Final Project

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1 Statistics and the LawArguement that the difference between the rates of mortgage application refusals of white applicants and minority applicatns constitued evidence of discrimination. Test (1) the data are sufficient evidence of discrimination to warrant corrective action and (2) the data are not sufficient.

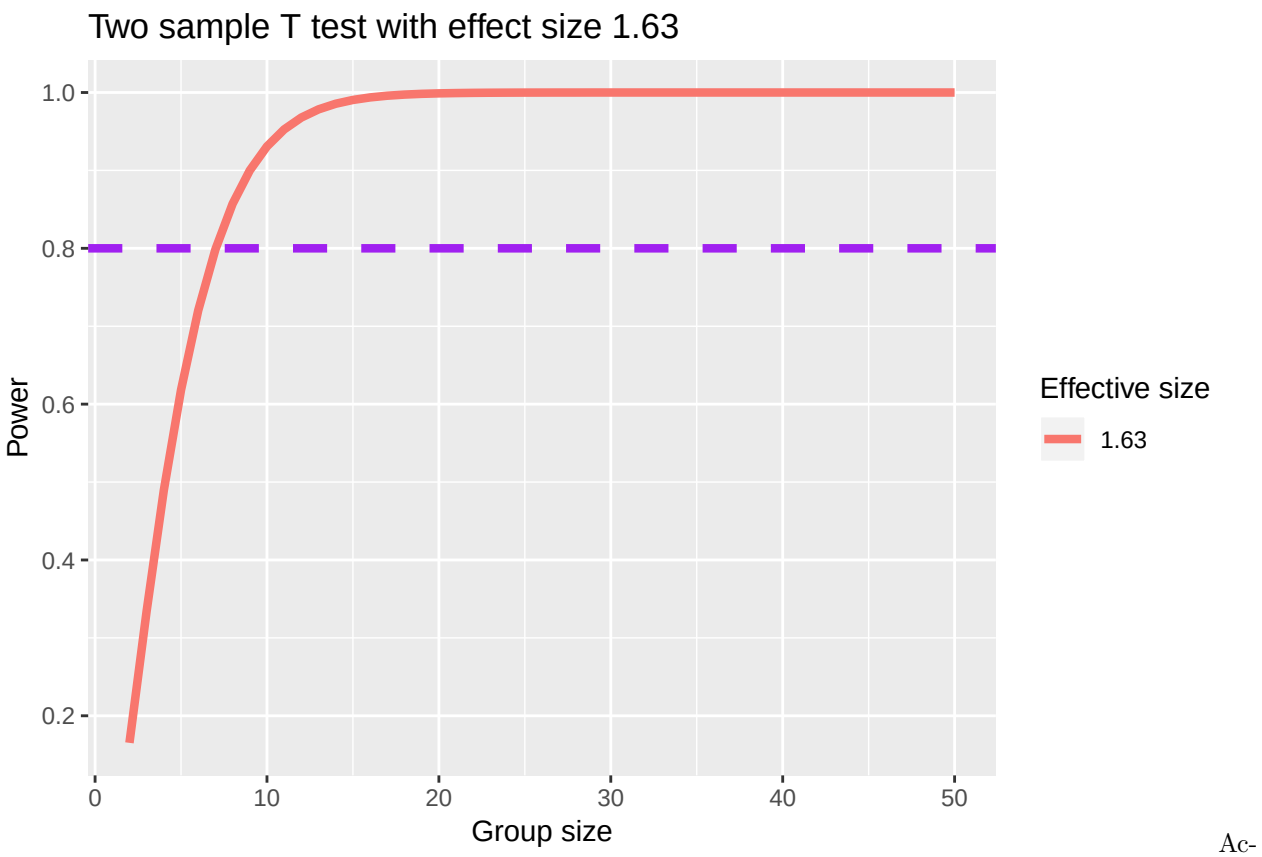
H0: morgage refusal rate of white applicants are the same as that of minority applicant H1: morgage refusal rate of white applicants are lower than that of minority applicant

```
#Store the data
Min<-c(20.90,23.23,23.10,30.40,42.70,62.20,39.5,38.40,26.20,55.90,49.70,44.60,36.40,32.00,10.60,34.30,42
White<-c(3.7,5.5,6.7,9.0,13.9,20.6,13.4,13.2,9.3,21.0,20.1,19.1,16.0,16.0,5.6,18.4,23.3,15.6,32.4,29.7)
data1 <- data.frame(
                group = rep(c("Min", "White"), each = 20),
                percent = c(Min,  White)
                )
# Two sample T-test to test whether the refusal rate for minority applicants is greater than the refusa
res<-t.test(percent ~ group, data = data1,
        var.equal = TRUE, alternative = "greater")
res$p.value
```

```
## [1] 1.279668e-07
```

