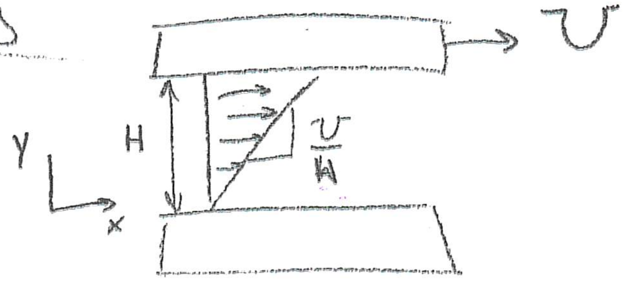
Materials Processing

Couette flow



$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial u}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right) + \rho g_x$$

$$\mu\frac{\partial^2 u_x}{\partial y^2} = 0 \qquad \frac{\partial^2 u_x}{\partial y^2} = 0 \qquad \frac{\partial u_x}{\partial y} = C_1 \qquad u_x = C_1 y + C_2$$

$y(0),\ u_x = 0$ $\qquad 0 = 0 + C_2,\ C_2 = 0$ $\qquad u_x = \frac{U}{H}y$

$y(H),\ u_x = U$ $\qquad U = HC_1,\ C_1 = \frac{U}{H}$

Material Removal Rate

for drilling $= \frac{\pi D^2}{4} fN$ volume/min (D: diameter)

$\frac{[mm]^2 [mm] [rev]}{[rev] [min]}$

D = diameter (mm)

f = feed $\left[\frac{mm}{rev}\right]$ 每转一周沿轴向移动的距离

N: rotations/min [rpm] [rev/min]

for turning (lathe) $= \pi D f d N$ Volume/min

$\frac{[mm][mm][mm][Rev]}{[Rev][MIN]}$

d = depth of cut.

格拉晓夫数 **Grashof**: $\frac{g\beta(T_s - T_\infty)D^3}{\nu^3} = G_r$ = $\frac{\text{buoyancy force}}{\text{viscous force}}$

β - coefficient of thermal expansion; T_s: surface; T_∞: Bulk; D: Pipe diameter; ν: kinematic viscosity

雷诺数 **Reynolds**: $\frac{\rho D v}{\mu}$ = $\frac{\text{inertial force}}{\text{viscous force}}$

ρ: density; D: diameter; v: velocity; μ: dynamic viscosity

Similar to Reynolds num

Brinkman: $\frac{v\mu}{k\Delta T}$ = $\frac{\text{flow viscous heating}}{\text{conduction}}$

k: thermal conductivity; ΔT: $T_w - T_o$ (T_w: wall temp, T_o: bulk fluid temp)

$B_r = P_r E_c$ (Prandtl num, Eckert num)

Pr: $\frac{\nu}{\alpha} = \frac{C_p \mu}{k}$ = $\frac{\text{momentum viscous diffusion}}{\text{thermal diffusivity}}$

C_p: Specific heat; α: thermal diffusivity

$\frac{\rho}{\mu} = \frac{1}{\nu}$

$E_c = \frac{v^2}{c_p \Delta T} = \frac{\text{advective transport}}{\text{Heat dissipation Potential}}$ (v: velocity)

ΔT diff between wall and local temp.

Manufacturing types for # of parts

a) 1 — rapid proto 快速原型制造 or hand machining 手工加工

b) 5 — CNC (Computer Numerical Control) or rapid prototyping

c) 100 — investment casting 投资铸造

d) 10,000 — die casting 压铸

Materials processing

Rake angle = the angle of orientation of tools rake surface from the reference plane and measured on some other plane. the angle of the surface of a cutting tool tip over which the removed chips flow

Chips: 切屑

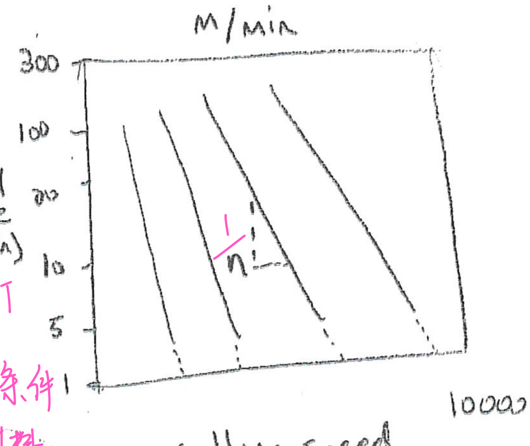
Continuous chips: ductile material 韧性材料
high rake angle
or high cutting speed

Built up edge: adverse effects on surface finish 金属粒子粘附在刀具上（粘着性）
reduce by increasing cutting speed
reduce depth of cut, increase rake angle

Serrated chips: metals w/ low k (thermal conductivity) 低热导率 热性能差
锯齿状（振动） or low strength (Breaks like glass) 低强度
or due to chatter

discontinuous chips: brittle workpeice, workpeice w/ inclusions
very high or low cutting speed 脆性材料

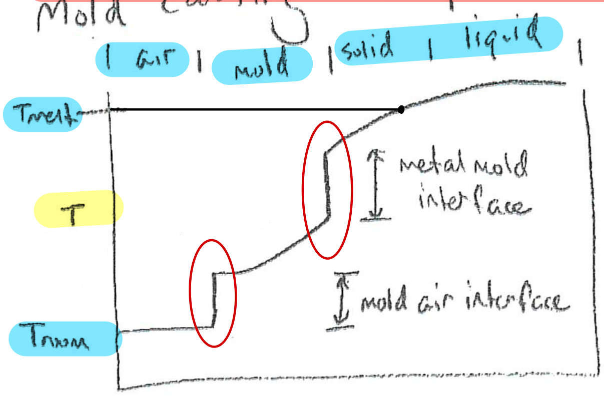
Taylor's tool life: $VT^n = C$



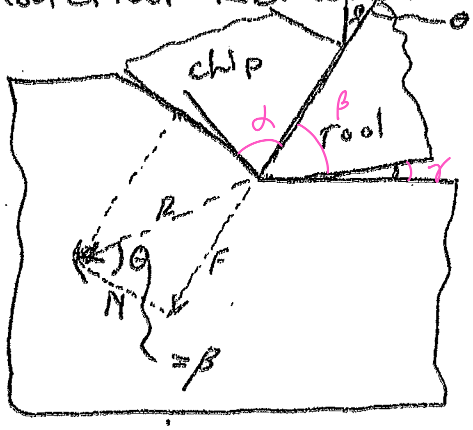
V = cutting speed 切削速度，m/min
T = tool life 刀具寿命，min
n = slope Tool life index
C = constant 1 min toollife
constant Machining index，取决于切削条件和材料

$$\log T = \frac{1}{n}(\log C - \log V)$$

Mold casting Temperature profile



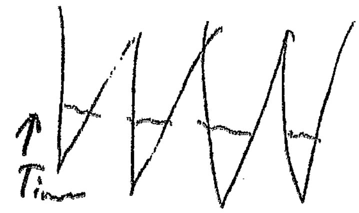
Material Removal



$\theta = \alpha \Rightarrow$ Rake ∠

Cutting Force

$F = R \sin\beta$



Taylor tool-wear $VT^n = C$

V = cutting speed, T = time in minutes to develope a flank wear land, n depends on cutting conditions, C is a constant

MMR - Material Removal Rate, Volume of mtl removed per unit time

* In Turning, $MMR = \pi D_{avg}\, d f N$

cutting time $t = \dfrac{l}{fN}$

完成一段给定长度的切削所需的时间

$V = \pi D_o N$ = turning speed.

切削速度 (m/min)

f = feed or distance tool travels in one revolution

N = # rev per min

l = workpiece length. 工件的长度

D_{avg} gives depth of cut

$D_{avg} = (D_o + D_{final})/2$

* In Milling

$V = \pi D N$

$t_c = 2f\sqrt{\dfrac{d}{D}}$

$f = \dfrac{v}{Nn}$

$t = \dfrac{l + l_c}{v}$

$MMR = \dfrac{lwd}{t}$

t_c = chip thickness, d = depth of cut

D = Diameter of cutter

N = rotational speed, f = feed per tooth of cutter, v = linear speed, n = # teeth

t = cutting time, l = length of workpiece

l_c = extent of cutters first contact

w = width of cut

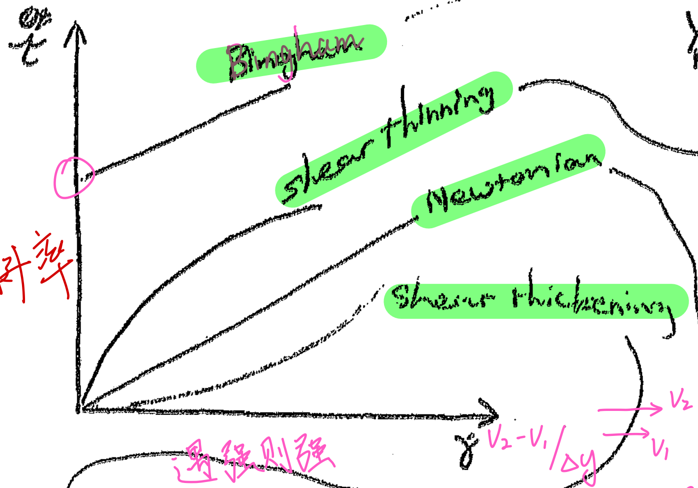
l: 刀具接触工件的长度

w: 切削宽度

d: 切削深度

$\eta = \tau/\dot{\gamma}$, viscosity, 斜率

Shear VS. Shear rate



看斜率

牙膏

Fluid remains rigid when magnitude of shear stress is smaller than yield stress, but flows like a Newtonian fluid when shear stress exceeds (τ_0)

化妆品

n<1, Pseudoplastic Fluids, As shear rate ↑, friction between layers ↓, causing long chain molecules to straighten + align w/ flow, which reduces viscosity

n=1, Viscosity constant + independent of shear rate.

n>1 As shear rate ↑ friction between layers ↑, which increases viscosity

流体层之间相对运动的速率

Metal Casting.

- Viscosity: 流动性 fluidity ↓ as 粘度 viscosity ↑
- Surface tension: fluidity ↓ w/ high surface tension
- Inclusions: insoluble particles 不溶的颗粒，阻碍流动 that can effect fluidity
- Solidification pattern of the alloy: how a material solidifies can influence fluidity. Fluidity is inversely prop. to the freezing range: Thus pure metals = ↑ fluidity. Alloys = ↓ fluidity
- Mold design: mold components can influence fluidity
- Mold Mtl + Surface: higher the thermal conductivity + rougher the surface, ↓ the fluidity. Heating mold improves fluidity, but increases grain size which lowers strength.
- Degree of Super Heat 过热度: ↑ fluidity by delaying solidification. Temp above meltin point
- Rate of pouring 流速: ↓ pouring rate, ↓ fluidity b/c faster cooling.
- Heat Transfer: affects viscosity + ∴ fluidity.

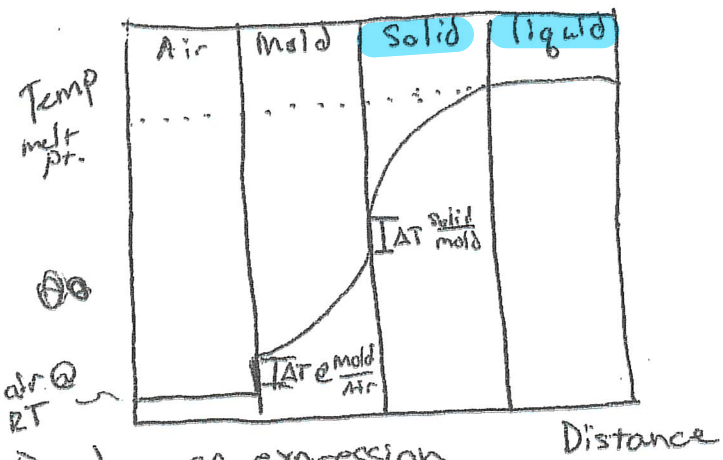
Testing for fluidity - not universal, but a molten mtl can be made to flow through a channel. The distance the mtl gets to b4 solidification can help define its fluidity.

