Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2222182

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【细节待修改】Market traders buy and sell volatile assets frequently, with a goal to maximize their total return. Traders are always looking to increase their returns by predicting asset ups and downs and minimizing the risk (lower returns or even bankruptcy) when investing. Gold and Bitcoin are two representative assets. In this paper, we use the attached data to predict the possible best investment scenarios, use the regret level as evidence to validate the strategy, adjust the commission to test the sensitivity of the scenarios to the cost, and write the model decision as a memo to the trader.

In this paper, we introduce a multi-stage fuzzy investment method, by analyzing the characteristics of gold and bitcoin, we decide to invest in stages, the investment decision is mainly based on the price data before the trading day (schedule LBMA-GOLD.csv, BCHAIN-MKPRU.csv), each investment change or maintain the current asset ratio (C,G,B), in order to achieve the purpose of reducing losses and increasing returns The result: investing $1000 on 9/11/2016 will result in $62,211 on 9/10/2021.

To ensure that the evidence is convincing, the degree of regret is used here as reliable evaluation evidence.

The sensitivity of the strategy to the transaction cost is analyzed by constantly adjusting the commission percentage. The transaction cost is mainly the commission charged for each purchase or sale of bitcoin or gold (αgold = 1%, αbitcoin = 2%). For this reason, we adjusted the commission ratio and reduced it to (αgold = 0.1%, αbitcoin = 0.2% αgold = 0.01%, αbitcoin = 0.02%) respectively, concluding that the reduction in transaction costs allowed for higher returns on upfront decisions, a higher percentage of bitcoins invested, and a small increase in final wealth.

Last but not least, we summarize the suggestions and write a memorandum to the trader to communicate our strategy, model, and results.

Key Words: TODO

现在还没有完成的：

1. 图片和表格的题注
2. 所有标为 xxx 的（例如：Figure xxx shows the result of xxxx）
3. 总结和备忘录
4. 参考文献和附录

Content

目录需要更新，选中-右键-更新域。

关于提交论文前对论文页眉的修改：

1. Page 后面的第 1 个数字不需要修改，表“当前页码”，它是正常自动变化设置的。

2. 总页数需要论文完成后手工修改；总页数不应该包含 Summary 页数；

所以上面页眉的总页数应该减去 1 才是正确的页数。例如，本文档页眉的“3”应该改为“2”，表明正文只有 2 页。

1. Introduction

1.1 Background

Nowadays, market investment is one of the most important means to realize wealth appreciation. Through the purchase or sale of various assets in the market, investors can obtain different returns. In order to maximize the total return to investors, we can optimize the investment strategy and decide the amount and proportion of assets to be bought or sold based on market conditions.

Different assets have different risks, rewards and trading rules. Generally speaking, gold is less risky and less profitable; while bitcoin has higher risks and higher returns. The commission cost per transaction costs γ % of the amount traded. Usually, γ_{gold} is smaller than $\gamma_{bitcoin}$. 为了方便起见，下文中所有的资产比例指的都是将资产折合为美元后的价值比例。

1.2 Problem Analysis

First of all, we note that this is a personal investment and the amount of investment is small, so the strategy should be in line with the market mechanism to reduce risks and increase returns.

As for gold trading, its price is affected by both short-term and long-term factors. Short-term factors, which are uncertain and contingent, let gold price fall more than its recent increase, so it is not recommended to invest in short-term timing. The medium and long-term timing model is influenced by the Federal Reserve's monetary policy the characteristics of economic cycle. These economic data can be made out obviously after their occurrence, but are hard to predict in advance.

Therefore, under the complex and unpredictable driving factors, it is recommended to allocate gold assets for a long time. In this way, no matter how time or market influencing factors changes, we can finally obtain long-term benefits. The price of such strategy is only to give up some band benefits that are difficult to grasp in the short term. We do not recommend that individual investors to take risks for this return. Therefore, for gold trading, we recommend maintaining long-term allocation of small positions, i.e. 5 % ~ 10 % of the position.（这部分写法有问题）

For bitcoin transactions, we prefer the automatic investment plan. In the stock market, automatic investment plan has been proved to be a suitable strategy for most people. By buying a certain amount of stock regularly, you can get the same or even higher returns with the market level with less time and energy cost. Since bitcoin's price change is also volatile, we think the

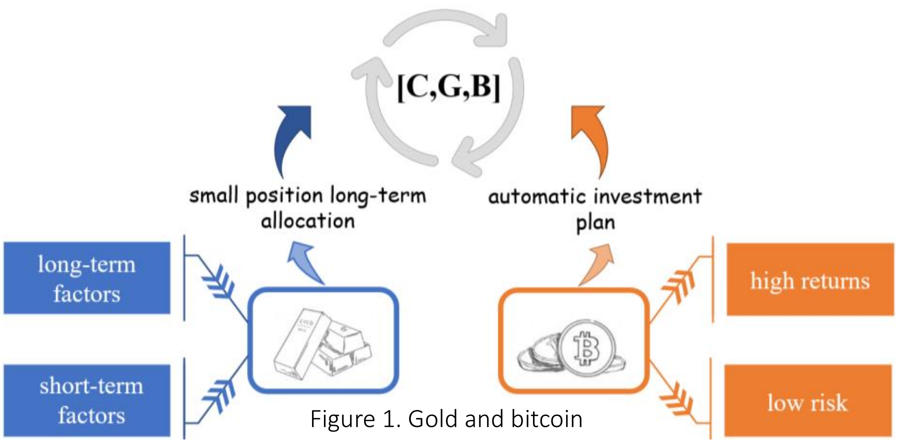


Figure 1. Gold and bitcoin

automatic investment plan apply equally to bitcoin. By timing purchase, we can effectively smooth out the price volatility, reduce holding costs and halve risk.

In a word, securities returns, which can only be estimated according to limited information, can be considered as fuzzy variables rather than just random variables. Since the trapezoidal fuzzy【这里引用 xxx】 number has been shown to be an appropriate measure of security returns, we decide to establish a multi-stage decision-making model, which helps us to regularly change the proportion of holding assets and reduce trading in other time to reduce risks and improve returns.

To verify the sensitivity of the model to transaction costs, we let the transaction cost changes in a certain range. The sensitivity of the model is captured by the change in the transaction amount at each stage and the final return.

In actual financial activities, considering investors are '**bounded rationality**', investors' psychological factors will affect their investment behavior. Therefore, we introduce specific measures of traders' regret psychology to prove that our model is the best model. When other conditions are the same, the lower the degree of traders' regret is, the more satisfied the decision is and the more successful the model is.

