Name: ____________________________

Problem 1

A beam with a T-shape cross-section is loaded by a distributed load as shown below.

i) Draw the shear (V) diagram. Label the axes with units and mark all local maxima and minima. For straight lines, label the slopes. For curves, indicate if they are quadratic, cubic, etc. Please don't draw the moment (M) diagram.

~~ii) Determine the location of the neutral axis (centroid of the cross-section).~~ I'll give the location of centroid.

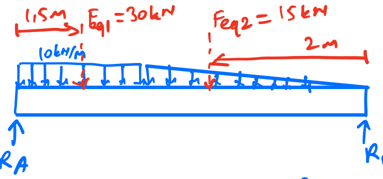
iii) Compute the area moment of inertia I_z of the cross-section.

iv) Use part i) to determine the critical section where the internal shear force is maximum along the beam and determine the maximum shear stress at that section.

(i)

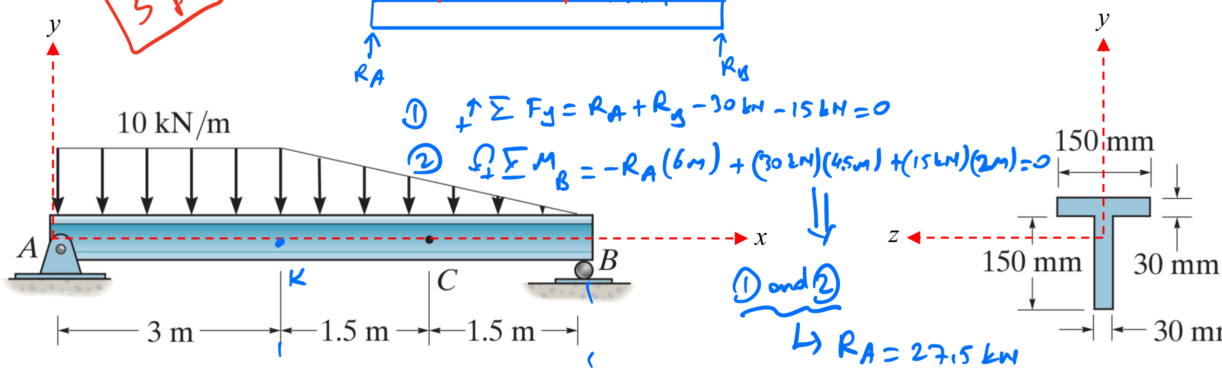
5 pts

FBD:



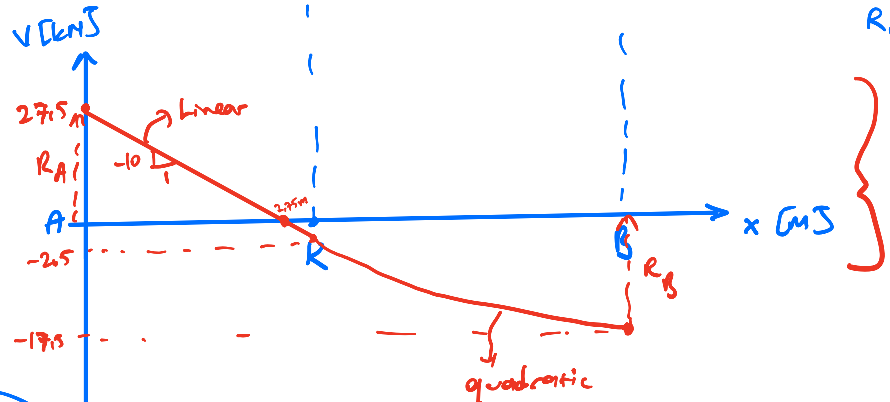
① $+\uparrow \sum F_y = R_A + R_B - 30\text{ kN} - 15\text{ kN} = 0$

② $\curvearrowleft + \sum M_B = -R_A(6\text{ m}) + (30\text{ kN})(4.5\text{ m}) + (15\text{ kN})(2\text{ m}) = 0$



