Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2222182

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1. Introduction

Memo 格式

1.1 Background

Nowadays, market investment is one of the most important means to realize wealth appreciation. Through the purchase or sale of various assets in the market, investors can obtain different returns. In order to maximize the total return to investors, we can optimize the investment strategy and decide the amount and proportion of assets to be bought or sold based on market conditions.

风险小

Different assets have different risks, rewards and trading rules. Generally speaking, gold is less risky and less profitable; while bitcoin has higher risks and higher returns. The commission cost per transaction costs α % of the amount traded. Usually, α_{gold} is smaller than $\alpha_{bitcoin}$. we should ....

1.2 Problem Analysis

First of all, we note that this is a personal investment and the amount of investment is small, so the strategy should be in line with the market mechanism to reduce risks and increase returns.

As for gold trading, its price is affected by both short-term and long-term factors. Short-term factors, which are uncertain and contingent, let gold price fall more than its recent increase, so it is not recommended to invest in short-term timing. The medium and long-term timing model is influenced by the Federal Reserve’s monetary policy the characteristics of economic cycle. These economic data can be made out obviously after their occurrence, but are hard to predict in advance.

Therefore, under the complex and unpredictable driving factors, it is recommended to allocate gold assets for a long time. In this way, no matter how time or market influencing factors changes, we can finally obtain long-term benefits. The price of such strategy is only to give up some band benefits that are difficult to grasp in the short term. We do not recommend that individual investors to take risks for this return. Therefore, for gold trading, we recommend maintaining long-term allocation of small positions, i.e. 5 % ~ 10 % of the position.

For bitcoin transactions, we prefer the automatic investment plan. In the stock market, automatic investment plan has been proved to be a suitable strategy for most people. By buying a certain amount of stock regularly, you can get the same or even higher returns with the market level with less time and energy cost. Since bitcoin's price change is also volatile, we think the automatic investment plan apply equally to bitcoin. By timing purchase, we can effectively smooth out the price volatility, reduce holding costs and halve risk.

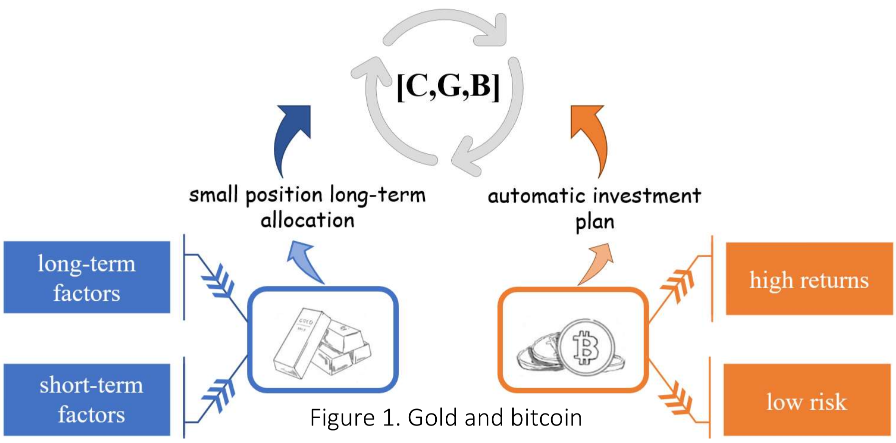

Figure 1. Gold and bitcoin

In a word, securities returns, which can only be estimated according to limited information, can be considered as fuzzy variables rather than just random variables. Since the trapezoidal fuzzy【这里引用 xxx】 number has been shown to be an appropriate measure of security returns, we decide to establish a multi-stage decision-making model, which helps us to regularly change the proportion of holding assets and reduce trading in other time to reduce risks and improve returns.

To verify the sensitivity of the model to transaction costs, we let the transaction cost changes in a certain range. The sensitivity of the model is captured by the change in the transaction amount at each stage and the final return.

In actual financial activities, considering investors are '**bounded rationality**', investors' psychological factors will affect their investment behavior. Therefore, we introduce specific measures of traders' regret psychology to prove that our model is the best model. When other conditions are the same, the lower the degree of traders' regret is, the more satisfied the decision is and the more successful the model is.

