NANYANG TECHNOLOGICAL UNIVERSITY
School of Electrical & Electronic Engineering

EE/IM4152 Digital Communications

Tutorial No. 10 (Sem 1, AY2016-2017)

1. Consider the coherent receiver of a binary communication system shown in Figure 1. The received signal is $r(t) = s_i(t) + n(t)$, $0 \le t \le T_b$, where the signal component $s_i(t)$ is equally likely to be $s_1(t)$ or $s_2(t)$, and $n(t)$ is additive white Gaussian noise (AWGN) with mean zero and two-sided power spectral density $N_0/2$. The signals are given by

$$s_1(t) = -s_2(t) = \begin{cases} A/2, & 0 < t \le T_b/2 \\ -A, & T_b/2 < t \le T_b \\ 0, & \text{otherwise} \end{cases},$$

where A is a constant and T_b denotes the bit duration. The reference signal $v(t) = s_1(t)$ is used to correlate the received signal $r(t)$. The test statistic $Z(T_b) = a_i + n_0$ is compared with the optimum threshold $\gamma_0 = (a_1 + a_2)/2$ to decide whether $s_1(t)$ or $s_2(t)$ has been sent at the transmitter. The noise component n_0 is due to AWGN and a_i is the signal component of $Z(T_b)$ when $s_i(t)$ is sent.

(a) Plot the waveforms of $s_1(t)$ and $s_2(t)$. Compute the average energy per bit E_b.

(b) Express a_1, a_2 and γ_0 in terms of E_b.

(c) Find the mean and variance of the noise component n_0 in terms of E_b and N_0.

(d) Derive the bit-error rate (BER) of the coherent system in terms of Q-function, E_b and N_0.

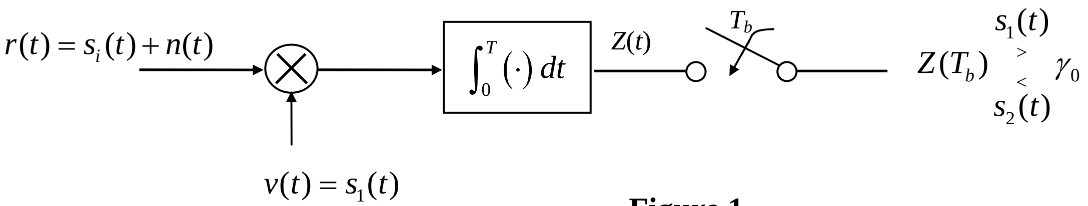

Figure 1

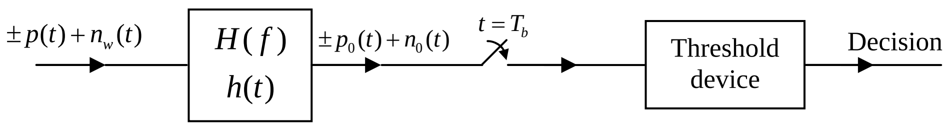

Figure 2

2. In our derivation of the matched filter, we have chosen polar signalling where only one basic pulse $p(t)$ is used, as shown in Figure 2. Generally, in binary communication, there are two waveforms $s_1(t)$ and $s_2(t)$ representing binary 1 and binary 0, respectively. The optimum receiver for this case is shown in Figure 3.

(a) If the input signal is $s_i(t)$, $i = 1, 2$, find the signal component a_i at the output.

(b) Determine the mean and variance of the output noise when additive white Gaussian noise (AWGN), with mean zero and two-sided power spectral density $N_0/2$, is applied at the input of the receiver.

(c) Suppose the threshold is $\gamma_0 = (a_1 + a_2)/2$. The decision is $s_1(t)$ if $r(T_b) > \gamma_0$. Otherwise, the decision is $s_2(t)$. Derive the error probability of the receiver.

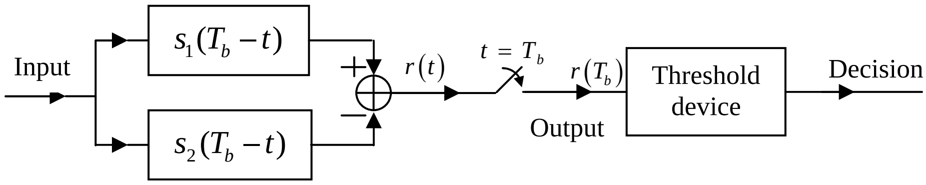

Figure 3

3. An information source generates 8 equiprobable messages every T_0 seconds, as shown in Figure 3. After modulation, the modulated waveform is transmitted over an additive white Gaussian noise (AWGN) channel. The two-sided power spectral density of AWGN is $N_0/2$. Each signal component is chosen from one of signal points $\{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_8\}$ in the signal space. Determine the optimum receiver and the corresponding error probability P_{eM} in terms of a, N_0 and Q-function.

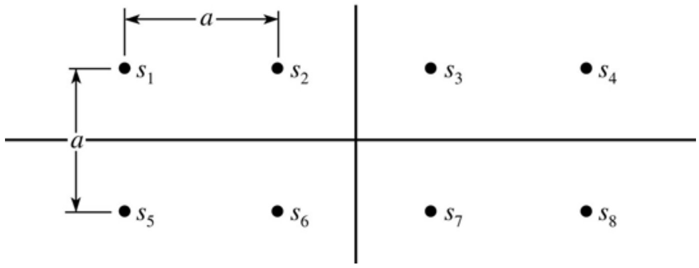

Figure 4