MA677 Hw7 Jiahao Xu

① X: 0.225, 0.262, 0.217, 0.240, 0.230, 0.229, 0.235, 0.217

Y: 0.209, 0.205, 0.196, 0.210, 0.202, 0.207, 0.224, 0.223, 0.220, 0.201

$\bar{X} = 0.232$, $\bar{Y} = 0.21$

$S_X^2 = \frac{\sum x_i^2}{n} - \bar{x}^2 = 0.054 - (0.232)^2 = 0.00013$

$S_Y^2 = \frac{\sum y_i^2}{n} - \bar{y}^2 = 0.0441 - (0.21)^2 = 0.00008$

$$t = \frac{\bar{x}-\bar{y}}{s_p} = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_x^2}{n_1}+\frac{s_y^2}{n_2}}} = \frac{0.232-0.21}{\sqrt{\frac{0.00013}{8}+\frac{0.00008}{10}}} \approx 4.33$$

$p = (t > 4.33) = 0.000342 << 0.05$

∴ Reject H_0 ⇒ they are written by different author

② $X_1 - X_n \sim N(M, 6^2)$

$\bar{x} = \frac{\sum x_i}{n} = 156.7$ $\quad 6x^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = 512.76$

$6x = \sqrt{512.7} \approx 22.6$

$sd = \frac{6x}{\sqrt{n}} = \frac{22.6}{\sqrt{20}} \approx 5.06$

$\Phi^{-1}(1-0.05) = \Phi^{-1}(0.95) = 1.729$ (with df = n-1 = 19)

CI: $\bar{x} \pm 1.729 \times 5.06 = [148.1, 165.6]$

③ $X_1 - X_n \sim N(\mu_1, \sigma^2)$

$Y_1 - Y_n \sim N(\mu_2, \sigma^2)$

$H_0: \mu_1 \geq \mu_2$, $H_1: \mu_1 < \mu_2$

$m = 8$, $n = 6$

$\bar{x} = 1.51$ $\bar{Y} = 1.67$

$S_x^2 = 0.18$, $S_Y^2 = 0.17$

$$U = \frac{(m+n-2)^{\frac{1}{2}}(\bar{X_1} - \bar{X_2})}{(\frac{1}{m}+\frac{1}{n})^{\frac{1}{2}}(S_{X_1}^2 + S_{X_2}^2)^{\frac{1}{2}}}$$

$$= \frac{\sqrt{8+6-2}(1.51-1.67)}{\sqrt{\frac{1}{8}+\frac{1}{6}} \times \sqrt{0.18 \times 0.17}}$$

≈ -1.69

$df = m+n-2 = 12$, $\alpha = 0.1$

$U \sim t_{12}(0.9) = 1.365$

We reject H_0 when $U \leq -t_{12}(0.9)$

∴ μ_1 is not greater or equal to μ_2