Two resistor values need to be determined in order ot properly design the differential amplifier. R_{REF} needs to be determined to design the current source. The two transistors M_{2A} and M_{2B} are matched and conduct a total current of I_{SS}. Thus, each conducts a current of $\frac{I_{SS}}{2}$. Because M_D is diode connected, $V_{DS} = V_{GS}$. Therefore, R_{REF} can be determined using Ohm's Law:

$$R_{REF} = \frac{V_{DD} - V_{GS}}{\frac{I_{SS}}{2}} \tag{1}$$

Ignoring channel-length modulation effects, the current through M_D can be acquired using $\frac{k_n'}{2}\frac{W}{L}(V_{GS} - V_{tn})^2 = \frac{I_{SS}}{2}$. Therefore, V_{GS} can acquired using:

$$V_{GS} = V_{tn} + \sqrt{\frac{I_{SS}}{k_n' \frac{W}{L}}} \tag{2}$$

Using equations (2) and (1), $R_{REF} = 15.14\text{k}\Omega$. This ensures that the I_{SS} =450.0µAspecification is met. To achieve maximum output swing, one must know the midpoint of V_{out}^{+} and V_{out}^{-}, referred to as $V_{midpoint}$. From this information, R_D can be acquired:

$$R_D = \frac{V_{DD} - V_{midpoint}}{\frac{I_{SS}}{2}} \tag{3}$$

$V_{midpoint}$ is the only unknown for this circuit. At either V_{out}^{+} or V_{out}^{-}, the highest value is the supply voltage V_{DD} when the transistors are cutoff. The lowest value is approximately the output voltage during the transition from triode to saturation. This occurs when $V_{DS} = V_{GS} - V_{tn}$ or equivalently when $V_D = V_{out}^{+,-} = V_G - V_{tn}$. The transistor is biased at $V_G = 2.5$Vand V_{tn} is known. Therefore, the midpoint of the output voltage at either side of the amplifier occurs at:

$$V_{midpoint} = \frac{V_{High} + V_{Low}}{2} = \frac{V_{DD} + (V_G - V_{tn})}{2} \tag{4}$$

Using equations (3) and (4), $V_{midpoint}$ =3.05Vand R_D =8.67kΩ. Figure (1) depicts the amplifier design with the DC operating point results.

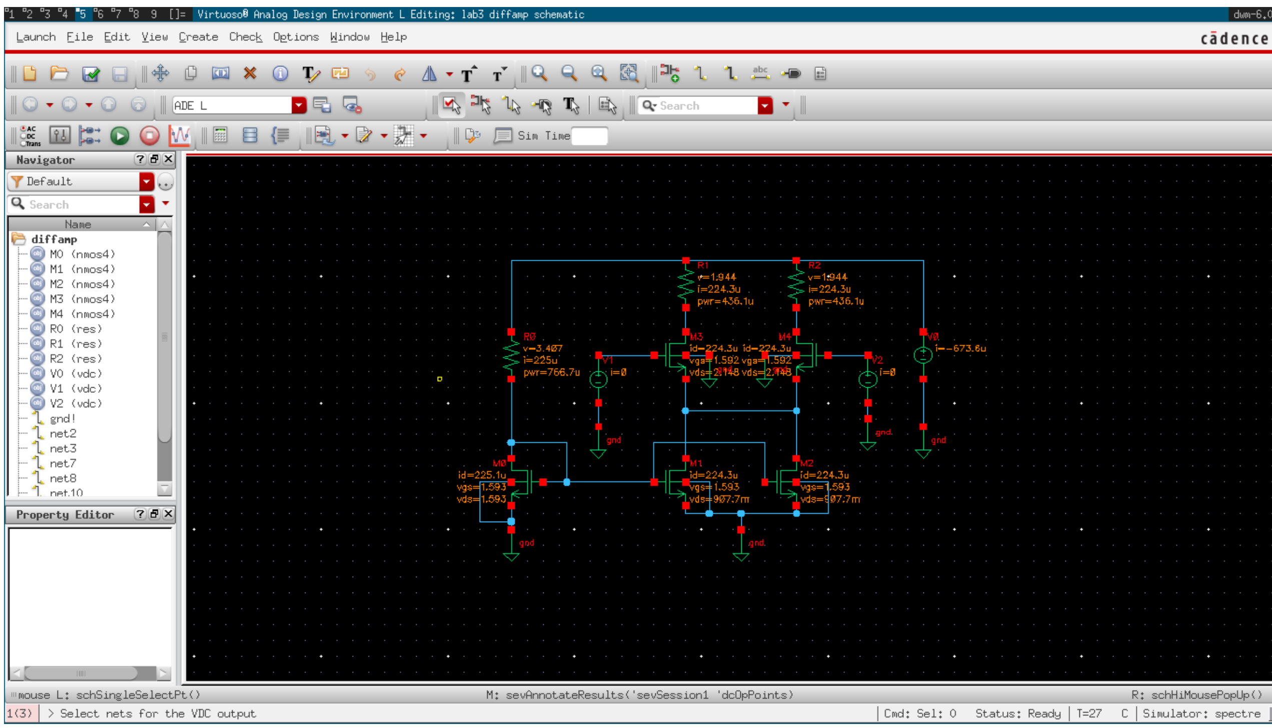

Figure 1: Differential Amplifier with DC Operating Point Results

All of the transistors are in saturation as expected. I_{SS} is about 448.6µA, which is slightly below the desired 450µA. This is a result of channel-length modulation effects causing the currents in the current source to depend on V_{DS}. Let $V_{DS,D}$ be V_{DS} for M_D and $V_{DS,AB}$ be V_{DS} for either M_{2A} or M_{2B}. To a first order approximation, the ratio of the current in M_{2A} or M_{2B} to the current in M_D is:

$$\frac{1 + \lambda_n V_{DS,AB}}{1 + \lambda_n V_{DS,D}} \tag{5}$$

If the current through M_D as well as other relevant parameters from figure (1) and the MOSFET model are used, then the expected current through M_{2A} or M_{2B} is about 224.3µA, which is precisely what is observed in the simulation. If R_{REF} is decreased slightly to 15.09kΩ, I_{SS} becomes exactly 450µA, determined from the sum of the currents through each current source. Figure (2) depicts the results for this second design iteration. It should be noted that this does not affect the output voltage swing since that is determined by R_D. Furthermore, all of the transistors remain in saturation.

