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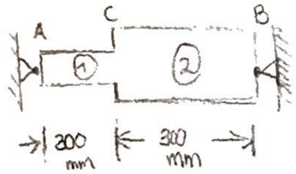
# Qualifying Exam - Mechanics

## ) Basic MoM:




a) Determine stress in AC & CB of the steel bar, currently @ -45°C, knowing that a close fit exists at both of the rigid supports when the temperature is 24°C.

Useful info: $E = 200$ GPA ; $\alpha = 11.7\times10^{-6}\ °C^{-1}$; $A_1 = 380\ mm^2$; $A_2 = 750\ mm^2$

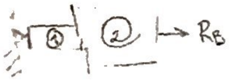
Beer, page 84 (7th ed)

Using superposition: Thermal expansion + Supports = complete problem

Thermal:

$$\delta_T = L\alpha\Delta T = (300\ mm + 300\ mm)\times(11.7\times10^{-6}\ °C^{-1})(-45°C - 24°C) = -0.484\ mm$$

Supports:




$$\delta_R = \frac{P_1L_1}{EA_1} + \frac{P_2L_2}{EA_2} \quad (\text{From inspection, } P_1 = P_2 = R_B)$$

$(1\ GPa = 1000\ N/mm^2)$

$$\delta_R = \frac{R_B}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2}\right) = \frac{R_B}{200\ GPa}\times\left(\frac{300\ mm}{380\ mm^2} + \frac{300\ mm}{750\ mm^2}\right) = R_B\times5.947\times10^{-6}\ \frac{mm}{N}$$

> Applying superposition + constraints: $\delta_T + \delta_R = 0 \Rightarrow 0 = -0.484 + R_B\times5.947\times10^{-6}\ \frac{mm}{N}$

$$\Rightarrow R_B = \frac{0.484\ mm}{5.947\times10^{-6}\ \frac{mm}{N}} = 81.39\ KN \qquad R_A = -81.39\ KN \text{ due to equilibrium.}$$

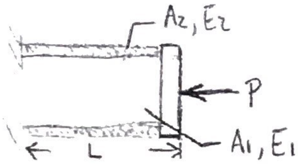
$$\sigma_1 = \frac{81.39\times10^3\ N}{380\ mm^2} = 214.2\ MPa;\quad \sigma_2 = 108.5\ MPa$$

b) Determine the strain in AC & BC, as well as the deformation of each section

$$\varepsilon_T = \frac{\delta_T}{L} = -8.07\times10^{-4} \qquad \varepsilon_1 = \frac{214\ MPa}{200\times10^3\ MPa} = 1.07\times10^{-3} \qquad \varepsilon_2 = \frac{108.5\ MPa}{200\times10^3\ MPa} = 5.43\times10^{-4}$$

$$\varepsilon_{AC} = \varepsilon_T + \varepsilon_1 = 2.63\times10^{-4} \qquad \varepsilon_{BC} = \varepsilon_T + \varepsilon_2 = -2.64\times10^{-4}$$

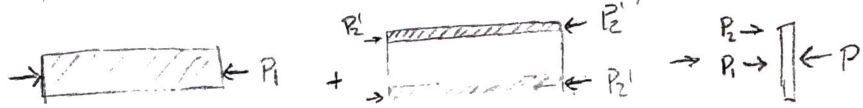
$$\delta_{AC} = \varepsilon_{AC}\cdot \underbrace{L_{AC}}_{300\ mm} = 0.079\ mm \qquad \delta_{BC} = \varepsilon_{BC}\cdot \underbrace{L_{BC}}_{300\ mm} = -0.079\ mm$$




cr, page 78 ($7^{th}$ ed)

a) Find the deformation of the tube and rod when force P is exerted in the rigid endcap as shown? (2)

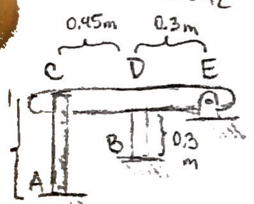
Distributed radially, summarized as $P_2$




$P = P_1 + P_2$ , $\delta_1 = \delta_2$ due to compatibility

$$\delta_1 = \frac{P_1 L}{E_1 A_1} = \frac{P_2 L}{E_2 A_2} = \delta_2 \Rightarrow P_1 = \frac{P_2 E_1 A_1}{E_2 A_2} \Rightarrow P = P_2\left(1 + \frac{E_1 A_1}{E_2 A_2}\right) \Rightarrow \boxed{P_2 = \frac{P}{\left(1+\frac{E_1 A_1}{E_2 A_2}\right)}}$$

$$\boxed{P_1 = \frac{P E_1 A_1}{\left(1+\frac{E_1 A_1}{E_2 A_2}\right) E_2 A_2}}$$




Rigid bar CDE, attached to a pin support @ E, rests on a 30 mm Ø brass cyl. BD. A 22 mm Ø steel rod AC passes through a hole in the bar, secured by a nut @ 20°C. The temp. of BD is raised to 50°C, while AC remains @ 20°C. Determine the stresses.

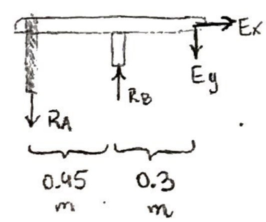
eer, page 87 ($7^{th}$ ed)

$E_{steel} = 200\ GPa$

$\alpha_{steel} = 11.7 \times 10^{-6}\ °C^{-1}$

$E_{brass} = 105\ GPa$

$\alpha_{brass} = 20.9 \times 10^{-6}\ °C^{-1}$




$$\Sigma M_E = -0.3m \cdot R_B + 0.75m \cdot R_A = 0$$

$$\Rightarrow \boxed{R_A = \frac{0.3}{0.75} R_B = 0.4 R_B}$$

$$\delta_{T_{brass}} = \Delta T \alpha L = (50-20)°C\,(0.3m)(20.9 \times 10^{-6}\ °C^{-1}) = 1.88 \times 10^{-4}\ m$$


Due to geometry

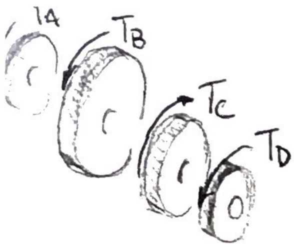
$$\frac{\delta c}{0.75} = \frac{\delta_D}{0.3} \Rightarrow \boxed{\delta_D = 0.4 \delta c}$$

$$\delta c = \frac{R_A L}{A_{steel} E_{steel}} = \frac{R_A \cdot (0.9m)}{\frac{\pi}{4}(22 \times 10^{-3} m)^2 \cdot 200 \times 10^9 Pa} = 1.184 \times 10^{-8} R_A$$

$$\delta_D = 0.4\,\delta c = 4.735 \times 10^{-9} R_A \qquad \delta_{BD} = \frac{R_B L}{E_{Brass} A_{Brass}} = \frac{R_B \cdot (0.3m)}{\frac{\pi}{4}(30 \times 10^{-3} m)^2 \times 105 \times 10^9 Pa} = 4.04 \times 10^{-9} R_B$$

$$\delta_1 = \delta_D + \delta_{BD} = \delta_T \Rightarrow 4.735 \times 10^{-9} R_A + 4.04 \times 10^{-9} R_B = 1.88 \times 10^{-4} m \qquad R_A = 0.4 R_B$$

$$\Rightarrow 5.934 \times 10^{-9} R_B = 1.88 \times 10^{-4} m \Rightarrow R_B = 31.68 \times 10^3 N \quad \text{thus } \sigma_{Brass} = \frac{31.68\ kN}{\frac{\pi}{4}(30 \times 10^{-3} m)^2} = 44.82\ MPa$$




$\overline{AB} = 0.9m$ $\quad T_A = 6KNm$
$\overline{BC} = 0.7m$ $\quad T_B = 14KNm$
$\overline{CD} = 0.5m$ $\quad T_C = 26KNm$
$\quad T_D = 6KNm$

Shaft BC is hollow w. $Ø_i = 90mm$ & $Ø_o = 120mm$.
Shafts AB & CD are solid of $Ø = d$ ..
Determine a) max. & min. shear stress in BC
b) Required value for d if $\tau_{all} = 65MPa$

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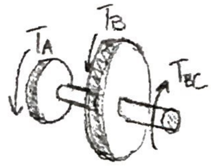
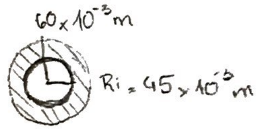
a)


$T_A = T_{AB} = 6KNm$ ;




$T_A + T_B - T_{BC} = 0$
$T_{BC} = T_A + T_B = 20KNm$

$\tau = \dfrac{Tr}{J}$ In Shaft BC

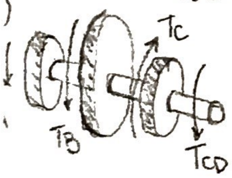



$J = \dfrac{\pi}{2}\left[(60\times10^{-3})^4 - (45\times10^{-3})^4\right] = 1.392\times10^{-5}\,m^4$

Max torque will be @ $Ø_o \Rightarrow r = 60\times10^{-3}m$ ; Min torque will be in $Ø_i \Rightarrow 45\times10^{-3}m = r$

$\tau_{max} = \dfrac{20KNm \cdot 60\times10^{-3}m}{1.392\times10^{-5}m^4} = 82.23MPa$ ; $\tau_{min} = \dfrac{20KNm \cdot 45\times10^{-3}m}{1.392\times10^{-5}m^4} = 64.67MPa$

b)




$T_{CD} + T_A + T_B - T_C = 0 \Rightarrow T_{CD} = T_C - T_A - T_B = 6KN$

$T_{AB} = T_{CD} \Rightarrow 65MPa = \dfrac{6KNm \cdot (d/2)}{\frac{\pi}{2}(d/2)^4} \Rightarrow \dfrac{6\times10^3 Nm}{\frac{\pi}{2}\cdot 65\times10^6 \frac{N}{m^2}} = (d/2)^3$

$\Rightarrow \dfrac{d}{2} = \sqrt[3]{5.876\times10^{-5}m^3} = 3.888\times10^{-2}m \Rightarrow d = 77.76\,mm$

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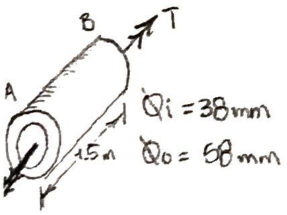


Shaft AB is made of elastoplastic steel with $G = 77GPa$ & $\tau_y = 145MPa$. A torque T is gradually increased in magnitude. Determine torque T and angle of twist $\phi$ at which a) Yield occurs b) Deformation is fully plastic..

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a) $\tau_y = \dfrac{T \cdot r}{J}$ ; $J = \dfrac{\pi}{2}\left[\left(\dfrac{58\times10^{-3}m}{2}\right)^4 - \left(\dfrac{38\times10^{-3}m}{2}\right)^4\right] = 9.063\times10^{-7}m^4$

Yield will first occur at outer radius, where $r = Ø_o/2$

Thus, $145\times10^6 Pa = \dfrac{T_o \cdot (58\times10^{-3}m/2)}{9.063\times10^{-7}m^4} \Rightarrow T_o = 4.532\,KN.m$ ; $\phi_o = \dfrac{\gamma_y L}{(Ø_o/2)} = \dfrac{L.\tau_y}{G(Ø_o/2)} = 9.74\times10^{-2}\,rad$

b) Deformation will be fully plastic when inner radius reaches $\tau_y$

$\phi_f = \dfrac{L.\tau_y}{\left(\frac{Ø_i}{2}\right).G} = 0.1487\,rad$ ; $T_y = 2\pi\displaystyle\int_{Ø_i/2}^{Ø_o/2} \rho^2 \tau_y\, d\rho = \dfrac{2\pi}{3}\rho^3\tau_y\Big|_{Ø_i/2}^{Ø_o/2} = \dfrac{2}{3}\pi\,(145\times10^6 Pa)\left[\left(\dfrac{58\times10^{-3}m}{2}\right)^3 - \left(\dfrac{38\times10^{-3}m}{2}\right)^3\right]$

$= 5.324\,KNm$

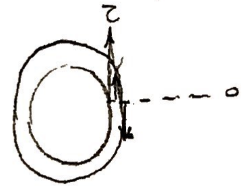
) determine the residual stresses and permanent angle of twist after the torque found in part b)c is removed.