2. Assumptions and Justifications

1. **There is no limit to the maximum and minimum transaction amount.**
   To simplify the model, we assume that the minimum amount per transaction is 0, with no upper limit. And no debit or credit occurs. This applies to transactions of both gold and bitcoin.
2. **Traders make their decisions to purchase or sell based only on data from attachment, which are independent from people and circumstances around them.**
   When people make decisions to buy or sell, they usually receive many factors, such as the evaluation from people around them, the current situation of life, and the news on the Internet. In this question, when making a trading decision, only the attached data matters with no other factors.
3. **Investors are more concerned about downside risks.**
   When studying the psychology of investor regret, we find that the intensity of emotions is greater about loss than gain【这里引用 xxx】.

3. Notations

Table 1 shows the notations commonly used in this paper and their description.

Table 1. Notations

Notations	Description
X_0	Initial assets owned by investors
t	An investment period
X_t	Total wealth at the end of period t
R_{it}	Fuzzy yield of risky assets i in period t
$\overline{svar}(X_t)$	The semi-variance of X_t
u_{it}	Investment amount to asset i on period t
u_{ft}	Investment amount to risk-free asset i on period t
l_{it}	The upper bound constraint of risk asset i on period t
p_{it}	The lower bound constraint of risk asset i on period t

4. Model Construction and Resolution

Based on the analysis above, with the '**time inconsistency**' of investment strategy brought into model construction, a **multi-stage mean semi-variance fuzzy portfolio** model is proposed. It is based on transaction cost, loan constraint, threshold constraint, revenue demand and cardinality constraint, and the optimal investment strategy of time consistency is studied. Due to transaction cost, revenue demand and cardinality constraints, the model provides a **mixed-integer semi-closed-loop dynamic optimization with path-dependence** to solve this problem. As for details, we'll use the discrete approximate iterative algorithm is to obtain the optimal time-consistent investment strategy in the following paper. Figure 2 is the flowchart of this process.

