

In the feedback system displayed in (1), an input $(1/s)$ is applied, and so $V_{out}(s)$ takes the raw form: (1).

Figure 1: Modified system with feedback gain

$$V_{out}(s) = \frac{1}{s} \frac{\frac{kC_1}{C_2s+1}}{1 + \frac{0.1kC_1}{C_2s+1}} \quad (1)$$

With some manipulation, this turns into (2).

$$V_{out}(s) = \frac{kC_1/C_2}{s} \frac{1}{s + \frac{1+0.1kC_1}{C_2}} \quad (2)$$

With an inverse Laplace transform, this turns into (3).

$$V_{out}(t) = \frac{kC_1/C_2}{\frac{1+0.1kC_1}{C_2}} (1 - e^{-(\frac{1+0.1kC_1}{C_2})t}) \quad (3)$$

Finally, this simplifies to (4).

$$V_{out}(t) = \frac{kC_1}{1 + 0.1kC_1} (1 - e^{-(\frac{1+0.1kC_1}{C_2})t}) \quad (4)$$

This implies that the steady state value is $\frac{kC_1}{1+0.1kC_1}$, and the time constant is $\frac{C_2}{1+0.1kC_1}$.