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Invest Wisely: a Mixed-integer Semi-closed-loop Dynamic Optimization Model

明智投资：一个混合整数半闭环动态优化模型

Investors are always eager to increase their returns and minimizing the risk by predicting the ups and downs of assets. Gold and bitcoin are two representative assets. In this paper, we build our model based on data in attachments to predict the possible best investment strategy. The performance of the model is evaluated by the investors' feelings when investing under the guidance of our strategy. Adjust the transaction costs to test its sensitivity to the cost, and finally write a memorandum including our strategy, model, and results to the trader.

投资者总是渴望通过预测资产的起起伏伏来增加他们的回报并最小化风险。黄金和比特币是两种具有代表性的资产。在本文中，我们基于附件中的数据建立模型，以预测可能的最佳投资策略。在我们的策略指导下，投资者在投资时的感受会对模型的性能进行评估。调整交易成本以测试其对成本的敏感性，最后写一份备忘录，包括我们的策略、模型和结果给交易者。

In this paper, we introduce the **multi-stage fuzzy investment** method. By analyzing the characteristics of gold and bitcoin, investing by period is a good choice. The investment decision is mainly based on the price data before the trading day. Each investment change or maintain the current asset ratio [C,G,B], to achieve the purpose of reducing losses and increasing returns. Based on these demands, we construct the **mixed-integer semi-closed-loop dynamic optimization** model and solve it through **discrete approximate iterative algorithm**. The result is that the initial $1000 on 9/11/2016 will finally be the wealth equivalent to $62,211 on 9/10/2021.

本文介绍了多阶段模糊投资法。通过分析黄金和比特币的特点，按周期投资是一个不错的选择。投资决策主要基于交易日前的价格数据。每次投资改变或保持流动资产比率[C，G，B]，达到减少损失、增加收益的目的。基于这些需求，我们构造了混合整数半闭环动态优化模型，并通过离散近似迭代算法进行求解。结果是，2016年9月11日最初的1000美元最终将相当于2021年9月10日的62，211美元。

To ensure that the model is convincing, regret value is used as reliable evaluation evidence. We set the risk level of the three assets (bitcoin: high-risk, gold: mid-risk and cash: risk-free). Then, plot the risk-return regret curve. The **regret value** of the adopted decision is compared with three other cases (all gold acquisition, all bitcoin acquisition, gold:bitcoin = 1:1). Here comes the conclusion: the regret level of the decisions made by the model is always at a low level (0.2~0.5). Regardless of the risk- return rate, the lower regret value is, the better the strategy is.

为确保模型的说服力，后悔值被用作可靠的评估证据。我们设定了三种资产（比特币：高风险，黄金：中等风险，现金：无风险）的风险等级。然后，绘制风险回报后悔曲

线。我们将所采纳决定的后悔值与其他三种情况(所有黄金收购、所有比特币收购、黄金:比特币= 1:1)进行了比较。由此得出结论:该模型所作决策的后悔水平始终处于较低的水平(0.2~0.5)。无论风险报酬率如何，后悔值越低，策略越好。

The adjustment of transaction costs is closely connected to changes in returns, which reflects the sensitivity of the strategy to costs. The cost in here is the commission (initially γ_{gold}=1%, $\gamma_{bitcoin}$=2%). Therefore, we adjust the commission percentage and reduce it to γ_{gold}=0.1%,$\gamma_{bitcoin}$= 0.2% andγ_{gold}=0.01%, $\gamma_{bitcoin}$=0.02%) respectively. It is obvious that the reduction in transaction costs leads to higher returns on the upfront decisions, an increase in the percentage of bitcoins invested, and a small increase in the final wealth.

交易成本的调整与收益的变化密切相关，反映了战略对成本的敏感性。这里的成本是佣金(最初=1%，=2%)。因此，我们调整佣金百分比并将其分别降低到=0.1%、= 0.2%和=0.01%、=0.02%)。很明显，交易成本的降低会提高前期决策的回报，提高比特币的投资比例，使最终财富小幅增长。$\gamma_{gold}\gamma_{bitcoin}\gamma_{gold}\gamma_{bitcoin}\gamma_{gold}\gamma_{bitcoin}$

Last but not least, we summarize the suggestions and write a memorandum to the trader to share our strategy, model, and results, hoping to bring a satisfying trading strategy.

最后，我们总结了这些建议，并写了一份备忘录给交易员，分享我们的策略、模型和结果，希望能带来一个令人满意的交易策略。

Key Words: trapezoidal fuzzy number, discrete approximate iterative algorithm, regret value,
关键词:梯形模糊数，离散逼近迭代算法，后悔值，
dynamic optimization
动态最佳化

内容

1. 介绍

Nowadays, market investment is one of the most important means to realize wealth appreciation. Through the purchase or sale of various assets in the market, investors can obtain different returns. In order to maximize the total return to investors, we can optimize the investment strategy and decide the amount and proportion of assets to be bought or sold based on market conditions.

目前，市场投资是实现财富增值的重要手段之一。投资者通过在市场上购买或出售各种资产，可以获得不同的收益。为了使投资者的总收益最大化，我们可以根据市场情况优化投资策略，确定拟购买或出售的资产的数量和比例。

Different assets have different risk levels, rewards and trading rules. Generally speaking, gold is less risky and less profitable; while bitcoin has higher risks and higher returns. The commission cost per transaction costs γ% of the amount traded. Usually, γ_{gold} is smaller than $\gamma_{bitcoin}$. For convenience, all "proportion of asset" in this paper refer to the proportion of the value of assets.

不同的资产有不同的风险水平、报酬和交易规则。一般来说，黄金风险较低，利润较低；而比特币具有更高的风险和回报。每笔交易的佣金成本为交易金额的%。通常，小于。为方便起见，本文中所有"资产比例"均指资产价值的比例。$\gamma\gamma_{gold}\gamma_{bitcoin}$

2. Assumptions and Justifications

3. 假设和理由

1. **There is no limit to the maximum and minimum transaction amount.**
2. **最高和最低交易金额没有限制。**

   To simplify the model, we assume that the minimum amount per transaction is infinitely close to 0, with no upper limit. And no debit or credit occurs. This applies to transactions of both gold and bitcoin.

   为了简化模型，我们假设每笔交易的最小金额无限接近于 0，没有上限。并且不发生借方或贷方。这适用于黄金和比特币的交易。
3. **Investors make their decisions to purchase or sell based only on data from attachment, which are independent from people and circumstances around them.**
4. **投资者仅根据附件中的数据做出买入或卖出的决定，这些数据独立于他们周围的人和环境。**

   Many factors exert influence on investors' decisions, such as the evaluation from people around them, the current situation of life, and the news on the Internet. In this question,

when making a trading decision, only the attached data matters with no other factors.
影响投资者决策的因素很多，比如周围人的评价、生活现状、互联网上的新闻等。在这个问题上，当作出交易决定时，只有所附的数据是重要的，没有其他因素。

5. **No major crises events affecting the world economy occurred during the investment period.**
6. **投资期内未发生影响世界经济的重大危机事件。**
For example, uncontrollable events such as world wars and major natural disasters will have a huge impact on the economy. These impacts do not fully comply with market rules, so they cannot be correctly predicted.
例如，世界大战和重大自然灾害等无法控制的事件将对经济产生巨大影响。这些影响并不完全符合市场规则，因此无法正确预测。

4. Notations

5. 记号

Table 1 shows the notations commonly used in this paper and their description.
表 1 显示了本文中常用的符号及其说明。

Table 1. Notations

桌子 2. Notations

Notations	Description
X_0	Initial assets owned by investors
t	An investment period
X_t	Total wealth at the end of period t
γ_i	The transaction cost of asset i
$R_{¿}$	Fuzzy yield of risky assets i in period t
$\overline{svar}(X_t)$	The semi-variance of X_t
$u_{¿}$	Investment amount to asset i on period t
u_{ft}	Investment amount to risk-free asset i on period t
$l_{¿}$	The upper bound constraint of risk asset i on period t
$p_{¿}$	The lower bound constraint of risk asset i on period t
记号	描述
X_0	投资者拥有的初始资产
t	投资期
X_t	期末财富总额t
γ_i	资产交易成本i
$R_{¿}$	风险资产期内模糊收益率it
$\overline{svar}(X_t)$	的半方差X_t
$u_{¿}$	投资金额至资产期间it
u_{ft}	投资金额至无风险资产之期间it
$l_{¿}$	风险资产对周期的上限约束it
$p_{¿}$	风险资产对周期的下限约束it

6. 模型构建与求解

6.1 Problem Analysis

6.2 问题分析

First of all, we note that this is a personal investment and the amount of investment is small, so the strategy should be in line with the market mechanism to reduce risks and increase returns.

首先，我们注意到这是一项个人投资，而且投资额很小，所以策略上要符合市场机制，降低风险，增加收益。

As for gold trading, its price is affected by both short-term and long-term factors. Short-term factors, which are uncertain and contingent, let gold price fall more than its recent increase, so it is not recommended to invest in short-term timing. The medium and long-term timing model is influenced by the Federal Reserve's monetary policy and the characteristics of economic cycle. These economic characteristics can be made out obviously after their occurrence, but are hard to predict in advance.

就黄金交易而言，其价格受短期和长期因素的影响。不确定性和或然性的短期因素会让金价下跌幅度超过近期涨幅，因此不建议在短期时机进行投资。中长期时间模型受美联储货币政策和经济周期特征的影响。这些经济特征在它们发生之后可以明显地表现出来，但很难预先预测。

Therefore, under the influence of complex and unpredictable driving factors, it is recommended to allocate gold assets for a long time. In this way, no matter how time flies or market influencing factors changes, we can finally obtain long-term benefits. The cost of such strategy is only to give up some band benefits that are difficult to grasp in the short term. It is not recommended that individual investors take risks for this kind of return. Therefore, for gold trading, we recommend maintaining long-term allocation of small positions, i.e. 5 % ~ 10 % of the position.

