final section of the chapter will discuss the validity of the REH, drawing on theoretical arguments as well as empirical evidence.

[1] The rational expectations hypothesis was originally introduced in a microeconomic setting by John Muth, 'Rational Expectations and the Theory of Price Movements', *Econometrica*, **29**, 1961, pp. 315 – 335. Later on, the REH was introduced into macroeconomic theory by Robert E. Lucas, 'Expectations and the Neutrality of Money', *Journal of Economic Theory*, **4**, 1972, pp. 103 –124, and by Thomas J. Sargent, 'Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment', *Brookings Papers on Economic Activity*, **2**, 1973, pp. 429 – 472; followed by many others.

21.1 Backward-looking versus forward-looking expectations

The case against backward-looking expectations

One way of justifying the assumption of rational expectations is to examine more carefully the implications of a macro model with backward-looking expectations. As we will illustrate in this section, in some circumstances the assumption of backward-looking expectations implies that economic agents are implausibly naïve.

We base our discussion on the model of aggregate demand and aggregate supply developed in Chapters 15 through 19 and used intensively in Chapter 20. We simplify by abstracting from fiscal stabilization policy ($g_t = \bar{g}$ and $\tau_t = \bar{\tau}$), so that demand shocks only stem from our 'confidence' variable v_t (this simplification is in no way critical for our results; it just reflects that we can illustrate our main points by focusing on monetary stabilization alone). In the usual notation, our AS–AD model may then be restated as follows:

Goods market equilibrium: $$y_t - \bar{y} = v_t - \alpha_2(r_t - \bar{r}), \qquad \text{(IS)}$$

Monetary policy rule: $$r_t = \bar{r} + h(\pi_t - \pi^*) + b(y_t - \bar{y}) + \hat{\rho}_t \qquad \text{(MP)}$$

Aggregate supply or price formation: $$\pi_t = \pi_t^e + \gamma(y_t - \bar{y}) + s_t, \qquad \text{(AS)}$$

Static expectations: $$\pi_t^e = \pi_{t-1}. \qquad \text{(SE)}$$

Equation (SE) assumes the particularly simple form of backward-looking (or adaptive) expectations called static expectations, postulating that the expected inflation rate for the current period equals the actual inflation rate observed during the previous period. This suffices (but is in no way essential) for illustrating the implications of backward-looking expectations we want to point out. To do this we now solve the model.

For the moment, we will simplify further by setting the exogenous aggregate demand and supply shock variables as well as the imprecision shock in monetary policy equal to their zero mean values, $v_t = s_t = \hat{\rho}_t = 0$. On this assumption, we find by substituting (MP) into (IS) (as we have seen several times before) the aggregate demand curve:

$$y_t - \bar{y} = a(\pi^* - \pi_t), \qquad a \equiv \frac{\alpha_2 h}{1 + \alpha_2 b}, \qquad (1)$$

while substitution of (SE) into (AS) yields the aggregate supply curve:

$$\pi_t = \pi_{t-1} + \gamma(y_t - \bar{y}). \qquad (2)$$

Inserting (1) into (2) and rearranging, we obtain the first-order linear difference equation:

$$\pi_t = \pi_{t-1} + \gamma a(\pi^* - \pi_t) \quad \Leftrightarrow \quad \pi_t - \pi^* = \pi_{t-1} - \pi^* - \gamma a(\pi_t - \pi^*) \quad \Leftrightarrow$$
$$\pi_t - \pi^* = \beta(\pi_{t-1} - \pi^*), \quad \beta \equiv \frac{1}{1+\gamma a}, \tag{3}$$

which has the solution:

$$\pi_t = \pi^* + \beta^t(\pi_0 - \pi^*), \quad t = 0,\ 1,\ 2,\ \ldots \tag{4}$$

where π_0 is the predetermined initial value of the inflation rate in period 0. Since we see from (3) that $0 < \beta < 1$, it follows from (4) that the inflation rate will converge monotonically towards its target rate π^* as t tends to infinity.

From (4) we may calculate the *inflation forecast error*, defined as the difference between the actual and the expected inflation rate. Given the assumption of static expectations, $\pi_t^e = \pi_{t-1}$, the inflation forecast error during the phase of adjustment to long-run equilibrium is:

$$\pi_t - \pi_t^e = \pi_t - \pi_{t-1} = [\pi^* + \beta^t(\pi_0 - \pi^*)] - [\pi^* + \beta^{t-1}(\pi_0 - \pi^*)] \quad \Leftrightarrow$$
$$\pi_t - \pi_t^e = \beta^{t-1}(\pi_0 - \pi^*)(\beta - 1). \tag{5}$$

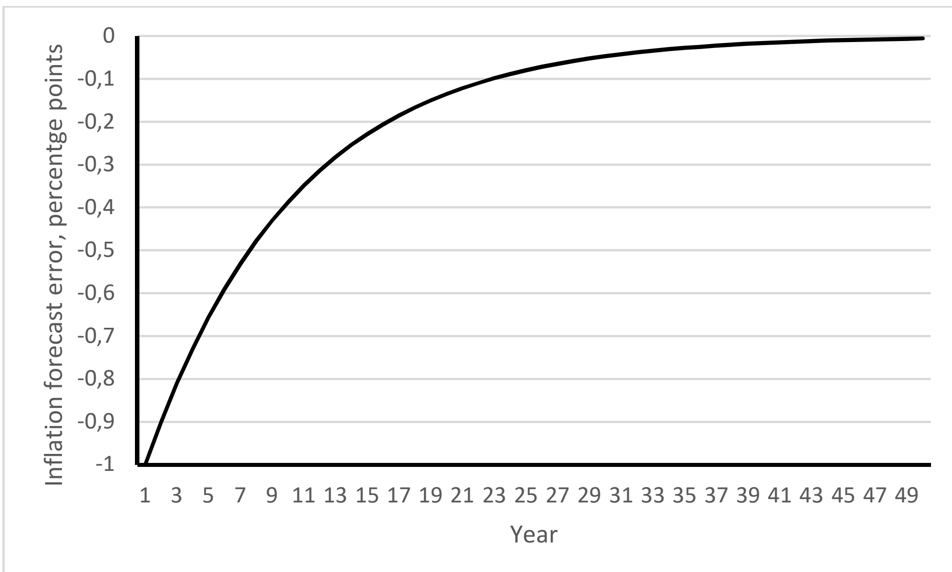
Suppose now that in period 0, the government appoints a 'tough' new central bank governor who announces a significant reduction in the target inflation rate from the start of period 1. For concreteness, suppose the inflation target π^* is reduced from 12 per cent to 2 per cent per year.[2] Using empirically plausible parameter values like those reported in Chapter 19, involving a β around 0.9 on annual basis, we may then simulate the evolution of the inflation forecast error implied by Equation (5), assuming that the initial inflation rate π_0 was equal to the previous inflation target of 12 per cent. Hence, $\pi_0 - \pi^* = 0.12 - 0.02 = 0.1$, and then from (5), $\pi_1 - \pi_1^e = 0.9^0 \cdot 0.1 \cdot (0.9-1) = -0.01$ or -1 percentage point, while $\pi_2 - \pi_2^e = 0.9^1 \cdot 0.1 \cdot (0.9-1) = -0.009$ or -0.9 percentage points etc. The result of the simulation is shown in Fig. 21.1.

We see that throughout the period of adjustment to the new inflation target of 2 per cent per year, *the public systematically overestimates the actual rate of inflation*. The reason for these systematic mistakes in forecasting inflation is that the public mechanically extrapolates last period's observed inflation rate into the future. Thus, even though the central bank announces its determination to bring inflation down by setting a

[2] The situation described is reminiscent of the one considered in Exercise 3 of Chapter 20, which continues in an exercise for this chapter.

high interest rate as long as $\pi_t > \pi^*$, people nevertheless continue to believe period after period that next year's inflation rate will be the same as this year's. Clearly, this behaviour is not very intelligent. Figure 21.1 illustrates how people keep on making forecast errors in the same *systematic* and *predictable* way. But if the expectation errors are predictable, why go on making them? Presumably, informed citizens will observe or at least gradually learn that the monetary policy regime has changed, and this should affect the way they form their expectations of inflation.

Figure 21.1 The inflation forecast error during a disinflation with static expectations (simulation)



This is a statement of the case against the assumption of backward-looking expectations: if important aspects of the economic environment (such as the economic policy regime) change, rational agents are likely to realize that the future path of the economy cannot be projected by simply observing how the economy behaved in the past. To put it another way, rational economic agents will utilize all relevant information when they form their expectations about the future, including information about changes in economic policy and other new developments which are likely to influence the course of the economy.

Defining rational expectations

The rational expectations hypothesis takes the above line of reasoning one step further by suggesting that *economic agents do not make systematic forecast errors* of the kind illustrated in Fig. 21.1. To be sure, since the economy is often hit by stochastic shocks (which were ignored for simplicity in Fig. 21.1), agents usually *do* make mistakes when they try to predict the future state of the economy. But according to the REH there will be *no systematic bias* in these forecast errors. For example, sometimes the rate of inflation will be overestimated, and sometimes it will be underestimated, but on average people's inflation forecasts will be correct. The justification for this assumption is that economic actions based on erroneous expectations cause losses of profits and utility, and hence agents have an incentive to minimize their forecast errors. If the forecast errors revealed a systematic pattern such as persistent overestimation or underestimation, rational agents should be able to detect this pattern and would have an incentive to revise their methods of expectations formation to weed out systematic biases in their guesses about the future.

To define rational expectations formally, suppose the economy may be described by relationships such as the IS and AS curve above, a specification of economic policy like the Taylor rule above, and various stochastic shock processes like for our v_t and s_t above. The economic variables about which agents form their expectations will then be stochastic variables because of the influence of the stochastic shocks, as we also saw in Chapter 19. The 'solution' for an economic variable X_t will not be a deterministic value, but will take different values with different probabilities. In such a setting the subjective expectations of the economic agents will then plausibly also be stochastic variables where, for instance, different values of the coming rate of inflation are expected with different probabilities. If we denote by $f_{t-1}^{e}(X_t^j)$ the probability that economic agents in period t-1 attach to the variable X_t taking the value X_t^j, the mean expected value (the mathematical expectation) of that variable will be $X_{t,t-1}^{e} \equiv \sum_j f_{t-1}^{e}(X_t^j)X_t^j$. Note that we now use a notation where $X_{t,t-1}^{e}$ specifies both when the expectation is formed (t-1) and for which period it is valid (t). As a simplification, we will assume that only the mean value $X_{t,t-1}^{e}$ of expectations matters to the agents, essentially an assumption of risk neutral agents. We can then formulate expectations as so-called 'point expectations' where the value $X_{t,t-1}^{e}$ is expected with certainty. This greatly simplifies matters, but is not essential to our results.

The economic model will yield an equilibrium value of each economic variable X_t as a function of the model's various exogenous variables including policy reaction parameters, the agents' subjective expectations, $X_{t,t-1}^{e}$, and the realizations of the model's stochastic shocks. In this way, X_t becomes a stochastic variable that with a probability distribution that depends on the probability distributions of the shock variables, and *this stochastic variable X_t depends on the subjective expectations*, $X_{t,t-1}^{e}$. Let I_{t-1} be the set of information that the agents have when they form their expectation, $X_{t,t-1}^{e}$. This will typically include realizations of economic variables up to and including period t-1 and the agents' knowledge of the economic structure and policy rules, but not the specific shock realizations of period t. Let $f(X_t^j \mid I_{t-1})$ be the *objective* probability for X_t taking the value X_t^j according to the model, conditional on the information contained in I_{t-1}. As mentioned, the objective probability distribution $f(X_t^j \mid I_{t-1})$ will depend on the agents' subjective expectations $X_{t,t-1}^e$. Finally, define the mean value (or mathematical expectation) of the objective distribution, $E[X_t \mid I_{t-1}] \equiv \sum_j f(X_t^j \mid I_{t-1}) X_t^j$, which again depends on $X_{t,t-1}^e$. Rational expectations are then defined by the requirement:

$$\overbrace{X_{t,t-1}^e}^{\substack{\text{subjective}\\\text{expectation}}} = \overbrace{E[X_t \mid I_{t-1}]}^{\substack{\text{objective}\\\text{conditional}\\\text{expectation}}} \tag{RE}$$

In words and in general, the hypothesis of rational expectations says that the *subjective* expectation of some economic variable X for time t must equal the *objective* mathematical expectation of X, conditional on all information available to the economic agents at the time the expectation is formed. Thus, if the expectation for period t is formed in period $t-1$, rational expectations are exactly expressed by (RE) above.

A more strict definition of rational expectations would require accordance between the entire subjectively expected distribution and the objective one, that is, $f_{t-1}^e(X_t^j) = f(X_t^j \mid I_{t-1})$ for all X_t^j from which (RE) would obviously follow. As mentioned, we make the simplification that only the mean value of their expectations matters to the agents in which case (RE) is the relevant formulation of rational expectations.

The specification in (RE) captures the idea that expectations are on average correct, even though they are hardly ever precisely correct, given the unpredictable stochastic shocks which are hitting the economy all the time. Formally, Equation (RE) states that economic agents know enough about the stochastic process determining X_t to be able to calculate the correct conditional mean value of X_t. In this sense, rational expectations are *model-consistent*: the rationally expected value of X_t is equal to the mean value of this variable that one would calculate from the correct stochastic economic model describing the determination of X_t. Thus agents form their expectations *as if* they knew the 'correct' model of the economy (or the model of the subsector of the economy in which they are interested), even though in practice they may have arrived at their forecasting rules through a more intuitive trial-and-error learning process. In summary:

THE RATIONAL EXPECTATIONS HYPOTHESIS (REH)

The REH says that agents optimally exploit all available information when they form their expectations of the values of future economic variables. Hence agents do not make systematic forecast errors, for if they did, they could use observations of past forecast errors to correct their forecasting rules to eliminate any *systematic* bias in their forecasts. Effectively the REH thus assumes that agents form their expectations as if they knew the 'true' structure of the economy.

In further defence of the obviously strong assumption of rational expectations there is also the view that at least it must be of interest to uncover the robustness of some economic prediction, say on the implications of a certain economic policy, to the state of affairs where agents do not make systematic expectations errors: if the predicted policy implications *only* hold if agents are assumed to make systematic expectations errors, these implications can be considered to be on shaky ground.

Later on, we will discuss the realism of the REH, but first we will explore some of its striking implications.

21.2 Rational expectations and stabilization policy: the debate on ineffectiveness vs. effectiveness and optimal inflation targeting

We will now use our model of aggregate demand and aggregate supply to study how rational expectations affect the scope for macroeconomic stabilization policy.

Our model will essentially consist of the relationships (IS), (MP) and (AS) above, now denoting the expected rate of inflation occurring in (AS) by $\pi^e_{t,t-1}$, combined with the assumption of rational expectations. As usual, the expected *future* rate of inflation that would now be denoted by $\pi^e_{t+1,t}$ and appear in the definition of the real interest rate, $r_t = i_t - \pi^e_{t+1,t}$, is omitted on the ground that the central bank and the public are assumed to have the same expectation. To complete the model, one must specify the stochastic properties of the exogenous demand, supply and policy shock variables, v_t, s_t and $\hat{\rho}_t$, respectively. For simplicity we assume that these variables are 'white noise', being distributed identically and independently over time and independently of each other, with zero means and constant variances σ^2_v, σ^2_s and σ^2_ρ, respectively.

The Policy Ineffectiveness Proposition

As a starting point, suppose that when the central bank sets the interest rate for period

t, it does not have more information than the private sector agents have when they negotiate wage contracts for period t. Thus the central bank does not know what the actual levels of inflation and output will be in period t when it sets the interest rate for that period. Instead, because macroeconomic data are only available with a time lag, the central bank has to base its interest rate decision on the *expected* inflation rate $\pi^e_{t,t-1}$ and on the *expected* general activity level $y^e_{t,t-1}$. With this modification of (MP) above our model becomes:

Goods market equilibrium: $$y_t - \bar{y} = v_t - \alpha_2(r_t - \bar{r}), \quad \text{(IS)}$$

Monetary policy rule: $$r_t = \bar{r} + h(\pi^e_{t,t-1} - \pi^*) + b(y^e_{t,t-1} - \bar{y}) + \hat{\rho}_t \quad \text{(MP')}$$

Aggregate supply or price formation: $$\pi_t = \pi^e_{t,t-1} + \gamma(y_t - \bar{y}) + s_t, \quad \text{(AS)}$$

Rational expectations: $$\pi^e_{t,t-1} = E(\pi_t \mid I_{t-1}), \quad y^e_{t,t-1} = E(\pi_t \mid I_{t-1}). \quad \text{(RE)}$$

To solve a model like this with rational expectations, we must derive the expectations $\pi^e_{t,t-1}$ and $y^e_{t,t-1}$ which are consistent with the model, given the information available at time $t-1$. We will solve the model by going through the following three steps:[3]

Step 1: Solve the model for the endogenous variables y_t and π_t as functions of the exogenous variables, the shocks and the expectations variables $y^e_{t,t-1}$ and $\pi^e_{t,t-1}$.

Step 2: Find the solutions for $y^e_{t,t-1}$ and $\pi^e_{t,t-1}$ by calculating the expected value of the expressions found in Step 1, given the information available at time t-1, that is, determine $y^e_{t,t-1}$ and $\pi^e_{t,t-1}$ by requiring $\pi^e_{t,t-1} = E(\pi_t \mid I_{t-1})$ and $y^e_{t,t-1} = E(\pi_t \mid I_{t-1})$, where y_t and π_t must be the functions found in Step 1.

Step 3: Insert the solutions for $y^e_{t,t-1}$ and $\pi^e_{t,t-1}$ into the functions found in Step 1 to obtain the final solutions for y_t and π_t in terms of the exogenous variables. Hereby, y_t and π_t become functions of exogenous variables and shocks alone.

[3] The solution procedure described here works in simple models like the present one, which only includes expectations relating to the current period. Models that are more general typically also include expected values of variables for one or several future time periods. The solution of such models requires more advanced techniques, which will be left for a future macro course. However, we give an illustration of these techniques in the subsection on 'announcement effects' below.

Let us illustrate the mechanics of this procedure.

Step 1. *Solve the model for y_t and π_t in terms of the exogenous variables, the shocks and the expectations variables.* Inserting (MP') into (IS), we find:

$$y_t = \bar{y} + v_t - \alpha_2[h(\pi^e_{t,t-1} - \pi^*) + b(y^e_{t,t-1} - \bar{y}) + \hat{\rho}_t]. \qquad (6)$$

If we substitute (6) into (AS), we get:

$$\pi_t = \pi^e_{t,t-1} - \gamma\alpha_2[h(\pi^e_{t,t-1} - \pi^*) + b(y^e_{t,t-1} - \bar{y}) + \hat{\rho}_t] + \gamma v_t + s_t. \qquad (7)$$

We have now expressed the actual values of y_t and π_t as functions of exogenous variables, shock realizations and the expected values of y_t and π_t, as required in Step 1.

Step 2. *Find $y^e_{t,t-1}$ and $\pi^e_{t,t-1}$ by taking expected values of the functions found in Step 1, here (6) and (7), conditional on information available at time t−1.* The rationally expected value of output in period t is the mean value of the expression on the right-hand side of (6), calculated on the basis of information available at time t-1. In calculating this conditional expectation, we may use the facts that $E[\pi^e_{t,t-1} \mid I_{t-1}] = \pi^e_{t,t-1}$ and $E[y^e_{t,t-1} \mid I_{t-1}] = y^e_{t,t-1}$. This simply says that agents do, of course, know at time $t-1$ what their own (point) expectations are at that time. Furthermore, since the shocks are assumed to be white noise, $E(v_t \mid I_{t-1}) = E(\hat{\rho}_t \mid I_{t-1}) = 0$. From (6) we then find by taking mathematical expectations $E(\cdot \mid I_{t-1})$ on both sides that the rationally expected value of output in period t is:

$$y^e_{t,t-1} = E(y_t \mid I_{t-1}) = \bar{y} - \alpha_2[h(\pi^e_{t,t-1} - \pi^*) + b(y^e_{t,t-1} - \bar{y})], \qquad (8)$$

given that agents are assumed to know the structure of the economy, including the values of the parameters appearing in Equation (19). In a similar way, since $E(v_t \mid I_{t-1}) = E(s_t \mid I_{t-1}) = E(\hat{\rho}_t \mid I_{t-1}) = 0$, we may use Equation (7) to calculate the rationally expected inflation rate for period t:

$$\begin{aligned} &\pi^e_{t,t-1} = E(\pi_t \mid I_{t-1}) = \pi^e_{t,t-1} - \gamma\alpha_2[h(\pi^e_{t,t-1} - \pi^*) + b(y^e_{t,t-1} - \bar{y})] \quad \Leftrightarrow \\ &h(\pi^e_{t,t-1} - \pi^*) + b(y^e_{t,t-1} - \bar{y}) = 0. \end{aligned} \qquad (9)$$

Inserting (9) into (8), we get:

$$y^e_{t,t-1} - \overline{y}. \tag{10}$$

which may then be substituted into (9) to give (assuming that $h \neq 0$):

$$\pi^e_{t,t-1} = \pi^*. \tag{11}$$

Step 3. *Find the final solutions for π_t and y_t by inserting the solutions for the rationally expected variables found in Step 2, here (10) and (11), into the expressions for the endogenous variables found in Step 1, here (6) and (7).* Taking this final step, we find:

$$y_t = \overline{y} + v_t - \alpha_2 \hat{\rho}_t. \tag{12}$$

$$\pi_t = \pi^* + \gamma v_t + s_t - \alpha_2 \hat{\rho}_t. \tag{13}$$

A first remark on this solution is that there *will* be expectational errors since, e.g., according to (10) and (12), $y_t - y^e_{t,t-1} = v_t - \alpha_2 \hat{\rho}_t$, which will typically be different from zero, but there will be no *systematic* expectational errors since $E(y_t - y^e_{t,t-1} \mid I_{t-1}) = 0$.

Second, the systematic policy parameters π^*, b and h do not appear in the solution for y_t given in (12) and only π^* appears in the solution for π_t in (13). Hence systematic monetary stabilization policy does not affect the evolution of real output! This is the famous *Policy Ineffectiveness Proposition (PIP)*, which says that *systematic demand management policies cannot influence real output and employment when expectations are rational*.

Third, the unsystematic and uncontrollable part of stabilization policy $\hat{\rho}_t$ *does* affect real output. This was a major point of the new-classical economists: even though one observes a clear negative correlation between interest rates and activity empirically, this cannot be used to stabilize output exactly because it is due to the unsystematic and unpredictable element of the stabilization rule.

To understand the intuition for PIP, note that the aggregate supply curve (AS) may be rearranged as:

$$y_t = \overline{y} + (1/\gamma)(\pi_t - \pi^e_{t,t-1}) + (1/\gamma)s_t. \tag{14}$$

Since $\overline{y}$ and s_t are exogenous, the supply curve (14) implies that monetary policy can only affect real output by influencing the inflation forecast error $\pi_t - \pi^e_{t,t-1}$, that is, by creating unanticipated inflation. But since wage and price setters have the same information on macroeconomic data as the central bank, and since they know the monetary policy rule (MP') as well as the way the interest rate affects the economy through its impact on aggregate demand, *private agents can perfectly anticipate the effect of systematic monetary policy on the current inflation rate*. Thus, systematic monetary policy (whether it takes the form of a Taylor rule or some other fixed policy rule) cannot generate surprise inflation, and hence it cannot cause output and employment to deviate from their natural rates.

THE POLICY INEFFECTIVENESS PROPOSITION

The Policy Ineffectiveness Proposition says that when expectations are rational, *systematic* monetary (or fiscal) policy cannot influence 'real' economic variables such as output, employment, real wages and the real interest rate. Demand management policy can only influence these variables by generating unanticipated inflation. When agents have rational expectations, they fully anticipate the effects of any systematic policy on the rate of inflation. Hence, systematic stabilization policy can generate no inflation surprises, and consequently it cannot affect real economic variables. Only the *unsystematic* element of stabilization policy can affect the real variables, but this is of no use for stabilization because it arises from the uncontrollable part of the policy rule.

Purely erratic changes in the interest rate set by the central bank could create unanticipated inflation and thereby affect real output. However, to be unpredictable, such policy changes would have to be purely random and completely unrelated to the state of the economy. Such a random policy would have negative welfare effects by creating expectational errors, inducing agents to make economic decisions they would probably regret *ex post*. Clearly, such central bank behaviour would not qualify as stabilization policy.

The PIP was a frontal attack on the conventional wisdom. Hence, it created a strong controversy among macroeconomists when it was initially presented.[4] Because of its important implications for public policy, we will now discuss the robustness of the PIP.

Policy effectiveness under rational expectations

The model of the previous section assumes that the interest rate as well as the wages and prices for period t have to be set at the end of period $t-1$, based on the information available at that time. In other words, it is assumed that the central bank cannot act on the basis of more updated information than wage and price setters in the private sector. This is hardly realistic. Nominal wages and prices are, as explained in earlier chapters, often

[4] The PIP was originally put forward by Thomas J. Sargent and Neil Wallace, 'Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule', *Journal of Political Economy*, **83**, 1975, pp. 241– 254; and in a paper by the same authors entitled: 'Rational Expectations and the Theory of Economic Policy', *Journal of Monetary Economics*, **2**, 1976, pp. 169 –183.

rigid: Wage contracts often fix the nominal wage (or specify the evolution of the nominal wage) for a considerable period ahead, and many firms only change their prices at infrequent intervals. By contrast, the central bank can change its interest rate quite quickly if it feels that new information on the state of the economy warrants such a change. This means that the central bank can act *after* many wages and prices have been set, so even if private agents fully understand the effects of the interest rate change on the economy, they may not have the opportunity to adjust their wages and prices immediately to offset the effect of the policy change on the real economy as they are temporarily locked into the existing nominal contracts.

A realistic macro model should therefore allow for the possibility that the central bank can react to economic developments occurring after (some of) the nominal wages and prices in the private sector have been set. To capture this, we now assume that the central bank interest rate can react to the *actual* levels of current output y_t and current inflation π_t, whereas wages and prices are set on the basis of *expectations* formed at the end of the previous period (so that $\pi^e_{t,t-1}$ is still the relevant expected inflation rate to include in the aggregate supply curve). Our full model will then be:

Goods market equilibrium: $$y_t - \bar{y} = v_t - \alpha_2(r_t - \bar{r}), \qquad \text{(IS)}$$

Monetary policy rule: $$r_t = \bar{r} + h(\pi_t - \pi^*) + b(y_t - \bar{y}) \qquad \text{(MP)}$$

Aggregate supply or price formation: $$\pi_t = \pi^e_{t,t-1} + \gamma(y_t - \bar{y}) + s_t, \qquad \text{(AS)}$$

Rational expectations: $$\pi^e_{t,t-1} = E(\pi_t \mid I_{t-1}). \qquad \text{(RE)}$$

where the monetary policy rule (MP) now takes the more familiar form and where we now omit the unsystematic part $\hat{\rho}_t$, since for the present analysis this would just work as another demand shock. In practice, the central bank may not have perfect information on the current levels of output and inflation, but (MP) is just a convenient way of modelling the fact that the central bank can react to observed economic developments occurring *after* (some of) the wages and prices in the private sector have been set. The qualitative results derived below will hold as long as the central bank can react to new information arriving after some of the wage and price setting decisions for the current period were made.

Expectations are still rational, so we must solve this revised model by going through the three steps described in the previous section, maintaining the assumption that all the shocks appearing are white noise. As we have seen several times in earlier chapters, if we insert (MP) into (IS) and rearrange, we get the aggregate demand curve:

$$y_t - \overline{y} = \frac{v_t}{1+\alpha_2 b} - \frac{\alpha_2 h}{1+\alpha_2 b}(\pi_t - \pi^*). \qquad \text{(AD)}$$

Step 1: *Solve the model for y_t and π_t in terms of the exogenous variables, the shocks and the expectations variables.* From (AS) we have:

$$\pi_t - \pi^* = \pi^e_{t,t-1} - \pi^* + \gamma(y_t - \overline{y}) + s_t, \qquad (15)$$

which may be inserted into (AD) to give:

$$y_t - \overline{y} = \frac{v_t - \alpha_2 h s_t - \alpha_2 h(\pi^e_{t,t-1} - \pi^*)}{1+\alpha_2(b+\gamma h)}. \qquad (16)$$

Furthermore, we may substitute (AD) into (15) to get:

$$\pi_t = \pi^* + \frac{(1+\alpha_2 b)(\pi^e_{t,t-1} - \pi^*) + \gamma v_t + (1+\alpha_2 b)s_t}{1+\alpha_2(b+\gamma h)}. \qquad (17)$$

Step 2: *Find $\pi^e_{t,t-1}$ by taking expected values in (17), conditional on information available at time t–1*. Performing this operation and remembering that $E(s_t|I_{t-1}) = E(v_t|I_{t-1}) = 0$, we get:

$$\pi^e_{t,t-1} = E(\pi_t \mid I_{t-1}) = \pi^* + \frac{(1+\alpha_2 b)}{1+\alpha_2(b+\gamma h)}(\pi^e_{t,t-1} - \pi^*), \qquad (18)$$

or

$$\pi^e_{t,t-1} - \pi^* = \frac{(1+\alpha_2 b)}{1+\alpha_2(b+\gamma h)}(\pi^e_{t,t-1} - \pi^*), \qquad (19)$$

which can only be fulfilled for:

$$\pi^e_{t,t-1} = \pi^*. \qquad (20)$$

Step 3: *Find the final solutions for π_t and y_t by inserting the solution for the rationally expected inflation rate (20) into (16) and (17).* Taking this last step, we obtain:

$$y_t = \overline{y} + \frac{v_t - \alpha_2 h s_t}{1 + \alpha_2 (b + \gamma h)}. \tag{21}$$

$$\pi_t = \pi^* + \frac{\gamma v_t + (1 + \alpha_2 b) s_t}{1 + \alpha_2 (b + \gamma h)}. \tag{22}$$

The important message from (21) is that the process for *real output is now influenced by systematic monetary policy*, since the policy parameters b and h appear in the solution for y_t. For example, we see that a more activist countercyclical policy (a higher value of b) will reduce the impact of demand and supply shocks on output. Hence, the Policy Ineffectiveness Proposition breaks down, even though expectations are rational. The reason has already been suggested above: since the expectations governing the formation of wages and prices are formed *before* the central bank sets the interest rate, monetary policy can react to new information, which was not available when the private sector formed its expectations for the current period. The inflation forecast error, $\pi_t - \pi^e_{t,t-1}$, is therefore affected by the central bank's systematic reactions to new events, and consequently systematic monetary policy will influence real output, according to the aggregate supply curve (27).

Hence, under our more realistic informational assumption, stabilization authorities can in fact affect and stabilize output by influencing the inflation forecast errors, but can it be advantageous to the economic agents in a welfare sense that policy makers affect the 'surprise inflation'? The answer is yes. If a shock that the economic agents could not foresee creates a deviation between the agents' expectations and actual realized values after the shock, the authorities can by their actions stabilize the actual values of the variables and thus make the expectational errors *smaller* than they would otherwise have been.

Since it seems realistic to assume that the central bank can indeed act after (some agents in) the private sector has temporarily locked itself into nominal contracts, most economists today consider the Policy Ineffectiveness Proposition as theoretically interesting, but not very relevant in practice. Summing up:

POLICY EFFECTIVENESS UNDER RATIONAL EXPECTATIONS

Since many nominal wages and prices are pre-set for a considerable period of time, the central bank can react to new information that was not available when (some of the) wage and price setters formed their inflation expectations for the current period. Hence, the systematic reaction of monetary policy to new information can generate deviations between actual and expected inflation, thereby influencing output and other real economic variables.

Persistency under rational expectations

In our AS–AD model with backward-looking expectations we have seen (in Chapter 19) how the gradual adjustment of expectations to observed changes in the actual inflation rate generates persistence in output and inflation. By contrast, in our model with rational expectations the expected inflation rate is pinned down by the central bank's target inflation rate, so there is no adjustment over time in expected inflation following a shock, and hence no persistence mechanism rooted in expectations formation.

Indeed, we see from (21) and (22) that in the rational expectations model that we prefer (because of its more plausible informational assumptions), output in period *t* will fluctuate randomly around the natural rate and inflation will fluctuate stochastically around the inflation target depending on shock realizations v_t and s_t in period *t*, but not on shock realizations in earlier periods. As soon as a temporary shock disappears, and assuming that no new shocks occur, the economy immediately returns to its long-run equilibrium ($y_t = \bar{y}$ and $\pi_t = \pi^*$) because there are no gradual shifts in the AS curve stemming from the updating of expectations. In other words, there will be no tendency that if output or inflation was 'high' in period *t*-1, it will also be high in period *t*, that is, the model does not generate any persistency in output or inflation. The same is true for the rational expectations model with the less convincing informational assumption, as can be seen from Equations (12) and (13).

Apparently, the assumption of rational expectations eliminates the kind of persistency in macroeconomic time series which was documented in Chapter 13. Doesn't the absence of such persistence in our simple models with rational expectations speak against rational expectations and in favour of the assumption of backward-looking expectations? Well, not necessarily. Just as policy ineffectiveness does not necessarily follow from rational expectations, absence of persistency does not either. Here are some plausible mechanisms that can create persistency also under rational expectations where persistency cannot be explained by sluggish expectations.

Persistency in the shocks themselves. Above we assumed that the shocks v_t and s_t were uncorrelated white noise. This is not really plausible. If consumer or business confidence as embodied in the demand shock v_t is above normal in one quarter, say, then most likely this optimism will not disappear in just one quarter, so probably confidence will also have been above normal the quarter before and will be again in the quarter to follow. Likewise, if productivity is unusually low in one quarter or some input prices unusually high creating an unfavourable supply shock $s_t > 0$, then probably also $s_{t+1} > 0$, since productivity and cost deviations are usually longer-lived than just one quarter. Hence, these shocks are perhaps better formulated as being autocorrelated as we also studied in Chapter 19: $v_t = \rho v_{t-1} + x_{t-1}$ and $s_t = \omega s_{t-1} + c_{t-1}$, where $0 < \rho < 1$, $0 < \omega < 1$ and x_{t-1} and c_{t-1} are white noise shocks. Interestingly, autocorrelated supply shocks (but not demand shocks) are capable of creating persistence in output and inflation if added to our preferred rational expectations model studied above. This is the subject of Exercise 4 at the end of this chapter.

The presence of capital. In our short run models we abstract from capital and other elements that would link the future state of the economy to its present state in a natural way. Assume that economic activity, including investment, has been high for some time as the result of purely random factors. A higher than normal level of current investment implies a faster accumulation of capital meaning that more capital will be available in the future, and this in turn will allow a higher future level of activity. Including capital explicitly in a model with rational expectations will therefore tend to create some persistency in output even if the underlying shocks are uncorrelated over time.

Habit formation in consumption. The theory of consumption demand presented in Chapter 16 assumed that the representative consumer only derives utility from the current level of consumption C_t, as expressed by the utility function $u(C_t)$. In real life, the 'normal' level of consumption that consumers are used to or experience that others generally obtain may also be important to them. If we call this habitual level of consumption C_t^h, the idea is that the consumer gets some satisfaction from the difference $C_t - C_t^h$ (and hence dissatisfaction if this is negative) on top of the satisfaction obtained by consumption C_t itself. Depending on how important consumption itself is relative to the importance of not falling short of one's habits, we can imagine that there is a given parameter, β say, $0 \le \beta \le 1$, such that in each period the consumer derives utility from the β-weighted average $(1-\beta)C_t + \beta(C_t - C_t^h)$ of consumption C_t and the 'keeping-up' part $C_t - C_t^h$. If $\beta = 0$, we are back to the standard case, and if $\beta = 1$ we have the extreme case where only the keeping-up part matters. Most plausibly, however, $0 < \beta < 1$. The instantaneous utility function will then be $u(C_t - \beta C_t^h)$. Assuming as in Chapter 16 that the consumer maximizes the discounted sum of instantaneous utilities over time, subject to her intertemporal budget constraint, and given our normal assumptions on *u* etc., a higher habitual level of consumption C_t^h will, ceteris paribus, pull current consumption C_t upwards, since a higher C_t^h means that the consumer is less satisfied with respect to the keeping-up part and will compensate for this. Formally, as $C_t - \beta C_t^h$ decreases, the marginal utility $u'(C_t - \beta C_t^h)$ of consumption in period *t* increases, which induces the consumer to shift some consumption to period *t*. Whether habitual consumption represents what the consumer is used to consume or usually see others consume, it is a natural assumption in our representative consumer framework to assume that $C_t^h = C_{t-1}$, so that the period-wise utility is $u(C_t - \beta C_{t-1})$, creating a tendency that higher past consumption gives higher current consumption demand. Obviously, this can work as a propagation mechanism: if a purely random shock gave above-normal activity and income in the previous period, this would also pull consumption up in that period, which will give a higher habitual consumption in the current period, in turn giving higher consumption demand and activity in the current period.

Summing up, the assumption of rational expectations itself does not prevent persistency, it just excludes sluggish, backward-looking expectations as a mechanism for persistency. But other natural mechanisms exist. In extended models containing these mechanisms, the basic point remains that the way expectations are formed will influence the dynamics of the macroeconomy.

Optimal stabilization policy under rational expectations

In the analysis of optimal stabilization policy that we now turn to we will abstract from the propagation mechanisms described above in order to illustrate some basic policy principles in the simplest possible way. Thus we will focus on the RE model with the informational assumption that we prefer, that is, the model consisting of Equations (IS), (MP), (AS) and (RE) above leading to the solutions stated in Equations (21) and (22). For convenience we restate the solutions in a slightly different form where they have the output and inflation *gaps* on the left hand sides:

$$\hat{y}_t \equiv y_t - \bar{y} = \frac{v_t - \alpha_2 h s_t}{1+\alpha_2(b+\gamma h)}. \tag{21'}$$

$$\hat{\pi}_t \equiv \pi_t - \pi^* = \frac{\gamma v_t + (1+\alpha_2 b) s_t}{1+\alpha_2(b+\gamma h)}. \tag{22'}$$

Our question will be: what is the *optimal* monetary stabilization policy under rational expectations? When using the model that leads to (21') and (22'), we have already assumed that the central bank has decided to follow a Taylor rule like (MP), which is reasonable since the empirical analysis in Chapter 17 showed that monetary policy in the most important countries seems to be quite well described by such a rule. By postulating that the central bank follows the Taylor rule (MP), we are assuming that the bank can observe and react to the current inflation and output gaps whereas it cannot directly observe the shocks currently hitting the economy. The question then becomes: What are the optimal values of the coefficients on the inflation and output gaps, *h* and *b*, in the Taylor rule when expectations are rational and we assume that the agents know the monetary policy rule, i.e., have the rule (MP) and its parameter values in their information sets?

Before turning to formal analysis, let us sum up and recap some features of (21') and (22'): First, these expressions show that monetary policy can in fact influence real output and employment even if expectations are rational.[5] Second, if the central bank chooses

[5] In Exercise 2, we ask you to demonstrate that systematic *fiscal* policy can also be used to stabilize output under rational expectations, provided fiscal policy makers can act on information obtained after nominal wages have been set.

$b = 0$, this does not imply that it neglects output stabilization. Equation (21') shows that the output gap will still depend on *h*, and this *h* could be chosen also from a concern of output stabilization. Third, the distributions of the output and inflation gaps do not depend on the past, but only on contemporaneous shocks and exogenous variables. This means that the central bank can safely optimize one period at a time; what it does today does not have any implications for its stabilization possibilities tomorrow (an implication, of course, of our simplifying, but not too realistic choice of avoiding persistency mechanisms in our model).

In its decision on the optimal Taylor rule we assume that the central bank attaches a social loss in each period to deviations in the output and inflation gaps from zero according to a social loss function as presented in Chapter 14 and used in Chapter 20:

$$SL_t = (y_t - \bar{y})^2 + \kappa(\pi_t - \pi^*)^2 = \hat{y}_t^2 + \kappa\hat{\pi}_t^2, \quad \kappa > 0, \tag{23}$$

The central bank chooses the parameters of its monetary policy rule so as to minimize the *expected average social loss* given by

$$E[SL_t] = E\left[\overbrace{(y_t - \bar{y})^2 + \kappa(\pi_t - \pi^*)^2}^{SL_t}\right] = \sigma_y^2 + \kappa\sigma_\pi^2 \tag{24}$$

where σ_y^2 and σ_π^2 are the variances of output and inflation (and of the output and inflation gaps), respectively.

We will first derive optimal monetary policy strictly on the premises of the model, but afterwards bring some realistic features into the picture, which will modify our conclusions. The period-wise social loss in (23) will be minimized by keeping the output and inflation gaps as close as possible to zero. In the case of demand shocks, the concerns about the two gaps do not contradict each other. If we assume $s_t = 0$ for a moment, we can see from (21') and (22') that we can bring both the output gap and the inflation gap arbitrarily close to zero by choosing *h* or *b* or both sufficiently large. So, assume that this has been decided. Demand shocks will then have been 'taken care of' and we can set them aside and focus on supply shocks, now assuming $v_t = 0$. From (21') and (22') one will then for given *h* and *b* have the following 'balance' between the output and inflation gaps in connection with supply shocks:

$$\frac{\hat{y}_t}{\hat{\pi}_t} \equiv \frac{y_t - \bar{y}}{\pi_t - \pi^*} = -\frac{\alpha_2 h}{1 + \alpha_2 b} \quad \Leftrightarrow \quad \hat{y}_t = -\frac{\alpha_2 h}{1 + \alpha_2 b}\hat{\pi}_t. \tag{25}$$

But what is the optimal balance? From the analysis of the previous section we know that the rational expectation of the inflation rate in our model is $\pi_{t,t-e}^e = \pi^*$. Inserting this into (AS) gives $\pi_t - \pi^* = \gamma(y_t - \bar{y}) + s_t$, which we can insert into our expression (23) for

SL_t to get:

$$SL_t = (y_t - \bar{y})^2 + \kappa[\gamma(y_t - \bar{y}) + s_t]^2 = \hat{y}_t^2 + \kappa(\gamma\hat{y}_t + s_t)^2. \tag{26}$$

The central bank wants to minimize $E(SL_t)$. For any given value of s_t, the first order condition for minimizing SL_t of (26) with respect to the output gap is:

$$\begin{aligned}&\frac{\partial SL_t}{\partial \hat{y}_t} = 2\hat{y}_t + \kappa 2(\gamma\hat{y}_t + s_t)\gamma = 2\hat{y}_t + 2\gamma\kappa\hat{\pi}_t = 0\\&\Leftrightarrow\\&\hat{y}_t = -\gamma\kappa\hat{\pi}_t\end{aligned} \tag{27}$$

and the second order condition for a minimum is easily seen to be fulfilled. Since this solution holds irrespective of the value of s_t, the optimal balance between the output and inflation gaps in connection with supply shocks is exactly $\hat{y}_t = -\gamma\kappa\hat{\pi}_t$. Comparing to (25), which states the balance that our model gives rise to, one sees that the optimal balance is obtained if h and b are chosen such that:

$$\frac{\alpha_2 h}{1+\alpha_2 b} = \gamma\kappa. \tag{28}$$

This condition does not contradict that one of h and b (or both) are large, but if that is the case the other one will also have to be large. Hence, strictly on basis of our RE macro model the optimal Taylor rule is described by the following principle: Choose both of h and b to be very large and ensure that they are chosen such that (28) is fulfilled. We have seen a similar conclusion before in Chapter 20, and the principle is again intuitive: Large values of h and b will roughly eliminate the effects of demand shocks. For supply shocks, larger values of γ and κ call for a higher value of h (the reaction to inflation deviations) relative to b (the reaction to output deviations), since a larger γ implies that a reduction of the numerical value of the inflation gap can be attained at a lower cost in terms of an increased numerical value of the output gap, and since a larger value of κ means that society puts greater emphasis on inflation stabilization relative to output stabilization.

Our derived optimal principle for monetary policy is correct on the premises of the model, but, as also noted in Chapter 20, strongly questionable in view of several real world features: First, due to incomplete information about the true values of the output and inflation gaps (due to measurement problems, delays in the gathering of statistical data etc.), one would be very cautious not to set the reaction parameters h and b too large. This contradicts the principle. Second, information about the output gap is probably more uncertain, comes with more delay and is subject to more revisions than information about inflation. Third, in their charters, certainly many central banks are more obliged to ensuring low and stable inflation than to stabilize output.

In view of these concerns it seems of interest to investigate what the best possible Taylor rule would look like under the additional restriction that the reaction parameter *b* must be kept smaller than a given limited size (which, in order to come close to the optimal balance in connection with supply shocks, would also require a reaction parameter *h* of limited size).

We will undertake such an investigation simplifying the analysis by setting the limiting value of the reaction to output deviations to $b = 0$. The Taylor rule with $b = 0$ is referred to as *strict* inflation targeting, whereas the same rule with both *h* and *b* strictly positive is called *flexible* inflation targeting. Thus, we will ask: what does optimal *strict* inflation targeting require, that is, what is the optimal *h* in the Taylor rule given that b = 0? As already noted, $b = 0$ does not mean that the concern of output stabilization is neglected. And the choice of *h* will imply a true trade-off: a larger *h* will mean more stabilization of both output and inflation in connection with demand shocks, but at the same time a large *h* means relatively much inflation stabilization and little output stabilization in connection with supply shocks, which sets a limit to how high *h* should be chosen.

Hence, given that $b = 0$, we want to identify the *h* that will minimize the $E(LS_t)$ of (24) above. We start by computing the variances present in (24) for the case $b = 0$. This is done by setting $b = 0$ in the expressions (21') and (22') for the output and inflation gaps and using standard rules for computing variances, remembering that here $E(v_t s_t) = 0$ due to or assumption that these shocks are uncorrelated. We obtain:

$$\sigma_y^2 \equiv E[(y_t - \bar{y})^2] = \frac{\sigma_v^2 + \alpha_2^2 h^2 \sigma_s^2}{(1+\alpha_2 \gamma h)^2}, \tag{29}$$

$$\sigma_\pi^2 \equiv E[(\pi_t - \pi^*)^2] = \frac{\gamma^2 \sigma_v^2 + \sigma_s^2}{(1+\alpha_2 \gamma h)^2}, \tag{30}$$

where σ_v^2 and σ_s^2 are the variances of the demand and the supply shock, respectively. Inserting (29) and (30) in (24), we get:

$$E[SL_t] = \sigma_y^2 + \kappa \sigma_\pi^2 = \frac{\sigma_v^2 + \alpha_2^2 h^2 \sigma_s^2 + \kappa(\gamma^2 \sigma_v^2 + \sigma_s^2)}{(1+\alpha_2 \gamma h)^2} \tag{31}$$

The optimal value of *h* that will minimize the expected social loss (31) from instability of output and inflation may be found from the first-order condition $\partial E(SL_t) / \partial h = 0$, which is equivalent to (only considering the numerator in the fraction resulting from differentiation):

$$(1+\alpha_2 h\gamma)^2 2\alpha_2^2 h\sigma_s^2 - [\sigma_v^2 + \alpha_2^2 h^2\sigma_s^2 + \kappa(\gamma^2\sigma_v^2 + \sigma_s^2)]2(1+\gamma\alpha_2 h)\gamma\alpha_2 = 0$$
$$\Leftrightarrow$$
$$(1+\alpha_2 h\gamma)\alpha_2 h\sigma_s^2 = [\sigma_v^2 + \alpha_2^2 h^2\sigma_s^2 + \kappa(\gamma^2\sigma_v^2 + \sigma_s^2)]\gamma$$
$$\Leftrightarrow$$
$$\alpha_2 h\sigma_s^2 + \alpha_2^2 h^2\gamma\sigma_s^2 = \gamma\sigma_v^2 + \alpha_2^2 h^2\gamma\sigma_s^2 + \gamma\kappa(\gamma^2\sigma_v^2 + \sigma_s^2) \tag{32}$$
$$\Leftrightarrow$$
$$\alpha_2 h\sigma_s^2 = \gamma\sigma_v^2 + \gamma\kappa(\gamma^2\sigma_v^2 + \sigma_s^2)$$
$$\Leftrightarrow$$
$$h = \frac{\gamma}{\alpha_2}\left[(1+\gamma^2\kappa)\frac{\sigma_v^2}{\sigma_s^2} + \kappa\right].$$

Equation (32) characterizes the *optimal inflation targeting rule under rational expectations*, with $b = 0$. We note the following general principles following from (32):

- Since (32) implies that $h > 0$, we see that the optimal monetary policy satisfies the Taylor principle discussed in earlier chapters.
- Further, a relatively large value of σ_v^2/σ_s^2 implies a relatively large value of h. When business fluctuations are mainly driven by demand shocks, the relative variance σ_v^2/σ_s^2 will be large, and a positive (negative) inflation gap will then typically be associated with a positive (negative) output gap, as (21') and (22') show. A strong interest rate reaction to the inflation gap will then typically serve to close the output gap as well as the inflation gap itself. This explains why (32) requires that h should be larger the larger the relative variance σ_v^2/σ_s^2.
- On the other hand, if supply shocks are important relative to demand shocks so that σ_v^2/σ_s^2 is low, a positive inflation gap will typically be associated with a negative output gap, and vice versa, as (21') and (22') again show. In this situation a rise in the interest rate in response to a positive inflation gap will serve to reduce that gap by reducing aggregate demand, but at the same time this fall in demand will further increase the size of the negative output gap numerically. Since the central bank cares about both of the gaps, (32) prescribes that the interest rate should only react moderately to the inflation gap when supply shocks are relatively important, i.e., when σ_v^2/σ_s^2 is low. This is a reflection of the trade-off between output stability and inflation stability in connection with supply shocks mentioned earlier.
- A stronger inflation response to a change in the output gap, i.e., a greater slope γ of the Phillips curve, calls for a stronger interest rate reaction to a change in the inflation gap. This is because the greater γ is, the less it costs in terms of lost output stability to stabilize inflation in connection with supply shocks as (AS) shows.
- A larger κ implies a larger h. It is very intuitive that the optimal interest rate reaction to the inflation gap is larger the more society loses from inflation volatility relative to

output volatility, that is, the greater the value of κ.

- The greater the sensitivity of aggregate demand to a change in the interest rate, that is, the higher the value of α_2, the smaller is the optimal interest rate response to a change in the inflation gap. The stronger the economy reacts to interest changes, the less the central bank needs to change the interest to achieve its desired impact on the economy.

To recap on optimal monetary stabilization policy:

OPTIMAL INFLATION TARGETING UNDER RATIONAL EXPECTATIONS

In theory the central bank interest rate should react strongly to both the inflation and output gaps (subject to the zero bound constraint on the nominal interest rate) in order to offset demand shocks, and the reaction parameters should be balanced in order to achieve the desired balance between output and inflation stability when the economy is hit by supply shocks. However, due to a number of real world imperfections less policy activism is warranted and more emphasis on reactions to inflation may be needed. Under strict inflation targeting the optimal interest rate reaction to the inflation gap is increasing in the variance of demand shocks relative to the variance of supply shocks, in the social loss society attaches to inflation volatility relative to output volatility and in the slope of the aggregate supply curve (the impact of output on inflation). It is decreasing in slope of the IS curve (the impact of the real interest rate on aggregate demand).

21.2 Rational expectations and stabilization policy: the Lucas critique and announcement effects

The Lucas Critique

The Policy Ineffectiveness Proposition played a prominent role in the so-called rational expectations revolution which swept through the field of macroeconomics in the 1970s. As we have seen, the PIP is not robust to plausible changes in assumptions, so although it is regarded as an interesting theoretical benchmark, it is not really taken literally by today's macroeconomists. However, the more general insight that policy implications may differ fundamentally if people form expectations rationally rather than adaptively is widely accepted.

Another more lasting influence of the rational expectations revolution is the so-called Lucas Critique of macroeconometric policy evaluation, advanced by Nobel Laureate

Robert Lucas.[6] The Lucas Critique says that *an econometric macro model with backward-looking expectations which was estimated under a previous economic policy regime cannot be used to predict economic behaviour under a new policy regime*. The reason is that a change in the policy regime will affect private sector behaviour, including the way in which expectations are formed, and this will change (some of) the parameters of the relevant economic model.

Our AS–AD model with rational expectations provides a simple illustration of the Lucas Critique. Consider Equation (20) which shows that the rationally expected inflation rate equals the central bank's target inflation rate π^*. Suppose now that the government appoints an 'inflation hawk' as a new central bank governor to implement a more anti-inflationary monetary policy, implying a fall in π^*. According to (20), rational agents will then immediately *reduce* their expected inflation rate. If the economic analyst does not allow for this effect of the change in policy regime on expectations, he or she will miscalculate the effect of the policy change on inflation and output. For example, if the analyst assumes static expectations, we saw in Figure 21.1 that he or she will predict a long period where agents will overestimate the inflation rate, following the downward revision of the inflation target. As a consequence, the analyst assuming static expectations will forecast a long and protracted recession. But if expectations are actually rational, and if the reduction in the official inflation target is considered to be credible, Equations (21) and (22) show that the cut in π^* can be implemented without any loss of output, because it is immediately translated into a corresponding fall in the expected inflation rate.

The Lucas Critique is relevant for structural policies as well as for stabilization policy. For example, the labour market models presented in Chapters 11, 12 and 18 imply that the natural rate of unemployment depends on the level of unemployment benefits, among other things. According to these models, a labour market reform involving a cut in unemployment benefits is likely to reduce structural unemployment. A macroeconomic model incorporating a Phillips curve estimated on historical data for periods when benefits were higher will then tend to overestimate the natural unemployment rate after the reform, leading to inaccurate economic forecasts.

The Lucas Critique is a warning that one cannot mechanically extrapolate past economic behaviour into the future. To avoid this problem, Lucas argued that the analyst must build economic models with rational expectations and explicit microfoundations. In such a setting it is possible to describe economic behaviour as functions of the government's policy instruments and of the 'deep' parameters representing tastes and technology which are not influenced by policy changes. Armed with such a micro-based rational expectations model, one can in principle predict how a change in the policy regime will affect the economy. This methodological approach has had a profound influence on the way macroeconomists are nowadays trying to evaluate economic policies. It also provides part of the motivation for our efforts in earlier chapters to offer a microtheoretical foundation for the behavioural relationships of our AS–AD model.

[6] See Robert E. Lucas, Jr, 'Econometric Policy Evaluation: A Critique', *Carnegie-Rochester Conference Series on Public Policy*, **1**, 1976, pp. 19 – 46.

Announcement effects

In Chapter 17 on aggregate demand and monetary policy we discussed the unconventional monetary policy instrument referred to as 'forward guidance', which is an example of an attempts to stabilize the economy via so-called *announcement effects*. The idea was that the central bank may stimulate economic activity by announcing that it will keep its policy interest rates low for a certain amount of time reaching into the future. If the public considers the central bank's announcement to be credible, it will expect low short-term interest rates for a certain amount of time, which will cause low longer-term interest rates here and now through the mechanism of the expectations hypothesis and hence boost demand. So, by announcing what it will do in the future, the central bank aims at obtaining a desired effect here and now. This requires that agents form expectations in a more sophisticated way than simply projecting past evolutions into the future as with backward-looking, adaptive expectations. For forward guidance to be successful, some form of forward-looking, rational expectations must prevail so that agents take their knowledge of future policy into account when forming expectations. As we also mentioned in Chapter 17, this is not enough for forward guidance to work: it only works if people believe that the central bank will keep its interest rate low even in a future situation where it would normally want it to be higher, which does not seem to accord well with rational expectations. This is one reason why forward guidance is probably a doubtful instrument.[7]

Forward guidance in monetary policy is just one example of announcement effects of economic policies. In economies with forward-looking expectations, markets will react to *new information* on factors that are likely to affect the future course of the economy. Credible announcements of future changes in economic policy – or even just official statements which are interpreted as credible *signals* of future policy changes – will therefore influence the state of the economy even before the policy changes are implemented. To illustrate, financial market participants typically pay close attention to the public statements of central bankers and immediately react when they believe that these statements indicate true future changes in monetary policy. A famous example of this was given on 5 December 1996, when the US Federal Reserve chairman Alan Greenspan gave a speech in which he argued that the booming stock prices at that time reflected the 'irrational exuberance' of stock market investors. This was interpreted as a sign that the Fed was ready to tighten monetary policy in order to bring stock prices down from an unsustainable level. As a result, stock prices immediately fell quite significantly, not only in the USA, but around the world, as illustrated in Figure 21.2.[8]

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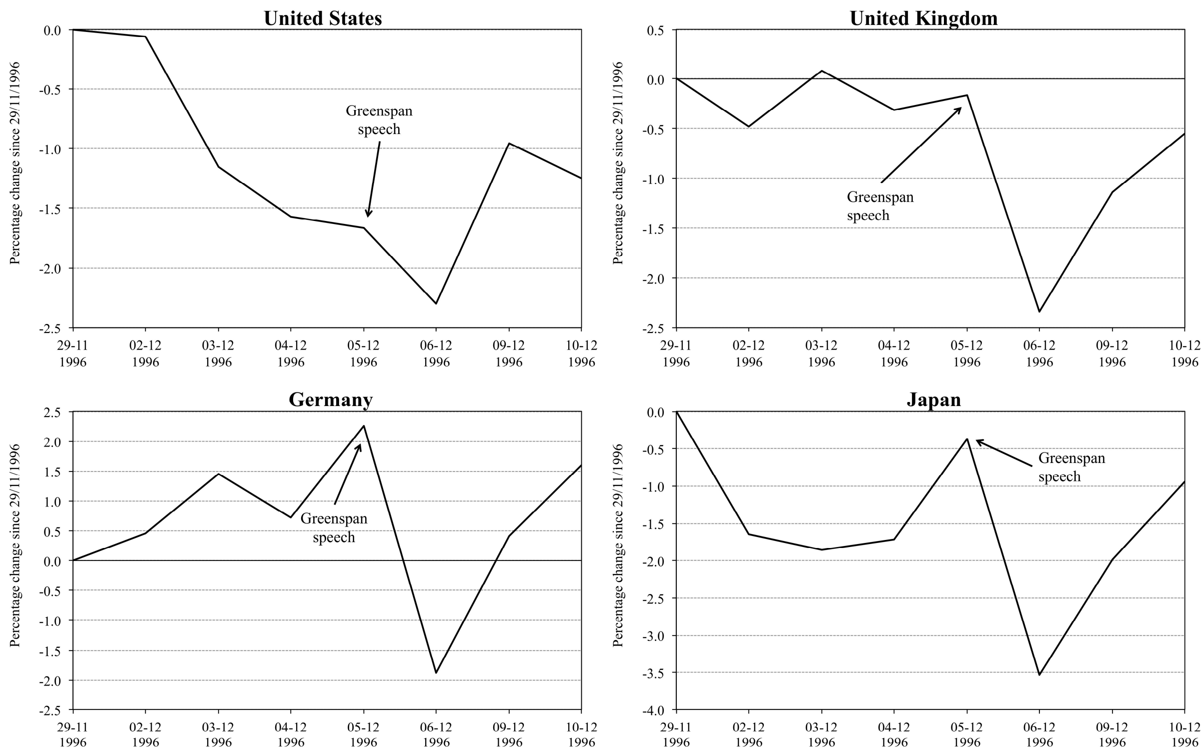
[7] Another reason is that some market participants may interpret the central bank's announcement of low future interest rates as an indication that the bank believes the economy will be weak for some time to come. This may make these market participants more pessimistic, thereby dampening aggregate demand.

[8] In the end, as you may know, the Fed was not willing to tighten monetary policy sufficiently to halt the stock market rally, so in the years after 1996 the stock market rose to even more exuberant heights before finally crashing in the year 2000.

Figure 21.2 Stock market reactions (in stock prices) to Alan Greenspan's speech on 'irrational exuberance' on 5 December 1996



Note: The following stock indices have been used: S&P500 for United States, FTSE 100 for United Kingdom, DAX 30 for Germany and Nikkei 225 for Japan.
Source: Yahoo Finance.

To show how forward-looking expectations give rise to announcement effects in financial markets, we will consider a simple model of the stock market. As you recall from Chapter 15, the (fundamental) market value V_t of shares outstanding at the start of period t is given by the discounted value of expected future dividends. Assuming that dividends are paid out at the end of each period and that the representative shareholder's real discount rate is expected to stay constant over time at the value r, we thus have:

$$V_t = \frac{D^e_{t,t}}{1+r} + \frac{D^e_{t+1,t}}{(1+r)^2} + \frac{D^e_{t+2,t}}{(1+r)^3} + \ldots, \tag{33}$$

where $D^e_{t+n,t}$ is the *after-tax* real dividend which the shareholder in period t expects to receive at the end of period $t + n$, given the information available at the beginning of period t. In all of the following we assume that expectations are formed rationally, that is, $D^e_{t+n,t} = E(D_{t+n} \mid I_t)$, where I_t is the set of information available at the beginning of period t, which we will assume includes knowledge of the dividend processes and taxation policy now to be described.

For simplicity we assume that the pre-tax real dividend d_t fluctuates stochastically around a constant mean value $\bar{d}$ so that:

$$d_{t+n} = \bar{d} + \varepsilon_{t+n}, \tag{34}$$

where ε_{t+n} is a stochastic 'white noise' variable with zero mean. Suppose further that dividends are initially taxed at the proportional rate τ_0. According to (34) and our informational assumptions, the rational expectation of the after-tax dividend received at the end of period $t + n$ is then given by:

$$D^e_{t+n,t} = (1-\tau_0)\bar{d}, \tag{35}$$

as long as the dividend tax rate is expected to stay constant. Inserting (35) into (33) and collecting terms, we get:[9]

$$V_t = \frac{(1-\tau_0)\bar{d}}{r}. \tag{36}$$

Now suppose that at time $t = t_0$ the government announces that it will permanently reduce the dividend tax rate to a lower level $\tau_1 < \tau_0$ from some future time $t_1 > t_0$ (with effect for the first time in period t_1). If the government is seen to be firmly committed to the future tax cut, economic agents will consider its announcement to be credible and

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[9] In deriving (53), we have used the general formula: $1 + a + a^2 + a^3 + \ldots = \dfrac{1}{1-a}$ for $-1 < a < 1$. In our particular case we have $a = 1/(1 + r)$.

will update their expectations regarding future tax policy accordingly. When the tax cut is implemented from time t_1 and onwards, the actual net dividends accruing after that time will be:

$$D_t = (1-\tau_1)(\bar{d} + \varepsilon_t) \quad \text{for } t \geq t_1. \tag{37}$$

For the time periods from $t = t_0$ and up until $t = t_1 - 1$ the rationally expected net dividend will still be given by (35) (with $t + n \leq t_1 - 1$), since the dividend tax cut does not take effect until period t_1. However, from (37) it follows that the rational expectation of net dividends paid out from period t_1 and onwards will be:

$$D^e_{t+n,t} = (1-\tau_1)\bar{d} \quad \text{for } t + n \geq t_1 \text{ and } t \geq t_0. \tag{38}$$

Inserting (35) for $t_0 \leq t + n < t_1$ and (38) for $t + n \geq t_1$ into (33), we obtain the value of the stock market in the time interval between t_0 and t_1:[10]

$$\begin{aligned} V_t &= \frac{(1-\tau_0)\bar{d}}{1+r} + \frac{(1-\tau_0)\bar{d}}{(1+r)^2} + \ldots + \frac{(1-\tau_0)\bar{d}}{(1+r)^{t_1-t}} + \frac{(1-\tau_1)\bar{d}}{(1+r)^{t_1+1-t}} + \frac{(1-\tau_1)\bar{d}}{(1+r)^{t_1+2-t}} + \ldots \Leftrightarrow \\ V_t &= \left\{ \left[1 - \frac{1}{(1+r)^{t_1-t}} \right](1-\tau_0) + \left[\frac{1}{(1+r)^{t_1-t}} \right](1-\tau_1) \right\} \frac{\bar{d}}{r} \quad \text{for } t_0 \leq t < t_1. \end{aligned} \tag{39}$$

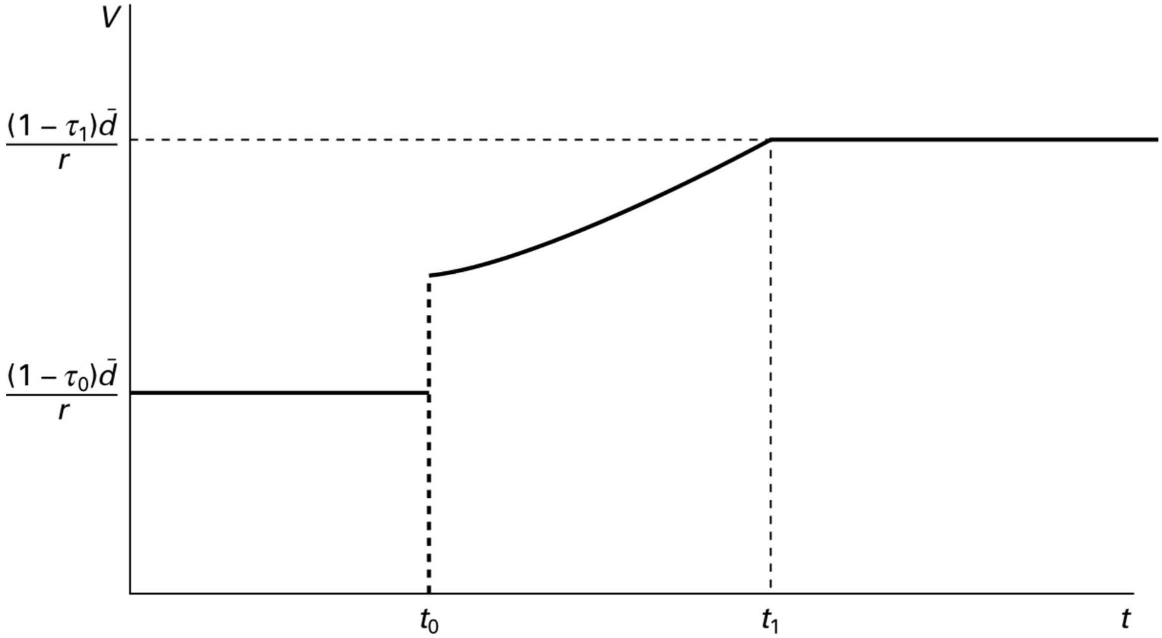
We now have a complete picture of the evolution of the stock market. Before time t_0 the value of the market is given by (36). Between the time of the policy announcement and the time when the tax cut takes effect, the market value is given by (39), and from the time the tax cut is implemented we have

$$V_t = \frac{(1-\tau_1)\bar{d}}{r} \quad \text{for } t \geq t_1, \tag{40}$$

by analogy to (36). Figure 21.3 illustrates the evolution of the stock market implied by these three equations.

[10] To derive (56), we use the formula in the previous footnote plus the general formula: $1 + a + a^2 + a^3 + \ldots + a^{n-1} = \frac{1-a^n}{1-a}$. In the present case we have $a = 1/(1 + r)$ and $n = t_1 - t$.

Figure 21.3 Effects of a dividend tax cut announced at time t_0 and implemented at t_1



Note that the market value of shares immediately jumps at time t_0 when the new information about future tax policy becomes available to the market, but *before* the new policy has actually been implemented. As the government (credibly) announces its intention to change the future tax rules, investors start to anticipate higher future after-tax dividends, and this expected rise in future earnings is immediately capitalized in stock prices as a result of the forward-looking behaviour of the market.

The magnitude of the initial jump in stock prices can be derived by setting $t = t_0$ in (39) and subtracting (36) from the resulting expression to find the rise in the value of the stock market between time $t_0 - 1$ and time t_0:

$$V_{t_0} - V_{t_0 - 1} = \frac{\tau_0 - \tau_1}{(1+r)^{t_1 - t_0}} \frac{\bar{d}}{r}. \tag{41}$$

This result is quite intuitive: the larger the future tax cut ($\tau_0 - \tau_1$), the greater is the expected rise in future net dividends, so the greater is the initial rise in stock prices.

On the other hand, the longer it takes before the dividend tax is cut (the greater the difference between t_1 and t_0), the more heavily the market discounts the rise in expected future earnings, so the smaller is the initial upward jump in stock prices.

The magnitude $1 - \tau$ is sometimes called 'the retention ratio' because it measures the fraction of the pre-tax dividend which the shareholder is allowed to retain for himself. From (39) we see that between the time of announcement and the time when the tax cut takes effect, shares are valued as if the current retention ratio were a weighted average of the lower initial retention ratio $1 - \tau_0$ and the new higher retention ratio $1 - \tau_1$ which will prevail after time t_1. The weight $1/(1+r)^{t_1-t}$ given to the new retention ratio will be heavier, the shorter the time interval $t_1 - t$ between the current period t and the time when the dividend tax is cut. Again this is intuitive: the sooner the dividend tax will be cut, the more heavily the tax cut is capitalized in stock prices. After the initial jump in stock prices at time t_0, the value of the stock market will therefore gradually rise as the date t_1 for the tax cut moves closer. When that day arrives, the tax cut has already been fully capitalized by the market, so from time t_1 and onwards there is no further increase in stock prices, as illustrated in Fig. 21.3.[11]

The general lesson from this analysis is the following:

ANNOUNCEMENT EFFECTS OF POLICY CHANGES

In an asset market with a flexible asset price and forward-looking expectations, a credibly announced change in a policy instrument affecting the return to the asset will cause an immediate 'jump' in its price even if the policy change does not take effect immediately. The magnitude of the initial price jump will be smaller, the smaller the change in the policy instrument and the longer the time span until the policy change takes effect. Following the initial jump, the asset price will gradually adjust to its new long-run equilibrium value during the time interval between the announcement and the implementation of the policy change. As a consequence, no further price jump will take place when the new policy takes effect.

For concreteness we have focused on the effects of anticipated changes in future economic policy. However, the method of analysis described in this subsection can also be used to study the effects of anticipated changes in other economic variables which are relevant for the valuation of assets with fully flexible prices. In all such cases the general rule is that asset prices 'jump' immediately at the time when new information becomes available, and after that time the asset price moves gradually and continuously towards its new long-run equilibrium value. In Exercise 5 we invite you to explore announcement effects in a simple model of the housing market.

21.3 Do people have rational expectations?

Arguments for and against the REH

[11] Of course, in practice the change in stock prices after time t_0 will influence aggregate private investment and consumption and this in turn may have feedback effects on corporate earnings and dividends which will also affect the evolution of stock prices. These complications are ignored here.

The REH assumes that people’s expectations accord with the predictions of the relevant economic model. The hypothesis does not postulate that ordinary people literally apply complicated economic models to form expectations about future economic conditions. This would obviously be a highly unrealistic assumption. But as we mentioned earlier, the economic forecasts produced by professional economists are widely publicized by the media and are thereby available to the general public. By using this information, people should be able to form unbiased forecasts of, say, the rate of inflation.

Critics have pointed out that economic experts have different views on how the economy works. Hence they often differ in their forecasts of future economic developments, so it is not obvious on which forecast ordinary persons should base their expectations. This is probably one of the most compelling criticisms of the REH. We know that the available economic models are wrong to some extent, since even the brightest economic experts do not fully understand how the economy works.

A related objection to the REH is that it takes time for people to learn enough about the structure of the economy to be able to form rational expectations. Learning from one’s past forecast errors and correcting them accordingly is a time-consuming process. Similarly, identifying the economic experts who make the most reliable forecasts of the relevant economic variables may take time. In the meantime, many agents are in a process of learning about the economic environment, and during this period they are likely to commit systematic forecast errors.

Defenders of the REH argue that since economic agents have an incentive to avoid making forecasting errors, it is not attractive to assume that their forecasts differ systematically from the forecasts of the preferred model of the analyst. Such an assumption would imply that the analyst could permanently hide his supposedly superior information from other agents, even though they would benefit from getting access to it. In the context of economic policy making, if policy makers applied an economic model which assumes that private sector expectations differ systematically from the model’s predictions, they would effectively be assuming that they could permanently fool the public. It may be a risky strategy to base economic policy on the premise that policy makers are systematically wiser than the private sector. As Abraham Lincoln said long before the rational expectations hypothesis in economics was suggested: ‘You can fool all of the people some of the time; you can even fool some of the people all of the time, but you can’t fool all of the people all of the time.’

Thus, although nobody really knows the ‘true’ structure of the economy in all of its detail, and although for this reason agents cannot literally calculate the true objective mean values of economic variables, defenders of the REH argue that it is not safe for the macroeconomic analyst to assume that the private sector is ignorant of the information embodied in his or her preferred economic model. From this perspective one may apply the REH as a sort of *robustness check* in economic policy evaluation: By asking whether the economic policy considered will have the desired effects if agents form their expectations in accordance with the economic model used by the policy adviser, one can check whether a proposed policy will work even if people fully understand its effects. Undertaking such a check seems useful even if expectations may not be fully rational in practice.

The importance of expectations for consumer behaviour

In the end, however, 'the proof of the pudding is in the eating': whether the REH is a fruitful hypothesis depends on the extent to which it helps us understand observed economic behaviour. In the rest of this chapter we will therefore discuss some empirical evidence bearing on the REH.

We will investigate whether the implications of the REH for the most important component of aggregate demand – private consumption – are consistent with the data for aggregate consumption. This will give us an opportunity illustrate the surprising consequences of the REH for consumer behaviour.

To simplify the exposition, we will follow the procedure in Chapter 16 and split the consumer's time horizon into two periods, period 1 ('the present') and period 2 ('the future'), but it should be stressed that our results carry over to a setting with many periods. The consumption levels of the representative consumer in periods 1 and 2 are denoted by C_1 and C_2, respectively, the consumer holds financial wealth V_1 at the beginning of period 1 and earns incomes Y_1 and Y_2 (after tax and transfers) in periods 1 and 2, respectively. For a given choice of C_1, the consumer will accumulate a financial wealth of $V_2 = V_1 + Y_1 - C_1$ in period 1, which will pay an interest income of rV_2, where r is the real interest rate. Hence, an amount of wealth $(1+r)V_2$ will be available for consumption in period 2, so consumption in that period will be

$$C_2 = (1+r)V_2 + Y_2 = (1+r)(V_1 + Y_1 - C_1) + Y_2 \tag{42}$$

which is a version of the consumer's intertemporal budget constraint. The consumer chooses consumption levels that maximize her lifetime utility subject to (42). In the notation of Chapter 16, the consumer's lifetime utility is $u(C_1) + u(C_2)/(1+\phi)$, where $u(C)$ is the instantaneous utility function fulfilling standard properties ($u' > 0,\ u'' < 0$), and ϕ is the exogenous rate of time preference.

From Chapter16 we know that maximization of lifetime utility will imply a certain degree of consumption smoothing over time depending on r, ϕ and the elasticity of intertemporal substitution in consumption, called σ in the notation of Chapter 16. The smaller σ, the more the consumer wants to smooth consumption, that is, come closer to $C_1 = C_2$. For simplicity we take this degree of consumption smoothing to the extreme, assuming *perfect consumption smoothing*, that is, we assume that the consumer wants $C_1 = C_2$ irrespective of r and ϕ. This arises for σ very small. We emphasize that the assumption of perfect consumption smoothing is not essential for our qualitative results; it just simplifies matters a great deal.

As an extension of the analysis in Chapter 16 we now allow for the fact that the consumer cannot know his or her future income with certainty when deciding on present consumption. Rather, she must base the choice of C_1 on expected future income, $Y^e_{2,1}$ (the income expected in period 1 to be obtained in period 2, using the notation for subjective expectations adopted in this chapter). The consumer is assumed to know the real interest rate r when he or she decides on current consumption. According to the budget constraint (42), which is a relation between *actual* values, a choice of present consumption C_1 implies an *expectation* of future consumption of $C^e_{2,1} = (1+r)(V_1 + Y_1 - C_1) + Y^e_{2,1}$. Now, the consumer's desire for complete consumption smoothing means that her current consumption will be $C_1 = C^e_{2,1}$, implying:

$$C_1 = (1+r)(V_1 + Y_1 - C_1) + Y^e_{2,1} \tag{43}$$

It follows from (42) and (43) that

$$C_2 - C_1 = Y_2 - Y^e_{2,1}, \tag{43}$$

Thus the rise (fall) in consumption over time equals the amount by which the consumer underestimated (overestimated) his or her future income when forming the expectations governing the first-period consumption.

From (43) we may derive the implications of alternative hypotheses regarding expectations formation. Suppose first that expectations are *static*, implying that expected future income equals actual current income, $Y^e_{2,1} = Y_1$. According to (43) the change in consumption over time will then be equal to the change in income:

$$C_2 - C_1 = Y_2 - Y_1. \tag{44}$$

Suppose alternatively that expectations of future income are *rational*, being an unbiased prediction of the actual future income so that $Y^e_{2,1} = E(Y_2 \mid I_1) = Y_2 - x$, $E(x) = 0$, where x is a stochastic variable with zero mean, reflecting that agents with rational expectations do not make *systematic* forecast errors. In this case it follows from (43) that

$$C_2 - C_1 = x. \tag{45}$$

We have derived these results under the simplifying assumptions of a two-period planning horizon and complete consumption smoothing. But the essence of our results hold under more general conditions: under rational expectations only the unpredictable component of a change in income causes a change in consumption, whereas under adaptive expectations all of an income change generates a consumption change. Hence, for any two periods we have:

Under static expectations: $\quad C_t - C_{t-1} = Y_t - Y_{t-1}, \qquad (46)$

Under rational expectations: $\quad C_t - C_{t-1} = x_t, \quad E[x_t] = 0. \qquad (47)$

Notice the striking result in (47): *under rational expectations the change in consumption* *over time is entirely unpredictable* (when consumption arises from intertemporal optimization with access to perfect credit markets). According to the REH, the best forecast of consumption tomorrow is the level of consumption observed today. Indeed, current consumption is the only unbiased forecast of future consumption. Because (47) implies that consumption may change by any amount in any direction, it is said that consumption follows a *random walk*. This implication of the REH was first derived in a seminal paper by American economist Robert Hall.[12]

Testing the random walk hypothesis

In summary, under rational expectations the change in consumption over time equals the *unpredictable* part of the change in income, whereas under static expectations the change in consumption equals the *total* change in income, including that part of the income change which could have been predicted by a forward-looking consumer. In another influential paper, economists John Campbell and Greg Mankiw proposed a simple way of testing the empirical relevance of these two competing hypotheses on consumption behaviour.[13] Campbell and Mankiw assumed that a fraction λ of aggregate income accrues to consumers who behave in accordance with (46), as if they have static expectations. The remaining fraction $1-\lambda$ of total income was assumed to accrue to consumers with rational expectations whose consumption follows a random walk, (47). From (46) and (47), the change in aggregate consumption will then relate to the change in aggregate income as:

$$C_t - C_{t-1} = \lambda(Y_t - Y_{t-1}) + (1-\lambda)x_t, \qquad 0 \le \lambda \le 1 \tag{48}$$

This equation spans the two alternative hypotheses on expectations: if $\lambda = 1$ all consumers have static expectations, and if $\lambda = 0$ they all hold rational expectations. Using data on consumption and income, Campbell and Mankiw provided an econometric estimate of the magnitude of the parameter λ in a number of countries. Their results are summarized in Table 21.1, where the figures in brackets indicate standard errors.[14]

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[12] See Robert E. Hall, 'Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence', *Journal of Political Economy*, **86**, December 1978, pp. 971– 987. At the time the random walk hypothesis was so provocative that when Hall first presented his paper, one prominent economist told him that he must have been on drugs when he wrote the paper!

[13] See John Y. Campbell and N. Gregory Mankiw, 'The Response of Consumption to Income – A Cross-Country Investigation', *European Economic Review*, **35**, 1991, pp. 723 – 767.

[14] Campbell and Mankiw actually measured the changes in income and consumption in logarithms and included a constant term on the right-hand side of Equation (48) to capture long-term growth. Since periods with surprise increases in income probably often coincide with periods with rapid growth in total income, the explanatory variable $Y_t - Y_{t-1}$ is likely to be positively correlated with the error term x_t. Readers trained in econometrics will know that the estimation method of Ordinary Least Squares will then generate an upward bias in the estimate for λ. To deal with this problem, Campbell and Mankiw used the method of Two-Stage Least Squares. The first stage of this estimation method involved the use of lagged changes in consumption (which are necessarily uncorrelated with x_t) in an OLS regression to predict $Y_t - Y_{t-1}$. In the

Table 21.1 The proportion of consumers with static expectations (λ)

Country	Sample period	Estimate of λ (standard errors in bracket)
Canada	1972(1)–1988(1)	0.225 (0.107)
France	1972(1)–1988(1)	0.401 (0.208)
Sweden	1972(2)–1988(1)	0.203 (0.092)
United Kingdom	1975(2)–1988(2)	0.351 (0.117)
United States	1953(1)–1985(4)	0.357 (0.173)

Source: J.Y. Campell and N.G. Mankiw, 'The Response of Consumption to Income', *European Economic Review*, **35**, 1991, p. 736.

In all of the countries, the magnitude of λ was found to be significantly greater than 0, ranging from about 0.2 to 0.4. In other words, a large part of consumption seems to be undertaken by consumers who are not forward-looking. This is clearly at odds with a strict interpretation of the REH. On the other hand, forward-looking consumers do seem to account for another large part of consumption in most countries, so the REH certainly cannot be dismissed as being irrelevant on this ground. Rather, it appears that consumers are divided into two groups; one which is forward-looking, and another which is backward-looking. Note that consumers obeying (44) do not necessarily all have static expectations. It is also possible that they are simply *myopic*, living from 'hand to mouth' or by 'rules of thumb', as discussed in Chapter 16, immediately consuming all of their current income without caring about the future. Indeed, this is the interpretation adopted by Campbell and Mankiw. However, in both cases the fact remains that these consumers do not behave as predicted by the REH.

Of course, the results of this analysis should be interpreted with some caution. As a potential objection, a positive value of λ may indicate that some consumers are *credit-constrained* rather than myopic. As discussed in Chapter 16, if a forward-looking consumer expects future income to be higher than current income, the consumer may want to borrow against the anticipated future income in order to smooth consumption over time. But in the absence of collateral, the consumer's bank may be unwilling to accommodate a credit demand for fear that the consumer may default on the loan if his or

second stage, the estimated values of Y_t - $Y_{t\text{-}1}$ were used as the explanatory variable in (48) to estimate the value of λ in another OLS regression.

her future income turns out to be lower than expected. Then the best thing such a credit-constrained consumer can do is to consume all of current income ($C_t = Y_t$), implying $C_t - C_{t-1} = Y_t - Y_{t-1}$, as if expectations were static.

Although the hypothesis of credit constraints may sound plausible, and evidence reported in Chapter 16 does support that a considerable fraction of consumers are credit constrained, probably some part of the estimated fraction λ is simply due to consumers with backward-looking expectations or myopic (not intertemporally optimizing) consumers. Hence, our preferred interpretation of the study by Campbell and Mankiw is that many consumers behave as if they have static expectations or are simply myopic, but at the same time another large part of total consumption seems to be governed by forward-looking rational expectations. In summary:

EXPECTATIONS, CONSUMPTION AND THE RANDOM WALK HYPOTHESIS

For consumers with static expectations (or myopic consumers who live 'from hand to mouth'), the change in consumption from one period to the next equals the total change in income. For consumers with rational expectations the change in consumption only equals the *unpredictable* part of the change in income. Hence, the REH implies that consumption will follow a random walk. Empirical evidence from a number of Western countries suggests that about 25 to 40 per cent of consumers behave as if they have static expectations (or are myopic), while the remaining fraction of consumers behave in accordance with the REH.

Towards a more general theory of expectations

Thus both of the competing hypotheses on expectations formation which you have encountered in this book – backward-looking expectations and rational expectations – appear to have some empirical relevance. One might therefore specify the expected inflation rate as:

$$\pi^e_{t,t-1} = \lambda \pi_{t-1} + (1-\lambda) E(\pi_t \mid I_{t-1}), \quad 0 \le \lambda \le 1. \tag{49}$$

Here $\pi^e_{t,t-1}$ is the average expected inflation rate calculated across the entire population, and $E(\pi_t \mid I_{t-1})$ is the rationally expected inflation rate, based on all information available up until the end of the previous period. Equation (49) assumes that a fraction λ of the population has static expectations, while the remaining fraction has rational expectations. If we go back to our model (IS), (MP), (AS) and (RE) and replace (RE) by (49), we obtain a model with partly rational expectations. To solve such a model, one still has to go through the three steps used to solve a model with purely rational expectations. Making the first two steps, we find the expected inflation rate for those agents who have rational expectations:[15]

$$E(\pi_t \mid I_{t-1}) = \pi^* + \left(\frac{\lambda}{\lambda + a\gamma}\right)(\pi_{t-1} - \pi^*), \quad a \equiv \frac{\alpha_2 h}{1+\alpha_2 b}. \tag{50}$$

The rational agents know that the expectations of the backward-looking agents have some influence on the actual inflation rate via the expectations-augmented Phillips curve. Hence the rational forecast of inflation stated in (40) accounts for the fact that the expectations held by the backward-looking agents are not anchored by the central bank's inflation target. This is why the term ($\pi_{t-1} - \pi^*$) enters the right-hand side of (50) with a positive coefficient which is larger the greater the fraction of the population with backward-looking expectations.

Using (49) and (50), one can show that our AS–AD model with partly rational expectations can be reduced to the following two difference equations in the output gap, $\hat{y}_t \equiv y_t - \bar{y}$, and the inflation gap, $\hat{\pi}_t \equiv \pi_t - \pi^*$:

$$\hat{y}_t = \left(\frac{\lambda}{\lambda + \gamma\alpha}\right)\hat{y}_{t-1} + \frac{z_t}{1+\gamma\alpha} - \frac{\lambda z_{t-1}}{\lambda+\gamma\alpha} - \frac{\alpha s_t}{1+\gamma\alpha}, \tag{51}$$

$$\hat{\pi}_t = \left(\frac{\lambda}{\lambda + \gamma\alpha}\right)\hat{\pi}_{t-1} + \frac{\gamma z_t}{1+\gamma\alpha} + \frac{s_t}{1+\gamma\alpha}, \qquad z_t \equiv (1+\alpha_2 b)v_t. \tag{52}$$

The model analysed in Chapter 19 is the special case of (51) and (52) where $\lambda = 1$ (if you compare (51) and (52) to Equations (41) and (42) in Chapter 19 and set $\lambda = 1$ in the first and $\phi = 0$ in the latter, implying that all of the population has static expectations, you will see that the two sets of equations are exactly identical). The model (IS), (MP), (AS) and (RE) studied in Section 21.2 of this chapter is the other special case where $\lambda = 0$ (in this case (51) and (52) collapse to (21) and (22), given that $a = \alpha_2 h/(1+\alpha_2 b)$ and $z_t = (1+a_2 b)v_t$). In the more general case where $0 < \lambda < 1$, (51) and (52) still have the same general form as the difference equations in the output and inflation gaps implied by our AS–AD model with purely static expectations. From an empirical perspective the solutions in (51) and (52) with $\lambda > 0$ has the attractive property that they imply persistence in the output and inflation gaps, that is, the current gaps depend positively on their own lagged values.

Note that substitution of (50) into (49) yields:

$$\pi^e_{t,t-1} = \varphi\pi^* + (1-\varphi)\pi_{t-1}, \quad 0 \le \varphi \equiv \frac{a\gamma(1-\lambda)}{\lambda + a\gamma} \le 1. \tag{53}$$

[15] Here we maintain our previous assumption that the expected inflation rate of the central bank is the same as that of the public, both equal to $\pi^e_{t+1,t}$, so for this reason this expectation does not appear explicitly in (IS) or (MP).

This shows that, in an economy with partly rational and partly static expectations, the average expected inflation rate is a weighted average of the central bank's inflation target and last period's actual inflation rate. When all agents are rational ($\lambda = 0$), we have $\varphi = 1$ and $\pi_t^e = \pi^*$, and when they all have static expectations ($\lambda = 1$), we get $\varphi = 0$ and $\pi_t^e = \pi_{t-1}$.

Equation (53) was derived on the assumption that a part of the population forms rational expectations in accordance with (50). As we have discussed, this assumption that (some) people behave as if they know the entire structure of the economy is indeed a strong one. But there is an alternative interpretation of (53), which does not require that agents are quite as sophisticated as postulated by the REH. Even if the forward-looking part of the population may not always have sufficient information to be able to form the strictly rational expectation given by (50), these people may at least be informed about the central bank's inflation target π^*. Therefore, if the central bank has credibility (an issue to which we return in the next chapter), it may make good sense for agents to assume that, on average, the inflation rate will correspond to the target rate of inflation. Thus we may interpret the parameter φ in (53) as the fraction of the population that is informed about (and has confidence in) the central bank's inflation target. Since this is also the long-run equilibrium rate of inflation, this forecasting behaviour may be called 'long-term rational expectations' or 'weakly rational expectations'. We will return to this hypothesis in Chapters 24 and 25 when we consider the open economy.

A MORE GENERAL THEORY OF EXPECTATIONS

If some consumers have static expectations while others have rational expectations, the expected inflation rate will be a weighted average of last year's actual inflation rate and the central bank's target inflation rate. The weight placed on the target inflation rate may be seen as an indicator of the fraction of the population which is informed about and has confidence in the inflation target, even if these people do not have strictly rational expectations.

Summary

1. The assumption of backward-looking (static or adaptive) expectations is hard to reconcile with rational behaviour because it implies that economic agents may make systematic forecast errors.
2. As an alternative to backward-looking expectations, economists have developed the rational expectations hypothesis (REH) according to which an agent's subjective expectation of an economic variable equals the objective mathematical expectation of the variable, calculated on the basis of all relevant information available at the time the expectation is formed.
3. The REH assumes that the information available to agents includes knowledge about the structure of the economy. Hence, rational expectations are *model-consistent*: they correspond to the predictions of the relevant economic model. This does not require that ordinary people are able to solve economic models, since the average person may rely on the publicly available forecasts of professional economists.
4. The REH has led to the Lucas Critique, which says that an econometric macro model based on backward-looking expectations which was estimated under a previous economic policy regime cannot be used to predict economic behaviour under a new policy regime. The reason is that a change in the policy regime will affect private sector behaviour, including the way in which expectations are formed.
5. Some macroeconomic models with rational expectations have led to the Policy Ineffectiveness Proposition (PIP) which claims that systematic demand management policy cannot affect real output and employment because the private sector will fully anticipate the effects of systematic policy on the rate of inflation.
6. The PIP is nowadays considered unrealistic, since the central bank can typically change its policy in reaction to new economic developments occurring after nominal wages (and prices) have been temporarily locked into existing contracts. Because nominal wages and prices only respond to economic shocks with a lag, systematic monetary policy can affect output and employment, even if the policy is fully anticipated by rational agents.
7. According the macroeconomic model with rational expectations model where the central bank realistically has an informational advantage over the public, the central bank interest rate should in theory react very strongly to both the inflation and output gaps (subject to the zero bound constraint on the nominal interest rate) to offset demand shocks. The two reaction parameters should then further be balanced to achieve the desired balance between output and inflation stability when supply shocks occur. However, due to a number of real world imperfections less policy activism is warranted and more emphasis on reactions to inflation may be relevant.
8. Under strict inflation targeting, the optimal monetary stabilization policy involves a

trade-off between stabilizing inflation and stabilizing output. The greater the variance of demand shocks relative to the variance of supply shocks, the more the central bank should raise the interest rate in response to a rise in the inflation gap. Furthermore, the optimal interest rate reaction to the inflation gap is increasing in the social loss society attaches to inflation volatility relative to output volatility and in the slope of the AS curve (the impact of output on inflation). It is decreasing in slope of the IS curve (the impact of the real interest rate on aggregate demand).

9. Under rational expectations, the announcement of future changes in economic policy will influence the economy already at the time of announcement, even before the new policy is implemented. In particular, the flexible prices of financial assets such as stocks will 'jump' instantaneously at the time of announcement and will then gradually adjust towards its new long-run equilibrium value as the date of implementation of the policy change comes closer.

