Index

accessible information, 321–323 Adami–Cerf lemma, 553–555 adjoint, 10–11 affine subspace, 48 Alberti's theorem, 160–163 algebra, 14, 17 anti-degradable channel, 598 anti-symmetric subspace, 435–438 antisymmetric subspace, 348 bag, 430–431 Bell inequality, 409–411 Bhattacharyya coefficient, 165 Borel function, 41–42 Borel measure, 42–43 Borel set, 41 Carathéodory's theorem, 48 Cauchy–Schwarz inequality, 4 chain rule for differentiation, 40 channel, 79–108 classical, 103 completely dephasing, 103–104, 237–239 completely depolarizing, 101, 237–238, 467–469 entanglement-breaking, 419 erasure, 599 extremal, 105–108 isometric, 100	replacement, 80, 101 representations of, 84–89 Schur, 239–242 self-complementary, 598 separable, 355–360 unital, 219–253, 467–469 unitary, 80, 100 Werner–Holevo, 179–181 Weyl-covariant, 231–239 channel discrimination, 178–181, 190–197 isometric channels, 194–197 unitary channels, 194–197 channel fidelity, see mapping fidelity χ-distribution, 61 Choi rank, 86 Choi representation, 85–86 classical capacity, 510–513 classical-to-quantum channel code, 523–525, 531–537 cloning of pure states, 464–466 closed set, 37 closure of a set, 37 coherent information, 520–522 commutant, 17 compact set, 38–39 complementary channels, 522 completely bounded trace norm, 181–213 basic properties, 186–189 of Hermitcity-preserving maps,
•	
LOCC, 355, 361–363	190–191
mixed-unitary, 220–230, 467–469 pinching, 221–223 product, 80 quantum-to-classical, 111–113	of tensor product maps, 189 semidefinite program, 203–207 spectral norm characterization, 207–209

complex Euclidean space, 1–7, 67
concave function, 47
conditional Shannon entropy, 276
conditional von Neumann entropy, 291
cone, 46
conjugate, 10–11
continuous function, 38
convex combination, 47
convex function, 47
convex hull, 47–48
convex set, 46
correlation operator, 406–409
•
decoding channel, 511
decoupling, 570–573
dense coding, 392–393, 401–405
dense set, 38
determinant, 16, 17
differentiable function, 39
direct sum, 5–6
of channels, 590–592
discrete Fourier transform, 233
discrete Weyl operators, 231–233
double commutant theorem, 443–445
Dvoretzky's theorem, 482-489
•
eigenvalue, 16–17
eigenvector, 16–17
emulation (of a channel), 510
encoding channel, 511
ensemble of states, 69
entanglement, 72, 339, 371–406
cost, 378–385
distillable, 378–392
entropy, 385
rank, 352–354, 360
transformation, 371–377
entanglement entropy, 491–493
entanglement fidelity, see mapping
fidelity
entanglement generation capacity,
562–568
entanglement-assisted classical capacity,
513–515

```
entanglement-assisted classical capacity
         theorem, 541-560
entanglement-assisted Holevo capacity,
         518-520, 557-559
entanglement-assisted quantum
         capacity, 568-570
environment-assisted channel correction,
         223-227
\varepsilon-net, 40
expected value, 55
extreme point, 48-49
Fannes-Audenaert inequality, 296-298
fidelity, 151-178
    between extensions, 170–171
    Bhattacharyya coefficient
         characterization, 165-168
    block operator characterization,
         156-160
    characterizations, 156–168
    joint convexity, 168–170
    monotonicity, 170
    semidefinite program, 159-160
    sum-of-squares, 171-173
Fubini's theorem, 46
Fuchs-van de Graaf inequalities, 175-178
gradient vector, 39
Haar measure, 450-455
Hayashi-Nagaoka operator inequality,
         530-531
Hoeffding's inequality, 58
Holevo capacity, 515–518
Holevo information, 323-324, 404
Holevo's theorem, 326-327
Holevo-Helstrom theorem, 139-141
Holevo-Schumacher-Westmoreland
         theorem, 523-540
    entanglement-assisted form,
         541-545
Horodecki criterion, 345-349
hyperplane separation theorem, 49
identically distributed random variables,
         57
```

611

image, 12	one-way LOCC, 366
independent random variables, 57	partial, 114–116
induced trace norm, 181–185	pretty good, 148–151
inner product	projective, 116–118
operator, 15–16	separable, 363–368
vector, 3–4	with respect to a basis, 117
instrument, 121	measurement operator, 109
integration, 43–46	midpoint concave function, 47
isotropic state, 347–349, 457–460	midpoint convex function, 47
isotropic twirling channel, 459	minimum output entropy, 493–503
I / : 1: F0	mixture of states, 68
Jensen's inequality, 58	Moore-Penrose pseudo-inverse, 32-33
Jordan–Hahn decomposition, 28	mutual information, 276
kernel, 12	
Klein's inequality, 293	Naimark's theorem, 118
Kraus representation, 86	natural representation, 84–85
unitary equivalence of, 92	Nayak's theorem, 330–333
Kullback–Leibler divergence, see relative	networks of channels, 193–194
entropy	Nielsen's theorem, 371–377
chiopy	non-additivity of Holevo capacity,
Lévy's lemma, 476–482	590–596
Lie Bracket, 17	non-classical correlations, 406–419
Lieb's concavity theorem, 301	norm
Lipschitz function, 38	Euclidean, 4
•	Frobenius, 35–36
majorization, 254–268	<i>p</i> -norm, 4
map	Schatten, 33–36
completely positive, 24, 90	spectral, 35
Hermiticity-preserving, 24, 93	trace, 36
on square operators, 22–24	numerical range, 195
positive, 24	
separable, 355–360, 363	open set, 37
trace-preserving, 24, 95–97	operator, 8–36
unital, 24, 95	density, 18
Weyl-covariant, 231–239	diagonal, 19
mapping fidelity, 173–175	Hermitian, 18, 20–21
Markov's inequality, 58	identity, 15
maximum output fidelity, 200–207	invertible, 15
semidefinite program, 203–207	isometry, 19
mean value, 55	matrix representation, 9–10
measurement, 109–127	normal, 18
extremal, 122–127	permutation invariant, 438–443, 446
information-complete, 119–120	positive definite, 18
LOCC, 363–370	positive semidefinite, 18, 21–22
nondestructive, 110, 120–122	PPT, 386–392