Applying torque in opposite direction assuming elastic behavior to model elastic recovery of material yields:

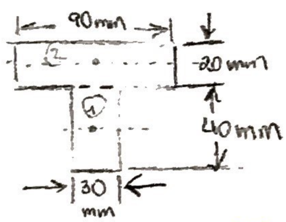
$$\tau_{max} = \frac{T_o(\varnothing_o/2)}{J} = \frac{5.324\times10^3\,Nm \cdot \left(\frac{58\times10^{-3}m}{2}\right)}{9.063\times10^{-7}\,m^4} = 170.4\ MPa$$

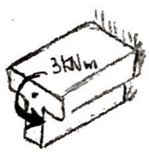
$$\tau_{min} = \frac{T_o(\varnothing_i/2)}{J} = 111.6\ MPa \qquad \phi = \frac{TL}{GJ} = \frac{5.324\times10^3 Nm \cdot 1.5m}{9.063\times10^{-7}m^4 \cdot 77\times10^9 Pa} = 0.1144\ rad$$

Thus $\tau_{r_i} = (145 - 111.6) = 33.4\ MPa$, $\tau_{r_o} = (145 - 170.4\ MPa) = -25.4\ MPa$




$$\phi_r = (0.1487 - 0.1144)\,rad = 0.0343\ rad$$




E = 165 GPa

Determine a) Max. tensile & Compressive stresses
b) Radius of curvature

Find centroid & Moment of inertia

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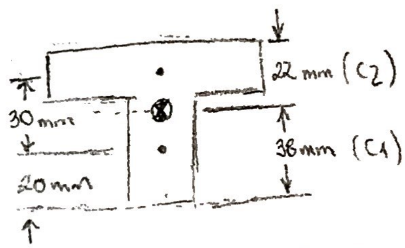
| | Area [mm²] | $\bar{y}$ [mm] | $A\bar{y}$ [mm³] |
|---|---|---|---|
| 1 | (40 x 30) = 1200 | 20 | 24 000 |
| 2 | (20 x 90) = 1800 | 50 | 90 000 |
| | 3000 | | 114 000 |

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 38\ mm$$




$$I_x = \Sigma(I_i + A_i d_i^2) = \frac{30\times40^3}{12} + 1200\cdot18^2 + \frac{90\times20^3}{12} + 1800\cdot12^2 = \boxed{8.68\times10^5\ mm^4} = 8.68\times10^{-7}\,m^4$$

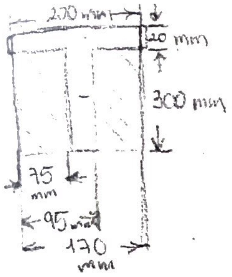
$d_1 = (38-20) = 18\ mm$

$d_2 = (22-10) = 12\ mm$

$$\sigma_T = \frac{Mc_2}{I_x} = \frac{3\times10^3\,Nm \cdot 2.2\times10^{-2}m}{8.67\times10^{-7}\,m^4} = 7.61\times10^7\,Pa = 76.1\ MPa$$

$$\sigma_c = -\frac{Mc_1}{I_x} = -\frac{3\times10^3\,Nm \cdot 3.8\times10^{-2}m}{8.67\times10^{-7}\,m^4} = -131.5\ MPa$$

$$\frac{1}{\rho} = \frac{M}{EI} \Rightarrow \rho = \frac{EI}{M} = \frac{165\times10^9 Pa \cdot 8.68\times10^{-7}m^4}{3\times10^3\,Nm} = 47.74\ m$$

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A T shaped steel beam has been strengthened by wood. $E_{wood} = 12.5\,GPa$ $E_{steel} = 200\,GPa$. A bending moment of $50\,KN.m$ is applied to the beam. Determine
a) Max. stress in the wood
b) Stress in the steel along top edge

$n = \dfrac{200}{12.5} = 16 \rightarrow$ Transform cross section (Horizontal)

Multiply steel width by 16 $\rightarrow$ Cross section made of wood

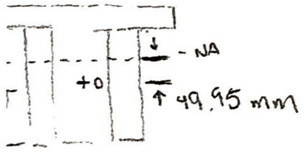
Centroid: Place origin @ height of 150 mm from bottom (i.e. middle of bottom rectangle)

$$\bar{Y} = \frac{\Sigma \bar{Y}A}{\Sigma A} = \frac{(150 + 20/2).\,[200 \times 16 \,.\, 20] + 0}{(2.(75) + 20.16)(300) + 20.(200 \times 16)} = 49.95\ mm$$

$$I_x = \Sigma(I_x + Ad^2) = \frac{(2.75 + 20.16)(300\,mm)^3}{12} + (2.75+20.16)(300)(49.95)^2 + \frac{(200.16)(20)^3}{12} + [200.16.20\,(150+10-49.95)^2$$

$$I_x = 1.409 \times 10^9\,mm^4 + 8.007 \times 10^8\,mm^4 = 2.210 \times 10^9\,mm^4 \ \frac{1\,m^4}{(1000\,mm)^4} = 2.21 \times 10^{-3}\,m^4$$

a) Max. Stress $\rightarrow$ Point furthest away from NA.




$c_1 = (150 + 20) - 49.95 = 120.05\ mm$

$c_2 = (150) + 49.95 = 199.95\ mm$

$$\sigma_w = \frac{M c_2}{I} = \frac{50 \times 10^3\,Nm \,.\, 199.95 \times 10^{-3}\,m}{2.21 \times 10^{-3}\,m^4}$$

$\sigma_w = 4.52\ MPa$

b) Stress in steel on top edge $\sigma_{s_t} = \dfrac{M c_1}{I} = \dfrac{50 \times 10^3\,Nm \,.\, 120.05 \times 10^{-3}\,m}{2.21 \times 10^{-3}\,m^4} = 2.72\,MPa$

Due to transformed cross-section $\rightarrow \sigma_s = \sigma_{s_t} \cdot 16 = 43.5\ MPa$

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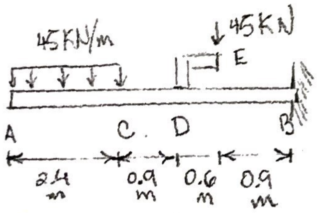


a) Draw shear & bending-moment diagrams. b) Determine the max. normal stress in the vicinity of point D.

De-couple structure in point D $\Rightarrow$ generates downward for of 45 KN & moment $0.6\,m \,.\, 45\,KN = 27\ KN.m$

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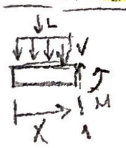
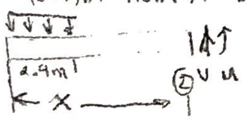



From A to C

$\Sigma F_y = 0 \Rightarrow V = L = 45\,KN/m \,.\, x$

$\Sigma M_1 = 0 \Rightarrow 45\,KN/m \,.\, x \,.\, \dfrac{x}{2} - M = 0 \Rightarrow M = -\dfrac{45}{2} x^2$

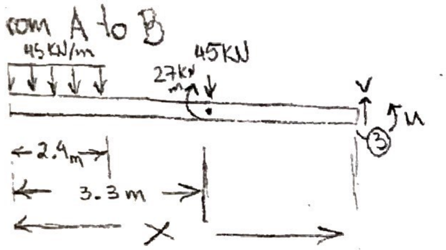
45 KN/m From A to D




$\Sigma F_y = 0 \Rightarrow 45\,\frac{KN}{m} \,.\, 2.4\,m - V = 0 \Rightarrow V = 108\ KN$

$\Sigma M_2 = 0 \Rightarrow -45\,\frac{KN}{m} \,.\, 2.4\,m \,.\, (x - 1.2\,m) + M = 0 \Rightarrow M = -108\,x + 129.6\ KNm$

rom A to B



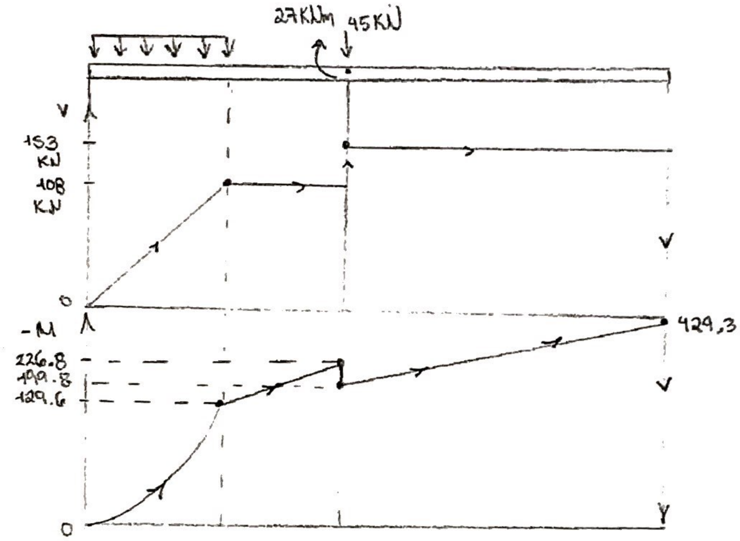

$$\Sigma F_y = 0 \Rightarrow -45\frac{KN}{m}\cdot 2.4m - 45KN + V = 0 \Rightarrow V = 153KN$$

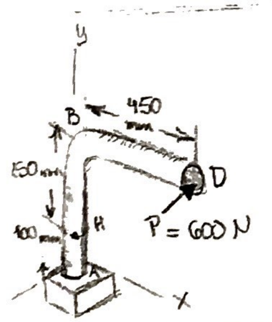
$$\Sigma M_3 = 0 \Rightarrow 45\frac{KN}{m}\cdot 2.4m\cdot(x-1.2m) - 27KNm + 45KN(x-3.3m) + M = 0$$

$$\Rightarrow M = -(108x - 129.6) + 27KNm - 45x + 148.5$$

$$= -153x + 305.1$$

Using equations found:




Load P is applied on lever as shown in the figure. Knowing that $\overline{AB}$ has a diameter of 30 mm, determine:

a) Normal & shearing stresses located @ H (located w. sides parallel to x & y axes)

b) Principal planes & stresses @ H

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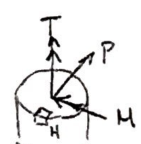
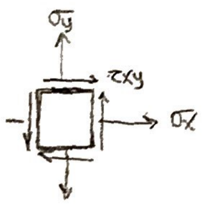
a) Load P will produce a momentum & torque in AB

$$T = 600N \cdot 0.45m = 270\ Nm$$

$$M = 600N \cdot 0.25m = 150\ Nm$$



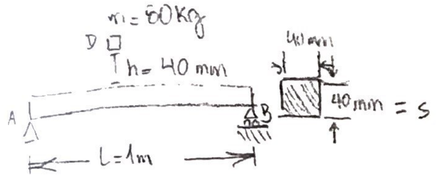

$$\sigma_y = \frac{Mc}{I} = \frac{150\,Nm \cdot 0.015m}{\frac{\pi}{4}(0.015m)^4} = 56.6\ MPa$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{270Nm \cdot 0.015m}{\frac{\pi}{2}(0.015m)^4} = 50.9\ MPa$$

b) $$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{0+56.6}{2} \pm \sqrt{\frac{56.6}{2} + 50.9} \Rightarrow \sigma_1 = 86.5\ MPa,\ \sigma_2 = -29.9\ MPa$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y} = -1.8 \Rightarrow 2\theta = \arctan(-1.8) = -60.93° \Rightarrow \theta = 30.46$$

(7)

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Block D of mass m is released from rest & falls distance h before striking midpoint C of the Al beam AB. Using E = 73 GPa, determine a) Max. deflection of C b) Max stress of beam.

Using beam deflection tables: $y_m = \frac{PL^3}{48EI}$ (*)

a) $EI = 73\times10^9\,Pa \cdot \frac{1}{12}(40\times10^{-3}m)(40\times10^{-3}m)^3 = 15.57\times10^3\,Nm^2$

At max. deflection point, $W = \underbrace{m.g}_{w}.(h + y_m)$ [Work on block] (A)

$w = 80\,kg \times 9.8\,\frac{m}{s^2} = 784\,N$  Strain energy at max. deflection point: $U = \frac{1}{2}P.y_m$ (B)

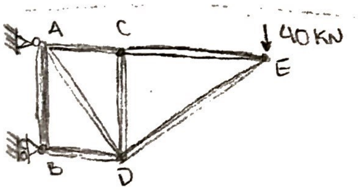
Using (*) => $P = \frac{48EIy_m}{L^3}$, thus, equating (A) & (B) due to energy balance assuming no losses:

$$\frac{24EIy_m^2}{L^3} = w.(h + y_m) \Rightarrow \frac{24EIy_m^2}{L^3} - wy_m - wh = 0 \Rightarrow 373.7\times10^3 y_m^2 - 784y_m - 31.36 = 0$$

Solving for $y_m$ → $1.027\times10^{-2}\,m$ Solution
→ $-8.171\times10^{-3}\,m$ → implies beam bent upwards, discard.

b) $y_m = \frac{PL^3}{48EI} = 1.027\times10^{-2}\,m$ => Solving for P => $P = 7675.4\,N$

$\sigma_{max} = \frac{Mc}{I}$ ; $M_{max} = \frac{1}{4}PL$ => $\sigma_{max} = \frac{PL.c}{4.\frac{1}{12}s^4} = \frac{7675.4N.(0.25m).(0.02m)}{\frac{1}{12}.(0.04m)^4} = 179.9\,MPa$



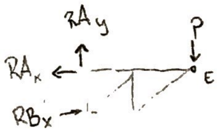

Members of the truss shown consist of sections of Al pipe with cross-sectional areas as shown. Using E = 73 GPa, determine the vertical deflection of point E caused by load P.

