Part III – Knowledge and Reasoning

- **6 Agents that Reason Logically**
  - Knowledge-based Agents. – Representations.
  - Propositional Logic. – The Wumpus World.
- **7 First-Order Logic**
  - Syntax and Semantics. – Using First-Order Logic.
  - Logical Agents. – Representing Changes.
  - Deducing Properties of the World.
  - Goal-based Agents.
- **8 Building a Knowledge Base**
  - Knowledge Engineering. – General Ontology.

Validity and Inference

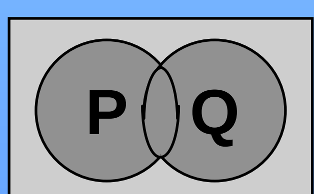
- **Testing for validity**
  - Using truth-tables, checking all possible configurations
    - e.g.: $((P \vee Q) \wedge \neg Q) \Rightarrow P$

P	**Q**	**$P \vee Q$**	**$\neg Q$**	**$(P \vee Q) \wedge \neg Q$**	**$((P \vee Q) \wedge \neg Q) \Rightarrow P$**
False	False	False	True	False	True
False	True	True	False	False	True
True	False	True	True	True	True
True	True	True	False	False	True

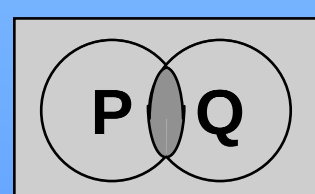
- **A method for sound inference**
  - Build and check a truth-table for *Premises* $\Rightarrow$ *Conclusion*

Models

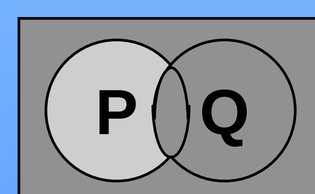
- **Definition**
  - *A world in which a sentence is true under a particular interpretation*
    - e.g. the Wumpus world example is a model for S(1,2) meaning “there is a stench in [1,2]”
  - Entailment: *KB |= α if the models of KB are all models of α*
- **Models as mathematical objects**
  - A mapping from propositional symbols to truth-values
- **Models as sets**



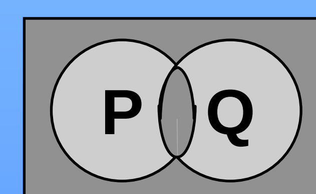
$P \vee Q$



$P \wedge Q$



$P \Rightarrow Q$



$P \Leftrightarrow Q$

Rules of Inference

- **Sound inference rules**
  - Pattern of inference, that occur again and again $\dfrac{\alpha}{\beta}$
  - Soundness proven once and for all (truth-table)
- **Classic rules of inference**
  - Implication-Elimination, or *Modus Ponens*
    - $$\frac{\alpha \Rightarrow \beta,\ \alpha}{\beta}$$

      e.g. Cloudy Λ Humid $\Rightarrow$ Rain
      Cloudy Λ Humid |= Rain

Rules of Inference

- **Classic rules of inference**
  - And-Elimination
    - $$\frac{\alpha_1 \land \alpha_2 \land \ldots \land \alpha_n}{\alpha_i}$$

    e.g. Cloudy $\land$ Humid |= Cloudy
    Cloudy $\Rightarrow$ NoSunTan
  - And-Introduction
    - $$\frac{\alpha_1, \alpha_2, \ldots, \alpha_n}{\alpha_1 \land \alpha_2 \land \ldots \land \alpha_n}$$

    e.g. Cloudy, Humid
    Cloudy $\land$ Humid $\Rightarrow$ Rain
  - Or-Introduction
    - $$\frac{\alpha_i}{\alpha_1 \lor \alpha_2 \lor \ldots \lor \alpha_n}$$
  - Double-Negation-Elimination
    - $$\frac{\neg\neg\alpha}{\alpha}$$

