Microeconomics III, Ex. Class 7: Problem Set 1[a]

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Department of Economics, University of Copenhagen

[a] slides created by Thor Donsby Noe, adapted for autumn 2022 semester

Outline

Welcome

About me

Malte Jacob Rattenborg
B.Sc.

- *University of Mannheim, Germany*

PhD (4+4) student:

- *Labor Economics, Behavioral Economics & Field Experiments*

My (unexpected) applications of game theory:

- *Bargaining models*
- *Models of bounded rationality*
- *Ex-ante predictions for field experiments*
- *...*

Contact Details:

- *Mail: malterattenborg@econ.ku.dk*
- *Office: 26.1.22*
- *Pigeon Box for Hand-ins: Hall in building 26, right hand-side coming from the main entrance (PhDs section)*
- *Note: Hand-ins of assignments can be done either via e-mail or in my pigeon box*

About you?
Your experience?

Motivation

Relevance of game theory

From the course description:

1. This course furthers the ***introduction of game theory*** and its applications in economic models.
2. The student who successfully completes the course will learn the basics of game theory and will be ***enabled to work further with advanced game theory***.

Relevance of game theory

From the course description:

1. This course furthers the ***introduction of game theory*** and its applications in economic models.
2. The student who successfully completes the course will learn the basics of game theory and will be ***enabled to work further with advanced game theory***.
3. The student will also learn how economic problems involving strategic situations can be ***modeled*** using game theory, as well as how these models are ***solved***.
4. The course intention is that the student ***becomes able to work with*** modern economic theory, for instance within the areas of industrial organization, macroeconomics, international economics, labor economics, public economics, political economics and financial economics.

Game Theory in the History of Economics

- Adam Smith: "By pursuing his own interest he frequently promotes that of the society more effectually than when he really intends to promote it." (Smith, 1887, p.184)
- First fundamental theorem of welfare economics: In an economic equilibrium with perfect competition, the allocation of goods will be pareto-optimal. (Walras, 1870, Edgeworth 1881, Pareto, 1906 and many others)
- John von Neumann: laid the mathematical groundwork for game theory (Von Neumann, 1959)
- John von Neumann, Oskar Morgenstern, John Nash: Made game theory applicable to economic problems (Von Neumann and Morgenstern, 1947; Nash, 1951)

$\rightarrow$ **Was Adam Smith right?**[1]

[1] For further interest: https://towardsdatascience.com/game-theory-history-overview-5475e527cb82

Overview of the course

Overview of the course

Form:

- 12 lectures + conclusion
- 12 problem sets in 12 exercise classes
- 3 mandatory assignments delivered via e-mail or in my pigeon box in the hall of building 26 no later than: Sep. 30, Oct. 28, Nov. 25 (Fridays)
- Note: Game theory is quite tedious to do in LaTeX. Feel free to write on paper to save you some effort. Please write readable :)

Course content

1. Static games with complete information (PS 1-3)
2. Dynamic games with complete information (PS 4-6)
3. Static games with incomplete information (PS 3, 8-10)
4. Dynamic games with incomplete information (PS 6-7, 10-11)
5. Psychological Game Theory (PS 12)

Exam

Exam content:

- ***Cook-book solutions*** from the problem sets
- ***Reflection*** and ***discussion***

Important to complete all problem sets - but also to attend the lectures and read the curriculum:

- Exercises will provide cook book solutions to different problems
- Many students struggle with: ***interpret, explain, and give examples*** from the real world

Concept of Exercises

Content of Exercises:

- Cookbook solutions
- Intuition behind models and real-world examples → These skills are essential for the exam

Structure of Exercises:

- There are A & B questions. A questions should be solved at home, B questions will be solved in class
- You will be given time to try B questions in class. Afterwards we will go through solutions together
- The exercises prepare you for the exam → practice of solution concepts + discussion of intuitions behind models

Your Participation:

- Read through the lecture slides
- Prepare A questions at home, read through B questions
- Participate during the exercise → It is essential to ask questions and participate
- The slides will be uploaded → I would advice you to take notes when we discuss intuitions + when we solve difficult questions together on the whiteboard

PS1, Ex. 1 + 2: Basics of game theory

PS1: Exercise 1

Have a look a the syllabus and flip through the Gibbons book. Re-read the learning outcomes for the course.

(a) What are the main two dimensions this course is structured by?

(b) In your own words, what are the learning outcomes you would need to achieve for an excellent completion of this course?

PS1: Exercise 1 - Solution

(a) What are the main two dimensions this course is structured by?

