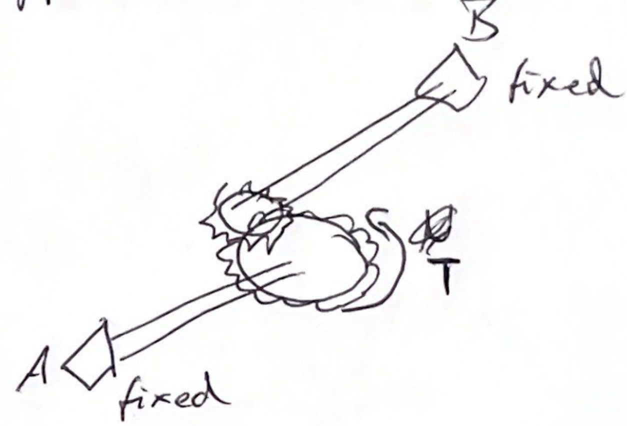
Mechanics Parla Vinter 2024 8)

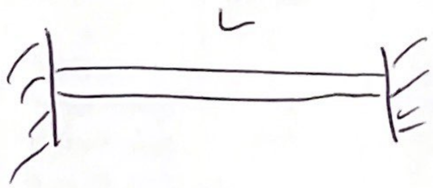
① Gear problem
sth like sample problem 3.4,
but the torque was applied to the gear

What is M at A & B ?



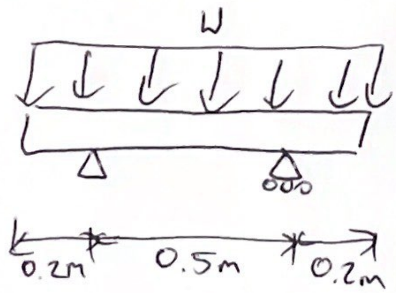
② Airy stress function,
with function with
power > 4 or 5
(didn't turn out
to be 0)

③



How much can ΔT be for the pipe to buckle ?

④



$\sigma_{all,ten} = 40\,MPa$

$\sigma_{all,com} = -130\,MPa$

① Find max w

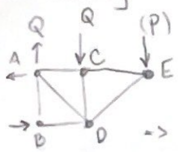
② Draw V, M - Diagram

$$U = \sum \frac{F_i^2 L_i}{2A_i E} = \frac{1}{2E} \sum \frac{F_i^2 L_i}{A_i} = \frac{4.752 \times 10^7 \, N^2 m}{2.73 \times 10^9 \, Pa} = 325 \, \frac{N}{m}$$

$$\frac{1}{2} P y_E = U \Rightarrow y_E = \frac{U \cdot 2}{P} = 0.01625 \, m$$

- Using the same setup as the previous problem, determine the vertical deflection of joint C.
(Beer 7th. edition, page 812)

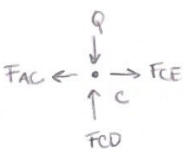
Add dummy load to joint C

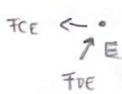


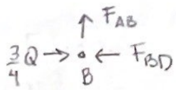
Per Castigliano's theorem: $y_C = \sum \left(\frac{F_i L_i}{A_i E} \right) \frac{\partial F}{\partial Q}$

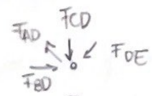
$\Sigma M_x = 0$

$\Rightarrow 0.6\,Q - 0.8\,R_{Bx} = 0 \Rightarrow R_{Bx} = \frac{0.6}{0.8} Q = \frac{3}{4} Q \Rightarrow R_{Ax} = 3/4\,Q$









$F_{CD} = Q$

$F_{AC} = 0$

$F_{DE} = 0$

$F_{CE} = 0$

$F_{AB} = 0$

$F_{BD} = \frac{3}{4} Q$

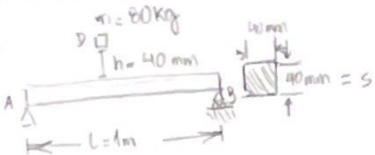
$F_{BD} - F_{AD}\left(\frac{0.6}{1}\right) - F_{DE}\left(\frac{1.5}{1.7}\right) = 0$

$\frac{(F_{BD})}{0.6} = F_{AD} = \frac{30}{24} Q = \frac{5}{4} Q$

Member	F_i	$\partial F_i / \partial Q$	L_i [m]	A_i [m^2]	$\left(\frac{F_i L_i}{A_i}\right) \frac{\partial F_i}{\partial Q}$
AB	0	0	0.8	600×10^{-6}	0
AC	75KN	0	0.6	"	0
AD	50 KN + 5/4 Q	5/4	1	"	$6.25 \times 10^4 + 3125\,Q$ ($\times 10^3$)
BD	-105 KN + 3/4 Q	3/4	0.6	1000×10^{-6}	$-6.3 \times 10^4 + 337.5\,Q$
CD	0 + Q	1	0.8	"	800 Q
CE	75KN + 0	0	1.5	500×10^{-6}	0
DE	-85KN + 0	0	1.7	1000×10^{-6}	0

$-500 \times 10^3 + 4263\,Q \Rightarrow y_C = -6.85 \times 10^{-6} \, m$

(?)



Beer (7th edition) page 793

Block D of mass m is released from rest & falls distance h before striking midpoint C of the Al beam AB. Using E = 73GPa, determine a) Max. deflection of C b) Max stress of beam.-

Using beam deflection tables: $y_m = \frac{PL^3}{48EI}$ (*)

a) $EI = 73\times10^9\,Pa \cdot \frac{1}{12}(40\times10^{-3}m)(40\times10^{-3}m)^3 = 15.57\times10^3\,Nm^2$

At max. deflection point, $W = \underbrace{m.g}_{w}.(h+y_m)$ [work on block] (A)

$w = 80Kg \times 9.8\frac{m}{s^2} = 784\,N$ Strain energy at max. deflection point: $U = \frac{1}{2}P.y_m$ (B)

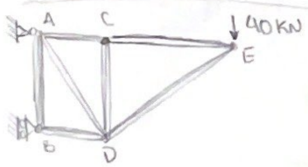
Using (*) $\Rightarrow P = \frac{48EIy_m}{L^3}$, thus, equating (A) & (B) due to energy balance assuming no losses:

$$\frac{24EIy_m^2}{L^3} = w.(h+y_m) \Rightarrow \frac{24EIy_m^2}{L^3} - wy_m - wh = 0 \Rightarrow 373.7\times10^3 y_m^2 - 784y_m - 31.36 = 0$$

Solving for y_m $\begin{cases} \to 1.027\times10^{-2}\,m \text{ Solution} \\ \to -8.171\times10^{-3}\,m \Rightarrow \text{implies beam bent upwards, discard.-}\end{cases}$

b) $y_m = \frac{PL^3}{48EI} = 1.027\times10^{-2}m \Rightarrow$ Solving for $P \Rightarrow P = 7675.4\,N$

$\sigma_{max} = \frac{Mc}{I}$; $M_{max} = \frac{1}{4}PL \Rightarrow \sigma_{max} = \frac{PL.c}{4.\frac{1}{12}s^4} = \frac{7675.4N.(0.25m).(0.02m)}{\frac{1}{12}.(0.04m)^4} = 179.9\,MPa$



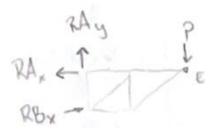
Members of the truss shown consist of sections of Al pipe with cross-sectional areas as shown. Using E = 73GPa, determine the vertical deflection of point E caused by load P.-

Beer (7th edition) Page 794

$\overline{AB} = \overline{CD} = 0.8m$ $A_{AB} = A_{AD} = A_{AC} = A_{CE} = 500\,mm^2$

$\overline{AC} = \overline{BD} = 0.6m$ $A_{BD} = A_{CD} = A_{DE} = 1000\,mm^2$

$\overline{CE} = 1.5m$

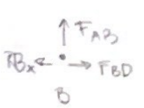


$\Sigma F = 0 \Rightarrow R_{Ay} = P = 40KN$

$R_{Ax} = R_{Bx} = 168KN$

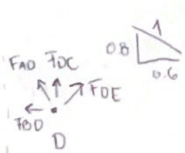
$\Sigma M_A = 0 \Rightarrow -(2.1m)P + (0.8m)R_{Bx} = 0$

$\Rightarrow R_{Bx} = \frac{2.1}{0.8}P = 105KN$



$F_{BD} = 105KN$

$F_{AB} = 0$



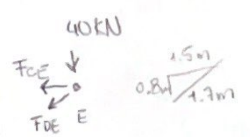
$F_{DE}\left(\frac{1.5}{1.7}\right) - F_{AD}\left(\frac{0.6}{1}\right) - F_{BD} = 0$ (?)

$\Rightarrow F_{BD} = -105KN$

$F_{AD}\left(\frac{0.8}{1}\right) + \overset{0}{F_{DC}} + F_{DE}\left(\frac{0.8}{1.7}\right) = 0$

$\Rightarrow F_{AD} = \frac{-F_{DE}}{1.7}$

$= 50KN$

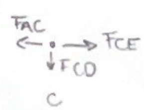


$\Sigma F_x = 0 \Rightarrow F_{CE} = -F_{DE}\left(\frac{1.5}{1.7}\right)$

$\Sigma F_y = 0 \Rightarrow F_{DE}\left(\frac{0.8}{1.7}\right) = -40$

$\Rightarrow F_{DE} = -85KN$

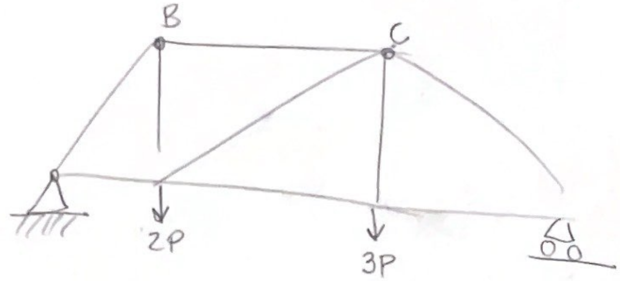
$\Rightarrow F_{CE} = 75KN$



$F_{CD} = 0$

$F_{AC} - F_{CE} = 0 \Rightarrow F_{AC} = 75KN$

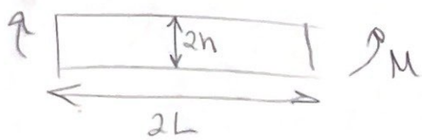
Gerardo's goals

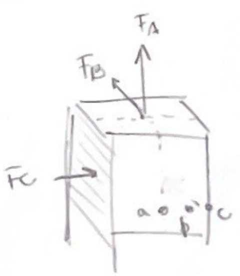


Determine horizontal & vertical deflections of B & C

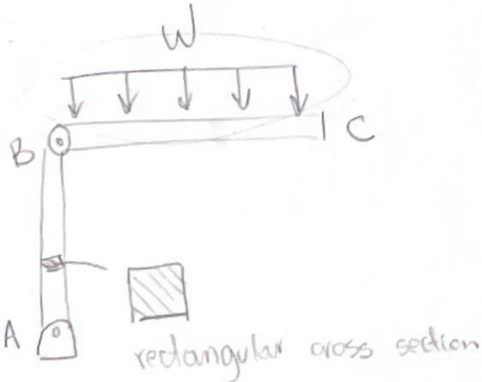
$\phi = Ay^3$

determine stress state & M using Airy stress function





Determine stresses for points a, b & c



Determine max w that won't cause AB to buckle in the pin-pin condition & fixed-fixed condition.

Mechanics

Tina Winter 2020

1. 2-dimensional, elastic body

State compatibility conditions in terms of strain field

equilibrium equations in terms of stress field

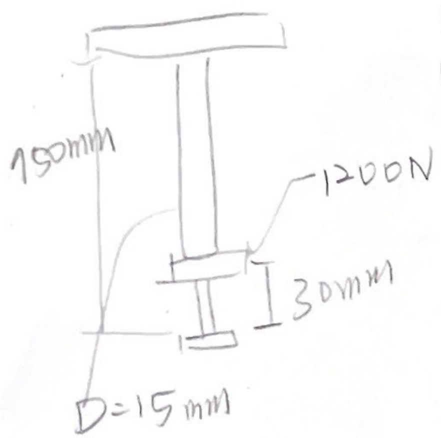
Verify given statements if they satisfy compatibility and equilibrium

2. Impact loading

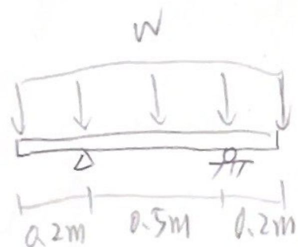
(a) release slowly

(b) max. deflection

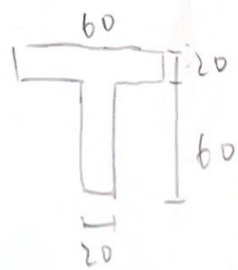
(c) dynamic force / max. normal stress



3.



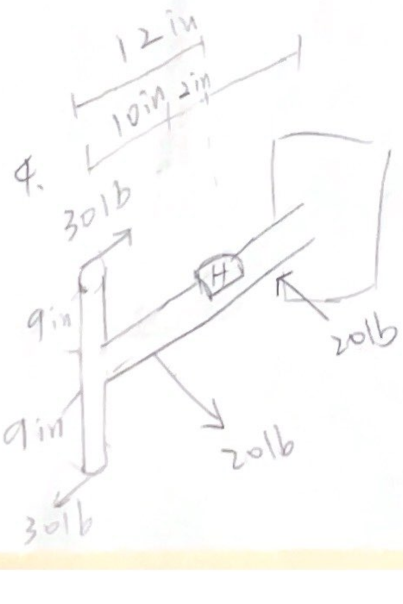
$\sigma_{all_{ten}} = 40$ MPa , $\sigma_{all_{comp}} = -130$ MPa



① Find max dist. load w

② Draw V and M diagram

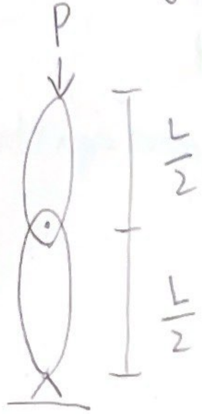
4.



① Find state of stress on element H

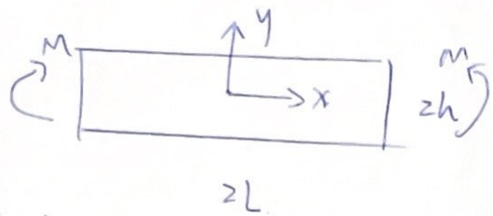
② Draw all nonzero stress on given graph

5. Buckling : 2 pin-end rigid bar. Derive the P_{CR} with Stiffness k (spring in the center)



Solid Mechanics 2022 Summer Zijie

1)



$\phi = Ay^3$

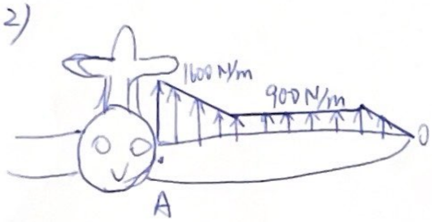
Find:

a) stress states

b) surface tractions, BCs

c) Moment with A, h, L

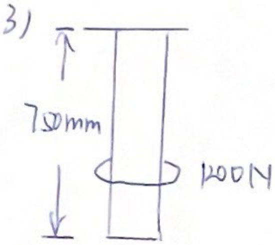
2)



Find moment and shear force at point A.

Draw V/M graph.

3)

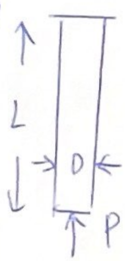


a) release slowly $\sigma = ?$ and $\delta = ?$

b) max axial displacement

c) dynamic force and max normal stress

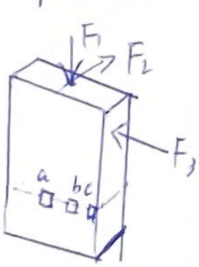
4)



Given: yield stress, Young's Modulus, permanent elongation δ_d, L, D

Find: P_{max} with δ_d

5)



Find normal and shear stress on a, b, and c.

$\phi = Ay^3$ 飞机.

yield stress

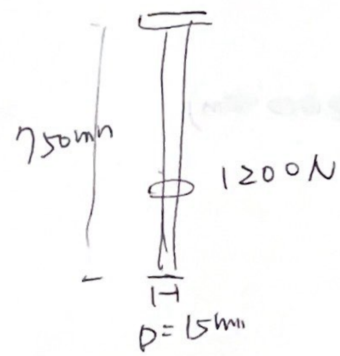
E P?

δ_P

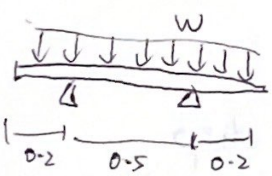
2021 Summer

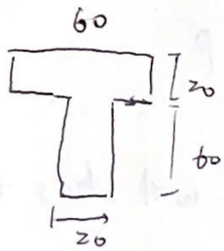
1. 2D. compatibility Eq. in terms of strain
equilibrium Eq. in terms of stress
verify given statements if they satisfy compatibility and equilibrium
(a) u_x, u_y, Displacements are given
(b) ε_x, ε_y Strains are given

2. Impact loading
(a) release slowly.
(b) Max axial displacement
(b) dynamic force / max normal stress
(Droped from 30 mm)



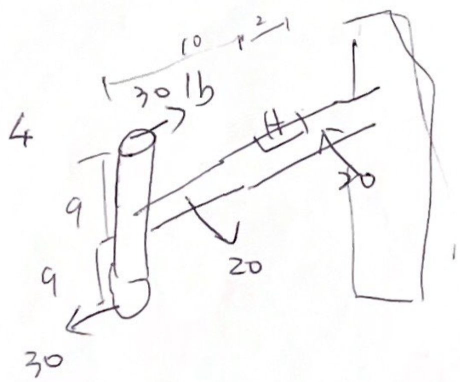
3. $\sigma_{all\,ten} = 40$, $\sigma_{all\,comp} = -130\,MPa$



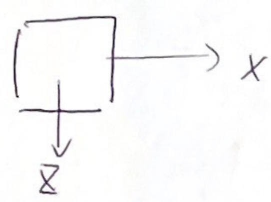


Find max w
and draw V and M

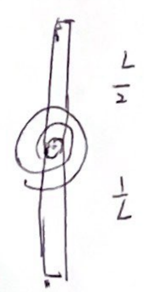
4.