Rules of Inference

- **The resolution rule of inference**
  - Unit Resolution
    - $$\frac{\alpha \vee \beta, \ \neg\beta}{\alpha}$$
  - Resolution
    - same as MP: $\dfrac{P \Rightarrow Q, \ P}{Q}$ i.e. $\dfrac{\neg\beta \Rightarrow \alpha, \ \neg\beta}{\alpha}$
    - $$\frac{\alpha \vee \beta, \ \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

e.g. monday $\vee$ tuesday, $\neg$ monday |= tuesday

Truth-table for the resolution

α	β	γ	$\alpha \vee \beta$	$\neg\beta \vee \gamma$	$\alpha \vee \gamma$
False	False	False	False	True	False
False	False	True	False	True	True
False	True	False	True	False	False
False	True	True	True	True	True
True	False	False	True	True	True
True	False	True	True	True	True
True	True	False	True	False	True
True	True	True	True	True	True

Equivalence Rules

- **Inference as implication**
  - Equivalent notations, e.g. MP:
    - $\dfrac{\alpha \Rightarrow \beta,\ \alpha}{\beta}$ $\qquad \alpha \Rightarrow \beta,\ \beta \ \vdash\ \beta \qquad ((\alpha \Rightarrow \beta) \wedge \alpha) \Rightarrow \beta$

      $\alpha \Rightarrow \beta,\ \beta \ \models\ \beta$

- **Equivalence rules**
  - Associativity: $\alpha \wedge (\beta \wedge \gamma) \Leftrightarrow (\alpha \wedge \beta) \wedge \gamma$

    $\alpha \vee (\beta \vee \gamma) \Leftrightarrow (\alpha \vee \beta) \vee \gamma$
  - Distributivity: $\alpha \wedge (\beta \vee \gamma) \Leftrightarrow (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$

    $\alpha \vee (\beta \wedge \gamma) \Leftrightarrow (\alpha \vee \beta) \wedge (\alpha \vee \gamma)$
  - De Morgan's Law: $\neg(\alpha \vee \beta) \Leftrightarrow \neg\alpha \wedge \neg\beta$

    $\neg(\alpha \wedge \beta) \Leftrightarrow \neg\alpha \vee \neg\beta$

Complexity of Inference

- **Proof by truth-table**
  - Complete
    - The truth-table can always be written.
  - Exponential time complexity
    - A proof involving ***N*** proposition symbols requires $\mathbf{2^N}$ rows.
    - In practice, a proof may refer only to a small subset of the KB.
- **Monotonicity**
  - Knowledge always increases

$$\textbf{if } KB_1 \models \alpha \textbf{ then } (KB_1 \cup KB_2) \models \alpha$$

    - Allows for local rules,
      e.g. Modus Ponens $\alpha \Rightarrow \beta, \alpha \vdash \beta$
    - Propositional and first-order logic are monotonic.

Horn Sentences

- **A particular sub-class of sentences**
  - Implication: $\mathbf{P_1 \wedge P_2 \wedge \ldots \wedge P_N \Rightarrow Q}$ where $P_1, \ldots P_N$, Q are non-negated atoms.
  - Particular cases:
    - $Q \Leftrightarrow (\text{True} \Rightarrow Q)$
    - $(P_1 \vee P_2 \vee \ldots \vee P_N \Rightarrow Q) \Leftrightarrow (P_1 \Rightarrow Q) \wedge \ldots \wedge (P_N \Rightarrow Q)$
    - $(P \Rightarrow Q_1 \wedge \ldots \wedge Q_N) \Leftrightarrow (P \Rightarrow Q_1) \wedge \ldots \wedge (P \Rightarrow Q_N)$
    - $(P \Rightarrow Q_1 \vee \ldots \vee Q_N)$ cannot be represented
- **Prolog, a logic programming language**
  - Horn sentences + Modus-Ponens **Q :– $\mathbf{P_1, P_2, \ldots, P_N}$**
  - Inference of <u>polynomial</u> time complexity