因此，在复杂不可预测的驱动因素影响下，建议长期配置黄金资产。这样，无论时间如何飞逝，或者市场影响因素如何变化，我们最终都能获得长远的利益。这种策略的代价只是放弃一些短期内难以把握的波段收益。不建议个人投资者为这种回报承担风险。因此，对于黄金交易，我们建议维持长期配置的小仓位，即 5 % ~ 10 %的仓位。

对于比特币交易，我们更喜欢自动投资计划。在股票市场上，自动投资计划已经被证明是一种适合大多数人的策略。通过定期购买一定数量的股票，你可以得到与市场水平相同的回报，甚至更高。由于比特币的价格变化无常，自动投资计划同样适用于比特币。通过选择购买时机，可以有效地平抑股价波动，降低持有成本，将风险减半。

Securities returns, which can only be estimated according to limited information, can be

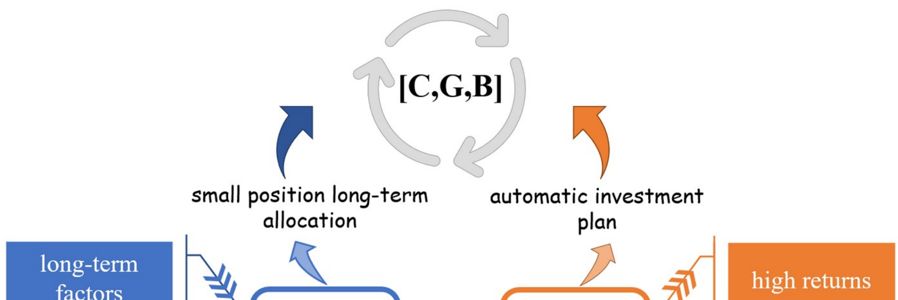


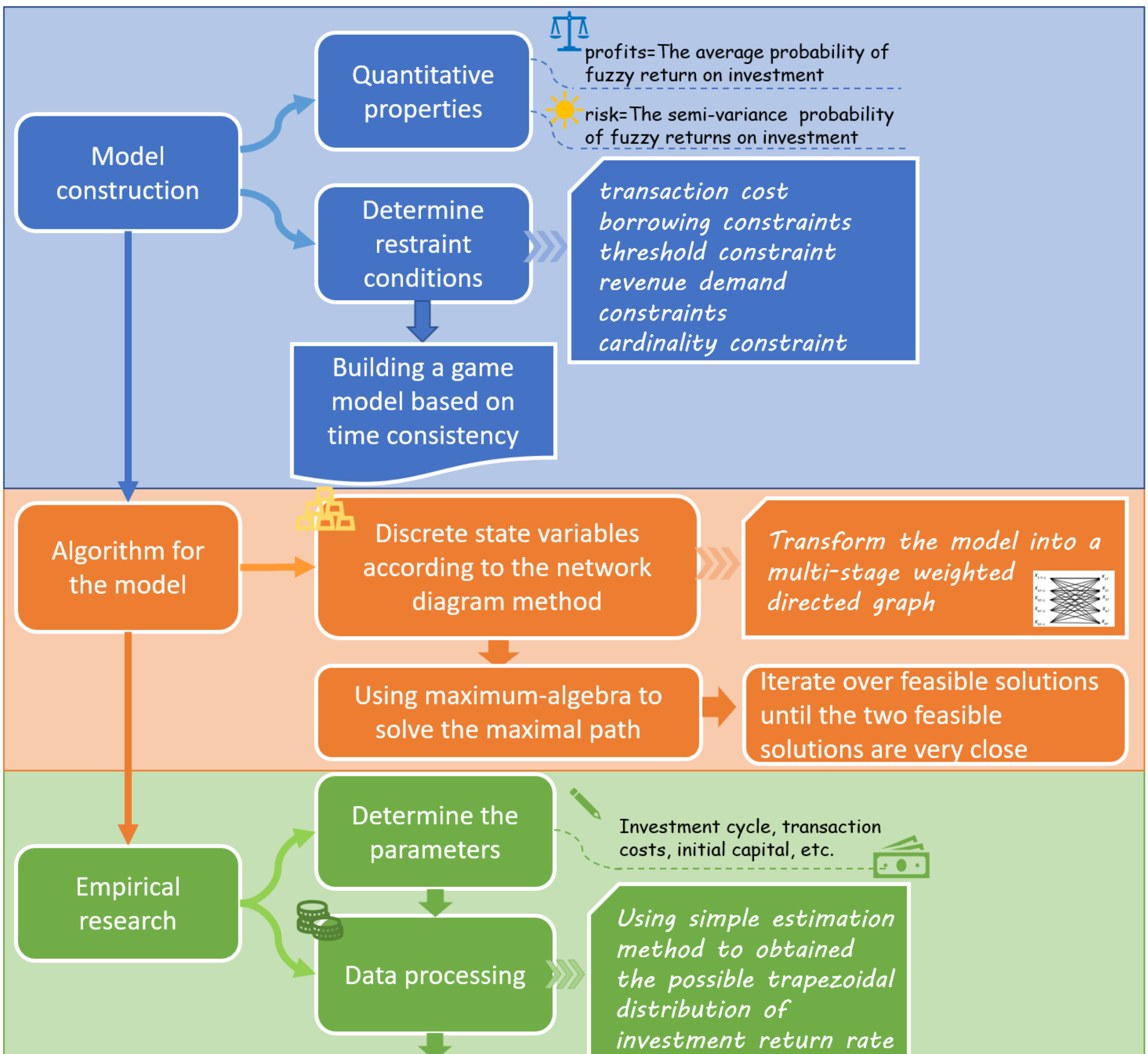
Figure 1. Gold and bitcoin

considered as fuzzy variables r ariables, which has been proved by many studies. Since the trapezoidal fuzzy number[1] has been shown to be an appropriate measure of security returns, we decide to predict the price of gold and bitcoin with fuzzy number. Based on this, establish a multi-stage decision-making model, which helps us to regularly change the proportion of holding assets and reduce trading in other time to reduce risks and improve returns.

证券收益只能根据有限的信息进行估计，可以认为是模糊变量，而不仅仅是随机变量，许多研究都证明了这一点。由于梯形模糊数[1]已被证明是衡量证券收益的适当指标，我们决定用模糊数来预测黄金和比特币的价格。在此基础上，建立多阶段决策模型，帮助我们定期改变持有资产的比例，减少在其他时间的交易，以降低风险，提高收益。

6.3 问题分析与建模思路

基于以上分析，将投资策略的“时间不一致性”引入模型构建，提出了一种多阶段均值半方差模糊投资组合模型。基于交易成本、贷款约束、门槛约束、收益需求和基数约束，研究了具有时间一致性的最优投资策略。考虑到交易成本、收益需求和基数约束，该模型提供了一个具有路径依赖的混合整数半闭环动态优化模型来解决该问题。至于具体细节，我们将使用离散近似迭代算法来获得本文中的最优时间一致投资策略。图 2 是这个过程的流程图。



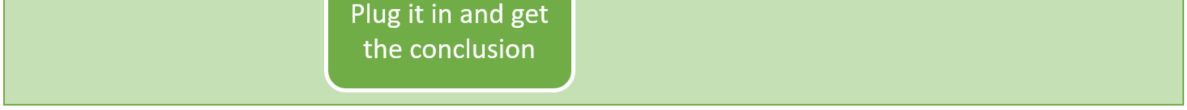


Figure 2. Model construction

6.4 Data Preprocessor

6.5 数据预处理器

在数据分析和决策之前，必须保证数据的质量，这就决定了模型的预测和泛化能力。为保证数据的准确性、完整性和可信度，我们对附件:BCHAIN-MKPRU.csv 和 LBMA-GOLD.csv 提供的数据进行了预处理。

Method *pandas.isnull.sum()* in python is used to process these two files. Some data is found in need of supplement. Consider a line as one item. Items without price data are included in the total but marked as incomplete. We find that there are 10 incomplete items out of a total of 1265 items in LBMA-GOLD.csv. In order to make the data smoother, we choose the mean interpolation method to supplement the data. For each incomplete item, the mean value of price on the day before and after the missing data day is supplemented into it. Table 2 shows the price data inserted into *LBMA-GOLD.csv*.

python 中的方法熊猫. isnull.sum()用于处理这两个文件。有些数据需要补充。将一行视为一个项目。没有价格数据的项目包括在总额中，但标记为不完整。我们发现，在 LBMA-GOLD.csv 的 1265 个项目中，有 10 个项目是不完整的，为了使数据更平滑，我们选择了均值内插法对数据进行补充。对于每个不完整的项目，在其中补充了缺失数据日前后一天的价格平均值。表 2 显示了插入 LBMA-GOLD.csv 的价格数据。

Table 3. Data inserted into file

桌子 4. Data inserted into file

Date	**12/23/16**	**12/30/16**	**12/22/17**	**12/29/17**	**12/24/18**	**12/31/18**	**12/24/19**	**12/31/19**	**12/24/20**	**12/31/20**
Price	**1132.98**	**1148.45**	**1272**	**1301.5**	**1263.1**	**1281**	**1469.8**	**1521**	**1874.65**	**1915.4**
日期	12/23/16	12/30/16	12/22/17	12/29/17	12/24/18	12/31/18	12/24/19	12/31/19	12/24/20	12/31/20
价格	1132.98	1148.45	1272	1301.5	1263.1	1281	1469.8	1521	1874.65	1915.4

6.6 The Necessity of Portfolio Investment

6.7 证券投资的必要性

Focusing on gold and bitcoin, we mainly analyze their trading risks and correlation of price trends in this part.

本部分主要以黄金和比特币为研究对象，分析了它们的交易风险以及价格走势的相关性。

With the development of market investment, people are more inclined to trade rationally and reduce risks while obtaining greater expected returns. Portfolio investment is a common way to avoid risks in investment management. By selecting different investment portfolios, we can maximize returns and minimize risks.

随着市场投资的发展，人们更倾向于理性交易，在降低风险的同时获得更高的预期收益。组合投资是投资管理中规避风险的一种常用方法。通过选择不同的投资组合，我们可以实现收益最大化和风险最小化。

By introducing DCCcoefficient[2]， we can study the dynamic correlation between gold and bitcoin in the market. DCC-GARCH model is widely used in this field. Its general form is:

通过引入系数[2]，我们可以研究市场上黄金与比特币的动态相关性。DCC-GARCH 模型在这一领域有着广泛的应用。其一般形式是:*DCC*

$$
\begin{aligned}
&\text{¿} r_t | F_{t-1} \ \ N(0, H_t) \\
&\text{¿} H_t = D_t R_t D_t \\
&\text{¿} Q_t = (1-\alpha-\beta)\overline{Q} + \alpha \mu_{t-1}\mu_{t-1}^T + \beta Q_{t-1} \\
&\text{¿} D_t = diag\left(\sqrt{h_{11,t}}, \sqrt{h_{22,t}}, \ldots \sqrt{h_{kk,t}}\right) \\
&\text{¿} R_t = (diag(Q_t))^{\frac{-1}{2}} Q_t (diag(Q_t))^{\frac{-1}{2}}
\end{aligned}
\quad (1)
$$

$$
\begin{aligned}
&\text{¿} r_t | F_{t-1} \ \ N(0, H_t) \\
&\text{¿} H_t = D_t R_t D_t \\
&\text{¿} Q_t = (1-\alpha-\beta)\overline{Q} + \alpha \mu_{t-1}\mu_{t-1}^T + \beta Q_{t-1} \\
&\text{¿} D_t = diag\left(\sqrt{h_{11,t}}, \sqrt{h_{22,t}}, \ldots \sqrt{h_{kk,t}}\right) \\
&\text{¿} R_t = (diag(Q_t))^{\frac{-1}{2}} Q_t (diag(Q_t))^{\frac{-1}{2}}
\end{aligned}
\quad (1)
$$

where r_t is the conditional revenue rate of k kinds of assets, F_{t-1} is the set of collected information, H_t is the conditional covariance matrix, R_t is the dynamic conditional correlation index matrix, Q_t is the covariance matrix and $\overline{Q}$ is the unconditional covariance matrix of standardized residual. α and β are positive parameters which meet $\alpha+\beta<1$.
其中是 k 类资产的条件收益率，是所收集信息的集合，是条件协方差矩阵，是动态条件相关指数矩阵，是协方差矩阵，是标准化残差的无条件协方差矩阵。并且是满足+的正参数。$r_t F_{t-1} H_t R_t Q_t \overline{Q} \alpha \beta \alpha \beta < 1$