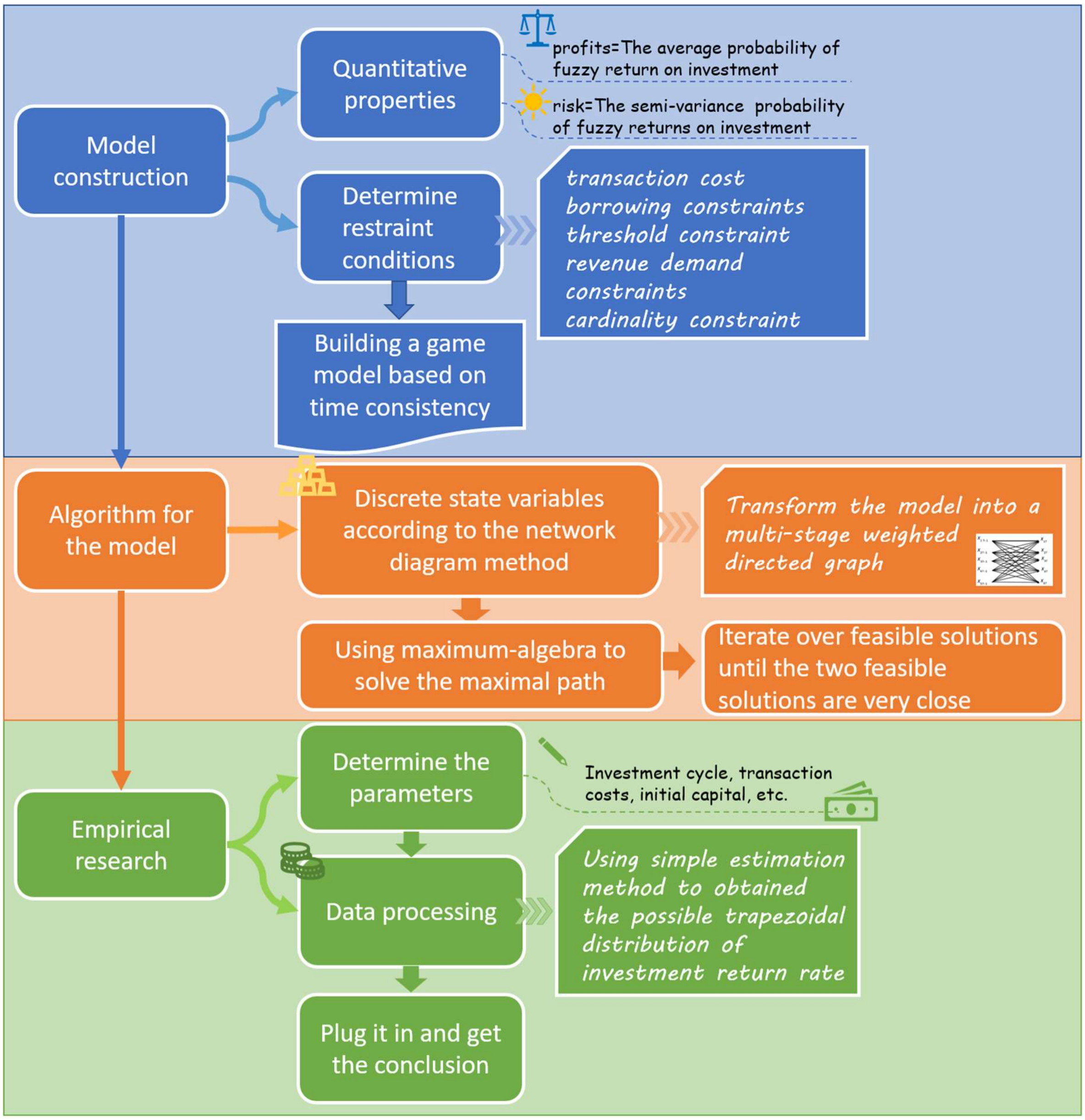

Figure 2. Model construction

4.1 Subtitle 预处理

Before data analysis and decision making, we must ensure its quality, which determines the prediction and generalization ability of the model. In order to ensure the accuracy, integrity and credibility of the data, we preprocess the data based on the attachments given: *BCHAIN-MKPRU.csv* and *LBMA-GOLD.csv*. We analyze the data in the table and delete useless data, supplement data that may be missing, and finally retain the revised data.

After analysis, we found that there is no illogicality data such as negative data, text or outliers in both files, so there is no need to delete any.

In order to make the data smoother, we choose the mean interpolation method to supplement the data. For each incomplete item, the mean value of price on the day before and after the missing data day is supplemented into it. Table 2 shows the price data inserted into *LBMA-GOLD.csv*.

Table 2. Data inserted into file

Date	12/30/16	12/23/16	12/22/17	12/29/17	12/24/18	12/31/18	12/24/19	12/31/19	12/24/20	12/31/20
Price	1132.98	1148.45	1272	1301.5	1263.1	1281	1469.8	1521	1874.65	1915.4

4.2 Subtitle 数据分析

4.3 Subtitle 参数说明和问题描述

Assume that there are n kinds of risk assets and 1 kind of risk-free asset for investors to choose. His initial wealth is X_0. Let t stand for investment period, with T stages. At the first stage of investment stage, the investor can only invest with X_0. The asset portfolios changes at the beginning of eachstage and investment last for T stages.

Let R_{it} stand for the fuzzy yield of risky assets i in period t, and $R_t = (R_{1t}, R_{2t}, \dots R_{nt})'$. Set X_t as the total wealth at the end of period t. In period t, set $\overline{svar}(X_t)$ as the semi-variance of X_t, u_{it} as the investment in asset i, u_{ft} as the investment in risk-free asset i, l_{it} and p_{it} as the upper and lower bound constraint of risk asset i on period t. Let $u\_t = (u_{1t}, u_{2t}, \dots, u_{nt}, u_{ft})'$.

4.4 Subtitle 参数量化

The securities market is a complex system with dynamic changes. It is difficult for people to obtain the overall information of the random distribution of securities returns. We can only estimate the return according to the historical information of securities. Thus, securities returns can be considered as fuzzy variables rather than random variables. In addition, considering the uncertainty of market environment, trapezoidal fuzzy number is often used to measure the yield of securities. Many scholars have discussed the fuzzy portfolio optimization problem deeply, such as Carlsson[xxx], Anne Trefethen[xxx], Zhang Peng and Zhang Weiguo[xxx], etc. 这里引用 xxx

The return and risk of asset portfolios are measured by the possibilistic mean value and the possibilistic standard semi-variance of asset fuzzy return, respectively. Clearly, the whole process is self-financing, as investors did not add additional funds during this period. Record the return rate of risk assets as R_{it}, where $R_{it} = (a_{it}, b_{it}, \alpha_{it}, \beta_{it})$.

