24.2 Example 2 (Proof by Induction)

Proof:
Conjecture: $P(n) : 0 + 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$
We will proceed with a proof by induction to prove $P(n)$ for all $n \in \mathbb{N}$.

Base Case: $P(0)$
When $n = 0$:

$$P(0) = 0 + 1 + 2 + \cdots + n = 0$$

$$\frac{n(n+1)}{2} = \frac{0(0+1)}{2} = 0$$

Therefore, the base case holds.

Inductive Case:
To prove the inductive case, we must prove that $P(k) \to P(k+1)$. We will do so with a direct proof.
Direct Proof:

Statement	Reasoning
$1 + 2 + \cdots + k = \frac{k(k+1)}{2}$	Assume inductive hypothesis
$1 + 2 + \cdots + k + (k+1) = \frac{k(k+1)}{2} + k + 1$	Add $k+1$ to both sides
$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$	Multiply term by 1
$= \frac{k(k+1)+2(k+1)}{2}$	Add fractions
$= \frac{(k+1)(k+2)}{2}$	Factor out $k+2$
$1 + 2 + \cdots + k + (k+1) = \frac{(k+1)(k+1+1)}{2}$	Substitute 2 with 1+1

Therefore, we have proven by direct proof that $P(k) \to P(k+1)$.

□

Conclusion:
Since we have shown that $P(n)$ is true for the case $n = 0$, and we have proven that $P(k) \to P(k+1)$ for any arbitrary k, then we have proven that $P(n)$ is true for all $n \in \mathbb{N}$ by mathematical induction.

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25 Lecture 25 (2024-03-08 Fri) - Induction (cont.)

25.1 Towers of Hanoi solution, inductively

Proof: Towers of Hanoi
Conjecture:
Solving the Towers of Hanoi problem for n disks, for any $n \in \mathbb{Z}^+$, takes $2^n - 1$ moves.

Proof:
We will proceed with a proof by induction to prove that solving the Towers of Hanoi problem for n disks, for any $n \in \mathbb{Z}^+$, takes $2^n - 1$ moves.

Base case:
When $n = 1$, then it takes one move to move the entire tower from one peg to another. Since $2^n - 1 = 2^1 - 1 = 1$, then the base case holds.

Inductive step: