Theory of Quantum Information

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Contents

Preface

This is a draft of a book that began as a set of course notes for a graduate course on the theory of quantum information that I have taught several times at the University of Waterloo.

The book is primarily intended for graduate students and researchers having some familiarity with quantum information and computation, such as would be covered in an introductory-level undergraduate or graduate course on the subject. The focus of the book is on the mathematical aspects of quantum information, with an emphasis on proofs. No attention is paid to motives for studying the theory of quantum information, as it is assumed that the reader has already been motivated—and is perhaps interested in proving new theorems on quantum information of his or her own. It should also be said that this is not a physics book: the Schrödinger equation will not be found herein, and the difficult technological challenge of building quantum information processing devices is blissfully ignored.

The selection of topics covered in this book is not intended to be fully representative of the diverse subject of quantum information science. There is, for example, no discussion of quantum cryptography, quantum error correcting codes and fault-tolerance, quantum algorithms and complexity theory, or topological quantum computing, which are among the topics within the theoretical branches of quantum information science having fundamental importance. Nevertheless, one is likely to encounter some of the core mathematical notions discussed in this book when studying these and other topics.

As the students who have taken my course on the theory of quantum information will attest, I sometimes choose to deviate from the standard conventions of quantum information and computation, particularly with respect to notation and terminology. I have exhibited this behavior once again when writing this book. For example, I have avoided the use of the commonly used Dirac notation, and in some cases I have changed the names and symbols associated with concepts as I have seen fit. I hope that readers who have previously grown familiar with the notation and conventions of quantum information that I have chosen not to follow will excuse me for this, and hope that they will find value in this book nevertheless.

Each chapter aside from the first includes a collection of exercises, some of which can reasonably be viewed as straightforward, and some of which are much more difficult. In some cases, these exercises have been derived from research papers that clearly reveal their solutions, and I have not attempted to disguise this fact or hide their source. While the exercises may potentially be useful to course instructors, their true purpose is to be useful to students of the subject; there is no substitute for the learning experience to be found in wrestling with (and ideally solving) a difficult problem.

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