9. The REH has been criticized for being unrealistic because economists differ in their views of the workings of the economy, making it difficult for the average person to base their expectations on expert forecasts. Defenders of the REH argue that it is not safe to base economic policy evaluation on the assumption that policy makers are systematically better informed than the private sector. According to this argument, policy makers should assume that the knowledge embodied in their economic models is also available to the private sector, as implied by the REH.

10. When consumers with rational expectations seek to smooth their consumption over time, private consumption will follow a random walk, changing only as new information about future incomes becomes available. The random walk hypothesis implies that the current consumption level is the best forecast of future consumption (adjusted for underlying trend growth) and that consumption changes only in response to unpredictable changes in income. By contrast, under static expectations the change in consumption corresponds to the change in the consumer's total income. Empirical evidence suggests that a large part of aggregate consumption follows a random walk, consistent with the REH, but at the same time many consumers behave as if they have static expectations.

11. In an economy where some consumers have rational expectations and others have static expectations, the average expected inflation rate may be written as a weighted average of the central bank's inflation target and last period's inflation rate. The weight given to the inflation target may be interpreted as the fraction of agents informed about the monetary policy target, even if these people do not have all the information needed to form strictly rational expectations regarding the short run.

Exercises

Exercise 1. Issues in rational expectations

1. Define and explain the concept of rational expectations and discuss the arguments for and against this hypothesis.
2. Explain the Policy Ineffectiveness Proposition and discuss its relevance.
3. Explain the content of the Lucas Critique and its implications for the evaluation of the effects of economic policy.
4. Discuss whether the central bank can pull the economy out of a liquidity trap by promising to keep future nominal interest rates low and/or by promising higher future inflation.
5. Explain the content of the random walk hypothesis for private consumption. Explain the difference between the dynamics of private consumption under static and under rational expectations.

Exercise 2. Monetary and fiscal policy under rational expectations

Suppose that the central bank does not always react systematically to changes in macroeconomic conditions so that monetary policy may be described by the interest rate rule:

$$r_t = \overline{r} + h(\pi^e_{t,t-1} - \pi^*) + a_t, \qquad h > 0, \tag{54}$$

where a_t is a 'white noise' stochastic variable reflecting the non-systematic part of monetary policy (at times called $\hat{\rho}_t$ in this book). Equation (54) states that the central bank bases its policy decisions on the *expected* inflation gap, since it does not have full information on the current inflation rate at the time when it sets the interest rate. For simplicity, we assume that the bank does not react to the expected output gap.

As usual, the economy's demand and supply sides are described by:

$$y_t - \overline{y} = z_t - \alpha_2(r_t - \overline{r}), \tag{IS}$$
$$\pi_t = \pi^e_{t,t-1} + \gamma(y_t - \overline{y}) + s_t, \tag{AS}$$

where z_t and s_t are white noise reflecting demand and supply shocks.

1. Assume that expectations are rational and find the model solution for real output by going through the three steps in the solution procedure described in the main text. On this basis, show that the variance of output is given by:

$$\sigma_y^2 \equiv E(y_t - \bar{y})^2] = \sigma_z^2 + \alpha_2^2 \sigma_a^2, \tag{55}$$

where σ_z^2 and σ_a^2 are the variances of z and a, respectively. Is monetary policy 'effective' in this model? What would be the effect of greater predictability of monetary policy? Would greater predictability be desirable? Discuss.

The variable $z_t = \alpha_1(g_t - \bar{g}) + v_t$ includes deviations of public spending from trend as well as the private demand shock variable v_t. Suppose now that fiscal policy reacts to the expected inflation and output gaps in the following systematic way, reflecting an intended countercyclical and anti-inflationary policy:

$$\begin{aligned} g_t - \bar{g} &= c_\pi(\pi^* - \pi_{t,t-1}^e) + c_y(\bar{y} - y_{t,t-1}^e) \quad \Leftrightarrow \\ z_t &= \alpha_1 c_\pi(\pi^* - \pi_{t,t-1}^e) + \alpha_1 c_y(\bar{y} - y_{t,t-1}^e) + v_t. \end{aligned} \tag{56}$$

Here we assume that v_t is white noise. To focus on fiscal policy, suppose further that monetary policy is passive, keeping the real interest rate at its natural level:

$$r_t = \bar{r}. \tag{57}$$

2. Assume rational expectations and demonstrate that the Policy Ineffectiveness Proposition holds in the model consisting of Equations (IS), (AS), (56) and (57). Explain why fiscal policy is ineffective.

Suppose instead that the fiscal authorities can react on current information on the actual output and inflation gaps whereas nominal wages are pre-set at the start of each period when inflation expectations can be based only on the information available up until the end of the previous period. Thus (57) is still valid, but (56) is replaced by:

$$z_t = \alpha_1 c_\pi(\pi^* - \pi_t) + \alpha_1 c_y(\bar{y} - y_t) + v_t. \tag{58}$$

3. Show that the Policy Ineffectiveness Proposition no longer holds in the model consisting of (IS), (AS), (57) and (58) even if expectations are rational. Explain why fiscal policy is now effective.

Exercise 3. Nominal GDP targeting with rational expectations

You are now asked to study the properties of an economy where the fiscal and monetary authorities pursue a target for the growth rate of nominal GDP. Specifically, we assume that

$$\overbrace{y_t^n - y_{t-1}^n}^{\substack{\text{actual growth rate}\\ \text{of nominal GDP}}} = \overbrace{\mu + \eta(\bar{y} - y_{t-1})}^{\substack{\text{target growth rate}\\ \text{of nominal GDP}}} + v_t, \qquad \eta > 0, \tag{59}$$

where y_t^n is the natural log of *nominal* GDP, and where the stochastic white noise variable v_t reflects that the authorities cannot perfectly control the growth in aggregate nominal demand. The first expression on the right-hand side of (59) shows that policy makers try to speed up the growth in nominal GDP if the previous period's real output y_{t-1} has been below the trend level of real output, and vice versa. Thus, the authorities follow a countercyclical demand management policy, but since they can only observe output with a lag, they react to the previous period's activity level rather than to current activity.

In our previous notation, $y_t^n \equiv p_t + y_t$, and $\pi_t \equiv p_t - p_{t-1}$, where p and y are the logs of the price level and of real output, respectively. By definition we thus have:

$$y_t^n - y_{t-1}^n = \pi_t + y_t - y_{t-1}. \tag{60}$$

Equations (59) and (60) describe the economy's demand side. The supply side is given by the AS-curve:

$$\pi_t = \pi_{t,t-1}^e + \gamma(y_t - \bar{y}) + s_t, \tag{61}$$

where the stochastic supply shock variable s_t is white noise.

1. Find the solutions for real output and inflation on the assumption that expectations are rational. Does the Policy Ineffectiveness Proposition hold in this model? Explain your results.

Suppose next that the authorities can react to current output so that the policy rule (88) is replaced by

$$y_t^n - y_{t-1}^n = \mu + \eta(\bar{y} - y_t) + v_t. \tag{62}$$

2. Find the solution for real output and check whether the Policy Ineffectiveness Proposition holds. Which of the scenarios (Question 1 versus Question 2) do you consider to be more realistic? Give reasons for your answer.

Exercise 4. Output persistence under rational expectations

In the main text we noted that our basic AS–AD model with rational expectations does not generate any persistence (autocorrelation) in the deviations of output from trend, in contrast to what we observe empirically. This exercise asks you to show that persistence in output will emerge if we allow for autocorrelation in our supply shock variable. Thus, we now describe the economy by the following equations, where we assume for simplicity that the central bank only reacts to the inflation gap:

Short-run aggregate supply:	$\pi_t = \pi^e_{t,t-1} + \gamma(y_t - \bar{y}) + s_t,$	(AS)
Goods market equilibrium:	$y_t - \bar{y} = z_t - \alpha_2(r_t - \bar{r}),$	(IS)
Monetary policy rule:	$r_t = \bar{r} + h(\pi_t - \pi^*), \quad h > 0,$	(MP)
Supply shock:	$s_t = \omega s_{t-1} + c_t, \quad 0 < \omega < 1,$	(63)
Demand shock:	$z_t = \rho z_{t-1} + x_t, \quad 0 \le \rho < 1.$	(64)
Rational expectations:	$\pi^e_{t,t-1} = E(\pi_t \mid I_{t-1}).$	(RE)

The stochastic variables c_t and x_t are assumed to be white noise. When the private agents form their inflation expectations for the current period, $\pi^e_{t,t-1}$, the information set available to them includes knowledge of the model above *plus* information on the shocks observed during the previous period, s_{t-1} and z_{t-1}. However, the private sector's information set does *not* include information on the 'innovations' to the shocks, c_t and x_t .

1. Show that the rational expectations solution for the output gap in the model is given by:

$$\hat{y}_t \equiv y_t - \bar{y} = \beta(x_t - \alpha_2 h c_t) - \frac{\omega s_{t-1}}{\gamma}, \quad \beta \equiv \frac{1}{1 + \gamma\alpha_2 h}. \tag{65}$$

Try to explain why the output gap is affected by s_{t-1}, but not by z_{t-1}.

2. Show that output displays persistence by using (63) and (65) to write the solution for the output gap in the form:

$$\hat{y}_t = a\hat{y}_{t-1} + \varepsilon_t \tag{66}$$

where $0 < a < 1$, and where ε_t is a (composite) white noise variable. Write the explicit expressions for a and ε_t. (Hint: start by lagging (65) by one period and then use the fact from (63) that $\omega s_{t-2} = s_{t-1} - c_{t-1}$ to write s_{t-1}/γ as a function of $\hat{y}_{t-1}$, x_{t-1} and c_{t-1}.

Then insert the resulting expression for s_{t-1}/γ into (65) and collect terms.) Discuss whether output persistence can be said to be endogenous or exogenous in this model.

Exercise 5. Policy announcement effects in the housing market

In the main text we saw that when expectations are forward-looking, announcements of future policy changes will have an effect on the stock market right from the time of announcement. Here we ask you to analyse announcement effects on the housing market. We consider a representative consumer who owns a fixed stock of housing yielding a flow of housing services whose real rental value h_t fluctuates stochastically around the constant mean value $\bar{h}$. We therefore assume that, during any period, the homeowner expects that the real value of his housing service will be $\bar{h}$. The homeowner pays a property tax which is levied in the real amount τ_t per square metre. For simplicity, the property tax is thus assumed to be unrelated to the market value of the house, but the tax may vary over time. Suppose that the consumer owns one unit of housing with a market price Q_t at the beginning of period t. Suppose further that, at the start of period t, the market price at the start of the next period is expected to be $Q^e_{t+1,t}$. If the real interest rate is r (assumed for convenience to be constant), the current market price of housing must then satisfy the following arbitrage condition for consumers to be willing to own the existing stock of owner-occupied housing:

$$\overbrace{rQ_t}^{\substack{\text{opportunity cost of}\\ \text{home-ownership}}} = \overbrace{\bar{h}-\tau_t}^{\substack{\text{after-tax value}\\ \text{of housing}\\ \text{service}}} + \overbrace{Q^e_{t+1,t}-Q_t}^{\substack{\text{expected capital}\\ \text{gain}}}. \tag{67}$$

The left-hand side of (67) measures the consumer's opportunity cost of owning his home rather than selling it at the going market price and investing the proceeds in the capital market, in which case he would earn an interest on the proceeds from the sale. The right-hand side of (67) is the return to home-ownership, consisting of the after-tax value of the housing service yielded by the consumer's housing wealth plus the expected capital gain on that wealth over the period considered. Rearranging (67), we get:

$$Q_t = \frac{\bar{h}-\tau_t + Q^e_{t+1,t}}{1+r}. \tag{68}$$

Rational homeowners know that an arbitrage condition similar to (68) must also hold in future periods; they just do not know with certainty what the future property tax will be. At the start of period t, the expectations of future housing prices will thus be given by:

$$Q^e_{t+1,t} = \frac{\bar{h}-\tau^e_{t+1,t} + Q^e_{t+2,t}}{1+r}, \quad Q^e_{t+2,t} = \frac{\bar{h}-\tau^e_{t+2,t} + Q^e_{t+3,t}}{(1+r)^2}, \ldots \text{ etc.} \tag{69}$$

where $\tau^e_{t+n,t}$ is the property tax rate expected in period t to prevail in period $t + n$. We assume that agents do not expect the real price of housing to rise systematically at a rate in excess of the real interest rate, so the expected housing price satisfies the boundary condition:

$$\lim_{n\to\infty} \frac{Q^e_{t+n,t}}{(1+r)^n} = 0. \tag{70}$$

1. Explain more carefully why the arbitrage condition (67) must hold for the housing market to be in equilibrium. Explain the economic mechanism, which establishes this equilibrium.

2. Show by using (68)–(70) that the current market price of housing is:

$$Q_t = \frac{\bar{h} - \tau_t}{1+r} + \frac{\bar{h} - \tau^e_{t+1,t}}{(1+r)^2} + \frac{\bar{h} - \tau^e_{t+2,t}}{(1+r)^3} + \ldots = \sum_{n=0}^{\infty} \frac{\bar{h} - \tau^e_{t+n,t}}{(1+r)^{n+1}}, \tag{71}$$

where the last equality in (71) exploits the fact that $\tau^e_{t,t} = \tau_t$, since the homeowner is assumed to know the current property tax rate from the start of the period. (Hint: you may use the same procedure as the one we used to derive the fundamental stock price (6) in Chapter 15). Give a verbal interpretation of the result in (71) and compare with the expression for the fundamental share price given in Equation (6) in Chapter 15.

Now assume that the real property tax is kept constant at the rate τ_0 in the periods between 0 and t_0:

$$\tau_t = \tau_0 \qquad \text{for } 0 < t = t_0. \tag{72}$$

At the start of period t_0, the government suddenly announces that it will cut the property tax to the lower constant level τ_1, taking effect from the start of the future period t_1. In other words,

$$\tau_t = \tau_1 < \tau_0 \qquad \text{for } t \geq t_1 > t_0. \tag{73}$$

3. Use (71)–(73) to show that:

$$Q_t = \frac{\bar{h} - \tau_0}{r} \qquad \text{for } 0 < t < t_0, \tag{74}$$

$$Q_t = \frac{\bar{h}}{r} - \frac{1}{r}\left\{\left[1 - \frac{1}{(1+r)^{t_1 - t}}\right]\tau_0 + \left[\frac{1}{(1+r)^{t_1 - t}}\right]\tau_1\right\} \qquad \text{for } t_0 \leq t \leq t_1, \tag{75}$$

$$Q_t = \frac{\bar{h} - \tau_1}{r} \quad \text{for } t \geq t_1. \tag{76}$$

(Hint: follow the same procedure as the one we used to derive announcement effects in the stock market, including the formulae in Footnotes 8 and 9). Draw a diagram to illustrate the evolution of the housing price from time 0 onwards. Give an interpretation of (75) and explain why the housing price reacts already at the time the future change in tax policy is announced. Would the same results emerge if expectations were static?

4. Use (74) and (75) to derive an expression for the size of the price jump $Q_{t_0} - Q_{t_0-1}$ between period $t_0 - 1$ and period t_0 when the future property tax cut is announced. Explain the factors determining the size of the initial price jump.

Exercise 6. Creating disinflation by monetary policy: the role of expectations – Exercise 3 of Chapter 20 continued

[To do this exercise you have to answer Exercise 3 of Chapter 20 first. The text below comes in immediate succession of the text of that exercise].

We now leave the assumption of static expectations and assume rational expectations. Equations (AS) and (AD) are still valid. First we drop the assumption $v_t = s_t = 0$, but assume that both shocks are white noise with zero means etc. The REH means that the subjective expectation π_t^e (which in the notation of this chapter would be $\pi_{t,t-1}^e$) formed in period t-1, equals the mean value of the distribution that the model generates for π_t conditional on the information I_{t-1} the economic agents have in period t-1. The latter is assumed to include the prevailing inflation target π^* of the central bank and the objective relations (AS) and (AD) and their parameters and the agents' own expectation π_t^e, but not the realizations of the shocks in period t. Denoting the model consistent conditional mean value by $E(\pi_t \mid I_{t-1})$, the REH is:

$$\pi_t^e = E(\pi_t \mid I_{t-1}) \tag{RE}$$

and the relevant model consists of (AS), (AD) and (RE).

7. Show that for a given expectation π_t^e, it follows from the model that:

$$y_t - y = \frac{v_t - \alpha_2 h s_t}{1+\alpha_2 h\gamma} - \frac{\alpha_2 h}{1+\alpha_2 h\gamma}(\pi_t^e - \pi^*) \tag{77}$$

$$\pi_t - \pi^* = \frac{1}{1+\alpha_2 h\gamma}(\pi_t^e - \pi^*) - \frac{\gamma v_t + s_t}{1+\alpha_2 h\gamma} \tag{78}$$

Then show from (78) that

$$\pi_t^e = E(\pi_t \mid I_{t-1}) = \pi^* \tag{79}$$

Finally, state the relevant solutions for the distributions of the output gap and the inflation gap.

Consider again the situation where, starting from a long-run equilibrium under the old inflation target, the central bank reduces the inflation target from π^{high} to π^{low} working from period 1 and credibly announced to the public in period 0 (zero). Now assume again $v_t = s_t = 0$ in all periods.

8. Explain and illustrate graphically in the $y-\pi$ diagram how output and inflation will evolve the periods 0, 1, 2, 3 … according to the mode with rational expectations. What is now the accumulated, relative output loss arising from the combat against inflation? On basis of your results in the full exercise, come up with some reflections on how best to conduct monetary policy aimed at creating a disinflation and on the size of the associated output loss.

Chapter 22

Limits to stabilization policy: Credibility and uncertainty

Introduction

Do monetary and fiscal policy makers have the ability to stabilize the macro economy, thereby reducing the social costs of business cycles? In the two previous chapters, our answer to this basic question in macroeconomics has been predominantly "yes". However, our formal analysis was based on some important simplifying assumptions. First, we assumed predominantly that policy makers have *perfect information* about the current inflation gap and most often also the current output gap and that they can immediately react on the basis of this information.. Second, in the case with forward-looking agents we postulated predominantly that any policy rule announced by policy makers is always considered fully *credible* by the public, implying, for example, that policy makers never have any problem convincing the public that they will stick to an anti-inflationary policy. Taken together, these assumptions are very optimistic and not very realistic. In this chapter we shall study the problems of stabilization policy when these strong assumptions are replaced by more realistic ones.

We will start by studying the problems of establishing the *credibility* of an anti-inflationary monetary policy. This part of our analysis will show how our AS–AD model combined with the hypothesis of rational expectations may provide a theoretical case for *delegation* of monetary policy to an *independent central bank*, as a lot of countries have actually done in recent decades. The subsequent part of the chapter investigates how *uncertainty* about the current state of the economy limits the scope for stabilization policy.

22.1 Policy rules versus discretion: the credibility problem

The time-inconsistency of optimal monetary policy

In the previous chapter, we saw how changes in the rational expectations of the private sector may sometimes offset the intended effects of stabilization policy. Another basic discovery made by the rational expectations school in macroeconomics was the insight that it may not be possible to implement an optimal economic policy because it lacks *credibility*. For example, when the previous chapter derived the rational expectations solution for expected inflation, $\pi^e = \pi^*$, it was assumed that private agents are confident that the central bank will always stick to the announced monetary policy rule with a target inflation rate, π^*. But such credibility of economic policy may be difficult to achieve when policy makers can undertake *discretionary* policy changes *after* private agents have formed their expectations.[1] If monetary policy makers announce that they will keep the inflation rate down to a certain target level, the private sector may not consider such a statement to be credible if the central bank can boost output and employment by generating unanticipated inflation 'down the road'.

We will now use our AS–AD model to illustrate the credibility problem arising under discretionary policy with rational expectations. To simplify (without invalidating our qualitative conclusions), we will set the aggregate supply curve parameter $\gamma = 1$. For the moment, we will also abstract from demand and supply shocks as well as monetary policy shocks ($v_t = z_t = s_t = \hat{\rho}_t = 0$). The expectations-augmented Phillips curve may then be written as:

$$\pi_t = \pi^e_{t,t-1} + y_t - \bar{y}. \qquad (1)$$

and the goods market equilibrium condition simplifies to:

$$y_t - \bar{y} = -\alpha_2 (r_t - \bar{r}). \qquad (2)$$

As before, we assume that the central bank can observe the expected rate of inflation (say, through consumer surveys or by observing the difference between the interest rates on indexed and non-indexed bonds) when it sets the nominal interest rate. Thus the central bank can set the *real* interest rate r_t after the private sector has formed its expectations of inflation. It then follows from (2) that the central bank can control the current output gap $y_t - \bar{y}$. According to (1) it can therefore determine the actual inflation rate, π_t, through its choice of $y_t - \bar{y}$, for any given expected inflation rate $\pi^e_{t,t-1}$.

We assume that monetary policy makers would like to minimize the social loss

[1] Remember from Chapter 20 that when policy is discretionary, policy makers do not follow a fixed policy rule like the Taylor rule. Instead, they can adjust their instruments in any way they believe will serve the goals of stabilization policy in a particular situation.

function:

$$SL_t = (y_t - y^*)^2 + \kappa\pi_t^2, \quad \kappa > 0. \tag{3}$$

According to (3) society loses welfare when output deviates from its target level y^* and when inflation deviates from its target rate which we now take to be zero, that is, $\pi^* = 0$, in our usual notation. The latter assumption is only for simplicity. The quadratic form implies that large deviations of output and inflation from their respective targets cause disproportionately larger losses than small deviations. The parameter κ indicates the strength of the social preference for price stability relative to output stability. Note that in the case where the target level of output equals natural output, $y^* = \bar{y}$, Equation (3) is just a version of the social loss function (1) in Chapter 20, where the target inflation rate has been set at zero.

As an important element of the present 'credibility analysis' we will assume that policy makers may have a more ambitious target for economic activity than the structural level of output or, equivalently, that they aim at a level of unemployment below the natural (structural) rate. In this case, the target output in the social loss function (3) will be larger than structural output, $y^* > \bar{y}$, or

$$y^* = \bar{y} + \omega, \quad \omega > 0. \tag{4}$$

There can be several reasons for such a situation. It may be that there is public pressure on policy officials (including central bankers) to secure a low level of unemployment below the natural rate, because of the serious social costs of unemployment we discussed in Chapter 14. This may be particularly relevant if the structural level of unemployment is relatively high due to badly functioning labour and product markets. It may also be that politicians compete for votes by expressing very high ambitions for output and income levels and thusbecome 'overambitious' in stabilization policy in their attempts to keep their promises. For these reasons it is relevant to analyse the *consequences* of a situation where $y^* > \bar{y}$. The parameter ω in (4) reflects the magnitude of 'over-ambition' and/or of the distortions in labour and product markets (including tax distortions) that keeps structural GDP below the policy makers' target for GDP.

From (1) it follows that $y_t = \bar{y} + \pi_t - \pi_{t,t-1}^e$, so according to (4) we have $y_t - y^* = \pi_t - \pi_{t,t-1}^e - \omega.$ Substituting this into (3), we get:

$$SL_t = (\pi_t - \pi_{t,t-1}^e - \omega)^2 + \kappa\pi_t^2. \tag{5}$$

Now suppose, but only for a moment, that the central bank follows a Taylor rule with a zero inflation target:

$$r_t = \bar{r} + h\pi_t + b(y_t - \bar{y}) \tag{6}$$

The public is assumed to have rational expectations and to know and (first) to believe in the policy rule (6). Recalling that $v_t = s_t = 0$, it then follows from the model consisting of (1), (2), (6) and the rational expectations (RE) requirement $\pi^e_{t,t-1} = E(\pi_t \mid I_{t-1})$ that the economy will end up in the following equilibrium (show this for practice, using the method explained in the previous chapter):

RE equilibrium under believed-in Taylor rule $\qquad \pi_t = \pi^e_{t,t-1} = \pi^* = 0, \quad y_t = \bar{y}. \qquad (7)$

But would the central bank actually want to stick to the Taylor rule? To investigate this, let us set the central bank free of (6) and simply assume that it chooses the interest rate r_t as it wants. As mentioned above, through Equation (2) this will create a certain output gap, which through (1) will then create a certain inflation rate π_t given the expectation $\pi^e_{t,t-1}$, that is, it will create a certain surprise inflation, $\pi_t - \pi^e_{t,t-1}$. Hence, if the central bank can choose its interest rate r_t after the expectation $\pi^e_{t,t-1}$ has been formed, then the central bank can create the surprise inflation, $\pi_t - \pi^e_{t,t-1}$ it wants. We will proceed as if the central bank directly chose π_t given $\pi^e_{t,t-1}$, although this happens indirectly through the choice of r_t. Now, starting from the situation (7) that the believed-in Taylor rule (6) would create, where $\pi_t = \pi^e_{t,t-1} = 0$, suppose the central bank were to deviate from the rule by generating surprise inflation after the private sector had formed its expectation $\pi^e_{t,t-1} = 0$. How would social welfare be affected by such a policy of 'cheating'? Calculating the derivative of the social loss function (5) with respect to the inflation rate at the initial point where $\pi_t = \pi^e_{t,t-1} = 0$ we find:

$$dSL/d\pi_t = -2\omega < 0. \tag{8}$$

Starting from the Taylor rule equilibrium (or any other equilibrium where $\pi_t = \pi^e_{t,t-1} = 0$), the social loss can thus be *reduced* if the central bank decides to increase inflation from zero and thus drive output closer to its desired level y^* by generating surprise inflation. The reason is that if $\pi_t = \pi^e_{t,t-1} = 0$, so that $y_t = \bar{y}$, Equations (3) and (4) imply that the marginal social cost of a slight rise in inflation is zero, whereas the marginal social benefit from a slight rise in output is positive. *Hence, a central bank which can engage in discretionary policy will not want to stick to a policy rule that generates price stability*.

Indeed, for any given expected inflation rate, the central bank will want to set the actual inflation rate such that the social loss function (5) is minimized. The first-order condition for the optimal choice of the rate of inflation is $\partial SL_t / \partial \pi_t = 0$, which implies:

$$\underbrace{2(\pi_t - \pi^e_{t,t-1} - \omega)}_{\substack{=2(y_t - y^*)=\text{marginal} \\ \text{reduction in } SL \text{ due to} \\ \text{higher output}}} + \underbrace{2\kappa\pi_t}_{\substack{\text{marginal} \\ \text{increase in } SL \\ \text{due to higher} \\ \text{inflation}}} = 0.$$

Solving this optimum condition for π_t gives:

$$\pi_t = \frac{1}{1+\kappa}\pi^e_{t,t-1} + \frac{\omega}{1+\kappa}. \tag{9}$$

Hence, if the central bank has first led the public to believe that it will ensure price stability, $\pi^e_{t,t-1} = 0$, it follows from (9) and (1) that the economy will actually end up with the

'Cheating outcome' with surprise inflation $\qquad \pi_t = \dfrac{\omega}{1+\kappa}, \quad y_t = \bar{y} + \dfrac{\omega}{1+\kappa}.$ (10)

If SL_R is the social loss incurred in the believed-in Taylor rule equilibrium (7), and SL_C is the social loss in the 'cheating' outcome (10), we can use (3), (4), (7) and (10) to calculate $SL_R = \omega^2$ and $SL_C = \kappa\omega^2 / (1+\kappa)$, and then the social welfare gain from cheating is:

Temptation to cheat $\qquad SL_R - SL_C = \dfrac{\omega^2}{1+\kappa}.$ (11)

Equation (11) is intuitively appealing: the greater the difference between natural output and desired output, ω, the greater is the temptation to create surprise inflation in order to raise output above the natural rate. On the other hand, the stronger the social aversion to inflation, κ, the smaller is the gain from surprise inflation.

The important point is that the central bank has no incentive to actually implement the policy $\pi_t = \pi^* = 0$, if it has made the private sector believe that it will implement this policy. In other words, over time the central bank will not want to act in a manner consistent with the rule it previously announced. Economists therefore say that a rule-based equilibrium like (7) with zero inflation is *dynamically inconsistent* or *time-inconsistent* when policy makers have discretion.[2]

[2] The problem of time inconsistency was first analysed by the Nobel Prize winners Finn E. Kydland and Edward C. Prescott, 'Rules Rather than Discretion: The Inconsistency of Optimal Plans', *Journal of Political Economy*, **88**, 1977, pp. 867 – 896. The problem was later elaborated in another famous paper by Robert J. Barro and David B. Gordon, 'A Positive Theory of Monetary Policy in a Natural Rate Model', *Journal of Political Economy*, **91**, 1983, pp. 589 – 610.

THE PROBLEM OF TIME CONSISTENCY

When the socially desired output level exceeds natural output and the initial inflation rate is zero, policy makers will have an incentive to boost output by creating surprise inflation. A promise by the central bank to maintain price stability is therefore said to be time-inconsistent when the bank can act in a discretionary manner, since it will not be optimal for the central bank to avoid inflation once private agents have formed their expectations and negotiated nominal wage contracts on the assumption that prices will be kept stable.

Time-consistent monetary policy

Of course, rational agents who know the preferences of policy makers will realize that the central bank will not really want to implement the policy $\pi_t = 0$. This is the *credibility problem*: if the central bank cannot make a *binding commitment* to stick to the Taylor rule or some other rule ensuring price stability, the announcement that the bank intends to follow a policy of price stability will not be credible and will hence fail to eliminate expectations of inflation. This is because private agents know that the central bank will have an incentive to deviate from price stability in an effort to stimulate output and employment. More precisely, rational agents will recognize that the central bank will set the interest rate so as to achieve the inflation rate implied by the first-order condition (9). Thus neither the rule-based equilibrium (7) nor the 'cheating' outcome (10) will be realized, since these outcomes are not true rational-expectations equilibria when agents know that policy makers have discretion and seek to minimize the social loss function (5). Instead, rational agents will form their expectations on the basis of (9). Inserting the RE requirement, $\pi^e_{t,t-1} = E(\pi_t \mid I_{t-1})$, here simplifying to $\pi^e_{t,t-1} = \pi_t$, gives $\pi^e_{t,t-1} = \pi_t = \omega/\kappa$. Inserting this solution for the expected inflation rate into (1), we obtain the

$$\textit{Time-consistent RE equilibrium:} \qquad \pi_t = \pi^e_{t,t-1} = \frac{\omega}{\kappa}, \quad y_t = \bar{y}. \tag{12}$$

The equilibrium in (12) is said to be *time consistent* because policy makers have no incentive to deviate *ex post* from the inflation rate $\pi_t = \omega/\kappa$, given that this rate of price increase is derived from the first-order condition (9). In the time-consistent equilibrium, expectations of inflation have driven the actual inflation rate up to a level which is so high that policy makers do not wish to generate further (surprise) inflation, even though they have the ability to do so through discretionary policy.

The results in (12) illustrate the unfortunate implications of the credibility problem under discretionary monetary policy. Though the realized inflation rate permanently exceeds the target inflation rate by the amount ω/κ, inflation is fully anticipated, so no output gains are obtained in return for the excess inflation. Clearly, this outcome is worse than the outcome (7) which would emerge if the central bank could make a binding commitment to stick to a policy of price stability. Let SL_D denote the social loss in the time-consistent equilibrium with discretionary policy. Inserting (4) and (12) into (3), we find that:

$$SL_D = \overbrace{\omega^2}^{\substack{\text{social loss}\\ \text{due to output}\\ \text{below target}}} + \overbrace{\frac{\omega^2}{\kappa}}^{\substack{\text{social loss}\\ \text{due to}\\ \text{inflation}}} . \qquad (13)$$

The second term on the right-hand side of (13) could be eliminated if the central bank could somehow commit itself to a rule of price stability as follows from $SL_R = \omega^2$. This term therefore represents the welfare loss from the inability to commit. We see that the presence of over-ambition or market distortions (ω) and the resulting temptation to generate surprise inflation in order to boost output creates an *inflation bias* under discretionary monetary policy.

The points made above are illustrated graphically in Figure 22.1, which is of the same type as Figure 20.1 in Chapter 20. The concentric ovals are social indifference curves showing alternative combinations of output and inflation, which generate a constant social loss. The equation for the indifference curve corresponding to the social loss $\overline{C}$ is found by setting the expression on the right-hand side of (3) equal to the constant $\overline{C}$.[3] The minimum social loss (= 0) is achieved at the 'bliss' point E^*, where $y = y^*$ and $\pi = 0$. Larger ovals further away from E^* correspond to higher levels of social loss. The *first-best* optimum E^* cannot be attained in an equilibrium where expectations are fulfilled, since it follows from (1) that $y_t = \overline{y}$ when $\pi^e_{t,t-1} = \pi_t$ (to put it another way, monetary policy cannot eliminate the imperfection reflected in $\omega = y^* - \overline{y}$). The *second-best* optimum E_R is the equilibrium, which emerges when the central bank can make a binding commitment to a policy of price stability. In that case the private sector will rationally expect stable prices, and the economy's short-run aggregate supply curve will be given by the AS curve marked SRAS ($\pi^e = 0$) corresponding to a zero expected rate of inflation. However, if the central bank can engage in discretionary policy so that the policy rule $\pi = 0$ is not truly binding, it will have an incentive to create surprise inflation to move the economy from point E_R to the 'cheating' outcome E_C, where the AS curve is tangent to a social indifference curve. Point E_C represents the lowest possible level of social loss, given a zero expected inflation rate. The trouble is that rational private agents anticipate

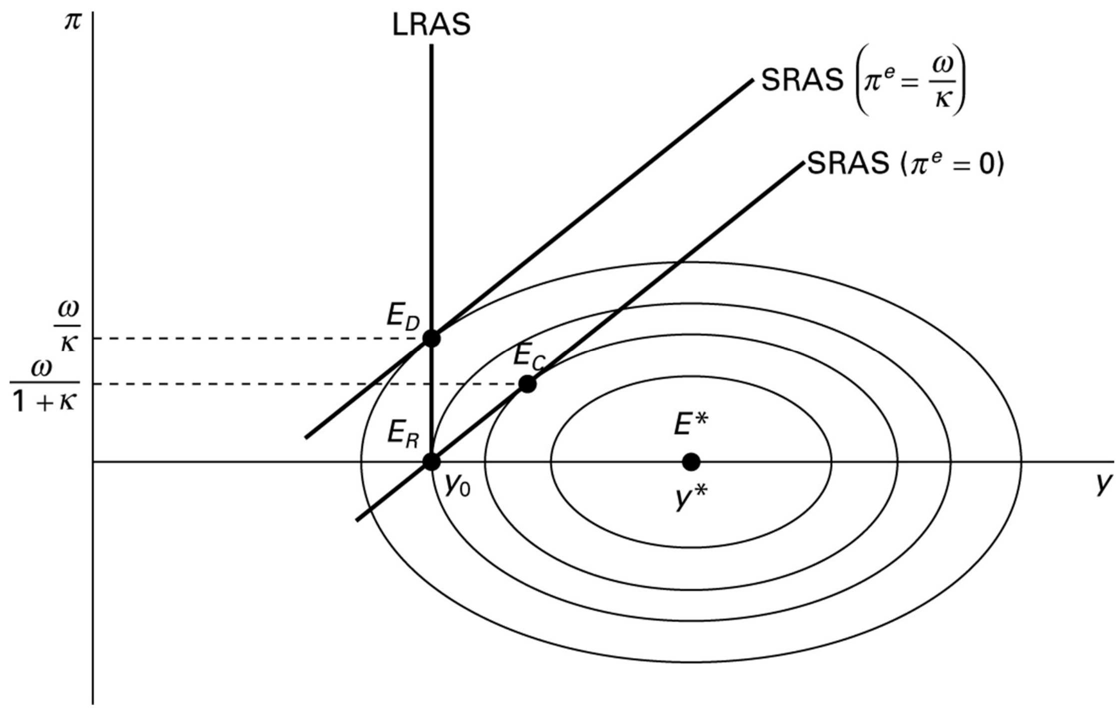
[3] Along a social indifference curve we thus have $dSL = 0$. According to (3) this implies

$$(y - y^*) \cdot dy + \kappa\pi \cdot d\pi = 0 \iff \frac{d\pi}{dy} = \frac{y^* - y}{\kappa\pi}.$$

Thus the slope of the indifference curves becomes 0 when $y = y^*$, whereas the slope tends to infinity when inflation tends to 0, as illustrated in Figure 22.1.

the central bank's incentive to cheat, so according to (12) they will expect an inflation rate equal to ω/κ under discretionary policy. The actual short-run aggregate supply curve will then be given by the curve SRAS ($\pi^e = \omega/\kappa$) in Figure 22.1, and the economy will end up in the time-consistent rational expectations equilibrium E_D. This is a *third-best* optimum where the social loss is minimized given the private sector's positive expected inflation rate.

Figure 22.1 Monetary policy in the Barro–Gordon model: the rule-based equilibrium (E_R), the 'cheating' outcome (E_C), and the time-consistent equilibrium (E_D).



The model of inflation summarized in Figure 22.1 is often referred to as the *Barro–Gordon model*, named after its inventors (see the reference in Footnote 2). It has had a strong influence on the way economists think about monetary policy, and it helps to explain why many economists have come to favour binding policy rules over discretionary policy. In particular, the Barro–Gordon model has motivated economists and policy makers to think about ways of securing commitment to policy rules in order to overcome the potential inflation bias in monetary policy.

TIME-CONSISTENT MONETARY POLICY

When the socially desired output exceeds natural output and monetary policy makers have discretion, rational private sector agents will anticipate that the central bank has an incentive to create surprise inflation if the private sector were to act on the assumption that prices will be stable. The private sector will therefore expect a positive inflation rate. A time-consistent macroeconomic equilibrium is reached when the expected and actual inflation rate is so high that the central bank no longer has an incentive to generate surprise inflation. In the time-consistent equilibrium output will be at its natural rate. A lower social loss could be secured if the central bank could convince the private sector that it will stick to a policy of price stability, but under policy discretion an announced policy of price stability will lack credibility because it is time-inconsistent.

22.2 Dealing with inflation bias: reputation-building and delegation of monetary policy

Building a reputation

One situation where a rule-based policy of price stability may be sustainable is when policy makers rely on *reputation*. The analysis in the previous section implicitly assumed that policy makers are short-sighted, caring only about the economic outcome in the current period. In that case, it is always rational for them to 'cheat' by creating surprise inflation if the initial inflation rate is sufficiently small, e.g., (close to) zero, given that they are over-ambitious in the sense that $\omega > 0$. Hence, rational agents will never consider an announced policy of price stability to be credible if they believe policy makers to be myopic. However, policy makers who interact with the private sector period after period will have an incentive to consider the implications of their current actions for the future behaviour of private agents. For example, if the creation of surprise inflation implies that the policy makers will face higher expected inflation rates in the future, they may prefer to stick to an announced policy of price stability. In this way, they will earn a reputation for being a reliable protector of monetary stability, and this will keep future expected and actual inflation rates down.[4]

To illustrate how such a mechanism of reputation-building might work, suppose that the public believes the inflation announcements of the central bank as long as the bank

[4] This idea was developed in another influential paper by Robert J. Barro and David B. Gordon, 'Rules, Discretion and Reputation in a Model of Monetary Policy', *Journal of Monetary Economics*, **12**, 1983, pp. 101–121. The simplified version of the dynamic Barro–Gordon model presented below is heavily inspired by Ben J. Heijdra and Frederick van der Ploeg, *Foundations of Modern Macroeconomics*, Oxford University Press, 2002, Section 10.1.3.

does not generate any unanticipated inflation, but, if it does, will not believe in the central bank for a period and instead form inflation expectations consistent with the bank being a 'cheater'. More precisely, if there were no inflation surprises last period, that is, $\pi_{t-1} = \pi^e_{t-1,t-2}$, the central bank has credibility in the current period. Monetary policy makers can use such credibility to eliminate expectations of inflation by announcing that they will follow a policy rule ensuring an inflation rate π_R equal to zero. We then have:

$$\pi^e_{t,t-1} = \pi_R = 0 \quad \text{if } \pi_{t-1} = \pi^e_{t-1,t-2}. \tag{14}$$

If the central bank 'cheats' in some period $t - 1$ by deviating from price stability, it loses its credibility for the subsequent period t. The public will then form its expectations for period t on the assumption that the central bank will pursue the optimal discretionary policy yielding the inflation rate ω/κ derived in (12) above. Hence we have:

$$\pi^e_{t,t-1} = \pi_D = \frac{\omega}{\kappa} \quad \text{if } \pi_{t-1} \neq \pi^e_{t-1,t-2}, \tag{15}$$

where π_D is the optimal inflation rate under discretion. Knowing that this is the public's expected inflation rate, the best thing the central bank can do in period t is to announce and implement the policy $\pi_D = \omega/\kappa$. In this way, it regains credibility in the next period (t+1) by carrying out its announced plan in the current period. We may say that the public is playing 'tit-for-tat' against the policy maker. If the central bank 'behaves well' by sticking to its promises, it is 'rewarded' by zero expected inflation in the next period, enabling it to avoid the social loss associated with inflation. If the central bank 'misbehaves' by creating surprise inflation, it is 'punished' by high expectations of inflation in the next period.

Suppose we start out in a period in which the policy maker has inherited credibility from the past and has announced $\pi_t = \pi_R = 0$ believed in by the public, $\pi^e_{t,t-1} = \pi_R = 0$. The policy maker must then decide whether to stick to the policy $\pi_R = 0$ generating the current-period social loss SL_R associated with the rule-based equilibrium (7), or whether to cheat in order to reduce the current-period social loss to the lower level SL_C associated with the 'cheating' outcome (10). If the policy maker decides to cheat, the net social gain in the current period will be $SL_R - SL_C$ as given by (11). However, in the next period the public will have no faith in the policy maker and will therefore expect the inflation rate of the time-consistent equilibrium (12), ω / κ, in which case the policy maker can do no better than exactly creating this inflation rate. Note that this will indeed restore the public's faith in the policy maker. Hence, in the period after the cheating, the economy will end up in the third-best equilibrium (12), whereas it could have ended up in the second-best equilibrium (7) if the policy maker had not cheated. This means that the cheating will generate a net social loss of $SL_D - SL_R$ in the period coming after the cheating period and only in that period. Since $SL_R = \omega^2$ and $SL_D = \omega^2 + \omega^2 / \kappa$, we have $SL_D - SL_R = \omega^2 / \kappa$.

If the central bank (or the society for which the central bank is here the policy maker) has a positive rate of time preference ρ, it will discount next period's social loss when comparing it to this period's social gain. Instead of Equation (11) which is relevant only for a myopic policy maker, we then get the following modified expression for the

$$\textit{Temptation to cheat} \qquad \overbrace{SL_R - SL_C}^{\substack{\text{current-period}\\ \text{gain from}\\ \text{cheating}}} - \frac{\overbrace{(SL_D - SL_R)}^{\substack{\text{next-period}\\ \text{loss from}\\ \text{cheating}}}}{1+\rho}. \tag{16}$$

Our previous expression (11) for the temptation to cheat is just the special case of (16) occurring when the policy maker is very short-sighted. In that case his or her rate of time preference ρ appoaches infinity so that the second term on the right-hand side of (16) vanishes.

If the expression in (16) is positive, the policy maker will always want to cheat. The rule-based policy of price stability will then be unsustainable, and the economy will end up in the third-best time-consistent equilibrium (12) in every period, just like before. But if (16) is negative, the policy maker has no incentive to deviate from price stability, and the rule-based second-best equilibrium (7) will then be sustainable and implemented every period. Using the expression (11) for $SL_R - SL_C$ and $SL_D - SL_R = \omega^2 / \kappa$ as just shown, we find that the temptation to cheat may be written as:

$$(SL_R - SL_C) - \frac{SL_D - SL_R}{1+\rho} = \frac{\omega^2(\kappa\rho - 1)}{\kappa(1+\kappa)(1+\rho)}. \tag{17}$$

Equation (17) shows that the policy maker will not want to cheat if $\kappa\rho < 1$. Hence, if the central bank's discount rate ρ is sufficiently low, that is, if society cares sufficiently about the future, it will always announce and create the inflation rate $\pi_t = \pi_R = 0$. With a low discount rate, the short-term gain from cheating will be outweighed by the future social loss from the inflation that follows from the loss of credibility. Furthermore, a low value of the inflation aversion parameter κ also helps to increase the likelihood that the policy maker will not want to cheat. The reason is that a low value of κ generates a high rate of inflation in the third-best equilibrium emerging when the policy maker has lost credibility. Hence, a low value of κ implies a high value of SL_D (see (13)) which makes the policy maker more eager to avoid a loss of credibility. On the other hand, if $\kappa\rho > 1$, it will always pay for the central bank to cheat, that is, if it has first made the public believe in a zero inflation rate it will actually inflate the economy. The public looks through this and therefore in all periods expects the inflation rate $\pi^e_{t,t-1} = \omega / \kappa$, in which case the central bank can do no better than creating exactly this inflation rate. Note that in both cases the public's expectations turn out to be correct or 'rational'.

Finally, it follows from (17) that the magnitude of over-ambition ($\omega > 0$) does not influence the *sign* of the expression for the temptation to cheat, since a higher value of ω

increases next period's loss as well as this period's gain from cheating.

DEALING WITH INFLATION BIAS THROUGH REPUTATION-BUILDING

If the public believes in an announced policy of price stability as long as the central bank does not in fact generate surprise inflation, policy makers may be able to establish credibility by building a reputation for sticking to a policy rule of price stability. Policy makers will stick to such a rule only if the short-run gain from creating surprise inflation is smaller than the discounted future social loss from the higher expected future inflation that follows from 'cheating' the private sector. If the time horizon of policy makers is too short and their discount rate consequently too high, a rule-based policy of price stability will be unsustainable, and the economy will end up in the third-best, time-consistent equilibrium emerging under policy discretion.

Delegation of monetary policy

The insight from the preceding analysis is that the inflation bias in monetary policy may be eliminated by the policy maker's incentive to build a reputation, provided the policy maker places sufficient weight on the future. But casual observation suggests that governments are often very preoccupied with the short term, perhaps because they are mainly concerned about winning the next election.[5] In other words, governments often seem to act as if they put little weight on the more distant future, indicating a high discount rate. For this reason, many economists doubt that the incentive for reputation-building will be sufficiently strong to eliminate the inflation bias if monetary policy is controlled directly by the government. As an alternative way of offsetting the inflation bias, it has therefore been suggested that politicians should *delegate* monetary policy to an *independent central bank* with a strong mandate to resist inflation.

In practice, central banks can be more or less independent of the government and the economic literature suggests various measures of the degree of central bank independence (CBI). The most common measure is *de jure* CBI, which bases the evaluation of CBI on studies of the formal rules governing the banks, that is, their statutes, charters, constitutions, regulatory laws and amendments etc. There are typically three dimensions of *de jure* CBI: personnel, financial, and policy independence. *Personnel independence* reflects the degree to which government officials are represented on the governing board of the central bank; the extent to which board members are appointed by the government; the length of the term of office of the governor(s), etc. *Financial*

[5] Of course, if voters actually care about the (more distant) future, politicians seeking re-election should also support policies which place a reasonable weight on the future. If actual policies nevertheless seem to be biased towards the short run, there must be some kind of imperfections in the political process whereby voter preferences get translated into political outcomes. Such imperfections are one of the subjects of the theory of Political Economy which we shall have to leave for another course.

independence focuses on the stringency of limitations on the ability of the government to borrow from the central bank. *Policy independence* has two subcomponents and measures the degree to which the central bank can set its policy instruments (*instrument independence*) and its policy goals (*goal independence*) without having to take instructions from the government. The three dimensions can be aggregated into an overall index of CBI.

One of the most comprehensive and recent evaluations of *de jure* CBI was created by the economist Ana Carolina Garriga.[6] Her data set covers 182 countries and the years between 1970 through 2012, providing for many of these country-year combinations a total index of CBI normalized to be between 0 and 1 with 1 representing maximal independence. Table 22.1 shows average CBI indices for the periods 1970-90 and 1991-2012 for the 'old' 22 OECD member states joining before 1989.

Table 22.1 Average central bank independence in 22 'old' OECD countries, 1970-1990 and 1991-2012.

	Average Central Bank Independence			**Average Central Bank Independence**	
	1970-1990	**1991-2012**		**1970-1990**	**1991-2012**
Australia	0.35	0.28	Luxembourg	0.33	0.69
Austria	0.62	0.78	Netherlands	0.42	0.72
Belgium	0.16	0.72	New Zealand	0.25	0.37
Canada	0.45	0.46	Norway	0.17	0.29
Denmark	0.50	0.50	Portugal	0.35	0.77
Finland	0.27	0.67	Spain	0.21	0.78
France	0.27	0.80	Sweden	0.29	0.29
Germany	0.69	0.81	Switzerland	0.54	0.65
Greece	0.55	0.74	Turkey	0.49	0.70
Italy	0.25	0.77	UK	0.28	0.56
Japan	0.18	0.36	USA	0.48	0.48

Source: Article by Ana Carolina Garriga mentioned in Footnote 6.

Note that for a given period, e.g., 1991-2012, there are notable differences in CBI between countries reaching from around 0.3 to around 0.8 (the maximal possible distance is, of course, 1), and differences between otherwise highly comparable countries of 0.2-0.3. Furthermore, there is a general, almost exception-free tendency across countries that

[6] Ana Carolina Garriga, 'Central Bank Independence in the World: A New Data Set', *International Interactions*, **42**, 2016, pp. 849–868.

CBI has increased from the earlier to the later period. We use Garriga's data on CBI further below to study the relationship between CBI and inflation levels.

However, first we will show formally how the delegation of monetary policy to a (more or less) independent and 'conservative' central bank may help to reduce the inflation bias.[7] Suppose the governor of the central bank considers the loss from instability of output and prices to be given by the loss function:

$$SL_t^B = (y_t - y^*)^2 + (\kappa + \varepsilon)\pi_t^2, \quad \varepsilon > 0, \tag{18}$$

where ε measures the degree to which the inflation aversion of the central banker *exceeds* the inflation aversion of the government. We may say that the parameter ε is a measure of the central banker's degree of 'conservativeness' (his or her 'excess' aversion to inflation, taking the preferences of the government as the benchmark).

Let β be an index of CBI which can assume a value between 0 (no independence) and 1 (full independence). The parameter β measures the degree to which the government has *delegated* monetary policy to the central bank. The greater the value of β, the greater is the weight of the central banker's preferences in the determination of monetary policy. Since the government's loss function is still given by (3), we may then assume that monetary policy is determined by minimization of the modified loss function:

$$\tilde{S}L_t = (1-\beta)\cdot SL_t + \beta \cdot SL_t^B = (y_t - y^*)^2 + (\kappa + \beta\varepsilon)\pi_t^2, \quad 0 \le \beta \le 1, \tag{19}$$

reflecting a compromise between the government and the central bank. The loss function (19) is similar to (3) except that the inflation aversion parameter κ has been replaced by $\kappa + \beta\varepsilon$. All the results from the previous section therefore carry over if we just substitute $\kappa + \beta\varepsilon$ for κ. From (12) we then obtain the *time-consistent rational expectations equilibrium with delegation of monetary policy*:

$$\pi_t = \pi_{t,t-1}^e = \frac{\omega}{\kappa + \beta\varepsilon}, \quad y_t = \bar{y}. \tag{20}$$

A comparison of (12) and (20) reveals that *by delegating monetary policy authority to a central banker who is more conservative than itself, the government can reduce the inflation bias in discretionary monetary policy*. Indeed, by appointing a very conservative central banker who cares only about price stability ($\varepsilon \to \infty$), the government can move from the third-best equilibrium (12) with positive inflation to the second-best equilibrium (7) with zero inflation. The reason is that a policy maker who is extremely averse to inflation has no incentive to cheat, since cheating involves the creation of inflation. The paradox is that, by voluntarily tying its own hands and delegating monetary policy to a policy maker whose preferences deviate from its own, the government can achieve an

[7] This idea was originally put forward by Kenneth Rogoff, 'The Optimal Degree of Commitment to an Intermediate Monetary Target', *Quarterly Journal of Economics*, **100**, 1985, pp. 1169 –1189.

outcome, which is more desirable from its own viewpoint. To see this, we may insert the results from (20) into the *government's* loss function (3) and use (4) to get the *social loss with delegation of monetary policy*:

$$SL = \omega^2 + \frac{\kappa\omega^2}{(\kappa + \beta\varepsilon)^2}. \tag{21}$$

Clearly this loss is smaller when policy is (partly) delegated to a conservative central banker ($\beta\varepsilon > 0$) than when monetary policy only reflects the government's own preferences ($\beta\varepsilon = 0$).

Note that *two* conditions are required to obtain this beneficial outcome:

1. The central bank must have some independence ($\beta > 0$).

2. The central banker must be more conservative than the government ($\varepsilon > 0$).

If the central bank has no independence at all, the preferences of its governor are irrelevant, since they will be completely overruled by the government. And if the central bank has the same preferences as the government, it will obviously make no difference whether policy is made by the bank or by the government itself.

This analysis suggests that, other things equal, we should expect to observe a lower average rate of inflation in countries with a higher degree of central bank independence.[8] A number of empirical studies have indeed found a clear negative correlation between CBI and inflation, using alternative indices of CBI. Here we will investigate the relationship between CBI and inflation using the Carriga data set for CBI described above and inflation data from OECD. We focus here on 'old' OECD member states joining before 1989, except that we exclude Turkey from the sample.[9] We thus have a (balanced) panel data set of CBI index values and inflation rates for 21 old OECD countries from 1970 to 2012.

We will base our investigation on the presumption that the inflation rate π_t^i in country i in year t, is well explained by the following factors: An independently, identically and normally distributed shock variable (or error term) ξ_t^i that captures the influences of supply and demand shocks; the inflation rate π_{t-1}^i in country i in the former period to capture inflation inertia and as an imperfect substitute for the fact that the shock variable

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[8] The usual caveat 'other things equal' is important here, since the government of a country with a more independent central bank might want to appoint a less conservative central bank governor when there is a trade-off between fighting inflation and minimizing the variability of output. Below we will explain the circumstances in which such a trade-off will arise.

[9] The 21 countries considered are thus Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States of America. Turkey is only excluded to make Figure 22.2 below nicer (some very big variations for Turkey would require another scaling of the axes if Turkey should be included), but the exclusion does not affect the reported results.

should perhaps, more realistically, have been assumed serially correlated; the CBI index CBI_t^i for country *i* in period *t*; and finally a country-specific, time-independent inflationary effect called α_i to account for the eventuality that some countries may simply be more prone to inflation than others. All in all, our hypothesis is expressed by

$$\pi_t^i = \beta \pi_{t-1}^i + \delta CBI_t^i + \alpha_i + \xi_t^i, \tag{22}$$

which is an equation often used in this kind of analysis. Our intention is to estimate the parameters of this equation using our panel data set for CBI and inflation, thus obtaining an estimate of δ, the parameter we are interested in. There are no data for the 'proneness-to-inflation' of a country, represented by α_i in (22), but as shown below we can overcome this difficulty by using a so-called fixed-effect method of regression analysis, which utilizes the assumption that the α_i's are constant over time. You will probably learn much more about estimation with fixed effects in your further studies.

Letting the first period of our panel data set be 0 and *T* the last, we can study Equation (22) over the periods *t* = 1, …, *T* (period *t* = 1 is the first for which we have π_{t-1}^i). If for each country *i* we add up both sides of (22) over the *T* periods from 1 to *T* and in the resulting equation divide on both sides by *T*, we get for each country *i*:

$$E[\pi^i] = \beta E[\pi_{-1}^i] + \gamma E[CBI^i] + \alpha_i + \zeta^i,$$

where $E[\pi^i] \equiv \sum_{t=1}^{T} \pi_t^i / T$, $E[\pi_{-1}^i] \equiv \sum_{t=0}^{T-1} \pi_t^i / T$, $E[CBI^i] \equiv \sum_{t=1}^{T} CBI_t^i / T$ and $\zeta^i \equiv \sum_{t=1}^{T} \xi_t^i / T$. Now, subtracting this second equation from (22) gives for each country-year combination:

$$\pi_t^i - E[\pi^i] = \beta(\pi_{t-1}^i - E[\pi_{-1}^i]) + \gamma(CBI_t^i - E[CBI^i]) + \eta_t^i, \tag{22'}$$

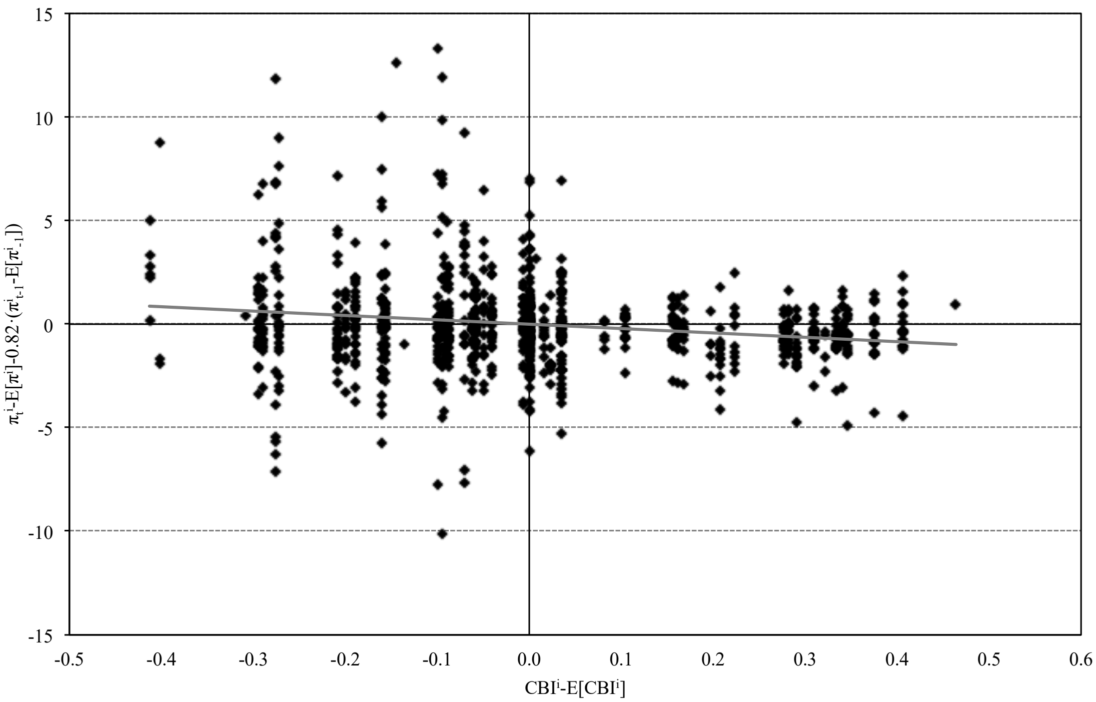
where $\eta_t^i \equiv \xi_t^i - \zeta^i$ is just a new error tem and *the country-specific factor* α_i *has disappeared*. This is exactly the point. Because of the panel structure of our data set and the time-independency of the country-specific proneness-to-inflation (which is, of course, really an assumption), we can get rid of α_i and still, by estimating (22'), obtain an estimate of the δ of (22), that isour parameter of interest. A direct estimation of (22') by OLS, where inflation is measured in percent, gives:

$$\pi_t^i - E[\pi^i] = 0.82 \cdot (\pi_{t-1}^i - E[\pi_{-1}^i]) - \underset{(se=0.45;\ t=-4.46)}{2.11} \cdot (CBI_t^i - E[CBI^i]) + \eta_t^i, \quad R^2 = 0.75$$

We see directly that the coefficient δ is estimated to be negative and significantly so. If this can be interpreted as a causal effect from CBI to inflation, our analysis points to a clear negative influence of CBI on inflation levels and it indicates the strength of this influence. If CBI goes all the way from 0 to 1 (an extreme case), the annual rate of inflation will, according to our estimation, go down by a bit more than 2 percentage points. From Table 22.1 a large change of CBI could be by 0.5, while a highly relevant one could be 0.2, which should, according to our analysis, give reductions of annual inflation rates by around 1 and 0.4 percentage points, respectively, which are still reductions of 'interesting' size.

Figure 22.2 gives a graphical illustration based on our estimation of (22') by plotting $\pi_t^i - E[\pi^i] - 0.82 \cdot (\pi_{t-1}^i - E[\pi_{-1}^i])$ against $CBI_t^i - E[CBI^i])$.

Figure 22.2 Central bank independence and inflation, 21 'old' OECD countries, 1970-2012



Sources: For CBI the article by Ana Carolina Garriga mentioned in Footnote 6; for inflation rates General Statistics database, OECD.

Influenced by this experience, many countries have moved towards a much higher degree of central bank independence over recent decades as Table 22.1 illustrates. For example, the constitutional framework for the European Monetary Union aims to secure a maximal degree of independence of the European Central Bank. In specifying the mandates for their central banks, governments in recent years have also been keen to stress that central bankers should act 'conservatively' by pursuing price stability (defined as a low rate of inflation) as their main goal.

DEALING WITH INFLATION BIAS THROUGH DELEGATION OF MONETARY POLICY TO AN INDEPENDENT CENTRAL BANK

If the government cannot firmly commit to a monetary policy rule of price stability, it can reduce the inflation bias arising under policy discretion by delegating monetary policy to an independent and 'conservative' central bank, which places more weight on the goal of price stability than the government itself. In this way, a more favourable macroeconomic equilibrium can be established even though the central bank does not minimize the 'true' social loss function. The prediction that countries with more independent central banks will experience lower average inflation is borne out by the evidence.

Credibility versus flexibility

So far our analysis might seem to indicate that the greater the degree of central bank independence and/or the stronger the conservatism of the central bank, the better is the macroeconomic outcome. However, things are not that simple when we allow for the possibility of aggregate supply shocks. To demonstrate this, let us replace the simple deterministic AS curve (1) by the more realistic supply curve:

$$\pi_t = \pi^e_{t,t-1} + y_t - \bar{y} + s_t, \quad E[s_t] = 0, \quad E[s_t^2] = \sigma_s^2, \tag{1'}$$

where s_t is a stochastic supply shock variable with zero mean and constant variance σ_s^2. We continue to assume that monetary policy is (partly) delegated to an independent and conservative central bank, so monetary policy is still determined by minimization of the modified loss function (19). Using (1') to eliminate y_t from (19) and recalling that $y^* = \bar{y} + \omega$, we may rewrite (19) as:

$$\tilde{S}L_t = (\pi_t - \pi^e_{t,t-1} - s_t - \omega)^2 + (\kappa + \beta\varepsilon)\pi_t^2. \tag{23}$$

Assuming that policy makers cannot credibly commit to a rule of price stability, they will set the inflation rate so as to minimize the loss function (23), given the private sector's expected rate of inflation, and given the current-period supply shock s_t which we assume to be observable at the time monetary policy has to be decided. Thus the first-order condition for the solution to the policy makers' problem under discretion is:

$$\begin{aligned} \frac{\partial \tilde{S}L_t}{\partial \pi_t} = 0 \;&\Rightarrow\; 2(\pi_t - \pi^e_{t,t-1} - s_t - \omega) + 2(\kappa + \beta\varepsilon)\pi_t = 0 \\ &\Rightarrow\; \pi_t = \frac{\pi^e_{t,t-1} + s_t + \omega}{1 + \kappa + \beta\varepsilon}. \end{aligned} \tag{24}$$

Rational private agents know that policy makers will set the inflation rate in accordance with (24), so the expected inflation rate is found by taking expected values on both sides of (24), remembering that $E[s_t] = 0$:

$$\pi^e_{t,t-1} = E(\pi_t \mid I_{t-1}) = \frac{\pi^e_{t,t-1} + \omega}{1+\kappa+\beta\varepsilon} \quad \Leftrightarrow \quad \pi^e_{t,t-1} = \frac{\omega}{\kappa+\beta\varepsilon} \tag{25}$$

Substituting (25) into (24), we get:

$$\pi_t = \frac{\omega}{\kappa+\beta\varepsilon} + \frac{s_t}{1+\kappa+\beta\varepsilon}, \tag{26}$$

and inserting (25) and (26) into (1') we find:

$$y_t = \bar{y} - \frac{\kappa+\beta\varepsilon}{1+\kappa+\beta\varepsilon} s_t. \tag{27}$$

We may now substitute (26) and (27) into the *government's* loss function $SL_t = (y_t - y^*)^2 + \kappa\pi_t^2$ to obtain (using $y^* = \bar{y} + \omega$):

$$\begin{aligned} SL &= \left(\frac{\kappa+\beta\varepsilon}{1+\kappa+\beta\varepsilon} s_t + \omega \right)^2 + \kappa \left(\frac{\omega}{\kappa+\beta\varepsilon} + \frac{s_t}{1+\kappa+\beta\varepsilon} \right)^2 \\ &= \left(\frac{\kappa+\beta\varepsilon}{1+\kappa+\beta\varepsilon} \right)^2 s_t^2 + \omega^2 + 2\omega \frac{\kappa+\beta\varepsilon}{1+\kappa+\beta\varepsilon} s_t + \kappa \left(\frac{\omega}{\kappa+\beta\varepsilon} \right)^2 + \frac{\kappa s_t^2}{(1+\kappa+\beta\varepsilon)^2} \\ &\quad + \frac{2\kappa\omega}{\kappa+\beta\varepsilon} \frac{1}{1+\kappa+\beta\varepsilon} s_t. \end{aligned} \tag{28}$$

To find the *average* loss experienced by the government, we calculate the *mean* value of the expression in (28), using $E[s_t] = 0$ and $E[s_t^2] = \sigma_s^2$. We then get:

$$E[SL] = \omega^2 \left[1 + \frac{\kappa}{(\kappa+\beta\varepsilon)^2} \right] + \sigma_s^2 \left[\left(\frac{\kappa+\beta\varepsilon}{1+\kappa+\beta\varepsilon} \right)^2 + \frac{\kappa}{(1+\kappa+\beta\varepsilon)^2} \right]. \tag{29}$$

The value of $\beta\varepsilon$ will be higher the greater the degree of CBI (β) and the stronger the conservativeness of the central banker (ε). We may say that $\beta\varepsilon$ measures the 'effective degree of central bank conservativeness'. If there are no supply shocks, that is, if $\sigma_s^2 = 0$, we see from (29) that a higher effective degree of central bank conservativeness will always reduce the government's loss. This is in line with our previous analysis. However, in the presence of supply shocks $(\sigma_s^2 \neq 0)$, a higher value of $\beta\varepsilon$ will not necessarily reduce the expected social loss. Taking the partial derivative of the expression in (29) with respect to $\beta\varepsilon$, we find after some manipulations (that you should check) that:

$$\frac{\partial E[SL]}{\partial(\beta\varepsilon)} = -\frac{\omega^2\kappa}{(\kappa+\beta\varepsilon)^3} + \frac{\beta\varepsilon\sigma_s^2}{(1+\kappa+\beta\varepsilon)^3}. \qquad (30)$$

If the variance of the aggregate supply shocks is high (if σ_s^2 is large), this expression may well be positive, implying that more CBI and/or a more conservative central bank will actually *increase* the government's social loss from instability of output and prices. To understand why, note from (26) that changes in s_t will only be allowed to affect prices to a limited degree when $\beta\varepsilon$ is high. Hence the supply shocks must be absorbed mainly by changes in output (you can verify from (27) that the coefficient on s_t in the solution for output will indeed be larger the larger the value of $\beta\varepsilon$). If the government is not too concerned about price stability so that κ is relatively small, it will incur a welfare loss if the central bank pursues price stability at the expense of output stability.

Thus, governments are faced with a trade-off. They may reduce the inflation bias by delegating monetary policy to an independent central bank that is given a mandate to pursue price stability. But if the central bank adheres rigidly to its goal of price stability, the result may be greater instability of output in periods when supply shocks are important. We may say that policy makers face an unpleasant trade-off between *credibility* and *flexibility*. To reduce the variability of output, policy makers might like to react to shocks in a flexible manner by allowing a greater rise in inflation when the economy is hit by an inflationary supply shock than when it is hit by an inflationary demand shock. But if the central bank does not take a tough anti-inflationary stance regardless of the cause of a rise in inflation, it is in danger of losing its credibility as a protector of price stability.

THE TRADE-OFF BETWEEN CREDIBILITY AND FLEXIBILITY

Delegating monetary policy to an independent and 'conservative' central bank will enhance the credibility of the monetary policy maker's commitment to the goal of price stability. However, if the economy is exposed to supply shocks with a high variance, a strong central bank emphasis on securing low inflation may increase the variance of output to such an extent that social welfare falls. Thus there is a trade-off between preserving flexibility (allowing a rise in inflation when a negative supply shock occurs) and strengthening the credibility of the central bank's commitment to price stability.

22.3 Groping in the dark: the implications of macroeconomic measurement errors

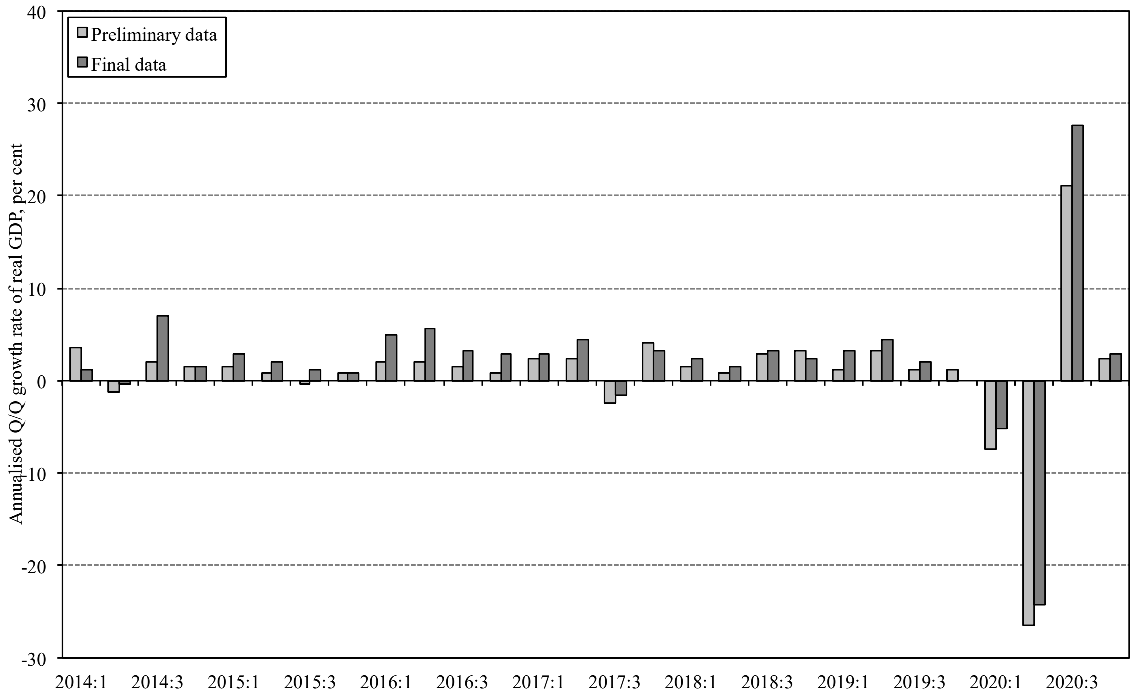
So far our discussion of stabilization policy has implicitly assumed that policy makers have full information on the current state of the economy, that they can instantaneously react to changes in macroeconomic conditions, and that they can fully predict how the economy will react to any change in policy. None of these assumptions is very realistic, and the present section will discuss the implications of relaxing them. Specifically, we will describe the difficulties of stabilizing the economy when there is uncertainty about the current state of the economy and when economic policy changes only take full effect with a time lag. We start by focusing on the former problem.

Measurement errors

In the real world macroeconomic policy makers have imperfect information about the current state of the economy when they have to make their policy decisions. This is because economic data typically only become available with a certain delay, and because the data are frequently revised – sometimes substantially so – at a later stage when statisticians have had time to check the numbers more carefully. Figure 22.3 illustrates this trivial but nevertheless important point. The figure shows the initial preliminary estimates of real GDP growth published by Statistics Denmark compared with the final official growth figures in the national income accounts when all revisions to the data have been completed. Ideally, the latter time series reflects the 'true' historical rate of growth, but the problem for policy makers is that they typically have to base their judgements and policy decisions on the preliminary estimates even though they know that these estimates may be subject to substantial revision.

To estimate the current output gap, policy makers do not only need reliable information on *actual* current output; they also face the difficult problem of estimating current *potential* output which we have also referred to as *trend*, *structural* or *natural output*. As you recall from Chapter 13, there are several ways of estimating potential output and they typically yield different results, leaving uncertainty regarding the true level of natural output.

Figure 22.3 Preliminary versus final data on real GDP growth in Denmark

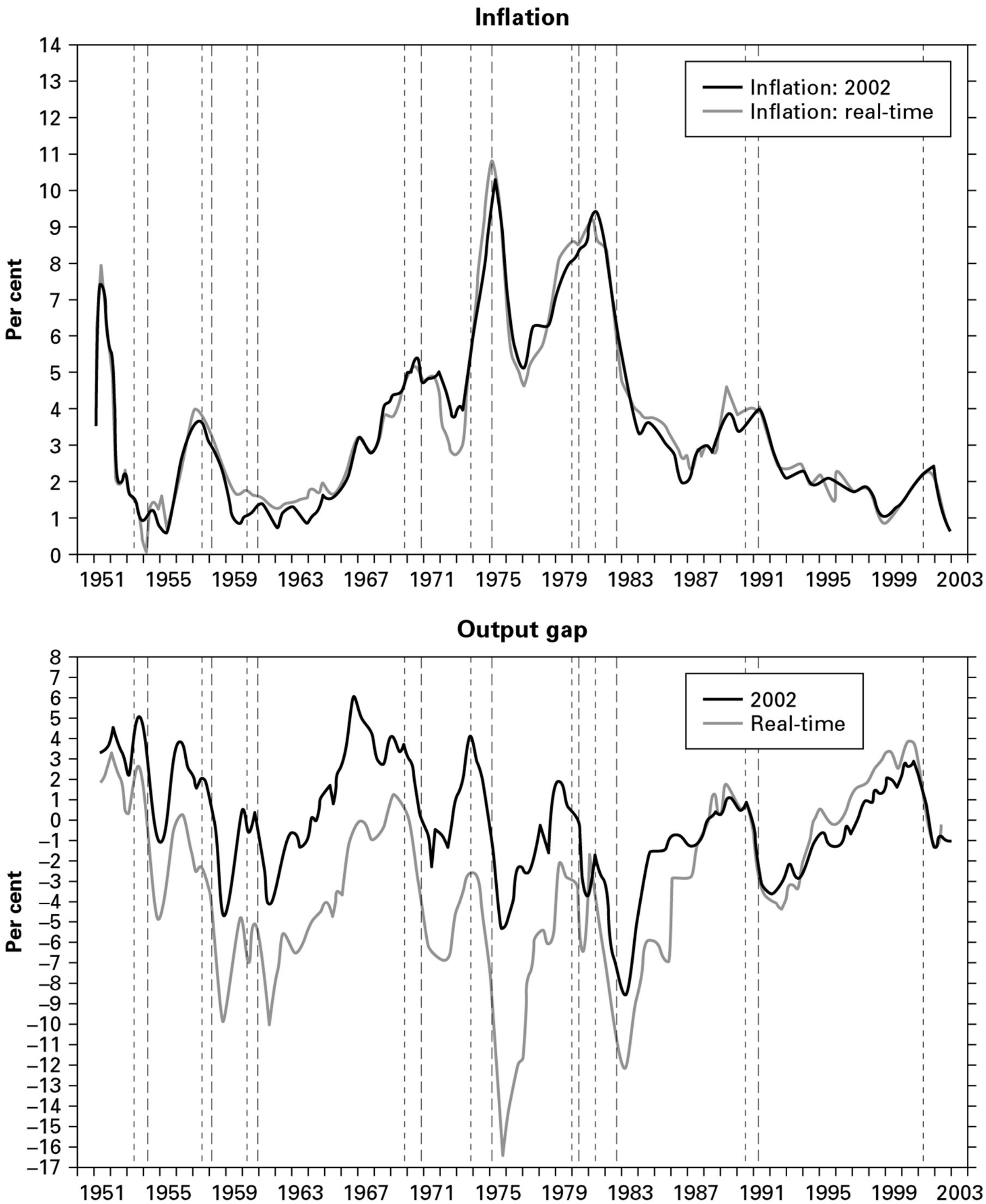


Note: Preliminary data are defined as the initially published estimates of GDP growth. Final data are the latest estimates of GDP growth.
Source: Konjunkturstatistik, Statistics Denmark.

The implications of measurement errors for stabilization policy

A dramatic example of the problems raised by macroeconomic measurement errors is given in Figure 22.4. The dotted graphs show the preliminary data, which were available to policy makers in 'real time' when they had to make their policy decisions, while the solid graphs represent the revised data available in 2002. The upper part of Figure 22.4 illustrates the sharp increase in the rate of inflation in the USA in the late 1960s and in the 1970s. The lower part of the figure gives a hint why policy makers failed to prevent this episode which is nowadays referred to as the Great Inflation. Assuming that the more recent revised numbers are the more correct ones, we see that during the 1960s and 1970s policy makers significantly overestimated the amount of slack in the economy. For example, in 1975 monetary and fiscal policy makers believed that US output was more than 16 per cent below its potential level, whereas the data available today indicate that output was in fact less than 5 per cent below potential in 1975. In particular, policy makers at the time were slow to recognize the increase in the natural unemployment rate and the slowdown in the trend rate of productivity growth which took place in the 1970s. For that reason they relaxed monetary and fiscal policy to fight the massive perceived slack in the economy, but because potential output was considerably lower than estimated at the time, the rate of inflation ended up at a much higher level than intended.

Figure 22.4 Real-time versus retrospective views of the US economy



Source: Reprinted from *Journal of Monetary Economics*, Vol. 50, Athanasios Orphanides, '*Historical Monetary Policy Analysis*', pp. 983 –1022, 2003 with permission from Elsevier.

In Chapter 20 we presented evidence to suggest that the Great Inflation in the USA was due to a failure of monetary policy to follow John Taylor's prescription of raising the real interest rate in response to a rise in inflation. In that analysis we had the benefit of hindsight by being able to use the revised macroeconomic data which are available today. However, recent research has shown that monetary policy in the 1970s actually followed a standard Taylor rule fairly closely, given the real-time data available when monetary policy was made.[10] Thus, the Great Inflation did not necessarily arise because policy makers were unusually 'soft' or irresponsible; the problem might well have been that the data available at the time indicated a very large negative output gap which called for an expansionary monetary policy.

What are the implications of measurement errors for the optimal monetary stabilization policy? We will now use our AS–AD model to analyse this question. We start by specifying the actual output gap and the actual inflation gap as:

$$\hat{y}_t \equiv y_t - \overline{y}, \tag{31}$$

$$\hat{\pi}_t \equiv \pi_t - \pi^*. \tag{32}$$

Suppose now that the *estimated* output gap $\hat{y}_t^e$ and the *estimated* inflation gap $\hat{\pi}_t^e$ deviate from the respective true gaps by some random measurement errors μ_t and ε_t:

$$\hat{y}_t^e = \hat{y}_t + \mu_t, \quad E[\mu_t] = 0, \quad E[\mu_t^2] = \sigma_\mu^2, \tag{33}$$

$$\hat{\pi}_t^e = \hat{\pi}_t + \varepsilon_t, \quad E[\varepsilon_t] = 0, \quad E[\varepsilon_t^2] = \sigma_\varepsilon^2, \tag{34}$$

The variable μ_t captures errors in the measurement of actual as well as potential output, while the variable ε_t represents errors in the measurement of the actual inflation rate. The constant variances σ_μ^2 and σ_ε^2 reflect the degree of uncertainty in measurement. Operating in 'real time', the central bank must base its interest rate policy on the *estimated* rather than the actual output and inflation gaps. Hence, we assume that the central bank follows a Taylor rule of the form:[11]

$$r_t = \overline{r} + h\hat{\pi}_t^e + b\hat{y}_t^e \quad \Rightarrow \quad r_t = \overline{r} + h\hat{\pi}_t + b\hat{y}_t + h\varepsilon_t + b\mu_t. \tag{35}$$

In Chapter 13 we saw that demand shocks seem to be the main driver of business cycles

[10] This is documented by Athanasios Orphanides, 'Historical Monetary Policy Analysis and the Taylor Rule', *Journal of Monetary Economics*, **50**, 2003, pp. 983 –1022.

[11] In so far AS μ reflects errors in the measurement of potential output, these errors will generally cause the central bank to mis-measure the equilibrium real interest rate $\overline{r}$. For simplicity we abstract from this complication here. In Exercise 4 you are invited to analyse the case where $\overline{r}$ is mis-measured.

in most countries. To keep the exposition simple, we will therefore abstract from aggregate supply shocks. As we have seen earlier, if the private sector has rational expectations and the central bank has full credibility, the expected inflation rate will be equal to the central bank's inflation target π^*. We may then specify the aggregate supply curve as:

$$\pi_t = \pi^* + \gamma(y_t - \bar{y}) \quad \Leftrightarrow \quad \hat{\pi}_t = \gamma \hat{y}_t. \tag{36}$$

Allowing for demand shocks z_t, the goods market equilibrium condition may be written as:

$$\hat{y}_t = z_t - \alpha_2(r_t - \bar{r}), \quad E[z_t] = 0, \quad E[z_t^2] = \sigma_z^2. \tag{37}$$

Equations (35)–(37) constitute a complete AS–AD model which may be solved to give the following expression for the output gap (you may want to derive this result as an exercise):

$$\hat{y}_t = \frac{z_t - \alpha_2 h \varepsilon_t - \alpha_2 b \mu_t}{1 + \alpha_2(b + \gamma h)}. \tag{38}$$

This expression shows that a negative bias in the measurement of the output gap ($\mu_t < 0$) or an underestimation of the inflation rate ($\varepsilon_t < 0$) will tend to generate a positive actual output gap (and hence a positive inflation gap) by inducing the central bank to set a lower interest rate, thereby stimulating aggregate demand. As we have seen above, there was in fact a negative bias in the estimated output gap in the USA during the Great Inflation of the 1970s.

Assuming that the stochastic variables z_t, ε_t and μ_t are all uncorrelated, and recalling that they all have zero mean values, it follows from (38) that the variance of the output gap is:

$$\sigma_y^2 \equiv E[\hat{y}_t^2] = \frac{\sigma_z^2 + \alpha_2^2 h^2 \sigma_\varepsilon^2 + \alpha_2^2 b^2 \sigma_\mu^2}{[1 + \alpha_2[b + \gamma h)]^2}. \tag{39}$$

Equation (39) shows that errors in the measurement of macroeconomic data contribute to macroeconomic instability because they make the central bank set the 'wrong' level of interest rates which is not adequately tuned to the true state of the economy.

From (36) we see that a monetary policy minimizing the variance of the output gap will also minimize the variance of the inflation gap in the present model without aggregate supply shocks. To minimize the social loss from variability in output and inflation, the central bank should therefore choose those values of the policy parameters h and b which will minimize the expression in (39). The first-order conditions for the solution to this optimization problem are:

$$\frac{\partial \sigma_y^2}{\partial h} = 0 \quad \Rightarrow \quad h\alpha_2\sigma_\varepsilon^2[1+\alpha_2(b+\gamma h)] = \gamma(\sigma_z^2 + \alpha_2^2 h^2 \sigma_\varepsilon^2 + \alpha_2^2 b^2 \sigma_\mu^2), \tag{40}$$

$$\frac{\partial \sigma_y^2}{\partial b} = 0 \quad \Rightarrow \quad b\alpha_2\sigma_\mu^2[1+\alpha_2(b+\gamma h)] = (\sigma_z^2 + \alpha_2^2 h^2 \sigma_\varepsilon^2 + \alpha_2^2 b^2 \sigma_\mu^2), \tag{41}$$

from which it follows straightforwardly dividing one by the other that:

$$\frac{h}{b} = \frac{\gamma\sigma_\mu^2}{\sigma_\varepsilon^2}. \tag{42}$$

Thus, the greater the uncertainty σ_μ^2 in the measurement of the output gap relative to the uncertainty σ_ε^2 in the measurement of inflation, the larger is the central bank's optimal response to the estimated inflation gap relative to its optimal response to the estimated output gap. Using (41) and (42), one can also show that:[12]

$$b = \frac{\sigma_z^2}{\alpha_2\sigma_\mu^2}, \tag{43}$$

$$h = \frac{\gamma\sigma_z^2}{\alpha_2\sigma_\varepsilon^2}. \tag{44}$$

These results are quite intuitive since they show that monetary policy should react more cautiously to the measured output and inflation gaps the greater the variances σ_μ^2 and σ_ε^2, that is, the greater the average errors in the measurement of the gaps.

As we have noted, policy makers in the USA (and elsewhere) tended to overestimate the size of the negative output gap for several years during the 1970s. This suggests that measurement errors are likely to display some degree of persistence. We may formalize this by assuming that the measurement errors μ_t and ε_t are given by the autoregressive processes:

$$\mu_t = \rho\mu_{t-1} + d_t, \quad 0 < \rho < 1, \quad E[d_t] = 0, \quad E[d_t^2] = \sigma_d^2, \tag{45}$$

$$\varepsilon_t = \theta\varepsilon_{t-1} + k_t, \quad 0 < \theta < 1, \quad E[k_t] = 0, \quad E[k_t^2] = \sigma_k^2, \tag{46}$$

where the parameters ρ and θ quantify the degree of persistence in the measurement errors, and where d_t and k_t are stochastic white noise variables. By successive substitutions, we find from (45) and (46) that

[12] We are assuming that the values of b and h implied by (43) and (44) do not lead to a violation of the non-negativity constraint on the nominal interest rate.

$$\mu_t = d_t + \rho d_{t-1} + \rho^2 d_{t-2} + \rho^3 d_{t-3} + \dots \tag{47}$$

$$\varepsilon_t = k_t + \theta k_{t-1} + \theta^2 k_{t-2} + \theta^3 k_{t-3} + \dots \tag{48}$$

Since d_t and k_t are white noise variables, it follows from (47) and (48) that $E[\mu_t] = E[\varepsilon_t] = 0$, as we assumed in (33) and (34). Recalling that $0 < \rho < 1$ and $0 < \theta < 1$, we also see from (47) and (48) that:

$$\sigma_\mu^2 \equiv E[\mu_t^2] = \sigma_d^2(1 + \rho + \rho^2 + \rho^3 + \dots) = \frac{\sigma_d^2}{1-\rho}, \tag{49}$$

$$\sigma_\varepsilon^2 \equiv E[\varepsilon_t^2] = \sigma_k^2(1 + \theta + \theta^2 + \theta^3 + \dots) = \frac{\sigma_k^2}{1-\theta}. \tag{50}$$

Inserting these expressions into (43) and (44), we obtain:

$$b = \frac{\sigma_z^2(1-\rho)}{\sigma_2 \sigma_d^2}, \tag{51}$$

$$h = \frac{\gamma \sigma_z^2(1-\theta)}{\alpha_2 \sigma_k^2}. \tag{52}$$

Quite intuitively, we see that the greater the degree of persistence in the measurement errors (the higher the values of ρ and θ), the smaller are the optimal values of b and h, that is, the more cautiously monetary policy should respond to the measured output and inflation gaps.

Let us sum up the insights from this analysis:

OPTIMAL STABILIZATION POLICY IN THE PRESENCE OF MEASUREMENT ERRORS

Errors in the measurement of output and inflation gaps contribute to macroeconomic instability and reduce the scope for activist stabilization policy. The greater the variance of and persistence in the measurement errors, the smaller is the optimal response of the policy interest rate to variations in the measured output and inflation gaps.

The difficulties of fine tuning aggregate demand

Apart from the problems caused by measurement errors and a possible lack of credibility, the scope for stabilization policy is also limited by the existence of time lags, that is, the delay from the time when the need for a change in some policy instrument arises until the time when the policy change achieves its maximum impact on the economy. In Chapter 20 we saw that the so-called outside lag in monetary policy could in principle be dealt with through a policy of inflation forecast targeting. However, the analysis in Chapter 20 was based on the optimistic assumptions that policy makers know the structure of the economy, including the exact length of the outside lag, and that there are no errors in the measurement of current macroeconomic conditions. In practice, there is uncertainty about the true structure of the economy, about current business cycle conditions, and about the length of the outside lag which may vary from one situation to another. For these reasons few economists nowadays believe in the possibility of 'fine tuning' aggregate demand through macroeconomic stabilization policy. We do not know exactly when and by how much a change in monetary (or fiscal) policy affects the economy. The inevitable time lags and our imperfect understanding of the economy imply a real danger that an over-ambitious stabilization policy actually ends up *de*stabilizing the economy, because it only takes full effect long after the economic shock it was meant to offset, or because its effect turns out to be stronger than intended. Many economists therefore believe that monetary policy, and in particular fiscal policy, should only be used actively when the economy is hit by significant shocks which are expected to last for a considerable amount of time.

Summary

1. With rational expectations, monetary policy may suffer from a credibility problem when the socially desired output level exceeds the natural level and the central bank can act in a discretionary manner after the private sector has formed its expectations. If monetary policy makers announce that they will keep the inflation rate down to a certain target level, the private sector may not consider such a statement to be credible when the central bank can boost output and employment by generating unanticipated inflation. The problem is that a rule-based policy of price stability is not time-consistent.
2. In a time-consistent rational expectations equilibrium with discretionary monetary policy, rational private agents anticipate the central bank's incentive to stimulate output by generating surprise inflation if inflation expectations are sufficiently low. The expected and actual inflation rate then becomes so high that the central bank does not wish to generate additional (unanticipated) inflation. The end result is an equilibrium with inflation above the target rate and an output level equal to the natural rate. Hence monetary policy suffers from an inflation bias under discretionary policy. A socially superior outcome could be achieved if the central bank could credibly commit to pursuing a rule-based policy of price stability.
3. If the policy makers care sufficiently about the future, they may not have an incentive to generate surprise inflation in the short run, because they realize that this will cause an unfavourable upward shift in the expected inflation rate in the longer run. In such a case the inflation bias in monetary policy may be held in check by the policy makers' desire to build up a reputation as defenders of price stability.
4. The inflation bias may also be reduced by delegating monetary policy to an independent and 'conservative' central bank which puts more emphasis on price stability than the government itself. However, if the variance of supply shocks is high and the central bank emphasis on price stability is strong, the result of such delegation may be that the variance of output increases to such an extent that social welfare falls. Thus there is a trade-off between gaining credibility in the fight against inflation and preserving the flexibility to allow inflation to rise when the economy is hit by a large negative supply shock.
5. Errors in the measurement of macroeconomic data such as the output and inflation gaps imply that monetary policy makers sometimes set the 'wrong' level of interest rates which is not adequately tuned to the true state of the economy. In this way measurement errors contribute to macroeconomic instability. The greater the average magnitude of the measurement errors, and the higher the degree of persistence in these errors, the smaller is the optimal interest rate response of the central bank to changes in the estimated output and inflation gaps.

22.5 Exercises

Exercise 1. Time-consistent monetary policy

When we analysed the credibility problem under discretionary monetary policy, we assumed for simplicity that the policy maker's target rate of inflation (π^*) was zero. Now we assume that, instead of being given by Equation (3) in the main text, the policy maker's loss function is:

$$SL_t = (y_t - y^*)^2 + \kappa(\pi_t - \pi^*)^2, \quad \pi^* > 0. \tag{53}$$

1. Discuss reasons why it may be reasonable to have a positive target rate of inflation (recall the arguments made in Chapter 14).

Suppose that there are no stochastic demand and supply shocks so that the economy may be described by the following equations:

Aggregate supply: $$\pi_t = \pi_{t,t-1}^e + y_t - \bar{y}, \tag{54}$$

Goods market equilibrium: $$y_t - \bar{y} = -\alpha_2(r_t - \bar{r}), \tag{55}$$

Announced monetary policy rule: $$r_t = \bar{r} + h(\pi_t - \pi^*) + b(y_t - \bar{y}), \tag{56}$$

Target output: $$y^* = \bar{y} + \omega, \quad \omega > 0, \tag{57}$$

2. Explain briefly why it may be reasonable to assume that target output exceeds trend output, as we have done in Equation (57).

3. What levels of output and inflation will emerge if private agents have rational expectations and if they believe that the central bank will stick to the announced monetary policy rule (56)? Will the monetary policy maker actually want to stick to this rule if she can act in a discretionary manner? Explain.

