Name: ___________________________

University of Wisconsin-Madison
EMA 303/ME 306 Mechanics of Materials
Final Exam

Instructions

- This exam has 10 pages. Possibly useful equations and beam table are given in the last 3 pages.
- This is a closed book exam.
- Calculators are allowed. No cell phone calculators. No laptops, tablets, etc.
- The University of Wisconsin-Madison holds each student's academic work to the highest standards of integrity. Receiving or giving aid on this exam is academic misconduct. Penalties for academic misconduct range from failing an exam/assignment to expulsion from the University.
- You have 90 minutes to complete the exam.
- Write out equations symbolically and substitute in values at the end.
- Assume all numbers given to you are accurate to 3 significant digits. This means you should report final numerical answers to 3 digits. Numerical answers also must include units.

<u>Please initial the statement below to show that you have read it.</u>
By affixing my name to this exam, I affirm that I will not, under any circumstance, discuss this exam or anything relating to this exam with anyone until the exam has been graded and returned to me.

Student Initials: ______________________________

Question	Score
1	_____/ 25
2	_____/ 35
3	_____/ 25
4-5-6	_____/ 15
Total	_____/ 100

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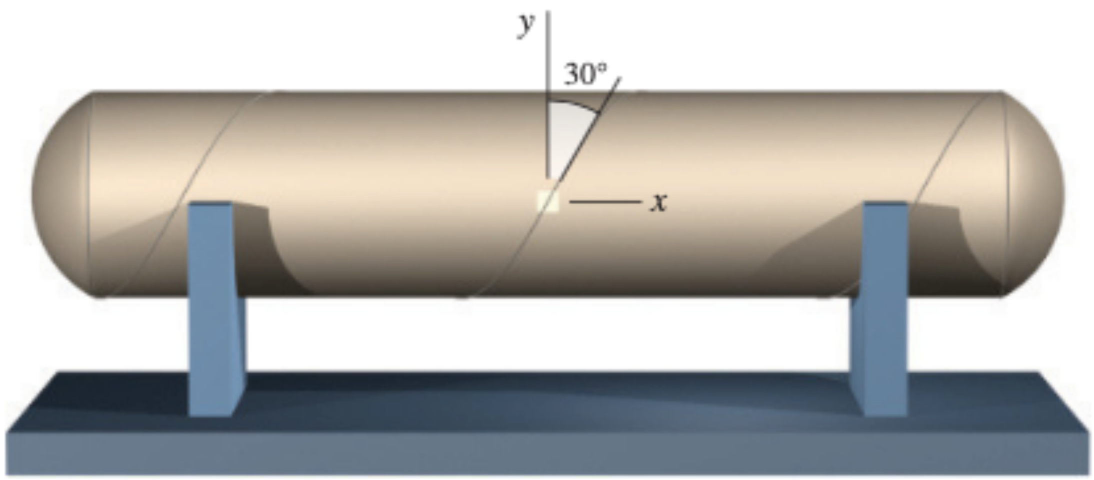
Problem 1

A cylindrical pressure tank of outer diameter 900 mm is made of 15 mm thick plates welded at 30º from the vertical axis *y* as shown below. Determine the normal and shear stresses experienced by the weld when the internal pressure of the tank is at 2.2 MPa (neglect the support forces and weight of the tank).



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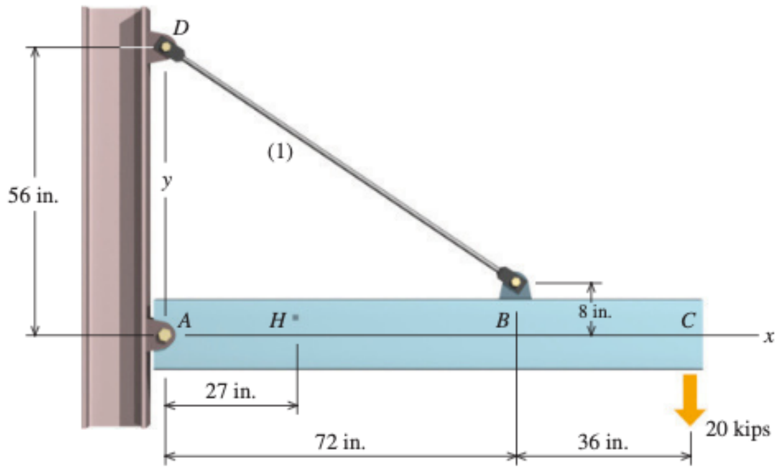
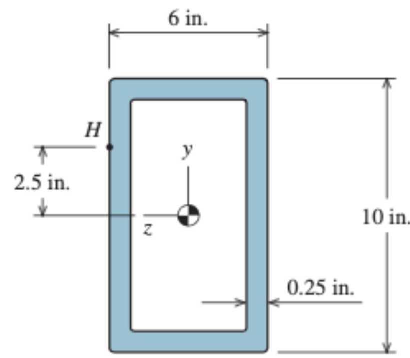
Problem 2