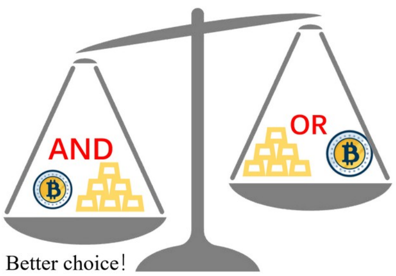


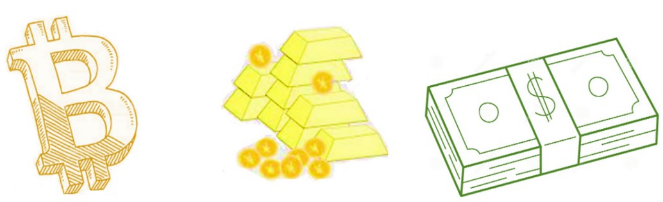
Figure 3. Investment choice

通过该模型，我们建立了比特币与黄金收益率序列的单变量 GARCH (1，1)模型，并得出比特币与黄金的动态相关性具有显著的持续性的结论。这些细节对于我们要解决的主要问题并不重要，因此我们在本文中省略了它们。简而言之，它应该同时投资黄金和比特币，以获得更高的回报，同时降低风险。

As for volatility, it describes the uncertainty of asset returns, which is used to reflect the risk level of financial assets. According to how volatility changes, assets can be devided into assets with different risks. Common types include high-risk assets, mid-risk assets, and risk-free assets.

波动性方面，描述了资产收益的不确定性，用于反映金融资产的风险水平。根据波动性的变化，资产可以分为不同风险的资产。常见类型包括高风险资产、中等风险资产和无风险资产。

与黄金和比特币不同，美元受多种因素影响，而且基本上处于受控状态，因此其价值可以长期持有，波动性很低。可以将其视为无风险资产。黄金的价格和波动性变化比美元更剧烈，但没有比特币剧烈。因此，黄金可以被视为中等风险资产，而比特币可以被视为高风险资产，如图 4 所示。



6.8 Definition of Parameters

6.9 参数定义

Assume that there are n kinds of risk assets and 1 kind of risk-free asset for investors to choose. The initial wealth is X_0. Let $t$stand for investment period, with T stages. At the first stage of investment stage, the investor can only invest with X_0. The asset portfolios changes at the beginning of eachstage and investment last for T stages.

假设有 2 种风险资产和 1 种无风险资产可供投资者选择。最初的财富是。让代表投资期，用阶段来代表。在第一阶段的投资阶段，投资者只能用。资产组合在每个阶段开始时发生变化，投资持续数个阶段。nX_0tTX_0T

Let $R_{¿}$ stand for the fuzzy yield of risky assets i in period t, and $R_t=\left(R_{1t},R_{2t},\ldots R_{nt}\right)'$. Set X_t as the total wealth at the end of period t. In period t, set $\overline{svar}\left(X_t\right)$ as the semi-variance of X_t, $u_{¿}$ as the investment in asset i, u_{ft} as the investment in risk-free asset i, $l_{¿}$ and $p_{¿}$ as the upper and lower bound constraint of risk asset i on period t. Let $u_t=\left(u_{1t},u_{2t},\ldots,u_{nt},u_{ft}\right)'$.

让我们来看看这段时期风险资产的模糊收益率。设置为期末总财富。在期间内，设定为的半方差，作为对资产的投资，作为对无风险资产的投资，并作为风险资产对期间的上下限约束。让 ⋯ ⋯ $R_{¿}itR_t=\left(R_{1t},R_{2t},\ldots R_{nt}\right)'X_ttt\overline{svar}\left(X_t\right)X_tu_{¿}iu_{ft}il_{¿}p_{¿}it$

$u_t=\left(u_{1t},u_{2t},\ldots,u_{nt},u_{ft}\right)'$

6.10 Parameter Quantization

6.11 参数量化

The securities market is a complex system with dynamic changes. It is difficult for people to obtain the overall information of the random distribution of securities returns. The return can only be estimated according to the historical information of securities. According to most of the research[3] in this field, securities returns can be considered as fuzzy variables rather than random variables. In addition, considering the uncertainty of market environment, trapezoidal fuzzy number is often used to measure the yield of securities. Many scholars have discussed the fuzzy portfolio optimization problem deeply, such as Carlsson[4], Anne Trefethen[5], Zhang Peng and Zhang Weiguo[6], etc.

证券市场是一个动态变化的复杂系统。人们很难获得证券收益随机分布的整体信息。回报只能根据证券的历史信息来估计。根据该领域的大多数研究[3]，证券收益可以被认为是模糊变量而不是随机变量。此外，考虑到市场环境的不确定性，通常采用梯形模糊数来衡量证券的收益率。许多学者对模糊投资组合优化问题进行了深入的探讨，如Carlson[4]，Anne Trefethen[5]，张鹏和张维国[6]等。

The return and risk of asset portfolios are measured by the possibilistic mean value and the possibilistic standard semi-variance of asset fuzzy return, additionally. Clearly, the whole process is self-financing, since the investor did not add additional funds during this period.

此外，资产组合的回报和风险由资产模糊回报的可能性均值和可能性标准半方差来衡量。显然，整个过程是自筹资金的，因为投资者在此期间没有增加额外资金。

Record the return rate of risk assets as $R_{¿}$, where $R_{¿}=(a_{¿},b_{¿},\alpha_{¿},\beta_{¿})$.

将风险资产的回报率记录为，其中$R_{¿}R_{¿}=(a_{¿},b_{¿},\alpha_{¿},\beta_{¿})$.

In period t, the possibilistic mean value r_{pt} of the assets $u_t=(u_{1t},u_{2t},\ldots u_{nt},u_{ft})$ can be calculated as:

在期间内，资产的可能性平均值可计算为:$t r_{pt} u_t=(u_{1t},u_{2t},\ldots u_{nt},u_{ft})$

$$r_{pt}\wedge ¿\sum_{i=1}^{n}\overline{M}(R_{¿})u_{¿}+r_{ft}(X_{t-1}-¿\sum_{i=1}^{n}u_{¿})-X_{t-1}¿\wedge ¿(r_{fi}-1)X_{t-1}+\sum_{i=1}^{n}\left(\frac{a_{¿}+b_{¿}}{2}+\frac{\beta_{¿}-\alpha_{¿}}{6}-r_{ft}\right)u_{¿},t=1,2,\ldots,T \quad (2)$$

$$r_{pt}\wedge ¿\sum_{i=1}^{n}\overline{M}(R_{¿})u_{¿}+r_{ft}(X_{t-1}-¿\sum_{i=1}^{n}u_{¿})-X_{t-1}¿\wedge ¿(r_{fi}-1)X_{t-1}+\sum_{i=1}^{n}\left(\frac{a_{¿}+b_{¿}}{2}+\frac{\beta_{¿}-\alpha_{¿}}{6}-r_{ft}\right)u_{¿},t=1,2,\ldots,T \quad (2)$$

本文假设交易成本为%。显然，资产在期间内的交易成本是，而资产组合的总成本是:$\gamma itu_{¿}\times\gamma\% u_t=(u_{1t},u_{2t},\ldots,u_{nt},u_{ft})$

$$C_t=\sum_{i=1}^{n}u_{¿}\times\gamma,t=1,2,\ldots,T \quad (3)$$

$$C_t=\sum_{i=1}^{n}u_{¿}\times\gamma,t=1,2,\ldots,T \quad (3)$$

Then, the net return ratio of asset portfolio X_t by the end of period t is:

那么，资产组合期末净收益率为:$X_t t$

$$r_{Nt}=(r_{ft}-1)X_{t-1}+\sum_{i=1}^{n}\left(\frac{a_{¿}+b_{¿}}{2}+\frac{\beta_{¿}-\alpha_{¿}}{6}-r_{ft}\right)u_{¿}-\sum_{i=1}^{n}\gamma_{¿}|u_{¿}| \quad (4)$$

$$r_{Nt}=(r_{ft}-1)X_{t-1}+\sum_{i=1}^{n}\left(\frac{a_{¿}+b_{¿}}{2}+\frac{\beta_{¿}-\alpha_{¿}}{6}-r_{ft}\right)u_{¿}-\sum_{i=1}^{n}\gamma_{¿}|u_{¿}| \quad (4)$$

and the total wealth held by investor by the end of period t is:

投资者期末持有的全部财富为:t

$$X_t=r_{Nt}+X_{t-1}=r_{ft}X_{t-1}+\sum_{i=1}^{n}\left(\frac{a_{¿}+b_{¿}}{2}+\frac{\beta_{¿}-\alpha_{¿}}{6}-r_{ft}\right)u_{¿}-\sum_{i=1}^{n}\gamma_{¿}|u_{¿}| \quad (5)$$

$$X_t = r_{Nt} + X_{t-1} = r_{ft} X_{t-1} + \sum_{i=1}^{n}\left(\frac{a_{¿}+b_{¿}}{2}+\frac{\beta_{¿}-\alpha_{¿}}{6}-r_{ft}\right)u_{¿} - \sum_{i=1}^{n}\gamma_{¿}\left|u_{¿}\right| \quad (5)$$

where $t=1,2,\ldots,T$.
在那里。$t=1,2,\ldots,T$

6.12 Determination of Constraint Conditions

6. 13 约束条件的确定

The threshold constraint of multi-stage portfolio is:
多阶段投资组合的门槛约束为:

$$l_{¿} \le u_{¿} \le p_{¿} \quad (6)$$

$$l_{¿} \le u_{¿} \le p_{¿} \quad (6)$$

where $l_{¿}$ and $p_{¿}$ as the upper and lower bound constraint of $u_{¿}$.
其中和作为约束的上限和下限。$l_{¿} p_{¿} u_{¿}$

Set the lower bound constrain of proportion of investment ratio on risk-free assets to be $u_{ft}^{b}\left(u_{ft}^{b} \le 0\right)$, then the borrowing constraints of risk-free assets in period t is:

将无风险资产的投资比例比例下限约束设置为，则无风险资产期内的借款约束为: $u_{ft}^{b}\left(u_{ft}^{b} \le 0\right) t$

$$u_{ft} = X_{t-1} - \sum_{i=1}^{n} u_{¿} \ge u_{ft}^{b} \quad (7)$$

$$u_{ft} = X_{t-1} - \sum_{i=1}^{n} u_{¿} \ge u_{ft}^{b} \quad (7)$$

According to formula 1, the semi-variance of asset portfolio $u_{¿}$ is:
根据公式 1，资产组合的半方差为: $u_{¿}$

$$\overline{svar}_t\left(u_t\right) = u_t' H_t^{-¿u_t¿} (8)$$

$$\overline{svar}_t\left(u_t\right) = u_t' H_t^{-¿u_t¿} (8)$$

where its standard semi-variance is $\sqrt{\overline{svar}_t\left(u_t\right)}$. Assume that $H_t^{-¿¿}$ is a semi-positive definite matrix.

