

Physics 239: Homework 3  
Fall 2020  
Due 11/6/20

In this assignment, you are going to write a code that simulates the interaction between an electron and a charged particle. You will then use that to calculate the acceleration of the electron and the frequency spectrum of radiation it emits during the interaction.<sup>1</sup>

1. Assume an ion of charge  $Ze$  is located at the origin and is fixed during the interaction. Define the  $x$  and  $y$  axes in units of the Bohr radius  $a_0$ . Inject an electron at some initial position many Bohr radii away from the origin (e.g. 100s of Bohr radii). Give it an initial velocity in the  $\hat{x}$  direction (make sure this is non-relativistic  $v \ll c$ , try something like  $10^7$  cm s<sup>-1</sup> for example).
2. Using a time interval that finely resolves the interaction (i.e. several thousands of steps), calculate the position and acceleration of the electron. To do this, at each time interval use the position of the electron to compute the force between the electron and the ion. Assume the force acts on the electron for time  $\Delta t$  that is equal to your time interval and compute what the velocity will be after the time interval. Then use the current position and the current velocity to calculate the position of the electron after the time interval.
3. Make the following plots for a single interaction:
  - Position in  $x$  and  $y$  at each time interval (i.e. show the path of the electron).
  - Velocity in  $x$  and  $y$  as a function of time.
  - Acceleration in  $x$  and  $y$  as a function of time.
4. Do a Fourier transform of the acceleration to show the power spectrum of the of the radiation as a function of frequency. (You'll need to consider how to include the acceleration in the  $x$  and  $y$  directions). Plot the power spectrum.
5. Run a series of simulations with different combinations of initial  $y$  positions, i.e. different "impact parameters"  $b$  and different initial velocities  $v$ . Plot how the frequency of the peak of the power spectrum varies with  $b$  and  $v$ .

Some advice: The calculation will only work in the case of small deviations, so you will have to be careful about the combination of initial position and velocity.

---

<sup>1</sup>This problem is based on one from Prof. Aaron Parsons who teaches Berkeley's Astro 207 class.