Beer (7th edition) page 794

$\overline{AB} = \overline{CD} = 0.8m$  $A_{AB} = A_{AD} = A_{AC} = A_{CE} = 500\,mm^2$

$\overline{AC} = \overline{BD} = 0.6m$  $A_{BD} = A_{CD} = A_{DE} = 1000\,mm^2$

$\overline{CE} = 1.5\,m$




$\Sigma F = 0 \Rightarrow RA_y = P = 40\,KN$

$RA_x = RB_x = 168\,KN$

$\Sigma M_A = 0 \Rightarrow -(2.1m)P + (0.8m)RB_x =$

$\Rightarrow RB_x = \frac{2.1}{0.8}P = 105\,KN$

B: $F_{AB}$, $RB_x$, $F_{BD}$

$F_{BD} = 105\,KN$

$F_{AB} = 0$

D: $F_{AD}$, $F_{DC}$, $F_{DE}$, $F_{BD}$ (0.8, 1, 0.6)

$F_{DE}\left(\frac{1.5}{1.7}\right) - F_{AD}\left(\frac{0.6}{1}\right) - F_{BD} = 0$ (?)

$\Rightarrow F_{BD} = -105\,KN$

$F_{AD}\left(\frac{0.8}{1}\right) + \cancel{F_{DC}} + F_{DE}\left(\frac{0.8}{1.7}\right) = 0$

$\Rightarrow F_{AD} = \frac{-F_{DE}}{1.7}$

$= 50\,KN$

E: 40 KN, $F_{CE}$, $F_{DE}$ (1.5m, 0.8m, 1.7m)

$\Sigma F_x = 0 \Rightarrow F_{CE} = -F_{DE}\left(\frac{1.5}{1.7}\right)$

$\Sigma F_y = 0 \Rightarrow F_{DE}\left(\frac{0.8}{1.7}\right) = -40$

$\Rightarrow F_{DE} = -85\,KN$

$\Rightarrow F_{CE} = 75\,KN$

C: $F_{AC}$, $F_{CE}$, $F_{CD}$

$F_{CD} = 0$

$F_{AC} - F_{CE} = 0 \Rightarrow F_{AC} = 75\,KN$

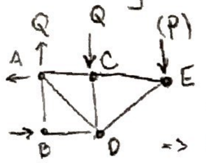
$$U = \sum \frac{F_i^2 L_i}{2A_iE} = \frac{1}{2E}\sum \frac{F_i^2 L_i}{A_i} = \frac{4.752 \times 10^7 \cdot N^2 m}{2.73 \times 10^9 \, Pa} = 325 \, \frac{N}{m}$$

$$\frac{1}{2} P y_E = U \Rightarrow y_E = \frac{U \cdot 2}{P} = 0.01625 \, m \, \#$$

---

Using the same setup as the previous problem, determine the vertical deflection of joint C._
(Beer 7th edition, page 812)

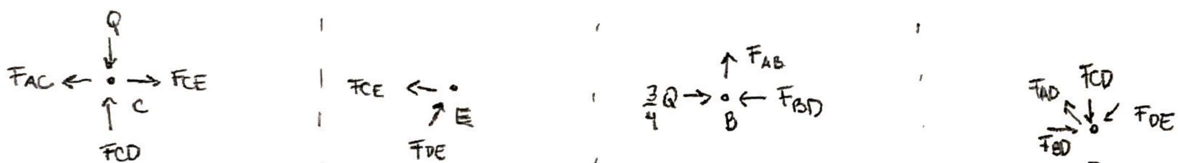
Add dummy load to joint C




Per Castigliano's theorem: $y_C = \sum \left(\frac{F_i L_i}{A_i E}\right)\frac{\partial F}{\partial Q}$

$\Sigma M_x = 0$

$\Rightarrow 0.6\,Q - 0.8\,R_{Bx} = 0 \Rightarrow R_{Bx} = \frac{0.6}{0.8} Q = \frac{3}{4} Q \, \# \Rightarrow R_{Ax} = 3/4\,Q$




$F_{CD} = Q$

$F_{AC} = 0$

$F_{DE} = 0$

$F_{CE} = 0$

$F_{AB} = 0$

$F_{BD} = \frac{3}{4} Q$

$F_{BD} - F_{AD}\left(\frac{0.6}{1}\right) - F_{DE}\left(\frac{1.5}{1.7}\right) = 0$

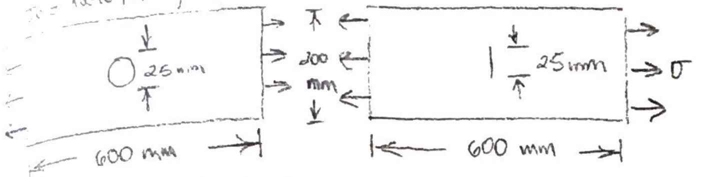
$\frac{(F_{BD})}{0.6} = F_{AD} = \frac{30}{24} Q = \frac{5}{4} Q \, \#$

| Member | $F_i$ | $\partial F_i/\partial Q$ | $L_i$ [m] | $A_i$ [$m^2$] | $\left(\frac{F_i L_i}{A_i}\right)\frac{\partial F_i}{\partial Q}$ |
|---|---|---|---|---|---|
| AB | 0 | 0 | 0.8 | $600 \times 10^{-6}$ | 0 |
| AC | 75KN | 0 | 0.6 | " | 0 |
| AD | 50 KN + 5/4 Q | 5/4 | 1 | " | $6.25 \times 10^4 + 3125\,Q$ ($\times 10^3$) |
| BD | -105 KN + 3/4 Q | 3/4 | 0.6 | $1000 \times 10^{-6}$ | $-6.3 \times 10^4 + 337.5\,Q$ |
| CD | 0 + Q | 1 | 0.8 | " | 800 Q |
| CE | 75KN + 0 | 0 | 1.5 | $500 \times 10^{-6}$ | 0 |
| DE | -85KN + 0 | 0 | 1.7 | $1000 \times 10^{-6}$ | 0 |

$-500 \times 10^3 + 4263\,Q \Rightarrow \boxed{y_C = -6.85 \times 10^{-4} \, m}$

(?)

$\sigma_u = 1290$ MPa ; $\sigma_y = 1150$ MPa ; $K_{IC} = 77$ MPa $m^{1/2}$




EMA 506 Homework 6, Fall 2018

Energy methods + Theory of elasticity + other topics (9)

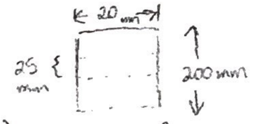
The two plates shown are each 20 mm thick and are made of steel. One plate contains a circular hole and the other a central crack. The applied stress $\sigma$ is caused by an axial force P.

a) Determine the force P that will cause yielding of the plate with the hole.

b) Determine the approx. force P that will cause the plate with the hole to break.

c) Determine the force that will break the plate with the crack to fail.

a) Using stress conc. factor: $\frac{2r}{D} = \frac{25}{200} = \frac{1}{8} \Rightarrow K_t = 2.66$




$\sigma_y = K_t \sigma = 2.66 \cdot \frac{P}{A}$ , $A = (200\times10^{-3} m - 25\times10^{-3} m)(20\times10^{-3} m) = 3.5\times10^{-3} m^2$

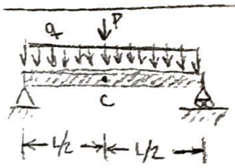
thus $1150 \times 10^6 Pa = \frac{2.66 \cdot P_y}{3.5\times10^{-3} m^2} \Rightarrow P_y = 1.51 \times 10^6 N$ //

b) Similarly: $\sigma_U = K_t \sigma = \frac{2.66 \, P_U}{3.5\times10^{-3} m^2} = 1290 \times 10^6 Pa \Rightarrow P_U = 1.69 \times 10^6 N$ //

c) $K_{IC} = \sigma_c \cdot y \sqrt{\pi a}$ Using geometry to determine y: $a = 12.5$ mm , $c = 100$ mm $\Rightarrow \frac{a}{c} = 0.125$

$y = 1.01 \Rightarrow 77 MPa\, m^{1/2} = 1.01 \cdot \sigma_c \cdot \sqrt{\pi \cdot 12.5\times10^{-3} m} \Rightarrow \sigma_c = 384.7 MPa = \frac{P}{(200 mm)(20 mm)}$

$\Rightarrow P = 1.54 \times 10^6 N$ //

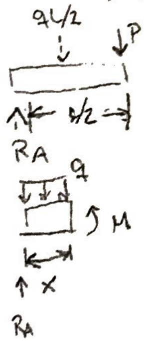



A simply supported beam has a uniform load of $q = 1.5$ Kips/ft over its length, as well as a conc. load $P = 5$ Kips that acts on the mid-point of the beam. $L = 8ft$, $E = 30 \times 10^6$ psi ; $I = 75$ in$^4$.

a) Find the complimentary strain energy of the beam and derive w.r.t P to determine the displacement of the mid-point.

b) Use Castigliano's 2nd theorem to determine the deflection of C.

Due to symmetry:




$\Rightarrow R_A = \frac{P}{2} + \frac{qL}{2} \Rightarrow M - R_A x + \frac{qx^2}{2} = 0 \Rightarrow M = R_A x - \frac{qx^2}{2}$

$= \frac{Px}{2} + \frac{qLx}{2} - \frac{qx^2}{2}$

$$U = \int \frac{M^2 dx}{2EI} = 2 \int_0^{L/2} \frac{1}{2EI} \left( \frac{Px}{2} + \frac{qLx}{2} - \frac{qx^2}{2} \right)^2 dx$$

Integrating (lengthy process)

$$= \frac{P^2 L^3}{96EI} + \frac{5PqL^4}{384EI} + \frac{q^2 L^5}{240EI}$$

$$\delta_c = \frac{\partial U}{\partial P} = \frac{PL^3}{48EI} + \frac{5qL^4}{384}$$ //

o) $\frac{\partial M}{\partial P} = \frac{x}{2} \Rightarrow \delta_c = \int \left(\frac{M}{EI}\right)\left(\frac{\partial M}{\partial P}\right) dx = 2\int_0^{L/2} \frac{1}{EI}\left(\frac{Px}{2} + \frac{qLx}{2} - \frac{qx^2}{2}\right)\left(\frac{x}{2}\right) dx$

$= \frac{2}{EI}\int_0^{L/2}\left(\frac{Px^2}{4} + \frac{qLx^2}{4} - \frac{qx^3}{4}\right) dx = \frac{2}{EI}\left[\frac{Px^3}{12} + \frac{qLx^3}{12} - \frac{qx^4}{16}\right]\Bigg|_0^{L/2}$

$= \frac{2}{EI}\left[\frac{PL^3}{96} + \frac{qL^4}{96} - \frac{qL^4}{256}\right] = \frac{2}{EI}\left[\frac{PL^3}{96} + \left(\frac{5q}{768}\right)L^4\right] = \frac{PL^3}{48EI} + \frac{5L^4q}{384EI}$

---

A plane displacement field has the components $u = -a_1(y^2 + \nu x^2)$ ; $v = 2a_1xy$ ($a_1 \equiv$ Constant)
Determine the stresses, and sketch them acting on boundaries $-a \le x \le a$ ; $-b \le y \le b$
(Homework 7 - EMA 506 Fall 2018)

1.