projection, 19
separable, 340–351
square, 14
swap, 102, 347
unitary, 19
operator square root, 29
operator-vector correspondence, 25
optimal measurement, 141, 144–148
criteria, 147–148
semidefinite program, 145–147
orthogonal, 5
orthogonal set, 5
orthonormal basis, 5
orthonormal set, 5
oratoriorima see, s
partial trace, 23–24, 74
Pauli operators, 232, 413
Pinsker's inequality, 287–289
polar decomposition, 31
PPT operator, see operator, PPT
probability measure, 42
probability vector, 47
product measure, 42
purification, 75–78
unitary equivalence of, 78
quantum canacity E61 E69
quantum capacity, 561–568 quantum capacity theorem, 570–588
quantum capacity theorem, 570–566 quantum channel, see channel
quantum de Finetti theorem, 460–464
quantum mutual information, 291 quantum Pinsker inequality, 308
quantum random access code, 327–333 quantum relative entropy, 290–293,
299–306, 308
joint convexity, 300–308
monotonicity, 305–306
qubit, 71
qubit, 71
random variable, 54–61
rank, 12
real Euclidean space, 7–8
register, 63–68
classical, 71, 104
trivial, 66, 81–83
relative entropy, 275–289

joint convexity, 284	
Schmacher's quantum source coding theorem, 316–320	
Schmidt decomposition, 32	
Schur mapping, 239–242	
Schwartz–Zippel lemma, 433	
semidefinite programming, 50–54	
Shannon entropy, 273–289	
concavity, 279	
subadditivity, 282	
Shannon's source coding theorem,	
311–314	
singular value decomposition, 29–31	
singular value theorem, 29–31	
Sion's mmin-max theorem, 50	
source coding, 309–333	
classical, 309–314	
classical into quantum, 320–333	
quantum, 315–320	
source coding scheme	
classical, 310	
quantum, 315	
spectral radius, 17	
spectral theorem, 26–28	
standard basis	
operators, 10	
vectors, 5	
standard Borel measure, 42 standard Gaussian measure, 59–61	
standard Gaussian measure, 39–61 standard normal random variable, 59	
standard normai random variable, 59 state	
classical, 65	
classical-quantum, 104	
completely mixed, 70	
exchangeable, 438, 441–443	
flat, 70	
isotropic, 347–349, 457–460	
maximally entangled, 75	
permutation invariant, 427–435,	
441–443	
PPT, 386–392	
probabilistic, 67	
product, 71	

pure, 69

613

quantum, 66-78 reduction of, 73-78 separable, 340-344 Werner, 347-349, 457-460 state discrimination, 135–151 by LOCC measurements, 368-370 by separable measurements, 367-368 convex sets of states, 142-143 ensembles of states, 144-151 unit sphere, 4 pairs of states, 138-141 probabilistic states, 136–138 Stinespring representation, 87 unitary equivalence of, 93 strongly typical string, 545–549 super-activation, 589-590, 596-607 swap operator, 102, 347 symmetric subspace, 348, 428–435 teleportation, 392-401 tensor product of maps, 23 of operators, 13-14

of vectors, 6-7

trace, 15-17

Toeplitz-Hausdorff theorem, 195-196

transpose, 10-11, 102, 184 Tsirelson's bound, 418 Tsirelson's theorem, 412-419 typical string, 312 joint distribution, 525-527

Uhlmann's theorem, 164-165 unextendable product set, 387-389 uniform spherical measure, 447-449, 454-455

vec mapping, see operator-vector correspondence von Neumann entropy, 289–298, 306–307 concavity, 294 continuity, 292 purification technique, 295 strong subadditivity, 306-307 subadditivity, 294-295

weak law of large numbers, 58 Werner state, 347-349, 457-460 Werner twirling channel, 458 Weyl-Brauer operators, 413 Winter's gentle measurement lemma, 154-156

Bibliography

- [1] A. Abeyesinghe, I. Devetak, P. Hayden, and A. Winter. The mother of all protocols: restructuring quantum information's family tree. *Proceedings of the Royal Society A*, 465(2108):2537–2563, 2009.
- [2] C. Adami and N. Cerf. Von Neumann capacity of noisy quantum channels. *Physical Review A*, 56(5):3470–3483, 1997.
- [3] D. Aharonov, A. Kitaev, and N. Nisan. Quantum circuits with mixed states. In *Proceedings of the 30th Annual ACM Symposium on Theory of Computing*, pages 20–30, 1998.
- [4] G. Alber, T. Beth, C. Charnes, A. Delgado, M. Grassl, and M. Mussinger. Stabilizing distinguishable qubits against spontaneous decay by detected-jump correcting quantum codes. *Physical Review Letters*, 86(19):4402–4405, 2001.
- [5] P. Alberti. A note on the transition probability over C*-algebras. *Letters in Mathematical Physics*, 7(1):25–32, 1983.
- [6] P. Alberti and A. Uhlmann. *Stochasticity and Partial Order*, volume 9 of *Mathematics and Its Applications*. D. Reidel Publishing Company, 1982.
- [7] P. Alberti and A. Uhlmann. Stochastic linear maps and transition probability. *Letters in Mathematical Physics*, 7(2):107–112, 1983.
- [8] A. Ambainis, A. Nayak, A. Ta-Shma, and U. Vazirani. Dense quantum coding and a lower bound for 1-way quantum automata. In *Proceedings of the Thirty-First Annual ACM Symposium on Theory of Computing*, pages 376–383, 1999.
- [9] A. Ambainis, A. Nayak, A. Ta-Shma, and U. Vazirani. Dense quantum coding and quantum finite automata. *Journal of the ACM*, 49(4):496–511, 2002.
- [10] T. Ando. Concavity of certain maps on positive definite matrices and applications to Haramard products. *Linear Algebra and Its Applications*, 26:203–241, 1979.
- [11] T. Apostol. Mathematical Analysis. Addison-Wesley, second edition, 1974.
- [12] H. Araki and E. Lieb. Entropy inequalities. Communications in Mathematical Physics, 18(2):160–170, 1970.