Find sate of Stress
(Normal, shear)
Draw stress on this element (



5.



torsionl sping K
Derive P_{CR}

Summer 2021

1. T/F Questions
   (plastics . metal)

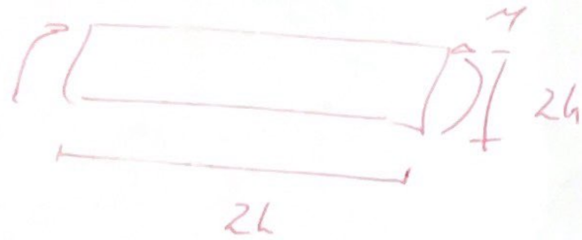
2. How are following items manufatured? materias?

3. Compare AM, to CNC, IM, casting

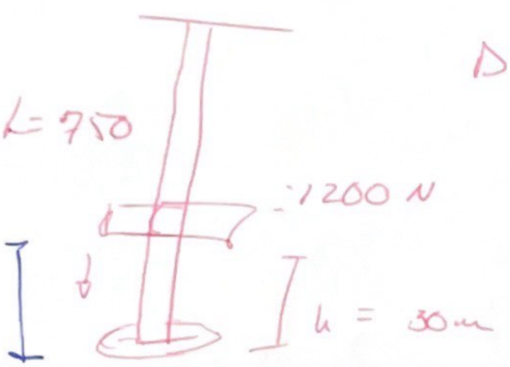
4. Pressure flow through a tube (Newtonian)
   $V_z(r)$ and Q

5. Metal Cutting for chip
   1. to, tc
   2. chang something? will has thick chip ..

① $\phi = Ay^3$ → Airy stress



②



D = 15 mm

① axial Load and deformation static.

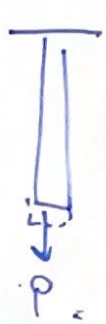
② deformation under impact Load.

③ Max Load and stress

- 1200N collar loaded slowly and then dropped from height h

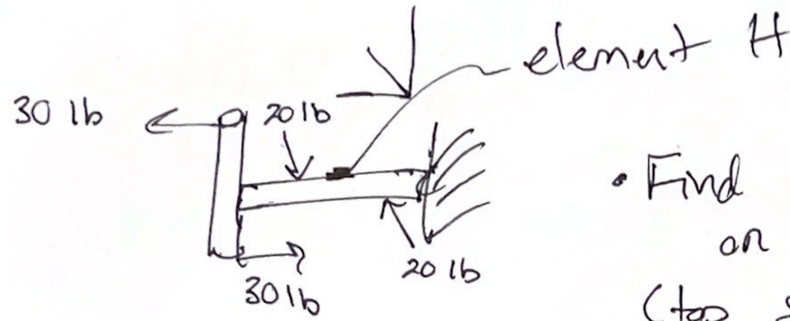
$U = mgh$

δ



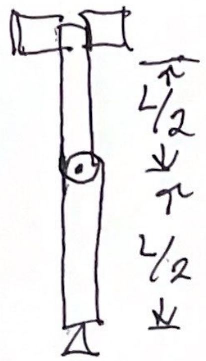
$\delta = \frac{PL}{AE}$

State of Stress



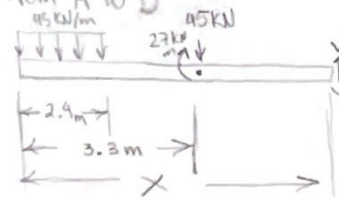
- Find state of stress on element H (top surface of a pipe)

Buckling



Define $P_{critical}$ for column with stiffness K and hinge in center.

From A to B



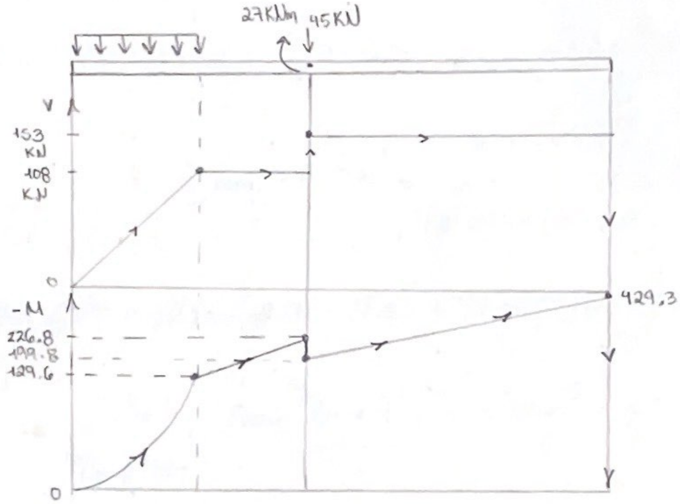
$\Sigma F_y = 0 \Rightarrow -45\frac{KN}{m} \cdot 2.4m - 45KN + V = 0 \Rightarrow V = 153KN$

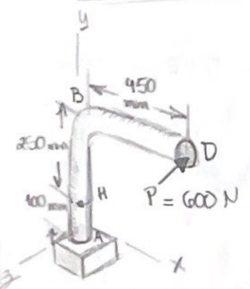
$\Sigma M_3 = 0 \Rightarrow 45\frac{KN}{m} \cdot 2.4m \cdot (X - 1.2m) - 27KNm + 45KN(x - 3.3m) + M = 0$

$\Rightarrow M = -(108x - 129.6) + 27KNm - 45x + 148.5$

$= -153X + 305.1$

Using equations found:





Beer (7th ed.) page 486

Load P is applied on lever as shown in the figure. Knowing that $\overline{AB}$ has a diameter of 30 mm, determine:

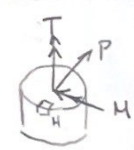
a) Normal & shearing stresses located @ H (located w. sides parallel to x & y axes)

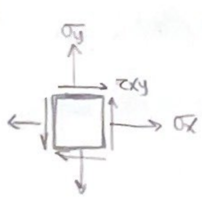
b) Principal planes & stresses @ H

a) Load P will produce a momentum & torque in AB

$T = 600N \cdot 0.45m = 270 Nm$

$M = 600 N \cdot 0.25m = 150 Nm$





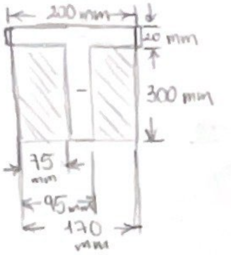
$\sigma_y = \frac{Mc}{I} = \frac{150 Nm \cdot 0.015m}{\frac{\pi}{4}(0.015m)^4} = 56.6 MPa$

$\tau_{xy} = \frac{Tc}{J} = \frac{270Nm \cdot 0.015m}{\frac{\pi}{2}(0.015m)^4} = 50.9 MPa$

b) $\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{0 + 56.6}{2} \pm \sqrt{\frac{56.6}{2} + 50.9} \Rightarrow \sigma_1 = 86.5 MPa$

$\sigma_2 = -29.9 MPa$

$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.8 \Rightarrow 2\theta = \arctan(-1.8) = -60.93° \Rightarrow \theta = 30.46$



Beer (7th ed.) page 265

A T shaped steel beam has been strengthened by wood. $E_{wood} = 12.5\,GPa$ $E_{steel} = 200\,GPa$. A bending moment of 50 KN.m is applied to the beam. Determine:

a) Max. stress in the wood

b) Stress in the steel along top edge

$$n = \frac{200}{12.5} = 16 \rightarrow \text{Transform cross section (Horizontal)}$$

Multiply steel width by 16 → Cross section made of wood

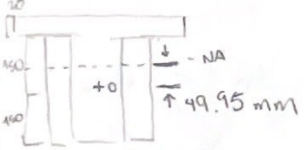
Centroid: Place origin @ height of 150 mm from bottom (i.e. middle of bottom rectangle)

$$\bar{Y} = \frac{\sum \bar{Y}A}{\sum A} = \frac{(150 + 20/2).\,[\underbrace{200 \times 16}_{b_n} . 20] + 0}{(2.(75) + 20.16)(300) + 20.(200 \times 16)} = 49.95\text{ mm}$$

$$I_x = \sum(I_x + Ad^2) = \frac{(2.75 + 20.16)(300\text{ mm})^3}{12} + (2.75 + 20.16)(300)(49.95)^2 + \frac{(200.16)(20)^3}{12} + [200.16\,20\,(150 + 10 - 49.95)^2]$$

$$I_x = 1.409 \times 10^9\text{ mm}^4 + 8.007 \times 10^8\text{ mm}^4 = 2.210 \times 10^9\text{ mm}^4 \; \frac{1\text{ m}^4}{(1000\text{ mm})^4} = 2.21 \times 10^{-3}\text{ m}^4$$

a) Max. Stress → Point furthest away from NA.



$$c_1 = (150 + 20) - 49.95 = 120.05\text{ mm}$$

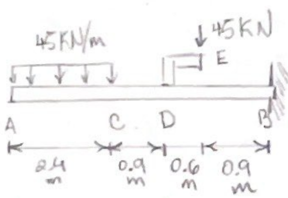
$$c_2 = (150) + 49.95 = 199.95\text{ mm}$$

$$\sigma_w = \frac{Mc_2}{I} = \frac{50 \times 10^3\text{ Nm}.\,199.95 \times 10^{-3}\text{ m}}{2.21 \times 10^{-3}\text{ m}^4}$$

$$\sigma_w = 4.52\text{ MPa}$$

b) Stress in steel on top edge $\sigma_{s_t} = \frac{Mc_1}{I} = \frac{50 \times 10^3\text{ Nm}.\,120.05 \times 10^{-3}\text{ m}}{2.21 \times 10^{-3}\text{ m}^4} = 2.72\text{ MPa}$

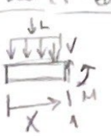
Due to transformed cross-section → $\sigma_s = \sigma_{s_t} \cdot 16 = 43.5\text{ MPa}$



Beer (7th edition) page 353

a) Draw shear & bending-moment diagrams. b) Determine the max. normal stress in the vicinity of point D.

De-couple structure in point D → generates downward for of 45 KN & moment $0.6\text{ m} \cdot 45\text{ KN} = 27\text{ KN.m}$

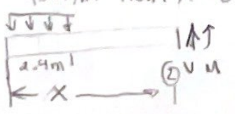


From A to C

$$\sum F_y = 0 \Rightarrow V = L = 45\,\text{KN/m}.x$$

$$\sum M_1 = 0 \Rightarrow 45\,\text{KN/m}.x.\frac{x}{2} - M = 0 \Rightarrow M = -\frac{45}{2}x^2$$

From A to D



$$\sum F_y = 0 \Rightarrow 45\frac{KN}{m} \cdot 2.4\text{ m} - V = 0 \Rightarrow V = 108\text{ KN}$$

$$\sum M_2 = 0 \Rightarrow -45\frac{KN}{m} \cdot 2.4\text{ m}.(x - 1.2\text{ m}) + M = 0 \Rightarrow M = -108x + 129.6\text{ KNm}$$

b) $\frac{\partial M}{\partial P} = \frac{x}{2} \Rightarrow \delta c = \int \left(\frac{M}{EI}\right)\left(\frac{\partial M}{\partial P}\right) dx = 2\int_0^{L/2} \frac{1}{EI}\left(\frac{Px}{2} + \frac{qLx}{2} - \frac{qx^2}{2}\right)\left(\frac{x}{2}\right) dx$

$$= \frac{2}{EI}\int_0^{L/2}\left(\frac{Px^2}{4} + \frac{qLx^2}{4} - \frac{qx^3}{4}\right)dx = \frac{2}{EI}\left[\frac{Px^3}{12} + \frac{qLx^3}{12} - \frac{qx^4}{16}\right]_0^{L/2}$$

$$= \frac{2}{EI}\left[\frac{PL^3}{96} + \frac{qL^4}{96} - \frac{qL^4}{256}\right] = \frac{2}{EI}\left[\frac{PL^3}{96} + \left(\frac{5q}{768}\right)L^4\right] = \frac{PL^3}{48EI} + \frac{5L^4q}{384EI}$$

A plane displacement field has the components $u = -a_1(y^2 + \nu x^2)$; $v = 2a_1xy$ ($a_1 \equiv$ Constant)
Determine the stresses, and sketch them acting on boundaries $-a \le x \le a$; $-b \le y \le b$
(Homework 7 - EMA 506 Fall 2018)

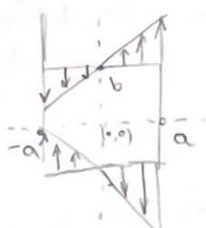
$\varepsilon_x = \frac{\partial u}{\partial x}$ $\quad \varepsilon_y = \frac{\partial v}{\partial y}$ $\quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

$\hookrightarrow -2\nu a_1 x$ $\quad \hookrightarrow 2a_1x$ $\quad \hookrightarrow -2a_1y + 2a_1y = 0$

$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = 0$; $\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{E}{1-\nu^2}(2a_1x - 2\nu^2 a_1 x) = \frac{E}{\cancel{1-\nu^2}}\left[(2a_1x)(\cancel{1-\nu^2})\right]$

$= 2Ea_1x$

$\tau_{xy} = G\gamma_{xy} = 0$



Examine the field below and see if it is a valid solution for a plane elasticity problem. If not, what condition isn't met? For simplicity, let Poisson's ratio be zero and assume a_1 is a constant. $u = a_1xy^2$; $v = -a_1x^2y$

$\varepsilon_x = \frac{\partial u}{\partial x}$ $\quad \varepsilon_y = \frac{\partial v}{\partial y}$ $\quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

$\hookrightarrow \varepsilon_x = a_1y^2$ $\quad \hookrightarrow \varepsilon_y = -a_1x^2$ $\quad \hookrightarrow \gamma_{xy} = \cancel{2a_1xy - 2a_1xy} = 0$

Check compatibility: $\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$ $\qquad \frac{\partial \varepsilon_x}{\partial y} = 2a_1y \Rightarrow \frac{\partial^2 \varepsilon_x}{\partial y^2} = 2a_1$

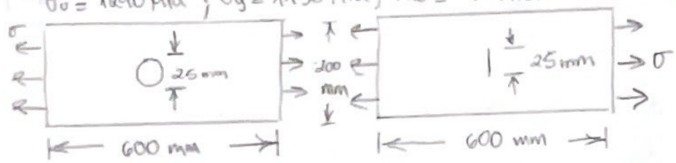
$2a_1 - 2a_1 = 0$ satisfies compatibility! $\qquad \frac{\partial \varepsilon_y}{\partial x} = -2a_1x \Rightarrow \frac{\partial^2 \varepsilon_y}{\partial x^2} = -2a_1$

Check equilibrium $\frac{\cancel{\partial \sigma_x}}{\partial x} + \frac{\partial \tau_{xy}}{\cancel{\partial y}} = 0$; $\frac{\partial \sigma_y}{\cancel{\partial y}} + \frac{\cancel{\partial \tau_{xy}}}{\partial x} = 0$ Satisfies equilibrium!

$\therefore$ VALID SOLUTION

Energy methods + Theory of elasticity + other topics (9)

$\sigma_u = 1290\,MPa$; $\sigma_y = 1150\,MPa$; $K_{IC} = 77\,MPa\,m^{1/2}$



The two plates shown are each 20 mm thick and are made of steel. One plate contains a circular hole and the other a central crack. The applied stress σ is caused by an axial force P.

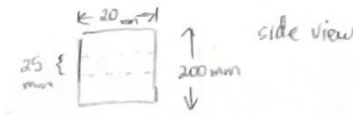
EMA 506 Homework 6, Fall 2018

a) Determine the force P that will cause yielding of the plate with the hole.

b) Determine the approx. force P that will cause the plate with the hole to break.

c) Determine the force that will break the plate with the crack to fail.

a) Using stress conc. factor: $\frac{2r}{D} = \frac{25}{200} = \frac{1}{8} \Rightarrow K_t = 2.66$



$\sigma_y = K_t\,\sigma = 2.66 \cdot \frac{P}{A}$; $A = (200\times10^{-3}m - 25\times10^{-3}m)(20\times10^{-3}m) = 3.5\times10^{-3}\,m^2$

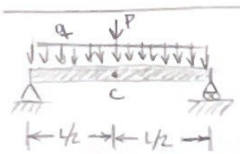
thus $1150\times10^6\,Pa = \frac{2.66 \cdot P_y}{3.5\times10^{-3}m^2} \Rightarrow P_y = 1.51\times10^6\,N$

b) Similarly: $\sigma_u = K_t\,\sigma = \frac{2.66\,P_u}{3.5\times10^{-3}m^2} = 1290\times10^6\,Pa \Rightarrow P_u = 1.69\times10^6\,N$

c) $K_{IC} = \sigma_c \cdot y\sqrt{\pi a}$ using geometry to determine y: $a = 12.5\,mm$; $c = 100\,mm \Rightarrow \frac{a}{c} = 0.125$

$y = 1.01 \Rightarrow 77\,MPa\,m^{1/2} = 1.01 \cdot \sigma_c \cdot \sqrt{\pi \cdot 12.5\times10^{-3}m} \Rightarrow \sigma_c = 384.7\,MPa = \frac{P}{(200\,mm)(20\,mm)}$

$\Rightarrow P = 1.54\times10^6\,N$

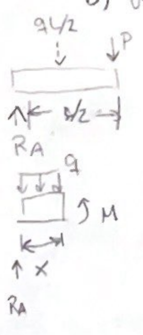


A simply supported beam has a uniform load of q = 1.5 Kips/ft over its length, as well as a conc. load P = 5 Kips that acts on the mid-point of the beam. L = 8 ft; E = 30×10^6 psi ; I = 75 in^4.

a) Find the complimentary strain energy of the beam and derive w.r.t P to determine the displacement of the mid-point.

b) Use Castigliano's 2nd theorem to determine the deflection of C.

