```
# Import dependencies
import numpy as np
import torch
import torchvision
from torch.utils.data.dataset import Dataset
from torchvision import datasets, transforms
from torch import nn, optim
import matplotlib.pyplot as plt
```

Problem 4.2 Determine appropriate mini-batch size = 32, epoch = 100

```
# ################# Part 1: Load data and create batch
##################
N_total = 600
N_train = 500
x = torch.unsqueeze(torch.linspace(0, 1, N_total), dim=1)
r = torch.randperm(N_total)
x = x[r, :]
y = 0.2 + 0.4 * torch.pow(x, 2) + 0.3 * x * torch.sin(15 * x) + 0.05 *
torch.cos(50 * x)

class CustomDataset(Dataset):
    def __init__(self, x, y):
        self.y = y
        self.x = x

    def __len__(self):
        return len(self.y)

    def __getitem__(self, idx):
        y1 = self.y[idx]
        x1 = self.x[idx]
        return (x1, y1)

# Change batch_size here to test different values
batch_size = 32  # Experiment with different batch sizes: 32, 64, 128
trainset = CustomDataset(x[0:N_train, :], y[0:N_train, :])
testset = CustomDataset(x[N_train:N_total, :], y[N_train:N_total, :])
train_loader = torch.utils.data.DataLoader(trainset,
batch_size=batch_size)
test_loader = torch.utils.data.DataLoader(testset,
batch_size=batch_size)

# ################# Part 2: Define Model #################
model = nn.Sequential(
    nn.Linear(1, 1024, bias=True),
    nn.ReLU(),
    nn.Linear(1024, 1, bias=True)
)
```

```python
def init_weights(m):
    if isinstance(m, nn.Linear):
        m.weight.data.uniform_(-1, 1)
        m.bias.data.uniform_(-1, 1)

model.apply(init_weights)

Sequential(
  (0): Linear(in_features=1, out_features=1024, bias=True)
  (1): ReLU()
  (2): Linear(in_features=1024, out_features=1, bias=True)
)

# ################# Part 3: Define Loss and Optimizer
##################
criterion = nn.MSELoss()
optimizer = optim.Adam(model.parameters(), lr=0.001)

# ################# Part 4: Training and Testing #################
def train_NN():
    model.train()
    for images, labels in train_loader:
        out = model(images)
        loss = criterion(out, labels)
        loss.backward()
        optimizer.step()
        optimizer.zero_grad()
    return loss

def test_NN(loader):
    model.eval()
    loss = 0
    with torch.no_grad():
        for images, labels in loader:
            out = model(images)
            loss += criterion(out, labels).item()
    loss = loss / len(loader)
    return loss

# Experiment with different numbers of epochs
N_epoch = 100   # Try values like 100, 200, 500
train_loss = np.zeros((N_epoch, 1))
test_loss = np.zeros((N_epoch, 1))

for epoch in range(N_epoch):
    train_NN()
    train_loss[epoch, 0] = test_NN(train_loader)
    test_loss[epoch, 0] = test_NN(test_loader)
    print(f'Epoch: {epoch+1:03d}, Train Loss: {train_loss[epoch,
0]:.7f}, Test Loss: {test_loss[epoch, 0]:.7f}')
```
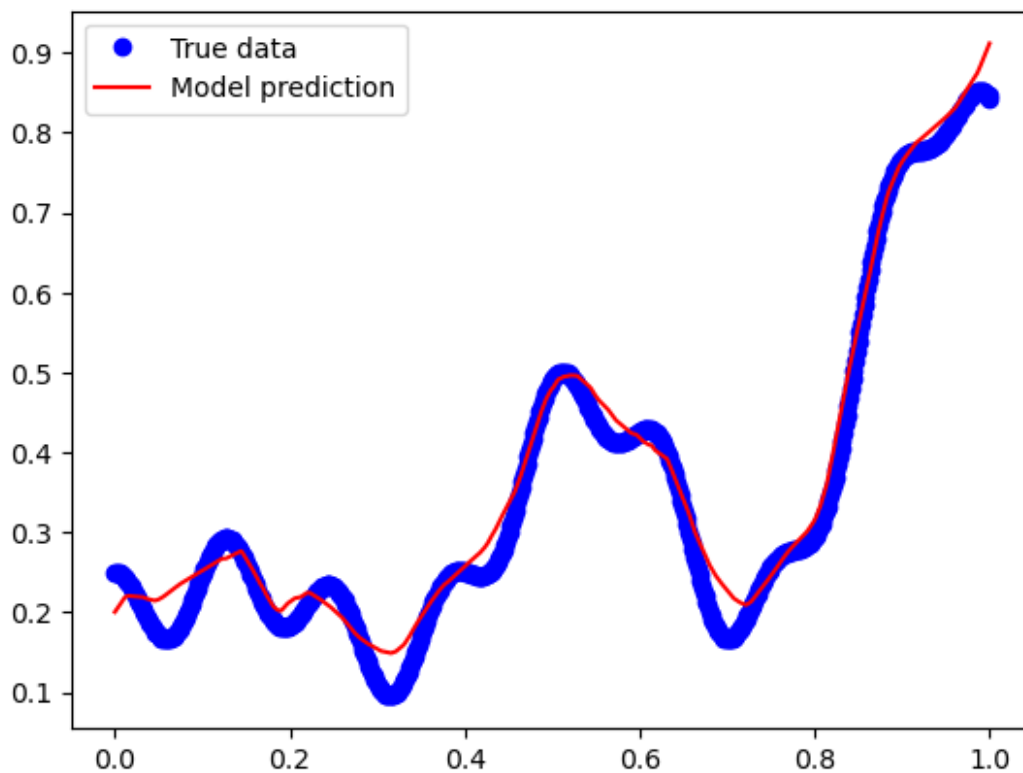
```
Epoch: 001, Train Loss: 2.4354299, Test Loss: 2.4755852
Epoch: 002, Train Loss: 3.9236410, Test Loss: 5.1143095
Epoch: 003, Train Loss: 0.5623158, Test Loss: 0.6476966
Epoch: 004, Train Loss: 0.2407783, Test Loss: 0.2889059
Epoch: 005, Train Loss: 0.1428621, Test Loss: 0.1905185
Epoch: 006, Train Loss: 0.0941955, Test Loss: 0.0943288
Epoch: 007, Train Loss: 0.0746617, Test Loss: 0.0724883
Epoch: 008, Train Loss: 0.0617504, Test Loss: 0.0571178
Epoch: 009, Train Loss: 0.0513076, Test Loss: 0.0466694
Epoch: 010, Train Loss: 0.0430132, Test Loss: 0.0386989
Epoch: 011, Train Loss: 0.0365885, Test Loss: 0.0324178
Epoch: 012, Train Loss: 0.0316494, Test Loss: 0.0277373
Epoch: 013, Train Loss: 0.0278367, Test Loss: 0.0241990
Epoch: 014, Train Loss: 0.0248465, Test Loss: 0.0215016
Epoch: 015, Train Loss: 0.0224383, Test Loss: 0.0193680
Epoch: 016, Train Loss: 0.0204374, Test Loss: 0.0176288
Epoch: 017, Train Loss: 0.0187305, Test Loss: 0.0161740
Epoch: 018, Train Loss: 0.0172499, Test Loss: 0.0149224
Epoch: 019, Train Loss: 0.0159318, Test Loss: 0.0138053
Epoch: 020, Train Loss: 0.0147235, Test Loss: 0.0127849
Epoch: 021, Train Loss: 0.0136354, Test Loss: 0.0118672
Epoch: 022, Train Loss: 0.0126465, Test Loss: 0.0110175
Epoch: 023, Train Loss: 0.0117301, Test Loss: 0.0102177
Epoch: 024, Train Loss: 0.0108768, Test Loss: 0.0094759
Epoch: 025, Train Loss: 0.0100872, Test Loss: 0.0087866
Epoch: 026, Train Loss: 0.0093559, Test Loss: 0.0081463
Epoch: 027, Train Loss: 0.0086779, Test Loss: 0.0075576
Epoch: 028, Train Loss: 0.0080505, Test Loss: 0.0070149
Epoch: 029, Train Loss: 0.0074707, Test Loss: 0.0065112
Epoch: 030, Train Loss: 0.0069354, Test Loss: 0.0060476
Epoch: 031, Train Loss: 0.0064402, Test Loss: 0.0056201
Epoch: 032, Train Loss: 0.0059825, Test Loss: 0.0052276
Epoch: 033, Train Loss: 0.0055611, Test Loss: 0.0048635
Epoch: 034, Train Loss: 0.0051722, Test Loss: 0.0045270
Epoch: 035, Train Loss: 0.0048144, Test Loss: 0.0042171
Epoch: 036, Train Loss: 0.0044843, Test Loss: 0.0039327
Epoch: 037, Train Loss: 0.0041800, Test Loss: 0.0036707
Epoch: 038, Train Loss: 0.0039007, Test Loss: 0.0034307
Epoch: 039, Train Loss: 0.0036431, Test Loss: 0.0032092
Epoch: 040, Train Loss: 0.0034060, Test Loss: 0.0030050
Epoch: 041, Train Loss: 0.0031879, Test Loss: 0.0028171
Epoch: 042, Train Loss: 0.0029874, Test Loss: 0.0026439
Epoch: 043, Train Loss: 0.0028028, Test Loss: 0.0024836
Epoch: 044, Train Loss: 0.0026331, Test Loss: 0.0023360
Epoch: 045, Train Loss: 0.0024772, Test Loss: 0.0021994
Epoch: 046, Train Loss: 0.0023339, Test Loss: 0.0020734
Epoch: 047, Train Loss: 0.0022023, Test Loss: 0.0019576
Epoch: 048, Train Loss: 0.0020812, Test Loss: 0.0018507
Epoch: 049, Train Loss: 0.0019697, Test Loss: 0.0017521
Epoch: 050, Train Loss: 0.0018672, Test Loss: 0.0016613
```

```
Epoch: 051, Train Loss: 0.0017732, Test Loss: 0.0015775
Epoch: 052, Train Loss: 0.0016867, Test Loss: 0.0015003
Epoch: 053, Train Loss: 0.0016074, Test Loss: 0.0014286
Epoch: 054, Train Loss: 0.0015344, Test Loss: 0.0013624
Epoch: 055, Train Loss: 0.0014672, Test Loss: 0.0013019
Epoch: 056, Train Loss: 0.0014049, Test Loss: 0.0012461
Epoch: 057, Train Loss: 0.0013476, Test Loss: 0.0011952
Epoch: 058, Train Loss: 0.0012948, Test Loss: 0.0011483
Epoch: 059, Train Loss: 0.0012462, Test Loss: 0.0011058
Epoch: 060, Train Loss: 0.0012013, Test Loss: 0.0010673
Epoch: 061, Train Loss: 0.0011601, Test Loss: 0.0010318
Epoch: 062, Train Loss: 0.0011223, Test Loss: 0.0009991
Epoch: 063, Train Loss: 0.0010875, Test Loss: 0.0009688
Epoch: 064, Train Loss: 0.0010555, Test Loss: 0.0009415
Epoch: 065, Train Loss: 0.0010264, Test Loss: 0.0009166
Epoch: 066, Train Loss: 0.0009998, Test Loss: 0.0008934
Epoch: 067, Train Loss: 0.0009755, Test Loss: 0.0008722
Epoch: 068, Train Loss: 0.0009533, Test Loss: 0.0008526
Epoch: 069, Train Loss: 0.0009333, Test Loss: 0.0008347
Epoch: 070, Train Loss: 0.0009149, Test Loss: 0.0008183
Epoch: 071, Train Loss: 0.0008983, Test Loss: 0.0008036
Epoch: 072, Train Loss: 0.0008834, Test Loss: 0.0007903
Epoch: 073, Train Loss: 0.0008702, Test Loss: 0.0007783
Epoch: 074, Train Loss: 0.0008586, Test Loss: 0.0007677
Epoch: 075, Train Loss: 0.0008485, Test Loss: 0.0007581
Epoch: 076, Train Loss: 0.0008397, Test Loss: 0.0007494
Epoch: 077, Train Loss: 0.0008321, Test Loss: 0.0007417
Epoch: 078, Train Loss: 0.0008260, Test Loss: 0.0007350
Epoch: 079, Train Loss: 0.0008210, Test Loss: 0.0007294
Epoch: 080, Train Loss: 0.0008174, Test Loss: 0.0007250
Epoch: 081, Train Loss: 0.0008148, Test Loss: 0.0007215
Epoch: 082, Train Loss: 0.0008130, Test Loss: 0.0007187
Epoch: 083, Train Loss: 0.0008119, Test Loss: 0.0007167
Epoch: 084, Train Loss: 0.0008114, Test Loss: 0.0007149
Epoch: 085, Train Loss: 0.0008116, Test Loss: 0.0007137
Epoch: 086, Train Loss: 0.0008123, Test Loss: 0.0007130
Epoch: 087, Train Loss: 0.0008134, Test Loss: 0.0007127
Epoch: 088, Train Loss: 0.0008145, Test Loss: 0.0007126
Epoch: 089, Train Loss: 0.0008154, Test Loss: 0.0007123
Epoch: 090, Train Loss: 0.0008156, Test Loss: 0.0007114
Epoch: 091, Train Loss: 0.0008154, Test Loss: 0.0007102
Epoch: 092, Train Loss: 0.0008145, Test Loss: 0.0007086
Epoch: 093, Train Loss: 0.0008130, Test Loss: 0.0007065
Epoch: 094, Train Loss: 0.0008103, Test Loss: 0.0007035
Epoch: 095, Train Loss: 0.0008066, Test Loss: 0.0006999
Epoch: 096, Train Loss: 0.0008016, Test Loss: 0.0006956
Epoch: 097, Train Loss: 0.0007956, Test Loss: 0.0006908
Epoch: 098, Train Loss: 0.0007884, Test Loss: 0.0006851
```

```
Epoch: 099, Train Loss: 0.0007800, Test Loss: 0.0006785
Epoch: 100, Train Loss: 0.0007706, Test Loss: 0.0006711

# ################# Final Prediction #################
x_test = torch.unsqueeze(torch.linspace(0, 1, 1999), dim=1)
y_test = model(x_test)

# Plot the results
plt.plot(x[0:N_total], y[0:N_total], 'bo', label='True data')
plt.plot(x_test, y_test.detach().numpy(), 'r', label='Model
prediction')
plt.legend()
plt.show()
```