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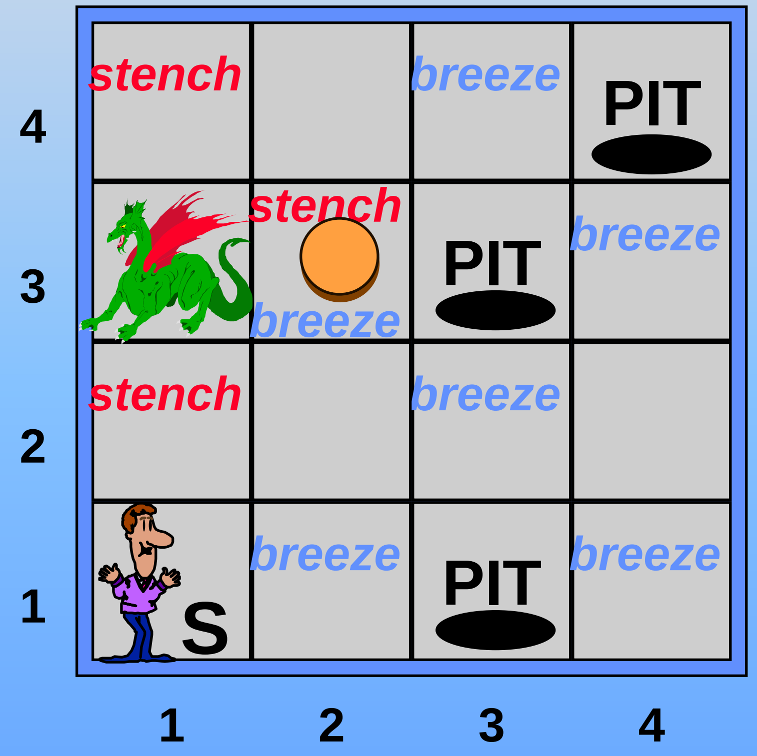
An Agent for the Wumpus World

- **A reasoning agent**
  - Propositional logic as the "programming language"
  - Knowledge base (KB) as problem representation
    - Percepts
    - Knowledge
    - Actions

    – sentences
  - Rule of inference (e.g. Modus Ponens) as the algorithm that will find a solution

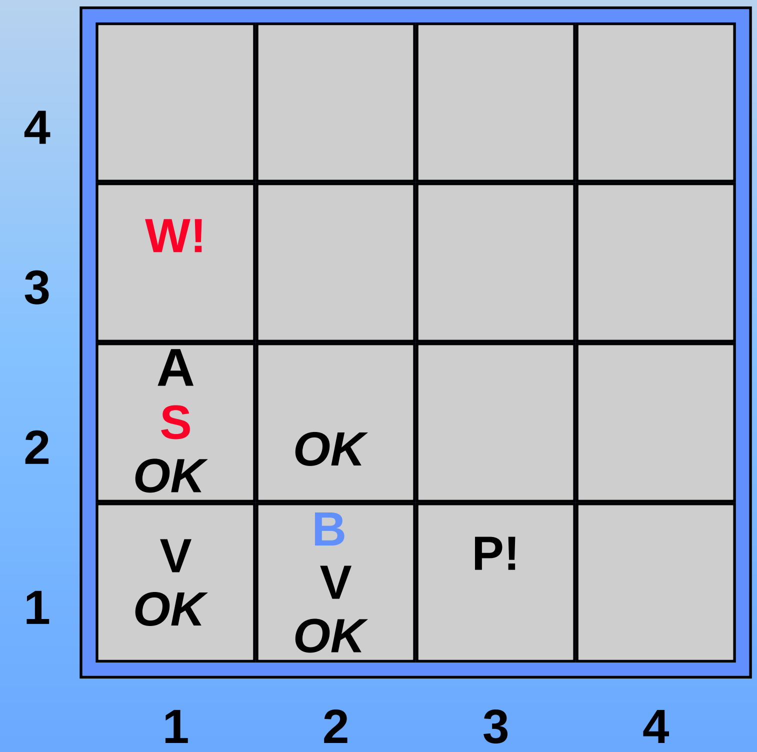


The Knowledge Base

- **TELLing the KB: percepts**
  - Syntax and semantics
    - Symbol **S11**, meaning "there is a stench at [1,1]"
    - Symbol **B12**, meaning "there is a breeze at [1,2]"

      ...
- **Percept sentences**
  - Partial list:
    - ¬ S11, ¬ B11, ¬ G11, …
      ¬ S21, B21, ¬ G21, …
      S12, ¬ B12, ¬ G11, …
      ...