A hollow rectangular beam AC is supported by a pin at point A and a steel cable at point B and loaded by a point load of 20 kips at point C. The cross-section of the beam has an area of A = 7.75 in.^{2}; area moment of inertia about z-axis of I_z = 107.04 in.^{4}; the first moment of area corresponding to point H of Q_H = 11.39 in.^{3}, and shear thickness of t_H = 0.5 in.

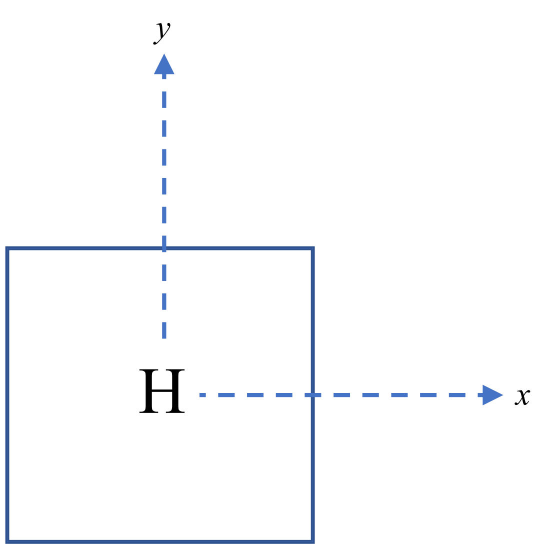




Cross-sectional dimensions.

(a) Determine the complete state of stress (normal and shear stresses) in the x-y coordinate system at point H, and

(b) Draw in all the normal and shear stresses acting at point H on a differential element as shown below (note the direction of the x and y axes). Indicate the actual direction / sign of the stress by the ***direction of your arrows***. Next to your arrows write the magnitude of the stress associated with each specific arrow.



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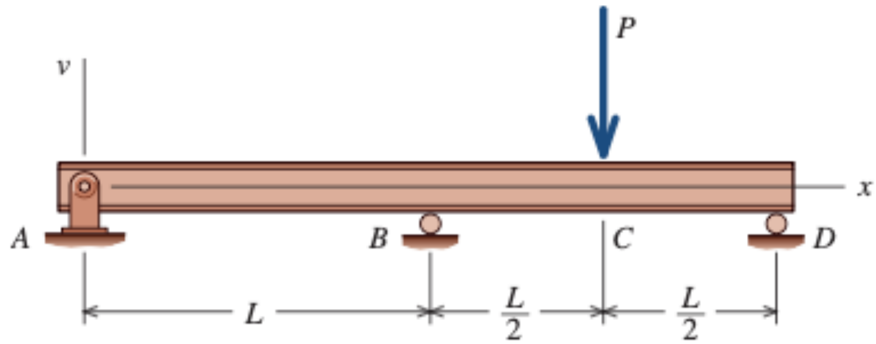
Problem 3

A wide-flange beam with a length [$L = 5$ m], Young's modulus [$E = 200$ GPa] and area moment of inertia [$I = 351 \times 10^6$ mm^4] is loaded by a point force [$P = 240$ kN] at point *C*. Use the beam tables and superposition, and determine the reaction force at roller support *B*.



