Optimal monetary policy

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Last time

Calvo pricing

- Sellers adjust prices with a given probability
- Very elegant solution to a complicated price setting problem

The New Keynesian Model

- Three equations to rule the world: PC, IS, TR
- Output responses to different shocks

Heterogeneity

- The representative agent model must be extended to allow for better policy analysis
- Different marginal propensities to consume are a good starting point

Today

Central bankers are (rational) people, too

- Policy makers have goal functions, but where do they come from?
- Can central bankers be expected to adhere to rules?

Policy and politics

- Assuming some exogenous rule for monetary policy is too simple
- The policy itself is an outcome of its environment: it is endogenous
- In this context, credibility and reputation will play key roles

Optimal Monetary policy

- Agents and the central bank play a non-cooperative "game" ($\Longrightarrow$ Nash)
- Outcome depends on the ability of the central bank to **commit** to a plan

Monetary policy makers care about their credibility

Isabel Schnabel

- "Instead, for monetary policy to remain credible in the current environment, it must not be an inflationary source itself."

Christine Lagarde

- "What became evident [during 2012] is that the perceived commitment of policymakers was a crucial variable in effective policymaking."

Janet Yellen

- "My remarks today will focus on the issue of credibility—in particular on the Federal Reserve's credibility regarding its announced commitment to maintaining price stability."

Mario Draghi

- "Within our mandate, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough."

Value judgements

Optimal monetary policy

- So far, all of our analyses have been positive
- Given the initial assumptions, they contained no value judgements, only descriptive conclusions
- Optimal monetary policy (i.e., what the central bank **should** do) requires us to do normative analysis

Rational expectations

- How trustworth/predictable the central bank's actions are is important
- Policy outcomes differ based on commitment/discretion

The model

Setup (Persson & Tabellini 15)

- The model is as simple as possible to isolate the channels we care about: the influence of central bank policy on output and inflation
- There is a relationship between output and inflation (PC) and an IS curve (reduced form)

The goal is to

- identify a rule for monetary policy that optimizes a loss function
- analyze how the economy's aggregates change under different assumption on credibility

Background

Unions

- In the Persson-Tabellini model, labor unions negotiate for some wage growth w such that

$$w = \omega + \pi^e$$

- Output growth, in turn, depends on the negotiated real wage:

$$x = \gamma - (w - \pi) - \varepsilon$$

- γ is a parameter, if wages are too high, output is too low
- supply shocks ε lower domestic output

Resulting output

$$x = \underbrace{(\gamma - \omega)}_{\theta} + (\pi - \pi^e) - \varepsilon$$

Equations

Phillips Curve

$$\pi_t = m_t + \underbrace{v}_{\text{Demand shock}} + \underbrace{\mu}_{\text{MP shock}}$$

Demand equation

$$x = \underbrace{\theta}_{\text{natural rate of output}} + (\pi_t - \pi^e) - \underbrace{\varepsilon}_{\text{Supply shock}}$$

- Inflation depends on money growth, unexpected demand and monetary policy mistakes
- Output depends on unexpected inflation, supply and its natural rate
- Expected inflation is $\mathbb{E}_t[\pi_t] \equiv \pi^e$
- All shocks are independent and 0 in expectation

Timing assumptions

Perfect commitment

1. Announcement of monetary rule
2. Everyone observes the natural level of output θ
3. Expectations π^e are formed, given the information about θ
4. Everyone observes v and ε
5. The central bank decides the money supply m
6. μ is realized, pinning down output x and inflation π

Consequences

- The central bank has an informational advantage (expectations are pinned down before the money supply is set)

Intuition

Monetary policy can move after expectations are formed

- This is a reduced form way to make monetary policy powerful
  $\Longrightarrow$ it can stabilize output against shocks
  $\Longrightarrow$ it can save the agents from themselves
- Monetary policy is decided every six weeks, wages are only renegotiated at longer intervals

Lucas again

- After θ realizes, only unexpected changes in monetary policy have an effect
- Moves in m can stabilize shocks to v and μ

$$\mathbb{E}_t[\pi_t|\theta] = \mathbb{E}_t[m_t|\theta]$$

$$x = \theta + (\underbrace{m_t + v_t + \mu_t}_{\pi_t} - \mathbb{E}_t[m_t|\theta]) + \varepsilon$$

