Problem 1. [20pts] Runge-Lenz vector redux

Consider a particle in **three dimensions** moving under the action of the Kepler potential $V(r) = -\frac{k}{r}$. Show that the Runge-Lenz vector

$$\mathbf{A} = \mathbf{p} \times \mathbf{L} - \frac{mk\mathbf{r}}{r}, \tag{1}$$

is an integral of motion (i.e., $\{\mathbf{A}, H\} = 0$).

Problem 2. [20 pts] Generators of canonical transformations

Consider a mechanical system with one degree of freedom. Obtain the generating function $F_3(p, Q)$ that generates the same canonical transformation as $F_2(q, P) = q^2 e^P$.

Problem 3. [20 pts] Building canonical transformations

Let

$$Q_1 = q_1^2, \quad Q_2 = q_1 + q_2, \quad P_i = P_i(q_j, p_j), \quad i = 1, 2 \tag{2}$$

be a canonical transformation of a system with two degrees of freedom.

(a) Complete the transformation by finding *the most general* expression for the P_i's.

(b) Find *a particular* choice of the P_i's that will reduce the Hamiltonian

$$H(q_i, p_i) = \left(\frac{p_1 - p_2}{2q_1}\right)^2 + p_2 + (q_1 + q_2)^2 \tag{3}$$

to

$$H_{\text{new}}(Q_i, P_i) = P_1^2 + P_2. \tag{4}$$

(c) Use $H_{\text{new}}(Q_i, P_i)$ to solve for $q_i(t)$.