HW4

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Throughout this assignment,
- n denotes the number of variables and m the number of clauses in context of CNF formula.
- n denotes the number of vertices plus edges in context of graph.

1. Prove that the following two statements are equivalent.

   - (ETH) For all integer $k \geq 3$, there exists $c > 0$ such that any algorithm for kSAT requires $\Omega(2^{c \cdot n} \cdot poly(m))$ time.
   - They instead require $\Omega(2^{c \cdot m} \cdot poly(m))$ time.

2. Assuming the ETH, prove the followings.

   (a) Any algorithm for Hamiltonian-Path = {Undirected Graph $G \mid G$ has a simple path that visits every vertex exactly once.} requires $2^{\Omega(n)}$ time.

   (b) Any algorithm for Planar-Hamiltonian-Path, Hamiltonian-Path restricted to planar graphs, requires $2^{\Omega(\sqrt{n})}$ time.

3. Recall that

   - For all integer $k \geq 1$, $s_k = \inf\{s|$ there is an $2^{s \cdot n}$ algorithm for kSAT$\}$.
   - Exponential Time Hypothesis(ETH): $s_3 > 0$
   - Strong Exponential Time Hypothesis(SETH): $\lim_{k \to \infty} s_k = 1$

   Prove that SETH implies ETH.

4. Get an AC(accepted) verdict on BOJ 20259 - Quality Monitoring. (Attach your submission link)

5. Prove that the existence of a kernelization of Vertex-Cover$_k$ with kernel of size $O(\log k)$ implies P=NP.