Policy

Quadratic loss function

$$\mathcal{L} = \frac{1}{2}\left[a(\pi - \overline{\pi})^2 + \lambda(x - \overline{x})^2\right]$$

- Loss function implies that the central bank dislikes deviations from some inflation benchmark $\overline{\pi}$, and deviations from some output target $\overline{x}$
- The degree of "pain" such deviations cause the banker are governed by the parameters a and λ
- The parameters a and λ are known to all agents in the economy

Policy rule

Linear policy rule

- With a quadratic objective and linear shock processes, it can be shown that a policy rule which is linear in the shocks is optimal
- It can achieve the minimization of the loss function given the realizations of the shocks

Assumes the rule

$$m = \varphi + \varphi_\theta \theta + \varphi_v v + \varphi_\varepsilon \varepsilon$$

- The central bank reacts to shocks to natural output θ, demand shocks v and productivity shocks ε
- By definition, it cannot do anything about μ
- Recall: θ is observed before expectations are formed, v and ε realize after

Perfect credibility

Credible policy rule

- If agents know the rule and it is perfectly credible, they will include it into their expectations
- Strong assumption: central bankers may have an incentive to deviate (more on that later)

Expectations

$$\mathbb{E}_t[m_t|\theta] = \varphi + \varphi_\theta \mathbb{E}[\theta|\theta] + \varphi_v \mathbb{E}[v|\theta] + \varphi_\varepsilon \mathbb{E}[\varepsilon|\theta]$$
$$\mathbb{E}_t[\pi_t|\theta] = \mathbb{E}[m_t|\theta] = \varphi + \varphi_\theta \theta$$

- The shocks are independent $\implies$ conditional expectations don't help
- Expected inflation only depends on the realization of θ
- Other shocks are 0 in expectation

Perfect credibility

$$\mathbb{E}_t[\pi_t] = \varphi + \varphi_\theta \theta$$

Realized inflation

$$\begin{aligned}\pi_t &= \underbrace{\varphi + \varphi_\theta\theta + \varphi_v v + \varphi_\varepsilon \varepsilon}_{m_t} + v + \mu \\ &= \varphi + \varphi_\theta \theta + (1+\varphi_v)v + \varphi_\varepsilon\varepsilon + \mu\end{aligned}$$

Realized output

$$\begin{aligned}x &= \theta + (\underbrace{\varphi + \varphi_\theta\theta + (1+\varphi_v)v + \varphi_\varepsilon\varepsilon + \mu}_{\pi_t} - \underbrace{\varphi - \varphi_\theta\theta}_{\pi_t^e}) - \varepsilon \\ &= \theta + (1+\varphi_v)v + (\varphi_\varepsilon - 1)\varepsilon + \mu\end{aligned}$$

What is the optimal policy?

Ex-ante optimality

- What parameters should be set for the policy rule **ex-ante**?
- Crucially: Implies optimal policy **in expectation**

Minimize the loss function

- If the rule is credible, then output and inflation will behave as on the previous slide
- Plug into the loss function
- Minimize **the expectation**

Expected loss

$$
\begin{aligned}
\mathbb{E}[\mathcal{L}] &= \frac{1}{2}\mathbb{E}\left[a(\pi-\overline{\pi})^2+\lambda(x-\overline{x})^2\right] \\
&= \frac{1}{2}\mathbb{E}\Bigg[a(\underbrace{\varphi+\varphi_\theta\theta+(1+\varphi_v)v+\varphi_\varepsilon\varepsilon+\mu}_{\pi}-\overline{\pi})^2 \\
&\quad +\lambda(\underbrace{\theta+(1+\varphi_v)v+(\varphi_\varepsilon-1)\varepsilon+\mu}_{x}-\overline{x})^2\Bigg] \\
&= \frac{1}{2}\mathbb{E}\left[a(A)+\lambda(B)\right]
\end{aligned}
$$

- Want to minimize $\implies$ take derivatives
- But: Expectation of square term is complicated, better multiply out first (next slide)