- [13] A. Arias, A. Gheondea, and S. Gudder. Fixed points of quantum operations. *Journal of Mathematical Physics*, 43(12):5872–5881, 2002.
- [14] W. Arveson. Subalgebras of C*-algebras. Acta Mathematica, 123(1):141–224, 1969.
- [15] R. Ash. Information Theory. Dover, 1990. Originally published in 1965 by Interscience Publishers.
- [16] G. Aubrun, S. Szarek, and E. Werner. Hasting's additivity counterexample via Dvoretzky's theorem. Communications in Mathematical Physics, 305(1):85–97, 2011.
- [17] K. Audenaert. A sharp Fannes-type inequality for the von Neumann entropy. *Journal of Physics A: Mathematical and Theoretical*, 40(28):8127–8136, 2007.
- [18] K. Audenaert and S. Scheel. On random unitary channels. *New Journal of Physics*, 10:023011, 2008.
- [19] S. Axler. Linear Algebra Done Right. Springer, second edition, 1997.
- [20] H. Barnum and E. Knill. Reversing quantum dynamics with near-optimal quantum and classical fidelity. *Journal of Mathematical Physics*, 43(5):2097–2106, 2002.
- [21] H. Barnum, E. Knill, and M. Nielsen. On quantum fidelities and channel capacities. *IEEE Transactions on Information Theory*, 46(4):1317–1329, 2000.
- [22] H. Barnum, M. Nielsen, and B. Schumacher. Information transmission through a noisy quantum channel. *Physical Reivew A*, 57(6):4153–4175, 1998.
- [23] J. Barrett. Nonsequential positive-operator-valued measurements on entangled mixed states do not always violate a Bell inequality. *Physical Review A*, 65(4):042302, 2002.
- [24] R. Bartle. The Elements of Integration. John Wiley & Sons, Inc., 1966.
- [25] D. Beckman, D. Gottesman, M. Nielsen, and J. Preskill. Causal and localizable quantum operations. *Physical Review A*, 64(5):52309, 2001.
- [26] V. Belavkin. Optimal multiple quantum statistical hypothesis testing. *Stochastics*, 1:315–345, 1975.
- [27] J. Bell. On the Einstein Podolsky Rosen paradox. Physics, 1(3):195–200, 1964.
- [28] A. Ben-Aroya and A. Ta-Shma. On the complexity of approximating the diamond norm. *Quantum Information and Computation*, 10(1):77–86, 2010.
- [29] I. Bengtsson and K. Życzkowski. *Geometry of Quantum States*. Cambridge University Press, 2006.

- [30] C. Bennett, H. Bernstein, S. Popescu, and B. Schumacher. Concentrating partial entanglement by local operations. *Physical Review A*, 53(4):2046–2052, 1996.
- [31] C. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. Wootters. Teleporting an unknown quantum state via dual classical and EPR channels. *Physical Review Letters*, 70(12):1895–1899, 1993.
- [32] C. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. Smolin, and W. Wootters. Purification of noisy entanglement and faithful teleportation via noisy channels. *Physical Review Letters*, 76(5):722–725, 1996.
- [33] C. Bennett, D. DiVincenzo, C. Fuchs, T. Mor, E. Rains, P. Shor, J. Smolin, and W. Wootters. Quantum nonlocality without entanglement. *Physical Review A*, 59:1070–1091, 1999.
- [34] C. Bennett, D. DiVincenzo, T. Mor, P. Shor, J. Smolin, and B. Terhal. Unextendible product bases and bound entanglement. *Physical Review Letters*, 82(26):5385–5388, 1999.
- [35] C. Bennett, D. DiVincenzo, and J. Smolin. Capacities of quantum erasure channels. Physical Review Letters, 78(16):3217–3220, 1997.
- [36] C. Bennett, D. DiVincenzo, J. Smolin, and W. Wootters. Mixed-state entanglement and quantum error correction. *Physical Reivew A*, 54(5):3824–3851, 1996.
- [37] C. Bennett, P. Hayden, D. Leung, P. Shor, and A. Winter. Remote preparation of quantum states. *IEEE Transactions on Information Theory*, 51(1):56–74, 2005.
- [38] C. Bennett and P. Shor. Quantum information theory. IEEE Transactions on Information Theory, 44(6):2724–2742, 1998.
- [39] C. Bennett, P. Shor, J. Smolin, and A. Thapliyal. Entanglement-assisted classical capacity of noisy quantum channels. *Physical Review Letters*, 83(15):3081–3084, 1999.
- [40] C. Bennett and S. Wiesner. Communication via one-and two-particle operators on Einstein–Podolsky–Rosen states. *Physical Review Letters*, 69(20):2881–2884, 1992.
- [41] R. Bhatia. Matrix Analysis. Springer, 1997.
- [42] O. Bratteli, P. Jorgensen, A. Kishimoto, and R. Werner. Pure states on O_d . *Journal of Operator Theory*, 43(1):97–143, 2000.
- [43] F. Buscemi. On the minimum number of unitaries needed to describe a random-unitary channel. *Physics Letters A*, 360(2):256–258, 2006.
- [44] C. Caves, C. Fuchs, and R. Schack. Unknown quantum states: the quantum de Finetti representation. *Journal of Mathematical Physics*, 43(9):4537–4559, 2002.

