Group Project
https://github.com/jhw-0/linear_alg_group_4

Project Report

MAS 3105
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1 Report

1.1 Introduction

1.1.1 Game board

The game board consists of five spaces: Space 1 (Start, S), Space 2 (A), Space 3 (B), Space 4 (C), and Space 5 (Final, F), as seen in Figure 1. The R space is not counted as a "space" since it returns the player to the Space 1. The game piece will begin at Space 1 and the players will follow the game rules to continue.

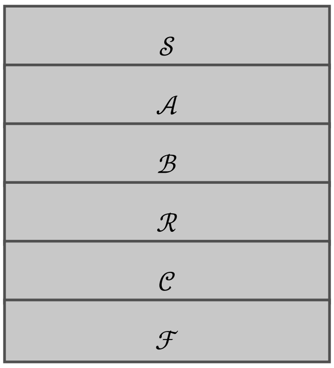

Figure 1: Game board

1.1.2 Game rules

The players will take turns moving the game piece, which begins on Space 1. Player 1 will spin the game spinner as shown in Figure 2. If the spinner lands on a whole number, the player will move the game piece forward that number of times. If the spinner lands on 1.5 or 2.5, the player will flip a coin. If the coin lands on heads, the player will add 0.5 to the number spun and move the game piece accordingly. If the coin lands on tails, the player will subtract 0.5 from the number spun and move the game piece accordingly. Once player 1 is done moving the game piece, player 2 will repeat these steps. If the game piece lands on the "R" space, the game piece will return to Space 1.

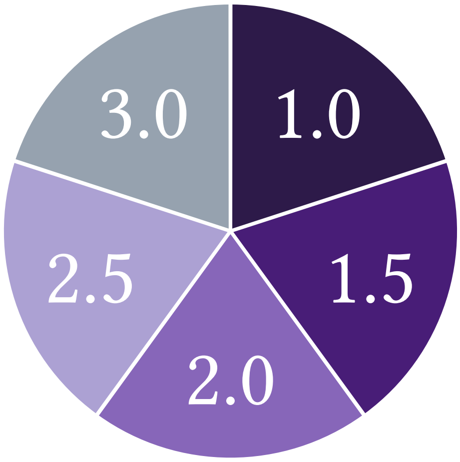

Figure 2: Game spinner

1.2 Procedure

1.2.1 Representing the game as a probabilistic automaton

The game can be represented by the probabilistic automaton $\mathsf{PA} = (\mathsf{NFA}, P, v)$, where NFA is the nondeterministic finite automaton $\mathsf{NFA} = (Q, \Sigma, \delta, q_0, F)$,

> Note [1]: A nondeterministic finite automaton is a 5 -tuple consisting of
> - a set of states Q,
> - a set of inputs Σ,
> - a transition function $\delta : Q \times \Sigma \times Q \to \{0, 1\}$,
> - an initial state $q \in Q$, and
> - a set of accepting states $F \subseteq Q$.

P is a function describing the probability of a state transition taking place, and v is a stochastic vector that redefines the initial state q_0 of NFA probabilistically. NFA is further defined as the 5-tuple containing the objects in Equation 1, and it is visualized in Figure 3

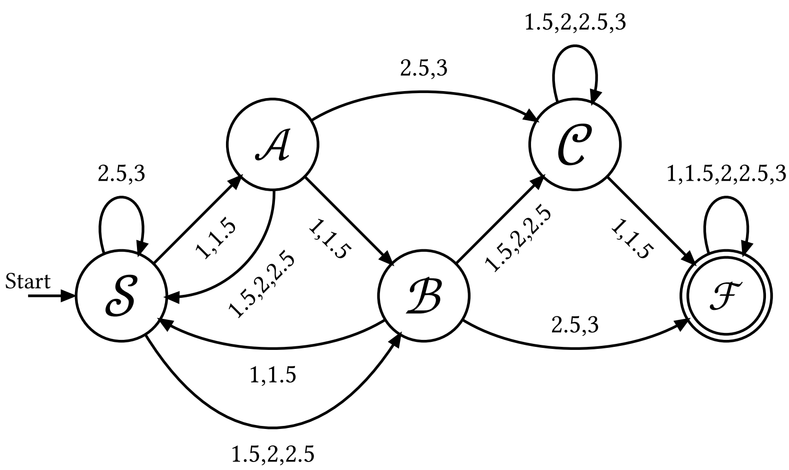

Figure 3: Non-deterministic automaton representing game states and moves. States represent board-game spaces and transitions represent board-game moves.