Algebra

$$
\begin{aligned}
A &= (\varphi + \varphi_\theta\theta + (1+\varphi_v)v + \varphi_\varepsilon\varepsilon + \mu - \overline{\pi})^2 \\
&= \varphi^2 + \varphi\varphi_\theta\theta + \varphi(1+\varphi_v)v + \varphi\varphi_\varepsilon\varepsilon + \varphi\mu - \varphi\overline{\pi} + \varphi\varphi_\theta\theta + \varphi_\theta^2\theta^2 + (1+\varphi_v)v\varphi_\theta\theta \\
&+ \varphi_\varepsilon\varepsilon\varphi_\theta\theta + \mu\varphi_\theta\theta - \overline{\pi}\varphi_\theta\theta + \varphi(1+\varphi_v)v + \varphi_\theta\theta(1+\varphi_v)v + (1+\varphi_v)^2v^2 \\
&+ \varphi_\varepsilon\varepsilon(1+\varphi_v)v + \mu(1+\varphi_v)v - \overline{\pi}(1+\varphi_v)v + \varphi_\varepsilon\varepsilon\varphi + \varphi_\theta\varphi_\varepsilon\varepsilon\theta + (1+\varphi_v)\varphi_\varepsilon\varepsilon v \\
&+ \varphi_\varepsilon^2\varepsilon^2 + \varphi_\varepsilon\varepsilon\mu - \varphi_\varepsilon\varepsilon\overline{\pi} + \varphi\mu + \varphi_\theta\mu\theta + (1+\varphi_v)\mu v + \varphi_\varepsilon\mu\varepsilon + \mu^2 - \mu\overline{\pi} \\
&+ \varphi\overline{\pi} + \varphi_\theta\overline{\pi}\theta + (1+\varphi_v)\overline{\pi}v + \varphi_\varepsilon\overline{\pi}\varepsilon + \overline{\pi}\mu - \overline{\pi}\overline{\pi}
\end{aligned}
$$

All shocks are independent!

- In expectation, shock terms multiplied by constants are zero, e.g. $\mathbb{E}[\varphi\varphi_\theta\theta] = \varphi\varphi_\theta\mathbb{E}[\theta] = 0$
- In expectation, cross-terms are zero: $\mathbb{E}[\varphi_\varepsilon\varepsilon\varphi_\theta\theta] = \varphi_\varepsilon\varphi_\theta\mathbb{E}[\varepsilon\theta] = \varphi_\varepsilon\varphi_\theta\mathbb{E}[\varepsilon]\mathbb{E}[\theta] = 0$

Light at the end of the tunnel

$$\begin{aligned}\mathbb{E}[A] &= \mathbb{E}[\varphi + \varphi_\theta \theta + (1+\varphi_v)v + \varphi_\varepsilon \varepsilon + \mu - \overline{\pi})^2] \\ &= \varphi^2 - \varphi\overline{\pi} + \varphi_\theta^2 \mathbb{E}[\theta^2] + (1+\varphi_v)^2 \mathbb{E}[v^2] + \varphi_\varepsilon^2 \mathbb{E}[\varepsilon^2] + \mathbb{E}[\mu^2] - \varphi\overline{\pi} + \overline{\pi}^2 \\ &= \varphi^2 - \varphi\overline{\pi} + \varphi_\theta^2 \sigma_\theta^2 + (1+\varphi_v)^2 \sigma_v^2 + \varphi_\varepsilon^2 \sigma_\varepsilon^2 + \sigma_\mu^2 - \varphi\overline{\pi} + \overline{\pi}^2\end{aligned}$$

Expectations depend on variances of shocks

- $\sigma_q^2 = \mathrm{Var}(q) = \mathbb{E}[(q - \overline{q})^2]$ – If mean of random variable is 0, the expectation of its square is the variance
- The zero-mean and independence assumptions are doing **a lot** of heavy lifting for us

Second square term

Apply the same principle to the square variable B

$$\begin{aligned}\mathbb{E}[B] &= \mathbb{E}[(\theta + (1+\varphi_v)v + (\varphi_\varepsilon - 1)\varepsilon + \mu - \overline{x})^2] \\ &= \mathbb{E}[\theta^2] + (\varphi_v + 1)^2\mathbb{E}[v^2] + (1-\varphi_\varepsilon)^2\mathbb{E}[\varepsilon^2] + \mathbb{E}[\mu^2] + \overline{x}^2 \\ &= \sigma_\theta^2 + (\varphi_v+1)^2\sigma_v^2 + (1-\varphi_\varepsilon)^2\sigma_\varepsilon^2 + \sigma_\mu^2 + \overline{x}^2\end{aligned}$$