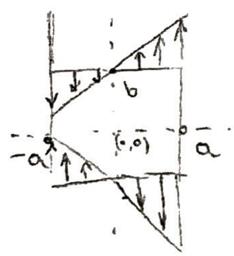
$\varepsilon_x = \frac{\partial u}{\partial x}$ $\quad \varepsilon_y = \frac{\partial v}{\partial y}$ $\quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

$\hookrightarrow -2\nu a_1 x \quad \hookrightarrow 2a_1x \quad \hookrightarrow -2a_1y + 2a_1y = 0$

$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = 0$ ; $\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{E}{1-\nu^2}(2a_1x - 2\nu^2 a_1 x) = \frac{E}{\cancel{1-\nu^2}}\left[(2a_1x)\cancel{(1-\nu^2)}\right]$

$= 2Ea_1x$

$\tau_{xy} = G\gamma_{xy} = 0$

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2. Examine the field below and see if it is a valid solution for a plane elasticity problem. If not, what condition isn't met? For simplicity, let Poisson's ratio be zero and assume $a_1$ is a constant. $u = a_1xy^2$ ; $v = -a_1x^2y$

$\varepsilon_x = \frac{\partial u}{\partial x}$ $\quad \varepsilon_y = \frac{\partial v}{\partial y}$ $\quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

$\hookrightarrow \varepsilon_x = a_1y^2 \quad \hookrightarrow \varepsilon_y = -a_1x^2 \quad \hookrightarrow \gamma_{xy} = 2a_1xy - 2a_1xy = 0$

Check compatibility: $\frac{\partial^2\varepsilon_x}{\partial y^2} + \frac{\partial^2\varepsilon_y}{\partial x^2} = \frac{\partial^2 \cancel{\gamma_{xy}}}{\partial x\partial y}$ $\qquad \frac{\partial\varepsilon_x}{\partial y} = 2a_1y \Rightarrow \frac{\partial^2\varepsilon_x}{\partial y^2} = 2a_1$

$2a_1 - 2a_1 = 0$ satisfies compatibility! $\qquad \frac{\partial\varepsilon_y}{\partial x} = -2a_1x \Rightarrow \frac{\partial^2\varepsilon_y}{\partial x^2} = -2a_1$

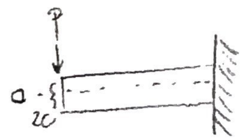
Check equilibrium $\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} = 0$ ; $\frac{\partial\sigma_y}{\partial y} + \frac{\partial\tau_{xy}}{\partial x} = 0$ Satisfies equilibrium!

∴ VALID SOLUTION

... stress in the cantilever beam is $\sigma_x = \frac{3Pxy}{2c^3}$

a) Use this equation & theory of elasticity to determine $\tau_{xy}$

b) What can be said about $\sigma_y$?



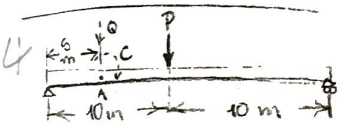
MAE6CG - Homework 7 - Fall 2018

a) Using equilibrium: $\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \Rightarrow -\frac{\partial \sigma_x}{\partial x} = \frac{\partial \tau_{xy}}{\partial y}$  $\frac{\partial \sigma_x}{\partial x} = \frac{3Py}{2c^3}$

thus $-\int \frac{3Py\,dy}{2c^3} = \boxed{\tau_{xy} = -\frac{3}{4}\frac{Py^2}{c^3} + \varphi(x)}$  Use BC: @ $y=c$, $\tau_{xy}=0$

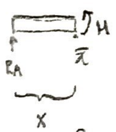
$0 = -\frac{3}{4}\frac{Pc^2}{c^3} + \varphi(x) \Rightarrow \frac{3P}{4c} = \varphi$ (constant) $\Rightarrow \tau_{xy} = -\frac{3Py^2}{4c^3} + \frac{3P}{4c}$ //

b) $\frac{\partial \tau_{xy}}{\partial x} = 0 \Rightarrow \sigma_y = \varphi(x)$ due to eq.  Use BC: @ $y=c$ $\sigma_y = 0 \Rightarrow \sigma_y = 0$ //

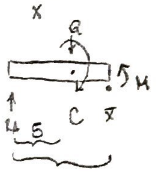
4 

Consider the simply supported beam with a load P applied at the center. Determine the deflection & rotation of point A.-

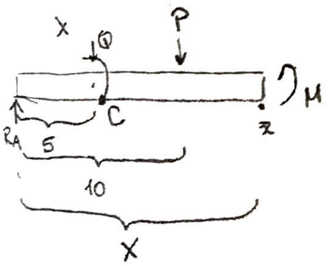
Add dummy moment C & dummy load Q in point A. Due to symmetry,



$\Sigma M_x = M - R_A x = 0 \Rightarrow M = R_A x$ (A)

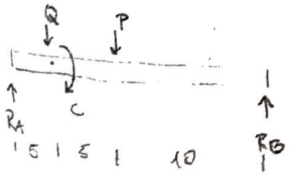


$\Sigma M_x = M - R_A x + (x-5)Q - C = 0 \Rightarrow M = R_A x - Q(x-5) + C$ (B)



$\Sigma M_x = M - C - R_A x + Q(x-5) + P(x-10) = 0.$

$\Rightarrow M = R_A x - Q(x-5) - P(x-10) + C$ (C)



$\Sigma M_B = 0 = -R_A L + Q \cdot \frac{3L}{4} + \frac{PL}{2} - C \Rightarrow \boxed{R_A = \frac{3Q}{4} + \frac{P}{2} - \frac{C}{L}}$ (*)

Substitute (*) into (A), (B) & (C)

$$* = \int_0^L \frac{M^2}{2EI} dx$$

$$= \int_0^5 \frac{\{[(P/2) + (3Q/4) - (C/L)]x\}^2 dx}{2EI} + \int_5^{10} \frac{\{[(P/2)+(3Q/4)-(C/L)]x - Q(x-5)+C\}^2 dx}{2EI}$$

$$+ \int_{10}^{20} \frac{\{[(P/2)+(3Q/4)-(C/L)]x - Q(x-5) - P(x-10)+C\}^2 dx}{2EI}$$

According to Castigliano's 2nd Theorem

$$\delta_A = \frac{\partial U^*}{\partial Q} \quad ; \quad \theta_A = \frac{\partial U^*}{\partial C}$$

$$\delta_A = \int_0^5 \frac{\cancel{2}\{[(P/2)+(3Q/4)-(C/L)]x\,(3/4x)\}\, dx}{\cancel{2}EI} + \int_5^{10} \frac{\{[(P/2)+(3Q/4)-(C/L)]x - Q(x-5)+C\}\{\frac{3}{4}x-(x-5)\}dx}{EI}$$

$$+ \int_{10}^{20} \frac{\{[(P/2)+(3Q/4)-(C/L)]x - Q(x-5) - P(x-10)+C\}\{\frac{3}{4}x-(x-5)\}dx}{EI}$$

Similar procedure for $\theta_A$..

QE, 19年12月:

## Solid Mechanics

1. Given an eqn. Begin with Airy stress Function.
   Check compatibility & Equilibrium
   Homeworks
2. Impact Loading
3. Energy method (11.7) ~~环~~
4. ~~Eo~~ Parallel axis theorem w/ moment of inertia
5. Beam question; 6. stress element of the surface of a pipe. draw stress elements
7. Distributed Load
   Draw the shear & moment diagrams
8. Bulking, dirive critical laws (1st section of this chap). 14.
(9. Bending & work & shear
   distributed Load)

## Materials Processing

1. Maching questions about terms
2. If you change some variables how that affect your chip formation & the geometry of chip
3. How to manufacture the parts. What materials to use. / processe
4. Comparing 3D to IM. CNC. die casting
5. Manufacturing; material forming; Newtonian simplification.

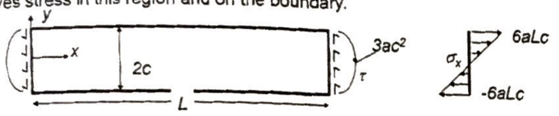
7.5 Plane Elasticity Polynomial Solutions in Cartesian Coordinates

Problem to be solved: Find F(x,y) that satisfies $\nabla^4 F=0$ and the boundary conditions.
Unfortunately, there is no single approach that will guarantee a closed-form solution to such a problem.
IDEA: guess at solutions & see what problems they solve! Use intuition, plus superposition. The text has several examples.

E.g. try $F(x,y) = a(xy^3 - 3c^2xy)$ [$a$ and $c$ are constants]

$$\therefore \sigma_x = \frac{\partial^2 F}{\partial y^2} = 6axy, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2} = 0, \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} = 3a(c^2 - y^2)$$

Take the problem domain to be $0 \le x \le L$, and $-c \le y \le c$. The solution above gives stress in this region and on the boundary.




Remarks:

(1) No assumptions have been made about the geometry of deformation - *e.g.* we haven't assumed that plane sections remain plane (they don't here)

(2) In terms of *stresses*, the solution happens to be **identical** to elementary (mechanics of materials) beam theory for an end-loaded cantilever. *i.e.* despite the simplifications made in beam theory, the MoM stress solution is *exact* for the given boundary conditions! (which actually are NOT the B.C.'s for a fixed beam). The beam theory assumptions were:

  (1) plane sections remain plane and $\perp$; and

  (2) bending and shear fields can be analyzed independently (one does not affect the other)

(3) To compute strains, use Hooke's Law: E.g. $\gamma_{xy} = \frac{1}{G}\tau_{xy} = \frac{2(1+\nu)}{E} 3a(c^2 - y^2)$

(4) Displacements can be computed by integrating the strains. This is a good exercise to carry out! If the beam is fixed at the right end, the deformed shape is:

Remarks:
(5) The B.C.'s for the elasticity solution require a parabolic distribution of shear stress on each end, and a symmetric linear distribution of normal stress at each end. This is not likely! However, the solution is useful because of St. Venant's principle:
At a distance ~2c away from the ends, the stress solutions for statically equivalent boundary conditions will be the same.
In general, polynomial solutions use a $n^{th}$-degree polynomial with unknown coefficients:

$F = a_0$ ⟵ gives no stress - omit
$+ a_1x + b_1y$ ⟵ gives no stress - omit
$+ a_2x^2 + b_2xy + c_2y^2$ ⟵ give constant stress
$+ a_3x^3 + b_3x^2y + c_3xy^2 + d_3y^3$ ⟵ gives linear stress
$+ \ldots$ ⟵ higher order terms must satisfy relations between coefficients $F=0$ that $\nabla^4\Phi=0$

terms up to 3rd order automatically satisfy the bi-harmonic eqn. Just adjust coeff's to match BC's

7.5 Plane Elasticity Polynomial Solutions in Cartesian Coordinates

Example:
Show that $F = -\frac{3P_1}{8c^2}\left(xy - \frac{xy^3}{3c^2}\right) + \frac{P_2}{8c^2}y^2$

is suitable for use as an Airy stress function and determine the stress components in the region $x>0$, $-c<y<c$. What kind of problem does this correspond to?
To be valid, we must show $\nabla^4 F = 0$

$$\nabla^4 F = \frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4}$$

$$\frac{\partial^4 F}{\partial x^4} = 0, \frac{\partial^4 F}{\partial x^2 \partial y^2} = 0, \frac{\partial^4 F}{\partial y^4} = 0 \therefore \text{valid}$$

Check the stresses:

$\sigma_x =$

$\sigma_y =$

$\tau_{xy} =$

**Think-Pair-Share**

Exercise: can you figure out what the loading situation is?
Hint: consider the domain $0<x<L$, $-c<y<c$

7.6 Calculating Displacements from Stress

Find the displacements that result from the Airy Stress Function: $F = -\frac{3P_1}{8c^2}\left(xy - \frac{xy^3}{3c^2}\right) + \frac{P_2}{8c^2}y^2$

$$\sigma_x = \frac{3P_1}{4c^4}xy + \frac{P_2}{4c^2}, \sigma_y = 0, \tau_{xy} = \frac{3P_1}{8c^2}\left(1 - \frac{y^2}{c^2}\right)$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \left(\frac{3P_1}{4Ec^4}xy + \frac{P_2}{4Ec^2}\right)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{-3\nu P_1}{4Ec^4}xy - \frac{\nu P_2}{4Ec^2}$$

$$\gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy} = \frac{3(1+\nu)P_1}{4Ec^2}\left(1 - \frac{y^2}{c^2}\right)$$

$$\frac{\partial u}{\partial x} = \varepsilon_x, \therefore u = \frac{3P_1}{8Ec^4}x^2y + \frac{P_2x}{4Ec^2} + f(y)$$

$$\frac{\partial v}{\partial y} = \varepsilon_y, \therefore v = \frac{-3\nu P_1}{8Ec^4}xy^2 - \frac{\nu P_2 y}{4Ec^2} + g(x)$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{xy}$$

$$\therefore \frac{3P_1}{8Ec^4}x^2 + f'(y) + \frac{-3\nu P_1}{8Ec^4}y^2 + g'(x) = \frac{3(1+\nu)P_1}{4Ec^2}\left(1 - \frac{y^2}{c^2}\right)$$

$$\left(\frac{3P_1}{8Ec^4}x^2 + g'(x)\right) + \left(\frac{3(2+\nu)P_1}{8Ec^4}y^2 + f'(y)\right) = \frac{3(1+\nu)P_1}{4Ec^2}$$