4. Derive the 'cheating' solutions for output and inflation if the public believes that the central bank follows the rule (56) whereas the bank actually follows the optimal discretionary policy. Compare your results with Equation (10) in the main text and explain the difference.

5. Explain the concept of time consistency. Derive the time-consistent rational expectations equilibrium values for output and inflation. Compare your results with Equation (12) in the main text and explain the difference. Draw a diagram analogous to Figure 22.1 and illustrate the Taylor rule equilibrium (where the private sector believes that the policy maker will follow the rule (56)), the 'cheating' outcome, and the time-consistent equilibrium in the present situation, where $\pi^* > 0$.

Exercise 2. Inflation bias and delegation of monetary policy

When we discussed the effects of delegating monetary policy to an independent central bank, we assumed that the central banker had the same output target y^* as the government. However, central bankers often claim that they are not targeting an output level above the natural rate, since they know that this will only lead to inflation.

Against this background, you are now asked to analyse the effects of delegating monetary policy in a setting where the central bank targets trend output $\bar{y}$, whereas elected politicians have a more ambitious output target $y^* > \bar{y}$. Thus we assume that the government's loss function is:

$$SL_t = (y_t - y^*)^2 + \kappa \pi_t^2, \quad \kappa > 0, \tag{58}$$

where

$$y^* = \bar{y} + \omega, \quad \omega > 0, \tag{59}$$

while the central bank's loss function is given by:

$$SL_t^B = (y_t - \bar{y})^2 + \kappa \pi_t^2. \tag{60}$$

Note that the government and the central bank are assumed to have the same degree of aversion to inflation (κ).

The central bank has been granted a degree of independence β from the government. Hence monetary policy is determined by minimization of the following loss function, representing a compromise between the preferences of politicians and the preferences of the central bank:

$$\tilde{S}L_t = (1-\beta) SL_t + \beta SL_t^B, \quad 0 \le \beta \le 1. \tag{61}$$

Finally, the rate of inflation is given by the SRAS curve:

$$\pi_t = \pi_{t,t-1}^e + y_t - \bar{y}. \tag{62}$$

By controlling the (real) interest rate, monetary policy can control the output gap $y_t - \bar{y}$ and hence indirectly the rate of inflation.

1. Use the procedure described in Section 22.1 to derive the time-consistent rational expectations equilibrium emerging when monetary policy is determined by minimizing (61). Compare the solutions for output and inflation with Equation (12) in Section 1 and comment on similarities and differences.

2. What is the socially optimal degree of independence for the central bank (the optimal value of β)? Give reasons for your answer. How may central bank independence be achieved in practice?

Exercise 3. The implications of macroeconomic measurement errors

In (43) and (44) in the main text we derived the following formulae for the optimal values of the monetary policy parameters *b* and *h* when there are errors in the measurement of the output and inflation gaps:

$$b = \frac{\sigma_z^2}{\alpha_2 \sigma_\mu^2}, \tag{63}$$

$$h = \frac{\gamma \sigma_z^2}{\alpha_2 \sigma_\varepsilon^2}. \tag{64}$$

Give an intuitive interpretation of these results in which you explain the role of all the parameters $\sigma_z^2, \sigma_\mu^2, \sigma_\varepsilon^2, \gamma$ and α_2.

Exercise 4. Optimal stabilization policy with measurement errors

In Section 22.4 we noted that when policy makers err in measuring the output gap, they are also likely to mismeasure the equilibrium ('natural') real interest rate. Suppose therefore that the economy is described by the following system, where $\overline{r}^e$ is the *estimated* equilibrium real interest rate, $\hat{y}_t^e$ and $\hat{\pi}_t^e$ are the *estimated* output and inflation gaps, and where the measurement errors a_t, μ_t and ε_t are stochastic white noise variables:

Aggregate supply: $$\hat{\pi}_t = \gamma \hat{y}_t, \quad \hat{\pi}_t \equiv \pi_t - \pi^*, \quad \hat{y}_t \equiv y_t - \overline{y} \tag{65}$$

Goods market equilibrium: $$\hat{y}_t = z_t - \alpha_2(r_t - \overline{r}), \quad E[z_t] = 0, \quad E[z_t^2] = \sigma_z^2, \tag{66}$$

Monetary policy: $$r_t = \overline{r}^e + h\hat{\pi}_t^e + b\hat{y}_t^e, \tag{67}$$

Estimate of $\overline{r}$: $$\overline{r}^e = \overline{r} + a_t, \quad E[a_t] = 0, \quad E[a_t^2] = \sigma_a^2, \tag{68}$$

Estimate of $\hat{y}_t$: $$\hat{y}_t^e = \hat{y}_t + \mu_t, \quad E[\mu_t] = 0, \quad E[\mu_t^2] = \sigma_\mu^2, \tag{69}$$

Estimate of $\hat{\pi}_t$: $$\hat{\pi}_t^e = \hat{\pi}_t + \varepsilon_t, \quad E[\varepsilon_t] = 0, \quad E[\varepsilon_t^2] = \sigma_\varepsilon^2. \tag{70}$$

Equation (65) assumes that the central bank's inflation target is credible so that rational agents have the expected inflation rate $\pi^e = \pi^*$. For simplicity, the model abstracts from supply shocks and from persistence in the measurement errors. Note that since (65) implies that the variance of the inflation gap (σ_π^2) is proportional to the variance of the output gap (σ_y^2), an optimal policy which minimizes the social loss function $SL = \sigma_y^2 + \kappa\sigma_\pi^2$ must minimize the variance of the output gap $\sigma_y^2 \equiv E[\hat{y}_t^2]$..

1. Solve the model (65)–(70) for the output gap and show that the variability of output is given by:

$$\sigma_y^2 = \frac{\sigma_z^2 + \alpha_2^2(h^2\sigma_\varepsilon^2 + b^2\sigma_\mu^2 + \sigma_a^2)}{[1+\alpha_2(b+\gamma h)]^2}.$$

Comment on this expression and explain the effects of the measurement errors, in particular the effect of the error in measuring the natural rate of interest.

2. Use the procedure described in Section 22.3 to show that the optimal values of b and h are given by the expressions

$$b = \frac{\sigma_z^2 + \alpha_2^2\sigma_a^2}{\alpha_2\sigma_\mu^2}.$$

$$h = \frac{\gamma(\sigma_z^2 + \alpha_2^2\sigma_a^2)}{\alpha_2\sigma_\varepsilon^2}.$$

Compare these expressions to those given in Equations (43) and (44) in the text and give an intuitive explanation for the differences.

PART 6

The Short-run Model for the Open Economy

Chapter 23

Aggregate demand and aggregate supply in the open economy

Introduction

In Part 6 we focused on the closed economy. Understanding the workings of the closed economy before moving on to study the open economy is useful because most of the key mechanisms in the closed economy are also present in the open economy. However, in some important respects openness to international trade in goods and capital does change the way the macroeconomy works. For this reason, openness may significantly affect the scope for and the effects of macroeconomic policy. It is therefore time now to put the spotlight on the open economy.

One key insight to emerge from our study of the open economy is that the short-run macroeconomic dynamics and the effects of monetary and fiscal policy will depend on the exchange rate regime. In Chapter 24 we will consider a fixed exchange rate regime where the central bank intervenes in the foreign exchange market to keep the nominal exchange rate fixed. Chapter 25 will study a flexible exchange rate regime where the exchange rate is determined by the forces of supply and demand in the foreign exchange market. We study both of these regimes under the assumption of free capital mobility between the country considered and the surrounding world.

In the present chapter, we derive some important economic relationships prevailing under both exchange rate regimes. The analysis will allow for the possibility that the exchange rate may change over time, as it typically does under flexible exchange rates (outside long-run equilibrium). If the exchange rate is actually fixed, one can simply set the nominal exchange rate equal to a constant in all the relationships presented in this chapter without invalidating any of our conclusions.

Back in Chapter 4, we documented the trend towards increased international trade and financial integration over the past half-century. In Section 23.1 we present some further evidence on the ongoing process of globalization and we lay out the key assumptions regarding the open economy. Section 23.2 then considers the implications of capital mobility for the formation of nominal and real interest rates, and Section 23.3 uses these insights to highlight how government attitudes towards capital mobility and the choice of exchange rate regime have changed over time. Following this, Section 23.4 explains how international trade and capital mobility affects the economy's aggregate demand curve, and Section 23.5 discusses the modelling of the aggregate supply side in the open economy. Section 23.6 confronts the aggregate supply curve with the aggregate demand curve to characterize the long-run equilibrium in the open economy.

The analysis in this chapter will set the stage for the next three chapters, 24, 25 and 26. Here we will first in Chapters 24 and 25 use our AS–AD model for the open economy to highlight the characteristics of two alternative exchange rate regimes: a fixed exchange rate with no autonomy in monetary policy, and a floating exchange rate with monetary policy guided by a Taylor rule. This will serve as a prelude to the discussion of the choice of exchange rate regime that we take up in Chapter 26, which naturally leads further to the discussion of the prerequisites for a monetary (currency) union like the euro area of the EU. This in turn leads to a discussion, still in Chapter 26, of the European sovereign debt crisis that took place in the aftermath of the great financial crisis starting in 2008-09.

23.1 Globalization and the small specialized economy

The internationalization of the economy

Beginning a decade or so after the Second World War, there has been a tremendous increase in cross-border economic transactions. In the first decades after the war this process of international economic integration mainly took the form of an increase in the volume of international trade. Thus, while the Western countries gradually liberalized their international trade regimes by reducing tariffs and eliminating quantitative restrictions on imports, they maintained substantial restrictions on the private export and import of capital. One motivation for this policy was that capital controls made it easier for governments to defend the fixed exchange rate parities under the so-called Bretton Woods system of fixed exchange rates established after the Second World War. Another motivation was that capital controls were necessary to implement the regulations of borrowing and lending which were seen by most governments as an essential part of their monetary policies.

However, during the 1970s the Bretton Woods system of fixed exchange rates broke down, and quantitative restrictions on international capital flows and on domestic credit came to be seen as increasingly ineffective and harmful to economic efficiency. In the

1980s, 1990s and 2000s, a large number of countries therefore abolished their capital controls. At the same time the rapid improvements in communication and information technologies significantly reduced the transactions costs associated with international investment. As a result, the last two decades of the twentieth century and the first two decades of the twenty-first witnessed a truly dramatic increase in international capital mobility.

Table 23.1 gives an impression of the strong increase in foreign trade relative to total output for eight representative OECD member states since 1960. Table 23.2 shows for the same countries the evolution of the sum of total foreign assets and liabilities relative to GDP since 1985. We see that the growth in this ratio has been even more spectacular than the growth in the foreign trade ratio since the 1980s.

Table 23.1 Ratio of foreign trade to GDP for selected countries, 1960-2019

	1960	**1980**	**2000**	**2019**
Denmark	33.3	32.7	41.5	54.6
Finland	21.7	31.5	37.4	40.0
France	13.6	21.8	27.9	32.3
Germany	-	20.9	30.8	44.0
Netherlands	47.9	49.9	62.8	78.1
Sweden	21.2	27.9	40.7	45.2
United Kingdom	20.9	25.5	26.1	32.1
United States	4.6	10.1	12.5	13.2

Note: The ratio of foreign trade to GDP is the average of nominal export of goods and services and nominal import of goods and services relative to nominal GDP.
Source: World Development Indicators, World Bank.

Table 23.2 Foreign assets and liabilities as percentage of GDP, Western countries, 1985-2015

	1985	**1995**	**2005**	**2015**
Denmark	-	165.8	372.1	584.1
Finland	81.6	132.2	391.6	657.3
France	77.2	175.0	423.7	593.9
Germany	95.6	123.7	339.5	466.1
Netherlands	218.1	273.4	1,326.2	2,293.7
Sweden	88.8	207.0	418.8	571.8
United Kingdom	300.7	360.4	815.9	993.4
United States	61.8	110.8	219.2	297.9

Note: Each row shows the sum of nominal foreign assets and liabilities divided by nominal GDP.
Source: Foreign assets and liabilities are from IMF's Balance of Payment and International Investment Position. Statistics (BOP/IIP); GDP from IMF's World Economic Outlook, April 2018.

The small, open and specialized economy with perfect capital mobility

In this chapter, we will show how our AS–AD framework can be extended to allow for international trade in goods and capital. Specifically, we will explain how one can model aggregate demand and aggregate supply in a small, specialized economy with perfect capital mobility. We will assume that this economy has its own currency, so it fits with countries like the UK, Sweden, Switzerland, Norway and Denmark. Other countries that are also small relative to the world economy such as Finland, the Netherlands, Belgium, Austria and even Italy, France and Spain do not have their own currencies , but share the euro as currency via their membership of the European Monetary Union (EMU). They will nevertheless fit into our picture by being extreme cases of a small open economy with a fixed exchange rate to be considered in the next chapter.

Openness of the economy means that commodities and capital (but not persons, we assume) can move freely across its borders: the households and firms of the 'home' country considered can import and export physical commodities as well as services, and they can acquire assets of other countries, e.g., foreign bonds and shares. Likewise, foreigners can export to, import from and acquire assets of the home country *and there are no legal or other hindrances to these types of movements*. On top of the basic assumption of openness, three other key assumptions characterize the economy considered.

The first one is that the domestic economy is so *small* that it cannot significantly affect macroeconomic conditions such as the level of interest rates, output and inflation in the 'world economy', i.e., in the rest of the world. For example, if a purely domestic recession strikes our small open economy, this will not affect its export market since foreign economic activity will remain the same. The reason is that if the domestic economy is small, its imports are only a very small fraction of total foreign output, so even if imports fall due to the domestic recession, this fall in demand will hardly be felt by the rest of the world. For most countries, the assumption that the domestic economy is small relative to the world economy is a reasonable first approximation. Exceptions could be economically very large countries like the USA and China or a large region like the euro zone.

Our second key assumption is that the small domestic economy is *specialized* in the sense that the goods produced domestically are to some extent different from the goods produced abroad and therefore domestic and foreign goods are *imperfect substitutes*. This means that the demand for net exports from the small open economy is less than perfectly elastic with respect to the relative price between domestic and foreign goods, implying that the price of domestic goods can vary relative to the price of foreign goods. Indeed, as we shall see in the next two chapters, the endogenous adjustment of the relative price between domestic and foreign goods is a basic mechanism through which the small specialized economy adjusts to a long-run macroeconomic equilibrium.[1] The

[1] The assumption that foreign and domestic goods are imperfect substitutes distinguishes our open economy AS–AD model from the open economy growth model set up in Chapter 4. In that chapter, where we just wanted to focus on the long run aspects of wealth accumulation, we made the simplifying assumption that domestic goods were perfect substitutes for foreign goods.

manufactured goods that make up the bulk of exports in most industrialized countries are typically somewhat differentiated from the manufactured goods produced in other countries. Hence, our assumption that countries specialize in the production of different goods also seems plausible.

Reflecting the deep international integration of capital markets prevailing today, our third key assumption is that *international capital mobility is perfect*. You may recall from Chapter 4 that under perfect capital mobility financial investors can instantaneously and costlessly switch between domestic and foreign assets. Further, in the absence of any country-specific risks, the assumption of perfect capital mobility is usually taken to mean that domestic and foreign financial assets are seen as perfect substitutes. From these assumptions it follows that domestic and foreign financial assets must yield the same expected rate of return.

However, since foreign and domestic assets are usually denominated in different currencies, and since market exchange rates may fluctuate, investment across borders normally involves exchange rate risk as the domestic-currency value of a foreign asset may vary in a stochastic manner. In these circumstances, domestic and foreign assets can only be perfect substitutes if investors are *risk neutral*, caring only about the expected (average) asset returns and not about the stochastic variability of returns.

A more realistic assumption would be that investors are risk averse. In a country that is a net creditor and has a need to invest part of national wealth abroad, the expected return on domestic investment would then tend to be lower than the expected return on foreign investment to induce domestic wealth owners to incur the exchange rate risk associated with holding foreign assets. By analogy, in a net debtor country that needs to attract foreign capital, the expected return on domestic investment would tend to exceed the expected return on investment abroad to provide an incentive for foreign investors to place part of their savings in the domestic economy. The resulting international rate-of-return differentials would represent risk premia needed to compensate for exchange rate risk. In the analysis in the rest of this book we could have included an exogenous (positive or negative) risk premium on international investment (as we did in Section 4.4 of Chapter 4), but this would not have affected any of our qualitative results. We have therefore chosen to simplify the exposition and combine the assumption of perfect capital mobility with an assumption of risk-neutral investors so that domestic and foreign assets effectively become perfect substitutes. This means that there can be no cross-country differences in expected asset returns. In fact, as we shall see below, differences in expected rates of return on financial investment do tend to be quite small across Western countries.

THE SMALL, OPEN AND SPECIALIZED ECONOMY WITH PERFECT CAPITAL MOBILITY

In a small specialized economy domestic economic developments do not significantly affect the world economy, but the price of domestic goods can vary relative to the price of foreign goods because the domestic economy specializes in the production of goods that are imperfect substitutes for goods produced abroad. Reflecting the deep international integration of capital markets, it is also often assumed that small economies are faced with perfect capital mobility, meaning that financial investors at home and abroad can instantaneously and costlessly switch between domestic and foreign assets. With risk-neutral investors, perfect capital mobility also means that domestic and foreign financial assets are seen as perfect substitutes. Together these assumptions imply that the expected rates of return on financial assets are equalized between the domestic and the foreign economy.

The next section derives some important implications from our key assumptions for the formation of nominal and real interest rates and for the relative price between domestic and foreign goods in the open economy.

23.2 Capital mobility, interest rate parity and purchasing power parity

Capital mobility and nominal interest rate parity

As already mentioned, foreign and domestic assets must yield the same expected rate of return when capital mobility is perfect. For example, if domestic bonds had a lower expected return than foreign bonds, investors would immediately sell domestic bonds in order to buy foreign bonds, thus driving down domestic bond prices and pushing up foreign bond prices until the expected rates of return were equalized. Given that this arbitrage can occur instantaneously and costlessly, the expected returns must be equal at any point in time.

Let us be more precise. Following common European practice, we define the *nominal exchange rate*, E, as the number of domestic currency units needed to buy one unit of the foreign currency. The nominal exchange rate is established through the clearing of demand and supply in the foreign exchange markets, the markets on which the currencies of different countries are traded against each other, e.g., the domestic currency against 'the foreign' (there will typically be many foreign currencies, but here we simplify 'the rest of the world' to being one country). The foreign exchange markets are highly effective and very large, worldwide markets. Where does the demand for the domestic currency on the foreign exchange market come from?

If a firm in the home country exports goods, it may receive payment in foreign currency and then have a need to change this payment into domestic currency to 'pay its bills' in its own currency, which means that it will demand domestic and supply foreign currency. It may as well receive payment in its own, domestic currency, but in this case the foreign importer must have bought the home country's currency against its own, again representing a demand for the domestic currency. In this way, exports give rise to demand for the domestic currency against the foreign. Likewise, the imports of the home country give rise to supply of the domestic currency. In the same way, income transfers going into the home country from abroad, e.g., returns on foreign assets, create a demand for domestic currency while transfers out of the home country give rise to supply of domestic currency. The difference between the demand and supply of domestic currency originating from the sources mentioned is the surplus on the current account of the balance of payments, which can be negative.

There are more sources of demand and supply of the domestic currency. All (autonomously motivated) capital movements into the home country, that is, all capital imports, e.g., purchases of domestic bonds and shares by foreign persons, firms or pension funds, give rise to demand for the domestic currency, while all capital movements out of the home country, capital exports, give rise to supply of the domestic currency. The difference between the demand and supply of domestic currency originating from all the sources mentioned, current ones or capital posts, is the surplus on the autonomous items on the balance of payments.

There is one final source of demand and supply of domestic against foreign currency: if the central bank of the home country buys (sells) foreign currency in order to increase (decrease) its foreign currency reserves, it supplies (demands) domestic currency. Likewise, interventions by foreign central banks in the currency markets can give rise to demand or supply of domestic against foreign currency. The difference between the demand and supply of domestic currency originating from all the sources mentioned is the surplus on the overall balance of payments. This has to be zero, since the exchange rate E adjusts to equilibrate the total demand and supply in the currency market.

Note that since the overall balance of payments must balance, a country that runs a surplus on the current account of the balance of payments must run a deficit on the capital account, which means that it must be exporting capital and hence accumulate foreign assets.

It is of some importance for our coming discussions that among the capital flows that affect the exchange rate are the speculative ones. Many capital movements are motivated by longer-run, portfolio management concerns of, e.g., pension funds. However, many other are motivated by the prospect of earning a short-run profit. For example, if some event makes international investors expect that investment in a particular country or group of countries will generate a relatively high rate of return for a period, capital will flow into these countries, and the resulting increase in the demand for their currencies will cause those currencies to appreciate. But experience shows that the moods of international investors can change quickly, and if they lose their confidence in a country, they will withdraw their capital from it by selling their assets of the country, which will create a downward pressure on the value of its currency. The changing moods of investors will imply the same pressures up or down on currencies whether they are well motivated or not. In this way, speculative capital movements can imply a volatility in exchange rates that is not necessarily rooted in true, fundamental changes in the economies concerned.

All economic agents must accept that exchange rates are determined by the clearing of supply and demand on the currency markets, but if the central bank of the home country, say, for some reason is not content with the exchange rate the market establishes it can intervene, in principle in two different ways. 1) If it finds the relative price of the domestic currency too low (E too high), it can use some of its foreign currency reserves to buy the domestic currency in the market thus creating a demand for it that will contribute to increase its value (reduce E). The size of the currency reserve sets a limit to how far it can go, of course, unless it obtains support from the central banks of other countries. If it finds the domestic currency too strong (E too low), it can sell its own currency against the foreign (without limits), thus creating a supply of the domestic currency to weaken it (increase E). 2) The central bank can also intervene by changing its interest rates. If it raises them it will, as described in Chapter 17, tend to increase both the shorter-term, money market interest rates and, through the yield curve, the longer-term interest rates of the home country, which will make it more advantageous to place wealth in the country, thus creating a demand for the domestic currency through the autonomous capital flows. This will strengthen the domestic currency (decease E). Likewise, a lowering of the central bank's interest rates can contribute to weaken the domestic currency (increase E) if wanted. The central bank may also use other unconventional measures like asset buying or selling to affect the general interest rates of the domestic economy as also described in Chapter 17, which will again have an effect on the exchange rate through demand and supply arising from the capital flows.

There are two 'clean' exchange rate regimes (and some in-between to be explained later). In a *floating exchange rate* regime the central bank makes no interventions in order to affect the exchange rate, neither intervention in the form of selling or buying currency nor interest rate adjustments with the purpose of affecting the exchange rate. It can then use its instruments for other purposes, in practice for stabilizing inflation and activity, and thus obtains autonomy over its monetary (stabilization) policy. In a *fixed exchange rate* regime, the central bank intervenes through selling or buying of currency and/or interest rate adjustments until a specific, pre-determined value of the nominal exchange rate E is obtained, thus 'pegging' the national currency to some other currency or basket of currencies. Here the central bank must use its instruments to support the decided exchange rate and therefore cannot use them for independent stabilization purposes. In practice, intervention aims at securing that the nominal exchange rate stays within a certain interval, the narrowness of which indicates how fixed the exchange rate is.

The implications of alternative exchange rate regimes for macroeconomic dynamics and for stabilization policies are the subject of the coming chapters. In the rest of this chapter we will explain some economic relationships which will hold under fixed as well as flexible exchange rates. We will start by considering the relationship between domestic and foreign interest rates, and we will allow for the possibility that the exchange rate can vary over time, since even a fixed exchange rate regime typically allows the exchange rate to vary within some narrow interval. The formulas derived below also encompass a regime with a completely fixed exchange rate (a monetary union) as a simple special case.

If i is the domestic nominal interest rate and i^f is the foreign nominal interest rate, perfect capital mobility implies the arbitrage condition:

$$1+i=(1+i^f)\left(\frac{E_{+1}^e}{E}\right), \tag{1}$$

where E is the nominal exchange rate at the start of the current period, and E_{+1}^e is the nominal exchange rate expected to prevail at the beginning of the next period. The left-hand side of (1) measures the amount of wealth accruing to an investor at the end of the current period if she invests one unit of the domestic currency in the domestic capital market at the beginning of the period. As an alternative to such a domestic investment, the investor could have bought $1/E$ units of the foreign currency at the start of the period for the purpose of investment in the foreign capital market. At the end of the period she would then have ended up with an amount of wealth $(1/E)(1 + i^f)$ in foreign currency. At the start of the period when the investment is made, the investor expects that this end-of-period wealth will be worth $(E_{+1}^e/E)(1+i^f)$ units of the domestic currency. Thus, Equation (1) says that domestic and foreign investment must generate the same expected end-of-period wealth and hence must yield the same expected rate of return.[2] If, for instance, one had $1+i>(1+i^f)E_{+1}^e/E$, investment in the home country would be more advantageous than investment abroad for domestic as well as foreign investors, which would make capital flow into the home country, pressing i and E downwards until equality of the expected rates of return is established.

If we take natural logarithms on both sides of (1) and use the approximation $\ln(1+x) \approx x$, we get:

$$i=i^f+e_{+1}^e-e, \quad e\equiv \ln E, \quad e_{+1}^e \equiv \ln E_{+1}^e. \tag{2}$$

The magnitude $(e_{+1}^e-e)\times 100$ is the expected percentage rate of depreciation of the domestic currency against the foreign currency. This is the expected *capital gain* on foreign bonds relative to domestic bonds over the period considered. Thus Equation (2) says that if the domestic currency is expected to depreciate by x per cent, the domestic nominal interest rate must exceed the foreign nominal interest rate by x percentage points to make domestic and foreign assets equally attractive.

Equation (2) (and its approximate equivalent (1)) is known as the condition for

[2] As already mentioned, (1) also covers the case where exchange rates are completely fixed, as indeed they are if the countries considered belong to a monetary union with a common currency. In that case we simply have $E_{+1}^e/E=1$.

uncovered interest rate parity. The term 'uncovered' refers to the fact that the investor has not covered her risk: when she invests in the foreign capital market, she expects a capital gain $e^e_{+1} - e$, but this gain is uncertain, so she is exposed to risk. If she wants to cover her risk at the time of investment, she can use the *forward market* for foreign exchange. In this market she can sell an amount of foreign currency $(1/E)(1 + i^f)$ for delivery one period from now at the known forward exchange rate $\tilde{E}_{+1}$ currently prevailing in the forward market. The one-period forward exchange rate $\tilde{E}_{+1}$ is the domestic-currency price of one unit of foreign currency delivered one period from now. Since $\tilde{E}_{+1}$ is known at the start of the current period when the foreign investment is made, the investor knows for sure that she will end up with $(\tilde{E}_{+1}/E)(1+i^f)$ units of the domestic currency at the end of the period if she covers her foreign investment via the forward market. For domestic and foreign investment to be equally attractive, we thus have the arbitrage condition:

$$1 + i = (1 + i^f)\left(\frac{\tilde{E}_{+1}}{E}\right). \tag{3}$$

Taking logs on both sides of (3), we get the approximate relationship:

$$i = i^f + \tilde{e}_{+1} - e, \qquad \tilde{e}_{+1} \equiv \ln \tilde{E}_{+1}. \tag{4}$$

Equations (3) and (4) are known as the condition for *covered interest rate parity*. Is it possible for covered and uncovered interest rate parity to hold at one and the same time? According to (2) and (4) the answer is 'yes', provided

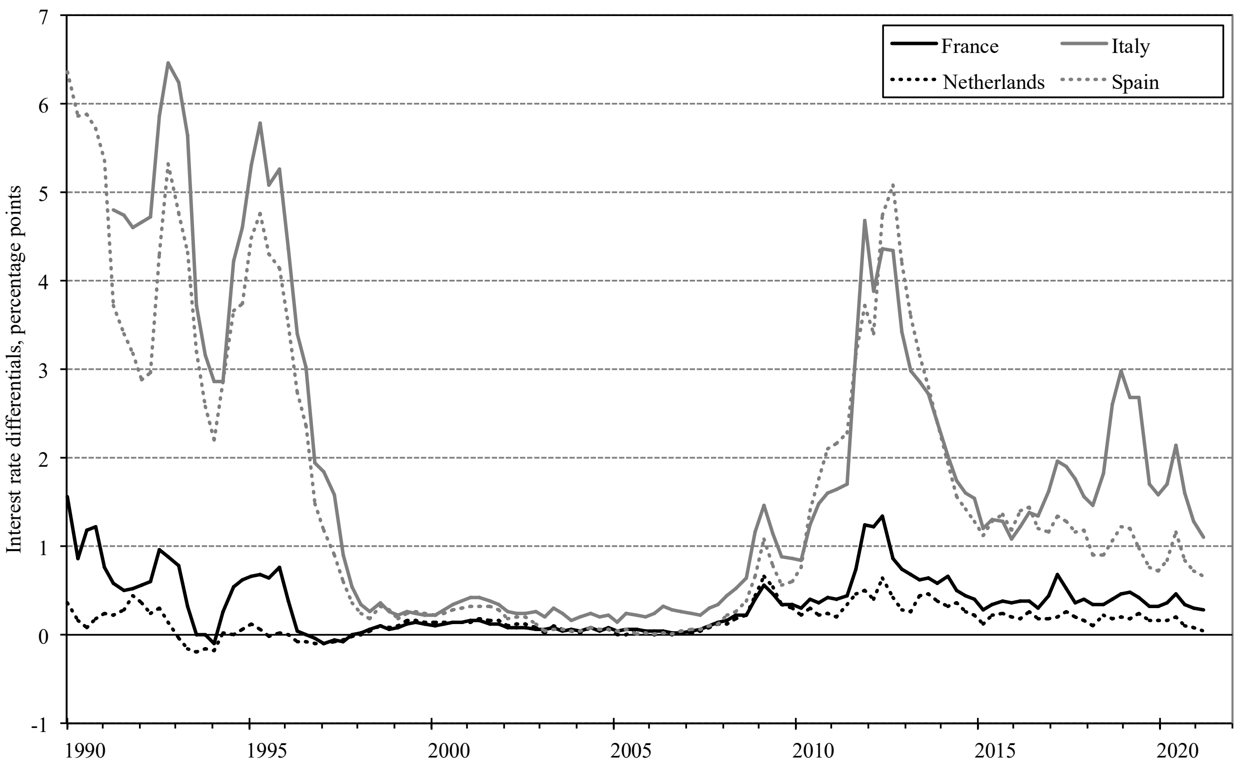
$$\tilde{e}_{+1} - e = e^e_{+1} - e \quad \Leftrightarrow \quad \tilde{e}_{+1} = e^e_{+1}. \tag{5}$$

The term on the left-hand side of the first equation in (5) is the *forward foreign exchange premium*, defined as the (percentage) difference between the price of forward exchange (the price of foreign exchange for delivery one period from now) and the current *spot* exchange rate E (the price of foreign exchange for immediate delivery). Equation (5) says that if the spot exchange rate is expected to increase by $(e^e_{+1} - e) \times 100$ per cent over the next period, then the forward foreign exchange premium should also equal $(e^e_{+1} - e) \times 100$ per cent. When investors are risk neutral, this condition must hold. To see this, suppose that $\tilde{e}_{+1} < e^e_{+1}$. In that case it would be possible to score an expected profit by buying foreign exchange in the forward market today and then selling the foreign currency in the spot market when it is delivered one period from now. Similarly, if $\tilde{e}_{+1} > e^e_{+1}$. risk-neutral investors will want to sell foreign exchange in the forward market today and buy foreign exchange in the spot market one period ahead when the foreign currency is to be delivered, thus scoring an expected profit of $\tilde{e}_{+1} - e^e_{+1}$. at the time of delivery. To rule out such arbitrage opportunities of pure profit making for risk-neutral investors, Equation (5) would have to be satisfied, so covered and uncovered interest rate parity will hold simultaneously *when investors are risk neutral*. With risk averse investors, which is more realistic, there would normally have to be a risk premium ε on the expected spot exchange rate, $e^e_{+1} = \tilde{e}_{+1} - \varepsilon$, where ε can be positive or negative, so that from (4) the covered, more realistic interest rate parity becomes $i = i^f + e^e_{+1} - e + \varepsilon$. Without difficulties, we could have chosen to bring risk premia like ε with us in all expressions, but for simplicity we will assume risk neutral investors, so the relevant nominal interest rate parity is (2), which we could also write as:

$$i = i^f + \Delta e^e_{+1}, \qquad \Delta e^e_{+1} \equiv e^e_{+1} - e. \tag{2'}$$

This suggests that when exchange rates can vary a lot, so that Δe^e_{+1} can be relatively large numerically, the interest rates of different countries can also deviate substantially from each other. However, when countries move towards greater fixity of exchange rates, the expected exchange rate changes should tend towards zero, forcing national interest rates into equality. This hypothesis is confirmed by Figure 23.1, which shows nominal interest rate differentials on government bonds between some European countries and Germany. Around the mid-1990s the exchange rates within the European Monetary System could in principle vary within an exchange rate band of ±15 per cent around the central parity, and countries like Italy and Spain with a history of devaluations against the German mark had relatively high nominal interest rates, reflecting the perceived probability of devaluation of the Italian and Spanish currencies. At the same time countries like, Netherlands and France, whose currencies were perceived to be strong, had nominal interest rates relatively close to the German level. However, when financial markets in the second half of the 1990s became convinced that political leaders in Europe were determined to establish a monetary union with completely fixed exchange rates (and ultimately a common currency), the nominal interest rates of the different EU countries quickly converged, as fears of substantial exchange rate movements vanished. From around 2008, the nominal interest rates on government bonds began to spread again, this time not because of expected exchange rate changes, which are impossible as long as the countries stay in the EMU, but because of fear of default on the government debt in some countries, here mainly Italy and Spain. These countries ran into deep trouble in the sovereign debt crisis arising in the aftermath of the great financial crisis of 2008-09 as we return to in Chapter 26. More recently, a considerable normalization has occurred as the perceived risk of defaults has diminished and exchange rate changes between the countries are still considered irrelevant.

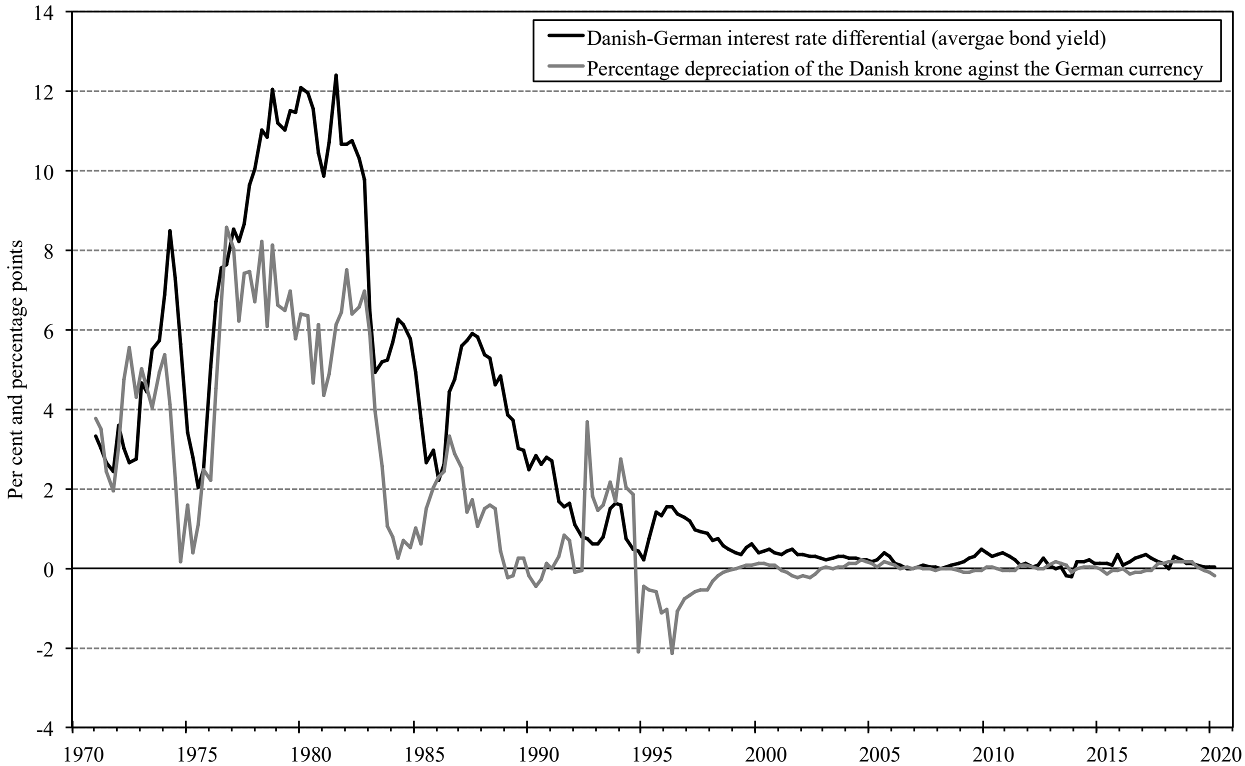
Figure 23.1 Nominal interest rate differentials in Europe, 1990Q1-2021Q1



Note: Difference between domestic ten-year government bond yield and the corresponding German ten-year government bond yield. The figure is based on quarterly data.
Source: Main Economic Indicators database, OECD.

Figure 23.2 tells a similar story about Denmark. From the early 1970s to the early 1980s, the Danish krone was devalued against the German mark on numerous occasions, and expectations of future devaluation kept the Danish nominal interest rate far above the German interest rate. But as the policy of systematic Danish devaluations was abandoned after 1982 and the krone became a much more stable currency, the Danish interest rate gradually converged towards the German level.

Figure 23.2 Interest rate differential and currency devaluation in Denmark, 1971Q1-2020Q1



Note: The figure is based on quarterly data. The interest rate differential is the difference between the Danish ten-year government bond yield and the corresponding German ten-year government bond yield. The percentage depreciation has been annualized and has been smoothen by taking a 9 quarters (4 backwards and 4 forwards) moving average.
Source: Long-term interest rates from MONA database, Danmarks Nationalbank & Main Economic Indicators database, OECD; exchange rates from the German Bundesbank.

NOMINAL INTEREST RATE PARITY

The hypothesis of *uncovered* interest rate parity assumes that investors are risk neutral. In that case, perfect capital mobility implies that foreign and domestic financial assets must yield the same expected rate of return. Hence, the domestic interest rate must equal the foreign interest rate plus the expected percentage rate of depreciation of the domestic currency (positive or negative). The condition for *covered* interest rate parity is a pure arbitrage condition ruling out the possibility of making risk-free gains via the foreign exchange market. Under covered interest parity, the domestic interest rate equals the foreign interest rate plus the percentage difference between the forward market exchange rate and the spot market exchange rate. If investors are risk neutral, the conditions for uncovered and covered interest parity will both hold.

We have so far focused on the implications of capital mobility for the cross-country relationship between *nominal* interest rates, but the aggregate demand for goods and services depends on the *real* interest rate, defined as the nominal interest rate minus the (expected) rate of inflation. To see what international economic integration implies for the cross-country link between real interest rates, we must therefore explore the link between national inflation rates created by international trade. This is the agenda for the next subsection.

The real exchange rate, relative purchasing power parity and real interest rate parity

A country's international competitiveness is often measured by the price of foreign goods relative to the price of domestic goods.[3] This important variable is called *the real exchange rate*, E^r, defined as

$$E^r \equiv \frac{EP^f}{P}, \tag{6}$$

where P^f is the price of foreign goods denominated in foreign currency, EP^f is the price of foreign goods measured in domestic currency, and P is the price of domestic goods in units of the domestic currency. The real exchange rate indicates the number of units of the domestic good that must be given up (paid) to acquire one unit of the foreign good. The higher the real exchange rate, the cheaper are domestic goods relative to goods produced abroad. The inverse of the real exchange rate ($1/E^r$) measures the quantity of foreign goods that can be obtained by giving up one unit of the domestic good, that is, how advantageous it is for the home country to trade domestic for foreign goods. This is referred to as the international *terms of trade*.

Hence, an increase in the real exchange rate (higher competiveness) implies a deterioration of the terms of trade (a lower commodity payment for one's exports), so is it good or bad to have a 'high' real exchange rate? The answer is that as long as a country does not have difficulties employing its resources (labour and capital) due to an inability to compete with foreign firms in the domestic and foreign markets, it is good to have a low real exchange rate because this means that the country can obtain many imported goods in exchange for its exports.

For later purposes it will be convenient to measure the real exchange rate in natural logarithms. We therefore define:

$$e^r \equiv \ln E^r = e + p^f - p, \quad p^f \equiv \ln P^f, \quad p \equiv \ln P. \tag{7}$$

[3] Strictly speaking, international competitiveness should be measured by the relative prices of those goods, which can be traded internationally. In our AS–AD model, we will make the simplifying assumption that all goods can be traded.

The log of the real exchange rate in the previous period is then given by:

$$e_{-1}^{r} = e_{-1} + p_{-1}^{f} - p_{-1}, \tag{8}$$

which may be subtracted from (7) to give:

$$\begin{aligned} &\Delta e^{r} = \Delta e + \pi^{f} - \pi, \\ &\Delta e^{r} \equiv e^{r} - e_{-1}^{r}, \quad \Delta e \equiv e - e_{-1}, \quad \pi^{f} \equiv p^{f} - p_{-1}^{f}, \quad \pi \equiv p - p_{-1}, \end{aligned} \tag{9}$$

where Δe is the percentage depreciation of the domestic currency, π^f is the foreign rate of inflation, and π is the domestic inflation rate. According to (9), the percentage depreciation of the real exchange rate is thus equal to the nominal exchange rate depreciation plus the inflation differential between the foreign and the domestic economy.

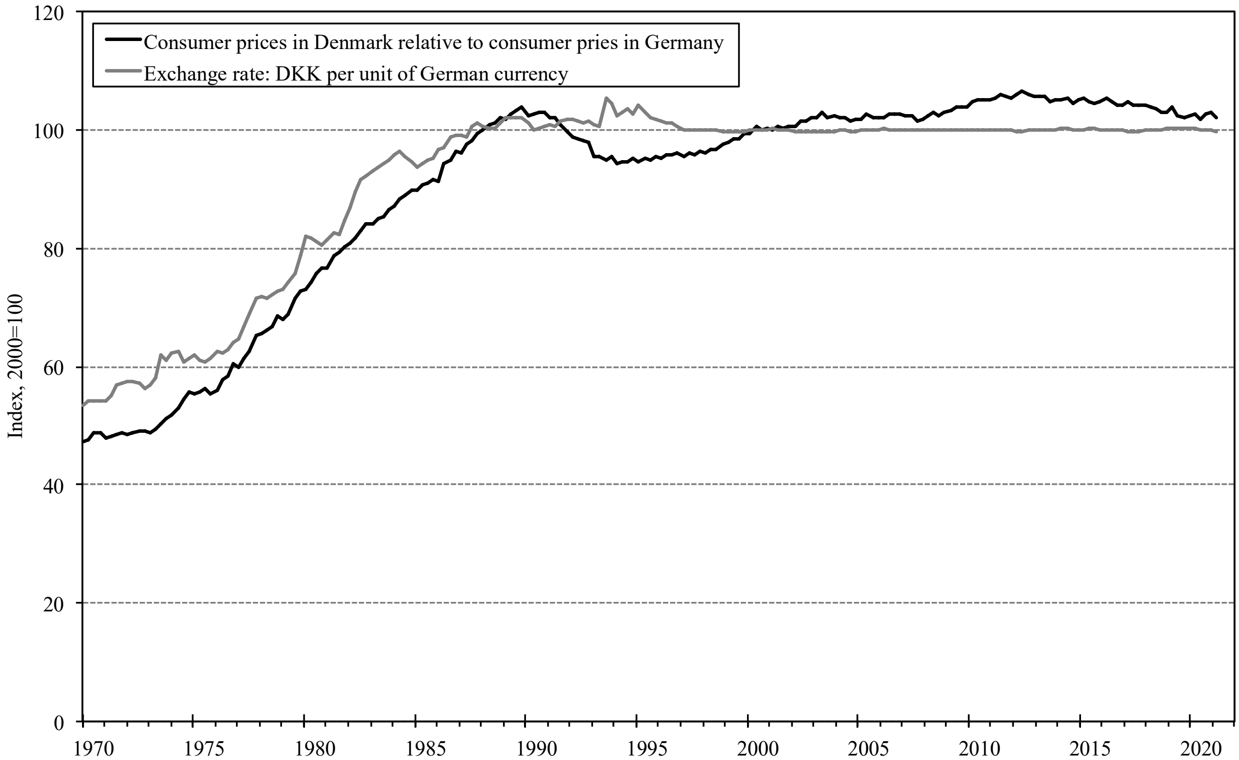
The definitional relationship (9) has an interesting implication when we combine it with the concept of a *long-run macroeconomic equilibrium*. The real exchange rate is an important determinant of the trade balance and macroeconomic activity. For the economy to be in long-run equilibrium, the real exchange rate therefore has to be constant. For example, a country cannot keep on having $\Delta e^{r} > 0$ over long periods since this would make it ever more competitive relative to other countries attracting more and more of the world's demand to it and in the end inevitably pulling the domestic price level upwards and hence e^{r} downwards. Therefore, as a long-run equilibrium condition we must have $\Delta e^{r} = 0$. Inserting this into (9), we get the *long-run equilibrium condition*:

$$\Delta e = \pi - \pi^{f}. \tag{10}$$

This condition is known as *relative purchasing power parity* (RPPP), while absolute (APPP) is $E^{r} = 1$ or $e^{r} = 0$. Relative PPP says that, over the long run, a country's rate of nominal exchange rate depreciation must correspond to the excess of the domestic over the foreign inflation rate. In this way, the country's international competitiveness (its real exchange rate) is kept constant over time, but only as a long-run feature. While changes in the real exchange rate over time will be an important part of our description of the economy's short-run adjustments, RPPP will play an important part in our characterization of long-run equilibrium in the open economy.

Figure 23.3 shows that the hypothesis of RPPP provides a reasonably good description of the long-run relationship between price levels and the nominal exchange rate between Denmark and her largest trading partner, Germany. For many years the Danish price level increased relative to the German price level, but this loss of Danish competitiveness was more or less offset by a corresponding depreciation of the exchange rate until some time in the 1980s when Denmark switched to a fixed exchange rate policy vis-à-vis Germany.

Figure 23.3 Bilateral exchange rate and relative prices between Denmark and Germany, 1970Q1-2021Q1



Note: The figure is based on quarterly data.
Source: Consumer prices from Main Economic Indicators database, OECD; exchange rates from the German Bundesbank.

Combined with our assumption of perfect capital mobility, relative purchasing power parity implies that in the long run the domestic real interest rate is tied to the real interest rate abroad. This may be seen as follows: as another condition for long-run equilibrium, the rate of exchange rate depreciation must be correctly anticipated, that is, the expected rate of depreciation, $\Delta e_{+1}^{e} = e_{+1}^{e} - e$, must equal the actual rate, $\Delta e_{+1} \equiv e_{+1} - e$. Hence, we may write the condition for uncovered nominal interest rate parity as $i = i^{f} + \Delta e_{+1}$, and the condition for relative purchasing power parity may be rewritten as $\pi_{+1} = \pi_{+1}^{f} + \Delta e_{+1}$. Subtracting the latter from the former equation gives:

$$i - \pi_{+1} = i^{f} - \pi_{+1}^{f}. \tag{11}$$

The left-hand side of (11) is the *ex post* domestic real interest rate, and the right-hand side is the foreign real interest rate measured *ex post*.[4] Equation (11) thus expresses *ex-post real interest rate parity* as a feature of long-run equilibrium.

As a final condition for long-run equilibrium we assume that inflation is correctly anticipated. In (11), we can therefore substitute the expected inflation rates, π_{+1}^{e} and $(\pi_{+1}^{f})^{e}$ for the actual ones and obtain:

$$i-\pi_{+1}^{e}=i^{f}-(\pi_{+1}^{f})^{e}. \tag{11'}$$

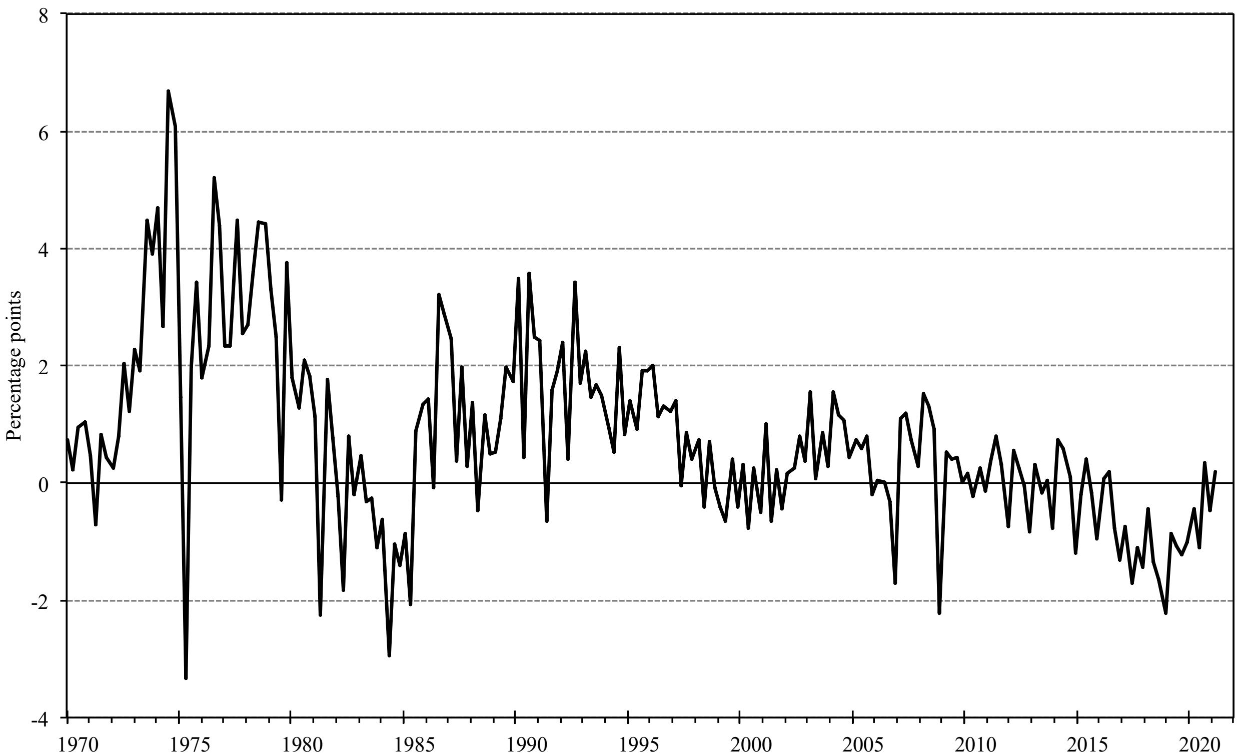
This is *ex-ante real interest rate parity* (based on expected inflation rates) saying that in the long run, the real interest rate in the small domestic economy will be equal to the exogenous foreign real interest rate. In other words, while capital mobility establishes a link between the *nominal* interest rates at home and abroad, the combination of capital mobility and foreign trade also implies a long-run link between the domestic and the foreign *real* rates of interest. Denoting real interest rates by *r*, so $r\equiv i-\pi_{+1}^{e}$ etc., we may write the real interest rate parity (11') as $r=r^{f}$, but note that since this relationship is only meant as a property of a long-run equilibrium, the correct statement is $\overline{r}=\overline{r}^{f}$, where the bars indicate long-run, structural levels.[5]

If real interest rate parity holds, the difference between the *ex-post* real interest rates across any two countries in the world economy with free commodity and capital mobility between them should tend to fluctuate around a zero mean value. Figure 23.4, focusing on the UK and the USA, suggests that this is indeed the case. The figure shows that the long-term real interest rate differential between these two countries has fluctuated quite a lot over time – indicating that the economy can fluctuate substantially around its long-run growth trend – but with a clear tendency to revert towards a mean value of zero, as implied by the hypothesis of real interest rate parity. Note that our derivation of (11) and (11') did not rely on any specific assumption regarding the exchange rate regime. Thus real interest rate parity should hold in the long run whether nominal exchange rates are fixed ($\Delta e = 0$) or flexible ($\Delta e \neq 0$).

[4] The definition of the real interest rate given here is the so-called real producer rate of interest, defined as the nominal interest rate minus the rate of increase of the price of domestic goods. This is the real rate of interest, which determines the profitability of investment from the viewpoint of domestic firms. From the viewpoint of consumers, the relevant real interest rate is the so-called real consumer rate of interest, defined as the nominal interest rate minus the rate of increase of consumer prices (which include the prices of imported consumer goods). By focusing on the real producer rate of interest, we are implicitly assuming that the interest rate affects aggregate demand mainly through its impact on business investment decisions, whereas household savings are relatively insensitive to interest rates.

[5] In Chapter 4, we directly assumed real interest rate parity. We have now provided a deeper understanding of this 'law' for the long run.

Figure 23.4 Long-term real interest rate differential, UK-USA, 1970Q1-2021Q1



Note: The figure shows the difference UK minus USA in real returns on 10-year government bonds on a quarterly basis computed by deflating the nominal interest rates with consumer price inflation according to the formula $1+\rho=(1+i)/(1+\pi)$, where i is the pro anno nominal interest rate in a quarter and π is the rate of inflation over the year preceding the quarter.
Source: Main Economic Indicators database, OECD.

PURCHASING POWER PARITY AND REAL INTEREST RATE PARITY

The real exchange rate is the price of foreign goods relative to the price of domestic goods. In a long-run equilibrium, the real exchange rate must be constant. This means that relative purchasing power parity (RPPP) must hold in the long run. Under RPPP the rate of nominal exchange rate depreciation equals the difference between the domestic and the foreign inflation rate. When combined with uncovered interest rate parity, RPPP implies real interest rate parity, that is, in the long run the domestic real interest rate must correspond to the foreign real interest rate, regardless of the exchange rate regime. The long-run prediction of real interest rate parity is supported by empirical evidence.

23.3 Capital mobility and exchange rate regimes: past and present

The macroeconomic trilemma and its historical resolution

The link between capital mobility and the formation of interest rates leads to a fundamental macroeconomic 'trilemma', also known as the 'Impossible Trinity'. The trilemma says that a macroeconomic policy regime can include at most two out of the following three policy goals:

1. Free cross-border capital flows,
2. A fixed exchange rate, and
3. An independent monetary policy.

The Impossible Trinity is easily understood by going back to the condition for uncovered interest parity (UCP) which assumes perfect capital mobility. As you recall from Equation (2), this condition may be stated as

$$\text{UCP:} \quad i = i^f + e^e_{+1} - e \equiv i^f + \Delta e^e_{+1}.$$

If a country fixes its nominal exchange rate while at the same time allowing free international capital flows, it follows from UCP that the domestic nominal interest rate i becomes tied to the foreign nominal interest rate i^f, since credibly fixed exchange rates imply that the expected rate of nominal exchange rate depreciation becomes zero, $\Delta e^e_{+1} = 0$,. Alternatively, if a country wants to be able to set its own interest rate independently of the foreign interest rate and thus enforcing a certain difference $i - i^f$, while allowing capital mobility, it follows from UCP that it must also allow its exchange rate to vary. As a final alternative, since UCP is enforced by capital mobility, a country must impose capital controls if it wants to pursue an independent interest rate policy while at the same time fixing its exchange rate.

Over the years, countries have resolved the fundamental macroeconomic trilemma in different ways. Table 23.3 provides a simplified summary of the historical experience with the trilemma, drawing on a study by economists Maurice Obstfeld and Alan Taylor. Historically the developed countries in the world have made several attempts to establish an international monetary system based on fixed exchange rates. One important and interesting example of a fixed exchange rate regime was the classical gold standard, which had its heyday from around 1870 up until the outbreak of the First World War in 1914. During this period, almost all advanced countries made their currencies convertible into gold at a fixed price. All agents in the private sector could thus exchange domestic currency for gold with their central bank at the quoted official price. This convertibility of currency into gold implied that exchange rates could only fluctuate within a fairly narrow band determined by the cost of shipping gold from one country to another. If the market exchange rate of some foreign currency were higher than the official gold price of that currency plus the cost of transporting gold, a domestic resident could gain by buying gold from the domestic central bank at the official domestic-currency price of gold, shipping the gold abroad, selling it to the foreign central bank at the official foreign-currency price of gold, and then selling the acquired foreign currency in the market in return for domestic currency. Thus, the demand for a currency tended to disappear at the same time as the supply of that currency tended to increase strongly if its market price (the exchange rate) became so high that it became profitable to transport gold across borders as part of the above-mentioned arbitrage activity. By a similar mechanism, the supply of a currency tended to disappear at the same time as the demand for it tended to infinity if it became so cheap that it was profitable to buy the currency in the market, sell it to the foreign central bank in return for gold, ship the gold home and sell it to the domestic central bank in return for domestic currency (convince yourself of this).

Table 23.3 The trilemma and major phases of capital mobility

	Resolution of trilemma – Countries choose to sacrifice:			
Era	**Activist policies**	**Capital mobility**	**Fixed exchange rate**	**Notes**
Gold standard	most	Few	Few	Broad consensus
Interwar (when off gold)	Few	Several	Most	Capital controls especially in Central Europe, Latin America
Bretton woods	Few	Most	Few	Broad consensus
Float	Few	Few	Many	Some consensus; except currency unions, currency boards, dollarization, etc.

Note: For an explanation of the concepts of 'currency boards' and 'dollarization', see the next subsection.
Source: Reprinted from Table 1, Chapter3: 'Globalization and Capital Markets', by Maurice Obstfeld and Alan Taylor, in Michael D. Bordo, Alan m. Taylor and Jeffrey G. Williamson (eds), *Globalization in Historical Perspective*. The University of Chicago Press, 2003.

One intriguing feature of the classical gold standard was that it tended to be self-regulating. Countries with a weak domestic currency (i.e. countries where the price of foreign currency was high) tended to experience an outflow of gold, which reduced the domestic monetary base, thereby driving up the domestic interest rate. Indeed, under the gold standard central banks typically raised their interest rates as soon as there was a slight tendency for gold to flow out of the country. Hence, it became more profitable for foreign and domestic portfolio investors to invest in domestic interest-bearing assets, and this tended to increase the supply of foreign currency as well as the demand for domestic currency. By lowering domestic economic activity, the higher interest rate also tended to lower the demand for foreign currency for the purpose of importing foreign goods. In this way, the outflow of gold tended to eliminate the excess demand for foreign exchange. In countries with a strong domestic currency and an inflow of gold, the resulting downward pressure on interest rates meant that similar forces worked in the opposite direction to eliminate the excess demand for the domestic currency.

During the era of the classical gold standard up until the First World War, there was a broad consensus among Western governments about the desirability of fixed exchange rates and free international mobility of capital. Thus, governments were willing to subordinate their monetary policies to the goal of protecting their gold reserves rather than using monetary policy to stabilize the domestic economy. Under the dominant laissez-faire philosophy of that time, policy makers did not perceive a need for activist stabilization policies since they tended to believe strongly in the self-regulating forces of the free market. But despite the self-regulating forces of the gold standard, the system broke down under the pressure of the First World War. Many governments found it impossible to finance the war effort without resorting to the age-old practice of printing money, so they abolished the convertibility of their currencies into gold in order to be able to expand the money supply.

After the First World War, most countries tried to restore the gold standard, but the system broke down again during the Great Depression of the 1930s when many countries started to use monetary policy to stimulate the domestic economy. To be able to do so, either they had to impose capital controls, or they had to abandon their fixed exchange rates and adopt a policy of devaluation or of flexible exchange rates. The latter route was chosen by a large number of countries, which were thereby able to recover more quickly from the depression than the countries that stuck to the gold standard.

After the Second World War, very few countries were willing to give up the possibility of pursuing an activist monetary stabilization policy, but at the same time, they feared a return to the beggar-thy-neighbour policies of the 1930s, where countries had tried to gain competitiveness at each other's expense through large aggressive devaluations. Consequently, a new international system of fixed exchange rates was established as part of the so-called Bretton Woods agreement. Under this system countries pegged their currencies to the US dollar, allowing their exchange rates to fluctuate within a narrow band of ±1 per cent against the dollar. The dollar itself was made convertible into gold at a fixed price and became the main international reserve currency used by central banks, reflecting the dominant role of the USA in the world economy. Countries were supposed to devalue their currencies only in the event of a so-called 'fundamental disequilibrium' on their balance of payments, and the International Monetary Fund was established to provide international credit to countries, which ran into temporary balance-of-payments crises. To secure some scope for an independent monetary policy, the great majority of countries during the Bretton Woods era imposed substantial restrictions on international capital flows.

However, over time capital controls became harder to maintain, as communication and information technologies developed and the volume of international trade transactions increased. The fixed exchange rates of the Bretton Woods system came under speculative attack in the late 1960s as international capital flows expanded, and the system broke down in a massive wave of speculation between 1971 and 1973. This illustrated the weakness of a fixed exchange rate regime where the exchange rate can be adjusted and capital can move across borders. Under such a system, speculation is virtually risk-free: if an investor moves out of a currency, which is expected to be devalued, she will obviously gain from this move if devaluation actually occurs, and if the exchange rate is maintained, she will lose nothing except a small transaction cost. A system of fixed but adjustable exchange rates with free capital mobility is thus highly vulnerable to speculative attacks.

Since the collapse of the Bretton Woods system, the major currencies in the world have been floating against each other, as the most important countries and economic areas such as the euro zone have been unwilling to give up an independent monetary policy. So, in recent times the following currencies of the world have been freely floating vis-à-vis each other: the euro, the US dollar, the UK pound, the Japanese yen, the Australian dollar, the Canadian dollar and many more. The member states of the European Union have made repeated efforts to create exchange rate stability within Europe. During the 1970s, most of the EU countries made a half-hearted attempt to keep their bilateral exchange rates within a fairly narrow band called the 'currency snake' (because the band could fluctuate vis-à-vis the currencies of third countries). From 1979 they established a more ambitious system of fixed but adjustable exchange rates in the form of the European Monetary System. In 1999 these efforts culminated in the formation of the European Monetary Union; the ultimate fixed exchange rate arrangement where member countries have irrevocably fixed their bilateral exchange rates by giving up their national currencies in favour of a common currency, the euro. At the same time, these countries have established fee capital mobility between them, so what the individual member states have given up in the Trilemma is independent monetary policy. However, the euro as a whole floats vis-à-vis all the other main currencies in the world and the European Central Bank (ECB) conducts an independent monetary policy on behalf of all the member states in common.

Exchange rate regimes in the world

Table 23.4 shows the exchange rate regimes of 26 developed countries as categorized by the International Monetary Fund (IMF). All the 13 members of the EMU in the left column have been marked 'EUR member – free floating'. The term free floating is appropriate in the sense that the currency of each of these countries, the euro, floats freely vis-à-vis most other currencies. However, as just explained, since exchange rate adjustment *within* the euro zone is impossible, one could as well say that these countries have absolutely fixed exchange rates *vis-à-vis each other*. Sharing a common currency can be seen as an extreme version of fixed exchange rates between the participating countries, the hardest possible peg, so to say. Other forms of fixed exchange rates are "conventional peg" mentioned for Denmark, whose central bank intervenes to keep the Danish krone within a very narrow band vis-à-vis the euro (and is potentially supported in this by the ECB), and "currency board" mentioned for Hong Kong. In a currency board, central bank is obliged by law to exchange domestic currency for a specified foreign currency at a completely fixed exchange rate, and domestic currency can only be issued against foreign currency so that the domestic monetary base is fully backed by foreign exchange reserves. A fourth form of a fixed exchange rate not mentioned in the table is 'dollarization', where a country formally adopts the currency of some other country, typically the US dollar.

Table 23.4 Exchange rate regimes in developed market economics 2018

Euro area		Other	
	Exchange arrangement		**Exchange arrangement**
Austria	EUR member - free floating	Australia	Free floating
Belgium	EUR member - free floating	Canada	Free floating
Estonia	EUR member - free floating	Denmark	Conventional peg
Finland	EUR member - free floating	Euro area	Free floating
France	EUR member - free floating	Hong Kong SAR	Currency board
Germany	EUR member - free floating	Japan	Free floating
Greece	EUR member - free floating	New Zealand	Floating
Ireland	EUR member - free floating	Norway	Free floating
Italy	EUR member - free floating	Singapore	Stabilized arrangement
Luxembourg	EUR member - free floating	Sweden	Free floating
Netherlands	EUR member - free floating	Switzerland	Floating
Portugal	EUR member - free floating	United Kingdom	Free floating
Spain	EUR member - free floating	United States	Free floating

Source: IMF's Annual Report on Exchange Arrangements and Exchange Restrictions 2018.

The 'floating' group contains economies whose exchange rate regime is described by the International Monetary Fund as either 'free floating' or just 'floating'. Under 'free floating' the exchange rate is market-determined. To the extent that the central bank intervenes in the foreign exchange market by buying or selling domestic currency against foreign currency or adjusts its interest rates, such interventions do not aim at affecting the exchange rate more than just to moderate undue fluctuations in it. Under 'floating' the central bank influences the movements of the exchange rate through active intervention in the foreign exchange market without specifying, or precommitting to, a predetermined path for the exchange rate. This is sometimes referred to as 'dirty' floating.
There are some 'intermediate' regimes not mentioned in the table falling between the harder pegs and the floating group. For example, some countries have set a central parity

for the exchange rate against a particular foreign currency or against a basket (a weighted average) of currencies, but the actual exchange rate is allowed to fluctuate within a fixed band around the parity. If this band is very narrow the regime is rightfully referred to as a 'conventional peg', but if it is rather broad, the regime is obviously in-between fixed and floating. Other countries operate a 'crawling band' where the band for the exchange rate is allowed to move over time, or a 'crawling peg' where the exchange rate is temporarily fixed, but where it is allowed to shift gradually over time.

Since the 1990s, there has been a remarkable polarization of the choice of exchange rate regimes away from the intermediate arrangements and towards either hard peg or free floating.[6] This is not coincidental, since almost all of the serious foreign exchange crises that occurred in the 1990s involved some form of intermediate exchange rate regime. This was true of the crisis in the European Monetary System in 1992 – 93 and of the crises in Mexico in 1994, Thailand, Indonesia and South Korea in 1997, Russia and Brazil in 1998, and Turkey in 2000. The background for these crises was the huge increase in international capital mobility during the 1990s, which greatly increased the scope for speculative attacks against 'soft' fixed exchange rate regimes. Alerted by the numerous foreign exchange crises during the 1990s, many countries with a preference for stable exchange rates moved towards a hard peg, whereas countries with a preference for monetary policy autonomy moved towards floating. Indeed, it is now widely believed that intermediate exchange rate regimes tend to be unsustainable in the long run in the modern world of high capital mobility.

THE MACROECONOMIC TRILEMMA AND ITS RESOLUTION THROUGH HISTORY

The link between capital mobility and interest rate formation leads to the 'Impossible Trinity': a macroeconomic policy regime can include at most two out of the following three policy goals: 1) free capital mobility, 2) a fixed exchange rate, and 3) an independent monetary policy. Under the classical gold standard prevailing prior to the First World War and during part of the interwar period, countries chose to sacrifice an independent monetary policy. Under the Bretton Woods system of fixed exchange rates established after the Second World War countries sacrificed free capital mobility, but as capital controls became increasingly hard to maintain, the major countries in the world decided instead to sacrifice the goal of a fixed exchange rate. However, recent decades have seen a polarization of exchange rate regimes, with countries moving either towards free floating or towards a 'hard peg' with completely fixed exchange rates.

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[6] See Stanley Fischer, 'Exchange Rates Regimes – Is the Bipolar View Correct?' *Journal of Economic Perspectives*, **15**, Spring 2001.

23.4 Aggregate demand in the open economy

Drawing on the analysis in Section 23.2, we will now study the determinants of aggregate demand in a small specialized economy with free capital mobility.

The trade balance and the real exchange rate

In the open and specialized economy, it is necessary to distinguish between domestic and foreign goods. Let us denote by *Y* the real domestic production or GDP measured in units of domestically produced goods, and by C^d and I^d, respectively, the domestic demand for private consumption and investment covered by domestically produced commodities. Furthermore, let *G* be the public demand for goods and services – this is assumed to be fully covered by domestically produced goods – and let *X* be the export demand for the domestically produced goods. Then obviously the condition for equilibrium in the goods market is $Y = C^d + I^d + G + X$. If we let *C* and *I* be the total domestic demands for consumption and investment, respectively, we must have $C = C^d + \hat{C}^f$ and $I = I^d + \hat{I}^f$, where $\hat{C}^f$ and $\hat{I}^f$ are the parts of domestic demand for consumption and investment, respectively, covered by foreign goods (imports), *measured in units of domestically produced goods*. However, the imports must physically take place in foreign goods. So, let C^f and I^f be the imports of foreign goods for domestic consumption and investment, respectively, *measured in units of foreign goods*. The conversion between these variables uses the relative price between domestic and foreign goods and hence the real exchange rate: $\hat{C}^f = EP^f C^f / P = E^r C^f$ and $\hat{I}^f = EP^f I^f / P = E^r I^f$. This corresponds to how aggregation is always done in economics, adding quantities of different commodities using relative prices as weights. It follows that $C^d = C - \hat{C}^f = C - E^r C^f$ and $I^d = I - \hat{I}^f = I - E^r I^f$, so the goods market equilibrium condition can be written: $Y = (C - E^r C^f) + (I - E^r I^f) + G + X$, or $Y = C + I + G + X - E^r (C^f + I^f)$. If we denote total domestic private demand by *D*, $D \equiv C + I$, total imports measured in units of foreign goods by *M*, $M = C^f + I^f$, we get $Y = D + G + X - E^r M$. If, finally, we denote net exports (or the trade balance) measured in units of domestic goods by *NX*, that is,

$$NX \equiv X - E^r M, \tag{12}$$

the equilibrium condition for domestically produced goods becomes:

$$Y = D + G + NX. \tag{13}$$

Thus, net exports add to the total demand for domestically produced goods. It is reasonable to assume that the volume of exports, X, depends positively on the international competitiveness of domestic producers as measured by the real exchange rate E^r. It is also natural to assume that the export volume depends positively on total real output Y^f in the rest of the world, since higher economic activity abroad leads to a larger export market for domestic producers. Hence we may write $X = X(E^r, Y^f)$, where X varies positively with both E^r and Y^f, or

$$X = X(\underset{+}{E^r}, \underset{+}{Y^f}),$$

where the signs below the variables indicate the signs of the partial derivatives.

In Chapter 17 we saw that total private demand for goods in the closed economy depends on domestic output Y, the net tax rate τ, the real interest rate r, and the state of confidence (the expected future growth rate) ε. There is no reason why this should be different in the open economy, but in addition private demand will here depend negatively on the real exchange rate:

$$D = D(\underset{+}{Y}, \underset{-}{\tau}, \underset{-}{r}, \underset{+}{\varepsilon}, \underset{-}{E^r}).$$

The latter is due to an income effect: Domestic money income stems from the money value of domestically produced goods, so a rise in the price of imported goods relative to the price of domestic good (an increase in E^r) erodes the purchasing power of domestic money income since a part of this income is spent on imported goods. This reduces total private demand for goods because domestic residents become poorer.

In the open economy, all of the variables included in D above are likely to affect imports in the same directions as they affect domestic demand, since some of the goods demanded by the private sector are imported from abroad. In addition to the negative dependence of imports on the real exchange rate due to the income effect just described, there will also be a substitution effect pulling in the same direction: a higher relative price of foreign goods induces consumers to substitute towards domestic goods. We may therefore specify the import function as:

$$M = M(\underset{-}{E^r}, \underset{+}{Y}, \underset{-}{\tau}, \underset{-}{r}, \underset{+}{\varepsilon}).$$

With these specifications of exports and imports, it follows from (12) that net exports are given by:

$$NX = X(\underset{+}{E^r}, \underset{+}{Y^f}) - E^r M(\underset{-}{E^r}, \underset{+}{Y}, \underset{-}{\tau}, \underset{-}{r}, \underset{+}{\varepsilon}), \tag{14}$$

We see that a rise in Y or ε will raise the level of imports, because an increase in these variables will stimulate total private demand for goods and services. On the other hand, a rise in τ or r will reduce imports by dampening total demand for goods.

A crucial question is how a change in the real exchange rate will affect the trade balance. To investigate this, we calculate the partial derivative of the net export function (14) with respect to E^r:

$$\frac{\partial NX}{\partial E^r} = \frac{\partial X}{\partial E^r} - E^r \cdot \frac{\partial M}{\partial E^r} - M = M\left(\frac{\partial X}{\partial E^r}\frac{1}{M} - \frac{\partial M}{\partial E^r}\frac{E^r}{M} - 1\right). \tag{15}$$

Let us denote the initial magnitudes of X, M and E^r by X_0, M_0 and E_0^r, respectively. As a benchmark case, we will assume that the trade balance is initially in equilibrium so that $X_0 = E_0^r M_0$. We may then rewrite Equation (15) as:

$$\begin{aligned} &\frac{\partial NX}{\partial E^r} = M_0\left(\frac{\partial X}{\partial E^r}\frac{E_0^r}{X_0} - \frac{\partial M}{\partial E^r}\frac{E_0^r}{M_0} - 1\right) \quad \Leftrightarrow \\ &\frac{\partial NX}{\partial E^r} = M_0(\eta_X + \eta_M - 1), \quad \eta_X \equiv \frac{\partial X}{\partial E^r}\frac{E^r}{X} > 0, \quad \eta_M \equiv \frac{\partial M}{\partial E^r}\frac{E^r}{M} > 0, \end{aligned} \tag{16}$$

where η_X is the elasticity of exports with respect to the real exchange rate, and η_M is the numerical elasticity of imports with respect to the real exchange rate. Equation (16) shows that a *real depreciation* of the domestic currency – that is, a rise in the real exchange rate E^r – will improve the trade balance provided the sum of the relative price elasticities of export and import demand is greater than 1, that is, $\eta_X + \eta_M > 1$. This important result is called the *Marshall– Lerner condition*, named after its discoverers. Numerous empirical studies for a large number of countries suggest that the Marshall–Lerner condition is almost always satisfied, at least when the time horizon is one year or longer, so that economic agents have had time to adjust to the price change. In the following, we will assume that this condition is in fact met. In the very short run, η_X and η_M may actually be so small that the Marshall–Lerner condition is violated, reflecting that firms and households do not instantaneously adjust their patterns of demand and supply to a relative price change. Thus, the trade balance tends to follow a 'J-curve' pattern over time, following a real depreciation: in the very short run, the trade balance tends to deteriorate due the worsening of the terms of trade, but as the quantities of exports and imports start to adjust, the trade balance gradually improves.

The aggregate demand curve in the open economy

We are now ready to derive the aggregate demand curve for the open economy. Recalling the factors influencing the trade balance and the private demand for goods, we may restate the goods market equilibrium condition (13) as:

$$
\begin{aligned}
Y &= D(Y,\tau,r,\varepsilon,E^r) + NX(E^r,Y^f,Y,\tau,r,\varepsilon) + G \\
&= \tilde{D}(Y,\tau,r,\varepsilon,E^r,Y^f) + G, \quad \tilde{D} \equiv D + NX.
\end{aligned} \tag{17}
$$

The magnitude $\tilde{D} \equiv D + NX$ measures the total private demand for domestic goods emanating from the domestic as well as the foreign private sector.

Since the foreign activity level, Y^f, only enters into total private demand for domestic output, $\tilde{D}$, through exports, obviously $\tilde{D}$ depends positively on Y^f:

$$
\tilde{D}_{Y^f} \equiv \frac{\partial \tilde{D}}{\partial Y^f} = \frac{\partial NX}{\partial Y^f} = \frac{\partial X}{\partial Y^f} > 0.
$$

An increase in domestic economic activity Y will stimulate total private demand for goods, D, but will also increase imports and hence reduce the trade balance NX. Thus, it would seem that the sign of the partial derivative $\tilde{D}_Y \equiv \partial \tilde{D}/\partial Y$ is ambiguous. However, since only a fraction of total demand for goods will be directed towards imports, the rise in imports will be less than the rise in total demand for goods. Hence, we have $\tilde{D}_Y \equiv \partial D/\partial Y + \partial NX/\partial Y = \partial D/\partial Y - E^r(\partial M/\partial Y) > 0.$ Maintaining our assumption from Chapter 17 that $\partial D/\partial Y < 1,$ it then follows that:

$$
0 < \tilde{D}_Y \equiv \frac{\partial D}{\partial Y} + \frac{\partial NX}{\partial Y} < 1.
$$

The fact that only a fraction of total demand is directed towards imports also implies that:

$$
\tilde{D}_r \equiv \frac{\partial D}{\partial r} + \frac{\partial NX}{\partial r} < 0.
$$

$$
\tilde{D}_\varepsilon \equiv \frac{\partial D}{\partial \varepsilon} + \frac{\partial NX}{\partial \varepsilon} > 0.
$$

$$
\tilde{D}_\tau \equiv \frac{\partial D}{\partial \tau} + \frac{\partial NX}{\partial \tau} < 0.
$$

Finally, we assume that:

$$
\tilde{D}_E \equiv \frac{\partial D}{\partial E^r} + \frac{\partial NX}{\partial E^r} > 0.
$$

Recall that $\partial NX/\partial E^r > 0$, when the Marshall–Lerner condition is met. However, because of the negative income effect of a higher real exchange rate, we have $\partial D/\partial E^r < 0.$ The assumption that $\tilde{D}_E > 0$ thus requires that the Marshall–Lerner condition is satisfied with a certain margin so that $\eta_X + \eta_M$ is sufficiently greater than 1. Empirically this has turned out to be a reasonable assumption (except for the very short run where the price elasticities in export and import demand are small, due to inertia in consumer reactions to relative price changes).

We now derive a log-linear approximation of the goods market equilibrium condition given in Equation (17) following a procedure similar to the one which was used to derive the aggregate demand curve for the closed economy in Chapter 17. We start by restating Equation (17) in shorter form:

$$Y = \tilde{D}(Y, \tau, r, \varepsilon, E^r, Y^f) + G \tag{18}$$

Assuming that we start out in a long-run equilibrium, and applying the notation for partial derivatives used above, we calculate the total differential of (18) (keeping the tax rate τ constant for simplicity) to get the linear approximation:

$$\begin{aligned} Y - \bar{Y} &= \tilde{D}_Y(Y - \tilde{Y}) + \tilde{D}_r(r - \bar{r}) + \tilde{D}_E(E^r - \bar{E}^r) + \tilde{D}_\varepsilon(\varepsilon - \bar{\varepsilon}) \\ &\quad + \tilde{D}_{Yf}(Y^f - \bar{Y}^f) + (G - \bar{G}) \quad \Leftrightarrow \\ Y - \bar{Y} &= \left(\frac{\tilde{D}_E}{1 - \tilde{D}_Y}\right)(E^r - \bar{E}^r) + \left(\frac{\tilde{D}_r}{1 - \tilde{D}_Y}\right)(r - \bar{r}) + \left(\frac{\tilde{D}_\varepsilon}{1 - \tilde{D}_Y}\right)(\varepsilon - \bar{\varepsilon}) \\ &\quad + \left(\frac{\tilde{D}_{Yf}}{1 - \tilde{D}_Y}\right)(Y^f - \bar{Y}^F) + \left(\frac{1}{1 - \tilde{D}_Y}\right)(G - \bar{G}), \end{aligned} \tag{19}$$

where long-run equilibrium values are indicated by a bar above the variables. It is natural to assume that, other things equal, an increase in world economic activity will increase the domestic economy's export market in proportion to the domestic economy's initial weight in the world economy. In that case we have:

$$\tilde{D}_{Yf} \equiv \frac{\partial NX}{\partial Y^f} = \frac{\bar{Y}}{\bar{Y}^f}. \tag{20}$$

When a relationship like (20) holds for all countries, it simply means that an increase in world output Y^f does not in itself change the individual country's share of the world market.

Our next step is to rewrite (19) in terms of *relative* changes of the various variables (except the real interest rate which is already expressed in percentage terms). Doing this, and using (20), we get:

$$\frac{Y-\overline{Y}}{\overline{Y}}=\left(\frac{\overline{E}^r\tilde{D}_E}{\overline{Y}(1-\tilde{D}_Y)}\right)\left(\frac{E^r-\overline{E}^r}{\overline{E}^r}\right)+\left(\frac{\tilde{D}_r}{\overline{Y}(1-\tilde{D}_Y)}\right)(r-\overline{r})+\left(\frac{\overline{\varepsilon}\tilde{D}_\varepsilon}{\overline{Y}(1-\tilde{D}_Y)}\right)\left(\frac{\varepsilon-\overline{\varepsilon}}{\overline{\varepsilon}}\right)$$
$$+\left(\frac{1}{1-\tilde{D}_Y}\right)\left(\frac{Y^f-\overline{Y}^f}{\overline{Y}^f}\right)+\left(\frac{\overline{G}}{\overline{Y}(1-\tilde{D}_Y)}\right)\left(\frac{G-\overline{G}}{\overline{G}}\right). \tag{21}$$

Using the definitions:

$$y\equiv\ln Y,\quad y^f\equiv\ln Y^f,\quad e^r\equiv\ln E^r,\quad g\equiv\ln G,$$

and remembering that the relative change in some variable x is approximately equal to the change in the natural logarithm of x, we may write (21) as:

$$y-\overline{y}=\beta_1(e^r-\overline{e}^r)-\beta_2(r-\overline{r})+\beta_3(g-\overline{g})+\beta_4(y^f-\overline{y}^f)+\beta_5(\ln\varepsilon-\ln\overline{\varepsilon}) \tag{22}$$

where

$$\beta_1\equiv\frac{\overline{E}^r\tilde{D}_E}{\overline{Y}(1-\tilde{D}_Y)},\quad \beta_2\equiv\frac{-\tilde{D}_r}{\overline{Y}(1-\tilde{D}_Y)},\quad \beta_3\equiv\frac{\overline{G}}{\overline{Y}(1-\tilde{D}_Y)},$$
$$\beta_4\equiv\frac{1}{1-\tilde{D}_Y},\quad \beta_5\equiv\frac{\overline{\varepsilon}\tilde{D}_\varepsilon}{\overline{Y}(1-\tilde{D}_Y)}$$

Equation (22) can be seen as the IS curve for the open economy. All of the coefficients β_1 through β_5 are considered constants. As we shall see in the next two chapters, the magnitude of the parameter β_1 is crucial for the open economy's adjustments to long-run equilibrium. It is therefore useful to express β_1 in a form, which will enable us to estimate its likely magnitude. From the above we have:

$$\tilde{D}_E\equiv\frac{\partial D}{\partial E^r}+\frac{\partial NX}{\partial E^r},\qquad \frac{\partial NX}{\partial E^r}=M_o(\eta_X+\eta_M-1).$$

Furthermore, let us specify the numerical elasticity of total domestic private demand with respect to the real exchange rate as:

$$\eta_D\equiv-\frac{\partial D}{\partial E^r}\frac{\overline{E}^r}{\overline{D}}.$$

Using these relationships, and choosing units such that the initial real exchange rate $\overline{E}^r=1$, you may verify that the expression for β_1 may be rewritten as:

$$\beta_1 = \frac{(M_O/\bar{Y})(\eta_X + \eta_M - 1) - (\bar{D}/\bar{Y})\eta_D}{1 - \tilde{D}_Y}. \tag{23}$$

This specification of β_1 will be used in the next two chapters.

We will now show that (22) , *ceteris paribus*, implies a negative relationship between domestic output and the domestic inflation rate, just as we found in the closed economy. First we recall from (9) that

$$e^r = e^r_{-1} + \Delta e + \pi^f - \pi. \tag{24}$$

Second, we note from the condition for uncovered interest parity (2) or (2') that the domestic (*ex ante*) real interest rate is given by

$$r = i - \pi^e_{+1} = i^f + \Delta e^e_{+1} - \pi^e_{+1}, \quad \Delta e^e \equiv e^e_{+1} - e. \tag{25}$$

We choose our units of measurement such that the real exchange rate $\bar{E}^r = 1$ in the initial long-run equilibrium, implying $\bar{e}^r \equiv \text{In}\, \bar{E}^r = 0$. Further, we know from the condition for long-run real interest rate parity (11) that the domestic long-run equilibrium interest rate, $\bar{r}$, equals the foreign equilibrium real interest rate which we denote by $\bar{r}^f$. Inserting (24) and (25) into (22) along with $\bar{e}^r = 0$ and $\bar{r} = \bar{r}^f$, we then obtain the following preliminary expression for the aggregate demand curve for the open economy:

$$y - \bar{y} = \beta_1(e^r_{-1} + \Delta e + \pi^f - \pi) - \beta_2(i^f - \pi^e_{+1} + \Delta e^e_{+1} - \bar{r}^f) + \tilde{z}, \tag{26}$$

$$\tilde{z} \equiv \beta_3(g - \bar{g}) + \beta_4(y - \bar{y}^f) + \beta_5(\ln \varepsilon - \ln \bar{\varepsilon}). \tag{27}$$

Equation (26) is preliminary since it does not constitute a complete theory of aggregate demand until we have described the formation of inflation expectations (π^e_{+1}) and exchange rate expectations (Δe^e). In the next two chapters, we shall argue that these expectations are likely to depend on the exchange rate regime, and we shall therefore postpone the specification of expectations until then. However, remembering that $\beta_1 > 0$, we can already see from (26) that an increase in domestic inflation will, *ceteris paribus*, reduce aggregate demand for domestic output. The reason is that *higher domestic inflation erodes the international competitiveness of domestic producers* by reducing the real exchange rate. Thus, a rise in π raises the relative price of domestic goods, thereby reducing net exports. To see that this is indeed the mechanism, note that the term $(e^r_{-1} + \Delta e + \pi^f - \pi)$ on the right-hand side of (26) is simply the (log of the) current real exchange rate, e_r. Furthermore, we have just shown that the response of aggregate demand to the real exchange rate depends on the price elasticities of export and import demand (η_X and η_M) which are incorporated in the parameter β_1. The higher the value of these elasticities, the stronger is the reaction of net exports to a change in the real exchange rate, and the flatter is the aggregate demand curve in (y, π) space.

A second important observation is that the preliminary AD curve defined by (26) will change position from one period to the next whenever relative purchasing power parity fails to hold, that is, whenever the economy is out of long-run equilibrium. This follows from (24) which shows that the real exchange rate will change over time whenever $\Delta e \neq \pi - \pi^f$. Hence, the value of the lagged real exchange rate, e^r_{-1}, appearing on the right-hand side of (26) will also change from one period to the next when the rate of exchange depreciation deviates from the inflation differential between the domestic and the foreign economy, and this will cause the AD curve to shift even if the other variables on the right-hand side of (26) are constant.[7] In the open economy, shifts in the AD curve are thus an inherent part of the adjustment to long-run equilibrium. This contrasts with our AS–AD model of the closed economy where the adjustment took place solely through shifts in the AS curve. In summary:

AGGREGATE DEMAND IN THE OPEN ECONOMY

Aggregate demand in the open economy varies negatively with the rate of domestic producer price inflation because a higher domestic inflation rate raises the price of domestic goods relative to the price of foreign goods, thereby eroding the competitiveness of domestic producers and reducing net exports. Whenever the rate of nominal exchange rate depreciation deviates from the differential between the domestic and the foreign inflation rate, the real exchange rate will change from one period to the next, and the aggregate demand curve will therefore also shift over time until a long-run equilibrium satisfying Relative Purchasing Power Parity is reached.

23.5 Aggregate supply in the open economy

To complete our AS–AD model for the open economy, we need to confront the aggregate demand curve with the aggregate supply curve. Fundamentally, the AS curve in the open economy will look just as in the closed economy and have the same kind of foundations as explained in Chapter 18 (in the main text based on a trade union model of the labour market and in the appendix on an efficiency wage model). So, the short run AS curve will be:

$$\text{SRAS:} \qquad \pi = \pi^e + \gamma(y - \bar{y}) + s, \quad \gamma > 0, \tag{28}$$

[7] The fact that the AD curve will be shifting over time as long as $\Delta e \neq \pi - \pi^f$ explains our earlier claim that a long-run equilibrium requires relative purchasing power to hold.

where π is the rate of *producer price* inflation and π^e is the rate of *producer price* inflation which was expected when wages and prices for the current period were set, and *s* is a supply shock variable capturing productivity shocks and possibly shocks to the wage and price mark-ups. However, modelling aggregate supply in the open and specialized economy requires some care because of the difference between producer prices and consumer prices, and hence between producer price inflation and consumer price inflation, that characterizes such an economy: whereas the economy *produces* the commodities it is specialized in, it *consumes* a mixture of domestic and foreign goods. The subtle issue is how producer price inflation and/or consumer price inflation should enter (28)? Should we not expect wage formation to be influenced by changes in import prices, and hence by changes in the real exchange rate, which could lead to consumer price inflation entering the AS curve in some way? As already suggested, we end up with an AS curve involving only producer price inflation; let us now explain in some more detail how.

Let us assume that the consumer price index for the open economy takes the form:

$$P^c = (EP^f)^{\psi} P^{1-\psi} = (E^r)^{\psi} P, \quad 0 < \psi < 1, \quad E^r \equiv EP^f / P, \tag{29}$$

In fact, one can show that if consumer preferences are given by a Cobb–Douglas utility function defined over domestic goods and imported goods with preference parameter ψ for foreign goods, then (29) is the appropriate consumer price index.

If wage setters seek to achieve a certain level of the *real consumer wage*, W / P^c, the nominal wage rate W deflated by the consumer price index P^c, an expected change in the real exchange rate will obviously induce wage setters to change their nominal wage claim.

However, if wage setters focus instead on *relative wages*, trying to maintain some target ratio between their own wage rate and the average *wage* level in the rest of the economy, an expected change in the real exchange rate caused by a change in import prices will not influence wage setting, since a movement in import prices does not disturb the existing pattern of relative wages.

The theories of wage formation developed in Chapters 11, 12 and used in Chapter 18 to derive the AS curve all imply that at some stage wages are set as a mark-up over the representative worker's 'outside option' which includes the rate of unemployment benefit and possibly the general wage level of the economy. Whether wage setters focus mainly on real or on relative wages therefore depends on the rules for indexation of nominal unemployment benefits. If nominal benefits are indexed to *consumer prices* to keep the real unemployment benefit constant, it follows from (29) that a change in the real exchange rate will affect the nominal unemployment benefit and therefore the nominal outside option. Hence it will also affect domestic nominal wages and prices because a higher nominal unemployment benefit will drive up nominal wage claims and lead to higher domestic prices through the mark-up price setting of firms. This means that the position of the economy's aggregate supply curve will depend on the (expected) level of the real exchange rate.

However, if nominal unemployment benefits are instead indexed to *nominal wages*, nominal benefits will not automatically change when the real exchange rate changes and the nominal outside option will only depend on the economy's nominal wage level. Consequently, in such a setting, wage setters will focus on their *relative* wage position even if the wage earners only derive utility from their real incomes. Therefore, when benefits are indexed to wages, the real exchange rate will not directly influence the formation of domestic wages and prices, and the AS curve for the open economy will have the same form as the AS curve for the closed economy with *producer price* inflation being the type of inflation entering it.

Here we will stick to the assumption that benefits are indexed to wages, partly because this is the most common practice in the real world, and partly because it leads to a simpler specification of aggregate supply as given by Equation (28), which is directly comparable to the SRAS curve for the closed economy.

THE AGGREGATE SUPPLY CURVE IN THE OPEN ECONOMY

When unemployment benefits are linked to the general wage level, wage setting will be characterized by 'relative wage resistance' in the sense that individual wage setters will wish to maintain a certain level of the wage rate relative to the general wage level. An efficiency wage model or a trade union model of wage setting then leads to a short-run aggregate supply curve for the open economy in terms of producer price inflation of exactly the same form as the SRAS curve for the closed economy.