- Now we have all the ingredients to fill in the expectation of the loss function

Minimize expected loss

Plugging in from the previous slides:

$$\min_{\varphi,\varphi_\theta,\varphi_v,\varphi_\varepsilon} \frac{1}{2}a\left(\varphi^2 - \varphi\overline{\pi} + \varphi_\theta^2\sigma_\theta^2 + (1+\varphi_v)^2\sigma_v^2 + \varphi_\varepsilon^2\sigma_\varepsilon^2 + \sigma_\mu^2 - \varphi\overline{\pi} + \overline{\pi}^2\right)$$
$$+ \frac{1}{2}\lambda\left(\sigma_\theta^2 + (\varphi_v+1)^2\sigma_v^2 + (1-\varphi_\varepsilon)^2\sigma_\varepsilon^2 + \sigma_\mu^2 + \overline{x}^2\right)$$

- A hypothetical social planner wants to set the rule (i.e., the parameters in the central bank's response function) to minimize this loss
- The rule is in place forever $\implies$ minimizing single period expectation of loss is the same as discounted infinite sum of all future periods' losses
- Take the derivatives w.r.t. $\varphi, \varphi_\theta, \varphi_v, \varphi_\varepsilon$

Minimum expected loss

$$\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial \varphi} : a(\varphi - \overline{\pi}) = 0 \implies \varphi = \overline{\pi}$$
$$\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial \varphi_\theta} : a\vartheta_\theta \sigma_\theta^2 = 0 \implies \varphi_\theta = 0$$
$$\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial \varphi_v} : \sigma_v^2 (a + \lambda)(1 + \varphi_v) = 0 \implies \varphi_v = -1$$
$$\frac{\partial \mathbb{E}[\mathcal{L}]}{\partial \varphi_\varepsilon} : \sigma_\varepsilon^2 (a\varphi_\varepsilon - \lambda(1 - \varphi_\varepsilon)) = 0 \implies \varphi_\varepsilon = \frac{\lambda}{a + \lambda}$$

Implications

- Anchor inflation where society wants it
- Shocks that are priced into expectations need no reaction (θ)
- Neutralize demand shocks
- Supply shocks: it depends. Countering supply shocks causes less deviations from $\overline{x}$, but at the cost of deviations from $\overline{\pi} \implies$ tradeoff

Equilibrium under commitment

Optimal rule

$$m_t = \overline{\pi} - v_t + \frac{\lambda}{a+\lambda}\varepsilon$$

Equilibrium inflation

$$\pi^C = \overline{\pi} + \frac{\lambda}{a+\lambda}\varepsilon + \mu$$

Equilibrium output

$$x^C = \theta - \frac{a}{a+\lambda}\varepsilon + \mu$$

- Output may fluctuate due to changes in the natural rate, supply shocks or policy errors
- Inflation only changes due to supply shocks and policy errors
- Depending on preferences, supply shocks will feed more into output, or more into inflation

Commitment conclusions

Benefits

- If the policy maker can commit to a rule, inflation and output are stable around their natural levels
- As we will see, this is the best possible outcome

Simplify the problem

- Demand shocks are neutralized $\implies$ we can ignore them
- Policy errors μ are not interesting to study because there is little we can do about them $\implies$ ignore for now
- **The only important shocks left are θ and ε**

Credibility

Credibility

Problems with rules

- Central bankers are not computers. They may want to exploit their informational advantage
- Once expectations are locked in, it's possible to decrease societal losses even further
- The rule may not be credible if bankers have **discretion** (i.e., ability) to deviate

Discretion

- It's more realistic to assume policy makers don't stick to a rule
- This feeds back into agents (rational) expectations
- Equilibrium outcomes are different without commitment

New timing

Discretion/Non-credible rule

1. ~~Announcement of monetary rule~~
2. Everyone observes the natural level of output θ
3. Expectations π^e are formed, given the information about θ
4. Everyone observes ε
5. The central bank decides the money supply m
6. Output x and inflation π are pinned down

Implications

- Without a (credible) rule, the central bank is free to do what it wants each period