① and ② $\Rightarrow$ $R_A = 27.5$ kN

$R_B = 17.5$ kN



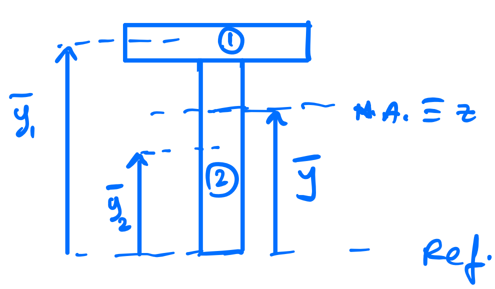
15 pts

10 pts: V-values consistent with FBD and $\frac{dV}{dx} = -q$

Labels (units, slopes, linear, quadratic) ⇒ 5 pts

ii and iii

	$\bar{y}_i$ (mm)	A_i (mm²)	$y_i A_i$ (mm³)	I_i^o (mm⁴)	d_i (mm)
①	165	30×150	165 × 30×150	$\frac{1}{12}(150)(30^3)$	45
②	75	150×30	75 × 150×30	$\frac{1}{12}(30)(150^3)$	45

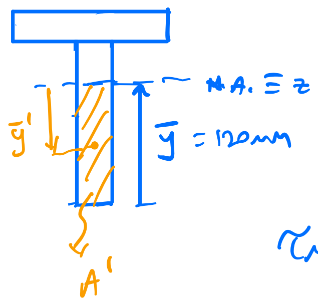


$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{(165+75)\times \cancel{150\times 30}}{2\times \cancel{150\times 30}} = \boxed{120\text{ mm}}$$

5 pts

$$I_z = \sum I_i^o + A_i d_i^2 = \frac{1}{12}(150)(30)\left[30^2 + 150^2\right] + 150\times 30\times 45^2 \times 2 = \boxed{27\times 10^6\text{ mm}^4}$$

(iv) Critical section around pt. A: $V_A = 27.5\,kN$



$$Q_{max} = A'\bar{y}' = (120\,mm)(30\,mm)(60\,mm) = 2.16 \times 10^5\ mm^3$$

$$t = 30\,mm$$

$$\tau_{max} = \underbrace{\frac{V_A\, Q_{max}}{I_z\, t}}_{\boxed{10\ pts}} = \frac{(27.5\,kN)(2.16 \times 10^5\,mm^3)}{(27 \times 10^6\,mm^4)(30\,mm)} = \boxed{7.33\ MPa}$$

→ Q_{max} → $\boxed{5\ pts}$

→ t → $\boxed{2\ pts}$

→ V_A and I_z consistent with above

⇓

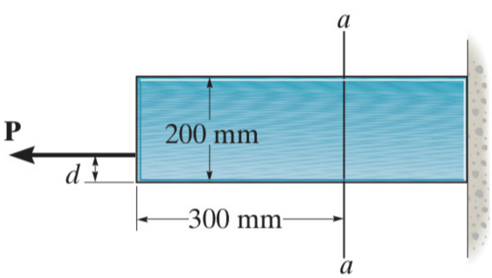
$\boxed{3\ pts}$

Name: ____________________________

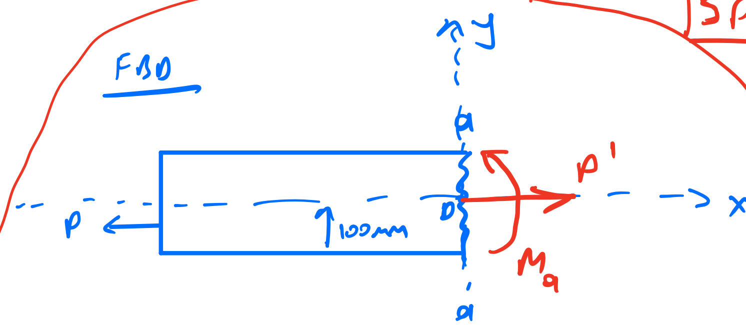
Problem 2

A plate has a rectangular cross-section of 20x200 mm^2, in thickness and height, respectively. A point force P is applied on the plate along the centerline in the thickness direction but off centered in the height direction as shown below. What is the smallest distance d to the plate's bottom edge at which the force can be applied so that no compressive stress occurs at section a-a of the plate?