Due to symmetry:

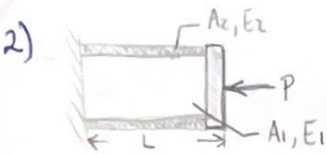


$\Rightarrow R_A = \frac{P}{2} + \frac{qL}{2} \Rightarrow M - R_A X + \frac{qx^2}{2} = 0 \Rightarrow M = R_A X - \frac{qx^2}{2} = \frac{Px}{2} + \frac{qLx}{2} - \frac{qx^2}{2}$

$$U = \int \frac{M^2 dx}{2EI} = 2\int_0^{L/2} \frac{1}{2EI}\left(\frac{Px}{2} + \frac{qLx}{2} - \frac{qx^2}{2}\right)^2 dx$$

Integrating (lengthy process)

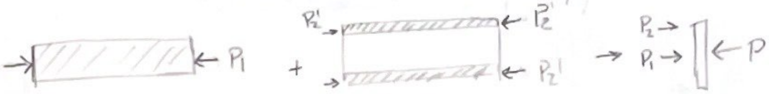
$$= \frac{P^2L^3}{96EI} + \frac{5PqL^4}{384EI} + \frac{q^2L^5}{240EI} \qquad \delta_c = \frac{\partial U}{\partial P} = \frac{PL^3}{48EI} + \frac{5qL^4}{384}$$

2) 

Beer, page 78 (7th ed)

a) Find the deformation of the tube and rod when force P is exerted in the rigid endcap as shown? (2)

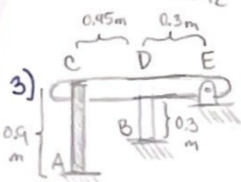
Distributed radially, summarized as P_2



$$P = P_1 + P_2 \,, \quad \delta_1 = \delta_2 \quad \text{due to compatibility}$$

$$\delta_1 = \frac{P_1 L}{E_1 A_1} = \frac{P_2 L}{E_2 A_2} = \delta_2 \Rightarrow P_1 = \frac{P_2 E_1 A_1}{E_2 A_2} \Rightarrow P = P_2\left(1 + \frac{E_1 A_1}{E_2 A_2}\right) \Rightarrow P_2 = \frac{P}{\left(1+\frac{E_1 A_1}{E_2 A_2}\right)}$$

$$P_1 = \frac{P E_1 A_1}{\left(1+\frac{E_1 A_1}{E_2 A_2}\right) E_2 A_2}$$

3) 

Rigid bar CDE, attached to a pin support @ E, rests on a 30 mm Ø brass cyl. BD. A 22 mm Ø steel rod AC passes through a hole in the bar, secured by a nut @ 20°C. The temp. of BD is raised to 50°C, while AC remains @ 20°C. Determine the stresses.-

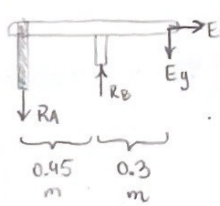
Beer, page 87 (7th ed)

$E_{steel} = 200\,GPa$

$\alpha_{steel} = 11.7 \times 10^{-6}\ °C^{-1}$

$E_{brass} = 105\,GPa$

$\alpha_{brass} = 20.9 \times 10^{-6}\ °C^{-1}$



$$\Sigma M_E = -0.3m \cdot R_B + 0.75m \cdot R_A = 0$$

$$\Rightarrow \boxed{R_A = \frac{0.3}{0.75} R_B = 0.4 R_B}$$

$$\delta_{T_{Brass}} = \Delta T \alpha L = (50-20)°C\,(0.3\,m)(20.9\times10^{-6}\,°C^{-1}) = 1.88\times10^{-4}\,m$$

Due to geometry $\frac{\delta_C}{0.75} = \frac{\delta_D}{0.3}$

$\Rightarrow \boxed{\delta_D = 0.4\,\delta_C}$

$$\delta_C = \frac{R_A L}{A_{steel} E_{steel}} = \frac{R_A \cdot (0.9m)}{\frac{\pi}{4}(22\times10^{-3}m)^2 \cdot 200\times10^9\,Pa} = 1.184\times10^{-8} R_A$$

$$\delta_D = 0.4\,\delta_C = 4.735\times10^{-9} R_A \qquad \delta_{BD} = \frac{R_B L}{E_{Brass} A_{Brass}} = \frac{R_B \cdot (0.3m)}{\frac{\pi}{4}(30\times10^{-3}m)^2 \times 105\times10^9\,Pa} = 4.04\times10^{-9} R_B$$

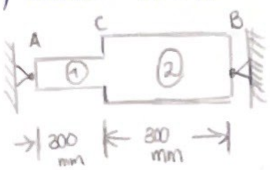
$$\delta_1 = \delta_D + \delta_{BD} = \delta_T \Rightarrow 4.735\times10^{-9} R_A + 4.04\times10^{-9} R_B = 1.88\times10^{-4}\,m \qquad R_A = 0.4 R_B$$

$$\Rightarrow 5.934\times10^{-9} R_B = 1.88\times10^{-4}\,m \Rightarrow R_B = 31.68\times10^3\,N \quad \text{thus } \sigma_{brass} = \frac{31.68\,kN}{\frac{\pi}{4}(30\times10^{-3}m)^2} = 44.82\,MPa$$

Gerardo A. Mazzei Capote

Qualifying Exam - Mechanics

1) Basic MoM:



a) Determine stress in AC & CB of the steel bar, currently @ -45°C, knowing that a close fit exists at both of the rigid supports when the temperature is 24°C.

Useful info: $E = 200\ GPA$; $\alpha = 11.7 \times 10^{-6}\ °C^{-1}$; $A_1 = 380\ mm^2$; $A_2 = 750\ mm^2$

Beer, page 84 (7th ed)

Using superposition: Thermal expansion + Supports = Complete problem

Thermal:

$$\delta_T = L\alpha\Delta T = (300\ mm + 300\ mm) \times (11.7\times10^{-6}\ °C^{-1})(-45°C - 24°C) = -0.484\ mm$$

Supports:

$$\delta_R = \frac{P_1L_1}{EA_1} + \frac{P_2L_2}{EA_2} \quad (\text{From inspection, } P_1 = P_2 = R_B)$$

$1GPa = 1000\ N/mm^2$

$$\delta_R = \frac{R_B}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2}\right) = \frac{R_B}{200GPa} \times \left(\frac{300\ mm}{380\ mm^2} + \frac{300\ mm}{750\ mm^2}\right) = R_B \times 5.947\times10^{-6}\ \frac{mm}{N}$$

=> Applying superposition + constraints: $\delta_T + \delta_R = 0 \Rightarrow 0 = -0.484 + R_B \times 5.947\times10^{-6}\ \frac{mm}{N}$

$$\Rightarrow R_B = \frac{0.484\ mm}{5.947\times10^{-6}\ \frac{mm}{N}} = 81.39\ KN \qquad R_A = -81.39\ KN \text{ Due to equilibrium...}$$

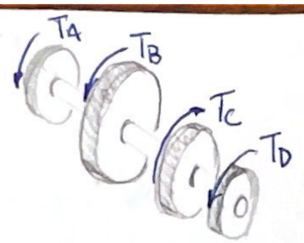
$$\sigma_1 = \frac{81.39\times10^3\ N}{380\ mm^2} = 214.2\ MPa; \quad \sigma_2 = 108.5\ MPa$$

b) Determine the strain in AC & BC, as well as the deformation of each section

$$\varepsilon_T = \frac{\delta_T}{L} = -8.07\times10^{-4} \quad \varepsilon_1 = \frac{214\ MPa}{200\times10^3\ MPa} = 1.07\times10^{-3} \quad \varepsilon_2 = \frac{108.5\ MPa}{200\times10^3\ MPa} = 5.43\times10^{-4}$$

$$\varepsilon_{AC} = \varepsilon_T + \varepsilon_1 = 2.63\times10^{-4} \quad \varepsilon_{BC} = \varepsilon_T + \varepsilon_2 = -2.64\times10^{-4}$$

$$\delta_{AC} = \varepsilon_{AC}\cdot \underbrace{L_{AC}}_{300\ mm} = 0.079\ mm \quad \delta_{BC} = \varepsilon_{BC}\cdot \underbrace{L_{BC}}_{300\ mm} = -0.079\ mm$$



$\overline{AB} = 0.9m$ $\quad T_A = 6KNm$

$\overline{BC} = 0.7m$ $\quad T_B = 14KNm$

$\overline{CD} = 0.5m$ $\quad T_C = 26KNm$

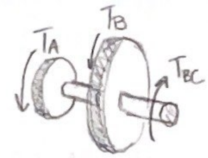
$T_D = 6KNm$

Shaft BC is hollow w. $\varnothing_i = 90mm$ & $\varnothing_o = 120mm$.
Shafts AB & CD are solid of $\varnothing = d$.-
Determine a) max. & min. shear stress in BC
b) Required value for d if $\tau_{all} = 65 MPa$

Beer (7th ed.) page 158

a)

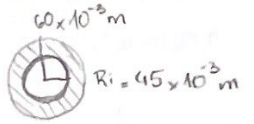
$T_A = T_{AB} = 6KNm$;



$T_A + T_B - T_{BC} = 0$

$T_{BC} = T_A + T_B = 20KNm$

$\tau = \frac{Tr}{J}$ In Shaft BC

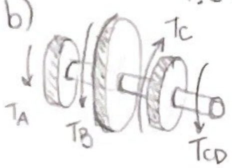


$J = \frac{\pi}{2}\left[(60\times10^{-3})^4 - (45\times10^{-3})^4\right] = 1.392\times10^{-5} m^4$

Max torque will be @ $\varnothing_o \Rightarrow r = 60\times10^{-3}m$; Min torque will be in $\varnothing_i \Rightarrow 45\times10^{-3}m = r$

$\tau_{max} = \frac{20KNm \cdot 60\times10^{-3}m}{1.392\times10^{-5}m^4} = 82.23MPa$; $\tau_{min} = \frac{20KNm \cdot 45\times10^{-3}m}{1.392\times10^{-5}m^4} = 64.67MPa$

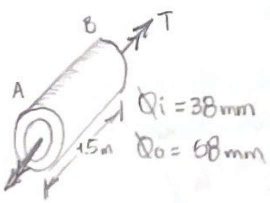
b)



$T_{CD} + T_A + T_B - T_C = 0 \Rightarrow T_{CD} = T_C - T_A - T_B = 6KN$

$T_{AB} = T_{CD} \Rightarrow 65MPa = \frac{6KNm \cdot (d/2)}{\frac{\pi}{2}(d/2)^4} \Rightarrow \frac{6\times10^3 Nm}{\frac{\pi}{2}\cdot 65\times10^6 \frac{N}{m^2}} = (d/2)^3$

$\Rightarrow \frac{d}{2} = \sqrt[3]{5.876\times10^{-5}m^3} = 3.888\times10^{-2}m \Rightarrow d = 77.76 mm$



Beer (7th ed) page 202

Shaft AB is made of elastoplastic steel with $G = 77GPa$ & $\tau_y = 145 MPa$. A torque T is gradually increased in magnitude. Determine torque T and angle of twist ϕ at which a) Yield occurs b) Deformation is fully plastic.-

a) $\tau_y = \frac{T \cdot r}{J}$; $J = \frac{\pi}{2}\left[\left(\frac{58\times10^{-3}m}{2}\right)^4 - \left(\frac{38\times10^{-3}m}{2}\right)^4\right] = 9.063\times10^{-7} m^4$

Yield will first occur at outer radius, where $r = \varnothing_o/2$

Thus, $145\times10^6 Pa = \frac{T_0 \cdot (58\times10^{-3}m/2)}{9.063\times10^{-7}m^4} \Rightarrow T_0 = 4.532 KN.m$; $\phi_0 = \frac{\gamma_y L}{(\varnothing_o/2)} = \frac{L \cdot \tau_y}{G(\varnothing_o/2)} = 9.74\times10^{-2} rad$

b) Deformation will be fully plastic when inner radius reaches τ_y

$\phi_f = \frac{L \cdot \tau_y}{\left(\frac{\varnothing_i}{2}\right)\cdot G} = 0.1487 rad$; $T_y = 2\pi \int_{\varnothing_i/2}^{\varnothing_o/2} \rho^2 \tau_y d\rho = \frac{2\pi}{3} \rho^3 \tau_y \Big|_{\varnothing_i/2}^{\varnothing_o/2} = \frac{2}{3}\pi (145\times10^6 Pa)\left[\left(\frac{58\times10^{-3}m}{2}\right)^3 - \left(\frac{38\times10^{-3}m}{2}\right)^3\right]$

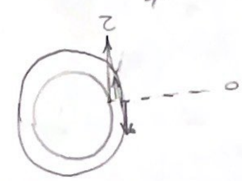
$= 5.324 KNm$

c) determine the residual stresses and permanent angle of twist after the torque found in part b) (4)
is removed.

Applying torque in opposite direction assuming elastic behavior to model elastic recovery of material yields:

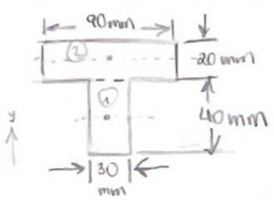
$$\tau_{max} = \frac{T.(D_o/2)}{J} = \frac{5.324\times10^3\,Nm.\left(\frac{58\times10^{-3}m}{2}\right)}{9.063\times10^{-7}\,m^4} = 170.4\text{ MPa}$$

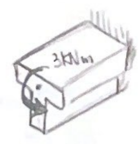
$$\tau_{min} = \frac{T_o\,(D_i/2)}{J} = 111.6\text{ MPa} \qquad \phi = \frac{TL}{GJ} = \frac{5.324\times10^3 Nm\,.\,1.5m}{9.063\times10^{-7}m^4\,.\,77\times10^9 Pa} = 0.1144\text{ rad}$$



Thus $\tau_{r_i} = (145-111.6) = 33.4\text{ MPa}$, $\tau_{r_o} = (145-170.4\text{ MPa}) = -25.4\text{ MPa}$

$\phi_r = (0.1487 - 0.1144)\,rad = 0.0343\text{ rad}$





E = 165GPa

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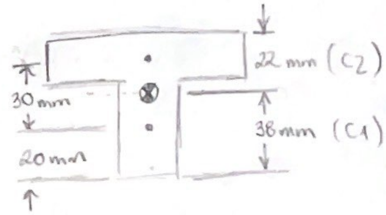
Determine a) Max. tensile & Compressive stresses
b) Radius of curvature

Find centroid & Moment of inertia

	Area [mm²]	$\bar{y}$ [mm]	$A\bar{y}$ [mm³]
1	(40 x 30) = 1200	20	24 000
2	(20 x 90) = 1800	50	90 000
	3000		114 000

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 38\text{ mm}$$



$$I_x = \Sigma(I_i + A_i d_i^2) = \frac{30\times40^3}{12} + 1200.18^2 + \frac{90\times20^3}{12} + 1800.12^2 = \boxed{8.68\times10^5\ mm^4}$$

$d_1 = (38-20) = 18$ mm
$d_2 = (22-10) = 12$mm
$= 8.68\times10^{-7}\,m^4$

$$\sigma_T = \frac{Mc_2}{I_x} = \frac{3\times10^3 Nm.\,2.2\times10^{-2}m}{8.67\times10^{-7}\,m^4} = 7.61\times10^7\,Pa = 76.1\text{ MPa}$$

$$\sigma_c = -\frac{Mc_1}{I_x} = \frac{-3\times10^3 Nm.\,3.8\times10^{-2}m}{8.67\times10^{-7}\,m^4} = -131.5\text{ MPa}$$

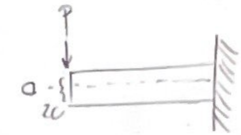
$$\frac{1}{\rho} = \frac{M}{EI} \Rightarrow \rho = \frac{EI}{M} = \frac{165\times10^9 Pa\,.\,8.68\times10^{-7}m^4}{3\times10^3\,Nm} = 47.74\text{ m}$$

(11)

Axial stress in the cantilever beam is $\sigma_x = \frac{3Pxy}{2c^3}$

a) Use this equation & theory of elasticity to determine τ_{xy}

b) What can be said about σ_y?