23.6 Long-run equilibrium in the open economy

Conditions for long-run equilibrium

As noted earlier, a complete AS–AD model, which is able to describe the *short-run* dynamics of the open economy must include a theory of the formation of inflation and exchange rate expectations, and these expectations are likely to depend on the exchange rate regime. In the next two chapters, we will study expectations formation and short-run macro dynamics under alternative exchange rate regimes. However, at this stage we can already characterize the *long-run* equilibrium on which the small open economy will converge if it is stable.

We have explained earlier that the real exchange rate must be constant in a long-run equilibrium. Otherwise, the AD curve (26) would keep on shifting over time. We have also seen that constancy of the real exchange rate implies relative purchasing power parity, which leads to long-run real interest rate parity when capital mobility is perfect. In the long run, the domestic real interest rate is thus tied to the exogenous foreign real interest rate, regardless of the exchange rate regime.

To illustrate a long-run equilibrium and the effects of permanent shocks for the open economy, we return to the IS curve (22) and the short-run AS curve (28), for convenience restated here:

$$y-\bar{y}=\beta_1(e^r-\bar{e}^r)-\beta_2(r-\bar{r})+\beta_3(g-\bar{g})+\beta_4(y^f-\bar{y}^f)+\beta_5(\ln\varepsilon-\ln\bar{\varepsilon}) \qquad \text{(IS)}$$

$$\pi=\pi^e+\gamma(y-\bar{y})+s, \quad \gamma>0, \qquad \text{(AS)}$$

These equations must hold in any period. As a pure normalization that we already used above, we assume $\bar{e}^r=0$. Focusing on the long run, that is, on a specific period after all domestic adjustments and with no temporary shocks, we insert that from long-run real interest rate parity we must have $r=r^f$ and, of course, $\bar{r}=\bar{r}^f$, which is long-run real interest rate parity itself. Hence: $r-\bar{r}=r^f-\bar{r}^f$. Note here that we allow for a foreign real interest rate shock, $r^f-\bar{r}^f\neq 0$. Since we focus on the long run, the interpretation of this shock is that the foreign structural interest rate permanently shifts from $\bar{r}^f$ to r^f. Inserting these relationships into (IS), we obtain the open economy's *long-run aggregate demand curve* (LRAD):

$$\begin{aligned} &y=\bar{y}+\beta_1 e^r+z, \\ &z\equiv-\beta_2(r^f-\bar{r}^f)+\beta_3(g-\bar{g})+\beta_4(y^f-\bar{y}^f)+\beta_5(\ln\varepsilon-\ln\bar{\varepsilon}), \end{aligned} \qquad (30)$$

where again, since we focus on the long run, the demand shock *z* stemming from the foreign and the domestic economy should be considered a long-run shock, for instance caused by a permanent increase in foreign activity from $\bar{y}^f$ to $y^f>\bar{y}^f$. Inserting the long-run equilibrium condition $\pi=\pi^e$ into the AS curve we get the *long-run aggregate supply curve* (LRAS):

$$y=\bar{y}-\frac{s}{\gamma}, \qquad (31)$$

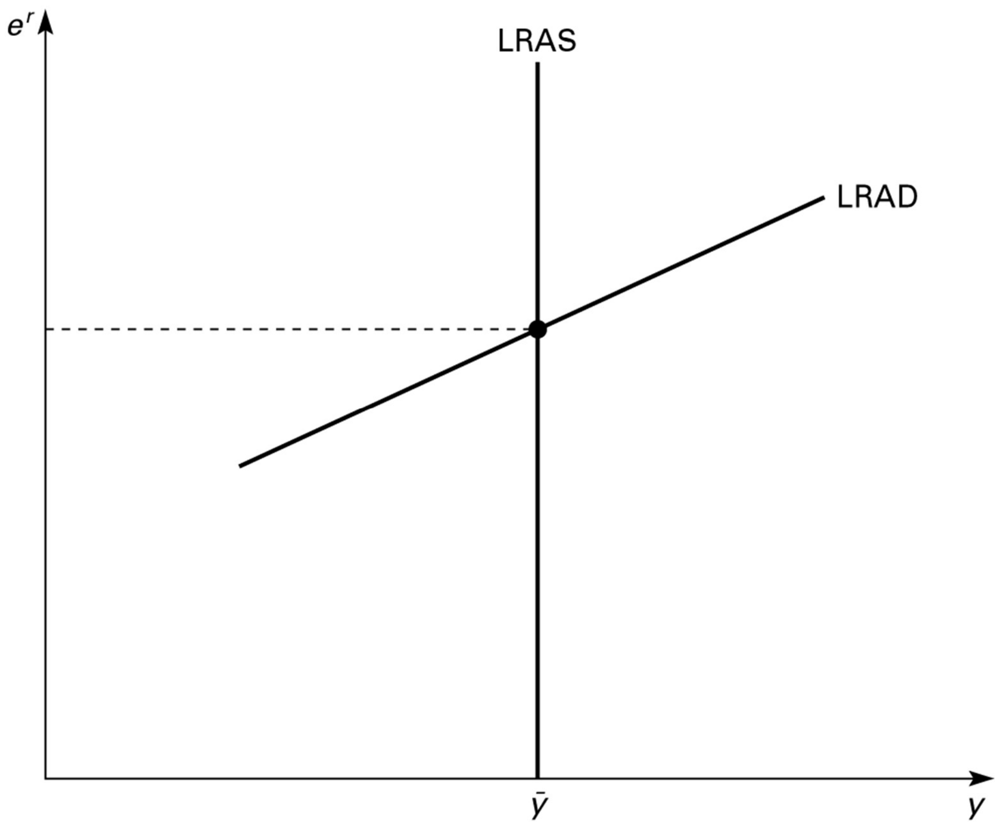
where *s* is to be viewed as a permanent shock.

To illustrate first an initial long-run equilibrium without any (permanent) shocks, assume $z=s=0$, so that (30) and (31) reduce to $y=\bar{y}+\beta_1 e^r$ and $y=\bar{y}$, respectively.

Figure 23.5 illustrates such an initial long-run equilibrium. Output is determined exclusively from the supply side, and the equilibrium real exchange rate is found where the upward-sloping LRAD curve crosses the vertical LRAS curve (the vertical axis should be viewed as having its zero at the e^r of the point of intersection).

Let us now consider a positive, permanent shock to aggregate demand, $z>0$, but no permanent shock to supply, $s=0$. In the closed economy we showed in Chapter 19 that this would cause an increase in the country's long-run, structural interest rate $\bar{r}$, but this cannot happen here since, due to long-run real interest rate parity, $\bar{r}$ is determined from abroad. What happens is that the LRAD curve in Figure 23.5 shifts upwards, so that the permanent change in z is fully absorbed by a change in the equilibrium real exchange rate, leaving the long-run output level unaffected. A positive (advantageous), permanent supply shock, $s < 0$, will, as (31) shows, shift the LRAS curve to the right by $-s/\gamma > 0$. This implies increases in both output and the real exchange rate in the long run: as the domestic economy permanently becomes more productive, its long-run real exchange rate has to fall – its commodities have to become relatively cheaper – to create the necessary demand for the increased supply.

Figure 23.5 Long-run equilibrium in the open economy



The long-run neutrality of the exchange rate regime

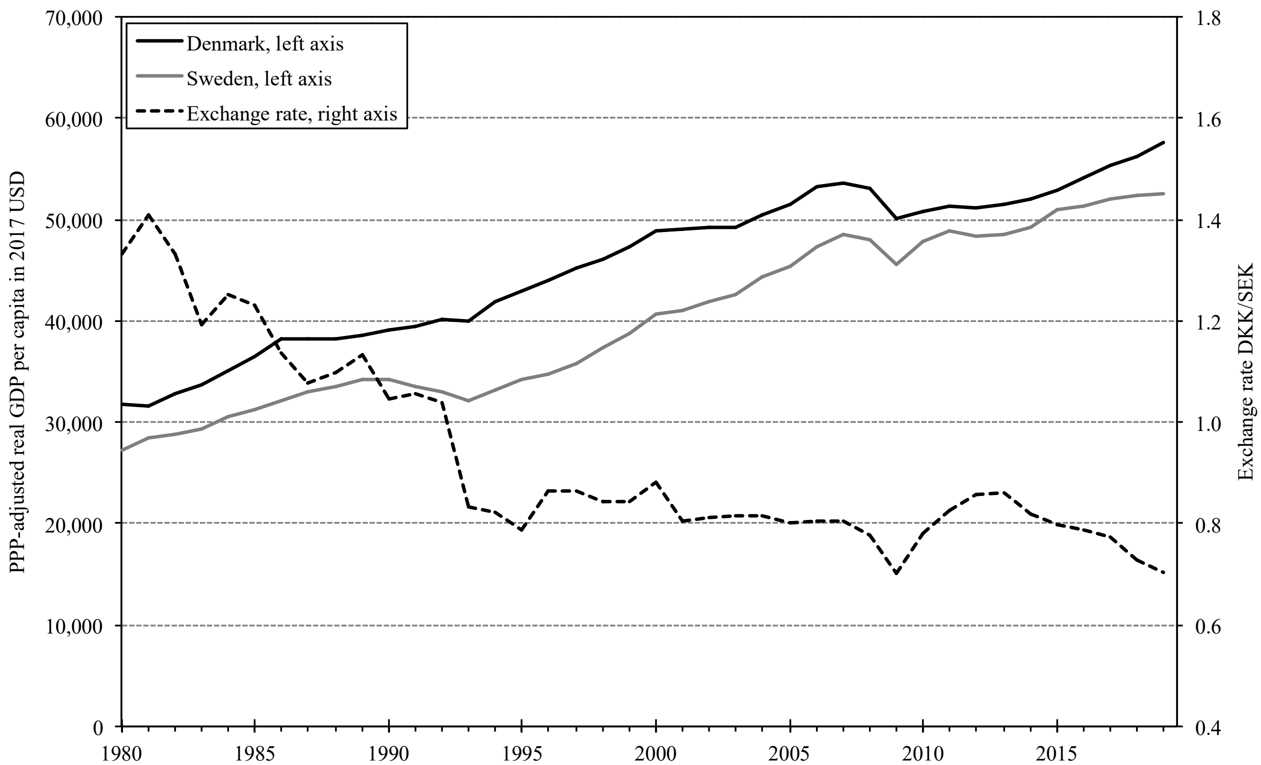
Our analysis of *long-run* equilibrium in the open economy has one striking implication, which is worth emphasizing: *the long-run equilibrium values of 'real' variables such as the real interest rate, the real exchange rate and real output are all independent of the exchange rate regime*. This follows from the simple fact that we did not have to make any assumption regarding the exchange rate regime when we derived the long-run equilibrium values of r, e^r and y. The analysis in Figure 23.5 is valid regardless of whether exchange rates are fixed or flexible.

Our analysis thus suggests that the exchange rate regime is *neutral* in the long run, since it has no impact on the long-run equilibrium values of real variables. This is a parallel to the long-run neutrality of money in the closed economy. In a closed economy setting, the authorities can control the nominal money supply M (or at least the monetary base), but in the long run they cannot control the real money supply M/P, because the price level is endogenous. Similarly, in the open economy policy makers can choose to fix the nominal exchange rate E rather than allowing it to float, but they cannot fix the long-run value of the real exchange rate, EP^f/P.

The idea of long-run neutrality of the exchange rate regime is well illustrated by the experience of Denmark and Sweden, two neighbouring countries heavily engaged in trade with one another. As shown in Figure 23.6, it took about 1.40 Danish kroner to purchase one Swedish krona in the early 1980s. By 2019, the price of one SEK was only around 0.70 DKK. Over the period considered the Swedish currency thus depreciated by a dramatic 50 per cent in nominal terms against the Danish currency. This reflected that Sweden pursued a policy of devaluation during part of the 1980s and later shifted to a floating exchange rate regime, whereas Denmark continued to fix its exchange rate vis-à-vis the German mark and later against the euro. Yet we see that the purchasing-power-parity adjusted GDP levels in the two countries have evolved in almost complete parallel. This is consistent with our theory that the choice of exchange rate regime does not affect real economic outcomes over the long run.

One should not conclude from this analysis that the choice of exchange rate regime is unimportant. As we shall see in the next two chapters, the mechanisms through which the economy adjusts towards its long-run equilibrium are different under fixed and flexible exchange rates. The choice of exchange rate regime may therefore make an important difference for the *short-run* and *medium-run* dynamics of the macro economy. Indeed, we shall see that this choice will determine whether or not monetary policy can be used as a tool of short-run stabilization policy.

Figure 23.6 The evolution of nominal exchange rates and PPP-adjusted real GDP per capita in Denmark and Sweden, 1980-2019



Note: Based on annual data.
Source: PPP-adjusted real GDP per capita from the World Bank's World Development Indicators; the exchange rate from Danmarks Nationalbank.

Moreover, there is not general agreement among economists that the exchange rate regime is really neutral in the long run. Some believe that the choice of exchange rate regime may have important structural effects in the long run. For example, some researchers have argued that the lower volatility of exchange rates under a fixed exchange rate regime may significantly promote international economic integration in trade as well as in capital movements by reducing the degree of risk and uncertainty relating to cross-border economic transactions, which may well have a positive impact on structural output. Others claim that the independent monetary stabilization policy that can be conducted under a floating exchange rate will indeed bring more stability in the short run and this stability can have a positive influence on structural output as discussed already back in Chapter 1. These conflicting views are part of the great and interesting debate over fixed vs. floating exchange rates that we return much to in the remaining chapters.

LONG-RUN EQUILIBRIUM IN THE OPEN ECONOMY

The long-run aggregate demand curve (LRAD) in the open economy shows the relationship between domestic GDP and the real exchange rate consistent with long-run equilibrium in the market for domestic goods. The LRAD curve is upward-sloping because a rise in the real exchange rate improves the competitiveness of domestic producers, thereby boosting demand for domestic goods via an increase in net exports. Under relative wage resistance the open economy's long-run aggregate supply curve (LRAS) is vertical at the natural rate of output. A long-run equilibrium is found where the LRAS and LRAD curves intersect. The resulting equilibrium is independent of the exchange rate regime, implying that the choice of exchange rate regime cannot affect real economic variables in the long run.

Wealth effects and the balance of payments in the long run

We explained above that whenever an open economy is running a surplus (deficit) on the current account of the balance of payments – that is, whenever it is exporting (importing) capital – it will accumulate (decumulate) foreign assets. When the current account imbalance stems from private sector transactions, it will therefore generate a change in the private sector's aggregate net wealth. In Chapter 16 we saw that changes in private wealth are likely to affect private consumption. In an economy with no long-run growth, a 'true' long-run equilibrium would then require that the real (inflation-adjusted) balance on the current account be zero. Otherwise, real private wealth would be changing over time, inducing changes in aggregate demand, which would be incompatible with long-run equilibrium.

However, in an economy with secular growth the real balance of payments must deviate from zero in the long run to allow real private wealth to grow in line with output and income (see Exercise 5). Moreover, real-world experience suggests that it takes a very long time before the economy fully adjusts to wealth effects stemming from current account imbalances. The Danish experience is a good example of this. As shown in Figure 23.7, Denmark ran a persistent current account deficit from 1975 to around 1990; in fact the period of substantial deficits started back in the late 1950s. The deficit was so high that the ratio of foreign debt to GDP was rising throughout most of this period. On the other hand, since 1990 Denmark has run a persistent current account surplus. The fact that the economy seems to respond very slowly to current account imbalances in the short and medium run indicates that it may be a reasonable simplification to ignore current account dynamics unless one wants to focus on the very long run. To keep matters simple, our model of the open economy therefore abstracts from wealth effects stemming from current account imbalances, just as we have previously ignored changes in private wealth arising from possible imbalances on the government budget.

Figure 23.7 The current account on the Danish balance of payment (% of GDP), 1975-2019

Note: Based on annual data.
Source: World Development Indicators, World Bank.

23.7 Summary

1. This chapter develops a model of aggregate demand and aggregate supply for an open economy. The economy is assumed to be so small that it cannot significantly affect macroeconomic conditions in the rest of the world. It is also assumed to be specialized in the sense that the goods produced domestically are imperfect substitutes for the goods produced abroad. This means that the price of domestic goods can vary relative to the price of foreign goods. A third important assumption is that capital is perfectly mobile across borders.
2. Under perfect capital mobility the arbitrage behaviour of risk-neutral investors will enforce uncovered nominal interest rate parity. This implies that the domestic nominal interest rate will be equal to the foreign nominal interest rate plus the expected percentage rate of depreciation of the domestic currency against the foreign

currency. If a group of countries moves towards credibly fixed exchange rates, the interest rate differentials (net of default risks) between them will therefore tend to vanish.

3. In the forward market for foreign exchange investors can buy or sell foreign currency for future delivery. The arbitrage behaviour in the forward market enforces covered nominal interest rate parity. This means that the domestic nominal interest rate equals the foreign nominal interest rate plus the forward foreign exchange premium. The latter is defined as the percentage difference between the forward exchange rate and the current spot market exchange rate. Covered and uncovered interest rate parity will hold simultaneously if the forward exchange premium equals the expected rate of change in the exchange rate over the period considered. This equality must hold if investors are risk neutral.

4. The link between capital mobility and interest rate formation means that macroeconomic policy choices are subject to the macroeconomic trilemma. The trilemma arises because a macroeconomic policy regime can include at most two out of the following three policy goals: (i) free cross-border capital flows, (ii) a fixed exchange rate, and (iii) an independent monetary policy. Under the classical gold standard before the First World War, most countries chose to sacrifice monetary autonomy. Under the Bretton Woods system of fixed exchange rates between 1945 and 1971 most countries maintained restrictions on international capital flows, but since the early 1970s the largest economies have abandoned fixed exchange rates while allowing capital mobility and pursuing independent monetary policies. More recently, the majority of EU member states have adopted a common monetary policy and a common currency as a means of ensuring irrevocably fixed exchange rates in a common market with free capital mobility. The member states have thus given up their individual independence in monetary policy, but have a common central bank, the ECB, that conducts a common monetary for them.

5. The various exchange rate regimes currently found in the world can be categorized into fixed, intermediate, and floating regimes. Under a fixed regime with a hard peg the exchange rate is fully fixed or kept within a rather narrow band, and the national currency may even have been abandoned as in the case of membership of a monetary union. Under floating the exchange rate is market determined, although the central bank may sometimes intervene in the foreign exchange market to moderate exchange rate fluctuations. The intermediate exchange rate regimes fall between the hard peg, fixed and the floating regimes by allowing some exchange rate flexibility within a broader (possibly shifting) band around the parity. In recent decades, there has been a worldwide tendency for countries to move away from intermediate regimes towards a hard peg or towards floating exchange rates.

6. The real exchange rate is the price of foreign goods relative to the price of domestic goods. The real exchange rate is inversely related to the international terms of trade. The percentage change in the real exchange rate equals the percentage rate of depreciation of the nominal exchange rate plus the difference between the foreign and the domestic rate of producer price inflation. In *long-run* equilibrium the real exchange rate must be constant, implying that relative purchasing power parity (RPPP) must hold. Under RPPP the rate of depreciation of the nominal exchange rate equals the difference between the domestic and the foreign inflation rate.

7. When capital is perfectly mobile, relative purchasing power parity implies that in the long run the domestic real interest rate is tied to the foreign real interest rate. This long-run relationship is referred to as real interest rate parity and tends to hold empirically in the long run.

8. The trade balance is the difference between exports and imports of goods and services, also denoted net exports. When the Marshall–Lerner condition is met, a depreciation of the real exchange rate will improve the trade balance. The Marshall–Lerner condition requires that the sum of the numerical price elasticities of export and import demand exceeds unity.

9. The short-run aggregate demand curve in the open economy implies a negative relationship between the rate of domestic producer price inflation and the aggregate demand for domestic goods. The reason is that higher domestic inflation will, *ceteris paribus*, erode the international competitiveness of domestic producers. The higher the price elasticities of export and import demand, the stronger is the reaction of net exports to a change in the real exchange rate, and the flatter is the aggregate demand curve in the output-inflation space. When the economy is out of long-run equilibrium, the short-run aggregate demand curve will gradually shift as the real exchange rate changes over time.

10. The properties of the short-run aggregate supply curve in the open economy will depend on whether there is relative wage resistance or real wage resistance. Under real wage resistance, wage setters have a target for the real consumer wage, defined as the nominal wage rate deflated by the consumer price index. Real wage resistance implies that the short-run aggregate supply curve will shift when the real exchange rate changes, as workers respond to changing import prices by adjusting their nominal wage claims. Real wage resistance will exist when nominal unemployment benefits are indexed to consumer prices.

11. Under relative wage resistance, individual wage setters seek to maintain a certain relation between their own wage rate and the wages set in the rest of the economy. Such behaviour emerges when nominal unemployment benefits are indexed to nominal wages, and it leads to an aggregate supply curve of the same form as the SRAS curve for the closed economy with producer price inflation as the relevant form of inflation entering the relationship. The specification of aggregate supply adopted here assumes relative wage resistance since indexation of unemployment benefits to wages is the most common international practice.

12. The open economy's long-run aggregate demand curve (LRAD) shows the relationship between real output and the real exchange rate which is consistent with long-run equilibrium in the market for domestic goods. The LRAD curve is upward-sloping in the output–real exchange rate space since a real exchange rate depreciation increases total demand for domestic goods by improving the international competitiveness of domestic producers.

13. Under relative wage resistance the open economy's long-run aggregate supply curve (LRAS) is vertical at the natural rate of output. The long-run equilibrium level of output is then uniquely determined by the position of the LRAS curve, and the long-run equilibrium real exchange rate is found where the LRAD curve intersects the LRAS curve.
14. Our AS–AD model of the open economy implies that the long-run equilibrium values of the real interest rate, the real exchange rate and real output and employment will be independent of the exchange rate regime. In the long run the exchange rate regime is thus neutral with respect to real variables. The proposition that the exchange rate regime is neutral in the long run should be seen only as an approximation, since the exchange rate regime may have some influence on the degree of international economic integration and on the macroeconomic stability of the economy, which in turn can have effects on the economy's structural output.
15. When there are wealth effects on aggregate demand, the accumulation of net foreign assets via the current account on the balance of payments will influence the evolution of the economy. In a long-run equilibrium without secular growth, the current account balance measured in units of domestic output must then be zero to ensure constancy of the real stock of net foreign assets. However, the evidence suggests that in practice it takes a very long time for current account imbalances to adjust via wealth effects of domestic demand, suggesting that these effects are quite weak. To simplify the analysis, our AS–AD model of the open economy therefore ignores wealth effects stemming from the current account.

23.8 Exercises

Exercise 1. Important concepts in the theory of the open economy

1. Explain what is meant by a 'small specialized economy'.
2. Define the concept of perfect capital mobility and explain the conditions for uncovered and covered interest rate parity. Which assumption is necessary for both parity conditions to hold at the same time? Are these parity conditions short-run relationships or long-run relationships?
3. Define the concepts of the real exchange rate and the international terms of trade. Explain the conditions for relative purchasing power parity (RPPP) and real interest rate parity. Are these conditions short-run relationships or long-run relationships?
4. Explain the Marshall–Lerner condition. Is fulfilment of this condition sufficient to ensure that a real exchange rate depreciation will raise the demand for domestically produced goods?
5. Suppose that individual domestic exporters are each selling a differentiated product

and that each of them therefore has a monopoly position in the international market (and thus faces a downward-sloping demand curve). Give a theoretical reason why, in these circumstances, the Marshall–Lerner condition must necessarily hold. (Hint: a monopoly firm will produce and sell up to the point where marginal revenue equals marginal cost. What does this imply for the size of the price elasticity of demand in the firm's optimum?).

6. Explain the concepts of relative wage resistance and real wage resistance and how these phenomena are related to the rules for indexation of unemployment benefits.

Exercise 2. Alternative monetary and exchange rate regimes

1. Explain the classification of alternative exchange rate regimes used by the International Monetary Fund. Discuss briefly why there has been a polarization of the choice of exchange rate regimes over recent decades.

2. Explain the nature of the macroeconomic 'trilemma'. Why is the trilemma inescapable, and how have countries tended to deal with it over time?

Exercise 3. Permanent shocks under relative wage resistance

In this exercise you are asked to undertake a graphical analysis of the long-run effects of permanent shocks in a small open economy with relative wage resistance. The conditions for long-run equilibrium in such an economy were summarized in Equations (30) and (31) in the text, and the long-run equilibrium was illustrated graphically in Figure 23.5. You may use these equations and diagram as a basis for answering the following questions.

1. Suppose the foreign real interest rate permanently increases. Give a graphical illustration of the long-run effects of this demand shock on the real exchange rate. Explain the effect.

2. Assume that the domestic government permanently increases public consumption. Illustrate and explain the long-run effect of this policy change on the real exchange rate.

3. Suppose that domestic productivity permanently increases so that the supply shock variable s permanently declines from zero to some negative amount. Illustrate and explain the long-run effects of this productivity shock on domestic output and on the real exchange rate.

Exercise 4. Permanent shocks under real wage resistance

This exercise invites you to study the long-run effects of permanent shocks in an open economy with real wage resistance and to compare these effects with those emerging under relative wage resistance. For this purpose you must first derive the aggregate supply curve under real wage resistance and characterize the economy's long-run equilibrium under this wage setting regime.

Consider a representative monopoly trade union, which sets the nominal wage rate W with the purpose of achieving a target real consumer wage. The nominal wage rate is set at a time when the union does not have perfect information on the average consumer price level. According to (29) in the main text the actual consumer price level is $P^c = (E^r)^{\psi}P$, where E^r is the real exchange rate and P is the price of domestically produced goods. Hence the expected consumer price level is $P^{ce} = (E^{re})^{\psi}P^e$, where E^{re} and P^e are the expected levels of E^r and P. We assume that the nominal rate of unemployment benefit is indexed to the consumer price level, so the real rate of unemployment benefit, b, is unaffected by changes in P^c. The union aims to obtain an expected real consumer wage, which is a mark-up, m^w, over the real unemployment benefit. Thus the nominal union wage claim is given by:

$$W = m^w b P^{ce} = m^w b (E^{re})^{\psi} P^e. \tag{32}$$

It is reasonable to assume that the union will moderate its wage claim when the rate of unemployment (u) rises. We may model this by specifying the union mark-up factor as:

$$m^w = \omega(1-u)^{\gamma}, \qquad \omega > 0, \qquad \gamma > 0, \tag{33}$$

where ω and γ are constants.

The representative domestic firm uses a linear production technology by which one unit of labour produces a units of output. Hence the marginal cost of production is W/a, and the firm sets its price as a mark-up, m^p, over marginal cost:

$$P = m^p \cdot \frac{W}{a}, \qquad m^p > 1. \tag{34}$$

Finally, we assume that the real unemployment benefit is tied to the 'normal' level of productivity, that is, the trend value of a, denoted $\overline{a}$:

$$b = \overline{c} \cdot \overline{a}. \tag{35}$$

This specification assumes that, over the long run, the unemployed are allowed to share in the real income gains generated by productivity growth.

1. Show by taking logarithms that Equations (32)–(35) lead to an expectations-augmented Phillips curve of the form:

$$\pi = \pi^e + \gamma(\overline{u} - u) + \psi e^{re} + \ln \overline{a} - \ln a, \tag{36}$$

$$\bar{u} \equiv \frac{\ln(m^p \omega \bar{c})}{\gamma}, \quad \pi \equiv \ln P - \ln P_{-1}, \quad \pi \equiv \ln P^e - \ln P_{-1}, \quad e^{re} \equiv \ln E^{re},$$

where π and π^e are, of course, the actual and the expected domestic inflation rate, respectively. (Hint: use the approximation $\ln(1 - u) \approx -u$.) Explain in economic terms why the (log of the) expected real exchange rate, e^{re}, appears in (36).

2. Use (36) along with

$$Y \equiv a(1-u)N \quad \Rightarrow \quad y \equiv \ln Y \approx \ln a + \ln N - u,$$

$$\bar{Y} \equiv \bar{a}(1-\bar{u})N \quad \Rightarrow \quad \bar{y} \equiv \ln \bar{Y} \approx \ln \bar{a} + \ln N - \bar{u},$$

to show that the short-run aggregate supply curve under real wage resistance takes the form:

$$\pi = \pi^e + \gamma(y - \bar{y}) + \psi e^{re} + s, \qquad s \equiv (1+\gamma)(\ln \bar{a} - \ln a), \tag{37}$$

where s is a supply shock variable reflecting productivity shocks.

3. Derive the long-run aggregate supply curve (LRAS) in an open economy with real wage resistance by inserting the long-run equilibrium conditions $\pi^e = \pi$ and $e^{re} = e^r \equiv \ln E^r$ into (37). Draw the resulting LRAS curve in a diagram with y along the horizontal axis and e^r along the vertical axis, and give an economic explanation for the slope of the LRAS curve. Draw the long-run aggregate demand curve (the LRAD curve given by Equation (30) in the text) in your (y, e^r)- diagram. Explain the slope of the LRAD curve and identify the economy's long-run equilibrium. Will y necessarily be equal to $\bar{y}$ in long-run equilibrium? Make a graphical comparison of the long-run equilibrium under real wage resistance and under relative wage resistance.

4. Assume that the domestic government permanently increases public consumption. Illustrate the long-run effects of this policy change on domestic output and on the real exchange rate under the two alternative scenarios of real wage resistance and relative wage resistance. Compare and explain the effects.

5. Suppose that domestic productivity permanently increases so that the supply shock variable $s \equiv (1+\gamma)(\ln \bar{a} - \ln a)$ permanently declines from zero to some negative amount. Illustrate the long-run effects of this productivity shock on domestic output and on the real exchange rate under the two alternative scenarios of real wage resistance and relative wage resistance. Compare and explain the effects.

Exercise 5. The open economy with wealth effects on aggregate demand

In this exercise you are asked to study an open economy with relative wage resistance and wealth effects on private consumption demand. We noted in the text that imbalances on the current account of the balance of payments may affect aggregate demand by changing the private sector's stock of wealth and the associated amount of capital income accruing to domestic residents.

To concentrate on the role of the current account in the process of wealth accumulation, we will ignore other sources of changes in private wealth. Thus, we will identify private wealth with the private sector's stock of net foreign assets, denoted by F and measured in units of domestic output (just as we have measured the trade balance in units of domestic output). *Ceteris paribus*, a rise in F will generate an increase in private consumption. Hence, we must respecify our previous goods market equilibrium condition (17) in the following way, where we do not explicitly include the exogenous confidence variable e and the exogenous foreign activity level Y_f:

$$Y = D(Y, \tau, r, E^r, F) + NX(Y, \tau, r, E^r, F) + G,$$
$$D_F = \frac{\partial D}{\partial F} > 0, \quad \frac{\partial NX}{\partial F} < 0, \quad D_F + \frac{\partial NX}{\partial F} > 0. \tag{38}$$

By stimulating private consumption, a rise in net foreign assets will cause a deterioration of the trade balance, because part of the rise in consumer demand will be directed towards imports. However, since the marginal propensity to import is less than 1, the net effect of a rise in F on the demand for domestic goods is positive, as indicated by the last inequality above.

By definition, we have:

$$F = \frac{EF^f}{P} \tag{39}$$

where F^f is the nominal stock of net foreign assets denominated in foreign currency. As an accounting identity, we also have:

$$E_{+1}F^f_{+1} - EF^f = P \cdot NX + \left(i^f + \frac{E_{+1} - E}{E} \right) EF^f \tag{40}$$

This equation says that the increase in the *nominal* stock of net foreign assets (the left-hand side) arises through a nominal surplus on the trade balance, $P \cdot NX$, through interest earnings on the existing foreign assets, $i^f \cdot EF^f$, or through exchange rate gains on the current asset stock, $((E_{+1} - E)/E)EF^f$. Dividing by $P_{+1} \equiv (1 + \pi_{+1})P$ in (40) and using (39) plus the approximation $\Delta e_{+1} = (E_{+1} - E)/E$, we find that the increase in foreign assets measured in units of domestic output is given by:

$$F_{+1} - F = \frac{NX + (i^f + \Delta e_{+1} - \pi_{+1})F}{1 + \pi_{+1}}. \tag{41}$$

We can now characterize the long-run equilibrium in the open economy with wealth effects. Since we are abstracting from long-term growth, a long-run equilibrium requires that the stock of net foreign assets be constant, since aggregate demand would otherwise keep on shifting over time. We also know that relative purchasing power parity and real interest rate parity must hold in the long run. Moreover, if we assume relative wage resistance, we know that the long-run equilibrium level of output is an exogenous constant, $\bar{Y}$. Hence we have the long-run equilibrium conditions:

$$\begin{gathered} F_{+1} = F, \quad Y = \bar{Y}, \quad r = r^f, \\ \Delta e_{+1} = \pi_{+1} - \pi^f_{+1} \Rightarrow i^f + \Delta e_{+1} - \pi_{+1} = i^f - \pi^f_{+1} = r^f, \end{gathered} \tag{42}$$

which may be inserted into (38) and (41) to give:

$$\bar{Y} = D(\bar{Y}, \tau, r^f, E^r, F) + NX(\bar{Y}, \tau, r^f, E^r, F) + G, \tag{43}$$

$$NX(\bar{Y}, \tau, r^f, E^f, F) + r^f F = 0. \tag{44}$$

Equations (43) and (44) determine the long-run equilibrium values of the two endogenous variables E^r and F. Notice from (44) that in the long run the current account has to balance in real terms to ensure constancy of the real stock of net foreign assets.

1. Use Equations (43) and (44) to illustrate the economy's long-run equilibrium in a diagram in which F is measured along the horizontal axis and E_r is measured along the vertical axis. The equilibrium combinations of F and E_r implied by (43) may be termed the 'GME curve' (GME for Goods Market Equilibrium). Show that the slope of the GME curve is:

$$\left(\frac{dE^r}{DF}\right)_{GME} = -\left(\frac{D_F + \dfrac{\partial NX}{\partial F}}{D_E + \dfrac{\partial NX}{\partial E^r}}\right) < 0, \tag{45}$$

which is seen to be negative since we assumed in the main text that $D_E + (\partial NX/\partial E^r) > 0$. Give an economic explanation for the negative slope.

The equilibrium combinations of F and E^r implied by (44) may be denoted the 'CAB curve' (CAB for Current Account Balance). Show that the slope of this curve is:

$$\left(\frac{dE^r}{DF}\right)_{CAB} = -\left(\frac{r_F + \dfrac{\partial NX}{\partial F}}{\dfrac{\partial NX}{\partial E^r}}\right) \tag{46}$$

Empirically, a value of r^f in the region of 5 per cent per annum is plausible. In many empirical studies based on annual data the magnitude of the derivative D_F in the numerator of (45) is also estimated to be close to 0.05. Assuming $D_F \approx r^f$, you can now determine the sign of the slope of the CAB curve and say whether it is steeper or flatter than the GME curve. (Hint: what is the sign of the derivative $D_E \equiv \partial D/\partial E^r$?).

Identify the long-run equilibrium values of F and E^r in your diagram.

2. Suppose the domestic government permanently raises public consumption. Give a graphical illustration of the long-run effects of this policy change on net foreign assets and the real exchange rate. Explain the effects.

3. Assume that the foreign real interest rate permanently increases. Illustrate and explain the long-run effects of this demand shock on net foreign assets and on the real exchange rate.

4. Assume alternatively that the domestic economy is hit by a positive supply shock which permanently increases domestic potential output $\bar{Y}$. Illustrate and explain the long-run effects of this supply shock on net foreign assets and on the real exchange rate.

Exercise 6 The sustainable balance of payments

In Exercise 5 we saw that in an economy with wealth effects on aggregate demand and with no real growth in the long run, the current account must balance in real terms in the long run to keep the stock of private wealth constant. However, if there is secular real growth, a long-run equilibrium no longer requires constancy of real private wealth; it only requires that real private wealth grows at the same rate as real GDP. This means that the real balance on the current account no longer has to be zero in the long run, although there is a limit to how much it can deviate from zero. You are now asked to illustrate these points by some simple numerical examples.

Consider an open economy where real GDP is expected to grow at an average annual rate of 2 per cent in the long run. Suppose that this economy starts out with a stock of net foreign debt equal to 25 per cent of GDP. Suppose further that the average inflation rate at home and abroad is expected to be 2 per cent per annum. Assume finally that the international real interest rate is 4 per cent per annum.

1. What is the magnitude of the nominal current account deficit relative to nominal GDP which the country can afford to sustain without increasing the ratio of foreign debt to GDP?

2. What is the magnitude of the real current account deficit (the deficit measured in units of domestic goods) which the country can afford to sustain without increasing the debt–GDP ratio?
3. What is the magnitude of the nominal trade balance relative to nominal GDP, which the country must sustain to maintain a constant ratio of foreign debt to GDP?

Chapter 24

The open economy with fixed exchange rates

Introduction

The AS–AD model of the open economy developed in the previous chapter implies that the long-run equilibrium values of real variables such as real output, the real interest rate and the real exchange rate are unaffected by the exchange rate regime. In this and the next chapter, we shall see that a country's choice of exchange rate regime is nevertheless a fundamental economic policy issue because of its influence on the economy's short-run dynamics. First, the choice of exchange rate regime will determine whether monetary policy can be used as a tool of macroeconomic stabilization policy. Second, the exchange rate regime will affect the way in which the economy adjusts to its long-run equilibrium, and how it responds to exogenous shocks in the short and medium term.

The rest of this book will elaborate on these points by analysing the workings of the macroeconomy under alternative exchange rate regimes. The present chapter focuses on a *small, open and specialized economy with perfect capital mobility* and *fixed exchange rate*. We start by studying the characteristics of fixed exchange rates as an economic policy regime and by discussing why a country might want to adopt a fixed nominal exchange rate. We then adapt the AS–AD model of the open economy set up in Chapter 23 to analyse how the economy adjusts to long-run equilibrium under fixed exchange rates. The final parts of the chapter use our AS–AD model to study the effects of macroeconomic policy under fixed exchange rates.

24.1 Fixed exchange rates as an economic policy regime

Under a *fixed exchange rate regime* the central bank fixes the nominal price of (some) foreign currency in terms of domestic currency. This official price of foreign currency is referred to as *the exchange rate parity*. In the purest version of a fixed exchange rate regime, the central bank stands ready to buy and sell unlimited amounts of foreign currency at the exchange rate parity so that the nominal exchange rate is completely fixed. A regime like that is referred to as a 'currency board'. In practice, most countries with 'fixed' exchange rates have allowed the market price of foreign currency to fluctuate within some band around the exchange rate parity and the central bank defends the parity through selling and buying of currency in the very short run and mainly through interest rate adjustments in the longer run. Of course, the wider the band within which the exchange rate is allowed to vary, the closer the 'fixed' exchange rate regime comes to a regime with freely floating exchange rates. Moreover, many countries with 'fixed' exchange rates have occasionally changed the official exchange rate parity, in effect following a policy referred to as *fixed but adjustable exchange rates*. The difference between fixed and flexible exchange rates is therefore a matter of degree, and the precise dividing line between the two types of regime is not clear-cut.

However, if a country and its central bank, possibly in cooperation with foreign central banks, defines a very narrow band for its currency and has a long historical record of not changing the parity rate and being ready to go far to keep the exchange rate very close to the parity rate, the country will come close to a regime of an effectively fixed exchange rate. Denmark vis-à-vis the euro and the ERM2 arrangement (European Exchange Rate Mechanism 2) between the Danish central Bank and the ECB is an example of this. However, as we return to later, there will always be a certain degree of vulnerability of such a regime.

In Chapter 23 we described the different types of exchange rate regimes found in the world and mentioned some important historical examples of fixed exchange rate arrangements with varying degrees of 'fixity' of the exchange rate. For analytical purposes, the present chapter will first focus mainly on the theoretical benchmark case where the nominal exchange rate is completely fixed, but at appropriate places explain the consequences of an expected de- or revaluation. Later in the chapter, we will consider a regime with fixed but adjustable exchange rates.

The impotence of monetary policy under fixed exchange rates and free capital mobility

Under a regime with fully fixed exchange rates and free capital mobility, it becomes impossible to use monetary policy for the purpose of stabilizing the domestic economy. The previous chapter already made this point, but it is sufficiently important to warrant a restatement. Recall from Chapter 23 that when international capital mobility is perfect, the following condition for uncovered interest parity must hold:

$$i = i^f + e^e_{+1} - e = i^f + \Delta e^e_{+1}, \quad e \equiv \ln E, \quad e^e_{+1} \equiv \ln E^e_{+1}, \quad \Delta e^e_{+1} \equiv e^e_{+1} - e. \tag{1}$$

Here i is the domestic nominal interest rate, i^f is the foreign nominal interest rate, E is the nominal exchange rate (the price of foreign currency in terms of domestic currency), and E^e_{+1} is the nominal exchange rate expected to prevail in the next period. Hence

$\Delta e_{+1}^e \equiv e_{+1}^e - e$ is the expected percentage rate of depreciation of the domestic currency against the foreign currency, that is, the expected capital gain on foreign bonds relative to domestic bonds over the period considered and $i^f + \Delta e_{+1}^e$ is the total expected nominal return on foreign assets. This must equal the nominal return *i* on domestic assets when the two asset types are perfect substitutes. Under a so-called hard peg where the exchange rate is credibly fixed, the expected exchange rate change will be zero, that is, $\Delta e_{+1}^e = 0$. It then follows from (1) that:

$$i = i^f. \tag{2}$$

In other words, *under credibly fixed exchange rates and perfect capital mobility the domestic nominal interest rate is tied to the foreign nominal interest rate*.

As far as the market-determined interest rates are concerned, the equality in (2) is enforced by the arbitrage between domestic and foreign assets described in the previous chapter.

Now, under a regime of fixed *but adjustable* exchange rates (where the exchange rate is not fully credibly fixed), the market participants may have an expectation of future changes in the exchange rate parity, that is, of de- or revaluations, $\Delta \overline{e}_{+1}^e \equiv \overline{e}_{+1}^e - \overline{e} \neq 0$, where $\overline{e}$ is the exchange rate parity in the current period and $\overline{e}_{+1}^e$ is the expected parity next period. Note that $\overline{e}_t$ is a *policy rate* decided by the relevant authorities for period *t*, and as long as no de- or revaluation occurs, $\overline{e}_t$ stays constant. On the other hand, e_t is the *market determined*, actual (log of the) exchange rate. These two are thus conceptually different, but as long as the fixed exchange rate policy is adhered to one will, of course, have $e_t = \overline{e}_t$, where for clarity we assume that the band within which the actual exchange rate is allowed to fluctuate has been brought down to a zero width. One will then also have $\Delta e_t = \Delta \overline{e}_t$ and $\Delta e_t^e = \Delta \overline{e}_t^e$. In this case the nominal interest rate parity will be:

$$i = i^f + \Delta \overline{e}_{+1}^e. \tag{2'}$$

Under a floating exchange rate there is no $\Delta \overline{e}_{+1}^e$, but possibly $\Delta e_{+1}^e \neq 0$. Equation (2) or (2') is enforced by the need to protect the country's foreign exchange reserves. If the central bank tries to keep the domestic nominal interest rate below the foreign short-term interest rate plus expected devaluation, $i < i^f + \Delta \overline{e}_{+1}^e$, capital will flow out of the domestic economy. As investors sell domestic currency and buy foreign currency to invest in foreign assets, the central bank will have to sell foreign currency and buy domestic currency to maintain the fixed exchange rate. In this way, the country's foreign exchange reserves will be exhausted if the central bank insists on keeping the domestic short-term interest rate below the foreign level. Indeed, the market will most likely interpret such an interest rate policy as a signal that the central bank is not committed to defending the exchange rate. In that case expectations of a future devaluation will arise, and a speculative attack on the domestic currency will occur, leading to an even faster depletion of foreign exchange reserves. Once the reserves are gone, the central bank can no longer intervene in the foreign exchange market to keep the exchange rate fixed, and the domestic currency will have to fall.

On the other hand, if for some reason the central bank keeps the domestic nominal interest rate above what is dictated by interest rate parity, $i > i^f + \Delta\overline{e}_{+1}^e$, a massive inflow of capital from abroad will occur. This will swell the foreign exchange reserves and create a constant upward pressure on the exchange rate. The huge capital inflow will also lead to a large expansion of domestic bank credit and a rise in domestic asset prices, which will generate strong inflationary pressures.

The upshot is that if the central bank wishes to maintain a fixed exchange rate, it cannot implement a nominal interest rate, which deviates from the nominal interest rate abroad (plus expected devaluation). The situation in Denmark illustrates this point very well. The Danish krone is pegged to the euro, and whenever the European Central Bank decides to change its leading interest rate, the Danish central bank interest rate is almost always changed by exactly the same amount within the same hour.

Monetary policy thus becomes *impotent* under fixed exchange rates and perfect capital mobility. When the exchange rate is fixed, the central bank cannot pursue an independent monetary policy, be it interest rate policy or a more unconventional policy, aimed at managing aggregate demand. Instead, monetary policy is fully bound by the commitment to defend the exchange rate. In contrast, if a country is willing to accept exchange rate variability, it can set its interest rate independently of the foreign interest rate. The interest rate parity of Equation (1) still has to hold, of course, which means that if the central bank chooses, e.g., $i < i^f$ to stimulate domestic activity and inflation, this must be reflected in an expected appreciation of the domestic currency, $\Delta e_{+1}^e < 0$. Chapter 25 will explain in detail how this can go on, but for now we briefly note that a relatively low domestic interest rate, $i < i^f$, will tend to create an outflow of capital from the country that will increase the supply of domestic currency and increase the demand for foreign currency, thereby driving up the relative price of foreign currency, $\Delta e > 0$,which can create an expectation that the exchange rate will fall back again to a more normal level in the next period, $\Delta e_{+1}^e < 0$.

In principle, a country with a fixed exchange rate can also pursue an independent monetary policy if it is able to maintain effective control over private cross-border capital flows, thereby preventing the arbitrage, which would otherwise drive the domestic interest rate into equality with the foreign interest rate. However, nowadays practically all the developed countries in the world have abandoned capital controls, partly because they are difficult to enforce, and partly because they are considered undesirable, since they prevent investors from seeking out the most profitable investment opportunities and from diversifying their portfolios to hedge against country-specific investment risks.[1] However, in a situation with a strong speculative attack on a pegged exchange rate, a

[1] In Chapter 4, we offered a more extensive discussion of the long-run benefits and risks associated with free capital mobility.

central bank that is ready to go far to defend the peg and wants to signal that fact to the world can resort to temporary constraints on free capital mobility.

The case for fixed exchange rates

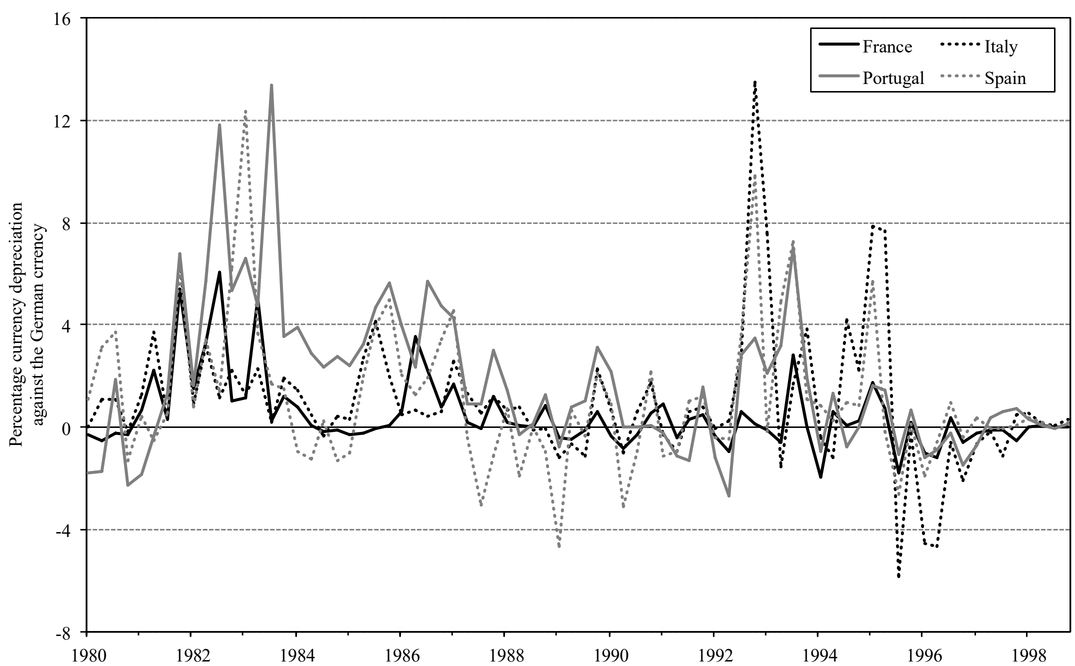
If capital controls are ineffective or are seen as undesirable, the price of maintaining fixed exchange rates is the loss of monetary policy autonomy. Given this fact, why would a country nevertheless choose a fixed exchange rate regime? Advocates of fixed exchange rates usually point to the disadvantages of exchange rate uncertainty and to the benefits of using a fixed exchange rate as a nominal anchor in the fight against inflation. Let us briefly consider these arguments.

1. The first argument is that the large exchange rate movements often observed under floating exchange rates may hamper international trade and investment by creating exchange rate uncertainty. To be sure, short-term foreign exchange risk can be covered through the forward exchange market at a modest transaction cost, but the opportunities for covering long-term foreign exchange risks are much more limited, and this might discourage long-term trade contracts and long-term international investments if the exchange rate is highly unstable.
2. A related argument starts from the observation that exchange rates often seem to 'overshoot' their long-run equilibrium values under floating exchange rates, as we shall see in the next chapter. Speculative capital movements may well and often do create large fluctuations in exchange rates that do not seem to be warranted by fundamental changes in the domestic or foreign economies. The concern is that such excessive exchange rate movements may cause an undesirable reallocation of resources and an unintended redistribution of real income across different sectors in the economy. For example, a sharp appreciation of the domestic currency will erode the international competitiveness of the domestic export industry and of domestic industries, which are competing against imports. Consequently, profits and employment in those sectors will be squeezed, whereas sectors relying on imported inputs and selling their output in the domestic market without any competition from abroad will benefit from the appreciation. If the real exchange rate overshoots its long-run equilibrium value due to excessive volatility of the nominal exchange rate, it may thus generate a costly and unwarranted reallocation of resources back and forth across sectors.
3. Another main argument is that a fixed exchange rate can provide a *nominal anchor* which may help to keep inflation down at a desirably low level. In the previous chapter we saw that relative purchasing power parity, $\pi = \pi^f + \Delta e$, is a condition for a long-run equilibrium because it is necessary for a stable real exchange rate. This implies that in the longer term a country can only maintain a fixed nominal exchange rate against a foreign trading partner if the domestic inflation rate does not systematically exceed or fall below inflation in the foreign country. Policy makers

may therefore signal a commitment to keep the domestic inflation rate low by announcing that they will peg the domestic currency to the currency of a foreign country with a history of low inflation. If this commitment is credible, it will keep domestic inflation expectations in check, and this in turn will make it easier to keep the actual inflation rate low. Thus, by pegging to a stable foreign currency, domestic policy makers may 'import' some of the credibility and discipline of foreign policy makers who have been successful in fighting inflation. Such a strategy is most likely to be credible and to succeed if the political costs of giving up the peg to the foreign currency are perceived to be considerable.

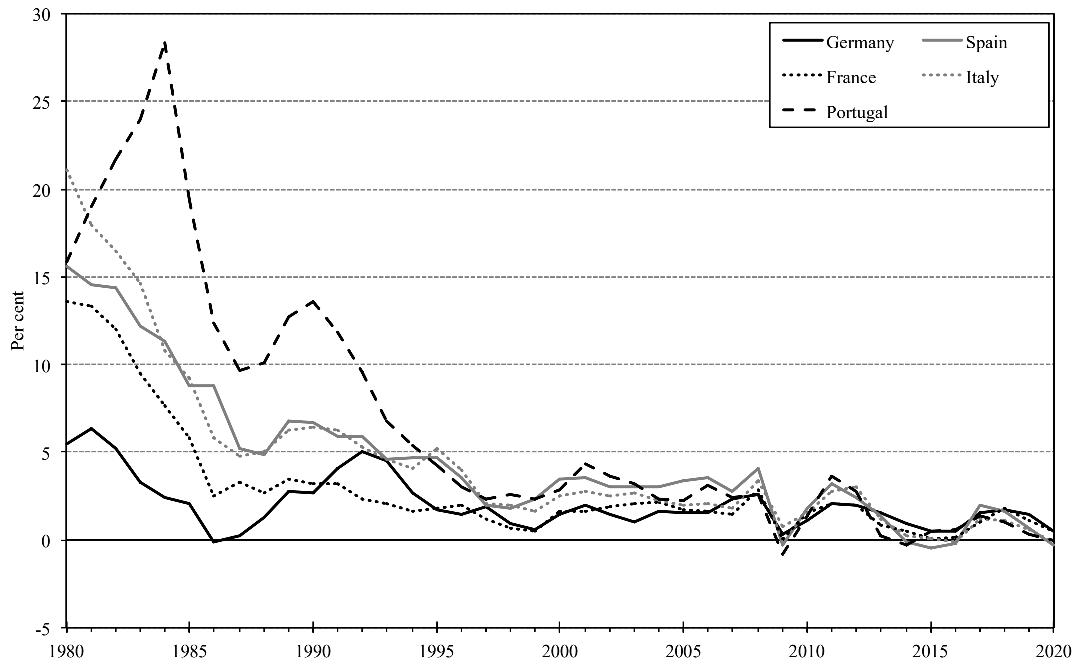
The belief that pegging to a stable foreign currency can help to bring down domestic inflation motivated many previous high-inflation countries in Europe to tie their currencies closer to the German D-mark after the mid 1980s. After a setback caused by the speculative attacks on the European Monetary System in the early 1990s (to which we shall return later in this chapter), this policy of pegging to the D-mark gradually led to lower exchange rate variability as shown by in Figure 24.1. After the establishment of the Economic and Monetary Union of the European Union (EMU) and the common currency, the euro, in 1999, there are, of course no further nominal exchange rate changes between the currencies of the countries in the euro area. The movement towards more and more – and eventually completely – fixed exchange rates did in fact gradually drive national inflation rates in the EU closer to the low German level, as shown in Figure 24.2.

Figure 24.1 Exchange rate variability in Europe, 1980-1999



Note: Percentage rate of currency depreciation against the German D-mark. Based on quarterly data.
Source: The German Bundesbank.

Figure 24.2 Inflation rates in Europe, 1980-2020



Note: Based on annual data.
Source: Main Economic Indicators database, OECD.

Our discussion illuminates the trade-offs involved in the choice of exchange rate regime: by fixing the exchange rate, a country may eliminate or at least reduce nominal exchange rate uncertainty and provide a nominal anchor for domestic prices, and it may also reap some gains from additional foreign trade and capital movements. But these benefits can only be achieved by sacrificing monetary policy autonomy, and giving up an independent monetary policy may weaken a country's ability to pursue an effective macroeconomic stabilization policy. Given this difficult trade-off, it is hardly surprising that different countries with different economic histories and different economic structures have chosen different exchange rate regimes. We will return to the great debate on fixed versus flexible exchange rates in Chapter 26.

FIXED EXCHANGE RATES AS AN ECONOMIC POLICY REGIME

Under fixed exchange rates and perfect capital mobility a small open economy cannot pursue an independent monetary policy, since the domestic interest rate will have to equal the interest rate on assets denominated in the foreign currency (possibly plus expected devaluation) to which the domestic currency is pegged. On the other hand, a fixed exchange rate may provide a 'nominal anchor' that helps to reduce domestic inflation if the domestic currency is pegged to the currency of a foreign country with a record of low and stable inflation. By reducing/eliminating exchange rate uncertainty, a fixed exchange rate regime may also stimulate foreign trade and capital movements, which may give rise to efficiency gains.

24.2 Model and macroeconomic adjustment under fixed exchange rates

Inflation and real exchange rate dynamics

Before analysing the effects of economic policy in an open economy with fixed exchange rates, it is useful to study the dynamic process through which such an economy adjusts to long-run equilibrium. As we shall see in this section, the evolution of the *real* exchange rate plays a key role in the macroeconomic adjustment process. From the previous chapter you will recall that the real exchange rate is defined as $E^r = EP^f/P$, where E is the nominal exchange rate, P^f is the foreign price level measured in foreign currency, and P is the price of domestic goods. From $E^r = EP^f/P$, we derived $\Delta e^r \equiv e^r - e^r_{-1} = \Delta e + \pi^f - \pi$, where $e^r \equiv \ln E^r$, $\Delta e \equiv \ln E - \ln E_{-1}$, $\pi^f \equiv \ln P^f - \ln P^f_{-1}$, $\pi \equiv \ln P - \ln P_{-1}$ (Equation (9) of Chapter 23). Hence, when the nominal exchange rate is fixed, it follows that the percentage change in the real exchange rate (the change in the log of the real exchange rate) is given by:

$$\Delta e^r \equiv e^r - e^r_{-1} = \pi^f - \pi, \tag{3}$$

where π^f and π are the foreign and the domestic inflation rate, respectively. Thus, with a fixed nominal exchange rate the real exchange rate can only remain stable over time if the domestic inflation rate corresponds to the foreign inflation rate. Since a constant real exchange rate is one condition for long-run equilibrium, it follows that the domestic inflation rate will converge on the foreign inflation rate if the long-run equilibrium is stable (as indeed it will be under the assumptions made below).

We will now illustrate the role of the identity (3) in our AS–AD model of the open economy with fixed exchange rates.

Aggregate demand and aggregate supply

For convenience, we restate Equation (26) of the previous chapter:

$$\begin{aligned} y - \bar{y} &= \beta_1(e^r_{-1} + \Delta e + \pi^f - \pi) - \beta_2(i^f + \Delta e^e_{+1} - \pi^e_{+1} - \bar{r}^f) + \tilde{z}, \\ \tilde{z} &\equiv \beta_3(g - \bar{g}) + \beta_4(y - \bar{y}^f) + \beta_5(\ln \varepsilon - \ln \bar{\varepsilon}), \end{aligned} \tag{4}$$

the general but preliminary expression for the aggregate demand curve in the open economy; all notation is as in Chapter 23. Allowing for the possibility of exchange rate flexibility, that equation included both the actual rate of nominal exchange rate depreciation, Δe, which affects the country's international competitiveness, and the expected rate of nominal exchange rate depreciation, Δe^e_{+1}, which influences the domestic rate of interest. Under a policy regime where nominal exchange rates are credibly fixed – that is, where the public believes in the government's declared commitment not to devalue or revalue the currency – the expected as well as the actual rate of nominal exchange rate depreciation will be zero. Inserting this condition ($\Delta e = \Delta e^e_{+1} = 0$) into the equation above, we get the following expression for the goods market equilibrium condition in an economy with credibly fixed exchange rates:

$$y - \bar{y} = \beta_1(e^r_{-1} + \pi^f - \pi) - \beta_2(i^f - \pi^e_{+1} - \bar{r}^f) + \tilde{z}, \tag{5}$$

where we recall that $\bar{r}^f$ is the structural, long-run foreign real interest rate, π^e_{+1} is the expected rate of domestic inflation between the current and the next period, and $\tilde{z}$ is a demand shock variable. From (3) we see that the magnitude $e^r_{-1} + \pi^f - \pi$ in (5) is simply the current real exchange rate, e^r. Thus, aggregate demand varies positively with the competitiveness of domestic producers, measured by the current real exchange rate. We also see that total demand varies negatively with the real interest rate, $i - \pi^e_{+1} = i^f - \pi^e_{+1}$. Note that if the exchange rate is not fully credibly fixed and there is a certain expectation of devaluation, $\Delta \bar{e}^e_{+1} \neq 0$, according to (2') all that is changed is that the second term on the right hand side of (4) should be: $-\beta_2(i^f + \Delta \bar{e}^e_{+1} - \pi^e_{+1} - \bar{r}^f)$.

In modelling aggregate supply, we will rely on the hypothesis of relative wage resistance explained in the previous chapter. As you recall, this hypothesis leads to a standard aggregate supply curve of the form:

$$\pi = \pi^e + \gamma(y - \bar{y}) + s. \tag{6}$$

To complete our AS–AD model with (credibly) fixed exchange rates, we now only need to specify the formation of inflation expectations.

Expectations formation under fixed exchange rates

As a starting point for doing so, let us return to the idea presented at the end of Chapter 21 that some people are backward-looking whereas others are forward-looking. Specifically, suppose the population is divided into two groups. The first group (Group 1) has purely backward-looking expectations and simply believes that this period's inflation rate will equal the rate of inflation observed in the previous period. This is, of course, the familiar hypothesis of static expectations.

The second group of people (Group 2) is more sophisticated. This group realizes that under fixed exchange rates domestic inflation cannot systematically exceed foreign inflation for long periods, since this would undermine the international competitiveness of domestic producers, lead to ever-decreasing demand for domestic goods and to an ever-increasing trade deficit. Falling demand would pull down domestic inflation itself and an ever-growing trade deficit would ultimately force the domestic authorities to bring down domestic inflation in order to protect the country's foreign exchange reserves. Similarly, Group 2 people also understand that if domestic inflation were systematically lower than foreign inflation, the persistent fall in the relative price of domestic goods would lead to ever-increasing demand for domestic output, which would ultimately pull up the domestic inflation rate. In short, Group 2 agents realize that on average over the longer run, the domestic inflation rate will have to equal the foreign inflation rate. In short, Group 2 people form inflation expectations in accordance with long-run purchasing power parity, $\Delta e^r = \Delta e + \pi^f - \pi = 0$, and fixed exchange rate, $\Delta e = 0$, giving $\pi = \pi^f$. However, they do not have enough information to predict the short-run fluctuations of the inflation rate. Instead, they assume that the domestic inflation rate for the current and next period will be at its average level given by the foreign inflation rate. We may say that these individuals have 'weakly rational' expectations, since they hold correct beliefs about the long-run equilibrium inflation rate but cannot perfectly predict the short-run inflation rate.

If Group 2 constitutes a fraction φ of the total population, these assumptions imply that the average expected inflation rate will be:

$$\pi^e = \varphi\, \pi^f + (1 - \varphi)\pi_{-1}, \qquad\qquad 0 \leq \varphi \leq 1.$$

In the following, we will consider the special case where all agents have weakly rational expectations, $\varphi = 1$, so that $\pi^e = \pi^e_{+1} = \pi^f$, where foreign inflation is assumed to be constant at the level π^f. We will concentrate on this case because it greatly simplifies the graphical exposition of our AS–AD model and allows us to focus on the role of real exchange rate dynamics in the adjustment to long-run macroeconomic equilibrium.

The assumption that $\pi^e = \pi^e_{+1} = \pi^f$ may be seen as a compromise between the case of purely backward-looking expectations and the strong postulate of strictly rational, forward-looking expectations. Our hypothesis on expectations formation is a simple way of formalizing the idea that, under credibly fixed exchange rates, the authorities have implicitly adopted an inflation target equal to the foreign inflation rate, and this inflation target serves as an anchor for domestic inflation expectations.

What if the exchange rate is not fully credibly fixed and there is an expected devaluation of $\Delta\bar{e}^e \neq 0$ formed in the previous period concerning a potential parity adjustment in the current one, or $\Delta\bar{e}^e_{+1} \neq 0$ formed in the current period concerning a potential adjustment in the next period? Would expected long-run relative PPP, $\Delta e^r = \Delta e + \pi^f - \pi = 0$, imply $\pi^e = \pi^f + \Delta\bar{e}^e$ and $\pi^e_{+1} = \pi^f + \Delta\bar{e}^e_{+1}$? The answer is no: relative PPP is a long-run 'law', so weakly rational agents would only expect PPP to be reestablished over a number of periods. Hence, an expected one-time adjustment of the exchange rate next period, $\Delta\bar{e}^e_{+1} > 0$ say, would be expected to create higher domestic inflation over a number of succeeding future periods, such that the change in expected inflation in any specific of these periods, $\partial\pi^e$ say, would be of the same sign as but numerically considerably smaller than $\Delta\bar{e}^e_{+1}$. So, for an expected one-time devaluation, $\Delta\bar{e}^e_{+1} > 0$, the weakly rational expectation would be $\pi^e_{+1} = \pi^f + \partial\pi^e$, where $\partial\pi^e > 0$, but $\partial\pi^e << \Delta\bar{e}^e_{+1}$, where << reads 'considerably smaller than'. Equivalently, for a one-time devaluation expected in the previous period, $\Delta\bar{e}^e > 0$, one would have $\pi^e = \pi^f + \partial\pi^e$, where $\partial\pi^e > 0$, but $\partial\pi^e << \Delta\bar{e}^e$.

The complete AS–AD model with fixed exchange rates

Our main focus for now is on fully credibly fixed exchange rates where expected inflation rates are $\pi^e = \pi^e_{+1} = \pi^f$. The term $-\beta_2(i^f - \pi^e_{+1} - \bar{r}^f)$ on the right hand side of (4) then becomes $-\beta_2(i^f - \pi^f - \bar{r}^f) = -\beta_2(r^f - \bar{r}^f)$. Hence, we may write (4) as: $y - \bar{y} = \beta_1(e^r_{-1} + \pi^f - \pi) - \beta_2(r^f - \bar{r}^f) + \tilde{z}$. Here we may choose to put the term $-\beta_2(r^f - \bar{r}^f)$, which is just a foreign real interest rate shock, into the shock variable, that is, to define a new shock variable $z \equiv \tilde{z} - \beta_2(r^f - \bar{r}^f)$ to get the AD curve: $y - \bar{y} = \beta_1(e^r_{-1} + \pi^f - \pi) + z$. Furthermore, we may also insert $\pi^e = \pi^f$ into the AS curve (5) and restate the real exchange rate dynamic (3) to obtain our complete model of the open economy with fixed exchange rates:

AD:
$$\begin{aligned} &y_t - \bar{y} = \beta_1(e^r_{t-1} + \pi^f - \pi_t) + z_t \quad \Leftrightarrow \\ &\pi_t = e^r_{t-1} + \pi^f - \frac{1}{\beta_1}(y_t - \bar{y} - z_t), \\ &z_t \equiv -\beta_2(r^f_t - \bar{r}^f) + \beta_3(g_t - \bar{g}) + \beta_4(y^f_t - \bar{y}^f) + \beta_5(\ln\varepsilon_t - \ln\bar{\varepsilon}), \end{aligned} \tag{7}$$

AS:
$$\pi_t = \pi^f + \gamma(y_t - \bar{y}) + s_t, \tag{8}$$

RERD (real exchange rate dynamic): $$e_t^r = e_{t-1}^r + \pi^f - \pi_t. \qquad (9)$$

We have now chosen to state time subscripts *t* explicitly. Equation (7) is the aggregate demand curve, which, for reasons that will soon become obvious, we state both in a version with the output gap on the left hand side and in a version with the domestic inflation rate on the left hand side. Since $\beta_1 > 0$, we see that a higher domestic inflation rate will *ceteris paribus* be associated with lower aggregate demand for domestic output. In the closed economy, the negative impact of inflation on demand stems from the fact that higher inflation induces the central bank to raise the real interest rate. In the open economy with fixed exchange rates, this mechanism is suppressed, because the domestic interest rate is tied to the foreign interest rate via perfect capital mobility. Instead, the reason for the negative slope of the AD curve (7) is that higher domestic inflation erodes the international competitiveness of domestic producers by reducing the real exchange rate. Thus, a rise in π raises the relative price of domestic goods, thereby reducing net exports.

The variable *z* in (7) captures so-called real shocks to aggregate demand. In the open economy, these shocks include changes in foreign real output, y^f, and in the foreign real interest rate, r^f, as well as changes in domestic real government spending, *g*, and in domestic private sector confidence, ε. A change in any of these variables will cause a shift in the aggregate demand curve. In addition, (7) shows that the domestic economy may be exposed to a so-called nominal shock in the form of a change in the foreign inflation rate, π^f. Such a shock will also shift the AD curve. In addition, a foreign inflation shock will shift the short-run aggregate supply curve (8) by changing the expected rate of domestic inflation.

Equations (7) and (8) are valid under fully credibly fixed exchange rates, which we focus on until we explicitly turn to de- and revaluations in section 24.4. There it will be explained how (7) and (8) are modified in the presence of expected devaluations, $\Delta\overline{e}_{t+1}^e \neq 0$ and/or $\Delta\overline{e}_t^e \neq 0$. For now we make some important observations on our model (7) through (9).

First, for $z_t = 0$, the AD curve (7) passes through $(\overline{y}, e_{t-1}^r + \pi^f)$ in a usual y_t, π_t-diagram: from (9), the domestic inflation rate $\pi_t = e_{t-1}^r + \pi^f$ is exactly the one that ensures a current real exchange rate at the structural level, $e_t^r = \overline{e}^r = 0$, which from (7) exactly ensures that there is demand for the structural output, $\overline{y}$, when $z_t = 0$. A demand shock $z_t \neq 0$ will shift the AD curve in the *y*-dimension by z_t (that is, to the right if $z_t > 0$ and to the left if $z_t < 0$), and by z_t / β_1 in the π-dimension.

Second, for $s_t = 0$, the AS curve (8) passes through $(\overline{y}, \pi^f)$, and a supply shock $s_t \neq 0$ will shift the AS curve vertically by s_t and horizontally by s_t / γ.

Third, as seen most clearly from the second writing of the AD curve in (7), *ceteris paribus*, the AD curve shifts from period *t* to *t*+1 vertically (upwards) in the y_t, π_t-

diagram by exactly $\Delta e_t^r \equiv e_t^r - e_{t-1}^r$, which from (9) equals $\pi^f - \pi_t$. Hence, from t to t+1 the AD curve shifts upwards by the inflation differential $\pi^f - \pi_t$, a simple consequence of the effect of this differential on the competitiveness of the domestic economy.

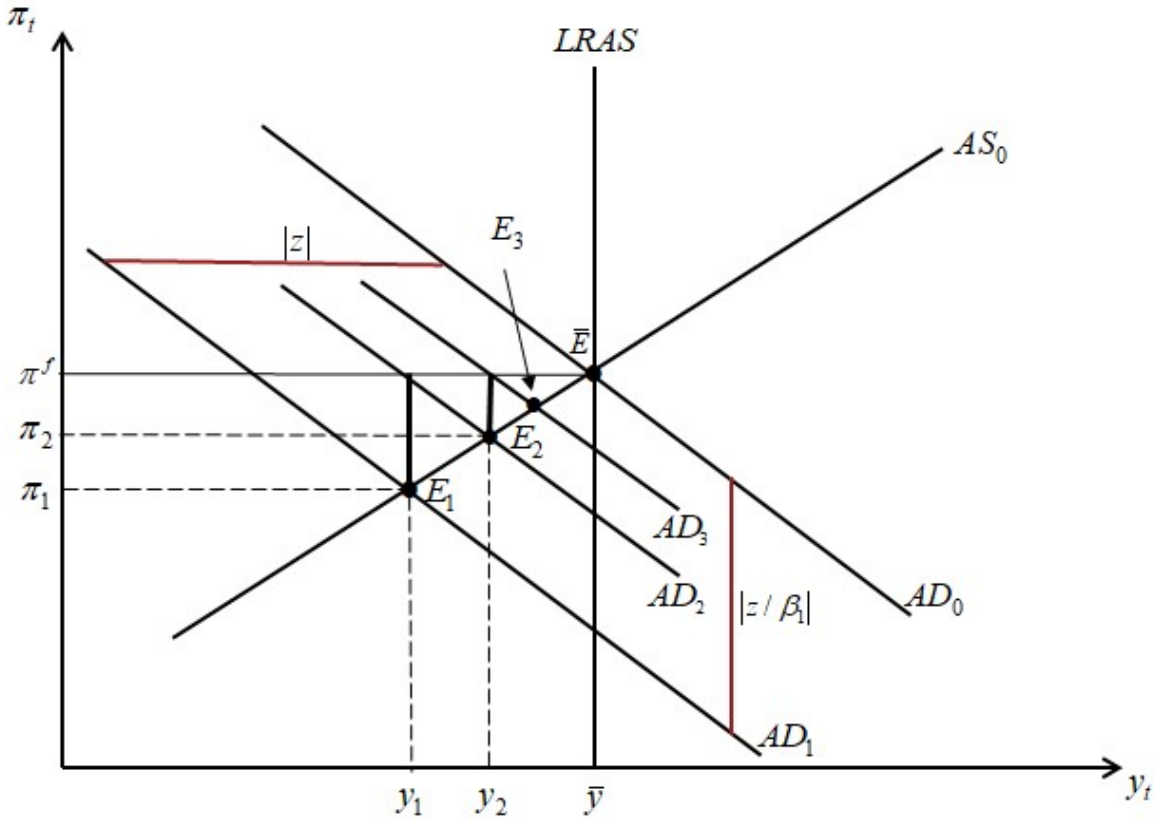
From short-run to long-run equilibrium

As the economy enters the current period t, last period's real exchange rate e_{t-1}^r is a predetermined constant. The current rates of output and inflation, y_t and π_t, are then determined simultaneously by the AD curve (7) and the AS curve (8), given π^f, z_t and s_t. The current level of inflation in turn determines the current real exchange rate via (9). If $\pi^f \neq \pi_t$, it follows that $e^r{}_t \neq e_{t-1}^r$, and the economy will then enter the next period with a new predetermined real exchange rate. According to (7) this means that the position of the AD curve will shift vertically from the current period to the next one by $\pi^f - \pi_t$, as just explained. This will generate new short-run values of output and inflation during the next period, which in turn will change the level of the real exchange rate, and so on.

Figure 24.3 illustrates this dynamic adjustment process. We start out from a long-run equilibrium in period 0, which is defined by no shocks in the period and the real exchange rate at its structural level the period before, that is, $z_t = s_t = 0$ and $e_{-1}^r = \bar{e}^r = 0$. Under these circumstances, both the AD curve and the AS curve pass through $(\bar{y}, \pi^f)$, as illustrated by AD_0 and AS_0 in the figure, making $\bar{E} = (\bar{y}, \pi^f)$ the short-run equilibrium of period 0, so the long-run and the short-run equilibria coincide. Note that this implies $e_0^r = \bar{e}^r = 0$ from (9). In period 1, we assume that a permanent negative demand shock occurs such that z_t shifts permanently from 0 down to $z < 0$ for $t \geq 1$, say because of a permanent drop in foreign activity, while $s_t = 0$ for all t. As (7) shows, this will shift the AD curve to the left by $|z|$ and downwards by $|z / \beta_1|$ from period 0 to 1 as illustrated by AD_1 in the figure. This effect is also present in the following periods, but here the AD curve will shift further in reaction to changes in the real exchange rate. The AS curve does not shift and remains at AS_0. This yields the short-run equilibrium E_1 in period 1 involving $y_1 < \bar{y}$ and $\pi_1 < \pi^f$, as illustrated in the figure.

Hence, the weakness of aggregate demand in period 1 means that the economy is in recession, with low levels of output and inflation. But this is only a short-run equilibrium: since domestic inflation is now lower than foreign inflation, the real exchange rate increases during period 1 so that $e_1^r = e_0^r + \pi^f - \pi_1 > e_0^r = 0$. Thus the international competitiveness of domestic producers improves by the amount $\pi^f - \pi_1 > 0$ during period 1, and this in itself shifts the AD curve upwards by exactly $\pi^f - \pi_1$ from period 1 to 2. Since the demand shock is unchanged $z < 0$ in period 2, all in all the AD curve shifts upwards by $\pi^f - \pi_1$ as the economy moves from period 1 to period 2, as illustrated by the AD curve AD_2 in Figure 24.3. As a result, domestic output and inflation increase to the levels y_2 and π_2 in period 2, where $y_1 < y_2 < \bar{y}$ and $\pi_1 < \pi_2 < \pi^f$, as illustrated by the short-run equilibrium E_2.

Figure 24.3 The adjustment to long-run equilibrium under fixed exchange rates



Since domestic inflation is still lower than foreign inflation in period 2, there is a further gain in domestic competitiveness, so the AD curve shifts upwards by the distance $\pi^f - \pi_2$ between periods 2 and 3, causing a further rise in output and inflation in period 3, and so on. In this way the gradual improvements in domestic competitiveness will continue to pull the economy up along the short-run aggregate supply curve until domestic output reaches its trend level, $\bar{y}$. At that point, domestic inflation catches up with foreign inflation so that no further changes in the real exchange rate will take place, that is, the economy will be in long-run equilibrium. After all adjustments, the initial and permanent drop in demand arising, e.g., as a result of lower foreign activity, has been exactly compensated by an increase in demand (mainly through higher net exports) caused by the increased competitiveness (real exchange rate) resulting from lower domestic inflation. The domestic economy thus produces the same amount of commodities, but has become poorer as a result of the worsened terms of trade associated with the improved competitiveness (larger real exchange rate).[2]

Notice the difference between the dynamic adjustment mechanism in the closed and in the open economy. In the closed economy, a recession will lead to falling inflation, which induces the central bank to cut the real interest rate so that aggregate demand recovers. Thus, economic policy is crucial for the dynamic adjustment of the closed economy. In the open economy with fixed exchange rates, a recession drives domestic inflation below foreign inflation, causing a gradual recovery of demand by lowering the relative price of domestic goods. This endogenous adjustment of the real exchange rate will pull the open economy towards long-run equilibrium even if economic policy remains passive.

Stability and speed of adjustment under fixed exchange rates

What determines the speed of adjustment to long-run equilibrium in the open economy with fixed exchange rates? To investigate this, we will now solve the model analytically. We start by defining the output gap and the inflation gap in the usual manner:

$$\hat{y}_t \equiv y_t - \bar{y}, \qquad \hat{\pi}_t \equiv \pi_t - \pi^f. \tag{10}$$

We may then rewrite the model consisting of (7), (8) and (9) as:

$$\hat{y}_t = \beta_1(e^r_{t-1} - \hat{\pi}_t) + z_t, \tag{11}$$

$$\hat{\pi}_t = \gamma \hat{y}_t + s_t, \tag{12}$$

$$e^r_t = e^r_{t-1} - \hat{\pi}_t. \tag{13}$$

Since we are now interested in adjustment of the economy back towards long-run equilibrium *after* some shocks have occurred, we set the shocks for the adjustment period equal to zero, $z_t = s_t = 0$. With this assumed, we insert (12) into (11) and solve for e^r_{t-1} to get:

$$e^r_{t-1} = \left(\frac{1+\gamma\beta_1}{\beta_1}\right)\hat{y}_t \tag{14}$$

Substituting (14) and the analogous expression for e^r_t into (13) along with the expression for $\hat{\pi}_t$ given in (12) with $s_t = 0$, we get the following first-order linear difference equation in the output gap:

[2] At this stage and as an exercise , try to analyze graphically the situation where the demand shock is temporary and only occurs in period 1.

$$\overbrace{\frac{1+\gamma\beta_1}{\beta_1}\hat{y}_{t+1}}^{e_t^r} = \overbrace{\frac{1+\gamma\beta_1}{\beta_1}\hat{y}_t}^{e_{t-1}^r} - \overbrace{\gamma\hat{y}_t}^{\hat{\pi}_t} \Leftrightarrow$$
$$\hat{y}_{t+1} = \beta\hat{y}_t, \quad \beta \equiv \frac{1}{1+\gamma\beta_1}. \tag{15}$$

Equation (15) has the same structure as the difference equation for the output gap in the closed economy given in Equation (11) in Chapter 19, repeated here for convenience: $\hat{y}_t = \beta\hat{y}_{t-1}, \beta \equiv 1/(1+a\gamma)$. Indeed, the only difference between the two equations is that the parameter a in the closed economy model is replaced by the parameter β_1 in the open economy model. This reflects the difference in the macroeconomic adjustment mechanisms: in the closed economy, the parameter a incorporates the central bank's interest rate response to changes in the inflation gap and in the output gap as well as the interest elasticity of the aggregate demand for goods (see Equation (9) of Chapter 19). In the open economy, the parameter β_1 includes the price elasticities of export and import demand (see Equation (23) of Chapter 23), which determine the effect of a change in domestic inflation on the demand for domestic goods.

The solution to (15) is:

$$\hat{y}_t = \hat{y}_0\beta^t, \qquad t = 0, 1, 2, ... \tag{16}$$

We see from (15) that the parameter $\beta \equiv 1/(1 + \gamma\beta_1)$ is positive but less than 1. According to (16) this ensures that the economy will indeed converge towards a long-run equilibrium. In other words, the open economy with fixed exchange rates is *stable*. The speed of convergence to long-run equilibrium will be higher, the smaller the value of β. Hence the adjustment to equilibrium will be faster the greater the value of β_1, that is, the larger the price elasticities in foreign trade. This is intuitive: if exports and imports are very sensitive to relative prices, a negative demand shock, which drives domestic inflation below foreign inflation, will lead to a large increase in net exports that will quickly pull the domestic economy out of recession. The magnitude of the price elasticities in foreign trade will depend on the structure of product markets. If competition in international markets is tough, the price elasticities will tend to be large, and the economy's adjustment to equilibrium will then be relatively fast.

Let us consider a numerical example to get a feel for the likely order of magnitude of the open economy's speed of adjustment. In Chapter 23, Equation (23), we showed that

$$\beta_1 = \frac{(M_0/\bar{Y})(\eta_X + \eta_M - 1) - (\bar{D}/\bar{Y})\eta_D}{1 - \tilde{D}_Y}, \qquad \eta_D \equiv -\frac{\partial D}{\partial E^r}\frac{E^r}{D},$$

where η_D is the numerical elasticity of domestic private demand for goods with respect to the real exchange rate, $M_0/\bar{Y}$ is the ratio of imports to GDP, $\tilde{D}_Y$ is the marginal propensity to spend domestic income, and $D/\bar{Y}$ is the ratio of total private demand to GDP (which in turn equals one minus the ratio of government consumption to GDP when foreign trade is balanced). Empirical studies indicate that if the length of the period is one year or longer, the sum of the relative price elasticities in export and import demand are greater than 1. For concreteness, suppose that

$$\eta_X + \eta_M = 3, \quad \eta_D = 0.3, \quad M_0/\bar{Y} = 0.3, \quad \bar{D}/\bar{Y} = 0.8, \quad \tilde{D}_Y = 0.5.$$

These parameter values are not implausible for a relatively open economy. The justification for the choice of the value of η_D is that if about 30 per cent of demand is directed towards imports, then a 1 per cent rise in the relative price of imported goods should erode the real purchasing power of domestic consumers by about 0.3 percentage points and lead to a similar percentage fall in demand. These values lead to $\beta_1 \approx 0.72$. In Chapter 19 we saw that a realistic value of the Phillips curve parameter on annual basis is $\gamma = 0.3$ when the length of the time period is one year. With $\beta_1 = 0.72$ and $\gamma = 0.3$ we find that:

$$\beta = \frac{1}{1+\gamma\beta_1} \approx 0.82.$$

From the analysis in Chapter 19 we know that (16) implies that the number of time periods t^h which must elapse before half of the economy's adjustment to long-run equilibrium is completed is given by:

$$t^h = -\frac{\ln 2}{\ln \beta} = -\frac{0.693}{\ln \beta}.$$

For $\beta = 0.82$ this implies that $t^h \approx 3.5$. In other words, given the parameter values assumed above, it will take about three and a half years before the economy has moved halfway back towards the long-run equilibrium after it has been disturbed by a shock. This is faster than assessed for the closed economy where we found a half-life around 6 years to be reasonable. The difference is due to the higher sensitivity of aggregate demand to inflation in the open vs. the closed economy, that is, the β_1 around 0.72 in contrast to the a around 0.4, that we have found reasonable.

MACROECONOMIC ADJUSTMENT UNDER FIXED EXCHANGE RATES

If exchange rates are credibly fixed, the expected domestic inflation rate is likely to equal the average foreign inflation rate. In that case the aggregate supply curve will not shift during the adjustment to long-run equilibrium. Instead, adjustment will take place via shifts in the aggregate demand curve. When the domestic economy is in recession (boom), the domestic inflation rate will be lower (higher) than the foreign inflation rate. Hence domestic competitiveness will gradually improve (deteriorate), and the AD curve will shift upwards (downwards) from one period to the next. In this way the economy will move up (down) along the SRAS curve until a long-run equilibrium is reached where output is at its natural rate and the domestic inflation rate equals the foreign inflation rate. For realistic parameter values, our AS–AD model suggests that it could take 3–4 years for the economy to complete half of the adjustment back to long-run equilibrium after the occurrence of a shock.

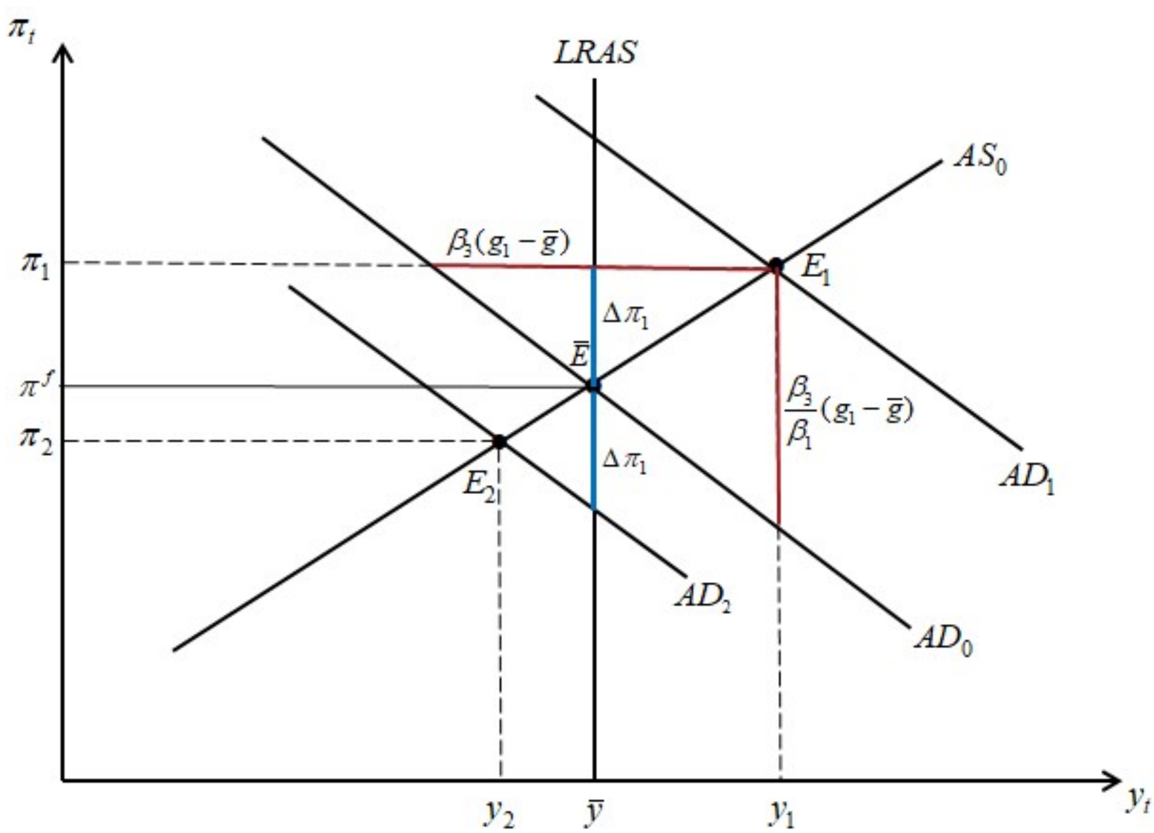
24.3 Fiscal policy

We will now use the AS–AD model developed above to study the effects of macroeconomic policy under fixed exchange rates. As we have already seen, when fixed exchange rates are coupled with high capital mobility, there is no room for an independent monetary policy. This leaves fiscal policy and discretionary changes in the exchange rate parity as the two major remaining instruments of macroeconomic stabilization policy. In Section 24.4 we shall consider the effects of exchange rate policy. The present section focuses on fiscal policy. We start by analysing the repercussions of a fiscal policy shock before moving on to consider the effects of a systematic countercyclical fiscal policy.

Unsystematic fiscal policy: a temporary fiscal shock

Figure 24.4 illustrates how an open economy with fixed exchange rates will react to a temporary fiscal expansion. In period 0 the economy is in long-run equilibrium at $\bar{E}$, where output is at its trend level and domestic inflation equals foreign inflation. Suppose then that the government temporarily increases its spending in period 1, but cuts spending back to its original level from period 2 onwards, so there is a fiscal shock $g_1 - \bar{g} > 0$ in period 1, but $g_t - \bar{g} = 0$ for all other *t*. In period 1 when the government raises its spending so that *z* goes up, the AD curve shifts to the right by $\beta_3(g_1 - \bar{g})$ and up by $(\beta_3 / \beta_1)(g_1 - \bar{g})$, from AD_0 to AD_1, and the economy moves to a new short-run equilibrium E_1 where domestic output and inflation are higher. In period 2 when the government cuts back its spending to its original level, one might think that the AD curve would simply fall back to its original position *AD0* so that long-run equilibrium would be restored from period 2 onwards. In fact, however, the AD curve for period 2 will fall to the position AD_2 *below* the original aggregate demand curve. The reason is that the higher rate of inflation in period 1 reduces the real exchange rate in that period (since $e_1^r = e_0^r + \pi^f - \pi_1 = \pi^f - \pi_1$) so the economy enters period 2 with a weaker international competitiveness. In formal terms, the lagged real exchange rate, e_{t-1}^r, in Equation (7) falls by the amount $\Delta\pi_1 \equiv \pi_1 - \pi^f$ between period 1 and period 2, thereby pulling down the aggregate demand curve by a similar vertical distance. As a result, the economy falls into *recession* in period 2 when the temporary fiscal stimulus has vanished and has left the economy with an inflated cost and price level.

Figure 24.4 Effects of a temporary fiscal expansion



From period 2 onwards, we see from Fig. 24.4 that domestic inflation is below the foreign inflation rate. This generates a gradual improvement of domestic competitiveness, which pulls domestic output and inflation back towards their long-run equilibrium levels through successive upward shifts in the AD curve.

When the economy has moved all the way back to point $\overline{E}$, all real variables (including the real exchange rate) are back at their original levels. The interesting point is that the temporary fiscal expansion takes the economy through a boom–bust cycle where a short-lived economic expansion is followed by a protracted period with below-normal activity. Of course, a similar adjustment pattern would occur if our demand shock variable *z* temporarily increased due to some event other than a change in fiscal policy.

In practice, it does not really make sense to stimulate the economy from a long-run equilibrium where activity is at the structural level, so the exercise above is just illustrative of temporary demand shocks in general. You are invited to perform a similar graphical analysis, where first, a longer-lasting, negative demand shock has pulled activity down and from that situation the government begins to counteract by stimulating fiscal policy.

Note that the aggregate demand effect of a fiscal stimulus, $\beta_3(g_1-\overline{g})$, may well be relative large compared to the closed economy because in the open economy with fixed exchange rate there is no counteracting monetary policy. In the model for the closed economy considered in Chapter 20, the corresponding stimulus would be $\alpha_1(g_1-\overline{g})/(1+\alpha_2 b)$, where α_1 corresponds to β_3 here and the term $\alpha_2 b$ in the denominator comes from monetary policy, that is, the reaction of the interest rate to increased activity. In the open economy with a fixed exchange rate fiscal policy may well be relatively effective.

Systematic fiscal policy

The analysis above focused on a fiscal policy *shock* rooted in exogenous political events. It is also of interest to study the effects of a *systematic* fiscal policy, which follows a *fixed policy rule*. Therefore, suppose that fiscal policy is *countercyclical* so that public spending is raised above trend when output is below the natural rate, and vice versa. In that case we have:

$$g_t-\overline{g}=b'(\overline{y}_t-y), \qquad b'>0, \tag{17}$$

where the policy parameter b' indicates how strongly fiscal policy reacts to changes in the output gap. Inserting (17) into the definition of *z* given in (7), we obtain a new equation for the AD curve:

$$\begin{aligned} &y_t-\overline{y}=\frac{\beta_1}{1+\beta_3 b'}(e^r_{t-1}+\pi^f-\pi_t)+\frac{\hat{z}_t}{1+\beta_3 b'} \quad \Leftrightarrow \\ &\pi_t=e^r_{t-1}+\pi^f-\frac{1+\beta_3 b'}{\beta_1}(y_t-\overline{y})+\frac{\hat{z}_t}{\beta_1}, \\ &\hat{z}_t\equiv-\beta_2(r^f_t-\overline{r}^f)+\beta_4(y^f_t-\overline{y}^f)+\beta_5(\ln\varepsilon_t-\ln\overline{\varepsilon}). \end{aligned} \tag{18}$$

We see that a countercyclical fiscal policy involving a positive value of b' makes the AD curve *steeper* , that is, aggregate demand becomes less sensitive to inflation, compared to a passive policy where $b'=0$, and by choosing b' sufficiently large, the AD curve can be made arbitrarily steep. The implications are illustrated in Figures 24.5 and 24.6 for demand shocks and supply shocks, respectively, focusing here on the effects in the shock period itself, and where we assume that the economy starts out in long-run equilibrium in period 0.