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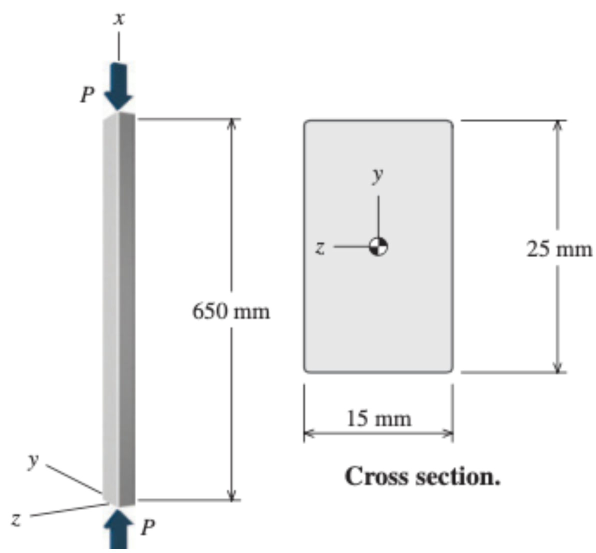
Short Problems

Problem 4

A beam with rectangular cross-section shown below is pinned at the ends and loaded in compression. On which plane does this beam more likely to buckle?

A) xz-plane (bending about y-axis) B) xy-plane (bending about z-axis)



Cross section.

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Problem 5

Beams with identical geometry, cross-sections and materials are supported differently as shown below in vertical and buckled configurations. Select the correct ordering of critical loads.

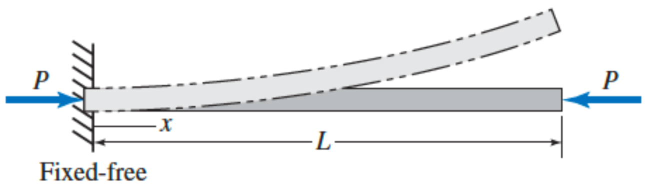
A) $P_{cr}^{I} > P_{cr}^{II} > P_{cr}^{III}$

B) $P_{cr}^{III} > P_{cr}^{II} > P_{cr}^{I}$

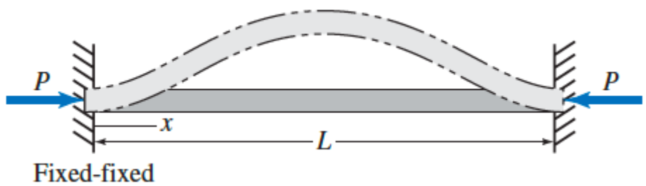
C) $P_{cr}^{II} > P_{cr}^{III} > P_{cr}^{I}$

D) $P_{cr}^{II} > P_{cr}^{I} > P_{cr}^{III}$

I.



II.



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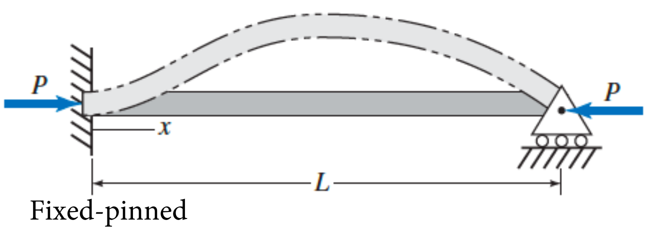

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Problem 6

A uniform cantilever beam is horizontal as unloaded (top figure). When the beam is loaded transversely by a combination of forces, which of the elastic curves drawn below can NOT belong to that cantilever beam? (Dashed lines in the choices show the unloaded horizontal configuration.)

A.

B.

C.

D.

Fundamental Mechanics of Materials Equations

Basic definitions

Average normal stress in an axial member: $\sigma_{avg} = \frac{F}{A}$

Average direct shear stress: $\tau_{avg} = \frac{V}{A_v}$

Average bearing stress: $\sigma_b = \frac{F}{A_b}$

Average normal strain in an axial member: $\varepsilon_{avg} = \frac{\delta}{L}$

Average normal strain caused by temperature change:

$$\varepsilon_T = \alpha \Delta T$$

Hooke's Law (one-dimensional): $\sigma = E\varepsilon$ and $\tau = G\gamma$

Poisson's ratio: $\upsilon = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$

Relationship between *E*, *G*, and *ν*: $G = \frac{E}{2(1+\upsilon)}$

Definition of allowable stress:

$$\sigma_{allow} = \frac{\sigma_{failure}}{\text{FS}} \text{ or } \tau_{allow} = \frac{\tau_{failure}}{FS}$$

Factor of safety:

$$\text{FS} = \frac{\sigma_{failure}}{\sigma_{actual}} \text{ or FS} = \frac{\tau_{failure}}{\tau_{actual}}$$