The p-value of this t test is 1.279668e-07, which is less than the significant level alpha = 0.05. We can conclude that refusal rate for minority applicants is significantly different from refusal rate for white applicants with a p-value = 1.279668e-07.

```
# Power analysis to show the sufficiency
# Calculate the effect size
effect_size=abs(mean(Min)-mean(White))/sd(Min)

ptab1 <- cbind(NULL)
n <- seq(2, 50, by = 1)
for (i in seq(2, 50, by = 1)) {
  pwrt1 <- pwr.t2n.test(
    n1 = i, n2 = i,
    sig.level = 0.05, power = NULL,
    d = effect_size, alternative = "two.sided"
  )
  ptab1 <- rbind(ptab1, pwrt1$power)
}
temp<-as.data.frame(ptab1)
colnames(temp)[1]<-"num"
ggplot(temp) +
  geom_line(aes(x = n, y = num, colour = "darkblue"), size = 1.5) +
  scale_color_discrete(name = "Effective size", labels = c(round(effect_size,2))) +
  geom_hline(yintercept = 0.8, linetype = "dashed", color = "purple", size = 1.5) +
  ylab("Power") + scale_y_continuous(breaks = seq(0, 1, by = 0.2)) + ggtitle("Two sample T test with ef
```



According to this power analysis plot, if we want to reach the general acceptable power 0.8, we need to have more than 7 data samples in each group under the effect size 1.63. And in our acorn data, we have 20 data samples in each group. Therefore,the data has sufficient evidence to show the discrimination to warrant corrective action.

2 Comparing Suppliers Revenue aside, which of the three schools produces the higher quality ornithopters, or are do they all produce about the same quality?

H0:They all produce about the same quality H1:They do not produce about the same quality

```
data2 <- matrix(c(12,23,89,8,12,62,21,30,119),ncol=3,nrow = 3,byrow=TRUE)
colnames(data2) <- c("dead","art","fly")
rownames(data2) <- c("Area51","BDV","Giffen")
fly <- as.table(data2)
chisq.test(data2,correct = F)
```

```
##
##  Pearson's Chi-squared test
##
## data:  data2
## X-squared = 1.3006, df = 4, p-value = 0.8613
```

The p-value of this chi-square test is 0.8613, which is much greater than the significant level alpha=0.05. Therefore, we fail to reject the null hypothesis. The data are sufficient to show that three schools produce

the same quality.

3 How deadly are sharks?

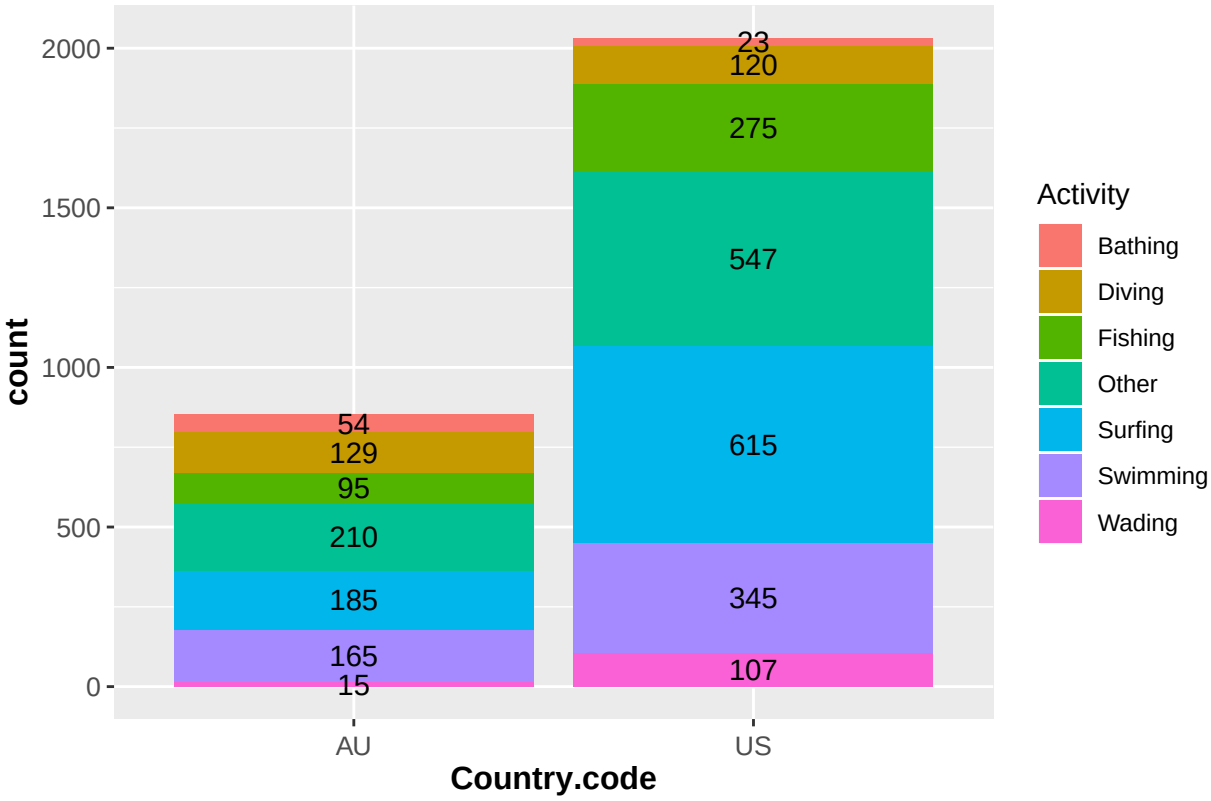
H0:Sharks in Australia were, on average, are the same as the sharks in the United States H1:Sharks in Australia were, on average, are more vicious lot than the sharks in the United States