其中其标准半方差为。假设是一个半正定矩阵。$\sqrt{\overline{svar}_t\left(u_t\right)} H_t^{-¿¿}$

$$H_t^{-¿=¿¿¿}$$
$$¿$$

$$H_t^{-¿=¿¿¿}$$

¿

$$Cov_t^{-¿(r_¿,r_{jt})=¿\frac{(b_¿-a_¿)(\beta_{jt}+\alpha_{jt})+(b_{jt}-a_{jt})(\beta_¿+\alpha_¿)}{12}¿} \wedge + \left[\frac{(\beta_¿+\alpha_¿)(\beta_{jt}+\alpha_{jt})}{36}\right] + \frac{(\beta_¿-\alpha_¿)(\beta_{jt}-\alpha_{jt})}{4} + \frac{\beta_¿\beta_{jt}}{18}$$

(10)

$$Cov_t^{-¿(r_¿,r_{jt})=¿\frac{(b_¿-a_¿)(\beta_{jt}+\alpha_{jt})+(b_{jt}-a_{jt})(\beta_¿+\alpha_¿)}{12}¿} \wedge + \left[\frac{(\beta_¿+\alpha_¿)(\beta_{jt}+\alpha_{jt})}{36}\right] + \frac{(\beta_¿-\alpha_¿)(\beta_{jt}-\alpha_{jt})}{4} + \frac{\beta_¿\beta_{jt}}{18}$$

(10)

The cardinality constraint is:

基数约束为:

$$\sum_{i=1}^{n} z_¿ \le K, z_¿ \in \{0,1\}$$

(11)

$$\sum_{i=1}^{n} z_¿ \le K, z_¿ \in \{0,1\}$$

(11)

where K is the maximum number of risk-free assets, and

其中是无风险资产的最大数量，以及K

$$z_¿ = \begin{cases} ¿\,1\, choose\ asset\ i \\ ¿\,0\, not\ choose\ asset\ i \end{cases}$$

(12)

$$z_¿ = \begin{cases} ¿\,1\, choose\ asset\ i \\ ¿\,0\, not\ choose\ asset\ i \end{cases}$$

(12)

6.14 博弈模型构建

The traditional multi-stage mean-standard semi-variance portfolio optimization model only considers the expected value of end-of-period wealth and standard semi-variance. In the real world, however, investors care not only about the expected value and standard semi-variance of end-of-period wealth, but also about which during the investment period. In other words, the expectation and the semi-variance of the portfolio are different in the $t$period. Therefore, this paper uses standard semi-variance to measure risk. Set the weight coefficient $w_t > 0$ and risk preference coefficient $\eta_t > 0, t = 1,2,..,T$.

传统的多阶段均值-标准半方差投资组合优化模型只考虑期末财富期望值和标准半方差。但在现实世界中，投资者关心的不仅仅是期末财富的期望值和标准半方差，更关心的是投资期间的期末财富是多少。换句话说，投资组合的期望和半方差在该期间是不同的。因此，本文采用标准半方差来度量风险。设定权重系数和风险偏好系数。t $w_t > 0 \eta_t > 0, t = 1,2,..,T$

Time-consistent strategy means when $t_1 < t_2$, the optimal policies based on these two stages are the same. Few scholars have studied the time consistency of the multi-stage mean-standard semi-variance portfolio model with transaction costs, borrowing constraints, threshold constraints, income demand and cardinality constraints. In this paper, we restate this problem as a game problem. In this part, we will study the optimal strategy of generalized multi-stage mean-quasi-semi-variance fuzzy portfolio model with time consistency under multiple realistic constraints.

时间一致策略是指当基于这两个阶段的最优策略相同时。很少有学者研究具有交易费用、借贷约束、门槛约束、收入需求和基数约束的多阶段均值-标准半方差投资组合模型的时间一致性。在这篇文章中，我们重申这个问题是一个博弈问题。在这一部分中，我们将研究多现实约束下具有时间一致性的广义多阶段均值-拟半方差模糊投资组合模型的最优策略。$t_1 < t_2$

According to Bjork and Murguci's research, a definition based on non-cooperative game was given:

根据比约克和Murguci的研究，给出了一个基于非合作博弈的定义:

Definition 1. Consider a fixed control law $u^{TC}(k-1)$. Set $u(k-1) = (u_{k-1}, u_k^{TC}, \ldots, u_{T-1}^{TC})$. u_{k-1}can be any control variable. For any $k(k=1,2,\ldots,T)$, if $u^{TC}(k-1)$is the same time-consistent strategy, then it comes the following result:

定义1。考虑一个固定的控制律。准备。可以是任何控制变量。对任何人来说，如果是同样的时间一致策略，那么就会得出下面的结果:$u^{TC}(k-1)$

$u(k-1) = (u_{k-1}, u_k^{TC}, \ldots, u_{T-1}^{TC}) u_{k-1} k (k=1,2,\ldots,T) u^{TC}(k-1)$

$$Max_{u_{k-1}} u_{k-1}(X_{k-1}, u(k-1)) = u_{k-1}(X_{k-1}, u^{TC}(k-1))$$

$$Max_{u_{k-1}} u_{k-1}(X_{k-1}, u(k-1)) = u_{k-1}(X_{k-1}, u^{TC}(k-1))$$

Such time-consistent law $u^{TC}(k-1)$ is called the **perfect Nash equilibrium strategy** of sub-game. Under a non-cooperative game frame, assume that a gamer starts at position $(k-1, X_{k-1})$. According to Definition 1, since he follows strategy $(u_k^{TC}, \ldots, u_{T-1}^{TC})$, he can only choose strategy u_{k-1} to maximize $u_{k-1}(X_{k-1}, u(k-1))$.

这样的时间一致性规律称为子博弈的完美纳什均衡策略。在非合作博弈框架下，假设一个玩家从位置开始。根据定义1，由于他遵循战略，他只能选择战略最大化。

$u^{TC}(k-1)(k-1, X_{k-1})(u_k^{TC}, \ldots, u_{T-1}^{TC}) u_{k-1} u_{k-1}(X_{k-1}, u(k-1))$

The following formula 13 shows a common equivalent transformation in time- consistent strategy:

下面的公式13显示了时间一致策略中的通用等效转换:

$$¿ w_T \left[\overline{M}_T(X_T) - \eta_T \sqrt{svar_{T-1}(X_T)} \right]$$

$$¿ = w_T ¿$$

$$¿$$

$$¿w_T\left[\overline{M}_T(X_T)-\eta_T\sqrt{\overline{svar}_{T-1}(X_T)}\right]$$
$$¿=w_T¿$$
$$¿$$

In actual investment, we can easily conclude the following conditions:
在实际投资中，我们可以很容易地得出以下条件:

- Expected return should be maximized;
- 预期回报应最大化；
- Investment to risk-free assets is a must;
- 投资无风险资产是必须的；

- 资产数量不超过；K
- The end-of-period wealth are supposed to be larger than X_{0T}
- 期末财富应该大于X_{0T}

Thus, base on formula 6, formula 8, formula 10, formula 11, formula 12, formula 13 and the conclusions above, we can get the following strategy model:

由此，基于公式6、公式8、公式10、公式11、公式12、公式13以及上述结论，我们可以得到如下策略模型:

$$Max\sum_{t=1}^{T}w_t¿¿$$
$$¿$$

$$Max\sum_{t=1}^{T}w_t¿¿$$
$$¿$$

$$s.t.\begin{cases} X_t=r_{ft}X_{t-1}+\sum_{i=1}^{n}\left(\frac{a_¿+b_¿}{2}+\frac{\beta_¿-\alpha_¿}{6}-r_{ft}\right)u_¿-\sum_{i=1}^{n}c_¿\left|u_¿-u_{i(t-1)}\right| \\ X_T\geq X_{0T} \\ X_{t-1}-\sum_{i=1}^{n}u_¿\wedge\geq u_{ft}^{b} \\ \sum_{i=1}^{n}z_¿\leq K,z_¿\in\{0,1\} \\ l_¿z_¿\leq u_¿\leq p_¿z_¿,i=1,\cdots,n,t=1,\cdots,T \end{cases}$$
(15)

$$s.t.\begin{cases} X_t=r_{ft}X_{t-1}+\sum_{i=1}^{n}\left(\frac{a_¿+b_¿}{2}+\frac{\beta_¿-\alpha_¿}{6}-r_{ft}\right)u_¿-\sum_{i=1}^{n}c_¿\left|u_¿-u_{i(t-1)}\right| \\ X_T\geq X_{0T} \\ X_{t-1}-\sum_{i=1}^{n}u_¿\wedge\geq u_{ft}^{b} \\ \sum_{i=1}^{n}z_¿\leq K,z_¿\in\{0,1\} \\ l_¿z_¿\leq u_¿\leq p_¿z_¿,i=1,\cdots,n,t=1,\cdots,T \end{cases}$$
(15)

This is the mixed-integer semi-closed-loop dynamic optimization model with path-dependence. It takes both returns and risks into consideration to help investors gain more while avoiding risks. The following parts will solve it and put it into practical application.

这是具有路径依赖的混合整数半闭环动态优化模型。它把回报和风险都考虑在内，帮助投资者在避免风险的同时获得更多收益。接下来的部分将解决这个问题，并将其投入实际应用。

6.15 Model Resolution

6.16 模型解析

In this part, we will use discrete approximate iterative algorithm[6] to solve this problem. First, discretize the state variables based on network graph methods. Through this way, we can turn the model into a multi-stage weighted directed graph. Next, solve the maximum path by the maximum algebraic method. Here we can get the feasible solution. Finally, based on the feasible solution, keep iterating until the last 2 solutions are very close.

在这一部分中，我们将使用离散近似迭代算法[6]来解决这个问题。首先，基于网络图方法对状态变量进行离散化。通过这种方法，我们可以将模型转化为一个多阶段加权有向图。接下来，用最大代数方法求最大路径。在这里我们可以得到可行的解决方案。最后，基于可行解，继续迭代，直到最后两个解非常接近。

The details are as follows:

详情如下:

Step 1. Disperse the interval $[X_t^{min}, X_t^{max}]$ into four equations. Therefore, discrete state variable $x_{¿}$ at stage t can be obtained. ($i=1,2,\ldots,5, t=1,2..,T$).

