MA677 Hw4 Jiahao Xu

1. MLE of $p = \frac{58}{70} = 0.83$

2. $X_1 - X_n \sim Bern(\theta) = \theta^{x}(1-\theta)^{1-x}$

$$L(\theta, X_1 - X_n) = \theta^{\sum x}(1-\theta)^{n-\sum x}$$

$$l(\theta, X_1 - X_n) = \sum x \log(\theta) + (n - \sum x)\log(1-\theta)$$

$$\frac{dl(\theta, X_1 - X_n)}{d\theta} = \frac{\sum x}{\theta} + \frac{(n - \sum x)}{1-\theta} = 0 \Rightarrow \hat{\theta} = \frac{\sum x}{n}$$

If every observed value is 0 or 1, then $\hat{\theta} = \frac{0}{n} = 0$ or $\frac{n}{n} = 1$

3. $X_1 - X_n \sim Poisson(\lambda) = \frac{\lambda^{x} e^{-\lambda}}{x!}$, λ is unknown and > 0

$$L(\lambda, X_1 - X_n) = \prod_{i=1}^{n} \exp(-\lambda) \frac{1}{x_i!} \lambda^{x_i}$$

$$l(\lambda, X_1 - X_n) = -n\lambda - \sum_{i=1}^{n} \log(x_i!) + \log(\lambda)\sum_{i=1}^{n} x_i$$

$$\frac{dl(\lambda, X_1 - X_n)}{d\lambda} = 0 \Rightarrow \hat{\lambda} = \frac{\sum x_i}{n} = \overline{X}_n$$

If every observed value is 0, then MLE of λ is 0

4. $X_1 - X_n \sim N(\mu, \sigma^2)$, μ is known and σ^2 is unknown

$$L(\mu, \sigma; X_1 - X_n) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x_i - \mu)\right)^2$$

$$l(\mu, \sigma; X_1 - X_n) = -n\log\sigma - \frac{n}{2}\log 2\pi - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2$$

$$\frac{dl(\mu, \sigma; X_1 - X_n)}{d\sigma} = -\frac{n}{\sigma} + \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{\sigma^3} = 0$$

$$\Rightarrow \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$