If we want to analyze the difference of viciousness between US sharks and AU sharks, we only care about the attacks in AU and US and the sharks are not be provoked first.

```
data3<-read.csv("sharkattack.csv")
data3<-data3%>%filter(Country.code=="US"|Country.code=="AU"&Type=="Unprovoked")
temp<-data3%>%group_by(Country.code,Activity)%>%summarise(count=n())%>% ungroup()%>%group_by(Country.co
kable(temp)
```

Country.code	Activity	count	percent
AU	Bathing	54	0.0633060
AU	Diving	129	0.1512309
AU	Fishing	95	0.1113716
AU	Other	210	0.2461899
AU	Surfing	185	0.2168816
AU	Swimming	165	0.1934349
AU	Wading	15	0.0175850
US	Bathing	23	0.0113189
US	Diving	120	0.0590551
US	Fishing	275	0.1353346
US	Other	547	0.2691929
US	Surfing	615	0.3026575
US	Swimming	345	0.1697835
US	Wading	107	0.0526575

```
ggplot(data = temp, aes(Country.code, count, group = Activity)) +
 geom_col(aes(fill = Activity)) +
 geom_text(aes(label = count), position = position_stack(vjust = 0.5))+theme(axis.title.x = element_tex
          axis.title.y = element_text(face="bold",  size=12),
          plot.title = element_text(size=12, face="bold"),
          axis.text.x  = element_text(vjust=0.5, size=10),axis.text.y = element_text(vjust=0.5, size=10
```



According to the bar plot and data frame, US sharks made more attacks in total and attacks in surfing had higher percentage than AU sharks.

```
# Transfer dataframe into matric to do chi-square test
data33<- matrix(c(23,120,275,547,615,347,107,54,129,95,210,186,165,12), nrow=2,
                dimnames = list(c("AU","US"),c("Bathing","Diving","Fishing","Other","Surfing","Swimming"
chisq.test(data33,correct = F)
```

```
##
##  Pearson's Chi-squared test
##
## data:  data33
## X-squared = 378.78, df = 6, p-value < 2.2e-16
```

The p-value of this chi-square test is much smaller than the significant level alpha=0.05. Therefore, we reject the null hypothesis. Sharks in Australia were, on average, are more vicious than the sharks in the United States.The statistical power of the chi-square test is 1.

4 Power analysis

The arcsine transformation is calculated as two times the arcsine of the square root of the proportion. Doing this can transform the proportional parameter from (0,1) to $(-/2,/2)$. The effect of the arcsine transformation is similar to the logit, in that it pulls out the ends of the distribution, but not to the extent that the logit does.

The power to detect the hypothetical parameters .65 and .45 is .48 while the power to detect the difference between hypothetical parameters .25 and .05 is .82, even though the difference between both pairs of values is .20. The reason why this happens is that 0.25 and 0.05 are at the end(extreme value) of the distribution. Therefore, it takes more power to detect and arcsine transformation can solve this problem.

