MA677HW1

Jiahao Xu

02/09/2019

Jiahao Xu

chapter 5.1

7. (a) $p(T) = \frac{5}{6}^{(T-1)} \cdot \frac{1}{6}$

(b) $P(T>3) = 1 - P(T=1) - P(T=2) - P(T=3)$

$= 1 - \frac{1}{6} - \frac{5}{6}\cdot\frac{1}{6} - (\frac{5}{6})^2 \frac{1}{6}$

$= \frac{125}{216}$

(c) $P(T>6 \mid T>3) = P(T>6-3) = P(T>3) = \frac{125}{216}$

10. (a) $h(N, n_1, n_2, k) = \frac{\binom{n_1}{k}\binom{N-n_1}{n_2-k}}{\binom{N}{n_2}}$

(b) At this time. $k = n_{12}$, then we want to find the max value of N

such that $\frac{h(N+1, n_1, n_2, n_{12})}{h(N, n_1, n_2, n_{12})}$ goes from positive to negative.

From the plot, we know that it happens when $N = \frac{n_1 n_2}{n_{12}}$

16. Let X be the number of calls that are missed.

$n = 5 \cdot 6^3$, $p = 0.01$, $\lambda = np = 1805$

$P(X \le 1) = P(X=0) + P(X=1) = \frac{e^{-3} \cdot 3^1}{1!} + \frac{e^{-3} \cdot 3^0}{0!} = \frac{4}{e^3}$

18. (a) Let X be the number of raisins in the cookie and follows Poisson distribution.

$p = \frac{1}{500}$ and $n = 600$, $\lambda = np = 1.2$, $P(X=0) = e^{-1.2} = 0.3$

(b) Let Y be the number of chocolate chips in the cookie and follows Poisson distribution.

$p = \frac{1}{500}$ and $n = 400$, $\lambda = np = 0.8$, $P(Y=2) = \frac{e^{-0.8} \cdot 2^2}{2!} = 0.144$

(c) $P(X+Y \ge 2) = 1 - P(X=0, Y=2) - P(X=1, Y=1) - P(X=2, Y=0)$

$= 1 - 0.595$

$= 0.405$

25. $\lambda = np = 100 \times 0.05 = 5$, Let x be the number of time without paying the parking

$E(x) = 2 \cdot P(x=2) + (2+5\times1) \cdot P(x=3) + \dots + (2+5\times98) \cdot P(x=100)$

$= 2 \cdot \frac{e^{-5} \cdot 5^2}{2!} + (2+5) \frac{e^{-5} \cdot 5^3}{3!} + \dots + (2+5\times98) \cdot \frac{e^{-5} \cdot 5^{100}}{100!}$

≈ 17.16

27. Let x be the number of accident happened in one year

$\lambda = np = 100 \cdot 0.001 = 0.1$

$P(x \geq 1) = 1 - P(x=0) = 1 - e^{-0.1} = 0.0952$

28. Let x be the number of passengers who bought the ticket but do not show up.

$\lambda = np = 100 \times 0.04 = 4$

And everyone has the seat

100 people bought tickets with only 98 seats $\Rightarrow$

$P(x \geq 2) = 1 - P(x=0) - P(x=1)$

$= 1 - e^{-4} - 4e^{-4} = 0.908$

38. (a) Let x be the number of defective ball and follows binomial distribution

$P(x=1) = \binom{5}{1} \left(\frac{5}{20}\right)^1 \cdot \left(\frac{15}{20}\right)^{5-1} = 0.396$

(b) Let x be the number of defective ball and follows hypergeometric distribution

$P(X=1) = h(20,5,5,1) = \frac{\binom{5}{1}\binom{20-5}{5-1}}{\binom{20}{5}} = 0.44$

Chapter 5.2

$F(U) \sim \frac{1}{b-a} = \frac{1}{1-0} = 1 \quad f(u) = \frac{x-0}{b-a} = \frac{x}{1} = x$

1. (a) $f(Y) = 1$ on $[2,3]$; $F(Y) = Y-2$ on $[2,3]$

(b) $f(Y) = \frac{1}{3} Y^{-2/3}$ on $[0,1]$; $F(Y) = Y^{1/3}$ on $[0,1]$

17. $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \sin^2(\pi x/2) & \text{if } 0 \le x \le 1 \\ 1 & \text{if } 1 < x \end{cases}$

(a) $f(x) = \int F(x) = \begin{cases} \int \sin^2(\pi x/2)\, dx = \frac{\pi}{2}\sin(\pi x) & 0 \le x \le 1 \\ \int 1\, dx = x & 1 < x \end{cases}$

(b) $P(x < \frac{1}{4}) = \sin^2(\frac{\pi}{8}) = 0.146$

21. $P(Y \le y) = P(F(X) \le y) = P(X \le F^{-1}(y)) = F(F^{-1}(y)) = y$ on $[0,1]$

37. $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$G'(y) = \frac{1}{y} F'(\ln y) \Rightarrow G'(y) = g(y)$; $F'(\ln y) = f(\ln y)$

$f(\ln y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{\ln y - \mu}{\sigma}\right)^2}$

$Y = e^X \Rightarrow Y > 0$

$\Rightarrow g(y) = \frac{1}{\sqrt{2\pi}\sigma y} e^{-\frac{1}{2\sigma^2}(\ln y - \mu)^2}$