

Table 1: The experimental numerical results of PFD-SQOM, PF-SQO and PRS-SQO-DSM in solving model (5.1), respectively.

q	PFD-SQOM ₀					PFD-SQOM _{0,5}					PFD-SQOM ₁				
	C_t	$N_{vit}/N_{it} = RA$	F_*	$RE_F(\%)$	$RE_C(\%)$	C_t	$N_{vit}/N_{it} = RA$	F_*	$RE_F(\%)$	$RE_C(\%)$	C_t	$N_{vit}/N_{it} = RA$	F_*	$RE_F(\%)$	$RE_C(\%)$
5	0.14	17/19=89%	664.8205	-0.00	-94.89	0.09	11/13=85%	664.8205	-0.00	-0.00	0.03	1/3=33%	664.8205	-0.00	-0.00
50	0.86	112/123=91%	-100653.91	-0.08	-90.72	0.64	83/94=88%	-100661.75	-0.07	-0.07	0.51	56/69=81%	-100681.85	-0.05	-0.05
100	1.33	121/135=90%	-84990.58	-0.22	-89.72	1.03	92/104=88%	-849518.79	-0.27	-0.27	0.80	61/74=82%	-849864.13	-0.23	-0.23
200	2.38	145/159=91%	-8584937.26	-0.19	-91.58	1.85	107/121=88%	-8584971.78	-0.19	-0.19	1.32	72/83=87%	-8573334.45	-0.32	-0.32
300	4.06	166/184=90%	-36275417.44	-0.06	-90.98	2.98	121/136=89%	-36257544.63	-0.11	-0.11	2.19	81/96=84%	-36259237.63	-0.10	-0.10
400	5.37	187/210=89%	-104013285.34	-0.01	-90.35	4.00	132/150=88%	-103987535.86	-0.04	-0.04	3.26	93/115=81%	-104011123.57	-0.02	-0.02
500	5.80	190/197=96%	-237336638.13	-0.66	-91.41	4.49	141/148=95%	-237392156.15	-0.64	-0.64	3.24	95/102=93%	-237454522.21	-0.61	-0.61
600	7.47	205/216=95%	-474411940.13	-0.15	-90.59	6.05	155/167=93%	-474619916.50	-0.11	-0.11	3.76	98/103=95%	-469877268.57	-1.10	-1.10
700	8.50	216/226=96%	-852298479.77	-0.18	-91.42	6.43	159/167=95%	-850842509.56	-0.35	-0.35	4.61	108/116=93%	-850938370.34	-0.34	-0.34
800	9.46	213/222=96%	-1420203875.26	-0.21	-91.24	8.31	170/179=95%	-1420308577.96	-0.20	-0.20	5.44	111/120=93%	-1420417660.43	-0.20	-0.20
900	11.83	241/250=96%	-2234220685.04	-0.19	-90.66	9.03	180/189=95%	-2234311731.30	-0.19	-0.19	6.61	121/132=92%	-2236570863.26	-0.09	-0.09
1000	13.18	249/258=97%	-3356197867.71	-0.18	-90.62	10.66	190/201=95%	-3359371301.83	-0.08	-0.08	7.15	125/133=94%	-3353871312.07	-0.25	-0.25
Total	70.38	2062/2199=94%				54.56	1541/1669=92%				38.92	1022/1146=89%			

Notions: C_t — CPU time (second); N_{it} — the number of iterations; N_{vit} — the number of iterations of valid QO splitting; F_* — the generated optimal solution;

$$RE_F = \frac{F_* \text{ generated by PFD-SQOM} - F_* \text{ generated by PF-SQOM}}{F_* \text{ generated by PF-SQOM}} \times 100\%;$$

$$RE_C = \frac{C_t \text{ consumed by PFD-SQOM}_0 - C_t \text{ consumed by PF-SQOM}}{C_t \text{ consumed by PF-SQOM}} \times 100\%.$$

Table 2: The experimental numerical results of PFD-SQOM, PF-SQOM and PRS-SQO-DSM in solving model (5.1), respectively (continued to Table 1).

q	PF-SQOM		PRS-SQO-DSM [32]		
	C_t	F_*	C_t	φ_{eq}	F_*
5	2.74	664.8205	33.53	0.05	664.4469
50	9.26	-100731.9991	5.21	0.10	-100805.95
100	12.97	-851848.4624	7.16	0.41	-852125.53
200	28.26	-8601024.8229	7.12	1.66	-8607951.18
300	45.03	-36296013.8192	8.64	3.79	-36349214.05
400	55.64	-104028525.8658	10.74	6.78	-104253825.26
500	67.50	-238919860.1155	8.59	10.54	-239606919.50
600	79.43	-475121472.8440	15.35	15.20	-476837139.87
700	98.96	-853814075.5936	12.30	20.62	-857511463.20
800	108.05	-1423211419.9339	12.87	26.85	-1430403970.65
900	126.73	-2238551531.1670	26.94	33.99	-2251518625.06
1000	140.47	-3362108667.9320	31.00	41.84	-3384003612.77
Total	775.04		179.45	161.83	

Notions: $\varphi_{eq} := \|A\tilde{x}_* + B\tilde{y}_* - b\|_\infty$.