5 Use the Method of Moments and MLE to find estimators as described in these three cases.

Case1 MLE of Exponential Distribution

$$f(x;\lambda) = \lambda e^{-\lambda x}$$

$$L(\lambda; X_1, ..., X_n) = \lambda e^{-\lambda X_1} \lambda e^{-\lambda X_2} ... \lambda e^{-\lambda X_n}$$

$$L(\lambda; X_1, ..., X_n) = \lambda^n e^{-\lambda \sum X_i}$$

$$l(\lambda; X_1, ..., X_n) = nlog(\lambda) - \lambda \sum X_i$$

$$\frac{dl(\lambda; X1, ..., Xn)}{d\lambda} = \frac{n}{\lambda} - \sum X = 0$$

$$\hat{\lambda} = \frac{n}{\sum X_i} = \frac{1}{\bar{X}_n}$$

Case2 Moment Estimator and MLE for new distribution θ

MOM:

$$\begin{aligned} E[X] &= \int_0^1 x((1-\theta) + 2\theta x)dx \\ &= (1-\theta)\int_0^1 xdx + \int_0^1 2\theta x^2 dx \\ &= (1-\theta)\frac{1}{2}x^2|_0^1 + 2\theta\frac{1}{3}x^3|_0^1 \\ &= \frac{1}{2} - \frac{1}{2}\theta + \frac{2}{3}\theta \\ &= \frac{1}{6}\theta + \frac{1}{2} \end{aligned}$$

MLE:

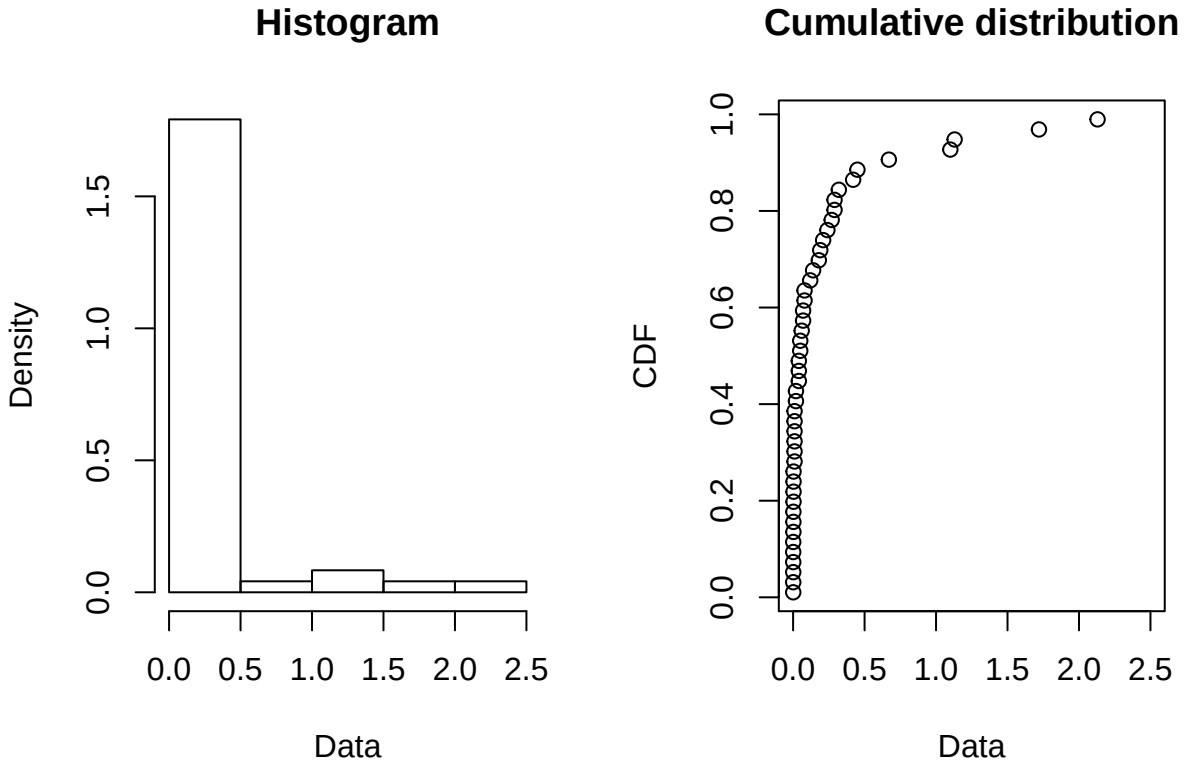
$$L(\theta; X_1, ..., X_n) = [(1-\theta) + 2\theta X_1]...[(1-\theta) + 2\theta X_n]$$

$$l(\theta; X_1, ..., X_n) = log[(1-\theta) + 2\theta X_1] + ... + log[(1-\theta) + 2\theta X_1]$$