In period t, the possibilistic mean value r_{pt} of the assets $u_t = (u_{1t}, u_{2t}, \dots u_{nt}, u_{ft})$ can be calculated as:

$$r_{pt} = \sum_{i=1}^{n} \bar{M}(R_{it})u_{it} + r_{ft}(X_{t-1} - \sum_{i=1}^{n} u_{it}) - X_{t-1}$$

$$= (r_{fi} - 1)X_{t-1} + \sum \left(\frac{a_{it} + b_{it}}{2} + \frac{\beta_{it} - \alpha_{it}}{6} - r_{ft}\right) u_{it}, t = 1,2, \dots, T$$

Assuming the transaction cost is $\gamma\%$ in this paper. Obviously, the transaction cost of asset i in period t is $u_{it} \times \gamma\%$, and the total cost of the asset portfolio $u_t = (u_{1t}, u_{2t}, \dots, u_{nt}, u_{ft})$ is :

$$C_t = \sum_{i=1}^{n} u_{it} \times \gamma, \quad t = 1,2, \dots, T$$

Then, the net income ratio of asset portfolio X_t by the end of period t is:

$$r_{Nt} = (r_{ft} - 1)X_{t-1} + \sum_{i=1}^{n} \left(\frac{a_{it} + b_{it}}{2} + \frac{\beta_{it} - \alpha_{it}}{6} - r_{ft}\right) u_{it} - \sum_{i=1}^{n} \gamma_{it}|u_{it}| \tag{5}$$

and the total wealth held by investor by the end of period t is:

$$X_t = r_{Nt} + X_{t-1} = r_{ft}X_{t-1} + \sum_{i=1}^{n} \left(\frac{a_{it} + b_{it}}{2} + \frac{\beta_{it} - \alpha_{it}}{6} - r_{ft}\right) u_{it} - \sum_{i=1}^{n} \gamma_{it}|u_{it}| \tag{6}$$

where $t = 1,2, \dots, T$.

4.5 Subtitle 确定约束条件

The threshold constraint of multi-stage portfolio is:

$$l_{it} \le u_{it} \le p_{it} \tag{7}$$

where l_{it} and p_{it} as the upper and lower bound constraint of u_{it}.

Set the lower bound constrain of proportion of investment ratio on risk-free assets to be $u_{ft}^{b}(u_{ft}^{b} \le 0)$, then the borrowing constraints of risk-free assets in period t is:

$$u_{ft} = X_{t-1} - \sum_{i=1}^{n} u_{it} \ge u_{ft}^{b} \tag{8}$$

According to formula xxx(13), the semi-variance of asset portfolio u_{it} is:

$$\overline{svar}_t(u_t) = u_t' H_t^- u_t \tag{9}$$

where its standard semi-variance is $\sqrt{\overline{svar}_t(u_t)}$. Assume that H_t^- is a semi-positive definite matrix.

$$H_t^- = \left(Cov_t^-(r_{it}, r_{jt})\right)_{n\times n} \tag{10}$$

$$Cov_t^-(r_{it}, r_{jt}) = \frac{(b_{it} - a_{it})(\beta_{jt} + \alpha_{jt}) + (b_{jt} - a_{jt})(\beta_{it} + \alpha_{it})}{12} + \left[\frac{(\beta_{it} + \alpha_{it})(\beta_{jt} + \alpha_{jt})}{36}\right] + \frac{(\beta_{it} - \alpha_{it})(\beta_{jt} - \alpha_{jt})}{4} + \frac{\beta_{it}\beta_{jt}}{18} \tag{11}$$

The cardinality constraint is:

$$\sum_{i=1}^{n} z_{it} \leq K, z_{it} \in \{0,1\} \tag{12}$$

where K is the maximum number of risk-free assets,

$$z_{it} = \begin{cases} 1 & choose\ asset\ i \\ 0 & not\ choose\ asset\ i \end{cases} \tag{13}$$

4.6 Subtitle 博弈模型确定

The traditional multi-stage mean-standard semi-variance portfolio optimization model only considers the expected value of end-of-period wealth and standard semi-variance. In the real world, however, investors care not only about the expected value and standard semi-variance of end-of-period wealth, but also about which during the investment period. In other words, the expectation and the semi-variance of the portfolio are different in the t period. Therefore, this paper uses standard semi-variance to measure risk. Set the weight coefficient $w_t > 0$ and risk preference coefficient $\eta_t > 0, t = 1,2,..,T$.