Area moment of inertia: $I = \int_A y^2 dA$

Area MOI for a rectangle: $I_{rect} = \frac{bh^3}{12}$

Area MOI for a circle: $I_{circ} = \frac{\pi c^4}{4} = \frac{\pi d^4}{64}$

Parallel axis theorem: $I_{x\prime} = I_{xc} + Ad^2$

Axial deformation

Deformation in axial members: $\delta = \frac{FL}{AE}$ or $\delta = \sum_i \frac{F_i L_i}{A_i E_i}$

Force-temperature-deformation: $\delta = \frac{FL}{AE} + \alpha \Delta T L$

Torsion

Max. torsion shear stress in a circular shaft: $\tau_{max} = \frac{Tc}{J}$

Polar moment of inertia for a circle:

$$J = \frac{\pi}{2}[R^4 - r^4] = \frac{\pi}{32}[D^4 - d^4]$$

Torsional shearing strain: $\gamma = \frac{\rho\phi}{L}$

Angle of twist in a circular shaft: $\phi = \frac{TL}{JG}$ or $\phi = \sum_i \frac{T_i L_i}{J_i G_i}$

Power transmission in a shaft: $P = T\omega$

Flexure

Flexure formula: $\sigma = -\frac{My}{I}$

Flexure strain: $\varepsilon = -\frac{y}{\rho}$

Horizontal shear stress associated with bending: $\tau_H = \frac{VQ}{It}$

Shear flow formula: $q = \frac{VQ}{I}$

Shear flow, fastener spacing, and fastener shear relationship: $qs \le n_f V_f = n_f \tau_f A_f$

First moment of area: $Q = \int_A y dA$

For circular cross sections, $Q = \frac{2}{3}[R^3 - r^3] = \frac{1}{12}[D^3 - d^3]$

Beam deflections

Elastic curve relations between *w*, *V*, *M*, *θ*, and *v* for constant *EI*:

$$\text{Deflection} = v$$

$$\text{Slope} = \frac{dv}{dx} = \theta$$

$$\text{Moment } M = EI\frac{d^2v}{dx^2}$$

$$\text{Shear } V = \frac{dM}{dx} = EI\frac{d^3v}{dx^3}$$

$$\text{Load } w = \frac{dV}{dx} = EI\frac{d^4v}{dx^4}$$

Plane stress transformations

Normal and shear stresses on an arbitrary plane:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal stress magnitudes:

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Orientation of principal planes: $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

Fundamental Mechanics of Materials Equations

Maximum in-plane shear stress magnitude:

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \text{ or } \tau_{max} = \frac{\sigma_{p1} - \sigma_{p2}}{2}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

Absolute maximum shear stress magnitude:

$$\tau_{abs\,max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Normal stress invariance:

$$\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'} = \sigma_{p1} + \sigma_{p2}$$

Plane strain transformations

Normal and shear strain in arbitrary directions:

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\varepsilon_1 = \varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy}\cos\theta_1 \sin\theta_1$$

Principal strain magnitudes:

$$\varepsilon_{p1,p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Orientation of principal strains: $\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$

Maximum in-plane shear strain:

$$\frac{\gamma_{max}}{2} = \pm\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \text{ or } \gamma_{max} = \varepsilon_{p1} - \varepsilon_{p2}$$

$$\varepsilon_{max} = \frac{\varepsilon_x + \varepsilon_y}{2}$$

Normal strain invariance:

$$\varepsilon_x + \varepsilon_y = \varepsilon_{x'} + \varepsilon_{y'} = \varepsilon_{p1} + \varepsilon_{p2}$$

Generalized Hooke's Law

Normal stress/normal strain relationships:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \upsilon\sigma_y - \upsilon\sigma_z\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \upsilon\sigma_z - \upsilon\sigma_x\right)$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \upsilon\sigma_x - \upsilon\sigma_y\right)$$

Shear stress/shear strain relationships:

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz} \qquad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

Pressure vessels

Axial stress in spherical pressure vessel: $\sigma_a = \frac{pr}{2t} = \frac{pd}{4t}$

Longitudinal and hoop stresses in cylindrical pressure vessels

$$\sigma_{\text{long}} = \frac{pr}{2t} = \frac{pd}{4t} \qquad \sigma_{\text{hoop}} = \frac{pr}{t} = \frac{pd}{2t}$$

Column buckling

Euler buckling load: $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$

Euler buckling stress: $\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}$

Radius of gyration: $r^2 = \frac{I}{A}$

End conditions effective length constants:

