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Early Exercise of Put Options on Stocks

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ABSTRACT

U.S. exchange-traded stock options are exercisable before expiration. While put options should frequently be exercised early to earn interest, they are not. In this paper, we derive an early exercise decision rule and then examine actual exercise behavior during the period January 1996 through September 2008. We find that more than 3.96 million puts that should have been exercised early remain unexercised, representing over 3.7% of all outstanding puts. We also find that failure to exercise cost put option holders $1.9 billion in forgone interest income and that this interest is systematically captured by market makers and proprietary firms.

STOCK OPTIONS TRADED ON exchanges in the U.S. may be exercised before contract expiration. While early exercise decisions by option holders have been examined empirically in a number of studies, most have focused on call options. The decision about whether to exercise a call early depends on the amount of any cash dividends paid during the option's life. Since exchange-traded options are unprotected from cash dividend payments on the underlying stock, holders of deep in-the-money calls may find it optimal to exercise their positions just prior to the ex-dividend day when the stock price drops by the amount of the cash dividend. If no cash dividends are paid, early exercise is suboptimal. In the most comprehensive investigation of call option early exercise behavior to date, Pool, Stoll, and Whaley (2008) find that, during the period January 1996 through September 2008, more than half of outstanding long call option positions that should be exercised on the day before the ex-dividend day went unexercised. They estimate that this failure to exercise has caused call option holders to lose over $491 million over the 10-year period. They attribute about half of the loss to transaction costs, but suggest that a substantial portion is attributable to call option holders' lack of awareness of the early exercise decision, inability to continually monitor their option positions, or irrationality.

Pool, Stoll, and Whaley (2008) focus on the exercise activity of call options on ex-dividend days. Other studies examine early exercise behavior of call options on days other than ex-dividend days—days on which early exercise is not rational. Poteshman and Serbin (2003), for example, investigate early

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exercises of Chicago Board Options Exchange (CBOE) call options and find that, while a large number of early exercises are irrational, the exercise decision varies with the type of investor. In particular, they show that customers of full-service and discount brokers exhibit irrational exercise behavior, while traders in large investment houses do not. Overdahl and Martin (1994) and Finucane (1997) also investigate the exercise behavior of stock option holders and find instances in which early exercise appears irrational.

The early exercise behavior of holders of long put options on stocks, on the other hand, has gone largely unexplored.[1] The reason is that the exercise decision is more complicated. Call options may be optimally exercised early only if the stock pays a cash dividend during the option's life, and even then only on the day before the ex-dividend day. Put options, in contrast, may be optimally exercised early for both dividend-paying and non-dividend-paying stocks, and on almost any day prior to the option's expiration day. The intuition underlying the put option early exercise decision is as follows. A deep in-the-money put has no time value remaining and is priced at its floor value. Upon exercise, the put option holder receives the exercise price in cash. Each day the put option holder defers exercising the deep in-the-money put, he forgoes the interest income that can be earned on the cash proceeds, but retains an option to exercise the put on the following day. The difference between forgone interest income and the value of future exercise opportunities determines whether the put should be exercised early or not. In this paper, we formalize this intuition and develop a rule for deciding whether to exercise the put early. Using a sample of put options on stocks during the period January 1996 through September 2008, we show that more than 3.96 million put options, or over 3.7% of all put options outstanding, are not exercised when it is optimal to do so and that the failure to exercise cost long put option holders over $1.9 billion.

The failure of long put option holders to exercise early has given rise to a trading game. Since the interest income being forfeited by long put option holders is being earned by short put option holders, the game involves capturing short open interest. The game, dubbed "short stock interest arbitrage," involves simultaneously buying and selling a large (relative to existing open interest), but equal, number of deep in-the-money puts and then immediately exercising the long puts. Since exercises are randomly assigned to open short positions, the arbitragers systematically capture the dominant share of the total short open interest and thereby earn the dominant share of the forfeit interest. Using actual exercise data, we document short stock interest arbitrage activity.

[1] To our knowledge, only two published studies have appeared and the generality of their results is limited. Overdahl and Martin (1994) use a relatively small sample of only 54 option classes over the period July 1990 to March 1991, and Engström, Nordén, and Strömberg (2000) investigate the early exercise of American-style put options in the Swedish stock option market. There is also an unpublished note by Duffie, Liu, and Poteshman (2005). On the other hand, a number of studies explore exercise activity of non-stock put options. Diz and Finucane (1993) consider early exercise of both puts and calls on the S&P 100 index, and Overdahl (1988) and Gay, Kolb, and Yung (1989) investigate exercise activity for T-bond futures options.

The purpose of this study is to develop an understanding of the exercise behavior of put option holders in U.S. stock option markets. In Section I, we begin with a formal description of the early exercise decision faced by the put option holder. We show how the critical stock price below which a put should be exercised immediately can be determined and define the concept of net interest income. We also show how cash dividend payments affect the early exercise policy of put option holders and the optimal timing of exercise decisions. Section II describes the data that form the basis of our analyses. Sections III and IV contain the main empirical results of the paper. In Section III, we examine actual early exercise decisions using plausible exercise cost assumptions. In Section IV, we discuss the short stock interest arbitrage used by market makers and proprietary traders to capture interest income on unexercised put option positions. Section V contains a summary and the main conclusions.

I. Early Exercise of Put Options on Stocks

The decision to exercise an American-style call option on a stock early is relatively straightforward. If the stock pays a dividend during the call's life, holders of deep in-the-money call option positions may find it optimal to exercise just prior to the ex-dividend day because of the impending stock price decline. Otherwise, early exercise is suboptimal. The decision to exercise an American-style put option on a stock early is less obvious. It may occur at almost any time, even for stocks that pay no dividends. In this section, we explain the economic motivation for exercising an American-style put option before expiration and derive an analytical decision rule for determining when to do so.

To understand the intuition for the early exercise of American-style puts, consider first a "quasi-American-style put" that can be exercised only at two points in time—immediately or at expiration. Assuming the underlying stock pays no dividends during the option's life, the critical stock price at which the put holder is indifferent about when to exercise, S_E^*, can be determined by equating the early exercise proceeds of the put to its value if left unexercised, that is,

$$X - S_E^* = p\left(S_E^*, X, T\right), \tag{1}$$

where $p(S_E^*, X, T)$is the value of a European-style put option with exercise price X and time to expiration T. If the current stock price is below S_E^*, the early exercise proceeds of the put exceed its value left alive,

$$X - S - p(S, X, T) > 0, \tag{2}$$

as is illustrated in Figure 1 in the region to the left of S_E^*, and the put should be exercised immediately. Conversely, if the current stock price is above S_E^*, the put is worth more alive than dead and exercise should be deferred to expiration.

An equivalent, albeit less obvious, early exercise decision rule to equation (2) can be developed by substituting European-style put-call parity for the value

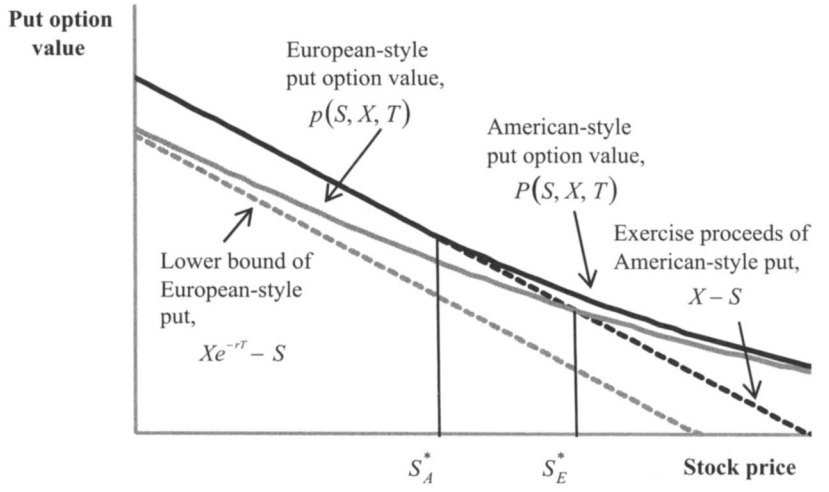

Figure 1. European- and American-style put option values as a function of stock price.

of the put. Rearranging, the net benefit from early exercise is

$$X(1 - e^{-rT}) - c(S, X, T) > 0. \tag{3}$$

Condition (3) says that, if the present value of the interest income that can be earned on the exercise price over the remaining life of the put exceeds the value of the corresponding call option, immediate exercise is optimal. The intuition underlying equation (3) is as follows. Assuming the stock price falls immediately to zero and stays there, the call is worthless and the benefit from immediately exercising the put reaches its maximum possible level, that is, the present value of the interest income on the exercise price over the remaining life of the put, $X(1 - e^{-rT})$. In general, however, the current stock price is nonzero and uncertain, and the call has some prospect of going in the money by expiration. The net benefit from exercising the put early is, therefore, the difference between the maximum interest income that can be earned and the value of the corresponding call.[2] The net benefit becomes smaller and smaller as the stock price approaches the critical stock price from below.

To develop the early exercise decision rule for an actual American-style put, we must solve for the critical stock price below which it is optimal to exercise the put early using

$$X - S_A^* = P(S_A^*, X, T), \tag{4}$$

[2] An ad hoc decision rule for exercising a deep in-the-money put is to do so when the present value of the interest on the exercise price over the remaining life of the option exceeds the price of the corresponding call. See http://www.asx.com.au/ products/options/trading_information/early_exercise.htm.

where $P(S_A^*, X, T)$ is the value of an American-style put. Unlike the solution for equation (1), equation (4) is satisfied by a range of stock prices for which the delta of the put on the right-hand side equals -1. The solution of interest, of course, is the highest level of stock price for which equation (4) is satisfied. Note that, since the right-hand side of equation (4) recognizes all possible early exercise opportunities rather than just one, $P(S, X, T) > p(S, X, T)$ and $S_A^* < S_E^*$ as is shown in Figure 1. But, these differences aside, the basic intuition underlying equations (1) and (4) is the same. On the left is the immediate exercise proceeds. On the right is the present value of the expected value of the put at the next available exercise opportunity. For equation (1), the next opportunity is at expiration. For equation (4), the next opportunity is the next instant in time.