Heat Transfer in A Mold

凝固时间公式

$$\text{Solidification time} = C\left(\frac{\text{Volume}}{\text{Surf. Area}}\right)^n$$

C = constant dependent on mold mtl
n = normally (2) (but is actually between 1.5 + 2)



热传导方程

Heat eq. for conduction

$$q'' = -k\frac{dT}{dx}$$

$C_p\,[J/kg\cdot C]$

$\rho \rightarrow$ density

Develope an expression

$$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} + \cancel{\dot{q}} = \rho C_p \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} = \rho C_p \frac{\partial T}{\partial t}$$

$$\frac{\partial T}{\partial x} = \rho C_p \frac{\partial T}{\partial t} x + c_1$$

$$T = \rho C_p \frac{\partial T}{\partial t}\frac{x^2}{2} + c_1 x + c_2$$

Material Selection

know mechanical properties
strength, Toughness, Ductility, hardness, resistance to fatigue, creep + impact.

know physical properties
density, melting pt., specific heat, conductivity, thermal expansion, magnetic properties.

know chem. properties
oxidation, corrosion

Mtls selected should

- exceed min requirements + specs.
- can mtl be replaced w/ cheaper mtl.
- have appropriate manufacturing characteristics.
- Raw mtls have standard shapes, tolerances, dimensions, surface finish
- reliable supply

材料选择标准

在选择材料时，需要确保材料能满足以下几个标准：

1、• **满足设计要求和规格**：选择的材料需要达到设计中对强度、耐久性、重量等方面的要求。

2、• **成本效益**：评估是否有更便宜的材料可以替代，同时达到相同的性能。

4、• **制造特性**：材料应适合预定的制造过程，如铸造、焊接、机加工等。

5、• **标准形状和尺寸**：选用的原材料应有标准的形状和公差，以减少制造成本和复杂性。

3、• **可靠的供应链**：确保所选材料的供应稳定，以免影响生产计划。

2. **物理特性**：

- **密度**：单位体积的质量。
- **熔点**：材料从固态转变为液态的温度。
- **比热容**：单位质量的材料升高单位温度所需的热量。
- **导热性**：材料传导热能的能力。
- **热膨胀性**：温度变化时，材料体积或长度变化的性质。
- **磁性特性**：材料响应磁场的特性。

3. **化学特性**：

- **抗氧化性**：材料抵抗在氧化环境中反应的能力。
- **耐腐蚀性**：材料抵抗环境因素，如湿度、酸、盐等引起的腐蚀的能力。

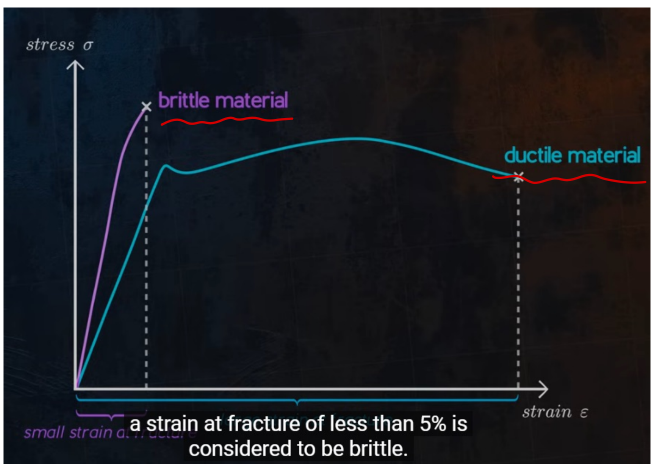
1. **强度 (Strength)**：材料在受到力时抵抗形变和破坏的能力。

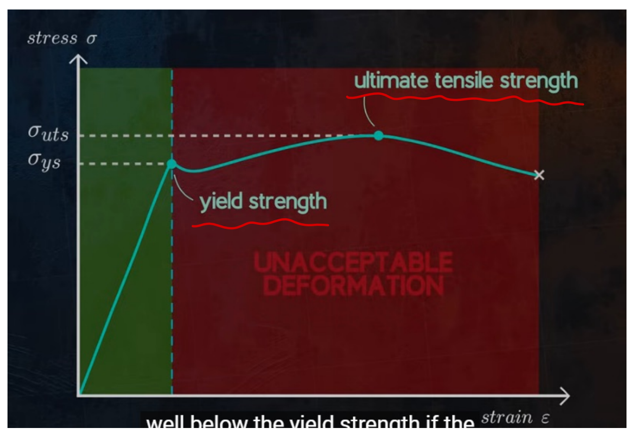
2. **韧性 (Toughness)**：材料吸收能量并且在受到冲击时抵抗断裂的能力。
3. **延展性 (Ductility)**：材料在受力后能够拉长的程度。 plastic range ↑
4. **硬度 (Hardness)**：材料抵抗硬物压入或划伤的能力。
5. **耐疲劳性 (Fatigue Resistance)**：材料抵抗反复应力或应变而不发生疲劳破坏的能力。
6. **抗蠕变性 (Creep Resistance)**：在高温下材料抵抗持续变形的能力。
7. **抗冲击性 (Impact Resistance)**：材料抵抗突然冲击或撞击的能力。

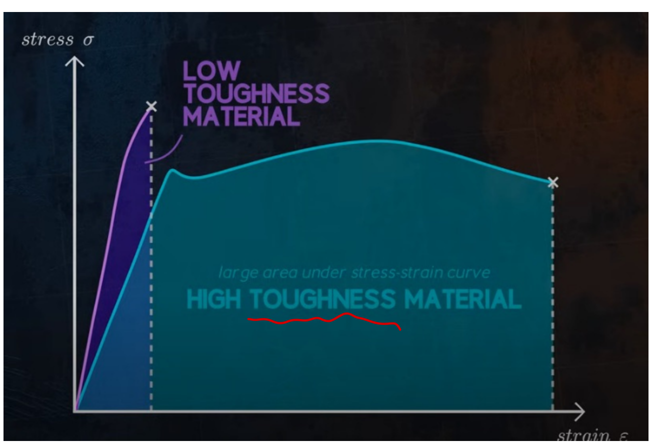
1. **Strength**: This is a measure of the maximum amount of stress a material can withstand while being stretched or pulled before failing or breaking. Strength is typically measured in terms of tensile strength (the resistance to being pulled apart) and compressive strength (the resistance to deformation under compression).

2. **Ductility**: Ductility refers to a material's ability to deform under tensile stress. Highly ductile materials can be stretched into a wire. Materials with high ductility, such as gold and copper, are typically capable of being shaped or molded without breaking, which is a crucial attribute for many manufacturing processes.

3. **Toughness**: Toughness is the ability of a material to absorb energy and plastically deform without fracturing. It combines strength and ductility, providing an indication of the energy a material can absorb before it breaks. A tough material can withstand impact and sudden forces well, making it valuable in applications where shock resistance is important.



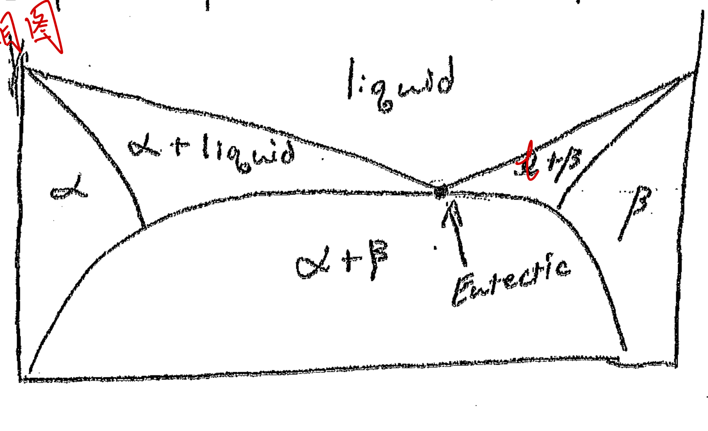




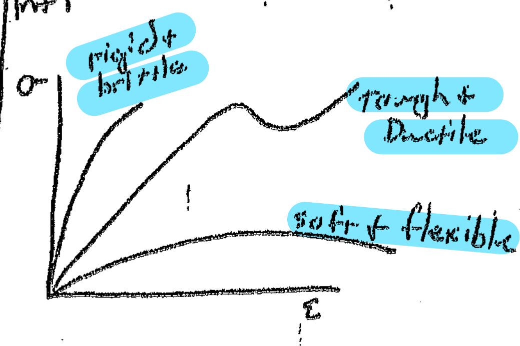
Phase diagrams

Eutectic point - point where liquid phase can transfer directly to solid

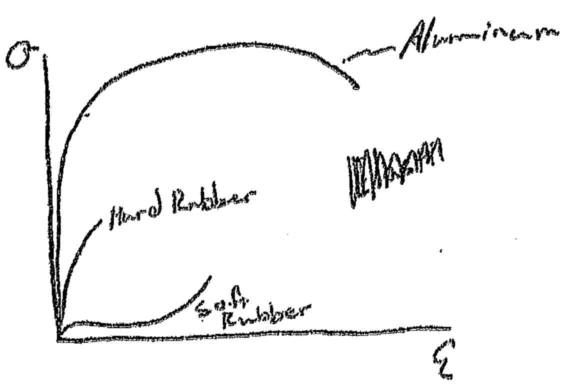
二元合金相图

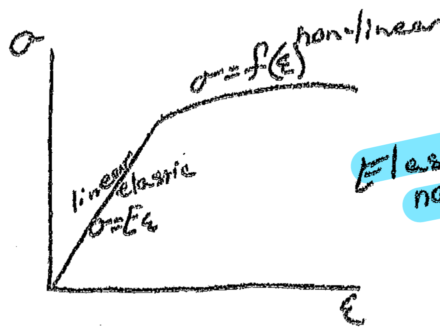


Stress-Strain, types of mat'l

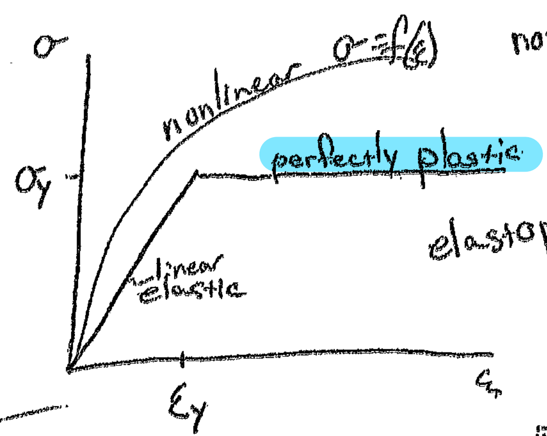


Stress-Strain Curves



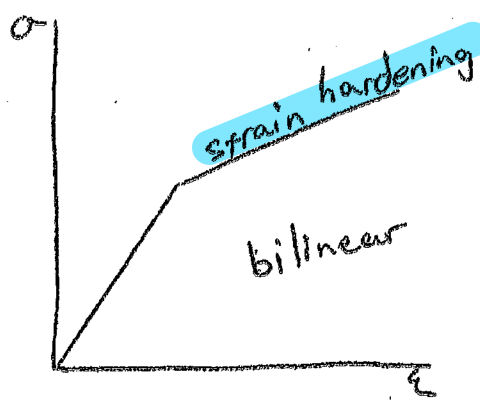


Elastic - nonlinear

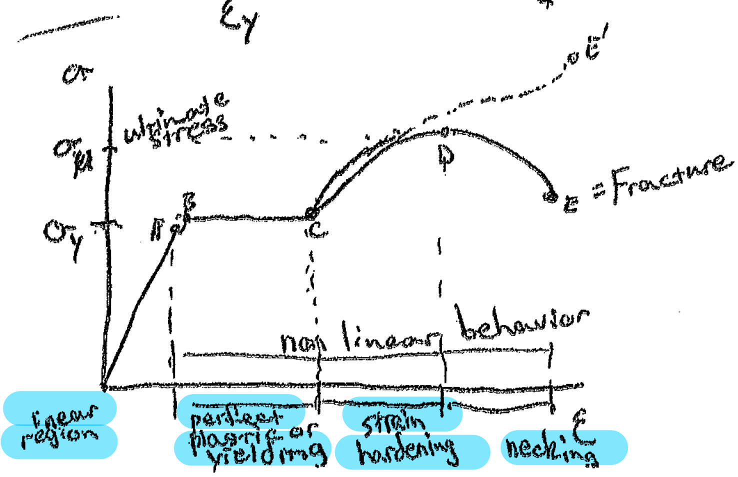


nonlinear

elastoplastic



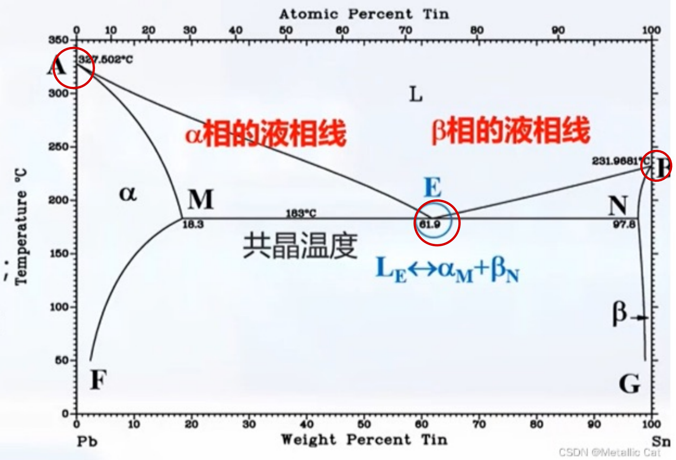
bilinear



↑ strain rate (like during tensile testing under variable speeds), ↑ strength.
And the sensitivity of strength to strain rate ↑ as temp ↑.