[Stench , nil, nil, nil, nil]



The Knowledge Base

- **TELLing the KB: knowledge**
  - Rules about the environment
    - "All squares adjacent to the wumpus have a stench."

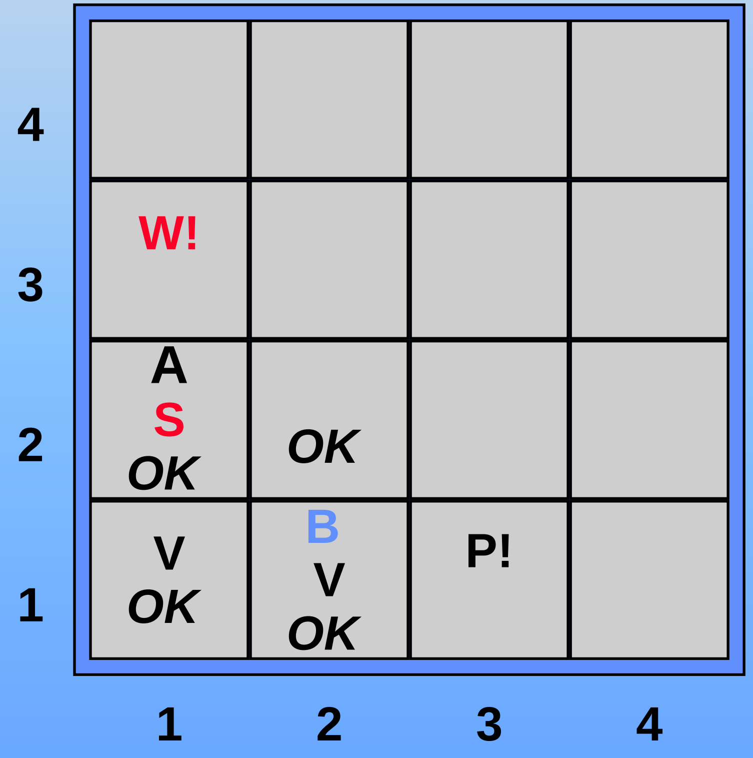
      $S12 \Rightarrow W11 \vee W12 \vee W22 \vee W13$
    - "A square with no stench has no wumpus and adjacent squares have no wumpus either."

      $\neg S11 \Rightarrow \neg W11 \wedge \neg W21 \wedge \neg W12$

      $\neg S21 \Rightarrow \neg W11 \wedge \neg W21 \wedge \neg W22 \wedge \neg W31$

      $\neg S12 \Rightarrow \neg W11 \wedge \neg W12 \wedge \neg W22 \wedge \neg W13$

[Stench , nil, nil, nil, nil]



Finding the Wumpus

- **Checking the truth-table**
  - Exhaustive check: every row for which KB is true also has W13 true
    - 12 propositional symbols, i.e. S11, S21, S12, W11, W21, W12, W22, W13, W31, B11, B21, B12
    - $2^{12} = 4096$ rows
  - > *possible, but lengthy* *impossible for the complete problem*

KB $\Rightarrow$ W13

- **Reasoning by inference**
  - Application of a sequence of inference rules (proof)
    - Modus Ponens, And-Elimination, and Unit-Resolution

Proof for “KB $\Rightarrow$ W13”

Knowledge Base

$\neg$ S11, $\neg$ B11, $\neg$ G11,
$\neg$ S21, B21, $\neg$ G21,
S12, $\neg$ B12, $\neg$ G11,