Step 2. 将间隔分散为四个等式。因此，可以获得阶段的离散状态变量。().
$[X_t^{min}, X_t^{max}] x_{¿} t i=1,2,\ldots,5, t=1,2..,T$

Step 3. 让我们代表多阶段加权有向图。解决…的边缘。构建如图 5 所示的图表。$F_t(j,k) F_t(j,k)$

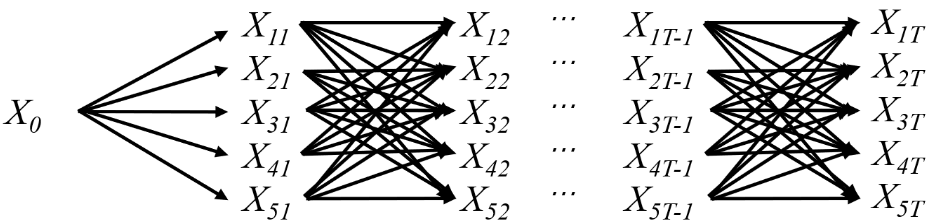


Figure 5.Multi-stage weighted directed graph

Step 4. Based on discrete approximate iterative algorithm, get the longest path $F^{(1)}$ of $F_t(j,k)$ after the first iteration:

Step 5. 基于离散近似迭代算法，得到第一次迭代后的最长路径:$F^{(1)} F_t(j,k)$

$$F^{(1)} = F_1^{(1)} \otimes F_2^{(1)} \otimes \ldots \otimes F_T^{(1)} \quad (16)$$

$$F^{(1)} = F_1^{(1)} \otimes F_2^{(1)} \otimes \ldots \otimes F_T^{(1)} \quad (16)$$

where $F_1^{(1)}=\left(F_1^{(1)}(1,j)\right)_{1\times 5}, F_2^{(1)}=\left(F_2^{(1)}(I,j)\right)_{5\times 5},\ldots,F_T^{(1)}=\left(F_T^{(1)}(I,j)\right)_{5\times 5}$

在哪里$F_1^{(1)}=\left(F_1^{(1)}(1,j)\right)_{1\times 5}, F_2^{(1)}=\left(F_2^{(1)}(I,j)\right)_{5\times 5},\ldots,F_T^{(1)}=\left(F_T^{(1)}(I,j)\right)_{5\times 5}$

Step 6. Continue iteration. The $(k+1)$-th iteration can be described as follows:

Step 7. 继续迭代。第次迭代可描述如下:$(k+1)$

Let the longest path of the k-th iteration $F^{(k)}$ be $X_0 \rightarrow X_{i_1 1}^{(k)} \rightarrow X_{i_2 2}^{(k)} \rightarrow \ldots \rightarrow X_{i_T T}^{(k)}$. The best solution to the longest path in Figure 5 is also the feasible solution to the multi-stage mean semi-variance fuzzy portfolio model. Based on this, disperse the variables from stage 1 to stage T into four equations, as the following steps shows.

让第次迭代的最长路径为。图 5 中最长路径的最佳解也是多阶段均值半方差模糊投资组合模型的可行解。基于此，将阶段 1 至阶段的变量分散为四个等式，如下步骤所示。$k F^{(k)} X_0 \rightarrow X_{i_1 1}^{(k)} \rightarrow X_{i_2 2}^{(k)} \rightarrow \ldots \rightarrow X_{i_T T}^{(k)} T$

1) Disperse X_2^{min} and $X_{i_2 2}^{(k)}$, $X_{i_2 2}^{(k)}$ and X_2^{max} into two internal structure which are the same. The 5 disperse points of X_2, which are $X_2^{min}, X_{22}^{(k+1)}, X_{i_1 2}^{(k+1)}, X_{32}^{(k+1)}$ and X_2^{max} can be solved.
2) 分散而成，又分成两个相同的内部结构。的 5 个分散点，它们是并且可以解决。$X_2^{min} X_{i_2 2}^{(k)} X_{i_2 2}^{(k)} X_2^{max} X_2 X_2^{min}, X_{22}^{(k+1)}, X_{i_1 2}^{(k+1)}, X_{32}^{(k+1)} X_2^{max}$
3) Based on $\left(X_{i_3 3}^{(k)},\ldots,X_{i_{T+1} T}^{(k)}\right)$, disperse the variables from stage 3 to stage T into 5 disperse points in the same way. The weight of stage t can also be easily solved.
4) 基于，以相同的方式将阶段 3 至阶段的变量分散为 5 个分散点。阶段的权重也很容易解决。$\left(X_{i_3 3}^{(k)},\ldots,X_{i_{T+1} T}^{(k)}\right) T t$
5) The longest path of $(k+1)$-th iteration $F^{(k+1)}$ and another feasible solution can be calculated :
6) 可以计算第次迭代的最长路径和另一个可行解:$(k+1) F^{(k+1)}$

$$F^{(k+1)}=F_1^{(k+1)} \otimes F_2^{(k+1)} \otimes \ldots \otimes F_T^{(k+1)} \tag{17}$$

$$F^{(k+1)}=F_1^{(k+1)} \otimes F_2^{(k+1)} \otimes \ldots \otimes F_T^{(k+1)} \tag{17}$$

where $F_1^{(k+1)}=\left(F_1^{(k+1)}(1,j)\right)_{1\times 5}, F_2^{(k+1)}=\left(F_2^{(k+1)}(I,j)\right)_{5\times 5} .. F_T^{(k+1)}=\left(F_T^{(k+1)}(I,j)\right)_{5\times 5}$.

在那里。$F_1^{(k+1)}=\left(F_1^{(k+1)}(1,j)\right)_{1\times 5}, F_2^{(k+1)}=\left(F_2^{(k+1)}(I,j)\right)_{5\times 5} .. F_T^{(k+1)}=\left(F_T^{(k+1)}(I,j)\right)_{5\times 5}$

If $\left|F^{(k+1)}-F^{(k)}\right|<10^{-6}$, the longest path $F^{(k+1)}$ is the approximate solution to our model. Else, keep iterating until it does.

如果，最长路径是我们模型的近似解。否则，继续迭代，直到迭代结束。

$\left|F^{(k+1)}-F^{(k)}\right|<10^{-6}F^{(k+1)}$

6.17 Application in Practical Investment

6. 18 在实际投资中的应用

In this part, based on the data in the attachments and the model above, we figure out how much the initial investment worth on 9/10/2021.

在这一部分中，基于附件中的数据和上面的模型，我们计算出 9/10/2021 的初始投资价值。

The first thing needs doing is clearing parameters:

首先需要做的是清除参数:

- According to the historical data on closing prices of gold and bitcoin from 09/11/2016 to 09/10/2021, consider a month as a period. Thus, $T=60$.
- 根据 2016 年 9 月 11 日至 2021 年 9 月 10 日黄金和比特币收盘价的历史数据，将一个月视为周期。因此……$T=60$
- Initial wealth $X_0=\$1000$.
- 初始财富。$X_0=\$1000$
- To ensure that investment is profitable, the ultimate wealth should be greater than the initial wealth. So $X_T>X_0=1000$.
- 为了确保投资是有利可图的，最终财富应该大于初始财富。所以…… $X_T>X_0=1000$
- There are 2 kinds of risk assets (gold, bitcoin) and 1 kind of risk-free asset(cash), thus, $n=2, K=2$.
- 有 2 种风险资产（黄金、比特币）和 1 种无风险资产（现金），因此，。$n=2, K=2$
- Set $i=1$stand for gold and $i=2$ stand for bitcoin. The transaction costs of gold and bitcoin are $\gamma_1=0.01, \gamma_2=0.02$.
- “Set”代表黄金，“stand”代表比特币。黄金和比特币的交易成本是。$i=1$ $i=2\gamma_1=0.01, \gamma_2=0.02$
- Since there will be no additional funds in the investment process, that is, no borrowings or arrears, the lower bound of the risk-free asset investment ratio is 0. Also, the amount of transactions in the period t is less than or equal to the total wealth in period $t-1$. Thus, $u_{ft}^b=0$, $X_{t-1}-\sum_{i=1}^{n} u_¿ \geq 0$.
- 由于在投资过程中不会有额外的资金，即没有借款或欠款，无风险资产投资比率的下限为 0。此外，该期间的交易金额小于或等于该期间的财富总额。因此……$tt-1u_{ft}^b=0X_{t-1}-\sum_{i=1}^{n} u_¿ \geq 0$
- 由于门槛限制，黄金的位置应保持在 5 %至 20 %之间。因此…… $0.05X_t z_{1t} \leq u_{1t} \leq 0.2X_t z_{1t}, t=1, \cdots, T$

The model described by formula 18 can be embodied as follows:

由公式 18 描述的模型可以体现如下:

$$Max\sum_{t=1}^{T} w_t ¿¿$$
$$¿$$

$$Max\sum_{t=1}^{T} w_t ¿¿$$
$$¿$$

$$s.t.\begin{cases} X_t = r_{ft} X_{t-1} + \sum_{i=1}^{2}\left(\frac{a_{¿}+b_{¿}}{2}+\frac{\beta_{¿}-\alpha_{¿}}{6}-r_{ft}\right)u_{¿} - \sum_{i=1}^{2} c_{¿}\left|u_{¿}-u_{i(t-1)}\right|(a) \\ X_T \ge 1000(b) \\ X_{t-1}-\sum_{i=1}^{2} u_{¿} \wedge \ge 0(c) \\ \sum_{i=1}^{2} z_{¿} \le 2, z_{¿} \in \{0,1\}(d) \\ 0.05 X_t z_{1t} \le u_{1t} \le 0.2 X_t z_{1t}, t=1,\cdots,T.(e) \\ \sum_{t=1}^{60} z_{1t} \ge 60 \times 60\%(f) \end{cases}$$

(19)

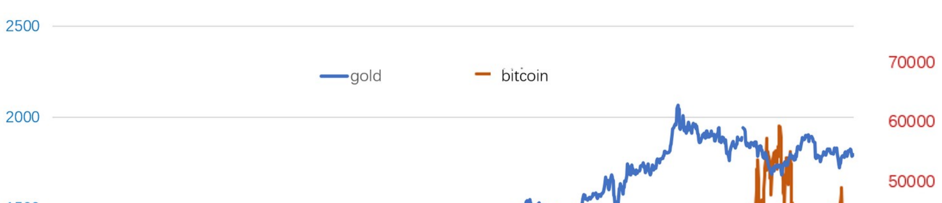
$$s.t.\begin{cases} X_t = r_{ft} X_{t-1} + \sum_{i=1}^{2}\left(\frac{a_{¿}+b_{¿}}{2}+\frac{\beta_{¿}-\alpha_{¿}}{6}-r_{ft}\right)u_{¿} - \sum_{i=1}^{2} c_{¿}\left|u_{¿}-u_{i(t-1)}\right|(a) \\ X_T \ge 1000(b) \\ X_{t-1}-\sum_{i=1}^{2} u_{¿} \wedge \ge 0(c) \\ \sum_{i=1}^{2} z_{¿} \le 2, z_{¿} \in \{0,1\}(d) \\ 0.05 X_t z_{1t} \le u_{1t} \le 0.2 X_t z_{1t}, t=1,\cdots,T.(e) \\ \sum_{t=1}^{60} z_{1t} \ge 60 \times 60\%(f) \end{cases}$$

(19)

Formula 19 is the wealth we hope to achieve, formula 19(a) corresponds to the iterative process, formula 19(b) corresponds to profit demand, formula 19(c) corresponds to the self-financing process, formula 19(d) corresponds to the product category and formula 19(e) corresponds to the threshold constraint of gold. In order to ensure that gold assets are in a low position for a long time, its low-position periods are not less than 60% of the total period, as formula 19(f) says.