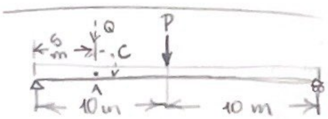
EMA506 - Homework 7 - Fall 2018

a) Using equilibrium: $\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \Rightarrow -\frac{\partial \sigma_x}{\partial x} = \frac{\partial \tau_{xy}}{\partial y}$ // $\frac{\partial \sigma_x}{\partial x} = \frac{3Py}{2c^3}$

thus $-\int \frac{3Py}{2c^3} dy = \boxed{\tau_{xy} = \frac{-3Py^2}{4c^3} + \varphi(x)}$ Use BC: @ $y = c$, $\tau_{xy} = 0$

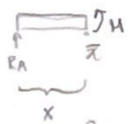
$0 = -\frac{3}{4}\frac{Pc^2}{c^3} + \varphi(x) \Rightarrow \frac{3P}{4c} = \varphi$ (constant) $\Rightarrow \tau_{xy} = -\frac{3Py^2}{4c^3} + \frac{3P}{4c}$ //

b) $\frac{\partial \tau_{xy}}{\partial x} = 0 \Rightarrow \sigma_y = \varphi(x)$ due to eq. Use BC: @ $y = c$ $\sigma_y = 0 \Rightarrow \sigma_y = 0$ //

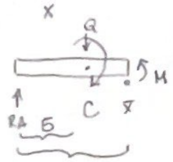


Consider the simply supported beam with a load P applied at the center. Determine the deflection & rotation of point A.-

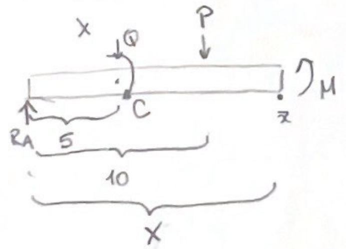
Add dummy moment C & dummy load Q in point A. Due to symmetry,



$\Sigma M_{\bar{x}} = M - R_A x = 0 \Rightarrow M = R_A x$ (A)

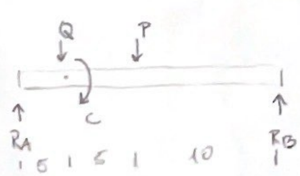


$\Sigma M_{\bar{x}} = M - R_A x + (x-5)Q - C = 0 \Rightarrow M = R_A x - Q(x-5) + C$ (B)



$\Sigma M_{\bar{x}} = M - C - R_A x + Q(x-5) + P(x-10) = 0$

$\Rightarrow M = R_A x - Q(x-5) - P(x-10) + C$ (C)



$\Sigma M_B = 0 = -R_A L + Q \cdot \frac{3}{4}L + \frac{P}{2}L - C \Rightarrow \boxed{R_A = \frac{3Q}{4} + \frac{P}{2} - \frac{C}{L}}$ (*)

Substitute (*) into (A), (B) & (C)

$$U^* = \int_0^L \frac{M^2}{2EI}\,dx$$

$$= \int_0^5 \frac{\{[(P/2) + (3Q/4) - (C/L)]x\}^2 dx}{2EI} + \int_5^{10} \frac{\{[(P/2)+(3Q/4)-(C/L)]x - Q(x-5)+C\}^2 dx}{2EI}$$

$$+ \int_{10}^{20} \frac{\{[(P/2) + (3Q/4) - (C/L)]x - Q(x-5) - P(x-10)+C\}^2 dx}{2EI}$$

According to Castigliano's 2nd Theorem

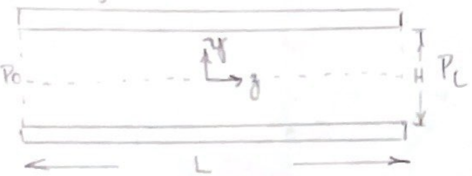
$$\delta_A = \frac{\partial U^*}{\partial Q} \quad ; \quad \theta_A = \frac{\partial U^*}{\partial C}$$

$$\delta_A = \int_0^5 \frac{\cancel{2}\{[(P/2)+(3Q/4)-(C/L)]x\,(3/4x)\,dx}{\cancel{2}\,EI} + \int_5^{10} \frac{\{[(P/2)+(3Q/4)-(C/L)]x - Q(x-5)+C\}\{\frac{3}{4}x-(x-5)\}\,dx}{EI}$$

$$+ \int_{10}^{20} \frac{\{[(P/2) + (3Q/4) - (C/L)]x - Q(x-5) - P(x-10)+C\}\{\frac{3}{4}x - (x-5)\}\,dx}{EI}$$

Similar procedure for θ_A.-

Rectangular slit



Assuming steady state: cartesian coord. for cont. & momentum

$\frac{\partial P}{\partial t} = 0$

Since there are no components to v_x & v_y

Continuity equation: $0 + 0 + 0 + \frac{\partial (\rho v_z)}{\partial z} = 0 \Rightarrow \boxed{\frac{\partial v_z}{\partial z} = 0}$

Similarly, on momentum balance $\frac{\partial v_z}{\partial t}, \frac{\partial v_z}{\partial x}, \frac{\partial v_z}{\partial z}, v_y = 0$

$$0 = -\frac{\partial P}{\partial z} + \mu \left[\frac{\partial^2 v_z}{\partial y^2}\right] \Rightarrow \frac{\partial P}{\partial z} = \mu \left[\frac{\partial^2 v_z}{\partial y^2}\right] \qquad \frac{\partial P}{\partial z} \approx \frac{\Delta P}{L}$$

$$\mu \left[\frac{\partial^2 v_z}{\partial y^2}\right] = \frac{\Delta P}{L} \Rightarrow \int \frac{\Delta P}{L\mu} dy = \frac{\partial v_z}{\partial y} \Rightarrow \int \left(\frac{\Delta P y}{L\mu} + C_1\right) dy = v_z$$

$$\Rightarrow v_z = \frac{\Delta P y^2}{2L\mu} + C_1 y + C_2$$

BC: $y = 1/2 H \Rightarrow v_z = 0$

$y = 0 \Rightarrow \frac{\partial v_z}{\partial y} = 0$

$\tau_{ij} = \mu \dot{\gamma}_{ij}$

$$0 = \frac{\Delta P (1/2 h)^2}{2L\mu} + \cancel{C_1 (1/2 h)}^{0} + C_2 \Rightarrow \boxed{C_2 = -\frac{\Delta P h^2}{8L\mu}}$$

$$0 = \frac{\Delta P (0)}{L\mu} + C_1 \Rightarrow \boxed{C_1 = 0}$$

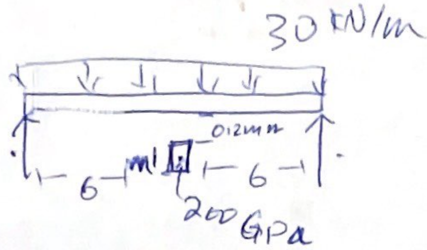
thus $\boxed{v_z = \frac{\Delta P y^2}{2L\mu} - \frac{\Delta P h^2}{8L\mu}}$

$$\hat{v}_z = \frac{1}{h/2} \int_0^{h/2} v_z \, dy = \frac{1}{h/2} \left(\frac{\Delta P y^3}{6L\mu} - \frac{\Delta P h^2 y}{8L\mu}\right)\Bigg|_0^{h/2}$$

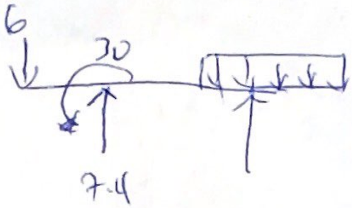
$$\hat{v}_z = \frac{2}{h} \left[\frac{\Delta P h^3}{6L\mu(8)} - \frac{\Delta P h^3}{8L\mu(2)}\right] = \frac{\Delta P h^2}{L\mu} \left[\frac{1}{24} - \frac{1}{8}\right] = \frac{\Delta P h^2}{L\mu} \left[\frac{1-3}{24}\right]$$

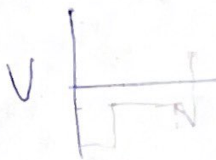
$$= -\frac{\Delta P h^2}{L\mu 12}$$

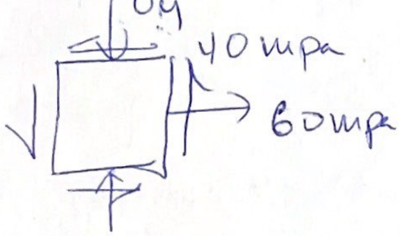
?



γ_A, R_B, R_C







~~τ_{xy}~~ $\tau_{max} < 45$ mpa

Drive P_{cr}

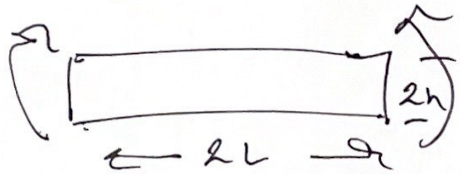
2D elastic body
State compatibility condition in terms of strain field
State equilibrium equation in terms of stress field
Verify given statement if they ~~sta~~ satisfy comp & eq

Airy Stress function

$$\Phi = A y^3$$

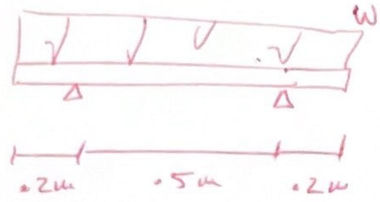
State of stresses, traction free on outer surfaces,

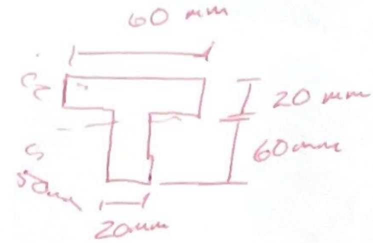
State moment in terms of ~~x, y~~, A, h, L



④

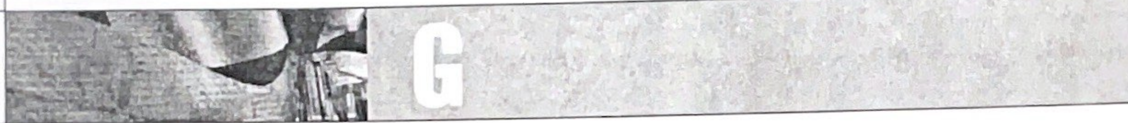
Find Max dist. Load w if





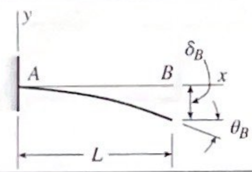
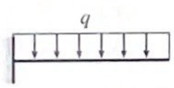
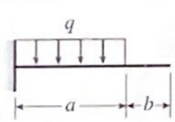
$\sigma_{all_{ten}} = 40\ MPa$

$\sigma_{all_{comp}} = -130\ MPa$

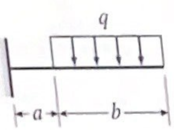


Deflections and Slopes of Beams

TABLE G-1 DEFLECTIONS AND SLOPES OF CANTILEVER BEAMS

		v = deflection in the y direction (positive upward) $v' = dv/dx$ = slope of the deflection curve $\delta_B = -v(L)$ = deflection at end B of the beam (positive downward) $\theta_B = -v'(L)$ = angle of rotation at end B of the beam (positive clockwise) EI = constant
1		$v = -\frac{qx^2}{24EI}(6L^2 - 4Lx + x^2) \qquad v' = -\frac{qx}{6EI}(3L^2 - 3Lx + x^2)$ $\delta_B = \frac{qL^4}{8EI} \qquad \theta_B = \frac{qL^3}{6EI}$
2		$v = -\frac{qx^2}{24EI}(6a^2 - 4ax + x^2) \qquad (0 \le x \le a)$ $v' = -\frac{qx}{6EI}(3a^2 - 3ax + x^2) \qquad (0 \le x \le a)$ $v = -\frac{qa^3}{24EI}(4x - a) \qquad v' = -\frac{qa^3}{6EI} \qquad (a \le x \le L)$ At $x = a$: $v = -\frac{qa^4}{8EI} \qquad v' = -\frac{qa^3}{6EI}$ $\delta_B = \frac{qa^3}{24EI}(4L - a) \qquad \theta_B = \frac{qa^3}{6EI}$

3



$$v = -\frac{qbx^2}{12EI}(3L + 3a - 2x) \qquad (0 \le x \le a)$$

$$v' = -\frac{qbx}{2EI}(L + a - x) \qquad (0 \le x \le a)$$

$$v = -\frac{q}{24EI}(x^4 - 4Lx^3 + 6L^2x^2 - 4a^3x + a^4) \qquad (a \le x \le L)$$

$$v' = -\frac{q}{6EI}(x^3 - 3Lx^2 + 3L^2x - a^3) \qquad (a \le x \le L)$$

At $x = a$: $\quad v = -\frac{qa^2b}{12EI}(3L + a) \qquad v' = -\frac{qabL}{2EI}$

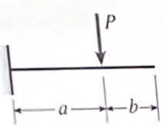
$$\delta_B = \frac{q}{24EI}(3L^4 - 4a^3L + a^4) \qquad \theta_B = \frac{q}{6EI}(L^3 - a^3)$$

4

$$v = -\frac{Px^2}{6EI}(3L - x) \qquad v' = -\frac{Px}{2EI}(2L - x)$$

$$\delta_B = \frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$$

5



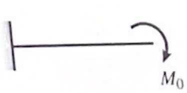
$$v = -\frac{Px^2}{6EI}(3a - x) \qquad v' = -\frac{Px}{2EI}(2a - x) \qquad (0 \le x \le a)$$

$$v = -\frac{Pa^2}{6EI}(3x - a) \qquad v' = -\frac{Pa^2}{2EI} \qquad (a \le x \le L)$$

At $x = a$: $\quad v = -\frac{Pa^3}{3EI} \qquad v' = -\frac{Pa^2}{2EI}$

$$\delta_B = \frac{Pa^2}{6EI}(3L - a) \qquad \theta_B = \frac{Pa^2}{2EI}$$

6

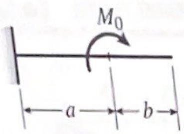


$$v = -\frac{M_0x^2}{2EI} \qquad v' = -\frac{M_0x}{EI}$$

$$\delta_B = \frac{M_0L^2}{2EI} \qquad \theta_B = \frac{M_0L}{EI}$$

(*Continued*)

7



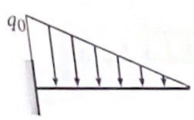
$$v = -\frac{M_0 x^2}{2EI} \qquad v' = -\frac{M_0 x}{EI} \qquad (0 \le x \le a)$$

$$v = -\frac{M_0 a}{2EI}(2x - a) \qquad v' = -\frac{M_0 a}{EI} \qquad (a \le x \le L)$$

$$\text{At } x = a\text{:} \quad v = -\frac{M_0 a^2}{2EI} \qquad v' = -\frac{M_0 a}{EI}$$

$$\delta_B = \frac{M_0 a}{2EI}(2L - a) \qquad \theta_B = \frac{M_0 a}{EI}$$

8

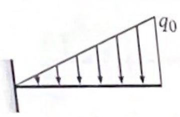


$$v = -\frac{q_0 x^2}{120LEI}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$$

$$v' = -\frac{q_0 x}{24LEI}(4L^3 - 6L^2 x + 4Lx^2 - x^3)$$

$$\delta_B = \frac{q_0 L^4}{30EI} \qquad \theta_B = \frac{q_0 L^3}{24EI}$$

9

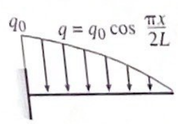


$$v = -\frac{q_0 x^2}{120LEI}(20L^3 - 10L^2 x + x^3)$$

$$v' = -\frac{q_0 x}{24LEI}(8L^3 - 6L^2 x + x^3)$$

$$\delta_B = \frac{11 q_0 L^4}{120EI} \qquad \theta_B = \frac{q_0 L^3}{8EI}$$

10

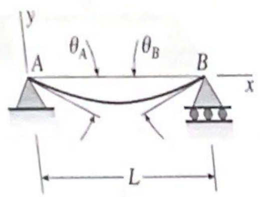


$$v = -\frac{q_0 L}{3\pi^4 EI}\left(48L^3 \cos\frac{\pi x}{2L} - 48L^3 + 3\pi^3 Lx^2 - \pi^3 x^3\right)$$

$$v' = -\frac{q_0 L}{\pi^3 EI}\left(2\pi^2 Lx - \pi^2 x^2 - 8L^2 \sin\frac{\pi x}{2L}\right)$$

$$\delta_B = \frac{2q_0 L^4}{3\pi^4 EI}(\pi^3 - 24) \qquad \theta_B = \frac{q_0 L^3}{\pi^3 EI}(\pi^2 - 8)$$

TABLE G-2 DEFLECTIONS AND SLOPES OF SIMPLE BEAMS



EI = constant

v = deflection in the y direction (positive upward)

$v' = dv/dx$ = slope of the deflection curve

$\delta_C = -v(L/2)$ = deflection at midpoint C of the beam (positive downward)

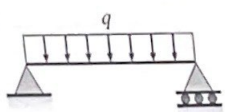
x_1 = distance from support A to point of maximum deflection

$\delta_{max} = -v_{max}$ = maximum deflection (positive downward)

$\theta_A = -v'(0)$ = angle of rotation at left-hand end of the beam (positive clockwise)

$\theta_B = v'(L)$ = angle of rotation at right-hand end of the beam (positive counterclockwise)