R1: $\neg S11 \Rightarrow \neg W11 \land \neg W21 \land \neg W12$

R2: $\neg S21 \Rightarrow \neg W11 \land \neg W21 \land \neg W22 \land \neg W31$

R3: $\neg S12 \Rightarrow \neg W11 \land \neg W12 \land \neg W22 \land \neg W13$

R4: $S12 \Rightarrow W11 \lor W12 \lor W22 \lor W13$

Inferences

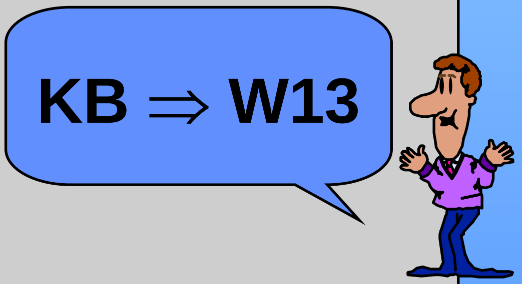
1. Modus Ponens: $\neg$ S11, **R1**
   $\vdash$ $\neg W11 \land \neg W21 \land \neg W12$
2. And-Elimination: ♦
   $\vdash$ $\neg W11$, $\neg W21$, $\neg W12$
3. Modus Ponens: $\neg$ S21, **R2**
   $\vdash$ $\neg W11 \land \neg W21 \land \neg W22 \land \neg W31$
4. And-Elimination: ♦
   $\vdash$ $\neg W11$, $\neg W21$, $\neg W22$, $\neg W31$

Proof for "KB ⇒ W13"

Knowledge Base	Inferences
$\neg$ S11, $\neg$ B11, $\neg$ G11, $\neg$ S21, B21, $\neg$ G21, S12, $\neg$ B12, $\neg$ G11, **R1:** $\neg S11 \Rightarrow \neg W11 \wedge \neg W21 \wedge \neg W12$ **R2:** $\neg S21 \Rightarrow \neg W11 \wedge \neg W21 \wedge \neg W22 \wedge \neg W31$ **R3:** $\neg S12 \Rightarrow \neg W11 \wedge \neg W12 \wedge \neg W22 \wedge \neg W13$ **R4:** $S12 \Rightarrow W11 \vee W12 \vee W22 \vee W13$	KB += $\neg$W11, $\neg$W21, $\neg$W12, $\neg$W22, $\neg$W31 5. Modus Ponens: S12, **R4** $\vdash$ $(W13 \vee W12 \vee W22) \vee W11$ 6. Unit-Resolution: ♦, $\neg$W11 $\vdash$ $(W13 \vee W12) \vee W22$

Proof for “KB ⇒ W13”

Knowledge Base	Inferences
$\neg$ S11, $\neg$ B11, $\neg$ G11, $\neg$ S21, B21, $\neg$ G21, S12, $\neg$ B12, $\neg$ G11, **R1:** $\neg S11 \Rightarrow \neg W11 \wedge \neg W21 \wedge \neg W12$ **R2:** $\neg S21 \Rightarrow \neg W11 \wedge \neg W21 \wedge \neg W22 \wedge \neg W31$ **R3:** $\neg S12 \Rightarrow \neg W11 \wedge \neg W12 \wedge \neg W22 \wedge \neg W13$ **R4:** $S12 \Rightarrow W11 \vee W12 \vee W22 \vee W13$	KB += $\neg$W11, $\neg$W21, $\neg$W12, $\neg$W22, $\neg$W31, $(W13 \vee W12) \vee W22$ 7. Unit-Resolution: ♦, $\neg$W22 $\vdash$ $W13 \vee W12$ 8. Unit-Resolution: ♦, $\neg$W12 $\vdash$ W13



From Knowledge to Actions

- **TELLing the KB: actions**
  - Additional rules
    - e.g. “if the wumpus is 1 square ahead then do not go forward”

