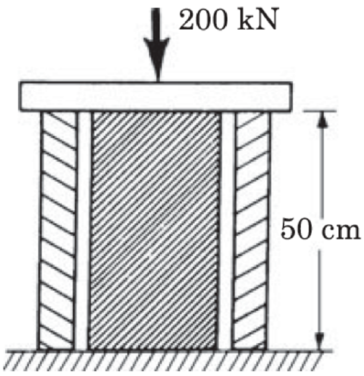
Problem 1

A composite post with a hallow steel cylinder surrounding a solid copper cylinder is applied an axial loading of 200 kN via a rigid plate as shown below. Both cylinders have the same length of 50 cm before loading and temperature changes. Determine the normal stress (σ) and deformation (δ) of the steel part after the load is applied and the temperature of the entire post is increased by 50 °C.

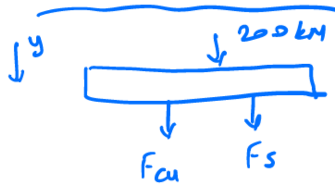
The cross-sectional areas, elastic moduli and coefficient of thermal expansions for steel and copper parts are listed in the table below.

Material/Part	Cross-section (cm^2)	Elastic modulus (GPa)	Coeff. of thermal expansion (/°C)
Steel	20	200	12×10^{-6}
Copper	60	100	1.7×10^{-6}



It's a statically indeterminate problem with thermal deformation. It's similar to the page 11 example in M3_L1 lecture slides, with the addition of thermal strain.

FBD + STAT. EQ. (10 pts)



$$\downarrow \Sigma F_y = F_{cu} + F_s + 200\text{ kN} = 0 \quad (1)$$

Please note that here we assume tension forces on the two cylinders. In this way such tension force will give the deformation in the same direction as that coming from the thermal effect, so that we can add them together next.

CONST. EQ. (5 pts)

$$(2) \quad \delta_{s,cu} = \left(\frac{FL}{EA}\right)_{s,cu} + \alpha_{s,cu} \Delta T L$$

COMP. EQ. $\quad \delta_s = \delta_{cu} \quad (3)$ (10 pts)

Solving those equations will give

$$F_s = -203.6 \text{ kN}$$

$$F_{cu} = 3.6 \text{ kN}$$

5 pts

And then

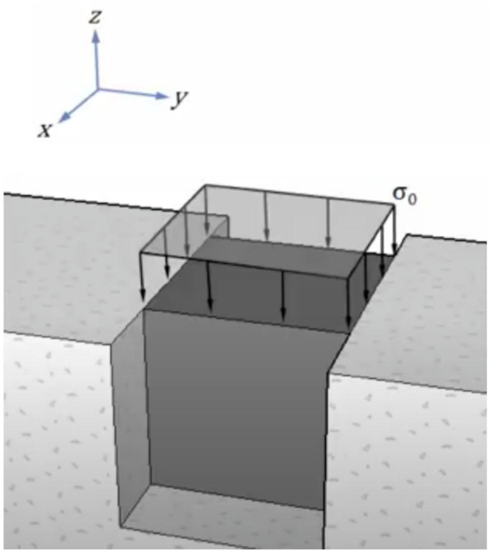
$$\sigma_s = \frac{F_s}{A} = \boxed{-101.8 \text{ MPa}}$$

5 pts

$$\delta_s = \left(\frac{F_s}{E_s A_s} + \alpha_s \Delta T\right) L = \boxed{45.5 \text{ mm}}$$

Problem 2

A block of material made of rubber (darker color in the figure) just fits between three rigid, smooth walls (one on each of two sides and one below). If the top of the block is loaded in compression ($\sigma_z = \sigma_0 = -100$ psi), as shown, find the strain in the z-direction (ε_z) given $E =$ 100ksi and $v = 0.4$. Neglect friction between the rubber and rigid walls.



We need to use 3D Hooke's law for this problem.

Use 3D Gen. Hookes Law

10 pts

① $\varepsilon_z = \frac{\sigma_z}{E} - \nu \frac{(\sigma_y + \sigma_x)}{E} = \frac{-100\ psi}{100\ ksi} - 0.4 \frac{\sigma_y}{100\ ksi}$

② $\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{(\sigma_x + \sigma_z)}{E} = \frac{\sigma_y}{100\ ksi} + 0.4 \frac{100\ psi}{100\ ksi} = 0$

4 pts

$\sigma_x = 0$ $\qquad$ $\varepsilon_y = 0$

We have these two expressions because x direction doesn't have force (but it will have deformation since nothing is blocking this direction), while y direction is blocked by walls so no deformation (but it will have force). Then

$\underline{\sigma_y = -40\ psi}$

2 pts

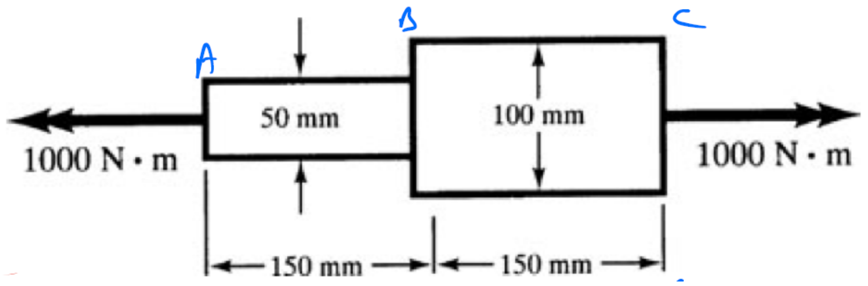
$\varepsilon_z = -\frac{100\ psi}{100\ ksi} + 0.4 \frac{40\ psi}{100\ ksi} = \boxed{-8.4 \times 10^{-4}}$