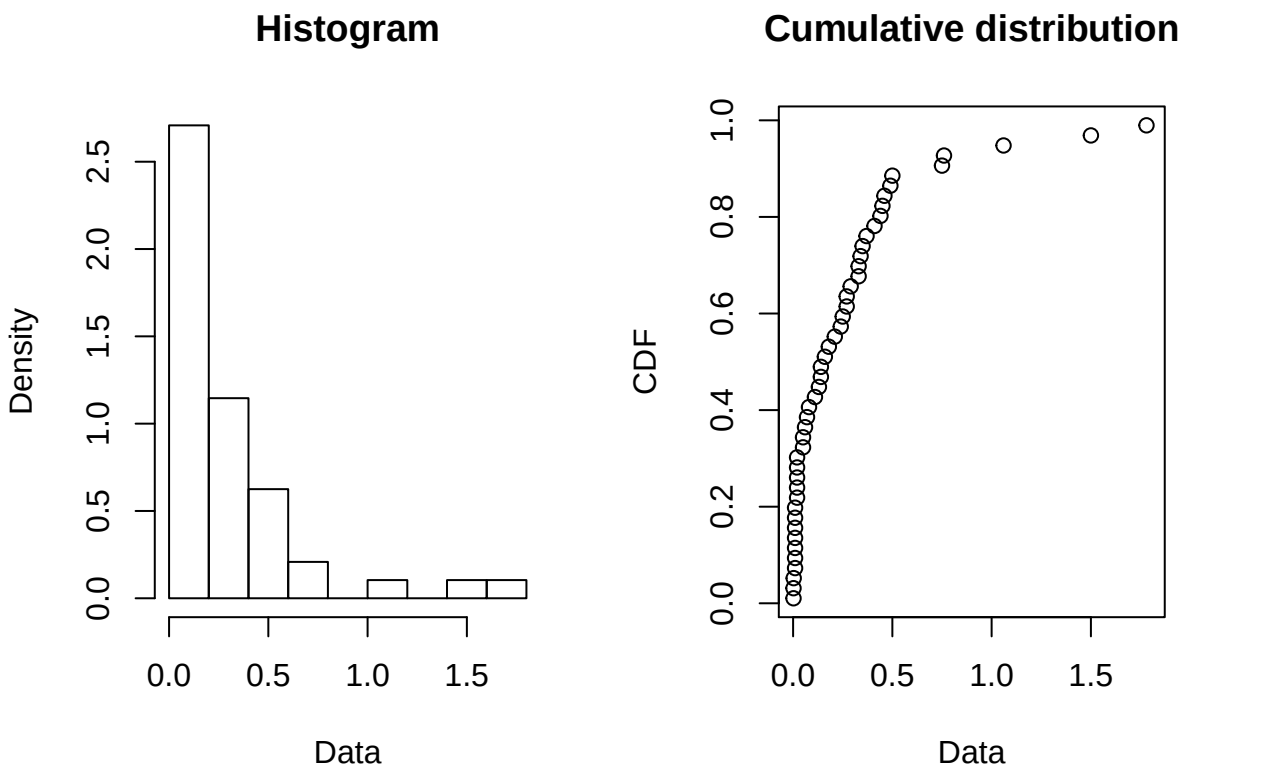
Under this situation, we cannot taking the derivative of $l(\theta; X_1, ..., X_n)$ to find the maximum value with corresponding θ.

Case3 Rainfall analysis (Working with PJ and Shannon)