1. 共晶反应

- 共晶（eutectic）：α与β同时自液相结晶；
- 共晶点E：液相线AE与BE相交点；
- 二元共晶为不变反应；
- 共晶转变线MEN：发生共晶相变的合金；包含三条共轭线。



~~(scribble)~~ flow (simple)

EOM: $\rho\left(\cancel{\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}}\right) = -\cancel{\frac{\partial P}{\partial x}} + \left(\cancel{\frac{\partial \tau_{xx}}{\partial x}} + \frac{\partial \tau_{yx}}{\partial y} + \cancel{\frac{\partial \tau_{zx}}{\partial z}}\right) + \cancel{\rho g_x}$

Re<<1 molten plastic

symmetry

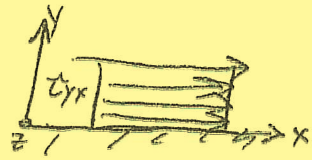
symmetry

in x component

X is flow direction

no. hydrodynamic or external applied forces

$\therefore\ 0 = \frac{\partial \tau_{yx}}{\partial y}$ $\qquad \tau_{yx} = C$



for newtonian fluids, $C = \eta\Delta = \mu\frac{\partial u_x}{\partial y}$ ← velocity gradient

viscosity

b/c of b.c. 1&2

$u_x = \frac{Vy}{h}$ $\qquad \frac{u_x}{V} = \frac{y}{h}$ $\qquad$ so, dimensionless velocity ∝ dimensionless position

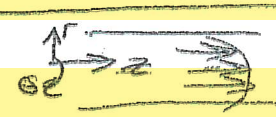
x-sec area

Q = volumetric flow rate $\Rightarrow Q = \langle u_x\rangle A_{cx}$

$Q = u_x(h \cdot w)$

Tube flow

continuity eq. $\frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{1}{r}\cancel{\frac{\partial u_\theta}{\partial \theta}}^{0} + \frac{\partial u_z}{\partial z} = 0$



$u_\theta = u_r = 0$, f.d.f.

$\frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{\partial u_z}{\partial z} = 0$ $\qquad$ if $u_r = 0$ then $\frac{\partial u_z}{\partial z} = 0$ f.d.f.

θ-sym

z(eom) $\rho\left(\cancel{\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}}\right) = -\frac{\partial P}{\partial z} + \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\cancel{\frac{\partial \tau_{\theta z}}{\partial \theta}} + \cancel{\frac{\partial \tau_{zz}}{\partial z}}\right) + \cancel{\rho g_z}$

Re<<1

neglect ⊥ stress for shear flow

$\frac{\partial P}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})$ $\qquad \frac{\partial P}{\partial z} = \frac{\Delta P}{L}$ $\qquad \tau_{rz} = \mu\left(\frac{\partial u_z}{\partial r}\right)$

$\frac{\partial u_z}{\partial r} = \frac{\Delta P r}{2\mu L}$ $\qquad u_z = \frac{\Delta P r^2}{4\mu L} - \frac{\Delta P R^2}{4\mu L}$

$Q = \int_0^R u_z(2\pi)r\,dr$

$Q = \frac{\Delta P}{4\mu L}(2\pi)\int_0^R [r^2 - R^2]\,r\,dr$

Pressure Driven Slit flow

$0 = -\frac{\partial P}{\partial z} + \left(\cancel{\frac{\partial \tau_{xz}}{\partial x}}^{0} + \frac{\partial \tau_{yz}}{\partial y} + \cancel{\frac{\partial \tau_{zz}}{\partial z}}^{0}\right) + \cancel{\rho g_z}^{0}$

$\frac{\partial P}{\partial z} = \frac{\partial \tau_{yz}}{\partial y}$ $\qquad \underline{P = \frac{\partial \tau_{yz}}{\partial y}(z) + C_1}$

Machining

- Cutting ⑥
- grinding ⑤
- milling ③
- polishing ④
- sanding ①
- turning ②

Heap Angle

堆积角

休止角

θ = angle of repose, which is the angle of a particular powder or particle pile based on its own friction

大量颗粒状物质被倾倒于水平面上堆积为锥体，堆积物的表面与水平面所成内角即为休止角，其与密度，颗粒的表面积和形状，及该物质的摩擦系数相关。

1. **锯割（Sawing）**
   - **应用**：初步将材料切割成大致需要的尺寸和形状，通常是加工过程的第一步，用于准备材料块。
   - **排序**：首步，因为通常先要将原材料切割成合适的工作尺寸。
2. **车削（Turning）**
   - **应用**：用于生成圆柱形或其他旋转体形状的部件，可以迅速去除较大的材料量。
   - **排序**：紧随锯割，尤其是在需要处理圆柱形或轴向对称的零件时。
3. **铣削（Milling）**
   - **应用**：进一步形状加工，适用于创建复杂的几何形状、槽口、孔和平面。
   - **排序**：在车削之后，用于在多个平面上或在不规则形状上进行精细加工。

STEM

GPC

4. **磨削（Grinding）**
   - **应用**：提供非常高的表面精度和光洁度，用于完成加工，确保零件符合严格的尺寸容差和表面光洁度要求。
   - **排序**：在大部分形状加工完成后进行，用于最终精加工表面。
5. **抛光（Polishing）**
   - **应用**：最终增强工件表面的美观和光滑度，通常作为最后一步进行，用以提高外观质量和减少表面缺陷。
   - **排序**：最后步骤，用于完成表面处理。
6. **切割（Cutting）**
   - **应用**：这里的"切割"可能特指某种特殊的切割方式，如激光切割、等离子切割等，用于精确切割或在需要非常精细的外形定义时使用。
   - **排序**：可能在任何需要精确形状定义的阶段使用，具体位置取决于具体的制造流程和零件要求。

Mtls.

Chap ① metals

Atomic structures of metals: bcc, fcc + hcp, or body-centered cubic, face-centered-cubic, hexagonal-close-packed.

more than one type of crystal in a metal is allotropism or polymorphism 多态性 异向性

plastic deformation, slipping due to shear stress & twinning

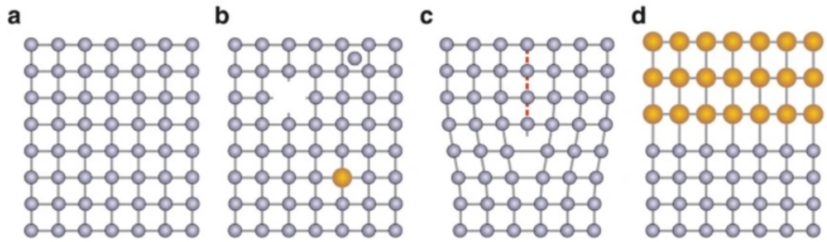
shear stress - ratio of shear force to x-sect. area being sheared

slip in single-crystal occurs along planes of max atomic density

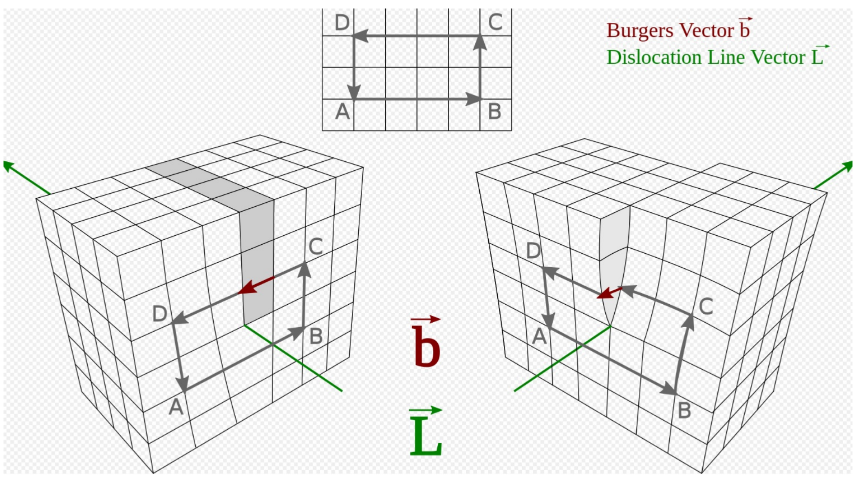
- Defects & impurities in crystal structures are the root cause of discrepancy in strength values between actual & theoretical calculations

- Point Defects, vacancies, interstitial atom, & impurities
- Linear Defects - dislocations (1D)
- Planer impurities (2D) - grain boundaries & phase boundaries
- Volume Impurities (3D) - voids, inclusions, cracks
- Dislocations: defects in the orderly arrangement of a metal's atomic structure linear defect
- Slip plane containing a dislocation requires less shear stress to allow slip.
- Entanglement & Impediments of dislocations increase shear stress required to cause slip. which results in overall ↑ in strength & hardness (strain or work hardening) ↑ deformation ⇒ ↑ # of entanglements ⇒ ↑ metal strength
- Work hardening: cold rolling sheetmetal for auto & aircraft bodies, forging & bolt head, x sec reduction to wire to increase strength called drawing
- Rapid cooling produces small grains (quench) & slow cooling makes large grains (rough surface & low strength, hardness ductility)
- ↑ # Grain boundaries ⇒ ↑ strength & ductility of metals as they interfere w/ the movement of dislocation & influence strain hardening

Fig. 1



(**a**) Two-dimensional representation of an ideal cubic lattice. (**b**) Point defects: a vacancy, an interstitial, and an impurity atom. (**c**) Line defect: cross section perpendicular to a dislocation line located at the end of the inserted lattice plane marked in *red*. (**d**) Planar defect: interface at the heterojunction of two dissimilar solids



left hand rule

位错的增加确实会降低单个位错滑移所需的局部剪切应力，但从宏观角度来看，材料的硬度和强度会提高。这是由于以下几个因素的相互作用：

1. **位错的相互作用**：当位错的密度（位错的数量和分布）增加时，它们之间的相互作用也增加。位错之间的相互阻碍会使得它们的移动更加困难。位错移动是材料塑性变形的主要机制，因此位错移动受阻意味着更大的外力（应力）是必需的才能使材料发生形变。

2. **位错与晶界的相互作用**：位错在穿越晶粒界时会遇到阻力。晶粒越小，晶界越多，位错在通过这些晶界时受到的阻碍也越多。因此，增加位错密度通常与提高晶界阻力相结合，可以有效提高材料的强度和硬度。

3. **应变硬化（Work Hardening）**：在塑性变形过程中，由于位错的增加和重新排列，材料的硬度和强度逐渐增加。这种现象称为应变硬化或加工硬化。位错密度的增加导致位错网络更复杂，任何进一步的变形都需要更大的应力。

总结来说，尽管位错的增加在局部降低了单个位错所需的剪切应力，但在宏观上，由于位错的相互作用和位错与晶界的相互阻碍，整体材料的硬度和强度实际上会提高。这是因为整体上位错的运动更加困难，需要更大的力来推动这些位错穿越材料。

Which depends on temp. deformation rate + type + amt of impurities along the grain boundaries

- Grain boundaries are more reactive than the grains themselves, b/c atoms along the boundaries are packed less efficiently ⇒ ↓ energy than atoms in the mtx + can be easily removed + chemically bonded to another atom.

- GB embrittlement: a normally ductile + strong metal wetted w/ a brittle material causing the entire mtx to crack when subjected to ↓ external stresses.

- During Plastic Deformation (tension or compression), the material exhibits ↑ strength b/c of the entanglements of dislocations w/ boundaries. The ↑ in strength depends on the degree of deformation (strain) ie, ↑ deformation ⇒ ↑ strength of the mtl.

- Plastic Deformation is caused by grain boundary sliding (creep)

- As strength ↑ ⇒ ductility ↓

- Recrystallization: the process in which (within a certain temp range) new equiaxed + strain free grains are formed (replacing old grains)

加热到再结晶温度

在冷加工后

- Cold working results in ↑ strength ⇒ ↓ ductility + causes anisotropy (preferred orientation or mechanical fibering) - properties are different in different directions.