公式 19 是我们希望实现的财富，公式 19(a)对应于迭代过程，公式 19(b)对应于利润需求，公式 19(c)对应于自筹资金过程，公式 19(d)对应于产品类别，公式 19(e)对应于黄金的门槛约束。为了确保黄金资产长期处于低位，其低位周期不低于总周期的 60%，如公式 19(f)所示。

根据第 4.7 节中的方法进行求解，得到各期资产收益率概率的梯形分布。



为了

示。将模

型的结果

Figure 6. Daily price of gold and bitcoin

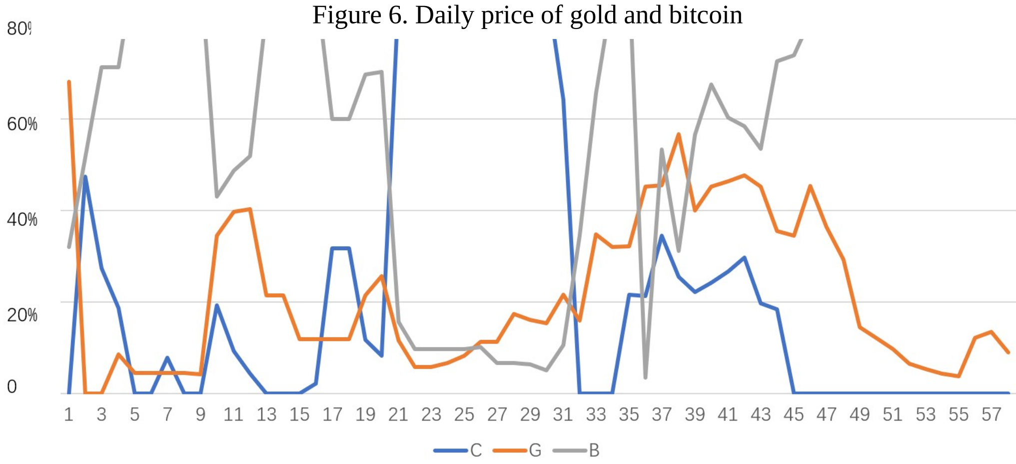


Figure 7. Proportion of assets in each period

Figure 7 shows the proportion of assets in total wealth at the end of each period. Blue, orange and gray lines represent cash, gold and bitcoin, respectively. We find that in periods 1 to 5 and 30 to 35, bitcoin is bought rapidly and massively. This is because the model judges that the price of bitcoin will continue to increase during this period and in the next several period, and the risk of decline in the short term is small, so a large number of purchases are made. In period 20 to 22, although we suffer some losses, the rapid sell-off of large amounts of gold and bitcoin helps us avoid the continuing decline. In period 34 and 35, most bitcoins are promptly and decisively sold, and gold, whose return is more stable, was invested. This further ensures the income when avoiding risks. In period 36 to 60, the slow purchase of bitcoin is due to its relatively low valuation caused by the rapid decline before. At this time, increasing the purchase is conducive to grasping the subsequent upward trend. Also, it is worth attention that although it is less likely to continue to fall after falling, that is, the risk is less, the model still suggests us to buy slowly to share the risk.

图 7 显示了每个时期结束时资产占总财富的比例。蓝色、橙色和灰色线条分别代表现金、黄金和比特币。我们发现，在 1 至 5 和 30 至 35 期间，人们会迅速而大量地购买比特币。这是因为该模型判断比特币的价格在此期间和今后几个期间将继续上涨，短期内下跌的风险较小，因此进行了大量的购买。在 20-22 年期间，尽管我们遭受了一些损失，但大量黄金和比特币的迅速抛售帮助我们避免了持续下跌。在第 34 和 35 阶段，大多数比特币都被迅速而果断地卖出，回报率更稳定的黄金也被投资。这进一步保证了在规避风险时的收益。在 36 至 60 年代期间，购买比特币的速度缓慢是由于其估值相对较低之前所造成的迅速下跌。这时，增加购买量有利于把握后续的上升趋势。此外，值得注意的是，尽管在下跌后继续下跌的可能性较小，即风险较小，但该模型仍然建议我们缓慢买入以分担风险。

The reasons and details of other transactions are similar to the above analysis, so they are not described too much here. Table 3 shows the total wealth and the ratio among the three assets in some periods of our investment. Full data can be found in Appendix A in this paper.

其他交易的原因和细节与上述分析类似，因此在此不做过多描述。表 3 显示了我们投资的一些时期的财富总额和三种资产之间的比率。完整数据见本文附录 A。

Table 5. Part of the investment

桌子 6. Part of the investment

Period	[C,G,B]			X_t
	Cash	Gold	Bitcoin	
1	0	680	320	959
2	478	0	522	1044
3	277	0	723	1059
4	190	86	724	1089
		…		
31	533	179	286	11025
32	350	126	522	12368
		…		
58	58	0	99	901
59	59	0	79	921
60	60	0	64	936

时期	[中、英、法]			X_t
	现金	金色的	比特币	
一	0	680	320	959
2	478	0	522	1044
3	277	0	723	1059
四	190	86	724	1089
		…		
31	533	179	286	11025
32	350	126	522	12368
		…		
58	58	0	99	901
59	59	0	79	921
60	60	0	64	936

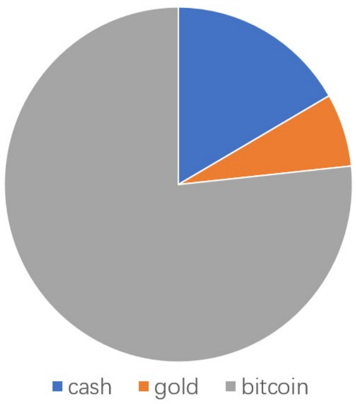


Figure 8. "Champion times" ratio

In order to more intuitively feel the key investment assets in different periods, count the assets with the highest proportion in each period, and call this asset the "champion" of this period. Count the "champion" times of the three assets. Bitcoin wins "champions" for 46 times, gold wins 10 times and cash wins 4 times. Their "champion times" ratio is shown in Figure 8.

为了更直观地感受不同时期的关键投资资产，统计各个时期中占比最高的资产，将该资产称为该时期的“冠军”。计算这三项资产的“冠军”倍数。比特币赢了 46 次“冠军”，黄金赢了 10 次，现金赢了 4 次。它们的“冠军时间”比率如图 8 所示。

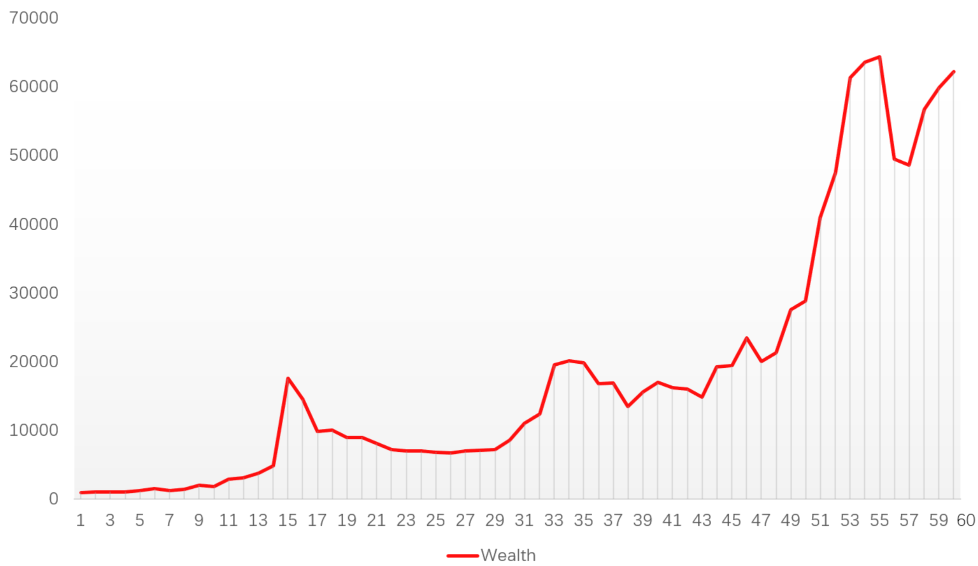


Figure 9. Total wealth

Figure 9 shows us our wealth intuitively. It can be found that the strategy based on the model helps us invest better, which means getting higher returns while avoiding risks. For example, in period 15 to 20 and 53 to 57, the market environment is quite unfavorable. Although there are losses, they are kept to a level that is far less than the market decline. And in period 46 to 55, we also keep up with the pace of market profits, obtained quite reliable income.

图 9 直观地向我们展示了我们的财富。可以发现，基于该模型的策略有助于我们更好地投资，这意味着在避免风险的同时获得更高的回报。例如，在 15 至 20 年和 53 至 57 年期间，市场环境相当不利。虽然有损失，但它们被保持在远低于市场下跌的水平。而在 46 至 55 年期间，我们也跟上了市场利润的步伐，获得了相当可靠的收入。

At the same time, it should be noted that compared with the losses in period 15 to 20 and 53 to 57, although the decline of the market in the latter is much larger than that in the former, the decline caused by investment based on the model is not much different. This shows that with the increase of historical data, the model’s resistance to sudden market deterioration continues to increase. If more historical data are available, the stronger our ability to predict and bear risks, the more radical strategies can be adopted to achieve higher returns.

同时，值得注意的是，与第 15 至 20 期和第 53 至 57 期的亏损相比，虽然后者的市场跌幅远大于前者，但基于该模型的投资造成的跌幅并无多大差异。这表明，随着历史数据的增加，模型对市场突然恶化的抵抗力持续增强。倘可获得的历史数据越多，我们预测及承担风险的能力越强，我们便可采取更激进的策略以取得更高回报。

In a nutshell, by 09/10/2021, the initial $1000 finally reaches assets equivalent to **$62211**. The rate of return has reached an impressive 100%

简而言之，到 2021 年 9 月 10 日，最初的 1000 美元最终会转化为相当于 62211 美元的资产。回报率已达到令人印象深刻的 100%

7. Model Analysis

8. 模型分析

8.1 Model Demonstration

8.2 模型演示

为了证明我们的模型的优越性，我们引入了交易者后悔心理的具体措施来证明我们的模型是最好的模型。在其他条件相同的情况下，交易者的后悔程度越低，决策越令人满意，模型越成功。

In actual financial activities, considering that investors are "bounded rational", investors' psychological factors will affect their investment behavior. In investment decision-making, any portfolio in the market is an alternative. Investors will compare the expected returns and risks of different portfolios, when investors have an unlimited number of alternative portfolios.