Time-consistent strategy means when $t_1 < t_2$, the optimal policies based on these two stages are the same. Few scholars have studied the time consistency of the multi-stage mean-standard semi-variance portfolio model with transaction costs, borrowing constraints, threshold constraints, income demand and cardinality constraints. In this paper, we restate this problem as a game problem. In this part, we will study the optimal strategy of generalized multi-stage mean-quasi-semi-variance fuzzy portfolio model with time consistency under multiple realistic constraints.

According to Bjork and Murguci's research, a definition based on non-cooperative game was given:

Definition xxx. Consider a fixed control law $u^{TC}(k-1)$. Set $u(k-1) = (u_{k-1}, u_k^{TC}, \dots, u_{T-1}^{TC})$. u_{k-1} can be any control variable. For any $k(k = 1,2,\dots,T)$, if $u^{TC}(k-1)$ is the same time-consistent strategy, then it comes the following result:

$$Max_{u_{k-1}} u_{k-1}\big(X_{k-1}, u(k-1)\big) = u_{k-1}(X_{k-1}, u^{TC}(k-1))$$

Such time-consistent law $u^{TC}(k-1)$ is called the **perfect nash equilibrium strategy** of sub-game. Under a non-cooperative game frame, assume that a gamer starts at position$(k-1, X_{k-1})$. According to Definition xxx, since he follows strategy $(u_k^{TC}, \dots, u_{T-1}^{TC})$, he can only choose strategy u_{k-1} to maximize $u_{k-1}\big(X_{k-1}, u(k-1)\big)$.

The following formula xxx shows a common equivalent transformation in time- consistent strategy:

$$w_T\left[\overline{M}_T(X_T)-\eta_T\sqrt{\overline{svar}_{T-1}(X_T)}\right]$$

$$=w_T\left[\begin{array}{c}\overline{M}_T\left(r_{fT}X_{T-1}+\sum_{i=1}^{n}(r_{iT}-r_{fT})u_{iT}-\sum_{i=1}^{n}C_{iT}|u_{iT}-u_{i(T-1)}|\right)\\ -\eta_T\sqrt{\overline{svar}_{T-1}\left(r_{fT}X_{T-1}+\sum_{i=1}^{n}(r_{iT}-r_{fT})u_{iT}-\sum_{i=1}^{n}C_{iT}|u_{iT}-u_{i(T-1)}|\right)}\end{array}\right]$$

$$=w_T\left[\begin{array}{c}\sum_{i=1}^{n}\left(\frac{a_{iT}+b_{iT}}{2}+\frac{\beta_{iT}-\alpha_{iT}}{6}-r_{fT}\right)u_{iT}-\sum_{i=1}^{n}C_{iT}|u_{iT}-u_{(i(T-1))}|\\ -r_{fT}X_{T-1}+\eta_T\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{\bar{ijT}}u_{iT}u_{jT}}\end{array}\right] \tag{25}$$

In actual investment, we can easily conclude the following conditions:

- Expected return should be maximized;
- Investment to risk-free assets is a must;
- The number of assets is no more than K;
- The end-of-period wealth are supposed to be larger than X_{0T}

Thus, base on formula xxx, formula xxx, formula xxx, formula xxx, formula xxx, formula xxx and the conclusions above, we can get the following strategy model:

$$Max\sum_{t=1}^{T}w_t\left[\begin{array}{c}r_{ft}X_{t-1}+\sum_{i=1}^{n}\left(\frac{a_{it}+b_{it}}{2}+\frac{\beta_{it}-\alpha_{it}}{6}-r_{ft}\right)u_{it}\\ -\sum_{i=1}^{n}c_{iT}|u_{it}-u_{i(t-1)}|-\eta_t\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{\bar{ijT}}u_{it}u_{jt}}\end{array}\right] \tag{26}$$

$$s.t.\begin{cases}X_t=r_{ft}X_{t-1}+\sum_{i=1}^{n}\left(\frac{a_{it}+b_{it}}{2}+\frac{\beta_{it}-\alpha_{it}}{6}-r_{ft}\right)u_{it}-\sum_{i=1}^{n}c_{it}|u_{it}-u_{i(t-1)}|\\ X_T\geq X_{0T}\\ X_{t-1}-\sum_{i=1}^{n}u_{it}\ \geq u_{ft}^{b}\\ \sum_{i=1}^{n}z_{it}\leq K, z_{it}\in\{0,1\}\\ l_{it}z_{it}\leq u_{it}\leq p_{it}z_{it}, i=1,\cdots,n, t=1,\cdots,T\end{cases} \tag{27}$$

This is a mixed-integer semi-closed-loop dynamic optimization model with path-dependence.