特定环节

温度通常高再结晶温度

- Annealing - heating the metal in a certain temp range for a period of time + allow successive process of recovery (recrystallization) + grain growth to take place

- Effects of cold working can be reversed by annealing the metal

chap ② Mechanical behavior

- Mechanical properties extracted from simple tension test: strength, ductility, toughness, E, strain hardening capability

- Ductility - plastic deformation that the mtl undergoes b4 fracture - Total area under stress strain curve

- Measure of duct. total elongation / reduction in area

Toughness - area under stress strain curve from ultimate strength to fracture

Yield stress -/- stress where elastic deformation ends & plastic deformation begins, In soft ductile mtls, yield stress is defined as the pt on the stress-strain curve that is offset by a strain of ~~2%~~ 0.2% elongation

$\sigma = \varepsilon E$

- E Young's - ratio of stress to strain in elastic region, Hook's law - stiffness, ↑E ⇒ ↑load to stretch ⇒ ↑ stiffness

横向应变 / 纵向应变

- ν, Poisson's - ratio of ~~lat~~ lateral strain to longitudinal strain
- ↑temp ⇒ ↑ duct & toughness ⇒ ↓ yield stress & ultimate tensile strength & E
- Deformation rate: speed @ which tension test occurs
- Strain rate - $\Delta\varepsilon / \Delta t$
- Torsion test 扭转测试 used to determine shear properties of a mtl.
- G, (Shear Modulus) ratio of shear stress / shear strain in elastic range.
- ↑ # twists in a round shaft in torsion prior to failure ⇒ better the forgeability of mtl.
- flexural test 抗弯强测试 conducted on brittle mtls to measure rupture strength, there mod of rupture or transverse rupture strength. on other mtls to get flexural strength.
- Hardness: 硬度测试 gives a general indication of the strength of the material & its resistance to scratch & wear
- Hardness is not a fundamental property b/c it depends on the shape of the indenter & applied load.

Fatigue - progressive & localized structural damage that occurs when a mtl is subjected to cyclic loading which ↓ yield limit, Ultimate Tensile Stress

- S-N curves - plot stress amplitude (S) & # of cycles (N) & are associated w/ fatigue

- Stress amplitude - max stress (tension & comp) to which the specimen is subjected

- endurance limit - max stress 无限循环负载下 to which the mtl can be subjected w/out fatigue failure

- creep - permanent elongation of a component 在恒定的静态载荷下,随时间发生的持续形变 under a static load maintained over a period of time. creep @ elevated temps is attributed to grain-boundary sliding

- resistance to creep ↑ as melt temp ↑

- stress relaxation - ↓ in magnitude of stresses 在持续固定形变的条件下, resulting from loading over a period of time, even though 材料内部应力随时间逐渐减小的现象 the dimensions of the component remains constant.

- Impact test: 冲击测试 consists of placing a notched specimen in an impact tester & breaking it w/ a pendulum (Izod & Charpy)

- Types of failure: brittle & ductile fracture (due to external cracks & buckling (compressive loading)

- Ductile fracture: plastic deformation that precedes failure & takes place along planes of max shear stress.

- In simple shear, failure occurs due to excessive slip along slip planes w/in the grains.

- factors affecting void formation - strength of bond between an inclusion & mtx & hardness of inclusions

- mechanical Fibering: alignment of inclusion during plastic deformation.

- Strain aging - a phenomenon where carbon in steels segregate to dislocations & pinning them down & ↑ resistance to dislocation movement

- Brittle fracture - no plastic deformation, fracture along crystal plane

- Improve fatigue Strength - induced comp. residual stresses on surfaces, case hardening, fine surface finish

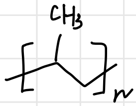
- Stress corrosion cracking - ductile mtl fails brittly by cracks over time

- Annealing could be used to help stress-relieve parts to avoid stress corrosion, however annealing can ↓ strength

- Hydrogen embitterment: presence of hydrogen can reduce ductility & cause failure

- Residual Stress: stresses that remain w/in a part after being formed & all external forces removed. reduced by annealing

ME 310

Polypropylene (PP) 

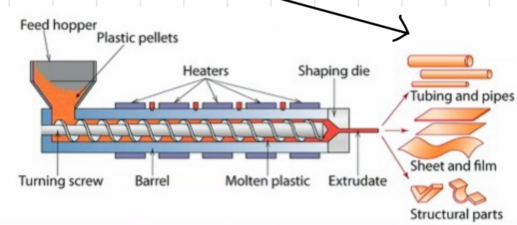
Co-polymers two or more monomer types

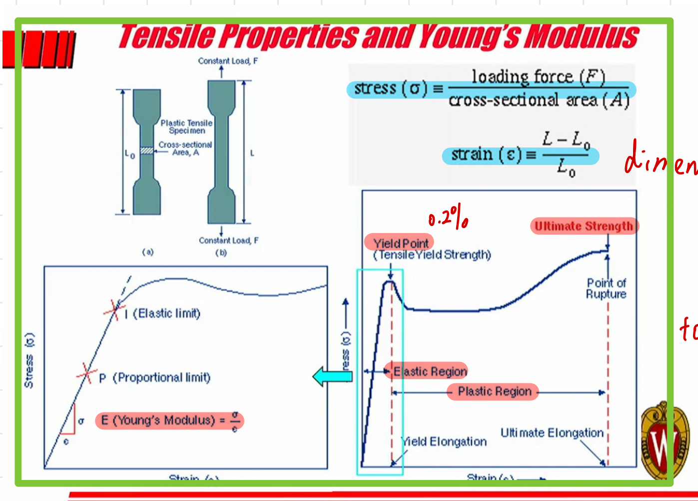
Polymerization

Polymer Blends mixing or blending two or more polymers

Polymer composites

Injection Mold —— thermoplastic materials

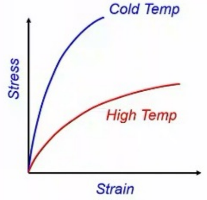




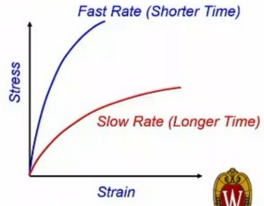
dimentionless

toughness — area under curve

Short-term Tensile Test (Effects of Temperature and Strain Rate)



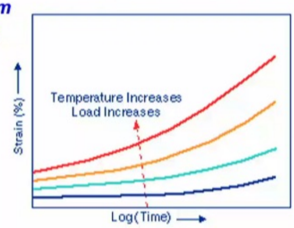
During a Normal Testing Timeframe

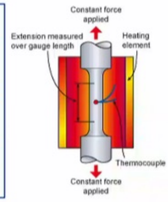


At a Normal Testing Temp.

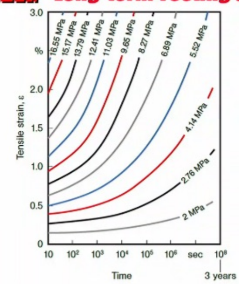
Long-term Testing: Creep

Long-term Testing

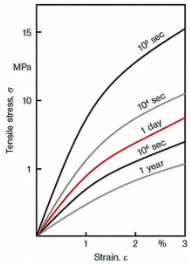




Long-term Testing and Material Behavior



Creep behavior of a propylene-ethylene copolymer at 20 °C



Isochronous stress-strain curves showing creep response and stress relaxation

Repeated Loading/Testing: Fatigue

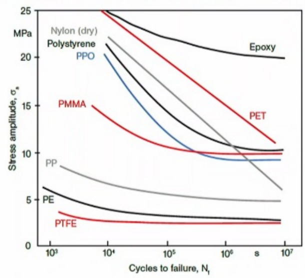
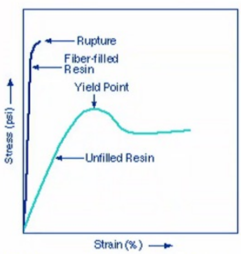
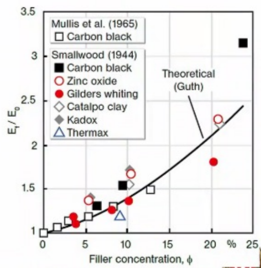


Figure 2.20 Stress-life (S-N) curves for several thermoplastic and thermoset polymers tested at a 30 Hz frequency about a zero mean stress

Mechanical Property of Filled Polymers



Stiffness and strength of polymers increase with reinforcing fillers



Modulus ratio of filled to unfilled natural rubber

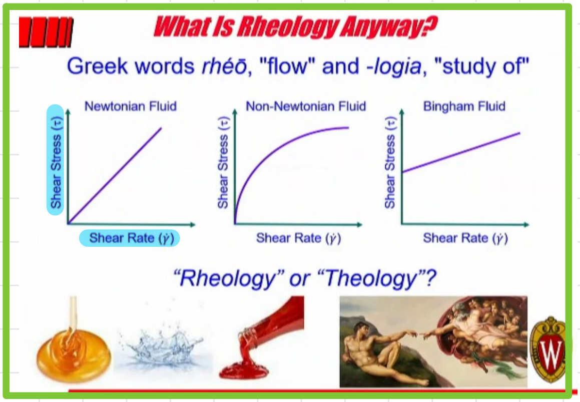
Rheology Example 3: Deborah Number

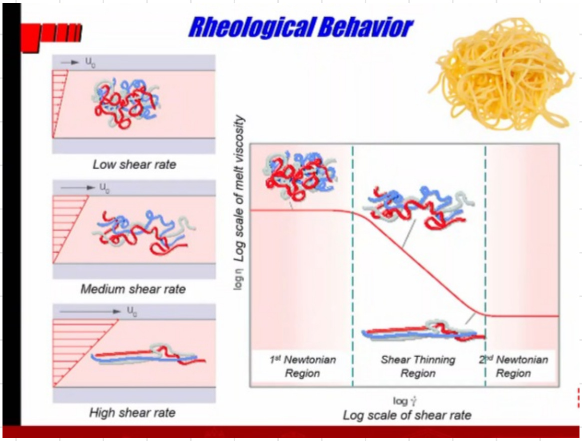
- **Deborah No. (*De*):**

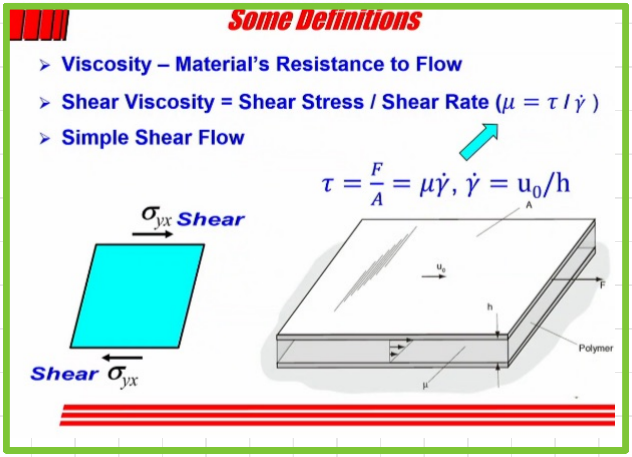
$$De = \frac{\text{Material's relaxation time}}{\text{Processing (Observation) time}}$$

0	⟵	De	⟶	∞
Viscous fluid	⟵	De	⟶	Elastic solid

Silly putty



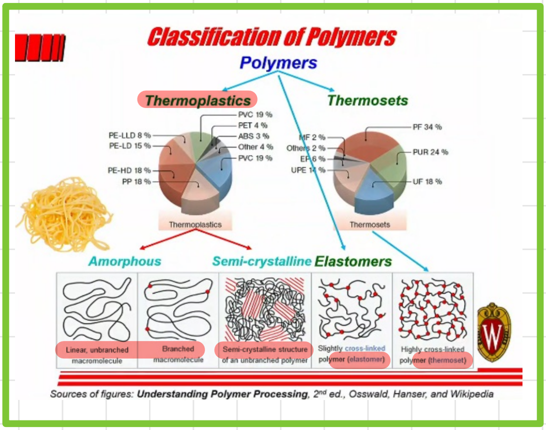


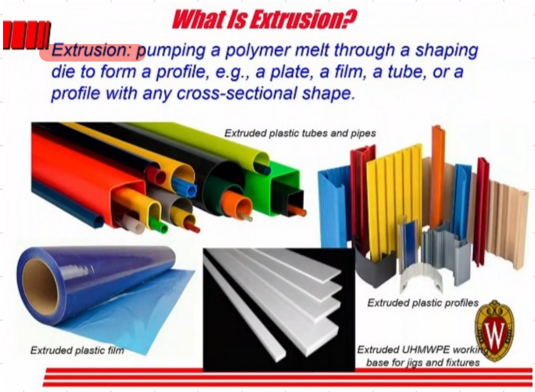


τ — shear stress

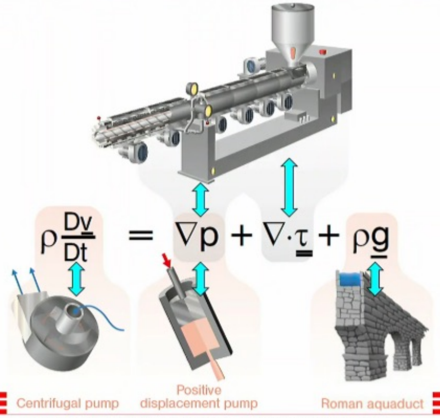
μ — 流体层的速度

$\dot{\gamma}$ — 不同层之间的速度梯度





Various Pumping Mechanisms (Principles)