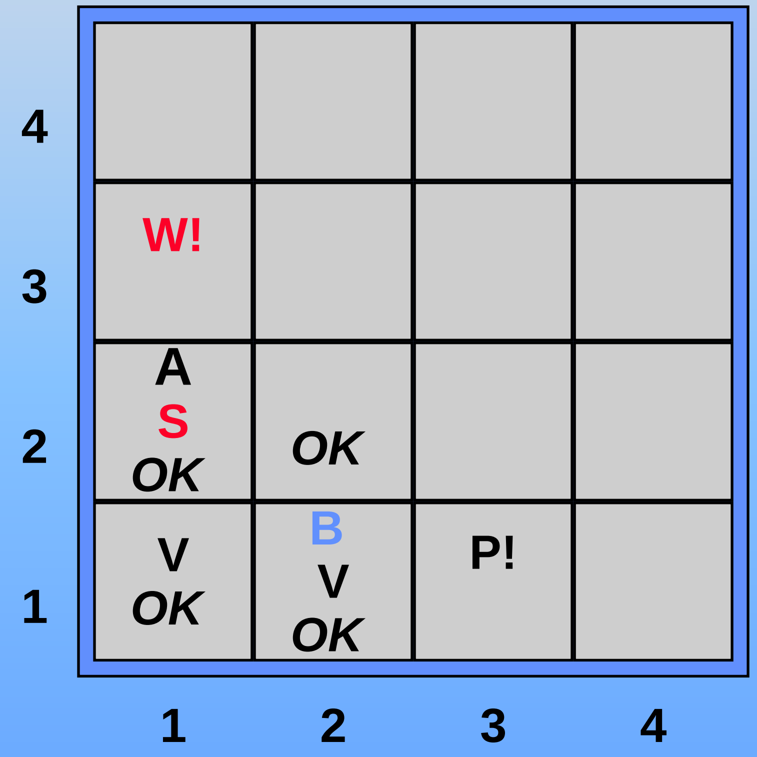
$A12 \wedge East \wedge W22 \Rightarrow \neg Forward$

$A12 \wedge North \wedge W13 \Rightarrow \neg Forward$

...

- **ASKing the KB**
  - Cannot ask “which action?” but “should I go forward?”

[Stench , nil, nil, nil, nil]



A Knowledge-Based Agent Using Propositional Logic

```
function Propositional-KB-Agent (percept) returns action
        static  KB,                 // a knowledge base
                t                   // a time counter, initially 0

        Tell (KB, Make-Percept-Sentence (percept, t))
        foreach action in the list of possible actions
        do
            if Ask (KB, Make-Action-Query (t, action)) then
                Tell (KB, Make-Action-Sentence (action, t))
                t ← t + 1
                return action
        end
```

The Limits of Propositional Logic

- **A weak logic**
  - Too many propositions to TELL the KB
    - e.g. the rule “if the wumpus is 1 square ahead then do not go forward” needs 64 sentences (16 squares x 4 orientations)!
    - Result in increased time complexity of inference
  - Handling change is difficult
    - Need time-dependent propositional symbols
      e.g. A11 means “the agent is in square [1,1]” - when?
      at t = 0: A11-0; at t = 1: A21-1;
      at t = 2: A11-2
    - Need to rewrite rules as time-dependent
      e.g. A12-0 $\wedge$ East-0 $\wedge$ W22-0 $\Rightarrow$ ¬Forward-0
      A12-2 $\wedge$ East-2 $\wedge$ W22-2 $\Rightarrow$ ¬Forward-2

Summary

- **Intelligent agents need …**
  - Knowledge about the world, so as to take good decisions.
- **Knowledge can be …**
  - Defined using a knowledge representation language.
  - Stored in a knowledge base in the form of sentences.
  - Inferred, using an inference mechanism and rules.
- **A representation language is defined by …**
  - A syntax, which specify the structure of sentences, and
  - A semantics, which specifies how the sentences relate to facts in the world.

Summary

- **Inference is …**
  - The process of deducing new sentences from old ones.
  - Sound if it derives true conclusions from true premises.
  - Complete if it can derive all possible true conclusions.
- **Logics …**
  - Make different commitments about what the world is made of and what kind of beliefs we can have about facts.
  - Are useful for the commitments they *do not* make.
- **Propositional logic …**
  - Commits only to the existence of facts.
  - Has simple syntax and semantics and is therefore limited.

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