```
# Import Dependencies
import numpy as np
import matplotlib.pyplot as plt
from sklearn.gaussian_process import GaussianProcessRegressor
from sklearn.gaussian_process.kernels import RBF, WhiteKernel,
ExpSineSquared, ConstantKernel
from sklearn.kernel_ridge import KernelRidge
```

Midterm 1 Project, Problem 2-2

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# Part 1: Generate noisy data as done previously
x = np.linspace(0, 50, 1000)  # Generate 1000 evenly spaced x data
points
y_true = np.cos(x)  # True function y = cos(x)

# Randomly select 40 points from the first 500 data points
np.random.seed(42)  # Ensure reproducibility
indices = np.random.choice(np.arange(500), size=40, replace=False)

# Add i.i.d. random noise (mean 0, variance 0.16) to the 40 selected
points
noise = np.random.normal(0, np.sqrt(0.16), size=40)
x_train = x[indices].reshape(-1, 1)  # Select 40 x points from the
first 500
y_train_noisy = y_true[indices] + noise  # Add noise to the
corresponding y points

# Part 2: Fit a Gaussian Process (GP) model using a periodic kernel
and a white noise kernel
# Define the kernel: periodic kernel (ExpSineSquared) + white noise
(WhiteKernel)
kernel = ExpSineSquared(length_scale=1.0, periodicity=1.0) +
WhiteKernel(noise_level=1.0)

# Create the Gaussian Process Regressor with the specified kernel
gp = GaussianProcessRegressor(kernel=kernel, n_restarts_optimizer=10)

# Fit the GP to the noisy data
gp.fit(x_train, y_train_noisy)

# Predict using the GP on the full x range
x_pred = x.reshape(-1, 1)
y_pred, y_std = gp.predict(x_pred, return_std=True)

# Extract the learned kernel hyperparameters
kernel_optimized = gp.kernel_
print("Optimized Kernel:", kernel_optimized)

# Extract the specific hyperparameters from the optimized kernel
p = kernel_optimized.k1.periodicity  # Periodicity (p)
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ell = kernel_optimized.k1.length_scale  # Length scale (ℓ)
sigma = np.sqrt(kernel_optimized.k2.noise_level)  # Noise level (σ)

# Report the hyperparameters
print(f"Optimized hyperparameters:")
print(f"Periodicity (p): {p}")
print(f"Length Scale (ℓ): {ell}")
print(f"Noise Level (σ): {sigma}")

# Plot the results
plt.figure(figsize=(10, 6))

# Plot the true function y = cos(x)
plt.plot(x, y_true, label=r"$y = \cos(x)$", color='blue')

# Plot the noisy training data points
plt.scatter(x_train, y_train_noisy, label="Noisy Training Data",
color='red', zorder=5)

# Plot the GP predictions with uncertainty bounds
plt.plot(x_pred, y_pred, label="GP Prediction", color='green')
plt.fill_between(x_pred.flatten(), y_pred - 1.96 * y_std, y_pred +
1.96 * y_std, alpha=0.2, color='green')

# Set labels, title, and legend
plt.title("Gaussian Process Regression with Periodic + White Noise
Kernel")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.grid(True)

# Show the plot
plt.show()
```

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Optimized Kernel: ExpSineSquared(length_scale=1.84, periodicity=6.45)
+ WhiteKernel(noise_level=0.14)
Optimized hyperparameters:
Periodicity (p): 6.447335368768318
Length Scale (ℓ): 1.8431891941054277
Noise Level (σ): 0.37480938853048
```

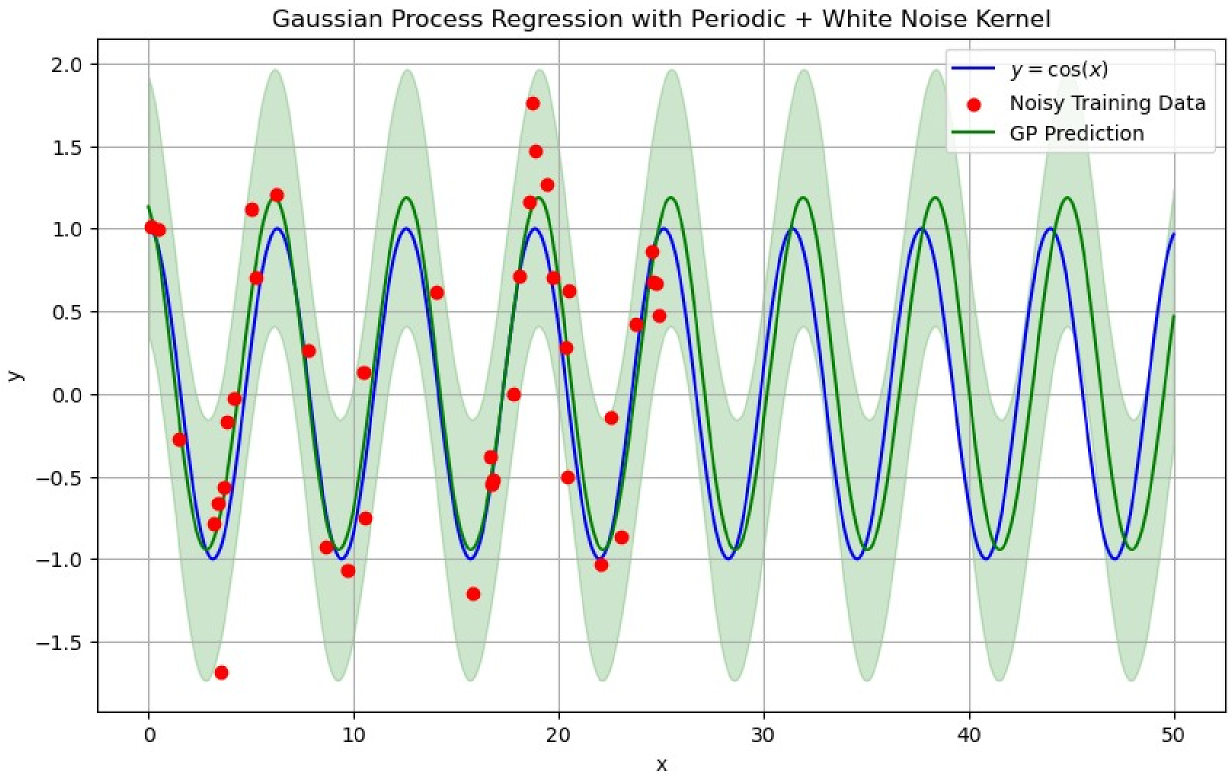
Gaussian Process Regression with Periodic + White Noise Kernel
$y = \cos(x)$
Noisy Training Data
GP Prediction
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
y
0
10
20
30
40
50
x