( function of x ) + ( function of y ) = constant

$$\therefore \frac{3P_1}{8Ec^4}x^2 + g'(x) = a_1, \therefore g(x) = -\frac{P_1}{8Ec^4}x^3 + a_1x + a_2$$

$$\frac{3(2+\nu)P_1}{8Ec^4}y^2 + f'(y) = \frac{3(1+\nu)P_1}{4Ec^2} - a_1, \therefore f(y) = -\frac{(2+\nu)P_1}{8Ec^4}y^3 + \frac{3(1+\nu)P_1}{4Ec^2}y - a_1y + a_3$$

# Gerardo's quals - Parte dos

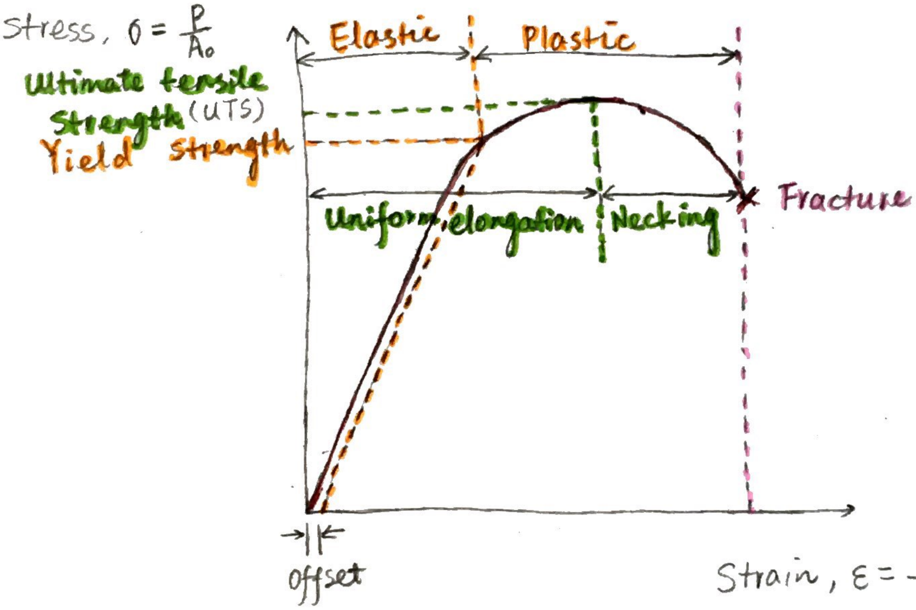
1) Compare Additive manufacturing techniques to a) CNC b) IM c) Die casting. Name pros & cons of each.-

2) a) Describe and draw the typical behaviour of a material tested under tension using a stress-strain curve. Name all properties and points of interest in the graph.-

b) Use your graph to define ductility and toughness.-

c) Draw a stress-strain diagram for a brittle material & a ductile material.-

2) a) Stress, $\sigma = \frac{P}{A_0}$



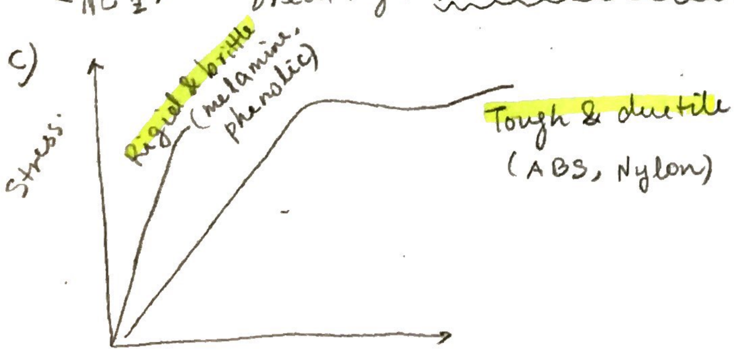

Strain, $\varepsilon = \frac{l - l_0}{l_0}$

b) $\boxed{\text{Ductility}}$: how much permanent strain the material undergoes before fracture, failure or breaking. Plastic deformation that the material undergoes before fracture.

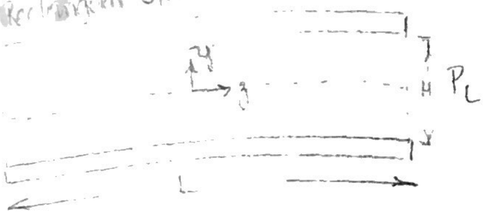
$\boxed{\text{Toughness}}$: Ability of the material to absorb energy before fracture, failure or breaking. Total area under the stress-strain curve.

<能量>

c)



Rectangular slit



Assuming steady state: cartesian coord. for cont. & momentum:

$\frac{\partial P}{\partial t} = 0$

Since there are no components to $v_x$ & $v_y$

Continuity equation: $0 + 0 + 0 + \frac{\partial (\rho v_z)}{\partial z} = 0 \Rightarrow \boxed{\frac{\partial v_z}{\partial z} = 0}$

Similarly, on momentum balance $\frac{\partial v_z}{\partial t}, \frac{\partial v_z}{\partial x}, \frac{\partial v_z}{\partial z}, v_y = 0$

$$0 = -\frac{\partial P}{\partial z} + \mu \left[\frac{\partial^2 v_z}{\partial y^2}\right] \Rightarrow \frac{\partial P}{\partial z} = \mu \left[\frac{\partial^2 v_z}{\partial y^2}\right] \qquad \frac{\partial P}{\partial z} \approx \frac{\Delta P}{L}$$

$$\mu \left[\frac{\partial^2 v_z}{\partial y^2}\right] = \frac{\Delta P}{L} \Rightarrow \int \frac{\Delta P}{L\mu} dy = \frac{\partial v_z}{\partial y} \Rightarrow \int \left(\frac{\Delta P y}{L\mu} + C_1\right) dy = v_z$$

$$\Rightarrow v_z = \frac{\Delta P y^2}{2L\mu} + C_1 y + C_2$$

BC: $y = \frac{1}{2}H \Rightarrow v_z = 0$ $\qquad \tau_{ij} = \mu \dot{\gamma}_{ij}$

$y = 0 \Rightarrow \frac{\partial v_z}{\partial y} = 0$

$$0 = \frac{\Delta P (1/2 h)^2}{2L\mu} + \cancel{C_1 (1/2 h)} + C_2 \Rightarrow \boxed{C_2 = -\frac{\Delta P h^2}{8L\mu}}$$

$$0 = \frac{\Delta P (0)}{L\mu} + C_1 \Rightarrow \boxed{C_1 = 0}$$

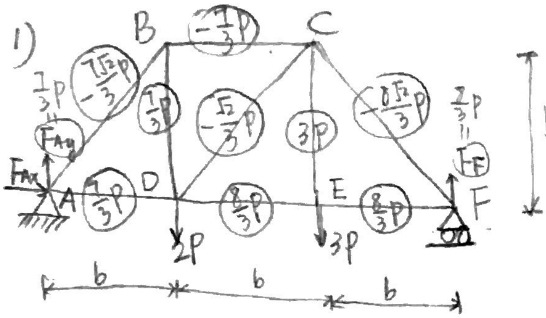
thus $\boxed{v_z = \frac{\Delta P y^2}{2L\mu} - \frac{\Delta P h^2}{8L\mu}}$ $\qquad \hat{v}_z = \frac{1}{h/2}\int_0^{h/2} v_z \, dy = \frac{1}{h/2}\left(\frac{\Delta P y^3}{6L\mu} - \frac{\Delta P h^2 y}{8L\mu}\right)\Big|_0^{h/2}$

$$\hat{v}_z = \frac{2}{h}\left[\frac{\Delta P h^3}{6L\mu(8)} - \frac{\Delta P h^3}{8L\mu(2)}\right] = \frac{\Delta P h^2}{L\mu}\left[\frac{1}{24} - \frac{1}{8}\right] = \frac{\Delta P h^2}{L\mu}\left[\frac{1-3}{24}\right]$$

$$= \overset{?}{-}\frac{\Delta P h^2}{L\mu 12}$$

Gerardo's quals' solution

1)




Q: 求 horizontal & vertical $\delta$ of B&C.

A: 整体 $F_{Ay}+F_F=5p$

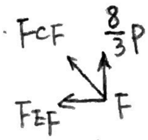
A点 (整) $\Sigma M=0=-2pb-3p\cdot 2b+F_F\cdot 3b$

$3bF_F-8bp=0$

$\boxed{F_F=\frac{8}{3}p}$

$\boxed{F_{Ay}=\frac{7}{3}p}$

F点



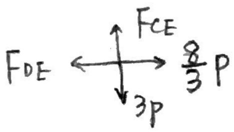

$\frac{F_{CF}}{\sqrt{2}}+F_{EF}=0$

$\frac{F_{CF}}{\sqrt{2}}+\frac{8}{3}p=0$

$\boxed{F_{EF}=\frac{8}{3}p}$

$\boxed{F_{CF}=-\frac{8\sqrt{2}}{3}p}$

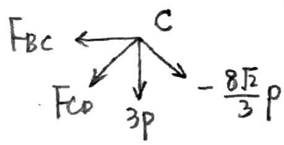
E点




$\boxed{F_{CE}=3p}$

$\boxed{F_{DE}=\frac{8}{3}p}$

C点




$F_{BC}+\frac{F_{CD}}{\sqrt{2}}=-\frac{8}{3}p$

$0=\frac{F_{CD}}{\sqrt{2}}+3p-\frac{8}{3}p$

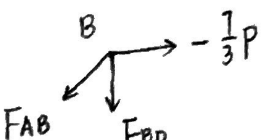
$\frac{F_{CD}}{\sqrt{2}}=\frac{8}{3}p-\frac{9}{3}p=-\frac{1}{3}p$

$\boxed{F_{CD}=-\frac{\sqrt{2}}{3}p}$

$F_{BC}-\frac{1}{3}p=-\frac{8}{3}p$

$\boxed{F_{BC}=-\frac{7}{3}p}$

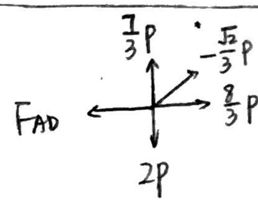
B点




$\boxed{F_{AB}=-\frac{7\sqrt{2}}{3}p}$

$\boxed{F_{BD}=\frac{7}{3}p}$

D点

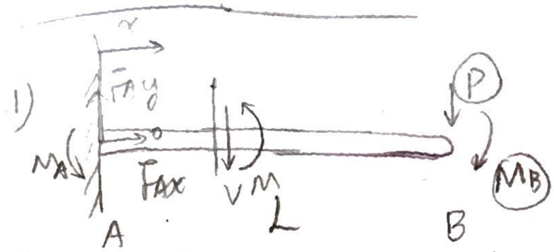



$\boxed{F_{AD}=-\frac{1}{3}p+\frac{8}{3}p=\frac{7}{3}p}$

| member | F | |
|---|---|---|
| AB | $-\frac{7\sqrt{2}}{3}p$ | |
| AD | $\frac{7}{3}p$ | |
| BC | $-\frac{7}{3}p$ | |
| BD | $\frac{7}{3}p$ | |
| CD | $-\frac{\sqrt{2}}{3}p$ | |
| CE | $3p$ | |
| CF | $-\frac{8\sqrt{2}}{3}p$ | |
| DE | $\frac{8}{3}p$ | |
| EF | $\frac{8}{3}p$ | |

solution

1)




$\boxed{Q}$ 求: $\delta_{v,B}$ 和 ~~$\theta_B$~~ $\theta_B$

$\boxed{A}$:

①

(卡氏)弯曲: $\delta = \int \frac{M(x)}{EI} \frac{\partial M(x)}{\partial F_i} dx$

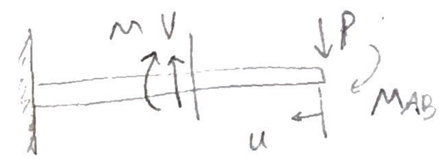
在AB段: 整体 $F_{Ax}=0$, $F_{Ay}=-P$, $M_A=M_B$

$$M = -F_{Ay}x + M_A$$
$$= Px + M_B$$
$$\frac{\partial M}{\partial P} = x$$
$$\delta_{v,B} = \int_0^L \frac{(Px+M_B)}{EI} \cdot x\,dx$$
$$= \frac{1}{EI}\int_0^L Px^2 + M_B x\,dx$$
$$= \frac{1}{EI}\left(\frac{1}{3}Px^3 + \frac{1}{2}M_B x^2\right)\Big|_0^L$$
$$= \frac{1}{EI}\left(\frac{1}{3}PL^3 + \frac{1}{2}M_B L^2\right)$$

②

$$\frac{\partial M}{\partial M_B} = 1$$
$$\theta_B = \int_0^L \frac{(Px+M_B)}{EI} \cdot 1\,dx$$
$$= \frac{1}{EI}\int_0^L (Px+M_B)\,dx$$
$$= \frac{1}{EI}\left(\frac{1}{2}PL^2 + M_B L\right)$$