- [45] A. Childs, D. Leung, L. Mančinska, and M. Ozols. A framework for bounding non-locality of state discrimination. *Communications in Mathematical Physics*, 323(3):1121–1153, 2013.
- [46] A. Childs, J. Preskill, and J. Renes. Quantum information and precision measurement. *Journal of Modern Optics*, 47(2–3):155–176, 2000.
- [47] G. Chiribella, G. D'Ariano, and P. Perinotti. Transforming quantum operations: Quantum supermaps. *Europhysics Letters*, 83(3):30004, 2008.
- [48] G. Chiribella, G. D'Ariano, and P. Perinotti. Theoretical framework for quantum networks. *Physical Review A*, 80(2):022339, 2009.
- [49] E. Chitambar, D. Leung, L. Mančinska, M. Ozols, and A. Winter. Everything you always wanted to know about LOCC (but were afraid to ask). *Communications in Mathematical Physics*, 328(1):303–326, 2014.
- [50] M.-D. Choi. Completely positive linear maps on complex matrices. *Linear Algebra and Its Applications*, 10(3):285–290, 1975.
- [51] M. Christandl, R. König, G. Mitchison, and R. Renner. One-and-a-half quantum de Finetti theorems. *Communications in Mathematical Physics*, 273(2):473–498, 2007.
- [52] J. Clauser, M. Horne, A. Shimony, and R. Holt. Proposed experiment to test local hidden-variable theories. *Physical Review Letters*, 23(15):880–884, 1969.
- [53] T. Cover and J. Thomas. Elements of Information Theory. Wiley Interscience, second edition, 2006.
- [54] E. Davies. On the repeated measurement of continuous observables in quantum mechanics. *Journal of Functional Analysis*, 6(2):318–346, 1970.
- [55] E. Davies and J. Lewis. An operational approach to quantum probability. *Communications in Mathematical Physics*, 17:239–260, 1970.
- [56] B. de Finetti. La prévision : ses lois logiques, ses sources subjectives. *Annales de l'institut Henri Poincaré*, 7(1):1–68, 1937.
- [57] J. de Pillis. Linear transformations which preserve Hermitian and positive semidefinite operators. *Pacific Journal of Mathematics*, 23(1):129–137, 1967.
- [58] I. Devetak. The private classical capacity and quantum capacity of a quantum channel. *IEEE Transactions on Information Theory*, 51(1):44–55, 2005.
- [59] P. Diaconis and D. Freedman. Finite exchangeable sequences. The Annals of Probability, 8(4):745–764, 1980.

- [60] P. Diaconis and M. Shahshahani. The subgroup algorithm for generating uniform random variables. Probability in the Engineering and Informational Sciences, 1(1):15–32, 1987.
- [61] D. DiVincenzo, P. Shor, and J. Smolin. Quantum-channel capacity of very noisy channels. *Physical Review A*, 57(2):830–839, 1998.
- [62] F. Dupuis. *The Decoupling Approach to Quantum Information Theory*. PhD thesis, Univversité de Montréal, 2009.
- [63] A. Dvoretzky. Some results on convex bodies and Banach spaces. In *Proceedings of the International Symposium on Linear Spaces (Held at the Hebrew University of Jerusalem, July 1960)*, pages 123–160, 1961.
- [64] F. Dyson. Statistical theory of the energy levels of complex systems. I. Journal of Mathematical Physics, 3(1):140–156, 1962.
- [65] F. Dyson. Statistical theory of the energy levels of complex systems. II. Journal of Mathematical Physics, 3(1):157–165, 1962.
- [66] F. Dyson. Statistical theory of the energy levels of complex systems. III. Journal of Mathematical Physics, 3(1):166–175, 1962.
- [67] T. Eggeling, D. Schlingemann, and R. Werner. Semicausal operations are semilocalizable. *Europhysics Letters*, 57(6):782–788, 2002.
- [68] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, 47(10):777–780, 1935.
- [69] Y. Eldar and D. Forney. On quantum detection and the square-root measurement. *IEEE Transactions on Information Theory*, 47(3):858–872, 2001.
- [70] Y. Eldar, A. Megretski, and G. Verghese. Designing optimal quantum detectors via semidefinite programming. *IEEE Transactions on Information Theory*, 49(4):1007–1012, 2003.
- [71] M. Fannes. A continuity property of the entropy density for spin lattic systems. *Communications in Mathematical Physics*, 31(4):291–294, 1973.
- [72] W. Feller. *An Introduction to Probability Theory and Its Applications*, volume I. John Wiley & Sons, Inc., third edition, 1968.
- [73] W. Feller. An Introduction to Probability Theory and Its Applications, volume II. John Wiley & Sons, Inc., second edition, 1971.
- [74] C. Fuchs and C. Caves. Mathematical techniques for quantum communication theory. *Open Systems & Information Dynamics*, 3(3):345–356, 1995.

- [75] C. Fuchs and J. van de Graaf. Cryptographic distinguishability measures for quantum-mechanical states. *IEEE Transactions on Information Theory*, 45(4):1216–1227, 1999.
- [76] M. Fukuda and M. Wolf. Simplifying additivity problems using direct sum constructions. *Journal of Mathematical Physics*, 48(7):072101, 2007.
- [77] V. Gheorghiu and R. Griffiths. Separable operations of pure states. *Physical Review A*, 78(2):020304, 2008.
- [78] A. Gilchrist, N. Langford, and M. Nielsen. Distance measures to compare real and ideal quantum processes. *Physical Review A*, 71(6):062310, 2005.
- [79] R. Goodman and N. Wallach. Representations and Invariants of the Classical Groups, volume 68 of Encyclopedia of Mathematics and Its Applications. Cambridge University Press, 1998.
- [80] M. Gregoratti and R. Werner. Quantum lost and found. *Journal of Modern Optics*, 50(67):915–933, 2003.
- [81] W. Greub. Mutilinear Algebra. Springer-Verlag, second edition, 1978.
- [82] L. Gurvits. Classical deterministic complexity of Edmonds' problem and quantum entanglement. In *Proceedings of the Thirty-Fifth Annual ACM Symposium on Theory of Computing*, pages 1–19, 2003.
- [83] L. Gurvits and H. Barnum. Largest separable balls around the maximally mixed bipartite quantum state. *Physical Review A*, 66(6):062311, 2002.
- [84] G. Gutoski and J. Watrous. Quantum interactive proofs with competing provers. In *Proceedings of the 22nd Symposium on Theoretical Aspects of Computer Science*, volume 3404 of *Lecture Notes in Computer Science*, pages 605–616. Springer, 2005.
- [85] G. Gutoski and J. Watrous. Toward a general theory of quantum games. In Proceedings of the 39th Annual ACM Symposium on Theory of Computing, pages 565–574, 2007.
- [86] R. Haag and D. Kastler. An algebraic approach to quantum field theory. *Journal of Mathematical Physics*, 5(7):848–861, 1964.
- [87] A. Haar. Der massbegriff in der theorie der kontinuierlichen gruppen. Annals of Mathematics (Second Series), 34(1):147–169, 1933.
- [88] P. Halmos. Measure Theory. Springer-Verlag, 1974. Originally published in 1950 by Litton Educational Publishing, Inc.
- [89] P. Halmos. *Finite-Dimensional Vector Spaces*. Springer–Verlag, 1978. Originally published in 1942 by Princeton University Press.

