Physics 239: Homework 2
Fall 2020
Due 10/23/20

In this assignment we are going computationally solve simple radiative transfer problems to reproduce the scenarios we went over during the lecture on October 8. Create a directory `hw2` in your github repository to store this work. Things to remember: comment your code, practice version control (check in things as you go), include axis labels on your plots. Create a script that generates the requested outputs below using the functions and programs you have made. When you are ready and have pushed your code to github, send me a message and I will clone it and run the code.

The basic set up for the problem is a cloud of depth $D = 100$ pc, with a uniform density of $n = 1$ cm^{-3} and a source function S_ν that is independent of frequency and depth in the cloud. An initial intensity $I_\nu(0)$ that is independent of frequency is entering the cloud at $s = 0$ and propagating towards the observer at $s = D$. We will assume there is no scattering.

1. What is the column density of the cloud in [cm^{-2}]? What would the cross section $\sigma_{\nu,0}$ have to be to have a total optical depth through the cloud of: a) 10^{-3}, b) 1, and c) 10^3? (Include in your script a line that prints out the answers to these questions to the command line or a file).

2. We will start the radiative transfer problem with a calculation at one frequency ν_0 and expand in a subsequent step to a range of frequencies. Write a program that takes as input $\sigma_{\nu,0}$, $I_\nu(0)$ and S_ν and calculates the specific intensity at $s = D$, i.e. $I_\nu(D)$. You can do this however you would like, but here are a few suggestions:

   (a) Define an array that contains the distance through the cloud going from 0 to D in a number of steps of size ds.

   (b) Define another array that uses those same steps and holds the specific intensity at each location. Fill in the first element (equivalent to $I_\nu(0)$).

   (c) Loop through the remaining elements using the equation of radiative transfer to adjust the specific intensity at each step using the optical depth τ_ν and the source function S_ν.

3. Next, make a function that generates a cross section σ_ν as a function of frequency. This function should take a vector of frequencies and some parameters that define the σ_ν dependence on frequency. You could imagine having a variety of shapes here (i.e. no dependence on frequency, a linear dependence on frequency, some other functional form). For this exercise, create a function that returns a σ_ν that has a Gaussian shape and a maximum equal to an input $\sigma_{\nu,0}$. The values you choose for the frequency range and line shape are arbitrary, but be sure that your frequency range in the vector is large enough that the Gaussian is contained mostly within that range, i.e. $\nu_2 - \nu_1 >> \Delta\nu_{\rm FWHM}$. Generate three plots of σ_ν as a function of ν using the $\sigma_{\nu,0}$ values you calculated in (1.) above.

4. Putting together the pieces of code you have created above reproduce the figures from lecture showing $I_\nu(D)$ as a function of ν for the following scenarios:

   (a) $I_\nu(0) = 0$ and $\tau_\nu(D) < 1$.

   (b) $I_\nu(0) > S_\nu$ and $\tau_\nu(D) < 1$.

   (c) $I_\nu(0) < S_\nu$ and $\tau_\nu(D) < 1$.

   (d) Optical depth at all frequencies $\tau_\nu(D) >> 1$.

   (e) $I_\nu(0) < S_\nu$ and $\tau_\nu(D) < 1$ while $\tau_{\nu,0}(D) > 1$.

   (f) $I_\nu(0) > S_\nu$ and $\tau_\nu(D) < 1$ while $\tau_{\nu,0}(D) > 1$.