To isolate the net benefit from early exercise of an American-style put, we assume that the early exercise decision made by the long put option holder (hereafter, the "long") is to either exercise the put immediately or defer the decision for one more trading day. Under this assumption, the net benefit from early exercise at the end of the next trading day may be written

$$(X - S)e^{r\Delta t} - \max\left[P(S_{\Delta t}, X, T - \Delta t), X - S_{\Delta t}\right], \tag{5}$$

where Δt is the number of calendar days between adjacent trading days.[3] The first term in equation (5) is the immediate exercise proceeds carried forward for one trading day at the risk-free interest rate. The second term is the value of the open put position left unexercised. Note that the second term explicitly recognizes that the stock price may rise above the critical stock price by the end of the next trading day. If so, the put is worth more alive than dead. To isolate the interest income component in equation (5), re-write the expression as

$$(X - S)e^{r\Delta t} - (X - S_{\Delta t}) - \max\left[P(S_{\Delta t}, X, T - \Delta t) - (X - S_{\Delta t}), 0\right]. \tag{6}$$

Note that the third term in equation (6) is the payoff of a call option whose terminal value at the end of one day is

$$c_{\Delta t} = \begin{cases} P(S_{\Delta t}, X, T - \Delta t) - (X - S_{\Delta t}) & \text{if} \quad S_{\Delta t} > S_{\Delta t}^* \\ 0 & \text{if} \quad S_{\Delta t} \le S_{\Delta t}^* \end{cases}. \tag{7}$$

Applying risk-neutral valuation, the present value of the long's expected net benefit from early exercise over the next trading day is

$$NII = X\left(1 - e^{-r\Delta t}\right) - c(P_{\Delta t}, X - S_{\Delta t}, \Delta t), \tag{8}$$

where $S = e^{-r\Delta t}E(S_{\Delta t})$ and $c(P_{\Delta t}, X - S_{\Delta t}, \Delta t)$ is the value of a one-day European-style call option on an American-style put (dubbed a "caput") with exercise price $X - S_{\Delta t}$. We define equation (8) as "net interest income." If $NII > 0$,

[3] Interest is earned on a daily basis, so, typically, the time increment is either one day or three days depending upon whether it falls within the trading week or over the weekend. Where necessary, we also adjust the time increment for holidays.

Table I

Illustration of Net Benefit of Early Exercise for an American-Style Put Option on a Non-Dividend-Paying Stock

The put option is assumed to have an exercise price of 50 and 30 days remaining to expiration. The stock volatility rate is 30%, and the risk-free interest rate is 5%.

Critical stock price	42.95
Present value of maximum interest income over life of option	0.20506
Present value of maximum daily interest income	0.00685

	Daily Early Exercise Opportunities			
	Put Value			
Stock Price	With Exercise Today	With Exercise Tomorrow	Value of Call on Put	Net Interest Income
37.50	12.50000	12.49315	0.00000	0.00685
38.00	12.00000	11.99315	0.00000	0.00685
38.50	11.50000	11.49315	0.00000	0.00685
39.00	11.00000	10.99315	0.00000	0.00685
39.50	10.50000	10.49315	0.00000	0.00685
40.00	10.00000	9.99315	0.00000	0.00685
40.50	9.50000	9.49315	0.00000	0.00685
41.00	9.00000	8.99315	0.00000	0.00685
41.50	8.50000	8.49320	0.00005	0.00680
42.00	8.00000	7.99361	0.00046	0.00639
42.50	7.50000	7.49549	0.00234	0.00451

the American-style put option should be exercised immediately. Failure to do so implies that the long chooses to forfeit (whether deliberately or not) today's *NII* to the short put option holder (hereafter, the "short"), only to face the same decision again tomorrow.

The economic intuition underlying equation (8) is easiest to understand from the perspective of someone who is short a deep in-the-money put and short the stock. If the long chooses to exercise early when it is optimal to do so, he receives X in cash and delivers the stock. The short is then assigned the exercise, pays X in cash, receives the stock, and covers his short stock position. If the long fails to exercise, the short defers the payment of X and thereby earns one day's interest income (i.e., the first term on the right-hand side of equation (8)). Since the put is deep in the money, the short put/short position is riskless for small changes in stock price (i.e., the deltas of the put and the stock positions sum to zero). If the stock price spikes upward, however, the put's delta rises above -1 and the hedge loses value. The second term on the right-hand side (i.e., the caput) is the present value of the expected cost of this contingency. Hence, we defined equation (8) to be *net* interest income.

The computation of the net interest income is illustrated in Table I. In the table, the put is assumed to have an exercise price of 50 and 30 days remaining to expiration. The stock underlying the put pays no dividends and the risk-free

interest rate is 5% on an annualized basis. The present value of the maximum interest income that can be earned over the life of the option is $50(1 - e^{-.05(30/365)})$, or 0.20506. The present value of the maximum daily interest income is $50(1 - e^{-.05(1/365)})$, or 0.00685. Note that $0.00685 \sum_{i=0}^{29} e^{-r(i/365)} = 0.20506$.

Next, we determine the critical stock price below which the put should be exercised immediately. While S_A^* can be determined under a variety of assumptions regarding the underlying stock price process, we use the Black-Scholes (1973)/Merton (1973) assumption of geometric Brownian motion. To value the put in equation (4), an option valuation approximation method is required. We use the Cox, Ross, and Rubinstein (1979) (hereafter, "CRR") binomial method.[4] Assuming the stock has a volatility rate of 30%, the critical stock price is 42.95, as is reported in Table I. Recall the solution to equation (4) is not unique. The *highest* level of stock price that satisfies equation (4) is 42.95.

In the main panel of Table I are option values at different levels of stock price. Since all of the assumed stock prices in the table are below the critical price, $S_A^* = 42.95$, we know that the put should be exercised immediately. Thus, all of the put option values reported in the column headed "With exercise today" equal their exercise proceeds. The adjacent column to the right is the present value of the American-style put whose first available exercise opportunity is in one day.[5] These values are uniformly less than the actual put value since the right to exercise immediately has been forfeited. The difference between the two values equals the difference between the present value of the maximum daily interest income and the value of the caput as shown in equation (8). For low levels of stock price, the value of the caput is negligible and the net interest income is the present value of the maximum daily interest income, 0.00685. The reason is simple. Since the put should optimally be exercised both today and tomorrow, the difference in the put values today is simply $X - S - e^{-r\Delta t}(X - E(S_{\Delta t}))$. Since under risk-neutral valuation $S = e^{-r\Delta t}E(S_{\Delta t})$, the net benefit from early exercise is $X(1 - e^{-r\Delta t})$. Indeed, looking down the column, the value of the caput remains zero until the stock price rises to a level close to the critical stock price. The first nonzero call option value to appear in Table I, 0.00005, is at a stock price level of 41.50. The rightmost column in the table is the net benefit from early exercise, or, equivalently, the net interest income earned by the short if the long fails to exercise today. It can be determined by taking the present value of the expected value of equation (5) or by (8). At a stock price level of 42.50, the former method is computed as $7.50000 - 7.49549 = 0.00451$. Under the latter method, the computation is $0.00685 - 0.00234 = 0.00451$. At the critical stock price 42.95, net interest income is zero. Recall

[4] The CRR binomial method is modified to account for cash dividend payments using the procedure described in Harvey and Whaley (1992).

[5] Mechanically, this put value is computed using a three-step procedure. First, we create a stock price lattice with 10 time steps over the next day. Second, for each of the terminal stock price nodes, we compute the value of an American-style put with 29 days remaining to expiration. Third, we discount these put price nodes back to the present in the usual manner.

that we showed earlier that an early exercise condition equivalent to $S < S_A^*$ is $NII > 0$.

Finally, cash dividends on the stock underlying the put affect the early exercise decision. While it may be optimal to exercise an American-style put on a non-dividend-paying stock at any time during its life, the same is not true for a put on a dividend-paying stock. The reason is straightforward. With an impending cash dividend, the anticipated stock price drop on the ex-dividend day may exceed the maximum interest income that can be earned on the exercise price between now and the ex-dividend day. If

$$D_t > X\left(e^{rt} - 1\right), \tag{9}$$

where t is the time to ex-dividend measured in years, early exercise is suboptimal. In such cases, the critical stock price below which the put option should be exercised immediately is zero. Equivalently, in an efficiently functioning option market, the put option price will exceed its immediate exercise proceeds.

II. Data and Sample Attributes

The empirical analyses in this study are based on all exchange-traded stock options traded in the U.S. during the period January 1996 through September 2008. The option data are drawn from OptionMetrics. The file includes closing bid-ask price quotes, trading volume, and closing open interest for each option series each day. Of particular importance is open interest information.[6] For put options that should be exercised immediately, this number measures failure to exercise. Closing stock price and dividend data are also from OptionMetrics. Closing bid-ask quotes for stocks are drawn from the Center for Research in Security Prices (CRSP) daily files.[7] The proxy for the risk-free interest rate is based on the zero-coupon yield curve of rates for overnight, 7-day, 30-day, 90-day, 180-day, and 1-year euro time deposits downloaded from Datastream. In addition to the information described above, an estimate of the expected future volatility rate is required to implement the early exercise decision rule. We use the historical volatility over the 60 trading days prior to the valuation date. Since OptionMetrics security data do not begin until January 1996, we use CRSP daily data to estimate historical volatility so as not to lose option classes during the first 60 trading days of 1996. We also use CRSP daily data to determine the closing bid-ask midpoint for each stock. To proxy for the amount and timing of the expected dividends paid during an option's life, we use the actual dividend payments. Naturally, the expected dividend stream is adjusted for any stock splits and stock dividends paid during the option's life. Stock split and stock dividend information come from the CRSP files.

[6] Users of OptionMetrics should note that, on and after November 28, 2000, the open interest reported for a particular option series at the end of day t is actually the open interest for the series at the end of day $t - 1$. Before November 28, 2000, the open interest reported for day t is correct.

[7] Where the CRSP daily file has missing bid-ask quotes, NYSE TAQ data are used.

Table II

Total Trading Volume of Calls, Puts, and Underlying Stocks during the Sample Period January 1996 through September 2008

To compute option volume to stock volume (i.e., relative volume), option volume is first multiplied by 100 (i.e., number of shares per contract). The number of option classes across years is 5,571.

Year	Number of Option Classes	Number of Option Contracts Traded		Number of Shares Traded	Relative Volumes		
		Calls	Puts		Calls to Stock (%)	Puts to Stock (%)	Calls to Puts
1996	2,030	126,271,114	52,695,360	175,415,452,204	7.2	3.0	2.396
1997	2,449	171,588,068	75,467,082	233,910,016,509	7.3	3.2	2.274
1998	2,709	206,253,694	94,104,381	305,623,899,940	6.7	3.1	2.192
1999	2,834	282,785,074	121,427,112	392,214,762,268	7.2	3.1	2.329
2000	2,766	396,520,175	196,020,627	593,363,325,833	6.7	3.3	2.023
2001	2,465	367,430,328	232,913,377	703,050,122,048	5.2	3.3	1.578
2002	2,299	335,964,811	229,320,023	720,160,681,678	4.7	3.2	1.465
2003	2,114	328,547,319	183,910,007	525,500,049,555	6.3	3.5	1.786
2004	2,313	527,925,596	312,652,804	733,334,390,539	7.2	4.3	1.689
2005	2,455	655,391,657	399,207,759	777,544,966,779	8.4	5.1	1.642
2006	2,663	847,756,208	531,267,614	916,929,217,958	9.2	5.8	1.596
2007	2,853	1,050,084,013	721,512,010	1,129,596,739,877	9.3	6.4	1.455
2008	2,800	855,788,882	691,678,594	1,123,220,207,306	7.6	6.2	1.237
Mean					7.2	4.1	1.820

Relatively few exclusionary criteria are applied to the data. First, for an option class to be included in the sample, it must trade at some time during the sample period. Second, we exclude options whose underlying stock experiences an unusual corporate event that alters the terms of the option contract during its life. Such events include capital gains distributions, special dividends, spin-offs, new equity issues, right offerings, and warrant issues.[8] After the exclusionary criteria are applied, the total number of option classes in the sample is 5,571.