法二, 从右往左



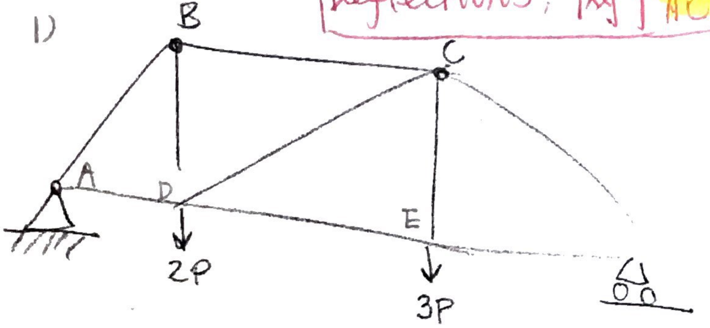

$\Sigma M = 0 = -M - M_{AB} - Pu$

$M = -Pu - M_B$

一样的(符号不一样)

Gerardo's goals
(看前页)

1) **Deflections, 网 | 能量法**




Determine horizontal & vertical deflections of B & C

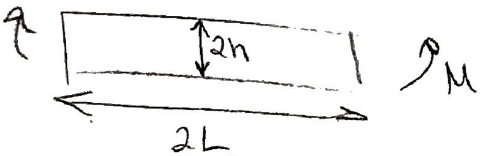
桁架问题用单位载荷法

$$\delta = \sum_{j=1}^{n} \frac{F_{Nj} l_j}{EA} \frac{\partial F_{Nj}}{\partial F_i}$$ ← 卡氏

**Airy stress Function**

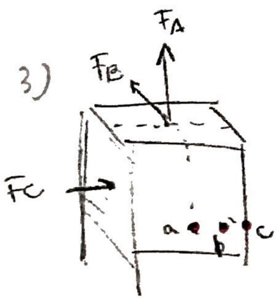
2) $\phi = Ay^3$

determine stress state & M using Airy stress function




**Combined Load**

3)

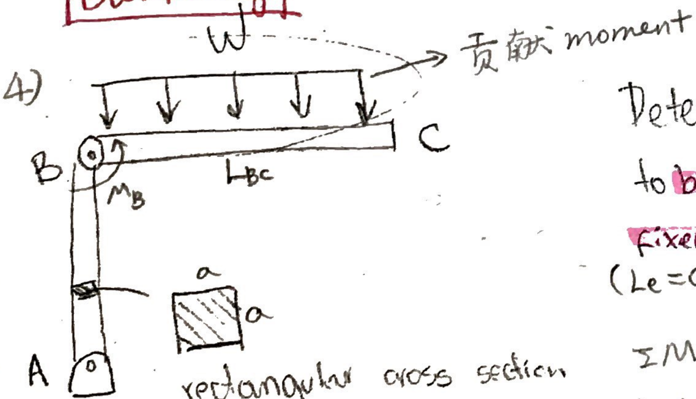



(看后面)

Determine stresses for points a, b & c

**Buckling**

4)




Determine max w that won't cause AB to buckle in the pin-pin condition & fixed-fixed condition. $(L_e = L)$ $(L_e = 0.5L)$

$$\Sigma M_C = 0 = wL_{BC}\left(\frac{L_{BC}}{2}\right) + M_B$$

$$\Rightarrow M_B = -\frac{1}{2} w L_{BC}^2$$

$$\sigma = \frac{My}{I} = \frac{\left(-\frac{1}{2} w L_{BC}^2\right)\frac{1}{2}a}{\frac{1}{12}a^4}$$

$$\parallel$$

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2}$$

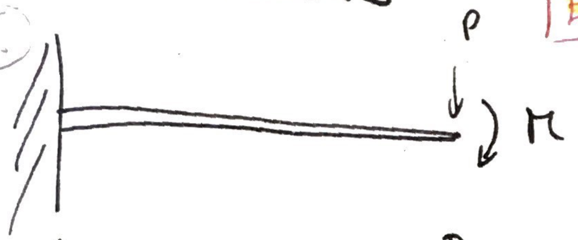
$$I = Ar^2$$

Sara's Quals – Mechanics

Summer 2019

$\boxed{\text{Energy method}}$

1)




**Determine** :

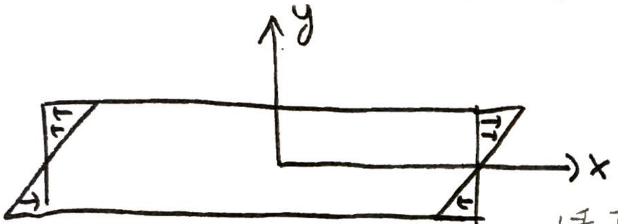
·) $\delta_{v,B}$ $\delta_{v,B}$ (B点 vertical)

·) $\phi_B$

组合变形卡氏: $\delta = \underbrace{\int \frac{F_N(x)}{EA}\frac{\partial F_N(x)}{\partial F_i}dx}_{\text{拉伸}} + \underbrace{\int \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial F_i}dx}_{\text{扭转}} + \underbrace{\int \frac{M(x)}{EI}\frac{\partial M(x)}{\partial F_i}dx}_{\text{弯曲}}$

$\boxed{\text{Airy stress Function}}$

2)




(看后面)

$\phi = Ay^3$

**Determine:**

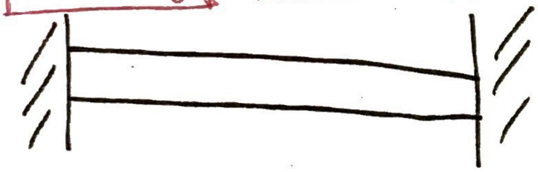
·) stresses

·) surface tractions , BCs boundary conditions
surface ~~tensions~~

·) Moment

$\boxed{\text{Bulkling}}$ & $\boxed{\text{Thermal effect}}$

3)



Pipe.

Fix-Fix: $L_e = 0.5L$

$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{4\pi^2 EI}{L^2}$, $\delta_p = \frac{P_{cr}L}{AE} = \frac{4\pi^2 EI\,L}{L^2 AE} = \frac{4\pi^2 I}{LA}$

$\delta_T = \alpha(\Delta T)L$

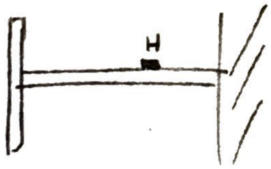
$\delta = 0 = \delta_T + \delta_P$

$0 = \alpha(\Delta T)L + \frac{4\pi^2 I}{LA} \Rightarrow \Delta T$

How much con (can) $\Delta T$ be to make pipe bulkle.

$\boxed{\text{Combined loads}}$

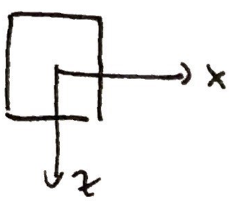
4)




(看最后一页)

4 Forces. Determine state of stress.
Draw state of stress on element.
state of stress

(看后面)




5) Impact Loading (看后面)

$\boxed{\text{Impact loads}}$

# Materials (100/100)

material:

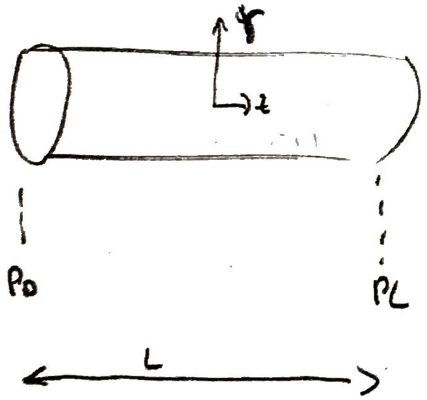
How are the following items now featured? manufactured which ~~navel~~ was used? (60 points)

- → Football Helmet: polycarbonate (strength & weight) // Injection molding
- → Coffee mug mug: ceramics // → Molding → Applying the handle → detailing → Glazing → Firing
- → PET Lego: ABS, Injection molding
- → Bottle: Injection molding / Blow molding // PET/HDPE/LDPE/PS
- → Dreamliner Fuselage Fuselage: composites // { Frames: Braiding; Resin transfer molding; compression molding; pultrusion; 还有 wet lay-out
  Innovative aluminum alloys, fiber-metal-laminates & graphite
- → Turbine Blade: Nickel-based $\boxed{\text{superalloys}}$ that incorporate chromium // investment casting
- → Toothbrush grip: plastics like PE // Injection molding
- → Bumper of car Injection molding // PC/ABS; PP; PVC; Nylon6; PS ...

⋮

2) Compare AM to CNC, IM, Die casting) (15 points)
Additive manufacturing
pros & cons

3) Pressure flow through a tube (Newtonian) (25 points)
Determine ~~[illegible]~~ $u_z(r)$ and Q.
$u_z(r)$




BPA Free

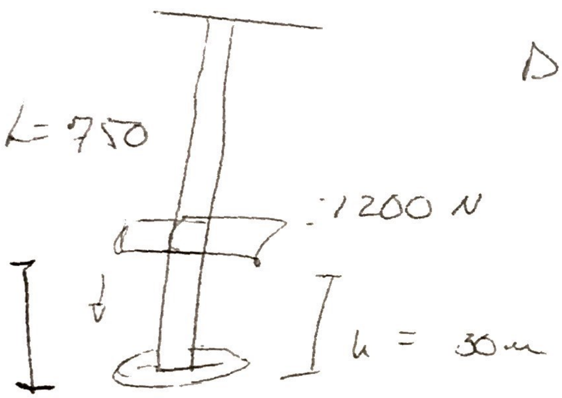
~~不是Polycarbonate的~~

PP

Acrylic

① $\phi = Ay^3$ → Max stress


②




D = 15 mm

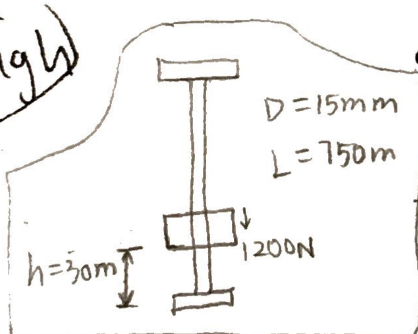
① ~~axial load and deformation (static)~~ deformation static

② ~~deformation~~ under ~~impact load~~ under impact Load

③ Max Load and stress

• 1200N collar loaded slowly and then dropped from height h

$U = mgh$




$\boxed{\text{Impact loads}}$

$ax^2+bx+c=0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$A = \frac{\pi}{4}(15)^2 = 176.71\ mm^2 = 176.71 \times 10^{-6}\ m^2$

$U_m = \frac{P_m^2 L}{2EA}$ ①   $\frac{1}{2} P_m \Delta_m = U_m$ ②

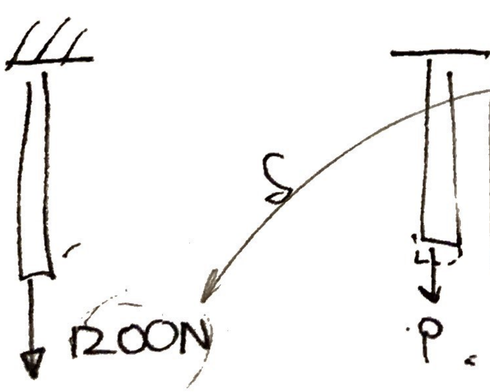
$H = h + \Delta m$ ③

$U_m = GH = G(h + \Delta m)$ ④

联立①②④

$\sigma_m = \frac{P_m}{A}$

联立①②④ $\begin{cases} U_m = \frac{P_m^2 L}{2EA} \\ \frac{1}{2} P_m \Delta_m = U_m \\ U_m = G(h+\Delta m) \end{cases}$




$\delta = \frac{PL}{AE}$

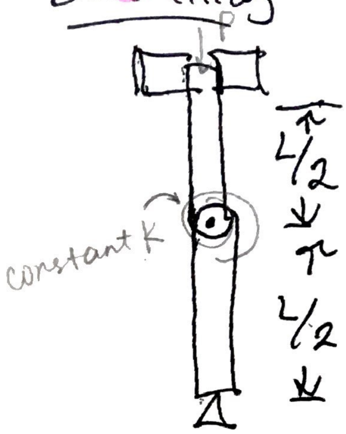
Static

$LP_m^2 - 2GLP_m - 2EAGh = 0$

$\Rightarrow P_m = \frac{2GL \pm \sqrt{4G^2L^2 - 8LEAGh}}{2L}$

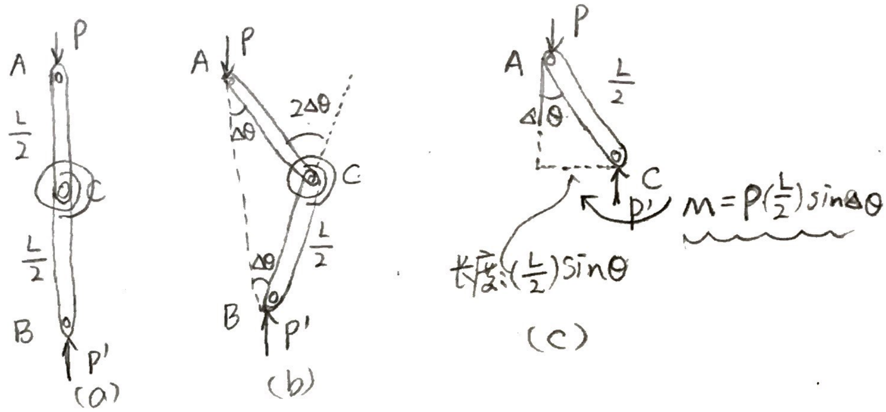
$\Rightarrow \Delta_m = \frac{2U_m}{P_m}$

③ Buckling




Define $P_{critical}$ for column with stiffness K and hinge in center.

