

Amdahl's LAW

Presented By
Dr. Banchhanidhi Dash

School of Computer Engineering
KIIT University

Amdahl's LAW

- performance gain that can be obtained by improving some portion of a computer can be calculated using Amdahl's Law.
- **Amdahl's Law states** that the performance improvement to be gained from using some faster mode of execution is limited by the fraction of the time the faster mode can be used.
- Amdahl's Law defines the **speedup** that can be gained by using a particular feature.

What is speedup?

Speedup is the ratio

$$\text{Speedup} = \frac{\text{Performance for entire task using the enhancement when possible}}{\text{Performance for entire task without using the enhancement}}$$

Alternatively,

$$\text{Speedup} = \frac{\text{Execution time for entire task without using the enhancement}}{\text{Execution time for entire task using the enhancement when possible}}$$

- The execution time using the original computer with the enhanced mode will be the time spent using the unenhanced portion of the computer plus the time spent using the enhancement:

$$\text{Execution time}_{\text{new}} = \text{Execution time}_{\text{old}} \times \left((1 - \text{Fraction}_{\text{enhanced}}) + \frac{\text{Fraction}_{\text{enhanced}}}{\text{Speedup}_{\text{enhanced}}} \right)$$

The overall speedup is the ratio of the execution times:

$$\text{Speedup}_{\text{overall}} = \frac{\text{Execution time}_{\text{old}}}{\text{Execution time}_{\text{new}}} = \frac{1}{(1 - \text{Fraction}_{\text{enhanced}}) + \frac{\text{Fraction}_{\text{enhanced}}}{\text{Speedup}_{\text{enhanced}}}}$$

Amdahl's law is used to find out overall speedup of the system when some part of the system is enhanced

Example: using Amdahl's Law

we have a system in which 40% operations are floating point. Suppose we enhance floating point unit such that it becomes 30 times faster. find overall speedup to the system

solution of Example-1 using Amdahl's Law

Solution:

$$\text{fraction}_{\text{enhancement}} = 0.4$$

$$\text{speedup}_{\text{enhancement}} = 30$$

$$\text{overall speedup} = 1 / ((1 - 0.4) + (0.4 / 30))$$

$$= 1 / ((0.6) + 0.014)$$

$$= 1 / (0.614)$$

$$= 1.62 \text{ (Ans)}$$

Example using Amdals Law

Assume that 30% instructions are data transfer instruction, 40 % instructions are ALU instruction and the rest are the control instruction. Each of data transfer, ALU and control instruction takes respectively 6clock cycle, 4clock cycle and 7 clock cycle. Find the CPI of the machine. If using latest hardware there found 3 times enhancement in the ALU instruction, then find the overall Speedup of the machine.

Solution:

$$CPI = 0.3 * 6 + 0.4 * 4 + 0.3 * 7 = 5.5$$

$$\text{Fraction Enhanced} = 40\% = 0.4$$

$$\text{Speedup Enhanced} = 3$$

$$\text{Overall speed-up} = 1 / [(1 - 0.4) + (0.4 / 3)] = 1.3636$$

Example :using Amdahl's law

Suppose that we want to enhance the processor used for web serving. The new processor is 10 times faster on computation in the web serving application than the old processor. Assuming that the original processor is busy with computation 40% of the time and is waiting for I/O 60% of the time, what is the overall speedup gained by incorporating the enhancement?

solution of Example-2 using Amdahl's Law

Solution:

$$\text{fraction}_{\text{enhancement}} = 0.4$$

$$\text{speedup}_{\text{enhancement}} = 10$$

$$\text{overall speedup} = 1 / ((1 - 0.4) + (0.4 / 10))$$

$$= 1 / ((0.6) + 0.04)$$

$$= 1 / (0.64)$$

$$= 1.5625 \text{ (Ans)}$$

Example :Quiz on Amdahl's law

Consider the enhancement to the processor of a web server the enhanced server is 40 times faster on search queries than old processor. old processor is busy with search queries 75% of the time then the speedup gained by integrating enhanced cpu is -----

- a) 4.72
- b) 3.72
- c) 2.79
- d) 4.72

Example :using Amdahl's law

a)4.72

b)3.72(Ans)

c)2.79

d)4.72

sol:

$$\begin{aligned}\text{Total speedup} &= 1 / ((1 - 0.75) + (0.75 / 40)) \\ &= 1 / (0.26875) \\ &= 3.72\end{aligned}$$

Example:using Amdahl's Law

Consider the cache memory which is 30 times faster than main memory and it can be used 90% of the total time. The overall speedup gain by cache memory is -----

Example:using Amdahl's Law

Solution:

$$\text{overall_speedup} = 1 / ((1 - 0.9) + (0.9 / 30))$$

$$= 1 / (0.1 + 0.03)$$

$$= 1 / (0.13)$$

$$= 7.69$$

Example:using Amdahl's Law

A common transformation required in graphics processors is square root. Implementations of floating-point (FP) square root vary significantly in performance, especially among processors designed for graphics. Suppose FP square root (FSQRT) is responsible for 20% of the execution time of a critical graphics benchmark. One proposal is to enhance the FSQRT hardware and speed up this operation by a factor of 10. The other alternative is just to try to make all FP instructions in the graphics processor run faster by a factor of 1.6; FP instructions are responsible for half of the execution time for the application. The design team believes that they can make all FP instructions run 1.6 times faster with the same effort as required for the fast square root. Compare these two design alternatives.

Answer :

We can compare these two alternatives by comparing the speedups:

Solution:

$$\text{fraction}_{\text{enhancement_FSQRT}} = 0.2$$

$$\text{speedup}_{\text{enhancement_FSQRT}} = 10$$

$$\text{overall speedup_FSQRT} = 1 / ((1 - 0.2) + (0.2 / 10))$$

$$= 1 / ((0.8) + 0.02)$$

$$= 1 / (0.82)$$

$$= 1.2195 \text{ (Ans)}$$

Solution:

$$\text{fraction}_{\text{enhancement_FP}} = 0.5$$

$$\text{speedup}_{\text{enhancement_FP}} = 1.6$$

$$\text{overall speedup_FP} = 1 / ((1 - 0.5) + (0.5 / 1.6))$$

$$= 1 / (0.8125)$$

$$= 1.23 \text{ (Ans)}$$

Question:

A system needs to be improved, and after analysis, we realize that the part that could be improved constitutes 45% of the system and it would allow for an improvement of 45%. What is the overall performance improvement in this case?