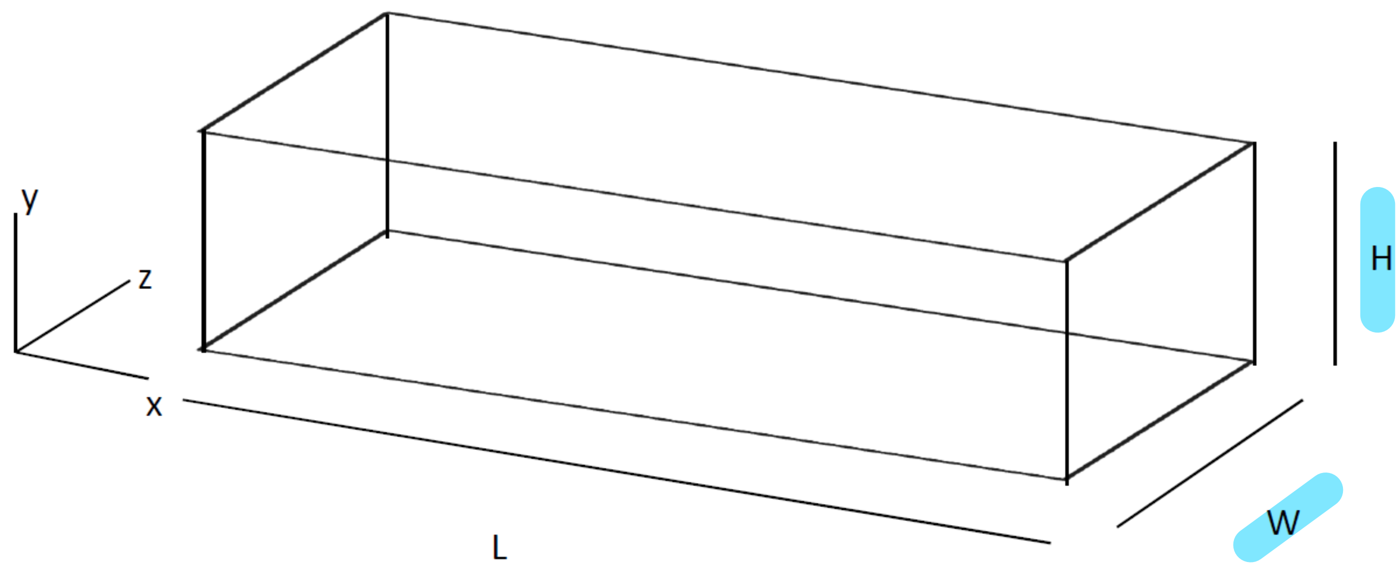
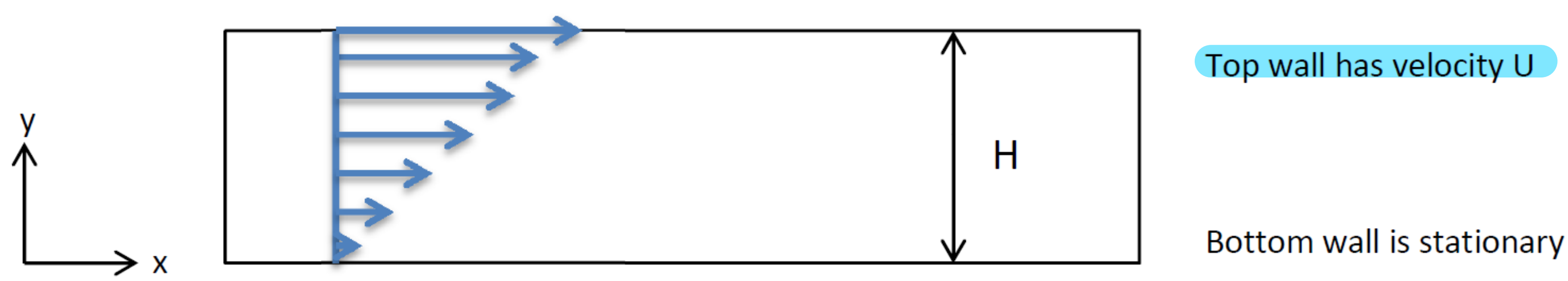
ME 314
Manufacturing Fundamentals

Discussion #3 - Simple Shear Flow

Simple Shear Flow

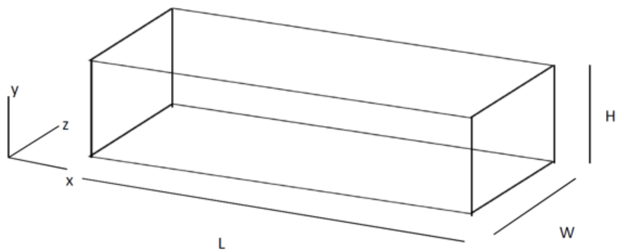


Orthormorphic Sketch with Coordinate System

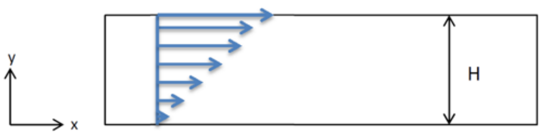


Assumptions

1. Incompressible fluid (ρ = constant)
2. Newtonian fluid (μ = constant)
3. Neglect gravity (Ps << 1)
4. Neglect Inertia (Re <<1)
5. Steady state (no $\partial/\partial t$)
6. Isothermal ($\Delta T = 0$)
7. No Slip at the Wall (Boundary Conditions)
8. Fully developed flow (no startup effects)
9. W >> H (no $\partial/\partial z$)
10. u_y is negligible ($u_y = 0$)
11. No flow movement in z direction; 2D problem ($u_z = 0$)



Orthormorphic Sketch with Coordinate System



Apply Equation of Continuity (mass balance)

$$\frac{\partial u_x}{\partial x} + \cancel{\frac{\partial u_y}{\partial y}} + \cancel{\frac{\partial u_z}{\partial z}} = 0$$

uy is negligible

No flow movement in z direction

- In *Main Course Handout* :
  - p37 - Table 2.1 – continuity Eqns.
  - P43 - table 2.2 - momentum eqns. in terms of τ
  - P45 - table 2.4 N-S eqns.
  - P48 - Table 2.5 energy eqns.