2. Assumptions and Justifications

1. **There is no limit to the maximum and minimum transaction amount.**
   To simplify the model, we assume that the minimum amount per transaction is 0, with no upper limit. And no debit or credit occurs. This applies to transactions of both gold and bitcoin.
2. **Traders make their decisions to purchase or sell based only on data from attachment, which are independent from people and circumstances around them.**
   When people make decisions to buy or sell, they usually receive many factors, such as the evaluation from people around them, the current situation of life, and the news on the Internet. In this question, when making a trading decision, only the attached data matters with no other factors.
3. **Investors are more concerned about downside risks.**
   When studying the psychology of investor regret, we find that the intensity of emotions is greater about loss than gain【这里引用 xxx】.这里有何作用？

3. Notations

Table 1 shows the notations commonly used in this paper and their description.

Table 1. Notations

Notations	Description
X_0	Initial assets owned by investors
t	An investment period
X_t	Total wealth at the end of period t
R_{it}	Fuzzy yield of risky assets i in period t
$\overline{svar}(X_t)$	The semi-variance of X_t
u_{it}	Investment amount to asset i on period t
u_{ft}	Investment amount to risk-free asset i on period t
l_{it}	The upper bound constraint of risk asset i on period t
p_{it}	The lower bound constraint of risk asset i on period t

4. Model Construction and Resolution

Based on the analysis above,（那里的分析？ ）with the '**time inconsistency**' of investment strategy brought into model construction, a **multi-stage mean semi-variance fuzzy portfolio** model is proposed. It is based on transaction cost, loan constraint, threshold constraint, revenue demand and cardinality constraint, and the optimal investment strategy of time consistency is studied. Due to transaction cost, revenue demand and cardinality constraints, the model provides a **mixed-integer semi-closed-loop dynamic optimization with path-dependence** to solve this problem. As for details, we'll use the discrete approximate iterative algorithm is to obtain the optimal time-consistent investment strategy in the following paper. Figure 2 is the flowchart of this process.

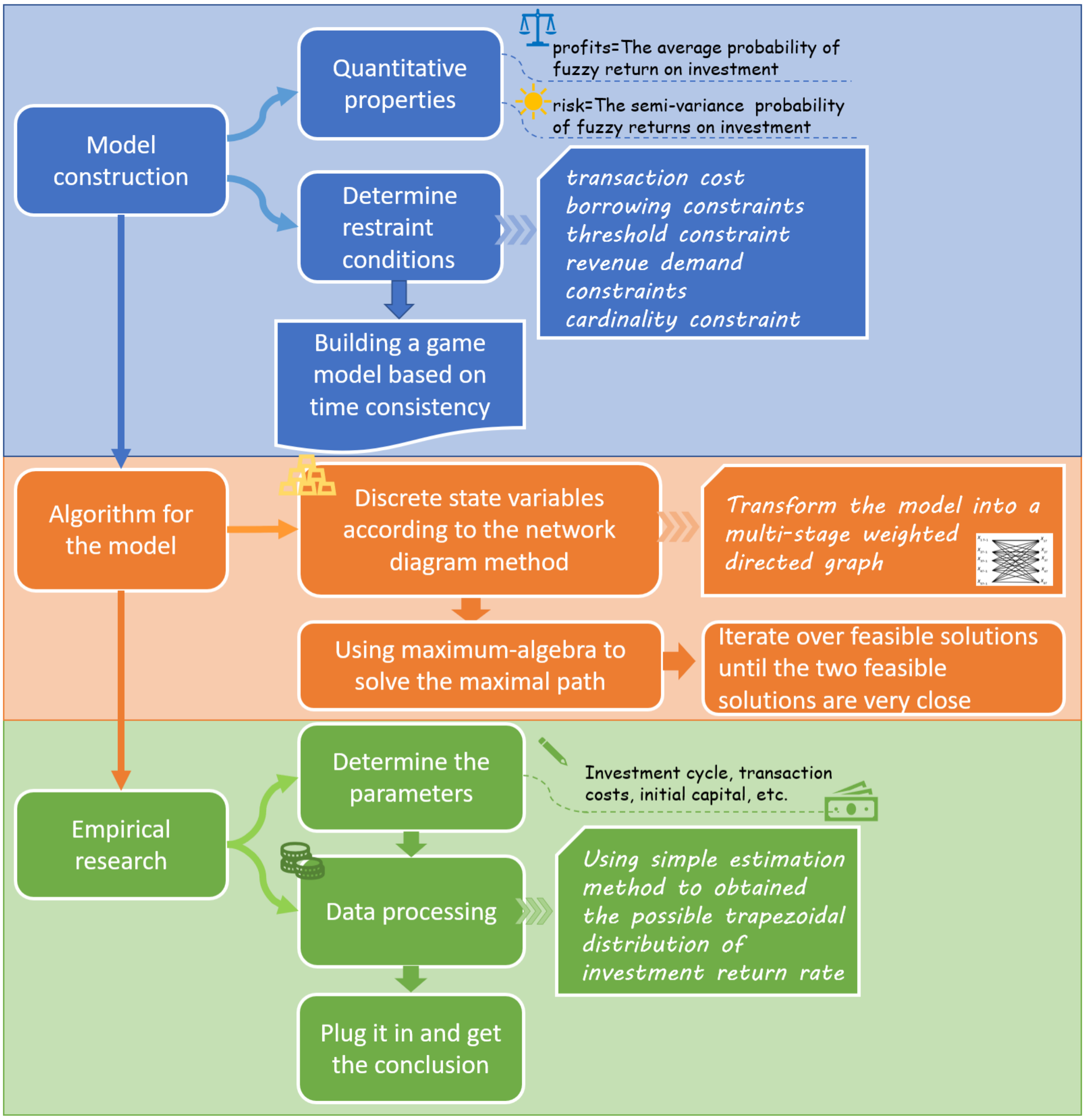


Figure 2. Model construction

4.1 Subtitle 预处理

Before data analysis and decision making, we must ensure its quality, which determines the prediction and generalization ability of the model. In order to ensure the accuracy, integrity and credibility of the data, we preprocess the data based on the attachments given: *BCHAIN-MKPRU.csv* and *LBMA-GOLD.csv*. We analyze the data in the table and delete useless data, supplement data that may be missing, and finally retain the revised data.

After analysis, we found that there is no illogicality data such as negative data, text or outliers in both files, so there is no need to delete any（没有就不需要写出来）.