4.7 Model Resolution

In this part, we will use discrete approximate iterative algorithm[xxx] to solve this problem. First, discretize the state variables based on network graph methods. Through this way, we can turn the model into a multi-stage weighted directed graph. Next, solve the maximum path by the maximum algebraic method. Here we can get the feasible solution. Finally, based on the feasible solution, keep iterating until the last 2 solutions are very close.

The details are as follows:

Step 1. Disperse the interval $[X_t^{min}, X_t^{max}]$ into four equations. Therefore, discrete state variable x_{it} at stage t can be obtained. ($i = 1,2,\dots,5, t = 1,2..,T$).

Step 2. Let $F_t(j,k)$ stand for the multi-stage weighted directed graph. Solve the edges of $F_t(j,k)$.Build the graph as Figure xxx shows

【这里插图】

Step 3. Based on discrete approximate iterative algorithm[xxx], get the longest path $F^{(1)}$ of $F_t(j,k)$ after the first iteration:

$$F^{(1)} = F_1^{(1)} \otimes F_2^{(1)} \otimes \ \dots \ \otimes F_T^{(1)}$$

where $F_1^{(1)} = \left(F_1^{(1)}(1,j)\right)_{1\times5}, F_2^{(1)} = \left(F_2^{(1)}(I,j)\right)_{5\times5}, \dots, F_T^{(1)} = \left(F_T^{(1)}(I,j)\right)_{5\times5}$

Step 4. Keep iteration. The $(k+1)$-th iteration can be described as follows:

Let the longest path of the k-th iteration $F^{(k)}$ be $X_0 \to X_{i_1 1}^{(k)} \to X_{i_2 2}^{(k)} \to \cdots \to X_{i_T T}^{(k)}$.

The best solution to the longest path in Figure xxx is also the feasible solution to the multi-stage mean semi-variance fuzzy portfolio model. Based on this, disperse the variables from stage 1 to stage T into four equations, as the following steps shows.

1) Disperse X_2^{min} and $X_{i_2 2}^{(k)}$, $X_{i_2 2}^{(k)}$ and X_2^{max} into two internal structure which are the same. The 5 disperse points of X_2 , which are $X_2^{min}, X_{22}^{(k+1)}, X_{i_1 2}^{(k+1)}, X_{32}^{(k+1)}, X_2^{max}$, can be solved.

2) Based on $\left(X_{i_3 3}^{(k)}, \dots, X_{i_{T+1} T}^{(k)}\right)$, disperse the variables from stage 3 to stage T into 5 disperse points in the same way. The weight of stage t can also be easily solved.

3) The longest path of $(k+1)$-th iteration $F^{(k+1)}$ and another feasible solution can be calculated :

$$F^{(k+1)} = F_1^{(k+1)} \otimes F_2^{(k+1)} \otimes \dots \otimes F_T^{(k+1)}$$

where $F_1^{(k+1)} = \left(F_1^{(k+1)}(1,j)\right)_{1\times5}, F_2^{(k+1)} = \left(F_2^{(k+1)}(I,j)\right)_{5\times5} .. F_T^{(k+1)} = \left(F_T^{(k+1)}(I,j)\right)_{5\times5}$.

If $\left|F^{(k+1)} - F^{(k)}\right| < 10^{-6}$, the longest path $F^{(k+1)}$ is the approximate solution to our model. Else, keep iterating until it does.

4.8 Subtitle 运算结果

Here, based on the data in the attachment and the model above, we figure it out that how much the initial investment worth on 9/10/2021.

The first thing needs doing is clearing parameters:

- According to historical data on closing prices of gold and ~~bitcoin from~~ 09/11/2016 to 09/10/2021, we consider a month as a cycle of 60 months, that is, 60 stages. Thus, $T = 60$.
- Initial wealth $X_0 = \$1000$.
- To ensure that investment is profitable, the ultimate wealth should be greater than the initial wealth. So $X_T > X_0 = 1000$.
- There are 2 kinds of risk assets(gold, bitcoin) and 1 kind of risk-free asset(cash), thus, $n = 2, K = 2$.
- Set $i = 1$ stand for gold and $i = 2$ stand for bitcoin. The transaction costs of gold and bitcoin are $\gamma_1 = 0.01, \gamma_2 = 0.02$.
- Since there will be no additional funds in the investment process, that is, no borrowings or arrears, the lower bound of the risk-free asset investment ratio is 0. Also, the amount of transactions in the period t is less than or equal to the total wealth in period $t-1$. Thus, $u_{ft}^b = 0,\ X_{t-1} - \sum_{i=1}^{n} u_{it} \geq 0$.
- Due to threshold constraints, the position of gold should be kept between 5 % and 10 %. Thus, $0.05X_t z_{1t} \leq u_{1t} \leq 0.1X_t z_{1t}, t = 1, \cdots, T$.