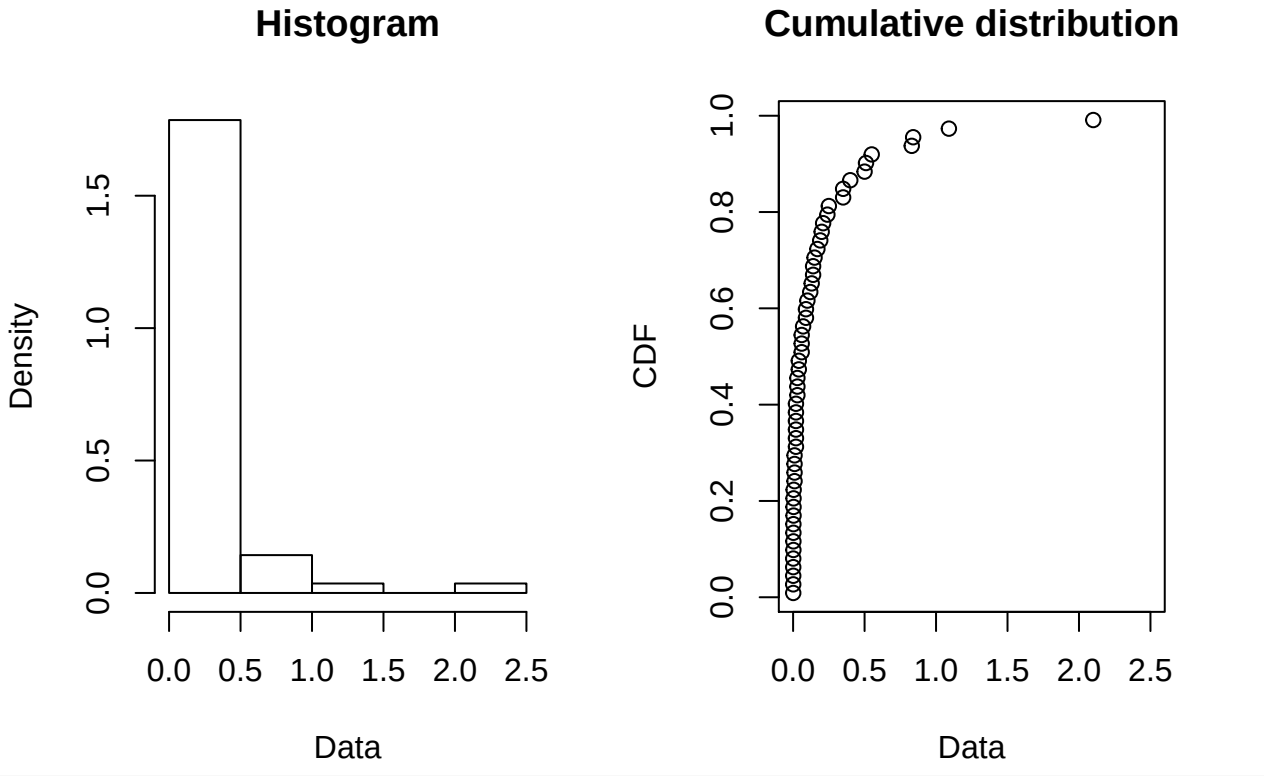
```
# read the data
data60 <- read.table("ill-60.txt", quote="\"", comment.char="")
data60 <-as.numeric(as.array(data60 [,1]))
data61 <- read.table("ill-61.txt", quote="\"", comment.char="")
data61<-as.numeric(as.array(data61[,1]))
data62 <- read.table("ill-62.txt", quote="\"", comment.char="")
data62<-as.numeric(as.array(data62[,1]))
data63<- read.table("ill-63.txt", quote="\"", comment.char="")
data63<-as.numeric(as.array(data63[,1]))
data64 <- read.table("ill-64.txt", quote="\"", comment.char="")
data64<-as.numeric(as.array(data64[,1]))

# explore the distribution of the rainfall data
plotdist(data60)
```