In order to make the data smoother（为什么要平滑？）, we choose the mean interpolation method to supplement the data. For each incomplete item, the mean value of price on the day before and after the missing data day is supplemented into it. Table 2 shows the price data inserted into *LBMA-GOLD.csv*.

Table 2. Data inserted into file

Date	**12/30/16**	**12/23/16**	**12/22/17**	**12/29/17**	**12/24/18**	**12/31/18**	**12/24/19**	**12/31/19**	**12/24/20**	**12/31/20**
Price	**1132.98**	**1148.45**	**1272**	**1301.5**	**1263.1**	**1281**	**1469.8**	**1521**	**1874.65**	**1915.4**

4.2 Subtitle 数据分析（这里有何作用？）

Focusing on gold and bitcoin, we mainly analyze their trading risks and correlation of price trends in this part.

With the development of market investment, people are more inclined to trade rationally and reduce risks while obtaining greater expected returns. Portfolio investment is a common way to avoid risks in investment management. By selecting different investment portfolios, we can maximize returns and minimize risks.

By introduce $\boldsymbol{DCC}$ coefficient【这里引用 xxx】, we can study the dynamic correlation between gold and bitcoin in the market. DCC-GARCH model is widely used in this field. Its general form is:

$$
\begin{aligned}
& r_t | F_{t-1} \sim N(0, H_t) \\
& \mathrm{H_t} = D_t R_t D_t \\
& Q_t = (1-\alpha-\beta)\overline{Q} + \alpha\mu_{t-1}\mu_{t-1}^T + \beta Q_{t-1} \\
& D_t = diag\left(\sqrt{h_{11,t}}, \sqrt{h_{22,t}}, \dots \sqrt{h_{kk,t}}\right) \\
& R_t = \left(diag(Q_t)\right)^{-\frac{1}{2}} Q_t \left(diag(Q_t)\right)^{-\frac{1}{2}}
\end{aligned}
\tag{1}
$$

where r_t is the conditional revenue rate of k kinds of assets, F_{t-1} is the set of collected information, H_t is the conditional covariance matrix, R_t is the dynamic conditional correlation index matrix, Q_t is the covariance matrix and $\overline{Q}$ is the unconditional covariance matrix of standardized residual. α and β are positive parameters which meet $\alpha+\beta < 1$.

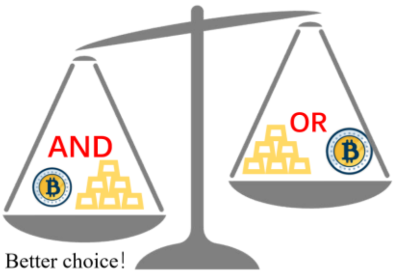


Figure 3. Investment choice

Through this model, we establish a single variable GARCH (1,1) model for the return ratio series of bitcoin and gold, and draw the conclusion that the dynamic correlation between bitcoin and gold is characterized by remarkable persistence. The details are not important to the main problem we want to solve, so we omitted them in this paper. In short, it is supposed to invest both gold and bitcoin to gain more return while lower the risks.

As for volatility, it describes the uncertainty of asset returns, which is used to reflect the risk level of financial assets. According to how volatility changes, assets can be devided into assets with different risks. Common types include high-risk assets, mid-risk assets, and risk-free assets.

Unlike gold and bitcoin, the U.S. dollars is affected by many factors and is largely controlled, so its value holds for a long time, and the volatility is very low. It can be considered as a risk-free asset. As for gold, its price and volatility changes more violent than U.S. dollar but less violent than bitcoin. Thus, gold can be considered as a mid-risk asset and bitcoin can

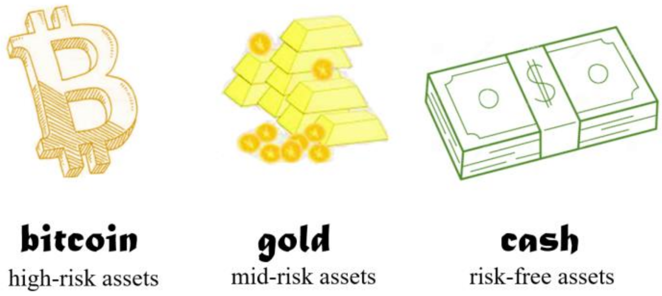


Figure 4. Risks of assets

be considered as a high-risk asset, as Figure 4 shows.

4.3 Subtitle 参数说明和问题描述

Assume that there are n kinds of risk assets and 1 kind of risk-free asset for investors to choose. His initial wealth is X_0. Let t stand for investment period, with T stages. At the first stage of investment stage, the investor can only invest with X_0. The asset portfolios changes at the beginning of eachstage and investment last for T stages.

Let R_{it} stand for the fuzzy yield of risky assets i in period t, and $R_t = (R_{1t}, R_{2t}, \dots R_{nt})'$. Set X_t as the total wealth at the end of period t. In period t, set $\overline{svar}(X_t)$ as the semi-variance of X_t, u_{it} as the investment in asset i, u_{ft} as the investment in risk-free asset i, l_{it} and p_{it} as the upper and lower bound constraint of risk asset i on period t. Let $u\_t = (u_{1t}, u_{2t}, \dots, u_{nt}, u_{ft})'$.

4.4 Subtitle 参数量化

The securities market is a complex system with dynamic changes. It is difficult for people to obtain the overall information of the random distribution of securities returns. We can only estimate the return according to the historical information of securities. Thus, securities returns can be considered as fuzzy variables rather than random variables. In addition, considering the uncertainty of market environment, trapezoidal fuzzy number is often used to measure the yield of securities. Many scholars have discussed the fuzzy portfolio optimization problem deeply, such as Carlsson[xxx], Anne Trefethen[xxx], Zhang Peng and Zhang Weiguo[xxx], etc. 这里引用 xxx

The return and risk of asset portfolios are measured by the possibilistic mean value and the possibilistic standard semi-variance of asset fuzzy return, respectively. Clearly, the whole process is self-financing, as investors did not add additional funds during this period. Record the return rate of risk assets as R_{it}, where $R_{it} = (a_{it}, b_{it}, \alpha_{it}, \beta_{it})$.