1

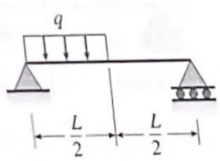


$$v = -\frac{qx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$v' = -\frac{q}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$\delta_C = \delta_{max} = \frac{5qL^4}{384EI} \qquad \theta_A = \theta_B = \frac{qL^3}{24EI}$$

2



$$v = -\frac{qx}{384EI}(9L^3 - 24Lx^2 + 16x^3) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

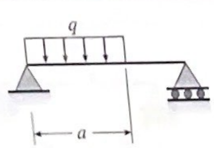
$$v' = -\frac{q}{384EI}(9L^3 - 72Lx^2 + 64x^3) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$v = -\frac{qL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3) \qquad \left(\frac{L}{2} \le x \le L\right)$$

$$v' = -\frac{qL}{384EI}(24x^2 - 48Lx + 17L^2) \qquad \left(\frac{L}{2} \le x \le L\right)$$

$$\delta_C = \frac{5qL^4}{768EI} \qquad \theta_A = \frac{3qL^3}{128EI} \qquad \theta_B = \frac{7qL^3}{384EI}$$

3



$$v = -\frac{qx}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 2a^2x^2 - 4aLx^2 + Lx^3) \qquad (0 \le x \le a)$$

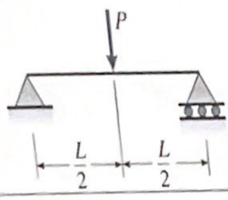
$$v' = -\frac{q}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 6a^2x^2 - 12aLx^2 + 4Lx^3) \qquad (0 \le x \le a)$$

$$v = -\frac{qa^2}{24LEI}(-a^2L + 4L^2x + a^2x - 6Lx^2 + 2x^3) \qquad (a \le x \le L)$$

$$v' = -\frac{qa^2}{24LEI}(4L^2 + a^2 - 12Lx + 6x^2) \qquad (a \le x \le L)$$

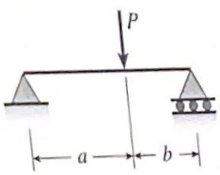
$$\theta_A = \frac{qa^2}{24LEI}(2L - a)^2 \qquad \theta_B = \frac{qa^2}{24LEI}(2L^2 - a^2)$$

(Continued)



$$v = -\frac{Px}{48EI}(3L^2 - 4x^2) \qquad v' = -\frac{P}{16EI}(L^2 - 4x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = \delta_{max} = \frac{PL^3}{48EI} \qquad \theta_A = \theta_B = \frac{PL^2}{16EI}$$



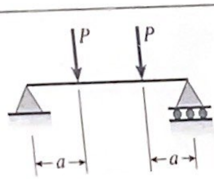
$$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \qquad v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) \qquad (0 \le x \le a)$$

$$\theta_A = \frac{Pab(L + b)}{6LEI} \qquad \theta_B = \frac{Pab(L + a)}{6LEI}$$

If $a \ge b$, $\delta_C = \dfrac{Pb(3L^2 - 4b^2)}{48EI}$ If $a \le b$, $\delta_C = \dfrac{Pa(3L^2 - 4a^2)}{48EI}$

If $a \ge b$, $x_1 = \sqrt{\dfrac{L^2 - b^2}{3}}$ and $\delta_{max} = \dfrac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}\, LEI}$

6

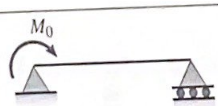


$$v = -\frac{Px}{6EI}(3aL - 3a^2 - x^2) \qquad v' = -\frac{P}{2EI}(aL - a^2 - x^2) \qquad (0 \le x \le a)$$

$$v = -\frac{Pa}{6EI}(3Lx - 3x^2 - a^2) \qquad v' = -\frac{Pa}{2EI}(L - 2x) \qquad (a \le x \le L - a)$$

$$\delta_C = \delta_{max} = \frac{Pa}{24EI}(3L^2 - 4a^2) \qquad \theta_A = \theta_B = \frac{Pa(L-a)}{2EI}$$

7

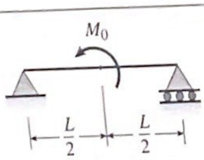


$$v = -\frac{M_0 x}{6LEI}(2L^2 - 3Lx + x^2) \qquad v' = -\frac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$$

$$\delta_C = \frac{M_0 L^2}{16EI} \qquad \theta_A = \frac{M_0 L}{3EI} \qquad \theta_B = \frac{M_0 L}{6EI}$$

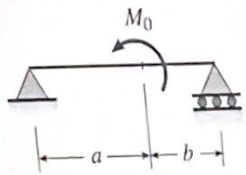
$$x_1 = L\left(1 - \frac{\sqrt{3}}{3}\right) \quad \text{and} \quad \delta_{max} = \frac{M_0 L^2}{9\sqrt{3}\, EI}$$

8



$$v = -\frac{M_0 x}{24LEI}(L^2 - 4x^2) \qquad v' = -\frac{M_0}{24LEI}(L^2 - 12x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = 0 \qquad \theta_A = \frac{M_0 L}{24EI} \qquad \theta_B = -\frac{M_0 L}{24EI}$$



$$v = -\frac{M_0 x}{6LEI}(6aL - 3a^2 - 2L^2 - x^2) \qquad (0 \le x \le a)$$

$$v' = -\frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2 - 3x^2) \qquad (0 \le x \le a)$$

At $x = a$: $v = -\dfrac{M_0 ab}{3LEI}(2a - L)$ $\qquad v' = -\dfrac{M_0}{3LEI}(3aL - 3a^2 - L^2)$

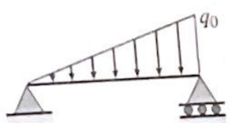
$$\theta_A = \frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2) \qquad \theta_B = \frac{M_0}{6LEI}(3a^2 - L^2)$$

10

$$v = -\frac{M_0 x}{2EI}(L - x) \qquad v' = -\frac{M_0}{2EI}(L - 2x)$$

$$\delta_C = \delta_{max} = \frac{M_0 L^2}{8EI} \qquad \theta_A = \theta_B = \frac{M_0 L}{2EI}$$

11



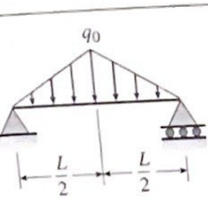
$$v = -\frac{q_0 x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$$

$$v' = -\frac{q_0}{360LEI}(7L^4 - 30L^2x^2 + 15x^4)$$

$$\delta_C = \frac{5q_0L^4}{768EI} \qquad \theta_A = \frac{7q_0L^3}{360EI} \qquad \theta_B = \frac{q_0L^3}{45EI}$$

$$x_1 = 0.5193L \qquad \delta_{max} = 0.00652\frac{q_0L^4}{EI}$$

12

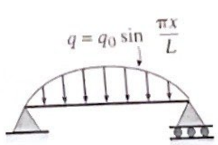


$$v = -\frac{q_0 x}{960LEI}(5L^2 - 4x^2)^2 \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$v' = -\frac{q_0}{192LEI}(5L^2 - 4x^2)(L^2 - 4x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = \delta_{max} = \frac{q_0L^4}{120EI} \qquad \theta_A = \theta_B = \frac{5q_0L^3}{192EI}$$

13



$$v = -\frac{q_0L^4}{\pi^4EI}\sin\frac{\pi x}{L} \qquad v' = -\frac{q_0L^3}{\pi^3EI}\cos\frac{\pi x}{L}$$

$$\delta_C = \delta_{max} = \frac{q_0L^4}{\pi^4EI} \qquad \theta_A = \theta_B = \frac{q_0L^3}{\pi^3EI}$$

$1\ lbf = 1\ lb \times g$

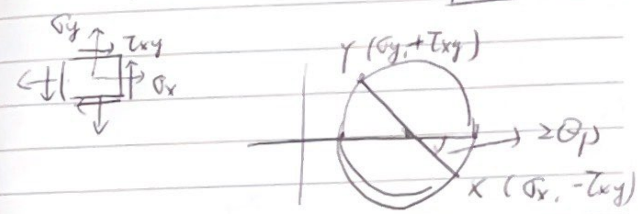
$= 1\ lb \cdot 9.8\ m/s^2\ /\ 0.3048\ \frac{m}{ft}$

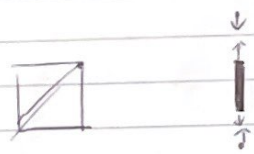
$\simeq 32.174 \left(\frac{ft\ lb}{s^2}\right)$

$\parallel$

pdl

Mohr's circle — p437





1 mile = 1760 yard.

1 yard = 3 foot

1 foot = ~~inch~~ 12 inch ~ 0.3048 m

(1 gallon = 4.621 L)

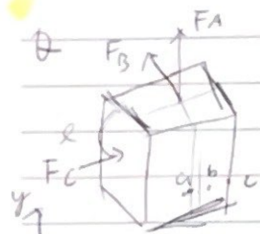
1 Pound (lb) = 16 oz (Ounce) ~454g

1 Ton = 2240 (lb)

1 Grain (gr) = $\frac{1}{7000}$ (lb)

prob 8.47

Q



Determine stresses for points a, b & c.

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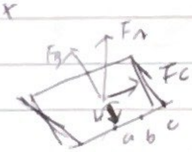


~~Ponto~~?

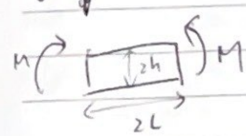
$M_x = F_C \cdot l$ $\qquad \sigma_y = \dfrac{F_A}{A} + \dfrac{|M_x| z}{I_x}$

a, b, c on the edge. F_B produces no shearing stress.

$\tau_{yx} = 0$

$$\tau_{yz} = \frac{F_C\, Q}{I_x\, t}$$

Q



$\phi = A y^3$

determine stress state & M using Airy Stress function.

$$\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2} = 6Ay$$

$$\sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2} = 0$$

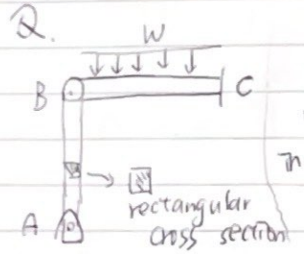
$$\sigma_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} = 0.$$

Also. $\sigma_{xx} = \dfrac{-My}{I} = -\dfrac{My}{\frac{1}{12} b (2h)^3}$ $\qquad b = 1?$

$$= -\frac{12M}{8h^3} y = -\frac{3M}{2h^3} y$$

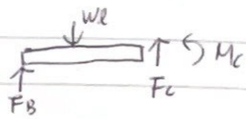
$$\rightarrow A = \frac{1}{6}\left(-\frac{3}{2}\right)\frac{M}{h^2} = -\frac{1}{4}\frac{M}{h^3}$$

Q.



Determine max w that won't cause AB to buckle in the pin-pin condition & fixed-fixed condition

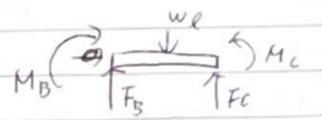
For pin-pin.



$+\circlearrowleft \Sigma M_C = 0, \rightarrow F_B = \frac{wl}{2}$

pin-pin. $P_{cr} = \frac{\pi^2 EI}{L^2} > \frac{wl}{2}$

$w < \frac{2\pi^2 EI}{L^2 l}$



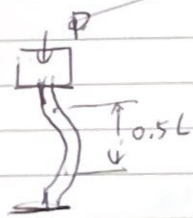
$+\circlearrowleft \Sigma M@_C = 0$

$-M_B - F_B \cdot l + wl \cdot \frac{l}{2} = 0$

$\Sigma F_y = 0: \; wl = F_B + F_C$

$+\circlearrowleft \Sigma M@_B = 0, \quad -wl \cdot \frac{l}{2} + F_C \cdot l + M_C = 0$

Fixed - Fixed ?



$\frac{\pi^2 EI}{Le^2} = \frac{\pi^2 EI}{(0.5L)^2}$

$= \frac{4\pi^2 EI}{L^2} > \frac{wl}{2}$

$w < \frac{8\pi^2 EI}{L^2 l}$

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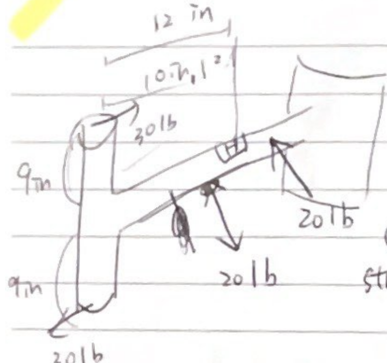
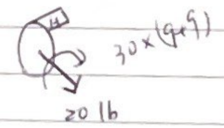


① Find state of stress on element H

② Draw all nonzero stress on given graph

$\boxed{H}$

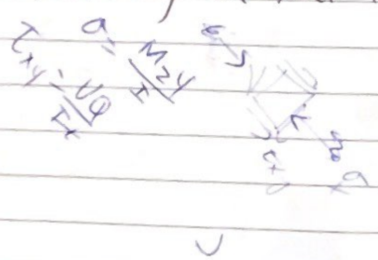


σ_{min}

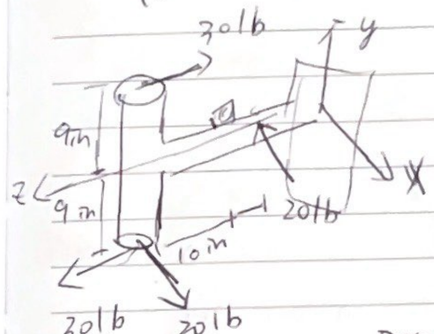
σ_{max}

τ_{max}

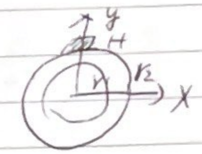
~~It seems only one force left.~~



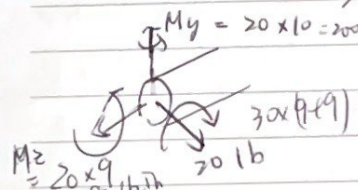
(solution : ~Problem 8.46 / 8.40)



Pipe-cross section



Determine state of stress in point H



Check Q, I for ⓞ

$$\tau_{zx} = \frac{VQ}{I(t)}$$

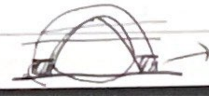
$$(\tau_H)_T = \frac{Tr}{J}$$

$$A = \pi(r_2^2 - r_1^2)$$

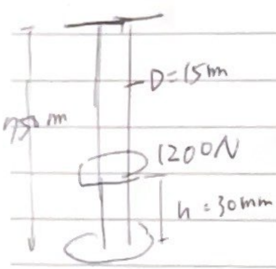
$$I = \frac{\pi}{4}(r_2^4 - r_1^4) \qquad (J = 2I)$$

for half pipe, $Q = \frac{2}{3}(r_2^3 - r_1^3)$

$2(r_2 - r_1)$



$t = 2$(shell thickness)

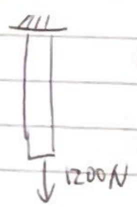


(a) release slowly
(b) max deflection
(c) dynamic force / max normal stress

Assume $E = 10\,GPa$

① axial load and deformation static
② deformation under impact load
③ Max Load and Stress

①



$$\delta = \frac{PL}{AE} = \frac{1200N \cdot 0.75}{\pi\left(\frac{0.015}{2}\right)^2 E} = \frac{900}{1.76\times10^{-4}\times10^{10}}$$

$= 5.11 \times 10^{-4}$ m

$= 0.51$ mm

②

$$\Delta PE = \Delta U$$

$$mgh = \frac{P^2L}{2AE}$$

$$1200 \cdot (0.03) = \frac{P^2(0.75)}{2\left(\pi\left(\frac{0.015}{2}\right)^2\right)E}$$

$$36 = \frac{P^2 \cdot 0.75}{3.52\times10^6}$$

$$P^2 = 168.96\times10^6$$

$$P = 13000 \text{ N}$$

$$\delta^{dynamic} = \frac{PL}{AE} = 5.525 \text{ mm}$$

$$\sigma = \frac{P}{A} = \frac{13000}{\pi\left(\frac{0.015}{2}\right)^2} = 73.39 \text{ MPa}$$

2D. elastic body

State compatibility conditions
in terms of strain field

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

State equilibrium equations in terms of ~~strain~~ field
stress

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + b_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + b_y = 0$$

Verify given statements if they satisfy compatibility and equilibrium

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E}$$

$$\varepsilon_y = -\frac{\nu \sigma_x}{E} + \frac{\sigma_y}{E}$$

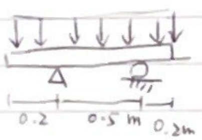
$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial u_y}{\partial y}$$

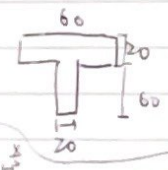
$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

(~ solution : problem 5.1>3)