$$
(1)\quad
\begin{aligned}
Q &= \text{the board spaces except } \mathcal{R} \\
\Sigma &= \text{the possibilities of the spinner} \\
\delta &= \text{as defined by the game rules} \\
q_0 &= \mathcal{S} \\
F &= \{\mathcal{F}\}
\end{aligned}
$$

The game rules (representing δ) are formalized by the recursive function Equation 2 where t is the current turn, $R : \{0, ..., 5\} \to Q$ bijectively in the order of the game board (descending from the top with respect to as it is shown in Figure 1), and b_t is the value of the spin, modified by the coin flip (were there one).

$$
(2)\quad q_{t+1} = \begin{cases} R(\varphi) & \text{if } \varphi \leq 5 \text{ and } \varphi \neq 3 \\ \mathcal{S} & \text{if } \varphi = 3 \\ q_t & \text{otherwise} \end{cases}
$$

$$
\text{where } \varphi = R^{-1}(q_t) + b_t
$$

1.2.2 Representing the game as a stochastic matrix

For each $a \in \Sigma$ of a probabilistic automata, there corresponds a right stochastic matrix P_a [1]. The matrix P_a for each input of the automata representing the game PA (Figure 3) can be found in Section 3.1.2; accordingly [2], if the matrices therein have any row summed, the answer will be 1. A singular stochastic matrix representing the expected input value is P_Σ (Equation 4), which because input probabilities are uniformly distributed, is calculated as the average P_a (Equation 3).

$$
(3)\quad P_\Sigma = \frac{1}{|\Sigma|} \sum P_a
$$

$$
(4)\quad P_\Sigma = \begin{bmatrix} 0.3 & 0.3 & 0.4 & 0 & 0 \\ 0.4 & 0 & 0.3 & 0.3 & 0 \\ 0.3 & 0 & 0 & 0.4 & 0.3 \\ 0 & 0 & 0 & 0.7 & 0.3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}
$$

Thus, the state of the game can be estimated by right-applications of P_Σ to the initial state vector $v = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$ as a row vector; or in other words, for the k^{th} turn the n^{th} value of the row vector given by $v(P_\Sigma)^k$ contains the probability that the the piece will be on the space designated on the board by $R(n-1)$.

1.3 Results

1.3.1 Original game

Figure 4 shows the smallest number of spins that gives 95% probability of finishing the game. The smallest number of spins is 14 spins.

Probability of Finishing the Game in n Spins	
n	Probability
2	0.12
3	0.258
4	0.3915
5	0.5122
6	0.6152
7	0.7004
8	0.7691
9	0.8236
10	0.8662
11	0.8991
12	0.9244
13	0.9435
14	0.958

Figure 4: Probability of finishing the game in n spins.

Figure 5 shows the expected number of times state each will be visited in a finished game for each starting state.

Expected Number of Times state *j* will be visited assuming you started at state *i*		
i	*j*	Number of Visits
1	1	2.3095
1	2	0.6928
1	3	1.1316
1	4	2.2017
2	1	1.1316
2	2	1.3395
2	3	0.8545
2	4	2.4788
3	1	0.6928
3	2	0.2079
3	3	1.3395
3	4	1.9938
4	1	0
4	2	0
4	3	0
4	4	3.3333

Figure 5: Expected number of times visiting each transient state j before a game ends. $R(j-1)$ is the state as shown on the game board.

Figure 6 shows that Space 4 is the transient state that was visited the most when summed over all starting positions and Space 2 is the transient state that was visited the least. This answer does seem to make sense intuitively because Space 4 has the largest chance of being landed on since there are a lot of different spins possible. However, Space 4 does have the property of being included in all games, regardless of starting position.

Total Visits to State *j*	
j	Total Visits
1	4.1339
2	2.2402
3	3.3256
4	10.0076

Figure 6: Total visits to state j (where $R(j-1)$ represents the state shown on the game board) summed across all initial starting positions.