In period t, the possibilistic mean value r_{pt} of the assets $u_t = (u_{1t}, u_{2t}, \dots u_{nt}, u_{ft})$ can be calculated as:

$$r_{pt} = \sum_{i=1}^{n} \bar{M}(R_{it})u_{it} + r_{ft}(X_{t-1} - \sum_{i=1}^{n} u_{it}) - X_{t-1}$$
$$= (r_{fi} - 1)X_{t-1} + \sum_{i=1}^{n} \left(\frac{a_{it} + b_{it}}{2} + \frac{\beta_{it} - \alpha_{it}}{6} - r_{ft} \right) u_{it}, t = 1,2,\dots,T \quad (2)$$

Assuming the transaction cost is $\gamma\%$ in this paper. Obviously, the transaction cost of asset i in period t is $u_{it} \times \gamma\%$, and the total cost of the asset portfolio $u_t = (u_{1t}, u_{2t}, \dots, u_{nt}, u_{ft})$ is :

$$C_t = \sum_{i=1}^{n} u_{it} \times \gamma, \quad t = 1,2,\dots,T \quad (3)$$

Then, the net income ratio of asset portfolio X_t by the end of period t is:

$$r_{Nt} = (r_{ft} - 1)X_{t-1} + \sum_{i=1}^{n} \left(\frac{a_{it} + b_{it}}{2} + \frac{\beta_{it} - \alpha_{it}}{6} - r_{ft} \right) u_{it} - \sum_{i=1}^{n} \gamma_{it} |u_{it}| \quad (4)$$

and the total wealth held by investor by the end of period t is:

$$X_t = r_{Nt} + X_{t-1} = r_{ft}X_{t-1} + \sum_{i=1}^{n} \left(\frac{a_{it} + b_{it}}{2} + \frac{\beta_{it} - \alpha_{it}}{6} - r_{ft} \right) u_{it} - \sum_{i=1}^{n} \gamma_{it} |u_{it}| \quad (5)$$

where $t = 1,2,\dots,T$.

4.5 Subtitle 确定约束条件

The threshold constraint of multi-stage portfolio is:

$$l_{it} \le u_{it} \le p_{it} \quad (6)$$

where l_{it} and p_{it} as the upper and lower bound constraint of u_{it}.

Set the lower bound constrain of proportion of investment ratio on risk-free assets to be $u_{ft}^{b}(u_{ft}^{b} \le 0)$, then the borrowing constraints of risk-free assets in period t is:

$$u_{ft} = X_{t-1} - \sum_{i=1}^{n} u_{it} \ge u_{ft}^{b} \quad (7)$$

According to formula xxx(13), the semi-variance of asset portfolio u_{it} is:

$$\overline{svar}_t(u_t) = u_t' H_t^- u_t \quad (8)$$

where its standard semi-variance is $\sqrt{\overline{svar}_t(u_t)}$. Assume that H_t^- is a semi-positive definite matrix.

$$H_t^- = \left(Cov_t^-(r_{it}, r_{jt}) \right)_{n \times n} \quad (9)$$

$$Cov_t^-(r_{it}, r_{jt}) = \frac{(b_{it} - a_{it})(\beta_{jt} + \alpha_{jt}) + (b_{jt} - a_{jt})(\beta_{it} + \alpha_{it})}{12} + \left[\frac{(\beta_{it} + \alpha_{it})(\beta_{jt} + \alpha_{jt})}{36} \right] + \frac{(\beta_{it} - \alpha_{it})(\beta_{jt} - \alpha_{jt})}{4} + \frac{\beta_{it}\beta_{jt}}{18} \quad (10)$$

The cardinality constraint is:

$$\sum_{i=1}^{n} z_{it} \leq K, z_{it} \in \{0,1\} \tag{11}$$

where K is the maximum number of risk-free assets, and

$$z_{it} = \begin{cases} 1 & choose\ asset\ i \\ 0 & not\ choose\ asset\ i \end{cases} \tag{12}$$

4.6 Subtitle 博弈模型确定

The traditional multi-stage mean-standard semi-variance portfolio optimization model only considers the expected value of end-of-period wealth and standard semi-variance. In the real world, however, investors care not only about the expected value and standard semi-variance of end-of-period wealth, but also about which during the investment period. In other words, the expectation and the semi-variance of the portfolio are different in the t period. Therefore, this paper uses standard semi-variance to measure risk. Set the weight coefficient $w_t > 0$ and risk preference coefficient $\eta_t > 0, t = 1,2,..,T$.

Time-consistent strategy means when $t_1 < t_2$, the optimal policies based on these two stages are the same. Few scholars have studied the time consistency of the multi-stage mean-standard semi-variance portfolio model with transaction costs, borrowing constraints, threshold constraints, income demand and cardinality constraints. In this paper, we restate this problem as a game problem. In this part, we will study the optimal strategy of generalized multi-stage mean-quasi-semi-variance fuzzy portfolio model with time consistency under multiple realistic constraints.

According to Bjork and Murguci's research, a definition based on non-cooperative game was given:

Definition 1. Consider a fixed control law $u^{TC}(k-1)$. Set $u(k-1) = (u_{k-1}, u_k^{TC}, \dots, u_{T-1}^{TC})$. u_{k-1} can be any control variable. For any $k(k = 1,2,\dots,T)$, if $u^{TC}(k-1)$ is the same time-consistent strategy, then it comes the following result:

$$Max_{u_{k-1}} u_{k-1}\big(X_{k-1}, u(k-1)\big) = u_{k-1}(X_{k-1}, u^{TC}(k-1))$$

Such time-consistent law $u^{TC}(k-1)$ is called the **perfect nash equilibrium strategy** of sub-game. Under a non-cooperative game frame, assume that a gamer starts at position $(k-1, X_{k-1})$. According to Definition xxx, since he follows strategy $(u_k^{TC}, \dots, u_{T-1}^{TC})$, he can only choose strategy u_{k-1} to maximize $u_{k-1}\big(X_{k-1}, u(k-1)\big)$.

The following formula xxx shows a common equivalent transformation in time- consistent strategy:

$$w_T\left[\overline{M}_T(X_T)-\eta_T\sqrt{\overline{svar}_{T-1}(X_T)}\right]$$
$$=w_T\left[\begin{array}{c}\overline{M}_T\left(r_{fT}X_{T-1}+\sum_{i=1}^{n}(r_{iT}-r_{fT})u_{iT}-\sum_{i=1}^{n}C_{iT}|u_{iT}-u_{i(T-1)}|\right)\\ -\eta_T\sqrt{\overline{svar}_{T-1}\left(r_{fT}X_{T-1}+\sum_{i=1}^{n}(r_{iT}-r_{fT})u_{iT}-\sum_{i=1}^{n}C_{iT}|u_{iT}-u_{i(T-1)}|\right)}\end{array}\right]$$
$$=w_T\left[\begin{array}{c}\sum_{i=1}^{n}\left(\frac{a_{iT}+b_{iT}}{2}+\frac{\beta_{iT}-\alpha_{iT}}{6}-r_{fT}\right)u_{iT}-\sum_{i=1}^{n}C_{iT}|u_{iT}-u_{(i(T-1))}|\\ -r_{fT}X_{T-1}+\eta_T\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{\bar{ij}T}u_{iT}u_{jT}}\end{array}\right] \tag{13}$$