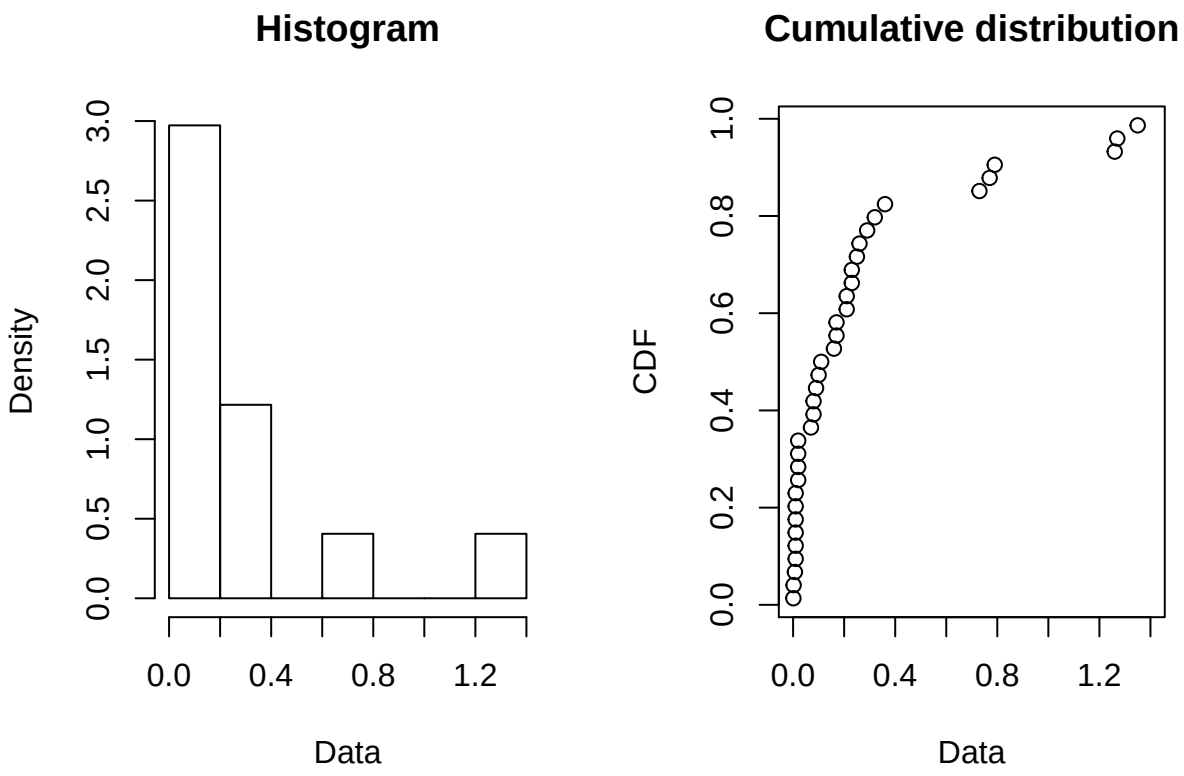

```
plotdist(data61)
```



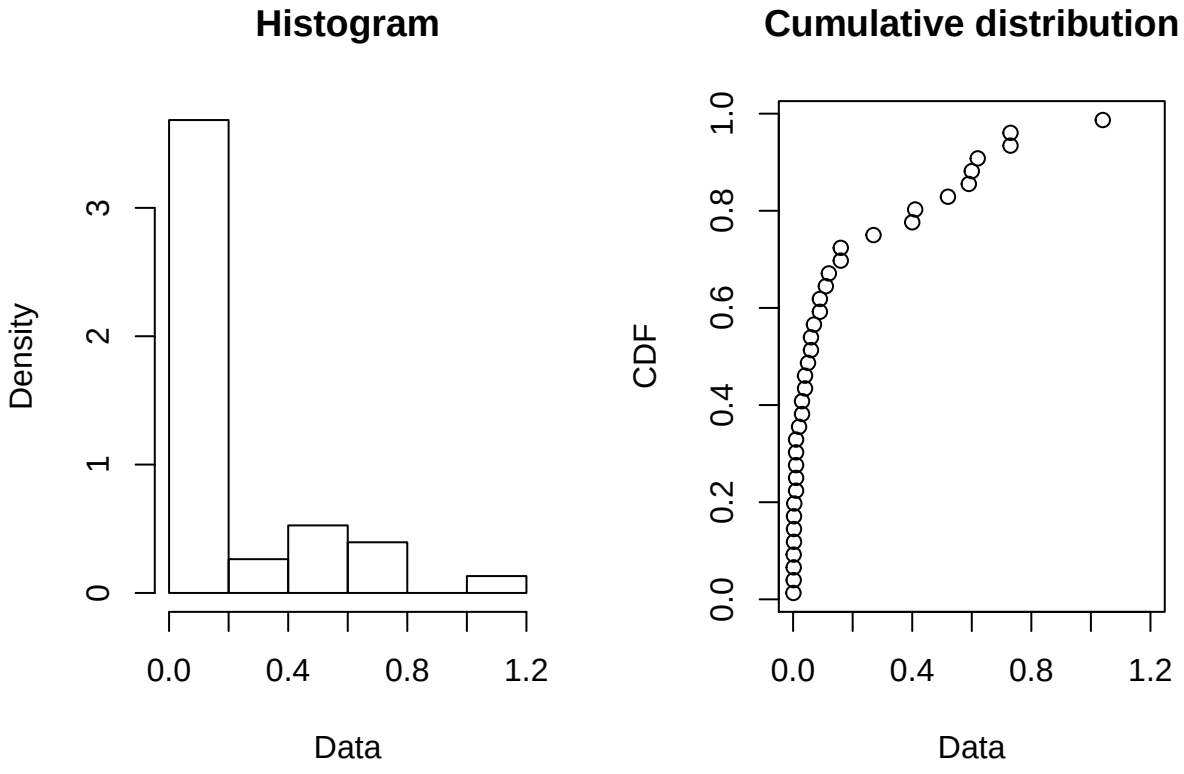

```
plotdist(data62)
```




```
plotdist(data63)
```




```
plotdist(data64)
```