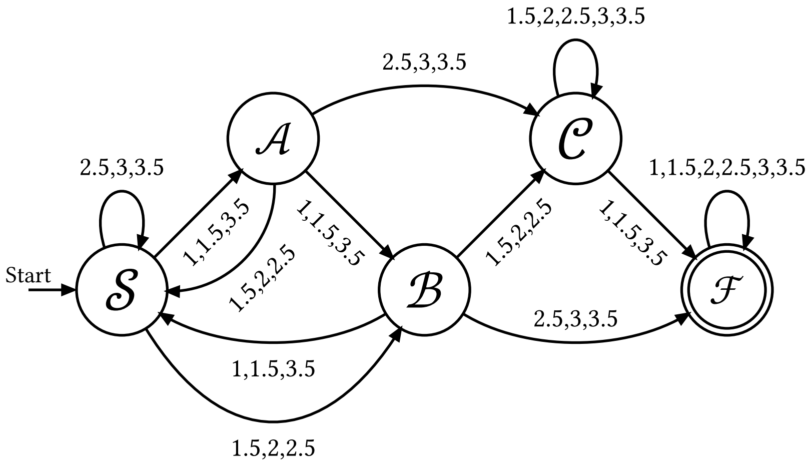

Figure 9: Non-deterministic automaton representing *modified* game inputs. The game has been modified from its original rule-set by allowing '3.5' to be spun.

Expected Number of Spins before Entering the Absorbing State starting at State *i*	
i	Expected Number of Spins
1	6.3356
2	5.8044
3	4.234
4	3.3333

Figure 7: Expected number of spins before the game ends. $R(i-1)$ is the state as shown on the game board.

1.3.2 Adding an input to the spinner

Figure 8 is the modified game spinner. Equation 5 is the resulting transition matrix from the aforementioned change. Figure 10 shows the probability of finishing the game in n spins with this new spinner (see Section 3.2.3 for MATLAB code). Figure 9 is the nondeterministic automata representing the game with the new spinner.

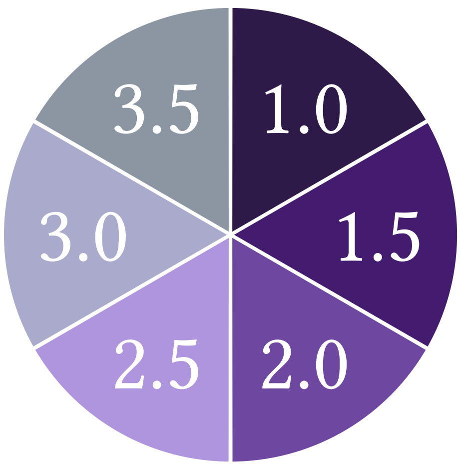

Figure 8: *Modified game spinner*

$$P_{\Sigma,m} = \begin{bmatrix} 0.33 & 0.33 & 0.33 & 0 & 0 \\ 0.33 & 0 & 0.33 & 0.33 & 0 \\ 0.33 & 0 & 0 & 0.33 & 0.33 \\ 0 & 0 & 0 & 0.67 & 0.33 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Probability of Finishing the Game in n Spins	
n	Probability
2	0.1089
3	0.2526
4	0.3838
5	0.525
6	0.6024
7	0.684
8	0.749
9	0.7999
10	0.8392
11	0.8693
12	0.892
13	0.9091
14	0.9219
15	0.9314
16	0.9384
17	0.9436
18	0.9473
19	0.9501

Figure 10: Probability of finishing the game in n spins given the modified game spinner.

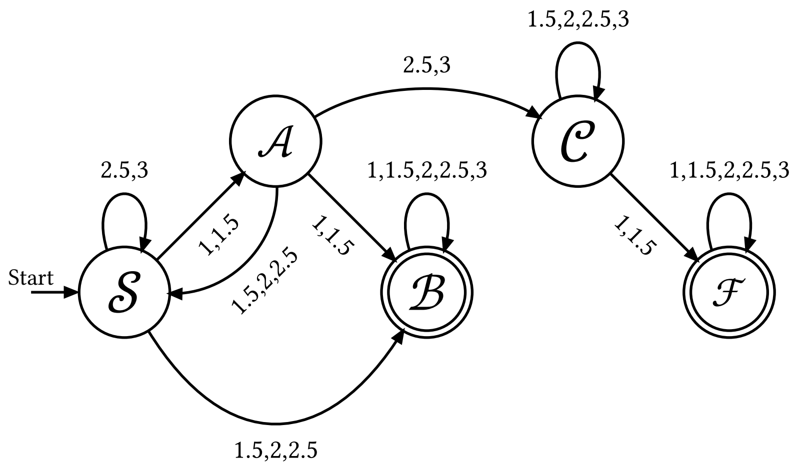

Figure 12: Non-deterministic automaton representing *modified* game states. The game has been modified from its original rule-set by designating $\mathcal{B}$ as an absorbing state loss.