Understanding Polymer Processing, 2nd ed., Osswald, Hanser

Single Screw Extruder and Three Screw Zones

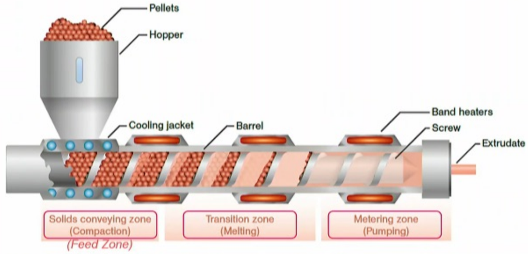
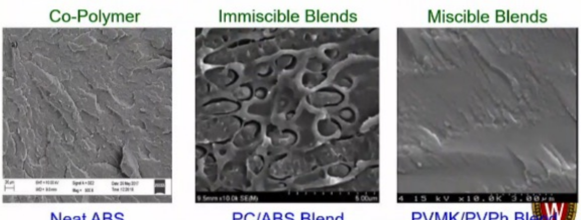


Figure 4.5 Schematic of a plasticating single screw extruder

Polymer Blends vs. Co-Polymers

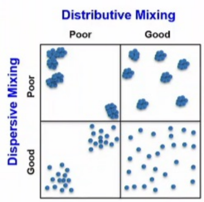
"Co-polymers contain two or more monomer types within a single polymer chain (this occurs during synthesis), whereas polymer blends are made via mixing techniques (combination of multiple polymers, which generally occurs after synthesis)"

www.sciencedirect.com



Journal of Physics Conference Series 1062(1):012016 | Polymer Korea 38(4):471-476 | Polymers **2014**, 6(11), 2752

Distributive vs Dispersive Mixing



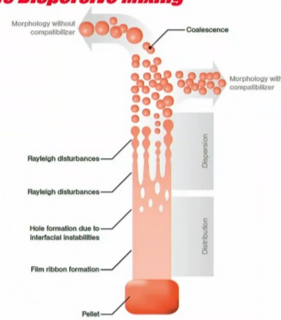
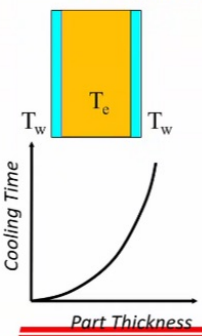


Figure 5.1 Mechanism for morphology development in polymer blends

Part Thickness and Cooling Time

Averaged cooling time for flat plates:



$$t_{cooing} = \frac{h^2}{\alpha\pi} ln\left[\frac{8}{\pi^2}\left(\frac{T_m - T_w}{T_e - T_w}\right)\right]$$

h: Part Thickness

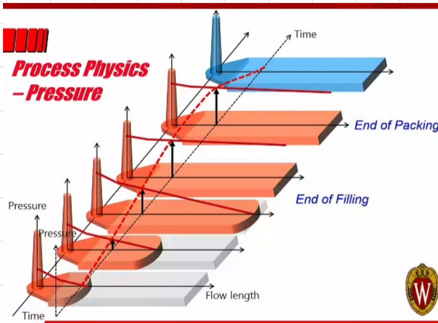
α: Thermal Diffusivity

T_m: Melt Temperature

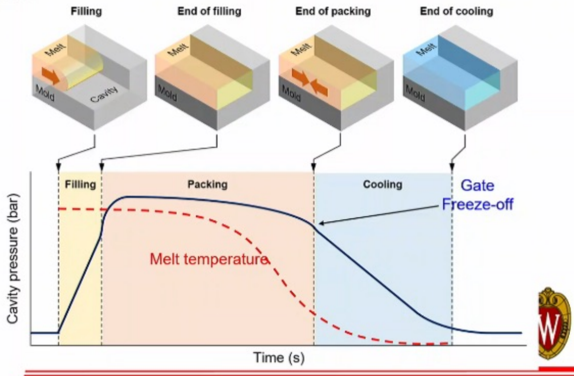
T_w: Mold-wall Temperature

T_e: Ejection Temperature

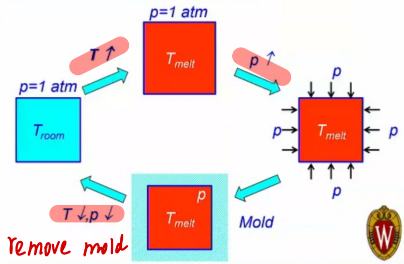
Process Physics – Pressure



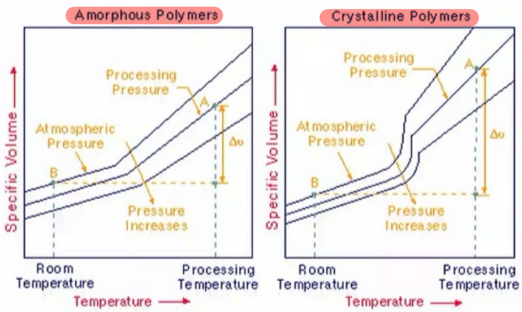
Process Physics – Pressure



Relationship among Pressure, Volume, and Temperature

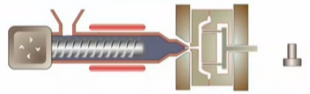


Schematic pvT Diagram



Manufacturing of Football Helmets

Injection Molding



Polymer Engineering Center
University of Wisconsin - Madison

Modeling for Materials Processing

A stainless steel turbine wheel

- What manufacturing method?
  - CNC
  - Die casting
  - Investment casting
  - Sand casting + machining
  - Additive manufacturing/ 3D printing
  - Injection molding (really?)
- It all depends
  - Quantity, quality, & time
  - Resources and facility

(1) Analytical Approach

Cartesian Coordinates (x, y, z):

$$\rho\left(\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\left[\frac{\partial}{\partial x}\tau_{xx}+\frac{\partial}{\partial y}\tau_{yx}+\frac{\partial}{\partial z}\tau_{zx}\right]+\rho g_x$$

Body force negligible

Cartesian Coordinates (x, y, z):

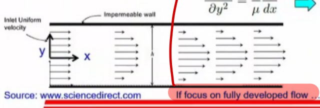
$$\rho\left(\cancel{\frac{\partial u_x}{\partial t}}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+\cancel{u_z}\frac{\partial u_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\cancel{\frac{\partial^2 u_x}{\partial z^2}}\right]+\cancel{\rho g_x}$$

Steady state | $u_z = 0$ | u_x independent of z

$$\rho\left(u_x\cancel{\frac{\partial u_x}{\partial x}}+\cancel{u_y}\frac{\partial u_x}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\cancel{\frac{\partial^2 u_x}{\partial x^2}}+\frac{\partial^2 u_x}{\partial y^2}\right)$$

$$\frac{\partial^2 u_x}{\partial y^2}=\frac{1}{\mu}\frac{dp}{dx} \Rightarrow u_x(y)=-\frac{h^2}{8\mu}\frac{dp}{dx}\left[1-\left(\frac{2y}{h}\right)^2\right]=\frac{h^2}{8\mu}\frac{\Delta p}{L}\left[1-\left(\frac{2y}{h}\right)^2\right]$$

$$Q = hW\bar{u}_z = \frac{Wh^3\Delta p}{12\mu L}$$



Source: www.sciencedirect.com

If focus on fully developed flow ...

"fully developed region" 是指在流体力学或传热学中，流动或温度场已经达到平衡的区域。在这个区域内，流体的速度或温度分布不再随流动方向变化。具体来说，这意味着：

1. **流体流动**：在管道或通道内，流体进入的最初阶段，流动会受到入口效应的影响，速度分布会随着距离逐渐变化，直到达到某一段距离后，速度分布稳定下来，即进入"充分发展区域"（fully developed region）。在这一区域，流体的速度分布不再随流动方向变化。
2. **传热过程**：类似地，在传热过程中，进入通道的流体温度分布会随着距离变化，直到达到某一段距离后，温度分布也会稳定下来，不再随流动方向变化，这一段距离之后的区域也称为"充分发展区域"。

T F A plastic component under a constant load will deform with time, a phenomenon called *stress relaxation*. creep

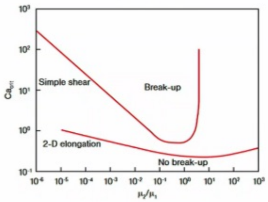
T F Thermoplastics can be repeatedly softened (or hardened) by an increase (or decrease) in temperature.

T F Plastics exhibit viscoelastic behavior. Therefore, their mechanical properties, such as the stress-strain curves, depend on the temperature and duration, but not on the rate at which the load is applied.

T F In thermoplastics injection molding, if one doubles the part thickness, the cooling time will also increase by approximately 100% (i.e., the cooling time will be twice as long).

T F For macromolecules like polymers, their weight-average molecular weight will always be lower than their number-average molecular weight.

The graph below shows the critical capillary number for drop break-up as a function of viscosity ratio in a simple shear flow and a 2-D elongational flow.

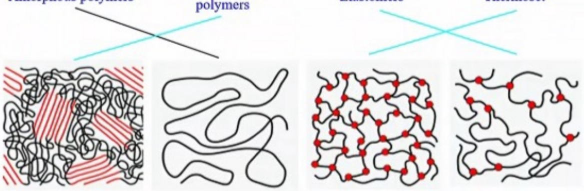


T F This graph shows that the simple shear flow is more effective in breaking up the droplets than the 2-D elongation flow.

T F The capillary number, Ca, can be considered as the ratio of flow stresses to droplet surface stresses, and it has a ~~unit~~ of [g·cm/sec^2]. dimensionless

2. Connect the polymers with their corresponding microstructure at the solid state. (3 points)

Amorphous polymers | Semi-crystalline polymers | Elastomers | Thermoset



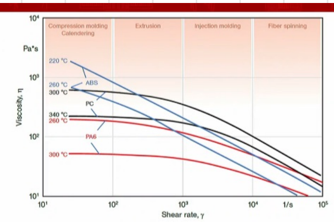


Figure 3.2 Viscosity curves for a selected number of thermoplastics

3(a) Which polymer exhibits the strongest shear thinning behavior at low shear rates? ABS

3(b) Which polymer has the highest shear viscosity at 300 °C and a shear rate of 100 s^{-1}? PC

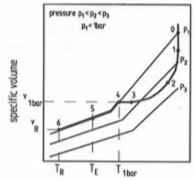
3(c) When one doubles the extrusion rate, which material allows the least torque % increase? ABS

4. Which one of the following rheometers can generate the highest shear rates within the sample?
   - ○ Melt flow indexer
   - ● Capillary rheometer
   - ○ Cone and plate rheometer
   - ○ Parallel plate rheometer

5. Which one of the following rheometers generates a constant-shear-rate throughout the sample?
   - ○ Melt flow indexer
   - ○ Capillary rheometer
   - ● Cone and plate rheometer
   - ○ Parallel plate rheometer

6. Which rheometer is often used to characterize a polymer and provide a simple and quick means of quality control?
   - ● Melt flow indexer
   - ○ Capillary rheometer
   - ○ Cone and plate rheometer
   - ○ Parallel plate rheometer

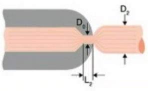
7. Shrinkage is a major concern for injection molding. Please select the one effective strategy for reducing part shrinkage.
   - ○ Increasing the part thickness
   - ● Increasing the pack/hold pressure
   - ○ Decreasing the pack/hold time
   - ○ Decreasing the melt temperature



Fill in the circle of all correct answers. There may be more than one correct answer. (4 points)