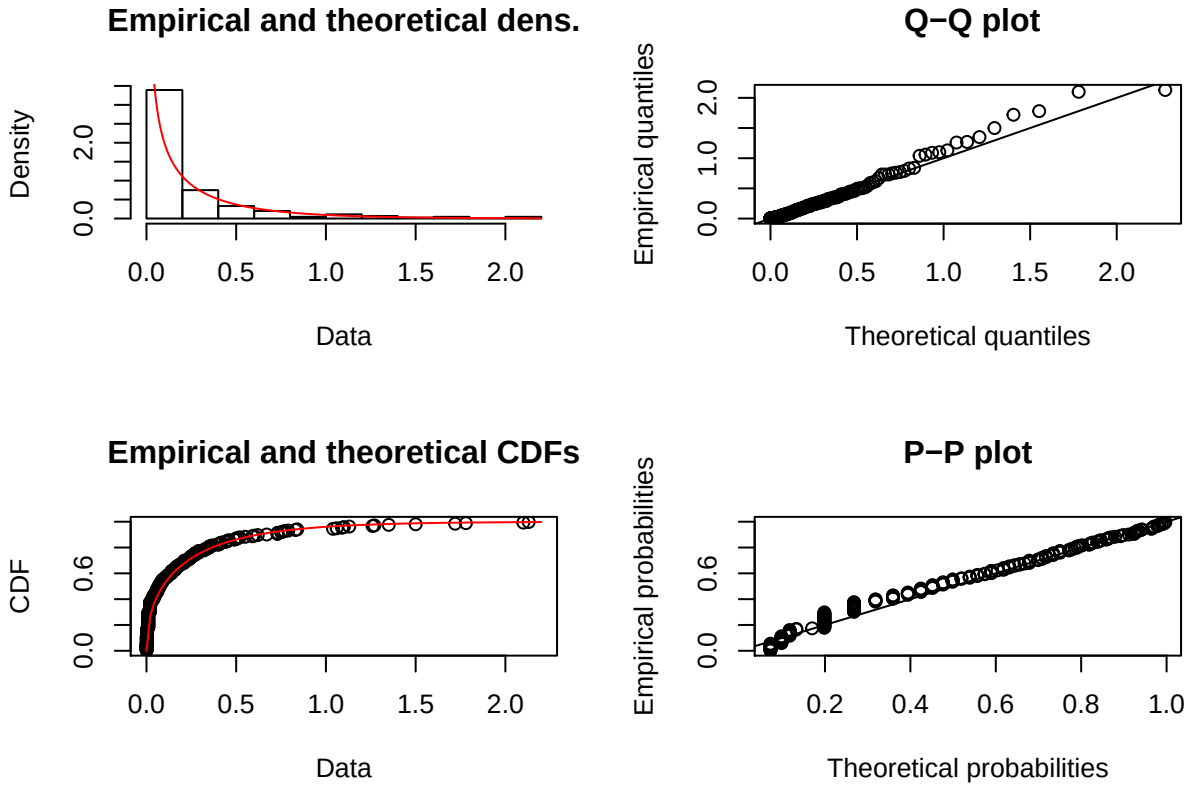



```
Rain<-matrix(c(sum(data60),sum(data60)/length(data60),sum(data61),sum(data61)/length(data61),sum(data62)

kable(Rain)
```

	1960	1961	1962	1963	1964
TotalRain	10.5740000	13.1970000	10.34600	9.7100000	7.1100000
AverageRain	0.2202917	0.2749375	0.18475	0.2624324	0.1871053

According to the distribution plot, five years are similar. 1961 is more wetter than others since it has the highest total rainfall. And from the row AverageRain in table, we can see that 1961 and 1963 have similar average rainfall in each storm. Therefore, it should be that storms produced more rain.

```
#Test whether the gamma distribution was a good fit for their data.
alldata<-c(data60,data61,data62,data63,data64)
test_gamma <- fitdist(alldata, "gamma")
plot(test_gamma)
```



According to the Empirical the theoretical CDFs and P-P plot, the data points almost excellent fit the gamma distribution.Although in Q-Q plot, the points after 1.0 theoretical quantiles are a little bit discrete from the gamma distribution. Generally, the gamma distribution was a good fit for their data. I will agree with Changnon and Hu.

```
# calculate MOM and MLE
set.seed(2019)
mom <- fitdist(alldata, "gamma",method = "mme")
boot_mom <- bootdist(mom)
summary(boot_mom)
```

```
## Parametric bootstrap medians and 95% percentile CI
##          Median      2.5%     97.5%
## shape 0.3902039 0.2777425 0.5301561
## rate  1.7497653 1.1801093 2.5899000
```

```
mle <- fitdist(alldata, "gamma",method = "mle")
boot_mle <- bootdist(mle)
summary(boot_mle)
```

```
## Parametric bootstrap medians and 95% percentile CI
##          Median      2.5%     97.5%
## shape 0.4447087 0.3808683 0.5170126
## rate  1.9913553 1.5616429 2.5430885
```

According to summary of MOM, the 95% confidence interval of shape from bootstrap sample is (0.27,0.53), the rate is (1.18,2.59). For MLE, the 95% confidence interval of shape from bootstrap sample is (0.38,0.51),the rate is (1.56,2.54).The MLE estimates have higher rate and more narraow CI, which means lower variance.Therefore, I would choose to present MLE as the estimator.

Analysis of decision theory article

```
#Derive equations (10a), (10b), (10c) in Section 3.2.2.
#Use R to reproduce the calculations in Table 1 which is explained in 3.2.3.
```

Totally no idea how to deal with this problem! In that paper, there are many mathematic symbols with no explanations.