1.3.3 Modifying accepting states

Figure 12 is the nondeterministic automata representing the game with a new absorbing state. Equation 6 is the resulting transition matrix from the aforementioned change. Figure 11 shows the probability of winning (0.1552) and the probability of losing (0.8448) with the corresponding MATLAB code in Section 3.3.3.

$$P_{\Sigma,m'} = \begin{bmatrix} 0.3 & 0.3 & 0.4 & 0 & 0 \\ 0.4 & 0 & 0.3 & 0.3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0.3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

```
>> P^400

ans =

    0.0000    0.0000    0.8448    0.0000    0.1552
    0.0000    0.0000    0.6379    0.0000    0.3621
         0         0    1.0000         0         0
         0         0         0    0.0000    1.0000
         0         0         0         0    1.0000
```

Figure 11: Probability of finishing the game in n spins.

2 Bibliography

[1] Wikipedia contributors, "Probabilistic automaton." Accessed: Dec. 06, 2024. [Online]. Available: https://en.wikipedia.org/wiki/Probabilistic_automaton

[2] Wikipedia contributors, "Stochastic matrix." Accessed: Dec. 06, 2024. [Online]. Available: https://en.wikipedia.org/wiki/Stochastic_matrix

3 Appendices

3.1 Original game

3.1.1 Membership matrices

$$\theta_{1.0} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$\theta_{1.5} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

$$\theta_{2.0} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

$$\theta_{2.5} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

$$\theta_{3.0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

3.1.2 Transition matrices

$$P_{1.0} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$P_{1.5} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$$P_{2.0} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

$$P_{2.5} = \begin{bmatrix} 0.5 & 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$P_{3.0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

3.1.3 MATLAB code

```
% Question 4
P = [0.3 0.3 0.4 0 0;
    0.4 0 0.3 0.3 0;
    0.3 0 0 0.4 0.3;
    0 0 0 0.7 0.3;
    0 0 0 0 1];

P^2
P^3
P^4
P^13
P^14

%% ans =
%%
%%      0.3300    0.0900    0.2100    0.2500    0.1200
%%      0.2100    0.1200    0.1600    0.3300    0.1800
%%      0.0900    0.0900    0.1200    0.2800    0.4200
%%           0         0         0    0.4900    0.5100
%%           0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%      0.1980    0.0990    0.1590    0.2860    0.2580
%%      0.1590    0.0630    0.1200    0.3310    0.3270
%%      0.0990    0.0270    0.0630    0.2710    0.5400
%%           0         0         0    0.3430    0.6570
%%           0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%      0.1467    0.0594    0.1089    0.2935    0.3915
%%      0.1089    0.0477    0.0825    0.2986    0.4623
%%      0.0594    0.0297    0.0477    0.2230    0.6402
%%           0         0         0    0.2401    0.7599
%%           0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%      0.0057    0.0025    0.0043    0.0439    0.9435
%%      0.0043    0.0019    0.0033    0.0364    0.9541
%%      0.0025    0.0011    0.0019    0.0228    0.9718
%%           0         0         0    0.0097    0.9903
%%           0         0         0         0    1.0000
```

```
%%
%%
%% ans =
%%
%%     0.0040    0.0017    0.0030    0.0332    0.9580
%%     0.0030    0.0013    0.0023    0.0273    0.9660
%%     0.0017    0.0007    0.0013    0.0171    0.9792
%%          0         0         0    0.0068    0.9932
%%          0         0         0         0    1.0000

% Question 5
Q = [0.3 0.3 0.4 0;
    0.4 0 0.3 0.3;
    0.3 0 0 0.4;
    0 0 0 0.7];

I = [1 0 0 0;
    0 1 0 0;
    0 0 1 0;
    0 0 0 1];

N = inv(I-Q);
N

sum_of_each_row = sum(N,2);
sum_of_each_row

%% N =
%%
%%     2.3095    0.6928    1.1316    2.2017
%%     1.1316    1.3395    0.8545    2.4788
%%     0.6928    0.2079    1.3395    1.9938
%%          0         0         0    3.3333
%%
%%
%% sum_of_each_row =
%%
%%     6.3356
%%     5.8045
%%     4.2340
%%     3.3333
```