8. What are the potential solutions to fix an incomplete fill problem for an injection molded tensile test sample?
   - ● Increase mold temperature
   - ● Use polymer materials with lower viscosities
   - ○ Enhance cooling in critical zones
   - ○ Lower holding pressure and shorten holding time
   - ● Increase gate size or shorten the runner length

9. During the extrusion process
   - ○ The extrudate swell parameter (χ) can be calculated as D_0/D_2
   - ● The extrudate swell can be reduced by increasing melt temperature
   - ● The extrudate swell can be reduced by increasing die length, L_2
   - ○ The extrudate swell can be reduced by increasing extrusion rate
   - ● The extrudate swell can be reduced by modifying die design



10. Below are four plastic products that were produced using various polymer processing methods. Fill in the blanks below with the most suitable process used. (6 points)

Milk jug from molded preform

Blow molding

Clamshell package

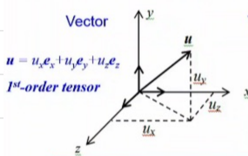
Thermoforming

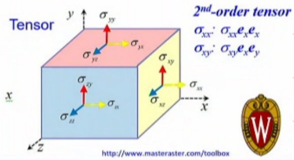
PVC elbow

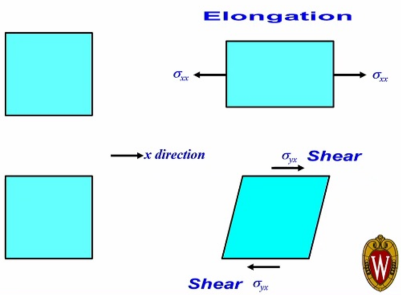
Injection molding

Basic Concept:

- **Scalar:** a value (no direction), e.g., pressure (p), temperature (T), speed, etc.
- **Vector:** both magnitude and direction, e.g., velocity, force, gradient of a scalar function (e.g., pressure gradient), etc.
- **Tensor:** "a multi-dimensional vector", e.g., 2nd-order surface traction (or stress tensor)







Basic Concept: Operators

- **Gradient operator: ∇( )**

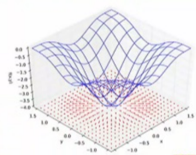
  Pressure gradient (∇p), temperature gradient (∇T), etc.

  p and T are scalars, but ∇p and ∇T are vectors

$$\nabla = \frac{\partial}{\partial x}\boldsymbol{e_x} + \frac{\partial}{\partial y}\boldsymbol{e_y} + \frac{\partial}{\partial z}\boldsymbol{e_z}$$

$$\nabla p = \frac{\partial p}{\partial x}\boldsymbol{e_x} + \frac{\partial p}{\partial y}\boldsymbol{e_y} + \frac{\partial p}{\partial z}\boldsymbol{e_z}$$

$$\nabla T = \frac{\partial T}{\partial x}\boldsymbol{e_x} + \frac{\partial T}{\partial y}\boldsymbol{e_y} + \frac{\partial T}{\partial z}\boldsymbol{e_z}$$



$\boldsymbol{e_x}$, $\boldsymbol{e_y}$, and $\boldsymbol{e_z}$: unit vectors

Basic Concept: Operators (Cont'd)

$$\nabla \cdot \boldsymbol{\sigma} = \left(\frac{\partial}{\partial x}\mathbf{e}_x + \frac{\partial}{\partial y}\mathbf{e}_y + \frac{\partial}{\partial z}\mathbf{e}_z\right) \cdot \left(\sigma_{xx}\,\mathbf{e}_x\mathbf{e}_x + \sigma_{xy}\,\mathbf{e}_x\mathbf{e}_y + \sigma_{xz}\,\mathbf{e}_x\mathbf{e}_z + \sigma_{yx}\,\mathbf{e}_y\mathbf{e}_x + \sigma_{yy}\,\mathbf{e}_y\mathbf{e}_y + \sigma_{yz}\,\mathbf{e}_y\mathbf{e}_z + \sigma_{zx}\,\mathbf{e}_z\mathbf{e}_x + \sigma_{zy}\,\mathbf{e}_z\mathbf{e}_y + \sigma_{zz}\,\mathbf{e}_z\mathbf{e}_z\right)$$

A vector operator　A 2nd order tensor

$$= \sigma_{xx}\,\mathbf{e}_x + \sigma_{xy}\,\mathbf{e}_y + \sigma_{xz}\,\mathbf{e}_z + \sigma_{yx}\,\mathbf{e}_x + \sigma_{yy}\,\mathbf{e}_y + \sigma_{yz}\,\mathbf{e}_z + \sigma_{zx}\,\mathbf{e}_x + \sigma_{zy}\,\mathbf{e}_y + \sigma_{zz}\,\mathbf{e}_z$$

$$= (\sigma_{xx}+\sigma_{yx}+\sigma_{zx})\mathbf{e}_x + (\sigma_{xy}+\sigma_{yy}+\sigma_{zy})\mathbf{e}_y + (\sigma_{xz}+\sigma_{yz}+\sigma_{zz})\mathbf{e}_z$$

A vector

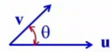
- Dot product reduces the "dimensional rank" of each of its two operands by one.
- Example here: Dot product of a vector and a 2nd order tensor yields a vector

Basic Concept: Operators

- "Dot" product operator for vectors/tensors: "•"

  $\mathbf{u} \cdot \mathbf{v}$, $\nabla \cdot \mathbf{u}$, $\nabla \cdot \boldsymbol{\sigma}$, etc.

  $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u} = |\mathbf{u}| \times |\mathbf{v}| \times \cos\theta$



  $\mathbf{u} \cdot \mathbf{v} = (u_x\mathbf{e}_x + u_y\mathbf{e}_y + u_z\mathbf{e}_z) \cdot (v_x\mathbf{e}_x + v_y\mathbf{e}_y + v_z\mathbf{e}_z)$

  $= u_x v_x + u_y v_y + u_z v_z$ ← A scalar

  Note: $\mathbf{e}_x \cdot \mathbf{e}_x = \mathbf{e}_y \cdot \mathbf{e}_y = \mathbf{e}_z \cdot \mathbf{e}_z = 1$

  $\mathbf{e}_x \cdot \mathbf{e}_y = \mathbf{e}_x \cdot \mathbf{e}_z = \mathbf{e}_y \cdot \mathbf{e}_z = 0$

- Dot product reduces the "dimensional rank" of each of its two operands by one.
- Note: scalars have a rank of zero, vectors have a rank of one, second order tensors have a rank of two, etc.
- Example here: Dot product of two vectors yields a scalar

Transient Derivative Term $\left(\frac{\partial(\,)}{\partial t}\right)$

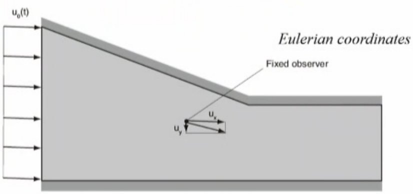


Figure 2.2: Flow system with a fixed obeserver.

In transient, $\frac{\partial u_x}{\partial t}, \frac{\partial u_y}{\partial t}, \frac{\partial u_z}{\partial t}, \frac{\partial T}{\partial t}$, etc.

Total, Material, or Substantial Derivative $\left(\frac{D(\,)}{Dt}\right)$

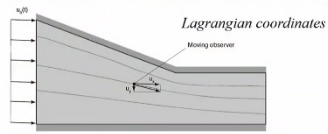


Figure 2.3: Flow system with an observer moving with a fluid particle on a given streamline.

$$\frac{D(\,)}{Dt} = \frac{\partial(\,)}{\partial t} + \mathbf{u}\cdot\nabla(\,) \Rightarrow$$

Material, Substantial, or Total Derivative

$$\frac{D\mathbf{u}}{Dt} = \left(\frac{\partial u_x}{\partial t} + u_x\cdot\frac{\partial u_x}{\partial x} + u_y\cdot\frac{\partial u_x}{\partial y} + u_z\cdot\frac{\partial u_x}{\partial z}\right)\mathbf{e}_x + \left(\frac{\partial u_y}{\partial t} + u_x\cdot\frac{\partial u_y}{\partial x} + u_y\cdot\frac{\partial u_y}{\partial y} + u_z\cdot\frac{\partial u_y}{\partial z}\right)\mathbf{e}_y + \left(\frac{\partial u_z}{\partial t} + u_x\cdot\frac{\partial u_z}{\partial x} + u_y\cdot\frac{\partial u_z}{\partial y} + u_z\cdot\frac{\partial u_z}{\partial z}\right)\mathbf{e}_z$$

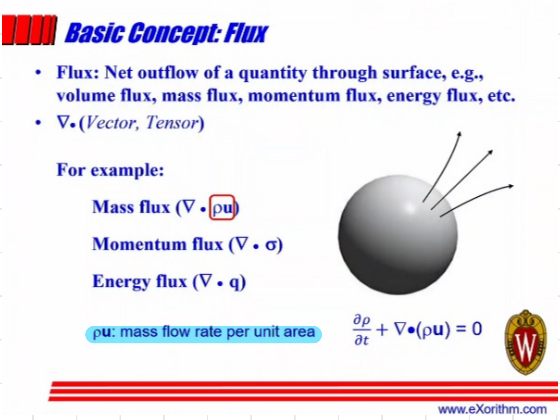
$$\frac{DT}{Dt} = \underbrace{\frac{\partial T}{\partial t}}_{\text{Transient term}} + \underbrace{u_x\cdot\frac{\partial T}{\partial x} + u_y\cdot\frac{\partial T}{\partial y} + u_z\cdot\frac{\partial T}{\partial z}}_{\text{Convention term}}$$

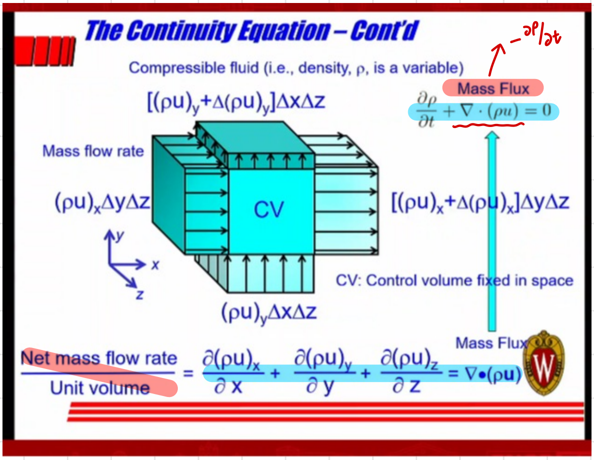
1. 固定观察者与随动观察者的区别

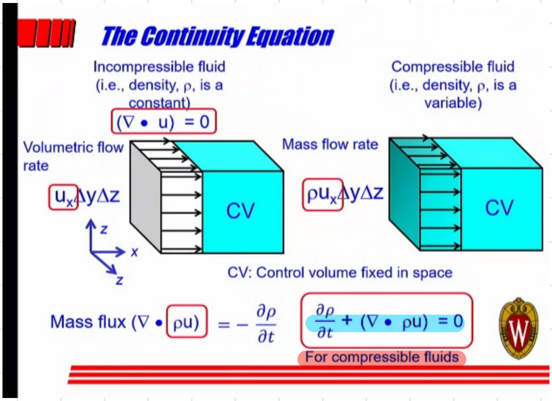
- **固定观察者**（Eulerian描述）：观察者固定在空间中的某一个点，记录流体在这个固定点的变化情况。例如，一个温度计放在房间的某个角落，记录这个点的温度变化。
- **随动观察者**（Lagrangian描述）：观察者跟随流体质点一起移动，记录流体质点随时间的变化情况。例如，一个漂浮在河流中的叶子，记录它随水流移动过程中所经历的温度变化。

2. 全导数的物理意义

- **全导数** ($\frac{D}{Dt}$) 也叫做物质导数或实质导数，是流体力学中用于描述随动观察者看到的物理量变化的工具。
- 它综合了两部分变化：时间变化和空间变化。
  - **时间变化** ($\frac{\partial}{\partial t}$)：在固定位置上，物理量随时间的变化。
  - **空间变化** ($\mathbf{u}\cdot\nabla$)：由于流体质点在空间中移动，物理量随空间位置的变化。