In actual investment, we can easily conclude the following conditions:

- Expected return should be maximized;
- Investment to risk-free assets is a must;
- The number of assets is no more than K;
- The end-of-period wealth are supposed to be larger than X_{0T}

Thus, base on formula xxx, formula xxx, formula xxx, formula xxx, formula xxx, formula xxx and the conclusions above, we can get the following strategy model:

$$Max\sum_{t=1}^{T}w_t\left[\begin{array}{c}r_{ft}\,X_{t-1}+\sum_{i=1}^{n}\left(\frac{a_{it}+b_{it}}{2}+\frac{\beta_{it}-\alpha_{it}}{6}-r_{ft}\right)u_{it}\\ -\sum_{i=1}^{n}c_{iT}|u_{it}-u_{i(t-1)}|-\eta_t\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{\bar{ij}T}u_{it}u_{jt}}\end{array}\right] \tag{14}$$

$$s.t.\begin{cases}X_t=r_{ft}X_{t-1}+\sum_{i=1}^{n}\left(\frac{a_{it}+b_{it}}{2}+\frac{\beta_{it}-\alpha_{it}}{6}-r_{ft}\right)u_{it}-\sum_{i=1}^{n}c_{it}|u_{it}-u_{i(t-1)}|\\ X_T\geq X_{0T}\\ X_{t-1}-\sum_{i=1}^{n}u_{it}\geq u_{ft}^{b}\\ \sum_{i=1}^{n}z_{it}\leq K,z_{it}\in\{0,1\}\\ l_{it}z_{it}\leq u_{it}\leq p_{it}z_{it},i=1,\cdots,n,t=1,\cdots,T\end{cases} \tag{15}$$

This is a mixed-integer semi-closed-loop dynamic optimization model with path-dependence.

这里需要小结一下模型的特点，以及大概的求解思路。

4.7 Model Resolution

In this part, we will use discrete approximate iterative algorithm[xxx] to solve this problem.

First, discretize the state variables based on network graph methods. Through this way, we can turn the model into a multi-stage weighted directed graph. Next, solve the maximum path by the maximum algebraic method. Here we can get the feasible solution. Finally, based on the feasible solution, keep iterating until the last 2 solutions are very close.

The details are as follows:

Step 1. Disperse the interval $[X_t^{min}, X_t^{max}]$ into four equations. Therefore, discrete state variable x_{it} at stage t can be obtained. ($i = 1,2, \ldots ,5, t = 1,2.., T$).

Step 2. Let $F_t(j,k)$ stand for the multi-stage weighted directed graph. Solve the edges of $F_t(j,k)$.Build the graph as Figure 5 shows.

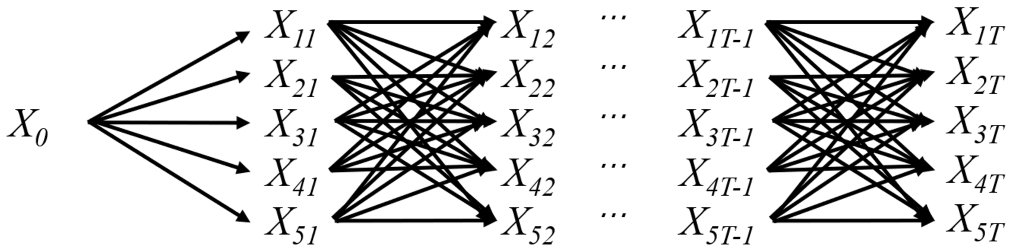


Figure 5.Multi-stage weighted directed graph

Step 3. Based on discrete approximate iterative algorithm[xxx], get the longest path $F^{(1)}$ of $F_t(j,k)$ after the first iteration:

$$F^{(1)} = F_1^{(1)} \otimes F_2^{(1)} \otimes \ \ldots \ \otimes F_T^{(1)} \tag{16}$$

where $F_1^{(1)} = \left(F_1^{(1)}(1,j)\right)_{1\times5}, F_2^{(1)} = \left(F_2^{(1)}(I,j)\right)_{5\times5}, \ldots, F_T^{(1)} = \left(F_T^{(1)}(I,j)\right)_{5\times5}$

Step 4. Keep iteration. The $(k+1)$-th iteration can be described as follows:

Let the longest path of the k-th iteration $F^{(k)}$ be $X_0 \to \mathrm{X}_{\mathrm{i}_1 1}^{(k)} \to X_{i_2 2}^{(k)} \to \cdots \to X_{i_T T}^{(k)}$.

The best solution to the longest path in Figure xxx is also the feasible solution to the multi-stage mean semi-variance fuzzy portfolio model. Based on this, disperse the variables from stage 1 to stage T into four equations, as the following steps shows.

1) Disperse X_2^{min} and $X_{i_2 2}^{(k)}$, $X_{i_2 2}^{(k)}$ and X_2^{max} into two internal structure which are the same. The 5 disperse points of X_2, which are $X_2^{min}, X_{22}^{(k+1)}, X_{i_1 2}^{(k+1)}, X_{32}^{(k+1)}$ and X_2^{max} can be solved.
2) Based on $\left(\mathrm{X}_{\mathrm{i}_3 3}^{(k)}, \ldots, X_{i_{T+1} T}^{(k)}\right)$, disperse the variables from stage 3 to stage T into 5 disperse points in the same way. The weight of stage t can also be easily solved.
3) The longest path of $(k+1)$-th iteration $F^{(k+1)}$ and another feasible solution can be calculated :

$$F^{(k+1)} = F_1^{(k+1)} \otimes F_2^{(k+1)} \otimes \ldots \otimes F_T^{(k+1)} \tag{17}$$

where $F_1^{(k+1)} = \left(F_1^{(k+1)}(1,j)\right)_{1\times5}, F_2^{(k+1)} = \left(F_2^{(k+1)}(I,j)\right)_{5\times5} .. F_T^{(k+1)} = \left(F_T^{(k+1)}(I,j)\right)_{5\times5}$.

If $\left|F^{(k+1)} - F^{(k)}\right| < 10^{-6}$, the longest path $F^{(k+1)}$ is the approximate solution to our model. Else, keep iterating until it does.

4.8 Subtitle 运算结果

Here, based on the data in the attachment and the model above, we figure it out that how much the initial investment worth on 9/10/2021.