3.2 Game with modified spinner

3.2.1 Membership matrices

$$\theta_{1.0} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

$$\theta_{1.5} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

$$\theta_{2.0} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

$$\theta_{2.5} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

$$\theta_{3.0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

$$\theta_{3.5} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

3.2.2 Transition matrices

$$P_{1.0} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{23}$$

$$P_{1.5} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

$$P_{2.0} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$

$$P_{2.5} = \begin{bmatrix} 0.5 & 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{26}$$

$$P_{3.0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

$$P_{3.5} = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{28}$$

3.2.3 MATLAB code

```
% Question 6

P_2 = [0.33 0.33 0.33 0 0;
    0.33 0 0.33 0.33 0;
    0.33 0 0 0.33 0.33;
    0 0 0 0.67 0.33;
    0 0 0 0 1];

P_2^2
P_2^3
P_2^4
P_2^5
P_2^6
P_2^7
P_2^8
P_2^9
P_2^10
P_2^11
P_2^12
P_2^13
P_2^14
P_2^15
P_2^16
P_2^17
P_2^18
P_2^19

%% ans =
%%
%%      0.3267    0.1089    0.2178    0.2178    0.1089
%%      0.2178    0.1089    0.1089    0.3300    0.2178
%%      0.1089    0.1089    0.1089    0.2211    0.4389
%%           0         0         0    0.4489    0.5511
%%           0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%      0.2156    0.1078    0.1437    0.2537    0.2526
%%      0.1437    0.0719    0.1078    0.2930    0.3626
%%      0.1078    0.0359    0.0719    0.2200    0.5478
%%           0         0         0    0.3008    0.6992
%%           0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%      0.1542    0.0712    0.1067    0.2530    0.3838
%%      0.1067    0.0474    0.0712    0.2556    0.4949
%%      0.0712    0.0356    0.0474    0.1830    0.6441
%%           0         0         0    0.2015    0.7985
%%           0         0         0         0    1.0000
%%
%%
```

```
%% ans =
%%
%%     0.1096    0.0509    0.0744    0.2282    0.5025
%%     0.0744    0.0352    0.0509    0.2104    0.6027
%%     0.0509    0.0235    0.0352    0.1500    0.7202
%%          0         0         0    0.1350    0.8650
%%          0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%     0.0775    0.0362    0.0530    0.1942    0.6024
%%     0.0530    0.0245    0.0362    0.1694    0.6889
%%     0.0362    0.0168    0.0245    0.1199    0.7813
%%          0         0         0    0.0905    0.9095
%%          0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%     0.0550    0.0256    0.0375    0.1595    0.6840
%%     0.0375    0.0175    0.0256    0.1335    0.7568
%%     0.0256    0.0119    0.0175    0.0939    0.8289
%%          0         0         0    0.0606    0.9394
%%          0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%     0.0390    0.0181    0.0266    0.1277    0.7490
%%     0.0266    0.0124    0.0181    0.1037    0.8093
%%     0.0181    0.0084    0.0124    0.0727    0.8657
%%          0         0         0    0.0406    0.9594
%%          0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%     0.0276    0.0129    0.0188    0.1003    0.7999
%%     0.0188    0.0088    0.0129    0.0795    0.8494
%%     0.0129    0.0060    0.0088    0.0555    0.8938
%%          0         0         0    0.0272    0.9728
%%          0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%     0.0196    0.0091    0.0134    0.0777    0.8392
%%     0.0134    0.0062    0.0091    0.0604    0.8799
%%     0.0091    0.0042    0.0062    0.0421    0.9150
%%          0         0         0    0.0182    0.9818
%%          0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%     0.0139    0.0065    0.0095    0.0595    0.8693
%%     0.0095    0.0044    0.0065    0.0455    0.9029
%%     0.0065    0.0030    0.0044    0.0316    0.9309
%%          0         0         0    0.0122    0.9878
%%          0         0         0         0    1.0000
%%
```

```
%%
%% ans =
%%
%%     0.0098    0.0046    0.0067    0.0451    0.8920
%%     0.0067    0.0031    0.0046    0.0341    0.9200
%%     0.0046    0.0021    0.0031    0.0237    0.9428
%%          0         0         0    0.0082    0.9918
%%          0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%     0.0070    0.0032    0.0048    0.0339    0.9091
%%     0.0048    0.0022    0.0032    0.0254    0.9328
%%     0.0032    0.0015    0.0022    0.0176    0.9517
%%          0         0         0    0.0055    0.9945
%%          0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%     0.0049    0.0023    0.0034    0.0254    0.9219
%%     0.0034    0.0016    0.0023    0.0188    0.9422
%%     0.0023    0.0011    0.0016    0.0130    0.9582
%%          0         0         0    0.0037    0.9963
%%          0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%     0.0035    0.0016    0.0024    0.0189    0.9314
%%     0.0024    0.0011    0.0016    0.0139    0.9492
%%     0.0016    0.0008    0.0011    0.0096    0.9630
%%          0         0         0    0.0025    0.9975
%%          0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%     0.0025    0.0012    0.0017    0.0140    0.9384
%%     0.0017    0.0008    0.0012    0.0102    0.9543
%%     0.0012    0.0005    0.0008    0.0070    0.9665
%%          0         0         0    0.0016    0.9984
%%          0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%     0.0018    0.0008    0.0012    0.0103    0.9436
%%     0.0012    0.0006    0.0008    0.0075    0.9581
%%     0.0008    0.0004    0.0006    0.0052    0.9691
%%          0         0         0    0.0011    0.9989
%%          0         0         0         0    1.0000
%%
%%
%% ans =
%%
%%     0.0012    0.0006    0.0009    0.0076    0.9473
%%     0.0009    0.0004    0.0006    0.0055    0.9608
%%     0.0006    0.0003    0.0004    0.0038    0.9710
%%          0         0         0    0.0007    0.9993
%%          0         0         0         0    1.0000
```

```
%%
%%
%% ans =
%%
%%     0.0009    0.0004    0.0006    0.0055    0.9501
%%     0.0006    0.0003    0.0004    0.0040    0.9628
%%     0.0004    0.0002    0.0003    0.0027    0.9724
%%          0         0         0    0.0005    0.9995
%%          0         0         0         0    1.0000
```