The model described by formula xxx can be embodied as follows:

$$Max \sum_{t=1}^{T} w_t \left[\begin{array}{l} r_{ft} X_{t-1} + \sum_{i=1}^{n} \left(\frac{a_{it}+b_{it}}{2} + \frac{\beta_{it}-\alpha_{it}}{6} - r_{ft} \right) u_{it} \\ - \sum_{i=1}^{n} c_{iT} \left| u_{it} - u_{i(t-1)} \right| - \eta_t \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{\bar{ij}T} u_{it} u_{jt}} \end{array} \right] \tag{26}$$

$$s.t. \begin{cases} X_t = r_{ft} X_{t-1} + \sum_{i=1}^{2} \left(\frac{a_{it}+b_{it}}{2} + \frac{\beta_{it}-\alpha_{it}}{6} - r_{ft} \right) u_{it} - \sum_{i=1}^{2} c_{it} \left| u_{it} - u_{i(t-1)} \right| & (a) \\ X_T \geq 1000 & (b) \\ X_{t-1} - \sum_{i=1}^{2} u_{it} \geq 0 & (c) \\ \sum_{i=1}^{2} z_{it} \leq 2, z_{it} \in \{0,1\} & (d) \\ 0.05X_t z_{1t} \leq u_{1t} \leq 0.1X_t z_{1t}, t = 1, \cdots, T. & (e) \end{cases} \tag{27}$$

Formula xxx is the wealth we hope to achieve, formula xxx(a) corresponds to the iterative process, formula xxx(b) corresponds to profit demand, formula xxx(c) corresponds to the self-financing process, formula xxx(d) corresponds to the product category and formula xxx(e) corresponds to the threshold constraint of gold.

Solve it according to the method in Section 4.7 and get the trapezoidal distribution of the probability of return on assets in each period. Visualize the results as Figure xxx shows.

【图片】

By 09/10/2021, We finally have assets equivalent to **$xxx**.

5. Model Analysis

5.1 Model Demonstration

In actual financial activities, considering that investors are "bounded rational", investors' psychological factors will affect their investment behavior. In investment decision-making, any portfolio in the market is an alternative, and investors will compare the expected returns and risks of different portfolios, when investors have an unlimited number of alternative portfolios. This part compares the investor's portfolio with the portfolio that is likely to receive the maximum return, to get the investor's "**regret value**" when the return is not optimal. Similarly, compare the investor's portfolio with the portfolio that is likely to have the least risk, to get the "regret value" when the portfolio does not meet the minimum risk. Because of "regret aversion" psychology, it is difficult for investors to maximize wealth and minimize risk from the perspective of absolute rationality when making investment decisions. Otherwise, they add regret psychology into investment decisions, hoping that the investment results will not bring regret to themselves.

We introduce regret psychology into the demonstration of the rationality of the model, which shows that the decision made by our model for different types of investors makes the degree of regret of investors lower than other choices, and makes investors more satisfied, that is, our model is more successful.

Due to the irrational behavior of investors in the process of investment, such as chasing up and down, overconfidence and regret aversion, many traditional portfolio models cannot explain the anomalies in financial markets well. Therefore, a large number of scholars began to develop and improve portfolio theory from the perspective of investor behavior. Chorus C.G proposed a **Generalized Random Regret Minimization** model in 2014. Based on Chorus's model, this part describes the investor's regret psychology from the perspective of return and risk.