The first thing needs doing is clearing parameters:

- According to historical data on closing prices of gold and bitcoin from 09/11/2016 to 09/10/2021, we consider a month as a cycle of 60 months, that is, 60 stages. Thus, $T = 60$.
- Initial wealth $X_0 = \$1000$.
- To ensure that investment is profitable, the ultimate wealth should be greater than the initial wealth. So $X_T > X_0 = 1000$.
- There are 2 kinds of risk assets(gold, bitcoin) and 1 kind of risk-free asset(cash), thus, $n = 2, K = 2$.
- Set $i = 1$ stand for gold and $i = 2$ stand for bitcoin. The transaction costs of gold and bitcoin are $\gamma_1 = 0.01, \gamma_2 = 0.02$.
- Since there will be no additional funds in the investment process, that is, no borrowings or arrears, the lower bound of the risk-free asset investment ratio is 0. Also, the amount of transactions in the period t is less than or equal to the total wealth in period $t-1$. Thus, $u_{ft}^{b} = 0$, $X_{t-1} - \sum_{i=1}^{n} u_{it} \geq 0$.
- Due to threshold constraints, the position of gold should be kept between 5 % and 10 %. Thus, $0.05 X_t z_{1t} \leq u_{1t} \leq 0.1 X_t z_{1t}, t = 1, \cdots, T$.

The model described by formula xxx can be embodied as follows:

$$Max \sum_{t=1}^{T} w_t \left[\begin{array}{l} r_{ft} X_{t-1} + \sum_{i=1}^{n} \left(\frac{a_{it} + b_{it}}{2} + \frac{\beta_{it} - \alpha_{it}}{6} - r_{ft} \right) u_{it} \\ - \sum_{i=1}^{n} c_{iT} \left| u_{it} - u_{i(t-1)} \right| - \eta_t \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ijT}^{-} u_{it} u_{jt}} \end{array} \right] \tag{18}$$

$$s.t. \begin{cases} X_t = r_{ft} X_{t-1} + \sum_{i=1}^{2} \left(\frac{a_{it} + b_{it}}{2} + \frac{\beta_{it} - \alpha_{it}}{6} - r_{ft} \right) u_{it} - \sum_{i=1}^{2} c_{it} \left| u_{it} - u_{i(t-1)} \right| & (a) \\ X_T \geq 1000 & (b) \\ X_{t-1} - \sum_{i=1}^{2} u_{it} \geq 0 & (c) \\ \sum_{i=1}^{2} z_{it} \leq 2, z_{it} \in \{0,1\} & (d) \\ 0.05 X_t z_{1t} \leq u_{1t} \leq 0.1 X_t z_{1t}, t = 1, \cdots, T. & (e) \end{cases} \tag{19}$$

Formula xxx is the wealth we hope to achieve, formula xxx(a) corresponds to the iterative process, formula xxx(b) corresponds to profit demand, formula xxx(c) corresponds to the self-financing process, formula xxx(d) corresponds to the product category and formula xxx(e) corresponds to the threshold constraint of gold.

Solve it according to the method in Section 4.7 and get the trapezoidal distribution of the probability of return on assets in each period.

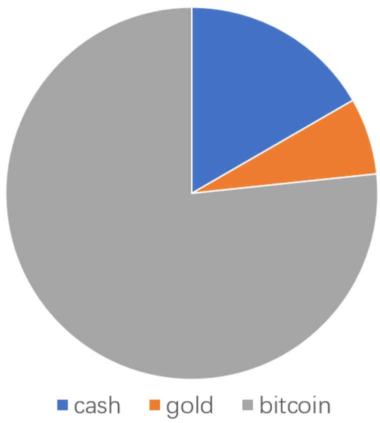
To better understand the transaction in each period, the price charts of gold and bitcoin are shown below as Figure xxx shows. Visualize the results of the model as the following Figure xxx, xxx and xxx show.

Figure 6. Daily price of gold and bitcoin

Figure xxx shows the proportion of assets in total wealth at the end of each period. Blue, orange and gray lines represent cash, gold and bitcoin, respectively. We find that in periods 1 to 5 and 30 to 35, bitcoin is bought rapidly and massively. This is because the model judges that the price of bitcoin will continue to increase during this period and in the next several period, and the risk of decline in the short term is small, so a large number of purchases are made. In period 20 to 22, although we suffer some losses, the rapid sell-off of large amounts of gold and bitcoin helps us avoid the continuing decline. In period 34 and 35, most bitcoins are promptly and decisively sold, and gold, whose return is more stable, was invested. This further ensures the income when avoiding risks. In period 36 to 60, the slow purchase of bitcoin is due to its relatively low valuation caused by the rapid decline before. At this time, increasing the purchase is conducive to grasping the subsequent upward trend. Also, it is worth attention that although it is less likely to continue to fall after falling, that is, the risk is less, the model still suggest us to buy slowly to share the risk.

The reasons and details of other transactions are similar to the above analysis, so they are not described too much here.

In order to more intuitively feel the key investment assets in different periods, count the assets with the highest proportion in each period, and call this asset the "champion" of this period. Count the "champion" times of the three assets. Bitcoin wins "champions" for 46 times,



gold wins 10 times and cash wins 4 times. Their "champion times" ratio is shown in Figure xxx.