Problem 4.2 Determine appropriate mini-batch size = 32, epoch = 200

```
# ################# Part 1: Load data and create batch
#################
N_total = 600
N_train = 500
x = torch.unsqueeze(torch.linspace(0, 1, N_total), dim=1)
r = torch.randperm(N_total)
x = x[r, :]
y = 0.2 + 0.4 * torch.pow(x, 2) + 0.3 * x * torch.sin(15 * x) + 0.05 *
torch.cos(50 * x)
```

```python
class CustomDataset(Dataset):
    def __init__(self, x, y):
        self.y = y
        self.x = x

    def __len__(self):
        return len(self.y)

    def __getitem__(self, idx):
        y1 = self.y[idx]
        x1 = self.x[idx]
        return (x1, y1)

# Change batch_size here to test different values
batch_size = 32  # Experiment with different batch sizes: 32, 64, 128
trainset = CustomDataset(x[0:N_train, :], y[0:N_train, :])
testset = CustomDataset(x[N_train:N_total, :], y[N_train:N_total, :])
train_loader = torch.utils.data.DataLoader(trainset,
batch_size=batch_size)
test_loader = torch.utils.data.DataLoader(testset,
batch_size=batch_size)

# ################# Part 2: Define Model #################
model = nn.Sequential(
    nn.Linear(1, 1024, bias=True),
    nn.ReLU(),
    nn.Linear(1024, 1, bias=True)
)

def init_weights(m):
    if isinstance(m, nn.Linear):
        m.weight.data.uniform_(-1, 1)
        m.bias.data.uniform_(-1, 1)

model.apply(init_weights)

# ################# Part 3: Define Loss and Optimizer
#################
criterion = nn.MSELoss()
optimizer = optim.Adam(model.parameters(), lr=0.001)

# ################# Part 4: Training and Testing #################
def train_NN():
    model.train()
    for images, labels in train_loader:
        out = model(images)
        loss = criterion(out, labels)
        loss.backward()
        optimizer.step()
```

```python
        optimizer.zero_grad()
    return loss

def test_NN(loader):
    model.eval()
    loss = 0
    with torch.no_grad():
        for images, labels in loader:
            out = model(images)
            loss += criterion(out, labels).item()
    loss = loss / len(loader)
    return loss

# Experiment with different numbers of epochs
N_epoch = 200  # Try values like 100, 200, 500
train_loss = np.zeros((N_epoch, 1))
test_loss = np.zeros((N_epoch, 1))

for epoch in range(N_epoch):
    train_NN()
    train_loss[epoch, 0] = test_NN(train_loader)
    test_loss[epoch, 0] = test_NN(test_loader)
    print(f'Epoch: {epoch+1:03d}, Train Loss: {train_loss[epoch,
0]:.7f}, Test Loss: {test_loss[epoch, 0]:.7f}')

# ################## Final Prediction ##################
x_test = torch.unsqueeze(torch.linspace(0, 1, 1999), dim=1)
y_test = model(x_test)

# Plot the results
plt.plot(x[0:N_total], y[0:N_total], 'bo', label='True data')
plt.plot(x_test, y_test.detach().numpy(), 'r', label='Model
prediction')
plt.legend()
plt.show()
```

```
Epoch: 001, Train Loss: 2.0267818, Test Loss: 2.1714951
Epoch: 002, Train Loss: 3.1269895, Test Loss: 3.0556849
Epoch: 003, Train Loss: 0.4870037, Test Loss: 0.5017008
Epoch: 004, Train Loss: 0.1158632, Test Loss: 0.1175885
Epoch: 005, Train Loss: 0.0763223, Test Loss: 0.0932067
Epoch: 006, Train Loss: 0.0526999, Test Loss: 0.0570447
Epoch: 007, Train Loss: 0.0422577, Test Loss: 0.0513590
Epoch: 008, Train Loss: 0.0347963, Test Loss: 0.0403812
Epoch: 009, Train Loss: 0.0297586, Test Loss: 0.0346408
Epoch: 010, Train Loss: 0.0259011, Test Loss: 0.0297962
Epoch: 011, Train Loss: 0.0229931, Test Loss: 0.0262312
Epoch: 012, Train Loss: 0.0207413, Test Loss: 0.0234615
Epoch: 013, Train Loss: 0.0189482, Test Loss: 0.0212690
Epoch: 014, Train Loss: 0.0174663, Test Loss: 0.0194782
```
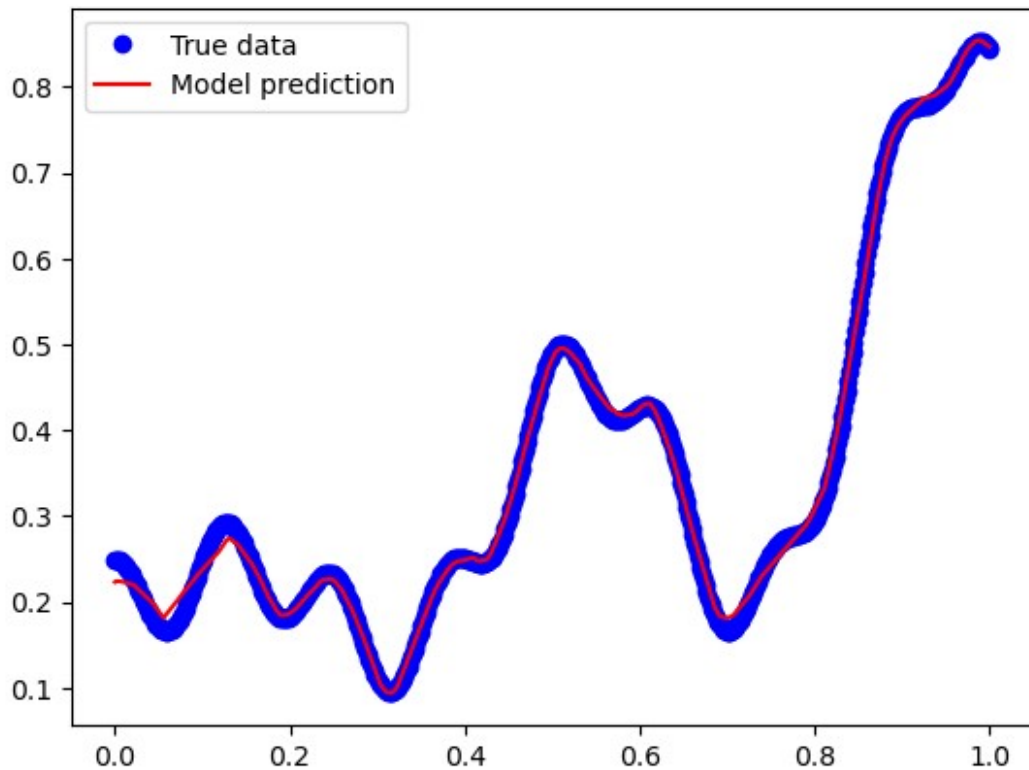
```
Epoch: 015, Train Loss: 0.0161937, Test Loss: 0.0179651
Epoch: 016, Train Loss: 0.0150643, Test Loss: 0.0166443
Epoch: 017, Train Loss: 0.0140387, Test Loss: 0.0154675
Epoch: 018, Train Loss: 0.0130912, Test Loss: 0.0143999
Epoch: 019, Train Loss: 0.0122091, Test Loss: 0.0134190
Epoch: 020, Train Loss: 0.0113824, Test Loss: 0.0125114
Epoch: 021, Train Loss: 0.0106074, Test Loss: 0.0116700
Epoch: 022, Train Loss: 0.0098817, Test Loss: 0.0108860
Epoch: 023, Train Loss: 0.0092031, Test Loss: 0.0101552
Epoch: 024, Train Loss: 0.0085675, Test Loss: 0.0094750
Epoch: 025, Train Loss: 0.0079730, Test Loss: 0.0088395
Epoch: 026, Train Loss: 0.0074178, Test Loss: 0.0082504
Epoch: 027, Train Loss: 0.0069005, Test Loss: 0.0077029
Epoch: 028, Train Loss: 0.0064193, Test Loss: 0.0071953
Epoch: 029, Train Loss: 0.0059723, Test Loss: 0.0067247
Epoch: 030, Train Loss: 0.0055574, Test Loss: 0.0062900
Epoch: 031, Train Loss: 0.0051734, Test Loss: 0.0058875
Epoch: 032, Train Loss: 0.0048172, Test Loss: 0.0055127
Epoch: 033, Train Loss: 0.0044882, Test Loss: 0.0051667
Epoch: 034, Train Loss: 0.0041846, Test Loss: 0.0048479
Epoch: 035, Train Loss: 0.0039042, Test Loss: 0.0045526
Epoch: 036, Train Loss: 0.0036460, Test Loss: 0.0042804
Epoch: 037, Train Loss: 0.0034088, Test Loss: 0.0040287
Epoch: 038, Train Loss: 0.0031904, Test Loss: 0.0037963
Epoch: 039, Train Loss: 0.0029893, Test Loss: 0.0035821
Epoch: 040, Train Loss: 0.0028049, Test Loss: 0.0033842
Epoch: 041, Train Loss: 0.0026359, Test Loss: 0.0032019
Epoch: 042, Train Loss: 0.0024810, Test Loss: 0.0030329
Epoch: 043, Train Loss: 0.0023388, Test Loss: 0.0028752
Epoch: 044, Train Loss: 0.0022087, Test Loss: 0.0027286
Epoch: 045, Train Loss: 0.0020896, Test Loss: 0.0025934
Epoch: 046, Train Loss: 0.0019807, Test Loss: 0.0024681
Epoch: 047, Train Loss: 0.0018808, Test Loss: 0.0023527
Epoch: 048, Train Loss: 0.0017889, Test Loss: 0.0022451
Epoch: 049, Train Loss: 0.0017044, Test Loss: 0.0021448
Epoch: 050, Train Loss: 0.0016269, Test Loss: 0.0020527
Epoch: 051, Train Loss: 0.0015557, Test Loss: 0.0019655
Epoch: 052, Train Loss: 0.0014901, Test Loss: 0.0018847
Epoch: 053, Train Loss: 0.0014295, Test Loss: 0.0018091
Epoch: 054, Train Loss: 0.0013736, Test Loss: 0.0017383
Epoch: 055, Train Loss: 0.0013218, Test Loss: 0.0016730
Epoch: 056, Train Loss: 0.0012734, Test Loss: 0.0016114
Epoch: 057, Train Loss: 0.0012284, Test Loss: 0.0015544
Epoch: 058, Train Loss: 0.0011865, Test Loss: 0.0015011
Epoch: 059, Train Loss: 0.0011472, Test Loss: 0.0014512
Epoch: 060, Train Loss: 0.0011105, Test Loss: 0.0014043
Epoch: 061, Train Loss: 0.0010760, Test Loss: 0.0013608
Epoch: 062, Train Loss: 0.0010437, Test Loss: 0.0013200
Epoch: 063, Train Loss: 0.0010134, Test Loss: 0.0012809
```

```
Epoch: 064, Train Loss: 0.0009853, Test Loss: 0.0012443
Epoch: 065, Train Loss: 0.0009589, Test Loss: 0.0012098
Epoch: 066, Train Loss: 0.0009340, Test Loss: 0.0011767
Epoch: 067, Train Loss: 0.0009104, Test Loss: 0.0011458
Epoch: 068, Train Loss: 0.0008881, Test Loss: 0.0011165
Epoch: 069, Train Loss: 0.0008671, Test Loss: 0.0010888
Epoch: 070, Train Loss: 0.0008472, Test Loss: 0.0010627
Epoch: 071, Train Loss: 0.0008283, Test Loss: 0.0010378
Epoch: 072, Train Loss: 0.0008103, Test Loss: 0.0010142
Epoch: 073, Train Loss: 0.0007933, Test Loss: 0.0009918
Epoch: 074, Train Loss: 0.0007771, Test Loss: 0.0009706
Epoch: 075, Train Loss: 0.0007616, Test Loss: 0.0009502
Epoch: 076, Train Loss: 0.0007468, Test Loss: 0.0009307
Epoch: 077, Train Loss: 0.0007327, Test Loss: 0.0009122
Epoch: 078, Train Loss: 0.0007192, Test Loss: 0.0008946
Epoch: 079, Train Loss: 0.0007063, Test Loss: 0.0008777
Epoch: 080, Train Loss: 0.0006939, Test Loss: 0.0008614
Epoch: 081, Train Loss: 0.0006819, Test Loss: 0.0008458
Epoch: 082, Train Loss: 0.0006702, Test Loss: 0.0008306
Epoch: 083, Train Loss: 0.0006590, Test Loss: 0.0008158
Epoch: 084, Train Loss: 0.0006481, Test Loss: 0.0008016
Epoch: 085, Train Loss: 0.0006375, Test Loss: 0.0007876
Epoch: 086, Train Loss: 0.0006272, Test Loss: 0.0007740
Epoch: 087, Train Loss: 0.0006170, Test Loss: 0.0007606
Epoch: 088, Train Loss: 0.0006071, Test Loss: 0.0007476
Epoch: 089, Train Loss: 0.0005973, Test Loss: 0.0007348
Epoch: 090, Train Loss: 0.0005876, Test Loss: 0.0007222
Epoch: 091, Train Loss: 0.0005781, Test Loss: 0.0007099
Epoch: 092, Train Loss: 0.0005687, Test Loss: 0.0006978
Epoch: 093, Train Loss: 0.0005594, Test Loss: 0.0006857
Epoch: 094, Train Loss: 0.0005501, Test Loss: 0.0006736
Epoch: 095, Train Loss: 0.0005409, Test Loss: 0.0006616
Epoch: 096, Train Loss: 0.0005317, Test Loss: 0.0006497
Epoch: 097, Train Loss: 0.0005227, Test Loss: 0.0006379
Epoch: 098, Train Loss: 0.0005137, Test Loss: 0.0006262
Epoch: 099, Train Loss: 0.0005048, Test Loss: 0.0006146
Epoch: 100, Train Loss: 0.0004961, Test Loss: 0.0006032
Epoch: 101, Train Loss: 0.0004873, Test Loss: 0.0005919
Epoch: 102, Train Loss: 0.0004787, Test Loss: 0.0005806
Epoch: 103, Train Loss: 0.0004702, Test Loss: 0.0005694
Epoch: 104, Train Loss: 0.0004616, Test Loss: 0.0005582
Epoch: 105, Train Loss: 0.0004533, Test Loss: 0.0005473
Epoch: 106, Train Loss: 0.0004450, Test Loss: 0.0005364
Epoch: 107, Train Loss: 0.0004370, Test Loss: 0.0005257
Epoch: 108, Train Loss: 0.0004291, Test Loss: 0.0005153
Epoch: 109, Train Loss: 0.0004213, Test Loss: 0.0005050
Epoch: 110, Train Loss: 0.0004138, Test Loss: 0.0004950
Epoch: 111, Train Loss: 0.0004064, Test Loss: 0.0004852
Epoch: 112, Train Loss: 0.0003991, Test Loss: 0.0004757
```

```
Epoch: 113, Train Loss: 0.0003921, Test Loss: 0.0004664
Epoch: 114, Train Loss: 0.0003851, Test Loss: 0.0004572
Epoch: 115, Train Loss: 0.0003784, Test Loss: 0.0004483
Epoch: 116, Train Loss: 0.0003718, Test Loss: 0.0004396
Epoch: 117, Train Loss: 0.0003653, Test Loss: 0.0004310
Epoch: 118, Train Loss: 0.0003589, Test Loss: 0.0004226
Epoch: 119, Train Loss: 0.0003527, Test Loss: 0.0004144
Epoch: 120, Train Loss: 0.0003466, Test Loss: 0.0004064
Epoch: 121, Train Loss: 0.0003406, Test Loss: 0.0003989
Epoch: 122, Train Loss: 0.0003347, Test Loss: 0.0003914
Epoch: 123, Train Loss: 0.0003289, Test Loss: 0.0003842
Epoch: 124, Train Loss: 0.0003233, Test Loss: 0.0003773
Epoch: 125, Train Loss: 0.0003177, Test Loss: 0.0003704
Epoch: 126, Train Loss: 0.0003123, Test Loss: 0.0003638
Epoch: 127, Train Loss: 0.0003070, Test Loss: 0.0003573
Epoch: 128, Train Loss: 0.0003018, Test Loss: 0.0003510
Epoch: 129, Train Loss: 0.0002967, Test Loss: 0.0003448
Epoch: 130, Train Loss: 0.0002918, Test Loss: 0.0003387
Epoch: 131, Train Loss: 0.0002869, Test Loss: 0.0003328
Epoch: 132, Train Loss: 0.0002821, Test Loss: 0.0003271
Epoch: 133, Train Loss: 0.0002774, Test Loss: 0.0003214
Epoch: 134, Train Loss: 0.0002728, Test Loss: 0.0003159
Epoch: 135, Train Loss: 0.0002683, Test Loss: 0.0003103
Epoch: 136, Train Loss: 0.0002639, Test Loss: 0.0003048
Epoch: 137, Train Loss: 0.0002596, Test Loss: 0.0002995
Epoch: 138, Train Loss: 0.0002554, Test Loss: 0.0002941
Epoch: 139, Train Loss: 0.0002512, Test Loss: 0.0002889
Epoch: 140, Train Loss: 0.0002472, Test Loss: 0.0002837
Epoch: 141, Train Loss: 0.0002432, Test Loss: 0.0002786
Epoch: 142, Train Loss: 0.0002393, Test Loss: 0.0002736
Epoch: 143, Train Loss: 0.0002354, Test Loss: 0.0002687
Epoch: 144, Train Loss: 0.0002317, Test Loss: 0.0002640
Epoch: 145, Train Loss: 0.0002280, Test Loss: 0.0002594
Epoch: 146, Train Loss: 0.0002244, Test Loss: 0.0002548
Epoch: 147, Train Loss: 0.0002209, Test Loss: 0.0002503
Epoch: 148, Train Loss: 0.0002174, Test Loss: 0.0002459
Epoch: 149, Train Loss: 0.0002139, Test Loss: 0.0002416
Epoch: 150, Train Loss: 0.0002105, Test Loss: 0.0002373
Epoch: 151, Train Loss: 0.0002072, Test Loss: 0.0002331
Epoch: 152, Train Loss: 0.0002040, Test Loss: 0.0002289
Epoch: 153, Train Loss: 0.0002008, Test Loss: 0.0002250
Epoch: 154, Train Loss: 0.0001977, Test Loss: 0.0002211
Epoch: 155, Train Loss: 0.0001946, Test Loss: 0.0002173
Epoch: 156, Train Loss: 0.0001916, Test Loss: 0.0002136
Epoch: 157, Train Loss: 0.0001887, Test Loss: 0.0002099
Epoch: 158, Train Loss: 0.0001858, Test Loss: 0.0002063
Epoch: 159, Train Loss: 0.0001829, Test Loss: 0.0002028
Epoch: 160, Train Loss: 0.0001801, Test Loss: 0.0001994
Epoch: 161, Train Loss: 0.0001774, Test Loss: 0.0001960
```

```
Epoch: 162, Train Loss: 0.0001747, Test Loss: 0.0001927
Epoch: 163, Train Loss: 0.0001720, Test Loss: 0.0001895
Epoch: 164, Train Loss: 0.0001694, Test Loss: 0.0001864
Epoch: 165, Train Loss: 0.0001668, Test Loss: 0.0001834
Epoch: 166, Train Loss: 0.0001643, Test Loss: 0.0001804
Epoch: 167, Train Loss: 0.0001618, Test Loss: 0.0001775
Epoch: 168, Train Loss: 0.0001594, Test Loss: 0.0001745
Epoch: 169, Train Loss: 0.0001570, Test Loss: 0.0001717
Epoch: 170, Train Loss: 0.0001547, Test Loss: 0.0001689
Epoch: 171, Train Loss: 0.0001524, Test Loss: 0.0001661
Epoch: 172, Train Loss: 0.0001501, Test Loss: 0.0001635
Epoch: 173, Train Loss: 0.0001479, Test Loss: 0.0001608
Epoch: 174, Train Loss: 0.0001457, Test Loss: 0.0001582
Epoch: 175, Train Loss: 0.0001436, Test Loss: 0.0001557
Epoch: 176, Train Loss: 0.0001414, Test Loss: 0.0001533
Epoch: 177, Train Loss: 0.0001394, Test Loss: 0.0001509
Epoch: 178, Train Loss: 0.0001373, Test Loss: 0.0001485
Epoch: 179, Train Loss: 0.0001353, Test Loss: 0.0001462
Epoch: 180, Train Loss: 0.0001333, Test Loss: 0.0001439
Epoch: 181, Train Loss: 0.0001314, Test Loss: 0.0001417
Epoch: 182, Train Loss: 0.0001295, Test Loss: 0.0001395
Epoch: 183, Train Loss: 0.0001276, Test Loss: 0.0001374
Epoch: 184, Train Loss: 0.0001257, Test Loss: 0.0001352
Epoch: 185, Train Loss: 0.0001239, Test Loss: 0.0001331
Epoch: 186, Train Loss: 0.0001221, Test Loss: 0.0001311
Epoch: 187, Train Loss: 0.0001204, Test Loss: 0.0001292
Epoch: 188, Train Loss: 0.0001186, Test Loss: 0.0001272
Epoch: 189, Train Loss: 0.0001169, Test Loss: 0.0001253
Epoch: 190, Train Loss: 0.0001153, Test Loss: 0.0001234
Epoch: 191, Train Loss: 0.0001136, Test Loss: 0.0001216
Epoch: 192, Train Loss: 0.0001120, Test Loss: 0.0001198
Epoch: 193, Train Loss: 0.0001104, Test Loss: 0.0001181
Epoch: 194, Train Loss: 0.0001089, Test Loss: 0.0001163
Epoch: 195, Train Loss: 0.0001073, Test Loss: 0.0001147
Epoch: 196, Train Loss: 0.0001058, Test Loss: 0.0001130
Epoch: 197, Train Loss: 0.0001043, Test Loss: 0.0001113
Epoch: 198, Train Loss: 0.0001029, Test Loss: 0.0001097
Epoch: 199, Train Loss: 0.0001014, Test Loss: 0.0001082
Epoch: 200, Train Loss: 0.0001000, Test Loss: 0.0001066
```