Figure 24.5 Short-run effects of a negative demand shock under a passive versus a countercyclical fiscal policy

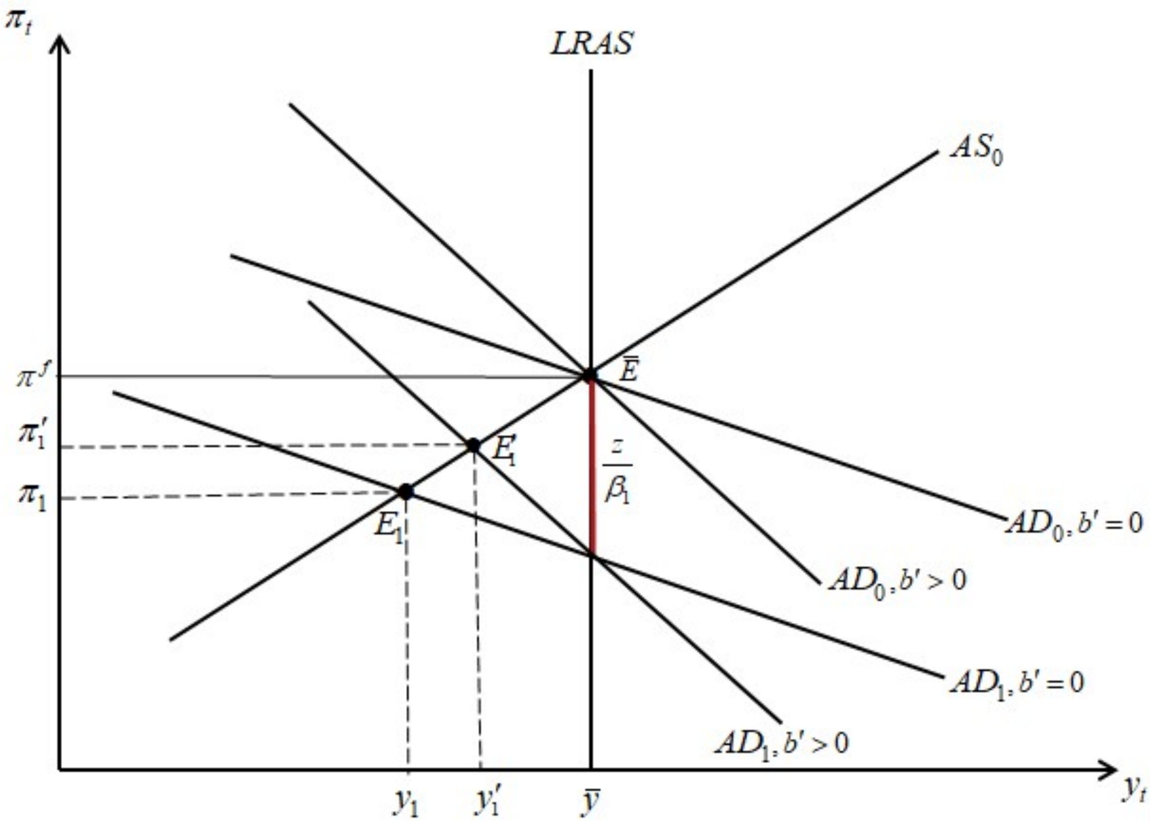


Figure 24.5 shows the short-run effects of a negative demand shock which lowers our variable $\hat{z}$ from zero to some negative amount in period 1, thereby shifting the AD curve (18) downwards by the vertical distance $\hat{z}/\beta_1$. Note from (18), that the (horizontal) shift in the y-dimension will be smaller the larger b' is, but the (vertical) shift in the π-dimension is the same irrespective of the size of b', and try to explain why this is so. In the case of a passive fiscal policy ($b'=0$), the economy will then end up in the short-run

equilibrium E_1 in period 1, but with an active countercyclical policy ($b' > 0$), the new short-run equilibrium will be given by point E_1'. In this case, we see that the steeper slope of the AD curve ensures a smaller drop in output as well as inflation. Of course, this is because the countercyclical policy implies an offsetting rise in public sector spending when the exogenous fall in aggregate demand reduces the level of output. In conclusion, the countercyclical fiscal policy rule dampens the immediate fluctuations in both activity and inflation in response to demand shocks.

Figure 24.6 Short-run effects of a negative supply shock under a passive versus a countercyclical fiscal policy

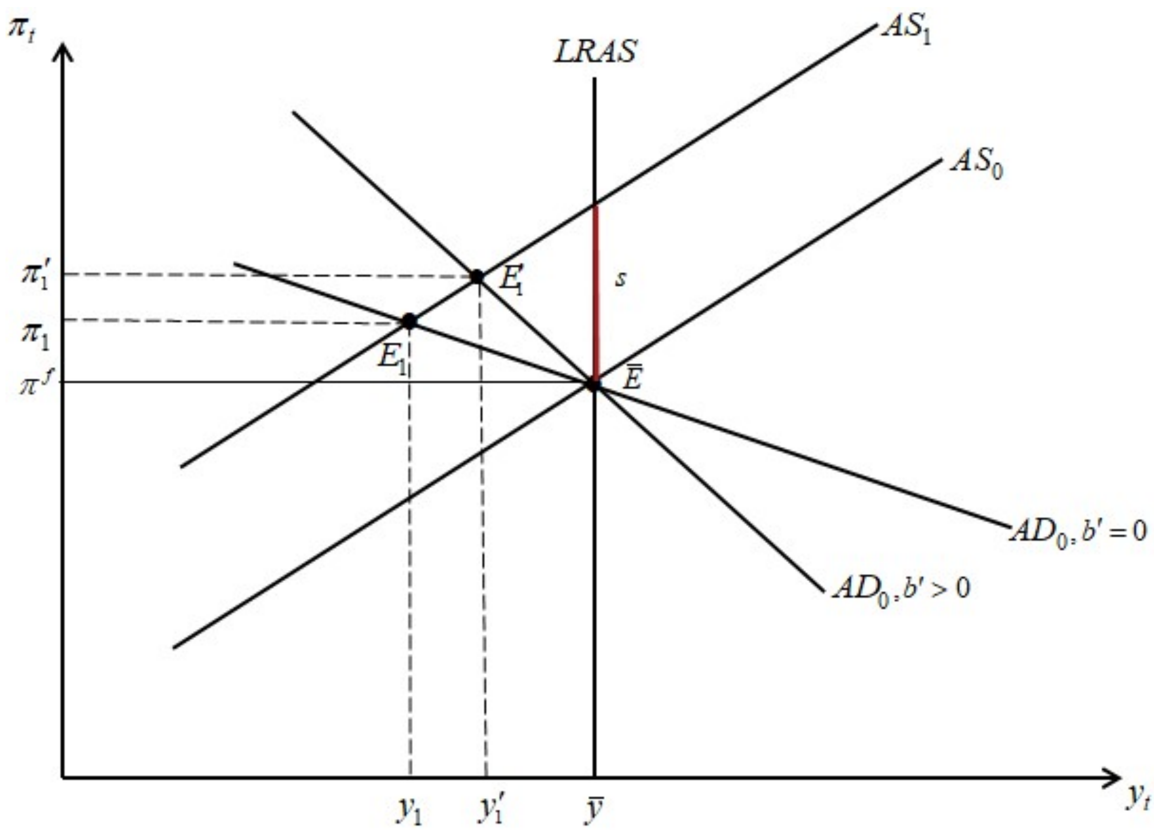


Figure 24.6 illustrates the case where the AS curve shifts upwards from AS_0 to AS_1 due to an unfavourable supply shock, $s > 0$, occurring in period 1. Again, we see that the steeper AD curve implied by a countercyclical fiscal policy will ensure a smaller drop in output, but at the same time it will imply a larger increase in domestic inflation. Thus, policy makers face a trade-off between reducing the volatility of output and reducing the variance of inflation when the economy is exposed to supply shocks. This dilemma is well known also from our analysis of the closed economy.

Overall, the active fiscal policy rule (17) implies more stabilization of both inflation and activity in the shock period at demand shocks than a passive fiscal policy, but in connection with supply shocks the greater stability of output is attained at the cost of less stable inflation. However, the latter is an artifact of the simple rule considered. If the government follows the more sophisticated rule, $g_t - \overline{g} = -h'(\pi_t - \pi^f) - b'(y_t - \overline{y}),\ h' > 0,\ b' > 0$, this active fiscal policy rule unambiguously implies more stability at demand shocks, and at supply shocks, any combination of output gap and inflation gap on the relevant part of the AS curve can be obtained for appropriate choices of the reaction parameters h' and b', including the mix arising from passive fiscal policy. You should convince yourself of this. (Hint: Show by deriving the appropriate AD curve that in this case any slope between zero and minus infinity can be obtained for the AD curve while at the same time the shift in it from demand shocks can be made arbitrarily small). Therefore, in this sense the active policy rule is better than passivity.

The countercyclical fiscal policy involves another dilemma: although it reduces the short-run output fluctuations caused by exogenous shocks, it also reduces the economy's speed of adjustment back to long-run equilibrium. If we compare our previous AD curve (7) to (18), we see that in the latter the term $\beta_1/(1+\beta_3 b')$ has taken the place of β_1 in the first. It is therefore obvious that the procedure used to derive Equation (15) will now, under the countercyclical fiscal policy, give the following dynamics of the output gap:

$$\hat{y}_{t+1} = \beta'\hat{y}_t, \quad \beta' \equiv \frac{1}{1+\dfrac{\beta_1}{1+\beta_3 b'}\gamma} = \frac{1+\beta_3 b'}{1+\beta_3 b' + \beta_1\gamma}. \tag{19}$$

As you can easily verify, the coefficient β' on the lagged output gap is *larger* the greater the value of b', that is, the stronger fiscal policy reacts to the output gap. A more vigorous countercyclical policy will therefore slow down the convergence towards long-run equilibrium. The reason is that once output starts to move back towards the natural rate after having been pushed into recession by a negative shock, fiscal policy is automatically tightened under a countercyclical policy. Obviously, this will reduce the speed at which aggregate demand and output are able to increase.

Yet a countercyclical fiscal policy will also reduce the initial displacement from long-run equilibrium when the economy is hit by a shock. With a quadratic social loss function of the form assumed in our analysis of stabilization policy in Part 4, governments are presumably willing to accept some reduction in the economy's speed of adjustment if they can thereby avoid large deviations of output from trend. This is because large deviations from the natural rate cause a disproportionately greater social loss than small deviations. But our analysis shows that governments cannot have it both ways: they cannot reduce the short-run impact of shocks through countercyclical policy and at the same time increase the economy's speed of adjustment.

In closing this discussion of fiscal policy, we should remind you of the warnings given in Chapter 20. The simple policy rule (17) assumes that fiscal policy makers can instantaneously react to the current output gap. This may not be very realistic, since the so-called inside lag in the fiscal policy process may sometimes be quite long due to the parliamentary procedures needed for approval of fiscal policy changes. Because of time lags, there is a danger that a too ‘fine tuning’ fiscal policy intended to be countercyclical end up being destabilizing because of inappropriate timing.

FISCAL POLICY UNDER FIXED EXCHANGE RATES

A positive but temporary fiscal shock will generate a boom–bust pattern of adjustment: the initial impact on economic activity will be positive, but the resulting rise in domestic inflation will reduce domestic competitiveness so that activity will fall below normal when the fiscal stimulus is withdrawn. As the recession drives domestic inflation below foreign inflation, competitiveness is gradually restored and domestic activity gradually returns to the natural rate. A systematic countercyclical fiscal policy, whether it reacts only to the output gap or to the output and the inflation gap, reduces the immediate fluctuations in the output and inflation gaps resulting from demand shocks. When the economy is hit by supply shocks, a countercyclical fiscal policy that only reacts to the output gap reduces the volatility of output, but at the same time increases the volatility of inflation. If the rule is such that the fiscal stimulus reacts to both the output and the inflation gap, any mix of output and inflation gap can be obtained, including the one arising with passive fiscal policy. A countercyclical fiscal policy rule reduces the speed of adjustment to long-run equilibrium.

24.4 Exchange rate policy

So far we have mainly analysed a so-called hard currency peg where the government does not use changes in the nominal exchange rate as an instrument of economic policy. We will now consider a ‘softer’ type of fixed exchange rate regime where the authorities may occasionally adjust the exchange rate. This is sometimes referred to as fixed but adjustable exchange rates. In practice, most countries with such a policy regime have tended to *devalue* their currencies from time to time in order to compensate for the impact of high domestic inflation on international competitiveness or to stimulate domestic economic activity in response to negative shocks.

When analysing the effects of exchange rate policy, it is crucial to distinguish between *anticipated* and *unanticipated* devaluations. When a devaluation is foreseen by the private sector, it will already have an anticipation effect on economic activity before it occurs, whereas an unanticipated devaluation will have no impact until it is implemented. The anticipation effect of an anticipated devaluation is predominantly negative, as will be explained below, so an unanticipated devaluation is ‘better’ than an anticipated one, but the act of a devaluation itself can create expectations of further future devaluations. In this sense, devaluation is a two-edged sword. Because of the negative anticipation effects, using the weapon of devaluation may well make the weapon less effective in the future. Therefore, in case of a devaluation the policy makers will often try to convince the public that this is only due to very particular circumstances and for this reason, no further devaluations should be expected in the future, a story that the public may find it hard to believe.

We will start by considering an unanticipated devaluation since this provides a useful benchmark when we move on to analyse the more complicated case of a devaluation which is (partly) anticipated.

An unanticipated devaluation "and we never do it again"

Suppose the economy is initially, in period 0, in a long-run equilibrium like the one illustrated by point $\overline{E}$ in Fig. 24.7. In period 1, a serious and longer-lasting negative demand shock occurs, $z<0$, for instance because of a loss of consumer and business confidence or a negative influence from foreign activity. According to (7), this shifts the AD curve from period 0 to 1 to the left by $|z|$ and downwards by $|z/\beta_1|$, as illustrated by the curve labelled AD_1 in the figure. This brings the economy to the short-run equilibrium E_1 in period 1, where there is a recession with $y_1<\overline{y}$ and $\pi_1<\pi^f$. Hence, there is a need for stabilization.

Suppose that the government finds that the unemployment rate corresponding to this equilibrium is too high and needs to be brought down quickly, possibly before the next election. As a 'quick fix', policy makers may then engineer a short-run improvement in international competitiveness through a devaluation of the domestic currency. We will assume that the devaluation is undertaken in period 1, so $\Delta e_1 \equiv e_1 - e_0 = \Delta\overline{e}_1 \equiv \overline{e}_1 - \overline{e}_0 > 0$, where (remember), $\overline{e}_t$ denotes the nominal exchange rate parity. We also assume that there are no other devaluations, so $\Delta e_t = \Delta\overline{e}_t = 0$ for $t \neq 1$. In this subsection, we assume heroically that 1) the devaluation is unanticipated, 2) the government and central bank solemnly declare that this is a unique, one-time event, which will never be repeated; from now on the government is fully committed to a solidly fixed exchange rate, and 3) the public actually believes the policy makers.

The devaluation will, of course, other things equal create a higher real exchange rate and thereby improved competiveness, $e_1^r = e_0^r + \Delta\overline{e}_1 + \pi^f - \pi_1$; this is the whole point of it. For all other periods $t \geq 2$, we will have the usual real exchange rate dynamics of (9) above, $e_t^r = e_{t-1}^r + \pi^f - \pi_t$.

Now, how will the devaluation in period 1 affect the AD and the short-run AS curve? Let us first consider aggregate demand and go all the way back to the temporary AD curve (4) above which is valid irrespective of the exchange rate regime and repeated here for convenience:

$$y_t - \overline{y} = \beta_1(e_{t-1}^r + \Delta e_t + \pi^f - \pi_t) - \beta_2(i^f + \Delta e_{t+1}^e - \pi_{t+1}^e - \overline{r}^f) + \tilde{z}_t. \tag{4}$$

Above we inserted $\Delta e_t = 0$ (due to the fixity of the exchange rate), $\Delta e_{t+1}^e = 0$ (due to the expected fixity of the exchange rate) and $\pi_{t+1}^e = \pi^f$ (due to expected long-run relative PPP), which gave us the AD curve (7) under a credibly fixed exchange rate.

Now, for period 1, where the devaluation occurs we should not set $\Delta e_1 = 0$, but $\Delta e_1 = \Delta \overline{e}_1 > 0$ and $\Delta e_t = 0$ for all other *t*. With our assumptions 1)-3) above, the devaluation will never cause the expected rate of exchange rate depreciation $\Delta e_{t+1}^e = \Delta \overline{e}_{t+1}^e$ to deviate from 0, because the devaluation is unanticipated, so we can insert $\Delta e_{t+1}^e = 0$ in the temporary AD curve. Finally, as argued above, the expected rate of inflation π_{t+1}^e will (from expected long-run PPP) go up above π^f, but only by a smaller amount in each of a larger number of periods after period 1, so for $t \geq 1$ we will have $\pi_{t+1}^e = \pi^f + \partial \pi_{t+1}^e$, where each $\partial \pi_{t+1}^e$ is small. For our analysis of an unanticipated devaluation we will as a first approximation neglect this effect on expected inflation, that is, we will assume $\partial \pi_{t+1}^e \approx 0$ and thus $\pi_t^e = \pi^f$ for all *t*. Inserting these features into the temporary demand curve gives the following AD curves for period 1 and for the succeeding periods, respectively:

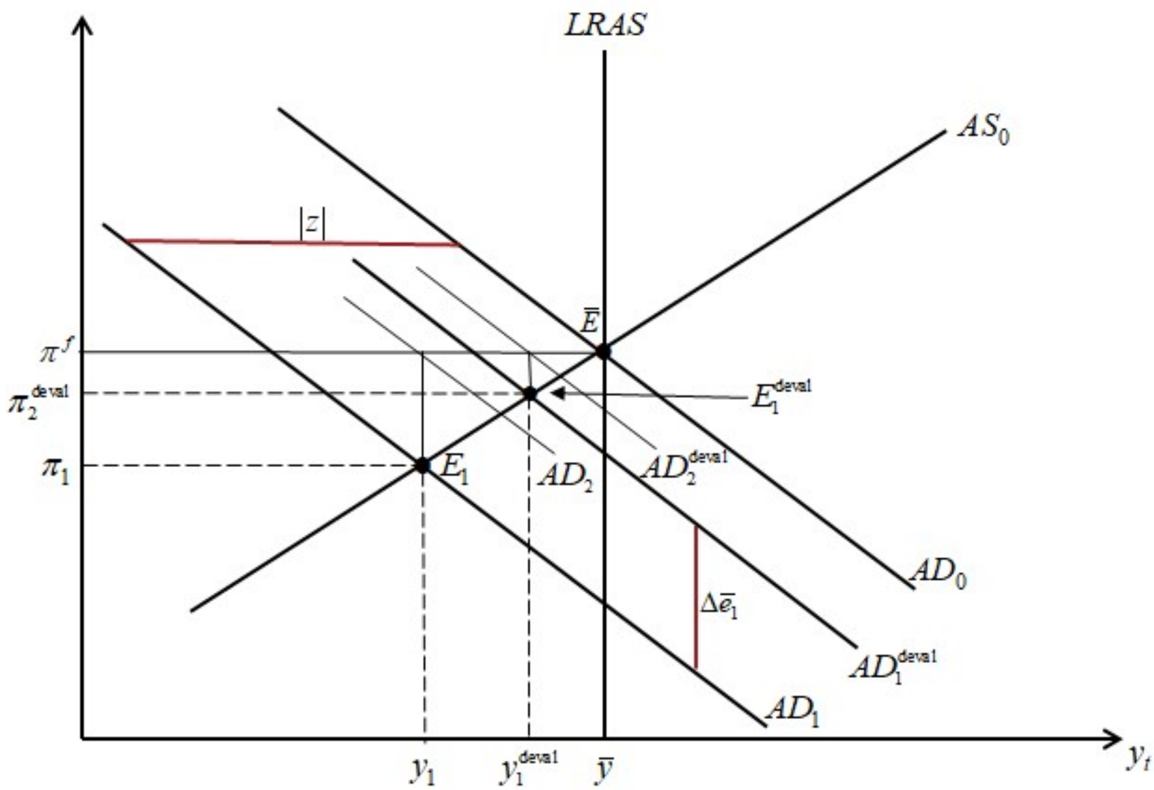
$$y_1 - \overline{y} = \beta_1(e_0^r + \Delta \overline{e}_1 + \pi^f - \pi_1) + z_1 \quad \Leftrightarrow \quad \pi_1 = e_0^r + \Delta \overline{e}_1 + \pi^f - \frac{1}{\beta_1}(y_1 - \overline{y}) + z_1, \qquad (20)$$

$$y_t - \overline{y} = \beta_1(e_{t-1}^r + \pi^f - \pi_t) + z_t \quad \Leftrightarrow \quad \pi_t = e_{t-1}^r + \pi^f - \frac{1}{\beta_1}(y_t - \overline{y}) + z_t. \qquad (21)$$

Under our simplifying assumption that $\pi_t^e = \pi^f$ for all *t*, the AS curve never changes from the expression of (8). Hence, all in all, the devaluation itself makes the AD curve shift upwards by $\Delta \overline{e}_1$ in period 1 (on top of the shift downwards caused by the permanent negative demand shock occurring first time in period 1), while in all periods coming after period 1, the AD curve will shift according to the real exchange rate dynamics of (9) implying upward shifts by $\pi^f - \pi_t$ from period *t* to *t*+1. In Figure 24.7, the AD curve in period 1 including the devaluation has been labelled AD_1^{deval}, which lies $\Delta \overline{e}_1$ above AD_1. We assume that the shift upwards caused by the devaluation is smaller than that caused by the demand shock; otherwise, the policy reaction would be overdosed. The short-run equilibrium with the devaluation thus becomes E_1^{deval}, in which both output and inflation are larger than in E_1, so the devaluation has improved the situation in period 1. The AD curves for period 2 relevant without and with the devaluation, respectively, have been sketched in the figure and labelled AD_2 and AD_2^{deval}. These show that also in period 2 the situation is better with than without the unanticipated devaluation and the same will be true for the succeeding periods. So, the devaluation mitigates the negative output and inflation gaps during the full adjustment and thus unambiguously improves the situation. In the long run, the economy converges towards one and the same long-run equilibrium devaluation or not, so in the long run a devaluation is *neutral*, having no effect on real variables such as output and the real exchange rate, but during the convergence process the devaluation alleviates the recession. Even if we had taken into account that the devaluation considered would have created a bit higher expected domestic inflation in each of a number of periods, this would only have had a small effect on the AD and AS curves for some periods, so the conclusion would remain the same.

This makes devaluation seem a wonderful weapon, but the effects are only as good as described because the devaluation was assumed to be unforeseen and not to give rise to anticipation of further devaluations.

Figure 24.7 Effects of an unanticipated devaluation



An anticipated devaluation

Our analysis of an unanticipated devaluation serves as a useful theoretical benchmark, but in the real world devaluations rarely take the private sector by complete surprise. This is especially true in countries that have devalued on several previous occasions. In such countries the government's declared commitment to a 'fixed' exchange rate will typically lack credibility, and expectations of a future devaluation may easily arise whenever the state of the macroeconomy provides (or is believed to provide) a temptation for policy makers to devalue the currency. Thus, if the authorities devalue the domestic currency from time to time, households and firms will start to incorporate the risk of future devaluation into their forecasts of inflation and asset returns. A realistic analysis of exchange rate policy must include such effects on expectations.

To illustrate this point, we will now perform an analysis of an anticipated devaluation that is highly stylized, but gives important insight into the real world effects of devaluations, exactly because it takes the anticipation effects into account. We consider an economy, which is in long-run equilibrium in period 0, and for simplicity we will not consider any demand or supply shocks in other periods either. The reason why policy makers might nevertheless want to undertake a devaluation could be that they consider the natural rates of output and employment in the long-run equilibrium to be unacceptably low and hope to obtain a temporary expansion of activity via a devaluation.[3] However, our qualitative results would still be valid if we assumed instead that the economy starts out in a short-run equilibrium where output is below trend due to a shock.

We consider the economy over the periods 0, 1, 2, 3, … and assume that it undertakes a devaluation in period 2. Because the country considered has devalued previously on several occasions, we assume that fears of a future devaluation arise in the private sector in period 1 already. The economy therefore goes through three stages of reaction to the devaluation. The first stage may be termed 'the anticipation stage' where the emergence of devaluation expectations starts to affect the real economy even before the devaluation occurs. The second stage may be called 'the implementation stage' and includes the short-run effects of the devaluation occurring in the period when it is implemented, while the third and final 'adjustment stage' includes all the subsequent macroeconomic adjustments which take the economy back to the long-run equilibrium. We will consider each of these stages in turn.

To be very specific, in period 2 an actual devaluation takes place, $\Delta\overline{e}_2 \equiv \overline{e}_2 - \overline{e}_1 > 0$. Since the parity rate is always enforced, $e_t = \overline{e}_t$ for all t, the change in the market exchange rate will be the same, $\Delta e_2 \equiv e_2 - e_1 = \Delta\overline{e}_2 \equiv \overline{e}_2 - \overline{e}_1 > 0$. Because the authorities have a record of devaluations, the period 2 devaluation is foreseen from period 1, but not necessarily perfectly so. The expected devaluation in period 2 as seen from period 1 is $\Delta\overline{e}_2^e \equiv \overline{e}_2^e - \overline{e}_1 > 0$, and this is also the expected change in the market exchange rate since the public knows that the parity rate is always enforced, $\Delta\overline{e}_2^e \equiv \overline{e}_2^e - \overline{e}_1 = \Delta e_2^e \equiv e_2^e - e_1$. As mentioned, we do not assume that necessarily $\Delta\overline{e}_2^e = \Delta\overline{e}_2$, but we will assume that the expected devaluation is not too far from the actual one. All in all:

$$0 < \Delta\overline{e}_2^e = \Delta e_2^e \approx \Delta\overline{e}_2 = \Delta e_2. \qquad (22)$$

[3] This is parallel to the situation, which can create a problem of time inconsistency and inflation bias in monetary policy in a closed economy, as we explained in Chapter 22.

There are no other actual or expected changes in the exchange rate parity than the ones described, that is, we focus on one single anticipated devaluation.

Stage 1: The anticipation effects of a devaluation

The anticipation stage coincides with period 1 where expectations of a future devaluation have arisen but where the devaluation has not yet been implemented.

To explain how the AD curve in period 1 shifts in reaction to the devaluation we start again from the temporary AD curve repeated for convenience but without the shock variable $\tilde{z}_t$ because we now abstract from shocks:

$$y_t - \bar{y} = \beta_1(e^r_{t-1} + \Delta e_t + \pi^f - \pi_t) - \beta_2(i^f + \Delta e^e_{t+1} - \pi^e_{t+1} - \bar{r}^f). \tag{4'}$$

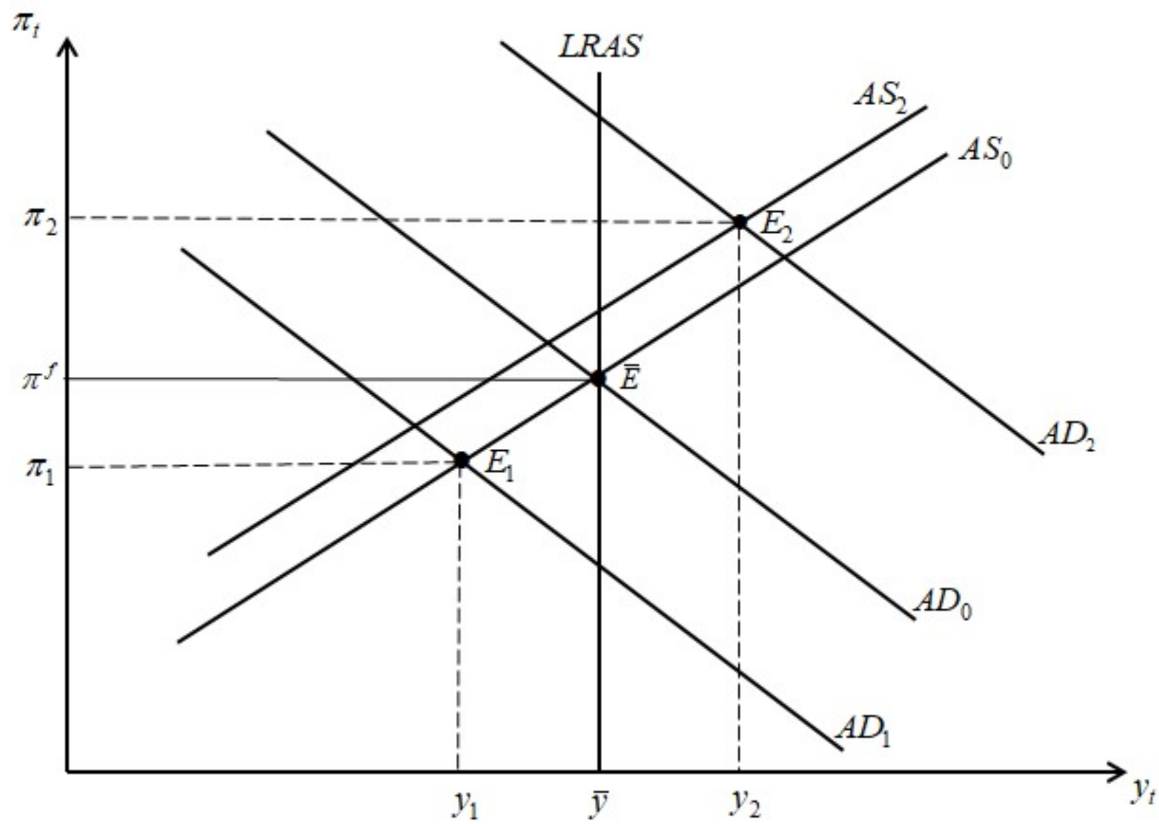
The AD curve in period 1 could be affected either through a real exchange rate effect associated with the first term on the right hand side of (22) or from a real interest rate effect associated with the second term. In period 1 the real exchange rate is $e^r_1 = e^r_0 + \Delta e_1 + \pi^f - \pi_1$. Since $\Delta e_1 = \Delta \bar{e}_1 = 0$, the devaluation causes no shift in the AD curve in period 1 from a real exchange rate effect. The domestic real interest rate in period 1 is $r_1 = i^f + \Delta e^e_2 - \pi^e_2$. Here, $\Delta e^e_2 = \Delta \bar{e}^e_2 > 0$, which will tend to pull the real interest rate upwards and hence to shift the AD curve in period 1 downwards. However, as argued above, it is also reasonable to assume that the expectation of a devaluation in period 2, $\Delta \bar{e}^e_2 > 0$, will push the future expected inflation rate up above the foreign inflation rate by a certain amount in each of a number of periods starting in period 1. This will tend to shift the AD curve in period 1 upwards. But since higher inflation is only expected to restore PPP gradually over many periods, the increase in future expected inflation in period 1 will be relatively small. Let us say that expected future inflation increases above foreign inflation by $\partial\pi^e > 0$ in period 1, so that $\pi^e_2 = \pi^f + \partial\pi^e$, where $\partial\pi^e << \Delta \bar{e}^e_2$. Inserting these features for period 1 in the temporary AD curve gives the AD curve for period 1:

$$\begin{aligned} &y_1 - \bar{y} = \beta_1(e^r_0 + \pi^f - \pi_1) - \beta_2(i^f + \Delta \bar{e}^e_2 - \pi^f - \partial\pi^e - \bar{r}^f) \quad \Leftrightarrow \\ &\pi_1 = e^r_0 + \pi^f - \frac{\beta_2}{\beta_1}(i^f + \Delta \bar{e}^e_2 - \pi^f - \partial\pi^e - \bar{r}^f) - \frac{1}{\beta_1}(y_1 - \bar{y}) . \end{aligned} \tag{23}$$

We see that the real interest rate effect will shift the AD curve by $-\beta_2(\Delta e^e_{t+1} - \partial\pi^e)$ horizontally and by $-(\beta_2 / \beta_1)(\Delta e^e_{t+1} - \partial\pi^e)$ vertically. Since $\partial\pi^e << \Delta \bar{e}^e_2$, the AD curve shifts considerably in- and downwards. In other words, when the anticipated devaluation is not expected to be fully and immediately passed through to the domestic inflation rate, which is the most reasonable assumption, *the domestic real interest rate will rise above the foreign real interest rate before the devaluation*, provided the devaluation is at least partly anticipated. This shifts the AD curve in period 1, the anticipation stage, in- and downwards.

The AS curve does not shift in period 1 due to the devaluation because the expected inflation rate in the AS curve for period 1 is $\pi_1^e = \pi^f$, an expectation that was formed in period zero when the devaluation in period 2 was not yet foreseen. Figure 24.8 illustrates the initial long-run equilibrium in period 0 labelled $\overline{E}$ as well as the shift in the AD curve in period 1 from AD_0 to AD_1 caused by the anticipation effect of the period 2 devaluation. This shift gives rise to the short-run equilibrium E_1 in period 1, where $y_1 < \overline{y}$ and $\pi_1 < \pi^f$. The conclusion is that the pure anticipation of a later devaluation creates a negative effect on activity and inflation here and now without any devaluation having actually taken place. The reason is that the expectation of future devaluations implies a higher real interest rate in the domestic economy, as the expected devaluation and depreciation of the domestic currency increases the domestic nominal interest rate (through nominal interest at parity) by more than it increases the dometic expected inflation rate. This is why policy makers are often eager to convince the public that they will never devaluate again.

Figure 24.8 The effects of an anticipated devaluation



Stage 2: The implementation effects of an anticipated devaluation

Now we turn to period 2, where the domestic currency is actually devalued by the amount $\Delta\bar{e}_2$. We study again how this affects the AD and the AS curve in period 2 in turn. We start with the AD curve departing again from the temporary AD curve as specified in (4'). The first thing to note is that the shift downwards in the AD curve that occurred in period 1 is now gone. This shift was caused by a future expected devaluation, but in period 2, there is no longer any future expected devaluation under our idealized assumptions of a *one-time* anticipated devaluation. This means that the shift in the AD curve that may occur in period 2 takes place from the initial AD curve AD_0. From this, the AD curve may shift in period 2 due to a real exchange rate effect or a real interest rate effect.

The real exchange rate in period 2 is $e_2^r = e_1^r + \Delta e_2 + \pi^f - \pi_2$, where it should be recalled that $e_1^r = e_0^r + \pi^f - \pi_1$ (since $\Delta e_1 = 0$) and we just found above that in period 1, $\pi^f - \pi_1 > 0$. Hence, in period 2 the real exchange rate is pushed upwards compared to e_0^r by the actual devaluation itself, $\Delta e_2 = \Delta\bar{e}_2 > 0$, *and* by the inflation differential in period 1, $\pi^f - \pi_1 > 0$, in total by $\Delta\bar{e}_2 + \pi^f - \pi_1$. *Ceteris paribus* this will shift the AD curve upwards by exactly that amount from the real exchange rate effect.

The real interest rate in any period *t* from period 2 and onwards is $r_t = i_t - \pi^e_{t+1} = i^f + \Delta e^e_{t+1} - \pi^e_{t+1}$. In all of these periods, $\Delta e^e_{t+1} = 0$ under our idealized assumptions since no further devaluations are expected. Hence, $r_t = i^f - \pi^e_{t+1}$ for $t \geq 2$. As also explained above, for period 1 and a number of periods after it is reasonable to assume that the expected future inflation rate from the present to the next period is pushed somewhat above the foreign inflation rate as higher inflation than abroad is expected to restore relative PPP gradually, so $\pi^e_{t+1} = \pi^f + \partial\pi^e_{t+1}$ for $t \geq 1$, whereeach $\partial\pi^e_{t+1}$ is relatively small. Hence, $r_t = i^f - \pi^f - \partial\pi^e_{t+1}$ for $t \geq 2$, so in period 2 and a number of periods coming after, the domestic real interest rate is pulled somewhat down below the foreign one. Ceteris paribus this also shifts the AD curve out- and upwards.

Inserting these features for period 2 into (4') gives the relevant AD curve for period 2:

$$
\begin{aligned}
& y_2 - \bar{y} = \beta_1[e^r_0 + (\pi^f - \pi_1) + (\Delta\bar{e}_2 + \pi^f - \pi_2)] - \beta_2(i^f - \pi^f - \partial\pi^e_3 - \bar{r}^f) \quad \Leftrightarrow \\
& \pi_2 = e^r_0 + (\pi^f - \pi_1) + \Delta\bar{e}_2 + \pi^f - \frac{\beta_2}{\beta_1}(i^f - \pi^f - \partial\pi^e_3 - \bar{r}^f) - \frac{1}{\beta_1}(y_2 - \bar{y}) .
\end{aligned}
\tag{24}
$$

We see that effects just described shift the AD curve in period 2 vertically by $\Delta\bar{e}_2 + (\pi^f - \pi_1) + (\beta_2/\beta_1)\partial\pi^e_3$ compared to AD_0, that is, the AD curve of period 2 lies above AD_0 by an amount slightly larger than $\Delta\bar{e}_2 + (\pi^f - \pi_1) > \Delta\bar{e}_2$. The shift is illustrated in Figure 24.8 by the AD curve labelled AD_2.

How does the AS curve shift in period 2 and later? The AS curve in absence of shocks is:

$$
\pi_t = \pi^e_t + \gamma(y_t - \bar{y}). \tag{25}
$$

Here again, for $t = 2$ and for a number of subsequent periods it is reasonable to assume that $\pi^e_t = \pi^f + \partial\pi^e_t$, where we recall that $\partial\pi^e_t << \Delta\bar{e}_2$. Hence, in period 2 (and a number of periods coming after), the AS curve may be shifted upwards compared to AS_0 as illustrated by the AS curve labelled AS_2 in Figure 24.8, but the shift is considerably smaller than the shift upwards in the AD curve from AD_0 to AD_2 as also illustrated. In the short-run equilibrium of period 2 labelled E_2 in the figure we must therefore have $y_2 > \bar{y}$ and $\pi_2 > \pi^f$.

Hence, in period 2, a considerable expansionary effect is obtained by the devaluation as intended, but at the cost of a prior contractionary effect in period 1 due to the anticipation of the devaluation. Under our idealized assumptions, this contractionary effect only occurs in period 1, since the actual devaluation in period 2 *is assumed* not to cause expectations of further devaluations. Therefore, the expansionary effect in period 2 is not counteracted by a new contractionary anticipation effect, but it would probably be more realistic to assume that the very act of devaluation often creates expectations of new later devaluations and hence implies new contractionary anticipation effects.

The basic insight from this analysis is that *anticipated devaluations may generate a 'bust–boom' movement in the domestic economy*. Before the adjustment of the exchange rate, when fears of a future devaluation build up, the impact on the economy is likely to be contractionary, as the perceived risk of devaluation drives up the domestic real interest rate. But when the devaluation occurs, the economy will expand, as domestic competitiveness improves and the domestic real interest rate falls. However, if the exchange rate adjustment fosters expectations of further imminent devaluations, the contractionary anticipation effect may well be tougher and long-lived.

Stage 3: The longer-term adjustment to an anticipated devaluation

While the devaluation will stimulate the economy in the period in which it occurs, its effects in the subsequent periods depend on the exact way in which the AD curve and the AS curve will shift over time. After the devaluation (in period 2), we see from Figure 24.8 that the domestic inflation rate is higher than the foreign inflation rate. This will gradually erode the domestic economy's initial competitive gain from the devaluation. Hence, the AD curve will gradually shift downward due to a falling real exchange rate. *Ceteris paribus*, this will tend to push domestic output and inflation back towards their original levels, $\bar{y}$ and π^f. As domestic inflation falls due to falling aggregate demand, and as the devaluation becomes an event of the past, firms and households will most likely reduce their estimate of the effect of the devaluation on next period's domestic inflation rate. This fall in expected inflation will cause the AS curve to shift downwards, back towards its original position. At the same time, the lower expected rate of inflation and the associated rise in the domestic real interest rate will push the AD curve further down. Because of these shifts in the two curves, the domestic inflation rate will continue to fall back towards the foreign inflation rate, and the downward-shifting AD curve will help to pull output back towards the natural rate.

Thus, although the devaluation does indeed raise the real exchange rate in the short run, it will be followed by periods in which the domestic inflation rate exceeds the foreign inflation rate, and this situation will continue until the real exchange rate is back at its original level.

THE EFFECTS OF DEVALUATION

In the long run a devaluation is neutral towards real economic variables such as output, employment, the real interest rate and the real exchange rate. An unanticipated devaluation will temporarily stimulate economic activity by improving domestic competitiveness, but over time the gain in competitiveness is eliminated by the higher domestic inflation generated by higher activity. Moreover, a devaluation is hardly ever completely unanticipated. A (partly) anticipated devaluation will tend to depress domestic activity before it occurs by driving up the domestic real interest rate. When the devaluation is undertaken, competitiveness temporarily improves and the domestic real interest rate falls, thereby stimulating domestic activity, but the gain in competitiveness is gradually eroded by higher domestic inflation, and in the long run the economy returns to the natural rate of output and employment.

Our arguments seem to go against the idea of devaluation. Yes, a surprise devaluation will have an expansionary effect, but once a country has obtained a reputation of devaluing, its nominal interest rate will increase because of expected future devaluations and nominal interest rate parity, and the expected rate of domestic inflation will also go up as the economic agents expect a return to relative PPP, but probably the nominal interest rate will go more up than inflation expectations. This creates a higher real interest rate as along with higher inflation expectations, which can have a significant contractionary effect (just draw for yourself a diagram with a downward shifting AD curve and an upward shifting AS curve). This contractionary effect may call for new devaluations so the country can end in a vicious spiral of low activity, high inflation and repeated devaluations.

This does not mean that a devaluation could *never* be the right medicine for a country with a fixed but adjustable exchange rate regime. For example, consider a country that has had a good economic performance and has maintained a fixed nominal exchange rate for a long time, and suppose this country is suddenly hit by a large negative economic shock due to external circumstances beyond its control. Suppose further that the shock is "asymmetric", hitting the country much harder than other countries. In that case a large one-time devaluation may well help the country to return more quickly to a normal activity level without igniting expectations of further future devaluations, and from the perspective of the country's trading partners, the devaluation would be justified by the country's need to improve its competitiveness after being hit by an asymmetric shock.

As another example, consider a country that has long suffered from the contractionary effects of anticipated devaluations because it has a record of trying to push output and employment above their high structural levels via devaluations. Now imagine that this country changes track and implements labour and product market reforms that can permanently increase the natural rates of output and employment. Imagine further that, at the same time as the reforms are implemented, in a situation with high unemployment, the government announces that it will undertake one last big devaluation to fasten the adjustment of economic activity to the new higher structural level, but that it intends to keep the exchange rate fixed forever after. If market participants consider the structural reforms to be effective and credible, they may actually believe that the government will not return to a policy of repeated future devaluations, and in that case the "final-and-we-will-never-do-it again" devaluation can help to increase aggregate demand and move the economy faster towards the new higher structural level of output. In other words, a large and claimed-to-be one-time devaluation may be the right medicine *if combined with strong structural policy measures acting to reduce structural unemployment*, although there is always a danger of bad anticipation effects if market participants are not convinced that the structural reforms will be effective.

24.5 Empirical illustration: the vulnerability of fixed exchange rates

The EMS crisis of 1992 – 93

The dramatic crisis in the European Monetary System (EMS) in 1992 – 93 provides some empirical support for our stylized analysis of an anticipated devaluation. Under the EMS system introduced in 1979, the participating EU countries had obliged themselves to keep their bilateral exchange rates within a band of ±2.5 per cent around the fixed exchange rate parities. Germany also participated in this arrangement, but the independent German central bank (Bundesbank) had indicated from the beginning that it would not compromise on its strong historical commitment to maintain a low German inflation rate.[4] Consequently, German monetary policy continued to be directed mainly towards the goal of domestic price stability, so the EMS system effectively implied that the other EU countries pegged their currencies to the D-mark and hence had to subordinate their interest rate policy to the policy followed by the Bundesbank. For a while during the 1980s, this arrangement worked quite well in the sense that the other EU countries were able to bring down their rates of inflation significantly by voluntarily subjecting themselves to the German monetary discipline (see Figure 24.2 above). Indeed, around 1990 Finland, Norway and Sweden all decided to unilaterally peg their currencies to the ECU (a unit of account consisting of a basket of EU currencies) in an effort to import the low rate of inflation prevailing in the EU area.

However, the reunification of East and West Germany in late 1990 initiated a strong construction boom concentrated in the former DDR and led to massive debt-financed income transfers from the German federal government to the new *Länder* in the East. Not surprisingly, this fiscal expansion tended to create excess demand in the German economy and induced the Bundesbank to raise its leading interest rate in order to stop German inflation from rising. This tightening of German monetary policy forced the other member countries in the EMS system to raise *their* interest rates to defend their exchange rates against the D-mark even though their economies were not exposed to any expansionary forces like those felt by the reunifying Germany. As a result of having to mimic the tight German monetary policy, the rest of the EU therefore fell into a recession that deepened as the high German interest rates persisted.

During the summer of 1992, a growing number of international financial market participants began to doubt that this situation would remain politically acceptable to the non-German members of the EMS. Speculations arose that several EU countries would

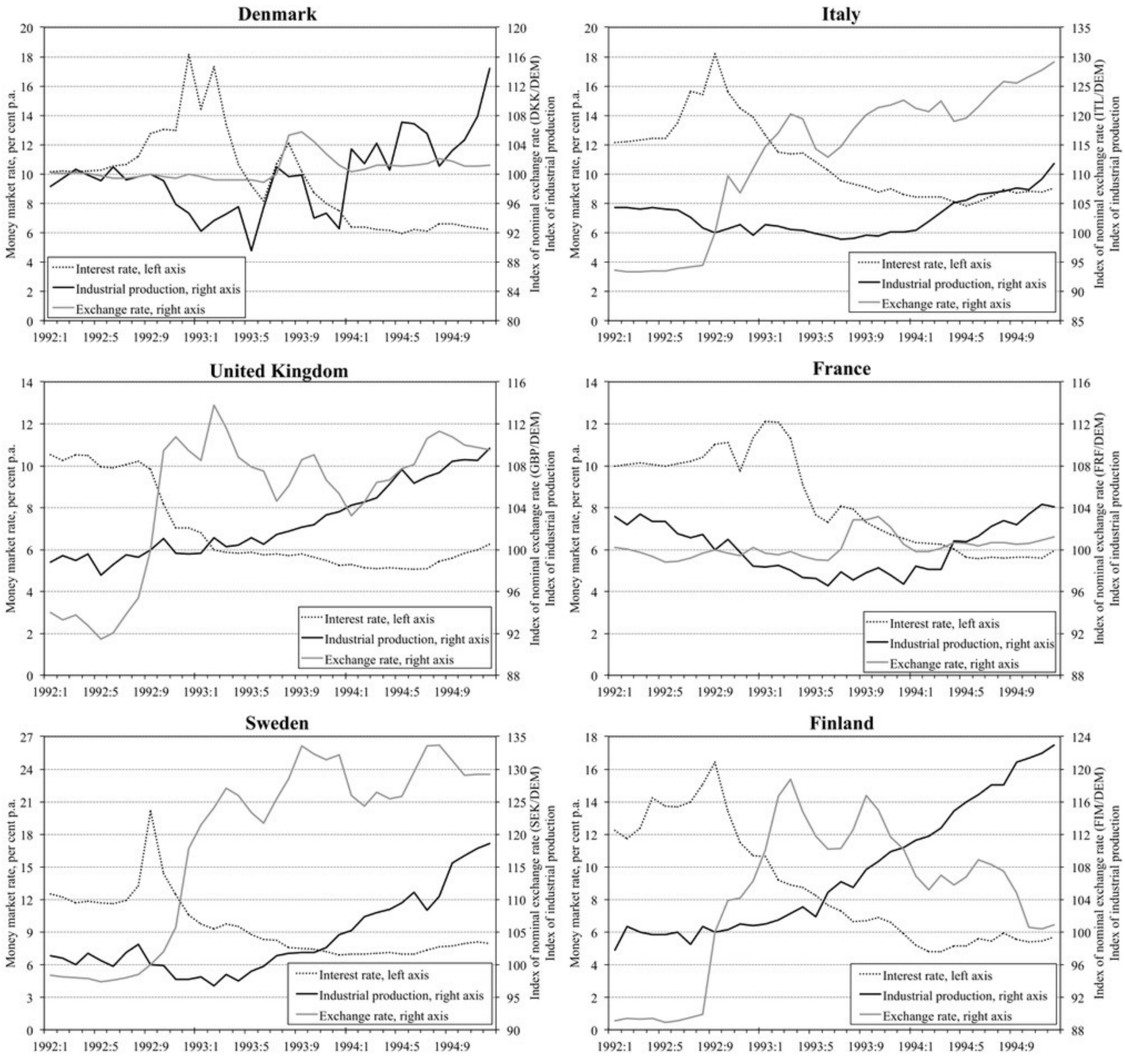
[4] In the aftermath of the two world wars of the twentieth century, Germany had experienced two episodes of devastating hyperinflation. Because of this traumatic experience, German voters and policy makers have given high priority to the goal of price stability in the postwar period.

be tempted to devalue their currencies against the D-mark in order to escape from the recession. These speculations were strengthened when Danish voters rejected the Maastricht Treaty on European Monetary Union in June 1992, and when French voters only barely accepted the Treaty with a very narrow majority in September of the same year. Given this voter skepticism, would European politicians remain committed to the creation of a monetary union with completely fixed exchange rates, or would they fall back on their earlier practice of using devaluations as a temporary relief during economic recessions?

As doubts about the policy makers' commitment to fixed exchange rates were growing, the currencies of most EMS member countries came under violent speculative attacks in September 1992. Within a few days Finland, Italy and the UK had to drop out of the EMS system and allow their currencies to float, and the Spanish peseta had to be devalued. A further devaluation of the peseta and the Portuguese escudo followed in November, and Sweden and Norway had to move to a floating exchange rate in November and December 1992, respectively. The spring of 1993 saw yet another round of devaluations of pesetas and escudos as well as a devaluation of the Irish pound. In the summer of 1993 the crisis culminated in a massive attack on practically all the remaining currencies in the EMS, so on 2 August the EU finance ministers felt forced to widen the exchange rate band around the EMS parities to ±15 per cent, effectively giving up fixed exchange rates for the time being.

The diagrams in Fig. 24.9 show the evolution of short-term interest rates, nominal exchange rates against the D-mark and industrial production from month to month in a number of European countries in the period from the start of 1992 until the end of 1994.

Figure 24.9 The behaviour of interest rates, exchange rates and output before and after the EMS crisis



Note: Short-term interest rates are based on three-month money market rates where available, or rates on similar financial instruments.
Source: Industrial production and interest rates are from General Statistics database, OECD; exchange rates are from German Bundesbank.

We see that, before the devaluations/switches to floating exchange rates, all countries had to raise their interest rates sharply in reaction to mounting expectations of a future devaluation/depreciation of their currencies. The high interest rate policy prevented output from growing and even forced a decline in industrial production in several countries. When the previous exchange rate parities were abandoned and the currencies were allowed to fall, interest rates could be lowered, and output started to increase in all countries. This accords with our graphical analysis of an anticipated devaluation in Fig. 24.8.

The vulnerability of a fixed exchange rate regime

The EMS crisis in 1992 – 93 illustrates that a fixed exchange rate regime coupled with free capital mobility is vulnerable to speculative attacks. When financial investors believe that some currency X may soon be devalued, they have an incentive to borrow a large amount in this currency, exchange it into securities in some other currency and convert the funds back into currency X later when a devaluation may have occurred. If currency X has actually been devalued in the meantime, this transaction will generate an exchange rate gain, and if no devaluation occurs, it will only generate a small transaction cost. Thus, speculation against a fixed exchange rate is virtually a one-way bet: you have a chance of scoring a large gain at the risk of losing very little.

An expectation that currency X could be devalued may therefore generate an extremely large additional supply of X, imposing a strong downward pressure on the exchange rate (an upward pressure on E). To be sure, the central bank issuing currency X may try to reduce the attractiveness of speculation by raising its interest rate and thereby increase the cost of borrowing in currency X and the gain from investing in it. However, if the market believes that a large devaluation is just around the corner, it may take an extremely high interest rate to offset the expected gain from speculation against the currency. For example, on 16 September 1992 the Swedish central bank raised its overnight lending rate to 500 per cent in an attempt to ward off a speculative attack on the Swedish krona. Very few governments can live with the economic and social consequences of such exorbitant interest rates for very long.[5] This is why a speculative attack may become *self-fulfilling* in the sense that it may force a government to let the currency fall even if policy makers would not have devalued if the attack had not occurred. The sustainability of a fixed exchange rate regime is therefore crucially dependent on the credibility of the government and the central bank. This credibility in turn depends on the ability of policy makers to avoid large macroeconomic imbalances, which are seen by the market as providing a temptation to devalue the currency. The historical experience shows that a speculative attack can occur very suddenly once the markets start to doubt a government's political will to defend its exchange rate.

The vulnerability of a fixed exchange rate regime to speculative attacks is an important reason why many countries have moved towards flexible exchange rates in recent decades when growing capital mobility has increased the scope for currency speculation. In the next chapter, we shall study how the macro economy works under flexible exchange rates.

[5] In September 1992 the Swedish central bank did, in fact, win the first battle against the speculators, but on 19 November of the same year, a renewed speculative attack forced the bank to abandon its fixed exchange rate and allow a sharp depreciation of the krona.

Summary

1. In an open economy with credibly fixed exchange rates and perfect capital mobility monetary policy is impotent in the sense that the domestic interest rate has to follow the foreign interest rate because of the condition for uncovered interest parity. Despite the loss of monetary autonomy, a country may nevertheless choose a fixed exchange rate regime to reduce the uncertainty associated with exchange rate fluctuations. There is some evidence that elimination of exchange rate uncertainty stimulates foreign trade. By pegging the domestic currency to the currency of a country with a history of low inflation, domestic policy makers may also succeed in bringing down the expected and actual rate of domestic inflation.
2. If exchange rates are credibly fixed, private agents are likely to realize that the domestic inflation rate cannot systematically deviate from the foreign inflation rate for an extended period of time. Our AS–AD model for the open economy with fixed exchange rates therefore assumes that the expected domestic inflation rate is tied to the foreign inflation rate. On this assumption, the economy's short-run aggregate supply curve will not shift during the adjustment to long-run macroeconomic equilibrium. Instead, adjustment will take place through shifts in the aggregate demand curve as the international competitiveness of domestic producers changes over time whenever the domestic inflation rate deviates from the foreign inflation rate.
3. A temporary fiscal expansion may generate a boom–bust cycle: at first output expands, but when the fiscal stimulus disappears the economy falls into a recession because of the loss of international competitiveness incurred during the previous expansion. The economy then gradually recovers from the recession as international competitiveness is gradually restored through a low domestic rate of inflation.
4. A systematic countercyclical fiscal policy will reduce the short-run output fluctuations generated by exogenous shocks to aggregate demand and supply, but it will also reduce the economy's speed of adjustment towards long-run equilibrium, since a countercyclical fiscal policy rule implies an automatic tightening of fiscal policy as the economy recovers from a recession, and an automatic relaxation of fiscal policy when the economy starts to move back towards a normal activity level following a boom.
5. An unanticipated devaluation will temporarily stimulate domestic output, but in the long run the economy settles in an equilibrium where all real variables are unaffected by the devaluation. Hence, a devaluation is neutral in the long run because the initial gain in international competitiveness is gradually lost again due to the higher domestic inflation rate following the devaluation. However, in a domestic recession, an unanticipated devaluation may speed up the adjustment to a normal activity level.

6. In practice, a devaluation is often (partly) anticipated. In that case, it will generate a bust–boom cycle: before the devaluation the economy will be pushed into recession because the fear of a future devaluation drives up the domestic real interest rate. When the devaluation occurs, output expands, even though domestic inflation also rises, in part because of a higher expected rate of inflation. Over time, the higher inflation rate gradually eliminates the gain in domestic competitiveness stemming from the devaluation, and the economy returns to the original long-run equilibrium.
7. A fixed exchange rate regime is vulnerable to speculative attacks in a world of high capital mobility. The vulnerability stems from the fact that currency speculation is virtually without any risk under such a regime. The historical experience shows that a speculative attack can occur very suddenly once financial markets start to doubt a government's commitment or ability to defend its fixed exchange rate.

Exercises

Exercise 1. Issues in the theory of the open economy with fixed exchange rates

1. Explain why monetary policy is impotent in an open economy with perfect capital mobility and (truly) fixed exchange rates. Discuss some reasons why a country might nevertheless want to adopt a fixed nominal exchange rate.
2. Explain the basic macroeconomic adjustment mechanism under fixed exchange rates and compare to the basic adjustment mechanism in the closed economy.
3. Explain why a devaluation is neutral in the long run.
4. Explain why a system of fixed but adjustable exchange rates may lead to anticipation effects in advance of a devaluation. Explain the nature of these anticipation effects and how they are likely to affect the economy.
5. Explain why a system of fixed exchange rates is vulnerable to speculative attacks. Explain briefly what is meant by a 'self-fulfilling' speculative attack.

Exercise 2. Temporary and permanent shocks in the open economy with fixed exchange rates

This exercise asks you to undertake a graphical analysis of the effects of temporary and permanent shocks, using the AS–AD model for the open economy with fixed exchange rates summarized in Equations (7)–(9) in the text. In all scenarios you may assume that the economy starts out in long-run equilibrium in period 0.

1. Do a graphical analysis of the effects on output and domestic inflation of a *permanent fiscal expansion*. Illustrate and explain the short-run effects as well as the economy's adjustment over time. Explain the macroeconomic adjustment mechanism under fixed exchange rates and make sure that your diagram indicates precisely how much the AD curve shifts from one period to the next (indicate the position of the AD curves for periods 0, 1, 2 and 3, and use arrows to indicate the movement of the AD curve after period 3). What happens to the real exchange rate in the long run?
2. Illustrate the effects on output and domestic inflation of a *temporary negative demand shock* hthat lasts for one period. Explain the short-run effects as well as the economy's adjustment over time. (Hint: it may be useful if you go through the analysis in Figure 24.4 once again.)
3. Illustrate the effects on output and domestic inflation of a *temporary positive supply shock* hthat lasts for one period. Explain the effects in the short run and over time. Do the effects of the shock go away as soon as the shock has died out?
4. Suppose now that *the positive supply shock is permanent*. Illustrate and explain the evolution of the economy from the initial long-run equilibrium to the new long-run equilibrium. What happens to the real exchange rate in the long run?

Exercise 3. The effects of a global recession

You are now asked to use the AS–AD model of the small open economy with fixed exchange rates to analyse the domestic effects of an international recession. In Questions 1– 3 you may assume for simplicity that the global recession only affects foreign output, y^f, and the foreign inflation rate, π^f, but *not* the foreign real interest rate, r^f. In all questions you may also assume that the domestic economy starts out in long-run equilibrium in period 0.

1. Suppose that the foreign economy is hit by a negative *demand* shock, which generates an international recession in period 1. The recession lasts for one period only (which may be thought of as a year), so all foreign macroeconomic variables return to their original values from period 2 onwards. Use the AS–AD diagram to illustrate how the domestic economy is affected over time by the temporary global recession. Explain the effects in period 1 as well as the subsequent adjustments. (Hint: start by explaining how y^f and π^f are affected by the global recession.)
2. Suppose alternatively that the international recession lasts for *two* periods so that y^f and π^f only return to their original values from period 3 onwards. Illustrate and explain the dynamic effects on the domestic economy in this scenario and compare with the scenario in Question 1 where the recession lasted only one period.
3. Now assume instead that the international recession is triggered by a negative *supply* shock, which occurs in period 1. Suppose further that y_f and π^f return to their normal values from period 2 onwards. Illustrate and explain the effects of the global recession on the domestic economy in the short run and over time. Compare to the effects found in Question 1. (Hint: start by explaining how y_f and π^f are affected in period 1 when the international recession originates from a negative supply shock.)
4. Discuss briefly how the answers to Questions 1– 3 are likely to be modified if we allow for the possibility that the international recession may affect the international real interest rate, r^f, and the confidence variable *e*. (Hint: how are foreign monetary policy and domestic confidence likely to react to the recession?)

Exercise 4. The effects of a fully anticipated devaluation

In this exercise you are invited to study the effects of a devaluation in the borderline scenario where the devaluation which occurs in period 2 is fully and correctly anticipated already from period 1, and the private sector expects that the rate of domestic inflation in period 2 will rise by the same amount as the percentage rate of devaluation (and that domestic inflation will fall back to the initial level π^f from period 3 onwards).

1. Use the AS–AD diagram for the open economy to illustrate the effects of a fully anticipated devaluation. (Hint: explain what happens to Equation (22) and to the

expected rate of inflation, π_2^e, from period 1 to 2). Will the devaluation have any effects on real economic variables? Will there be any difference between the short-run and the long-run effects of the devaluation? Discuss the realism of the assumptions underlying this scenario.

Exercise 5. The open economy with fixed exchange rates and rational expectations

The model in the main text of this chapter assumed a form of 'weakly' rational expectations where the expected domestic inflation rate is anchored by the foreign inflation rate, $\pi^e = \pi^f$, provided the exchange rate is credibly fixed. In this exercise, we assume instead that expectations are *strictly rational* in the sense defined in Chapter 21. Thus the expected domestic inflation rate, $\pi_{t,t-1}^e$, is the expected inflation rate for period t predicted by our AS–AD model, given all the information available up until the end of period $t-1$. The model of the small open economy with fixed exchange rates consists of the following equations, where the stochastic demand and supply shock variables are white noise, and where the foreign inflation rate, π^f, is assumed to be constant:

Goods market equilibrium: $$y_t - \bar{y} = \beta_1 e_t^r + z_t, \quad E[z_t] = 0, \quad E[z_t^2] = \sigma_z^2, \tag{26}$$

AS: $$\pi_t = \pi_{t,t-1}^e + \gamma(y_t - \bar{y}) + s_t, \quad E[s_t] = 0, \quad E[s_t^2] = \sigma_s^2, \quad E[z_t s_t] = 0. \tag{27}$$

Real exchange rate dynamics: $$e_t^r = e_{t-1}^r + \pi^f - \pi_t. \tag{28}$$

1. Show that under rational expectations the solutions to the model (35)–(37) are given by:

$$y_t - \bar{y} = \frac{z_t - \beta_1 s_t}{1 + \gamma\beta_1}, \tag{29}$$

$$\pi_t = \pi^f + e_{t-1}^r + \frac{s_t + \gamma z_t}{1 + \gamma\beta_1}. \tag{30}$$

(Hint: solve the model by the procedure outlined in Section 2 of Chapter 21). Discuss whether this model has plausible persistence properties. (Hint: does the model display persistence in output?)

Now suppose instead that a fraction λ of the population has 'weakly' rational expectations, expecting the domestic inflation rate to equal the foreign inflation rate, while the remaining fraction has strictly rational expectations. The average expected inflation rate is then given as:

$$\pi_t^e = \lambda\pi^f + (1-\lambda)\pi_{t,t-1}^e, \qquad 0<\lambda<1. \tag{31}$$

and the SRAS curve becomes:

$$\pi_t = \pi_t^e + \gamma(y_t - \bar{y}) + s_t, \tag{32}$$

where π_t^e is given by (31). The agents with rational expectations form their expectations for period t knowing that the structure of the economy is given by the equations (26), (27), (31) and (32), and using all information available up until the end of period $t-1$.

2. Show that the rate of inflation expected by the 'strictly rational' part of the population is:

$$\pi_{t,t-1}^e = \pi^f + \frac{\gamma\beta_1}{\lambda+\gamma\beta_1}e_{t-1}^r. \tag{33}$$

3. Show by using (33) that the output gap may be written as:

$$y_t - \bar{y} = \left(\frac{\lambda\beta_1}{\lambda+\gamma\beta_1}\right)e_{t-1}^r + \frac{z_t - \beta_1 s_t}{1+\gamma\beta_1}. \tag{34}$$

Use this result along with (28), (31), (32) and (33) to show that the output gap $\hat{y}_t \equiv y_t - \bar{y}$ evolves according to the difference equation:

$$\hat{y}_{t+1} = \left(\frac{\lambda}{\lambda+\gamma\beta_1}\right)\hat{y}_t + \left(\frac{1}{\lambda+\gamma\beta_1}\right)z_{t+1} - \left(\frac{\lambda}{\lambda+\gamma\beta_1}\right)z_t - \left(\frac{\beta_1}{1+\gamma\beta_1}\right)s_{t+1}. \tag{35}$$

(Hints: use (31), (32) and (33) to find an expression for $\pi_t - \pi^f$. Use this expression along with (34) to rewrite (28) in terms of $\hat{y}_{t+1}$, $\hat{y}_t$ and the shock variables z_{t+1}, z_t and s_{t+1}. Then collect terms to get (35).)

4. Does the model with $0 < \lambda < 1$ have more plausible persistence properties than the model with strictly rational expectations where $\lambda = 0$? How is the economy's speed of adjustment to long-run equilibrium affected by a higher degree of rationality in expectations formation (a lower value of λ)? Try to give an economic explanation for your answer.

Chapter 25

The open economy with floating exchange rates and the choice of exchange rate regime

Introduction

As we have seen, a fixed exchange rate regime is vulnerable to speculative attacks in a world with high capital mobility. In recent decades many governments have learned this lesson the hard way. We have already reported how the fixed exchange rate regime of the Europan Union (the European Monetary System) came under heavy attack in 1992 – 93. A few years later, a number of other countries, which pegged their currencies to the US dollar, were attacked by currency speculators. This happened to Mexico in 1994, to the East Asian countries in 1997, to Russia in 1998, to Brazil in 1999, and to Argentina in 2000 – 01, just to mention some spectacular examples.

Countries have reacted in different ways to the growing vulnerability of fixed exchange rate regimes arising from the increasing mobility of capital. Contrary to the prediction of many observers at the time, the EMS crisis in 1992 – 93 strengthened the resolve of European political leaders to move towards the ultimate fixed exchange rate regime by forming a monetary union, the EMU, where the use of a common currency rules out the possibility of currency speculation. The commitment to one single currency for the 19 euro zone member states has been sufficiently strong to ensure that the euro has survived for all 19 countries even under the hard pressures from the great financial crisis evolving from 2008 and from the subsequent European sovereign debt crisis heavily affecting many euro member states, as we shall see in the next chapter.

However, during the 1990s some Western European countries, e.g,, the UK, Sweden and Norway, as well as a large number of emerging market economies and developing countries reacted to the growing frequency of speculative attacks against fixed exchange rates by moving towards floating exchange rates where currency speculation is much more risky because there is no fixed exchange rate parity against which to speculate. Thus, these countries have followed the lead of big economies like the USA and Japan, which have allowed their currencies to float right from the breakdown of the Bretton Woods system of fixed exchange rates in the early 1970s. Likewise, the common currency of the euro zone countries, the euro, floates vis-à-vis most other currencies including all the large ones.

Against this background, the present chapter studies the workings of an open economy with floating exchange rates. We continue to focus on a small, specialized economy, and in line with the previous chapter we assume perfect capital mobility, since significant capital controls are nowadays very rare among developed countries.

The first section of the chapter defines the characteristics of a floating exchange rate regime and describes how a number of Western countries have designed their monetary policies under this regime. This part of the chapter also highlights the role of exchange rate expectations under floating exchange rates. In the second section, we adapt our AS–AD model to a regime with floating exchange rates and identify the similarities and differences in the macroeconomic adjustment process under floating and fixed exchange rates. The third section of the chapter then studies how an open economy with flexible exchange rates reacts to demand and supply shocks and compares these reactions to those occurring under fixed exchange rates. Thus, in our exposition of the theory for a floating exchange rate regime, we will all the time have the theory for a fixed exchange rate regime as summarized in the model consisting of Equations (7) through (9) of Chapter 24 as a point of comparison. At the end of this chapter we will summarize the arguments in the great economic debate on ‘fixed versus floating exchange rates’.

25.1 Floating exchange rates as an economic policy regime

Inflation targeting as a nominal anchor

A crucial characteristic of a floating exchange rate regime is that it allows the domestic central bank to pursue its own monetary policy even if capital mobility is perfect. Given perfect capital mobility, the condition for uncovered interest parity with which you are already familiar must hold:

$$i = i^f + e^e_{+1} - e = i^f + \Delta e^e_{+1}, \quad e \equiv \ln E, \quad e^e_{+1} \equiv \ln E^e_{+1}, \quad \Delta e^e_{+1} \equiv e^e_{+1} - e. \tag{1}$$

According to (1), the domestic central bank may set the domestic nominal interest rate, i, independently of the foreign nominal interest rate, i^f, provided the bank is willing to accept whatever magnitude of the expected percentage exchange rate depreciation,

$\Delta e_{+1}^{e} \equiv e_{+1}^{e} - e$, which is necessary to make the holding of domestic and foreign interest-bearing assets equally attractive.

Since the expected change in the exchange rate depends on the current level of the (log of the) exchange rate e, we see from (1) that the domestic central bank can pursue a fully independent interest rate policy only if it is willing to leave the determination of the exchange rate completely to the forces of the market. If, for instance, from a situation where (1) is fulfilled, it chooses to increase i, this will *ceteris paribus* make it more profitable to place capital in the domestic economy, which gives a pressure for capital inflow and hence appreciation of the domestic currency, that is, a pressure downwards on e. For a given expectation e_{+1}^{e}, a lower e will give a higher expected depreciation $\Delta e_{+1}^{e} \equiv e_{+1}^{e} - e$, which *ceteris paribus* tends to restore the nominal interest rate parity. When investors expect that the exchange rate will tend to revert towards some "normal" level, the current exchange rate e can change without inducing a similar change in the expected future exchange rate e_{+1}^{e}. A higher domestic interest rate can then be enforced by accepting a lower exchange rate e now, since this will lead to a higher expected rate of depreciation $\Delta e_{+1}^{e} \equiv e_{+1}^{e} - e$ in the future.

Obviously, this means that the exchange rate cannot serve as a nominal anchor, in contrast to a fixed exchange rate regime where pegging to the currency of a low-inflation country can help to keep the expected and actual domestic inflation rate low.

As an alternative way of providing a nominal anchor for inflation expectations, most countries with freely floating exchange rates have therefore adopted a monetary policy regime of *inflation targeting*. Under inflation targeting, policy makers specify a target for the rate of inflation which is considered to be consistent with an acceptable degree of price stability. Monetary policy is then given the task of ensuring that the actual inflation rate stays close to the target. Typically, the target inflation rate is quite low, say, 2 per cent per annum. Realizing that monetary policy cannot perfectly control the rate of inflation, inflation targeting countries often state explicitly that the target is meant to be fulfilled on average over the medium term or they specify a 'tolerance band', that is, an acceptable range of fluctuation of the actual inflation rate around the target rate. Indeed, some countries only specify a target *range* for the inflation rate without explicitly setting a particular target *rate*.[1]

To make the policy goal of a low inflation rate as credible as possible, most inflation-targeting countries have delegated monetary policy to an independent central bank. In Chapter 22 we saw that such delegation may be a way of overcoming the credibility problem arising from the possible inflation bias in monetary policy. As you recall, an

[1] It is possible for a country to have a floating exchange rate without an inflation target (or other explicitly announced target). This will mean that the central bank (possibly in cooperation with the government) does *not* decide on its various instruments, in particular its interest rates, in order to influence the exchange rate, but on the other hand also abstains from declaring any other particular goal of its monetary policy. The central bank (and the government) will then decide on monetary policy from day to day 'in view of the situation' without the central bank announcing any systematic way in how the situation will affect its policy. Nowadays, however, a floating exchange rate regime almost always involves some kind of inflation-targeting.

inflation bias may exist when policy makers with a short time horizon are tempted to stimulate output by creating surprise inflation.

More generally, the creation of an independent central bank may be a means of convincing the public that the goal of price stability will not be compromised by a government trying to manipulate monetary policy to its own short-term political advantage. However, to ensure that monetary policy makers can be held accountable for their actions by democratically elected politicians, many inflation-targeting countries require the central bank to justify its policy decisions at regular intervals in reports to the government and/or parliament and to the general public. Through such a reporting procedure to promote transparency in monetary policy making, it becomes easier to check whether the central bank adheres to the goal of price stability in a satisfactory manner. This may strengthen the credibility of the inflation target and at the same time make the delegation of monetary policy democratically acceptable.

The monetary policy regimes in USA, the euro area, UK, Japan, Canada, Australia, New Zealand, Switzerland, Sweden and Norway closely resemble inflation targeting as described here. Table 25.1 lists inflation targets, tolerance bands and actual inflation levels for these countries and areas. We see that all of them have chosen inflation targets of 2 – 2.5 per cent per year, with a typical tolerance band of ± ½-1 percentage points.

Table 25.1 Inflation targets and inflation in main countries and areas with inflation-targeting monetary policy, 2017

	Target for annual inflation rate, per cent	**Tolerance band, percentage points**	**Average annual inflation rate 2000-2020, per cent**
United States	2.0 [a]	-	2.1
Euro area	2.0 [b]	-	1.7
United Kingdom	2.0	1-3	2.0
Japan	2.0	-	0.1
Canada	2.0	1-3	1.9
Australia	2.5 [c]	2-3	2.6
New Zealand	2.0	1-3	2.1
Switzerland	-	< 2	0.4
Sweden	2.0	-	1.3
Norway	2.5	-	2.1

[a] From August 2020 officially described as “average inflation targeting of 2 per cent”, formerly as “inflation target of 2 per cent”.

[b] The official target of the ECB was formally “below but close to 2%”, but has changed to “a symmetric 2 per cent target over the medium term” as part of the ECB’s new monetary policy strategy from July 2021.

[c] Inflation target implied by midpoint of tolerance band. Only the tolerance band is publicly announced.

Source: Bulletin Vol. 80, No. 8, August 2017, Reserve Bank of New Zealand; actual average inflation rate from Main Economic Indicators database, OECD.

As from July 2021, the European Central Bank (ECB) has adopted a symmetric 2 per cent inflation target over the medium term (formerly it aimed to keep inflation in the euro area below but close to 2 per cent). The ECB bases its monetary policy on an analysis of a wide range of economic and financial variables, but the purpose of this analysis is to evaluate whether a change in monetary policy and the interest rate is needed to maintain a low and stable rate of inflation. The ECB's monetary policy regime is therefore (similar to) a regime of inflation targeting.

In the USA, the official, congressionally assigned goals of the Federal Reserve System are to promote maximum employment, stable prices and moderate long-term interest rates. However, at least since the early 1980s the actions of the Fed have revealed that it attaches great weight to the maintenance of a low and stable rate of inflation, and in a recent update of its 'Statement on Longer-Run Goals and Monetary Policy Strategy', the FOMC (the Federal Open Market Committee) of the Fed states: **"**The Committee judges that longer-term inflation expectations that are well anchored at 2 percent foster price stability and moderate long-term interest rates and enhance the Committee's ability to promote maximum employment in the face of significant economic disturbances. In order to anchor longer-term inflation expectations at this level, the Committee seeks to achieve inflation that averages 2 percent over time, and therefore judges that, following periods when inflation has been running persistently below 2 percent, appropriate monetary policy will likely aim to achieve inflation moderately above 2 percent for some time".[2] Although the USA is not by congressional mandate an inflation targeting country, US monetary policy is fairly well described as inflation-targeting.

Monetary policy under inflation targeting

The fact that there is an inflation target does not necessarily mean that the central bank only has an obligation concerning or only reacts to the rate of inflation. In macroeconomic theory, one distinguishes between 'strict' and 'flexible' inflation targeting. Under strict inflation targeting, monetary policy only reacts to the inflation rate, whereas flexible inflation targeting implies that monetary policy also reacts to the evolution of real output and employment. As also explained in earlier chapters, if a central bank performs strict inflation targeting and only reacts to inflation, it does not mean that it cannot be concerned also with, e.g., the output gap: in the choice of how strongly to react to inflation it may take into account what this means for output stabilization. In practice, however, inflation-targeting central banks have tended to adhere

[2] "2020 Statement on Longer-Run Goals and Monetary Policy Strategy" as amended effective August 27, 2020, quoted from https://www.federalreserve.gov/monetarypolicy/review-of-monetary-policy-strategy-tools-and-communications-statement-on-longer-run-goals-monetary-policy-strategy.htm

to a flexible regime where the rate of unemployment or the output gap may modify how strongly they react to inflation. In general therefore, we will assume that the inflation-targeting domestic central bank through its instruments seeks to establish a nominal interest rate in accordance with the policy reaction function:

$$i = r^f + \pi^e_{+1} + h(\pi - \pi^*) + b(y - \bar{y}), \quad h > 0, \quad b \geq 0, \tag{2}$$

where π^* is the target inflation rate, π is the actual current inflation rate, π^e_{+1} is the expected rate of inflation between the current and the next period and $\bar{y}$ is the structural or long-run equilibrium rate of output. As before, Equation (2) assumes that the central bank can observe the public's expected inflation rate, say, through consumer and business surveys, or by observing the difference between the interest rates on indexed and non-indexed bonds. Through its control over the nominal interest rate, *i*, the central bank can therefore also control the *ex ante* real interest rate, $i - \pi^e_{+1}$, in the short run. As (2) shows, the target real interest rate for the central bank, i.e., the real interest rate it aims at when both domestic inflation and output are right on target, $\pi = \pi^*$ and $y = \bar{y}$, is assumed to be the current foreign real interest rate, r^f. Here is the reason. The central bank recognizes that in long-run equilibrium the domestic real interest rate must equal the foreign real interest rate,. This is just long-run real interest rate parity as explained in Chapter 23, which can be written $\bar{r} = \bar{r}^f$. This could itself speak for having $\bar{r}^f$ as target real interest rate in (2), but what is assumed in (2) is rather that if, e.g., the foreign real interest rate temporarily rises above its structural level, $r^f > \bar{r}^f$, then the domestic central bank follows up by increasing *i*, thereby aiming at establishing r^f as the domestic real interest rate in a normal situation with $\pi = \pi^*$ and $y = \bar{y}$. If instead it aimed at $\bar{r}^f$ and did not change *i*, it would temporarily impose a domestic current real interest rate below the current foreign rate, which would *ceteris paribus* imply a capital outflow pressing for a depreciation of the domestic currency, that is, a higher *e*. This would imply stronger competitiveness in a situation where stronger competitiveness is not needed. Hence, by shadowing the current foreign real interest rate, the central bank avoids some undesirable exchange rate volatility. On the other hand, by increasing the nominal interest rate, the central bank imposes a domestic real interest rate that is higher than suited for the domestic economy. Hence, in case of a foreign real interest rate deviation, $r^f - \bar{r}^f \neq 0$, the domestic central bank must choose between two evils, a 'wrong' real exchange rate or a 'wrong' real interest rate. In a very open economy, the first may well be the worst. This motivates the appearance of r^f in the monetary policy rule (2).

Thus, the policy rule (2) says that *ceteris paribus* the central bank raises the domestic real interest rate above the foreign level when the inflation rate exceeds its target or output is above normal, and vice versa. Strict inflation targeting is just the special version of the Taylor rule (2), where the coefficient on the output gap has been set at zero, $b = 0$:

$$i = r^f + \pi^e_{+1} + h(\pi - \pi^*). \tag{3}$$

For simplicity, the main text of this chapter will to start with consider a regime of strict inflation targeting. As we shall see, even strict inflation targeting may reduce the volatility of output compared to a regime with fixed exchange rates, when business cycles are driven mainly by demand shocks. However, we will follow up on how flexible inflation targeting will modify our conclusions, and Exercise 3 will ask you to analyse the regime of flexible inflation targeting in detail.

In practice, aggregate demand and inflation only react to a change in the interest rate with a certain time lag. Inflation targeting central banks must therefore base their interest rate policy on a *forecast* for the inflation rate expected to prevail one or two years ahead, as we saw in Chapter 20, and as we will explain in more detail later in this chapter. However, since there is some persistence in the inflation rate, a rise in the current inflation rate, π, will typically increase the central bank's forecast of future inflation, thereby triggering a rise in the interest rate, as assumed in (2) or (3).

Specifying the inflation target

Table 25.1 shows that the advanced inflation targeting economies have chosen roughly the same target for the annual inflation rate close to 2 per cent. This suggests that inflation targeting economies tend to choose an inflation target which is equal to the average rate of inflation around them, say in the OECD area, either because they see the foreign inflation rate as an appropriate nominal anchor, or simply because there is a strong international consensus on what the appropriate level of inflation is. In other words, it seems realistic to assume that the domestic target inflation rate is roughly equal to the foreign target inflation rate, $\pi^* = (\pi^*)^f$. Apart from the empirical justification for this, there is also a theoretical rationale again related to exchange rate stability. By having the same inflation target as abroad and assuming that the domestic and the foreign economy are able to hold their actual inflation rates close to their targets on average, the domestic economy should achieve that on average, actual domestic inflation will be close to actual foreign inflation, that is, $\pi = \pi^f$. As you recall from Chapter 23, a long-run equilibrium requires fulfilment of the condition for *relative purchasing power parity* to ensure constancy of the real exchange rate:

$$\Delta e \equiv e - e_{-1} = \pi - \pi^f. \tag{4}$$

In the long run an inflation-targeting country which manages to keep the domestic inflation rate equal to the foreign inflation rate will therefore also be able to maintain a stable nominal exchange rate, since $\pi = \pi^f$ implies $\Delta e = 0$ according to (4). Figure 25.1 illustrates, and international experience more generally suggests, that although inflation-targeting countries do not pursue a specific target for the nominal exchange rate, and over the medium term allow large swings in the exchange rate, countries usually try to avoid policies that lead to systematic depreciation or appreciation of their currencies over very long periods.

Figure 25.1 The dollar-euro exchange rate, 1980M1-2021M5

Note: Based on monthly data. Before 1999 the graph shows the exchange rate of the dollar against a weighted average of the currencies of the countries in the euro area.
Source: Bloomberg.

Our assumption of $\pi^* = (\pi^*)^f$ combined with the feature that on average the domestic as well as the foreign economy should be able to hold their actual inflation rates close to their targets, $\pi = \pi^*$ and $\pi^f = (\pi^*)^f$, is our way of modelling this apparent policy preference for long-run stability of the nominal exchange rate.

For simplicity, we treat the foreign inflation rate π^f as an exogenous constant in our analysis and as just noted we assume $\pi^f = (\pi^*)^f$. Hence, taken together this amounts simply to specify the domestic inflation target as the foreign inflation level, that is, $\pi^* = \pi^f$.

Inflation expectations under inflation targeting

Under these assumptions it is reasonable to assume about inflation expectations that:

$$\pi_{+1}^{e} = \pi^{e} = \pi^{*} = \pi^{f} = (\pi^{f})^{e}. \tag{5}$$

Thus, in parallel to the previous chapter, we assume that agents have 'weakly' rational inflation expectations. They can form a correct estimate of the longer-run inflation rate even though they do not have enough information to predict the short-run fluctuations in inflation, and the economic agents understand that under inflation targeting, on average the inflation rate has to equal the central bank's official inflation target. This in turn equals the foreign inflation target, for simplicity assumed to always being equal to actual and expected inflation abroad.

Equation (5) is the simplest way of formalizing our assumption that the central bank's inflation target has credibility. It is the natural analogue to the analysis in the previous chapter where our specification $\pi_{+1}^{e} = \pi^{e} = \pi^{f}$ reflected the assumption that the central bank's commitment to a fixed exchange rate was credible.

Exchange rate expectations and interest rates

Consider next the formation of exchange rate expectations. From (1) we see that these expectations are crucial for the link between interest rates and exchange rates. Whereas the central bank's official inflation target provides a natural anchor for inflation expectations, there is no similar policy-determined (or otherwise determined) focal point for exchange rate expectations, since there is no official exchange rate target under freely floating exchange rates. Our theory of exchange rate expectations is therefore based on a different line of reasoning than the one underlying our simple theory of inflation expectations. Specifically, we will adopt a hypothesis of 'regressive' exchange rate expectations, which is often used in the literature on the open economy. This hypothesis postulates that the exchange rate is expected to rise if it is currently below its perceived normal level, and vice versa. If the expected exchange rate for the next period is e_{t+1}^{e} and the perceived 'normal' exchange rate is $\overline{e}_t$, the hypothesis of regressive expectations thus says that:

$$e_{t+1}^{e} - e_t = \theta(\overline{e}_t - e_t) \quad \Leftrightarrow \quad e_{t+1}^{e} = \theta\overline{e}_t + (1-\theta)e_t, \quad \theta > 0. \tag{6}$$

In other words, agents believe that there is some possibly time-dependent normal or average level towards which the exchange rate tends to move over time. If, e.g., $e_t < \overline{e}_t$, the expected 'correction' of the exchange rate from period *t* to *t*+1 is $\theta(\overline{e}_t - e_t)$, where we see that the parameter $\theta > 0$ measures the degree of correction. If $\theta = 1$, the correction is immediate, $e^e_{t+1} = \overline{e}_t$. We do not restrict the value of θ to be less than 1. If $\theta > 1$, agents expect that the exchange rate will tend to *overshoot* its normal level in the short run. As we will explain later in this chapter, exchange rates do in fact tend to overshoot their long-run equilibrium values in the short term. The assumption that $\theta > 0$ implies that a lower value of e_t implies a higher value of $e^e_{t+1} - e_t$, which is exactly the sluggishness or reversion effect we alluded to in connection with Equation (1) above. Now, if $0 < \theta < 1$, then e^e_{t+1} goes down if e_t goes down, but e^e_{t+1} goes down by less so that $e^e_{t+1} - e_t$ increases. If $\theta > 1$, then e^e_{t+1} goes up if e_t goes down, so that $e^e_{t+1} - e_t$ increases even more.

The perceived normal exchange rate is likely to depend on the actual exchange rates observed in the past, with greater weight being attached to the more recent observations. As a rough approximation capturing this idea, we will assume that the 'normal' exchange rate is simply identified with last period's observed exchange rate:

$$\overline{e}_t = e_{t-1}. \tag{7}$$

We emphasize that none of the results derived in this chapter depend crucially on this strong simplification. In Exercise 1 we ask you to derive a very similar AS–AD model from an alternative theory where the perceived normal exchange rate is simply treated as exogenous. However, the link between interest rates and exchange rates implied by (7) does find some empirical support, as we turn to now.

Inserting (7) into (6) we get:

$$e^e_{t+1} - e_t = -\theta(e_t - e_{t-1}) \quad \Leftrightarrow \quad e^e_{t+1} = \theta e_{t-1} + (1-\theta)e_t \quad \Leftrightarrow \quad \Delta e^e_{t+1} = -\theta \Delta e_t \tag{8}$$

which says that if the exchange rate was rising (falling) between the previous and the current period, it is expected to fall (rise) over the next period. Combining this version of regressive exchange rate expectations, $\Delta e^e_{t+1} = -\theta \Delta e_t$, with uncovered nominal interest rate parity (1), $i_t = i^f_t + \Delta e^e_{t+1}$, it follows that:

$$i_t - i^f_t = -\theta \Delta e_t \quad \Leftrightarrow \quad \Delta e_t = -\frac{1}{\theta}(i_t - i^f_t) \tag{9}$$

Equation (9) can be read both ways as the two writings suggest. If we start out assuming that the domestic interest rate is raised above the foreign interest rate, $i_t - i^f_t > 0$, then (9) says that the domestic currency will appreciate, $\Delta e_t < 0$, and vice versa. The reason is that when the doestic nominal interest rate is raised above the foreign rate, domestic assets tend to become more attractive than foreign assets, other things being equal. To maintain a capital market equilibrium where all assets are equally attractive, the domestic currency must be expected to depreciate over the next period so that the higher interest yield on domestic assets is offset by an expected exchange rate loss. To create this expectation of depreciation it is necessary for the domestic currency to *actually appreciate*, since this is exactly what will create an *expectation* that the domestic currency will *depreciate* according to our version of regressive exchange rate expectations.

We may also start out assuming that the domestic currency has actually depreciated, $\Delta e_t > 0$. This will according to regressive expectations create an expectation of a coming appreciation, $\Delta e^e_{t+1} < 0$, which *ceteris paribus* makes investment in the domestic economy more attractive giving a tendency that capital will flow into the domestic economy. This will tend to press the domestic nominal interest rate below the foreign one, which works to reestablish nominal interest rate parity.

Without presenting direct evidence for this, we will simply state that the regularity of Equation (9), that a relatively high interest rate in a country (with liberalized capital movements) tends to go hand in hand with an appreciation of the country's currency, is empirically plausible. Taking some version of nominal interest rate parity as given – and it is hard not to believe in this law between countries that allow free capital movements between them – regressive exchange rate expectations will exactly create this regularity. In this sense, the empirics can be said to support the hypothesis of some form of regressive exchange rate expectations.

'Dirty' floating

The specifications above relate to a regime of 'clean' floating where policy makers do not in any way try to influence the market-determined exchange rate. Yet historically many countries have practised so-called 'dirty' floating by intervening in the foreign exchange market in order to reduce the fluctuations in the exchange rate. The exchange rate can usually be controlled over a few days by relying on 'sterilized' foreign exchange market interventions where the central bank buys or sells foreign exchange without changing the interest rate. The evidence shows, however, that to have a persistent effect on the exchange rate, foreign exchange interventions must be 'unsterilized', that is, they must involve a change in the interest rate that supports the movement in the exchange rate that the central bank is trying to achieve.

To model a policy regime where inflation targeting is supplemented by unsterilized foreign exchange intervention to smooth fluctuations in the exchange rate, we might modify the monetary policy rule (3) in the following way:

$$i - \pi^e_{+1} = r^f + h(\pi - \pi^*) + \lambda(e - e_{-1}), \quad h > 0, \quad \lambda > 0. \tag{10}$$

According to (10) an observed depreciation of the exchange rate ($e > e_{-1}$) induces the central bank to raise the domestic interest rate. In this way, the bank may generate a capital inflow which will increase the supply of foreign exchange and the demand for domestic currency, thereby moderating the depreciation. By analogy, when the bank observes a tendency for the domestic currency to appreciate ($e < e_{-1}$), it reduces the interest rate to induce a capital outflow which will increase the supply of domestic currency and the demand for foreign currency. Such a policy is sometimes described as 'leaning against the wind', because the central bank goes against the tendencies in the foreign exchange market.

In Exercise 2 we will ask you to show that the AS–AD model emerging under the policy of dirty floating specified in (10) is qualitatively similar to the model implied by the clean floating regime (3), although the quantitative properties of the model will be affected by a policy of leaning against the wind. In the rest of the chapter we will maintain the assumption of clean floating, partly because it slightly simplifies the exposition, and partly because many countries with floating exchange rates have in fact moved towards 'cleaner' floating by reducing their interventions to affect exchange rates.

Let us sum up what we have learned so far:

FLOATING EXCHANGE RATES WITH INFLATION TARGETING

To provide a nominal anchor for inflation expectations, many countries with floating exchange rates have adopted explicit inflation targets, typically around 2–2.5 per cent on an annual basis. The fact that so many countries have announced similar inflation targets suggests that they seek to keep domestic inflation close to the average foreign inflation rate to avoid a systematic depreciation or appreciation of their exchange rates over the longer run. Our AS–AD model of the open economy with floating exchange rates therefore assumes that the expected inflation rate equals the target inflation rate, which in turn equals the (average) foreign inflation rate.