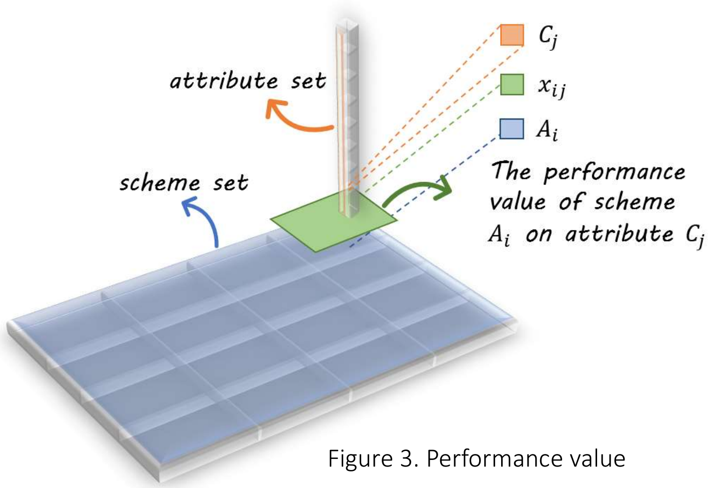

Figure 3. Performance value

Assume that there are n feasible schemes. $A = \{A_1, A_2, \dots, A_n\}$ is the set of schemes. $C = \{C_1, C_2, \dots, C_m\}$ is the set of attributes. x_{ij} is the performance value of scheme A_i on attribute C_j. Performance values of different schemes can be compared under the same attribute.

In the financial market, investors use the initial wealth to invest. If the final wealth is less than the maximum wealth available, investors may regret. This kind of regret value can be calculated as follows:

$$R_1 = ln\{\gamma + e^{\beta_1(r_{max}-r)}\}$$

where r_{max} is max available yield rate, r is the actual yield, γ is the investor's degree of regret and β_1 is the investor's sensitivity to yield rate.

In the process of investment, investors hope to reduce the risk by diversifying investment. If the portfolio risk is greater than the minimum risk that investors can bear, investors will regret to do so. This kind of regret value can be calculated as follows:

$$R_2 = ln\{\gamma + e^{\beta_2(\delta-\delta_{min})}\}$$

where δ_{min} is the lowest acceptable risk, δ is the actual risk, γ is the investor's degree of regret and β_2 is the investor's sensitivity to risks.

Usually, the higher the expected return of an investor, the greater the risk he is willing to take. Thus, investor's sensitivity to yield rate and risks can be considered as nearly negative correlation. Let

$$\beta_1 + \beta_2 = 1$$

If $\beta_1 > 0.5$, then the investor is more sensitive to yield. If $\beta_2 > 0.5$, then the investor is more sensitive to risks. Otherwise, the investor is equally sensitive to yield and risks. The total regret value is:

$$R = R_1 + R_2$$

In this paper, cash is considered as risk-free asset, gold is considered as mid-risk asset and bitcoin is considered as high-risk asset, according to their fluctuation. Investors are divided into conservative investors, normal investors and radical investors, according their investment style. As for conservative investors, $\beta_1 > 0.5$. As for normal investors, $\beta_1 = 0.5$. As for racial investors, $\beta_1 < 0.5$.

According to the model, we can get an investment proposal. Next, we will explore the regret value R of investors with different styles when investing based on this proposal. The score is used to measure the success of the model, and the results are compared with the results of *equal proportional investment*[1].

【图片】

5.2 Sensitive Analysis

When ~~transaction costs fluctuate~~, the investment decisions given by the model will also be affected. When transaction with costs rise, even if the investment phase remains unchanged, trades will be reduced or even stopped in some stages; when transaction costs fall, trades rise in some stages.

Therefore, we can change transaction costs and then observe the changes in investment suggestions given by the model. In this way, the sensitivity of the model to transaction costs is explored. We set three pairs of transaction costs of gold and bitcoin: $\gamma_1 = 0.0001$ and $\gamma_2 = 0.0002$, $\gamma_1 = 0.001$ and $\gamma_2 = 0.002$, $\gamma_1 = 0.01$ and $\gamma_2 = 0.02$. Here are the result of our model.

[1] equal proportional investment means purchasing at $[C, G, B] = [333,333,333]$ on 09/11/2016.

We can clearly see that with the increase of transaction cost, the transaction amount and times in most of the stages are gradually decreasing. From this, we can draw the conclusion that our model is very sensitive to transaction cost.

5.3 Strengths and Weakness

6. Conclusion

7. Memorandum

//TODO

References

[1] Vercher E, BermUdez J D. A Posibilistic Mcan-Downside Risk-Skewness Model f-or Efficient Portfolio Selection[J]. IEEE Transactions On Fuzzy Systems, 2013.3:585—595.
[2] Test
[3] test
[4] test
[5] test
[6] stest
[7] test
[8] test

Appendix A ?????

Appendix B ?????