NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2016/17 Semester 1 MH2500 Probability and Introduction to Statistics Tutorial 2

As 25 August is union day, tutorials for groups T1–T8 are cancelled. Please work on these problems on your own and refer to the solution file that will be uploaded on NTULearn. For tutorial groups T9–T11, classes will be as usual.

- Ex. 1.8.52, 56, 60, 64, 68, 72.

Ex. 1.8.52. A couple has two children. What is the probability that both are girls given that the oldest is a girl? What is the probability that both are girls given that one of them is a girl?

Ex. 1.8.56. (modified) Suppose that 5 cards are dealt from a 52-card deck.

(i) What is the probability of at least two kings given the first card one is a king?
(ii) What is the probability of at least two kings given there is at least one king?

Ex. 1.8.60. A factory runs three shifts. In a given day, 1% of the items produced by the first shift are defective, 2% of the second shift's items are defective, and 5% of the third shift's items are defective.

(i) If the shifts all have the same productivity, what percentage of the items produced in a day are defective?
(ii) If an item is defective, what is the probability that it was produced by the third shift?

Ex. 1.8.64. If B is an event with $P(B) > 0$, show that the set function $Q(A) = P(A|B)$ satisfies the axioms for a probability measure. Thus, for example,

$$P(A \cup C|B) = P(A|B) + P(C|B) - P(A \cap C|B).$$

Ex. 1.8.68. If A is independent of B and B is independent of C, then A is independent of C. Prove this statement or give a counterexample if it is false.

Ex. 1.8.72. Suppose that n components are connected in series. For each unit, there is a backup unit and the system fails if and only if both a unit and its backup fail. Assuming that all the units are independent and fail with probability p, what is the probability that the system works?