Problem 4.2 Determine appropriate mini-batch size = 32, epoch = 300

```python
# ################# Part 1: Load data and create batch
##################
N_total = 600
N_train = 500
x = torch.unsqueeze(torch.linspace(0, 1, N_total), dim=1)
r = torch.randperm(N_total)
x = x[r, :]
y = 0.2 + 0.4 * torch.pow(x, 2) + 0.3 * x * torch.sin(15 * x) + 0.05 *
torch.cos(50 * x)

class CustomDataset(Dataset):
    def __init__(self, x, y):
        self.y = y
        self.x = x

    def __len__(self):
        return len(self.y)

    def __getitem__(self, idx):
        y1 = self.y[idx]
        x1 = self.x[idx]
        return (x1, y1)

# Change batch_size here to test different values
```

```python
batch_size = 32  # Experiment with different batch sizes: 32, 64, 128
trainset = CustomDataset(x[0:N_train, :], y[0:N_train, :])
testset = CustomDataset(x[N_train:N_total, :], y[N_train:N_total, :])
train_loader = torch.utils.data.DataLoader(trainset,
batch_size=batch_size)
test_loader = torch.utils.data.DataLoader(testset,
batch_size=batch_size)

# ################# Part 2: Define Model #################
model = nn.Sequential(
    nn.Linear(1, 1024, bias=True),
    nn.ReLU(),
    nn.Linear(1024, 1, bias=True)
)

def init_weights(m):
    if isinstance(m, nn.Linear):
        m.weight.data.uniform_(-1, 1)
        m.bias.data.uniform_(-1, 1)

model.apply(init_weights)

# ################# Part 3: Define Loss and Optimizer
#################
criterion = nn.MSELoss()
optimizer = optim.Adam(model.parameters(), lr=0.001)

# ################# Part 4: Training and Testing #################
def train_NN():
    model.train()
    for images, labels in train_loader:
        out = model(images)
        loss = criterion(out, labels)
        loss.backward()
        optimizer.step()
        optimizer.zero_grad()
    return loss

def test_NN(loader):
    model.eval()
    loss = 0
    with torch.no_grad():
        for images, labels in loader:
            out = model(images)
            loss += criterion(out, labels).item()
    loss = loss / len(loader)
    return loss

# Experiment with different numbers of epochs
N_epoch = 300  # Try values like 100, 200, 500
```

```python
train_loss = np.zeros((N_epoch, 1))
test_loss = np.zeros((N_epoch, 1))

for epoch in range(N_epoch):
    train_NN()
    train_loss[epoch, 0] = test_NN(train_loader)
    test_loss[epoch, 0] = test_NN(test_loader)
    print(f'Epoch: {epoch+1:03d}, Train Loss: {train_loss[epoch,
0]:.7f}, Test Loss: {test_loss[epoch, 0]:.7f}')

# ################# Final Prediction #################
x_test = torch.unsqueeze(torch.linspace(0, 1, 1999), dim=1)
y_test = model(x_test)

# Plot the results
plt.plot(x[0:N_total], y[0:N_total], 'bo', label='True data')
plt.plot(x_test, y_test.detach().numpy(), 'r', label='Model
prediction')
plt.legend()
plt.show()
```

```
Epoch: 001, Train Loss: 1.6943940, Test Loss: 1.3576902
Epoch: 002, Train Loss: 0.2147134, Test Loss: 0.1538809
Epoch: 003, Train Loss: 0.2042176, Test Loss: 0.3091932
Epoch: 004, Train Loss: 0.0874204, Test Loss: 0.0568524
Epoch: 005, Train Loss: 0.0379086, Test Loss: 0.0642589
Epoch: 006, Train Loss: 0.0275121, Test Loss: 0.0280169
Epoch: 007, Train Loss: 0.0204654, Test Loss: 0.0293006
Epoch: 008, Train Loss: 0.0172203, Test Loss: 0.0208747
Epoch: 009, Train Loss: 0.0140973, Test Loss: 0.0179452
Epoch: 010, Train Loss: 0.0118216, Test Loss: 0.0144192
Epoch: 011, Train Loss: 0.0099079, Test Loss: 0.0119318
Epoch: 012, Train Loss: 0.0083451, Test Loss: 0.0099552
Epoch: 013, Train Loss: 0.0070816, Test Loss: 0.0083374
Epoch: 014, Train Loss: 0.0060404, Test Loss: 0.0070783
Epoch: 015, Train Loss: 0.0051920, Test Loss: 0.0060603
Epoch: 016, Train Loss: 0.0044988, Test Loss: 0.0052423
Epoch: 017, Train Loss: 0.0039305, Test Loss: 0.0045852
Epoch: 018, Train Loss: 0.0034660, Test Loss: 0.0040514
Epoch: 019, Train Loss: 0.0030858, Test Loss: 0.0036177
Epoch: 020, Train Loss: 0.0027742, Test Loss: 0.0032636
Epoch: 021, Train Loss: 0.0025181, Test Loss: 0.0029728
Epoch: 022, Train Loss: 0.0023065, Test Loss: 0.0027344
Epoch: 023, Train Loss: 0.0021306, Test Loss: 0.0025380
Epoch: 024, Train Loss: 0.0019837, Test Loss: 0.0023759
Epoch: 025, Train Loss: 0.0018604, Test Loss: 0.0022404
Epoch: 026, Train Loss: 0.0017561, Test Loss: 0.0021269
Epoch: 027, Train Loss: 0.0016667, Test Loss: 0.0020320
Epoch: 028, Train Loss: 0.0015893, Test Loss: 0.0019518
Epoch: 029, Train Loss: 0.0015223, Test Loss: 0.0018834
```

```
Epoch: 030, Train Loss: 0.0014633, Test Loss: 0.0018240
Epoch: 031, Train Loss: 0.0014109, Test Loss: 0.0017724
Epoch: 032, Train Loss: 0.0013638, Test Loss: 0.0017273
Epoch: 033, Train Loss: 0.0013214, Test Loss: 0.0016872
Epoch: 034, Train Loss: 0.0012829, Test Loss: 0.0016511
Epoch: 035, Train Loss: 0.0012472, Test Loss: 0.0016184
Epoch: 036, Train Loss: 0.0012142, Test Loss: 0.0015887
Epoch: 037, Train Loss: 0.0011835, Test Loss: 0.0015614
Epoch: 038, Train Loss: 0.0011547, Test Loss: 0.0015359
Epoch: 039, Train Loss: 0.0011278, Test Loss: 0.0015123
Epoch: 040, Train Loss: 0.0011025, Test Loss: 0.0014909
Epoch: 041, Train Loss: 0.0010786, Test Loss: 0.0014706
Epoch: 042, Train Loss: 0.0010560, Test Loss: 0.0014513
Epoch: 043, Train Loss: 0.0010346, Test Loss: 0.0014326
Epoch: 044, Train Loss: 0.0010143, Test Loss: 0.0014152
Epoch: 045, Train Loss: 0.0009953, Test Loss: 0.0013994
Epoch: 046, Train Loss: 0.0009774, Test Loss: 0.0013845
Epoch: 047, Train Loss: 0.0009608, Test Loss: 0.0013707
Epoch: 048, Train Loss: 0.0009458, Test Loss: 0.0013587
Epoch: 049, Train Loss: 0.0009324, Test Loss: 0.0013483
Epoch: 050, Train Loss: 0.0009206, Test Loss: 0.0013392
Epoch: 051, Train Loss: 0.0009103, Test Loss: 0.0013318
Epoch: 052, Train Loss: 0.0009018, Test Loss: 0.0013260
Epoch: 053, Train Loss: 0.0008949, Test Loss: 0.0013218
Epoch: 054, Train Loss: 0.0008893, Test Loss: 0.0013190
Epoch: 055, Train Loss: 0.0008846, Test Loss: 0.0013167
Epoch: 056, Train Loss: 0.0008807, Test Loss: 0.0013146
Epoch: 057, Train Loss: 0.0008774, Test Loss: 0.0013129
Epoch: 058, Train Loss: 0.0008744, Test Loss: 0.0013115
Epoch: 059, Train Loss: 0.0008713, Test Loss: 0.0013094
Epoch: 060, Train Loss: 0.0008677, Test Loss: 0.0013065
Epoch: 061, Train Loss: 0.0008635, Test Loss: 0.0013021
Epoch: 062, Train Loss: 0.0008585, Test Loss: 0.0012968
Epoch: 063, Train Loss: 0.0008522, Test Loss: 0.0012894
Epoch: 064, Train Loss: 0.0008445, Test Loss: 0.0012801
Epoch: 065, Train Loss: 0.0008352, Test Loss: 0.0012685
Epoch: 066, Train Loss: 0.0008244, Test Loss: 0.0012546
Epoch: 067, Train Loss: 0.0008122, Test Loss: 0.0012392
Epoch: 068, Train Loss: 0.0007990, Test Loss: 0.0012226
Epoch: 069, Train Loss: 0.0007850, Test Loss: 0.0012049
Epoch: 070, Train Loss: 0.0007704, Test Loss: 0.0011862
Epoch: 071, Train Loss: 0.0007554, Test Loss: 0.0011672
Epoch: 072, Train Loss: 0.0007402, Test Loss: 0.0011476
Epoch: 073, Train Loss: 0.0007247, Test Loss: 0.0011273
Epoch: 074, Train Loss: 0.0007091, Test Loss: 0.0011066
Epoch: 075, Train Loss: 0.0006935, Test Loss: 0.0010863
Epoch: 076, Train Loss: 0.0006782, Test Loss: 0.0010661
Epoch: 077, Train Loss: 0.0006631, Test Loss: 0.0010459
Epoch: 078, Train Loss: 0.0006481, Test Loss: 0.0010261
```
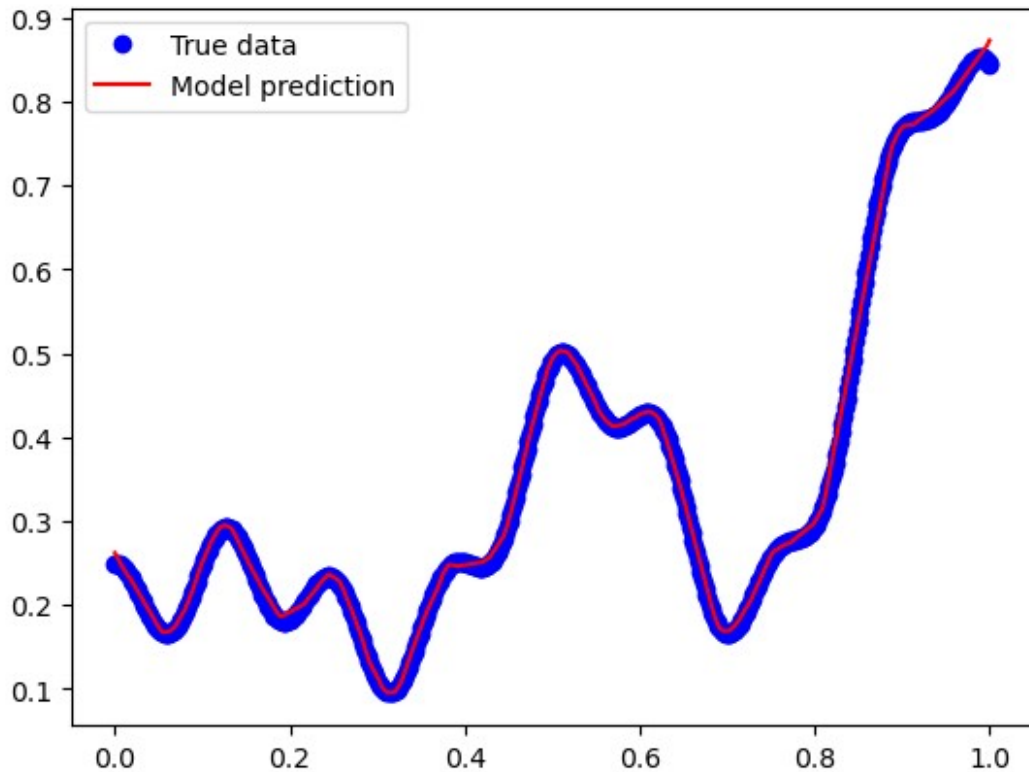
```
Epoch: 079, Train Loss: 0.0006338, Test Loss: 0.0010068
Epoch: 080, Train Loss: 0.0006197, Test Loss: 0.0009879
Epoch: 081, Train Loss: 0.0006058, Test Loss: 0.0009691
Epoch: 082, Train Loss: 0.0005923, Test Loss: 0.0009509
Epoch: 083, Train Loss: 0.0005792, Test Loss: 0.0009331
Epoch: 084, Train Loss: 0.0005665, Test Loss: 0.0009158
Epoch: 085, Train Loss: 0.0005542, Test Loss: 0.0008988
Epoch: 086, Train Loss: 0.0005423, Test Loss: 0.0008825
Epoch: 087, Train Loss: 0.0005309, Test Loss: 0.0008666
Epoch: 088, Train Loss: 0.0005196, Test Loss: 0.0008507
Epoch: 089, Train Loss: 0.0005087, Test Loss: 0.0008352
Epoch: 090, Train Loss: 0.0004978, Test Loss: 0.0008199
Epoch: 091, Train Loss: 0.0004873, Test Loss: 0.0008051
Epoch: 092, Train Loss: 0.0004773, Test Loss: 0.0007911
Epoch: 093, Train Loss: 0.0004676, Test Loss: 0.0007772
Epoch: 094, Train Loss: 0.0004582, Test Loss: 0.0007639
Epoch: 095, Train Loss: 0.0004490, Test Loss: 0.0007508
Epoch: 096, Train Loss: 0.0004400, Test Loss: 0.0007378
Epoch: 097, Train Loss: 0.0004314, Test Loss: 0.0007254
Epoch: 098, Train Loss: 0.0004229, Test Loss: 0.0007132
Epoch: 099, Train Loss: 0.0004147, Test Loss: 0.0007013
Epoch: 100, Train Loss: 0.0004065, Test Loss: 0.0006894
Epoch: 101, Train Loss: 0.0003986, Test Loss: 0.0006778
Epoch: 102, Train Loss: 0.0003910, Test Loss: 0.0006663
Epoch: 103, Train Loss: 0.0003836, Test Loss: 0.0006553
Epoch: 104, Train Loss: 0.0003764, Test Loss: 0.0006444
Epoch: 105, Train Loss: 0.0003693, Test Loss: 0.0006336
Epoch: 106, Train Loss: 0.0003625, Test Loss: 0.0006231
Epoch: 107, Train Loss: 0.0003558, Test Loss: 0.0006129
Epoch: 108, Train Loss: 0.0003493, Test Loss: 0.0006030
Epoch: 109, Train Loss: 0.0003429, Test Loss: 0.0005935
Epoch: 110, Train Loss: 0.0003367, Test Loss: 0.0005842
Epoch: 111, Train Loss: 0.0003306, Test Loss: 0.0005750
Epoch: 112, Train Loss: 0.0003247, Test Loss: 0.0005660
Epoch: 113, Train Loss: 0.0003188, Test Loss: 0.0005571
Epoch: 114, Train Loss: 0.0003130, Test Loss: 0.0005484
Epoch: 115, Train Loss: 0.0003073, Test Loss: 0.0005398
Epoch: 116, Train Loss: 0.0003018, Test Loss: 0.0005312
Epoch: 117, Train Loss: 0.0002964, Test Loss: 0.0005229
Epoch: 118, Train Loss: 0.0002911, Test Loss: 0.0005147
Epoch: 119, Train Loss: 0.0002859, Test Loss: 0.0005066
Epoch: 120, Train Loss: 0.0002809, Test Loss: 0.0004986
Epoch: 121, Train Loss: 0.0002760, Test Loss: 0.0004908
Epoch: 122, Train Loss: 0.0002711, Test Loss: 0.0004830
Epoch: 123, Train Loss: 0.0002664, Test Loss: 0.0004754
Epoch: 124, Train Loss: 0.0002617, Test Loss: 0.0004678
Epoch: 125, Train Loss: 0.0002572, Test Loss: 0.0004604
Epoch: 126, Train Loss: 0.0002527, Test Loss: 0.0004534
Epoch: 127, Train Loss: 0.0002484, Test Loss: 0.0004465
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Epoch: 128, Train Loss: 0.0002441, Test Loss: 0.0004396
Epoch: 129, Train Loss: 0.0002399, Test Loss: 0.0004330
Epoch: 130, Train Loss: 0.0002358, Test Loss: 0.0004263
Epoch: 131, Train Loss: 0.0002317, Test Loss: 0.0004199
Epoch: 132, Train Loss: 0.0002278, Test Loss: 0.0004134
Epoch: 133, Train Loss: 0.0002239, Test Loss: 0.0004071
Epoch: 134, Train Loss: 0.0002201, Test Loss: 0.0004008
Epoch: 135, Train Loss: 0.0002164, Test Loss: 0.0003946
Epoch: 136, Train Loss: 0.0002127, Test Loss: 0.0003885
Epoch: 137, Train Loss: 0.0002092, Test Loss: 0.0003825
Epoch: 138, Train Loss: 0.0002057, Test Loss: 0.0003766
Epoch: 139, Train Loss: 0.0002022, Test Loss: 0.0003709
Epoch: 140, Train Loss: 0.0001989, Test Loss: 0.0003652
Epoch: 141, Train Loss: 0.0001956, Test Loss: 0.0003594
Epoch: 142, Train Loss: 0.0001924, Test Loss: 0.0003538
Epoch: 143, Train Loss: 0.0001892, Test Loss: 0.0003482
Epoch: 144, Train Loss: 0.0001862, Test Loss: 0.0003427
Epoch: 145, Train Loss: 0.0001832, Test Loss: 0.0003373
Epoch: 146, Train Loss: 0.0001802, Test Loss: 0.0003319
Epoch: 147, Train Loss: 0.0001773, Test Loss: 0.0003267
Epoch: 148, Train Loss: 0.0001745, Test Loss: 0.0003217
Epoch: 149, Train Loss: 0.0001717, Test Loss: 0.0003167
Epoch: 150, Train Loss: 0.0001690, Test Loss: 0.0003118
Epoch: 151, Train Loss: 0.0001664, Test Loss: 0.0003070
Epoch: 152, Train Loss: 0.0001638, Test Loss: 0.0003023
Epoch: 153, Train Loss: 0.0001613, Test Loss: 0.0002977
Epoch: 154, Train Loss: 0.0001589, Test Loss: 0.0002932
Epoch: 155, Train Loss: 0.0001565, Test Loss: 0.0002888
Epoch: 156, Train Loss: 0.0001542, Test Loss: 0.0002843
Epoch: 157, Train Loss: 0.0001520, Test Loss: 0.0002800
Epoch: 158, Train Loss: 0.0001499, Test Loss: 0.0002758
Epoch: 159, Train Loss: 0.0001479, Test Loss: 0.0002718
Epoch: 160, Train Loss: 0.0001459, Test Loss: 0.0002678
Epoch: 161, Train Loss: 0.0001440, Test Loss: 0.0002637
Epoch: 162, Train Loss: 0.0001422, Test Loss: 0.0002597
Epoch: 163, Train Loss: 0.0001405, Test Loss: 0.0002557
Epoch: 164, Train Loss: 0.0001389, Test Loss: 0.0002519
Epoch: 165, Train Loss: 0.0001374, Test Loss: 0.0002482
Epoch: 166, Train Loss: 0.0001360, Test Loss: 0.0002446
Epoch: 167, Train Loss: 0.0001347, Test Loss: 0.0002412
Epoch: 168, Train Loss: 0.0001335, Test Loss: 0.0002379
Epoch: 169, Train Loss: 0.0001325, Test Loss: 0.0002348
Epoch: 170, Train Loss: 0.0001316, Test Loss: 0.0002318
Epoch: 171, Train Loss: 0.0001309, Test Loss: 0.0002290
Epoch: 172, Train Loss: 0.0001303, Test Loss: 0.0002263
Epoch: 173, Train Loss: 0.0001301, Test Loss: 0.0002240
Epoch: 174, Train Loss: 0.0001300, Test Loss: 0.0002218
Epoch: 175, Train Loss: 0.0001301, Test Loss: 0.0002199
Epoch: 176, Train Loss: 0.0001305, Test Loss: 0.0002183
```

```
Epoch: 177, Train Loss: 0.0001312, Test Loss: 0.0002170
Epoch: 178, Train Loss: 0.0001322, Test Loss: 0.0002161
Epoch: 179, Train Loss: 0.0001336, Test Loss: 0.0002155
Epoch: 180, Train Loss: 0.0001353, Test Loss: 0.0002153
Epoch: 181, Train Loss: 0.0001374, Test Loss: 0.0002155
Epoch: 182, Train Loss: 0.0001399, Test Loss: 0.0002159
Epoch: 183, Train Loss: 0.0001427, Test Loss: 0.0002168
Epoch: 184, Train Loss: 0.0001459, Test Loss: 0.0002180
Epoch: 185, Train Loss: 0.0001494, Test Loss: 0.0002198
Epoch: 186, Train Loss: 0.0001531, Test Loss: 0.0002216
Epoch: 187, Train Loss: 0.0001568, Test Loss: 0.0002234
Epoch: 188, Train Loss: 0.0001605, Test Loss: 0.0002250
Epoch: 189, Train Loss: 0.0001639, Test Loss: 0.0002264
Epoch: 190, Train Loss: 0.0001669, Test Loss: 0.0002274
Epoch: 191, Train Loss: 0.0001691, Test Loss: 0.0002277
Epoch: 192, Train Loss: 0.0001706, Test Loss: 0.0002274
Epoch: 193, Train Loss: 0.0001710, Test Loss: 0.0002261
Epoch: 194, Train Loss: 0.0001702, Test Loss: 0.0002237
Epoch: 195, Train Loss: 0.0001683, Test Loss: 0.0002205
Epoch: 196, Train Loss: 0.0001651, Test Loss: 0.0002160
Epoch: 197, Train Loss: 0.0001609, Test Loss: 0.0002108
Epoch: 198, Train Loss: 0.0001558, Test Loss: 0.0002048
Epoch: 199, Train Loss: 0.0001500, Test Loss: 0.0001982
Epoch: 200, Train Loss: 0.0001438, Test Loss: 0.0001914
Epoch: 201, Train Loss: 0.0001375, Test Loss: 0.0001844
Epoch: 202, Train Loss: 0.0001311, Test Loss: 0.0001775
Epoch: 203, Train Loss: 0.0001252, Test Loss: 0.0001708
Epoch: 204, Train Loss: 0.0001196, Test Loss: 0.0001646
Epoch: 205, Train Loss: 0.0001144, Test Loss: 0.0001588
Epoch: 206, Train Loss: 0.0001096, Test Loss: 0.0001533
Epoch: 207, Train Loss: 0.0001052, Test Loss: 0.0001482
Epoch: 208, Train Loss: 0.0001011, Test Loss: 0.0001436
Epoch: 209, Train Loss: 0.0000975, Test Loss: 0.0001392
Epoch: 210, Train Loss: 0.0000941, Test Loss: 0.0001351
Epoch: 211, Train Loss: 0.0000910, Test Loss: 0.0001312
Epoch: 212, Train Loss: 0.0000882, Test Loss: 0.0001276
Epoch: 213, Train Loss: 0.0000855, Test Loss: 0.0001242
Epoch: 214, Train Loss: 0.0000831, Test Loss: 0.0001211
Epoch: 215, Train Loss: 0.0000809, Test Loss: 0.0001182
Epoch: 216, Train Loss: 0.0000790, Test Loss: 0.0001156
Epoch: 217, Train Loss: 0.0000771, Test Loss: 0.0001128
Epoch: 218, Train Loss: 0.0000753, Test Loss: 0.0001105
Epoch: 219, Train Loss: 0.0000738, Test Loss: 0.0001083
Epoch: 220, Train Loss: 0.0000722, Test Loss: 0.0001059
Epoch: 221, Train Loss: 0.0000709, Test Loss: 0.0001040
Epoch: 222, Train Loss: 0.0000696, Test Loss: 0.0001021
Epoch: 223, Train Loss: 0.0000683, Test Loss: 0.0001001
Epoch: 224, Train Loss: 0.0000671, Test Loss: 0.0000982
Epoch: 225, Train Loss: 0.0000661, Test Loss: 0.0000966
```

```
Epoch: 226, Train Loss: 0.0000651, Test Loss: 0.0000951
Epoch: 227, Train Loss: 0.0000641, Test Loss: 0.0000934
Epoch: 228, Train Loss: 0.0000631, Test Loss: 0.0000918
Epoch: 229, Train Loss: 0.0000622, Test Loss: 0.0000903
Epoch: 230, Train Loss: 0.0000614, Test Loss: 0.0000889
Epoch: 231, Train Loss: 0.0000605, Test Loss: 0.0000875
Epoch: 232, Train Loss: 0.0000597, Test Loss: 0.0000862
Epoch: 233, Train Loss: 0.0000590, Test Loss: 0.0000849
Epoch: 234, Train Loss: 0.0000582, Test Loss: 0.0000837
Epoch: 235, Train Loss: 0.0000575, Test Loss: 0.0000825
Epoch: 236, Train Loss: 0.0000568, Test Loss: 0.0000813
Epoch: 237, Train Loss: 0.0000561, Test Loss: 0.0000802
Epoch: 238, Train Loss: 0.0000555, Test Loss: 0.0000791
Epoch: 239, Train Loss: 0.0000548, Test Loss: 0.0000780
Epoch: 240, Train Loss: 0.0000542, Test Loss: 0.0000770
Epoch: 241, Train Loss: 0.0000536, Test Loss: 0.0000760
Epoch: 242, Train Loss: 0.0000530, Test Loss: 0.0000750
Epoch: 243, Train Loss: 0.0000525, Test Loss: 0.0000739
Epoch: 244, Train Loss: 0.0000519, Test Loss: 0.0000730
Epoch: 245, Train Loss: 0.0000514, Test Loss: 0.0000720
Epoch: 246, Train Loss: 0.0000509, Test Loss: 0.0000710
Epoch: 247, Train Loss: 0.0000503, Test Loss: 0.0000701
Epoch: 248, Train Loss: 0.0000498, Test Loss: 0.0000692
Epoch: 249, Train Loss: 0.0000493, Test Loss: 0.0000683
Epoch: 250, Train Loss: 0.0000488, Test Loss: 0.0000674
Epoch: 251, Train Loss: 0.0000483, Test Loss: 0.0000666
Epoch: 252, Train Loss: 0.0000479, Test Loss: 0.0000657
Epoch: 253, Train Loss: 0.0000474, Test Loss: 0.0000649
Epoch: 254, Train Loss: 0.0000470, Test Loss: 0.0000641
Epoch: 255, Train Loss: 0.0000465, Test Loss: 0.0000633
Epoch: 256, Train Loss: 0.0000461, Test Loss: 0.0000625
Epoch: 257, Train Loss: 0.0000457, Test Loss: 0.0000617
Epoch: 258, Train Loss: 0.0000453, Test Loss: 0.0000610
Epoch: 259, Train Loss: 0.0000449, Test Loss: 0.0000603
Epoch: 260, Train Loss: 0.0000445, Test Loss: 0.0000596
Epoch: 261, Train Loss: 0.0000441, Test Loss: 0.0000589
Epoch: 262, Train Loss: 0.0000437, Test Loss: 0.0000582
Epoch: 263, Train Loss: 0.0000433, Test Loss: 0.0000575
Epoch: 264, Train Loss: 0.0000430, Test Loss: 0.0000568
Epoch: 265, Train Loss: 0.0000426, Test Loss: 0.0000562
Epoch: 266, Train Loss: 0.0000423, Test Loss: 0.0000556
Epoch: 267, Train Loss: 0.0000420, Test Loss: 0.0000550
Epoch: 268, Train Loss: 0.0000417, Test Loss: 0.0000544
Epoch: 269, Train Loss: 0.0000414, Test Loss: 0.0000539
Epoch: 270, Train Loss: 0.0000411, Test Loss: 0.0000533
Epoch: 271, Train Loss: 0.0000408, Test Loss: 0.0000527
Epoch: 272, Train Loss: 0.0000405, Test Loss: 0.0000522
Epoch: 273, Train Loss: 0.0000403, Test Loss: 0.0000517
Epoch: 274, Train Loss: 0.0000400, Test Loss: 0.0000511
```

```
Epoch: 275, Train Loss: 0.0000398, Test Loss: 0.0000506
Epoch: 276, Train Loss: 0.0000395, Test Loss: 0.0000501
Epoch: 277, Train Loss: 0.0000392, Test Loss: 0.0000496
Epoch: 278, Train Loss: 0.0000389, Test Loss: 0.0000491
Epoch: 279, Train Loss: 0.0000386, Test Loss: 0.0000486
Epoch: 280, Train Loss: 0.0000384, Test Loss: 0.0000481
Epoch: 281, Train Loss: 0.0000381, Test Loss: 0.0000476
Epoch: 282, Train Loss: 0.0000378, Test Loss: 0.0000471
Epoch: 283, Train Loss: 0.0000374, Test Loss: 0.0000466
Epoch: 284, Train Loss: 0.0000370, Test Loss: 0.0000461
Epoch: 285, Train Loss: 0.0000366, Test Loss: 0.0000455
Epoch: 286, Train Loss: 0.0000362, Test Loss: 0.0000450
Epoch: 287, Train Loss: 0.0000358, Test Loss: 0.0000445
Epoch: 288, Train Loss: 0.0000353, Test Loss: 0.0000439
Epoch: 289, Train Loss: 0.0000349, Test Loss: 0.0000434
Epoch: 290, Train Loss: 0.0000344, Test Loss: 0.0000429
Epoch: 291, Train Loss: 0.0000341, Test Loss: 0.0000424
Epoch: 292, Train Loss: 0.0000337, Test Loss: 0.0000419
Epoch: 293, Train Loss: 0.0000333, Test Loss: 0.0000415
Epoch: 294, Train Loss: 0.0000330, Test Loss: 0.0000411
Epoch: 295, Train Loss: 0.0000327, Test Loss: 0.0000407
Epoch: 296, Train Loss: 0.0000325, Test Loss: 0.0000403
Epoch: 297, Train Loss: 0.0000323, Test Loss: 0.0000400
Epoch: 298, Train Loss: 0.0000321, Test Loss: 0.0000396
Epoch: 299, Train Loss: 0.0000319, Test Loss: 0.0000394
Epoch: 300, Train Loss: 0.0000317, Test Loss: 0.0000390
```