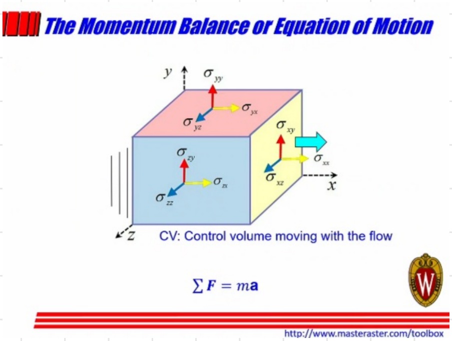


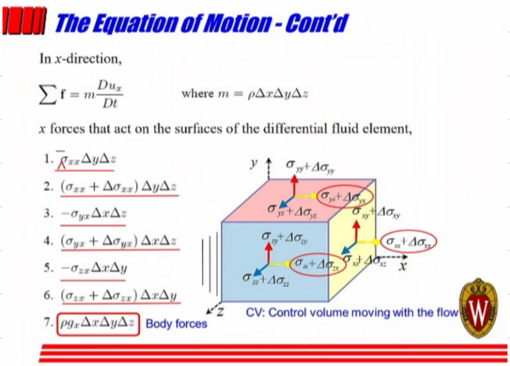
1. ρ (rho) ：

- **密度**：ρ 表示流体的密度，即单位体积的质量。它通常以千克每立方米（kg/m³）为单位。

2. $\mathbf{u}$ (u) ：

- **速度向量**：$\mathbf{u}$ 表示流体的速度向量，描述了流体在空间中的移动速度和方向。它的单位通常是米每秒（m/s）。





The Equation of Motion - Cont'd

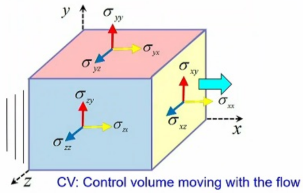
After adding the forces, dividing by the element's volume, and letting the volume go to zero, the force balance results in

$$\frac{\overbrace{\rho\Delta x\Delta y\Delta z\,\frac{Du_x}{Dt}}^{ma}}{\underbrace{\Delta x\Delta y\Delta z}_{\text{Volume}}} = \frac{\overbrace{\Delta\sigma_{xx}\,\Delta y\Delta z+\Delta\sigma_{yx}\,\Delta x\Delta z+\Delta\sigma_{zx}\,\Delta x\Delta y\,+\,\rho g_x\Delta x\Delta y\Delta z}^{\sum F}}{\underbrace{\Delta x\Delta y\Delta z}_{\text{Volume}}}$$

$$\underbrace{\rho\frac{Du_x}{Dt}}_{\text{Inertia}} = \underbrace{\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{yx}}{\partial y}+\frac{\partial\sigma_{zx}}{\partial z}}_{\text{Surface forces}} + \underbrace{\rho g_x}_{\text{Body forces}}$$

x-dir component of the momentum flux ($\nabla \bullet \sigma$)

The Momentum Balance (Equation of Motion)

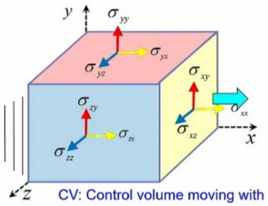


CV: Control volume moving with the flow

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{yx}}{\partial y}+\frac{\partial\sigma_{zx}}{\partial z}=\nabla\bullet(\sigma)$$

http://www.masteraster.com/toolbox

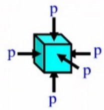
The Momentum Balance (Equation of Motion)



CV: Control volume moving with the flow

$$\boxed{\rho\frac{Du}{Dt}} = \underbrace{\rho a}_{\text{Material's Acceleration/volume}} = ma/V = \underbrace{\Sigma F/V}_{\text{Net forces/volume}} = \boxed{\underbrace{\nabla\bullet(\sigma)}_{\text{Surface forces/volume}} + \underbrace{\rho g}_{\text{Body forces/volume}}}$$

Momentum Equation (Conservation of Momentum)



"—" compress

$$\sigma = -pI + \tau$$

$$\rho\frac{Du}{Dt} = \nabla\bullet(\sigma) + \rho g$$

$$\rho\frac{Du}{Dt} = -\nabla p + \nabla\bullet(\tau) + \rho g$$

$\tau = \eta\dot{\gamma}$ *Constitutive equation*

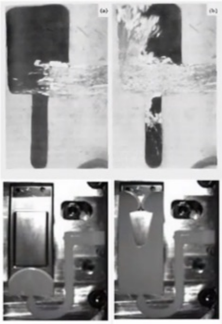
Note my notion:

$\eta = \eta(T,\dot{\gamma},p)$ or $\eta(x,y,z)$ *Non-Newtonian Fluids* $\quad \rho\frac{Du}{Dt} = -\nabla p + \nabla\bullet(\eta\dot{\gamma}) + \rho g$

μ = *A constant Newtonian Fluids* $\quad \rho\frac{Du}{Dt} = -\nabla p + \mu\nabla^2 u + \rho g$

Navier-Stokes equations

Die Casting vs. Injection Molding



C: 268 ms D: 794 ms

Die Casting w/ Liquid Metals

$$\underbrace{\rho\frac{Du}{Dt}}_{\text{Inertia}} = \overbrace{-\nabla p}^{\text{Pressure}} + \underbrace{\cancel{\nabla\cdot\tau}}_{\text{Surface forces w/o pressure}} + \overbrace{\rho g}^{\text{Body forces}}$$

Injection Molding w/ Plastic Melts

$$\underbrace{\cancel{\rho\frac{Du}{Dt}}}_{\text{Inertia}} = \overbrace{-\nabla p}^{\text{Pressure}} + \underbrace{\nabla\cdot\tau}_{\text{Surface forces w/o pressure}} + \overbrace{\cancel{\rho g}}^{\text{Body forces}}$$

压铸（Die Casting with Liquid Metals）

涉及液态金属的压铸过程，公式如下：

$\rho\frac{Du}{Dt} = -\nabla p + \cancel{\nabla\cdot\tau} + \rho g$

- $\rho\frac{Du}{Dt}$：表示惯性力。
- $-\nabla p$：表示压力梯度。
- $\cancel{\nabla\cdot\tau}$：表面力项被忽略，因为液态金属的流动主要受到压力和重力影响。
- ρg：表示重力影响。

The Energy Balance or Equation of Energy

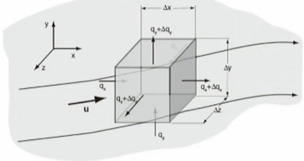


Figure 2.7: Heat flux across a differential fluid element during flow.

An energy balance around a moving fluid element can be written as,

$$\rho C_p \frac{DT}{Dt} = -\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} - \frac{\partial q_z}{\partial z} + \dot{Q} + \dot{Q}_{\text{viscous heating}}$$

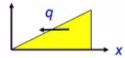
"−" leaving

where the left hand term represents the transient and convective effects and the right hand the conduction terms, arbitrary heat source ($\dot{Q}$), and viscous dissipation ($\dot{Q}_{\text{viscous heating}}$).

The Equation of Energy – Cont'd

Derivation of heat conduction term. $-\nabla\bullet(\boldsymbol{q})$

Using Fourier's law for heat conduction



$$q_i = -k_i \frac{\partial T}{\partial x_i}$$

and assuming an isotropic material, $k_x = k_y = k_z = k$, we can write

$$\rho C_p \frac{DT}{Dt} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \dot{Q} + \dot{Q}_{\text{viscous heating}}$$

where k_i is heat conduction coefficient.

$$\rho C_p \frac{DT}{Dt} = -\nabla\bullet(\mathbf{q}) + \dot{Q} + \dot{Q}_{\textit{viscous heating}}$$

Continuity Equation (Conservation of Mass)

$$\frac{\partial \rho}{\partial t} + \nabla\bullet(\rho\mathbf{u}) = 0$$

Momentum Equation (Conservation of Momentum)

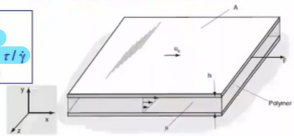
$$\rho\frac{D\mathbf{u}}{Dt} = \nabla\bullet(\sigma) + \rho\mathbf{g}$$

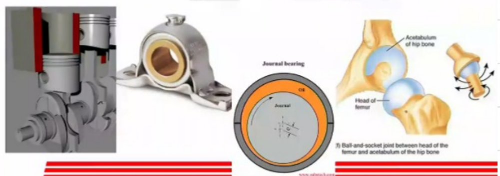
Energy Equation (Conservation of Energy)

$$\rho C_p \frac{DT}{Dt} = -\nabla\bullet(\mathbf{q}) + \dot{Q}_{\textit{viscous heating}} + \dot{Q}$$

Simple Shear Flow and Examples

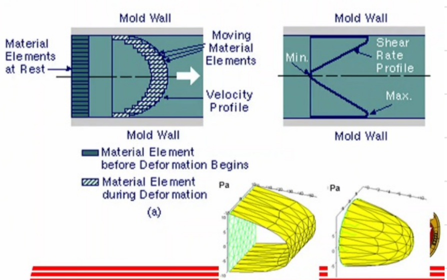
Shear stress (τ, σ_{yx}) = F/A
Shear rate ($\dot\gamma$, or $\dot\gamma_a$) = u_0/h
Shear viscosity (μ, or η) = $\tau/\dot\gamma$
Or τ = F/A = $\mu\dot\gamma$ = μ (u_0/h)





Sources of images: blenderartists.org, baldor.com, substech.com, pinterest.com

Flow between Plates or inside a Tube



Source: C-MOLD Design Guid

The Equation of Energy - Cont'd

$$\rightarrow \quad \frac{F}{A} = \mu\frac{u_0}{h}$$

In the system, the *rate of energy input* is given by

$$Fu_0 = \mu\frac{u_0}{h}Au_0$$

and the rate of energy input per unit volume is represented by

$$\frac{Fu_0}{Ah} = \mu\left(\frac{u_0}{h}\right)\left(\frac{u_0}{h}\right)$$

or

$$Q_{\text{viscous heating}} = \mu\left(\frac{\partial u_x}{\partial y}\right)\left(\frac{\partial u_x}{\partial y}\right)$$

Continuity Equation (Conservation of Mass)

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{u}) = 0$$

Table 2.1: Continuity Equation

$$\nabla = \frac{\partial}{\partial x}\mathbf{e_x} + \frac{\partial}{\partial y}\mathbf{e_y} + \frac{\partial}{\partial z}\mathbf{e_z}$$

Cartesian Coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0$$

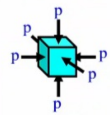
Cylindrical Coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{\partial}{\partial z}(\rho u_z) = 0$$

Spherical Coordinates (r, θ, ϕ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 u_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho u_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho u_\phi) = 0$$

Momentum Equation (Conservation of Momentum)



$$\sigma = -pI + \tau \qquad \sigma_{ij} = \begin{pmatrix} -p+\tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & -p+\tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & -p+\tau_{zz} \end{pmatrix}$$

$$\boxed{\rho \frac{D\mathbf{u}}{Dt} = \nabla \bullet (\sigma) + \rho \mathbf{g}}$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla \bullet (\tau) + \rho \mathbf{g}$$

$$\tau = \eta \dot{\gamma} \qquad \textit{Constitutive equation}$$

Note my notion:

$\eta = \eta(x,y,z)$ Non-Newtonian Fluids

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla \bullet (\eta \dot{\gamma}) + \rho \mathbf{g}$$

μ = A constant Newtonian Fluids

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g}$$

Navier-Stokes equations

Momentum Equation (Conservation of Momentum)

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla \bullet (\tau) + \rho \mathbf{g}$$

Table 2.2: Momentum Equation in terms of τ

Cartesian Coordinates (x, y, z):

$$\rho\left(\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \left[\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right] + \rho g_x$$

$$\rho\left(\frac{\partial u_y}{\partial t} + u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + u_z\frac{\partial u_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \left[\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right] + \rho g_y$$

$$\rho\left(\frac{\partial u_z}{\partial t} + u_x\frac{\partial u_z}{\partial x} + u_y\frac{\partial u_z}{\partial y} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \left[\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z$$

For non-Newtonian fluids like polymer melts, ketchup, silly putty, and paints

Energy Equation (Conservation of Energy)

$$\rho C_p \frac{DT}{Dt} = -\nabla \bullet (\mathbf{q}) + \dot{Q}_{\textit{viscous heating}} + \dot{Q}$$

$$\nabla \bullet (\mathbf{q}) \Downarrow k\nabla^2 T \qquad \dot{Q}_{\textit{viscous heating}} \Downarrow \tau : \dot{\gamma}$$

Table 2.5: Energy Equation for a Newtonian Fluid with Constant Properties

Cartesian Coordinates (x, y, z):

$$\rho C_p\left(\frac{\partial T}{\partial t} + u_x\frac{\partial T}{\partial x} + u_y\frac{\partial T}{\partial y} + u_z\frac{\partial T}{\partial z}\right) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \mu\Phi_v + \dot{Q}$$