3.3 Game with modified states

3.3.1 Membership matrices

$$\theta_{1.0} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{29}$$

$$\theta_{1.5} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{30}$$

$$\theta_{2.0} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{31}$$

$$\theta_{2.5} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{32}$$

$$\theta_{3.0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{33}$$

3.3.2 Transition matrices

$$P_{1.0} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{34}$$

$$P_{1.5} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{35}$$

$$P_{2.0} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{36}$$

$$P_{2.5} = \begin{bmatrix} 0.5 & 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{37}$$

$$P_{3.0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{38}$$

3.3.3 MATLAB code

```
>> % Question 7
P = [0.3 0.3 0.4 0 0;
    0.4 0 0.3 0.3 0;
    0 0 1 0 0;
    0 0 0 0.7 0.3;
    0 0 0 0 1];

P^14
P^40

%% ans =
%%
%%     0.0001    0.0001    0.8447    0.0037    0.1514
%%     0.0001    0.0000    0.6378    0.0050    0.3570
%%          0         0    1.0000         0         0
%%          0         0         0    0.0068    0.9932
%%          0         0         0         0    1.0000
%%

%% ans =
%%
%%     0.0000    0.0000    0.8448    0.0000    0.1552
%%     0.0000    0.0000    0.6379    0.0000    0.3621
%%          0         0    1.0000         0         0
%%          0         0         0    0.0000    1.0000
%%          0         0         0         0    1.0000
%%
>> P^200

%% ans =
%%
%%     0.0000    0.0000    0.8448    0.0000    0.1552
%%     0.0000    0.0000    0.6379    0.0000    0.3621
%%          0         0    1.0000         0         0
%%          0         0         0    0.0000    1.0000
%%          0         0         0         0    1.0000
%%
>> P^400

%% ans =
```

```
%%
%%     0.0000    0.0000    0.8448    0.0000    0.1552
%%     0.0000    0.0000    0.6379    0.0000    0.3621
%%          0         0    1.0000         0         0
%%          0         0         0    0.0000    1.0000
%%          0         0         0         0    1.0000
%%
%% >>
```