# HW5

Aeren

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1. Prove that Dominating-Set$_k$ is W[2]-hard.

2. Prove that FPT=W[0].

3. Consider the following problem

   | DetTM-Empty-String-Acceptance$_k$<br>Instance: A deterministic turing machine $M$, a non-negative integer $n$ in unary form, and a non-negative integer $k$.<br>Question: Does $M$ accept the empty string in time $\leq n^k$? |
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   (a) Prove that FPT $\neq$ XP. (Diagonalization Argument)

   (b) Prove that DetTM-Empty-String-Acceptance $\in$ XP.

   (c) Prove that DetTM-Empty-String-Acceptance $\in$ XP-hard and conclude that DetTM-Empty-String-Acceptance $\in$ XP$-$FPT.

4. Let $A_k$ and $B_k$ be parameterized problems. A $k$-linear (parameterized) reduction from $A_k$ to $B_k$ is a parameterized reduction such that an instance $(x, k)$ is mapped to $(x', k')$ where $k' \in O(k)$.
   Let $f$ be an arbitrary computable function, and assume ETH.

   (a) Let $A_k$ be a parameterized problem such that there exists a $k$-linear reduction from Clique$_k$ to $A_k$. Prove that $A_k$ requires $f(k) \cdot n^{\Omega(k)}$ time to solve.

   (b) Show that the following problem requires $f(k) \cdot n^{\Omega(k)}$ time to solve.

   | Partial-Vertex-Cover$_k$<br>Instance: A graph $G$, and non-negative integers $k$ and $l$.<br>Question: Is there a set of vertices of size $k$ that covers $l$ edges? |
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