Assignment #2: Variants of SAT

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Modified: October 3, 2022

1. Decide each problem in P or NP-Complete. If it is in P, decribe a polynomial time algorithm. You can assume that SAT, 3SAT, Cycle-through-2-vertices are NP-Complete (Note that this is **NOT** the easiest problem of the assignment).

   - *SAT*. Given a boolean formula with CNF form. Decide if it is satisfiable.
   - *3SAT*. Given a boolean formula with CNF form, and every clause of the formula has at most three literal. Decide if it is satisfiable.
   - *Cycle-through-2-vertices*. Given directed graph $G = (V, E)$ and $s, t \in V$. Decide if there is a simple cycle in G which contains two vertices s, t.

   (a) *Horn-SAT*. Given a boolean formula with CNF form, and every clause of the formula has at most one positive literal. Decide if it is satisfiable.

   (b) *Set-Cover*. Given $n, m, k \in \mathbb{N}$ and m sets $A_1, A_2, .., A_m \subset \{1, 2, .., n\}$. Decide if it exists a subset $S \subset \{1, 2, .., n\}$ such that $S \cap A_j \neq \phi$ and $|S| \leq k$.

   (c) *Set-Cover-2*. Given $n, m, k \in \mathbb{N}$ and m sets $A_1, A_2, .., A_m \subset \{1, 2, .., n\}$. Decide if it exists a subset $S \subset \{1, 2, .., m\}$ such that $\cup_{i \in S} A_i = \{1, 2, .., n\}$ and $|S| \leq k$.

   (d) *Cycle-1-mod-3*. Given directed graph $G = (V, E)$. Decide if there is a simple cycle in G which its length is 1 modulo 3.

   (e) *Cycle-K*. Given directed graph $G = (V, E)$ and an integer $K \geq |V|^2$. Decide if there is a closed walk in G which its length is equal to K. You can use arbitrary source(research, wiki, etc..) for solve this problem.

2. We didn't look GAP-SAT in the lecture. **GAP-3SAT[ρ, 1]** is the gap version of 3SAT such that an instance is given in the form of 3CNF, decide if it is satisfiable or there is no assignment that satisfies ρ of the clauses(It is guaranteed that input is included in those two cases).

   - Let GAP-E3SAT is almost same to GAP-3SAT but all clauses has exactly 3 literals and there is **no two same** literals in a clause. Show that GAP-E3SAT[$7/8$, 1] is in P.

3. Show that 3SAT $\leq_p$ MAX-2SAT.

4. We skipped some procedure in the proof of Schafer's Dichotomy Theorem. Prove the following lemma.

   - If relation R is not bijunctive, then $Rep(\{R, [x \neq y], [x \vee y]\})$ contains the relation "exactly one of x,y,z"

5. Show that NAE-E3SAT is NP-Complete using Schaefer's Dichotomy Theorem.