Bulkling




由图(C)

$P, P'$

$M = P \cdot (\frac{L}{2}) \sin\Delta\theta$

∵ The angle of deflection of the spring is $2\Delta\theta$

∴ the moment of couple $M = K(2\Delta\theta)$

Critical load: $P_{cr} (\frac{L}{2}) \sin\Delta\theta = K(2\Delta\theta)$

∵ $\sin\Delta\theta \approx \Delta\theta$ when the displacement of C is very small.

$$P_{cr} = \frac{4K}{L}$$

∴ System is stable when $P < P_{cr}$, unstabe when $P > P_{cr}$

---

上接右侧

中间处. $V = F_c - wx$

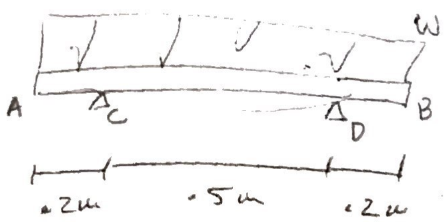
$M = F_c(x - 0.2) - \frac{1}{2}wx^2$

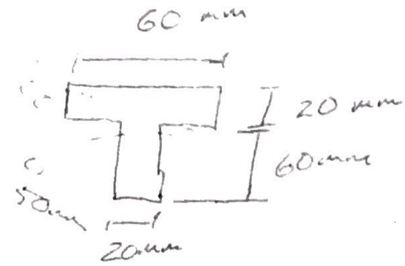
$$\begin{cases} x_1 = 0.45 \\ V_1 = F_c - 0.45w \\ M_1 = F_c(0.25) - \frac{1}{2}w\,0.45^2 = 0.25F_c - 0.10125w \end{cases}$$

$$\left.\begin{aligned} -130\text{MPa} = \sigma_上 &= \frac{M_1 y_上}{I_x} = \frac{(0.25F_c - 0.10125w)(30\text{mm})}{1.36\times10^6\,\text{mm}^4} \\ 40\text{MPa} = \sigma_下 &= \frac{M_1 y_下}{I_x} = \frac{(0.25F_c - 0.10125w)(50\text{mm})}{1.36\times10^6\,\text{mm}^4} \end{aligned}\right\} 得 \begin{cases} w = \\ F = \end{cases}$$

④

Find max dist. Load (?)



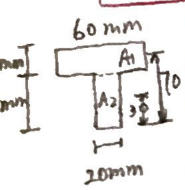




$\sigma_{all_{ten}} = 40\ MPa$

$\sigma_{all_{comp}} = -130\ MPa$

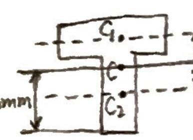
## T shape

## Distributed load




| | $A, mm^2$ | $\bar{y}_i, mm$ | $A_i\bar{y}_i, mm^3$ | $d, mm$ |
|---|---|---|---|---|
| 1 | $(20)(60)=1200$ | 70 | $84\times10^3$ | 20 |
| 2 | $(60)(20)=1200$ | 30 | $36\times10^3$ | 20 |
| $\Sigma$ | $\sum_i A_i = 2400$ | | $\sum_i A_i\bar{y}_i = 120\times10^3$ | |

$$\bar{Y} = \frac{\sum_i A_i\bar{y}_i}{\sum_i A_i} = \frac{120\times10^3\,mm^3}{2.4\times10^3\,mm^2} = 50\,mm$$



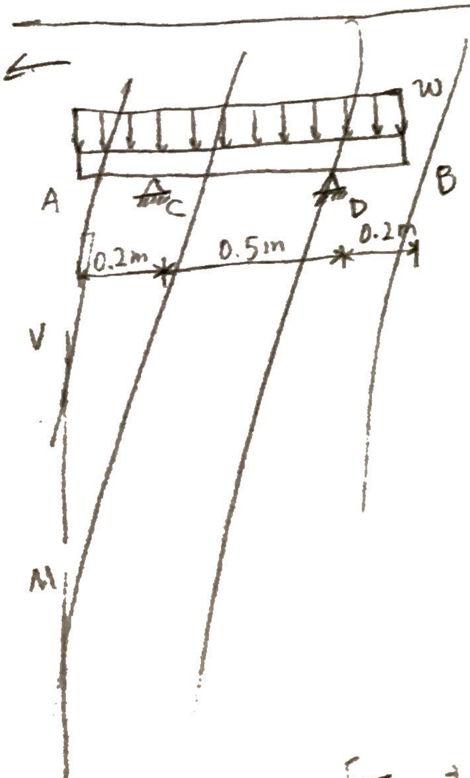

$d_1 = 10+10 = 20\,mm$

$d_2 = 50-30 = 20\,mm$

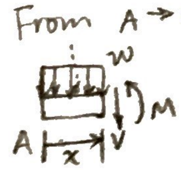
$$\begin{cases}(\bar{I}_{x'})_1 = \frac{1}{12}bh^3 = \frac{1}{12}(60mm)(20mm)^3 = 40\times10^3\,mm^4 \\ (I_x)_1 = (\bar{I}_{x'})_1 + A_1d_1^2 = 40\times10^3 + (1200)(20)^2 = 520\times10^3\,mm^4\end{cases}$$

$$\begin{cases}(\bar{I}_{x''})_2 = \frac{1}{12}bh^3 = \frac{1}{12}(20)(60)^3 = 360\times10^3\,mm^4 \\ (I_x)_2 = (\bar{I}_{x''})_2 + A_2d_2^2 = 360\times10^3 + (1200)(20)^2 = 840\times10^3\,mm^4\end{cases}$$

$$\therefore \bar{I}_x = (I_x)_1 + (I_x)_2 = 520\times10^3 + 840\times10^3 = 1360\times10^3\,mm^4$$
$$= 1.36\times10^6\,mm^4$$




From A → C:

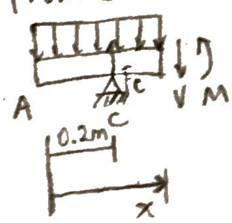



$\Sigma F_y = 0 = -wx - V \Rightarrow \boxed{V = -wx}$

$\Sigma M = 0 = M + (wx)\cdot(\frac{x}{2}) \Rightarrow \boxed{M = -\frac{1}{2}wx^2}$

(对右边求弯矩，因V未知)

From C → D:

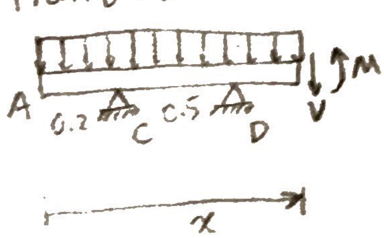



$\Sigma F_y = 0 = -wx + F_c - V \Rightarrow \boxed{V = F_c - wx}$

$\Sigma M = 0 = +\frac{1}{2}wx^2 - F_c(x-0.2) + M$

$\Rightarrow \boxed{M = F_c(x-0.2) - \frac{1}{2}wx^2}$

From D → B:




$\Sigma F_y = 0 = -V - wx + F_c + F_D$

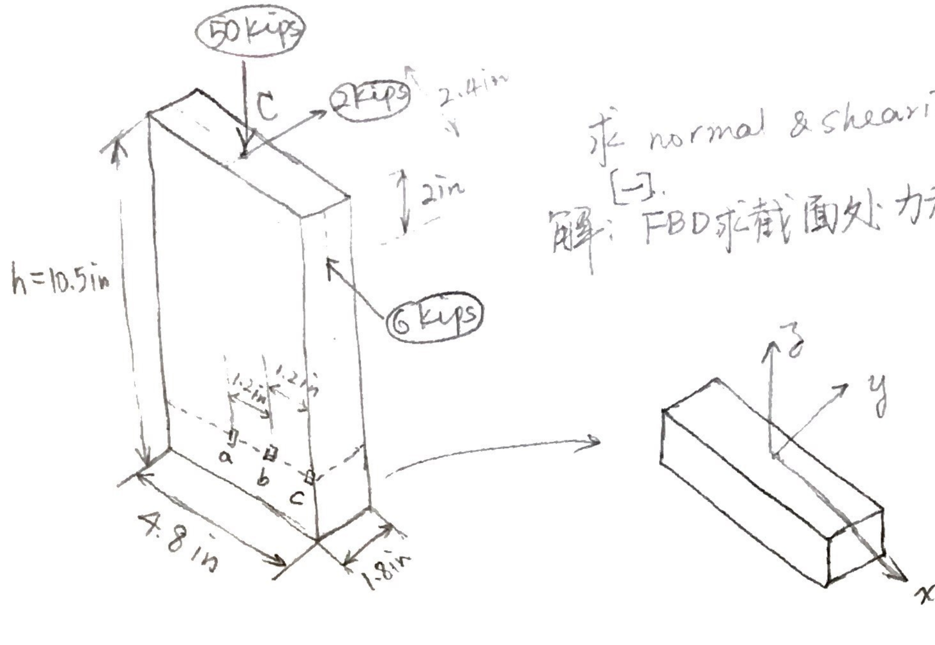
$\Rightarrow \boxed{V = F_c + F_D - wx}$

$\Sigma M = 0 = \frac{1}{2}wx^2 + M - F_D(x-0.7) - F_c(x-0.2)$

$\Rightarrow \boxed{M = F_D(x-0.7) + F_c(x-0.2) - \frac{1}{2}wx^2}$

[下接前页]

③




求 normal & shearing stresses @ a, b, c.

[一] 解：FBD求截面处力和弯矩

$A = (4.8in)(1.8in) = 8.64 in^2$

$I_x = \frac{1}{12}(4.8in)(1.8in)^3 = 2.3328 in^4$

$I_y = \frac{1}{12}(1.8in)(4.8in)^3 = 16.5888 in^4$

$$\begin{cases} \Sigma F_z = 0 = -50 kips + F_z & \Rightarrow F_z = 50 kips \\ \Sigma F_y = 0 = 2 kips + F_y & \Rightarrow F_y = -2 kips \\ \Sigma F_x = 0 = -6 kips + F_x & \Rightarrow F_x = 6 kips \end{cases}$$

$$\begin{cases} \Sigma M_x = 0 = (-2 kips)(10.5 in) + M_x & \Rightarrow M_x = 21 kip\cdot in \\ \Sigma M_y = 0 = (-6 kips)(8.5 in) + M_y & \Rightarrow M_y = 51 kip\cdot in \\ \Sigma M_z = 0 = M_z & \Rightarrow M_z = 0 \end{cases}$$

[二] ① Force $F_x \rightarrow$ transverse shear

$$\boxed{\tau_{zx}(a)} = \frac{F_x Q_a}{I_y t} = \frac{(6 kips)(5.184 in^3)}{(16.5888 in^4)(1.8 in)} = 1.042 ksi$$

$$Q_a = A_a y_a = (2.4in)(1.8in)(1.2in) = 5.184 in^3$$

$$\boxed{\tau_{zx}(b)} = \frac{F_x Q_b}{I_y t} = 0.781 ksi$$

$$Q_b = A_b y_b = (1.2in)(1.8in)(1.8in) = 3.888 in^3$$

$$\boxed{\tau_{zx}(c)} = 0$$

② Force $F_y \rightarrow$ transverse shear

$\tau_{zy}(a) = \tau_{zy}(b) = \tau_{zy}(c) = 0$

③ Force $F_z \rightarrow$ uniform axial

$\boxed{\sigma_1} = \frac{F_z}{A} = \frac{50 \text{ kips}}{8.64 \text{ in}^2}$

④ Moment $M_x \rightarrow$ bending

$\boxed{\sigma_2(a)} = \frac{M_x y_a}{I_x} = \frac{(21 \text{ kip}\cdot\text{in})(0.9 \text{ in})}{2.3328 \text{ in}^4}$

