Homework # 1: Report

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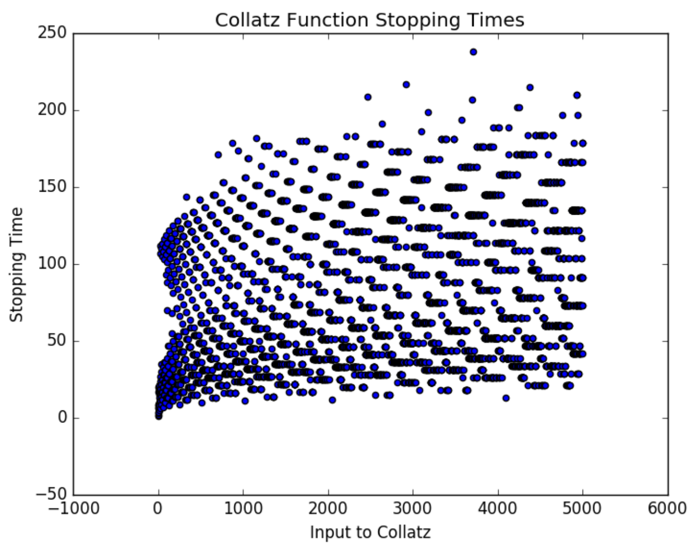
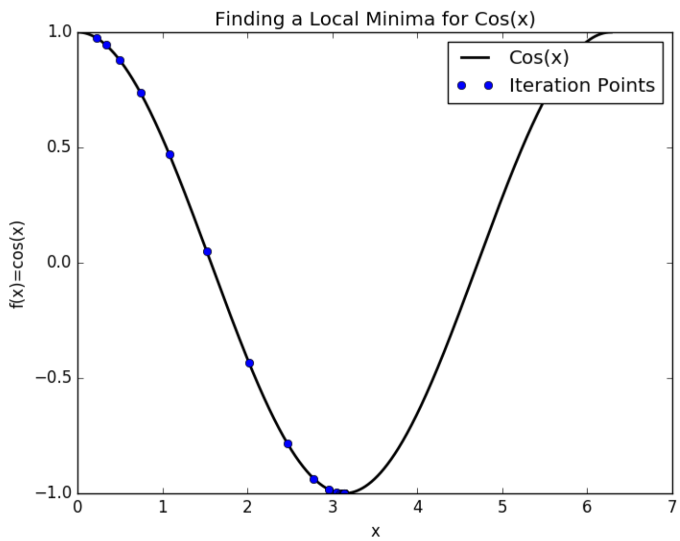
1.



It does seem likely that the Collatz conjecture is true although this specific plot doesn't do much to convince me. I have seen more convincing plots (histograms) that show that as $n \to \infty$ the stopping times essentially go to zero. Although there do appear to be less larger stopping times than smaller, at least to my eye this plot doesn't do much to convince me that the larger stopping times will continue to become less and less.

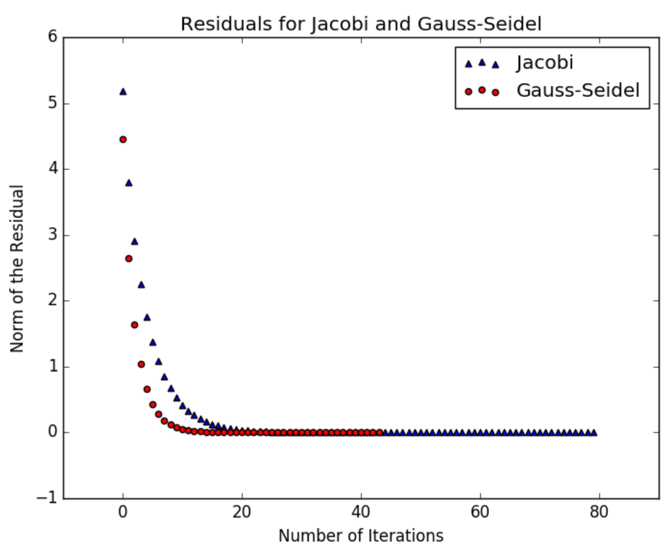
This type of conjecture is one that numerical evidence will likely continue to fail to produce a definitive answer, since (as it stands now) a computer cannot simulate all the way to infinity and it is possible that there will always be a large stopping time as n gets larger and larger. Though the amount of large stopping times might become less and less there is no guarantee that they will stop happening. Thus I choose to stay agnostic on the likelihood of the conjecture solely based on numerical evidence.

2.



One problem that could arise from allowing $\sigma > 1$ is that the gradient-descent algorithm can end up missing a local minima. It is possible that the algorithm will run into an “over-shooting” loop and get trapped bouncing back and forth about a minima.

3.



As expected, we see that the Gauss-Seidel algorithm converges quicker than the Jacobi algorithm. When $\epsilon =$ 1e-20, both algorithms take about double the time, i.e. the number of iterations assymptote to a residual that is essentially zero and sit there for a significant amount of iterations. Of course the real issue is that as epsilon becomes small enough the tolerance is beyond machine precision, and although the algorithm “converges” it is a toss up on the validity of being at the proscribed tolerance.