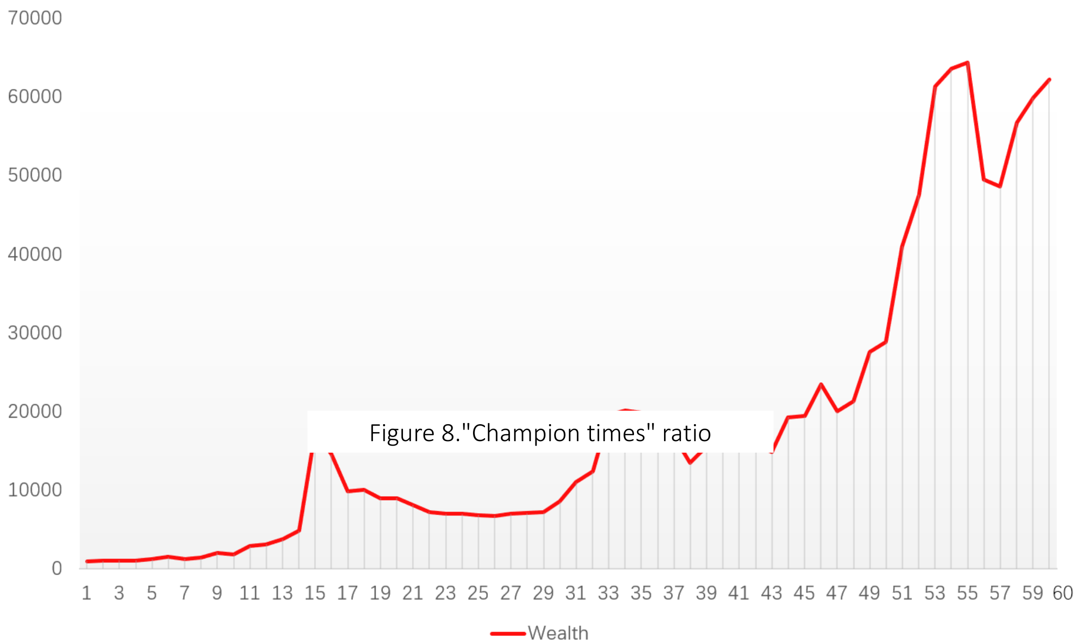


Figure 8."Champion times" ratio

Figure xxx shows us our wealth intuitively. It can be found that the strategy based on the model helps us invest better, which means getting higher returns while avoiding risks. For example, in period 15 to 20 and 53 to 57, the market environment is quite unfavorable. Although there are losses, they are kept to a level that is far less than the market decline. And in period 46 to 55, we also keep up with the pace of market profits, obtained quite reliable income.

At the same time, it should be noted that compared with the losses in period 15 to 20 and 53 to 57, although the decline of the market in the latter is much larger than that in the former, the decline caused by investment based on the model is not much different. This shows that with the increase of historical data, the model's resistance to sudden market deterioration continues to increase. If more historical data are available, the stronger our ability to predict and bear risks, the more radical strategies can be adopted to achieve higher returns.

In a nutshell, by 09/10/2021, the initial $1000 finally reaches assets equivalent to **$62211**. The rate of return has reached an impressive 100%

5. Model Analysis

5.1 Model Demonstration

In actual financial activities, considering that investors are "bounded rational", investors' psychological factors will affect their investment behavior. In investment decision-making, any portfolio in the market is an alternative, and investors will compare the expected returns and

Figure 9. Total wealth

risks of different portfolios, when investors have an unlimited number of alternative portfolios. This part compares the investor's portfolio with the portfolio that is likely to receive the maximum return, to get the investor's "**regret value**" when the return is not optimal. Similarly, compare the investor's portfolio with the portfolio that is likely to have the least risk, to get the "regret value" when the portfolio does not meet the minimum risk. Because of "regret aversion" psychology, it is difficult for investors to maximize wealth and minimize risk from the perspective of absolute rationality when making investment decisions. Otherwise, they add regret psychology into investment decisions, hoping that the investment results will not bring regret to themselves.

We introduce regret psychology into the demonstration of the rationality of the model, which shows that the decision made by our model for different types of investors makes the degree of regret of investors lower than other choices, and makes investors more satisfied, that is, our model is more successful.

Due to the irrational behavior of investors in the process of investment, such as chasing up and down, overconfidence and regret aversion, many traditional portfolio models cannot explain the anomalies in financial markets well. Therefore, a large number of scholars began to develop and improve portfolio theory from the perspective of investor behavior. Chorus C.G proposed a **Generalized Random Regret Minimization** model in 2014. Based on Chorus's model, this part describes the investor's regret psychology from the perspective of return and risk.

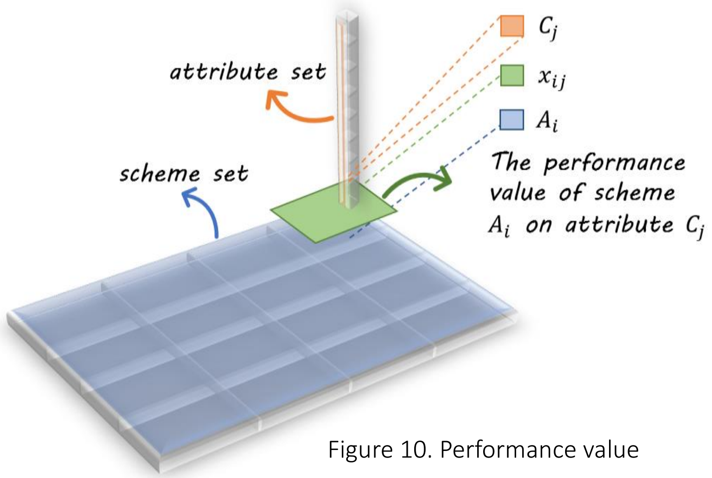


Figure 10. Performance value

Assume that there are n feasible schemes. $A = \{A_1, A_2, \dots, A_n\}$ is the set of schemes. $C = \{C_1, C_2, \dots, C_m\}$ is the set of attributes. x_{ij} is the performance value of scheme A_i on attribute C_j. Performance values of different schemes can be compared under the same attribute.

In the financial market, investors use the initial wealth to invest. If the final wealth is less than the maximum wealth available, investors may regret. This kind of regret value can be calculated as follows:

$$R_1 = l\,n\{\gamma + e^{\beta_1(r_{max} - r)}\} \tag{20}$$

where r_{max} is max available yield rate, r is the actual yield, γ is the investor's degree of regret and β_1 is the investor's sensitivity to yield rate.

In the process of investment, investors hope to reduce the risk by diversifying investment. If the portfolio risk is greater than the minimum risk that investors can bear, investors will regret to do so. This kind of regret value can be calculated as follows:

$$R_2 = l\,n\{\gamma + e^{\beta_2(\delta - \delta_{min})}\} \tag{21}$$

where δ_{min} is the lowest acceptable risk, δ is the actual risk, γ is the investor's degree of regret and β_2 is the investor's sensitivity to risks.

Usually, the higher the expected return of an investor, the greater the risk he is willing to take. Thus, investor's sensitivity to yield rate and risks can be considered as nearly negative correlation. Let

$$\beta_1 + \beta_2 = 1 \tag{22}$$

If $\beta_1 > 0.5$, then the investor is more sensitive to yield. If $\beta_2 > 0.5$, then the investor is more sensitive to risks. Otherwise, the investor is equally sensitive to yield and risks. The total regret value is:

$$R = R_1 + R_2 \tag{23}$$

In this paper, cash is considered as risk-free asset, gold is considered as mid-risk asset and bitcoin is considered as high-risk asset, according to their fluctuation. Investors are divided into conservative investors, normal investors and radical investors, according their investment style. As for conservative investors, $\beta_1 > 0.5$. As for normal investors, $\beta_1 = 0.5$. As for racial investors, $\beta_1 < 0.5$.