1. Static games with complete information (PS 1-3)
2. Dynamic games with complete information (PS 4-6)
3. Static games with incomplete information (PS 3, 8-10)
4. Dynamic games with incomplete information (PS 6-7, 10-11)
5. Psychological Game Theory (PS 12)

(b) In your own words, what are the learning outcomes you would need to achieve for an excellent completion of this course?

Learning outcome

From the course description:

Knowledge:

1. Formally ***state the definition*** of a game and ***explain*** the key differences between games of different types.
2. ***In detail account for*** the equilibrium (solution) concepts that are relevant for these games (Nash Equilibrium, Subgame Perfect Nash Equilibrium, Bayes-Nash Equilibrium, Perfect Bayesian Equilibrium).
3. ***Identify*** a number of special games and particular ***issues*** associated with them, such as repeated games (including infinitely repeated games), auctions and signaling games.

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3. ***Identify*** a number of special games and particular ***issues*** associated with them, such as repeated games (including infinitely repeated games), auctions and signaling games.

Skills:

1. Explicitly ***solve*** for the equilibria of these games.
2. ***Explain*** the relevant steps in the reasoning of the solution.
3. ***Interpret*** the outcomes of the analysis.
4. Apply equilibrium ***refinements*** and ***discuss*** the solution concepts

Competencies:

1. ***Analyze*** strategic situations by ***modeling*** them as formal games.
2. ***Set up, prove, analyze and apply*** the theories and methods used in the course in an ***independent*** manner.
3. ***Evaluate and discuss*** the crucial assumptions underlying the theory.

PS1: Exercise 2

Write down three real-world situations for which you think game theory could be useful for and briefly explain why. Choose different ones than what you saw in the lecture.

- Game theory is useful in complex situations of cooperation or competition. Analyzing these situations through modelling helps us the find an unambigious solutions using logic

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Examples:

1. Bargaining with your boss about your salary
2. Deciding on which university to go to
3. Discussing with your friends where to go tonight
4. Deciding on how much to bid in an auction
5. ...

PS1, Ex. 3 + 4: Ingredients of a game

PS1: Exercise 3

What is game theory and why do we do it? To answer this, briefly discuss the following questions:

(a) Write down the definition of game theory
(b) Why do you think it is practical to analyze problems as games?
(c) How do we analyze games?
(d) What are the ingredients of a (normal form) game?

Please take 15 min. to work on Ex. 3 & Ex. 4

PS1: Exercise 3 - Solution

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Answers:

(a) Defintion of Game Theory
- "The study of mathematical models of conflict and cooperation between intelligent rational decision-makers" (R. Myerson)
- "The study of multi-person decision problems" (R. Gibbons)

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(c) Define a solution concept, state the assumptions it relies on, and its possible limitations.
E.g. Iterative Elimination of Strictly Dominated Strategies (IESDS) requires common knowledge of rationality but is not always sufficient to find the Nash Equilibrium.

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E.g. Iterative Elimination of Strictly Dominated Strategies (IESDS) requires common knowledge of rationality but is not always sufficient to find the Nash Equilibrium.

(d) A normal form game consists of:
1. The set of players i
2. The possible strategy sets $S_i \in \{s_1, s_2, \ldots, s_n\}$ for each player i
3. Each players utility (payoff) function $u_i(s_1, s_2, \ldots, s_n)$

PS1, Ex. 4: Iterative Elimination of Strictly Dominated Strategies (IESDS)

PS1: Exercise 4

Solve this game by Iterative Elimination of Strictly Dominated Strategies (IESDS):

	t_1	t_2	t_3
s_1	5, 0	3, 3	1, 1
s_2	3, 4	2, 2	3, 1
s_3	2, 2	1, 1	0, 5

PS1: Exercise 4 - Solution

Solve this game by Iterative Elimination of Strictly Dominated Strategies (IESDS):

	t_1	t_2	t_3
s_1	5, 0	3, 3	1, 1
s_2	3, 4	2, 2	3, 1
s_3	2, 2	1, 1	0, 5

Player 1: s_3 is strictly dominated by s_1 as well as s_2 and can be eliminated, giving us the reduced form game ($s_1 \succ_1 s_3$, $s_2 \succ_1 s_3$):

	t_1	t_2	t_3
s_1	5, 0	3, 3	1, 1
s_2	3, 4	2, 2	3, 1

PS1: Exercise 4 - Solution

Solve this game by Iterative Elimination of Strictly Dominated Strategies (IESDS):

	t_1	t_2	t_3
s_1	5, 0	3, 3	1, 1
s_2	3, 4	2, 2	3, 1
~~s_3~~	~~2, 2~~	~~1, 1~~	~~0, 5~~