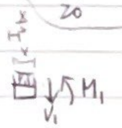
$\sigma_{all\,ten} = 40\ MPa$

$\sigma_{all\,comp} = -130\ MPa$



① Find max dist. load w

② Draw V and M diagram



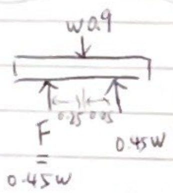
$+\uparrow \Sigma F_y = 0$

$V_1 = -wx$

$+\circlearrowleft \Sigma M_1 = 0$

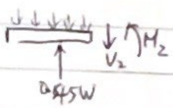
$wx \cdot \frac{1}{2}x + M_1 = 0$

$M_1 = -\frac{1}{2}wx^2$



$F \cdot 0.5 = w(0.9) \cdot 0.25$

$F = 0.45w$



$+\uparrow \Sigma F_y = 0$

$-wx + 0.45w - V_2 = 0$

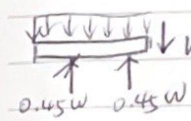
$V_2 = w(0.45 - x)$

$+\circlearrowleft \Sigma M_2 = 0$

$wx \cdot \frac{x}{2} - 0.45w(x - 0.2) + M_2 = 0$

$M_2 = w\left(0.45x - 0.09 - \frac{x^2}{2}\right)$

$+\uparrow \Sigma F_y = 0$



$-wx + 0.9w - V_3 = 0$

$V_3 = w(0.9 - x)$

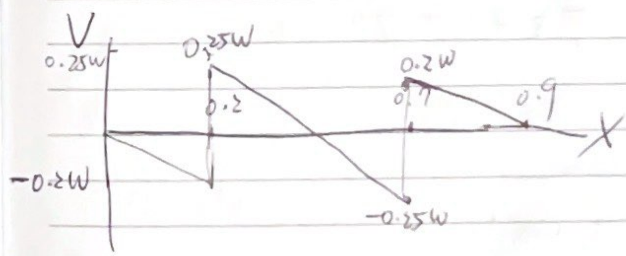
$+\circlearrowleft \Sigma M_3 = 0$

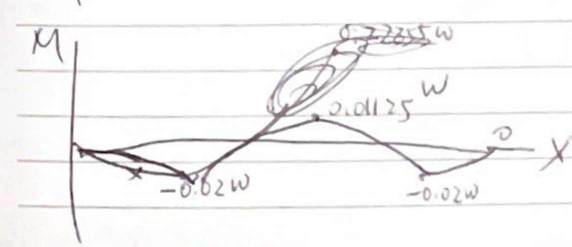
$wx \cdot \frac{x}{2} - 0.45w(x - 0.2)$

$- 0.45w(x - 0.7) + M_3 = 0$

$M_3 = 0.45w(2x - 0.9) - w\frac{x^2}{2}$

$= w\left(0.9x - 0.405 - \frac{x^2}{2}\right)$





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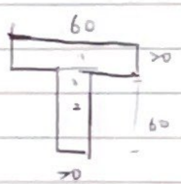
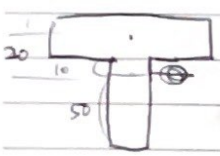
If the section is symmetric, isotropic and is not curved before a bend occurs, then the neutral axis is at the geometric centroid.



	area	$\bar{y}$	$\bar{y}A$
1	1200	70	84000
2	1200	30	36000
	$\Sigma A = 2400$		$\Sigma \bar{Y}A = 120000$

$\bar{Y} \Sigma A = \Sigma \bar{y} A = 120000$

$\bar{Y} = \frac{120000}{2400} = 50$



$I = \Sigma(\frac{1}{12} bh^3 + Ad^2)$

$= \frac{1}{12} 60 \cdot (20)^3 + 60 \times 20 \times 20^2$

$+ \frac{1}{12} 20 \cdot (60)^3 + 20 \times 60 \times 10^2$

$= 40000 + 480000 + 5184000 + 120000$

$= 5824000 \ mm^4$

$= 5.824 \cdot 10^{-6} \ m^4$

$\sigma = -\frac{Mc}{I}$ also need to consider 0.01125W

tension compres

$= \frac{-(-0.02W) \cdot 30 \text{ or } (-50)}{5.824 \cdot 10^{-6}}$

$= \frac{0.6W \text{ or } -W}{5.824 \times 10^{-6}}$

$\frac{0.6W}{5.824 \times 10^{-6}} < 40 \times 10^6$ ($\sigma_{all\ ten}$)

$W < 388.267 \ \frac{N}{m}$

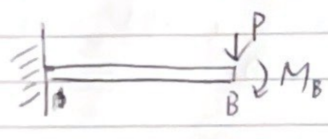
$-\frac{W}{5.824 \times 10^{-6}} < -130 \times 10^6$ ($\sigma_{all\ compre}$)

$W < 757.12 \ \frac{N}{m}$

$\frac{-0.3375W}{5.824 \times 10^{-6}} < -130 \times 10^6$ $\quad W < 2243 \ \frac{N}{m}$

$\frac{0.5625W}{5.824 \times 10^{-6}} < 40 \times 10^6$ $\quad W < 414.15 \ \frac{N}{m}$

page P14



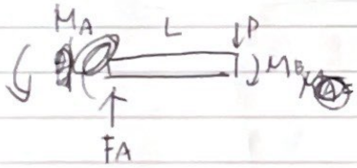
Determine

① y_B

② θ_B

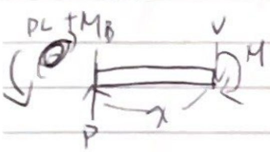
Deflection at B.

$$y_B = \frac{\partial U}{\partial P} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial P}\,dx$$



$M_A = PL + M_B$

$F_A = P$



$$M = \cancel{M_B + PL + PX} = M_B + P(L-X)$$

$$= \cancel{M_B + P(L+X)}$$

$$\frac{\partial M}{\partial P} = \cancel{(L+X)}\ (L-X)$$

$x' =$

$$y_B = \frac{1}{EI}\int_0^L (L-X)(M_B + P(L-X))\,dx$$

$=$

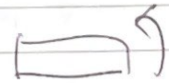
$$= \frac{1}{EI}\int_0^L \left(LM_B + PL^2 - xPL - xM_B - PLX + PX^2\right)dx$$

$$= \frac{1}{EI}\left((LM_B + PL^2)x - \frac{(L + M_B + PL)x^2}{2} + \frac{Px^3}{3}\right)\Bigg|_0^L$$

$$= \frac{1}{EI}\left(L^2M_B + PL^3 - \frac{PL^3 + M_BL^2 + PL^3}{2} + \frac{PL^3}{3}\right)$$

$$= \frac{1}{EI}\left(\frac{(L^2M_B + PL^3)}{2} - \frac{PL^3}{2} + \frac{PL^3}{3}\right)$$

$$= \frac{1}{EI}\left(\frac{L^2M_B}{2} + \frac{PL^3}{3}\right)$$

☆  M 方向 | 定義 小心.

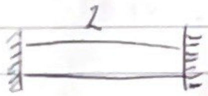
$$\frac{\partial M}{\partial M_B} = 1$$

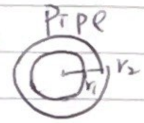
θ_B

$$\theta_B = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_B}\,dx$$

$$= \int_0^L \frac{(M_B + P(L-X))}{EI}\,dx = \frac{(M_BL + PL^2 - \frac{PL^2}{2})}{EI}$$

Q





How much ΔT be to make pipe buckle.

Both ends fixed.

$L_e = 0.5L$

$I = \frac{\pi}{4} c^4$ page 743

$I = \frac{\pi}{4}(r_2^4 - r_1^4)$

$$P_{cr} = \frac{\pi^2 E I}{L_e^2}$$

$\epsilon_T = \alpha \Delta T$

$$\text{So} \quad E\alpha\Delta T > \frac{P_{cr} L}{AE} > \frac{\pi^2 E \frac{\pi}{4}(r_2^4 - r_1^4)}{(0.5L)^2}$$

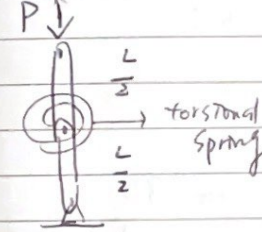
$$\Delta T > \frac{1}{\alpha}\ \pi^3 (r_2^4 - r_1^4)$$

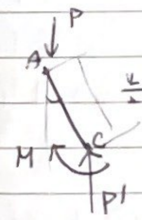
Q. page 607

Buckling : 2 pin-end rigid bar.

(Derive the PCR with stiffness K (spring in the center))

↑ check torsional or spring





$M = P \cdot \frac{L}{2} \sin\theta$

for stiffness K in the center

$M = K 2\theta$

So $\quad P_{cr} (L/2) \sin\theta = K 2\theta$

$$P_{cr} = \frac{4K}{L} \frac{\theta}{\sin\theta}$$

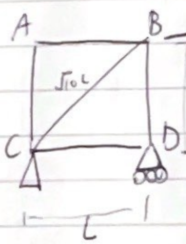
If $\theta \simeq \sin\theta$

$$P_{cr} = \frac{4K}{L}$$

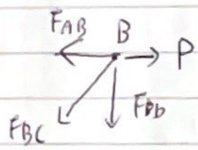
solution 11.71
solution 11.72
Q

P17/8

should be 3/4 L



Determine the horizontal displacement in B.



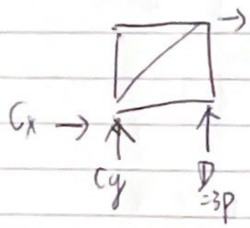
$F_{BD} = 3P$

$F_{BC} \cdot \frac{1}{\sqrt{10}} = 3P$

$F_{BC} = 3\sqrt{10}$

$-F_{AB} - 3P + 3P = 0$

$F_{AB} = 0$



$\circlearrowleft^{+} M_C = 0$

$P \cdot 3L = D \cdot L$

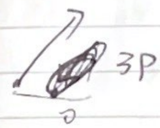
$D = 3P$

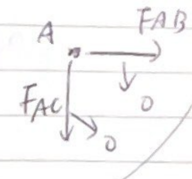
$C_y = -3P$

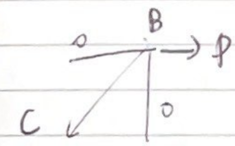
$C_x = -P$

$F_{BD} = 3P$

$F_{CD} = 0$







$F_{BC} \cdot \frac{1}{\sqrt{10}} = P$

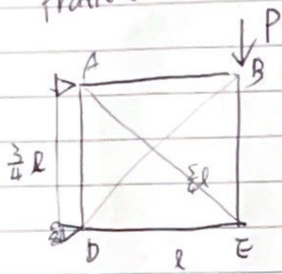
$F_{BC} = \sqrt{10}P$

$F_{BC} \cdot \frac{3}{\sqrt{10}} = 3P$ (check ✓)

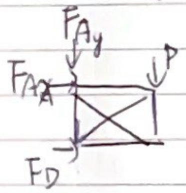
	F	$\frac{\partial F}{\partial P}$
AB	0	0
AC	0	0
BC	$\sqrt{10}P$	$\sqrt{10}$
BD	3P	3
CD	0	

$$x_B = \sum \frac{F_i L_i}{A_i E}\frac{\partial F}{\partial P} = \frac{\sqrt{10}PL\sqrt{10}}{AE} = \frac{10PL}{AE}$$

Pratice



determine the force F_{AB}



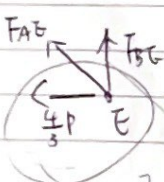
$\circlearrowleft M_A = 0$

$-Pl + \frac{3}{4}l F_D = 0$

$F_D = \frac{4}{3}P$

$F_{Ax} = -\frac{4}{3}P$

$F_{Ay} = -P$



有問題

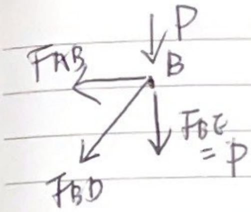
$-F_{AE}\cdot\frac{l}{\frac{5}{4}l} - \frac{4}{3}P = 0$

$F_{AE} = -\frac{4}{3}P \times \frac{5}{4}$

$= -\frac{5}{3}P$

$-\frac{5}{3}P\cdot\frac{\frac{3}{4}l}{\frac{5}{4}l} + F_{BE} = 0$

$F_{BE} = P$



$-F_{BD}\cdot\frac{\frac{3}{4}l}{\frac{5}{4}l} - P - P = 0$

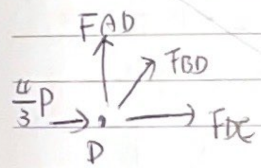
$F_{BD} = -\frac{5}{3}(2P)$

$= -\frac{10}{3}P$

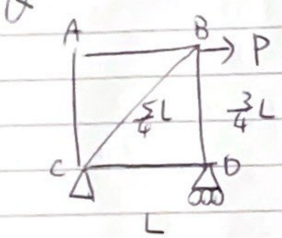
$-F_{AB} - (-\frac{10}{3})P\,\frac{l}{\frac{5}{4}l} = 0$

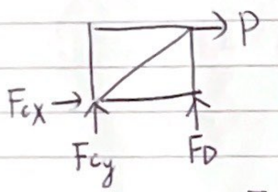
$F_{AB} = \frac{10}{3}P\cdot\frac{4}{5}$

$= \frac{8}{3}P$



Q





$\circlearrowleft + M_C = 0$

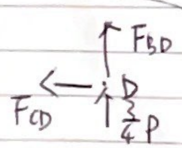
$F_D \cdot L - P \cdot \frac{3}{4}L = 0$

$F_D = \frac{3}{4}P$

$\Sigma F_x = 0 \quad F_{cx} = -P$

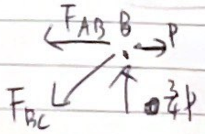
$\Sigma F_y = 0 \quad F_{cy} + F_D = 0$

$F_{cy} = -F_D = -\frac{3}{4}P$



$F_{CD} = 0$

$F_{BD} = -\frac{3}{4}P$



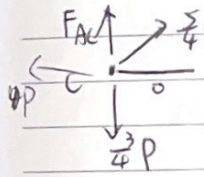
$-F_{BC} \cdot \frac{\frac{3}{4}L}{\frac{5}{4}L} + \frac{3}{4}P = 0$

$F_{BC} = \frac{3}{4}P \cdot \frac{5}{3} = \frac{5}{4}P$

$-F_{AB} + P - \frac{5}{4}P \cdot \frac{L}{\frac{5}{4}L} = 0$

$-F_{AB} + P - P = 0$

$F_{AB} = \cancel{\frac{1}{4}P} = 0$



$\frac{5}{4}P \cdot \frac{L}{\frac{5}{4}L} - P = 0$ ✓ check

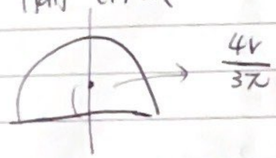
$\frac{5}{4}P \cdot \frac{\frac{3}{4}L}{\frac{5}{4}L} + F_{AC} - \frac{3}{4}P = 0$

$F_{AC} = 0$

	F	$\frac{\partial F}{\partial P}$	L
AC	0	0	$\frac{3}{4}L$
AB	$\cancel{\frac{1}{4}P}$ 0	$\cancel{\frac{1}{4}}$ 0	L
BC	$\frac{5}{4}P$	$\frac{5}{4}$	$\frac{5}{4}L$
BD	$-\frac{3}{4}P$	$-\frac{3}{4}$	$\frac{3}{4}L$
CD	0	0	L

$x_B = \Sigma \frac{F_i L_i}{A_i E} \frac{\partial F}{\partial P} = \frac{PL}{AE}\left(\frac{125}{64} + \frac{27}{64}\right) = \frac{PL}{AE}\left(\cancel{\frac{38}{16}}\ \frac{152}{64}\right) = 2.375 \frac{PL}{AE}$

Half circle

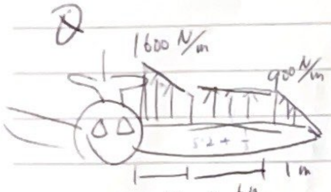


$$A = \frac{\pi r^2}{2}$$

$$Q = A\bar{y} = \frac{\pi r^2}{2} \cdot \frac{4r}{3\pi}$$

$$= \frac{2}{3} r^3$$

Determine moment and shear force at point A.