Table 3: The structures of 20 mid-scale instances obtained by copying the 5-unit system

No.	Unit					Total Units N	No.	Unit					Total Units N
	1	2	3	4	5			1	2	3	4	5	
1	1	2	3	2	2	10	11	20	24	27	20	19	110
2	3	3	3	3	3	15	12	22	26	29	22	21	120
3	4	4	4	4	4	20	13	26	30	30	22	22	130
4	5	6	7	7	5	30	14	30	33	32	25	30	150
5	5	10	10	5	10	40	15	34	37	36	29	34	170
6	8	11	12	9	10	50	16	36	39	38	30	37	180
7	10	14	16	15	15	70	17	40	44	41	34	41	200
8	13	18	18	13	18	80	18	44	48	45	38	45	220
9	12	20	25	20	13	90	19	48	52	48	40	52	240
10	18	22	25	18	17	100	20	50	54	50	42	54	250

Table 4: Numerical results of four methods in solving 20 ED instances without valve-point effect

No.	OPTI			Alg. 2.1 (PFD-SQOM)			MS-SQOM [31]			PRS-SQO-DSM [32]		
	C _t	F _*		C _t	F _*	RE(%)	C _t	F _*	RE(%)	C _t	F _*	RE(%)
1	1.53	1243485.2		0.72	1243169.69	-0.0254	2.48	1243611.17	0.0101	1.45	1243402.72	-0.0066
2	2.68	1833617.3		0.96	1833206.75	-0.0224	2.66	1833808.32	0.0104	1.55	1833482.67	-0.0073
3	5.83	2444823.1		1.17	2444306.86	-0.0211	2.94	2445089.70	0.0109	1.62	2445089.42	0.0109
4	28.84	3650316.9		1.67	3649583.38	-0.0201	3.94	3650940.12	0.0171	1.73	3650600.21	0.0078
5	29.57	5083735.1		2.08	5081810.86	-0.0379	4.69	5084391.63	0.0129	2.49	5084610.9	0.0172
6	43.50	6192152.2		2.95	6190828.63	-0.0214	6.86	6193185.51	0.0167	3.79	6193348.75	0.0193
7	464.27	8636967.3		5.34	8635338.33	-0.0189	17.11	8638532.86	0.0181	5.30	8637012.3	0.0005
8	654.49	9973328.8		6.42	9970531.52	-0.0280	20.99	9974943.64	0.0162	4.95	9973855.19	0.0053
9	1004.45	11035233.4		7.61	11032974.66	-0.0205	25.50	11037322.45	0.0189	8.45	11035931.18	0.0063
10	1005.87	12291433.0		7.91	12288864.36	-0.0209	30.03	12293741.41	0.0188	10.41	12292648.26	0.0099
11	1004.15	13513839.1		9.07	13511013.22	-0.0209	36.58	13516385.17	0.0188	11.39	13515264.88	0.0106
12	1009.19	14736246.1		10.22	14733163.45	-0.0209	43.06	14739033.12	0.0189	13.01	14737910.31	0.0113
13	1009.17	15975567.9		11.87	15972451.32	-0.0195	50.93	15978719.24	0.0197	14.99	15977941.18	0.0149
14	1009.88	18492204.7		14.60	18488435.24	-0.0204	65.53	18495641.46	0.0186	18.55	18494616.78	0.0130
15	1015.06	20937025.3		17.71	20932730.70	-0.0205	90.34	20940974.18	0.0189	22.48	20939847.58	0.0135
16	1015.32	22197414.8		19.22	22192857.81	-0.0205	99.25	22201586.65	0.0188	25.53	22200467.4	0.0138
17	1015.29	24659160.1		22.71	24654095.49	-0.0205	126.55	24663850.50	0.0190	30.87	24662733.12	0.0145
18	1014.70	27103981.1		26.37	27098395.25	-0.0206	165.28	27109222.63	0.0193	36.56	27108310.74	0.0160
19	1015.93	29641688.2		32.59	29635556.80	-0.0207	202.02	29647408.79	0.0193	41.66	29646844.00	0.0174
20	1021.03	30864098.3		35.82	30857707.43	-0.0207	223.97	30870121.91	0.0195	45.45	30870148.92	0.0196
Total	13370.75			237.01			1220.7			302.68		

Notions:

$$RE = \frac{F_* \text{ generated by Alg. 2.1} - F_* \text{ generated by OPTI}}{F_* \text{ generated by OPTI}} \times 100\%.$$

Table 5: Numerical results of four methods in solving 6 non-convex ED instances with valve-point effect

No.	OPTI			Alg. 2.1 (PFD-SQOM)			MS-SQO [31]			PRS-SQO-DSM [32]		
	C _t	F _*		C _t	F(P [*])	RE(%)	C _t	F _*	RE(%)	C _t	F _*	RE(%)
1	3.99	1243486.63		0.69	1242430.80	-0.0849	3.15	1243613.78	0.0102	1.60	1243404.31	-0.0066
6	669.00	6192158.64		3.33	6187183.24	-0.0804	6.96	6193193.34	0.0167	3.55	6193353.42	0.0193
10	1009.04	12291445.47		9.67	12281422.07	-0.0815	22.56	12293758.39	0.0188	11.91	12292659.53	0.0099
14	1024.90	18492222.66		18.77	18477663.86	-0.0787	45.11	18497628.33	0.0292	18.48	18494618.1	0.0130
17	1114.85	24659184.09		28.24	24638774.51	-0.0828	101.11	24667160.66	0.0323	31.08	24662682.15	0.0142
20	1133.98	30864170.56		44.16	30839509.37	-0.0799	181.82	30874962.95	0.0350	51.39	30870016.43	0.0189
Total	4955.8			104.9		-0.4882	360.7		0.1422	118.01		0.0687