Player 1: s_3 is strictly dominated by s_1 as well as s_2 and can be eliminated, giving us the reduced form game:

	t_1	t_2	t_3
s_1	5, 0	3, 3	1, 1
s_2	3, 4	2, 2	3, 1

Player 2: t_3 is strictly dominated by t_2 and can be eliminated ($t_2 \succ_2 t_3$)

Giving us a new reduced form game:

	t_1	t_2
s_1	5, 0	3, 3
s_2	3, 4	2, 2

PS1: Exercise 4 - Solution

Solve this game by Iterative Elimination of Strictly Dominated Strategies (IESDS):

	t_1	t_2	t_3
s_1	5, 0	3, 3	1, 1
s_2	3, 4	2, 2	3, 1
~~s_3~~	~~2, 2~~	~~1, 1~~	~~0, 5~~

Player 1: s_3 is strictly dominated by s_1 as well as s_2 and can be eliminated, giving us the reduced form game:

	t_1	t_2	~~t_3~~
s_1	5, 0	3, 3	~~1, 1~~
s_2	3, 4	2, 2	~~3, 1~~

Player 2: t_3 is strictly dominated by t_2 and can be eliminated.

Giving us a new reduced form game:

	t_1	t_2
s_1	5, 0	3, 3
s_2	3, 4	2, 2

Player 1: s_2 is strictly dominated by s_1 and is eliminated. Reduced form game ($s_1 \succ_1 s_2$):

	t_1	t_2
s_1	5, 0	3, 3

PS1: Exercise 4 - Solution

Solve this game by Iterative Elimination of Strictly Dominated Strategies (IESDS):

	t_1	t_2	t_3
s_1	5, 0	3, 3	1, 1
s_2	3, 4	2, 2	3, 1
~~s_3~~	~~2, 2~~	~~1, 1~~	~~0, 5~~

Player 1: s_3 is strictly dominated by s_1 as well as s_2 and can be eliminated, giving us the reduced form game:

	t_1	t_2	~~t_3~~
s_1	5, 0	3, 3	~~1, 1~~
s_2	3, 4	2, 2	~~3, 1~~

Player 2: t_3 is strictly dominated by t_2 and can be eliminated.

Giving us a new reduced form game:

	t_1	t_2
s_1	5, 0	3, 3
~~s_2~~	~~3, 4~~	~~2, 2~~

Player 1: s_2 is strictly dominated by s_1 and is eliminated. Reduced form game:

	t_1	t_2
s_1	5, 0	3, 3

Player 2: t_1 is strictly dominated by t_2 and is eliminated. I.e. IESDS provides the unique strategy profile (s_1, t_2), implying that this is also the Nash Equilibrium $(t_2 \succ_2 t_1)$:

	t_2
s_1	3, 3

PS1, Ex. 5: The Travelers' Dilemma

PS1: Exercise 5

The Travelers' Dilemma:

"An airline loses two suitcases belonging to two different travelers. Both suitcases look identical and contain identical items. An airline manager tasked to settle the claims of both travelers explains that the airline is liable for a maximum of $100 per suitcase, and in order to determine an honest appraised value of the antiques the manager separates both travelers so they can't confer, and asks them to write down the amount of their value no less than $0 and no larger than $100. He also tells them that if both write down the same number, he will treat that number as the true dollar value of both suitcases and reimburse both travelers that amount.

However, if one writes down a smaller number than the other, this smaller number will be taken as the true dollar value, and both travelers will receive that amount along with the following: $1 extra will be paid to the traveler who wrote down the lower value and a $1 fine imposed on the person who wrote down the higher amount."

(a) Write down the normal form of this game: players, strategy sets, payoffs

(b) Can you solve this game by IESDS?

(c) What number do you think each traveler will write down? Why? An informal discussion of the reasoning will suffice.

PS1: Exercise 5

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(b) Can you solve this game by IESDS?

(c) What number do you think each traveler will write down? Why? An informal discussion of the reasoning will suffice.

(d) (Discussion Point): Was Adam Smith right?

Please take 20 min. to work on Ex. 5 & Ex. 6

PS1: Exercise 5 - Solution

(a) Write down the normal form of this game: players, strategy sets, payoffs

(b) Can you solve this game by IESDS?

(c) What number do you think each traveler will write down? Why? An informal discussion of the reasoning will suffice.