According to the model, we can get an investment proposal. Next, we will explore the regret value R of investors with different styles when investing based on this proposal. The score is used to measure the success of the model, and the results are compared with the results of *equal proportional investment*[1].

【图片】

5.2 Sensitive Analysis

When transaction costs fluctuate, the investment decisions given by the model will also be affected. When transaction costs rise, even if the investment phase remains unchanged, trades will be reduced or even stopped in some stages; when transaction costs fall, trades rise in some stages.

Therefore, we can change transaction costs and then observe the changes in investment suggestions given by the model. In this way, the sensitivity of the model to transaction costs is explored. We set three pairs of transaction costs of gold and bitcoin: $\gamma_1 = 0.01$ and $\gamma_2 = 0.02$, $\gamma_1 = 0.001$ and $\gamma_2 = 0.002, \gamma_1 = 0.0001$ and $\gamma_2 = 0.0002$. Here are the results of our model.

When $\gamma_1 = 0.01$ and $\gamma_2 = 0.02$, the result is clearly displayed in 4.8.

When $\gamma_1 = 0.001$ and $\gamma_2 = 0.002$, Table xxx shows the proportion of assets [C,G,B] and the total wealth X_t in different period.

Table 3. Total wealth and proportion of assets

Period	[C,G,B]			X_t
	Cash	Gold	Bitcoin	
1	0	680	320	998
2	317	0	683	1067
3	160	91	749	1069
4	0	43	957	1157
5	0	43	957	1157
		…		
60	0	46	954	63157

[1] equal proportional investment means purchasing at $[C, G, B] = [333,333,333]$ on 09/11/2016.

When $\gamma_1 = 0.0001$ and $\gamma_2 = 0.0002$, Table xxx shows the proportion of assets [C,G,B] and the total wealth X_t.

Table 4. Total wealth and proportion of assets

Period	[C,G,B]			X_t
	Cash	Gold	Bitcoin	
1	0	680	320	998
2	317	0	683	1067
3	160	91	749	1069
4	0	43	957	1157
5	0	43	957	1157
		…		
60	0	46	954	63157

We can clearly see that with the increase of transaction cost, the transaction amount and times in most of the stages are gradually decreasing. From this, we can draw the conclusion that our model is very sensitive to transaction cost.

5.3 Strengths and Weakness

Strengths:

- **Comprehensiveness**. What our model takes into account are not only returns, but also risks investors to take when investing. It takes as little risk as possible while making the investor's return as large as possible.
- **Innovativeness**. Introduce a specific measure of investor regret psychology to demonstrate the superiority of the model, and use it to measure the success of the model.
- **Applicability**. Theoretically, in the absence of major international events, the model can continuously update itself over long periods of time and is highly applicable.

Weakness:

- Because the risks taken investors are considered, the decisions proposed are on the conservative side, which may cause investors to miss out on certain gains.
- The decision is mainly influenced by the price data without taking other factors into account such as international situation, policy implications, etc. The factors are relatively homogeneous and may lead to results that differ from the actual optimal ones.

6. Conclusion

TODO

7. Memorandum

//TODO

这里打算放个封面什么的

Background

这是一个关于个人小金额投资的模型报告。
·投资对象：黄金、比特币
·投资目标：低风险、高收益、后悔程度低

Descriptions

Strategy

1.黄金小仓位长期持有，仓位保持在5%~10%
2.比特币定期投入一定比例的资金

Reasons

1.黄金价格的影响因素较为复杂且难以预测
2.比特币的波动大，收益高的同时风险高，定期投资尽管会错过部分收益但可以平滑损失

Descriptions

how to model?

总的来说就是，先预测，再规划
规划目标是确保无借款，黄金仓
位保持，风险低，收益高

how to use this model?

Strengths and weakness of the model

Strengths

1.收益高的同时风险较低，投资者
后悔程度低
2.易于操作

Weakness

为了降低风险可能错过一部分收益

The result of the model

这里会是结果	这里会是比例曲线图

Questions you may have

Q1:如何证明模型的优越性?
A:我们引入了对投资者的后悔心理的具体量度，通过与其他方案分析比较可以得出结论：我们的模型给出的投资决定，在实现了风险尽量小的情况下，收益尽量大，让投资者后悔程度尽量低。这是一个优越的模型
Q2:对成本敏感吗?
A:敏感，····
Q3:可以改变周期吗,我想2个月进行一次投资?
A:可以，但决定后不再更改

References

[1] Vercher E, BermUdez J D. A Posibilistic Mcan-Downside Risk-Skewness Model f-or Efficient Portfolio Selection[J]. IEEE Transactions On Fuzzy Systems, 2013.3:585—595.
[2] Test
[3] test
[4] test
[5] test
[6] stest
[7] test
[8] test

Appendix A

Appendix B

Period	[C, G, B] Cash	Gold	Bitcoin	X_t	Period	[C, G, B] Cash	Gold	Bitcoin	X_t
1	0	680	320	959	31	533	179	286	11025
2	478	0	522	1044	32	350	126	522	12368
3	277	0	723	1059	33	0	128	872	19531
4	190	86	724	1089	34	0	259	741	20097
5	0	45	955	1243	35	0	259	741	19864
6	0	45	955	1512	36	179	376	445	16778
7	79	46	875	1263	37	177	374	449	16878
8	0	46	954	1439	38	345	466	209	13524
9	0	44	956	2074	39	282	326	252	15994
10	219	392	489	1856	40	163	335	502	17005
11	95	407	498	2892	41	184	355	461	16221
12	44	419	537	3110	42	202	359	439	15997
13	0	204	796	3765	43	231	352	417	14851
14	0	204	796	4869	44	154	277	568	19264
15	0	108	892	17597	45	156	274	570	19452
16	20	113	867	14556	46	0	356	644	23517
17	306	114	579	9877	47	0	344	656	20001
18	306	114	579	10013	48	0	261	739	21356
19	113	208	679	8992	49	0	137	862	27567
20	79	245	676	9015	50	0	117	883	28859
21	763	101	136	8113	51	0	94	906	41005
22	862	51	86	7235	52	0	65	935	47598
23	862	51	86	6997	53	0	54	946	61357
24	852	61	86	7004	54	0	38	962	63571
25	833	75	92	6841	55	0	35	965	64401
26	841	100	59	6755	56	0	127	873	49560
27	841	100	59	6998	57	0	139	861	48650
28	791	152	56	7112	58	0	99	901	56751
29	890	157	53	7199	59	0	79	921	59934
30	882	161	57	8549	60	0	64	936	62211