Problem 4.2 Determine appropriate mini-batch size = 32, epoch = 400

```python
# ################# Part 1: Load data and create batch
##################
N_total = 600
N_train = 500
x = torch.unsqueeze(torch.linspace(0, 1, N_total), dim=1)
r = torch.randperm(N_total)
x = x[r, :]
y = 0.2 + 0.4 * torch.pow(x, 2) + 0.3 * x * torch.sin(15 * x) + 0.05 *
torch.cos(50 * x)

class CustomDataset(Dataset):
    def __init__(self, x, y):
        self.y = y
        self.x = x

    def __len__(self):
        return len(self.y)

    def __getitem__(self, idx):
        y1 = self.y[idx]
        x1 = self.x[idx]
        return (x1, y1)

# Change batch_size here to test different values
```

```python
batch_size = 32  # Experiment with different batch sizes: 32, 64, 128
trainset = CustomDataset(x[0:N_train, :], y[0:N_train, :])
testset = CustomDataset(x[N_train:N_total, :], y[N_train:N_total, :])
train_loader = torch.utils.data.DataLoader(trainset,
batch_size=batch_size)
test_loader = torch.utils.data.DataLoader(testset,
batch_size=batch_size)

# ################# Part 2: Define Model #################
model = nn.Sequential(
    nn.Linear(1, 1024, bias=True),
    nn.ReLU(),
    nn.Linear(1024, 1, bias=True)
)

def init_weights(m):
    if isinstance(m, nn.Linear):
        m.weight.data.uniform_(-1, 1)
        m.bias.data.uniform_(-1, 1)

model.apply(init_weights)

# ################# Part 3: Define Loss and Optimizer
#################
criterion = nn.MSELoss()
optimizer = optim.Adam(model.parameters(), lr=0.001)

# ################# Part 4: Training and Testing #################
def train_NN():
    model.train()
    for images, labels in train_loader:
        out = model(images)
        loss = criterion(out, labels)
        loss.backward()
        optimizer.step()
        optimizer.zero_grad()
    return loss

def test_NN(loader):
    model.eval()
    loss = 0
    with torch.no_grad():
        for images, labels in loader:
            out = model(images)
            loss += criterion(out, labels).item()
    loss = loss / len(loader)
    return loss

# Experiment with different numbers of epochs
N_epoch = 400  # Try values like 100, 200, 500
```

```python
train_loss = np.zeros((N_epoch, 1))
test_loss = np.zeros((N_epoch, 1))

for epoch in range(N_epoch):
    train_NN()
    train_loss[epoch, 0] = test_NN(train_loader)
    test_loss[epoch, 0] = test_NN(test_loader)
    print(f'Epoch: {epoch+1:03d}, Train Loss: {train_loss[epoch,
0]:.7f}, Test Loss: {test_loss[epoch, 0]:.7f}')

# ################# Final Prediction #################
x_test = torch.unsqueeze(torch.linspace(0, 1, 1999), dim=1)
y_test = model(x_test)

# Plot the results
plt.plot(x[0:N_total], y[0:N_total], 'bo', label='True data')
plt.plot(x_test, y_test.detach().numpy(), 'r', label='Model
prediction')
plt.legend()
plt.show()
```

```
Epoch: 001, Train Loss: 56.2691040, Test Loss: 54.2276945
Epoch: 002, Train Loss: 0.4105022, Test Loss: 0.3816116
Epoch: 003, Train Loss: 2.4412567, Test Loss: 1.9652464
Epoch: 004, Train Loss: 0.3370843, Test Loss: 0.2396506
Epoch: 005, Train Loss: 0.2370436, Test Loss: 0.2311195
Epoch: 006, Train Loss: 0.1464007, Test Loss: 0.1341241
Epoch: 007, Train Loss: 0.1239662, Test Loss: 0.1118252
Epoch: 008, Train Loss: 0.1073626, Test Loss: 0.1034263
Epoch: 009, Train Loss: 0.0966345, Test Loss: 0.0970294
Epoch: 010, Train Loss: 0.0872553, Test Loss: 0.0897591
Epoch: 011, Train Loss: 0.0792265, Test Loss: 0.0830397
Epoch: 012, Train Loss: 0.0721541, Test Loss: 0.0765846
Epoch: 013, Train Loss: 0.0658721, Test Loss: 0.0703609
Epoch: 014, Train Loss: 0.0602676, Test Loss: 0.0646378
Epoch: 015, Train Loss: 0.0552482, Test Loss: 0.0592907
Epoch: 016, Train Loss: 0.0507279, Test Loss: 0.0542920
Epoch: 017, Train Loss: 0.0466342, Test Loss: 0.0497256
Epoch: 018, Train Loss: 0.0429211, Test Loss: 0.0455847
Epoch: 019, Train Loss: 0.0395484, Test Loss: 0.0418451
Epoch: 020, Train Loss: 0.0364827, Test Loss: 0.0384666
Epoch: 021, Train Loss: 0.0336970, Test Loss: 0.0354084
Epoch: 022, Train Loss: 0.0311635, Test Loss: 0.0326435
Epoch: 023, Train Loss: 0.0288581, Test Loss: 0.0301402
Epoch: 024, Train Loss: 0.0267556, Test Loss: 0.0278737
Epoch: 025, Train Loss: 0.0248367, Test Loss: 0.0258231
Epoch: 026, Train Loss: 0.0230815, Test Loss: 0.0239607
Epoch: 027, Train Loss: 0.0214760, Test Loss: 0.0222691
Epoch: 028, Train Loss: 0.0200044, Test Loss: 0.0207338
Epoch: 029, Train Loss: 0.0186555, Test Loss: 0.0193401
```

```
Epoch: 030, Train Loss: 0.0174163, Test Loss: 0.0180688
Epoch: 031, Train Loss: 0.0162783, Test Loss: 0.0169079
Epoch: 032, Train Loss: 0.0152326, Test Loss: 0.0158442
Epoch: 033, Train Loss: 0.0142707, Test Loss: 0.0148664
Epoch: 034, Train Loss: 0.0133845, Test Loss: 0.0139693
Epoch: 035, Train Loss: 0.0125673, Test Loss: 0.0131427
Epoch: 036, Train Loss: 0.0118132, Test Loss: 0.0123794
Epoch: 037, Train Loss: 0.0111169, Test Loss: 0.0116740
Epoch: 038, Train Loss: 0.0104719, Test Loss: 0.0110181
Epoch: 039, Train Loss: 0.0098745, Test Loss: 0.0104100
Epoch: 040, Train Loss: 0.0093202, Test Loss: 0.0098442
Epoch: 041, Train Loss: 0.0088056, Test Loss: 0.0093153
Epoch: 042, Train Loss: 0.0083272, Test Loss: 0.0088192
Epoch: 043, Train Loss: 0.0078820, Test Loss: 0.0083534
Epoch: 044, Train Loss: 0.0074675, Test Loss: 0.0079162
Epoch: 045, Train Loss: 0.0070800, Test Loss: 0.0075051
Epoch: 046, Train Loss: 0.0067175, Test Loss: 0.0071191
Epoch: 047, Train Loss: 0.0063783, Test Loss: 0.0067554
Epoch: 048, Train Loss: 0.0060614, Test Loss: 0.0064129
Epoch: 049, Train Loss: 0.0057648, Test Loss: 0.0060897
Epoch: 050, Train Loss: 0.0054865, Test Loss: 0.0057849
Epoch: 051, Train Loss: 0.0052253, Test Loss: 0.0054977
Epoch: 052, Train Loss: 0.0049806, Test Loss: 0.0052282
Epoch: 053, Train Loss: 0.0047508, Test Loss: 0.0049751
Epoch: 054, Train Loss: 0.0045350, Test Loss: 0.0047357
Epoch: 055, Train Loss: 0.0043327, Test Loss: 0.0045102
Epoch: 056, Train Loss: 0.0041436, Test Loss: 0.0042988
Epoch: 057, Train Loss: 0.0039664, Test Loss: 0.0040999
Epoch: 058, Train Loss: 0.0037994, Test Loss: 0.0039132
Epoch: 059, Train Loss: 0.0036429, Test Loss: 0.0037387
Epoch: 060, Train Loss: 0.0034962, Test Loss: 0.0035750
Epoch: 061, Train Loss: 0.0033584, Test Loss: 0.0034210
Epoch: 062, Train Loss: 0.0032289, Test Loss: 0.0032763
Epoch: 063, Train Loss: 0.0031068, Test Loss: 0.0031411
Epoch: 064, Train Loss: 0.0029911, Test Loss: 0.0030154
Epoch: 065, Train Loss: 0.0028821, Test Loss: 0.0028971
Epoch: 066, Train Loss: 0.0027796, Test Loss: 0.0027863
Epoch: 067, Train Loss: 0.0026830, Test Loss: 0.0026817
Epoch: 068, Train Loss: 0.0025916, Test Loss: 0.0025827
Epoch: 069, Train Loss: 0.0025055, Test Loss: 0.0024902
Epoch: 070, Train Loss: 0.0024243, Test Loss: 0.0023950
Epoch: 071, Train Loss: 0.0023482, Test Loss: 0.0023057
Epoch: 072, Train Loss: 0.0022759, Test Loss: 0.0022201
Epoch: 073, Train Loss: 0.0022070, Test Loss: 0.0021394
Epoch: 074, Train Loss: 0.0021416, Test Loss: 0.0020635
Epoch: 075, Train Loss: 0.0020795, Test Loss: 0.0019912
Epoch: 076, Train Loss: 0.0020203, Test Loss: 0.0019224
Epoch: 077, Train Loss: 0.0019641, Test Loss: 0.0018569
Epoch: 078, Train Loss: 0.0019101, Test Loss: 0.0017936
```

```
Epoch: 079, Train Loss: 0.0018579, Test Loss: 0.0017323
Epoch: 080, Train Loss: 0.0018076, Test Loss: 0.0016742
Epoch: 081, Train Loss: 0.0017590, Test Loss: 0.0016183
Epoch: 082, Train Loss: 0.0017117, Test Loss: 0.0015642
Epoch: 083, Train Loss: 0.0016660, Test Loss: 0.0015128
Epoch: 084, Train Loss: 0.0016223, Test Loss: 0.0014626
Epoch: 085, Train Loss: 0.0015804, Test Loss: 0.0014136
Epoch: 086, Train Loss: 0.0015402, Test Loss: 0.0013675
Epoch: 087, Train Loss: 0.0015007, Test Loss: 0.0013222
Epoch: 088, Train Loss: 0.0014620, Test Loss: 0.0012785
Epoch: 089, Train Loss: 0.0014246, Test Loss: 0.0012374
Epoch: 090, Train Loss: 0.0013883, Test Loss: 0.0011981
Epoch: 091, Train Loss: 0.0013533, Test Loss: 0.0011611
Epoch: 092, Train Loss: 0.0013196, Test Loss: 0.0011257
Epoch: 093, Train Loss: 0.0012875, Test Loss: 0.0010922
Epoch: 094, Train Loss: 0.0012568, Test Loss: 0.0010610
Epoch: 095, Train Loss: 0.0012277, Test Loss: 0.0010316
Epoch: 096, Train Loss: 0.0011999, Test Loss: 0.0010039
Epoch: 097, Train Loss: 0.0011737, Test Loss: 0.0009786
Epoch: 098, Train Loss: 0.0011487, Test Loss: 0.0009550
Epoch: 099, Train Loss: 0.0011251, Test Loss: 0.0009329
Epoch: 100, Train Loss: 0.0011027, Test Loss: 0.0009119
Epoch: 101, Train Loss: 0.0010814, Test Loss: 0.0008923
Epoch: 102, Train Loss: 0.0010612, Test Loss: 0.0008737
Epoch: 103, Train Loss: 0.0010418, Test Loss: 0.0008565
Epoch: 104, Train Loss: 0.0010234, Test Loss: 0.0008400
Epoch: 105, Train Loss: 0.0010060, Test Loss: 0.0008246
Epoch: 106, Train Loss: 0.0009893, Test Loss: 0.0008098
Epoch: 107, Train Loss: 0.0009735, Test Loss: 0.0007962
Epoch: 108, Train Loss: 0.0009584, Test Loss: 0.0007833
Epoch: 109, Train Loss: 0.0009439, Test Loss: 0.0007713
Epoch: 110, Train Loss: 0.0009298, Test Loss: 0.0007596
Epoch: 111, Train Loss: 0.0009162, Test Loss: 0.0007490
Epoch: 112, Train Loss: 0.0009030, Test Loss: 0.0007388
Epoch: 113, Train Loss: 0.0008902, Test Loss: 0.0007289
Epoch: 114, Train Loss: 0.0008779, Test Loss: 0.0007199
Epoch: 115, Train Loss: 0.0008660, Test Loss: 0.0007112
Epoch: 116, Train Loss: 0.0008544, Test Loss: 0.0007030
Epoch: 117, Train Loss: 0.0008431, Test Loss: 0.0006953
Epoch: 118, Train Loss: 0.0008321, Test Loss: 0.0006876
Epoch: 119, Train Loss: 0.0008214, Test Loss: 0.0006806
Epoch: 120, Train Loss: 0.0008110, Test Loss: 0.0006737
Epoch: 121, Train Loss: 0.0008009, Test Loss: 0.0006670
Epoch: 122, Train Loss: 0.0007910, Test Loss: 0.0006606
Epoch: 123, Train Loss: 0.0007815, Test Loss: 0.0006546
Epoch: 124, Train Loss: 0.0007721, Test Loss: 0.0006486
Epoch: 125, Train Loss: 0.0007629, Test Loss: 0.0006428
Epoch: 126, Train Loss: 0.0007540, Test Loss: 0.0006371
Epoch: 127, Train Loss: 0.0007452, Test Loss: 0.0006314
```

```
Epoch: 128, Train Loss: 0.0007366, Test Loss: 0.0006260
Epoch: 129, Train Loss: 0.0007282, Test Loss: 0.0006208
Epoch: 130, Train Loss: 0.0007201, Test Loss: 0.0006156
Epoch: 131, Train Loss: 0.0007121, Test Loss: 0.0006107
Epoch: 132, Train Loss: 0.0007041, Test Loss: 0.0006058
Epoch: 133, Train Loss: 0.0006964, Test Loss: 0.0006010
Epoch: 134, Train Loss: 0.0006888, Test Loss: 0.0005964
Epoch: 135, Train Loss: 0.0006813, Test Loss: 0.0005919
Epoch: 136, Train Loss: 0.0006739, Test Loss: 0.0005873
Epoch: 137, Train Loss: 0.0006666, Test Loss: 0.0005827
Epoch: 138, Train Loss: 0.0006594, Test Loss: 0.0005782
Epoch: 139, Train Loss: 0.0006524, Test Loss: 0.0005737
Epoch: 140, Train Loss: 0.0006454, Test Loss: 0.0005692
Epoch: 141, Train Loss: 0.0006385, Test Loss: 0.0005647
Epoch: 142, Train Loss: 0.0006317, Test Loss: 0.0005604
Epoch: 143, Train Loss: 0.0006250, Test Loss: 0.0005561
Epoch: 144, Train Loss: 0.0006183, Test Loss: 0.0005519
Epoch: 145, Train Loss: 0.0006118, Test Loss: 0.0005476
Epoch: 146, Train Loss: 0.0006053, Test Loss: 0.0005433
Epoch: 147, Train Loss: 0.0005989, Test Loss: 0.0005390
Epoch: 148, Train Loss: 0.0005925, Test Loss: 0.0005348
Epoch: 149, Train Loss: 0.0005863, Test Loss: 0.0005305
Epoch: 150, Train Loss: 0.0005801, Test Loss: 0.0005262
Epoch: 151, Train Loss: 0.0005739, Test Loss: 0.0005221
Epoch: 152, Train Loss: 0.0005678, Test Loss: 0.0005179
Epoch: 153, Train Loss: 0.0005618, Test Loss: 0.0005137
Epoch: 154, Train Loss: 0.0005557, Test Loss: 0.0005095
Epoch: 155, Train Loss: 0.0005498, Test Loss: 0.0005053
Epoch: 156, Train Loss: 0.0005438, Test Loss: 0.0005010
Epoch: 157, Train Loss: 0.0005380, Test Loss: 0.0004966
Epoch: 158, Train Loss: 0.0005322, Test Loss: 0.0004923
Epoch: 159, Train Loss: 0.0005265, Test Loss: 0.0004879
Epoch: 160, Train Loss: 0.0005208, Test Loss: 0.0004835
Epoch: 161, Train Loss: 0.0005152, Test Loss: 0.0004791
Epoch: 162, Train Loss: 0.0005095, Test Loss: 0.0004748
Epoch: 163, Train Loss: 0.0005040, Test Loss: 0.0004704
Epoch: 164, Train Loss: 0.0004985, Test Loss: 0.0004660
Epoch: 165, Train Loss: 0.0004930, Test Loss: 0.0004617
Epoch: 166, Train Loss: 0.0004875, Test Loss: 0.0004573
Epoch: 167, Train Loss: 0.0004821, Test Loss: 0.0004529
Epoch: 168, Train Loss: 0.0004767, Test Loss: 0.0004485
Epoch: 169, Train Loss: 0.0004713, Test Loss: 0.0004440
Epoch: 170, Train Loss: 0.0004661, Test Loss: 0.0004394
Epoch: 171, Train Loss: 0.0004608, Test Loss: 0.0004349
Epoch: 172, Train Loss: 0.0004556, Test Loss: 0.0004303
Epoch: 173, Train Loss: 0.0004505, Test Loss: 0.0004259
Epoch: 174, Train Loss: 0.0004453, Test Loss: 0.0004214
Epoch: 175, Train Loss: 0.0004402, Test Loss: 0.0004170
Epoch: 176, Train Loss: 0.0004351, Test Loss: 0.0004125
```
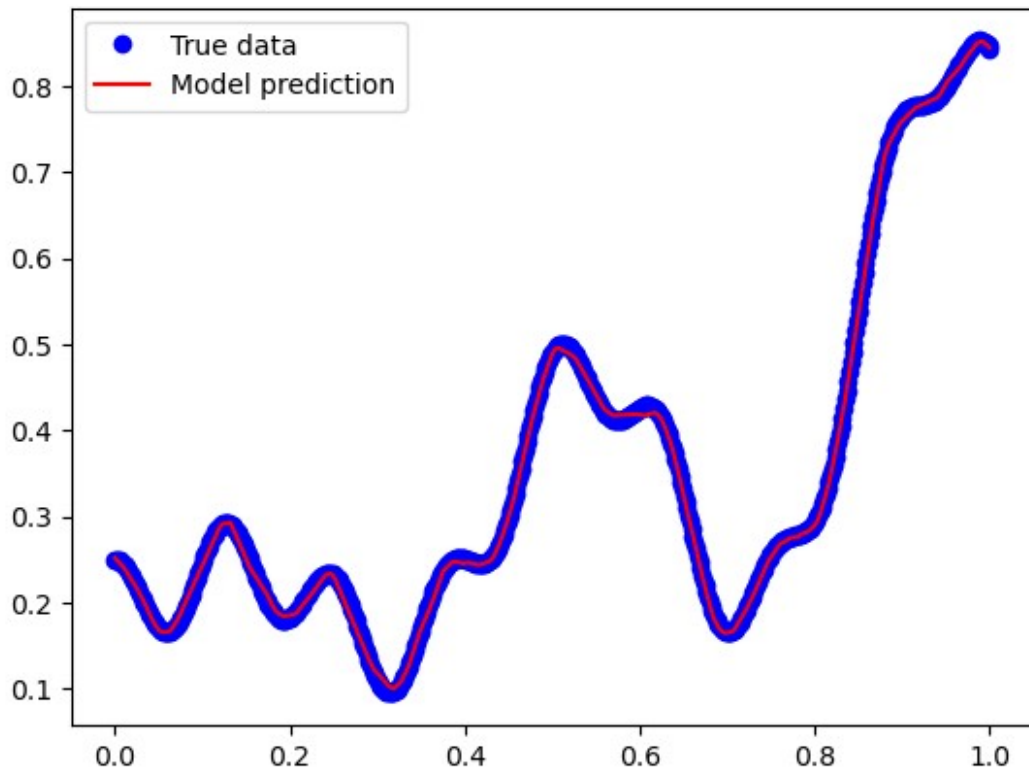
```
Epoch: 177, Train Loss: 0.0004301, Test Loss: 0.0004082
Epoch: 178, Train Loss: 0.0004251, Test Loss: 0.0004038
Epoch: 179, Train Loss: 0.0004202, Test Loss: 0.0003995
Epoch: 180, Train Loss: 0.0004153, Test Loss: 0.0003952
Epoch: 181, Train Loss: 0.0004105, Test Loss: 0.0003911
Epoch: 182, Train Loss: 0.0004056, Test Loss: 0.0003870
Epoch: 183, Train Loss: 0.0004009, Test Loss: 0.0003829
Epoch: 184, Train Loss: 0.0003962, Test Loss: 0.0003789
Epoch: 185, Train Loss: 0.0003915, Test Loss: 0.0003749
Epoch: 186, Train Loss: 0.0003869, Test Loss: 0.0003709
Epoch: 187, Train Loss: 0.0003823, Test Loss: 0.0003669
Epoch: 188, Train Loss: 0.0003777, Test Loss: 0.0003630
Epoch: 189, Train Loss: 0.0003732, Test Loss: 0.0003591
Epoch: 190, Train Loss: 0.0003688, Test Loss: 0.0003551
Epoch: 191, Train Loss: 0.0003644, Test Loss: 0.0003513
Epoch: 192, Train Loss: 0.0003600, Test Loss: 0.0003474
Epoch: 193, Train Loss: 0.0003557, Test Loss: 0.0003436
Epoch: 194, Train Loss: 0.0003514, Test Loss: 0.0003398
Epoch: 195, Train Loss: 0.0003471, Test Loss: 0.0003360
Epoch: 196, Train Loss: 0.0003429, Test Loss: 0.0003322
Epoch: 197, Train Loss: 0.0003387, Test Loss: 0.0003285
Epoch: 198, Train Loss: 0.0003345, Test Loss: 0.0003248
Epoch: 199, Train Loss: 0.0003304, Test Loss: 0.0003211
Epoch: 200, Train Loss: 0.0003263, Test Loss: 0.0003176
Epoch: 201, Train Loss: 0.0003223, Test Loss: 0.0003140
Epoch: 202, Train Loss: 0.0003183, Test Loss: 0.0003106
Epoch: 203, Train Loss: 0.0003144, Test Loss: 0.0003070
Epoch: 204, Train Loss: 0.0003105, Test Loss: 0.0003036
Epoch: 205, Train Loss: 0.0003066, Test Loss: 0.0003002
Epoch: 206, Train Loss: 0.0003028, Test Loss: 0.0002968
Epoch: 207, Train Loss: 0.0002990, Test Loss: 0.0002934
Epoch: 208, Train Loss: 0.0002952, Test Loss: 0.0002900
Epoch: 209, Train Loss: 0.0002915, Test Loss: 0.0002867
Epoch: 210, Train Loss: 0.0002878, Test Loss: 0.0002833
Epoch: 211, Train Loss: 0.0002842, Test Loss: 0.0002799
Epoch: 212, Train Loss: 0.0002806, Test Loss: 0.0002766
Epoch: 213, Train Loss: 0.0002770, Test Loss: 0.0002733
Epoch: 214, Train Loss: 0.0002735, Test Loss: 0.0002700
Epoch: 215, Train Loss: 0.0002699, Test Loss: 0.0002668
Epoch: 216, Train Loss: 0.0002665, Test Loss: 0.0002636
Epoch: 217, Train Loss: 0.0002630, Test Loss: 0.0002604
Epoch: 218, Train Loss: 0.0002596, Test Loss: 0.0002573
Epoch: 219, Train Loss: 0.0002562, Test Loss: 0.0002542
Epoch: 220, Train Loss: 0.0002529, Test Loss: 0.0002511
Epoch: 221, Train Loss: 0.0002496, Test Loss: 0.0002481
Epoch: 222, Train Loss: 0.0002463, Test Loss: 0.0002452
Epoch: 223, Train Loss: 0.0002431, Test Loss: 0.0002423
Epoch: 224, Train Loss: 0.0002399, Test Loss: 0.0002394
Epoch: 225, Train Loss: 0.0002367, Test Loss: 0.0002366
```

```
Epoch: 226, Train Loss: 0.0002335, Test Loss: 0.0002336
Epoch: 227, Train Loss: 0.0002304, Test Loss: 0.0002309
Epoch: 228, Train Loss: 0.0002273, Test Loss: 0.0002281
Epoch: 229, Train Loss: 0.0002242, Test Loss: 0.0002252
Epoch: 230, Train Loss: 0.0002212, Test Loss: 0.0002225
Epoch: 231, Train Loss: 0.0002182, Test Loss: 0.0002197
Epoch: 232, Train Loss: 0.0002152, Test Loss: 0.0002169
Epoch: 233, Train Loss: 0.0002123, Test Loss: 0.0002143
Epoch: 234, Train Loss: 0.0002094, Test Loss: 0.0002116
Epoch: 235, Train Loss: 0.0002065, Test Loss: 0.0002090
Epoch: 236, Train Loss: 0.0002037, Test Loss: 0.0002063
Epoch: 237, Train Loss: 0.0002008, Test Loss: 0.0002037
Epoch: 238, Train Loss: 0.0001980, Test Loss: 0.0002011
Epoch: 239, Train Loss: 0.0001953, Test Loss: 0.0001986
Epoch: 240, Train Loss: 0.0001925, Test Loss: 0.0001962
Epoch: 241, Train Loss: 0.0001898, Test Loss: 0.0001937
Epoch: 242, Train Loss: 0.0001871, Test Loss: 0.0001912
Epoch: 243, Train Loss: 0.0001845, Test Loss: 0.0001890
Epoch: 244, Train Loss: 0.0001818, Test Loss: 0.0001864
Epoch: 245, Train Loss: 0.0001793, Test Loss: 0.0001841
Epoch: 246, Train Loss: 0.0001767, Test Loss: 0.0001818
Epoch: 247, Train Loss: 0.0001742, Test Loss: 0.0001795
Epoch: 248, Train Loss: 0.0001717, Test Loss: 0.0001772
Epoch: 249, Train Loss: 0.0001692, Test Loss: 0.0001750
Epoch: 250, Train Loss: 0.0001668, Test Loss: 0.0001727
Epoch: 251, Train Loss: 0.0001643, Test Loss: 0.0001704
Epoch: 252, Train Loss: 0.0001619, Test Loss: 0.0001683
Epoch: 253, Train Loss: 0.0001595, Test Loss: 0.0001661
Epoch: 254, Train Loss: 0.0001572, Test Loss: 0.0001639
Epoch: 255, Train Loss: 0.0001548, Test Loss: 0.0001618
Epoch: 256, Train Loss: 0.0001526, Test Loss: 0.0001597
Epoch: 257, Train Loss: 0.0001503, Test Loss: 0.0001576
Epoch: 258, Train Loss: 0.0001481, Test Loss: 0.0001556
Epoch: 259, Train Loss: 0.0001459, Test Loss: 0.0001536
Epoch: 260, Train Loss: 0.0001437, Test Loss: 0.0001517
Epoch: 261, Train Loss: 0.0001416, Test Loss: 0.0001498
Epoch: 262, Train Loss: 0.0001395, Test Loss: 0.0001478
Epoch: 263, Train Loss: 0.0001374, Test Loss: 0.0001459
Epoch: 264, Train Loss: 0.0001353, Test Loss: 0.0001440
Epoch: 265, Train Loss: 0.0001333, Test Loss: 0.0001422
Epoch: 266, Train Loss: 0.0001313, Test Loss: 0.0001404
Epoch: 267, Train Loss: 0.0001294, Test Loss: 0.0001386
Epoch: 268, Train Loss: 0.0001274, Test Loss: 0.0001368
Epoch: 269, Train Loss: 0.0001255, Test Loss: 0.0001351
Epoch: 270, Train Loss: 0.0001236, Test Loss: 0.0001333
Epoch: 271, Train Loss: 0.0001217, Test Loss: 0.0001315
Epoch: 272, Train Loss: 0.0001199, Test Loss: 0.0001299
Epoch: 273, Train Loss: 0.0001181, Test Loss: 0.0001282
Epoch: 274, Train Loss: 0.0001164, Test Loss: 0.0001265
```