Table II summarizes the trading volume of calls, puts, and their underlying stocks in our sample by calendar year. Generally speaking, neither calls nor puts trade nearly as actively as their underlying stocks. For the overall sample, the average call option trading volume is 7.2% of stock trading volume. For puts, the average is 4.1%. These levels also reflect the fact that, on average, call options are 82.0% more actively traded than puts.[9] The asymmetry between

[8] For an example of how corporate actions may affect option valuation, see Barraclough, Stoll, and Whaley (2012).

[9] This asymmetry is just the opposite of that typically observed for S&P 500 index options, where put volume generally surpasses call volume. During the first 10 months of 2008, for example, S&P 500 put option trading volume exceeded call option volume by more than 70%. See http://www.optionsclearing.com/market/volume/volbyproduct_form.jsp. This asymmetry is driven largely by institutional demand for portfolio insurance. Bollen and Whaley (2004) show how net buying pressure on S&P 500 puts affects implied volatilities (option prices). Gârleanu, Pederson,

call and put stock option trading volume has become less in recent years, however. In 2008, for example, call activity surpassed put activity by only 23.7%. Another way of showing the increased interest in trading puts is by comparing put activity to stock activity. In 1996, puts were about 3.0% as active as stocks. By 2008, the relative activity had more than doubled to 6.2%.

For the subperiod July 2001 through September 2008, we also analyze actual exercise behavior for 4,011 stock option classes.[10] The Options Clearing Corporation (OCC) provided us data on the number of contract exercises for each option series each day. The exercise data are classified by customer (C), market maker (M), and firm (F), which makes it possible to observe the behavior of different market participants. The customer category consists of retail traders and hedge funds. Retail customers use brokerage services ranging from discount to full-service brokers. Hedge funds often operate through a prime broker. Although commission rates are higher, they are provided soft dollar services such as access to data, trading software, and so on. The firm category refers to proprietary trading by various financial institutions.

Table III summarizes the exercise behavior of call and put options for the three different classes of market participants during the subperiod July 2001 through September 2008. Calls are reported in Panel A. In total, over 671 million contracts are exercised during the sample period. Of those, 58.4% occurred at expiration and 41.6% occurred prior to expiration. As noted earlier, the only time other than at expiration when it may be optimal to exercise a call is on the day before the stock goes ex-dividend. The table shows that 34.1% of the contracts exercised occur on the day before the ex-dividend day. Of the market participants, market makers seem to be the most expert at determining whether early exercise is optimal,[11] and account for 70.9% of all call option exercises on the day before the ex-dividend day. Customers account for a meager 3.1%. This evidence is consistent with Pool, Stoll, and Whaley (2008). The total call option exercises on days other than the day before ex-dividend or at expiration, 7.5%, remains substantial, however. These exercises are irrational and are the focus of Poteshman and Serbin (2003).

Put option exercise activity is reported in Panel B of Table III. In total, 492 million contracts are exercised during the sample period. Of those, 52.2% occur at expiration and 47.8% occur prior to expiration. Dividend-motivated

and Poteshman (2009) use a proprietary data set to document the open interest of S&P call and put option series by trader type.

[10] The OCC was unable to recover the historical exercise data for the months of November 2001, January and July 2002, and January 2006. Since exercise data before July 2001 are unavailable, the number of stock option classes is fewer.

[11] The number of call options exercised on the day before ex-dividend is artificially inflated. Pool, Stoll, and Whaley (2008) show that the failure of call option holders to exercise their positions has given rise to dividend spread arbitrage, which produces abnormally high levels of trading volume and exercise activity around ex-dividend days. This strategy involves arbitragers simultaneously buying and selling in-the-money call options on the day before ex-dividend. At the end of the day the arbitrager exercises the long position and hopes to capture a portion of the open interest when some call option holders fail to exercise. When the stock goes ex-dividend the following day, the arbitrager experiences a windfall gain on the short position.

Table III

Number of Call and Put Option Contracts Exercised by Customers, Market Makers, and Proprietary Firms during the Sample Period July 2001 through September 2008

Contracts exercised are classified by market participant, and the classifications are customer, market maker, and proprietary firm. The number of option classes is 4,011. The months of November 2001, January and July 2002, and January 2006 are missing.

Panel A: Total Number of Call Option Contracts Exercised

	Other Exercises		On Day before Ex-Dividend		At Expiration		Total
Participant	Number	Percent	Number	Percent	Number	Percent	Exercises
Customer							
Number	12,561,557	9.3%	7,126,044	5.3%	114,691,451	85.3%	134,379,052
Percent (%)	24.8		3.1		29.3		20.0
Market maker							
Number	30,499,061	8.0%	162,128,389	42.3%	190,458,556	49.7%	383,086,006
Percent (%)	60.2		70.9		48.6		57.1
Proprietary firm							
Number	7,561,972	4.9%	59,278,677	38.6%	86,610,004	56.4%	153,450,653
Percent (%)	14.9		25.9		22.1		22.9
All	50,622,590	7.5%	228,533,110	34.1%	391,760,011	58.4%	670,915,711

Panel B: Total Number of Put Option Contracts Exercised

	Other Exercises		On Day before Ex-Dividend		At Expiration		Total
Participant	Number	Percent	Number	Percent	Number	Percent	Exercises
Customer							
Number	27,351,684	24.5%	954,739	0.9%	83,390,042	74.7%	111,696,465
Percent (%)	12.0		11.9		32.5		22.7
Market maker							
Number	107,182,307	48.8%	5,297,364	2.4%	107,341,290	48.8%	219,820,961
Percent (%)	47.2		66.0		41.9		44.7
Proprietary firm							
Number	92,587,290	57.9%	1,779,310	1.1%	65,672,286	41.0%	160,038,886
Percent (%)	40.8		22.2		25.6		32.6
All	227,121,281	46.2%	8,031,413	1.6%	256,403,618	52.2%	491,556,312

exercises are trivial in number, accounting for only 1.6% of early exercises. In contrast, 46.2% of put option exercises are motivated by interest income. Of the different categories of market participants, market makers account for 47.2% of exercises while proprietary firms account for 40.8%. It seems that either customers cannot exercise and exit their positions profitably after trading costs, are unaware of the optimality of early exercise, cannot continuously monitor their positions, or are irrational.

III. Early Exercise Behavior of Puts

In Section II, we described the data that form the basis of our analysis and documented the trading activity and exercise frequency of stock options during the sample period. In this section, we focus on analyzing the early exercise behavior of puts. The section has four parts. In the first, for each day during the sample period, we determine whether each in-the-money put option series should be exercised, and, if the put is not exercised when it should be, the net interest income forfeited by the long (or, equivalently, earned by the short) in the absence of exercise costs. These results are particularly relevant to market makers and proprietary firms who can arrange to trade with each other within prevailing bid-ask quotes, pay only clearing fees, and have favorable margins. In the second part, we develop estimates of exercise/trading costs for retail customers, and in the third part we examine the effects of these costs on the early exercise decision. In the fourth and final part, we extend the analysis to document both exercise when none should occur and failure to exercise when exercise should occur.

A. *Early Exercise Decisions in the Absence of Exercise Costs*

The analysis in this section begins by showing that the total forgone net interest income from the failure to exercise puts early is substantial. For expositional convenience, we define an "option series day" as a trading day on which a put option series has nonzero open interest. Each option series day is earmarked as being (a) out-of-the-money (*OTM*), (b) in-the-money but suboptimal to exercise (*ITM-S*), or (c) in-the-money and optimal to exercise (*ITM-O*). *OTM* puts are those whose exercise price is below the end-of-day stock price quote midpoint, and *ITM* puts are those whose exercise price is above. The early exercise decision is based on whether *NII*, as determined by equation (8), is positive, which, of course, is equivalent to whether the prevailing stock price is below its critical level as determined by equation (4). Rather than determining S_A^* through a cumbersome iterative search procedure, we compute the difference between the present value of the interest income received over the next trading day $X(1 - e^{-r\Delta t})$ and the value of a one-day caput (i.e., the European-style call option on an American-style put with exercise price $X - S_{\Delta t}$). See equation (8). If the put is in the money and $NII > 0$, it is earmarked as *ITM-O*; if not, it is classified as *ITM-S*.

The present value of the maximum potential gain from the early exercise of the put is $X(1 - e^{-rT})$. In a sense, it determines the size of the pie held by the long. If the long fails to exercise today, a slice of the pie is eaten by the short, with the size of each slice determined by equation (8). If, on the next trading day, the long again fails to exercise when it remains optimal to do so, the short eats another slice, and so on through the remaining life of the option. In the event the long steadfastly refuses to exercise, the short fully consumes the pie.

To illustrate these mechanics, consider Table IV. In the table, we consider all available January 2006 put option series on Wal-Mart Stores, Inc.'s common

Table IV

Maximum Interest Income from Optimal Early Exercise and Net Daily Interest Income Forfeit from Failure to Exercise for January 2006 Wal-Mart Put Option Series on August 17, 2005

Closing bid-ask stock price quotes are 47.11 and 47.12. The number of days to expiration is 156 and the risk-free rate over the life of the option (overnight) is 3.93% (3.50%).