THE LINK BETWEEN EXCHANGE RATES AND INTEREST RATES

The hypothesis of regressive exchange rate expectations says that the exchange rate is expected to rise when it is currently below its perceived normal level, and vice versa. The perceived normal exchange rate is likely to depend on the actual exchange rates observed in the recent past. When regressive exchange rate expectations interact with the condition for uncovered interest rate parity under floating exchange rates, the domestic exchange rate will tend to appreciate when the domestic interest rate is above the foreign interest rate, and vice versa. The evidence supports this hypothesis.

25.2 Model and macroeconomic adjustment under floating exchange rates

We are now ready to set up our version of the AS–AD model for the open economy with floating exchange rates. After having done so, we will study the economy's adjustment to long-run equilibrium and compare the adjustment process to the one characterizing a fixed exchange rate regime. Through this comparison we will gain a deeper understanding of the special features of the two alternative exchange rate regimes.

The aggregate demand curve

We start by restating the preliminary aggregate demand curve in the open economy, Equation (26) of Chapter 23 in a slightly different form, now again dropping the explicit time subscripts:

$$\begin{aligned} y-\bar{y} &= \beta_1 e^r - \beta_2(r-\bar{r}^f)+\tilde{z}, \\ \tilde{z} &\equiv \beta_3(g-\bar{g})+\beta_4(y-\bar{y}^f)+\beta_5(\ln\varepsilon-\ln\bar{\varepsilon}), \end{aligned} \tag{11}$$

All notation is as in the former chapters. Note that in the second term on the right hand side we have chosen this time not to write $i^f+\Delta e^e_{+1}-\pi^e_{+1}$ for the domestic real interest rate r (as we could from nominal interest rate parity, (1)). Likewise, we have chosen to write the first term on the right hand side simply as $\beta_1 e^r$, but we recall that as an identity the current real exchange rate is:

$$e^r = e^r_{-1}+\Delta e+\pi^f-\pi. \tag{12}$$

These rewritings make the derivations to follow simpler.

The effect on aggregate demand running through e^r is referred to as the real exchange rate channel. The effect on aggregate demand running through the real interest rate differential, $r-\bar{r}^f$, is referred to as the real interest rate channel.

From (3), which expresses our assumption of strictly inflation-targeting monetary policy, $i=r^f+\pi^e_{+1}+h(\pi-\pi^*)$, two relationships follow. First, since $r=i-\pi^e_{+1}$:

$$r=r^f+h(\pi-\pi^f). \tag{13}$$

Second, since $\pi^e_{+1}=\pi^f$, we get $i=r^f+\pi^f+h(\pi-\pi^*)$ and since we also assume $(\pi^e_{+1})^f=\pi^f$, we get $i=i^f+h(\pi-\pi^*)$, or:

$$i-i^f=h(\pi-\pi^f). \tag{14}$$

From (13) the real interest rate channel becomes:

$$r-\bar{r}^f=h(\pi-\pi^f)+r^f-\bar{r}^f. \tag{15}$$

For the real exchange rate channel we get by starting from the identity (12), then inserting that from (9) $\Delta e = -1/\theta(i - i^f)$, and finally using (14):

$$
\begin{aligned}
e^r &= e^r_{-1} + \Delta e + \pi^f - \pi = e^r_{-1} - \frac{1}{\theta}(i - i^f) + \pi^f - \pi \\
&= e^r_{-1} - (\pi - \pi^f) - \frac{h}{\theta}(\pi - \pi^f).
\end{aligned} \tag{16}
$$

We note that the domestic inflation rate affects the real exchange rate, or domestic competitiveness, through two effects. A higher π directly gives more expensive domestic products and thereby a lower real exchange rate. This is what the term $-(\pi - \pi^f)$ in (16) expresses. A higher π also implies a higher domestic nominal interest rate from a tightening of monetary policy, which leads to a lower nominal exchange rate (through the mechanism in (9)) and thereby a lower real exchange rate. This is what the last term $-(h/\theta)(\pi - \pi^f)$ in (16) expresses.

When we insert (15) and (16) into the temporary AD curve (11) we get:

$$
\begin{aligned}
y - \bar{y} &= \beta_1[e^r_{-1} - (\pi - \pi^f) - \frac{h}{\theta}(\pi - \pi^f)] - \beta_2[h(\pi - \pi^f) + r^f - \bar{r}^f] + \tilde{z} \\
&= \beta_1 e^r_{-1} - [\beta_1 + \beta_1\frac{h}{\theta} + \beta_2 h](\pi - \pi^f) - \beta_2(r^f - \bar{r}^f) + \tilde{z}
\end{aligned} \tag{17}
$$

This shows that domestic inflation, π, or the inflation differential, $\pi - \pi^f$, affects domestic aggregate demand through three mechanisms: 1) through the direct effect on the real exchange rate captured by the term $-\beta_1(\pi - \pi^f)$, and through two different effects both associated with monetary policy, 2) the effect on the real exchange rate from monetary policy captured by the term $-(\beta_1 h/\theta)(\pi - \pi^f)$, and 3) the effect on the real interest rate from monetary policy captured by $-\beta_2 h(\pi - \pi^f)$. We return to these effects soon below.

The term $-\beta_2(r^f - \bar{r}^f)$ on the right hand side of (17) is just a foreign real interest shock that we may prefer to write into the shock variable $\tilde{z}$. This gives us the final expression for *the AD curve under floating exchange rates with strict inflation targeting*:

$$
\begin{aligned}
& y - \bar{y} = \beta_1 e^r_{-1} - \hat{\beta}_1(\pi - \pi^f) + z \quad \Leftrightarrow \\
& \pi = \frac{\beta_1}{\hat{\beta}_1} e^r_{-1} + \pi^f - \frac{1}{\hat{\beta}_1}(y - \bar{y}) + \frac{z}{\hat{\beta}_1}, \\
& \hat{\beta}_1 \equiv \beta_1 + \beta_1\frac{h}{\theta} + \beta_2 h = \beta_1 + h(\frac{\beta_1}{\theta} + \beta_2) > \beta_1, \\
& z \equiv -\beta_2(r^f - \bar{r}^f) + \beta_3(g - \bar{g}) + \beta_4(y^f - \bar{y}^f) + \beta_5(\ln\varepsilon - \ln\bar{\varepsilon}).
\end{aligned} \tag{18}
$$

For comparison, Equation (7) in Chapter 24 gives the analogous expression for *the AD curve under fixed exchange rates*:

$$
\begin{aligned}
&y-\bar{y}=\beta_1 e^r_{-1}-\beta_1(\pi-\pi^f)+z \quad \Leftrightarrow \\
&\quad \pi=e^r_{-1}+\pi^f-\frac{1}{\beta_1}(y-\bar{y})+\frac{z}{\beta_1},
\end{aligned}
\tag{19}
$$

where the shock z has the same definition as in (18). We see that the AD curve has the same structure under the two exchange rate regimes. Under both regimes the demand shock variable z includes the same disturbances, and a rise in the domestic inflation rate lowers aggregate demand. However, since $\hat{\beta}_1 > \beta_1$, we see from (18) and (19) that a rise in domestic inflation causes a *larger* fall in aggregate demand under flexible exchange rates with strict inflation targeting than under fixed exchange rates implying that the AD curve is flatter in the usual $y-\pi$ -diagram.

By taking a closer look at the expression for $\hat{\beta}_1$ in (18), we see that it reflects the various effects generated by a rise in domestic inflation under a floating exchange rate also explained above but now in more detail: First, there is a direct effect as the rise in π reduces net exports by eroding the economy's international competitiveness. This direct effect – which is the *only* effect arising under *fixed* exchange rates – is reflected in the term β_1 in the expression for $\hat{\beta}_1$. Under floating exchange rates a rise in π generates two additional effects stemming from the fact that a floating exchange rate provides scope for an independent domestic monetary policy. As domestic inflation rises, the central bank reacts by raising the domestic interest rate. This creates additional downward pressure on aggregate demand, partly through the interest rate channel (reflected in the term $h\beta_2$ in the expression for $\hat{\beta}_1$), and partly through the exchange rate channel (captured by the term $\beta_1 h/\theta$ in the definition of $\hat{\beta}_1$). This explains why the AD curve is *flatter* under floating exchange rates with strict inflation targeting than under fixed exchange rates. As a consequence, the economy will respond differently to exogenous shocks under the two exchange rate regimes, as we shall see later on.

Real exchange rate dynamics

As was the case under fixed exchange rates, we see from (18) that the AD curve under floating exchange rates includes the lagged value of the real exchange rate, e^r_{-1}. As long as the economy is out of long-run equilibrium, the dynamics of the real exchange rate will therefore cause the short-run AD curve to shift from one period to the next. Under flexible exchange rates, these shifts in the AD curve are driven by (16) above, which can be rewritten as:

$$e^r = e^r_{-1} + \overbrace{\pi^f - \pi}^{\substack{\text{direct effect on} \\ \text{competitiveness}}} + \overbrace{\frac{h}{\theta}(\pi^f - \pi)}^{\substack{\text{monetary policy} \\ \text{effect}}} \quad \Leftrightarrow \tag{20}$$
$$\Delta e^r \equiv e^r - e^r_{-1} = (1 + \frac{h}{\theta})(\pi^f - \pi).$$

Thus the real exchange rate will depreciate (that is, e^r will rise) whenever the foreign inflation rate exceeds the domestic inflation rate. Part of this real depreciation is due to the direct effect on competitiveness. The other part is due to a monetary policy effect: when domestic inflation falls below the inflation target π^f, the central bank cuts the domestic interest rate, inducing a depreciation of the nominal exchange rate.

Under fixed exchange rates the corresponding real exchange rate dynamics was (we recall from Equation (9) of Chapter 24):

$$e^r_t = e^r_{t-1} + \pi^f - \pi_t \quad \Leftrightarrow \quad \Delta e^r \equiv e^r - e^r_{-1} = \pi^f - \pi. \tag{21}$$

Hence, for a given inflation differential, the real exchange rate shifts (numerically) more from the previous period to the current one under floating exchange rates than under fixed rates, reflecting the additional monetary policy response under flexible exchange rates.

The complete AS–AD model with floating exchange rates

We may now summarize our complete AS–AD model of the open economy with floating exchange rates and strict inflation targeting. Following the previous chapter, we continue to use a specification of the aggregate supply side based on the hypothesis of relative wage resistance. Stating the time subscripts explicitly, we end up with the following model (where RERD stands for Real Exchange Rate Dynamics):

AD:
$$\begin{aligned} & y_t - \bar{y} = \beta_1 e^r_{t-1} - \hat{\beta}_1(\pi_t - \pi^f) + z_t \quad \Leftrightarrow \\ & \pi_t = \frac{\beta_1}{\hat{\beta}_1} e^r_{t-1} + \pi^f - \frac{1}{\hat{\beta}_1}(y_t - \bar{y}) + \frac{z_t}{\hat{\beta}_1}, \\ & \hat{\beta}_1 \equiv \beta_1 + \beta_1 \frac{h}{\theta} + \beta_2 h > \beta_1, \\ & z_t \equiv -\beta_2(r^f_t - \bar{r}^f) + \beta_3(g_t - \bar{g}) + \beta_4(y^f_t - \bar{y}^f) + \beta_5(\ln \varepsilon_t - \ln \bar{\varepsilon}), \end{aligned} \tag{22}$$

AS:
$$\pi_t = \pi^f + \gamma(y_t - \bar{y}) + s_t, \tag{23}$$

RERD:
$$e^r_t = e^r_{t-1} + (1 + \frac{h}{\theta})(\pi^f - \pi_t). \tag{24}$$

For $z_t = 0$ and $e^r_{t-1} = 0$ (the real exchange rate at its structural level in the previous period), the AD curve (22) passes through $(\bar{y}, \pi^f)$ in a usual y_t, π_t-diagram. A demand shock $z_t \neq 0$ will shift the AD curve in the *y*-dimension by z_t (the same as under fixed exchange rates), and by $z_t / \hat{\beta}_1$ in the π-dimension (which is numerically smaller than under fixed exchange rates where the shift is z_t / β_1). For $s_t = 0$, the AS curve (8) passes through $(\bar{y}, \pi^f)$, and a supply shock $s_t \neq 0$ will shift the AS curve vertically by s_t and horizontally by s_t / γ.

The structure of the model (22) through (24) is similar to the structure of the model with fixed exchange rates (you may want to compare with Equations (7)–(9) in Chapter 24). At the start of each period, the previous period's real exchange rate e^r_{t-1} is predetermined, so the current rates of output and inflation, y_t and π_t, are determined by the intersection of the aggregate demand curve (22) and the short-run aggregate supply curve (23), where the position of the AD curve depends on the lagged real exchange rate, the foreign inflation rate π^f and the real demand shock variable z_t, while the position of the AS curve depends on π^f and on the supply shock variable s_t. The current level of inflation, π_t, then determines the current real exchange rate through (24). If $\pi^f \neq \pi_t$, it follows that $e^r_t \neq e^r_{t-1}$, and the economy will then enter the next period with a new predetermined real exchange rate, causing a shift in the position of the AD curve. This will generate new short-run values of output and inflation during the next period, which in turn will change the level of the real exchange rate, and so on.

The vertical shift (the shift in the π-dimension) in the AD curve from period *t* to *t*+1 will according to (22) and (24) be exactly:

$$\frac{\beta_1}{\hat{\beta}_1} e^r_t - \frac{\beta_1}{\hat{\beta}_1} e^r_{t-1} \equiv \frac{\beta_1}{\hat{\beta}_1} \Delta e^r_t = \frac{\beta_1}{\hat{\beta}_1} (1 + \frac{h}{\theta})(\pi^f - \pi_t).$$

Inserting here the expression for $\hat{\beta}_1$ from (22) gives the following expression for the vertical shift:

$$\frac{\beta_1}{\hat{\beta}_1} \Delta e^r_t = \frac{\beta_1}{\beta_1 + \beta_1 \dfrac{h}{\theta} + \beta_2 h} (1 + \frac{h}{\theta})(\pi^f - \pi_t) = \frac{\beta_1 (1 + \dfrac{h}{\theta})}{\beta_1 (1 + \dfrac{h}{\theta}) + \beta_2 h} (\pi^f - \pi_t), \qquad (25)$$

We see that the shift is a constant fraction strictly smaller than 1 of the inflation differential $\pi^f - \pi_t$ and therefore strictly smaller than $\pi^f - \pi_t$. Hence, the AD curve shifts vertically by (numerically) less than the inflation differential $\pi^f - \pi_t$ and thus for a given inflation differential less than the AD curve shifts under fixed exchange rates.

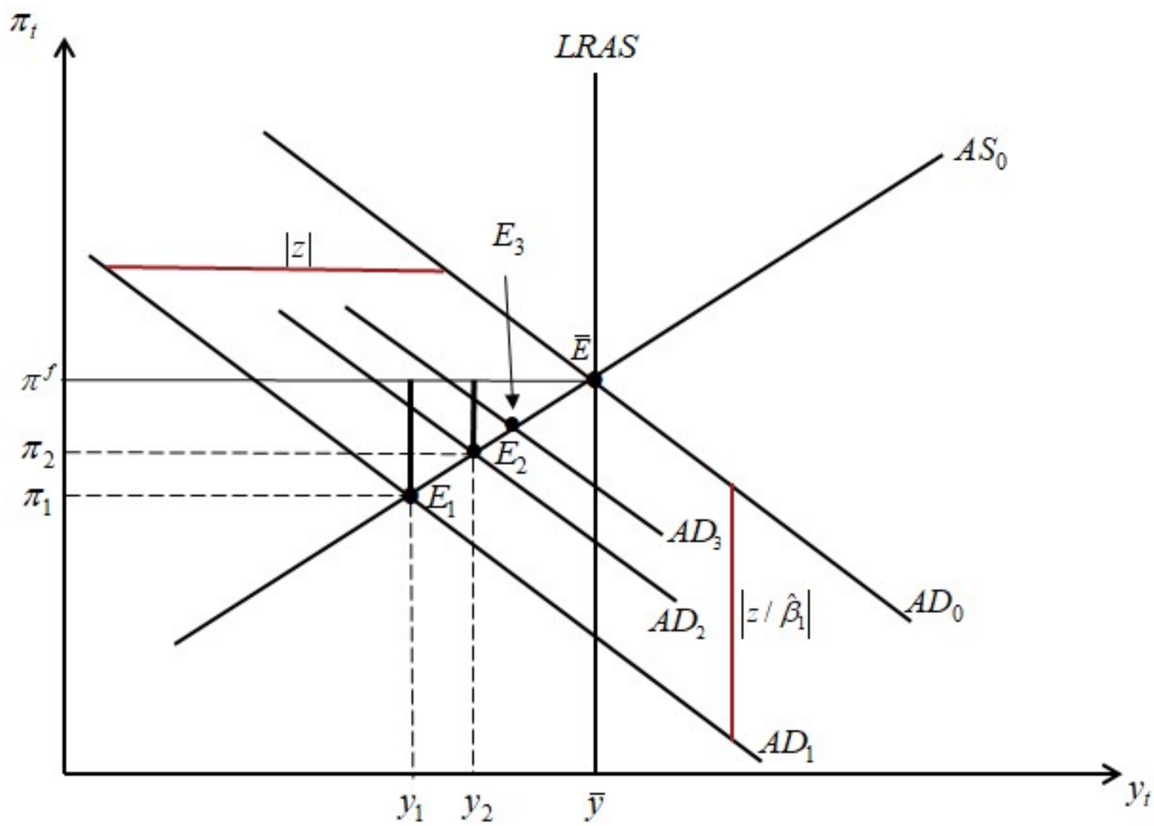
From short-run to long-run equilibrium

Figure 25.2 illustrates this adjustment process. We start out from a long-run equilibrium in period 0, with no shocks in the period and the real exchange rate at its structural level the period before, $z_t = s_t = 0$ and $e_{-1}^r = \bar{e}^r = 0$. The AD curve and the AS curve will then both pass through $(\bar{y}, \pi^f)$, as illustrated by AD_0 and AS_0 in the figure. Hence, the short-run and the long-run equilibrium coincide in period 0, both equal to $\bar{E} = (\bar{y}, \pi^f)$. This implies $e_0^r = \bar{e}^r = 0$ from (24). In period 1, a permanent (very long-lasting) negative demand shock $z < 0$ occurs, say, due to prolonged lower business confidence, while $s_t = 0$ for all *t*. As noted above, this will shift the AD curve to the left by $|z|$ and downwards by $|z / \hat{\beta}_1|$ from period 0 to 1, as illustrated by AD_1 in the figure. This effect remains in the following periods, but here the AD curve will shift further in reaction to changes in the real exchange rate. The AS curve remains at AS_0. This yields the short-run equilibrium E_1 in period 1 involving $y_1 < \bar{y}$ and $\pi_1 < \pi^f$, as illustrated in the figure.

Since domestic inflation is lower than foreign inflation in period 1, the real exchange rate increases from period 0 to period 1 from zero up to $e_1^r = (1 + \frac{h}{\theta})(\pi^f - \pi_1) > 0$ according to (24), meaning that the international competitiveness of domestic producers improves by this amount during period 1. According to (25) this in isolation shifts the AD curve upwards from period 1 to 2 by the amount $[\beta_1(1 + \frac{h}{\theta}) / (\beta_1(1 + \frac{h}{\theta}) + \beta_2 h)](\pi^f - \pi_1)$, which is smaller than $\pi^f - \pi_1$. Since the demand shock is unchanged at $z < 0$ in period 2, all in all the AD curve shifts upwards from period 1 to period 2 by this amount. This is illustrated by the AD curve AD_2 in Figure 25.2, where we note that the upward shift from AD_1 to AD_2 is smaller than the inflation differential $\pi^f - \pi_t$. Domestic output and inflation therefore increase to the levels y_2 and π_2 of the short-run equilibrium E_2 in period 2, where $y_1 < y_2 < \bar{y}$ and $\pi_1 < \pi_2 < \pi^f$.

In period 2, domestic inflation is still lower than foreign inflation, so the AD curve shifts upwards again from period 2 to period 3, and again by a smaller amount than the inflation differential in period 2, $\pi^f - \pi_2$. This is illustrated by AD_3 in the figure giving rise to the short-run equilibrium E_3 and so on. Since the AD curve from each period to the next shifts upwards by a certain fraction (less than one) of the period's inflation differential, it is bound to converge towards AD_0 in the long run. Therefore, the economy gradually converges towards the long-run equilibrium $\bar{E}$, where $y_t = \bar{y}$ and $\pi_t = \pi^f$ in all periods.

Figure 25.2 The adjustment to long-run equilibrium under floating exchange rates



After all adjustments have taken place, the initial and permanent drop in demand arising, e.g., as a result of lower business confidence, has been exactly compensated by an increase in demand (mainly through higher net exports) caused by the increased competitiveness (real exchange rate) resulting from lower domestic inflation. Since inflation is back at its target level π^f in long-run equilibrium, strict inflation targeting implies that the real interest rate will be back at its structural level $\bar{r}^f$, but the real exchange rate, which has increased throughout the adjustment period, will be higher in the new long-run equilibrium than in the old one. The domestic economy produces the same amount of commodities as before, but is now poorer as a result of the worsened terms of trade associated with the improved competitiveness (larger real exchange rate).

The speed of adjustment under alternative exchange rate regimes

As mentioned, under fixed exchange rates the *vertical shift* in the AD curve from one period to the next equals the *full* distance $\pi^f - \pi_t$, while under floating exchange rates with strict inflation targeting it is only a fraction of this distance. Hence, one might be tempted to conclude that the adjustment towards long-run equilibrium is faster under fixed than under floating exchange rates. However, this would be wrong since we have also shown that the *slope* of the AD curve is *flatter* under floating exchange rates with strict inflation targeting than under fixed exchange rates. It is easy to visualize from Figure 25.2 that with a flatter AD curve, a given vertical shift in this curve will cause output and inflation to move closer towards their long-run equilibrium values.

In economic terms, there are the following two opposing effects on the relative speed of adjustment under fixed versus floating exchange rates, say, after a long-lasting negative demand shock:

Under floating exchange rates with strict inflation targeting a given change in inflation has a larger impact on demand (as mirrored in the flatter AD curve), which pulls in the direction that aggregate demand recovers faster and thereby that the economy converges faster towards long-run equilibrium.

However, under floating exchange rates monetary policy is gradually tightened all through the adjustment process – after the initial relaxation due to the first fall in inflation – as inflation increases back towards its long-run level (which is mirrored in the smaller vertical shifts of the AD curve). This pulls in the direction of slower recovery of aggregate demand and thereby slower convergence to long-run equilibrium.

Thus we cannot say *a priori* whether the economy's speed of adjustment is higher under fixed than under floating exchange rates.

Let us now identify the parameters which are crucial for the relative speed of adjustment under the two exchange rate regimes. For this purpose we introduce our familiar 'gap' variables:

$$\hat{y}_t \equiv y_t - \bar{y}, \qquad \hat{\pi}_t \equiv \pi_t - \pi^f,$$

and write the model (22)–(24) as:

$$\hat{y}_t = \beta_1 e^r_{t-1} - \hat{\beta}_1 \hat{\pi}_t + z_t, \tag{26}$$

$$\hat{\pi}_t = \gamma \hat{y}_t + s_t, \tag{27}$$

$$e^r_t = e^r_{t-1} - (1 + \frac{h}{\theta})\hat{\pi}_t. \tag{28}$$

As in the convergence analysis in Chapter 24, since we are interested in adjustment of the economy *after* some shocks have occurred, we set the shocks for the adjustment period equal to zero, $z_t = s_t = 0$. Using (27) to eliminate $\hat{\pi}_t$ from (26) and solving for e^r_{t-1}, we get:

$$e^r_{t-1} = \left(\frac{1+\hat{\beta}_1\gamma}{\beta_1} \right) \hat{y}_t. \tag{29}$$

By substituting (29) and the corresponding expression for e^r_t into (28) along with the expression for $\hat{\pi}_t$ given in (27), and then inserting the definition of $\hat{\beta}_1$ from (22), you may verify that the evolution of the output gap under *floatingh exchange rates* is given by the following difference equation:

$$\hat{y}_{t+1} = \beta^{\text{float}} \hat{y}_t, \quad \beta^{\text{float}} \equiv \frac{1+\beta_2 h\gamma}{1+\hat{\beta}_1\gamma} = \frac{1+\beta_2 h\gamma}{1+\beta_2 h\gamma + \beta_1(1+\dfrac{h}{\theta})\gamma} < 1. \tag{30}$$

The solution to (30) is:

$$\hat{y}_t = \hat{y}_0 (\beta^{\text{float}})^t, \qquad t = 0, 1, 2, \ldots \tag{31}$$

From (30) we see that the coefficient β^{float} is positive and strictly smaller than 1. According to (31) this guarantees that the economy with floating exchange rates is stable and converges monotonically on its long-run equilibrium, as illustrated in Figure 25.2 above.

For comparison, the counterpart of Equation (30) above under *fixed exchange rates* is the previous chapter's Equation (15):

$$\hat{y}_{t+1} = \beta^{\text{fixed}} \hat{y}_t, \quad \beta^{\text{fixed}} \equiv \frac{1}{1+\beta_1\gamma} < 1. \tag{32}$$

We see that $\beta^{\text{fixed}} = \beta^{\text{float}}$ for $h = 0$: the difference in the speed of adjustment under the two exchange rate regimes arises from the fact that only a flexible exchange rate leaves scope for an independent monetary policy.

The speed of adjustment to long-run equilibrium will be lower (higher) under fixed than under floating exchange rates if $\beta^{\text{fixed}} > \beta^{\text{float}}$ ($\beta^{\text{fixed}} < \beta^{\text{float}}$). However, due to the two counteracting effects described above, there is no clear-cut evaluation of which of the two β's is the largest. Hence, comparing fixed exchange rates to floating exchange rates with *strict* inflation targeting, we have to say that it 'depends on parameters' which regime gives the fastest convergence. Indeed, from the definitions in (30) and (32) and the definition of $\hat{\beta}_1$, one can show that: $\beta^{\text{fixed}} > \beta^{\text{float}} \Leftrightarrow 1/\theta > \gamma\beta_2$.

Given $h > 0$, the relative magnitude of the parameters θ and $\gamma\beta_2$ will determine whether the net effect of the two offsetting mechanisms is to speed up or to delay the convergence to long-run equilibrium under a floating exchange rate regime compared to a fixed exchange rate regime. As the next subsection shows, the model with floating exchange rates and *flexible* inflation targeting is different from the one with strict inflation targeting in important aspects. The conclusion that the relative speed of convergence depends on parameters remains nevertheless.[3]

To recap:

MACROECONOMIC ADJUSTMENT UNDER FLOATING EXCHANGE RATES

A rise in domestic inflation causes a greater drop in aggregate demand under floating exchange rates with strict inflation targeting than under fixed exchange rates, so the AD curve is flatter under the first regime. Under both regimes higher inflation induces a fall in demand arising from reduced competitiveness, but under floating exchange rates with strict inflation targeting the higher inflation also triggers a rise in the domestic interest rate. This has a negative effect on demand from a higher real interest rate as well as from the resulting appreciation of the domestic currency. Over time the economy with floating exchange rates adjusts towards long-run equilibrium via successive shifts in the AD curve, as the real interest rate and the real exchange rate continue to adjust so long as domestic inflation deviates from foreign inflation. As a matter of theory one cannot say whether the speed of adjustment will be higher under floating exchange rates with strict inflation targeting than under fixed exchange rates.

Even though we cannot say for sure whether deviations of output and employment from their natural rates will last longer under fixed than under floating exchange rates, theory *does* allow us to predict the relative magnitude of the short-run fluctuations in output and inflation occurring under the two exchange rate regimes when the economy is hit by shocks. This is important in connection with stabilization policy. The immediate, short-run deviations in output and inflation occurring *on impact* when a shock hits the economy are larger than the deviations *during the adjustments to long-run equilibrium* coming after the shock period and, as mentioned many times earlier in this book, larger fluctuations are for good reasons considered disproportionally more costly for society than smaller fluctuations.

The short-run fluctuations occurring on impact and their stabilization under fixed and floating exchange rates are the topic for the next section. However, since it is of importance for some of our conclusions there whether inflation targeting under floating exchange rates is strict or flexible, we will first explain how our model is modified in the

[3] Strict inflation targeting is the limiting case of flexible inflation targeting for $b \to 0$, so a conclusion obtained for $b = 0$ will still hold for $b > 0$ but small (basically a continuity argument).

latter case.

The model with flexible inflation targeting

So, now we will set up the more complicated model arising if we do not assume $b = 0$ in (2), so that the Taylor rule is $i = r^f + \pi^e_{t+1} + h(\pi - \pi^f) + b(y - \bar{y})$ with $b > 0$. We will first go through steps corresponding to the ones of taken above that led to the AD curve under strict inflation targeting.

From the Taylor rule that we now consider and the definition $r = i - \pi^e_{+1}$, we get the equation corresponding to (13) above:

$$r = r^f + h(\pi - \pi^f) + b(y - \bar{y}), \tag{13'}$$

and further using $\pi^e_{+1} = \pi^f$ and $(\pi^e_{+1})^f = \pi^f$, so that $r^f + \pi^e_{t+1} = i^f$, we get the equation corresponding to (14):

$$i - i^f = h(\pi - \pi^f) + b(y - \bar{y}). \tag{14'}$$

From (13') the real interest rate channel becomes:

$$r - \bar{r}^f = h(\pi - \pi^f) + b(y - \bar{y}) + r^f - \bar{r}^f, \tag{15'}$$

For the real exchange rate channel we use again (9), $\Delta e = -1/\theta(i - i^f)$, and (14') to get:

$$\begin{aligned} e^r &= e^r_{-1} + \Delta e + \pi^f - \pi = e^r_{-1} - \frac{1}{\theta}(i - i^f) + \pi^f - \pi \\ &= e^r_{-1} - (\pi - \pi^f) - \frac{1}{\theta}[h(\pi - \pi^f) + b(y - \bar{y})]. \end{aligned} \tag{16'}$$

Inserting (15') and (16') into the temporary AD curve (11), $y - \bar{y} = \beta_1 e^r - \beta_2(r - \bar{r}^f) + \tilde{z}$, gives:

$$\begin{aligned} y - \bar{y} &= \beta_1 \left\{ e^r_{-1} - (\pi - \pi^f) - \frac{1}{\theta}[h(\pi - \pi^f) + b(y - \bar{y})] \right\} - \beta_2[h(\pi - \pi^f) + b(y - \bar{y}) + r^f - \bar{r}^f] + \tilde{z} \\ &= \beta_1 e^r_{-1} - [\beta_1 + \beta_1 \frac{h}{\theta} + \beta_2 h](\pi - \pi^f) - [\beta_1 \frac{b}{\theta} + \beta_2 b](y - \bar{y}) - \beta_2(r^f - \bar{r}^f) + \tilde{z} \end{aligned}$$

Isolating here $y - \bar{y}$ on the left hand side, putting the foreign real interest rate shock into the shock variable *z* and stating the time subscripts explicitly then gives the equation corresponding to (18) above, *the AD curve under floating exchange rates with flexible inflation targeting*:

$$y_t - \bar{y} = \frac{\beta_1}{1+\beta_1\dfrac{b}{\theta}+\beta_2 b} e_{t-1}^r - \frac{\hat{\beta}_1}{1+\beta_1\dfrac{b}{\theta}+\beta_2 b}(\pi_t - \pi^f) + \frac{z_t}{1+\beta_1\dfrac{b}{\theta}+\beta_2 b} \quad \Leftrightarrow$$

AD: $$\pi_t = \frac{\beta_1}{\hat{\beta}_1} e_{t-1}^r + \pi^f - \frac{1+\beta_1\dfrac{b}{\theta}+\beta_2 b}{\hat{\beta}_1}(y_t - \bar{y}) + \frac{z_t}{\hat{\beta}_1}, \tag{33}$$

$$\hat{\beta}_1 \equiv \beta_1 + \beta_1 \frac{h}{\theta} + \beta_2 h > \beta_1,$$

where z_t has the same definitions as in (18) and (19).

The short-run aggregate supply curve is unchanged from (23),

AS: $$\pi_t = \pi^f + \gamma(y_t - \bar{y}) + s_t, \tag{34}$$

while from (16') the real exchange rate dynamics become:

RERD: $$e_t^r = e_{t-1}^r + (1+\frac{h}{\theta})(\pi^f - \pi_t) - \frac{b}{\theta}(y_t - \bar{y}). \tag{35}$$

The complete model under floating exchange rates with flexible inflation targeting thus consists of Equation (33), (34) and (35). Exercise 3 asks you to analyse this model in some detail. Here we will limit ourselves to a few remarks of importance for the next section.

First, it is no longer true that the AD curve under floating exchange rates is necessarily flatter than the one under fixed exchange rates, which is $y - \bar{y} = \beta_1 e_{-1}^r - \beta_1(\pi - \pi^f) + z$ from (19). The coefficient to $\pi - \pi^f$ in the first line of (33) becomes numerically smaller, that is, the AD curve becomes steeper in the y, π -diagram, the larger b is. In particular, for a given h, flexible inflation targeting, $b > 0$, implies a steeper AD curve than does strict inflation targeting, $b = 0$. The reason is that monetary policy counteracts the expansionary effects of a lower inflation more the larger b is. Indeed, the coefficient to $\pi - \pi^f$ can be anything between zero and minus infinity, the first for h small and b large, the second for h large and b small. Try to explain this intuitively. Hence, for a large monetary policy response to inflation deviations and a small response to output deviations, the AD curve will indeed be relatively flat and may well be flatter than under fixed exchange rates. However, for a small response to inflation deviations and a large one to output deviations the AD curve will be relatively steep and may well be steeper than under fixed exchange rates. It follows that for appropriate choices of h and b, the slope of the AD curve will be the same under fixed exchange rates and floating exchange rates with flexible inflation targeting.

Second, comparing Equations (22) and (33), we see that a demand shock z will imply a smaller (horizontal) shift in the AD curve in the y-dimension under flexible than under strict inflation targeting, but the (vertical) shift in the π-dimension is the same in the two cases. Try again to explain why this is so.

25.3 Stabilization under fixed versus floating exchange rates: is a floating exchange rate a shock absorber or an amplifier of shocks?

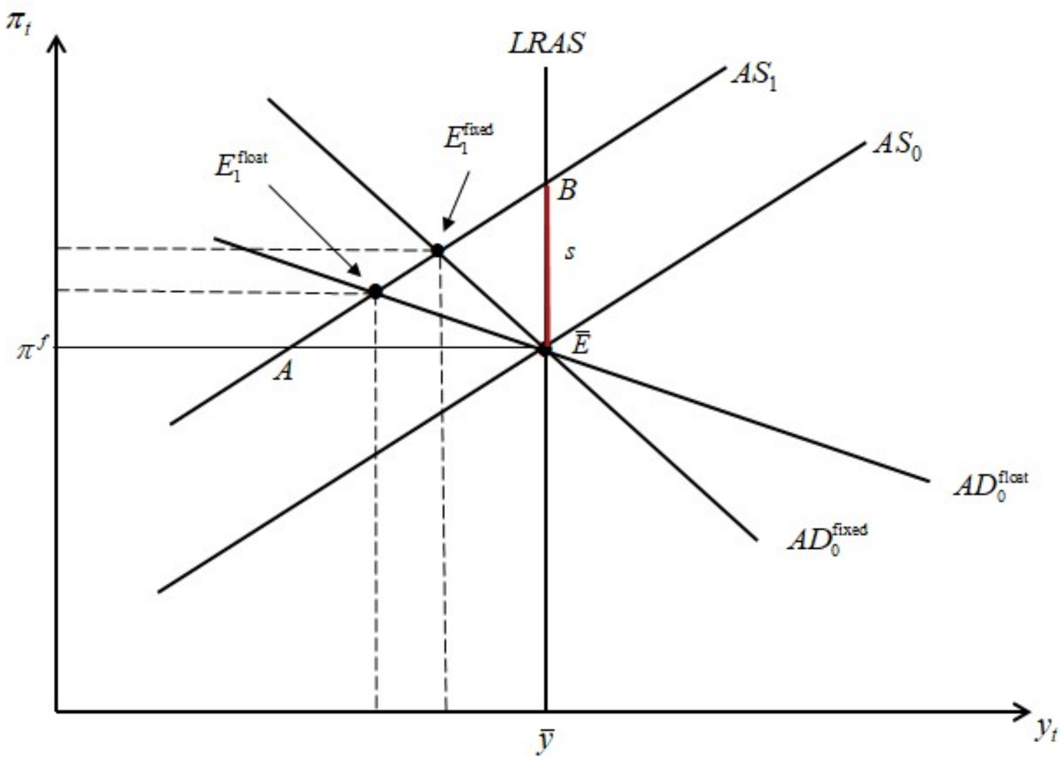
An important issue is whether a floating exchange rate can help to absorb shocks to the economy so that short-run fluctuations in output and inflation are reduced, or whether the response of a floating exchange rate will actually tend to amplify the effects of shocks on output and inflation? We will now use our AS–AD model of the open economy to analyse this question.

The short-run effects of aggregate supply shocks

We start by considering the short-run effects of aggregate *supply shocks* under alternative exchange rate regimes, first comparing fixed exchange rates to floating exchange rates with *strict* inflation targeting. We know that in this case the AD curve is flatter under floating than under fixed exchange rates. We depart from a long-run equilibrium in period 0 as illustrated in Figure 25.3 by the point $\overline{E}$, the intersection between the AS curve AS_0 for $s_0 = 0$ and the AD curve for $e^r_{-1} = z_0 = 0$, the latter in a relatively steep version for fixed exchange rates, AD_0^{fixed}, and a relatively flat version valid for floating exchange rates, AD_0^{float}.

Suppose now that the economy is hit by an unfavourable supply shock $s > 0$ in period 1, which shifts the AS curve in period 1 upwards by s from AS_0 to AS_1 as illustrated. We do not consider any demand shock, so the AD curve does not shift, and we only consider the effects in the shock period itself. As illustrated in the figure by the two alternative short-run equilibria in period 1, E_1^{fixed} and E_1^{float}, the unfavourable supply shock will have a larger negative short-run effect on output and a smaller positive impact on inflation under floating exchange rates with strict inflation targeting than under fixed exchange rates.

Figure 25.3 Short-run effects of a negative supply shock under fixed versus flexible exchange rates



The explanation is that whereas the interest rate stays constant under fixed exchange rates, under floating exchange rates with strict inflation targeting the domestic central bank responds to higher inflation by raising the interest rate, thereby depressing aggregate demand via the interest rate channel and the exchange rate channel.

Thus we conclude that under a floating exchange rate regime with strict inflation targeting, adjustments in the interest rate and in the nominal exchange rate help to dampen the fluctuations in inflation arising from supply shocks, but only at the cost of increased fluctuations in the level of output compared with a fixed exchange rate regime. Hence, one cannot conclude which of the two regimes is best for stabilization; this depends on society's preferences for output versus inflation stability.

This conclusion, however, hinges on the fact that we consider *strict* inflation targeting. Under floating exchange rates with *flexible* inflation targeting, the AD curve may well be steeper than under fixed exchange rates. For a given reaction parameter h, this will be the case for a sufficiently large b, as explained above. One can easily imagine from Figure 25.3 that in this case, output falls less and inflation increases more under floating exchange rates than under fixed exchange rates in response to a negative supply shock.

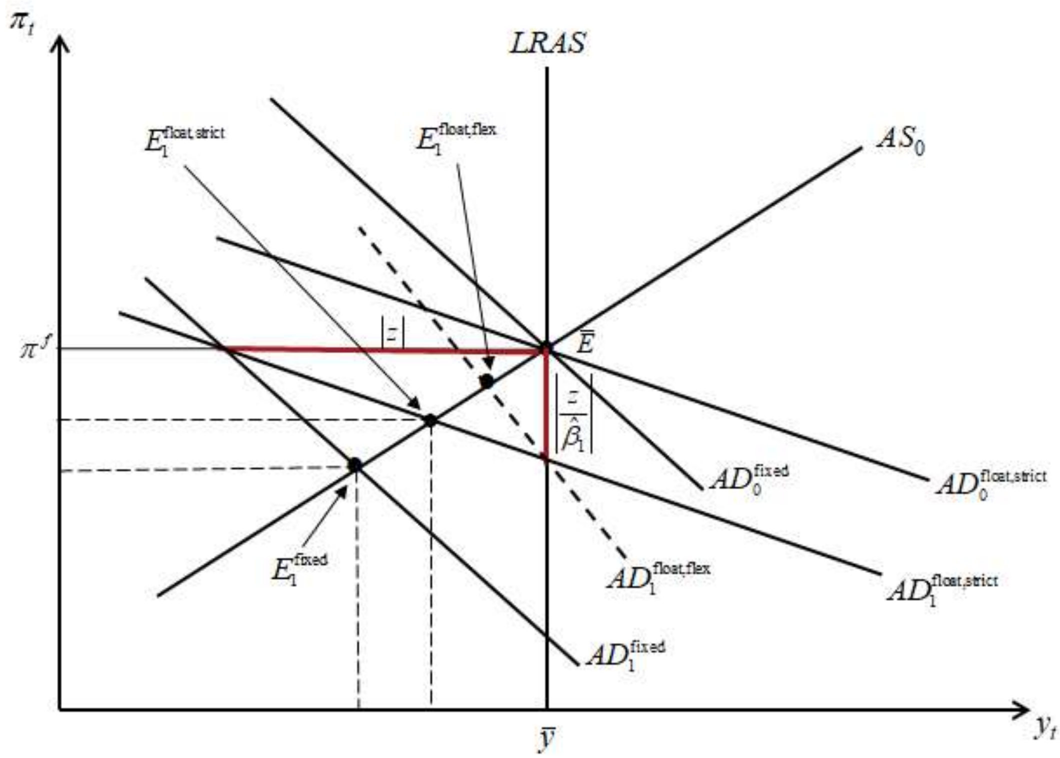
In fact, as was also explained above, any slope of the AD curve can be obtained with flexible inflation targeting and therefore any point on AS_1 between the points A and B in the figure can be obtained as a short-run equilibrium in period 1, in particular the point E_1^{fixed} obtained under fixed exchange rates.[4] Therefore, the 'right' conclusion on the premises of our model is that a regime of floating exchange rates (with possibly flexible inflation targeting) is better for stabilization than a regime of fixed exchange rates in connection with supply shocks. However, as we learned in Chapter 24, if we include systematic *fiscal* stabilization policy in the argument, anything that can be obtained under floating exchange rates can also be obtained under fixed exchange rates. Still, since monetary policy makers can often react more quickly than fiscal policy makers, as discussed in Chapter 20, the conclusion remains that floating exchange rates probably dominate a fixed exchange rate regime when it comes to stabilization of supply shocks.

The short-run effects of aggregate demand shocks

Let us next compare the short-run effects of aggregate *demand shocks* under alternative exchange rate regimes again first considering fixed versus floating exchange rates with *strict* inflation targeting. We start again from a long-run equilibrium in period 0 as illustrated by $\bar{E}$ in Figure 25.4. The AD curve in period 0 with a floating exchange rate and strict inflation targeting labelled $AD_0^{\text{float,strict}}$ is steeper than the one for a fixed exchange rate labelled AD_0^{fixed}. In period 1, a negative demand shock $z < 0$ occurs. As explained above this will shift the AD curve from period 0 to 1 by the same distance in the y-dimension under the two regimes, that is, the shock will give the same horizontal shift to the left, as illustrated in the figure by AD_1^{fixed} and $AD_1^{\text{float,strict}}$. This leads to the two, alternative short-run equilibria E_1^{fixed} and $E_1^{\text{float,strict}}$ in period 1. As seen from the figure, since the AD curve is flatter under the floating exchange rate, the fall in both output and inflation will be smaller than under a fixed exchange rate. The reason is that under the floating exchange rate, but not the fixed one, the central bank reacts to the lower inflation by lowering the interest rate, which stimulates demand through the real interest rate as well as the real exchange rate channel and thereby reduces the initial falls in output and inflation. Hence, for demand shocks, a floating exchange rate is unambiguously better for stabilization than a fixed one on the premises of our model.

[4] An exercise will ask you to show algebraically what we saw here graphically.

Figure 25.4 Short-run effects of a negative demand shock under fixed versus flexible exchange rates



Could this conclusion be invalidated if one considers *flexible* inflation targeting? For a given value of the reaction parameter *h*, a strictly positive *b* will make the AD curve steeper and possibly steeper than the one for fixed exchange rates. However, the horizontal shift in the AD curve will also be different for $b > 0$ (flexible inflation targeting) than for $b = 0$ (strict inflation targeting). As explained above, and as follows from comparing Equation (22) to Equation (33), the horizontal shift is numerically smaller for $b > 0$ than for $b = 0$, but the vertical shift is the same (for a given value of *h*). Hence, in case of flexible inflation targeting under a floating exchange rate, the appropriate shift of the AD curve in Figure 25.4 is down to the dashed one labelled $AD_1^{\text{float,flex}}$, which has shifted down by the same distance as $AD_1^{\text{float,flex}}$ (from an initial position in period 0 not indicated in the figure). Hence, output and inflation are stabilized even more under flexible than under strict inflation targeting and the conclusion that floating exchange rates give more stabilization than fixed rates (in the shock period) in connection with demand shocks remains.[5] The reason is again that floating exchange rates allow the central bank to take stabilizing monetary actions, while fixed exchange rates do not.

[5] An exercise will ask you to show algebraically what we saw here graphically.

Summing up on the effects of supply and demand shocks strictly on the premises of in our AS-AD model, we have:

STABILIZATION UNDER FIXED VERSUS FLOATING EXCHANGE RATES

When the economy is hit by supply shocks, any short-run deviations in output and inflation that may arise under fixed exchange rates can also be obtained under floating exchange rates with flexible inflation targeting for appropriate choices of the reaction parameters in the Taylor rule, but potentially preferred ones can also be obtained. When demand shocks hit, floating exchange rates unambiguously imply smaller short-run deviations in output and inflation than do fixed exchange rates and hence unambiguously more stabilization. All in all, on the assumptions of our model floating exchange rates give more and better stabilization than fixed exchange rates.

Seemingly, floating exchange rates are simply better when it comes to stabilization of business cycles. Why then do may economists and practitioners prefer fixed exchange rates? First, this may be due to the long-run, structural advantages of fixed exchange rates that we explained in Chapter 24 and will return to in Chapter 26, and which should be weighed against the possible short-run, stabilization advantages of floating exchange rates. However, and second, many economists doubt that floating exchange rates really give more macroeconomic stability than fixed rates on theoretical as well as empirical ground. This may simply be because of the *fiscal* stabilization policy that is possible – and particularly effective – under fixed exchange rates, but even disregarding this, many economists still think that fixed exchange rates give more macroeconomic stability. This is why we added 'strictly on the premises of the model' at several places above when we stated that floating exchange rates performed better with respect to stabilization. We now turn to features that have not yet been included in the model and to empirics which may speak in favour of fixed exchange rates as the most stabilizing exchange rate regime.

Is the exchange rate really a shock absorber?

To explain why many economists question whether a flexible exchange rate is really an effective shock absorber rather than an independent source of shocks, let us first illustrate *how* a floating exchange rate could be a source of shocks. Equation (9) above, saying $\Delta e = -(1/\theta)(i - i^f)$, was derived combining (8), a version of regressive exchange rate expectations, $\Delta e_{+1}^e = -\theta \Delta e$, with (1), *uncovered* nominal interest rate parity, $i = i^f + \Delta e_{+1}^e$. The latter assumes that investors are risk neutral and that foreign and domestic bonds are perfect substitutes. But suppose that financial investors are risk averse and that foreign and domestic bonds are considered to have different risk characteristics because they are issued in different political jurisdictions using different currencies. In equilibrium, the expected (average) returns to domestic and foreign bonds must therefore differ to compensate for the different risk characteristics of the two assets. This difference is the 'risk premium', which may consist of a 'systematic' component, v, plus a stochastic component, ε, which fluctuates around a zero mean value, reflecting, respectively, systematic and random shifts in the market's evaluation of the riskiness of the two assets or shifts in the 'appetite' for risk-taking. The relevant arbitrage condition is then the *covered* nominal interest rate parity (in which risk is 'covered'):

$$i = i^f + \Delta e^e_{+1} + v + \varepsilon, \quad E[\varepsilon] = 0. \tag{36}$$

Combining this again with (8), $\Delta e^e_{+1} = -\theta \Delta e$, gives instead of (9):

$$\Delta e = -\frac{1}{\theta}\Delta e^e_{+1} = -\frac{1}{\theta}(i - i^f) + \frac{1}{\theta}(v + \varepsilon). \tag{37}$$

Note that the systematic component v in the risk premium can be either positive or negative. For example, if the risk characteristics of domestic bonds are seen as more attractive than those of foreign bonds – say, because historically foreign bond prices have been more volatile than the prices of domestic bonds – v will tend to be negative.

Experience shows that risk premia in financial markets can vary quite a lot, as unexpected economic or political events motivate international investors to revise their evaluation of the relative riskiness of assets issued in different countries, sometimes without a fundamental change justifying the revision. For a given stance of domestic and foreign monetary policy – that is, for given values of i and i^f – we see from (37) that fluctuations in the stochastic component of the risk premium ε as well as changes in the systematic component v must generate fluctuations in the exchange rate. This is just a more formal statement of what we explained back in Chapter 23: the changing moods of international investors may well imply speculative capital movements that create a volatility in exchange rates which is not rooted in true, fundamental changes in the economies concerned.

This volatility in the nominal exchange rates may in turn contribute to instability in aggregate demand and thereby in output and inflation through its effect on the real exchange rate. The latter is still as an identity given by: $e^r = e^r_{-1} + \Delta e + \pi^f - \pi$. Inserting (37), second line gives: $e^r = e^r_{-1} - \frac{1}{\theta}(i - i^f) + \frac{1}{\theta}(v + \varepsilon) + \pi^f - \pi$. As also used in the derivation of the AD curve above, strict inflation targeting, $i = r^f + \pi^e_{+1} + h(\pi - \pi^f)$, and the assumption $\pi^e_{+1} = \pi^f = (\pi^f)^e$ implies $i - i^f = h(\pi - \pi^f)$. Inserting this in the expression for the real exchange rate then gives:

$$e^r = e^r_{-1} - (1+\frac{h}{\theta})(\pi - \pi^f) + \frac{v+\varepsilon}{\theta}. \quad (38)$$

This shows how the risk shocks in the covered nominal interest rate parity are passed on to the real exchange rate. When (38) is inserted for e^r in the temporary AD curve (11), $y - \overline{y} = \beta_1 e^r - \beta_2 (r - \overline{r}^f) + \tilde{z}$, then obviously a new shock appears on the right hand side: $\beta_1 (v+\varepsilon)/\theta$. This is what happens: If, say, investment in the assets of the domestic economy is suddenly considered more attractive, meaning $v+\varepsilon < 0$ (possibly without any fundamental change justifying this), capital will tend to flow into the domestic economy for given nominal interest rates, which will make the domestic currency appreciate, $\Delta e < 0$, until this – through regressive expectations – has created an expectation of a future depreciation, $\Delta e^e_{+1} = -\theta \Delta e > 0$, that restores covered nominal interest rate parity. For given inflation rates, monetary policy and hence interest rates will be unchanged, so the appreciation of the nominal exchange rate will work itself into the real exchange rate as shown by (38), causing the negative impact $\beta_1 (v+\varepsilon)/\theta$ on demand.

If the shifts in risk premia are frequent and significant, a floating exchange rate may thus become a source of excessive macroeconomic instability and not only an absorber of shocks arising elsewhere in the economy. The additional demand shock $\beta_1 (v+\varepsilon)/\theta$ will tend to create more volatility in output and inflation under a floating exchange rate, an effect that has to be weighed against the stabilizing effect of the stabilization policy possible under floating exchange rates.

Hence, what speaks for more macroeconomic stability under floating than under fixed exchange rates is the independent monetary stabilization policy possible under floating rates. Note, however, that this argument only applies in case of so-called asymmetric shocks. If a country with a fixed exchange rate is hit by the same (symmetric) shocks as the foreign economy against which it pegs its currency, then copying the monetary policy of the foreign country (setting the same nominal interest rate) should give just the 'right' stabilization. By 'right' we mean the same stabilization that the country could obtain through an independent monetary policy of its own. Hence, it is only – or at least mainly – if the shocks that the economies are exposed to are primarily asymmetric that an independent monetary policy may give an advantage with respect to macroeconomic stability. [6]

What speaks against more macroeconomic stability under floating exchange rates are the destabilizing effects of the excessive volatility in nominal and real exchange rates that

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[6] We assume here that the countries considered have roughly the same policy preferences regarding the stabilization of their output and inflation gaps. If these policy preferences differ a lot, the domestic and the foreign country may wish to react differently even if they are hit by the same shocks, so in this case it may be advantageous to have flexible exchange rates that allow the countries to pursue different monetary policies even under symmetric shocks. However, the fact that so many advanced economies have chosen similar inflation targets suggests that policy preferences regarding stabilization policies are rather similar across countries.

may occur under floating exchange rates and the fact that fiscal stabilization policy becomes more effective and may to some degree substitute for monetary stabilization policy under fixed exchange rates. It is thus an empirical matter whether one regime seems to give more macroeconomic stability than the other.

Here Sweden and Denmark constitute an interesting case for comparison. The two neighbouring countries are similar in many respects: they are not too different in size, they have similar political institutions and democratic rule, they share history and culture and are both examples of small and very open economies etc. However, with respect to exchange rate regime they differ fundamentally. Since late 1992, Sweden has had a floating exchange rate allowing an independent monetary policy with (flexible) inflation targeting. Denmark has pursued a strict fixed exchange rate policy, first as part of the European Monetary System, EMS, later by fixing its exchange rate vis-a-vis the euro as part of the European Exchange Rate Mechanism, ERM 2. If a floating exchange rate really gives substantially more macroeconomic stability than a fixed rate, this should be observable in the experiences of Sweden and Denmark.[7] In Table 25.2, we measure the (real) instability of GDP by the standard deviation of the output gap (computed through HP filtering) and the (nominal) instability of inflation by the coefficient of variation of the inflation rate. The table also reports on the average inflation levels and on the instability of the real effective exchange rate of the two countries. The period considered is 1996Q1-2020Q1 for the GDP and 1997Q1-2021Q1 for the other variables during which the exchange regimes in the two countries have been well established.

Table 25.2 Macroeconomic stability and inflation in Denmark and Sweden, 1996Q1-2021Q1

Country	Standard deviation of output gap, percentage points*	Coefficient of variation of inflation	Average annual inflation rate, per cent	Standard deviation of index for real effective exchange rate with 1997Q1 = 100
Denmark	1.4	1.3	1.4	3.0
Sweden	1.6	1.4	1.4	7.8

Note: Based on quarterly data. The output gaps have been estimated by the Hodrick-Prescott filter with $\lambda = 1600$ and cover 1996Q1-20020Q1 as four end observations have been excluded due to the end point problem of HP filtering. Quarterly, annualized inflation rates are calculated from the Harmonized Index of Consumer Prices over four quarters backwards and cover 1997Q1-2021Q1. The coefficient of variation is the standard deviation divided by the mean. Real effective exchange rates cover 1997Q1-2021Q1.
Sources: Real GDP from the National Accounts Statistics database, OECD; inflation and real effective exchange rates from the International Financial Statistics database, IMF.

[7] There is one proviso here: If the industrial structures of the two countries are very different, the sensitivity of their domestic economic activity to the international business cycle could differ in which case the differences in the volatility of their output and inflation gaps would not only stem from differences in their exchange rate regimes. A deeper scientific comparison of the macroeconomic experiences of Denmark and Sweden would have to control for differences in industrial structures, but as mentioned, the two countries are in fact similar in many important respects which makes our simple comparison interesting.

If anything, there seems to be more macroeconomic stability in Denmark than in Sweden, but the differences in the variabilities of output and inflation are small and insignificant. The average inflation levels are also very similar in the two countries. The real difference between them lies in the instability of the real exchange rate, which is substantially higher in Sweden than in Denmark.

Some advocates of floating exchange rates argue that it is not so much in connection with normal, everyday fluctuations that a floating exchange rate and the independent monetary policy going with it has its advantages, but in connection with the 'big swings' in economic activity. Therefore, Figure 25.5 shows the output gaps of Sweden and Denmark over the period also considered in Table 25.2, a period that indeed involved some big swings: a serious overheating in the run-up to the financial crisis starting in 2008-09 and afterwards the deep downturn of the financial crisis itself. The figure bears no evidence that the large swings should be smaller in Sweden than in Denmark.

Figure 25.5 Danish and Swedish output gap, 1996Q1-2020Q1

Note: Based on the same estimates of the output gaps as Table 25.3
Source: As Table 25.2.

All in all, the empirics of Table 25.2 and Figure 25.5 suggest: 1) A fixed exchange rate and an inflation target (under a floating exchange rate) seem to serve as nominal anchors for inflation expectations to about the same degree. 2) The independent monetary stabilization policy possible under a floating exchange rate seems to be counteracted by excessive variability of the real exchange rate to a degree giving about the same macroeconomic instability as under a fixed exchange rate.

If this is the general lesson, the argument that remains is that a fixed exchange rate probably gives some long-run, structural advantages mostly through intensified trade that a floating rate cannot deliver. This would leave fixed exchange rates as the exchange rate regime to prefer. However, we should add that economists disagree on this issue and there are other empirical investigations in the literature of the relative macroeconomic stability implied by fixed versus floating exchange rates that point to different conclusions than those obtained from the analysis performed here.[8] The issue whether fixed or floating exchange rates give most stability is therefore unsettled. For now, we recap:

THE ROLE OF EXCHANGE RATE FLUCTUATIONS UNDER FLEXIBLE EXCHANGE RATES

A flexible exchange rate can help to absorb the effects of shocks to the economy by allowing an independent monetary stabilization policy, but shifting expectations and risk premia in foreign exchange markets mean that exchange rate fluctuations can also be an independent source of shocks. There is evidence that supports that instability of the exchange rate is indeed a source of shocks under floating exchange rates and sufficiently important to compensate to a large degree or even fully for the higher stability resulting from an independent monetary stabilization policy.

25.4 Fixed versus floating exchange rates

Over the Chapters 23 through 25 we have explained the features of fixed and floating exchange rate regimes, the latter with inflation targeting, and presented arguments for and against each regime. Let us now sum up the important debate on fixed versus floating exchange rates.

- There is broad consensus that monetary policy should create a 'nominal anchor' by adopting some rule that helps to keep inflation expectations stable at a low, but no too low, level. The reason is that such 'anchored' inflation expectations will help to stabilize actual inflation around the same low level because expected inflation tends to work itself into actual inflation through the economy's nominal wage and price

[8] These investigations often involve many countries, which can be an analytical advantage, but at the same time, many of these countries will typically not be as comparable as Sweden and Denmark and will often not have as 'clean' exchange rate regimes.

contracts reflecting expected inflation. In Chapter 14 we explained many reasons why a stable and low, but not too low, inflation rate is preferable mainly for long-run efficiency reasons, but anchored inflation expectations are also good for short-run stability reasons. If medium-term inflation expectations are solidly anchored at a certain level (say 2 percent per year), then in a recession where short-run actual inflation and output drop, inflation expectations will not drop so much and therefore the real interest rate will be kept down, which helps to keep up demand and activity, and vice versa in a boom. Thus anchored inflation expectations are good for many reasons, but both fixed exchange rates (where the exchange rate is fixed against the currency of a low-inflation area) and floating exchange rates with inflation targeting can provide such an anchor. It is not easy to say which one does it best, so on this ground there is no clear winner.

- There is also broad consensus that, *ceteris paribus*, it is good to have relatively stable actual and expected nominal exchange rates, since this promotes trade and financial integration, which can increase prosperity by improving the allocation of resources. This seems to speak for fixed exchange rates based on long-run efficiency concerns, but this statement must be qualified, as the next bullet point will explain.
- As long as a country has its own currency, a "fixed" exchange rate may be adjusted in the future. Hence, just to declare a regime of fixed exchange rates does not necessarily provide stable actual and expected exchange rates. This requires that the exchange rate parity is not changed, and is not believed to be changed, every now and then. For instance, if fiscal and structural policies in a country with a declared fixed exchange rate regime generate a high inflation rate at the rates of unemployment that the country can accept, then this country will lose competitiveness as long as the exchange rate is kept fixed, which will create expectations of future devaluations. This in turn will imply a pressure on the exchange rate from speculative capital movements and therefore probably actual devaluations, so that instability of exchange rates results. On the other hand, if fiscal and structural policies support the fixed level of the exchange rate, and the parity is kept fixed for long times, then stable actual and expected exchange rates should result. So, the qualified statement is: from purely long-run structural concerns, a *credible* fixed exchange rate regime seems to be preferable to a regime of floating exchange rates.
- However, macroeconomic stability in the short run is also important, and *ceteris paribus* the independent monetary stabilization policy possible under floating exchange rates, but not under fixed, speaks for more stability under floating exchange rates with inflation targeting than under fixed exchange rates.[9] But here several reservations are relevant. First, it is mainly to the extent that the economic shocks hitting the economy are asymmetric, hitting the country considered differently than the area towards which it would fix its exchange rate, that the independent monetary stabilization policy really gives a stabilization advantage. Second, under fixed exchange rates fiscal stabilization policy can to some degree substitute for the

[9] If short-run stability contributes to a good long-run performance, the arguments concerning stability are only reinforced.

monetary stabilization policy not possible, and fiscal policy is relatively effective under fixed exchange rates. Yet the consensus seems to be that if it were only for these two reservations, then from stabilization concerns, a floating exchange rate with inflation targeting would be the preferable regime. Asymmetric shocks *are* typically of importance unless the country considered is very deeply integrated with the area against which it would potentially fix its exchange rate,[10] and monetary stabilization policy *does* have some advantages over fiscal stabilization policy mainly arising from the shorter time lags involved. But, there is a third reservation:

- The stabilization argument for floating exchange rates assumes that the shocks hitting the country considered are the same irrespective of the exchange rate regime, but there is evidence that floating exchange rates can be an independent source of shocks through the impulses to real exchange rates that fluctuating speculative capital movements can generate under such a regime. Whether this is sufficiently important to eliminate the potential stability advantage of floating exchange rates may well depend on the country and time period considered, and there is substantial disagreement among economists about this.
- A final issue concerns credibility. An ultimate *target* of a fixed exchange rate regime is the stable and relatively low level of actual and expected inflation it is intended to create. The *instruments* are (conventionally) the central bank's interventions in the exchange markets and its short-term interest rates and (less conventionally) its longer-term interest rates and, e.g., its asset purchase programs. These instruments are what the central bank can actually decide on, whereas it cannot decide directly on inflation, and it is not easy to assess from observations of actual inflation whether the central bank has done a good job in stabilizing inflation, since inflation is affected by many factors beyond the central bank's control. A fixed exchange rate serves as a so-called *intermediate target*,meaning that the central bank adjusts its instruments until a certain exchange rate (the parity rate) results. The intended effect is to stabilize inflation and inflation expectations which cannot be directly controlled, but on the other hand it is easy to check whether the central bank actually sticks to its declared monetary policy; this is simply a matter of observing to what degree the nominal exchange rate is stable around its parity rate. This can help to make the central bank's monetary policy credible, which in turn will help to bring about its intended effects. Hence, an advantage of a fixed exchange rate regime is that its intermediate target is relatively easy to control and very easy to observe. For a monetary policy of floating exchange rates with inflation targeting the instruments and the target (low and stable inflation) are the same, but is there an intermediate target, and if so, is it as observable as the one under fixed exchange rates? From a first consideration there does not seem to be an intermediate target as the intention of the central bank is to adjust its instruments until a certain target inflation rate results. However, the instruments of the central bank do not affect the inflation rate immediately, so the bank has to adjust its instruments to create a certain inflation *forecast*, its expectation of the future inflation rate generated by its current policy decisions. Inflation targeting thus

[10] We return to this issue in the next chapter on optimal currency areas and monetary union.

becomes *inflation forecast targeting* in reality, and the central bank's inflation forecast effectively becomes the intermediate target of its monetary policy. However, even though inflation targeting central banks publish their inflation forecasts, it is not easy for the government and the general public to hold them accountable for the outcome of their policies since the time lag in the effect of monetary policy on inflation may vary over time, and since inflation is affected by many other factors in the short and medium run. By contrast, p it is much easier to hold the central bank accountable under fixed exchange rates where one can immediately observe whether the bank manages to keep the exchange rate fixed. Proponents of fixed exchange rates argue that this may make it easier for the central bank to establish credibility which is alpha and omega in monetary policy.

There are many pros and cons in this discussion and little overall agreement. For a small area, state or country, that is strongly integrated with its surroundings with respect to trade, ownership of financial instruments and mobility of persons, like, e.g., the American state of Iowa, the Danish island Funen, the French department Loire, the countries of San Marino or Andorra, no one would argue that it should have its own, floating currency. Likewise, hardly anyone argues in favour of worldwide fixed exchange rates nowadays (since the breakdown of the Bretton-Woods system) or a worldwide currency union. However, as soon as one considers a country of reasonable size with a people, history, culture, geography, climate and political institutions etc. of its own, the opinions on advantages and drawbacks of fixed versus floating exchange rates are divided. On this background, it is no wonder that different countries have taken different routes with a tendency towards polarization of the regimes. The big economic areas of the world like USA, the Euro Zone, Japan, Australia, Canada etc. have own currencies that float freely vis-à-vis each other. Many smaller countries have chosen either a monetary regime of freely floating currencies like the UK, Sweden, Norway and Switzerland, or a rather strict fixed exchange rate regime. The latter can be in the form of an own currency, but with a hard peg like Denmark vis-à-vis the euro, or in the ultimate form of a currency union as for all the American states and all the countries in the Euro Zone.

The question of when it is sensible or even optimal for a country to take the ultimate step towards fixed exchange rates by joining a currency union is, of course, of particular importance. This question is the starting point of our next and final chapter on the economics of monetary union.

Summary

1. A crucial characteristic of a floating exchange rate regime is that it enables the domestic central bank to pursue an independent monetary policy even if capital mobility is perfect.
2. Under a floating exchange rate regime the nominal exchange rate cannot serve as a nominal anchor for inflation expectations. To provide an alternative nominal anchor, many countries with floating exchange rates have adopted a monetary policy regime of inflation targeting where the central bank seeks to keep inflation within a narrow band around some target rate which is typically set to 2 – 2.5 per cent. Under strict inflation targeting the central bank's interest rate reacts only to deviations of inflation from the target, whereas flexible inflation targeting implies that the central bank also reacts to the evolution of output and employment. The fact that inflation-targeting countries in the OECD have chosen roughly the same inflation target suggests that these countries try to avoid a systematic depreciation or appreciation of their nominal exchange rates over the longer run.
3. Most inflation-targeting countries have delegated monetary policy to an independent central bank to strengthen the credibility of the inflation target. When the target is credible, it will serve as an anchor for domestic inflation expectations. The model in this chapter assumes that agents have weakly rational expectations in the sense that the expected inflation rate equals the central bank's inflation target. Given that countries choose (roughly) the same inflation target, this means that the expected domestic inflation rate is (roughly) equal to the foreign inflation rate.
4. Under floating exchange rates and perfect capital mobility the difference between the domestic and the foreign nominal interest rate is given by the expected rate of change in the nominal exchange rate. According to the hypothesis of regressive exchange rate expectations, the exchange rate is expected to rise if it is currently below its perceived normal level, and vice versa. The perceived normal exchange rate is likely to depend on the actual exchange rates observed in the past. The model in this chapter makes the simplifying assumption that the perceived normal exchange rate equals last period's actual exchange rate. From this assumption plus the assumption of uncovered interest rate parity it follows that a positive differential between the domestic and the foreign interest rate will cause an appreciation of the domestic currency, whereas a negative interest rate differential will generate a depreciation. The empirical evidence lends some support to this hypothesis.
5. Under 'clean' floating the central bank does not intervene in the foreign exchange market. Under 'dirty' floating the central bank intervenes with the purpose of reducing fluctuations in the exchange rate. In the case of 'sterilized' interventions the central bank buys or sells foreign currency without changing the interest rate. To have a lasting impact on the exchange rate, interventions have to be 'unsterilized',

involving a change in the interest rate which supports the movement in the exchange rate that the central bank is trying to achieve. When the central bank systematically raises (lowers) the interest rate in response to a depreciation (appreciation) of the domestic currency, it is said to follow a policy of 'leaning against the wind'.

6. In a floating exchange rate regime monetary policy affects aggregate demand both through the interest rate channel and through the exchange rate channel. The interest rate channel is the direct impact of a change in the nominal interest rate on the real one and thereby on investment and consumption demand. The exchange rate channel is the impact through the foreign exchange market: when the interest rate is lowered, there is a tendency for the exchange rate to depreciate so that net exports increase.

7. Under floating exchange rates with strict inflation targeting a fall in the domestic rate of inflation boosts aggregate demand through three effects. First, there is a direct positive effect on net exports as domestic competitiveness improves. Second, a lower rate of inflation induces an inflation-targeting central bank to reduce the interest rate, thereby stimulating investment and consumption. Third, the lower interest rate causes a depreciation which gives a further stimulus to net exports. The latter two effects are absent under fixed exchange rates. Hence the aggregate demand curve is flatter under floating exchange rates with strict inflation targeting than under fixed exchange rates. Under floating exchange rates with flexible inflation targeting, the AD curve can be flatter or steeper than the AD curve under fixed exchange rates because of the central bank's reactions to output deviations.

8. The AS–AD model for the open economy has a similar structure under fixed and under floating exchange rates, but the quantitative properties of the economy will differ under the two regimes. *A priori* one cannot say whether convergence on long-run equilibrium will be faster under one or the other regime.

9. Under floating exchange rates with strict inflation targeting, supply shocks will cause larger short-run fluctuations in output but smaller fluctuations in inflation than under fixed exchange rates. However, under floating exchange rates with flexible inflation targeting, supply shocks may cause larger or smaller short-run fluctuations in output and smaller or larger fluctuations in inflation than under fixed exchange rates depending on the policy reaction parameters of the Taylor rule. In particular, one can have the same fluctuations as under fixed exchange rates. Our AS–AD model also predicts that demand shocks (including fiscal policy shocks) will generate smaller fluctuations in output as well as inflation under floating than under fixed exchange rates, because a floating exchange rate allows the central bank to counteract demand shocks through its interest rate policy.

10. When domestic and foreign assets have different risk characteristics and financial investors are risk averse, fluctuations in required risk premia can cause fluctuations in nominal and real exchange rates under floating exchange rates. Empirical evidence suggests that this can be an important, independent source of macroeconomic instability that to a large degree, and possibly fully, revokes the increased stability that obtained by independent monetary stabilization policy.

Exercises

Exercise 1. The aggregate demand curve under floating exchange rates

The derivation of the AD curve in the main text assumed that the expected 'normal' exchange rate is equal to the actual exchange rate observed during the last period. This exercise asks you to demonstrate that under certain assumptions, one can derive a similar AD curve under floating exchange rates by simply assuming that the expected normal exchange rate is exogenous.

We start by assuming that the real interest rate affects aggregate demand mainly through its influence on business investment. Assuming that it only takes one period for firms to adjust their capital stock to its desired level K^d, and abstracting from depreciation on the existing capital stock, total gross investment in the current period, I, will equal the desired increase in the capital stock:

$$I_t = K_t^d - K_{t-1}^d. \tag{39}$$

Reflecting the underlying growth in aggregate demand, the desired capital stock varies positively with time, but at the same time it varies negatively with the real interest rate. Adopting a linear specification for convenience, we thus have:

$$K_t^d = at - br_t \quad \Rightarrow \quad I_t = a - b(r_t - r_{t-1}), \tag{40}$$

where a and b are constants. In other words, the *level* of investment depends on the *change* in the real rate of interest. Dropping the time subscripts for the current period, we may therefore approximate the goods market equilibrium condition by:

$$\begin{aligned} y - \bar{y} &= \beta_1 e^r - \beta_2 (r - r_{-1}) + \tilde{z}, \\ \tilde{z} &\equiv \beta_3 (g - \bar{g}) + \beta_4 (y^f - \bar{y}^f) + \beta_5 (\ln \varepsilon - \ln \bar{\varepsilon}), \end{aligned} \tag{41}$$

where we apply the usual notation (the constant a has dropped out because (41) considers a deviation from trend, and the parameter b is incorporated in β_2).

Now suppose that the central bank targets the foreign inflation rate by raising the real interest rate when domestic inflation is above foreign inflation, and vice versa:

$$r - r_{-1} = h(\pi - \pi^f). \tag{42}$$

In addition we have the condition for uncovered interest rate parity:

$$i = i^f + e^e_{+1} - e, \qquad (43)$$

where we assume that exchange rate expectations are regressive:

$$e^e_{+1} - e = \theta(\overline{e} - e), \qquad \theta > 0. \qquad (44)$$

Here we treat the expected normal exchange rate, $\overline{e}$, as an exogenous variable, although it may change from time to time.

By definition, the *ex ante* domestic real interest rate is $r \equiv i - \pi^e_{+1}$. The central bank has credibility, so the expected domestic inflation rate equals the central bank's inflation target, that is, $\pi^e_{+1} = \pi^f_{+1} = \pi^f$. Hence we have:

$$r = i - \pi^f \qquad (45)$$

Finally, we have the familiar definition of the foreign real interest rate,

$$r^f \equiv i^f - \pi^f, \qquad (46)$$

and the bookkeeping identity for the current real exchange rate:

$$e^r = e^r_{-1} + e - e_{-1} + \pi^f - \pi. \qquad (47)$$

1. Use Equations (41)–(47) to show that the aggregate demand curve takes the following form:

$$y - \overline{y} = \beta_1 e^r_{-1} - \hat{\beta}_1(\pi - \pi^f) + z, \qquad \hat{\beta}_1 \equiv \beta_1 + h(\beta_2 + \beta_1\theta^{-1}), \qquad (48)$$

$$z \equiv \beta_1 \Delta\overline{e} + \beta_1\theta^{-1}\Delta r^f + \beta_3(g - \overline{g}) + \beta_4(y^f - \overline{y}^f) + \beta_5(\ln\varepsilon - \ln\overline{\varepsilon}). \qquad (49)$$

where $\Delta\overline{e} \equiv \overline{e} - \overline{e}_{-1}$ and $\Delta r^f \equiv r^f - r^f_{-1}$ (Hint: as an intermediate step, use (43)–(46) to derive an expression for $r - r_{-1}$ in terms of $e - e_{-1}$, $\Delta\overline{e}$, and Δr^f. Then insert (42) to obtain an expression for $e - e_{-1}$ in terms of $\pi - \pi^f$, $\Delta\overline{e}$, and Δr^f.) Make a brief comparison between these results and the AD curve (18) (or (22)) in the main text.

2. Suppose that expectations of a permanent weakening of the domestic currency arise. How will this affect the aggregate demand curve? Give some examples of economic events which might generate expectations of a permanent depreciation of the domestic currency.

Exercise 2. 'Leaning against the wind' under floating exchange rates

This exercise invites you to explore the implications of a monetary policy regime where the central bank follows a policy of 'leaning against the wind' by setting the interest rate in accordance with the following policy rule, explained in the section on 'dirty floating':

$$i - \pi_{+1}^e = r^f + h(\pi - \pi^*) + \lambda(e - e_{-1}), \qquad h > 0, \qquad \lambda > 0. \tag{50}$$

Because of perfect capital mobility and risk neutrality, the condition for uncovered interest parity must hold:

$$i = i^f + e_{+1}^e - e, \qquad e \equiv \ln E, \qquad e_{+1}^e \equiv \ln E_{+1}^e. \tag{51}$$

We also continue to assume regressive exchange rate expectations so that:

$$e_{+1}^e - e = -\theta(e - e_{-1}), \tag{52}$$

and we maintain the assumption that the central bank's inflation target is credible and equal to the foreign inflation rate:

$$\pi_{+1}^e = \pi^e = \pi^* = \pi^f. \tag{53}$$

The goods market equilibrium condition, corresponding to Equation (11) in the main text, can be written as:

$$y - \bar{y} = \beta_1 \overbrace{(e_{-1}^r + \Delta e + \pi^f - \pi)}^{e^r} - \beta_2(i^f - \pi_{+1}^e + e_{+1}^e - e - \bar{r}^f) + \tilde{z}, \tag{54}$$

$$\tilde{z} \equiv \beta_3(g - \bar{g}) + \beta_4(y^f - \bar{y}^f) + \beta_5(\ln\varepsilon - \ln\bar{\varepsilon}).$$

Finally, we continue to work with an aggregate supply curve of the form:

$$\pi = \pi^e + \gamma(y - \bar{y}) + s. \tag{55}$$

1. Discuss briefly why the authorities might want to adopt a policy of 'leaning against the wind'.
2. Use Equations (50)–(55) to show that the economy's aggregate demand curve takes the form:

$$\pi = \pi^f + \left(\frac{\beta_1}{\tilde{\beta}_1}\right) e^r_{-1} - \left(\frac{1}{\tilde{\beta}_1}\right)(y - \bar{y} - z), \qquad \tilde{\beta}_1 \equiv \beta_1 + \frac{h(\beta_1 + \theta\beta_2)}{\theta + \lambda},$$

$$z \equiv -\beta_2(r^f - \bar{r}^f) + \beta_3(g - \bar{g}) + \beta_4(y_f - \bar{y}_f) + \beta_5(\ln\varepsilon - \ln\bar{\varepsilon}). \tag{56}$$

(Hint: follow the procedure described in Section 25.2). Explain the economic mechanisms underlying the negative slope of the AD curve. Explain how the policy of leaning against the wind affects the slope of the AD curve.

3. Derive the equations determing the change over time in the nominal and in the real exchange rate. Does the policy of leaning against the wind amplify or dampen the impact of the inflation gap $\pi - \pi^f$ on nominal and real exchange rates? Explain. (Hints: use (50)–(53) to express $e - e_{-1}$ in terms of $\pi - \pi^f$ and the parameters h, θ, and λ.) Give a graphical illustration of the economy's adjustment to long-run equilibrium and explain the adjustment mechanisms.

4. Does leaning against the wind slow down or speed up the economy's speed of adjustment to long-run equilibrium? (Hint: derive a difference equation of the same form as (30) in the main text and investigate by differentiation how the parameter λ affects the coefficient on the lagged output gap). Try to provide an economic explanation for your result.

5. Use the procedure described in Section 25.3 of the main text to investigate how the policy of leaning against the wind affects the economy's short-run reaction to supply and demand shocks. Does leaning against the wind amplify or dampen the short-run effects of the shocks? Give an economic explanation for your findings.

6. Demonstrate that if the policy of leaning against the wind is very aggressive (so the parameter λ tends to infinity), the economy will work approximately as if the exchange rate were fully fixed. (Hint: compare the difference equation derived in Question 4 for the case of $\lambda \to \infty$ with the corresponding difference equation for the output gap under fixed exchange rates, given in Equation (15) of Chapter 24 or (32) of the present). Give an intuitive explanation for your finding.

Exercise 3 A floating exchange rate regime with flexible inflation targeting

In the main text we assumed that the domestic central bank pursued so-called strict inflation targeting, reacting only to changes in the inflation gap. This exercise studies a regime of so-called flexible inflation targeting where interest rate policy follows a standard Taylor rule with a positive coefficient on the output gap. Our open economy with floating exchange rates is thus described by the following equations (in our usual notation):

Goods market: $$y-\bar{y}=\beta_1\overbrace{(e_{-1}^r+e-e_{-1}+\pi^f-\pi)}^{e_t^r}-\beta_2(i-\pi_{+1}^e-\bar{r}^f)+\tilde{z}, \quad (57)$$

Inflation expectations: $$\pi_{+1}^e=\pi^e=\pi^f, \quad (58)$$

Monetary policy: $$i=r^f+\pi_{+1}^e+h(\pi-\pi^f)+b(y-\bar{y}), \quad b>0, \quad (59)$$

Uncovered interest rate parity: $$i=i^f+e_{+1}^e-e, \quad (60)$$

Exchange rate expectations: $$e_{+1}^e-e=-\theta(e-e_{-1}), \quad (61)$$

Foreign real interest rate: $$r^f=i^f-\pi^f, \quad (62)$$

AS: $$\pi=\pi^e+\gamma(y-\bar{y})+s, \quad (63)$$

Real exchange rate: $$e^r=e_{-1}^r+e-e_{-1}+\pi^f-\pi. \quad (64)$$

1. Use Equations (57)–(62) to show that the economy's aggregate demand curve is given by (this was done in a subsection of Section 25.2, but now do it yourself):

$$\pi=\frac{\beta_1}{\hat{\beta}_1}e_{-1}^r+\pi^f-\frac{1+\beta_1\dfrac{b}{\theta}+\beta_2 b}{\hat{\beta}_1}(y-\bar{y})+\frac{z}{\hat{\beta}_1},$$
$$\hat{\beta}_1\equiv\beta_1+\beta_1\frac{h}{\theta}+\beta_2 h>\beta_1, \quad z\equiv-\beta_2(r^f-\bar{r}^f)+\tilde{z}. \quad (65)$$

Explain in economic terms how the central bank's reaction to the output gap affects the slope of the AD curve.

2. Use a (y, π) diagram to undertake a graphical analysis of the way the economy reacts to demand and supply shocks in the short run (i.e. in the first period), assuming that the economy is in long-run equilibrium in period 0. Illustrate and explain what difference it makes for your results that b is positive rather than zero. Can you think of a situation where policy makers would not want to choose a positive b?

3. Use (58)–(62) to show that

$$e^r=e_{-1}^r(1+\frac{h}{\theta})(\pi-\pi^f)-\frac{b}{\theta}(y-\bar{y}). \quad (66)$$

Explain in economic terms why positive inflation and output gaps generate a real exchange rate appreciation (a fall in e^r).

Defining $\hat{y}_t \equiv y_t - \bar{y}$ and $\hat{\pi}_t \equiv \pi_t - \pi^f$ and using (58), (63), (65) and (66), we may summarize our model as follows:

AD: $$\hat{\pi}_t = \frac{\beta_1}{\hat{\beta}_1} e^r_{t-1} - \frac{1+\beta_1 \dfrac{b}{\theta} + \beta_2 b}{\hat{\beta}_1} \hat{y}_t + \frac{z_t}{\hat{\beta}_1}, \tag{67}$$

SRAS: $$\hat{\pi}_t = \gamma \hat{y}_t + s_t, \tag{68}$$

Real exchange rate: $$e^r_t = e^r_{t-1} - (1+\theta^{-1}h)\hat{\pi}_t - \theta^{-1} b \hat{y}_t. \tag{69}$$

4. Use (67)–(69) to show that the model may be condensed to the following difference equation in the output gap:

$$\hat{y}_{t+1} = a\hat{y}_t + \beta(z_{t+1} - z_t) - \beta\hat{\beta}_1(s_{t+1} - s_t) - \beta\beta_1(1+\theta^{-1}h)s_t,$$

$$\beta \equiv \frac{1}{1+\gamma\hat{\beta}_1 + b(\beta_2 + \theta^{-1}\beta_1)}, \qquad a \equiv \beta[1+\beta_2(b+\gamma h)]. \tag{70}$$

Is the economy's long-run equilibrium stable? Does an increase in the value of b have an unambiguous effect on the economy's speed of adjustment? Try to give an economic explanation for your conclusion on the latter question.

Exercise 4. Analytical solutions and main policy implications

In this exercise you are asked to analyse three different models of the small, open and specialized economy with free capital mobility, but only in the short run for which reason the equations describing real exchange rate dynamics are not considered. The models are 1) that for fixed exchange rates, (7) and (8) of Chapter 24, 2) the model for floating exchange rates with strict inflation targeting, (22) and (23) of this chapter, and 3) the model for floating exchange rates with flexible inflation targeting, (33) and (34) of this chapter. For each model we consider a period t for which the economy was in long-run equilibrium the period before implying $e^e_{t-1} = 0$.

1. For each of the three models, solve the model for the output gap, $y_t - \bar{y}$, and the inflation gap, $\pi_t - \pi^f$, so that the solutions express how these gaps depend on the

the shock variables z_t and s_t and on parameters. (You may want to take advantage of the fact that the second model is the particular case of the third one where $b = 0$, so you really only have to solve the first and the third model).

2. Use your solutions to verify all the short-run effects of stabilization policy under fixed versus floating exchange rates that were derived graphically in Section 25.3, among these: For supply shocks, the combination of output and inflation gap that arises under a fixed exchange rate can also be obtained under a floating exchange rate with flexible inflation targeting, but so can combinations on both sides. For demand shocks, a floating exchange rate with either strict or flexible inflation targeting gives unambiguously smaller fluctuations in both output and inflation than does a fixed exchange rate.
3. Explain why, despite of these analytical results, fixed exchange rates may nevertheless imply more macroeconomic stability than floating exchange rates.

Exercise 5. Symmetric versus asymmetric shocks

1. Explain how a fixed exchange rate, where the foreign anchor country has a tradition for low and stable inflation, can contribute to establish a low and stable domestic inflation rate.
2. Explain the difference between symmetric and asymmetric shocks in open economies. Explain why a fixed nominal exchange rate may be problematic if shocks are predominantly asymmetric.
3. Discuss the potential role of fiscal policy in a country with fixed exchange rates exposed to asymmetric shocks.
4. Assume that a group of countries with fixed but adjustable exchange rates and strong international trade links are exposed to a symmetric negative demand shock. What would happen if all (or most) countries tried to escape from the resulting recession by devaluing their currencies? What would be the most rational economic policy response from an international perspective?

Chapter 26

The economics of monetary union and the European sovereign debt crisis

Introduction

In Chapter 24 we saw how the growing international mobility of capital in recent decades has made a fixed exchange rate regime increasingly vulnerable to speculative attacks. To overcome this vulnerability, a number of European countries with a preference for fixed exchange rates have taken the far-reaching step of abandoning their national currencies by forming a European Monetary Union with a common currency, the euro, and a common monetary policy delegated to an independent European Central Bank. In this way the euro zone countries have created a greater degree of macroeconomic interdependence and hence a greater need for coordination of their economic policies. In this chapter, we discuss the interesting economic theory of "optimal currency areas" which explains when it is optimal for a country to adopt the ultimate form of a fixed exchange rate regime by joining a monetary union in which all union member states share a common currency and a common monetary policy. Against this background, we review the experience of the European Monetary Union (EMU) formed in 1999, which is the most important real-world example of a currency union. We describe the background for and the workings of the EMU with a special focus on the great European debt crisis of the early 2010s that had a profound impact on economic and political developments in Europe during the last decade, pitching debtor countries against creditor countries, generating serious social unrest in the worst affected countries, and causing the fall of many governments. In the final part of the chapter we discuss what macroeconomists have learned from the European debt crisis and from its forerunner, the great international financial and economic crisis of 2007-2009. As we go through these dramatic economic events of recent decades, we will illustrate how optimal currency area theory as well as our AS-AD model of the open economy can help to understand what happened.

26.1 Economics of monetary union: the theory of optimum currency areas

In Chapter 23 we noted that greater exchange rate stability may stimulate international trade by reducing its riskiness. Although some empirical studies have failed to identify such an effect, a large number of studies have indeed found that greater exchange rate volatility tends to hamper trade.[1] A stable exchange rate could also increase economic welfare by reducing the risk associated with cross-border investment. According to the theory of optimum currency areas, the choice of exchange rate regime then becomes a matter of trading off such structural microeconomic benefits against the potential cyclical macroeconomic costs of fixing the exchange rate. This theory has greatly influenced the continuing debate on the European Monetary Union (EMU) and its design. It is therefore worthwhile to review the theory of optimal currency areas (OCA) before we move on to consider the actual design and experience of the EMU.

The theory of optimal currency areas in a nutshell

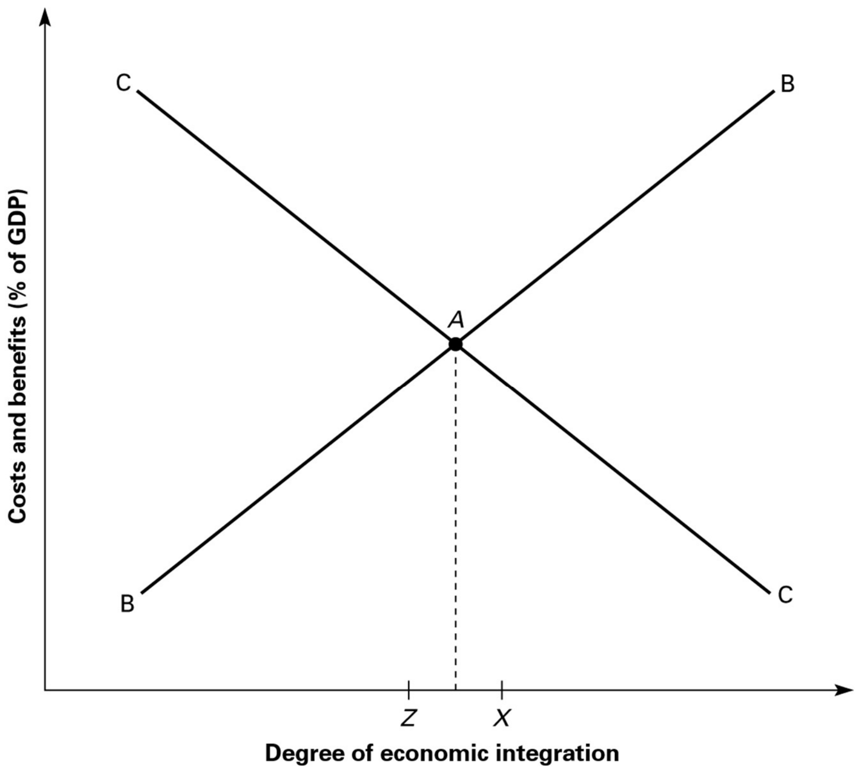
The theory of optimal currency areas may be summarized using Figure 26.1.[2] The diagram illustrates a country's total costs and benefits (relative to GDP) of moving from a flexible to an irrevocably fully fixed exchange rate, possibly by adopting the same currency as the major trading partners. The costs and benefits depend on the country's degree of economic integration with its trading partners measured, say, by the ratio of foreign trade and investment to GDP. The BB curve indicates the *microeconomic benefits* from exchange rate stability. One such benefit is that a credibly fixed exchange rate reduces the riskiness of foreign trade and investment. This is welfare-improving when economic agents are risk averse. Further benefits are gained if exchange rate stability is achieved through a common currency. In that case society saves the real resources (mostly labour time and computer software) which are needed to exchange one currency into another and to keep books in different currencies. It has also been argued that the adoption of a common currency will generate a welfare gain from stronger international

[1] See the article by Ilhan Ozturk, 'Exchange Rate Volatility and Trade: A Literature Survey', *International Journal of Applied Econometrics and Quantitative Studies,* **3**, 2006, pp. 85-102.

[2] The theory of optimum currency areas was pioneered by the later Nobel Prize winner Robert A. Mundell in 'A Theory of Optimum Currency Areas', *American Economic Review*, **51**, 1961, pp. 657 – 665. Other important contributions to the early OCA theory were made by Ronald I. McKinnon in 'Optimum Currency Areas', *American Economic Review*, **52**, 1963, pp. 712 – 725, and by Peter B. Kenen, 'The Theory of Optimum Currency Areas: An Eclectic View', in Robert A. Mundell and Alexander K. Swoboda (eds.), *Monetary Problems of the International Economy*, 1969, Chicago: University of Chicago Press.

price competition, because a single currency facilitates the comparison of prices across borders. Moreover, by increasing the volume of transactions carried out in the same currency, a monetary union tends to increase the liquidity of the markets for financial assets and paves the way for more competition and greater economies of scale in the financial sector.[3]

Figure 26.1 The costs and benefits of a common currency



The greater the volume of international transactions relative to the size of the economy, the larger are the microeconomic benefits from a stable exchange rate and a common currency relative to GDP. This is why the BB curve slopes upwards in Figure 26.1. Since several of the benefits mentioned above can only be reaped by adopting a common currency, the position and positive slope of the BB curve will be higher when the fixed exchange rate is achieved by entering a monetary union.