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Epoch: 275, Train Loss: 0.0001146, Test Loss: 0.0001249
Epoch: 276, Train Loss: 0.0001129, Test Loss: 0.0001233
Epoch: 277, Train Loss: 0.0001112, Test Loss: 0.0001217
Epoch: 278, Train Loss: 0.0001095, Test Loss: 0.0001202
Epoch: 279, Train Loss: 0.0001079, Test Loss: 0.0001186
Epoch: 280, Train Loss: 0.0001063, Test Loss: 0.0001172
Epoch: 281, Train Loss: 0.0001047, Test Loss: 0.0001156
Epoch: 282, Train Loss: 0.0001032, Test Loss: 0.0001142
Epoch: 283, Train Loss: 0.0001016, Test Loss: 0.0001127
Epoch: 284, Train Loss: 0.0001001, Test Loss: 0.0001113
Epoch: 285, Train Loss: 0.0000987, Test Loss: 0.0001098
Epoch: 286, Train Loss: 0.0000972, Test Loss: 0.0001084
Epoch: 287, Train Loss: 0.0000958, Test Loss: 0.0001070
Epoch: 288, Train Loss: 0.0000944, Test Loss: 0.0001056
Epoch: 289, Train Loss: 0.0000930, Test Loss: 0.0001043
Epoch: 290, Train Loss: 0.0000916, Test Loss: 0.0001029
Epoch: 291, Train Loss: 0.0000903, Test Loss: 0.0001015
Epoch: 292, Train Loss: 0.0000890, Test Loss: 0.0001002
Epoch: 293, Train Loss: 0.0000877, Test Loss: 0.0000989
Epoch: 294, Train Loss: 0.0000865, Test Loss: 0.0000977
Epoch: 295, Train Loss: 0.0000853, Test Loss: 0.0000964
Epoch: 296, Train Loss: 0.0000841, Test Loss: 0.0000952
Epoch: 297, Train Loss: 0.0000829, Test Loss: 0.0000940
Epoch: 298, Train Loss: 0.0000818, Test Loss: 0.0000929
Epoch: 299, Train Loss: 0.0000807, Test Loss: 0.0000918
Epoch: 300, Train Loss: 0.0000796, Test Loss: 0.0000907
Epoch: 301, Train Loss: 0.0000786, Test Loss: 0.0000896
Epoch: 302, Train Loss: 0.0000776, Test Loss: 0.0000886
Epoch: 303, Train Loss: 0.0000766, Test Loss: 0.0000875
Epoch: 304, Train Loss: 0.0000757, Test Loss: 0.0000866
Epoch: 305, Train Loss: 0.0000748, Test Loss: 0.0000857
Epoch: 306, Train Loss: 0.0000739, Test Loss: 0.0000847
Epoch: 307, Train Loss: 0.0000731, Test Loss: 0.0000839
Epoch: 308, Train Loss: 0.0000723, Test Loss: 0.0000831
Epoch: 309, Train Loss: 0.0000716, Test Loss: 0.0000823
Epoch: 310, Train Loss: 0.0000709, Test Loss: 0.0000816
Epoch: 311, Train Loss: 0.0000703, Test Loss: 0.0000809
Epoch: 312, Train Loss: 0.0000698, Test Loss: 0.0000803
Epoch: 313, Train Loss: 0.0000692, Test Loss: 0.0000798
Epoch: 314, Train Loss: 0.0000688, Test Loss: 0.0000793
Epoch: 315, Train Loss: 0.0000685, Test Loss: 0.0000789
Epoch: 316, Train Loss: 0.0000683, Test Loss: 0.0000787
Epoch: 317, Train Loss: 0.0000682, Test Loss: 0.0000786
Epoch: 318, Train Loss: 0.0000682, Test Loss: 0.0000786
Epoch: 319, Train Loss: 0.0000683, Test Loss: 0.0000787
Epoch: 320, Train Loss: 0.0000686, Test Loss: 0.0000790
Epoch: 321, Train Loss: 0.0000691, Test Loss: 0.0000794
Epoch: 322, Train Loss: 0.0000697, Test Loss: 0.0000801
Epoch: 323, Train Loss: 0.0000705, Test Loss: 0.0000809
```

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Epoch: 324, Train Loss: 0.0000714, Test Loss: 0.0000818
Epoch: 325, Train Loss: 0.0000725, Test Loss: 0.0000830
Epoch: 326, Train Loss: 0.0000738, Test Loss: 0.0000843
Epoch: 327, Train Loss: 0.0000752, Test Loss: 0.0000857
Epoch: 328, Train Loss: 0.0000768, Test Loss: 0.0000873
Epoch: 329, Train Loss: 0.0000784, Test Loss: 0.0000889
Epoch: 330, Train Loss: 0.0000800, Test Loss: 0.0000906
Epoch: 331, Train Loss: 0.0000816, Test Loss: 0.0000923
Epoch: 332, Train Loss: 0.0000831, Test Loss: 0.0000939
Epoch: 333, Train Loss: 0.0000845, Test Loss: 0.0000953
Epoch: 334, Train Loss: 0.0000856, Test Loss: 0.0000965
Epoch: 335, Train Loss: 0.0000863, Test Loss: 0.0000973
Epoch: 336, Train Loss: 0.0000867, Test Loss: 0.0000977
Epoch: 337, Train Loss: 0.0000866, Test Loss: 0.0000976
Epoch: 338, Train Loss: 0.0000860, Test Loss: 0.0000971
Epoch: 339, Train Loss: 0.0000850, Test Loss: 0.0000961
Epoch: 340, Train Loss: 0.0000837, Test Loss: 0.0000947
Epoch: 341, Train Loss: 0.0000820, Test Loss: 0.0000929
Epoch: 342, Train Loss: 0.0000800, Test Loss: 0.0000909
Epoch: 343, Train Loss: 0.0000778, Test Loss: 0.0000887
Epoch: 344, Train Loss: 0.0000755, Test Loss: 0.0000863
Epoch: 345, Train Loss: 0.0000731, Test Loss: 0.0000838
Epoch: 346, Train Loss: 0.0000707, Test Loss: 0.0000812
Epoch: 347, Train Loss: 0.0000683, Test Loss: 0.0000787
Epoch: 348, Train Loss: 0.0000660, Test Loss: 0.0000762
Epoch: 349, Train Loss: 0.0000639, Test Loss: 0.0000739
Epoch: 350, Train Loss: 0.0000617, Test Loss: 0.0000716
Epoch: 351, Train Loss: 0.0000597, Test Loss: 0.0000694
Epoch: 352, Train Loss: 0.0000579, Test Loss: 0.0000674
Epoch: 353, Train Loss: 0.0000562, Test Loss: 0.0000655
Epoch: 354, Train Loss: 0.0000545, Test Loss: 0.0000637
Epoch: 355, Train Loss: 0.0000530, Test Loss: 0.0000619
Epoch: 356, Train Loss: 0.0000516, Test Loss: 0.0000604
Epoch: 357, Train Loss: 0.0000503, Test Loss: 0.0000588
Epoch: 358, Train Loss: 0.0000490, Test Loss: 0.0000573
Epoch: 359, Train Loss: 0.0000478, Test Loss: 0.0000559
Epoch: 360, Train Loss: 0.0000467, Test Loss: 0.0000546
Epoch: 361, Train Loss: 0.0000455, Test Loss: 0.0000533
Epoch: 362, Train Loss: 0.0000445, Test Loss: 0.0000520
Epoch: 363, Train Loss: 0.0000434, Test Loss: 0.0000508
Epoch: 364, Train Loss: 0.0000425, Test Loss: 0.0000496
Epoch: 365, Train Loss: 0.0000415, Test Loss: 0.0000485
Epoch: 366, Train Loss: 0.0000407, Test Loss: 0.0000474
Epoch: 367, Train Loss: 0.0000398, Test Loss: 0.0000464
Epoch: 368, Train Loss: 0.0000390, Test Loss: 0.0000454
Epoch: 369, Train Loss: 0.0000382, Test Loss: 0.0000444
Epoch: 370, Train Loss: 0.0000374, Test Loss: 0.0000434
Epoch: 371, Train Loss: 0.0000366, Test Loss: 0.0000424
Epoch: 372, Train Loss: 0.0000359, Test Loss: 0.0000415
```

