

$$V(r, m, g) = \max_{c_1, c_2} \ln(c_1) + \beta \ln(c_2) + \beta^2 \ln(c_3)$$

$$C_1 + \frac{C_2}{1+r} + \frac{C_3}{(1+r)^2} \leq m + \frac{(1+g)m}{1+r} + \frac{(1+g)^2 m}{(1+r)^2}$$

a) g betegnør den positive eller negative værdi i anden og tredje periode.

b) Lagrange opstilles og maksimeringen løses.

$$L \doteq \ln(c_1) + \beta \ln(c_2) + \beta^2 \ln(c_3) - h(C_1 + \frac{C_2}{1+r} + \frac{C_3}{(1+r)^2} - m - \frac{(1+g)m}{1+r} - \frac{(1+g)^2 m}{(1+r)^2})$$

$$\frac{\partial L}{\partial C_1} = \frac{1}{c_1} - h \rightarrow \frac{1}{c_1} = h$$

$$\frac{\partial L}{\partial C_2} = \frac{\beta}{c_2} - \frac{h}{1+r} \rightarrow \frac{\beta}{c_2} = \frac{h}{1+r}$$

$$\frac{\partial L}{\partial C_3} = \frac{\beta^2}{c_3} - \frac{h}{(1+r)^2} \rightarrow \frac{\beta^2}{c_3} = \frac{h}{(1+r)^2}$$

Sætter
 $\frac{\partial L}{\partial C_1} = \frac{1}{c_1} = \frac{h}{1+r} \rightarrow \frac{C_1}{h} = \frac{1}{1+r}$
 $\frac{\partial L}{\partial C_2} = \frac{\beta}{c_2} = \frac{h}{1+r} \rightarrow \frac{C_2}{h} = \frac{\beta}{1+r}$
 $C_2 = (1+r)C_1\beta$

Sætter

$$\frac{\partial L}{\partial C_3} = \frac{1}{c_3} = \frac{h}{(1+r)^2} \rightarrow \frac{C_3}{h} = \frac{1}{(1+r)^2} \rightarrow C_3 = C_1\beta(1+r)^2$$

$$C_1 + \frac{(1+r)C_1\beta}{1+r} + \frac{(1+r)^2 C_1\beta^2}{(1+r)^2} = m + \frac{(1+g)m}{1+r} + \frac{(1+g)^2 m}{(1+r)^2}$$

$$C_1 + C_1\beta + C_1\beta^2 = m + \frac{(1+g)m}{1+r} + \frac{(1+g)^2 m}{(1+r)^2}$$

$$C_1(1 + \beta + \beta^2) = m + \frac{(1+g)m}{1+r} + \frac{(1+g)^2 m}{(1+r)^2}$$

$$C_1^* = \frac{1}{1 + \beta + \beta^2} \left(m + \frac{(1+g)m}{1+r} + \frac{(1+g)^2 m}{(1+r)^2} \right)$$

c)

Den marginale forbrugstilbøjelighed er givet ved

$$\frac{dc_t}{dm} = \frac{1}{1+B+B^2} \left(1 + \frac{1+g}{1+r} + \frac{(1+g)^2}{(1+r)^2} \right)$$

d)

$$v(r, m, g) = \max \sum_{t=1}^T B^{t-1} \ln(c_t)$$

o.b.b

$$\sum_{t=1}^T \frac{c_t}{(1+r)^{t-1}} \leq \sum_{t=1}^T \frac{(1+g)^{t-1}}{1+r} m$$

Vi ved fra b) at

$$L = B$$

$$C_t^* = \frac{1}{1+B+B^2} \left(m + \frac{(1+g)m}{1+r} + \frac{(1+g)^2 m}{(1+r)^2} \right)$$

Dette kan opstilles til en geometrisk række

$$C_t^* \underset{t \rightarrow \infty}{\lim} = \frac{1}{1+B+B^2} \left(m + \frac{(1+g)m}{1+r} + \frac{(1+g)^2 m}{(1+r)^2} + \frac{(1+g)^3 m}{(1+r)^3} \right)$$

Vha løsningsmetoden for geometrisk række

$$\frac{a}{1-k} = \frac{m}{1-\frac{1+g}{1+r}} = \frac{m}{\frac{1+r}{1+r} - \frac{1+g}{1+r}} = \frac{m}{\frac{r-g}{1+r}} = \frac{m(1+r)}{r-g}$$

Sammensænk

bruges ved $\frac{a}{k-B} = \frac{1}{1-B}$ som sætter tilbøje incl i

$$\frac{1}{1-B} = 1-B \quad \text{Deler er } C_t^* \text{ når } t \rightarrow \infty$$

$$\text{er } \frac{dc_t}{dm} = (1-B) \left(\frac{1+r}{r-g} \right) \quad \text{mens forbrugstilbøjeligheten}$$