MA677 HW6 Hypothesis Testing I   Jiahao Xu

1. (a)

If $\beta \geq 1$

$$power = \alpha(\beta) = P(X \geq 1) = 1-(1-e^{-\lambda}) = e^{-\lambda} = e^{-\frac{1}{\beta}}$$

If $\beta < 1$

$$power = \beta(\beta) = 1 - P(X<1) = e^{-\frac{1}{\beta}}$$

$\therefore$ power function $= e^{-\frac{1}{\beta}}$

(b) size of the test is the prob of type I error, $\alpha = \sup_{\beta \geq 1} e^{-\frac{1}{\beta}} = 1$

2. If $p = 0.2$

$$power = \alpha(p) = P(Y \geq 7) + P(Y \leq 1) = 1 - P(Y<7) + P(Y \leq 1) \approx 0.156$$

If $p \neq 0.2$

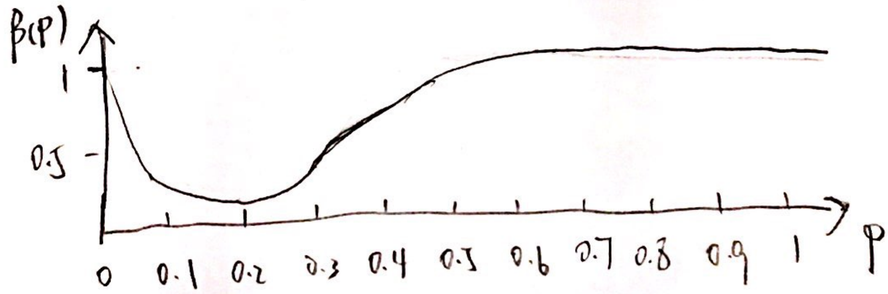
$$Power = \beta(p) = 1 - P(1<Y<7) = 1 - \sum_{i=1}^{6} C_{20}^{i} p^{i} (1-p)^{20-i}$$

Calculated by R: $p=0$, $\beta(p)=1$; $p=0.1$, $\beta(p)=0.394$; $p=0.2$, $\beta(p)=0.156$

$p=0.3$, $\beta(p)=0.3996$; $p=0.4$, $\beta(p)=0.75$; $p=0.5$, $\beta(p)=0.942$

$p=0.6$, $\beta(p)=0.994$; $p=0.7$, $\beta(p)=0.999$; $p=0.8$, $\beta(p)=0.999$

$p=0.9$ & $p=1$, $\beta(p)=1$



Size of the test is the prob of typ I error, $\alpha = 0.156$

3. the size of the test is prob of type I error.

$\alpha(\mu) = P(T(x) > c) = P(|\bar{x}_n - \mu_0| > c) = 0.05$

$\Rightarrow P\left(\frac{|\bar{x}_n - \mu_0|}{\sqrt{\frac{\sigma^2}{n}}} > \sqrt{n}c\right) = 0.05$

$\sqrt{n}c = 1.96$, $n = 25$

$\therefore c = 0.392$

4. (a) $P(Y \le c_1 | p = 0.4) + P(Y \ge c_2 | p = 0.4) < 0.1$

$= \sum_{i=0}^{c_1} C_9^i \, 0.4^i \, 0.6^{9-i} + 1 - \sum_{i=0}^{c_2-1} C_9^i \, 0.4^i \, 0.6^{9-i}$

$= 1 - \sum_{i=c_1+1}^{c_2-1} C_9^i \, 0.4^i \, 0.6^{9-i}$

when $c_1 = 1$ and $c_2 = 7$, calculated in R

$P(Y \le c_1 | p = 0.4) + P(Y \ge c_2 | p = 0.4) = 0.0956$, which is closest to 0.1

(b) The size of the test is Type I error. $\alpha(0.4) = 0.0956$