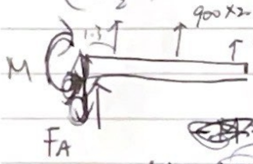


Draw moment - shear diagram

$\left(\frac{1600+900}{2}\right)\times 2.6 = 3250$

$900\times 2.6 = 2340$

$450\times 1 = 450$



$-M + 3200\times 1.3 + 2340\times 3.9 + 450\times 5.7 = 0$

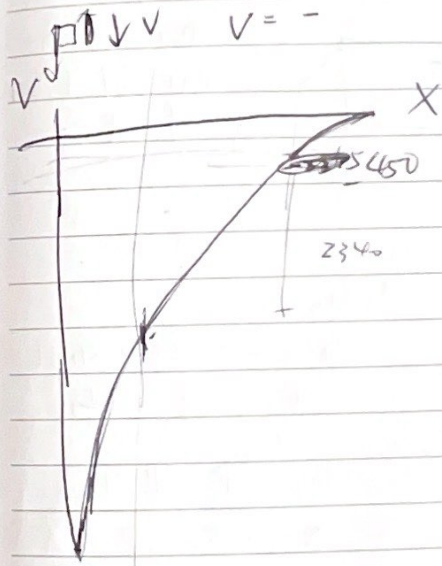
$M = (4160 + 9126 + 2565)$
$= 15851$ N.m

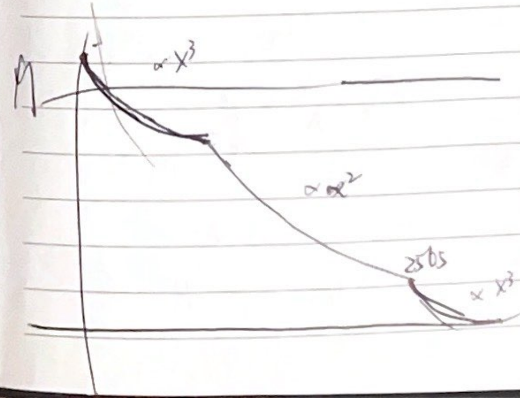
$F_A = -6040$

$\frac{dV}{dx} = -w$

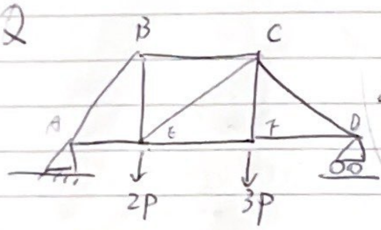
wx

$V = -$

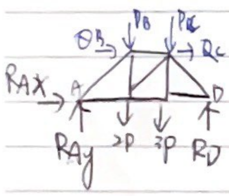




Q



Determine horizontal & vertical deflections of B & C



$\circlearrowleft M_A = 0$

$-2P \cdot L - 3P(2L) + R_D \cdot 3L = 0$

$R_D = \frac{8PL}{3L} = \frac{8}{3}P$

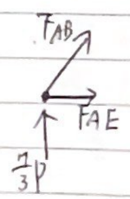
$\Sigma F_y = 0$

$R_{Ay} + \frac{8}{3}P - 2P - 3P = 0$

$R_{Ay} = 5P - \frac{8}{3}P = \frac{7}{3}P$

$\Sigma F_x = 0$

$R_{Ax} = 0$

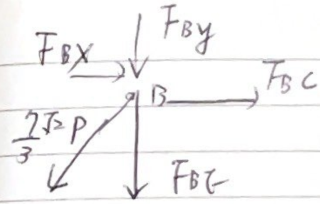


$F_{AB} \cdot \frac{1}{\sqrt{2}} = \frac{7}{3}P$

$F_{AB} = \frac{7\sqrt{2}}{3} P$

$F_{AB} \cdot \frac{1}{\sqrt{2}} + F_{AE} = 0$

$F_{AE} = -\frac{7}{3}P$

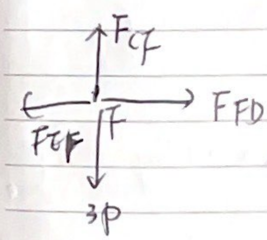


$\frac{7}{3}\sqrt{2}P \cdot \frac{1}{\sqrt{2}} + F_{BE} + F_{By} = 0$

$F_{BE} = -F_{By} - \frac{7}{3}P$

$-\frac{7}{3}\sqrt{2}P \cdot \frac{1}{\sqrt{2}} + F_{Bx} + F_{BC} = 0$

$F_{BC} = \frac{7}{3}P - F_{Bx}$

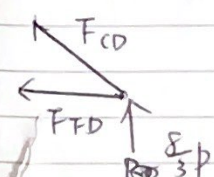


$F_{CF} = 3P$

$F_{EF} = F_{FD} = \frac{8}{3}P$

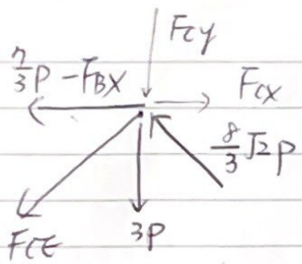
$F_{CD} \cdot \frac{1}{\sqrt{2}} + \frac{8}{3}P = 0$

$F_{CD} = -\frac{8}{3}\sqrt{2}P$



$-\frac{8}{3}\sqrt{2}P \cdot \frac{1}{\sqrt{2}} + F_{FD} = 0$

$F_{FD} = \frac{8}{3}P$



$\Sigma F_y = 0$ | $-F_{CE}\cdot\frac{1}{\sqrt{2}} - 3P + \frac{8}{3}\sqrt{2}P\cdot\frac{1}{\sqrt{2}} - F_{Cy} = 0$

$$\frac{1}{\sqrt{2}}F_{CE} = -3P + \frac{8}{3}P - F_{Cy}$$

$$= -\frac{P}{3} - F_{Cy}$$

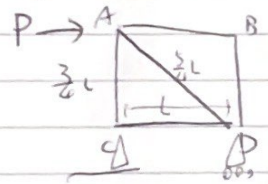
$$F_{CE} = -\frac{\sqrt{2}}{3}P - \sqrt{2}F_{Cy}$$

$\Sigma F_y = 0$

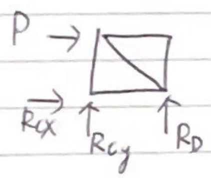
$$-\frac{7}{3}P + F_{BX} + F_{Cx} + \frac{1}{3}P + F_{Cy} - \frac{8}{3}P = 0$$

$$F_{BX} = \frac{14}{3}P - F_{Cx} - F_{Cy}$$

Pratice 11.84, 11.71



horizontal deflection



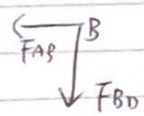
$\circlearrowleft \Sigma M_C = 0$

$-P \cdot \frac{3}{4}L + R_D \cdot L = 0$

$R_D = \frac{3}{4}P$

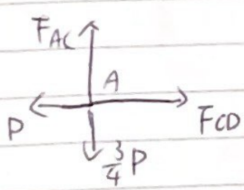
$R_{cy} = -\frac{3}{4}P$

$R_{cx} = -P$



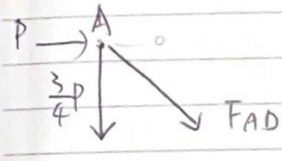
$\Rightarrow F_{AB} = 0$

$F_{BD} = 0$



$F_{AC} = \frac{3}{4}P$

$F_{CD} = P$



$F_{AD} \cdot \frac{\frac{3}{4}L}{\frac{5}{4}L} + \frac{3}{4}P = 0$

$F_{AD} = -\frac{3}{4}P \cdot \frac{5}{3}$

$= -\frac{5}{4}P$

$\left(F_{AD} \cdot \frac{L}{\frac{5}{4}L} + P = 0 \right.$ check

$\left. F_{AD} = \frac{-5}{4}P \right)$ ✓

	F	$\frac{\partial F}{\partial P}$	L_i
AD	0	0	L
~~BD~~			
AC	$\frac{3}{4}P$	$\frac{3}{4}$	$\frac{3}{4}L$
AD	$-\frac{5}{4}P$	$-\frac{5}{4}$	$\frac{5}{4}L$
BD	0	0	$\frac{3}{4}L$
CD	P	1	L

$$x_A = \Sigma \frac{F L_i}{AE} \frac{\partial F}{\partial P} = \frac{PL}{AE} \left(\frac{27}{64} + \frac{125}{64} + 1 \right)$$

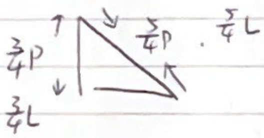
$$= \frac{PL}{AE} \left(\frac{216}{64} \right) \quad \frac{152}{64}$$

$$= 3.375 \frac{PL}{AE}$$

249
75

200

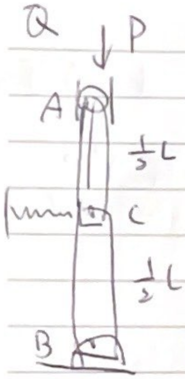
80



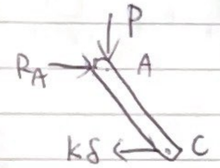
Φ to be valid, must show $\nabla^4 \Phi = 0$

$$\nabla^4 \Phi = \frac{\partial^4 \Phi}{\partial x^4} + 2 \frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi}{\partial y^4} = 0$$

Q



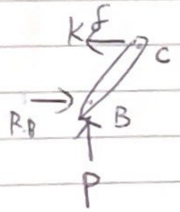
Assume deflection of point C is δ



$+\circlearrowleft \Sigma M_C = 0$

$-\frac{1}{2} L R_A + P\delta = 0$

$R_A = \frac{2P\delta}{L}$



$+\circlearrowleft \Sigma M_C = 0$

$R_B \cdot \frac{1}{2} L - P\delta = 0$

$R_B = \frac{2P\delta}{L}$

For AC & AB.

$+\Sigma F_x = 0$, $R_A + R_B - K\delta = 0$

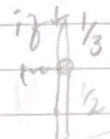
$\frac{4P\delta}{L} - k\delta = 0$

$\left(\frac{4P}{L} - K\right)\delta = 0$ $\Rightarrow P_{cr} = \frac{\quad}{L}$

$\Rightarrow P = \frac{KL}{4}$,

$P_{cr} = \frac{2KL}{9}$

if



Q Plane - Stress state

a) Demonstrate that

$$\nabla^2 (\sigma_{xx} + \sigma_{yy}) = 0$$

(absence of body forces)

$$\frac{\partial^2 \sigma_{xx}}{\partial x^2} + \frac{\partial^2 \sigma_{xx}}{\partial y^2} + \frac{\partial^2 \sigma_{yy}}{\partial x^2} + \frac{\partial^2 \sigma_{yy}}{\partial y^2} = 0$$

$$\begin{cases} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 & — ① \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{y}}{\partial y} = 0 & — ② \end{cases}$$

$$\cancel{\frac{\partial ①}{\partial x} + \frac{\partial ②}{\partial y} + \frac{\partial ②}{\partial x} + \frac{\partial ②}{\partial y} = 0}$$

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\begin{cases} \epsilon_x = \frac{1}{E}(\sigma_x - \nu \sigma_y) \\ \epsilon_y = \frac{1}{E}(\sigma_y - \nu \sigma_x) \\ \gamma_{xy} = \frac{1}{G} \tau_{xy} = \frac{2(1+\nu)}{E} \tau_{xy} \end{cases} \quad \left(G = \frac{E}{2(1+\nu)} \right.$$

$$\frac{1}{E}\left(\frac{\partial \sigma_x}{\partial y^2} - \nu \frac{\partial^2 \sigma_y}{\partial y^2}\right) + \frac{1}{E}\left(\frac{\sigma_y}{\partial x^2} - \nu \frac{\sigma_x}{\partial x^2}\right) = \frac{1}{G} \frac{\partial \tau_{xy}}{\partial x \partial y}$$

$$\frac{1}{E}\left(\frac{\partial \sigma_x}{\partial y^2} - \nu \frac{\partial \sigma_x}{\partial x^2}\right.$$

$$\frac{\partial \sigma_x}{\partial x^2} + \frac{\partial \sigma_x}{\partial y^2} - (1+\nu)\frac{\sigma_x}{\partial x^2} + \frac{\partial \sigma_y}{\partial x^2} + \frac{\partial \sigma_y}{\partial y^2} - (1+\nu)\frac{\sigma_y}{\partial y^2} = \frac{E}{G}\frac{\partial \tau_{xy}}{\partial x \partial y}$$

$2(1+\nu)$

$$\begin{cases} \frac{\partial ①}{\partial x} = \frac{\partial \sigma_{xx}}{\partial x^2} + \frac{\partial \tau_{xy}}{\partial xy} = 0 \\ \frac{\partial ②}{\partial y} = \frac{\partial \tau_{xy}}{\partial x \partial y} + \frac{\partial \sigma_y}{\partial y^2} = 0 \end{cases}$$

So $$\frac{\partial \sigma_x}{\partial x^2} + \frac{\partial \sigma_x}{\partial y^2} + \frac{\partial \sigma_y}{\partial x^2} + \frac{\partial \sigma_y}{\partial y^2} = 0$$

Proved!

$$\nabla^2(\sigma_x + \sigma_y) = 0$$

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} + V$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} + V$$

if no body force

$$\Rightarrow \frac{\partial^2 F}{\partial x^2 \partial y^2} + \frac{\partial V}{\partial x^2} + \frac{\partial^2 F}{\partial y^4} + \cancel{\frac{\partial V}{\partial y^2}}$$

$$+ \frac{\partial^2 F}{\partial x^4} + \cancel{\frac{\partial V}{\partial x^2}} + \frac{\partial^2 F}{\partial x^2 \partial y^2} + \cancel{\frac{\partial V}{\partial y^2}} = 0$$

$$2\left(\frac{\partial^2 F}{\partial x^2 \partial y^2} + \frac{\partial V}{\partial x^2} + \frac{\partial V}{\partial y^2}\right) + \frac{\partial^2 F}{\partial x^4} + \frac{\partial^2 F}{\partial y^4} = 0$$

$$\nabla^4 F = 0$$

Materials

1). How are the following items now produced?

→ Football Helmet
Injection molding. PC
Molded polycarbonate shells with foam padding

→ Coffee mugs
casting
ceramic

→ Lego
Injection Molding, ABS

→ Bottle
PET, stretch blow Molding

→ Dreamliner fuselage
CFRP composites
Carbon-fiber epoxy resin
cured in huge autoclaves
Autoclaving process

→ Turbine Blades
Investment casting
Nickel-based super alloys

→ Toothbrushes
PP. PE
Injection Molding

→ Car Bumper
PC/ABS, PP. PA. PUR with fibers
Injection Molding

→ Hammer
steel
Made by hot forging

→ Iphone case
PC . PP → Injection Molding
Silicone cases → Compression Molding

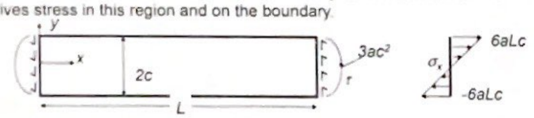
7.5 Plane Elasticity Polynomial Solutions in Cartesian Coordinates

Problem to be solved: Find F(x,y) that satisfies $\nabla^4 F=0$ and the boundary conditions.
Unfortunately, there is no single approach that will guarantee a closed-form solution to such a problem.
IDEA: guess at solutions & see what problems they solve! Use intuition, plus superposition. The text has several examples.

E.g. try $F(x,y)=a(xy^3-3c^2xy)$ [a and c are constants]

$$\therefore \sigma_x=\frac{\partial^2 F}{\partial y^2}=6axy, \qquad \sigma_y=\frac{\partial^2 F}{\partial x^2}=0, \qquad \tau_{xy}=-\frac{\partial^2 F}{\partial x \partial y}=3a(c^2-y^2)$$

Take the problem domain to be $0 \le x \le L$, and $-c \le y \le c$. The solution above gives stress in this region and on the boundary.



Remarks:

(1) No assumptions have been made about the geometry of deformation - e.g. we haven't assumed that plane sections remain plane (they don't here)

(2) In terms of *stresses*, the solution happens to be **identical** to elementary (mechanics of materials) beam theory for an end-loaded cantilever. *i.e.* despite the simplifications made in beam theory, the MoM stress solution is *exact* for the given boundary conditions! (which actually are NOT the B.C.'s for a fixed beam). The beam theory assumptions were:

   (1) plane sections remain plane and ⊥; and

   (2) bending and shear fields can be analyzed independently (one does not affect the other)

(3) To compute strains, use Hooke's Law: E.g. $\gamma_{xy}=\frac{1}{G}\tau_{xy}=\frac{2(1+\nu)}{E}3a(c^2-y^2)$

(4) Displacements can be computed by integrating the strains. This is a good exercise to carry out! If the beam is fixed at the right end, the deformed shape is:

Remarks:

(5) The B.C.'s for the elasticity solution require a parabolic distribution of shear stress on each end, and a symmetric linear distribution of normal stress at each end. This is not likely! However, the solution is useful because of St. Venant's principle.
At a distance ~2c away from the ends, the stress solutions for statically equivalent boundary conditions will be the same.
In general, polynomial solutions use a n^{th}-degree polynomial with unknown coefficients:

$F=a_0$ ←——— gives no stress - omit
$+a_1x+b_1y$ ←——— gives no stress - omit
$+a_2x^2+b_2xy+c_2y^2$ ←——— give constant stress
$+a_3x^3+b_3x^2y+c_3xy^2+d_3y^3$ ←——— gives linear stress
$+\ldots$ ←——— higher order terms must satisfy relations between coefficients so that $\nabla^4\Phi=0$

terms up to 3rd order automatically satisfy the bi-harmonic eqn. Just adjust coeff's to match BC's

F=0

7.5 Plane Elasticity Polynomial Solutions in Cartesian Coordinates

Example:

Show that $F=-\frac{3P_1}{8c^3}\left(xy-\frac{xy^3}{3c^2}\right)+\frac{P_2}{8c^2}y^2$

is suitable for use as an Airy stress function and determine the stress components in the region $x>0$, $-c<y<c$. What kind of problem does this correspond to?