New optimality

Ex-post optimality

- When the CB could commit to a credible rule, that rule was **ex-ante** optimal: $\mathbb{E}[\frac{\mathcal{L}}{\partial m}] = 0$
- Without a rule, policy will be ex-post optimal: $\frac{\partial \mathcal{L}}{\partial m} = 0$
- What seems like a small difference has big consequences

Nash-equilibrium

- Central bank and consumers play a game. In equilibrium nobody wants to deviate from decision
- Solve by backwards induction

Central bank optimum under discretion (second stage)

$$\mathcal{L} = \frac{1}{2}\mathbb{E}\left[a(\pi_t - \overline{\pi})^2 + \lambda(x_t - \overline{x})^2\right]$$

$$\pi_t = m_t \qquad \text{remember: } v \text{ and } \mu \text{ set to } 0$$

$$x_t = \theta + (\pi_t - \pi_t^e) - \varepsilon_t$$

- Since $\pi_t = m_t$, just assume that the CB sets π_t directly
- Recall that the CB takes π_t^e as given

$$\frac{\mathcal{L}}{\pi_t} : a(\pi_t - \overline{\pi}) + \lambda(\theta + (\pi_t - \pi_t^e) - \varepsilon_t - \overline{x}) = 0$$

$$\implies \pi = \frac{a}{a+\lambda}\overline{\pi} + \frac{\lambda}{a+\lambda}(\pi_t^e - \theta + \varepsilon + \overline{x})$$

- Note: If we plug in the result from the commitment equilibrium $\pi = \pi^e = \overline{\pi}$, $\frac{\mathcal{L}}{\pi_t} > 0 \implies$ CB can do better!

Consumer expectation under discretion (first stage)

Take expectation of central banks decision function

$$\begin{aligned}\mathbb{E}[\pi|\theta] &= \frac{a}{a+\lambda}\overline{\pi} + \frac{\lambda}{a+\lambda}\mathbb{E}[(\mathbb{E}[\pi|\theta] - \theta + \varepsilon + \overline{x})|\theta] \\ &= \overline{\pi} + \underbrace{\frac{\lambda}{a}(\overline{x} - \theta)}_{\text{Inflation bias}}\end{aligned}$$

- Expected inflation is higher than in the commitment case
- Because θ is known to consumers, they know what the CB will do. If $\theta < \overline{x}$: increase m, if $\theta > \overline{x}$: decrease m
- However, these actions are pointless, because prices adjust. As always, if higher m is expected, p (and therefore π) adjusts, and x stays constant

Realized values of inflation and output

Output

$$x^D = \theta - \frac{a}{a+\lambda}\varepsilon$$

- Output is the same as under commitment!

Inflation

$$\pi^D = \overline{\pi} + \frac{\lambda}{a}(\overline{x} - \theta) + \frac{\lambda}{a+\lambda}\varepsilon$$

- Inflation is higher and more volatile

Giving central banks discretion leaves output constant, but $\mathcal{L}$ is actually lower than it could be

Reputation

Longer time horizon

Single period

- The commitment and discretion cases before are single-period games
- In reality, central banks make decisions all the time

$\Longrightarrow$ Current decisions affect future reputation

Multi-period game

- The central bank makes decisions every period, proclaiming a rule
- Consumers decide whether they trust the bank or not
- Trust can never be rebuilt

Longer run optimality

Infinite loss function

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^j \mathcal{L}(\pi_{t+j}, x_{t+j})$$

- The central bank now cares about all future periods

Simplifying assumption

$$\mathcal{L}(\pi_t, x_t) = \frac{\pi_t^2}{2} - \lambda x$$
$$\implies \pi^C = 0, \quad \pi^D = \lambda, \quad x^C = x^D = \theta - \varepsilon$$

- As always: this contains the most important intuition
- Using the double square loss function is much more messy
- Inflation volatility is costly $\implies$ CB let's ε only affect output

Betraying trust

Inflation expectations

$$\pi_t^e = 0 \text{ if } \pi_{t-1} = \pi_{t-1}^e$$
$$\pi_t^e = \lambda \text{ otherwise}$$

- If realized inflation was in line with the agents expectations yesterday, the bank has not deviated from its rule
- In this case: keep trusting the central bank
- In any other case the bank has deviated $\implies$ don't trust the CB ever again

The central bank's problem

Adhere to rule or break trust?