Integrating both sides:

$$\int \frac{\partial u_x}{\partial x} = \int 0$$

$$u_x = C$$

Therefore, u_x is not a function of x.

Apply Equation(s) of Momentum (Momentum Balance)

u: 流速

p: 压力

τ: 粘性应力 (摩擦力)

ρg_x: 重力

X-Direction:

$$\rho\left(\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\left[\frac{\partial}{\partial x}\tau_{xx}+\frac{\partial}{\partial y}\tau_{yx}+\frac{\partial}{\partial z}\tau_{zx}\right]+\rho g_x$$

在x方向上的加速度

沿x方向的流动

压力在x方向上的变化

由于粘性而产生的摩擦力

重力在x方向的分量

Cartesian Coordinates (x, y, z):

$$\tau_{xx}=2\eta\frac{\partial u_x}{\partial x}\qquad \tau_{yy}=2\eta\frac{\partial u_y}{\partial y}$$

$$\tau_{zz}=2\eta\frac{\partial u_z}{\partial z}\qquad \tau_{xy}=\tau_{yx}=\eta\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)$$

$$\tau_{yz}=\tau_{zy}=\eta\left(\frac{\partial u_z}{\partial y}+\frac{\partial u_y}{\partial z}\right)\qquad \tau_{xz}=\tau_{zx}=\eta\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right)$$

Apply Equation(s) of Momentum (Momentum Balance)

X-Direction:

continuity　　　No pressure gradient　　　Neglect gravity

$$\rho\left(\cancel{\frac{\partial u_x}{\partial t}} + \cancel{u_x\frac{\partial u_x}{\partial x}} + \cancel{u_y\frac{\partial u_x}{\partial y}} + \cancel{u_z\frac{\partial u_x}{\partial z}}\right) = \cancel{-\frac{\partial p}{\partial x}} + \left[\cancel{\frac{\partial}{\partial x}\tau_{xx}} + \frac{\partial}{\partial y}\tau_{yx} + \cancel{\frac{\partial}{\partial z}\tau_{zx}}\right] + \cancel{\rho g_x}$$

Steady state　　No u_y　　No u_z　　continuity　　No z-dependence

与时间无关

2D

Cartesian Coordinates (x, y, z):

$$\tau_{xx} = 2\eta\cancel{\frac{\partial u_x}{\partial x}}$$

continuity

$$\tau_{yy} = 2\eta\frac{\partial u_y}{\partial y}$$

$$\tau_{zz} = 2\eta\frac{\partial u_z}{\partial z}$$

No u_y

$$\tau_{xy} = \tau_{yx} = \eta\left(\frac{\partial u_x}{\partial y} + \cancel{\frac{\partial u_y}{\partial x}}\right)$$

$$\tau_{yz} = \tau_{zy} = \eta\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right)$$

$$\tau_{xz} = \tau_{zx} = \eta\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)$$

EOM Continued

$$\frac{\partial}{\partial y}\left(\tau_{yx}\right) = 0$$

$$\tau_{yx} = \mu\left(\frac{\partial u_x}{\partial y} + \cancel{\frac{\partial u_y}{\partial x}}\right)$$

Newtonian viscosity → μ is constant. Can pull out of derivative:

$$\mu \frac{\partial}{\partial y}\frac{\partial u_x}{\partial y} = 0$$

EOM Continued

- Divide both sides by μ: $\frac{\partial}{\partial y}\frac{\partial u_x}{\partial y} = 0$
- Integrate twice with respect to y

$$\int\int \frac{\partial}{\partial y}\frac{\partial u_x}{\partial y} = \int\int 0$$

$$u_x = C_1 y + C_2$$

- Apply Boundary Conditions (BC's):

$$BC1\colon\ u_x = 0 \ @\ y = 0$$

$$0 = 0y + C_2;\ \ \boldsymbol{C_2 = 0}$$

$$BC2\colon\ u_x = U \ @\ y = H$$

$$U = C_1 H;\ \ \boldsymbol{C_1 = \frac{U}{H}}$$

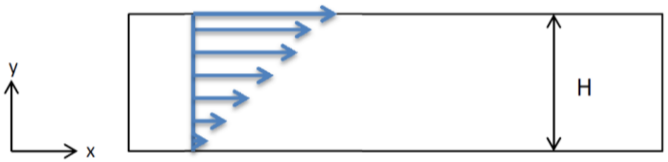
$\int \partial u_x / \partial y = \int 0$

$u_x = C_1$

$\int \partial u_x / \partial y = \int C_1$

$u_x = C_1 y + C_2$

EOM Continued



Velocity Profile is…

$$u_x = \frac{U}{H} y$$

Integrate and find Volumetric Flow Rate Q:

$$Q = \int_0^H u_x W dy = \int_0^H \frac{U}{H} yW dy = \frac{UW}{H} \int_0^H y dy = \frac{UW}{H} \frac{H^2}{2} = \frac{UWH}{2}$$

$$\frac{1}{2} y^2 \Big|_0^H$$

Access code for quiz 2: **Quiz2+Access**