- [90] A. Harrow, P. Hayden, and D. Leung. Superdense coding of quantum states. *Physical Review Letters*, 92(18):187901, 2004.
- [91] M. Hastings. Superadditivity of communication capacity using entangled inputs. *Nature Physics*, 5(4):255–257, 2009.
- [92] P. Hausladen, R. Jozsa, B. Schumacher, M. Westmoreland, and W. Wootters. Classical information capacity of a quantum channel. *Physical Review A*, 54(3):1869–1876, 1996.
- [93] P. Hausladen and W. Wootters. A 'pretty good' measurement for distinguishing quantum states. *Journal of Modern Optics*, 41(12):2385–2390, 1994.
- [94] M. Hayashi and H. Nagaoka. General formulas for capacity of classical-quantum channels. IEEE Transactions on Information Theory, 49(7):1753–1768, 2003.
- [95] P. Hayden, M. Horodecki, A. Winter, and J. Yard. A decoupling approach to the quantum capacity. *Open Systems & Information Dynamics*, 15(1):7–19, 2008.
- [96] P. Hayden, D. Leung, P. Shor, and A. Winter. Randomizing quantum states: constructions and applications. *Communications in Mathematical Physics*, 250(2):371–391, 2004.
- [97] P. Hayden, D. Leung, and A. Winter. Aspects of generic entanglement. Communications in Mathematical Physics, 265(1):95–117, 2006.
- [98] P. Hayden, P. Shor, and A. Winter. Random quantum codes from Gaussian ensembles and an uncertainty relation. *Open Systems & Information Dynamics*, 15(1):71–89, 2008.
- [99] P. Hayden and A. Winter. Counterexamples to the maximal p-norm multiplicativity conjecture for all p > 1. Communications in Mathematical Physics, 284(1):263–280, 2008.
- [100] C. Helstrom. Detection theory and quantum mechanics. *Information and Control*, 10:254–291, 1967.
- [101] C. Helstrom. Quantum Detection and Estimation Theory. Academic Press, 1976.
- [102] F. Hiai, M. Ohya, and M. Tsukada. Sufficiency, KMS condition and relative entropy in von Neumann algebras. *Pacific Journal of Mathematics*, 96(1):99–109, 1981.
- [103] K. Hoffman and R. Kunze. Linear Algebra. Prentice-Hall, Inc., second edition, 1971.
- [104] A. Holevo. An analogue of statistical decision theory and noncommutative probability theory. *Trudy Moskovskogo Matematicheskogo Obshchestva*, 26:133–149, 1972.
- [105] A. Holevo. Bounds for the quantity of information transmitted by a quantum communication channel. *Problemy Peredachi Informatsii*, 9(3):3–11, 1973.
- [106] A. Holevo. Information-theoretical aspects of quantum measurement. Problemy Peredachi Informatsii, 9(2):31–42, 1973.

- [107] A. Holevo. Statistical decision theory for quantum systems. *Journal of Multivariate Analysis*, 3:337–394, 1973.
- [108] A. Holevo. Statistical problems in quantum physics. In Proceedings of the Second Japan-USSR Symposium on Probability Theory, volume 330 of Lecture Notes in Mathematics, pages 104–119. Springer, 1973.
- [109] A. Holevo. A note on covariant dynamical semigroups. Reports on Mathematical Physics, 32(2):211–216, 1993.
- [110] A. Holevo. Covariant quantum Markovian evolutions. *Journal of Mathematical Physics*, 37(4):1812–1832, 1996.
- [111] A. Holevo. The capacity of the quantum channel with general signal states. *IEEE Transactions on Information Theory*, 44(1):269–273, 1998.
- [112] A. Holevo. On entanglement-assisted classical capacity. *Journal of Mathematical Physics*, 43(9):4326–4333, 2002.
- [113] A. Horn. Doubly stochastic matrices and the diagonal of a rotation matrix. *American Journal of Mathematics*, 76(3):620–630, 1954.
- [114] R. Horn and C. Johnson. *Matrix Analysis*. Cambridge University Press, 1985.
- [115] K. Horodecki, Ł. Pankowski, M. Horodecki, and P. Horodecki. Low-dimensional bound entanglement with one-way distillable cryptographic key. *IEEE Transactions* on *Information Theory*, 54(6):2621–2625, 2008.
- [116] M. Horodecki, P. Horodecki, and R. Horodecki. Separability of mixed states: necessary and sufficient conditions. *Physics Letters A*, 223(1):1–8, 1996.
- [117] M. Horodecki, P. Horodecki, and R. Horodecki. Mixed-state entanglement and distillation: is there a "bound" entanglement in nature? *Physical Review Letters*, 80(24):5239–5242, 1998.
- [118] M. Horodecki, J. Oppenheim, and A. Winter. Quantum state merging and negative information. *Communications in Mathematical Physics*, 269(1):107–136, 2007.
- [119] P. Horodecki. Separability criterion and inseparable mixed states with positive partial transposition. *Physics Letters A*, 232(5):333–339, 1997.
- [120] P. Horodecki. From entanglement witnesses to positive maps: towards optimal characterisation of separability. In A. Gonis and P. Turchi, editors, *Decoherence and its Implications in Quantum Computing and Information Transfer*, volume 182 of *NATO Science Series III: Computer and System Sciences*, pages 299–307. IOS Press, 2001.
- [121] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki. Quantum entanglement. Reviews of modern physics, 81(865):865–942, 2009.