- Each period, the CB faces a choice
- If it deviates, it can decrease its loss function today
- But at the cost of never being able to do so ever again

Determinants of decision

- Because the CB is a rational agent, it computes the one-time benefits of deviating and compares the to the future costs
- Whichever is more attractive is the equilibrium outcome

Contemporary benefit of deviating

Loss in case of exploitation

$$\pi_t = \lambda$$

$$x_t = \theta + \lambda - \varepsilon$$

$$\mathcal{L}(\lambda, \theta + \lambda - \varepsilon) = +\frac{1}{2}\lambda^2 - \lambda(\theta + \lambda - \varepsilon)$$

Loss in case of continuous commitment

$$\pi_t = 0$$

$$x_t = \theta - \varepsilon$$

$$\mathcal{L}(0, \theta - \varepsilon) = -\lambda(\theta - \varepsilon)$$

One-time loss from deviating

$$\mathcal{L}(\lambda, \theta + \lambda - \varepsilon) - \mathcal{L}(0, \theta - \varepsilon) = -\frac{1}{2}\lambda^2 \text{ (loss is lower)}$$

Long-run cost of deviating

Loss in case of deviating (starting at period $t = s + 1$—tomorrow)

$$\pi_s = \lambda, \quad x_s = \theta - \varepsilon$$

$$\mathbb{E}\left[\sum_{t=s+1}^{s+T} \beta^{t-s} \mathcal{L}(\lambda, \theta - \varepsilon)\right] = \sum_{t=s+1}^{s+T} \beta^{t-s} \left(\frac{1}{2}\lambda^2 - \lambda \mathbb{E}[(\theta - \varepsilon)]\right)$$

Loss in case of continuous commitment

$$\pi_s = 0, \quad x_s = \theta - \varepsilon$$

$$\mathbb{E}\left[\sum_{t=s+1}^{s+T} \beta^{t-s} \mathcal{L}(0, \theta - \varepsilon)\right] = -\sum_{t=s+1}^{s+T} \beta^{t-s} \lambda \mathbb{E}\left[(\theta - \varepsilon)\right]$$

Long-run loss from deviating

$$\mathbb{E}\left[\sum_{t=s+1}^{s+T} \beta^{t-s} \mathcal{L}(\lambda, \theta - \varepsilon)\right] - \mathbb{E}\left[\sum_{t=s+1}^{s+T} \beta^{t-s} \mathcal{L}(0, \theta - \varepsilon)\right] = \beta \frac{1}{2} \frac{(1 - \beta^{T-1})}{1 - \beta} \lambda^2$$

Overall cost-benefit analysis

Add up single-period and long-run losses from deviating

$$Q = \beta\frac{1}{2}\frac{(1-\beta^{T-1})}{1-\beta}\lambda^2 - \frac{1}{2}\lambda^2$$

- If $Q < 0$, the effect of deviating on the loss function (contemporaneous + long-run) is negative $\implies$ desirable! Smaller loss means gain
- If $Q > 0$, the loss is positive (that's bad) and the CB does not want to deviate
- The central bank will deviate if:

$$\frac{1}{2}\lambda^2\left(\beta\frac{(1-\beta^{T-1})}{1-\beta} - 1\right) < 0 \iff \beta\frac{(1-\beta^{T-1})}{1-\beta} < 1$$

Intuition

$$\beta\frac{(1-\beta^{T-1})}{1-\beta} < 1$$

Special cases

- If the world end tomorrow $(T = 1)$, $0 < 1$ implies that the central bank will deviate with certainty
- If the world never ends, we need $\beta < 0.5$ for the CB to find deviating attractive
- If the discount factor is low (0.5 is very low), the CB doesn't care about the future and will deviate

Implications

- The repeated game nature of this example, together with the threat of higher inflation forever, keep the central bank honest
- Once the CB has deviated, the economy can never go back

Institutions

Policy mandates

European Central Bank

- The primary objective of the European System of Central Banks (hereinafter referred to as 'the ESCB') shall be to maintain price stability. (Article 127, TFEU)
- In pursuing price stability, the ECB seeks to hold inflation below but close to 2 percent over a medium-term horizon.
- (...) support the general economic policies in the Union with a view to contributing to the achievement of the objectives of the Union as laid down in Article 3 of the Treaty on European Union.