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(a) A normal form game consists of:

1. The set of players: Traveler 1 and Traveler 2.

PS1: Exercise 5 - Solution

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(a) A normal form game consists of:

1. The set of players: Traveler 1 and Traveler 2.
2. Strategy sets: $S_i = \{0; 0.01; \ldots; 99 : 99; 100\}$ for $i = 1; 2$

PS1: Exercise 5 - Solution

(a) Write down the normal form of this game: players, strategy sets, payoffs

(b) Can you solve this game by IESDS?

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1. The set of players: Traveler 1 and Traveler 2.
2. Strategy sets: $S_i = \{0; 0.01; ...; 99 : 99; 100\}$ for $i = 1; 2$
3. Payoffs for player $i \neq j$:

$$u_i(s_i, s_j) = \begin{cases} s_i & \text{if} \quad s_i = s_j \\ s_i + 1 & \text{if} \quad s_i < s_j \\ s_j - 1 & \text{if} \quad s_i > s_j \end{cases}$$

PS1: Exercise 5 - Solution

(a) Write down the normal form of this game: players, strategy sets, payoffs

(b) Can you solve this game by IESDS?

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(b) No, as no strategy is always dominated by one other strategy no matter what the other traveler plays.

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(b) No, as no strategy is always dominated by one other strategy no matter what the other traveler plays.

(c) Given common knowledge of rationality each traveler will avoid getting "underbid" by the other, i.e. the Nash Equilibrium is $s_i, s_j = (0, 0)$ as there is no incentive to deviate.

PS1, Ex. 6: IESDS

PS1: Exercise 6

Solve these games by iterative elimination of strictly dominated strategies:

	t_1	t_2	t_3
s_1	5, 0	2, 3	1, 1
s_2	2, 4	2, 2	3, 1
s_3	2, 2	1, 1	0, 5

	t_1	t_2	t_3
s_1	5, 0	2, 3	1, 1
s_2	2, 4	2, 2	3, 1
s_3	2, 2	1, 1	1, 5

PS1: Exercise 6 - Solution

Solve these games by iterative elimination of strictly dominated strategies:

	t_1	t_2	t_3
s_1	5, 0	2, 3	1, 1
s_2	2, 4	2, 2	3, 1
~~s_3~~	~~2, 2~~	~~1, 1~~	~~0, 5~~

	t_1	t_2	t_3
s_1	5, 0	2, 3	1, 1
s_2	2, 4	2, 2	3, 1
s_3	2, 2	1, 1	1, 5

1. Player 1: s_3 is strictly dominated by s_1 and is eliminated
2. Player 2: After eliminating s_3, t_3 is strictly dominated by t_2 and is eliminated, giving us the reduced form game:

	t_1	t_2
s_1	5, 0	2, 3
s_2	2, 4	2, 2

PS1, Ex. 7: The higher number wins

PS1: Exercise 7

Mikael and Jonas are playing a game instead of working. The game has the following rules: Both secretly pick a (natural) number between 1 and 5. Then they reveal the numbers to each other. If both have picked the same number, nobody gets anything. If Jonas' number is higher than Mikael's number, Mikael has to pay Jonas 1 kr. If Mikael's number is higher than Jonas', Jonas has to pay 10 kr. to Mikael

(a) Does this game seem fair to you?

(b) Write the game in bimatrix form.

(c) Are there any strictly dominated strategies? Solve the game by iterated elimination of strictly dominated strategies.

(d) What is the outcome of the game if both Mikael and Jonas are rational, know that the other is rational, know that the other knows that they are rational etc.?

Please take 20 min. to work on Ex. 7 & Ex. 8

PS1: Exercise 7 - Solution

(a) Does this game seem fair to you?

(b) Write the game in bimatrix form.

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(a) How do you define "fair"? Symmetric payoffs and/or equal chance of winning?

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(a) How do you define "fair"? Symmetric payoffs and/or equal chance of winning?

(b) In bimatrix form:

	"1"	"2"	"3"	"4"	"5"
"1"	0, 0	-1, 1	-1, 1	-1, 1	-1, 1
"2"	10, -10	0, 0	-1, 1	-1, 1	-1, 1
"3"	10, -10	10, -10	0, 0	-1, 1	-1, 1
"4"	10, -10	10, -10	10, -10	0, 0	-1, 1
"5"	10, -10	10, -10	10, -10	10, -10	0, 0

PS1: Exercise 7 - Solution

(a) Does this game seem fair to you?
(b) Write the game in bimatrix form.
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(d) What is the outcome of the game if both Mikael and Jonas are rational, know that the other is rational, know that the other knows that they are rational etc.?