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Epoch: 373, Train Loss: 0.0000351, Test Loss: 0.0000406
Epoch: 374, Train Loss: 0.0000344, Test Loss: 0.0000396
Epoch: 375, Train Loss: 0.0000337, Test Loss: 0.0000387
Epoch: 376, Train Loss: 0.0000330, Test Loss: 0.0000378
Epoch: 377, Train Loss: 0.0000323, Test Loss: 0.0000369
Epoch: 378, Train Loss: 0.0000316, Test Loss: 0.0000361
Epoch: 379, Train Loss: 0.0000310, Test Loss: 0.0000352
Epoch: 380, Train Loss: 0.0000303, Test Loss: 0.0000344
Epoch: 381, Train Loss: 0.0000297, Test Loss: 0.0000336
Epoch: 382, Train Loss: 0.0000291, Test Loss: 0.0000328
Epoch: 383, Train Loss: 0.0000285, Test Loss: 0.0000320
Epoch: 384, Train Loss: 0.0000279, Test Loss: 0.0000312
Epoch: 385, Train Loss: 0.0000274, Test Loss: 0.0000304
Epoch: 386, Train Loss: 0.0000268, Test Loss: 0.0000297
Epoch: 387, Train Loss: 0.0000263, Test Loss: 0.0000290
Epoch: 388, Train Loss: 0.0000258, Test Loss: 0.0000283
Epoch: 389, Train Loss: 0.0000253, Test Loss: 0.0000276
Epoch: 390, Train Loss: 0.0000249, Test Loss: 0.0000270
Epoch: 391, Train Loss: 0.0000244, Test Loss: 0.0000263
Epoch: 392, Train Loss: 0.0000240, Test Loss: 0.0000257
Epoch: 393, Train Loss: 0.0000236, Test Loss: 0.0000252
Epoch: 394, Train Loss: 0.0000232, Test Loss: 0.0000246
Epoch: 395, Train Loss: 0.0000229, Test Loss: 0.0000241
Epoch: 396, Train Loss: 0.0000226, Test Loss: 0.0000237
Epoch: 397, Train Loss: 0.0000223, Test Loss: 0.0000232
Epoch: 398, Train Loss: 0.0000219, Test Loss: 0.0000228
Epoch: 399, Train Loss: 0.0000217, Test Loss: 0.0000224
Epoch: 400, Train Loss: 0.0000214, Test Loss: 0.0000220
```

Problem 4.2 Determine appropriate mini-batch size = 32, epoch = 500