4 pts

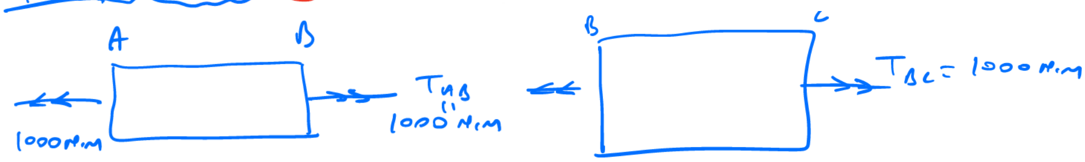
Problem 3

A solid 300-mm long steel shaft has a circular cross-section of diameter 50 mm over the left and of diameter 100 mm over the right half, as shown in the figure below. Each end of the shaft is loaded by a torque of 1000 N·m as indicated by the double-headed arrows. Take the shear modulus of steel as G=80 GPa, and determine

a) the angle of twist between the ends of the shaft, and
b) maximum shearing stress in the shaft.



FBD + STAT. EQ. (5 pts)



CONST. EQ. (10 pts)

$$\phi = \left(\frac{TL}{GJ}\right)_{AB} + \left(\frac{TL}{GJ}\right)_{BC} = \boxed{3,25 \times 10^{-3} \text{ rad.} \equiv 0,186^\circ}$$ (4 pts)

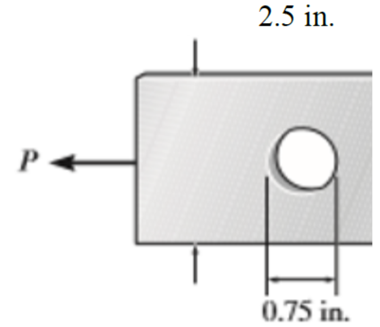
$$\tau_{max} = \tau_{AB} = \frac{T_{AB}\, c_{AB}}{J_{AB}} =$$

(6 pts)

$$\Rightarrow \boxed{\tau_{max} = 40,74 \text{ MPa}}$$

You can also check the maximum shear stress in shaft BC and it will be a about 5 MPa, less than that in shaft AB.

Problem 4

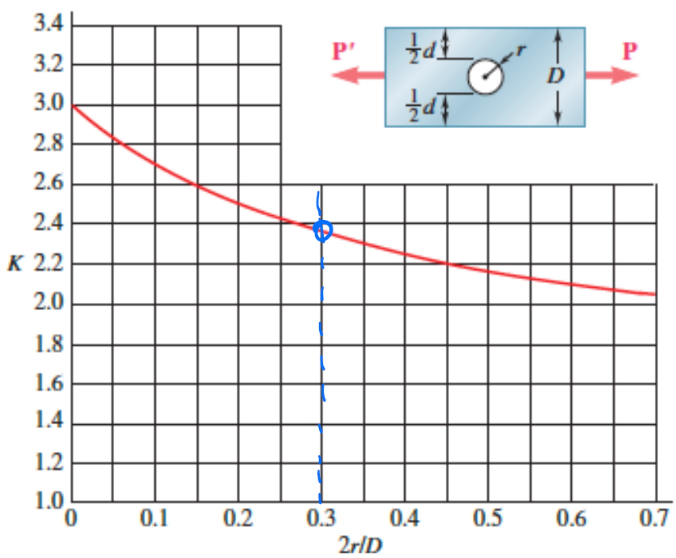


For the bar shown above, the stress concentration factor is closest to (circle one answer)

A) 2.0 B) 2.2 C) 2.4 D) 2.6 E) 2.8

10 points, right or wrong

$$\frac{2r}{D} = \frac{0.75}{2.5} = 0.3$$



Problem 5

Study the stress-strain diagram of an aluminum alloy given below and estimate the following material properties (state the units clearly):

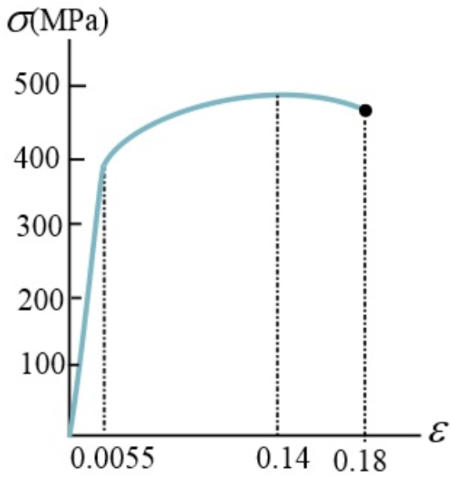
2.5 pts

i) Young's (elastic) Modulus: $\frac{400\ \text{MPa}}{0.0055} = 72727\ \text{MPa} = 72.7\ \text{GPa}$

ii) Yield Strength: ≈ 400 MPa

iii) Ultimate Strength: ≈ 500 MPa

iv) Fracture (rupture) strength: ≈ 460 MPa



Yield strength is the stress level at the end of linear part. Ultimate strength is that at the top point. Fracture strength is that of the end point.