The CC curve in Figure 26.1 indicates the *macroeconomic costs* of switching from a flexible to a fully fixed exchange rate, measured as a percentage of GDP. The fact that this curve lies above the horizontal axis reflects the assumption of conventional OCA

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[3] A 'liquid' market is a market with a large and fairly steady volume of day-to-day transactions. In a liquid market, a seller in need of cash can normally expect to be able to sell his or her asset without having to accept a significant cut in its price.

theory that the short-run volatility of output and inflation will tend to be lower under flexible than under fixed exchange rates. As you recall from the previous chapter, this is consistent with our AS–AD model, as long as fluctuations in floating exchange rates do not become too much of an independent source of shocks to the economy. More generally, the assumption of positive macroeconomic costs of fixing the exchange rate is based on the idea that the ability to pursue an independent monetary policy under flexible exchange rates makes it easier to stabilize the economy. But as indicated in the diagram, these macroeconomic costs will be smaller relative to GDP the more the domestic economy is integrated with the economies of its trading partners.

There are at least three reasons for the negative slope of the CC curve. First, and most important, the gain from nominal exchange rate flexibility is greater the more often the country is exposed to *asymmetric shocks* requiring a significant adjustment of its real exchange rate, as we explained in Chapter 26. In the case of symmetric shocks a common interest rate policy will be equally appropriate for the domestic and for the foreign economy (assuming that the two countries or regions have roughly the same social preferences for output stability relative to inflation stability), so there is no need for nominal exchange rate flexibility to allow for different national monetary policies. Now comes the point: as the domestic and the foreign economies become more integrated, it is more likely that they will be exposed to the same type of shocks.[4] Hence, they have less need for exchange rate flexibility as the degree of international economic integration increases. For this reason, the macroeconomic cost of moving to a fixed exchange rate will fall as we move to the right on the horizontal axis in Figure 26.1.

The second reason for the negative slope of the CC curve is that a higher degree of economic integration among countries is reflected inter alia in larger financial capital flows across borders. With a deeper integration of national capital markets it becomes easier for domestic economic agents to diversify their portfolios and reduce their exposure to asymmetric shocks to the domestic economy by investing some of their savings abroad or by borrowing from abroad. In other words, deeper financial integration makes it easier for domestic agents to smooth their consumption over time in the face of asymmetric economic shocks. In this way, an increase in financial market integration can work as a form of (partial) *insurance* against asymmetric shocks which reduces the social cost of these shocks.

Third, as the domestic and foreign economies become more integrated in terms of trade and investment, the international mobility of labour is also likely to increase, since cross-border economic transactions tend to reduce the information barriers and cultural

[4] Actually this relationship is not self-evident, since increased international trade may lead different countries to specialize in different types of production subject to different industry-specific shocks (so-called inter-industry trade). However, most of the trade among the OECD countries takes the form of intra-industry trade, that is, exchange of different variants of the same type of products. For example, France may export Citroen cars to Germany while Germany exports BMWs to France. Hence, both France and Germany are vulnerable to shocks hitting the car industry. In the following influential paper, the authors did indeed find that countries with closer trade links tend to have more tightly correlated business cycles: Jeffrey A. Frankel and Andrew K. Rose, 'The Endogeneity of the Optimum Currency Area Criteria', *The Economic Journal*, **108**, 1998, pp. 1009-1025.

barriers to migration. Higher international labour mobility means that if the economy is hit by a negative asymmetric shock, which creates unemployment, some domestic workers will emigrate to look for jobs abroad. Hence, it becomes less urgent to deal with the unemployment problem through a depreciation of the domestic currency, so the cost of giving up exchange rate flexibility falls to the extent that increased economic integration implies increased labour mobility. Moreover, if the economy is hit by a positive asymmetric demand shock, an import of labour from other parts of the currency union can help to avoid an overheating of the domestic economy, thereby reducing the need for an independent national monetary policy that can dampen aggregate demand.

At a level of economic integration corresponding to point A in Figure 26.1, the microeconomic benefits from a fixed exchange rate/common currency are just offset by the macroeconomic costs. At a higher degree of economic integration, the domestic country would benefit in economic terms from fixing its exchange rate or from entering a currency union with its trading partners (remember that the position of the BB curve will be higher and its slope steeper in the latter case). At lower degrees of integration, the macroeconomic costs of exchange rate fixity exceed the microeconomic benefits.

Optimum currency area theory does not offer a quantitative method for estimating whether a particular country is to the right or to the left of the critical point A in Figure 26.1. But the theory does help us to focus on the factors which are important for evaluating whether fixing the exchange rate or joining a currency union is a good or a bad idea. The theory also has one important qualitative long-run implication: if international economic integration continues to deepen, moving a country like the one considered in Figure 26.1 from a point like Z to a point like X, then a growing number of countries should find it in their economic interest to form/join a currency union with their most important trading partners.

Let us sum up what we have learned about optimum currency areas so far:

THE THEORY OF OPTIMUM CURRENCY AREAS

The theory says that the choice of permanently fixing the exchange rate (possibly by joining a currency union) involves a trade-off between microeconomic benefits and macroeconomic costs. The microeconomic benefits increase with the degree of economic integration between the domestic and the foreign economy, since the gains from exchange rate stability and the use of a common currency increase with the volume of cross-border transactions. At the same time the macroeconomic cost of sacrificing exchange rate flexibility falls with the degree of economic integration because 1) tighter integration with the foreign economy reduces the likelihood that the domestic economy is hit by asymmetric shocks; 2) financial market integration can help to insure countries in a currency union against asymmetric shocks; and 3) higher international labour mobility can serve as a substitute for exchange rate changes as a means of absorbing asymmetric shocks. When the degree of economic integration becomes sufficiently deep, a country can therefore gain economically from fixing its exchange rate against its main trading partners.

Trade-offs in a currency union and the endogeneity of the OCA criteria

In the reasoning above, we have taken the degree of international economic integration as given, but the adoption of a common currency may in itself tend to increase the degree of integration by stimulating cross-border trade and investment. Hence, one may conceive that a country which starts out from a point like *Z* in Figure 26.1 may end up at a point like *X* after having joined a currency union with its trading partners. In other words, even though union membership seems to imply an economic loss *ex ante*, it turns out to generate a net economic gain *ex post*. Much of the economic debate on the Economic and Monetary Union in Europe can be understood in this way. Those economists who were/are sceptical about EMU membership have tended to argue that economic integration among the (potential) EMU countries has not yet proceeded far enough to justify the adoption of a common currency, at least as far as the more peripheral EU member states are concerned. By contrast, those in favour of monetary union have tended to argue that, in so far as the preconditions for successful monetary unification are not already present, the adoption of a common currency will in itself promote economic integration to the point where the economic gains outweigh the costs.

The latter idea can be formalized by the simple equation $I = I_0 + \Delta I$, where I_0 is the degree of economic integration *before* the country considered joins a currency union, I is the degree of integration *after* the country has joined, and ΔI is the additional integration caused by membership of a monetary union with a common currency.[5] Building on this idea, we can now illustrate some economic trade-offs involved in the decision to join a monetary union and how these trade-offs are likely to be affected by a decision to join. Drawing on our discussion in the previous subsection, we may assume that a country's net economic benefit (benefit minus cost) from joining a monetary union with a common currency, B, is given by the function

$$B = \alpha_I I + \alpha_F F + \alpha_S S - \bar{B}, \tag{1}$$

where F is an index of the flexibility of labour and product markets, and S is a measure of the degree of symmetry of the economic shocks hitting the country considered and the other countries in the union. The assumption of linearity in (1) is made only for simplicity; all that matters for our qualitative results below is the sign of the partial derivatives of B with respect to I, F, and S. The larger the degree of nominal wage and price flexibility, and the larger the degree of labour mobility across borders, the higher is the value of the flexibility index F, and the greater a country's flexibility, the lower is the macroeconomic cost of giving up the exchange rate instrument and monetary policy independence, so the parameter α_F in (1) is positive. The symmetry of shocks, S, may be measured by, say, the degree of correlation between the output gaps of the country considered and its (potential) partners in the monetary union, since the output gaps of

[5] Integration is a time-consuming process, so ΔI will be larger, the longer the time horizon. The only thing that matters for our analysis here is that, at any future point in time, the degree of economic integration will be higher if the country has joined the currency union than if it has not.

different countries will tend to move together when they are simultaneously hit by the same shocks. As argued above, a higher degree of symmetry reduces the macroeconomic cost of union membership, so in (1) we have $\alpha_S > 0$. Moreover, we have seen that a higher degree of economic integration increases the microeconomic benefits from a common currency, implying $\alpha_I > 0$. The parameter $\bar{B}$ in (1) is positive, since the net benefits from joining a monetary union will surely be negative if there is neither integration, flexibility nor symmetry ($I = F = S = 0$).

This reasoning explains the signs of the partial derivatives in (1), but there is more to the story since we have argued above that a higher degree of economic integration will also tend to increase the degree of labour market flexibility by increasing the mobility of labour and to increase the degree of symmetry of shocks as countries expand their trade with one another. Maintaining our assumption of linearity for simplicity, we may therefore write

$$F = F_0 + \beta_F I, \qquad \beta_F > 0, \tag{2}$$

$$S = S_0 + \beta_S I, \qquad \beta_S > 0, \tag{3}$$

where F_0 and S_0 capture those parts of F and S which are not driven endogenously by the degree of economic integration but are instead determined by other structural and institutional features of the economy. Inserting (2) and (3) along with our earlier assumption $I = I_0 + \Delta I$ in (1), and setting $B = 0$, we now get the

$$\text{OCA plane:} \quad (\alpha_I + \alpha_F\beta_F + \alpha_S\beta_S)(I_0 + \Delta I) + \alpha_F F_0 + \alpha_S S_0 - \bar{B} = 0. \tag{4}$$

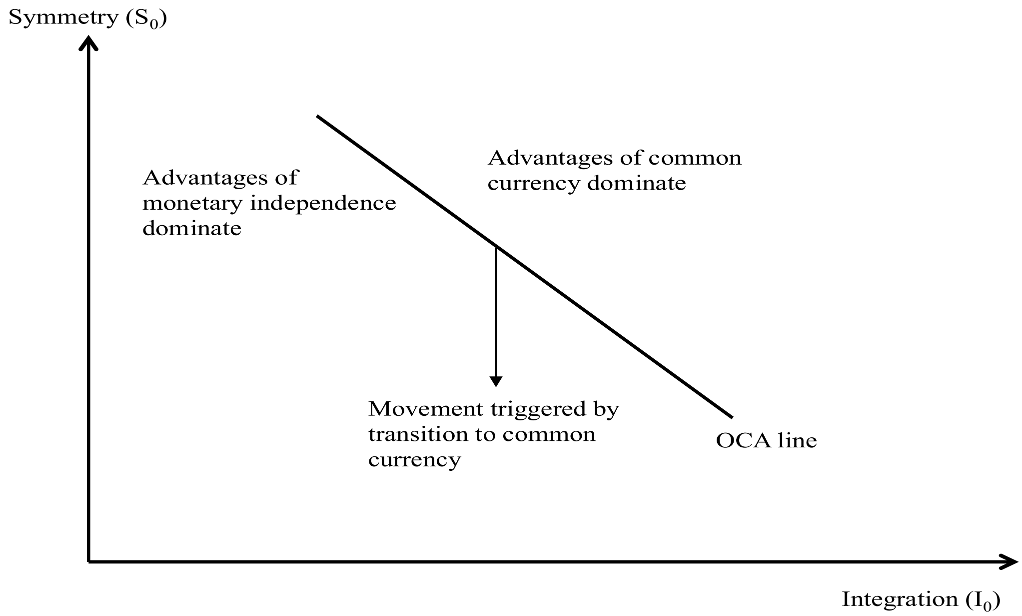
The OCA plane (4) describes the alternative combinations of integration, flexibility and symmetry which will ensure that the net benefit from joining a monetary union is exactly zero. We can rearrange (4) to give

$$S_0 = \bar{b} - \gamma_I \Delta I - \gamma_I I_0 - \gamma_F F_0, \tag{5}$$

$$\bar{b} \equiv \frac{\bar{B}}{\alpha_S} > 0, \quad \gamma_I \equiv \frac{\alpha_I + \alpha_F\beta_F + \alpha_S\beta_S}{\alpha_S} > 0, \quad \gamma_F \equiv \frac{\alpha_F}{\alpha_S} > 0.$$

In Figure 26.2 we have used (5) to draw an OCA line showing the combinations of symmetry and prior integration that will keep the economy on the OCA plane for a *given* degree of flexibility, F_0. The variable measured along the horizontal axis is the degree of integration prevailing *before* the country enters the monetary union, I_0, and the variable on the vertical axis is the part of symmetry that is *not* driven by integration itself, S_0. The OCA line is downward-sloping since a higher degree of integration increases the microeconomic benefits from a common currency which means that the country can still obtain a zero net benefit from monetary union even if the macroeconomic cost of losing monetary independence increases due to more asymmetric shocks.

Figure 26.2 The trade-off between integration and symmetry of shocks in a monetary union



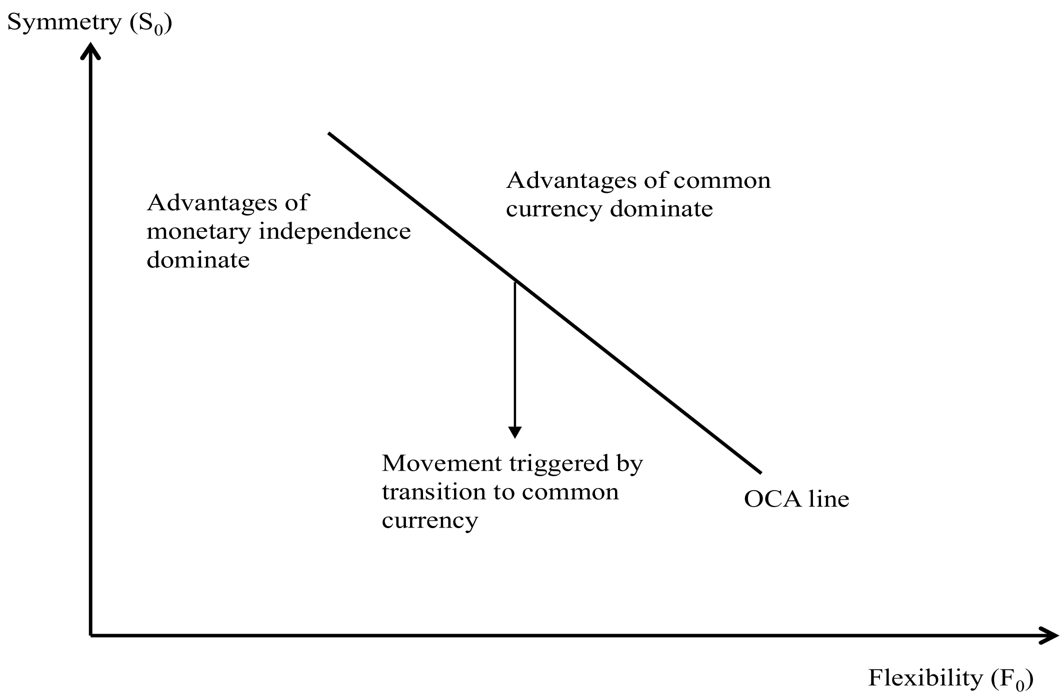
For all combinations of integration and symmetry to the northeast of the OCA line, the microeconomic benefits of a common currency dominate the macroeconomic costs, making it optimal for the country considered to join the monetary union, whereas all combinations of integration and symmetry southwest of the OCA line imply a net welfare cost of giving up monetary independence.[6]

Figure 26.2 also illustrates the idea that the switch to a common currency will itself increase the degree of integration by stimulating cross-border trade and investment. In terms of Equation (5), the transition to a common currency will increase the term ΔI from zero to some positive number which will increase over time as the integration effect of the common currency strengthens. After joining a monetary union, a country will therefore experience a gradual downward shift of its OCA line, as implied by (5) and indicated in Figure 26.2, and this may help to push the country from below to above the OCA line.

Instead of fixing the exogenous part of flexibility F_0 in (5), we may keep the initial degree of integration I_0 constant and use (5) to derive another OCA line showing the combinations of exogenous symmetry (S_0) and exogenous flexibility (F_0) that will keep the economy on its OCA plane.

[6] The idea of illustrating the trade-offs in optimum currency areas by OCA lines like those in Figures 26.2 and 26.3 is due to the Belgian economist Paul De Grauwe who has used this graphical technique in successive editions of his leading textbook on *The Economics of Monetary Union* (Oxford University Press). Here we have elaborated a bit on his analysis via the mathematical apparatus (1) through (5) which distinguishes between the exogenous and the endogenous components of integration and flexibility.

Figure 26.3 The trade-off between flexibility and symmetry of shocks in a monetary union



This is done in Figure 26.3, which shows the trade-off between symmetry and flexibility in a monetary union. As labour and product markets become more flexible, with more nominal wage and price flexibility or more labour mobility, the macroeconomic costs of asymmetric shocks become smaller, as the economy can more easily and quickly adjust to the shocks. Hence the country can maintain a zero net benefit of monetary union even if shocks become less symmetric, so once again the OCA line is downward sloping, just as joining the union is welfare-improving northeast of the line and welfare-decreasing southwest of the line. According to (5), a switch to a common currency will still increase ΔI from zero to some (gradually increasing) positive number, causing the downward shift of the OCA line illustrated in Figure 26.3, and increasing the likelihood that membership of the monetary union will some day become optimal for the country even if it were inoptimal in the beginning.

The trade-off between fiscal union and flexibility

When we discussed the need for labour and product market flexibility in a monetary union, we did not distinguish between temporary and permanent economic shocks, but this distinction is important for another issue that has frequently popped up in the debate on monetary union, namely the need for some degree of fiscal union. We will now dig into this issue.

As already mentioned, a permanent shock to the economy creates a need for a permanent adjustment of the real exchange rate. The analysis of economic dynamics under fixed exchange rates in Chapter 24 (see Equations (14) and (15) there) showed that the adjustment to a new long run equilibrium real exchange rate will occur more quickly – hence ensuring a faster closure of the output and inflation gaps – the larger the value of the parameter γ in the AS curve, that is, the higher the degree of nominal wage and price flexibility. Indeed, one can use our AS-AD model with fixed exchange rates to prove analytically that a larger value of γ means that the size of the output and unemployment gaps caused by permanent shocks to demand or supply will be smaller during the entire adjustment phase to the economy's new long-run equilibrium (see Exercise 2). That is why more wage and price flexibility is valuable from a macroeconomic perspective in a currency union which is hit by permanent asymmetric shocks.

Matters are different if the shocks hitting the economy are temporary. In that case, the shocks do not require permanent adjustments of the real exchange rate, which reduces the importance of the economy's ability to adjust quickly to a new long-run equilibrium. Moreover, a higher degree of nominal wage flexibility is not unambiguously desirable when shocks are temporary. To illustrate, suppose the economy is hit by a temporary positive shock to demand. When the shock hits, the short-run impact on the output gap will be smaller the higher the value of γ (the steeper the AS curve), since a larger part of the shock will then be absorbed by a rise in the inflation gap. This impact effect is desirable if policy makers are mainly concerned about stabilizing output and employment. However, the stronger short-run reaction of the inflation gap means that the economy is left with a weaker competitiveness and hence a lower level of aggregate demand when the temporary demand impulse goes away. In our AS-AD model, economic activity will therefore be weaker in several periods after the shock has disappeared (see Exercise 3).

For these reasons wage and price flexibility is not necessarily an important and effective way of stabilizing an economy faced with recurrent positive and negative temporary demand shocks arising from the shifting wawes of pessimism and optimism that seem to be an important driver of business cycles. Instead, economists have pointed out that macroeconomic stability in a monetary union can be strengthened if the member countries of the union establish a *common fiscal budget* that can be used as a means of transferring resources from countries benefiting from positive asymmetric temporary shocks to countries hit by negative asymmetric temporary shocks. For example, union member countries could decide to establish a common unemployment insurance fund based on contributions from member countries, say, in proportion to their GDP. The fund could then finance (a part of) the unemployment benefits paid out by member states in times of recession. Member states that are particularly hard hit by recession would then experience a larger (relative) fall in their contributions to and larger transfers from the fund than member states experiencing a milder recession or even a boom. In this way, a common fiscal budget operating along such lines could serve as an *automatic stabilizer* against *temporary asymmetric* shocks, thereby reducing the macroeconomic cost for individual member states of giving up the exchange rate instrument and monetary independence. One potential objection against such a scheme is that it would tend to serve as a mechanism for a systematic permanent redistribution from member countries with low to member countries with high structural unemployment, which might reduce the incentive for the latter countries to undertake structural reforms to bring down their natural rate of unemployment. However, in principle this problem of *moral hazard* could be avoided if transfers from the common unemployment insurance fund were only triggered when a country's current unemployment rate exceeds its long-term average unemployment rate.[7]

More generally, a common fiscal budget where the contributions from (and/or transfers to) member states have some positive (negative) correlation with their national economic activity will work as partial insurance against asymmetric temporary shocks and will thereby reduce their macroeconomic costs of adopting a common currency. That is why many economists have stressed that some element of *fiscal union* – effectively requiring some form of *political union* – may be necessary to ensure the long-run sustainability of a monetary union.

This analysis implies that Equation (1) should be generalized to the following expression for the net benefit from entering a monetary union,

$$B = \alpha_I I + \alpha_F F + \alpha_S S + \alpha_{CB} CB - \bar{B}, \qquad (6)$$

where *CB* is the size of the common budget in the union measured, say, by the amount of money allocated to the budget relative to the total GDP of all member states combined. In so far as the rules for contributions to and transfers from the common budget provide some insurance against asymmetric shocks to the national economies of member states, the coefficient α_{CB} in (6) will be positive.

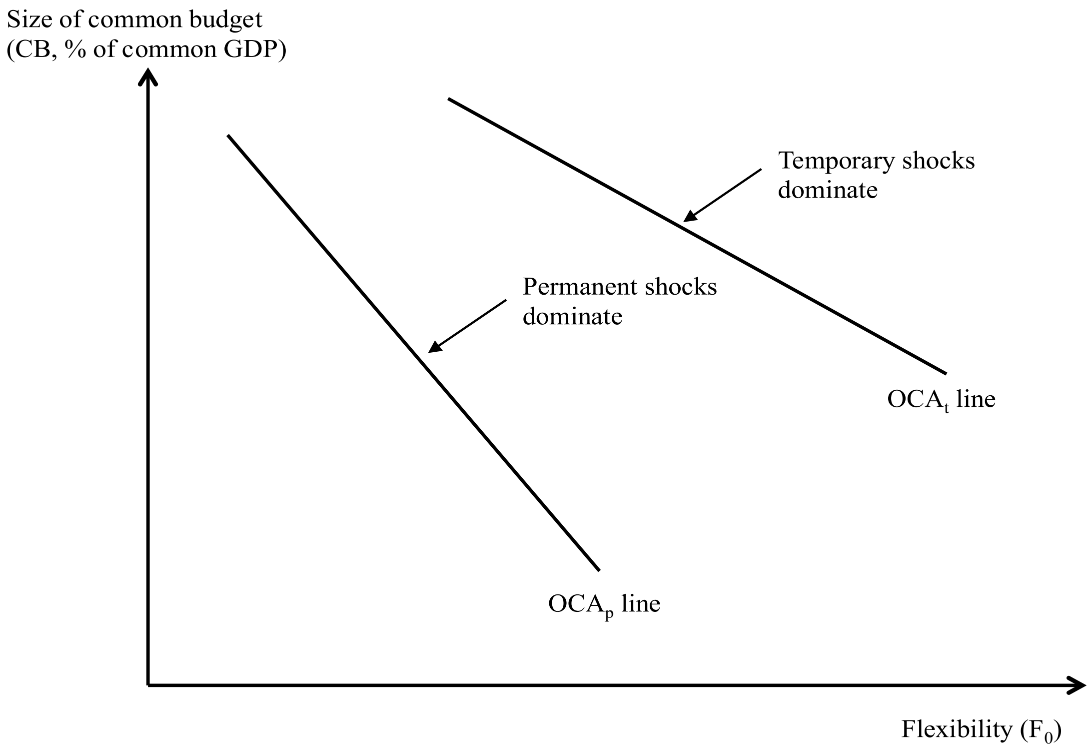
Proceeding in the same way as in the previous subsection, we can set $B = 0$ in (6), insert (2) and (3), and fix the variables I_0 and S_0 to obtain the OCA lines in Figure 26.4 depicting yet another trade-off in an optimum currency area: that between flexibility and fiscal union.[8] Flexibility is measured along the horizontal axis, and the degree of fiscal union is measured on the vertical axis by the size of the common union budget relative to total union GDP. Again the slope of the OCA line is negative, being equal to $-\alpha_F/\alpha_{CB}$, since a lower flexibility which reduces the ability to adjust to shocks at a low macroeconomic cost can be compensated by the insurance gain from an appropriate increase in the common budget so as to keep the net benefit of monetary union equal to zero. As before, it is not optimal for a country to join a monetary union if it is located

[7] The term *moral hazard* is generally used to describe a situation where insurance against some undesirable event may reduce the incentive of the insured agent to undertake actions to reduce the risk that the event will occur.

[8] The graphical analysis in Figure 26.4 is borrowed from Paul De Grauwe and Yuemei Ji, 'Flexibility versus Stability – A difficult trade-off in the euro zone', CEPS Working Document No. 422, April 2016, Centre for European Policy Studies, Brussels.

below its OCA line, whereas it is optimal to join if it is placed above the OCA line.[9]

Figure 26.4 The trade-off between flexibility and fiscal union in a monetary union



The lower line OCA_p in Figure 26.4 illustrates a situation where (asymmetric) economic shocks are predominantly permanent in nature. In that case, we have seen that labour and product market flexibility clearly helps a country to adjust to the shocks without having to go through long periods with large output and inflation gaps. Hence the marginal benefit from flexibility (α_F) is relatively high, and the marginal benefit from the insurance provided by the common budget (α_{CB}) is relatively low (since insurance is less needed), which is why the OCA_p line is relatively steep: it only takes a small increase in flexibility to compensate for any given reduction of the common budget.

By contrast, the upper line OCA_t in Figure 26.4 shows a case where (asymmetric) shocks are mostly temporary. In that situation we noted that wage and price flexibility does not provide a very effective cushion against the shocks, implying a relatively small value of α_F, whereas the insurance function of a common budget becomes more valuable because there is more need for it, implying a relatively large value of α_{CB}. This explains why the OCA_t line is relatively flat: now it takes a large increase in flexibility to compensate for a given cut in the common budget, since flexibility is now only valuable

[9] Equations (2), (3) and (6) imply that the two OCA lines in Figure 26.4 start from the same point on the vertical axis if $F_0 = 0$. For any $F_0 > 0$ the line OCA_t will therefore lie above the line OCA_p, since the latter line has a steeper slope, as explained below.

in the relatively few cases where shocks are permanent.

TRADE-OFFS IN AN OPTIMUM CURRENCY AREA

The net economic benefits from membership of a currency union are larger,
- the larger the degree of economic integration among the member countries of the union;
- the greater the symmetry of the economic shocks hitting the member countries;
- the higher the degree of labour and product market flexibility in member countries, and
- the greater the common fiscal budget of the union.

To some extent, the degrees of symmetry and flexibility of member countries depend endogenously and positively on their degree of economic integration. Yet, for a country for which membership of a currency union is on the borderline of being optimal, there is a trade-off between integration and the symmetry of shocks: the macroeconomic costs of more asymmetric shocks must be compensated by the microeconomic benefits from a higher degree of integration. There is also a trade-off between symmetry and flexibility: a lower degree of symmetry must be compensated by higher flexibility improving a country's ability to adjust to asymmetric shocks. Flexibility is particularly valuable when the shocks hitting the economy are permanent, but less valuable when shocks are temporary. A further trade-off is that between flexibility and the size of the common fiscal budget for the union: less flexibility must be compensated by a larger common budget allowing larger temporary transfers to member countries hit by temporary asymmetric shocks. Transition to a common currency may in itself relax (but will not eliminate) these trade-offs, as it will tend to increase the degree of economic integration among members of the monetary union.

From Mundell I to Mundell II

When the later Nobel Laureate Robert Mundell introduced the theory of optimum currency areas in his seminal 1961 article referred to in footnote 2, he emphasized how the co-existence of asymmetric shocks and nominal wage rigidity increased the value of exchange rate flexibility as a way of adjusting to the shocks and creating room for an independent monetary policy to stabilize the economy. In line with the Keynesian spirit of the time, Mundell was rather optimistic about the potential for an activist stabilization policy to keep the economy close to full employment, and he was also optimistic about the usefulness of an adjustable exchange rate as a shock absorber, whereas he was pessimistic about the possibility of creating more nominal wage flexibility, especially in the downward direction. He therefore believed that a considerable degree of labour mobility would be needed to compensate for the loss of the exchange rate instrument and an independent monetary policy in a currency union.

This approach to optimum currency areas is still shared by many economists today and has been dubbed "Mundell I". However, in a less well-known paper published in 1973, Mundell developed an alternative analytical approach that led him to reassess the net benefits from a common currency.[10] Around 1960, when Mundell drafted his first paper on OCA, international capital mobility was still quite limited, as countries maintained strict capital controls to defend their exchange rates and to obtain some scope for an independent monetary policy. Hence, there was little experience with fluctuating capital flows that could create unwarranted pressures in the foreign exchange markets. By the early 1970s, international capital mobility had increased, partly in the form of trade credits associated with the growing volumes of international trade, and Mundell anticipated that with growing capital mobility, a flexible or adjustable exchange rate would increasingly become an independent source of shocks rather than a shock absorber. This is in line with the view explained in Chapter 25 that capital flows can sometimes shift abruptly due to changes in expectations that are not necessarily fully rooted in changes in economic fundamentals, thereby creating "excess" volatility in foreign exchange markets. If the exchange rate is increasingly a source of rather than an absorber of shocks, the value of maintaining a national currency obviously decreases.

In his 1973 paper, Mundell made another important observation. In a world with separate national currencies, exchange rate risk stemming from the uncertainty about future exchange rates will hamper the international integration of capital markets, as firms and households can avoid the risk by investing or borrowing in the home country rather than abroad. By adopting a common currency, countries can eliminate exchange rate risk and achieve deeper capital market integration. This will improve the ability of capital markets to provide (partial) insurance against asymmetric shocks and allow greater portfolio diversification protecting against other risks. Mundell also pointed out that member countries in a currency union can draw on their common pool of reserves of the currencies of non-member countries when they are hit by a negative asymmetric shock. This ability to borrow from the common central bank provides an additional form of insurance of individual countries in a currency union.

In summary, Mundell's 1973 paper offered a rather different and more optimistic view of the net benefits from a currency union than his original 1961 paper. In the subsequent OCA literature, this later view became known as "Mundell II", and it had a considerable impact on the debate in the run-up to the formation of the European Monetary Union to which we now turn.

26.2 The foundation and design of the European Monetary Union

The European Monetary Union (EMU) is the world's largest currency union formed by largely sovereign nation states. It is considered by many as a bold historical experiment with economic and political integration among countries that used to wage war against

[10] See Robert A. Mundell, 'Uncommon Arguments for Common Currencies', in Harry G. Johnson and Alexander K. Swoboda (eds.), *The Economics of Common Currencies: Proceedings of the Madrid Conference on Optimum Currency Areas*, 1973, London: Allen & Unwin.

each other not so long ago. Studying the early history of the EMU allows us to illustrate the real-world relevance of several aspects of the theory of Optimum Currency Areas and provides further insights into the workings of a monetary union and the importance of its institutional design. Reviewing the early experience of the EMU also gives us an opportunity to discuss what macroeconomists have learned from the dramatic financial crises in recent decades.

The legal foundation for the EMU was laid by the Treaty on European Union signed in Maastricht in February 1992, which led to important amendments to the Treaty on the Functioning of the European Union. The Maastricht Treaty provided for the EMU to be implemented in three stages, with the third and final stage to be reached in 1999 with the irrevocable locking of exchange rates and later adoption of a common currency. A detailed timetable for the transition to EMU was decided at an EU Summit in Madrid in December 1995 where it was also agreed that the new common currency should be named the "euro".

The Maastricht Treaty specified a number of economic "convergence criteria" an EU country must meet in order to qualify for membership of the third stage of the EMU. To be allowed into the currency union, a country should have: 1) An annual rate of consumer price inflation no higher than 1.5 percentage points above the average inflation rate in the three EU member states with the lowest inflation. 2) Exchange rate stability, defined as the country not having devalued its currency against the euro during the previous two years and having participated in the Exchange Rate Mechanism (the EU's system of fixed exchange rates for countries outside the euro zone) for at least as long. 3) A nominal interest rate on 10-year government bonds no higher than 2 percentage points above the average interest rate in the three EU member states with the lowest rates. 4) Sound and sustainable public finances, interpreted as a government budget deficit no higher than 3 percent of GDP unless special circumstances prevail (such as a deep recession), and a government debt-to-GDP ratio no higher than 60 percent, with the proviso that a temporarily higher debt ratio is acceptable if it is approaching the reference value of 60 percent at a satisfactory pace.[11]

When the EMU entered its third stage by January 1, 1999, the following 11 countries became members of the currency union: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain. In the subsequent years (indicated in brackets), a number of other EU countries joined the euro zone: Greece (2001), Slovenia (2007), Cyprus (2008), Malta (2008), Slovakia (2009), Estonia (2011), Latvia (2014), and Lithuania (2015). During the first three years of the EMU, member states still used their national currencies, but at completely fixed exchange rates against each other. From January 1, 2002, the national currencies were replaced by the euro.

The essential institutional features of the EMU are a common monetary and exchange rate policy combined with independent national fiscal policies, subject to some budgetary rules and coordination procedures. The common monetary policy is implemented by the European System of Central Banks (ESCB) consisting of the European Central Bank

[11] A "satisfactory pace" was later defined as an annual reduction of the debt ratio of at least 1/20 of the difference between the actual debt ratio and the 60 percent reference value.

(ECB) located in Frankfurt and the national central banks (NCBs) of euro zone member states which serve as local branches of the ESCB. The ESCB is governed by the two decision-making bodies of the ECB, which are its six-member Executive Board and the Governing Council consisting of the Executive Board plus the governors of the NCBs. The votes of all members of the governing bodies count with equal weights, and the NCBs are obliged to implement the decisions of the governing bodies in individual member states. Thus, the ESCB has a "federal" structure, but below we will follow the common practice of referring to the EMU's central banking system as the "ECB", for convenience. The ECB is legally independent of EU governments and hence cannot take any instructions from them. In line with the Maastricht Treaty, the primary objective of the ECB is to maintain price stability in the euro zone. Until recently, the ECB interpreted "price stability" to mean an annual rate of consumer price inflation below, but close to, 2 percent over the medium term, but in July 2021, it changed its definition to mean simply an average inflation rate of 2 percent over the medium term. Subject to the primary objective of price stability, the ECB shall also support the general economic policies in the EMU. To be able to pursue an independent monetary policy, the ECB allows the exchange rate of the euro to float freely against the currencies of countries outside the EU. The currencies of EU countries outside the euro zone but aiming at future membership are held roughly fixed within the Exchange Rate Mechanism mentioned above, but it is mainly the responsibility of the central banks of those countries to maintain the exchange rate parity.

While monetary policy is centralized in the EMU, fiscal policies are left largely in the hands of member state governments, although they are supposed to respect the constraints on the size of government budget deficits and government debt mentioned above. In an effort to ensure this, the Stability and Growth Pact (SGP) agreed by EU member states in 1997 laid down rules for a common surveillance of their budgetary positions and specified a procedure for correcting "excessive deficits". Unless special circumstances such as an unusually deep recession prevail, a budget deficit is deemed to be excessive if it exceeds 3 percent of GDP. To prevent such deficits, the SGP includes "Medium-Term Budgetary Objectives (MTOs)" which are country-specific limits on the acceptable *structural* budget deficit, typically equal to 0.5 or 1 percent of structural GDP. If an excessive budget deficit nevertheless occurs in a member state of the euro zone, the SGP requires that the member state be penalized by a fine of up to 0.5 percent of its GDP if it fails to correct the deficit, unless a qualified majority in the European Council of Finance Ministers votes against a fine.

Another provision aimed at securing fiscal discipline was the so-called "no bailout clause" in Article 125 of the Treaty on the Functioning of the European Union. According to this article, member states cannot take on the debts of another member state, that is, each member state is responsible for its own government debt.

The fiscal rules in the EMU are motivated by the fact that an irresponsible fiscal policy in a member state may have negative spill-over effects on the rest of the union. For example, an overly loose fiscal policy in some member states can drive up the average rate of inflation in the union, thereby forcing the ECB to tighten monetary policy to the detriment of member states pursuing a stricter (more responsible) fiscal policy. Excessive levels of deficits and public debt in a member state can also threaten financial stability in the country, which in turn can undermine stability in other member states due to the deep integration of capital markets.

Importantly, the Maastricht Treaty also largely left individual member states with the responsibility for financial supervision and deposit insurance and for handling threats to financial stability due to ailing or failing banks. This would turn out to be a serious weakness of the EMU, as we shall see.

It is illuminating to compare the Maastricht design of the EMU to the features of another large currency union, that of the United States of America. Both unions have a common monetary policy and allow floating exchange rates against the rest of the world. However, U.S. fiscal policy is centralized to a high degree via a large federal budget providing some insurance against asymmetric shocks to individual U.S. states: if a U.S. state is hit by a negative economic shock, the federal income tax bill of its citizens will fall, and some of its citizens will receive more social transfers which are fully or partly financed by the federal government. Due to these automatic stabilizers, states that are hit particularly hard by a recession will receive some net transfers from other states via the federal budget. Furthermore, the bonds issued by the U.S. federal government represent common safe assets in which investors from all states can invest, so in times of financial panic there is no tendency for capital to flee from "unstable" to "stable" states in the union. By contrast, since the common EU budget only makes up about 1 percent of the union's GDP, fiscal policy in the EMU is largely a national affair, and since there is no common safe asset (a common union bond), capital tends to flow from the most crisis-stricken countries to the more stable countries in times of financial trouble, thereby exacerbating the difficulties for the worst-affected countries. The U.S. also has a more centralized system of financial supervision and deposit insurance than the EMU, so in America the responsibility for winding up large failing banks or recapitalizing them is not left to individual states that may not have the fiscal capacity to be up to the task. Finally, labour mobility was and still is significantly higher among U.S. states than among the member states of the EMU, providing yet another additional U.S. buffer against asymmetric shocks within the American currency union.

Against this background, many economists (especially from the U.S.) around the time of the signing of the Maastricht Treaty feared that the macroeconomic costs of a European currency union would be too large, even though they recognized the microeconomic benefits from a common currency. In particular, many commentators argued that a sustainable monetary union requires a common fiscal policy based on a substantial common budget. This in turn led them to the conclusion that some form of a political union was a prerequisite for monetary union – a viewpoint stressed not only by U.S. economists, but also by many German economists.[12]

However, according to an opposing view held by many other European economists, the very absence of a large common budget and of high labour mobility made it both

[12] As early as 1970, a centralized fiscal policy was described as an important prerequisite for a well-functioning Economic and Monetary Union in Europe. This view was expressed in a report from a group of experts led by the Prime Minister and Minister of Finance of Luxembourg Pierre Werner, prepared upon request from the European Council.

necessary and desirable to leave fiscal policy in the hands of national governments: since the member states of the EMU could no longer use national monetary policy as a tool of dealing with asymmetric shocks, it was all the more important that they be given freedom to use fiscal policy for this purpose, as long as they maintained sustainable public finances in the long run. At the time it was widely believed that the fiscal rules laid down by the Maastricht Treaty and the Stability and Growth Pact would be sufficient to ensure sound public finances. Indeed, some observers argued that the switch to a common currency combined with the no bail out clause would actually *strengthen* fiscal discipline. When countries with excessive fiscal deficits and overheating economies could no longer devalue and thereby erode their public debts via inflation, and when they could not expect to be bailed out of their debt problems by other member states, they would have to adopt more responsible fiscal policies – or so it was thought.

Furthermore, some optimistic commentators argued that when devaluations could no longer be used as a temporary remedy against weak international competitiveness caused by overexpansionary policies, the governments of hitherto inflation-prone countries would have a stronger incentive to carry out structural labour and product market reforms that could make them more productive and competitive. In addition, many observers pointed out that the adoption of a common currency would in itself promote economic integration, thereby increasing the likelihood of positive net economic benefits from monetary union.

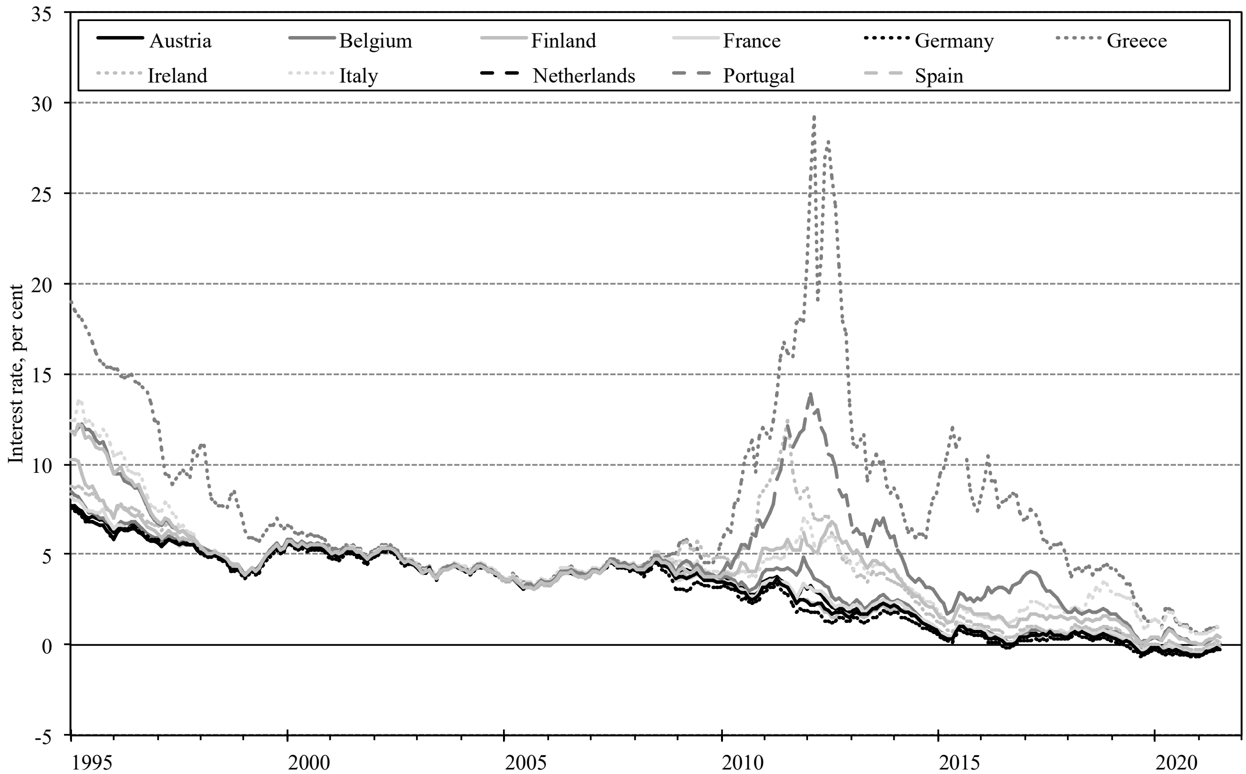
As we shall see below, things did not quite turn out that way, but before we move on, we want to stress an important point. The European Union is as much (if not more) a political as an economic project aimed at promoting peace and prosperity in Europe. Although our discussion in the present and the following sections may suggest that the formation of the EMU happened too early or included too many countries from the start seen from a purely economic perspective, it is also much too early to evaluate the long-run economic and political consequences of this hugely ambitious experiment in European integration. Future historians may conclude that it was an indispensable element on the way towards a richer and more cohesive European Union, despite the early difficulties. The purpose of our discussion here is just to illustrate how macroeconomic theory, including OCA theory, can help to understand why the EMU ran into economic trouble after a period of apparent initial success, and what kinds of institutional reforms (some of which have already been implemented or are underway) could help to make it more robust in the future.

26.3 The early boom years of the EMU: Sowing the seeds of crisis

Following the EU Summit in Madrid in 1995 where governments confirmed their commitment to form a monetary union and laid out a detailed timetable for implementing it, there was a remarkable convergence of interest rates on national government bonds.

Impressed by the strong signals from politicians, markets quickly became convinced that EU member states with a history of high inflation were determined to achieve nominal convergence with the low-inflation countries and avoid future devaluations to qualify for EMU membership already from the beginning of its third and final stage in 1999. As expectations of devaluations waned, the interest rates in all the original 11 EMU countries converged on the low German level already before the start of the final stage of EMU, as illustrated in Figure 26.5. Greece also managed to obtain near-perfect convergence of its interest rate before entering the EMU from the start of 2002. Notably, the almost perfect convergence of government bond yields indicated that the markets considered all EMU governments to be equally worthy of credit. We will return to this issue later.

Figure 26.5. Yield on 10-year government bonds



Note: Based on monthly data.
Source: Eurostat.

In the following, we will focus on economic developments in the largest EMU economy (and largest creditor country), Germany, and four countries in the south and the west of the union that later had to ask for external financial assistance to avoid an economic collapse: Greece, Ireland, Portugal, and Spain (the GIPS countries)[13].

[13] Cyprus also had to be bailed out, but since it is a very small economy that did not join the EMU until the international financial crisis had already started, we do not focus on Cyprus here.

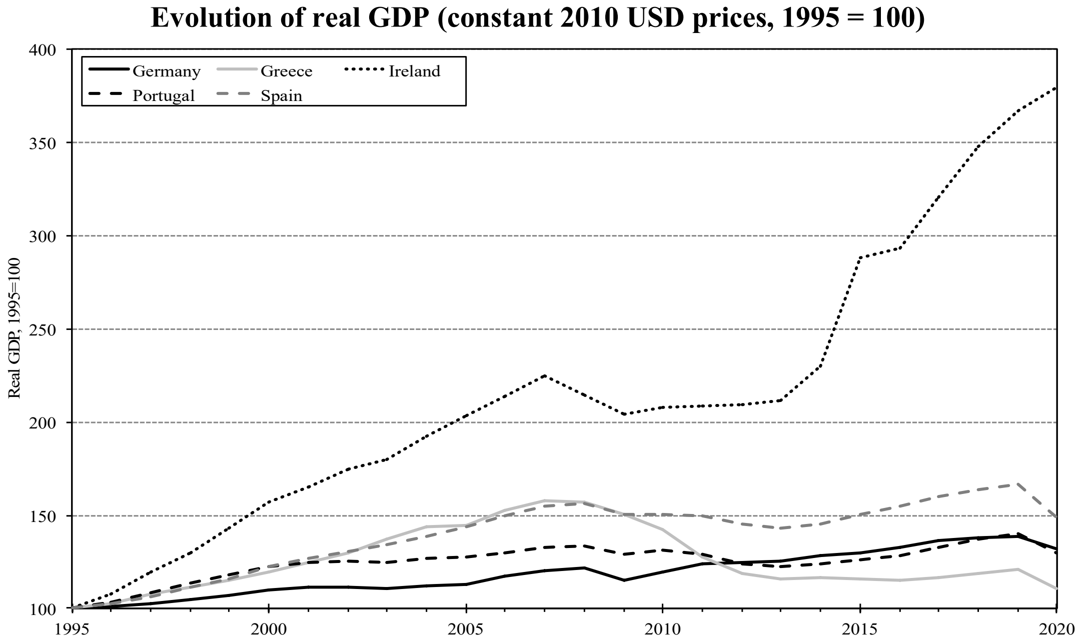
The sharp drop in interest rates gave a strong boost to the economies of the GIPS countries in the late 1990s and during the early years of the EMU. In a general wave of optimism, many investors in the Northern part of the EMU and elsewhere saw new attractive investment opportunities in the EMU periphery after the elimination of exchange rate risks, and since some of the peripheral countries started out from relatively low levels of income and capital per worker, they were expected to have good growth prospects via a process of catching up with the rest of the EMU countries. Hence, capital started to flow into the GIPS countries in large quantities, and these countries did in fact experience solid growth in real GDP and a significant drop in unemployment up until the time of the international financial crisis in the late 2000s, as shown in Figure 26.6. By contrast, growth was very slow and unemployment quite high in Germany during that period, in large part because German competitiveness was hampered for a while by the integration of the relatively low-productive former DDR into the German economy following the country's reunification.

The boom in the GIPS countries was associated with a massive credit expansion that greatly outpaced the growth of the real economy. This can be seen in Figure 26.7, which shows that domestic credit to the private sector rose dramatically relative to GDP in the first part of the 2000s. A lot of this credit was granted to the housing sector, and because the relatively inelastic supply of land and housing could not keep pace with the growing demand, countries like Spain, Greece, and Ireland experienced a formidable increase in residential property prices that even surpassed the simultaneous U.S. housing bubble, as illustrated in the bottom part of Figure 26.7.

The economic upswing in the GIPS countries was accompanied by rapidly rising wages, despite a relatively weak productivity growth in some of these countries. The weak productivity performance reflected in part that much of the inflow of capital was channeled into relatively low-productive investments in real estate where a large share of the private return to investment took the form of capital gains due to rising property prices. Some of the wage inflation was also driven by large increases in public sector wages, facilitated by the governments' easy access to credit. Because of these developments, domestic inflation (measured by the GDP deflator) and unit labour costs rose much faster in the GIPS countries than in the rest of the EMU, in particular Germany, as shown in Figure 26.8.

The high inflation rates coupled with interest rate parity with low-inflation countries like Germany drove down the real interest rates in the GIPS countries, which contributed further to the boom in demand and the house price bubbles. High inflation also eroded the international competitiveness of these countries. This combined with the rapidly rising aggregate demand to create huge deficits on the current account of the balance of payments, reaching exorbitant levels of 10-15 percent og GDP in Greece, Portugal, and Spain, as illustrated in the upper part of Figure 26.9. At the same time, Germany experienced a growing current-account surplus, roughly equal in absolute terms to the combined deficits of the GIPS countries. Because of the large current-account deficits, these countries accumulated a rapidly growing ratio of net foreign debt to GDP, as shown in the bottom part of Figure 26.9.

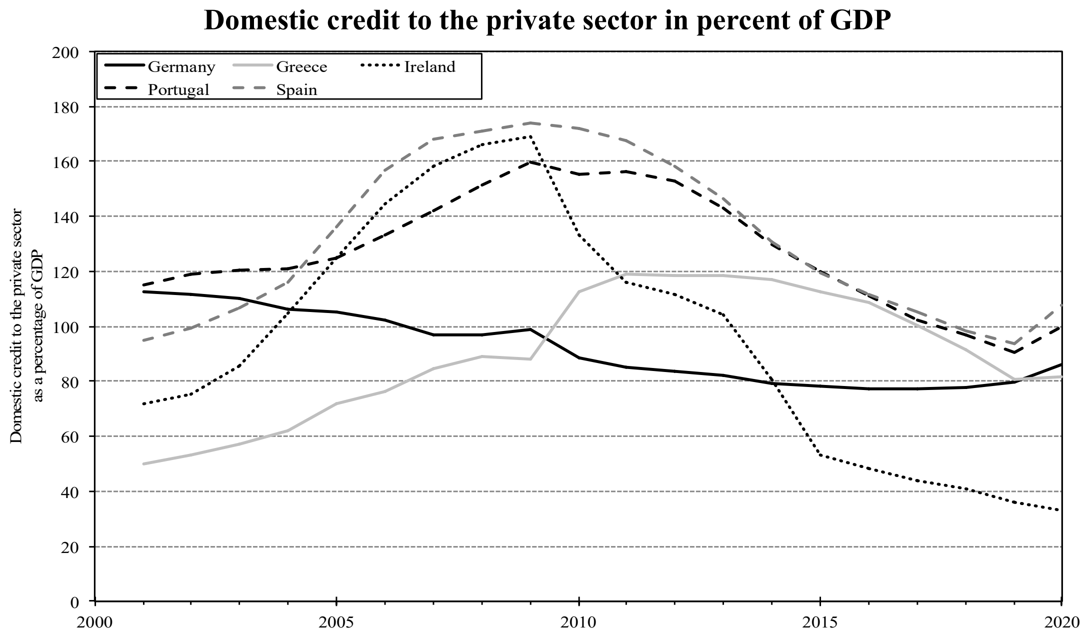
Figure 26.6. Growth and unemployment in selected EMU countries



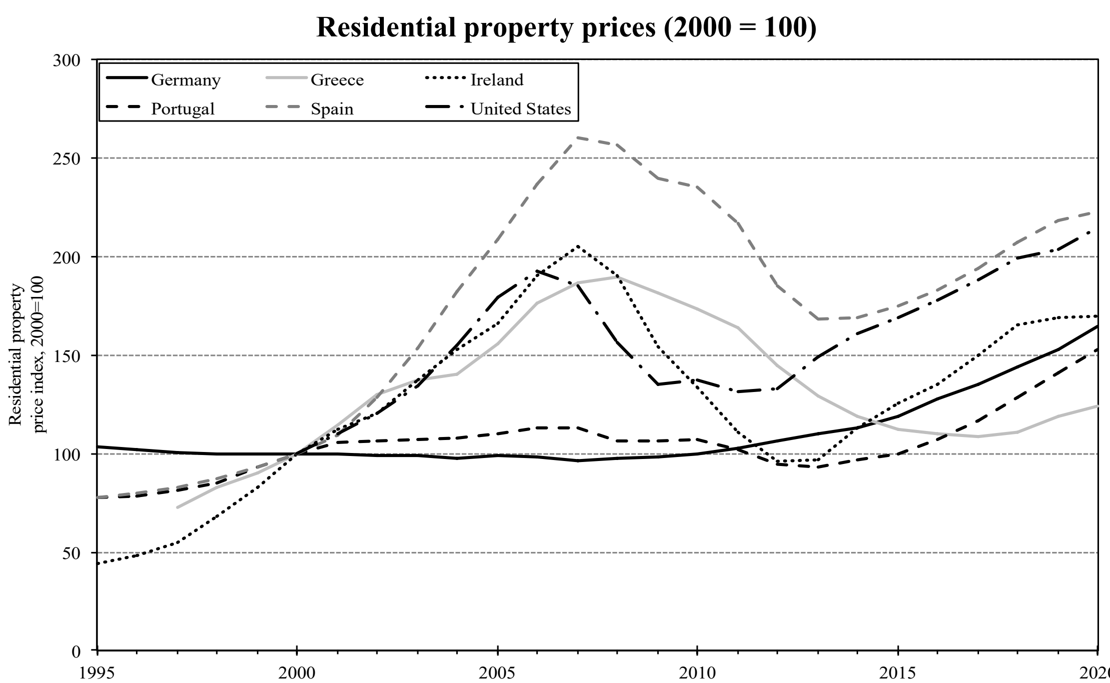
Note: Based on annual data.
Source: World Development Indicators, World Bank.

Note: Based on annual data.
Source: Eurostat.

Figure 26.7. Credit expansion and property prices in selected EMU countries

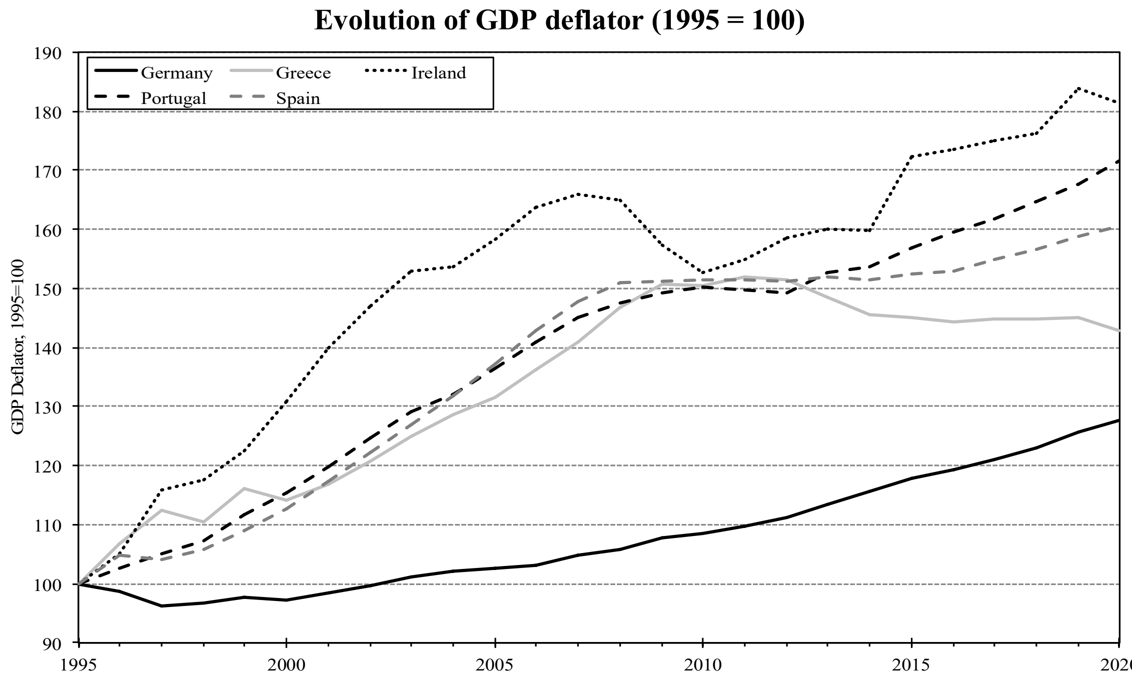


Note: Based on annual data.
Source: World Development Indicators, World Bank.



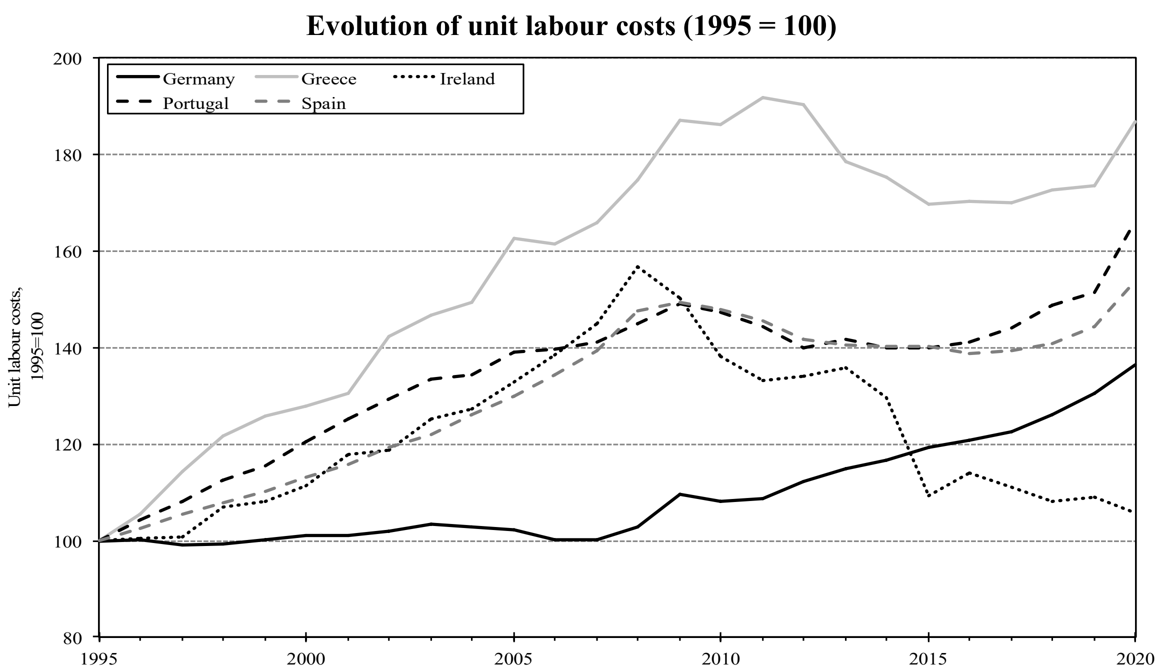
Note: The residential property prices in the United States is based on the US Case-Shiller 20-City Index. All series are based on annual data.
Source: OECD; Federal Reserve Bank of St. Louis.

Figure 26.8. Inflation and unit labour costs in selected EMU countries



Note: Based on annual data.

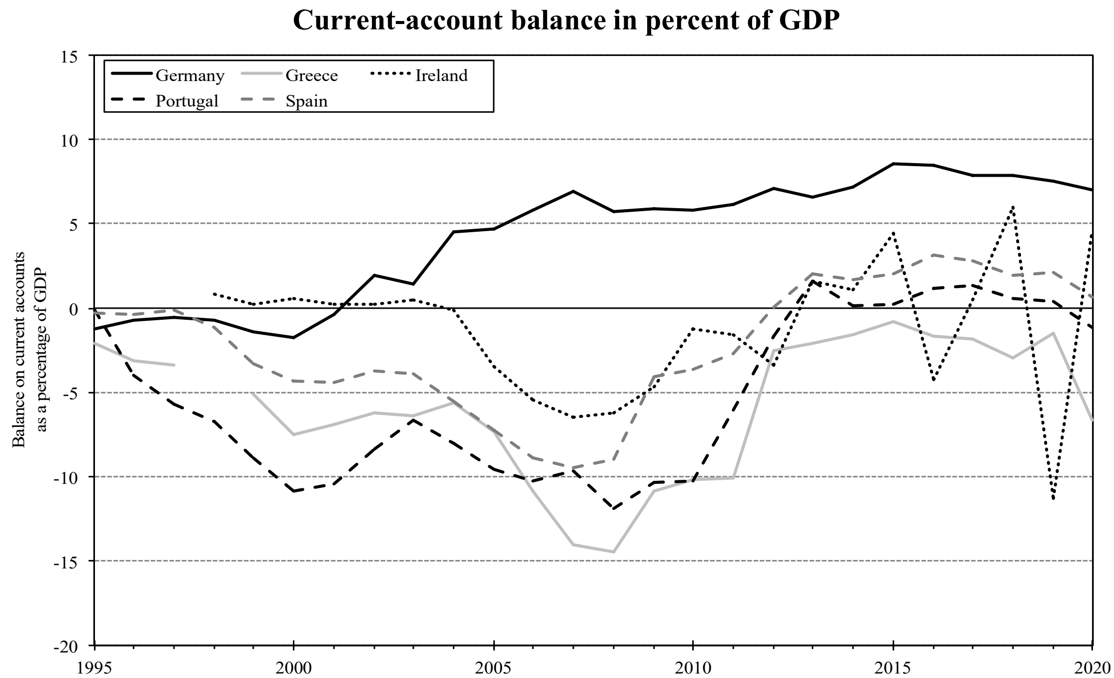
Source: Eurostat.



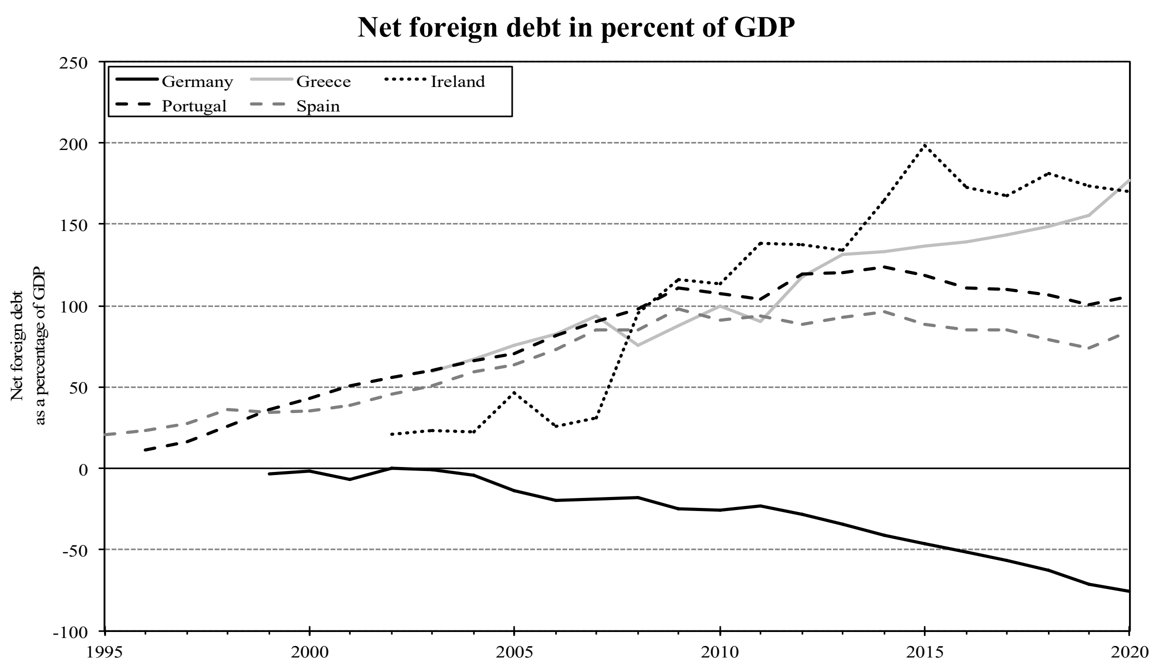
Note: Based on annual data.

Source: OECD.

Figure 26.9. Current-account balances and foreign debt in selected EMU countries



Note: Based on annual data. No data is available for Ireland before 1997 and for Greece in 1998.
Source: Eurostat (Ireland); World Development Indicators, World Bank.



Note: Based on annual data. Net foreign debt is measured by the (negative of the) country's Net International Investment Position.
Source: Eurostat.

To see how unsustainable the situation for the GIPS countries became, note that a country cannot continue to service its foreign debt if the ratio of debt to GDP keeps on increasing, since interest payments to foreigners will then end up absorbing all of its GDP. At some point a country will therefore have to stabilize its debt-GDP ratio, defined as D^n/Y^n, where D^n is nominal net foreign debt, and Y^n is nominal GDP. A nominal current-account deficit, CA^n, generates a corresponding increase in nominal net foreign debt,[14] that is, $D^n_{+1} - D^n = CA^n$, where variables without a subscript refer to the current period, and variables with the subscript "+1" refer to the next period. By definition, we also have $Y^n_{+1} = (1 + g^n)Y^n$, where g^n is the rate of growth of nominal GDP. From these identities it follows that constancy of the foreign debt-GDP ratio requires

$$\frac{D^n_{+1}}{Y^n_{+1}} - \frac{D^n}{Y^n} = 0 \implies \frac{D^n + CA^n}{(1+g^n)Y^n} = \frac{D^n}{Y^n} \iff$$

$$\frac{CA^n}{Y^n} = g^n \frac{D^n}{Y^n}. \tag{7}$$

From the bottom part of Figure 26.9 we see that by the late 2000s, the GIPS countries had accumulated a stock of net foreign debt roughly equal to 100 percent of their GDPs. Assuming a plausible nominal interest rate of 4 percent, this meant that these countries had to transfer about 4 percent of their GDPs annually to their foreign creditors; a substantial amount. Suppose they could nevertheless live with this 100 percent debt-GDP ratio ($\frac{D^n}{Y^n} = 1$) as a permanent condition. According to (7) they would then be able to run a permanent current-account deficit relative to GDP equal to the growth rate of their nominal GDP, which averaged around 7 percent per year in the boom period from 1995 to 2007. But as we see from the upper part of Figure 26.9, the Greek current-account deficit reached almost 15 percent of GDP around 2007, and the deficits of Portugal and Spain hovered around 10 percent of GDP around the same time. This indicates that all of these countries were on an unsustainable path towards an ever-increasing ratio of foreign debt to GDP, even if one assumes that a constant foreign debt ratio of as much as 100 percent would be economically and politically viable in the long run. The latter is highly questionable since such a high foreign debt ratio makes a country very vulnerable to shocks to the level of international interest rates. Moreover, after 2007 the trend growth rate of nominal GDP fell substantially in the GIPS countries, emphasizing the unsustainability of their current account deficits up until that time.

When the GIPS countries plunged into crisis after 2007, many commentators attributed all of their troubles to an overly lax if not downright irresponsible fiscal policy up until that time. A glance at Figure 26.10 indicates that reality was more nuanced. The governments of Greece did in fact run huge fiscal deficits between 5 and 10 percent of GDP in the years before 2007 (although the true size of the deficits was not revealed until

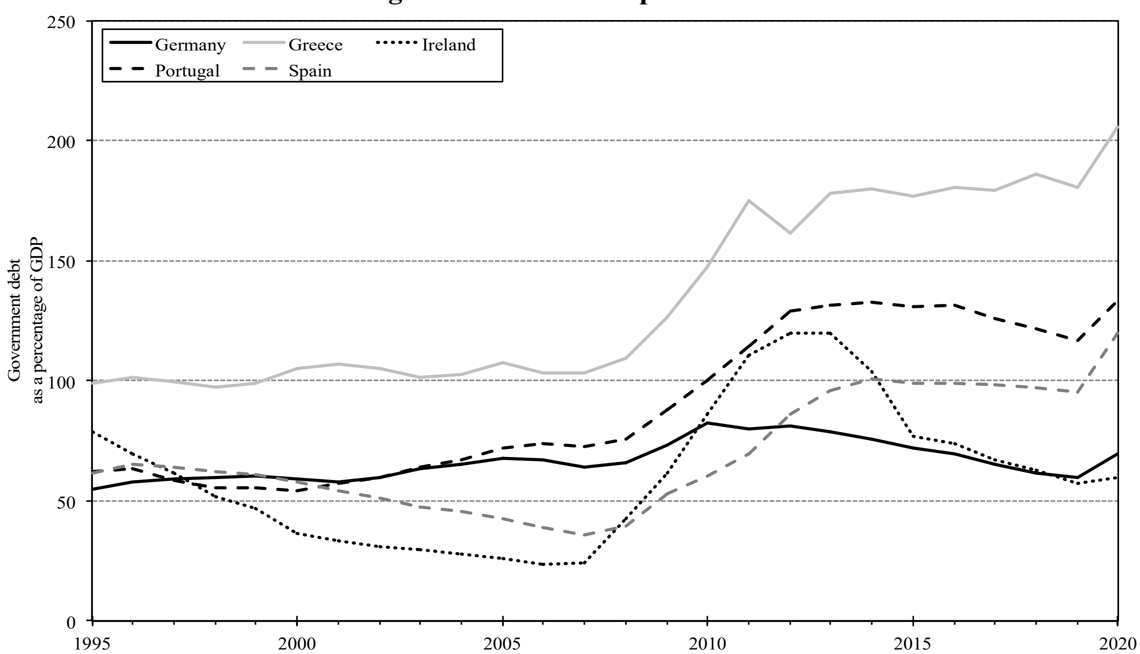
[14] Net foreign debt can also change as a result of revaluation of existing foreign assets and liabilities due to changes in asset prices, but in the long run such revaluation effects tend to wash out, so for simplicity we ignore them here.

Figure 26.10. Fiscal deficits and sovereign debt in selected EMU countries

Total government budget balance in percent of GDP

Note: Based on annual data.
Source: Eurostat.

Total government debt in percent of GDP



Note: Based on annual data.
Source: Eurostat.

late 2009, as we explain in the next subsection). These large deficits were maintained despite a booming economy and despite the fact that the Greek government debt-to-GDP ratio hovered around a very high level of 100 percent. No wonder that this fiscal behavior came to be seen as irresponsible. Portugal also ran high fiscal deficits of around 5 percent of GDP during most of the years before 2007, and these deficits gradually drove the Portuguese debt-GDP ratio well above the Maastricht limit of 60 percent. However, Ireland and Spain actually ran a surplus on their government budgets in several of the years before the international financial crisis, and both countries had a government debt-to-GDP ratio well below the Maastricht limit and much lower than the German debt ratio up until that time. Still, the small budget surpluses in Ireland and Spain before the crisis reflected their overheating economies which generated temporarily high tax revenues, and both countries actually ran structural budget deficits exceeding their Medium-Term Budgetary Objectives according the the Stability and Growth Pact. Hence there is a strong argument that Ireland and Spain should have pursued a tighter fiscal policy during the good years before the crisis to cool down their economies and create more room for a fiscal expansion when needed in the future. The evidence nevertheless suggests that fiscal profligacy was not the only reason for the economic troubles of Ireland and Spain during the euro crisis.

Let us now study how the crisis unfolded.

26.4 The euro crisis breaks out: sudden stops, capital flight and the doom loop

Like Ireland and Spain and several other European countries, the United States experienced a housing bubble in the first part of the 2000s, but the U.S. bubble culminated in 2005 and American house prices started to decline after that time. In the summer of 2007, it became clear that many so-called asset-backed securities backed inter alia by subprime mortgage loans to less credit-worthy American homeowners had been overvalued, as the falling house prices made many of them insolvent and created a growing number of defaults and foreclosures. These problems in the U.S. market for subprime mortgage credit created stress in the international interbank market for lending and borrowing among banks, as many banks throughout the world (including several large European banks) held assets whose values depended on the (failing) ability of subprime U.S. home-owners to pay their mortgage bills.

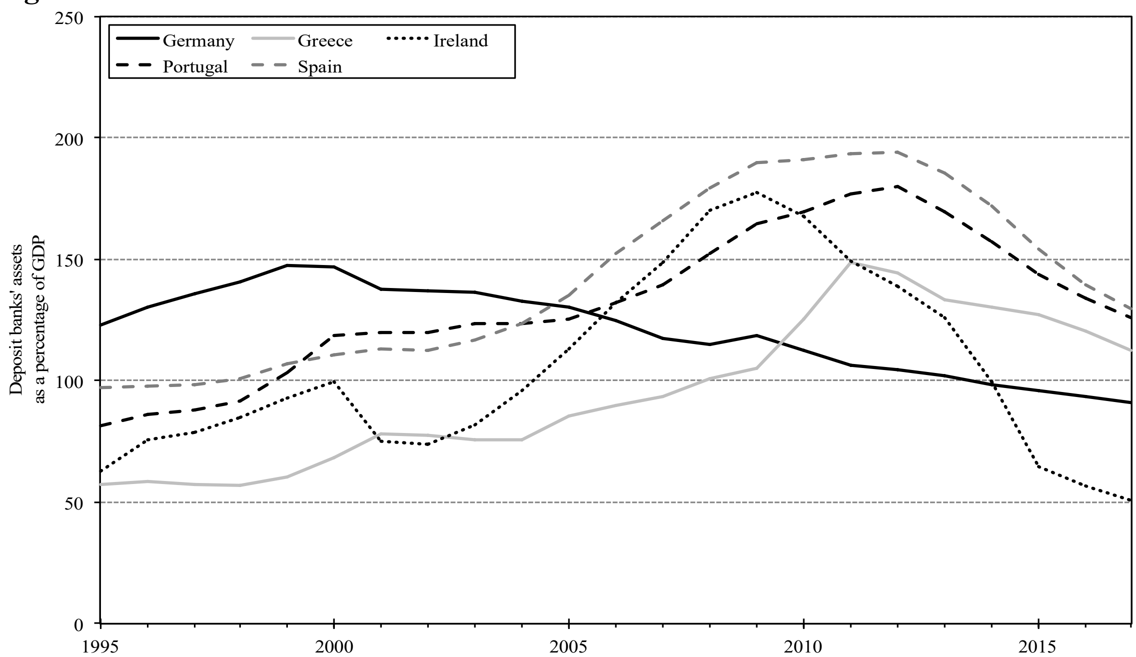
The stress in the interbank market in the summer of 2007 was temporarily relieved by injections of liquidity from central banks, but it rose to unprecedented heights when the U.S. investment bank Lehman Brothers failed on September 15, 2008, creating a true panic in international financial markets and triggering a serious global economic recession. The crisis led investors across the world to reassess their risks, and government bonds issued by the EMU countries with high levels of deficits and debt suddenly came to be seen as more risky than German government bonds. Therefore, a positive spread between the required yields on GIPS country bonds and German bonds emerged, as illustrated in Figure 26.5. Other countries with high ratios of government debt to GDP such as Italy also saw their yield spreads increase.

However, initially the euro zone countries were generally not affected worse than many other countries by the international financial crisis until some shocking news arrived from Greece in late 2009. Following a general election in October 2009, the new Greek government announced a revised 2009 budget deficit forecast of 12.7 percent of GDP which was more than double the previous estimate of 6.0 percent. It turned out that the Greek national statistical office had been manipulating the budget statistics for years, yielding to political pressure to make the Greek government budget look more solid than it actually was. The blow to investor confidence was devastating, and the yield spread on Greek government bonds started to take to the skies, as shown in Figure 26.5. As markets became more nervous, they also required much higher spreads than before on the government bonds of other GIPS countries (and of Italy) which were now seen as highly vulnerable.

The private sectors in the European periphery also felt the credit crunch. The large flows of new credit from Northern Europe and elsewhere suddenly stopped, and in many cases the private capital flows were reversed as lenders refused to roll over existing debts. The inflated property markets of Greece, Ireland and Spain had already started to turn down prior to these events, but now the downturn was made much worse as interest rates shot up and credit dried out. Consequently, banks and other financial institutions exposed to the real estate markets of these countries incurred massive losses as many property developers and homeowners failed and the property values seized by the creditors plunged. The sharp drops in credit flows and property prices can be seen in Figure 26.7.

A phenomenon that came to be known as the "doom loop" or the "diabolic loop" between banks and governments contributed to the steady worsening of the crisis. Following a long tradition, banks in the GIPS countries (and in several other EU countries) held large stocks of the bonds issued by their home governments. These bond holdings were used as collateral for the liquidity and credit granted to them by the ECB. As the required yields on the government bonds of the GIPS countries soared, the market value of the bonds took a deep dive, thereby eroding the capital (net equity) of the private banks and the value of the collateral they could mobilize. This threatened both the liquidity and the solvency of many banks, which stoked investor fears that their home governments might have to support them with capital injections or relieve them of their many non-performing loans to prevent an uncontrollable banking crisis. But since such government support to the banking sector would further increase the already high levels of government debt and leave governments with many "bad assets" of dubious value, it made markets even more nervous that some governments might not be able to service their debt, leading to further increases in yield spreads and thereby to further drops in the market value of government bonds. This in turn exacerbated the problems of ailing banks, and so on in a vicious "doom loop" circle. More broadly, the market reactions driving the doom loop were fed by fears that the balance sheets of banks had become so large relative to the size of their home country's economy and had come to include so many bad assets that the home governments of the banks would not have the fiscal and borrowing capacity to bail out systemically important banks threatened by collapse. As shown in Figure 26.11, the ratio of bank assets to GDP in the GIPS countries rose dramatically in the years before the crisis. As the crisis hit, this created serious doubts whether the governments of these countries could afford to save ailing banks that were deemed “too big to fail”, which further undermined confidence in the capacity of governments to service their debt. Thus the sheer size of the banking sector also contributed to setting the doom loop in motion.

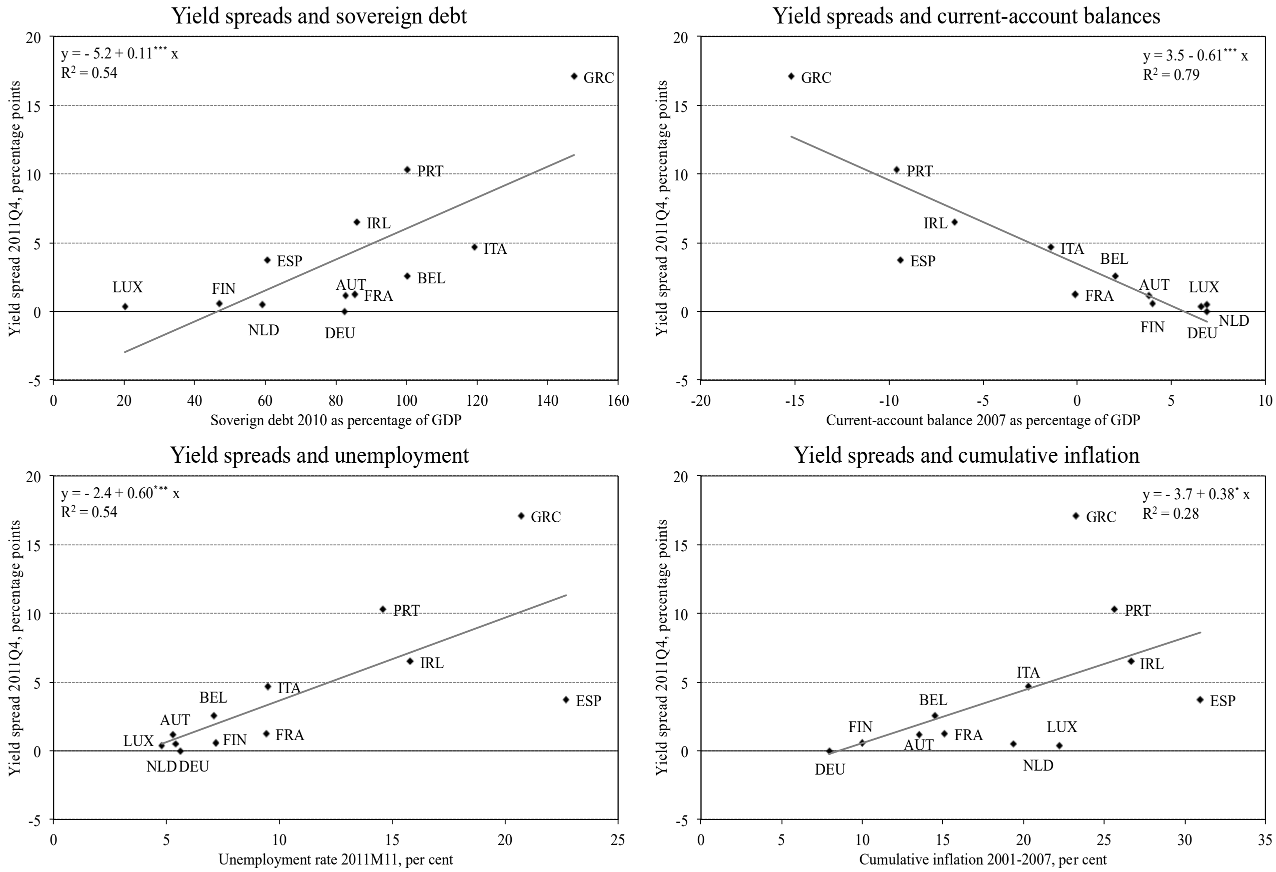
Figure 26.11. Bank assets relative to GDP



Note: Based on annual data.
Source: Global Financial Development, World Bank.

The deepening crisis in the sovereign bond markets and in the banking sectors of the GIPS countries in 2010-2012 took a terrible toll on their real economies, as documented in Figure 26.6. Their GDPs fell, and the unemployment rates in Greece and Spain rose to more than 25 percent, with youth unemployment reaching levels about twice as high as that. Ireland and Portugal also suffered unemployment rates of more than 15 percent in the trough of their recessions. By contrast, unemployment in Germany actually fell during these years, supported by a steady improvement of the country’s international competitiveness and the search by investors for new investment opportunities as they withdrew their capital from the European periphery and chose to invest in more stable countries like Germany instead.

Figure 26.12. Ten-Year Yield Spread Indicators (2011Q4) in EMU countries



Note: *** indicates statistical significance at the 1 percent level and * indicates statistical significance at the 10 percent level.
Source: Eurostat; World Development Indicators, World Bank.

Figure 26.12 shows correlations suggestive of the factors that drove up the interest rates on government bonds during the euro crisis.[15] The OLS regression analysis includes the 11 original EMU member countries plus Greece. The dependent variable in all regressions is the yield spread on 10-year government bonds over the yield on German government bonds, measured as an average over the last quarter of 2011, not long before the culmination of the crisis. Whilst correlation does not establish causation, the diagrams in Figure 26.12 nevertheless suggest that a larger ratio of government debt to GDP, a larger current-account deficit relative to GDP, a higher unemployment rate, and weaker international competitiveness all contributed to a higher risk premium on government debt. The impacts of the first three factors were all statistically significant at the 1 percent level. Interestingly, the strongest correlation (measured by R^2) was that between the

[15] The figure uses updated data to revise the estimates in Figure 8 in the following highly recommendable article on the causes of the euro crisis: Jay C. Shambaugh, 'The Euro's Three Crises', *Brookings Papers on Economic Activity*, Spring 2012, pp. 157-210.

current-account deficit and the yield spread. This indicates that the euro crisis was not just a sovereign debt crisis, but a broader crisis involving excessive private as well as public debt. The sovereign debt ratio and the unemployment rate correlated with the yield spread to the same extent, suggesting that investors worried just as much about weak growth and the potentially destabilizing political effects of mass unemployment as about high government debt. This has implications for the great debate on fiscal austerity policies to which we return.

The horizontal axis of the fourth diagram in Figure 26.12 measures the cumulative increase from 2001 to 2007 in the GDP deflators of the 12 countries considered. The larger the cumulative inflation rate, the more had the country's international competitiveness weakened up to the eve of the crisis. A loss of competitiveness seems to have had some impact on the risk premium included in government bond yields, indicating that investors also worried to some extent about the effect of competitiveness on the government's ability to service its debt, although the effect is statistically weaker than that of the other yield spread indicators.

Market failure versus government failure?

Before moving on to describe how policy makers tried to manage the euro crisis and what we might learn from that experience, it is worthwhile to review some elements of the fierce debate on the causes of the crisis that were not covered above.

Many commentators saw the euro crisis as yet another example of the failure of private capital markets to allocate resources efficiently over time and across countries. Economic history is indeed ripe with examples of how capital markets tend to fluctuate between periods of great optimism and times of undue pessimism, and how they sometimes create waves of speculation in particular assets followed by bursting asset price bubbles and financial panic. With the benefit of hindsight, it is clear that private investors tended to overestimate the earnings potential and underestimate the risks associated with investments in the GIPS countries prior to the great financial crisis and the subsequent euro crisis. At the same time, by suddenly stopping new investments and withdrawing many of their previous investments in the European periphery, investors may have overreacted, but all of this is just a variant of a well-known old story from the annals of market-based economies.[16]

Without denying this historical experience, several other economists argued that the euro crisis was just as much an example of the failure of government regulation.[17] Their first observation was that the ECB as well as national bank regulators considered government bonds issued by all EMU member states to be equally safe. As collateral for liquidity provision and credit from the ECB to private banks, Greek government bonds promising to pay a certain amount were considered just as valuable as German

[16] See, for example, the comprehensive evidence gathered by Carmen M. Reinhart and Kenneth S. Rogoff, *This Time is Different – Eight Centuries of Financial Folly*, Princeton University Press, 2009.
[17] One of the clearest expositions of this view was given by Hans-Werner Sinn, *The Euro Trap*, Oxford University Press, 2014. This book also gives a detailed account of the euro crisis and its causes.

government bonds promising to pay a similar amount. Similarly, as all government bonds issued by EMU governments were considered to be completely safe assets with a zero risk of default, banks were not required to increase their equity capital as insurance against potential losses when they acquired government bonds, regardless of the country that had issued the bonds. To the extent that markets considered, say, Greek bonds to be more risky than German bonds and therefore required a higher yield on Greek bonds, the regulatory practices thus made it more attractive for banks to hold the Greek bonds, since these bonds offered a higher return while having the same value as collateral and offering the same exemption from capital (equity) requirements.[18] In this way ECB practice and bank regulations effectively counteracted any attempts by markets to enforce different risk premia on government bonds issued by different EMU countries. Anyway, why should private investors care about different potential risks when the authorities considered all government bonds to be equally safe and were willing to act on this premise?

Another observation made by those who blamed the euro crisis on government as well as market failure was that governments never bothered to specify the meaning of the no-bailout clause of the Maastricht Treaty. As mentioned, this article of the treaty just said that member states cannot take on the debts of another member state. When the Treaty was adopted, it was hoped that this clause would incentivize private investors to consider the different default risks associated with the government bonds issued by different countries and differentiate their required risk premia accordingly. If a government accumulated debt that seemed unsustainable, markets would then respond by raising their required yield on its bonds, and this would induce that government to tighten its fiscal policy and bring its debt ratio onto a sustainable path. At least that was the theory. But to enable investors to calculate rational risk premia, policy makers would have had to clarify the no-bailout clause by implementing directives specifying what would happen if a government in the EMU became insolvent and unable to service its debt, explaining the sequence in which different groups of creditors would have to accept “haircuts” (write-downs of their claims) and laying out principles for a rescheduling of the debt. Since no such directives and explanations were given, it was tempting for investors to assume that the authorities did not really think the no-bail out clause would ever have to be activated. This interpretation was supported by the fact that the ECB and bank regulators treated all government bonds as if they were equally safe. Whatever the reason, the near-perfect convergence of government bond yields following the formation of the EMU is evidence that investors did not take the no-bailout clause of the treaty framework seriously, so the hoped-for fiscal discipline effect of this clause did not materialize.

As we shall discuss later, there were also other features of the institutional design of the EMU that made the union vulnerable to shocks, but we will be in a better position to discuss these features after having reviewed the experience with the management of the

[18] One might think that if the banks shared the view of other investors that Greek bonds were in fact more risky than German ones, banks would shy away from holding Greek bonds. But the business model of banks, relying on a combination of a low equity ratio and limited liability for shareholders, makes it attractive for them to hold a relatively high share of risky assets with a correspondingly high expected rate of return in their balance sheets.

euro crisis.

26.5 Managing the European debt crisis

Doing "whatever it takes"

As we have seen, all of the GIPS countries ran massive current-account deficits in the run-up to the crisis. These deficits reflected an excess of imports over exports as well as interest payments on the existing foreign debt. When the euro crisis hit, foreign private investors were no longer willing to finance the deficits via continued flows of new capital into the GIPS countries; in fact, on many occasions they withdrew their existing investments in those countries by not renewing existing loans.

With a sudden stop of capital inflows and nothing to replace them, the GIPS countries would have had to eliminate their current-account deficits immediately. The option of currency devaluation was no longer available, and because of nominal wage and price rigidities, a quick improvement of international competitiveness via a process of domestic deflation was not realistic on the scale required. Only a truly catastrophic fall in economic activity that would drive down imports could then have secured an immediate equilibration of the current accounts. The current-account improvements that actually took place in the GIPS countries after the crisis (as illustrated in Figure 26.9) did in fact mainly reflect a deep fall in output and incomes and a sharp rise in unemployment, but unless a total collapse in economic activity takes place, correcting a large current-account deficit is a time-consuming process.

The pre-crisis flow of capital to the GIPS countries was primarily intermediated by the banking system. Banks in the GIPS countries borrowed from banks in other countries, often taking loans with relatively short terms to maturity. These funds were then relent to the domestic non-financial sector or invested in domestic government bonds. When the GIPS country banks could no longer borrow abroad, they faced a serious liquidity crisis, as much of their lending was tied up in long-term assets such as mortgage loans. The mechanics of the doom loop between banks and governments started to be felt, and banks in other EMU countries that had lent to the ailing GIPS country banks also faced increasing risks.

In the early stage of the euro crisis it therefore fell upon the ECB to close the credit gap represented by the remaining current-account deficits in the GIPS countries and reflected in a funding gap in the banking systems of these countries. Effectively the ECB stepped in as a 'lender of last resort' when the foreign private investors moved out. This was done via a series of unconventional ECB measures improvised as the crisis deepened. While the details of these successive emergency measures varied, the underlying principles were straightforward: banks were gradually allowed to borrow increasing amounts of money from the ECB at longer maturities and against collateral of lower quality. In this way, the banks of the GIPS countries were able to obtain liquidity and maintain some of the flow of credit to their domestic customers and hold on to their stocks of domestic government bonds.[19]

The ECB also allowed the national central banks within the European System of Central Banks (ESCB) to offer so-called Emergency Liquidity Assistance (ELA) to their domestic private banks on terms that the NCBs could to some extent determine themselves. Formally, the extra credit granted via the ELA programme was provided at the risk of the individual NCBs, which had to bear the losses in case the loans were not repaid. But if the losses of an NCB became so large that it had to be recapitalized by its home government, and if that government were not able to raise the necessary funds in the market, the loss would ultimately have to be carried by the ESCB.