	Put Option Prices							Present Value of	
Exercise Price	Bid	Ask	Midpoint	Exercise Proceeds	Market-Based Rule	Open Interest	Delta	Maximum Interest Income over Life	Net Interest Income over Day
25.00	0.00	0.05	0.025	0	0.025	55	0	0	0
30.00	0.00	0.05	0.025	0	0.025	385	0	0	0
35.00	0.00	0.10	0.050	0	0.050	1,481	−0.0009	0	0
40.00	0.20	0.30	0.250	0	0.250	11,978	−0.0385	0	0
42.50	0.50	0.55	0.525	0	0.525	22,496	−0.1161	0	0
45.00	1.05	1.10	1.075	0	1.075	30,652	−0.2670	0	0
47.50	1.95	2.10	2.025	0.385	1.640	17,747	−0.4842	0	0
50.00	3.40	3.50	3.450	2.885	0.565	42,901	−0.7192	0	0
55.00	7.80	7.90	7.850	7.885	−0.035	30,095	−1	2,758,628	15,872
60.00	12.80	13.00	12.900	12.885	0.015	23,329	−1	2,332,830	13,425
65.00	17.80	18.00	17.900	17.885	0.015	4,578	−1	495,935	2,854
70.00	22.80	22.90	22.850	22.885	−0.035	4,200	−1	489,985	2,820
75.00	27.80	27.90	27.850	27.885	−0.035	0	−1	0	0
80.00	32.80	32.90	32.850	32.885	−0.035	0	−1	0	0
Total								6,077,377	34,970

stock as of the close on August 17, 2005. The options have 156 days remaining to expiration. The closing bid and ask stock price quotes are 47.11 and 47.12, respectively, the stock volatility rate (over the previous 60 trading days) is 15.80%, the risk-free interest rate over the life of the options is 3.93%, and the overnight risk-free rate is 3.50%. As the table shows, the number of available put option series on that day is 14. The series with exercise prices of 45 and below are out of the money. The 47.50-exercise price option is closest to being at the money (slightly in the money), and has a delta of −0.4842. While the 50-put is in the money, its delta, −0.7192, is above −1, which indicates that early exercise is suboptimal. The puts with exercise prices of 55 and above are all in the money and should be exercised immediately, as is reflected by the fact that they all have deltas equal to −1. For such options, all open interest should disappear.

For the puts with exercise prices between 55 and 70, open interest does not disappear. The series with the 65 exercise price, for example, has 4,578 contracts outstanding. If the longs had exercised immediately, they would have collectively earned up to 495,935 (i.e., $65(1 - e^{-0.0393(156/365)})$ times 100 shares per contract times the open interest) of interest income over the remaining life of the option, and the reported open interest would have fallen to zero. But the longs have another exercise opportunity on the following trading day. By not exercising on August 17, the longs forfeit only 15,872 (i.e., expression (8) times 100 shares per contract times the open interest) to the shorts. Across the put option series on August 17, the longs could have earned up to 6,077,377 in interest income from immediate exercise over the remaining life of the option.

Table V

Total Put Option Contracts Outstanding, Total Contracts that should be Exercised Early, and Forgone Net Interest Income during the Sample Period January 1996 through September 2008

The average interest rate is the average 30-day euro rate across all trading days in each year. The number of option classes is 5,571.

Year	Total Contracts Outstanding	Total Number of Series with with Failed Exercise	Total Contracts with Failed Exercise	Total Forgone Net Interest Income	Average Interest Interest Rate
			Panel A: Model-Based Rule		
1996	1,085,561,323	228,716	42,295,177	26,601,147	0.0540
1997	1,634,115,599	345,898	63,873,723	43,204,128	0.0566
1998	2,256,237,120	591,748	93,704,631	65,678,494	0.0559
1999	3,177,505,949	541,498	116,361,789	75,320,483	0.0530
2000	4,565,690,003	926,859	221,587,188	239,185,634	0.0646
2001	5,558,526,528	857,214	243,916,947	127,985,604	0.0390
2002	6,792,243,580	739,008	262,298,717	44,699,877	0.0177
2003	7,078,914,066	288,615	118,772,457	12,704,494	0.0119
2004	11,479,962,762	715,706	324,415,804	49,664,312	0.0149
2005	13,785,885,948	1,055,310	538,051,126	185,464,902	0.0341
2006	15,518,138,225	1,066,471	484,671,996	290,802,984	0.0513
2007	19,692,753,188	1,221,214	781,290,261	474,654,290	0.0530
2008	15,400,003,638	978,821	668,464,125	231,939,701	0.0297
Total	108,025,537,929	9,557,078	3,959,703,941	1,867,906,049	
			Panel B: Market-Based Rule		
Total	108,025,537,929	9,531,903	4,085,997,061	n/a	

By choosing not to exercise on August 17, they forfeit only 34,970 in net interest income and have an opportunity to exercise on the next trading day.

To measure the total amount of forgone net interest income, we sum across all put option series within each option class each day, and then across all option classes across all days of the sample period. Panel A of Table V contains summary statistics. In all, put options on 5,571 stocks are considered. The total put option open interest across all days in the sample period is 108.0 billion. Of these contracts, 4.0 billion (or approximately 3.7%), should have been exercised but were not. As a result, $1.87 billion of net interest income is forfeited. The table also shows that total net interest income varies widely from year to year. The two single highest values are reported for 2006 and 2007. The reasons are twofold. First, put option trading volume (see Table II) and open interest are highest in 2006 and 2007 (see Table V, Panel A). Second, and most importantly, the level of interest rates (as reflected by the average of the daily 30-day euro rates during the year) is over 5%. Recall that a key driver of the level of *NII* as specified by equation (8) is the interest rate. The third highest value of forgone benefit is in 2000, when the tech bubble burst and the stock

market, as measured by the S&P 500 index portfolio, dropped by 12.7% in the last four months alone. While the drop in stock prices undoubtedly contributed to the optimality of early exercise, the 6.5% interest rate (the highest during our sample period) did also. At the other extreme, the total value forfeited by failure to exercise is only $12.7 million in 2003, when the average short-term interest rate is a meager 1.19%. Overall, the results of Panel A of Table V show that the failure of the longs to exercise their deep in-the-money puts resulted in extraordinary gains for the shorts.

Finally, a brief discussion of *market-based* early exercise decision rules is warranted. In the illustration contained in Table IV, the early exercise decision is based on a *model-based* rule whereby the put should be exercised immediately if *NII*, as defined by equation (8), is greater than zero. Other model-based decision rules can also be used. The put should be exercised early, for example, if its delta equals -1. The problem with this criterion is that it sheds no light on the amount of economic benefit that will be realized if the put is exercised early. Indeed, while *NII* is zero in equation equation (8), the put satisfies the criterion that delta is -1 but economic benefit is zero. In other words, the long should exercise but receives no marginal benefit.

The same is true for *market-based* rules. Market-based rules use observed prices to determine whether the put should be exercised early. If the put price equals its early exercise proceeds, it should be exercised immediately. As such, it is the empirical equivalent to the delta decision rule. And, like the delta rule, it offers no insight into the amount of economic benefit that early exercise brings. The reason is, of course, that the market-based rule (and the delta rule) is satisfied for all stock prices below the critical stock price while the economic benefit from early exercise varies with the level of stock price. In order to estimate the economic benefit (i.e., *NII* in equation (8)), a model is needed. Since a key objective in this study is to estimate the magnitude of the cost of failure to exercise, we use a model-based approach in our analyses.

Before returning to our model-based analyses, however, we test the robustness of the early exercise decisions made using the model-based rule against the market-based rule across all put option series days in the sample period. Applying the market-based rule is no easy task, however, given the practical limitations imposed by price discreteness, minimum price increments, and bid-ask spreads. To illustrate, reconsider Table IV. The fourth and fifth columns are the put's end-of-day bid-ask quote midpoint and exercise proceeds, respectively. The exercise proceeds equal the exercise price less the stock's bid-ask quote midpoint. The sixth column contains the computations underlying the market-based rule for the different option series. Note first that none of the values reported in this column equal zero. This is attributable, in part, to price discreteness in the option market. During the latter half of our sample (which includes August 17, 2005), low-priced options were traded in nickels and high-priced options in dimes. This discreteness makes it highly unlikely that the bid-ask midpoint is exactly equal to the true value of the put. To mitigate the problem, the market-based rule can be extended to include all puts

whose prices are *less than* or equal to their exercise proceeds; however, this criterion fails too. Under such a rule, the puts with exercise prices of 55, 70, 75, and 80 should be exercised immediately but not those with exercise prices of 60 and 65. Of course, this makes no sense. If the 55-put should be exercised immediately, so should the 60 and 65. To further refine the market-based criterion, we incorporate some stylized facts about the stock option market. During our sample period, option prices were quoted in decimals after April 29, 2001 and traded in nickels, with the minimum price increment being \$0.05 for options whose prices were below \$3.00 and \$0.10 for options whose prices were \$3.00 or more. In the early part of the sample period, options traded in eighths and the minimum price was one-eighth for options whose prices were below \$3.00 and one-quarter for those priced above. Our market-based rule then becomes one in which the put should be exercised if the put's bid-ask midpoint less one-half the minimum price increment (given the put price and the date) is less than or equal to its exercise proceeds. Under such a rule, all of the puts in Table IV with exercise prices of 55 and above should be exercised immediately.

To understand how this market-based rule behaves across the sample period, we apply it to all of the option series days used to generate Table V. Panel B reports the early exercise decision results. While some comfort is drawn from the fact that the total number of series with failed exercises and the total contracts with failed exercise under the market-based rule are very close to the model-based levels (i.e., 9,531,903 versus 9,557,078 and 4,085,997,061 versus 3,959,703,941, respectively), the market-based rule is ad hoc and fails to account for other factors such as minimum price increments and the size of the bid-ask spread in the option market and price discreteness, minimum price increments, and bid-ask spreads in the stock market. Consequently, we move forward using only the model-based criterion.

B. Exercise Cost Estimates

The evidence provided thus far does not account for the trading/exercise costs faced by the long put option holders. Broadly speaking, traders can be categorized into two groups—hedgers and speculators.[12] Hedgers who buy puts, for example, typically have a long position in the underlying stock. In frictionless markets, they should exercise their puts when $NII > 0$, delivering the stock to the short put and receiving the exercise price in cash. Markets are not frictionless, however, and exercising the put may be encumbered by a number of practical considerations. Taxes, for example, may cause the hedger to be reluctant to exercise since he may realize a capital gain on his stock. Alternatively, the stock position may be part of a restricted stock plan whose vesting period

[12] Lakonishok et al. (2006) document equity option market activity by different classes of investors from 1990 through 2001. For a subsample of the data (one discount brokerage firm over the first half of the sample), they find that volatility trading strategies account for only a small fraction of overall option trading activity, suggesting that the primary motivations for trading equity options for non-market makers are speculating or hedging underlying stock price movements.

has not yet elapsed. Similarly, if the stock position is held by a top executive of a firm, the sale of stock may be effectively restricted since such sales are generally regarded as a negative signal by the marketplace. Finally, if the shareholder attaches value to the voting rights, he may be reluctant to liquidate his stock position. Naturally, all of these individuals can liquidate their option position by buying additional shares to deliver against the put's exercise or by selling (reversing) the put position in the marketplace; however, in both instances, significant trading costs may be incurred. Assuming hedgers are not restricted to holding the underlying stock, "exercise" can be accomplished by either (a) exercising the put and delivering the stock, or (b) reversing the put in the marketplace. In the former case, the hedger pays a fixed commission per exercise/assignment, independent of the number of contracts. In the latter case, the hedger pays an option trade execution commission as well as half of the put's bid-ask spread in selling the put and a stock trade execution commission as well as half the stock's bid-ask spread in selling the stock.[13]

Speculators who are long puts have the directional view that the stock price will fall and want to profit from their prediction. Assuming they are correct in their prediction and that the stock price happens to fall below its critical level, the speculator, like the hedger, can exercise or reverse. To exercise, the speculator must buy the stock, exercise the put, and then deliver the stock. The total costs include paying an option exercise commission, and paying a stock commission and half the stock's bid-ask spread to acquire the stock for delivery. To reverse, the total costs include paying the option trade execution commission and half the put's bid-ask spread.