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"3"	10, -10	10, -10	0, 0	-1, 1	-1, 1
"4"	10, -10	10, -10	10, -10	0, 0	-1, 1
"5"	10, -10	10, -10	10, -10	10, -10	0, 0

(c) In the initial game playing "2", "3", or "4" is only weakly dominated by playing "5". However, "1" is strictly dominated by "5". After removing "1" for each player, "2" is strictly dominated by "5" and is removed for each player as well. Continue till only ("5","5") survive IESDS.

PS1: Exercise 7 - Solution

(a) Does this game seem fair to you?
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(d) What is the outcome of the game if both Mikael and Jonas are rational, know that the other is rational, know that the other knows that they are rational etc.?

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"2"	10, -10	0, 0	-1, 1	-1, 1	-1, 1
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"4"	10, -10	10, -10	10, -10	0, 0	-1, 1
"5"	10, -10	10, -10	10, -10	10, -10	0, 0

(c) In the initial game playing "2", "3", or "4" is only weakly dominated by playing "5". However, "1" is strictly dominated by "5". After removing "1" for each player, "2" is strictly dominated by "5" and is removed for each player as well. Continue till only ("5","5") survive IESDS.

(d) Common knowledge of rationality should lead the players to play the IESDS solution("5"; "5"). They can use common knowledge of rationality to see that "1" is not a rational strategy for their counterpart, thus allowing them to see the "2" is also strictly dominated for themselves, etc.

PS1, Ex. 8: Three player game

PS1: Exercise 8

We can also write games with more than two players. Consider the game below where player 1 chooses the bi-matrix (A or B), player 2 chooses the row (C or D), and player 3 chooses the column (E or F). In each cell, the first number gives the payoff of Player 1, the second number the payoff of Player 2, and the third number the payoff of Player 3.

	E	F
C	0, 2, 2	2, 1, 1
D	0, 1, 1	3, 0, 0

A

	E	F
C	1, 0, 1	3, 1, 2
D	1, 1, 0	5, 2, 1

B

PS1: Exercise 8 - Solution

	E	F
C	0, 2, 2	2, 1, 1
D	0, 1, 1	3, 0, 0

A

	E	F
C	1, 0, 1	3, 1, 2
D	1, 1, 0	5, 2, 1

B

1st step: Player 1: A is strictly dominated by B, thus, matrix A can be eliminated:

	E	F
C	1, 0, 1	3, 1, 2
D	1, 1, 0	5, 2, 1

B

PS1: Exercise 8 - Solution

	E	F
C	0, 2, 2	2, 1, 1
D	0, 1, 1	3, 0, 0

A

	E	F
C	1, 0, 1	3, 1, 2
D	1, 1, 0	5, 2, 1

B

1^{st} step: Player 1: A is strictly dominated by B, thus, matrix A can be eliminated:

	E	F
C	1, 0, 1	3, 1, 2
D	1, 1, 0	5, 2, 1

B

2^{nd} step: Player 2: After matrix A is eliminated, C is strictly dominated by D and we eliminate C.

PS1: Exercise 8 - Solution

	E	F
C	0, 2, 2	2, 1, 1
D	0, 1, 1	3, 0, 0

A

	E	F
C	1, 0, 1	3, 1, 2
D	1, 1, 0	5, 2, 1

B

1st step: Player 1: A is strictly dominated by B, thus, matrix A can be eliminated:

	E	F
~~C~~	~~1, 0, 1~~	~~3, 1, 2~~
D	1, 1, 0	5, 2, 1

B

2nd step: Player 2: After matrix A is eliminated, C is strictly dominated by D and we eliminate C.

3rd step: Player 3: After matrix A and row C is eliminated, E (payoff $= 0$) is strictly dominated by F (payoff $= 1$) and we eliminate E.

The unique pure strategy profile that survives IESDS is $(B; D; F)$.

Closing Remarks

- We have a replacement class on the 23.09 (Friday) at 12-15 in this room
- For those who cannot make it, please have a look at my solution slides.
- I will stay a bit longer after the next exercise class, so you can ask questions if anything is unclear

Closing Remarks

- We have a replacement class on the 23.09 (Friday) at 12-15 in this room
- For those who cannot make it, please have a look at my solution slides.
- I will stay a bit longer after the next exercise class, so you can ask questions if anything is unclear
- The first mandatory assignment is due on Sep. 30, 12 pm (noon). Please start having a look at this.

References

References

Nash, John (1951). "Non-cooperative games". In: Annals of mathematics, pp. 286–295.

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