10.8.2 Example 2

Is $\{1, 2\} \subseteq \{1, 2, 3, 4\} \subseteq \mathbb{Z}$? Yes.
Is $\{-1, 0, 1\} \subseteq \mathbb{N}$? No. -1 is NOT a member of $\mathbb{N}$.
Is $1 \subseteq \{1, 2, 3\}$? No. To be a subset, something must be a set.
Is $\{1\} \subseteq \{1, 2, 3\}$? Yes.
Is $\{1\} \subseteq \{\mathbb{Z}\}$? No. 1 is an element of an element of B, but is not in B. B has only one element.

10.9 Proper subsets

The symbol for proper subset is $A \subset B$. At least one item in B must NOT be in A.

10.9.1 Example

If $S \subseteq S$, then $\emptyset \subseteq S$. How do you prove that A is a subset of B? Prove that $x \in A \to x \in B$. So, for this example, we can use a vacuous proof. x is NEVER $\subseteq \emptyset$, so the hypothesis of the conditional is never satisfied, and the conditional is always valid.

10.10 Ending examples

Contrast the following:

Name	Set (set builder)	Set (list)
A	$\{3x \mid x \in \mathbb{Z}\}$	$\{0, 3, 6, 9, ...\}$
B	$\{x \mid 3x \in \mathbb{Z}\}$	$\{\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, ...\}$

Find A, B such that $A \in B \land A \subseteq B$.
$A = \emptyset$ $B = \{\emptyset\}$

11 Lecture 11 (2024-02-02 Fri) - Set Theory (cont.)

"$x \in A$" means "x is an element of A"
"$x \subseteq B$" means "A is a subset of B", or "All elements of A are also elements of B"

Warning!
Do NOT say that A is **in** B if you really mean that A is a subset of B. Although it may seem natural, the wording is ambiguous between "is an element of" and "is a subset of".

Example:

$$\text{Let } U = \{1, 2, 3, 4, \{1, 2\}, \{\emptyset\}\}$$

Element	$x \in U$	$x \subseteq U$
1	Y	N
$\emptyset$	N	Y
$\{1\}$	N	Y
$\{\emptyset\}$	Y	N
$\{1, 2\}$	Y	Y
$\{1, 2, 3\}$	N	Y
$\{\{\emptyset\}\}$	N	Y
$\{1, 2, \emptyset\}$	N	N

11.1 Power sets