- [122] R. Hudson and G. Moody. Locally normal symmetric states and an analogue of de Finetti's theorem. Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete, 33(4):343–351, 1976.
- [123] L. Hughston, R. Jozsa, and W. Wootters. A complete classification of quantum ensembles having a given density matrix. *Physics Letters A*, 183(1):14–18, 1993.
- [124] R. Jain. Distinguishing sets of quantum states. Unpublished manuscript. Available as arXiv.org e-Print quant-ph/0506205, 2005.
- [125] A. Jamiołkowski. Linear transformations which preserve trace and positive semidefiniteness of operators. Reports on Mathematical Physics, 3(4):275–278, 1972.
- [126] N. Johnston, D. Kribs, and V. Paulsen. Computing stabilized norms for quantum operations. *Quantum Information and Computation*, 9(1):16–35, 2009.
- [127] R. Jozsa. Fidelity for mixed quantum states. Journal of Modern Optics, 41(12):2315–2323, 1994.
- [128] N. Killoran. *Entanglement quantification and quantum benchmarking of optical communication devices*. PhD thesis, University of Waterloo, 2012.
- [129] A. Kitaev. Quantum computations: algorithms and error correction. Russian Mathematical Surveys, 52(6):1191–1249, 1997.
- [130] A. Kitaev, A. Shen, and M. Vyalyi. Classical and Quantum Computation, volume 47 of Graduate Studies in Mathematics. American Mathematical Society, 2002.
- [131] O. Klein. Zur quantenmechanischen begründung des zweiten hauptsatzes der wärmelehre. Zeitschrift für Physik, 72(11–12):767–775, 1931.
- [132] R. Klesse. A random coding based proof for the quantum coding theorem. *Open Systems & Information Dynamics*, 15(1):21–45, 2008.
- [133] R. König and R. Renner. A de Finetti representation for finite symmetric quantum states. *Journal of Mathematical Physics*, 46(12):122108, 2005.
- [134] K. Kraus. General state changes in quantum theory. *Annals of Physics*, 64:311–335, 1971.
- [135] K. Kraus. States, Effects, and Operations: Fundamental Notions of Quantum Theory. Springer-Verlag, 1983.
- [136] D. Kretschmann, D. Schlingemann, and R. Werner. The information-disturbance tradeoff and the continuity of Stinespring's representation. *IEEE Transactions on In*formation Theory, 54(4):1708–1717, 2008.
- [137] D. Kretschmann and R. Werner. Tema con variazioni: quantum channel capacity. *New Journal of Physics*, 6(1):26, 2004.

- [138] D. Kribs. Quantum channels, wavelets, dilations and representations of O_n . *Proceedings of the Edinburgh Mathematical Society (Series 2)*, 46:421–433, 2003.
- [139] S. Kullback and R. Leibler. On information and sufficiency. Annals of Mathematical Statistics, 22(1):79–86, 1951.
- [140] B. Kümmerer and H. Maassen. The essentially commutative dilations of dynamical semigroups on M_n . Communications in Mathematical Physics, 109(1):1–22, 1987.
- [141] L. Landau. Das dämpfungsproblem in der wellenmechanik. Zeitschrift für Physik, 45:430–441, 1927.
- [142] L. Landau and R. Streater. On Birkhoff's theorem for doubly stochastic completely positive maps of matrix algebras. *Linear Algebra and Its Applications*, 193:107–127, 1993.
- [143] O. Lanford and D. Robinson. Mean entropy of states in quantum-statistical mechanics. *Journal of Mathematical Physics*, 9(7):1120–1125, 1968.
- [144] M. Ledoux. The concentration of measure phenomenon, volume 89 of Mathematical Surveys and Monographs. American Mathematical Society, 2001.
- [145] P. Lévy. Problémes concrets d'analyse fonctionelle. Gauthier Villars, 1951.
- [146] E. Lieb. Convex trace functions and the Wigner-Yanase-Dyson conjecture. Advances in Mathematics, 11(3):267–288, 1973.
- [147] E. Lieb and M. Ruskai. Proof of the strong subadditivity of quantum-mechanical entropy. *Journal of Mathematical Physics*, 14(12):1938–1941, 1973.
- [148] G. Lindblad. Expectation and entropy inequalities for finite quantum systems. *Communications in Mathematical Physics*, 39(2):111–119, 1974.
- [149] G. Lindblad. A general no-cloning theorem. Letters in Mathematical Physics, 47(2):189– 196, 1999.
- [150] S. Lloyd. Capacity of the noisy quantum channel. Physical Review A, 55(3):1613–1622, 1997.
- [151] H.-K. Lo and S. Popescu. Concentrating entanglement by local actions:âĂČbeyond mean values. *Physical Review A*, 63(2):022301, 2001.
- [152] M. Marcus. Finite Dimensional Multilinear Algebra, volume 1. Marcel Decker, 1973.
- [153] M. Marcus. Finite Dimensional Multilinear Algebra, volume 2. Marcel Decker, 1975.
- [154] A. Marshall, I. Olkin, and B. Arnold. *Inequalities: Theory of Majorization and Its Applications*. Springer, second edition, 2011.