在实际的金融活动中，考虑到投资者是“有限理性”的，投资者的心理因素会影响其投资行为。在投资决策中，市场上的任何一种投资组合都是一种选择。当投资者有无限多个可供选择的投资组合时，投资者会比较不同投资组合的预期回报和风险。

This part compares the investor's portfolio with the portfolio that is likely to receive the maximum return, to get the investor's "**regret value**" when the return is not optimal. Similarly, compare the investor's portfolio with the portfolio that is likely to have the least risk, to get the "regret value" when the portfolio does not meet the minimum risk. Because of the psychology of regret aversion, it is difficult for investors to maximize wealth and minimize risk from the perspective of absolute rationality when making investment decisions. Thus, add the psychology of regret aversion into investment decisions, hoping that the investment results will not be too regretful to be accepted.

该部分将投资者的投资组合与可能获得最大回报的投资组合进行比较，得出当回报不是最优时投资者的“后悔值”。类似地，比较投资者的投资组合与可能具有最小风险的投资组合，当投资组合不满足最小风险时，得到“后悔值”。由于后悔厌恶心理的存在，投资者在进行投资决策时很难从绝对理性的角度实现财富最大化和风险最小化。因此，在投资决策中加入厌恶后悔的心理，希望投资结果不会太过后悔而不被接受。

We introduce regret psychology into the demonstration of the rationality of the model. It shows that the decision made by our model makes the degree of regret of investors lower than other choices. In other words, it makes investors more satisfied, which means our model is more successful. It works for different types of investors.

我们将后悔心理引入到模型合理性的论证中。结果表明，我们的模型所做出的决策使得投资者的后悔程度低于其他决策。换句话说，它让投资者更满意，这意味着我们的模式更成功。它适用于不同类型的投资者。

Due to the irrational behavior of investors in the process of investment, such as chasing up and down, overconfidence and regret aversion, many traditional portfolio models cannot explain the anomalies in financial markets well. Therefore, a large number of scholars began to develop and improve portfolio theory from the perspective of investor behavior. Chorus C.G proposed a **Generalized Random Regret Minimization** model[7] in 2014. Based on Chorus's model, this part describes the investor's regret psychology from the perspective of return and risk.

由于投资者在投资过程中的非理性行为，如上下追逐、过度自信、后悔厌恶等，许多传统的投资组合模型不能很好地解释金融市场中的异象。因此，大量学者开始从投资者行为的角度发展和完善投资组合理论。Chorus C.G .在2014年提出了一个广义随机后悔最小化模型[7]。基于Chorus模型，从收益和风险的角度描述了投资者的后悔心理。

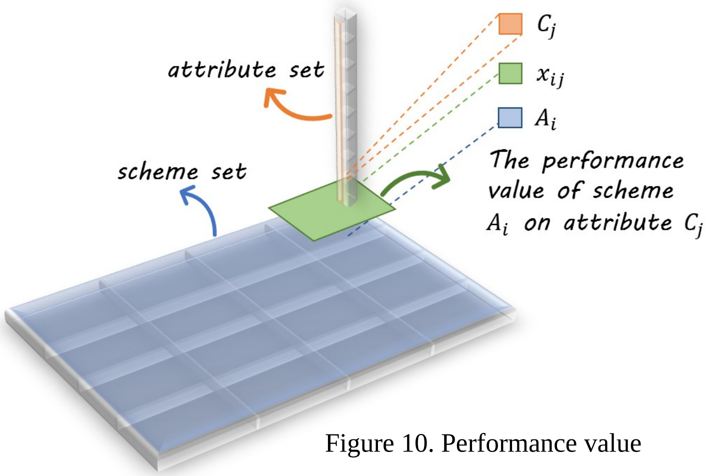


Figure 10. Performance value

假设有可行方案。就是这套方案。是一组属性。是属性上方案的性能值。不同方案的性能值可以在同一属性下进行比较。n $A=\{A_1,A_2,\ldots,A_n\}$ $C=\{C_1,C_2,\ldots,C_m\}$ x_{ij} A_i C_j

In the financial market, investors use the initial wealth to invest. If the final wealth is less than the maximum wealth available, investors may regret. This kind of regret value can be calculated as follows:

在金融市场上，投资者用最初的财富进行投资。如果最终财富少于可获得的最大财富，投资者可能会后悔。这种后悔值可计算如下：

$$R_1=ln\{\gamma+e^{\beta_1(r_{max}-r)}\} \quad (20)$$

$$R_1=ln\{\gamma+e^{\beta_1(r_{max}-r)}\} \quad (20)$$

where r_{max} is max available yield rate, r is the actual yield, γ is the investor's degree of regret and β_1 is the investor's sensitivity to yield rate.

其中是最大可用收益率，是实际收益率，是投资者的后悔程度，是投资者对收益率的敏感性。$r_{max} r \gamma \beta_1$

在投资过程中，投资者希望通过多元化投资来降低风险。如果投资组合风险大于投资者所能承受的最小风险，投资者就会后悔。这种后悔值可计算如下：

$$R_2=ln\{\gamma+e^{\beta_2(\delta-\delta_{min})}\} \quad (21)$$

$$R_2 = ln\left\{\gamma + e^{\beta_2(\delta - \delta_{min})}\right\} \quad (21)$$

where δ_{min} is the lowest acceptable risk, δ is the actual risk, γ is the investor's degree of regret and β_2 is the investor's sensitivity to risks.

其中是最低可接受风险，是实际风险，是投资者的后悔程度，是投资者对风险的敏感性。$\delta_{min} \delta \gamma \beta_2$

Usually, the higher the expected return of an investor, the greater the risk he is willing to take. Thus, investor's sensitivity to yield rate and risks can be considered as nearly negative correlation. Let

通常，投资者的预期回报越高，他愿意承担的风险就越大。因此，投资者对收益率和风险的敏感性几乎可以被认为是负相关的。让

$$\beta_1 + \beta_2 = 1 \quad (22)$$

$$\beta_1 + \beta_2 = 1 \quad (22)$$

If $\beta_1 > 0.5$, then the investor is more sensitive to yield. If $\beta_2 > 0.5$, then the investor is more sensitive to risks. Otherwise, the investor is equally sensitive to yield and risks. The total regret value is:

如果是，那么投资者对收益率更为敏感。如果是，那么投资者对风险更为敏感。否则，投资者对收益率和风险同样敏感。总后悔值为:$\beta_1 > 0.5 \beta_2 > 0.5$

$$R = R_1 + R_2 \quad (23)$$

$$R = R_1 + R_2 \quad (23)$$

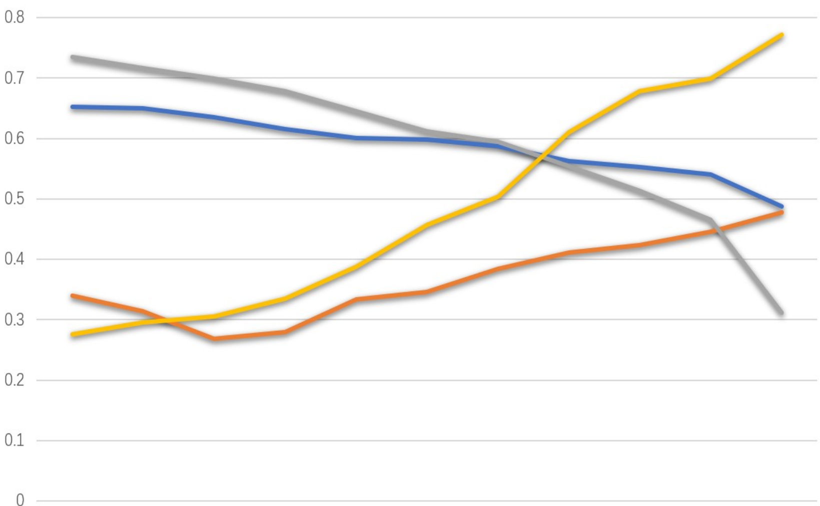
In this paper, cash is considered as risk-free asset, gold is considered as mid-risk asset and bitcoin is considered as high-risk asset, according to their fluctuation. Investors are divided into conservative investors, normal investors and radical investors, according their investment style.

根据波动性将现金视为无风险资产，黄金视为中风险资产，比特币视为高风险资产。投资者根据其投资风格分为保守型投资者、一般型投资者和激进型投资者。

As for conservative investors, $\beta_1 > 0.5$. As for normal investors, $\beta_1 = 0.5$. As for racial investors, $\beta_1 < 0.5$.

至于保守的投资者。至于普通投资者。至于种族投资者。$\beta_1 > 0.5 \beta_1 = 0.5 \beta_1 < 0.5$

根据这个模型，我们可以得到一个投资建议。接下来，我们将探讨不同风格的投资者在基于这一建议进行投资时的后悔价值。评分用于衡量模型的成功程度，并将结果与等比例投资的结果进行比较R[1]。



[1] equal proportion

Equal weight investments Our model All in bitcoin All in gold

Figure 11. Regret value of different kinds of investments

图 11 显示了不同投资策略下投资者的后悔值对收入变化的敏感性。其中，水平轴为，垂直轴为遗憾值。由于实际上可以逼近 0，因此该数字实际上是真实情况的部分表示。这是因为大多数投资者对风险和收益的考虑相对平衡，有些投资者只能接受最小的风险，但很少有投资者愿意忽视风险。因此，可以认为通常不小于 0.1。这个数字显示了大多数情况。蓝线代表初始时刻平均投资 1，000 美元现金、黄金和比特币的策略，橙色线代表根据我们的模型制定的策略，灰色线代表仅投资比特币的策略，黄色线代表仅投资黄金的策略。可以发现，在大多数情况下，我们的模型给投资者带来的遗憾较少。换句话说，我们的模型在大多数情况下是最好的。$\beta_1/\beta_2\beta_2\beta_2$

8.3 Sensitive Analysis

8.4 敏感性分析

To verify the sensitivity of the model to transaction costs, we let the transaction cost changes in a certain range. The sensitivity of the model is captured by the change in the transaction amount at each stage and the final return.

为了验证模型对交易费用的敏感性，我们让交易费用在一定范围内变动。模型的敏感度由每个阶段的交易金额变化和最终回报来捕捉。

When transaction costs fluctuate, the investment decisions given by the model will also be affected. Trades will reduce or even stop in some stages if transaction costs rise, even if the investment phase remains unchanged. And it will rise in some stages with the falling of transaction costs.

当交易成本波动时，模型给出的投资决策也将受到影响。如果交易成本上升，即使投资阶段保持不变，交易也会减少，甚至在某些阶段停止。在某些阶段，随着交易成本的下降，它还会上升。

Therefore, we can change transaction costs and then observe the changes in investment suggestions given by the model. In this way, the sensitivity of the model to transaction costs is explored. We set three pairs of transaction costs of gold and bitcoin: $\gamma_1=0.01$ and $\gamma_2=0.02$, $\gamma_1=0.001$ and $\gamma_2=0.002$,$\gamma_1=0.0001$ and $\gamma_2=0.0002$. Here are the results of our model.