Federal Reserve

- (...) so as to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates.

Central bank appointments

Guidance

- Both ECB and Fed have been given clear guidelines on what to focus their policies on
- ECB: Price stability – everything else is secondary
- Fed: Dual mandate – more in line with the formulas above

Doves or Hawks

- However, it is impossible for a central bank to credibly commit to a rule
- Still, governments can at least appoint the right person to head the central bank
- Who are they?

Finding the right central banker

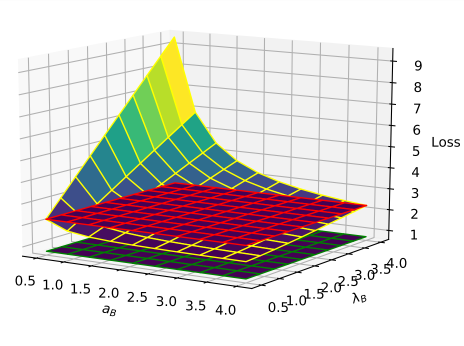
$$x_B^D = \theta - \frac{a_B}{a_B + \lambda_B}\varepsilon$$
$$\pi_B^D = \overline{\pi} + \frac{\lambda_B}{a_B}(\overline{x} - \theta) + \frac{\lambda_B}{a_B + \lambda_B}\varepsilon$$

- Each central banker has their own a_B and λ_B
- Which one should be chosen to make decisions?

$\Longrightarrow$ minimize conditional loss function

$$\mathbb{E}\left[\mathcal{L}(x_B^D, \pi_B^D)\right] = \mathbb{E}\left[\frac{1}{2}a\left(\frac{\lambda_B}{a_B}(\overline{x} - \theta) + \frac{\lambda_B}{a_B + \lambda_B}\varepsilon\right)^2 + \frac{1}{2}\lambda\left(\theta - \frac{a_B}{a_B + \lambda_B}\varepsilon - \overline{x}\right)^2\right]$$

Finding the right central banker



- red: discretion; green: commitment; yellow: central banker
- Large values of a_B and small values of λ_B approach the optimum
- Inflation hawks minimize the loss function

Conclusion

Commitment and Discretion

- Commitment to a rule leads to lowest inflation
- Discretion creates an inflationary bias—output is unchanged

$\Longrightarrow$ Credibility is important

Repeated game

- Interaction across many periods can keep the central bank in check
- Future costs of deviating make optimum more attractive

The ideal central banker

- Inflation hawks lead to a lower loss function
- Can approach commitment optimum

Algebra

$$
\begin{aligned}
&\sum_{t=s+1}^{s+T} \beta^{t-s}\left(\frac{1}{2}\lambda^2 - \lambda\mathbb{E}[(\theta-\varepsilon)]\right) + \sum_{t=s+1}^{s+T} \beta^{t-s}\lambda\mathbb{E}\left[(\theta-\varepsilon)\right] \\
&= \sum_{t=s+1}^{s+T} \beta^{t-s}\frac{1}{2}\lambda^2 = \frac{1}{2}\lambda^2 \sum_{t=s+1}^{s+T} \beta^{t-s} \\
&= \frac{1}{2}\lambda^2 \sum_{t=s+1}^{s+T} \beta^{t-s} = \frac{1}{2}\lambda^2\beta \sum_{t=s}^{s+T-1} \beta^{t-s} = \frac{1}{2}\lambda^2\beta \sum_{j=0}^{T-1} \beta^j \\
&= \frac{1}{2}\lambda^2\beta\left(\sum_{j=0}^{\infty} \beta^j - \sum_{j=T-1}^{\infty} \beta^j\right) = \frac{1}{2}\lambda^2\beta\left(\frac{1}{1-\beta} - \sum_{j=T-1}^{\infty} \beta^j\right) \\
&= \frac{1}{2}\lambda^2\beta\left(\frac{1}{1-\beta} - \beta^{T-1}\sum_{j=0}^{\infty} \beta^j\right) \\
&= \frac{1}{2}\lambda^2\beta\frac{1-\beta^{T-1}}{1-\beta}
\end{aligned}
$$