To be valid, we must show $\nabla^4 F=0$

$$\nabla^4 F=\frac{\partial^4 F}{\partial x^4}+2\frac{\partial^4 F}{\partial x^2 \partial y^2}+\frac{\partial^4 F}{\partial y^4}$$

$$\frac{\partial^4 F}{\partial x^4}=0,\ \frac{\partial^4 F}{\partial x^2 \partial y^2}=0,\ \frac{\partial^4 F}{\partial y^4}=0 \ \therefore \text{valid}$$

Think-Pair-Share

Check the stresses:

$\sigma_x=$

$\sigma_y=$

$\tau_{xy}=$

Exercise: can you figure out what the loading situation is? Hint: consider the domain $0<x<L$, $-c<y<c$

7.6 Calculating Displacements from Stress

Find the displacements that result from the Airy Stress Function $F=-\frac{3P_1}{8c^3}\left(xy-\frac{xy^3}{3c^2}\right)+\frac{P_2}{8c^2}y^2$

$$\sigma_x=\frac{3P_1}{4c^4}xy+\frac{P_2}{4c^2},\ \sigma_y=0,\ \tau_{xy}=\frac{3P_1}{8c^2}\left(1-\frac{y^2}{c^2}\right)$$

$$\varepsilon_x=\frac{1}{E}(\sigma_x-\nu\sigma_y)=\frac{3P_1}{4Ec^4}xy+\frac{P_2}{4Ec^2}$$

$$\varepsilon_y=\frac{1}{E}(\sigma_y-\nu\sigma_x)=\frac{-3\nu P_1}{4Ec^4}xy-\frac{\nu P_2}{4Ec^2}$$

$$\gamma_{xy}=\frac{2(1+\nu)}{E}\tau_{xy}=\frac{3(1+\nu)P_1}{4Ec^2}\left(1-\frac{y^2}{c^2}\right)$$

$$\frac{\partial u}{\partial x}=\varepsilon_x,\ \therefore u=\frac{3P_1}{8Ec^4}x^2y+\frac{P_2x}{4Ec^2}+f(y)$$

$$\frac{\partial v}{\partial y}=\varepsilon_y,\ \therefore v=\frac{-3\nu P_1}{8Ec^4}xy^2-\frac{\nu P_2y}{4Ec^2}+g(x)$$

$$\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=\gamma_{xy}$$

$$\therefore \frac{3P_1}{8Ec^4}x^2+f'(y)+\frac{-3\nu P_1}{8Ec^4}y^2+g'(x)=\frac{3(1+\nu)P_1}{4Ec^2}\left(1-\frac{y^2}{c^2}\right)$$

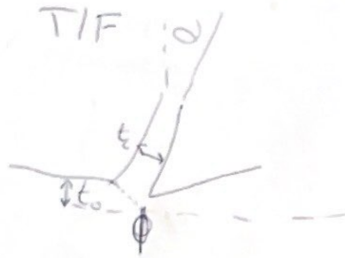
$$\left(\frac{3P_1}{8Ec^4}x^2+g'(x)\right)+\left(\frac{3(2+\nu)P_1}{8Ec^4}y^2+f'(y)\right)=\frac{3(1+\nu)P_1}{4Ec^2}$$

(function of x) + (function of y) = constant

$$\therefore \frac{3P_1}{8Ec^4}x^2+g'(x)=a_1,\ \therefore g(x)=-\frac{P_1}{8Ec^4}x^3+a_1x+a_2$$

$$\frac{3(2+\nu)P_1}{8Ec^4}y^2+f'(y)=\frac{3(1+\nu)P_1}{4Ec^2}-a_1,\ \therefore f(y)=-\frac{(2+\nu)P_1}{8Ec^4}y^3+\frac{3(1+\nu)P_1}{4Ec^2}y-a_1y+a_3$$

/ Materials



What is t_0, t_c

If ~~ϕ is~~ chip thickess ↑ at same cutting depth, what happens to cutting force? ↑

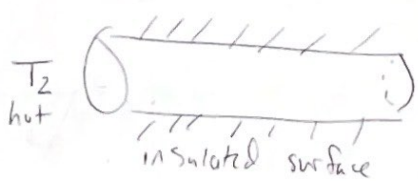
How to change material properties or machining params. to make discontinuous chips.

Draw and label $\sigma - \epsilon$. Define and show toughness and ductility.

Draw $\sigma - \epsilon$ for brittle vs. extremely ductile mat'l.

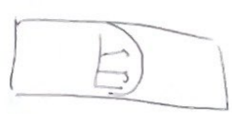
Compare Pros/cons for additive manufacturing vs. processes like CNC / die casting (consider production rate, cost,

Label schematics of common mfg. processes: turning, milling, drilling, broaching, planing, shaping, sawing

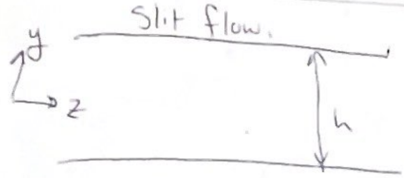


How does T profile look at steady state?

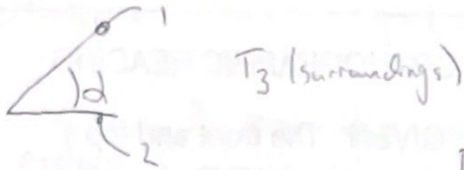
What are B.C.'s?



How does P profile look? what are B.C.'s?



given Cons. momentum equation. Find $v_z(y)$

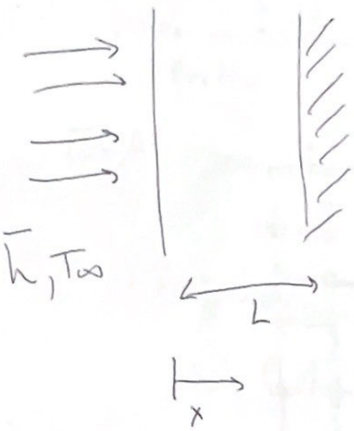


given T_1, T_3, ϵ_1, ϵ_2

Find T_2

If $\epsilon_1 \downarrow$, T_2 ?

If $\epsilon_2 \downarrow$, T_2 ?



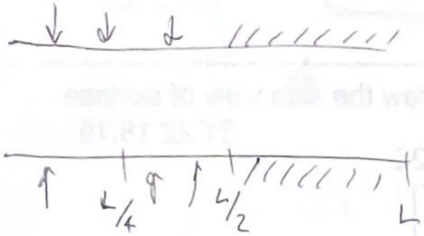
Given $\bar{h}$, T_∞, L, k, ρ, c_p

Find t where X_L feels temp change

Draw T vs. x for that t

Find $T_{x=0}$ for long t

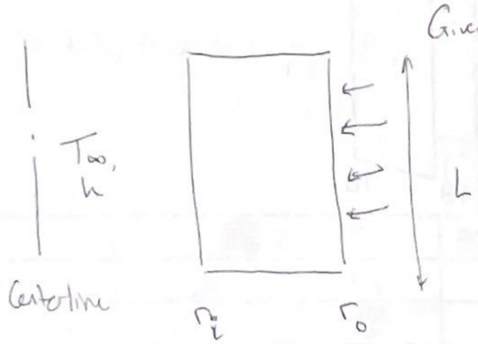
Draw T vs. x for long t



Constant heat flux 0 to L/2 thermally fully developed at L/4

adiabatic L/2 to L

Draw $T_m(x)$, $T_s(x)$



Given $\dot{q}''$, r_i, r_o, L, T_∞, h. No conv at r_o

Draw resistance network and calculate

Find T_{out}. Draw T vs. r

If radiation emitted at r_o w/ $\epsilon_o = 0.05$

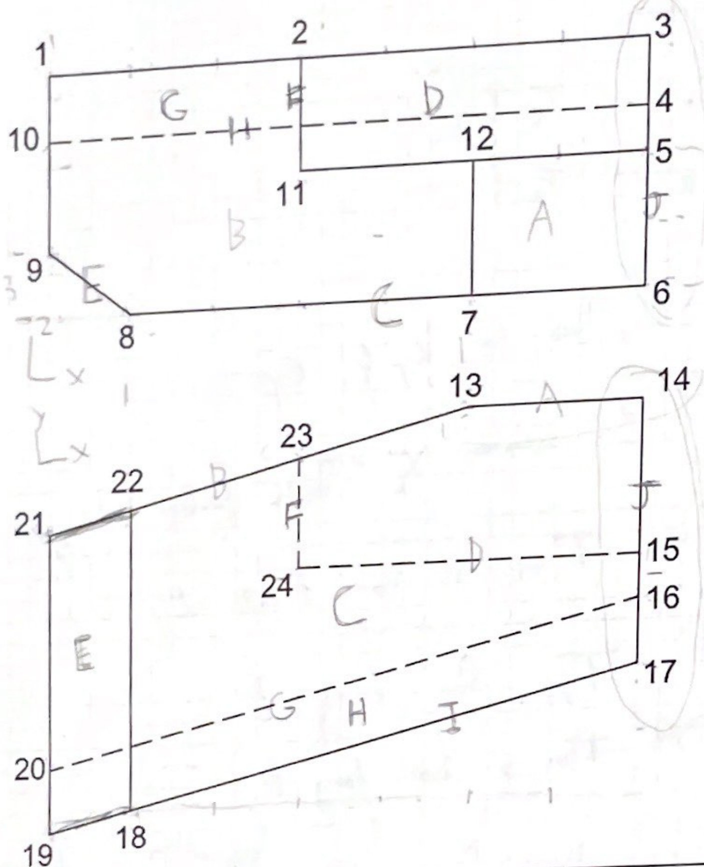
~~Find~~ Draw radiation resistance network

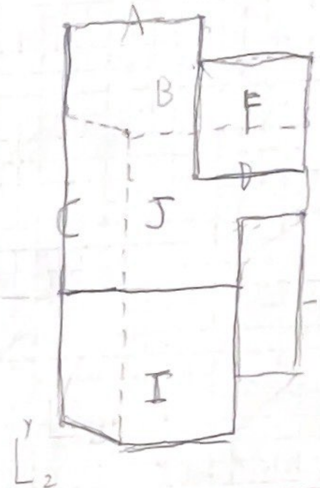
Is radiation from r_o significant? Calculate?

ORTHOGRAPHIC READING

GIVEN: The front and top views of an object.

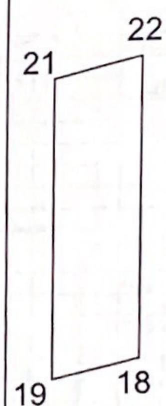
REQUIRED: Draw the right side view and complete the bottom half as instructed.





1. Draw the side view of surface 14,17.

2. Draw the side view of surface 21,22,18,19.



21,13	B	SURFACE	7, 8, 9, 1, 2, 11, 12
9,8	E	"	18, 19, 21, 22
1,3		"	15, 16, 20, 21, 23, 24
2,11	about F	EDGE	23 or 24
3,4		"	16
10,4		"	20,16 or 19, 17
9,8		EDGE	18,19 or 21,22

NAME:	3c-1

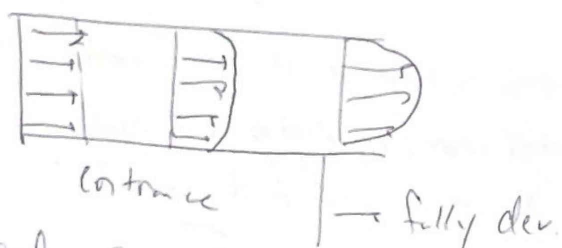
1) Rectangular Duct, $y = 0.2m$, $Z = 0.4m$, x is long direction

(a) simplify x momentum equation

(b) explain the terms of (a) in words

(c) from (a) find sign of $\frac{dp}{dy}$

2)

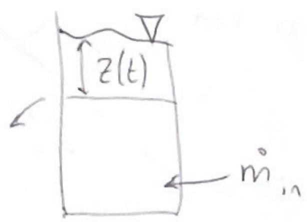


~~Find~~ See if both terms of Lagrangian derivative are zero or nonzero in

(a) entrance

(b) fully developed region

3)



given $u_{leak} = \sqrt{2g\,z(t)}$

Find expression for $\frac{dz(t)}{dt}$

(b) if a steady-state solution for $z(t)$ exists, find it. If not, find inequality?

4) Smooth, turbulent pipe flow, $f \propto \frac{1}{\sqrt{Re}}$

Find $P \propto Q\Delta p \propto v^n$ find n

(b) very rough, very turbulent,

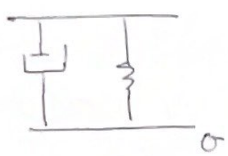
Find $P \propto v^n$

· Meaning + expression
Meaning

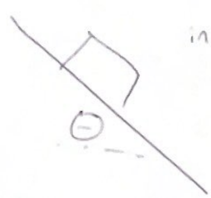
Brinkman, Newton, Reynolds, Froude, Fourier

String, Scale, Swimming pool

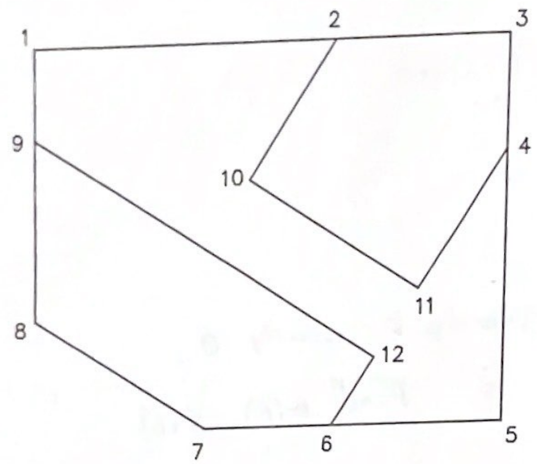
balls thrown into trunk of car with velocity v, initially 0.
balls have velocity u, mass rate $\dot{m}$. Find $v(t)$, $x(t)$
Assume balls stay in trunk.



Find $\sigma(\epsilon)$



initial velocity v_0. Find v at long time
$\mu = \tan\theta$



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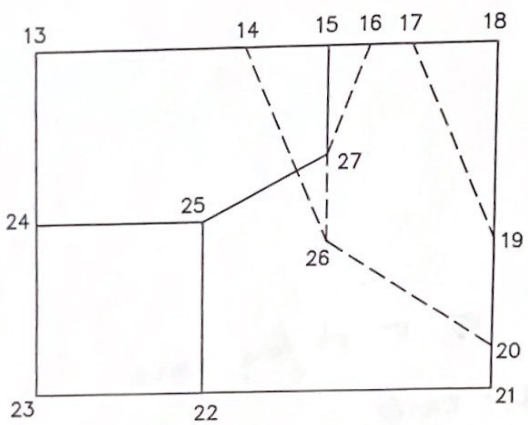
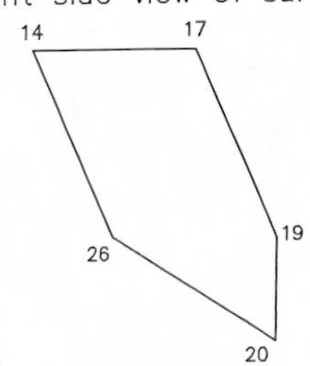


1. Draw the right side view of surface 14,17,19,20,26.



2,10	SURFACE	
9,12,6,7,8	"	
13,18	"	
4	EDGE	
3,4	"	
8,7	"	

NAME:	3d–1

$$\dot{m}(u-v) = (m+\dot{m}t)a \qquad \eta = \cancel{u-v}\ v-u \qquad v(0)=0$$

$$a = \frac{dv}{dt} = \frac{d\eta}{dt} \qquad \frac{d\eta}{dt} = \frac{dv}{dt}$$

$$v-u = \eta = -\frac{m+\dot{m}t}{\dot{m}}\frac{d\eta}{dt}$$

$$-\frac{\dot{m}\,dt}{m+\dot{m}t} = \frac{d\eta}{\eta}$$

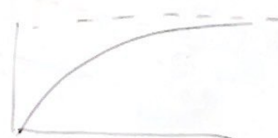
$$-\ln(m+\dot{m}t) = \ln\eta + \ln C = \ln C\eta \qquad \eta = -u \text{ at } t=0$$

$$-\ln(m) = \ln\frac{1}{m} = \ln -uC \qquad \therefore C = \frac{1}{-um}$$

$$\ln\left(\frac{1}{m+\dot{m}t}\right) = \ln -\frac{\eta}{um}$$

$$\frac{1}{m+\dot{m}t} = -\frac{\eta}{um} = -\frac{v-u}{um}$$

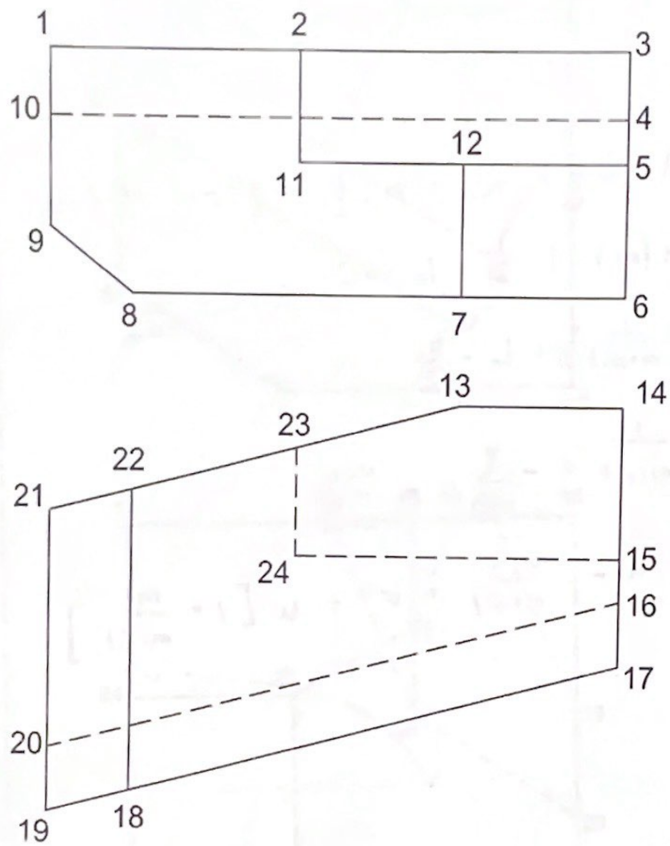
$$u - \frac{um}{m+\dot{m}t} = v = u\left[1 - \frac{m}{m+\dot{m}t}\right]$$



ORTHOGRAPHIC READING

GIVEN: The front and top views of an object.

REQUIRED: Draw the right side view and complete the bottom half as instructed.



1. Draw the side view of surface 14,17.

2. Draw the side view of surface 21,22,18,19.

21,13	SURFACE	
9,8	"	
1,3	"	
2,11	EDGE	
3,4	"	
10,4	"	

NAME: 3c-1