- [155] B. Maurey and G. Pisier. Séries de variables aléatoires vectorielles indépendantes et propriétés géométriques des espaces de Banach. Studia Mathematica, 58(1):45–90, 1976.
- [156] M. Mehta. Random Matrices. Elsevier, 2004.
- [157] V. Mil'man. New proof of the theorem of A. Dvoretzky on intersections of convex bodies. Functional Analysis and Its Applications, 5(4):288–295, 1971.
- [158] V. Milman and G. Schechtman. Asymptotic theory of finite dimensional normed spaces, volume 1200 of Lecture Notes in Mathematics. Springer, 1986.
- [159] M. Naimark. On a representation of additive operator set functions. *Doklady Akademii Nauk SSSR*, 41:359–361, 1943.
- [160] M. Nathanson. Distinguishing bipartitite orthogonal states using LOCC: Best and worst cases. *Journal of Mathematical Physics*, 46(6):062103, 2005.
- [161] A. Nayak. Lower bounds for Quantum Computation and Communication. PhD thesis, University of California, Berkeley, 1999.
- [162] A. Nayak. Optimal lower bounds for quantum automata and random access codes. In 40th Annual IEEE Symposium on Foundations of Computer Science, pages 369–376, 1999.
- [163] M. Nielsen. Conditions for a class of entanglement transformations. *Physical Review Letters*, 83(2):436–439, 1999.
- [164] M. Nielsen. Probability distributions consistent with a mixed state. *Physical Review A*, 62(5):052308, 2000.
- [165] M. Nielsen and I. Chuang. Quantum Computation and Quantum Information. Cambridge University Press, 2000.
- [166] M. Nielson. Quantum Information Theory. PhD thesis, University of New Mexico, 1998.
- [167] K. Parthasarathy. Extremal decision rules in quantum hypothesis testing. *Infinite Dimensional Analysis, Quantum Probability and Related Topics*, 2(4):557–568, 1999.
- [168] V. Paulsen. Completely Bounded Maps and Operator Algebras. Cambridge Studies in Advanced Mathematics. Cambridge University Press, 2002.
- [169] A. Peres. Quantum Theory: Concepts and Methods. Kluwer Academic Publishers, 1993.
- [170] A. Peres. Separability criterion for density matrices. *Physical Review Letters*, 77(8):1413–1415, 1996.

- [171] A. Peres and W. Wootters. Optimal detection of quantum information. *Physical Review Letters*, 66(9):1119–1122, 1991.
- [172] D. Perez-Garcia, M. Wolf, D. Petz, and M. Ruskai. Contractivity of positive and trace-preserving maps under L_p norms. *Journal of Mathematical Physics*, 47(8):083506, 2006.
- [173] M. Pinsker. Information and Information Stability of Random Variables and Processes. Holden–Day, 1964.
- [174] E. Rains. Entanglement purification via separable superoperators. Unpublished manuscript. Available as arXiv.org e-Print quant-ph/9707002, 1997.
- [175] R. T. Rockafellar. Convex Analysis. Princeton University Press, 1970.
- [176] R. Rosenkrantz, editor. E. T. Jaynes: Papers on Probability, Statistics and Statistical Physics. Kluwer Academic Publishers, 1989.
- [177] B. Rosgen and J. Watrous. On the hardness of distinguishing mixed-state quantum computations. In *Proceedings of the 20th Annual Conference on Computational Complex*ity, pages 344–354, 2005.
- [178] W. Rudin. Principles of Mathematical Analysis. McGraw-Hill, 1964.
- [179] B. Russo and H. Dye. A note on unitary operators in C*-algebras. *Duke Mathematical Journal*, 33(2):413–416, 1966.
- [180] E. Schrödinger. Die gegenwärtige situation in der quantenmechanik. Die Naturwissenschaften, 23(48):807–812, 1935.
- [181] E. Schrödinger. Die gegenwärtige situation in der quantenmechanik. Die Naturwissenschaften, 23(49):823–828, 1935.
- [182] E. Schrödinger. Die gegenwärtige situation in der quantenmechanik. Die Naturwissenschaften, 23(50):844–849, 1935.
- [183] E. Schrödinger. Discussion of probability relations between separated systems. *Mathematical Proceedings of the Cambridge Philosophical Society*, 31(4):555–563, 1935.
- [184] E. Schrödinger. Probability relations between separated systems. Mathematical Proceedings of the Cambridge Philosophical Society, 32(3):446–452, 1936.
- [185] B. Schumacher. Quantum coding. Physical Review A, 51(4):2738–2747, 1995.
- [186] B. Schumacher. Sending entanglement through noisy quantum channels. *Physical Review A*, 54(4):2614–2628, 1996.
- [187] B. Schumacher and M. Nielsen. Quantum data processing and error correction. *Physical Review A*, 54(4):2629–2635, 1996.

- [188] B. Schumacher and M. Westmoreland. Sending classical information via noisy quantum channels. *Physical Review A*, 56(1):131–138, 1997.
- [189] I. Schur. Über eine klasse von mittelbildungen mit anwendungen auf die determinantentheorie. Sitzungsberichte der Berliner Mathematischen Gesellschaft, 22:9–20, 1923.
- [190] J. Schur. Bemerkungen zur theorie der beschränkten bilinearformen mit unendlich vielen veränderlichen. Journal für die reine und angewandte Mathematik, 140:1–28, 1911.
- [191] C. Shannon. A mathematical theory of communication. *The Bell System Technical Journal*, 27:379–423, 1948.
- [192] P. Shor. Equivalence of additivity questions in quantum information theory. *Communications in Mathematical Physics*, 246(3):453–472, 2004.
- [193] B. Simon. *Trace ideals and their applications*, volume 35 of *London Mathematical Society Lecture Note Series*. Cambridge University Press, 1979.
- [194] G. Smith and J. Yard. Quantum communication with zero-capacity channels. *Science*, 321(5897):1812–1815, 2008.
- [195] R. Smith. Completely bounded maps between C*-algebras. Journal of the London Mathematical Society, 2(1):157–166, 1983.
- [196] R. Spekkens and T. Rudolph. Degrees of concealment and bindingness in quantum bit commitment protocols. *Physical Review A*, 65(1):012310, 2001.
- [197] W. F. Stinespring. Positive functions on C*-algebras. Proceedings of the American Mathematical Society, 6(2):211–216, 1955.
- [198] E. Størmer. Positive linear maps of operator algebras. Acta Mathematica, 110(1):233–278, 1963.
- [199] M. Talagrand. The Generic Chaining: Upper and Lower Bounds of Stochastic Processes. Springer, 2006.
- [200] B. Terhal and P. Horodecki. Schmidt number for density matrices. *Physical Review A*, 61(4):040301, 2000.
- [201] R. Timoney. Computing the norms of elementary operators. Illinois Journal of Mathematics, 47(4):1207–1226, 2003.
- [202] S. Tregub. Bistochastic operators on finite-dimensional von Neumann algebras. Izvestiya Vysshikh Uchebnykh Zavedenii Matematika, 30(3):75–77, 1986.
- [203] M. Tribus and E. McIrvine. Energy and information. Scientific American, 225(3):179– 188, 1971.