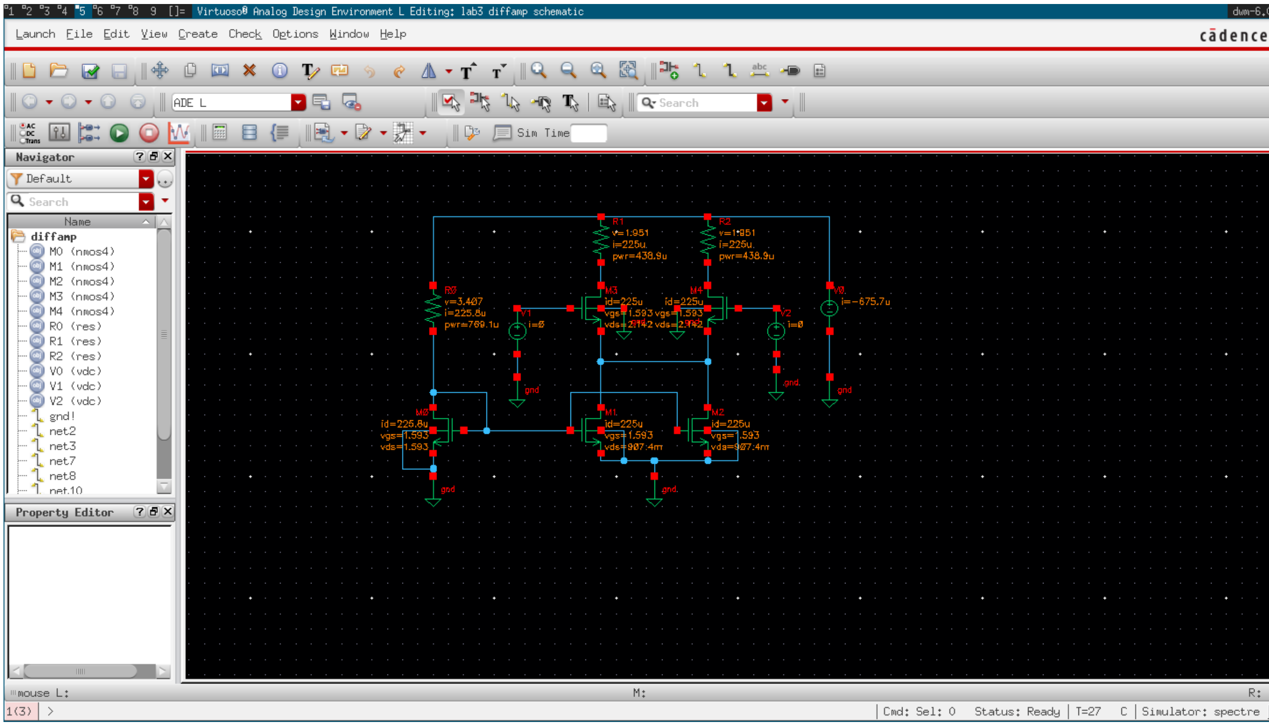

Figure 2: Second Differential Amplifier Design Iteration

Assume the small-signal parameters of the circuit are known. For only differential mode signals, the source of M_{1A} and M_{1B} becomes an AC ground. Therefore, the half-circuit is simply a common-source amplifier with drain resistor R_D. The negative sign disappears since the negative and positive differential inputs are flipped in the circuit. $g_{m1A,B}$ is g_m for both M_{1A} and M_{1B} which turns out to be the same. $r_{o1A,B}$ is the output resistance of M_{1A} and M_{1B}.

$$A_{dm} = g_{m1A,B}(R_D || r_{o1A,B}) \tag{6}$$

The common-mode gain can be acquired by recognizing that the current sources act as MOSFETs with the gate and source grounded since only DC signals exist in the reference part of the circuit. So, the small-signal model of these transistors reduces to the output resistance of each of the MOSFETs, which turn out to be identical. $r_{o2A,B}$ is the output resistance of M_{2A} and M_{2B}. Note that $r_{o1A,B} = r_{o2A,B}$. By analyzing the half-circuit with Kirchhoff's Laws, the following expression is acquired for the common-mode gain:

$$A_{cm} = -\frac{g_{m1A,B} R_D r_{o1A,B}}{R_D + g_{m1A,B} r_{o1A,B}^2} \tag{7}$$

g_m can be calculated using $\frac{2I'_D}{V_{GS}-V_{tn}}$, where I'_D is the drain current excluding channel-length modulation effects. Here, assume $I_D \approx I'_D$ since channel-length modulation effects are negligible. r_o can be calculated using $\frac{1}{\lambda_n I'_D}$. It should be noted that these gain calculations use the values in figure (2).

Table 1: Gains for Part 1 Second Iteration Amplifier

Differential Mode Gain [V/V]	Common Mode Gain [V/V]	Common-Mode Rejection Ratio
20.01	-0.01	2052.54