```python
# ################# Part 1: Load data and create batch
#################
N_total = 600
N_train = 500
x = torch.unsqueeze(torch.linspace(0, 1, N_total), dim=1)
r = torch.randperm(N_total)
x = x[r, :]
y = 0.2 + 0.4 * torch.pow(x, 2) + 0.3 * x * torch.sin(15 * x) + 0.05 *
torch.cos(50 * x)

class CustomDataset(Dataset):
    def __init__(self, x, y):
        self.y = y
        self.x = x

    def __len__(self):
        return len(self.y)

    def __getitem__(self, idx):
        y1 = self.y[idx]
        x1 = self.x[idx]
        return (x1, y1)

# Change batch_size here to test different values
```

```python
batch_size = 32  # Experiment with different batch sizes: 32, 64, 128
trainset = CustomDataset(x[0:N_train, :], y[0:N_train, :])
testset = CustomDataset(x[N_train:N_total, :], y[N_train:N_total, :])
train_loader = torch.utils.data.DataLoader(trainset,
batch_size=batch_size)
test_loader = torch.utils.data.DataLoader(testset,
batch_size=batch_size)

# ################# Part 2: Define Model #################
model = nn.Sequential(
    nn.Linear(1, 1024, bias=True),
    nn.ReLU(),
    nn.Linear(1024, 1, bias=True)
)

def init_weights(m):
    if isinstance(m, nn.Linear):
        m.weight.data.uniform_(-1, 1)
        m.bias.data.uniform_(-1, 1)

model.apply(init_weights)

# ################# Part 3: Define Loss and Optimizer
#################
criterion = nn.MSELoss()
optimizer = optim.Adam(model.parameters(), lr=0.001)

# ################# Part 4: Training and Testing #################
def train_NN():
    model.train()
    for images, labels in train_loader:
        out = model(images)
        loss = criterion(out, labels)
        loss.backward()
        optimizer.step()
        optimizer.zero_grad()
    return loss

def test_NN(loader):
    model.eval()
    loss = 0
    with torch.no_grad():
        for images, labels in loader:
            out = model(images)
            loss += criterion(out, labels).item()
    loss = loss / len(loader)
    return loss

# Experiment with different numbers of epochs
N_epoch = 500  # Try values like 100, 200, 500
```

```python
train_loss = np.zeros((N_epoch, 1))
test_loss = np.zeros((N_epoch, 1))

for epoch in range(N_epoch):
    train_NN()
    train_loss[epoch, 0] = test_NN(train_loader)
    test_loss[epoch, 0] = test_NN(test_loader)
    print(f'Epoch: {epoch+1:03d}, Train Loss: {train_loss[epoch,
0]:.7f}, Test Loss: {test_loss[epoch, 0]:.7f}')

# ################# Final Prediction #################
x_test = torch.unsqueeze(torch.linspace(0, 1, 1999), dim=1)
y_test = model(x_test)

# Plot the results
plt.plot(x[0:N_total], y[0:N_total], 'bo', label='True data')
plt.plot(x_test, y_test.detach().numpy(), 'r', label='Model
prediction')
plt.legend()
plt.show()
```

```
Epoch: 001, Train Loss: 1.8591078, Test Loss: 1.8779527
Epoch: 002, Train Loss: 3.3536461, Test Loss: 3.5683156
Epoch: 003, Train Loss: 0.6762136, Test Loss: 0.6476456
Epoch: 004, Train Loss: 0.3699355, Test Loss: 0.3676183
Epoch: 005, Train Loss: 0.2734823, Test Loss: 0.3078416
Epoch: 006, Train Loss: 0.2020788, Test Loss: 0.2080572
Epoch: 007, Train Loss: 0.1507670, Test Loss: 0.1631345
Epoch: 008, Train Loss: 0.1131928, Test Loss: 0.1192604
Epoch: 009, Train Loss: 0.0844115, Test Loss: 0.0875499
Epoch: 010, Train Loss: 0.0629654, Test Loss: 0.0637244
Epoch: 011, Train Loss: 0.0472580, Test Loss: 0.0465967
Epoch: 012, Train Loss: 0.0359955, Test Loss: 0.0346528
Epoch: 013, Train Loss: 0.0280716, Test Loss: 0.0263436
Epoch: 014, Train Loss: 0.0225176, Test Loss: 0.0206735
Epoch: 015, Train Loss: 0.0186077, Test Loss: 0.0167733
Epoch: 016, Train Loss: 0.0158094, Test Loss: 0.0140455
Epoch: 017, Train Loss: 0.0137526, Test Loss: 0.0120892
Epoch: 018, Train Loss: 0.0121942, Test Loss: 0.0106317
Epoch: 019, Train Loss: 0.0109726, Test Loss: 0.0095144
Epoch: 020, Train Loss: 0.0099813, Test Loss: 0.0086213
Epoch: 021, Train Loss: 0.0091465, Test Loss: 0.0078800
Epoch: 022, Train Loss: 0.0084258, Test Loss: 0.0072504
Epoch: 023, Train Loss: 0.0077910, Test Loss: 0.0067061
Epoch: 024, Train Loss: 0.0072258, Test Loss: 0.0062259
Epoch: 025, Train Loss: 0.0067190, Test Loss: 0.0058017
Epoch: 026, Train Loss: 0.0062623, Test Loss: 0.0054227
Epoch: 027, Train Loss: 0.0058486, Test Loss: 0.0050836
Epoch: 028, Train Loss: 0.0054735, Test Loss: 0.0047807
Epoch: 029, Train Loss: 0.0051327, Test Loss: 0.0045074
```