$\boxed{5pts}$



FBD



$A = 20 \times 200 \; mm^2$

$I_z = \frac{1}{12}(20\,mm)(200\,mm)^3 = \frac{4}{3} \times 10^7 \, mm^4$

$c = \frac{h}{2} = 100 \; mm$

$\xrightarrow{+} \Sigma F_x = -P + P' = 0 \Rightarrow P' = P$

$\circlearrowleft + \; \Sigma M_O = M_a - P(100\,mm - d) = 0$

$\Rightarrow M_a = P(100\,mm - d)$

(1) $\sigma_x^a = \frac{P}{A} - \frac{M_a c}{I_z} = \frac{P}{4 \times 10^3 \, mm^2} - \frac{P(100\,mm - d)\,100\,mm}{\frac{4}{3} \times 10^7 \, mm^4}$

$\boxed{10 pts}$

$\sigma_x^a = 0$ will give us the smallest d:

(1) $\Rightarrow \frac{\frac{4}{3} \times 10^7 \, mm^4}{4 \times 10^5 \, mm^3} = 100\,mm - d \Rightarrow d = 100 - \frac{100}{3} = \boxed{66.7 \; mm}$

$\boxed{5pts}$

Name: ____________________________

Problem 3

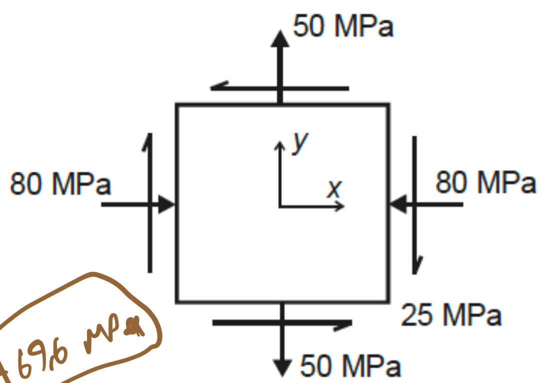
A point in a deformable material experiences the state of plane stress as shown below.

i) Draw the Mohr's circle. → $\boxed{15\text{ pts}}$

ii) Determine the principal stresses and planes. → $\boxed{8\text{ pts}}$

iii) Determine the maximum in-plane shear stress and planes. → $\boxed{4\text{ pts}}$

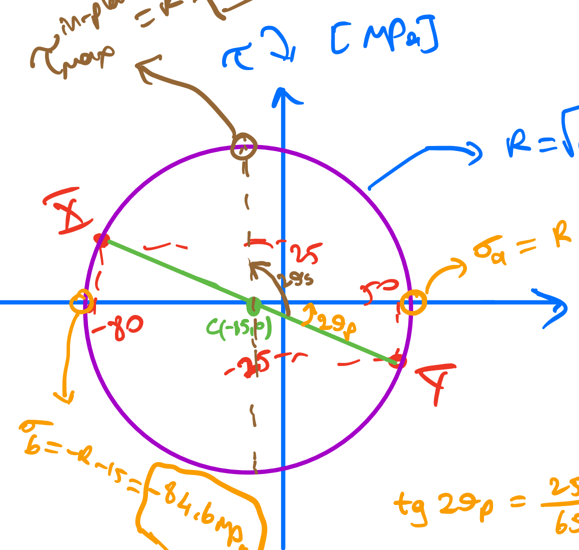
iv) Determine the absolute maximum shear stress. → $\boxed{3\text{ pts}}$