In addition, the ECB started in May 2010 to buy up government bonds issued by the GIPS countries and Italy as another way to inject liquidity into the euro zone economy and to support the bond markets of the distressed countries. The launch of this Securities Markets Programme (SMP) generated fierce debate within the ECB itself, as a minority of members of the ECB Executive Board saw it as a violation of the no-bailout clause of the Maastricht Treaty.

As the ECB emergency measures became increasingly controversial, it became clear that further support to the crisis-ridden member countries of the EMU would have to be granted by the governments of the other EU countries and/or by the International Monetary Fund (IMF), and that is what ultimately happened. The EU was the main provider of financial assistance to the GIPS countries, but the IMF also contributed substantial amounts. The loans to the governments of distressed countries were granted on the condition that they implemented economic adjustment programmes involving a combination of fiscal austerity and structural reforms. The terms of these programmes were negotiated between the government of the debtor country and a “Troika” consisting of representatives of the European Commission, the ECB and the IMF. Initially, the loans to the distressed countries were granted via the European Financial Stability Facility (EFSF), an institution that could issue bonds backed by guarantees given by the euro area member states in proportion to their share in the paid-up equity capital of the ECB. The EFSF was soon supplemented by the European Financial Stability Mechanism (EFSM), another emergency funding programme raising funds in financial markets and guaranteed by the European Commission using the budget of the European Union as collateral. Via the EFSM all EU countries, including those outside the euro zone, indirectly agreed to back up some of the financial assistance to the GIPS countries. In 2012, both the EFSF and the EFSM were replaced by the European Stability Mechanism (ESM) which is a permanent rescue funding programme backed by euro area countries and operating according to the same principles as its predecessor, the EFSF.

In May 2010, Greece became the first EMU country to receive official external

[19] One of the deviations from normal practice was that the ECB started to accept so-called asset-backed securities (ABS) as collateral for loans to the banking sector. The cash flows from ABS are financed by payments from a set of different debt contracts that sometimes take a curious form. For example, it was found that a Spanish ABS security accepted as collateral by the ECB was based in part on a loan to Real Madrid to purchase the football player Christiano Ronaldo! Apparently, the ECB had great confidence in the future of Real Madrid or, more likely, it had stopped worrying too much about the riskiness of the assets it accepted as collateral.

assistance, as the country was effectively shut out of the government bond market. The total loans from the EU and the IMF in this first bailout programme for Greece amounted to 110 billion euros over three years. In November 2010, Ireland had to be rescued, as the Irish government was threatened by default after having guaranteed depositors and bondholders in the six main Irish-based banks that had financed the country's enormous property bubble and had ended up with massive losses. The loan from the EU and IMF totaled 67.5 billion euros. Next in line was Portugal whose government received a loan package of 78 billion euros in April 2011. Then the wheel turned back to the Greeks whose economy sank ever deeper into depression. In February 2012, Greece was granted a second bailout loan of 130 billion euros to prevent a total economic collapse, but this time the Troika insisted that private holders of Greek government bonds should take a haircut by accepting a 53.5 percent write-down of their nominal claims on the government, which really amounted to a controlled government default on debt. By June 2012 the Spanish government, trapped in the dynamics of the afore-mentioned doom loop, faced difficulty in accessing the bond markets and sought official funding. The governments of the euro zone responded by granting a financial support package of up to 100 billion euros. The funds were earmarked for recapitalization of the Spanish banks that were threatened by failure due to their many non-performing loans to the Spanish real estate sector.

Despite the bailouts and haircuts, the euro crisis refused to go away, reflected by continuing high yield spreads on the government bonds of the ailing EMU countries, including large countries like Italy and Spain, as shown in Figure 26.5. Financial markets did not calm down until the ECB again took centre stage. On 26 July 2012, ECB President Mario Draghi gave a speech which earned him the title of "savior of the euro" from many commentators. In his speech, Draghi declared: "*Within our mandate, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough*." This statement had an immediate impact on markets, as the spreads on government bond yields started to decline right away. Shortly after, on 6 September 2012, the ECB followed up on Draghi's speech by announcing a new "Outright Monetary Transactions" (OMT) programme, which offered massive yield-lowering government bond purchases by the ECB for all euro zone countries involved in a sovereign state bailout programme from the EFSF/ESM. This gave credibility to Draghi's statement while emphasizing that the ECB would only purchase bonds issued by a government that accepted the lending conditions required by the EFSF/ESM. In this way, the ECB signaled that it would not engage in unconditional bailouts that could create serious moral hazard by encouraging future irresponsible fiscal policies. The announcement of the OMT programme contributed further to the decline of yield spreads in the following months and years, as can be seen from Figure 26.5.

The summer of 2012 thus marked a turning point in the euro crisis. After that time, the economies of the GIPS countries started to recover, most remarkably in Ireland, as shown in Figure 26.6. However, the recovery was much slower in the Southern European countries, and the Greek drama continued. In 2015 a new Greek government formed by the left-wing, anti-austerity Syriza party staged a show-down with international creditors, and on June 30, 2015, the Greek government missed a repayment of 1.6 billion euros of debt to the IMF, thereby becoming the first developed country to default to the Fund (payment was made by a 20-day delay). But the Syriza government ended up accepting a new programme of austerity and economic reform demanded by the EU in return for a third bailout loan of 86 billion euros in August 2015. By that time, a split between the IMF and the EU had opened, as the IMF refused to contribute more funds to Greece unless the country's creditors agreed to provide significant debt relief. When Greece received the final loan tranch of its third bailout programme in August 2018, thereafter exiting from the programme, the IMF maintained that the country would likely need further future debt relief. By that time Greece owed roughly 290 billion euros to the EU and the IMF, and its public debt had reached 180 percent of GDP while its GDP had shrunk by 25 percent since the start of the crisis. To finance its debt, it was foreseen that Greece would have to run a government budget surplus up until 2060.

A fundamental dilemma for policy makers: Liquidity crisis or insolvency?

As early as 20 January 2009, the leading American economic historian Barry Eichengreen wrote an article on the euro crisis that began with the following observations: *"What started as the Subprime Crisis in 2007 and morphed into the Global Credit Crisis in 2008 has become the Euro Crisis in 2009. Sober people are now contemplating whether a euro area member such as Greece might default on its debt."* The alternative to default, he wrote, would be "… *fiscal retrenchment, wage reductions, and assistance from the EU and the IMF for the cash-strapped government."* [20]

Eichengreen then continued, prophetically: *"To be sure, this alternative will be excruciatingly painful. No one will like it except possibly the IMF, which will relish the opportunity of reasserting its role as lender to developed countries. There will be demonstrations against the fiscal cuts and wage reductions. Politicians will lose support and governments will fall. The EU will resist providing financial assistance for its more troublesome members. But, ultimately, everyone will swallow hard and proceed, much as the U.S. Congress, having played rejectionist once, swallowed hard and passed the $700 billion bank bailout bill when disaster loomed."*

With these predictions, Eichengreen anticipated the dramatic events as well as the fierce debates that would follow. Well before it happened, many economists argued that Greece should be allowed to default on its debt (forcing a haircut on its creditors) to enable the country to get on its feet again and avoid continuing social misery that would not save the creditors, as a future default was inevitable anyway. They also argued that a quick default would eliminate the deep uncertainty regarding the future economic and political developments in Greece, which made international financial markets nervous to the detriment of all countries. On the other hand, the opponents to a Greek default feared that a default would have serious negative contagion effects on other EMU countries by

[20] The article can be found here: https://voxeu.org/article/was-euro-mistake. VoxEU.org is a web publication set up by the London-based Centre for Economic Policy Research to promote research-based policy analysis and commentary by leading economists.

stoking investor fears that other distressed member states might follow in the footsteps of Greece. Many opponents also worried that allowing the Greeks to write down their debt would create moral hazard by encouraging further reckless fiscal policies in Greece and elsewhere in the EMU.

The disagreement also reflected a difficult analytical problem arising during financial crises: evaluating whether a distressed debtor is just suffering from a *liquidity crisis* due to unfavourable temporary circumstances, or whether the debtor is *insolvent*, that is, unable to service the debt in the long run under any circumstances. In the case of Greece, insolvency would mean that the present value of the future primary government budget surpluses (surpluses before interest payments) that the country would realistically be able to run could not cover the existing stock of government debt. In that situation a default would sooner or later be inevitable, and rather than drawing out the pain, it would be better for all parties to resolve the situation here and now by negotiating a realistic write-down of the debt. By contrast, if Greece were indeed solvent, being able to pay its bills in the long run unless it would have to pay an "unreasonably" high interest rate on its debt, and just experiencing temporary difficulties of borrowing due to an overly nervous financial market, the country "only" suffered from a liquidity crisis from which it could escape if it were granted additional temporary credit at a "reasonable" interest rate. This was the assumption made by the Troika when they offered the first loan package to Greece in May 2010.

Modern macro-financial theory does indeed provide a strong rationale for official lenders to step in when a government that can reasonably be deemed to be solvent has landed in a liquidity crisis. The reason is that debt markets can have multiple equilibria, some of which are "good" in the sense of involving relatively low interest rates motivated by a low risk of default, whereas other equilibria are "bad", involving very high interest rates combined with high risks of default. A good equilibrium may be interpreted as a "normal" case, while a bad equilibrium can be interpreted as a liquidity crisis, but in the latter case an outside intervention may pull the debtor out of the crisis by pushing the market back to a good equilibrium. We will now illustrate this important insight from modern macro-financial theory with a very simple model.[21]

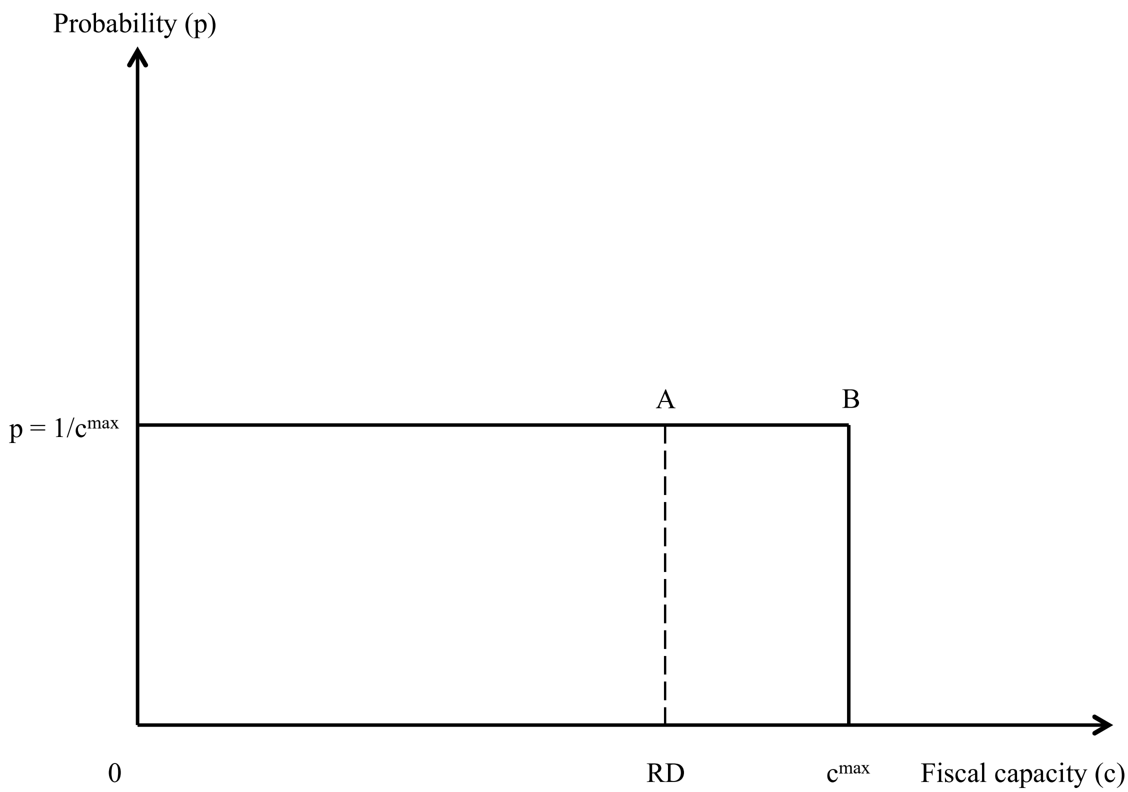
The model includes two time periods representing "the present" and "the future". The agents in the model are a government that enters the first period with a predetermined stock of debt, D, and the government's private creditors to whom it is obliged to repay the amount $(1 + r)D$ in the second period, where r is the interest rate on the debt. Whether the government will actually be able to service its debt depends on its future "fiscal capacity", c, which could be measured by the future primary budget surplus it is able to run. Since the future is uncertain, the government's future fiscal capacity is a stochastic variable. For simplicity, we assume that the fiscal capacity follows a uniform probability distribution defined over the continuous interval from zero to c^{max}, where the

[21] The model is inspired by section 5.1 in the paper by Markus K. Brunnermeier and Ricardo Reis, "A Crash Course on the Euro Crisis", National Bureau of Economic Research Working Paper 26229, September 2019. A richer model of sovereign debt crises can be found in the influential article by Guillermo Calvo, "Servicing the Public Debt: The Role of Expectations", *American Economic Review* 78 (4), 1988, pp. 647-661.

maximum fiscal capacity c^{max} represents the largest possible primary budget surplus the government can run under the most favourable future circumstances.

The density function for the uniform distribution of fiscal capacity is depicted in Figure 26.13 where p denotes the common probability that each of the possible realizations of fiscal capacity will occur. Since the sum of the probabilities that c will lie in the interval $[0, c^{max}]$ is 1, the area of the rectangle $0pBc^{max}$ in Figure 26.13 is also 1, implying $p = 1/c^{max}$. On the horizontal axis we have marked the point RD, where $R \equiv 1 + r$. RD is the total sum the government has promised to pay its creditors in the future. The probability that the realized fiscal capacity will be less than this amount is given by the area of the rectangle $0pARD$ which is $pRD = RD/c^{max}$. This is the probability that the government will default on its debt. The probability that the fiscal capacity will be larger than or equal to RD is $1 - (RD/c^{max})$, i.e., the area of the rectangle $RDABc^{max}$. This is the probability that the government will be solvent.

Figure 26.13. Probability distribution of fiscal capacity



If the government defaults, its creditors will have to negotiate the size of the haircut they must take. This not only involves negotiations between creditors and the government; typically it will also pitch different groups of creditors against each other as they may not agree on the acceptable haircut and on the distribution of the remaining diminished payments from the government. One problem for the creditors is that while they may know the true probability distribution illustrated in Figure 26.13, enabling them to calculate a rational risk premium in their required return ex ante before they lend to the government, they may not be able to observe the government's actual realized fiscal capacity ex post. When the future arrives, the government may therefore claim that its fiscal capacity is only c^{small}, without its creditors being able to verify it, whereas in fact the ex post fiscal capacity is $c^{actual} > c^{small}$. Indeed, if negotiations break down because the government finds the claims of the creditors quite unreasonable, the government may refuse to pay anything at all.

For these reasons a default is a costly process typically involving large legal costs and other transactions cost for creditors plus the risk of a complete breakdown of negotiations due to domestic political pressure on the government. If the realized fiscal capacity is c^{actual}, the creditors can therefore only expect to receive an amount αc^{actual} in case of default, where $0 \leq \alpha < 1$. In the following we assume for simplicity that $\alpha = 0$, i.e., that the creditors simply get nothing in case of default, since this is not important for our analysis.

Recall from our analysis of Figure 26.13 that the government will be solvent with probability $1 - (RD/c^{max})$. Hence the creditors' *expected* future payment from the government is $[1 - (RD/c^{max})]RD$. Suppose the creditors require an expected rate of return $\bar{r}$ for being willing to lend to the government, thus demanding an expected future repayment of $\bar{R}D$, where $\bar{R} \equiv 1 + \bar{r}$. The equilibrium gross interest rate R that will generate the expected yield required by investors must then satisfy the condition

$$[1 - (RD/c^{max})]RD = \bar{R}D \quad \Leftrightarrow$$

$$R^2 + aR + b = 0, \qquad a \equiv -\frac{c^{max}}{D}, \quad b \equiv \frac{c^{max}\bar{R}}{D}. \tag{8}$$

Equation (8) is a second-order polynomial with the two roots

$$R^{HIGH} = -\frac{a}{2} + \left(\frac{a^2}{4} - b\right)^{\frac{1}{2}} = \frac{c^{max}}{2D} + \left(\frac{(c^{max})^2}{4D^2} - \frac{c^{max}\bar{R}}{D}\right)^{\frac{1}{2}}, \tag{9}$$

$$R^{LOW} = -\frac{a}{2} - \left(\frac{a^2}{4} - b\right)^{\frac{1}{2}} = \frac{c^{max}}{2D} - \left(\frac{(c^{max})^2}{4D^2} - \frac{c^{max}\bar{R}}{D}\right)^{\frac{1}{2}}. \tag{10}$$

For economically meaningful solutions, we assume that $c^{max} > 4\bar{R}D$ which ensures that the roots R^{HIGH} and R^{LOW} are both positive real numbers. The interesting implication of (9) and (10) is that the government bond market has two equilibria. According to (10) the market can clear at a relatively low interest rate involving a low promised payment to creditors, $R^{LOW}D$, which in turn implies a low probability $R^{LOW}D/c^{max}$ that the government will default. This is a "good" equilibrium from a social viewpoint since it implies that the risk of incurring the various transactions costs of a default is low, and since it means that the government does not have to run a very large primary budget surplus to finance its interest expenses and can therefore spend more of its tax revenue on activities that benefit its citizens. But the market can also clear at the high gross interest rate given by (9) which carries with it a high default risk $R^{HIGH}D/c^{max}$. In this “bad” equilibrium there is a high probability that the costs of default will materialize and, furthermore, the government must spend a large part of its tax revenue on interest payments that do not benefit the citizens (note that in the bad equilibrium creditors are no better off than in the good equilibrium, since the high interest rate in the bad equilibrium just compensates them for the higher risk of default and hence does not imply a higher expected rate of return).

But what factors determine whether the government bond market ends up in the good or in the bad equilibrium? A plausible hypothesis is that this will depend on investor expectations, or more precisely, on the expectations of individual investors regarding the expectations of other investors. Suppose individual investors believe that the other investors believe that the market can reach an equilibrium at the low gross interest rate R^{LOW}, implying that the risk premium required by other investors is expected to be low. In that case each individual investor will believe that the risk of default, $R^{LOW}D/c^{max}$, will be low and will therefore be willing to lend to the government at the gross interest rate R^{LOW}. The market will then end up in the good equilibrium. But suppose alternatively that individual investors believe that the other investors are somewhat nervous, believing that the market will converge on the equilibrium with the high gross interest rate R^{HIGH}. Individual investors will then perceive a relatively high default risk $R^{HIGH}D/c^{max}$ which will induce them to require a high risk premium that actually forces the market into the bad equilibrium corresponding to our notion of a liquidity crisis.

In his seminal 1936 book on *The General Teory of Employment, Interest and Money*, John Maynard Keynes used a famous metaphor to explain how financial markets can have multiple equilibria depending on individual investor beliefs about the beliefs of other investors. Keynes imagined a beauty contest where the readers of a newspaper were asked to choose the six most attractive faces from hundred photographs. The readers who picked the most popular faces would be eligible for a prize. To maximize his chance of winning, a rational participant would not choose the faces that he himself found most handsome; rather, he would try to guess which faces would be considered most handsome by the other readers. Indeed, he would realize that the other readers probably reasoned in the same way, trying to guess what faces the readers of the newspaper would believe to be most popular. In Keynes’ own words: *“It is not a case of choosing those faces that, to the best of one’s judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligence to anticipating what average opinion expects the average opinion to be.”*

The analogy to our model of the government bond market is that it is not rational for an individual bond investor to base his investment decision on his own subjective judgment on what the interest rate “ought” to be, given his own evaluation of the government’s creditworthiness. Rather, to calculate the risk premium that will satisfy his required expected return, the individual investor must estimate what other investors consider to be a plausible equilibrium interest rate, since this will determine the actual equilibrium interest rate and the associated risk of default. According to this analysis, it may be possible for policy makers to shift a financial market from a bad to a good equilibrium if it can somehow shift investor beliefs about average market opinion. In the case of the government bond market, such a shift of expectations could occur if the central bank intervenes with purchases of government bonds to drive down the interest rate and convince the market that it intends to move the market towards the good equilibrium.

The famous statement by ECB President Mario Draghi on 26 July 2012 that the ECB would do "whatever it takes" to save the euro and the subsequent announcement of the ECB Outright Monetary Transactions (OMT) programme for government bond purchases are often mentioned as an example of a successful central bank move to shift the financial market from a bad to a good equilibrium by managing investor expectations. The yield spreads on the government bonds of distressed euro zone countries fell significantly immediately after Draghi's speech, and *in the end the ECB did not have to spend a single euro on government bond purchases under the OMT programme, as the market itself drove interest rates back towards much lower equilibrium levels*. The declaration that the ECB was willing to act as a lender of last resort to euro zone governments was sufficient to calm the market and make investors coordinate on a lower level of interest rates.

This experience seems to justify the ECB intervention in the euro crisis in the summer of 2012, despite opposition from the German Bundesbank.[22] However, as we have seen, Greece nevertheless had to receive a third bailout in 2015 in which the IMF refused to participate, as the Fund believed that Greece needed further debt relief to be able to stage a sustainable economic recovery. This reflects the difficult judgment policy makers have to make in such situations. In particular, it may be very difficult to estimate a debtor country's maximum long-run fiscal capacity, c^{max}. Perhaps the IMF believed that even if Greece were granted official loans at a risk-free interest rate r^{safe}, the country's maximum fiscal capacity would nevertheless fall short of $r^{safe}D$ because of the immense size of the Greek debt. In that case, Greece was surely insolvent, and a new write-down of the debt would sooner or later become inevitable. But maybe Greece's EU partners had a more optimistic view of the country's fiscal capacity which implied that Greece was not insolvent, but only suffered from yet another liquidity crisis that could be overcome through a new round of official lending at a low interest rate.

Distinguishing between insolvency and illiquidity has turned out to be a recurrent challenge for policy makers during financial crises, and the challenge is particularly big when the debtor is a sovereign government whose long-run fiscal capacity will depend on social and political as well as economic developments.

[22] The Bundesbank appealed to the German Constitutional Court that the ECB had overstepped its mandate by launching the OMT programme. The Bundesbank economists argued that the high spreads on government bond yields in the distressed EMU countries reflected macroeconomic fundamentals and were necessary to preserve fiscal discipline. Thus the Bundesbank did not acknowledge the possibility that the bond market could end up in a bad equilibrium that was not warranted by fundamentals. However, on 21 June 2016 the German Constitutional Court declared that the OMT programme did not violate the German Constitution, and that the Bundesbank was allowed to participate in the programme, as long as the programme complied with the conditions under EU law, as pronounced by the European Court of Justice. In this way a serious constitutional conflict within the EU was avoided.

Austerity: A double-edged sword

The financial assistance from the Troika to the GIPS countries was granted on the condition that they adopted economic stabilization programmes, which included a combination of fiscal tightening and structural reforms tailored to the specific circumstances in each country. The required fiscal retrenchment was considerable, and the demands for fiscal discipline in the EU were also reflected in the European Fiscal Compact agreed in 2011. The Fiscal Compact was a tightened version of the Stability and Growth Pact aimed at preventing future sovereign debt crises. Among other things, the Fiscal Compact strengthened the procedure for imposing penalties in case a country violated the 3 percent fiscal deficit or the 60 percent debt rules.

However, the austerity policy imposed on the GIPS countries soon came under attack for being ineffective in achieving its aim, which was to reduce the ratio of government debt to GDP. To understand this dispute which was one of the greatest economic debates of the 2010s, it is useful to start out from the government's budget constraint:

$$D_{+1}^{n} - D^{n} = (i + \varepsilon)D^{n} + G^{n} - \tau Y^{n}, \qquad 0 < \tau < 1. \tag{11}$$

Here D^n is nominal government debt, G^n is nominal government spending on goods and services, τ is the average net tax rate, τY^n is the difference between total nominal tax revenue and total nominal transfers, assumed to depend positively on nominal GDP, Y^n, i is the nominal interest rate on safe government bonds, and ε is a risk premium in the interest rate, $\varepsilon \geq 0$, reflecting the perceived risk of government default. The difference $G^n - \tau Y^n$ is the nominal deficit on the primary government budget (the deficit before interest expenses), and the entire right-hand side of (11) is the total budget deficit which must generate a similar increase in net government debt (the left-hand side). By definition, we also have the following link between the nominal GDP in the current and in the next period,

$$Y_{+1}^{n} = (1 + \pi)(1 + g)Y^{n}, \tag{12}$$

where π is the rate of domestic inflation (the rate of increase in the GDP deflator), and g is the growth rate of real GDP. Dividing by Y^n on both sides of (11) and using (12), and noting that $\frac{D^n}{Y^n} = \frac{D}{Y}$ and $\frac{G^n}{Y^n} = \frac{G}{Y}$, where D, G and Y are the *real* levels of debt, government spending and GDP, respectively, we get the following expression for next period's ratio of government debt to GDP:

$$d_{+1} = \left[\frac{1}{(1+\pi)(1+g)}\right]\left[(1 + i + \varepsilon)\frac{D}{Y} + \frac{G}{Y} - \tau\right], \qquad d_{+1} \equiv \frac{D_{+1}}{Y_{+1}}. \tag{13}$$

We will now use (13) to analyze the conditions under which a tightening of fiscal policy in the form of a cut in real government spending will succeed in bringing down next period's debt-GDP ratio. Let $m \equiv \frac{dY}{dG} \geq 0$ denote the first-year fiscal multiplier, and let $m_{+1} \equiv dY_{+1}/dG \geq 0$ indicate the second-year multiplier, i.e., the effect on next period's real GDP of a one unit change in real government spending on goods and services in the current period. We can then write

$$Y = Y(G), \quad Y' = m \geq 0, \qquad Y_{+1} = Y_{+1}(G), \quad Y'_{+1} = m_{+1} \geq 0, \tag{14}$$

$$g(G) \equiv \frac{Y_{+1}(G)-Y(G)}{Y(G)} \quad \Rightarrow \quad g' = \frac{Y(m_{+1}-m)-mgY}{Y^2} = \frac{m_{+1}-m(1+g)}{Y}, \tag{15}$$

where (15) shows how the real GDP growth rate g depends on the first- and second-year multipliers. We can also write the inflation rate as a function of the fiscal multiplier, using our standard AS-curve for the open economy:

$$\pi(G) = \gamma\left(\frac{Y(G)-\bar{Y}}{\bar{Y}}\right) + \pi^f, \quad \pi' = \frac{\gamma m}{\bar{Y}} \geq 0. \tag{16}$$

Finally, while a rise in the debt ratio may not have a noticeable impact on the risk premium at moderate levels of debt, it is likely to generate an increase in the risk premium at high levels of debt, since the risk of government default increases with the debt ratio. Hence, we assume that the risk premium in the government bond yield is a non-decreasing function of the debt-GDP ratio and a strictly increasing function at high levels of debt (the upper left panel of Figure 26.13 could indicate such a non-linear relationship). Since the debt-GDP ratio $d \equiv D/Y$ depends on real government spending via the fiscal multiplier, the risk premium ε then also becomes a function of G which can be written as

$$\varepsilon = \varepsilon(d) = \varepsilon\left(\frac{D}{Y(G)}\right), \quad \varepsilon' \geq 0 \quad \Rightarrow$$

$$\frac{d\varepsilon}{dG} = \frac{-mD\varepsilon\prime}{Y^2} = -\frac{m}{Y}d\varepsilon' = -\frac{m\varepsilon\eta}{Y}, \quad \eta \equiv \varepsilon'\frac{d}{\varepsilon} \geq 0, \tag{17}$$

where η is the elasticity of the risk premium with respect to the debt ratio. Equation (17) shows that the risk premium may be written as a function $\varepsilon(G)$, since the debt level in the current period is predetermined by the accumulated past budget deficits.

Using the functions defined in (14) through (16) and the fact that $\varepsilon = \varepsilon(G)$, we can rewrite (13) as

$$d_{+1} = \left[\frac{1}{(1+\pi(G))(1+g(G))}\right]\left[(1+i+\varepsilon(G))\frac{D}{Y(G)} + \frac{G}{Y(G)} - \tau\right]. \tag{18}$$

By differentiating the expression on the right-hand side of (18), we can analyze how an austerity policy in the form of a cut in real government spending on goods and services will affect next period's debt-GDP ratio. Carrying out the differentiation, using the expressions for the derivatives in (14) through (17), and rewriting the resulting expression in a convenient way by means of the definition $(1+g^n) \equiv (1+\pi)(1+g)$, we find that (Exercise 4 gives you hints how to derive this result):

$$\frac{\partial d_{+1}}{\partial G} Y = \left(\frac{1}{1+g^n}\right)(A1 + A2 - A3 - A4), \tag{19}$$

$$A1 \equiv 1 - m\frac{G}{Y}, \qquad A2 \equiv \left(\frac{D}{Y} + \frac{\Delta}{Y}\right)\left(m - \frac{m_{+1}}{1+g}\right), \tag{20}$$

$$A3 \equiv m\frac{D}{Y}[1 + i + \varepsilon(1+\eta)], \qquad A4 \equiv \left(\frac{D}{Y} + \frac{\Delta}{Y}\right)\frac{\gamma m(Y/\bar{Y})}{1+\pi}. \tag{21}$$

$$\frac{\Delta}{Y} \equiv \frac{(i+\varepsilon)D + G - \tau}{Y},$$

where Δ/Y is the total government budget deficit relative to GDP. In the long run our standard AS-AD model implies that the fiscal multiplier is zero, $m = m_{+1} = 0$, as the economy converges on the exogenous natural rate of output. In the long term, a permanent fiscal tightening will therefore succeed in reducing the debt ratio, according to (19) through (21). However, in the short and medium run the fiscal multipliers m and m_{+1} are both positive in case of a permanent change in G. From (19) through (21) it is then uncertain if next period's debt ratio will actually fall when G falls, and if it does not, this may be a serious problem since a rise in the debt ratio will scare financial investors and drive up the risk premium even further.

Let us take a closer look at (20) and (21) to study the mechanisms at work under an austerity policy. The variable $A1$ in (20) captures the effect of the fall in G on the ratio of government spending to GDP, G/Y. A fall in G lowers the absolute size of the budget deficit, which works to reduce next period's debt, but the multiplier effect of the fall in G also reduces current output which means that the fall in the ratio of spending to GDP becomes smaller. However, for realistic parameter values such as those given below, the ratio of spending to GDP will indeed fall, thereby contributing to a fall in next period's debt ratio.

The variable $A2$ in (20) accounts for the fact that the first-year multiplier effect of the fiscal contraction is likely to be larger than the second-year multiplier effect. In that case, the negative effect on economic activity will be larger in the current year than in the next year. This means that the fiscal tightening will *increase* the *growth rate* of real GDP between the current and the next year, albeit from a lower *level* of GDP: the economy will gradually recover from the negative fiscal shock, even if the tightening is permanent. Ceteris paribus, this increase in the real growth rate will tend to reduce next year's ratio of debt to GDP. Moreover, if there is a tendency towards positive real growth, $g > 0$, this will also tend to lower next year's debt ratio by dampening next year's negative multiplier effect relative to GDP. That is why m_{+1} is deflated by the growth factor $(1+g)$ in the variable $A2$. On the other hand, if the economy is in a protracted recession where $g < 0$, it is uncertain whether next year's multiplier effect will in fact be smaller relative to GDP than the current year's effect. However, with the plausible parameter values assumed below, we will find that $A2 > 0$, which means that the effect of the fiscal contraction on the real growth rate will work in favour of a fall in next year's debt ratio.

But there are mechanisms working in the opposite direction, reflecting the negative effect of fiscal tightening on economic activity in the *current* year. Ceteris paribus, the fall in GDP caused by the cut in government spending will raise the debt burden and the interest cost of serving it, measured relative to GDP. By raising the *current* ratio of debt to GDP, D/Y, the cut in G will also drive up the current risk premium and increase the budget deficit via higher interest expenses. These effects, which will all tend to increase next period's ratio of debt to GDP, are picked up by the variable $A3$ in (21).

Furthermore, the fall in output caused by the fiscal contraction reduces inflation via the standard Phillips curve mechanisms, which in turn works to reduce the growth rate of nominal GDP, thereby tending to increase next year's nominal debt relative to nominal GDP. This effect is represented by the variable $A4$ in (21).[23]

Because of these offsetting mechanisms, we can only determine the likely sign of the right-hand side of (19) by inserting empirically plausible parameter values. For concreteness, let us consider the following stylized parameters that roughly characterized the situation in Greece in the crisis years 2010-2012 where the country experienced negative growth rates of real and nominal GDP:[24]

$$g = -0.06, \quad \pi = 0.01, \quad g^n = -0.05, \quad \frac{D}{Y} = 1.7, \quad \frac{\Delta}{Y} = 0.18,$$

$$i = 0.03, \quad \varepsilon = 0.15, \quad \frac{G}{Y} = 0.2, \quad \frac{Y}{\bar{Y}} = 0.8, \quad \gamma = 0.3.$$

Further, we use the following estimates of the first-year and second-year fiscal multipliers based on macroeconometric models for Greece[25]:

$$m = 0.87, \quad m_{+1} = 0.74.$$

To apply the formula (19), we also need an estimate of the elasticity of the risk premium with respect to the debt ratio, $\eta \equiv \varepsilon' \frac{d}{\varepsilon}$. For this purpose we go back to the first panel in Figure 26.13 where the slope of the regression line provides us with the estimate $\varepsilon' =$

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[23] The formulas (19) through (21) include the terms $\left(\frac{1}{1+g^n}\right)$ and $\left(\frac{D}{Y} + \frac{\Delta}{Y}\right)$ which do not affect the signs, but only the magnitudes of the various effects discussed above. Exercise 4 invites you to reflect on the economic intuition for the presence of these terms.

[24] We use the value of the parameter γ estimated in Chapter 18, which does not necessarily hold for Greece. The other parameter values provide a fairly accurate picture of the Greek situation in the worst crisis years.

[25] These estimates of the Greek multipliers for a permanent change in government consumption are taken from Table 5 in Juha Kilponen, Massimiliano Pisani, Sebastian Schmidt, Vesna Corbo, Tibor Hledik, Josef Hollmayr, Samuel Hurtado, Paulo Júlio, Dmitry Kulikov, Matthieu Lemoine, Matija Lozej, Henrik Lundvall, José R. Maria, Brian Micallef, Dimitris Papageorgiou, Jakub Rysanek, Dimitrios Sideris, Carlos Thomas and Gregory de Walque, *Comparing Fiscal Multipliers Across Models and Countries in Europe*, Working Paper Research No. 278, March 2015, National Bank of Belgium.

0.11. From this plus the assumed values of d and ε we then get $\eta = 1.247$. Finally, to facilitate the interpretation of our numerical results, we introduce the elasticity of the government debt ratio with respect to real government spending, defined as

$$\epsilon_G^d \equiv \frac{\partial d_{+1}}{\partial G}\frac{G}{d} = \frac{\partial d_{+1}}{\partial G}\frac{G/Y}{d}Y. \tag{22}$$

From (19) and (22) we see that ϵ_G^d can be found by multiplying the right-hand side of (19) by $\frac{G/Y}{d}$. Doing this, and using the parameter values above, we obtain

$$A1 = 0.826, \quad A2 = 0.156, \quad A3 = 2.022, \quad A4 = 0.389, \quad \epsilon_G^d = -0.177.$$

The estimate for ϵ_G^d implies that a 1 percent cut in real government spending on goods and services will actually *increase* next period's debt ratio by 0.177 percent relative to the debt ratio in the current period. With an initial debt ratio of 1.7, as we assumed for Greece, this means that a drastic 10 percent cut in real government spending would increase next period's debt ratio by 3 percentage points, that is, the debt ratio would rise from 1.7 to 1.73! This perverse effect is not due to our assumption that a higher debt ratio in the current period drives up the risk premium on government debt ($\varepsilon' > 0$). If we set $\varepsilon' = 0$, we still find that a 10 percent cut in G will raise next period's debt ratio by 2.4 percentage points. The reason for the perverse impact of austerity is that the effects of lower output and inflation captured by our variables $A3$ and $A4$ tend to drive up the debt ratio, and these effects dominate the sum of the effects $A1$ and $A2$ that work to reduce debt ratio. As this numerical example indicates, there was a real danger that an austerity policy would be counterproductive in the Greek crisis situation, as indeed it seemed to be at the time.

However, the Greek crisis was extreme, reflecting the country's enormous macroeconomic imbalances before the euro crisis broke out. Let us therefore consider the following stylized parameter values, which are more descriptive of the macroeconomic situation in the other GIPS countries during 2010-2012 (although the situation did of course vary somewhat across countries):

$$g = -0.02, \quad \pi = 0.0, \quad g^n = -0.02, \quad \frac{D}{Y} = 1.0, \quad \frac{\Delta}{Y} = 0.08, \quad \varepsilon = 0.10.$$

Combining these macroeconomic parameters with the other parameter values from the previous numerical example, our formulas (19) through (22) now yield $\epsilon_G^d = -0.053$. With the assumed initial debt ratio of 100 percent, this result implies that a 10 percent cut in G would increase next period's debt ratio from 100 to 100.53 percent; still a slightly perverse effect.

It is worth noting that this analysis may actually give an overoptimistic picture of the effect of a fiscal tightening, since we assumed that the risk premium in the government bond yield only depends on the debt ratio. The third panel in Figure 26.13 clearly indicates that the risk premium also tends to go up if unemployment increases, presumably because investors fear that high unemployment reduces the government's fiscal capacity. A fiscal contraction creating higher unemployment may therefore induce a higher risk premium that burdens the government budget and further reduces the effectiveness of fiscal tightening as a means of cutting the debt ratio.

In any case, these numerical examples suggest why the austerity policies imposed on Greece and the other GIPS countries during the crisis years did not deliver the hoped-for significant cuts in debt ratios. On the contrary, we see from the bottom part of Figure 26.11 that the government debt ratios of these countries rose dramatically during the crisis years as a consequence of the sharp drop in economic activity that weakened the government budgets via the automatic stabilizers.

At the time many observers argued that the GIPS countries did not have any other choice but to cut their spending and raise their taxes since they could no longer borrow in the government bond market. However, since the Troika decided to provide financial assistance to prevent sovereign defaults that could seriously destabilize the entire EMU, it became a political decision by the creditor countries how the economic stabilization programmes for the GIPS countries should balance fiscal austerity against structural reforms of labour and product markets. Many economists argued that the Troika should have placed less weight on fiscal tightening and more weight on structural reforms, which would have a faster positive impact on growth if the economies of the GIPS countries were less depressed. The IMF itself also came to the conclusion that austerity policies could be overdone.[26] Austerity took a particularly heavy toll on the Southern European countries, while Ireland fared somewhat better for reasons we will discuss in the next subsection.

The competitiveness crisis: is there a way out?

As illustrated in Figure 26.8, the international competitiveness of the GIPS countries was seriously eroded in the run-up to the euro crisis. This problem became acute when these countries had to improve their trade balance quickly as they were no longer able to borrow large amounts abroad to finance their huge current-account deficits. To get a feel for the enormous scale of the GIPS competitiveness problem and why the problem has persisted in Southern Europe, we can use some elements of our AS-AD model of the open economy with fixed exchange rates plus some simple accounting relationships.

A permanent improvement of a country's competitiveness requires a permanent increase in its real exchange rate, $E^r \equiv EP^f/P$, where E is the nominal exchange rate, P^f is the foreign price level, and P is the domestic price level. In a currency union the nominal exchange rate is fixed[27], so to attain a required relative increase in the real

[26] For example, this was acknowledged in the following article by the Director of the Fiscal Affairs Department of the IMF, Carlo Cottarelli: https://voxeu.org/article/fiscal-adjustment-too-much-good-thing.
[27] For EMU member countries E is actually not quite fixed, since the EMU has a flexible exchange rate vis á vis third countries. Here we assume for simplicity that the inflation differential between the EMU and the rest of the world is roughly zero so that (by relative purchasing power parity) the nominal euro exchange rate stays roughly constant over the long run. This is consistent with Figure 25.1 in the previous chapter,

exchange rate, denoted as $T \equiv dE^r/E^r$, over a period of n years, given the annual foreign inflation rate π^f, the domestic inflation rate must satisfy the relationship

$$\frac{(1+\pi^f)^n}{(1+\pi)^n} - 1 = T \quad \Longrightarrow \quad n[\ln(1+\pi^f) - \ln(1+\pi)] = \ln(1+T) \quad \Longrightarrow$$

$$n(\pi^f - \pi) \approx \ln(1+T), \tag{23}$$

where we have used the approximation $\ln(1+x) \approx x$ to arrive at the left-hand side of (23) (this approximation will be close to perfect when the inflation rates do not deviate "too much" from zero). From the AS curve of our AS-AD model of the open economy we have

$$\pi = \gamma(y - \bar{y}) + \pi^f \iff \pi^f - \pi = \gamma(\bar{y} - y). \tag{24}$$

Inserting (24) in (23) (taking the approximation to be perfect) and rearranging, we obtain the number of years it will take to achieve the required improvement in international competitiveness, given the percentage negative output gap $\bar{y} - y$ the government can accept during this adjustment period:

$$n = \frac{\ln(1+T)}{\gamma(\bar{y}-y)}. \tag{25}$$

Equation (25) shows that a higher degree of nominal wage and price flexibility (a larger value of γ) reduces the time it takes to attain the needed improvement of international competitiveness. But (25) also reveals a trade-off: the faster the government wants to achieve the desired gain in competitiveness (the smaller the value of n), the larger is the negative output gap (and hence the higher is the unemployment rate) it will have to tolerate during the n years until the target change in the real exchange rate is attained.

With a low foreign inflation rate, a quick improvement of domestic competitiveness may require domestic deflation, that is, $\pi < 0$. But deflation is a painful and risky process. Since nominal wages tend to be particularly rigid in the downward direction, it may take a very high rate of unemployment to generate deflation. Falling prices will also increase the real debt burden of debtors, since debt contracts are written in nominal terms, so protracted deflation may cause many firms and households to default on their debts. This may impose large losses on banks and other credit institutions, thereby threatening financial stability. For these reasons a government may want to avoid deflation, so suppose the government aims at maintaining a stable domestic price level ($\pi = 0$) during the adjustment period, thus obtaining a gradual improvement of competitiveness through the positive foreign inflation rate. Inserting $\pi = 0$ in (23) and (24), and assuming that the adjustment period starts in year $t = 0$, we then get

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which shows that the euro-dollar nominal exchange rate has fluctuated around a roughly flat trend. It is also in line with the fact that the inflation targets of the ECB and the Federal Reserve are roughly similar.

$$n = \frac{\ln(1+T)}{\pi^f}, \qquad \bar{y} - y_t = \frac{\pi^f}{\gamma} \quad \text{for } t = 0,1,\dots,n. \tag{26}$$

Let us now consider the magnitude of the required improvement of competitiveness (T) in the GIPS countries after they plunged into the euro crisis. For this purpose, we will have to go through a number of analytical steps, but it will be worthwhile as the end result will be thought-provoking. We start from the fact that the increase in nominal net foreign debt, $D_{+1}^n - D^n$, equals the nominal deficit on the current account of the balance of payments which is given by the nominal interest payments on the existing foreign debt, $(i+\varepsilon)D^n$, minus the current surplus on the nominal trade balance, B^n:

$$D_{+1}^n - D^n = (i+\varepsilon)D^n - B^n. \tag{27}$$

In (27) we have included a risk premium ε on top of the nominal interest rate i on safe assets. Exploiting the relationship between the nominal GDP (Y^n) in the current and the next period, $Y_{+1}^n = (1+g^n)Y^n$, where g^n is the rate of nominal GDP growth, we can rewrite (27) as

$$d_{+1} = \left(\frac{1+i+\varepsilon}{1+g^n}\right) d - \frac{b}{1+g^n}, \qquad d \equiv \frac{D^n}{Y^n}, \quad b \equiv \frac{B^n}{Y^n}. \tag{28}$$

Setting $d_{+1} = d$ in (28) and solving for b, we obtain the ratio of trade balance to GDP that will stabilize the ratio of foreign debt to GDP:

$$b = (i+\varepsilon-g^n)d. \tag{29}$$

The trade balance ratio determined by (29) depends on the steady-state foreign debt ratio the country can accept. From the bottom part of Figure 26.9 we see that the foreign debt ratio in the GIPS countries had reached roughly 100 percent in 2009-2010 when the euro crisis hit. Many observers would consider such a debt ratio to be dangerously high, but suppose this was a debt level the GIPS countries could live with if it could be kept from rising further. Setting $d = 1$ in (29), we then find that these countries would need to keep their ratio of trade balance to GDP at the following level to stabilize their debt-GDP ratio:

$$b = i + \varepsilon - g^n. \tag{30}$$

In the good years 2001-2007, nominal GDP in the GIPS countries grew at an average annual rate of about 7 percent ($g^n \approx 0.07$), and their governments could borrow at a rate roughly equal to the interest rate on safe German government bonds, $i \approx 0.04$. Some of the foreign debt of the GIPS countries was private and carried a positive risk premium, but that premium was low, say, $\varepsilon \approx 0.02$. With these stylized numbers, Equation (30) implies that the GIPS countries could actually run a small trade deficit of about 1 percent of GDP and still service their foreign debt without increasing their debt ratio. However, in 2007-2008, shortly before the euro crisis struck, the average trade deficits in Greece, Portugal and Spain was more like 8 percent of GDP (somewhat larger in Greece, and somewhat smaller in Spain).[28] So already before the crisis, the Southern European countries had a strong need for improvement of their trade balances to prevent an unsustainable growth in their foreign debt. After the crisis hit, and the possibilities for borrowing from foreign private lenders dried up, the trade deficits of the GIPS countries became even more unsustainable, for two reasons. First, from the 2000s to the 2010s, the trend growth rate of nominal GDP in these countries fell from the afore-mentioned 7 percent per year to roughly 1.5 percent per year, according to the official national accounts. Second, the risk premium on foreign private lending to the GIPS governments as well as to their private sectors shot up to a permanently higher level which more or less neutralized the effect of the fall in the risk-free interest rate that took place during the 2010s. Against this background, let us insert $g^n \approx 0.015$ and maintain our previous assumption that $i + \varepsilon \approx 0.06$. Equation (30) then implies that the Southern European countries would have to run a trade *surplus* amounting to around 4.5 percent of their GDP to secure sustainability of their foreign debt positions. Recalling that these countries started out with a trade *deficit* of around 8 percent of GDP, these numbers mean that, on average, the Southern European countries needed to improve their trade balances by a massive 12.5 percent of GDP to sustain their foreign debts, even if they had no intention to bring down their foreign debt ratio from the very high level of 100 percent that made them vulnerable to new shocks to the international capital markets.

What is a realistic magnitude of the rate of real exchange rate depreciation (the value of our variable T in (25) and (26)) that could bring about the 12.5 percent permanent improvement of the trade balance ratio that the Southern European countries needed for a sustainable recovery from the euro crisis? We can use our AS-AD model of the open economy to seek an answer to this question. Remember from Chapter 23 that the trade balance, B, measured in units of the domestically produced good is $B = X - E^r M$, where X is the quantity of domestically produced goods that is exported, and M is the quantity of foreign-produced imported goods. Both X and M depend inter alia on the real exchange rate E^r, so

$$\frac{\partial B}{\partial E^r} = \frac{\partial X}{\partial E^r} - E^r \frac{\partial M}{\partial E^r} - M. \tag{31}$$

Since $b \equiv \frac{B}{Y}$, we have $b = \frac{X - E^r M}{Y}$, which can be rearranged to give $M = \frac{X - bY}{E^r}$. Using this expression for the initial level of imports, we can restate (31) as

$$\frac{\partial B}{\partial E^r} = M\left[\frac{\partial X}{\partial E^r}\left(\frac{E^r}{X - bY}\right) - \frac{E^r}{M}\frac{\partial M}{\partial E^r} - 1\right] \quad \Rightarrow$$

[28] We exclude Ireland here, because the country's trade statistics are atypical, being strongly affected by the royalties received by the resident affiliates of the many multinational companies that have assigned the intellectual property rights of the multinational group to affiliates in Ireland to benefit from the low Irish corporation tax.

$$\frac{\partial B}{\partial E^r} = M\left[\eta_X\left(\frac{x}{x-b}\right) + \eta_M - 1\right], \quad \eta_X \equiv \frac{\partial X}{\partial E^r}\frac{E^r}{X}, \quad \eta_M \equiv -\frac{\partial M}{\partial E^r}\frac{E^r}{M}, \quad x \equiv \frac{X}{Y}, \tag{32}$$

where η_X and η_M are the numerical price elasticities of exports and imports, respectively, and x is the ratio of exports to GDP. In the long run output converges on its exogenous natural rate $\bar{Y}$. The long-run effect of a real exchange rate depreciation dE^r on the trade balance ratio will then be

$$db = \frac{1}{\bar{Y}}\frac{\partial B}{\partial E^r}dE^r. \tag{33}$$

Without loss of generality, we may choose our units of measurement such that the initial real exchange rate (marked by a zero subscript) is $E_0^r = 1$. Doing this, inserting (32) in (33), and noting that $b \equiv x - m$, where m is the ratio of imports to GDP, we now get (check this):

$$\frac{dE^r}{E_0^r} = \frac{db}{x\eta_X + m(\eta_M - 1)}, \qquad m \equiv \frac{E^r M}{Y}. \tag{34}$$

Equation (34) gives the value of our variable T, that is, the rate of real exchange rate deprecation needed to attain a permanent improvement db of the ratio of the trade balance to GDP. As we have seen, the situation for the Southern GIPS countries around 2007-2008 was that $db \approx 0.125$, and their foreign trade positions at that time could roughly be described as $m \approx 0.34$ and $x \approx 0.26$, a trade deficit of about 8 percent of GDP, with some variation across the three countries. The price elasticities η_X and η_M are also country-specific, depending on the composition of exports and imports, but as noted in Chapter 24, a plausible approximate sum of these elasticities for many countries is $\eta_X + \eta_M = 3$, so for concreteness we assume $\eta_X = \eta_M = 1.5$. Inserting these parameter values in (34), we obtain $\frac{dE^r}{E_0^r} \approx 0.22$. In other words, our simple analysis suggests that the Southern GIPS countries were in need of a 22 percent depreciation of their real exchange rate on the eve of the euro crisis.

We emphasize that this estimate should not be taken too literally, since it relies on highly stylized parameter values. A serious estimate of the magnitude of the Southern competitiveness problem would require a more careful statistical analysis accounting for cross-country differences. However, our simplified estimate is actually fairly close to estimates offered by other analysts using more sophisticated methods. In fact, some studies have found that Greece, Portugal and Spain needed an even larger real depreciation on the order of 25-35 percent relative to the euro zone average.[29]

What are the macroeconomic consequences of a need for a 22 percent real

[29] Some of these studies are discussed in Hans-Werner Sinn, 'Austerity, Growth and Inflation: Remarks on the Eurozone's Unresolved Competitiveness Problem', *The World Economy*, 2014, pp. 1-13. According to these studies, countries like France and Italy were also in need of improved competitiveness.

depreciation? Consider Equation (25) and insert our estimate $T = \frac{dE^r}{E_0^r} = 0.22$ plus our estimate $\gamma = 0.3$. Suppose further that the government wishes to achieve the 22 percent real depreciation over a period of five years, i.e., $n = 5$ (recall the urgency of the euro crisis at the time). According to (25) the country would then have to live with an average annual negative output gap $(\bar{y} - y)$ equal to 13 percent over that five-year period; a dramatic protracted recession which would involve an annual rate of deflation of almost 2 percent according to our AS curve (24) with $\gamma = 0.3$ and the 2 percent eurozone target inflation rate $\pi^f = 0.02$.

Suppose alternatively that the government does not want to incur the risks of deflation that we explained earlier, aiming instead at a zero inflation rate during the adjustment period. According to (26) it would then take almost 10 years to attain the needed improvement of competitiveness, and the government would still have to accept an average negative annual output gap of 6.7 percent of potential GDP over an entire decade. For comparison, over the period 2009-2019 the IMF has estimated that Portugal and Spain "only" had an average negative output gap equal to 3.5 percent and 3.7 percent, respectively. This helps to explain why these countries did not manage to achieve any substantial improvement of their competitiveness relative to Germany during the 2010s, as witnessed by the evolution of GDP deflators and unit labour costs depicted in Figure 26.9.

Greece, on the other hand, did suffer from a devastating average annual output gap of minus 9.9 percent over the period 2009-2019, according to IMF estimates. The negative Greek output gaps were particularly large in the first half of the 2010s, and this did indeed force a fall in the Greek price level and the country's unit labour costs until around 2015, as shown in Figure 26.8. However, because the initial competitiveness problem was larger in Greece than in the other GIPS countries, it remained very large by the end of the 2010s where unit labour costs even started to increase again.

The scale of the macroeconomic imbalances in Greece can be grasped by considering the lower panel in Figure 26.6 along with the upper panel in Figure 26.9. Despite an unemployment rate of almost 17 percent by the end of the 2010s, Greece still ran a current-account deficit of 3-4 percent of GDP by that time. In Spain, the current account was roughly in balance, but the unemployment rate was almost as high as in Greece, indicating that Spain remained unable to check the growth of its foreign debt unless the country was willing to accept a very high rate of joblessness.

The Irish experience in the aftermath of the euro crisis was much different. During the Irish property price boom up until 2007, the country's competitiveness weakened and its current-account deficit shot up, but already after the bursting of the property bubble, the Irish price level and unit labour costs started to come down again, and the downward adjustment continued for a while after the outbreak of the euro crisis. As a consequence of this flexibility, the Irish economy recovered relatively quickly after the bailout package from the Troika in November 2010, as illustrated in Figure 26.6. By the end of the 2010s, the Irish unemployment rate had fallen to a relatively low level around 5 percent, and the country's current account hovered around zero.

These diverse experiences indicate that the flexibility of nominal wages and prices was significantly higher in Ireland than in Southern Europe. In terms of our AS curve (24), the parameter γ was a lot higher in Ireland, and according to (25) this meant that the country faced a more favourable trade-off between the number of years it takes to achieve a required improvement of competitiveness and the size of the negative output gap that must be tolerated during those years.

The difference in the evolution of public sector wages illustrates one aspect of the different degrees of flexibility in the Southern and Western periphery of the EMU. In Spain, public sector wages rose by 11 percent from 2007 to 2012, despite rising mass unemployment in the country. The Greek government likewise raised public sector wage rates by 11 percent from 2007 to 2011. In Portugal public wages stayed constant, whereas the Irish government cut public sector wages by 13 percent from 2007 to 2012. Since the public and the private sector compete for labour, the politically determined evolution of public sector wages matters for the evolution of wages in sectors exposed to international competition. Obviously, the Irish approach to public sector wage formation was more supportive of the needed wage adjustment in the private sector.

The analysis above takes us back to the great debate of the 2010s on the way out of the euro crisis. All participants in the debate agreed that a strong improvement of the competitiveness of the GIPS countries was a necessary condition for their ability to recover permanently from the crisis. But competitiveness can be strengthened in very different ways. One way is through an austerity policy that creates a large negative output gap via a cut in aggregate demand. Such a policy played a major role in the response to the euro crisis, but it comes with a huge social cost in terms of prolonged high unemployment and lost output. An alternative is to pursue structural reforms of labour and product markets. In terms of our AS-AD model of the open economy, structural reforms can work in three different ways.

First, if they succeed in creating more flexible labour markets and stronger competition in product markets, they may increase the degree of nominal wage and price flexibility, thereby increasing the size of our parameter γ in the AS curve. As we have seen, this will reduce the size of the negative output and employment gaps needed to attain any required gain in competitiveness over a given time period.

Second, as we explained in Part 4, structural reforms in product and labour markets can lower the natural rate of unemployment, thereby increasing the natural rate of output ($\bar{y}$) in the AS curve. From an economic viewpoint, this is a much more attractive way of moderating wage and price inflation, since it creates a negative output gap without reducing the current levels of output and employment.

Third, another attractive way of improving competitiveness (again, from an economic perspective) is to carry out structural reforms that increase the natural rate of output by improving productivity. However, two conditions must be met to ensure that a productivity gain actually improves a country's competitiveness by lowering its unit labour costs and increasing its real exchange rate: (i) Nominal wages must increase by less than the rise in productivity. This may require a period where the "outside option" for employed workers (unemployment benefits and other transfers to people out of work) rises (temporarily but not permanently) by less than the rate of labour productivity growth. (ii) Assuming condition (i) is met, the cut in unit labour costs stemming from higher productivity must be passed on to consumers of domestically produced goods via a cut in the prices of these goods. To ensure a full pass-on, structural reforms that increase the degree of competition in product markets may be required.

Although structural reforms seem much more attractive than austerity from a macroeconomic perspective, they face two hurdles. One is that many structural policies only work slowly, so if a quick improvement of the balance of payments is needed in a member country of a monetary union where devaluation is ruled out, a fiscal contraction may be the only realistic short-term option. Another hurdle is that the people who are supposed to be more "flexible" and "productive" may not catch on to the idea. "Structural reform" often means that many people will have to change established ways of behaviour and may feel uncertain about how they will fare under the new rules of the game. Typically, structural reforms will also affect the distribution of income and create opposition from those who stand to lose. Therefore, even though a reform may increase the total "cake" available for distribution in the long run, the government may not have the additional resources to compensate those who lose in the short run, because it takes time before the productivity gains from the reform materialize.

That said, the GIPS countries did carry out several structural reforms in response to the crisis, but as the data show, they were not sufficient to resolve the competitiveness problems of the Southern GIPS countries during the 2010s. The macroeconomic imbalances in Greece were particularly severe, and during the euro crisis many observers argued that the only realistic solution for Greece was an exit from the eurozone and a return to a national currency that would allow the country to devalue to regain competitiveness. For a moment in the summer of 2015, the Greek government did seem to play with the idea of a "Grexit" from the euro, but in the end it chose to stay in the currency union and accept the terms dictated by the Troika for receiving a third bailout.

Already in 2007, the American economic historian Barry Eichengreen (whom we have already met) gave the following explanation why it is very difficult for a country to leave the Eurozone:[30] *"Reintroducing the national currency would require essentially all contracts - including those governing wages, bank deposits, bonds, mortgages, taxes, and almost everything else - to be redenominated in the domestic currency. The legislature could pass a law requiring banks, firms, households and governments to redenominate their contracts in this manner. But in a democracy this decision would have to be preceded by very extensive discussion. And for it to be executed smoothly, it would have to be accompanied by detailed planning. Computers will have to be reprogrammed. Vending machines will have to be modified. Payment machines will have to be serviced to prevent motorists from being trapped in subterranean parking garages. Notes and coins will have to be positioned around the country. One need only recall the extensive planning that preceded the introduction of the physical euro."* Eichengreen then pointed out that the very motivation for a country to leave the Eurozone would be to devalue the new national currency, and that market participants would be aware of this fact. He argued that households and firms anticipating that domestic bank deposits would be redenominated into the new national currency, which would then lose value against the

[30] See https://voxeu.org/article/eurozone-breakup-would-trigger-mother-all-financial-crises.

euro, would shift their deposits to other Eurozone banks, creating a system-wide bank run. Investors anticipating that their claims on a government about to abandon the euro would be redenominated into a new depreciating currency would shift into claims on other Eurozone governments, leading to a bond-market crisis, and if the government was already in a weak fiscal position, it would not be able to borrow to bail out the banks and buy back its debt. According to Eichengreen, *"This would be the mother of all financial crises."*

Thus, since the cost of an exit from the Eurozone is likely to be catastrophic, a decision to adopt the euro is essentially irreversible. This makes it all the more important to consider how the EMU can be made more resilient against future debt crises and other economic shocks. We now turn to this issue.

26.6 Lessons from the European debt crisis

Several lessons can be learned from our study of the European debt crisis in the early 2010s.

Lesson 1: Above all, it is important that governments consolidate their public finances when the economy is booming, so they have enough fiscal space to carry out a fiscal expansion when the economy turns down. Otherwise, they run the risk of having to tighten fiscal policy during a recession in an effort to stem unsustainable deficits and debt. Fiscal policy should be countercyclical – not procyclical. Many EMU countries failed to follow this principle in the run-up to the euro crisis.

Lesson 2: When a government has lost the confidence of sovereign bond investors because its fiscal policy is unsustainable, some action to improve government finances and the general trust in them is inevitable. The road towards fiscal sustainability will typically involve some combination of fiscal tightening and structural reforms that reduce the *structural* budget deficit by increasing the tax base and reducing structural unemployment. As we have seen, fiscal austerity may be an ineffective way of reducing the government debt ratio in a recession; austerity may even increase the debt-GDP ratio in the short run. In an economic downturn, structural reforms that help to curb the structural budget deficit and to solve a country's competitiveness problem may be a better way. However, for a distressed government wishing to send a credible signal to its creditors that it is taking serious action to address its macroeconomic imbalances, an element of austerity in fiscal policy may be inescapable despite its painful consequences. International assistance can be one way to reduce the pain. This observation takes us to the next lesson.

Lesson 3: There may be crisis situations where a government has not yet had time to cut its debt ratio significantly, and where slow-working structural reforms are insufficient to calm the markets. A distressed government faced with difficulties of borrowing from private investors may therefore need temporary financial assistance from other EMU countries. It is desirable to have an established institution through which such financial assistance can be channeled, subject to transparent and pre-announced principles that help to reduce uncertainties for all parties involved. Such predictability will allow faster assistance and will help to reduce panic among investors. The EU has heeded this lesson by making the European Stability Mechanism (ESM) described earlier a permanent institution and by combining it with the ECB's Outright Monetary Transactions programme which enables the ECB to purchase bonds issued by distressed governments that subject themselves to a stabilization programme negotiated with the ESM.

Lesson 4: During a financial crisis, policy makers face the difficult task of distinguishing between insolvency and illiquidity. An indebted sovereign government is insolvent if its fiscal capacity (its ability to run future primary budget surpluses) is insufficient to enable it to service its debt even in the best of circumstances. In that case it is better for all parties involved to acknowledge the insolvency right away and negotiate the necessary debt relief. But if the government bond market has landed in a "bad" equilibrium with a very high interest rate because individual investors believe that the other investors have very pessimistic expectations about the government's ability to service its debt, a promise of external financial assistance to the government may push the bond market back to a "good" equilibrium with a much lower interest rate that enables the government to honour its debt. In such a case the government is not insolvent, but suffers instead from a temporary liquidity crisis that can be overcome by official lending (or just a promise thereof) from partner countries at a "normal" interest rate.

Lesson 5: The EMU experience shows that the no-bailout clause of the Maastricht Treaty was not sufficient to ensure a rational differentiation of risk premia in government bond yields that could help to enforce fiscal discipline. In the boom years before the euro crisis, markets apparently believed that an EMU government threatened by default would be bailed out by the rest of the union; a belief that was underpinned by the practice of the ECB to treat all government bonds issued in the EMU as fully safe and was ultimately vindicated by the actual bailouts during the crisis. This experience suggests that a currency union with decentralized fiscal policy does need some institutional bulwark against irresponsible fiscal behavior in individual member countries. The original Stability and Growth Pact and its successor, the European Fiscal Compact, were attempts to create such a bulwark by enforcing the 3 percent deficit rule and the 60 percent debt rule of the Maastricht Treaty and by requiring countries to live up to their MTOs (medium term objectives) for the structural government deficit. However, these rules have been widely criticized for being too rigid, leaving too little space for fiscal expansion in a deep recession and too little room for productive public investment. Creating fiscal rules and institutions that strike the optimal balance between discipline and flexibility is still work in progress for the EMU.

Lesson 6: Imposing restrictions on government borrowing may not be enough to prevent a debt crisis in a currency union. The elimination of exchange rate risk may allow the private sector to borrow much more from abroad than would be possible in a world with national (adjustable) currencies, meaning that a member country of a currency union can accumulate a much larger current-account deficit and level of foreign debt before markets start to react. A big current-account deficit in a country that balances its government budget – as Ireland and Spain roughly did before the crisis – indicates that the private sector is building up an unsustainable level of debt, and when the markets realize the problem, they may react with a "sudden stop" to further lending that triggers a financial and economic crisis. This is not only a problem for the debtor country itself, but also for the other members of the currency union which have to come to the rescue. Having learned this lesson the hard way, the EU countries agreed in the fall of 2011 to introduce the so-called Macroeconomic Imbalance Procedure (MIP); an annual surveillance procedure intended to highlight and correct large macroeconomic imbalances in member states, including large current-account deficits and high levels of foreign debt. According to the MIP, an external imbalance is likely to exist if a country runs an average current-account *deficit* exceeding 4 percent of GDP or an average current-account *surplus* in excess of 6 percent of GDP over a three-year period. Yet the MIP has not prevented countries like Germany, the Netherlands and Denmark from running persistent current-account surpluses of more than 6 percent of GDP. Critics have pointed out that these countries could help to solve the Southern European competitiveness problem if they accepted a period with higher domestic inflation that would bring down their high current-account surpluses. In short, to ensure a harmonious economic development within a currency union, all member countries must be willing to coordinate their macroeconomic policies, and that coordination is still rather imperfect in the EMU.

Lesson 7: A policy "trilemma" seems to exist within the EMU between (i) cross-border financial integration, (ii) financial stability, and (iii) fiscal policy flexibility.[31] An EMU country cannot achieve all three things at the same time. This lesson is based on the EMU experience with the "doom loop" between banks and governments explained earlier. With the deep financial integration and the expansion of cross-border capital flows that followed the introduction of the euro, the banking sectors in many individual EMU countries have grown so large relative to the national economy that their domestic governments may not have the fiscal capacity to bail out a domestic bank system on the brink of failure. When a financial crisis looms in an EMU country, investors may therefore lose faith in the creditworthiness of its government, which may in turn force it to tighten fiscal policy in an effort to regain the confidence of investors and prevent a capital flight. In this way, fiscal policy flexibility is lost, and the fall in economic activity caused by the austerity policy may further weaken the banking system. A country might theoretically be able to retain fiscal policy flexibility and financial stability by withdrawing from the single financial market and imposing capital controls, but this is not a legal option in the EMU. Hence, the only viable way out of the trilemma seems to be to shift the responsibility for deposit insurance, bank supervision and bank resolution from individual EMU member states to the supra-national level of the ECB or the EU.

Since 2012, European policy makers have in fact moved towards a European banking union. The banking union includes all Eurozone countries, but is also open to other EU countries that wish to join. It is based on three pillars: a Single Supervisory Mechanism, a

[31] This new trilemma, which comes on top of the "Impossible Trinity" explained in Chapter 23, was formulated by a leading U.S. international economist in the following article: Maurice Obstfeld, 'Finance at Center Stage: Some Lessons of the Euro Crisis', *European Economy, Economic Papers* 493, April 2013.

Single Resolution Mechanism, and a European Deposit Insurance Scheme. Through the Single Supervisory Mechanism, agreed in 2013, the ECB was made responsible for the direct supervision of the largest banking groups in the EMU, while national supervisors continue to monitor all other banks, under the ultimate responsibility of the ECB. Via the Single Resolution Mechanism, agreed in 2014, a new independent EU agency called the Single Resolution Board was given the authority to initiate the resolution of any bank within the union deemed by the Board to be insolvent. Such a bank can either be wound up or be recapitalized by drawing on a Single Resolution Fund financed by mandatory contributions from banks in the union countries. As a backstop, the Single Resolution Fund can also draw on funds from the European Stability Mechanism. The common European Deposit Insurance Scheme was proposed by the EU Commission in 2015 as the third pillar of the banking union. It is supposed to be financed by contributions from the banks. At the time of writing, it is still being negotiated within the EU. Thus, the full banking union remains unfinished business, but a large step has been taken towards a more solid framework for bank regulation within the EMU.

In the aftermath of the euro crisis scholars and policy makers have also debated the need for introducing a common European safe asset such as a bond guaranteed by all EU or EMU member governments. In times of financial turmoil, investors move their capital out of countries deemed to be particularly vulnerable to financial distress and into countries considered to be more stable (e.g., Germany in the European context). Such capital flight only makes matters worse and tends to make the expectations of overly pessimistic investors self-fulling by pushing the distressed economies into the "bad equilibrium" described by our model of multiple financial market equilibria. . If there were a common safe European asset, investors could just move into that asset in times of crisis instead of having to move their capital out of distressed countries. This could significantly reduce the problem of capital flight when an EMU country is hit by a negative shock. In 2020 the EU countries did in fact decide to allow the European Commission to issue commonly guaranteed bonds up to an amount of 100 billion euros as part of the financing of the big stimulus package to help member states recovering from the economic fallout of the coronavirus pandemic. However, a common political will to issue such common euro bonds on a broader and more permanent basis is still lacking, as politicians and voters in some member states fear that guaranteeing common borrowing could lead to irresponsible behaviour in other member countries.

The corona crisis: Towards a new paradigm for public debt?

Frightened by the experience of several governments staggering on the brink of default, European leaders at first reacted to the euro crisis by tightening the fiscal rules of the EMU through the Fiscal Compact agreed in 2011. However, during the second half of the 2010s, a gradual change in the attitude towards budget deficits and government debt seemed to take place among many economists and policy makers in Europe and elsewhere. In part, this may be explained by the disappointing experience with the European austerity policy that did not deliver the hoped-for reductions in government debt ratios in the short and medium term and which was seen by many as an important reason why the economic recovery from the financial crisis was much slower in Europe than in the United States.

But the reassessment of government deficits and debt was also motivated by the experience of a long period with very low interest rates in the aftermath of the crisis.[32] The long-run implications of government budget deficits depend crucially on the long-run relationship between the interest rate on government debt and the economy's trend rate of economic growth. To see this, we can reinterpret Equation (29), repeated here for convenience:

$$b = (i + \varepsilon - g^n)d. \tag{29}$$

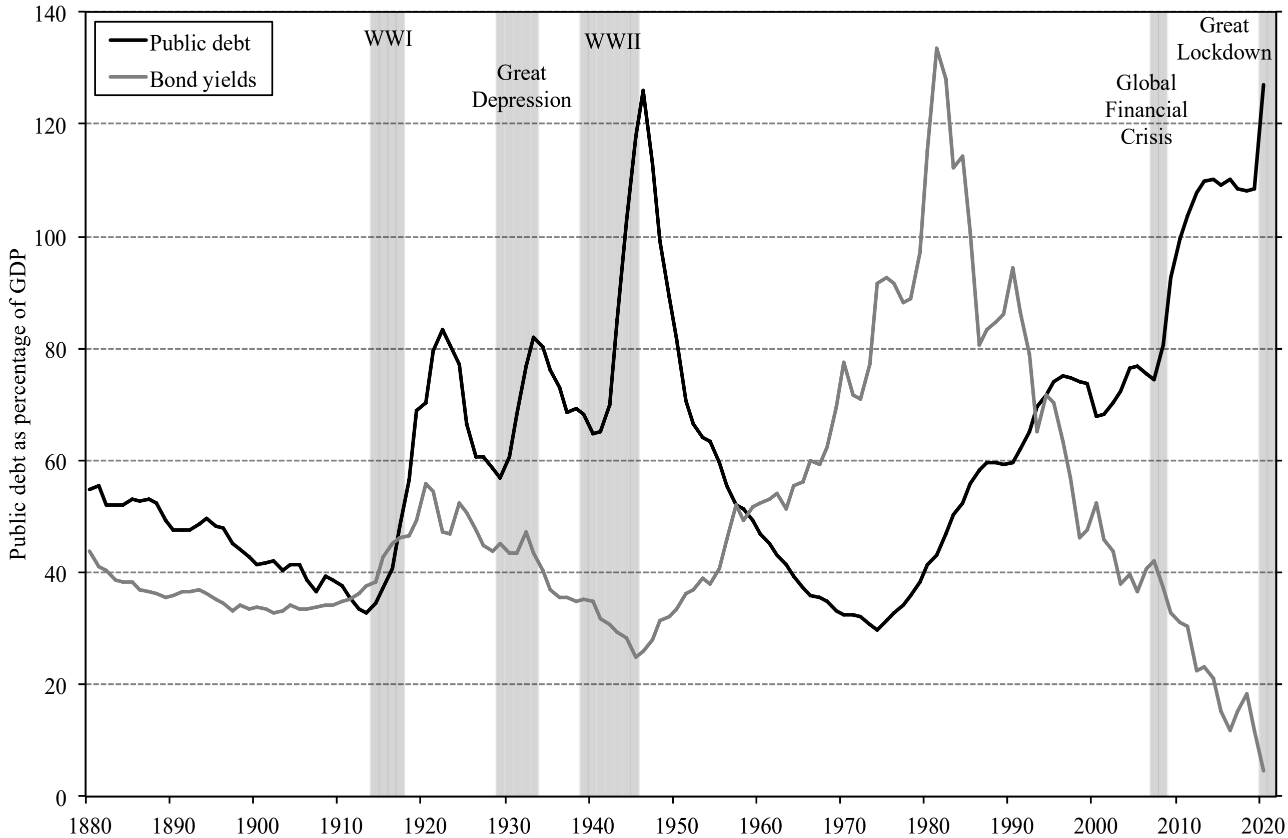
Now we define b as the balance on the government's primary budget (the budget before interest payments) measured relative to GDP, while d is the government debt-GDP ratio, i is the nominal interest rate on AAA-rated (completely safe) government bonds, ε is a possible risk premium, and g^n is the trend growth rate of nominal GDP. With this interpretation, (29) gives the relationship between the primary budget deficit and the government debt level which will ensure that the debt-GDP ratio stays constant over the long run. In "normal" times most government bonds are seen as safe assets which therefore do not carry a risk premium (or only a tiny one), $\varepsilon \approx 0$. If the trend growth rate exceeds the "normal" interest rate, $g^n > i$, it then follows from (29) that a government can run a persistent primary budget deficit, $b < 0$, without increasing the government debt ratio. Moreover, if the safe interest rate has fallen to a new long-run equilibrium level close to zero, the government's interest expenses will also converge on zero, so the total government budget deficit will converge on the primary budget deficit, which can be (almost) as high as the nominal growth rate when measured relative to GDP. If the long-run real growth rate is about 2 percent per year, and inflation hovers around the 2 percent target rate of many countries, it then follows that the government budget deficit can be (almost) as high as 4 percent of GDP without increasing the government debt ratio. In such circumstances, the requirement of the Stability and Growth Pact and the EU Fiscal Compact that the structural government budget deficit should not exceed 1 percent of GDP (and only ½ percent of GDP in high-debt countries) seems much too restrictive. During the corona pandemic of 2020-21, EU governments did in fact activate the "escape clause" of the Stability and Growth Pact that allows a departure from the normal fiscal rules in the event of a severe economic downturn.

This enabled the EU countries to respond to the corona crisis with massive fiscal stimulus packages which took public debt levels to new historic highs. In the United States, the Congress reacted to the pandemic with an even larger fiscal stimulus. As a consequence of the unprecedented fiscal expansion during the Great Lockdown, the

[32] An indication of the trend towards a new view of government debt was the following Presidential Lecture at the annual meeting of the American Economic Association, given by a leading macroeconomist and former Chief Economist of the IMF: Olivier Blanchard, 'Public Debt and Low Interest Rates', *American Economic Review,* 2019, 109(4), pp. 1197-1229.

average ratio of public debt to GDP in the advanced economies shot up to the highest level ever seen, exceeding even the debt accumulated during World War II, as shown in Figure 20.14. However, since the average government bond yield in the advanced economies also sank to a historically low level in the late 2010s and early 2020s, the ratio of government interest expenses to GDP was actually lower in 2020-2021 than during the 2000s, despite the much higher debt level, according to IMF estimates.

Figure 26.14. Public Debt and Bond Yields in Advanced Economies, 1880–2020



Note: Based on annual data.
Source: Fiscal Monitor April 2021, IMF.

With a nominal government bond yield close to zero and inflation expectations corresponding to a target inflation rate of 2 percent per year, the "safe" real interest rate in the advanced economies is close to minus 2 percent at the time of writing. From a historical perspective it is hard to imagine a "natural" real interest rate at such a low level. If or when interest rates return to a more normal level, governments will again start to feel the burden of a high debt level and will have to adjust their budgets. One of the big macroeconomic issues of the 2020s will be how this process will play out.

Summary

1. The theory of optimum currency areas explains when it is optimal for a country to have a completely fixed exchange rate vis á vis its main trading partners and possibly to adopt their currency. The theory sees the choice of exchange rate regime as a trade-off between the microeconomic benefits and the macroeconomic costs of a fully and irrevocably fixed exchange rate. One microeconomic benefit is that a credibly fixed exchange rate reduces the riskiness of foreign trade and investment. Further benefits are gained if exchange rate stability is achieved by entering a currency union where the adoption of a common currency reduces international transactions costs, improves market transparency and increases the liquidity of financial markets. The macroeconomic costs arise from the fact that a fixed exchange rate/common currency excludes the possibility of an independent national monetary policy to stabilize the domestic economy.

2. The microeconomic benefits of a fixed exchange rate/common currency increase with the degree of international economic integration whereas the macroeconomic costs decrease with economic integration. When integration proceeds beyond a certain point, it therefore becomes optimal to switch from a flexible exchange rate to a fixed rate/common currency. It has also been argued that even if joining a currency union is not optimal *ex ante*, it may become optimal *ex post* because the adoption of a common currency will in itself promote economic integration.

3. Optimum currency area (OCA) theory suggests that the macroeconomic costs of giving up exchange rate flexibility within a group of trading partners will be relatively small if there is a low frequency of asymmetric shocks, a high degree of labour mobility across countries, and an international transfer mechanism securing a transfer of resources from countries hit by positive shocks to those hit by negative shocks. OCA theory also implies that the microeconomic benefits of a fixed exchange rate/common currency will be greater the greater the volume of trade and investment across borders.

4. For a country for which membership of a currency union is on the borderline of being optimal, there is a trade-off between integration and the symmetry of shocks: the macroeconomic costs of more asymmetric shocks must be compensated by the microeconomic benefits from a higher degree of integration. There is also a trade-off between symmetry and flexibility: a lower degree of symmetry must be compensated by higher flexibility of labour and product markets improving a country's ability to adjust to asymmetric shocks. Flexibility is particularly valuable when the shocks hitting the economy are permanent, but less valuable when shocks are temporary. A further trade-off is that between flexibility and the size of the common fiscal budget for the union: less flexibility must be compensated by a larger common budget allowing larger temporary transfers to member countries hit by temporary asymmetric shocks.

5. In 1999 a group of EU countries entered the third and final stage of the European

Monetary Union (EMU) by irrevocably locking their exchange rates and subsequently adopting a common currency, the euro. The Eurozone is the world's largest currency union among largely independent nation states. In the early years after the locking of exchange rates, the elimination of exchange rate risk led to large inflows of capital into the Southern and Western periphery of the EMU which stimulated economic growth in these countries. However, in the wake of the international financial crisis of 2007-2008, a dramatic public and private debt crisis in Greece, Ireland, Portugal and Spain (the GIPS countries) followed in 2009-2012 as capital flows to these countries suddenly stopped. The crisis initiated a "doom loop" where financial distress in the government bond market and financial distress in the banking system mutually reinforced each other. To stem the crisis, the ECB had to step in as a lender of last resort, and the GIPS countries had to receive official financial assistance from the IMF and from the rest of the Eurozone via the European Stability Mechanism. Several lessons can be learned from the debt crisis.

6. *First lesson:* It is important that governments consolidate their public finances when the economy is booming, so they have enough fiscal space to carry out a fiscal expansion when the economy turns down.

7. *Second lesson:* Although fiscal tightening is necessary in the medium and long run when a country has accumulated an unsustainable level of public debt, fiscal austerity can be an ineffective and overwhelmingly painful way of reducing the government debt ratio in a recession; in fact, austerity may even increase the debt-GDP ratio in the short run.

8. *Third lesson:* A distressed government faced with difficulties of borrowing from private investors may need temporary financial assistance from other EMU countries. It is desirable to have an established institution through which such financial assistance can be channeled, subject to transparent and pre-announced principles that help to reduce uncertainties for all parties involved. The EU has heeded this lesson by making the European Stability Mechanism a permanent institution.

9. *Fourth lesson:* In a financial crisis it is important to distinguish between insolvency and illiquidity. An indebted government is insolvent if its fiscal capacity is insufficient to enable it to service its debt even in the best of circumstances. It is then better to acknowledge the insolvency right away and negotiate the necessary debt relief. But if the government bond market has ended up in a "bad" equilibrium with a very high interest rate because individual investors believe that the other investors have very pessimistic expectations about the government's ability to service its debt, a promise of external financial assistance to the government may push the bond market back to a "good" equilibrium with a much lower interest rate that enables the government to honour its debt. In such a case the government is not insolvent, but suffers instead from a temporary liquidity crisis that can be overcome by official lending (or just a promise thereof) from partner countries at a "normal" interest rate.

10. *Fifth lesson:* A currency union with decentralized fiscal policy does need some institutional bulwark against irresponsible fiscal behavior in individual member countries. The Stability and Growth Pact and the European Fiscal Compact are attempts to create such a bulwark by enforcing the 3 percent deficit rule and the 60 percent debt rule of the Maastricht Treaty. However, these rules have been widely criticized for being too rigid, leaving too little space for fiscal expansion in a deep recession and too little room for productive public investment. There is an ongoing debate on how to create fiscal rules and institutions that strike the optimal balance between discipline and flexibility within the EMU.

11. *Sixth lesson:* Imposing restrictions on government borrowing may not be enough to prevent a debt crisis in a currency union. The elimination of exchange rate risk may allow the private sector to borrow much more from abroad than would be possible in a world with national currencies, meaning that a member country of a currency union can accumulate a much larger current-account deficit and level of foreign debt before markets start to react. To ensure a harmonious economic development within a currency union, all member countries must therefore be willing to coordinate their macroeconomic policies. For that purpose, the EU countries have introduced an annual surveillance procedure called the Macroeconomic Imbalance Procedure, but despite this procedure several EMU countries are still running large surpluses on their current accounts which must be balanced by deficits elsewhere.

12. *Seventh lesson:* The EMU experience with the "doom loop" between banks and governments suggests the existence of a policy trilemma within the EMU between (i) cross-border financial integration, (ii) financial stability, and (iii) fiscal policy flexibility. An EMU country cannot achieve all three things at the same time. With the large increase in capital flows that has followed the introduction of the euro, the banking systems in many EMU countries have grown so large relative to the national economy that their governments may not have the fiscal capacity to rescue systemically important banks in times of financial crisis if this is deemed necessary to avoid economic collapse. A way out of this trilemma is to shift the responsibility for deposit insurance, bank supervision and bank resolution from the national to the supra-national European level. Since 2012 the EMU countries have moved in this direction by working towards a banking union.

Exercises

Exercise 1. Optimum currency area theory

1. Explain and discuss the nature of the microeconomic benefits from a fully fixed exchange rate/common currency and how these benefits depend on the degree of international economic integration. What difference does it make whether the exchange is fixed or whether national currencies are abandoned in favour of a common currency?

2. Explain and discuss the nature of the macroeconomic costs of a fully fixed exchange rate and how these costs depend on the degree of economic integration.
3. Many economists have argued that even though a fixed exchange rate/common currency among a group of countries may not seem optimal *ex ante*, it may nevertheless turn out to be optimal *ex post*. Explain and discuss this argument.
4. For a country for which membership of a currency union is on the borderline of being optimal, there is a trade-off between (i) integration and the symmetry of shocks, (ii) symmetry of shocks and labour and product market flexibility, and (iii) flexibility and the size of the common fiscal budget for the union. Explain why these trade-offs exist.
5. Discuss the relevance of optimum currency area theory for (the debate on) monetary integration in Europe.

Exercise 2. Nominal wage and price flexibility and the economy's reaction to permanent asymmetric shocks under completely fixed exchange rates

According to the theory of optimum currency areas, greater nominal wage and price flexibility improves the economy's ability to adjust to permanent asymmetric economic shocks and thereby makes entry into a currency union with trading partners more attractive, ceteris paribus. In this exercise you are asked to use our AS-AD model with fixed exchange rates from Chapter 24 to study how more nominal wage and price flexibility affects the economy's response to permanent demand and supply shocks. This will give you a deeper understanding of how greater nominal flexibility increases the attractiveness of joining a monetary union when shocks are permanent.

Using our usual notation for the output gap $\hat{y} \equiv y - \bar{y}$ and the inflation gap $\hat{\pi} \equiv \pi - \pi^f$, we may write our AS-AD model of the open economy with fixed exchange rates and labour mobility in the following way, where z_t is a demand shock, s_t is a supply shock, and e_t^r is the real exchange rate:

$$\hat{y}_t = \beta_1(e_{t-1}^r - \hat{\pi}_t) + z_t, \tag{35}$$

$$\hat{\pi}_t = \gamma\hat{y}_t + s_t, \tag{36}$$

$$e_t^r = e_{t-1}^r - \hat{\pi}_t. \tag{37}$$

The parameter γ in the AS curve (36) reflects how fast the rate of inflation responds to the output gap, so it is an indicator of the degree of nominal wage and price flexibility. In the following, you will be asked to analyze how a change in this parameter affects the size of the output gap and the inflation gap caused by permanent shocks to demand and supply.

1. Abstract for the moment from supply shocks by setting $A_t = \bar{A}_t$, implying $s_t = 0$, but suppose that, after having been in long-run equilibrium in period -1 with $e^r_{-1} = 0$, the economy is hit by a demand shock in period 0 which permanently changes the value of the shock variable z_t from zero to some value $\bar{z} \neq 0$ for all $t = 0,1,2,....$Derive the effect of this shock on the output gap in period 0, $\hat{y}_o$.

2. Use the model (35) through (37) plus your answer to Question 1 to show that, when the economy has been hit by a permanent demand shock $\bar{z}$ in period 0, the output gap from that period and onwards will be

$$\hat{y}_t = \bar{z}\left(\frac{1}{1+\gamma\beta_1}\right)^{t+1}, \qquad t = 0,1,2,3,... \tag{38}$$

How does greater nominal wage and price flexibility (a higher γ) affect the numerical size of the output gap in each period after the shock, and how does it affect the economy's speed of adjustment to the long-run equilibrium? Explain the economic intuition for your findings.

3. Use (36) and (38) to analyze how greater nominal wage and price flexibility will affect the numerical size of the inflation gap $\hat{\pi}_t$ caused by the permanent demand shock for all $t = 0,1,2,....$ (Hints: You should find that for $t \geq 1$, there are offsetting effects of a change in γ on the inflation gap and that the net effect will depend on the size of γ and β_1 as well as the time period *t*. In your analysis, you may use the estimates from Chapter 24 that $\gamma = 0.3$ and $\beta_1 = 0.72$ on an annual basis. You can then calculate the time *t* when the effect of a change in γ on the inflation gap changes sign). Explain the economic intuition for the offsetting effects on the inflation gap occurring from period 1 and onwards.

4. Now abstract from demand shocks by setting $z_t = 0$, but suppose the economy is hit by a permanent supply shock in period 0, still after having been in long-run equilibrium in period -1 with $e^r_{-1} = 0$. The supply shock is a productivity shock that permanently changes the (log of the) natural rate of output by the amount $\Delta\bar{y}$ without affecting the structural labour force. In terms of the AS curve (36) we can thus write the supply shock as $s_t = -\gamma\Delta\bar{y}$ for all $t = 0,1,2,....$, and after the shock we have the new level of structural output $\bar{y}^{new} = \bar{y} + \Delta\bar{y}$ and the corresponding new output gap $\hat{y}_t \equiv y_t - \bar{y}^{new} = y_t - \bar{y} - \Delta\bar{y}$. Since the AD curve is not affected by the supply shock, it is given by the equation $y_t - \bar{y} = \beta_1(e^r_{t-1} - \hat{\pi}_t)$. From period zero and onwards, we can therefore write the AD and AS curves as follows, where $\hat{y}_t \equiv y_t - \bar{y}^{new}$:

$$\hat{y}_t = \beta_1(e^r_{t-1} - \hat{\pi}_t) - \Delta\bar{y}, \tag{39}$$

$$\hat{\pi}_t = \gamma\hat{y}_t. \tag{40}$$

The real exchange rate still evolves according to (37). Use (37), (39) and (40) to show that, following the permanent supply shock, the output gap will be

$$\hat{y}_t = -\Delta\bar{y}\left(\frac{1}{1+\gamma\beta_1}\right)^{t+1}, \qquad t = 0,1,2,3,\ldots \tag{41}$$

5. Use (40) and (41) to analyze how greater nominal wage and price flexibility will affect the numerical size of the output gap $\hat{y}_t$ and the inflation gap for all $t = 0,1,2, \ldots$. Explain the economic intuition for your findings.

Exercise 3. Nominal wage and price flexibility and the economy's reaction to temporary asymmetric shocks under completely fixed exchange rates

You are now invited to use our AS-AD model of an open economy with fully fixed exchange rates to study how increased nominal wage flexibility affects the economy's reaction to a *temporary* shock throughout the entire adjustment period until it has returned to its initial long-run equilibrium. To limit the scope of the analysis, you are asked to focus on the effects of a temporary demand shock on the output gap. Setting the supply shock s_t equal to zero, we thus have the following model in the usual notation:

$$\hat{y}_t = \beta_1(e^r_{t-1} - \hat{\pi}_t) + z_t, \tag{42}$$

$$\hat{\pi}_t = \gamma\hat{y}_t, \tag{43}$$

$$e^r_t = e^r_{t-1} - \hat{\pi}_t. \tag{44}$$

1. Suppose that, after having been in long-run equilibrium in period -1 with $e^r_{-1} = 0$, the economy is hit by a temporary demand shock in period $t = 0$ which changes the value of the shock variable z from zero to some value $\bar{z} \neq 0$ for in that period. Derive the effect of this shock on the output gap in period 0, $\hat{y}_o$.

2. Use your answers to Question 1 to show analytically how greater nominal wage and price flexibility (a higher γ) affects the *numerical* magnitude of the output gap generated by the demand shock in period 0. Explain the economic intuition for your finding.

3. Since the demand shock in period 0 is temporary and short-lived, we have $z_t = 0$ for all $t = 1,2,3, \ldots$ Use the model (42) through (44) and your answer to Question 1 to derive the solution for $\hat{y}_t$ as a function of time for all $t = 1,2,3, \ldots$

4. Use your solution for $\hat{y}_t$ in Question 3 to analyze formally how greater nominal wage and price flexibility affects the magnitude of the *numerical* output gap in the adjustment periods from $t = 1$ and onwards, following the temporary demand shock in period 0. Contrast your findings for period $t = 0$ and for $t \geq 1$ and explain why greater nominal wage and price flexibility is not necessarily an unambiguous blessing (Hint: You should find that, for our previous plausible parameter values, $\beta_1 = 0.72$ and $\gamma = 0.3$ on an annual basis, a higher value of γ will increase the numerical output gap for several years after period 0).

Exercise 4. Fiscal austerity and government debt

In this exercise you are asked to derive the effects of a change in real government consumption on the government debt-GDP ratio reported in equations (19) through (21).

1. Use Equation (18) to calculate the total derivative of next period's debt-GDP ratio, d_{+1}, with respect to real government consumption and investment in the current period, G (Hint: You are not supposed to introduce the auxiliary variables A_1 through A_4 at this stage; just calculate a "raw" expression for $\partial d_{+1}/\partial G$ from (18)).

2. Now use your result in Question 1 plus the specification of the various derivatives in Equations (14) through (17) to rewrite your expression for $\partial d_{+1}/\partial G$ in terms of the auxiliary variables A_1 through A_4 defined in Equations (20) and (21) to that you end up with Equation (19).

3. Explain (briefly) some factors that will determine the size of the multipliers m and m_{+1} in the formula for $\partial d_{+1}/\partial G$.

4. Inspecting Figure 26.12, do you see any mechanisms that are left out from the theory underlying the result in (19) but which should be included in a richer theory? Motivate your answer.