因此，我们可以改变交易成本，然后观察模型给出的投资建议的变化。以此方式，探讨了该模型对交易成本的敏感性。我们设定了三对黄金和比特币的交易成本：and、and、and。以下是我们模型的结果。$\gamma_1=0.01\gamma_2=0.02\gamma_1=0.001\gamma_2=0.002$ $\gamma_1=0.0001\gamma_2=0.0002$

When $\gamma_1=0.01$ and $\gamma_2=0.02$, the result is clearly displayed in 4.8.

当和时，结果清楚地显示在 4.8 中。$\gamma_1=0.01\gamma_2=0.02$

When $\gamma_1=0.001$ and $\gamma_2=0.002$, Table 4 shows the proportion of assets [C,G,B] and the total wealth X_t in different period.

当与时，表 4 显示不同时期的资产[C，G，B]比例和财富总额。$\gamma_1=0.001\gamma_2=0.002$ X_t

Table 7. Total wealth and proportion of assets

桌子 8.

Period	[C,G,B]			X_t
	Cash	Gold	Bitcoin	
1	0	680	320	998
2	317	0	683	1067
3	160	91	749	1069

4	0	43	957	1157
5	0	43	957	1157
		…		
60	0	46	954	63157

时期	[中、英、法]			X_t
	现金	金色的	比特币	
一	0	680	320	998
2	317	0	683	1067
3	160	91	749	1069
四	0	43	957	1157
5	0	43	957	1157
		…		
60	0	46	954	63157

When $\gamma_1=0.0001$ and $\gamma_2=0.0002$, Table 5 shows the proportion of assets [C,G,B] and the total wealth X_t.

当和时，表 5 显示资产[C，G，B]的比例和总财富。$\gamma_1=0.0001\gamma_2=0.0002X_t$

Table 9. Total wealth and proportion of assets

桌子 10.

Period	[C,G,B]			X_t
	Cash	Gold	Bitcoin	
1	0	680	320	998
2	317	0	683	1067
3	160	91	749	1069
4	0	43	957	1157
5	0	43	957	1157
		…		
60	0	46	954	63157

时期	[中、英、法]			X_t
	现金	金色的	比特币	
一	0	680	320	998
2	317	0	683	1067
3	160	91	749	1069
四	0	43	957	1157
5	0	43	957	1157
		…		
60	0	46	954	63157

我们可以清楚地看到，随着交易成本的增加，大部分阶段的交易金额和交易次数都逐渐减少。由此，我们可以得出结论:我们的模型对交易成本非常敏感。

9. Strengths and Weakness

10. 优势与劣势

Strengths:

优势:

- **Comprehensiveness**. What our model takes into account are not only returns, but also risks investors to take when investing. It takes as little risk as possible while making the investor's return as large as possible.
- 全面。我们的模型考虑的不仅是回报，也包括投资者在投资时要承担的风险。它采取尽可能少的风险，同时使投资者的回报尽可能大。
- **Innovativeness**. Introduce a specific measure of investor regret psychology to demonstrate the superiority of the model, and use it to measure the success of the model.
- 创新。引入一个具体的衡量投资者后悔心理的指标来论证该模型的优越性，并以此来衡量该模型的成功与否。
- **Applicability**. Theoretically, in the absence of major international events, the model can continuously update itself over long periods of time and is highly applicable.
- 适用范围。从理论上讲，在没有重大国际事件的情况下，该模型可以在很长一段时间内不断更新，具有很高的适用性。

Weakness:

弱点:

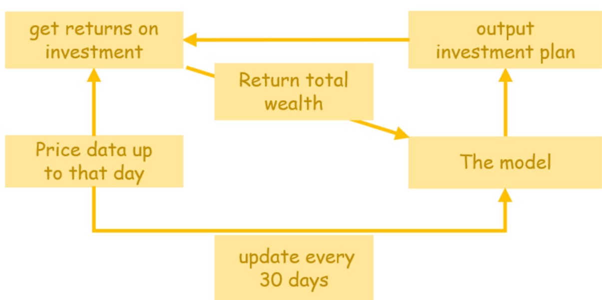
- Because the risks taken investors are considered, the decisions proposed are on the conservative side, which may cause investors to miss out on certain gains.
- 由于考虑了投资者承担的风险，因此所提决策较为保守，可能会导致投资者错过某些收益。
- The decision is mainly influenced by the price data without taking other factors into account such as international situation, policy implications, etc. The factors are relatively homogeneous and may lead to results that differ from the actual optimal ones.
- 有关决定主要受价格数据影响，并无计及其他因素，如国际形势、政策影响等。这些因素相对而言是同质的，可能导致不同于实际最佳结果的结果。

Descriptions

how to construct the model?

In a nutshell, predict first, plan next, make decisions last. Make sure it balances returns and risks.

how to use this model?



Strengths and weakness of the model

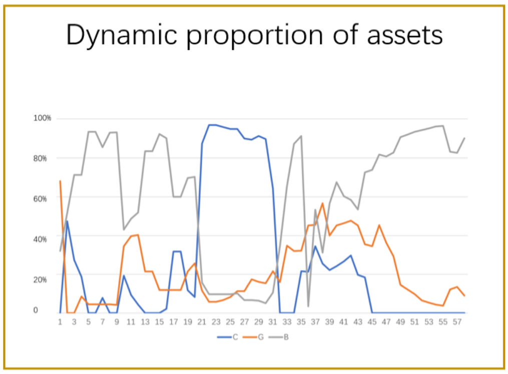
Strengths

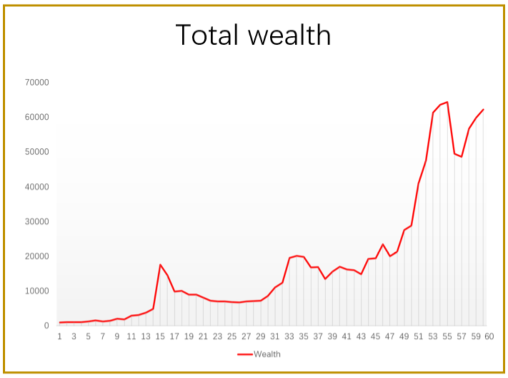
- High return with low risks
- Low regret value of investors
- Easy to handle

Weakness

- May give up some return to lower the risks

The result of the model





Questions you may have

Q1: How to prove the superiority of the model?

A: We introduce the specific measurement of the degree of investors' unsatisfaction. By comparing with other choices, we draw the conclusion that decisions made by our model make investors regret less, gain more and take less risks. That is how we prove its superiority.

Q2: It is sensitive to transaction costs?

A: Yes. T

Q3: Is t

A: Yes. E

参考

[1] Vercher E, BermUdez J D. A Posibilistic Mcan-Downside Risk-Skewness Model f-or Efficient Portfolio Selection[J]. IEEE Transactions On Fuzzy Systems, 2013.3:585—595.
[2] 引用该论文王志平，王志平，王志平，王志平，王志平，等. IEEE 模糊系统交易，2013. 3:585—595。
[3] Abbasbandy S, Hajjari T. A new approach for ranking of trapezoidal fuzzy numbers[J]. Computers & mathematics with applications, 2009, 57(3): 413-419.
[4] 引用该论文王志平，王志平，王志平，王志平，王志平. 《计算机与数学及其应用》，2009，57(3)： 413-419。
[5] Barnes R. Earnings volatility and market valuation: An empirical investigation[J]. Available at SSRN 335380, 2002.
[6] [1]巴恩斯. 盈余波动与市场估值:一项实证研究[J]。可于 SSRN 335380 查阅，二零零二年。
[7] Carlsson C, Fullér R. Fuzzy multiple criteria decision making: Recent developments[J]. Fuzzy sets and systems, 1996, 78(2): 139-153.
[8] 引用该论文王志平，王志平，王志平，王志平，王志平. 模糊集与系统，1996，78(2)： 139-153。
[9] Carlsson-Kanyama A. Weighted average source points and distances for consumption origin-tools for environmental impact analysis?[J]. Ecological Economics, 1997, 23(1): 15-23.
[10] 卡尔森-Kanyama . 消费起源的加权平均来源点和距离-环境影响分析的工具？[J]。生态经济学，1997，23(1)： 15-23。
[11] Zhang P, Zhang W G. Multiperiod mean absolute deviation fuzzy portfolio selection model with risk control and cardinality constraints[J]. Fuzzy sets and systems, 2014, 255: 74-91.
[12] [1]张平，张伟国. 带风险控制和基数约束的多期均值绝对偏差模糊投资组合选择模型[J]。模糊集与系统，2014，255： 74-91。
[13] Tian M, Yan S, Tian X. Discrete approximate iterative method for fuzzy investment portfolio based on transaction cost threshold constraint[J]. Open Physics, 2019, 17(1): 41-47.
[14] [1]田敏，闫 s，田 x . 基于交易成本门槛约束的模糊投资组合离散近似迭代算法[J]。开放物理学，2019，17(1)： 41-47。
[15] Chorus C G. A generalized random regret minimization model[J]. Transportation research part B: Methodological, 2014, 68: 224-238.
[16] 引用该论文王志平，王志平，王志平，王志平，王志平. 交通研究 B 部分:方法论，2014，68： 224-238。

Appendix A

附录 A

时期	[中、英、法]			X_t	时期	[中、英、法]			X_t
	现金	金色的	比特币			现金	金色的	比特币	
一	0	680	320	959	31	533	179	286	11025
2	478	0	522	1044	32	350	126	522	12368
3	277	0	723	1059	33	0	128	872	19531
四	190	86	724	1089	34	0	259	741	20097
5	0	45	955	1243	35	0	259	741	19864
6	0	45	955	1512	36	179	376	445	16778
七	79	46	875	1263	37	177	374	449	16878
8	0	46	954	1439	38	345	466	209	13524

9	0	44	956	2074	39	282	326	252	15994
10	219	392	489	1856	40	163	335	502	17005
11	95	407	498	2892	41	184	355	461	16221
12	44	419	537	3110	42	202	359	439	15997
13	0	204	796	3765	43	231	352	417	14851
14	0	204	796	4869	44	154	277	568	19264
15	0	108	892	17597	45	156	274	570	19452
16	20	113	867	14556	46	0	356	644	23517
17	306	114	579	9877	47	0	344	656	20001
18	306	114	579	10013	48	0	261	739	21356
19	113	208	679	8992	49	0	137	862	27567
20	79	245	676	9015	50	0	117	883	28859
21	763	101	136	8113	51	0	94	906	41005
22	862	51	86	7235	52	0	65	935	47598
23	862	51	86	6997	53	0	54	946	61357
24	852	61	86	7004	54	0	38	962	63571
25	833	75	92	6841	55	0	35	965	64401
26	841	100	59	6755	56	0	127	873	49560
27	841	100	59	6998	57	0	139	861	48650
28	791	152	56	7112	58	0	99	901	56751
29	890	157	53	7199	59	0	79	921	59934
30	882	161	57	8549	60	0	64	936	62211