- [204] J. Trimmer. The present situation in quantum mechanics: a translation of Schrödinger's "cat paradox" paper. Proceedings of the American Philosophical Society, 124(5):323–338, 1980.
- [205] B. Tsirel'son. Quantum analogues of the Bell inequalities. the case of two spatially separated domains. *Journal of Soviet Mathematics*, 36:557–570, 1987.
- [206] A. Uhlmann. Sätze über dichtematrizen. Wissenschaftliche Zeitschrift der Karl-Marx-Universitat Leipzig. Mathematisch-naturwissenschaftliche Reihe, 20(4/5):633–653, 1971.
- [207] A. Uhlmann. Endlich-dimensionale dichtematrizen I. Wissenschaftliche Zeitschrift der Karl-Marx-Universitat Leipzig. Mathematisch-naturwissenschaftliche Reihe, 21(4):421– 452, 1972.
- [208] A. Uhlmann. Endlich-dimensionale dichtematrizen II. Wissenschaftliche Zeitschrift der Karl-Marx-Universitat Leipzig. Mathematisch-naturwissenschaftliche Reihe, 22(2):139– 177, 1973.
- [209] A. Uhlmann. The "transition probability" in the state space of a *-algebra. *Reports on Mathematical Physics*, 9(2):273–279, 1976.
- [210] A. Uhlmann. Relative entropy and the Wigner–Yanase–Dyson–Lieb concavity in an interpolation theory. *Communications in Mathematical Physics*, 54(1):21–32, 1977.
- [211] H. Umegaki. Conditional expectations in an operator algebra IV (entropy and information). *Kodai Mathematical Seminar Reports*, 14(2):59–85, 1962.
- [212] V. Vedral, M. Plenio, M. Rippin, and P. Knight. Quantifying entanglement. *Physical Review Letters*, 78(12):2275–2278, 1997.
- [213] J. von Neumann. Thermodynamik quantenmechanischer gesamtheiten. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen*, 1(11):273–291, 1927.
- [214] J. von Neumann. Wahrscheinlichkeitstheoretischer aufbau der mechanik. *Nach-richten von der Gesellschaft der Wissenschaften zu Göttingen*, 1(11):245–272, 1927.
- [215] J. von Neumann. Zur algebra der funktionaloperationen und theorie der normalen operatoren. *Mathematische Annalen*, 102(1):370–427, 1930.
- [216] J. von Neumann. Die einfuhrung analytischer parameter in topologischen gruppen. *Annals of Mathematics (Second Series)*, 34(1):170–179, 1933.
- [217] J. von Neumann. Mathematical Foundations of Quantum Mechanics. Princeton University Press, 1955. Originally published in German in 1932 as Mathematische Grundlagen der Quantenmechanik.
- [218] J. Walgate, A. Short, L. Hardy, and V. Vedral. Local distinguishability of multipartite orthogonal quantum states. *Physical Review Letters*, 85(23):4972–4975, 2000.

- [219] J. Watrous. Notes on super-operator norms induced by Schatten norms. *Quantum Information and Computation*, 5(1):58–68, 2005.
- [220] J. Watrous. Distinguishing quantum operations having few Kraus operators. *Quantum Information and Computation*, 8(9):819–833, 2008.
- [221] J. Watrous. Mixing doubly stochastic quantum channels with the completely depolarizing channel. *Quantum Information and Computation*, 9(5/6):406–413, 2009.
- [222] J. Watrous. Semidefinite programs for completely bounded norms. *Theory of Computing*, 5(11), 2009.
- [223] J. Watrous. Simpler semidefinite programs for completely bounded norms. *Chicago Journal of Theoretical Computer Science*, 2013:8, 2013.
- [224] A. Weil. *L'intégration dans les groupes topologiques et ses applications*. Hermann, second edition, 1979. Originally published in 1940.
- [225] R. Werner. Quantum states with Einstein–Podolsky–Rosen correlations admitting a hidden-variable model. *Physical Review A*, 40(8):4277–4281, 1989.
- [226] R. Werner. Optimal cloning of pure states. Physical Review A, 58(3):1827–1832, 1998.
- [227] R. Werner. All teleportation and dense coding schemes. Journal of Physics A: Mathematical and General, 34(35):7081–7094, 2001.
- [228] H. Weyl. The Theory of Groups and Quantum Mechanics. Dover Publications, 1950. Originally published in German in 1929.
- [229] M. Wilde. Quantum Information Theory. Cambridge University Press, 2013.
- [230] A. Winter. Coding theorem and strong converse for quantum channels. *IEEE Transactions on Information Theory*, 45(7):2481–2485, 1999.
- [231] H. Wolkowicz, R. Saigal, and L. Vandenberge, editors. Handbook of Semidefinite Programming: Theory, Algorithms, and Applications. Kluwer Academic Publishers, 2000.
- [232] S. Woronowicz. Positive maps of low dimensional matrix algebras. Reports on Mathematical Physics, 10(2):165–183, 1976.
- [233] D. Yang, M. Horodecki, R. Horodecki, and B. Synak-Radtke. Irreversibility for all bound entangled states. *Physical Review Letters*, 95(19):190501, 2005.
- [234] H. Yuen, R. Kennedy, and M. Lax. On optimal quantum receivers for digital signal detection. *Proceedings of the IEEE*, 58(10):1770–1773, 1970.
- [235] H. Yuen, R. Kennedy, and M. Lax. Optimum testing of multiple hypotheses in quantum detection theory. *IEEE Transactions on Information Theory*, 21(2):125–134, 1975.

- [236] V. Zarikian. Alternating-projection algorithms for operator-theoretic calculation. Linear Algebra and Its Applications, 419(2–3):710–734, 2006.
- [237] K. Życzkowski, P. Horodecki, A. Sanpera, and M. Lewenstein. Volume of the set of separable states. *Physical Review A*, 58(2):883–892, 1998.