The most cost-effective means of exercising the put can be determined by examining actual costs. For commission levels, we use the online customer rates currently charged by Charles Schwab.[14] For option exercise/assignment, Schwab charges a flat fee of $8.95 for option exercise/assignment, and for option trade execution, they charge $8.95 plus $0.75 per contract. For stock trade execution, they charge a flat fee of $8.95. Since we require trading costs to be on a per-share basis, we compute the commission per-share based on different assumptions regarding the number of contracts exercised. In the case in which the hedger exercises 20 puts and delivers the stock, the commission rate per-share is less than half a penny. In the case in which the speculator exercises 20 put contracts and buys and delivers the appropriate number of shares of stock, the commission rate per-share is less than a penny.

[13] If the long put option holder is also long the underlying stock, an alternative to exercising or reversing the put is to sell a call option with the same exercise price. In this case, the option holder would incur a commission plus half the bid-ask spread for selling the call. In practice, however, most deep out-of-the-money calls have bid prices equal to zero, indicating that market makers are unwilling to buy.

[14] Developing accurate estimates of brokerage commission rates is virtually impossible since rates are negotiable and often embed services other than trade execution (e.g., an online trading platform with real-time data). Moreover, rates have undoubtedly fallen over the sample period due to increased competition in securities markets.

	Commission per Share		
Number of contracts	1	10	20
Hedger exercises put and delivers stock	0.0895	0.0090	0.0045
Hedger reverses put and sells stock	0.0970	0.0165	0.0120
Speculator exercises put, and buys and delivers stock	0.1790	0.0179	0.0090
Speculator reverses put	0.0970	0.0165	0.0120

To account for the effects of bid-ask spreads, we use actual closing bid-ask price quotes for put options and their underlying stocks each day during the period January 1996 through September 2008.[15] To get a general sense of the magnitudes of spreads, we compute the average bid-ask spreads each year for in-the-money puts that should optimally be exercised immediately and that have the potential to be exercised (i.e., have positive open interest). We also compute the average bid-ask spreads for their underlying stocks. The results, reported in Table VI, are quite compelling. Average put option spreads are many times higher than average spreads in the stock market. The average put option spread across all years in the sample is 0.4143, for example, nearly three times the average stock spread of 0.1446. But, what is even more interesting is that the effect has become more pronounced since 2001, when the move from fractional to decimal pricing in the securities industry was completed.[16] While competition in the options market reduces the average put spread from 0.4792 in the subperiod 1996 to 2000 to 0.3737 in the subperiod 2001 to 2008 (or by 22.0%) as the SEC had hoped, the average stock spread drops by 81.3%. As recently as 2008, the typical bid-ask spread for an exercisable put is more than 10 times that of the underlying stock. Combining our estimates of commissions and bid-ask spreads and the average bid-ask spreads from the overall sample period, we find that the average exercise costs per-share for retail customers are as follows:

	Total Costs per Share		
Number of contracts	1	10	20
Hedger exercises put and delivers stock	0.0895	0.0090	0.0045
Hedger reverses put and sells stock	0.4660	0.3049	0.2959
Speculator exercises put, and buys and delivers stock	0.2513	0.0902	0.0813
Speculator reverses put	0.3042	0.2236	0.2191

[15] The closing bid-ask spreads for options are from the OptionMetrics database, and the closing stock spreads are from the CRSP daily file. Where the CRSP daily file has missing stock price quotes, we use NYSE's TAQ quotes.

[16] The transition to decimal pricing was ordered by the SEC on July 24, 2000. All equities and options markets were to begin phasing in decimal pricing on August 28, 2000 and end with full implementation on or before April 29, 2001. See http://www.sec.gov/rules/other/decimalp.htm.

Clearly, reversing the put in the marketplace is more expensive from a trading cost standpoint. Put option bid-ask spreads are simply too high. Consequently, in the subsequent analysis we use the trading costs associated with exercising the put and ignore the alternative of reversing.

Table VI

Per Share Bid-Ask Spreads for All Put Options and Underlying Stock during the Sample Period January 1996 through September 2008

The number of option classes is 5,571. Spreads for stock options are from the OptionMetrics file, and spreads for stock are from the CRSP daily file. The subperiods 1996 to 2000 and 2001 to 2008 represent before and after the move to decimal pricing in the stock and option markets, respectively.

		Put Option		Stock		Average Ratio	
Year	No. of Observations	Average Price	Average Spread	Average Price	Average Spread	Option Price to Stock Price	Option Spread to Stock Spread
1996	227,361	9.67	0.4356	24.89	0.3340	0.55	1.82
1997	343,094	9.16	0.4433	23.80	0.2793	0.62	2.73
1998	586,359	11.27	0.5021	22.74	0.2968	0.82	3.29
1999	533,599	11.93	0.4632	24.55	0.3078	0.81	3.33
2000	898,668	23.52	0.5519	25.89	0.2289	1.56	5.83
2001	832,172	24.37	0.4716	20.08	0.1423	2.84	14.88
2002	721,504	17.08	0.3491	17.07	0.0977	2.61	14.08
2003	264,035	11.99	0.2572	23.81	0.0350	0.92	13.26
2004	660,341	11.88	0.3016	26.21	0.0198	0.74	15.40
2005	971,599	11.75	0.3369	30.09	0.0233	0.63	17.78
2006	996,407	12.14	0.3390	34.28	0.0248	0.52	19.60
2007	1,192,666	12.59	0.4330	35.25	0.0406	0.73	20.89
2008	965,055	21.61	0.5011	31.94	0.0497	1.72	28.45
All years	9,192,860	14.54	0.4143	26.20	0.1446	1.16	12.41
1996–2000	2,589,081	13.11	0.4792	24.38	0.2894	0.87	3.40
2001–2008	6,603,779	15.43	0.3737	27.34	0.0542	1.34	18.04

C. Early Exercise Decisions in the Presence of Exercise Costs

We now turn to re-examining the forgone economic benefits when exercise costs are considered. To do so is not simply a matter of comparing the per-share net interest income of each option series on each day with plausible estimates of exercise costs. The reason is that the forgone net interest income is computed over a single trading day while the realized benefits from early exercise accrue over the remaining life of the option. Put differently, the only difference between exercising today and exercising tomorrow from a cost perspective is not the amount of the exercise costs from immediate exercise per se but the present value of the interest that can be earned on the exercise costs by deferring payment of the costs for one day. In other words, exercise costs must be incorporated not only in the interest income component of equation (8) but also in the caput component. The decision rule for exercising the put in the presence

Table VII

Total Number of Series Days on Which Put Option Should Be Exercised, Total Open Interest across Series Days, and Total Forgone Net Interest Income during the Sample Period January 1996 through September 2008

The number of option classes is 5,571. The cost of the hedger exercising and delivering the stock is assumed to be \$0.0895 per share. The cost of a speculator exercising, and buying and delivering the stock is assumed to be \$0.1790 per share plus half of the prevailing bid-ask spread.

Trading Cost per Share Assumption	Total Number of Series with Failed Exercise	Total Contracts with Failed Exercise	Total Forgone Net Interest Income
No exercise costs	9,557,078	3,959,703,941	1,867,906,049
Hedger exercises and delivers stock	9,383,407	3,831,981,849	1,823,059,269
Speculator exercises, and buys and delivers stock	9,378,630	3,829,674,250	1,815,550,790

of exercise costs is

$$NII(k) = (X-k)\left(1-e^{-r\Delta t}\right) - c\left(P_{\Delta t}(k), X-k-S_{\Delta t}, \Delta t\right) > 0, \qquad (10)$$

where k is the per-share exercise cost.[17] For a hedger who exercises the put, k is assumed to be \$0.0895, based on the information provided above. Note that we are assuming a worst-case scenario in which the hedger exercises only a single contract (i.e., the minimum possible order size). For a speculator who exercises the put and buys and delivers the stock, k is assumed to be \$0.1790 (also a worst-case) plus half the prevailing bid-ask spread for the stock. So, while the net interest income forfeit over a single trading day is reduced, the reduction is only a small fraction of the per-share execution costs. We now turn to applying equation (10) to determine the effect on total forgone interest income.

Table VII summarizes the total number of contracts with failed exercise and the total forgone net interest benefits in the presence of exercise costs. The first row of the table matches the figures reported in Table V. The second and third rows document the effects of exercise costs. As the results show, the effects are relatively modest. Accounting for the exercise costs of a hedger, the number of contracts with failed exercise drops by only 128 million, with the total forgone net interest income remaining at an astonishing \$1,823 million. Increasing exercise costs to those of a speculator, the number of failed exercises drops by only another two million. In other words, failure to exercise is pervasive. Even after accounting for plausible exercise costs, the longs forfeit \$1,816 million over the 13-year sample period.

Based on the results in this part of the section, we are prepared to take exercise/trading costs off the table as being an important explanation for the apparent failures to exercise. This does not imply, however, that the remaining

[17] When k is zero, equation (10) reverts back to equation (8).

long option holders are irrational. As we have already noted, there are at least two other rational explanations. First, nonprofessional traders may decide that the cost of developing an early exercise decision rule is too high. Indeed, a key contribution of this paper is to provide such a decision rule and reduce the cost. Second, nonprofessional traders may not be willing or able to incur the cost of constantly monitoring their positions. In both cases, the decisions are rational. The words of Stigler (1967, p. 291) summarize it best:

> There is no "imperfection" in a market possessing incomplete knowledge if it would not be remunerative to acquire (produce) complete knowledge: information costs are the costs of transportation from ignorance to omniscience, and seldom can a trader afford to take the entire trip.

D. Actual Exercise Behavior

The evidence provided thus far in this section suggests that long put option holders are failing to exercise when they should even after accounting for exercise/trading costs. As a result, a substantial amount of money is "left on the table." While this evidence is important, it does not provide a complete picture of exercise behavior. To complete the picture, we return to our subsample of 4,011 option classes for which we have actual exercise data during the period July 2001 through September 2008, and determine whether observed put option exercises are, in fact, optimal and whether other put option series, which should have been exercised, are not.[18] Table VIII contains a summary of our results.