$\parallel$

$\boxed{\sigma_2(b)} = \frac{M_x y_b}{I_x}$

$\parallel$

$\boxed{\sigma_2(c)}$

⑤ Moment $M_y \rightarrow$ bending

$\sigma_3(a) = 0$

$\boxed{\sigma_3(b)} = \frac{M_y y_b}{I_y} = \frac{(51 \text{ kip}\cdot\text{in})(1.2 \text{ in})}{(16.5888 \text{ in}^4)}$

$\boxed{\sigma_3(c)} = \frac{M_y y_c}{I_y} = \frac{(51 \text{ kip}\cdot\text{in})(2.4 \text{ in})}{(16.5888 \text{ in}^4)}$

[三] 求和

point a: $\sigma_a = \frac{50}{8.64} + \frac{(21)(0.9)}{(2.3328)} = 2.31 \text{ ksi}$

$\tau_a = \tau_{zx}(a) = 1.042 \text{ ksi}$

point b: $\sigma_b = \frac{50}{8.64} + \frac{(21)(0.9)}{2.3328} + \frac{(51)(1.2)}{(16.5888)} = 6. \text{ ksi}$

$\tau_b = \tau_{zx}(b) = 0.781 \text{ ksi}$

point c: $\sigma_c = \frac{50}{8.64} + \frac{(21)(0.9)}{2.3328} + \frac{(51)(2.4)}{16.5888} = 9.69 \text{ ksi}$

$\tau_c = 0$

Materials (.100 /in.n)

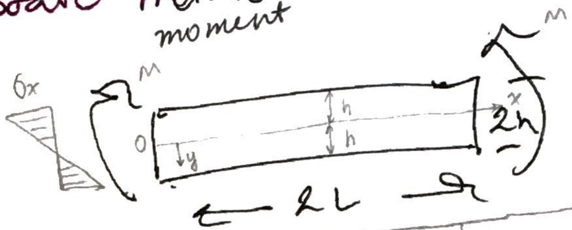
# Airy Stress function

$$\phi = Ay^3$$

State ~~of~~ (of) stresses, traction free on outer surfaces in terms of A, h, L

State moment

moment




$2L \times 2h \times 1$

$\phi = Ay^3$

2)

3)

求应力:

$$\begin{cases} \sigma_x = \dfrac{\partial^2 \phi}{\partial y^2} = 6Ay \\ \sigma_y = \dfrac{\partial^2 \phi}{\partial x^2} = 0 \\ \tau_{xy} = -\dfrac{\partial^2 \phi}{\partial x \partial y} = 0 \end{cases} \quad (a)$$

①先校核主要边界(大边界)

$y = \pm h$: (大边界)

$$\begin{cases} \tau_{xy(y=\pm h)} = 0 & (x \text{ direction}) \\ \sigma_{y(y=\pm h)} = 0 & (y \text{ direction}) \end{cases} \quad (b)$$

从(a)可见,(b)满足

②在校核次要边界(小边界)

[圣维南原理,积分]

$x = 0, 2L$: (小边界)

$$\begin{cases} (\tau_{xy})_{x=0,L} = 0 \text{ 满足 (y方向BC)} \\ \sigma_x \text{边界无法精确满足 (x方向BC)} \end{cases} \quad (C)$$

☆ ㊂

$$\begin{cases} \int_{-h}^{h} (\sigma_x)_{x=0,2L}\, dy = F_N = 0 \\ \int_{-h}^{h} (\sigma_x)_{x=0,2L}\, y\, dy = M \end{cases} \quad (d)$$

$\because \sigma_x = 6Ay$, $\therefore \int_{-h}^{h} (6Ay)_{x=0,2L}\, dy = 3Ay^2\Big|_{-h}^{h} = 0 \overset{F_N=0}{=}$ 自然满足

由($d_2$), $\int_{-h}^{h} (\sigma_x)_{x=0,2L} \cdot y\, dy$

$= \int_{-h}^{h} (6Ay)_{x=0,2L}\, y\, dy$

$= \int_{-h}^{h} 6Ay^2 dy = 2Ay^3\Big|_{-h}^{h} = 2Ah^3 + 2Ah^3 = 4Ah^3 = M$

**Airy stress Function**

$\therefore M = 4Ah^3$ ← moment

$\therefore A = \dfrac{M}{4h^3}$, 代回到应力中,

$$\therefore \begin{cases} \sigma_x = 6Ay = \dfrac{3M}{2h^3}\, y \\ \sigma_y = 0 \\ \tau_{xy} = 0 \end{cases}$$

State of stresses

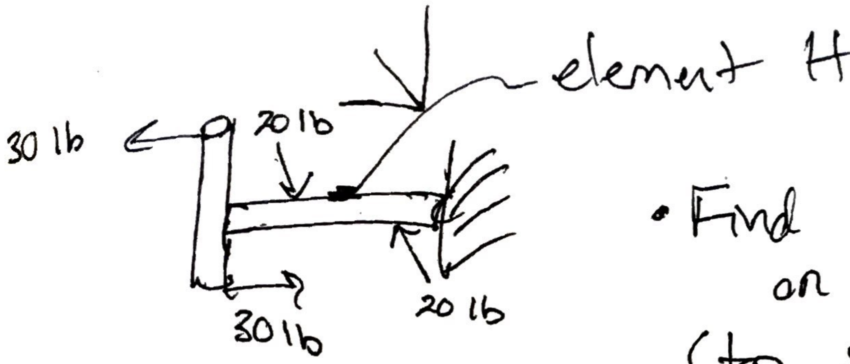
检查:

$$\left( \begin{aligned} I &= \tfrac{1}{12} \times 1 \times (2h)^3 \\ &= \tfrac{2}{3} h^3 \\ \therefore \sigma_x &= \tfrac{M}{I} y \end{aligned} \right)$$

注: BCs:

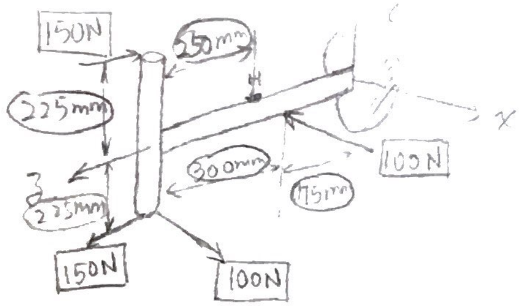
$$\begin{cases} x \text{ direction:} \\ \bar{\phi}_x = \sigma_x l + \tau_{xy} m \\ y \text{ direction:} \\ \bar{\phi}_y = \tau_{xy} l + \sigma_y m \end{cases}$$

State of Stress (看下一页)




- Find state of stress on element H (top surface of a pipe)

Combined Loads




$D_{inner} = 36mm = d_i$ $(c_i = 18mm)$

$D_{outer} = 42mm = d_o$ $(c_o = 21mm)$

求 normal & shear stresses @ H

↑ Top surface of the pipe

解: $t_1 = c_o - c_i = 3mm$

$t = 2t_1 = 6mm$

一、FBD:

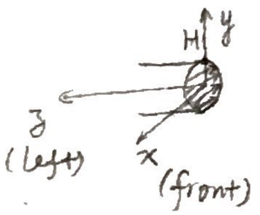
At the cross-section containing point H，(针对H点左侧的力)

$$\begin{cases} \Sigma F_x = 0 = 100N + F_x & \Rightarrow \boxed{F_x = -100N} \quad \text{(transverse shear)} ✓ \\ \Sigma F_y = 0 & \Rightarrow F_y = 0 \quad \text{(transverse shear)} × \\ \Sigma F_z = 0 = -150N + 150N = 0 & \Rightarrow F_z = 0 \quad \text{(uniform axial)} × \end{cases}$$

$$\begin{cases} \Sigma M_x = 0 = -150N \cdot 225mm - 150N \cdot 225mm + M_x & \Rightarrow \boxed{M_x = 67.5 N\cdot m} \quad \text{(bending)} ✓ \\ \Sigma M_y = 0 = 100N \cdot 250mm + M_y & \Rightarrow \boxed{M_y = -25 N\cdot m} \quad \text{(bending)} ✓ \\ \Sigma M_z = 0 = 100N \cdot 225mm + M_z & \Rightarrow \boxed{M_z = 22.5 N\cdot m} \quad \text{(torsion)} ✓ \end{cases}$$

一、φ 求 σ, σ, τ

二、① Force $F_x$ — transverse shear




Shear stress @ H

$$\boxed{\tau_{zx}^{1}(H)} = \frac{F_x \cdot Q}{I_y t}$$

$$= \frac{(-100N)(2.286 \times 10^{-6} m^3)}{(70.30 \times 10^{-9} m^4)(6 \times 10^{-3} m)}$$

$$= -0.54 \text{ MPa}$$

$A = \pi(c_o^2 - c_i^2)$

$= 367.57 \times 10^{-6} m^2$

$I = \frac{\pi}{4}(c_o^4 - c_i^4)$

$= 70.30 \times 10^{-9} m^4$

$J = 2I = 140.59 \times 10^{-9} m^4$

For half-pipe,

$Q = \frac{2}{3}(c_o^3 - c_i^3)$

$= 2.286 \times 10^{-6} m^3$

☆ 看Q及其符号怎样

② Moment $M_x$: bending

H点: compression

$$\boxed{\sigma_z^1(H)} = \frac{-M_x y}{I_x} = \frac{-(67.5 N\cdot m)(21 \times 10^{-3} m)}{(70.30 \times 10^{-9} m^4)} = -20.2 \text{MPa}$$

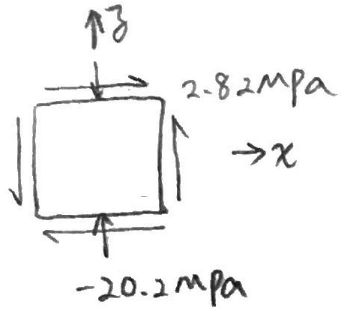
③ Moment $M_y$: bending

$$\sigma_z^2(H) = 0 \quad \text{(中轴线)}$$

④ Moment $M_z$: torsion:

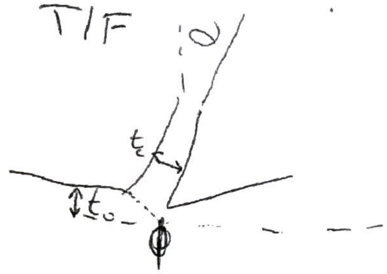
$$\boxed{\tau_{zx}^2(H)} = \frac{M_z \cdot c}{J} = + \frac{(22.5 N\cdot m)(21 \times 10^{-3} m)}{(140.59 \times 10^{-9} m^4)} = +3.36 \text{MPa}$$

$$\therefore \begin{cases} \sigma_z(H) = -20.2\,\text{Mpa} = \sigma_H \\ \tau_{zx}(H) = \tau_{zx}^{1}(H) + \tau_{zx}^{2}(H) = -0.54\,\text{Mpa} + 3.36\,\text{Mpa} = \underset{\tau_H}{\underset{\parallel}{2.82}}\,\text{Mpa} \end{cases}$$



Materials

T/F




① What is $t_o$, $t_c$

If ~~$\phi$ is~~ chip thickess ↑ at same cutting depth, what happens to cutting force? ↑

② How to change material properties or machining params, to make discontinuous chips.

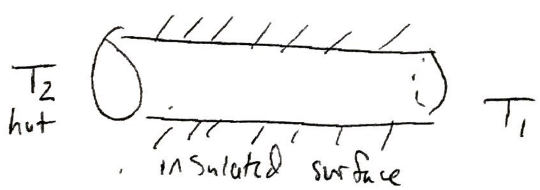
③ Draw and label $\sigma - \epsilon$. Define and show toughness and ductility.

④ Draw $\sigma - \epsilon$ for brittle vs. extremely ductile mat'l.

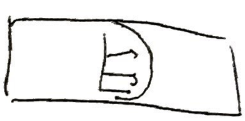
⑤ Compare Pros/cons for additive manufacturing vs. processes like CNC / die casting (consider production rate, cost,

⑥ Label schematics of common mfg. processes: turning, milling, drilling, broaching, planing, shaping, sawing

⑦

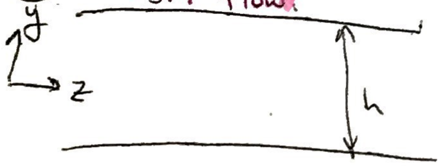



How does T profile look at steady state? What are B.C.'s?



How does P profile look? What are B.C.'s?

⑧ Slit flow




given Cons. momentum equation. Find $v_z(y)$