



**6.1** Being able to prove the validity of logical sentences is the key to sound reasoning.

(a) Use *truth tables* to show that the following logical equivalences hold.

$$(i) \quad P \Rightarrow Q \Leftrightarrow \neg P \vee Q$$

$$(ii) \quad P \Leftrightarrow Q \Leftrightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

(b) Prove *without using a truth table* that the following equivalence holds.  
(Hint: try applying and rewriting well-known logical equivalences.)

$$(iii) \quad P \Leftrightarrow Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$$

**6.2** Amy, Bob, Cal, Don, and Eve were invited to a party last night. Whenever there is a party, Cal will always go if Amy and Bob go. But Cal will not go if Don goes, and conversely. We know that Amy went to the party with Eve. And that Bob goes to every party that Eve goes to. Use *Propositional Logic* and the *Modus Ponens* to infer logically whether Don went to the party or not.

**6.3** If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Translate the above knowledge in *Propositional Logic* and use the *Modus Ponens* rule of inference to prove, if possible, that the unicorn is mythical, magical, and horned. State clearly any assumption or hypothesis that you may require to do so.