- Pinned-pinned: $K = 1$
- Fixed-pinned: $K = 0.7$
- Fixed-fixed: $K = 0.5$
- Fixed-free: $K = 2$

Strain energy

Strain energy density: $u = \int \sigma_x d\varepsilon_x$ or $u = \int \tau_x d\gamma_{xy}$

Strain energy: $U = \int u\, dV = \int \frac{\sigma_x^2}{2E}\, dV$ or $\int \frac{\tau_{xy}^2}{2G}\, dV$

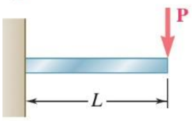
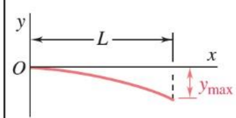
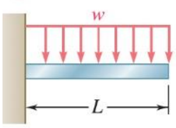
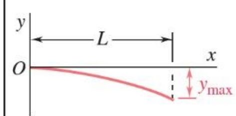
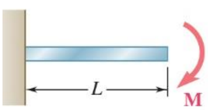
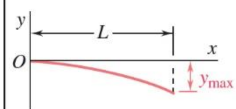
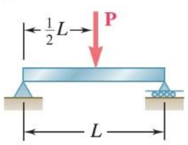
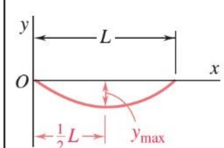
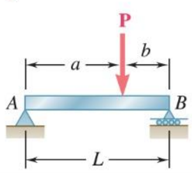
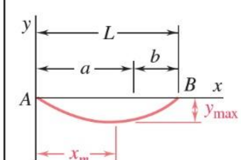
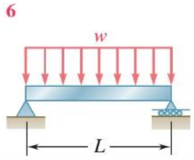
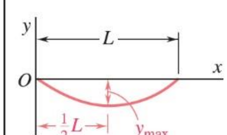
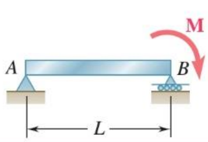
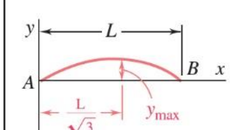
Strain energy due to axial load (const. P,L,E,A): $U = \frac{P^2 L}{2EA}$

Strain energy due to torsion (const. T,L,G,J): $U = \frac{T^2 L}{2GJ}$

Strain energy due to bending moment: $U = \int_0^L \frac{M^2}{2EI} dx$

Kinetic energy: $KE = \frac{1}{2}mv^2$

Gravitational potential energy: $PE = mgh$

Beam and Loading	Elastic Curve	Maximum Deflection	Slope at End	Equation of Elastic Curve
1 		$-\dfrac{PL^3}{3EI}$	$-\dfrac{PL^2}{2EI}$	$y = \dfrac{P}{6EI}(x^3 - 3Lx^2)$
2 		$-\dfrac{wL^4}{8EI}$	$-\dfrac{wL^3}{6EI}$	$y = -\dfrac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
3 		$-\dfrac{ML^2}{2EI}$	$-\dfrac{ML}{EI}$	$y = -\dfrac{M}{2EI}x^2$
4 		$-\dfrac{PL^3}{48EI}$	$\pm\dfrac{PL^2}{16EI}$	For $x \le \frac{1}{2}L$: $y = \dfrac{P}{48EI}(4x^3 - 3L^2x)$
5 		For $a > b$: $-\dfrac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL}$ at $x_m = \sqrt{\dfrac{L^2 - b^2}{3}}$	$\theta_A = -\dfrac{Pb(L^2 - b^2)}{6EIL}$ $\theta_B = +\dfrac{Pa(L^2 - a^2)}{6EIL}$	For $x < a$: $y = \dfrac{Pb}{6EIL}[x^3 - (L^2 - b^2)x]$ For $x = a$: $y = -\dfrac{Pa^2b^2}{3EIL}$
6 		$-\dfrac{5wL^4}{384EI}$	$\pm\dfrac{wL^3}{24EI}$	$y = -\dfrac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$
7 		$\dfrac{ML^2}{9\sqrt{3}EI}$	$\theta_A = +\dfrac{ML}{6EI}$ $\theta_B = -\dfrac{ML}{3EI}$	$y = -\dfrac{M}{6EIL}(x^3 - L^2x)$