Table VIII has two panels. Panel A contains actual numbers of contracts exercised and separates them by whether they "Should be exercised" or "Should not be exercised." The first row of Panel A reports the number of option series days, and the second reports the total open interest. Of the put options that should be exercised, about 6.2 million series had open interest totaling 3.2 billion contracts.[19] Recall that, since $NII > 0$ for these series, all remaining long (and, hence, implicitly short) open interest should disappear. Equally perplexing is the trading volume for the *ITM-O* puts—more than 201 million contracts. These trades are unlikely to be attributable to hedgers/speculators reversing their option positions since, as we have already documented, bid-ask spreads are so much higher in the option market than in the stock market. It is much cheaper to exercise than reverse. At the same time, the trades are unlikely to be newly established positions. Since these options are trading at their floor values, it is more cost-effective to short the stock than to buy the put. In the next section of the paper, we show that this trading volume is largely attributable not to position reversals or new positions but rather to an activity called short stock interest arbitrage.

[18] A small number of exercise observations are lost since this step involves matching the OCC exercise data to the option price data in OptionMetrics data, and the OptionMetrics data have missing information.

[19] This total, of course, includes open put positions that should have been exercised before day t.

Table VIII

Optimal and Sub-Optimal Put Option Exercises during the Sample Period July 2001 through September 2008

The number of option classes is 4,011. The months of November 2001, January and July 2002, and January 2006 are missing.

Panel A: Actual Exercises

	Should Be Exercised ($S_t < S^*$)		Should Not Be Exercised ($S_t \geq S^*$)			
	ITM-O		ITM-S		OTM	
No. of option series days	6,198,164		49,117,376		51,594,734	
Total open interest	3,191,601,879		25,619,699,835		60,393,758,470	
Previous day's open interest	3,247,528,057		25,493,688,859		59,790,672,931	
No. of contracts traded	201,454,795		860,709,189		1,902,760,394	
No. of contracts exercised by:						
Customer	18,325,606	9.9%	8,244,992	18.8%	125,687	21.6%
Market maker	87,748,104	47.4%	21,776,742	49.7%	275,824	47.5%
Proprietary firm	78,860,356	42.6%	13,772,026	31.4%	179,699	30.9%
Total	184,934,066		43,793,760		581,210	

Panel B: Dividend-Related Exercises

	Should Not Be Exercised ($D > X(e^{rt} - 1)$)
No. of option series days	5,431,304
Total open interest	6,504,790,185
No. of contracts traded	148,747,220

Market Participant	No. of Option Series Days		No. of Contracts Exercised	
	Number	% of Total	Number	% of Total
Customer	5,408	33.6	722,246	42.0
Market maker	9,597	59.6	661,778	38.4
Proprietary firm	1,088	6.8	337,545	19.6
Total	16,093		1,721,569	

Panel A of Table VIII also summarizes the numbers of contracts exercised by market participant. Of the contracts that should be exercised, only 184.9 million contracts are exercised, accounting for less than 6% of the previous day's open interest. Of the contracts exercised, market makers are the most active of market participants, exercising 87.7 million contracts, or 47.4% of all exercised puts, in the *ITM-O* category. Proprietary firms are almost as active, exercising 78.9 million contracts or 42.6%. Finally, retail customers account for only 9.9% of exercise activity, exercising only 18.3 million contracts.

These stylized facts are consistent with the rational trader explanations offered earlier. Retail customers, as nonprofessional traders, are not as sophisticated as professional traders (i.e., market makers and proprietary firms), so they may be unaware that early exercise is optimal, either because they do not fully understand the decision rule or do not have time to constantly monitor their positions. That is not to say they are irrational. They simply believe that the marginal cost of learning the early exercise mechanics or monitoring their option position is just too high in light of the marginal benefit.

The exercise summary for the *ITM-S* and *OTM* put options is also informative. About 44 million puts that are in the money but not optimal to exercise are in fact exercised. Of these, retail customers exercised about 8.2 million puts that are suboptimal to exercise, accounting for 18.8% of the exercise activity in the *ITM-S* category. Proprietary firms exercised around 13.8 million contracts, accounting for 31.4% of in-the-money puts that are exercised when it is not optimal to do so. Market makers exercised around 21.8 million puts, or 49.7%, a similar level to the *ITM-O* category. About 0.6 million out-of-the-money puts are exercised. Of these, 21.6% are by retail customers, 47.5% by market makers, and 30.9% proprietary firms. On the one hand, the relative frequency of exercise reflects the fact that professional traders know the early exercise decision rule and constantly monitor their positions. It is their primary line of business. On the other hand, the fact that exercises take place in this *ITM-S* category is worthy of note. One possible reason is that we use end-of-day stock price quote midpoint in determining whether exercise is optimal. It may be the case that it was optimal to exercise the put earlier in the day when the stock price was lower.[20] Another reason is that there may be noise in our model-based exercise decision rule due to option valuation model misspecification and/or noise in the estimate of expected future volatility.

Finally, the evidence in Panel A of Table VIII does document irrationality. Of the total number of actual exercises, 581,210 are in the out-of-the-money put category (*OTM*). Reassuringly, however, this number is inconsequential compared to the 3.19 billion failed exercises (0.0018%) during the sample period.

Panel B of Table VIII provides additional evidence regarding irrational exercises. In Panel B, actual exercises of option series days in which (a) the dividend condition (9) applies, (b) the put is in the money, and (c) the put has nonzero open interest are reported. Recall that, if the amount of the impending dividend exceeds the interest income that may be earned on the exercise proceeds of the put before the dividend payment, early exercise is suboptimal. The table reports that there are 5.4 million option series days in which the dividend condition (9) is satisfied, and the total open interest on these days is 6.5 billion

[20] While the OCC only does exercise assignment at the end of the day, the long put option holder can, in effect, exercise his option earlier in the day by buying the stock. When she exercises at the end of the day, the exercise proceeds equal the difference between the exercise price and the closing stock price plus the difference between the closing stock price and purchase price of the stock earlier in the day. For this reason, we repeat the analysis using the lowest daily stock price rather than the closing price to determine whether early exercise is optimal and still find results are nearly the same.

contracts. Of the total 5.4 million option series days, only on 16,093 (0.3%) option series days did exercises actually occur. And, of the total open interest of 6.5 billion contracts across option series days, only 1.7 million (0.03%) contracts are exercised. Again, while irrational exercises do appear, the number is relatively inconsequential.

IV. Short Stock Interest Arbitrage

Section III shows, among other things, a pervasive failure to exercise put options on stocks when it is optimal to do so. During the period July 2001 through September 2008, long put option holders forfeit $1.87 billion in net interest income. Even after plausible exercise costs, the losses amount to nearly $1.82 billion. This section examines who gathers up the forgone interest income and how they go about doing so. Specifically, we show that market makers and proprietary firms engage in a trading strategy called "short stock interest arbitrage." This strategy allows them to systematically capture short open interest of deep in-the-money puts and thereby capture the interest income being left on the table by long put option holders. The section is divided into two parts. In the first, we describe short stock interest arbitrage and illustrate its use using actual trading activity/exercise behavior of a single option series over a three-month period. In the second, we show that, over the past 13 years, short stock interest arbitrage accounts for more than half of all put option exercises. These exercises are unequivocally rational.

A. Trading Game

The failure of long put option holders to exercise deep in-the-money puts has given rise to what amounts to a trading game—short stock interest arbitrage (hereafter, "SSIA"). SSIA involves "... the purchase, sale and exercise of in-the-money options of the same class" on the same trading day.[21] For puts whose *NII* is greater than zero on a given day, open interest should completely disappear. To the extent it does not, the long is acting suboptimally and forgoes interest income. The total amount of interest income that is earned by the short at the expense of the long varies directly with open interest. The trading game involves capturing the open interest from the existing shorts, thereby garnering the abnormal interest income.

Capturing open interest works as follows. Assume that a deep in-the-money put has open interest of m contracts. Absent any new market activity during the trading day, the longs' failure to exercise provides the existing shorts the opportunity to earn interest on the exercise proceeds of the put over the next day. Anticipating the longs' failure to exercise, arbitragers step in front of the existing shorts to earn the forgone net interest income by simultaneously buying and selling n deep in-the-money puts with the same exercise price (where

[21] See NASDAQ OMX PHLX, Inc. Fee schedule, February 2, 2009. Available at http://nasdaqomxtrader.com/content/marketregulation/membership/phlx/feesched.pdf.

n is significantly greater than m) and then immediately exercising the n long puts.[22] Under clearinghouse rules, exercises are randomly assigned to open short positions at the end of the trading day. Before assignment, the total number of open short positions is $m+n$. After assignment, the total number of open shorts is m under the assumption that none of the pre-existing long put positions are exercised. Under random assignment, the pre-existing shorts' proportionate share of the total open interest is expected to drop from 100% to $\frac{m}{m+n}$, while the new shorts' goes from 0% to $\frac{n}{m+n}$. If the pre-existing open interest is 1,000 contracts and the new shorts buy and sell 9,000 contracts, for example, 90% of the open interest (and hence net interest income) is captured. At 19,000 contracts, the new shorts capture 95% of the open interest, and so on. The larger the trade, the greater the capture. The new shorts are "arbitragers" in the sense that the strategy is risk-free for small changes in the stock price. In exercising the long put position, the arbitrager receives X per-share in cash and a short position in the stock. Since any open short puts have a delta equal to one, the arbitrager is perfectly hedged. For large changes in the stock price, however, the delta-hedge may be imperfect. If the stock price spikes upward, the delta of the put grows larger than -1 and the value of the hedged position falls. Recall that we account for the expected cost of insurance against this contingency through the caput premium in the net interest income computation (8).

Before showing an actual example of the execution of SSIA, it is worthwhile to note that many stock option exchanges implicitly encourage this type of trading activity by capping fees. On the NASDAQ OMX PHLX, for example, the exchange fees for nonelectronic trades are \$0.25 per contract. Thus, to buy and sell n contracts, the total cost would be \$0.50n. The NASDAQ OMX PHLX, however, caps the fee on SSIA trades at \$1,000.[23] Consequently, for trades greater than 2,000 contracts, the per-contract cost of executing the SSIA spread begins to fall. So, not only does a very large trade garner more short open interest, but it also provides cost savings.

To illustrate the practice of short stock interest arbitrage, consider the daily trading/exercise activity of a single deep in-the-money put option series—the January 2006 65-put written on Wal-Mart's stock—during the period August 17 through November 9, 2005. Table IX contains a summary. For all days during the period, the put should be exercised immediately. But, as the table shows, the open interest of the series does not disappear. Indeed, it remains in the thousands of contracts throughout the period.