```
Epoch: 030, Train Loss: 0.0048227, Test Loss: 0.0042612
Epoch: 031, Train Loss: 0.0045403, Test Loss: 0.0040395
Epoch: 032, Train Loss: 0.0042820, Test Loss: 0.0038380
Epoch: 033, Train Loss: 0.0040467, Test Loss: 0.0036555
Epoch: 034, Train Loss: 0.0038313, Test Loss: 0.0034894
Epoch: 035, Train Loss: 0.0036342, Test Loss: 0.0033396
Epoch: 036, Train Loss: 0.0034531, Test Loss: 0.0032029
Epoch: 037, Train Loss: 0.0032870, Test Loss: 0.0030781
Epoch: 038, Train Loss: 0.0031342, Test Loss: 0.0029640
Epoch: 039, Train Loss: 0.0029932, Test Loss: 0.0028602
Epoch: 040, Train Loss: 0.0028638, Test Loss: 0.0027645
Epoch: 041, Train Loss: 0.0027448, Test Loss: 0.0026763
Epoch: 042, Train Loss: 0.0026346, Test Loss: 0.0025947
Epoch: 043, Train Loss: 0.0025336, Test Loss: 0.0025183
Epoch: 044, Train Loss: 0.0024404, Test Loss: 0.0024477
Epoch: 045, Train Loss: 0.0023545, Test Loss: 0.0023832
Epoch: 046, Train Loss: 0.0022755, Test Loss: 0.0023234
Epoch: 047, Train Loss: 0.0022027, Test Loss: 0.0022678
Epoch: 048, Train Loss: 0.0021361, Test Loss: 0.0022171
Epoch: 049, Train Loss: 0.0020744, Test Loss: 0.0021698
Epoch: 050, Train Loss: 0.0020178, Test Loss: 0.0021262
Epoch: 051, Train Loss: 0.0019658, Test Loss: 0.0020859
Epoch: 052, Train Loss: 0.0019177, Test Loss: 0.0020485
Epoch: 053, Train Loss: 0.0018729, Test Loss: 0.0020134
Epoch: 054, Train Loss: 0.0018310, Test Loss: 0.0019808
Epoch: 055, Train Loss: 0.0017918, Test Loss: 0.0019499
Epoch: 056, Train Loss: 0.0017547, Test Loss: 0.0019213
Epoch: 057, Train Loss: 0.0017197, Test Loss: 0.0018938
Epoch: 058, Train Loss: 0.0016864, Test Loss: 0.0018670
Epoch: 059, Train Loss: 0.0016540, Test Loss: 0.0018414
Epoch: 060, Train Loss: 0.0016223, Test Loss: 0.0018156
Epoch: 061, Train Loss: 0.0015910, Test Loss: 0.0017893
Epoch: 062, Train Loss: 0.0015597, Test Loss: 0.0017621
Epoch: 063, Train Loss: 0.0015289, Test Loss: 0.0017344
Epoch: 064, Train Loss: 0.0014994, Test Loss: 0.0017073
Epoch: 065, Train Loss: 0.0014720, Test Loss: 0.0016823
Epoch: 066, Train Loss: 0.0014473, Test Loss: 0.0016595
Epoch: 067, Train Loss: 0.0014250, Test Loss: 0.0016396
Epoch: 068, Train Loss: 0.0014053, Test Loss: 0.0016218
Epoch: 069, Train Loss: 0.0013873, Test Loss: 0.0016060
Epoch: 070, Train Loss: 0.0013705, Test Loss: 0.0015925
Epoch: 071, Train Loss: 0.0013545, Test Loss: 0.0015798
Epoch: 072, Train Loss: 0.0013391, Test Loss: 0.0015669
Epoch: 073, Train Loss: 0.0013242, Test Loss: 0.0015551
Epoch: 074, Train Loss: 0.0013099, Test Loss: 0.0015430
Epoch: 075, Train Loss: 0.0012960, Test Loss: 0.0015313
Epoch: 076, Train Loss: 0.0012826, Test Loss: 0.0015198
Epoch: 077, Train Loss: 0.0012697, Test Loss: 0.0015088
Epoch: 078, Train Loss: 0.0012571, Test Loss: 0.0014979
```

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Epoch: 079, Train Loss: 0.0012448, Test Loss: 0.0014874
Epoch: 080, Train Loss: 0.0012331, Test Loss: 0.0014774
Epoch: 081, Train Loss: 0.0012218, Test Loss: 0.0014680
Epoch: 082, Train Loss: 0.0012110, Test Loss: 0.0014592
Epoch: 083, Train Loss: 0.0012007, Test Loss: 0.0014508
Epoch: 084, Train Loss: 0.0011907, Test Loss: 0.0014430
Epoch: 085, Train Loss: 0.0011814, Test Loss: 0.0014356
Epoch: 086, Train Loss: 0.0011726, Test Loss: 0.0014287
Epoch: 087, Train Loss: 0.0011644, Test Loss: 0.0014223
Epoch: 088, Train Loss: 0.0011566, Test Loss: 0.0014162
Epoch: 089, Train Loss: 0.0011491, Test Loss: 0.0014103
Epoch: 090, Train Loss: 0.0011420, Test Loss: 0.0014047
Epoch: 091, Train Loss: 0.0011351, Test Loss: 0.0013993
Epoch: 092, Train Loss: 0.0011285, Test Loss: 0.0013941
Epoch: 093, Train Loss: 0.0011221, Test Loss: 0.0013891
Epoch: 094, Train Loss: 0.0011157, Test Loss: 0.0013841
Epoch: 095, Train Loss: 0.0011091, Test Loss: 0.0013784
Epoch: 096, Train Loss: 0.0011025, Test Loss: 0.0013720
Epoch: 097, Train Loss: 0.0010956, Test Loss: 0.0013654
Epoch: 098, Train Loss: 0.0010886, Test Loss: 0.0013584
Epoch: 099, Train Loss: 0.0010812, Test Loss: 0.0013509
Epoch: 100, Train Loss: 0.0010736, Test Loss: 0.0013433
Epoch: 101, Train Loss: 0.0010657, Test Loss: 0.0013352
Epoch: 102, Train Loss: 0.0010574, Test Loss: 0.0013267
Epoch: 103, Train Loss: 0.0010488, Test Loss: 0.0013174
Epoch: 104, Train Loss: 0.0010399, Test Loss: 0.0013078
Epoch: 105, Train Loss: 0.0010307, Test Loss: 0.0012976
Epoch: 106, Train Loss: 0.0010211, Test Loss: 0.0012871
Epoch: 107, Train Loss: 0.0010113, Test Loss: 0.0012763
Epoch: 108, Train Loss: 0.0010014, Test Loss: 0.0012653
Epoch: 109, Train Loss: 0.0009913, Test Loss: 0.0012542
Epoch: 110, Train Loss: 0.0009810, Test Loss: 0.0012428
Epoch: 111, Train Loss: 0.0009705, Test Loss: 0.0012311
Epoch: 112, Train Loss: 0.0009600, Test Loss: 0.0012192
Epoch: 113, Train Loss: 0.0009494, Test Loss: 0.0012072
Epoch: 114, Train Loss: 0.0009388, Test Loss: 0.0011951
Epoch: 115, Train Loss: 0.0009283, Test Loss: 0.0011831
Epoch: 116, Train Loss: 0.0009178, Test Loss: 0.0011711
Epoch: 117, Train Loss: 0.0009074, Test Loss: 0.0011594
Epoch: 118, Train Loss: 0.0008969, Test Loss: 0.0011475
Epoch: 119, Train Loss: 0.0008865, Test Loss: 0.0011357
Epoch: 120, Train Loss: 0.0008762, Test Loss: 0.0011238
Epoch: 121, Train Loss: 0.0008661, Test Loss: 0.0011123
Epoch: 122, Train Loss: 0.0008562, Test Loss: 0.0011008
Epoch: 123, Train Loss: 0.0008463, Test Loss: 0.0010894
Epoch: 124, Train Loss: 0.0008366, Test Loss: 0.0010784
Epoch: 125, Train Loss: 0.0008269, Test Loss: 0.0010675
Epoch: 126, Train Loss: 0.0008174, Test Loss: 0.0010567
Epoch: 127, Train Loss: 0.0008082, Test Loss: 0.0010463
```

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Epoch: 128, Train Loss: 0.0007992, Test Loss: 0.0010359
Epoch: 129, Train Loss: 0.0007904, Test Loss: 0.0010257
Epoch: 130, Train Loss: 0.0007817, Test Loss: 0.0010159
Epoch: 131, Train Loss: 0.0007732, Test Loss: 0.0010060
Epoch: 132, Train Loss: 0.0007649, Test Loss: 0.0009965
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Epoch: 135, Train Loss: 0.0007406, Test Loss: 0.0009686
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Epoch: 138, Train Loss: 0.0007175, Test Loss: 0.0009424
Epoch: 139, Train Loss: 0.0007102, Test Loss: 0.0009340
Epoch: 140, Train Loss: 0.0007029, Test Loss: 0.0009258
Epoch: 141, Train Loss: 0.0006958, Test Loss: 0.0009176
Epoch: 142, Train Loss: 0.0006888, Test Loss: 0.0009096
Epoch: 143, Train Loss: 0.0006819, Test Loss: 0.0009017
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Epoch: 152, Train Loss: 0.0006244, Test Loss: 0.0008346
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Epoch: 159, Train Loss: 0.0005840, Test Loss: 0.0007864
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Epoch: 161, Train Loss: 0.0005734, Test Loss: 0.0007731
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Epoch: 176, Train Loss: 0.0005032, Test Loss: 0.0006834
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Epoch: 185, Train Loss: 0.0004671, Test Loss: 0.0006344
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Epoch: 191, Train Loss: 0.0004452, Test Loss: 0.0006036
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Epoch: 204, Train Loss: 0.0004060, Test Loss: 0.0005443
Epoch: 205, Train Loss: 0.0004036, Test Loss: 0.0005402
Epoch: 206, Train Loss: 0.0004012, Test Loss: 0.0005362
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Epoch: 212, Train Loss: 0.0003897, Test Loss: 0.0005150
Epoch: 213, Train Loss: 0.0003882, Test Loss: 0.0005118
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Epoch: 223, Train Loss: 0.0003767, Test Loss: 0.0004818
Epoch: 224, Train Loss: 0.0003751, Test Loss: 0.0004784
Epoch: 225, Train Loss: 0.0003733, Test Loss: 0.0004747
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Epoch: 236, Train Loss: 0.0003167, Test Loss: 0.0003975
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Epoch: 239, Train Loss: 0.0002899, Test Loss: 0.0003668
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Epoch: 247, Train Loss: 0.0002557, Test Loss: 0.0003285
Epoch: 248, Train Loss: 0.0002562, Test Loss: 0.0003285
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Epoch: 254, Train Loss: 0.0002619, Test Loss: 0.0003290
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Epoch: 261, Train Loss: 0.0002573, Test Loss: 0.0003140
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Epoch: 264, Train Loss: 0.0002536, Test Loss: 0.0003055
Epoch: 265, Train Loss: 0.0002522, Test Loss: 0.0003024
Epoch: 266, Train Loss: 0.0002508, Test Loss: 0.0002994
Epoch: 267, Train Loss: 0.0002493, Test Loss: 0.0002963
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Epoch: 269, Train Loss: 0.0002463, Test Loss: 0.0002903
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Epoch: 274, Train Loss: 0.0002396, Test Loss: 0.0002764
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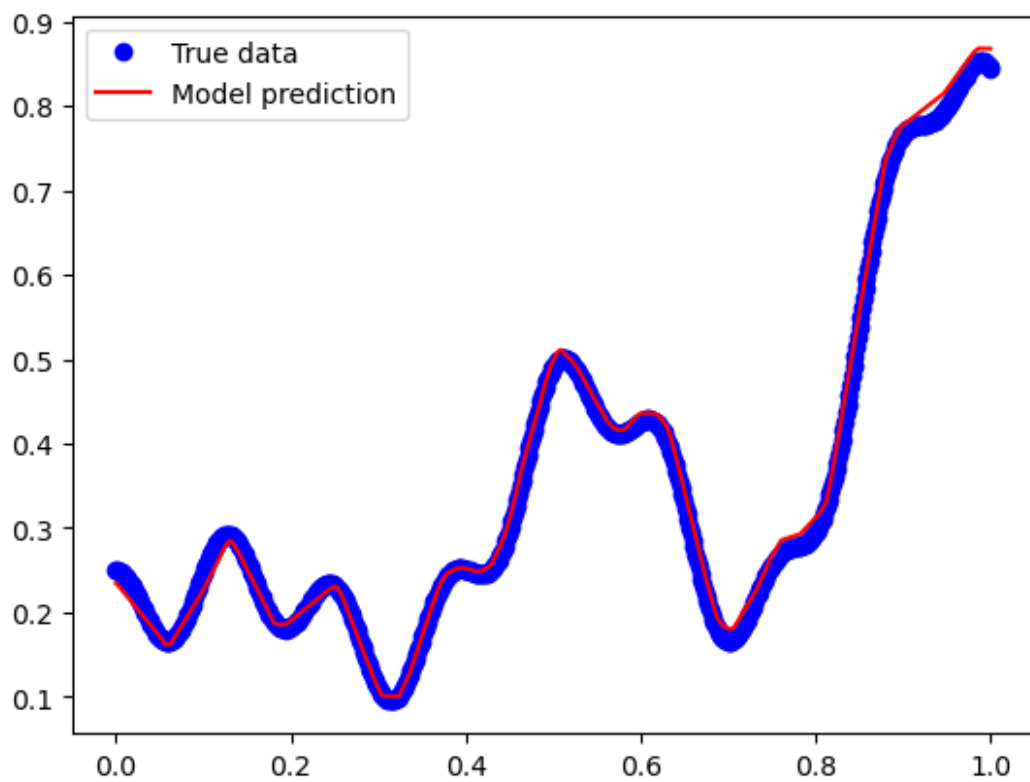
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Epoch: 321, Train Loss: 0.0001257, Test Loss: 0.0001476
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Epoch: 323, Train Loss: 0.0001220, Test Loss: 0.0001442
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Epoch: 481, Train Loss: 0.0004419, Test Loss: 0.0004295
Epoch: 482, Train Loss: 0.0004317, Test Loss: 0.0004185
Epoch: 483, Train Loss: 0.0004278, Test Loss: 0.0004136
Epoch: 484, Train Loss: 0.0004216, Test Loss: 0.0004060
Epoch: 485, Train Loss: 0.0004094, Test Loss: 0.0003926
Epoch: 486, Train Loss: 0.0003774, Test Loss: 0.0003604
Epoch: 487, Train Loss: 0.0003084, Test Loss: 0.0002938
Epoch: 488, Train Loss: 0.0002278, Test Loss: 0.0002182
Epoch: 489, Train Loss: 0.0001745, Test Loss: 0.0001699
Epoch: 490, Train Loss: 0.0001789, Test Loss: 0.0001777
Epoch: 491, Train Loss: 0.0002409, Test Loss: 0.0002447
Epoch: 492, Train Loss: 0.0004290, Test Loss: 0.0004521
Epoch: 493, Train Loss: 0.0020765, Test Loss: 0.0021151
Epoch: 494, Train Loss: 0.0006884, Test Loss: 0.0006391
Epoch: 495, Train Loss: 0.0001314, Test Loss: 0.0001394
Epoch: 496, Train Loss: 0.0001207, Test Loss: 0.0001310
Epoch: 497, Train Loss: 0.0001498, Test Loss: 0.0001607
Epoch: 498, Train Loss: 0.0002108, Test Loss: 0.0002221
Epoch: 499, Train Loss: 0.0002303, Test Loss: 0.0002353
Epoch: 500, Train Loss: 0.0001442, Test Loss: 0.0001489
```

Determine appropriate number of epoch = 400