Table 2.6: Viscous Dissipation Function Φ_v for Incompressible Newtonian Fluids

Cartesian Coordinates (x, y, z):

$$\Phi_v = 2\left[\left(\frac{\partial u_x}{\partial x}\right)^2 + \left(\frac{\partial u_y}{\partial y}\right)^2 + \left(\frac{\partial u_z}{\partial z}\right)^2\right] + \left[\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right]^2 + \left[\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right]^2 + \left[\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right]^2$$

Simplifying the Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0$$

Steady State $\Rightarrow$ $\frac{\partial u_x}{\partial t}, \frac{\partial u_y}{\partial t}, \frac{\partial u_z}{\partial t}, \frac{\partial T}{\partial t}$, etc. $= 0$

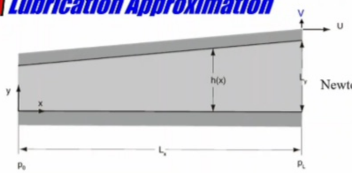
Constant Material Properties $\Rightarrow$ $\frac{\partial \rho}{\partial t} = 0 \quad \frac{\partial u_z}{\partial z} + \frac{\partial u_y}{\partial y} + \frac{\partial u_x}{\partial x} = \frac{\partial u_i}{\partial x_i} = 0 \quad \nabla \cdot u = 0$

Fully Developed Flow $\Rightarrow$

$\frac{\partial u_x}{\partial x} = 0$

Dimensions of Geometry $\Rightarrow$ *If $u_x = u_x(x, y)$, then* $\frac{\partial u_x}{\partial z} = 0$

Lubrication Approximation



Newtonian fluid is assumed

Continuity Equation (Conservation of Mass)

Cartesian Coordinates (x, y, z):

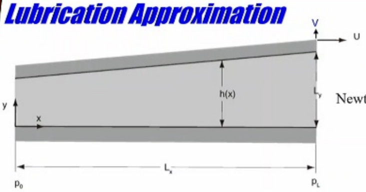
$$\cancel{\frac{\partial \rho}{\partial t}} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0$$

Steady state *Incompressible fluid*

$$\cancel{\frac{\partial u_z}{\partial z}} + \frac{\partial u_y}{\partial y} + \frac{\partial u_x}{\partial x} = 0 \Rightarrow \boxed{\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0}$$

$u_z = 0$

Lubrication Approximation



Newtonian fluid is assumed

Momentum Equation (Conservation of Momentum)

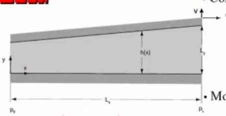
$$\rho\left(\cancel{\frac{\partial u_x}{\partial t}} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + \cancel{u_z\frac{\partial u_x}{\partial z}}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \cancel{\frac{\partial^2 u_x}{\partial z^2}}\right] + \cancel{\rho g_x}$$

Steady state | $u_z = 0$ | Independent of z | g_x negligible

$$\rho\left(\cancel{\frac{\partial u_y}{\partial t}} + u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + \cancel{u_z\frac{\partial u_y}{\partial z}}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \cancel{\frac{\partial^2 u_y}{\partial z^2}}\right] + \cancel{\rho g_y}$$

Steady state | $u_z = 0$ | Independent of z | g_y negligible

Lubrication Approx. – Order of Magnitude



- Continuity equation:

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0$$

$$\frac{U}{L_x} \approx \frac{V}{L_y} \Rightarrow \frac{U}{V} \approx \frac{L_x}{L_y}$$

$\because L_x \gg L_y$ ⇓ $\boxed{U \gg V}$

- Momentum equations:

$$\rho\left(\cancel{u_x\frac{\partial u_x}{\partial x}} + \cancel{u_y\frac{\partial u_x}{\partial y}}\right) = -\frac{\partial p}{\partial x} + \mu\left(\cancel{\frac{\partial^2 u_x}{\partial x^2}} + \frac{\partial^2 u_x}{\partial y^2}\right)$$

$$\rho U\frac{U}{L_x} \approx \rho V\frac{U}{L_y} \qquad -\frac{\Delta p}{L_x} \qquad \mu\frac{U}{L_x^2} \ll \mu\frac{U}{L_y^2}$$

(1) (2) (3) (4) (5)

if $Re\frac{L_y}{L_x} \ll 1$ then (5) ≫ (1)

$$\frac{\rho U\frac{V}{L_x}}{\mu\frac{V}{L_y^2}} = \frac{\rho U L_y}{\mu}\frac{L_y}{L_x}$$

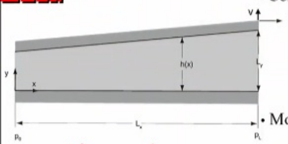
$$\rho\left(\cancel{u_x\frac{\partial u_y}{\partial x}} + \cancel{u_y\frac{\partial u_y}{\partial y}}\right) = -\frac{\partial p}{\partial y} + \mu\left(\cancel{\frac{\partial^2 u_y}{\partial x^2}} + \frac{\partial^2 u_y}{\partial y^2}\right)$$

$$\rho U\frac{V}{L_x} \approx \rho V\frac{V}{L_y} \qquad -\frac{\Delta p}{L_y} \qquad \mu\frac{V}{L_x^2} \ll \mu\frac{V}{L_y^2}$$

(1)' (2)' (3)' (4)' (5)'

if $Re\frac{L_y}{L_x} \ll 1$ then (5)' ≫ (1)'

Lubrication Approx. – Order of Magnitude



- Continuity equation:

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0$$

$L_x \gg L_y \Rightarrow \boxed{U \gg V}$

- Momentum equations:

$$\rho\left(\cancel{u_x\frac{\partial u_x}{\partial x}} + \cancel{u_y\frac{\partial u_x}{\partial y}}\right) = -\frac{\partial p}{\partial x} + \mu\left(\cancel{\frac{\partial^2 u_x}{\partial x^2}} + \frac{\partial^2 u_x}{\partial y^2}\right)$$

(1) (2) (3) (4) (5)

$$\frac{(5)}{(5)'} = \frac{\mu\frac{U}{L_y^2}}{\mu\frac{V}{L_y^2}} = \frac{U}{V}$$

$\because U \gg V \Rightarrow \boxed{(5) \gg (5)'}$

(3) ≈ (5)

$$\rho\left(\cancel{u_x\frac{\partial u_y}{\partial x}} + \cancel{u_y\frac{\partial u_y}{\partial y}}\right) = -\frac{\partial p}{\partial y} + \mu\left(\cancel{\frac{\partial^2 u_y}{\partial x^2}} + \frac{\partial^2 u_y}{\partial y^2}\right)$$

(1)' (2)' (3)' (4)' (5)'

(3)' ≈ (5)'

$\Rightarrow$ (3)' $\frac{\Delta p}{L_y} \ll \frac{\Delta p}{L_x}$ (3)

$$\Rightarrow -\frac{\partial p}{\partial y} \ll -\frac{\partial p}{\partial x}$$

Pressure Driven Flow of a Newtonian Fluid Through a Sli

X come out of the plane

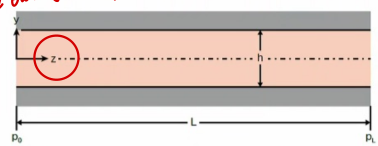


Figure 9.18 Schematic diagram of pressure flow through a slit

Assume incompressible Newtonian fluid; unidirectional and steady fully developed flow (a flow where the entrance effects are ignored).

Pressure Driven Flow Through a Slit – Cont'd

- Continuity equation: $u_y = 0$ $u_x = 0$

$$\frac{\partial u_z}{\partial z} + \cancel{\frac{\partial u_y}{\partial y}} + \cancel{\frac{\partial u_x}{\partial x}} = \frac{\partial u_i}{\partial x_i} = 0 \qquad \boxed{\frac{du_z}{dz} = 0}$$

- Momentum equations:

Table 2.4: Navier-Stokes Equations

Cartesian Coordinates (x, y, z):

$u_x = 0$

$$\rho\left(\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right] + \rho g_x \Rightarrow -\frac{\partial p}{\partial x} = 0$$

$u_y = 0$

$$\rho\left(\frac{\partial u_y}{\partial t} + u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + u_z\frac{\partial u_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right] + \rho g_y \Rightarrow -\frac{\partial p}{\partial y} = 0$$

$u_x = 0$ $u_y = 0$ u_z independent of x

$$\rho\left(\cancel{\frac{\partial u_z}{\partial t}} + \cancel{u_x\frac{\partial u_z}{\partial x}} + \cancel{u_y\frac{\partial u_z}{\partial y}} + u_z\cancel{\frac{\partial u_z}{\partial z}}\right) = -\frac{\partial p}{\partial z} + \mu\left[\cancel{\frac{\partial^2 u_z}{\partial x^2}} + \frac{\partial^2 u_z}{\partial y^2} + \cancel{\frac{\partial^2 u_z}{\partial z^2}}\right] + \cancel{\rho g_z}$$

Steady state | u_z independent of x | Fully developed flow, u_z independent of z | Fully developed flow, u_z independent of z | Body force negligible

$$-\frac{\partial p}{\partial z} + \mu\frac{\partial^2 u_z}{\partial y^2} = 0$$

Pressure Driven Flow Through a Slit – Cont'd

- Continuity equation:

$$\frac{du_z}{dz} = 0$$

- Momentum equations:

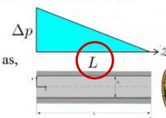
$$-\frac{\partial p}{\partial x} = 0 \qquad -\frac{\partial p}{\partial y} = 0$$

F(z) G(y) ⇒F(z) = G(y) = a constant

$$-\frac{\partial p}{\partial z} + \mu\frac{\partial^2 u_z}{\partial y^2} = 0$$

The total pressure is a function of z alone. Additionally, since u does not vary with z, the pressure gradient, $\partial p/\partial z$, must be a constant. Therefore,

$$\frac{dp}{dz} = -\frac{\Delta p}{L}$$



The momentum equation can now be written as,

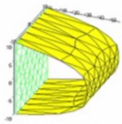
$$-\frac{1}{\mu}\frac{\Delta p}{L} = \frac{\partial^2 u_z}{\partial y^2}$$

$$\frac{\partial^2 u_z}{\partial y^2} = -\frac{\Delta p}{\mu L}$$

$$\frac{d^2 u_z}{dy^2} = -\frac{\Delta p}{\mu L}$$

$$\int \frac{d^2 u_z}{dy^2}\, dy = \int -\frac{\Delta p}{\mu L}\, dy$$

$$\frac{du_z}{dy} = -\frac{\Delta p}{\mu L} y + C_1$$



$Since\ \frac{du_z}{dy} = 0\ at\ y = 0\ (symm.\ condition),\ hence\ C_1 = 0$

$$\int \frac{du_z}{dy}\, dy = \int -\frac{\Delta p}{\mu L} y\, dy$$

$$u_z(y) = -\frac{\Delta p}{2\mu L} y^2 + C_2$$

Pressure Driven Flow Through a Slit – Cont'd

$Since\ u_z(y) = 0\ at\ y = \frac{h}{2}$ (no slip boundary condition), hence $C_2 = \cdots$

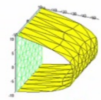
Boundary conditions, two no-slip conditions given by, $u_z(y) = -\frac{\Delta p}{2\mu L} y^2 + C_2$

$$u_z(\pm h/2) = 0$$

Integrating twice and evaluating the two integration constants with the boundary conditions gives,

$$u_z(y) = -\frac{h^2}{8\mu}\frac{dp}{dz}\left[1 - \left(\frac{2y}{h}\right)^2\right]$$

$$\boxed{= \frac{h^2}{8\mu}\frac{\Delta p}{L}\left[1 - \left(\frac{2y}{h}\right)^2\right]}$$



Note that the same profile will result if one of the non-slip boundary conditions is replaced by a symmetry condition at $y = 0$, namely $du_z/dy = 0$.

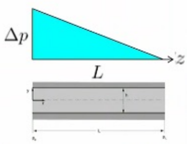
Pressure Driven Flow Through a Slit – Cont'd

The mean velocity in the channel is obtained integrating the above equation,

$$\bar{u}_z = \frac{2}{h}\int_0^{h/2} u_z(y)\, dy = -\frac{h^2}{12\mu}\frac{dp}{dz} \qquad \frac{dp}{dz} = -\frac{\Delta p}{L}$$

and the volumetric flow rate,

$$Q = hW\bar{u}_z = \frac{Wh^3}{12\mu}\frac{\Delta p}{L}$$



where W is the width of the channel.

Can you see the relationship among the required pressure, Δp, flow rate, Q, material viscosity, μ, and part dimensions, W, h, L?