To help interpret the contents of the table, begin with the first row. On August 17, 2005, the January 65-put had 156 days to expiration. The stock price closed at 47.11, so the put was deep in the money and well into its optimal early

[22] It is worth noting that SSIA may involve buying and exercising n puts at one exercise price and selling puts in the same option class but at a different exercise price. As long as both put series are deep in the money and should be exercised, the game may be played. Since we focus only on SSIA activity where the exercise prices are the same, we understate the amount of SSIA activity taking place.

[23] While the CBOE also caps fees at \$1,000, ARCA and AMEX cap fees at \$750.

Table IX

Trading Volume, Open Interest, and Number of Exercises of the January 2006 65-Put Option on Wal-Mart during the Period August 17 through November 9, 2005

Exercises are classified by market participant, and the classifications are customer, market maker, and proprietary firm.

Date	Closing Stock Price	Days to Expiration	Trading Volume	Open Interest	Change in Open Interest	Term Interest Rate	Overnight Interest Rate	Present Value of Maximum Interest Income	Net Interest Income	Number of Contracts Exercised		
										Customer	Market maker	Proprietary firm
8/17/2005	47.11	156	0	4,578	0	0.03932	0.03501	495,935	2,854	0	0	0
8/18/2005	47.24	155	10,002	4,578	0	0.03976	0.03548	498,160	2,893	0	0	10,002
8/19/2005	46.58	154	2,100	6,678	2,100	0.03978	0.03532	722,439	12,600	0	0	0
8/22/2005	46.67	151	1,002	6,678	0	0.03967	0.03548	706,506	4,220	0	0	1,002
8/23/2005	46.34	150	0	6,678	0	0.03996	0.03533	706,953	4,201	0	0	0
8/24/2005	45.55	149	0	6,678	0	0.03962	0.03533	696,455	4,201	0	0	0
8/25/2005	45.29	148	0	6,678	0	0.03971	0.03548	693,357	4,220	0	0	0
8/26/2005	45.70	147	0	6,678	0	0.03989	0.03564	691,774	12,713	0	0	0
8/29/2005	45.65	144	0	6,678	0	0.03992	0.03564	678,224	4,239	0	0	0
8/30/2005	45.19	143	0	6,678	0	0.03987	0.03596	672,715	4,276	0	0	0
8/31/2005	44.96	142	0	6,678	0	0.03976	0.03596	666,217	4,276	0	0	0
9/1/2005	45.00	141	0	6,678	0	0.03907	0.03628	650,264	4,314	0	0	0
9/2/2005	44.55	140	0	4,578	−2,100	0.03842	0.03556	435,308	11,594	0	2,100	0
9/6/2005	45.69	136	0	4,292	−286	0.03839	0.03596	396,220	2,748	0	286	0
9/7/2005	45.86	135	0	4,292	0	0.03869	0.03533	396,388	2,700	0	0	0
9/8/2005	45.86	134	0	4,292	0	0.03899	0.03548	396,504	2,712	0	0	0
9/9/2005	45.89	133	0	4,292	0	0.03914	0.03564	395,006	8,171	0	0	0
9/12/2005	45.89	130	0	4,292	0	0.03917	0.03548	386,490	2,712	0	0	0
9/13/2005	45.07	129	15,850	4,292	0	0.03925	0.03548	384,280	2,712	0	10	15,840
9/14/2005	44.70	128	8,115	6,405	2,113	0.03932	0.03533	570,173	4,029	0	0	6,002
9/15/2005	44.32	127	0	4,305	−2,100	0.03938	0.03786	380,773	2,902	0	2,100	0
9/16/2005	43.87	126	4,002	4,305	0	0.03952	0.03743	379,167	8,608	0	0	4,002
9/19/2005	44.01	123	0	4,224	−81	0.03976	0.03770	365,389	2,836	0	81	0

9/20/2005	43.21	122	12,057	4,224	0	0.03963	0.03754	361,300	2,824	0	4,019	8,038
9/21/2005	42.49	121	10,502	4,224	0	0.04009	0.03770	362,508	2,836	0	500	10,002
9/22/2005	43.19	120	0	4,224	0	0.03992	0.03786	357,988	2,848	0	0	0
9/23/2005	43.20	119	0	4,224	0	0.04022	0.03817	357,684	8,613	0	0	0
9/26/2005	43.11	116	0	4,224	0	0.04048	0.03802	350,938	2,860	0	0	0
9/27/2005	43.10	115	12,657	4,224	0	0.04062	0.03865	349,118	2,907	0	4,219	8,438
9/28/2005	43.13	114	10,002	4,224	0	0.04060	0.03802	345,967	2,860	0	0	10,002
9/29/2005	43.54	113	18,000	4,224	0	0.04113	0.03834	347,390	2,884	0	6,000	12,000
9/30/2005	43.82	112	12,002	4,224	0	0.04112	0.03907	344,231	8,816	0	0	12,002
10/3/2005	43.76	109	40	4,211	−13	0.04133	0.03897	335,760	2,922	0	40	0
10/4/2005	43.85	108	25,002	4,211	0	0.04141	0.03802	333,309	2,851	0	4,000	21,002
10/5/2005	43.50	107	14,040	4,211	0	0.04139	0.03770	330,121	2,827	0	4,019	10,021
10/6/2005	43.93	106	6,002	4,211	0	0.04154	0.03802	328,176	2,851	0	0	6,002
10/7/2005	44.03	105	11	4,220	9	0.04157	0.03823	326,084	8,617	0	0	0
10/10/2005	44.54	102	0	4,220	0	0.04152	0.03802	316,403	2,857	0	0	0
10/11/2005	45.02	101	23,559	4,220	0	0.04163	0.03834	314,204	2,881	0	4,519	19,040
10/12/2005	44.94	100	10,002	4,220	0	0.04164	0.03786	311,120	2,845	0	0	10,002
10/13/2005	44.76	99	0	2,720	−1,500	0.04193	0.03818	199,931	1,849	1,500	0	0
10/14/2005	45.04	98	0	2,720	0	0.04191	0.03839	197,839	5,577	0	0	0
10/17/2005	45.24	95	0	2,720	0	0.04186	0.03865	191,567	1,872	0	0	0
10/18/2005	45.13	94	14,459	2,720	0	0.04200	0.03818	190,185	1,849	0	2,819	11,640
10/19/2005	45.99	93	0	2,720	0	0.04181	0.03802	187,345	1,841	0	0	6,002
10/20/2005	45.60	92	0	2,720	0	0.04196	0.03770	186,002	1,826	0	0	0
10/21/2005	45.72	91	0	2,711	−9	0.04194	0.03838	183,299	5,558	0	9	0
10/24/2005	46.21	88	0	2,711	0	0.04203	0.03818	177,665	1,843	0	0	0

(continued)

Table IX—*Continued*

Date	Closing Stock Price	Days to Expiration	Trading Volume	Open Interest	Change in Open Interest	Term Interest Rate	Overnight Interest Rate	Present Value of Maximum Interest Income	Net Interest Income	Number of Contracts Exercised		
										Customer	Market maker	Proprietary firm
10/25/2005	45.39	87	5,438	2,711	0	0.04201	0.03786	175,591	1,828	0	2,719	2,719
10/26/2005	45.58	86	7,002	2,711	0	0.04215	0.03818	174,141	1,843	0	0	7,002
10/27/2005	44.74	85	0	2,711	0	0.04257	0.03913	173,846	1,889	0	0	0
10/28/2005	45.50	84	0	2,711	0	0.04253	0.04013	171,642	5,811	0	0	0
10/31/2005	47.31	81	0	2,711	0	0.04249	0.04071	165,385	1,965	0	0	0
11/1/2005	46.99	80	14,002	2,711	0	0.04242	0.04071	163,059	1,965	0	4,000	10,002
11/2/2005	47.56	79	6,002	2,711	0	0.04267	0.04071	161,996	1,965	0	0	6,002
11/3/2005	47.45	78	0	2,711	0	0.04251	0.0404	159,369	1,950	0	0	0
11/4/2005	47.69	77	0	2,711	0	0.04257	0.0406	157,552	5,880	0	0	0
11/7/2005	48.05	74	11,002	2,711	0	0.04259	0.0404	151,488	1,950	0	2,500	8,502
11/8/2005	47.61	73	6,002	2,711	0	0.04275	0.0404	150,021	1,950	0	0	6,002
11/9/2005	48.20	72	0	311	−2,400	0.04272	0.0404	16,962	224	0	0	2,400

exercise region. Although the put did not trade on that day, its open interest at the end of the day on August 17 was 4,578 contracts. This implies that the 4,578 longs behaved suboptimally. The present value of the maximum interest income that they could have earned over the remaining life of the option if they had exercised immediately was

$$4{,}578 \times 100 \times 65 \times \left(1 - e^{-.03932(156/365)}\right) = 495{,}935.$$

As noted earlier, this amount can be thought of as defining the potential size of the pie that the longs are making available to the shorts to share as of the close on August 17.

The column headed "Net interest income" contains the size of each slice of the pie. The net interest income is defined by equation (8) and is the interest income that can be earned on the exercise price over the next trading day less than the value of the caput. Standing at the close on August 17, the present value of the interest income is $65(1 - e^{-0.03501(1/365)})$ or 0.00623 and the value of the caput (not reported in the table) is zero. Hence, the total daily net interest income by the longs is

$$NII = 4{,}578 \times 100 \times (0.00623 - 0.00000) = 2{,}854.$$

The shorts earn this amount. Given the nature of publicly available data, there is no means of identifying whether the shorts are public customers, market makers, or proprietary firms at the beginning of the time series reported in the table.

On August 18, short stock interest arbitrage takes place. SSIA has a number of tell-tale signs. One sign is that trading volume is high but open interest remains unchanged from the previous day. Since daily trading volume is defined as the number of contracts purchased during the day (or, equivalently, the number of contracts sold), the only way open interest can remain unchanged is if two or more market participants arrange a trade whereby each market participant (a) buys and sells an equal number of 65-puts and then (b) immediately exercises the long puts. While determining exactly how many market participants are involved in the trading on August 18 is not possible from the available data, a likely scenario is that Proprietary Firm A simultaneously bought and sold 5,001 65-puts from Proprietary Firm B at, say, the midpoint between the bid and ask prices.[24] This activity produces a daily trading volume of 10,002 contracts.[25] Absent other considerations, open interest should increase by 10,002 contracts. But the trading game also involves both firms immediately exercising their long positions, an activity documented in the rightmost column of the table.

The fact that trading volume equals the number of exercised contracts on that day is a second tell-tale sign. Since the number of new positions equals

[24] Such trades cannot be accommodated in electronic markets like the ISE and must be executed on an exchange floor like CBOE or NASDAQ OMX PHLX.