$X\ (\sigma_x, -\tau_{xy}) \equiv (-80, 25)$

$Y\ (\sigma_y, \tau_{xy}) \equiv (50, -25)$

$\tau_{max}^{in\text{-}plane} = R = \boxed{69.6\text{ MPa}}$



$R = \sqrt{65^2 + 25^2} = 69.6$

$\sigma_a = R - 15 = \boxed{54.6\text{ MPa}}$

$\sigma_b = -R - 15 = \boxed{-84.6\text{ MPa}}$

$tg\ 2\theta_p = \frac{25}{65} \Rightarrow 2\theta_p = 21°$

$\boxed{\theta_p = 10.5°}$

$\boxed{\theta_p = 10.5° + 90° = 100.5°}$

$2\theta_s = 90° + 2\theta_p = 90° + 21° = 111°$

$\boxed{\theta_s = 55.5°}$

$\boxed{\theta_s = 55.5° + 90° = 145.5°}$

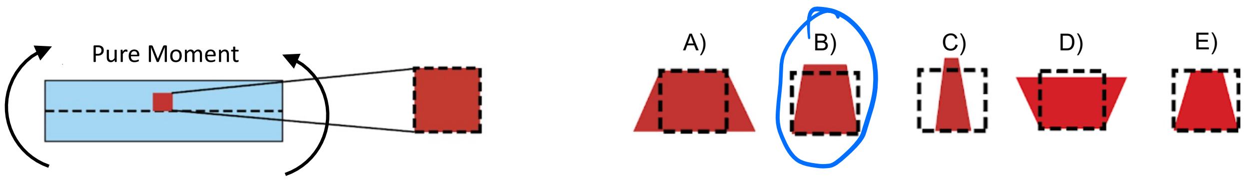
$\tau_{max}^{abs} = \tau_{max}^{in\text{-}plane} = \boxed{69.6\text{ MPa}}$

Name: ____________________________

Short Problems $\boxed{\text{5pts each}}$

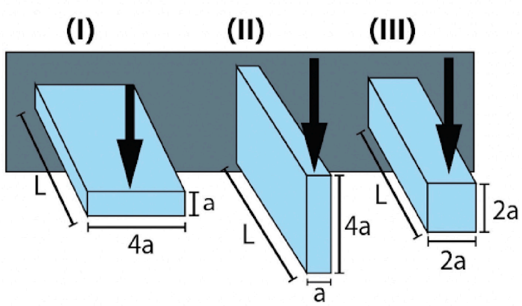
Problem 4

A homogenous, isotropic, linear elastic bar with a positive Poisson's ratio is horizontally aligned in the original unloaded configuration and then, pure moment is applied at the ends as shown in the figure below. Horizontal dashed line passes through the centroid of the bar's cross-section. A square element occupying ¼ of the bar's height is drawn in the unloaded configuration. Note that the bottom side of the square element is on the centroid line. What is the shape of that square element after pure moment is applied? (at each choice dashed squares denote the undeformed square element and are given only for reference; **circle one answer**).



Problem 5

Three cantilevered bars shown below (I, II and III) are made of the same material, have the same length and cross-sectional areas but different cross-sections. The same vertical force is applied at the free ends of those bars. Let $\Delta = \sigma_{max}$ denote the maximum normal stress due to bending at the cantilevered end of the bars. Circle the correct ranking of those Δ values.



A) $\Delta_I = \Delta_{II} = \Delta_{III}$

B) $\Delta_I > \Delta_{II} > \Delta_{III}$

C) $\Delta_I < \Delta_{III} < \Delta_{II}$

D) $\Delta_I > \Delta_{III} > \Delta_{II}$

E) $\Delta_I = \Delta_{II} > \Delta_{III}$

$$\sigma_M = \frac{M}{S}$$

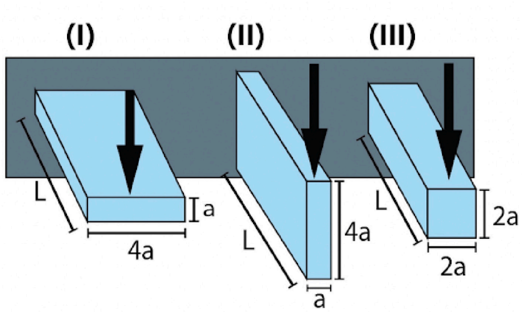
$$S = \frac{1}{6} A h$$

$$\Rightarrow S_I < S_{III} < S_{II}$$

$$\Rightarrow \sigma_{MI} > \sigma_{MIII} > \sigma_{MII}$$

Problem 6

Three cantilevered bars shown below (I, II and III) are made of the same material, have the same length and cross-sectional areas but different cross-sections. The same vertical force is applied at the free ends of those bars. Let $\Delta = \tau_{max}$ denote the maximum shear stress due to internal shear forces at the cantilevered end of the bars. Circle the correct ranking of those Δ values.



A) $\Delta_I = \Delta_{II} = \Delta_{III}$

B) $\Delta_I > \Delta_{II} > \Delta_{III}$

C) $\Delta_I < \Delta_{III} < \Delta_{II}$

D) $\Delta_I > \Delta_{III} > \Delta_{II}$

E) $\Delta_I = \Delta_{II} > \Delta_{III}$

$$\tau_{max} = \frac{3V}{2A}$$