Assignment #3 : NP-Hardness via SAT and Planar SAT

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Due : October 14 Fri, 2022

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1. Vertex Cover-DEG3 is a variant of Vertex Cover where maximum degree is at most 3. Planar Vertex Cover-DEG3 is Vertex Cover-DEG3 but restricted to planar graphs. Induced Subgraph Vertex Cover is a variant of Vertex Cover where a vertex cover should induce a connected subgarph.

   (a) Prove that 3SAT $\leq_p$ Vertex Cover-DEG3.

   (b) Prove that Planar 3SAT $\leq_p$ Planar Vertex Cover-DEG3, using the proof of (a).

   (c) Prove that Planar Vertex Cover-DEG3 $\leq_p$ Planar Induced Subgraph VC-DEG4.

2. (a) Consider a variant of Planar 3SAT where for every variable vertex, positive edges and negative edges can be separated. Prove that the problem still remains NP-complete. You may read Theorem 2.9.3 for help.

   (b) Using the gadgets in the textbook, prove that Planar 3SAT $\leq_p$ Planar 3COL.

   (c) Classify Planar n-COL for every $n \in \mathbb{N}$. In other words, which of them are NP-complete and which of them are in P?

3. (a) Prove that Planar Not-all-equal 3SAT $\leq_p$ Planar MaxCut.

   (b) Describe a polynomial-time algorithm for Planar MaxCut, and conclude that Planar Not-all-equal 3SAT is in P. (Hint : Observe that it is equivalent as solving Chinese Postman Problem on its dual graph.)

   (c) Recall Schaefer's Dichotomy Theorem from the last week. Could we have used the theorem to prove that Planar Not-all-equal 3SAT is in P? Why or why not?

4. Consider a variant of Planar SAT where there are at least k distinct variables for each clauses.

   (a) Determine whether the problem is NP-complete or in P when $k = 5$.

   (b) Determine whether the problem is NP-complete or in P when $k = 4$. (Challenging!)

5. We saw that the variant of Planar 3SAT where all variables can be connected as a cycle, is still NP-complete. If we add one extra condition that all clauses can be connected as a path, prove that this problem is in P.