[25] Proprietary firms tend to execute stock interest arbitrage in order sizes ending in the digit "1" so as to easily separate short stock interest positions from other positions in their books.

the number of exercises, the level of open interest does not change. With the clearinghouse randomly assigning the exercises to the shorts, the newcomers (in this case the proprietary firms) wind up with about $\frac{10,002}{4,578+10,002} = 68.6\%$ of the open interest after assignment,[26] and consequently capture about 68.8% of the forgone interest income, $2,893. While we do not know the identity of the shorts on August 17, we know that on August 18 about 69% are proprietary firms that will continue to earn their proportionate share of the forgone interest income provided that long positions remain open and no one steps in front of them to capture their open interest.

On the next day, 2,100 contracts are traded, no contracts are exercised, and open interest rises to a level of 6,678 (by 2,100 contracts from the previous day). Because open interest rises, this trading does not reverse existing positions—it adds to them. In theory, the buyer(s) of these contracts should exercise immediately. Yet no contracts are exercised. This trading activity seems irrational in the sense that it simply increases the available pool of forgone interest income available to the shorts.

The activity on August 22 is, again, short stock interest arbitrage. Trading volume matches the number of exercises, and open interest does not change from the previous day. Proprietary firms bought and sold 1,002 contracts. By immediately exercising the longs, they capture about $\frac{1,002}{6,678+1,002} = 13.0\%$ of the open interest and hence 13.0% of the forgone interest income, $4,220.

From August 22 through September 1, the activity in the 65-put is dormant—no trading and no exercises. This means that on August 22 the short put option holders are allowed to earn interest on the exercise proceeds of the 6,678 open long contracts for 10 days. On September 2, 2,100 long contracts held by market makers are exercised. Since there is no trading volume on this day, the exercise activity is normal and not part of a short stock interest arbitrage. Similarly, the 286 contracts exercised on September 6 are due to normal optimal exercise activity on the part of market makers with open long positions. But what remains a mystery is why the market maker waited so long to exercise. The market maker was holding the long put position at least as far back as August 23. By exercising earlier, more interest income would have been earned. Normal, but inexplicably delayed, exercises also appear on September 19, October 13, and November 9.

Aside from the normal exercises noted in the previous paragraph, all other exercise activity appears to be short stock interest arbitrage executed by both market makers and proprietary firms. In nearly all cases, trading volume equals the sum of the number of exercises across market makers and proprietary firms and open interest remains unchanged from the previous day. Proprietary firms are the single largest player of the game. Only once during the period did market makers play at an equal level. The trading activity on October 25 appears to be an arranged short stock interest arbitrage between a market maker and a proprietary firm in that the number of exercises by each

[26] With open interest remaining at 4,578 contracts, the proprietary firms were assigned delivery on 5,422 of the 10,002 put option contracts that they, themselves, exercised.

Table X

Trading Volume, Open Interest, Number of Exercises, and Number of Long Contracts Held by Pre-Existing Shorts for the January 2006 65-Put Option on Wal-Mart during the Period September 27 through October 6, 2005

Original shorts hold 4,224 contracts at the close on September 26, 2005.

		Number of Contracts Exercised				Proportion of Open Interest		
Date	Open Interest	Customer	Market Maker	Proprietary Firm	Total	New Positions	Old Positions	Old Shorts
9/27/2005	4,224	0	4,219	8,438	12,657	0.750	0.250	1,057
9/28/2005	4,224	0	0	10,002	10,002	0.703	0.297	314
9/29/2005	4,224	0	6,000	12,000	18,000	0.810	0.190	60
9/30/2005	4,224	0	0	12,002	12,002	0.740	0.260	16
10/3/2005	4,211	0	40	0	40	0.009	0.991	15
10/4/2005	4,211	0	4,000	21,002	25,002	0.856	0.144	2
10/5/2005	4,211	0	4,019	10,021	14,040	0.769	0.231	1
10/6/2005	4,211	0	0	6,002	6,002	0.588	0.412	0

was 2,719 and the total trading volume was 5,438. Once during the period, on September 14 and 15, the arbitrage activity stretches over two days. The trading/exercise activity on September 14 appears to be short stock interest arbitrage, but only 6,002 of the 8,115 contracts traded are exercised. Of the 2,113 long positions left open, however, 2,100 are exercised on the following day, leaving a net increase in open interest of only 13 contracts.

The speed and effectiveness of short stock interest arbitrage at capturing open interest is implied by the trading/exercise activity from September 27 through October 6. See Table X. For the sake of argument, assume that, at the close of trading on September 26, all 4,224 open short contracts are held by retail customers. On September 27, the short stock interest arbitrage of 12,657 contracts implies that newcomers (one-third market makers and two-thirds proprietary firms) capture about $\frac{12,657}{4,224+12,657} = 75.0\%$ of the open interest, leaving pre-existing shorts (i.e., the retail customers) with about $0.250 \times 4,224 = 1,057$ contracts as shown in the last column of the table. On September 28, short stock interest arbitrage reduces the number to about $0.297 \times 1,057 = 314$ contracts, and so on. By October 6, none of the original shorts (i.e., retail customers) have open positions. Proprietary firms (predominantly) and market makers are now the exclusive owners of the forgone interest income that the longs continue to provide.

B. Aggregate Short Stock Interest Arbitrage Activity

Table IX suggests that proprietary firms are the most active short stock interest arbitragers. The evidence is modest, however, considering the table contains only one put option series class for a three-month period. We now turn to examining aggregate SSIA exercise behavior across all put option series in

Table XI

Number of Put Option Contracts Exercised Early by Market Participant and Trading Activity during the Sample Period July 2001 through September 2008

Panel A contains the total number of exercises. Panel B contains number of exercises associated with trades earmarked as short stock index arbitrage (SSIA) trades. Panel C contains numbers of exercises in trades not associated with SSIA. The column headings "Less than $1,000" and "Greater than or equal to $1,000" refer to the size of the net interest income of the option series on that day. The number of option classes is 4,011.

	No Constraint		Less than $1,000		Greater than or Equal to $1,000	
	Number	Percent (%)	Number	Percent (%)	Number	Percent (%)
			Panel A: Total Exercises			
Customer	18,325,606	9.9	9,283,416	13.3	9,042,190	7.9
Market maker	87,748,104	47.4	35,081,494	50.2	52,666,610	45.8
Proprietary firm	78,860,356	42.6	25,588,148	36.6	53,272,208	46.3
Total	184,934,066		69,953,058		114,981,008	
			Panel B: SSIA Exercises			
Customer	2,209,460	2.1	459,501	2.1	1,749,959	2.0
Market maker	44,913,815	41.9	7,714,900	35.6	37,198,915	43.5
Proprietary firm	60,142,396	56.1	13,506,313	62.3	46,636,083	54.5
Total	107,265,671		21,680,714		85,584,957	
			Panel C: Non-SSIA Exercises			
Customer	16,116,146	20.7	8,823,915	18.3	7,292,231	24.8
Market maker	42,834,289	55.2	27,366,594	56.7	15,467,695	52.6
Proprietary firm	18,717,960	24.1	12,081,835	25.0	6,636,125	22.6
Total	77,668,395		48,272,344		29,396,051	

all option classes across all days during the sample period July 2001 through September 2008. To do so, we need to apply a rule for identifying SSIA trades. The rule that we use is that the number of contracts exercised for a particular put option series on a given day is earmarked as SSIA trading activity if (a) the total number of exercises by market makers and proprietary firms exceeds 1,000 contracts and (b) the trading volume exceeds 1,000 contracts. Note that this rule accurately identifies all SSIA trades in Table IX.

The aggregate exercise activity results are reported in Table XI. The "No constraint" columns correspond to the case in which the arbitrager pays no exchange fees. The results are telling in two respects. First, note that, of the total exercises reported in Panel A, 107,265,671 (see Panel B) or 58.0% are associated with SSIA activity. In other words, nearly six-tenths of all put option exercises are "game playing," designed to capture the short open interest from existing shorts. Second, the game is being played almost exclusively by market makers (41.9%) and proprietary firms (56.1%).

The columns headed "Less than $1,000" and "Greater than or equal to $1,000" categorize the number of exercises by net interest income, where $1,000 corresponds to the exchange fee cap discussed earlier. The most noteworthy result is that SSIA activity grows as the net interest income grows, which should come as no surprise. Proprietary firms are the single largest market participant, followed by market makers. For non-SSIA trades, market makers tend to exercise most frequently.

In summary, long put option holders fail to exercise when they should and leave substantial sums of money on the table. Knowing this, market makers and proprietary firms step in front of the holders of existing short open interest to earn the net interest income proceeds, and, from the analysis in this section, are very successful in their efforts.

V. Summary and Conclusions

In the United States, exchange-traded stock options may be exercised at any time before contract expiration. While a number of studies examined early exercise decisions of long call option holders, the early exercise behavior of long put option holders has gone largely unexplored. The reason is that the decision to exercise a put option early is more complicated than it is for a call. Unlike call options, which may be exercised optimally only on the day before the ex-dividend day, put options may be optimally exercised on any day up to and including the expiration day. This study develops an analytical put option early exercise decision rule and then reconciles the rule in light of actual early exercise behavior in U.S. stock options markets.

Using a sample of put options on stocks over the period January 1996 through September 2008, we find that over 3.96 million put option contracts remain unexercised on days when they should be and that the cost to long put option holders was nearly $1.9 billion. One possible explanation for the failed exercises is exercise/trading costs; however, based on our decision rule and plausible estimates of trading costs, this explanation accounts for only a small fraction of the total forgone net interest income—nearly $1.82 billion remains. We also provide indirect evidence that the net interest income is being largely forfeited by customers (i.e., nonprofessional traders). By engaging in short stock index arbitrage, both market makers and proprietary firms demonstrate that they know the early exercise decision rule and apply it in a timely and appropriate fashion. That is not to say that the nonprofessional traders are behaving irrationally. The costs of learning the early exercise decision rule and constantly monitoring open put option positions may be too high relative to the perceived benefits.

Whatever the reasons for the failure to exercise, however, $1.9 billion is money left on the table. We find that the main beneficiaries of this forgone net interest income are professional traders (i.e., market makers and proprietary firms) who know when to exercise and constantly monitor their positions. By simultaneously buying (and exercising) and selling thousands of deep

in-the-money put options, they systematically capture available short open interest and earn the interest income being forfeited by the longs. Thus, not only are the longs implicitly paying a premium for the ability to exercise early that they rarely use, but the potential gains to the original shorts are being appropriated by market makers and proprietary firms. Among other things, this raises fundamental concerns regarding contract design and market integrity. If many option buyers pay for the right to early exercise but either cannot or do not take advantage of it as a result of exercise costs, unawareness of appropriate decision rules, inability to continually monitor open positions, or irrationality, would the integrity of the market not be better preserved with stock option contracts that are European-style?

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