Compiler Techniques

3. Syntax Analysis

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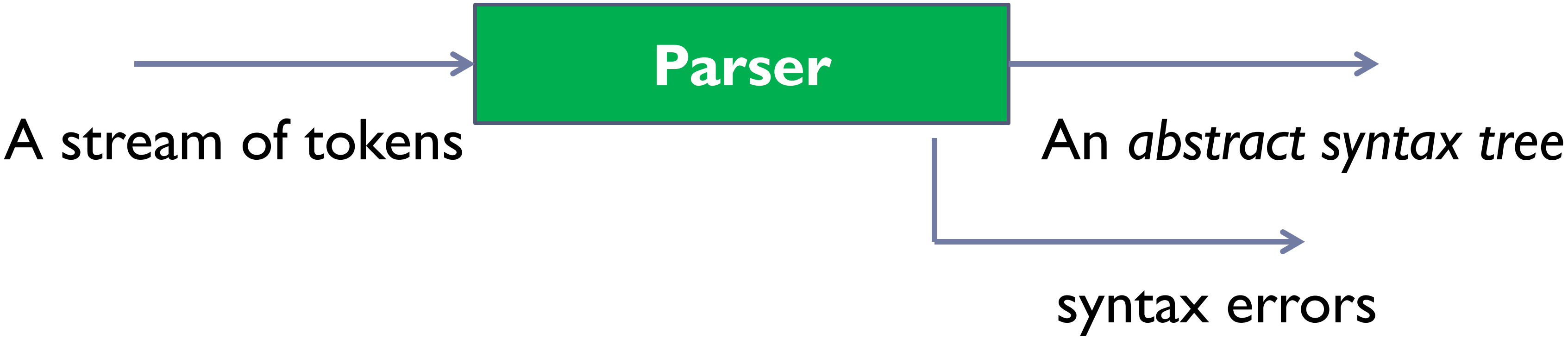
Overview

- The syntax analyzer (parser) checks that the program is **syntactically well-formed** and transforms it from a sequence of tokens into an ***abstract syntax tree (AST)***.



For example,

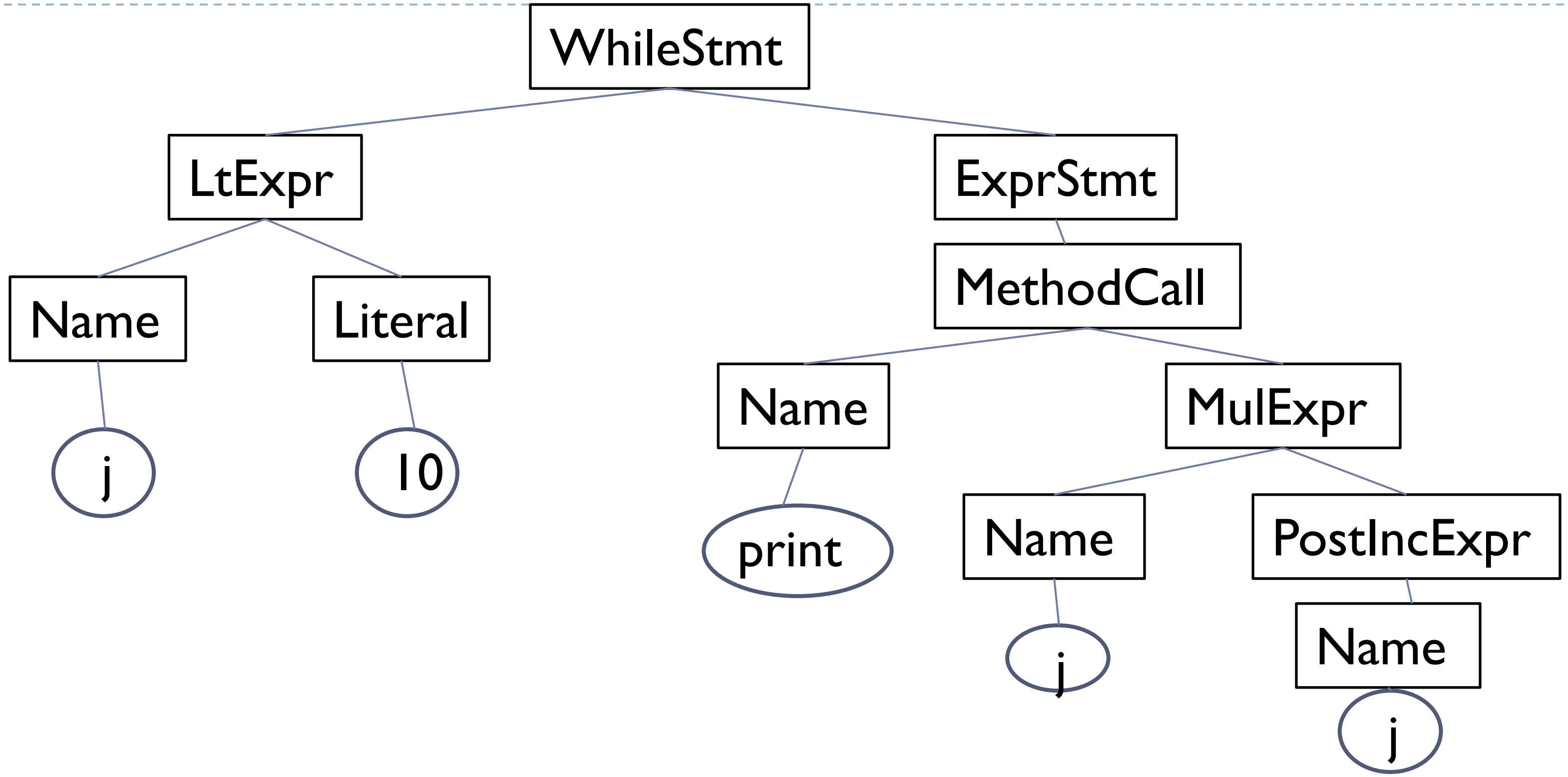
while (j < 10) print (j * (j ++)) ;

WHILE	LPAR	ID	LT	LIT	RPAR	ID	LPAR	ID	MUL	LPAR

…

back

AST for while (j < 10) print (j * (j ++)) ;



- The abstract syntax tree (AST) captures the **structure of the program.**
- Later stages of the compiler use the AST for **semantic analysis** and **code generation**.

Overview

- To check whether a program is well-formed, we need a tool – The grammar of a language.
- A language's grammar serves as a concise definition of how meaningful sentences in a language can be constructed.
- In this chapter, we will learn
  - Context-free grammar
  - The parsing problem
    - Top-down parsing
    - Bottom-up parsing

Context-free Grammars

- Programming language syntax can be described by a **context-free grammar** (can we use regular expressions?).

- A context-free grammar $G = (N, T, P, S)$ consists of four components:

1. A finite set T of **terminals** (token types)
2. A finite set N of **nonterminals** such that $T \cap N = \varnothing$
3. A **start symbol** $S \in N$
4. A finite set P of **rules** of the form $A \to s_1 \ldots s_n$ where $A \in N$, $n \geq 0$, and $\forall i \in \{1, \ldots, n\}$, $s_i \in T \cup N$.

   If $n = 0$, we write the rule as $A \to \lambda$.

Example of a context free grammar

Expr → *Expr* plus *Term*
| *Expr* minus *Term*
| *Term*
Term → *Term* mul *Factor*
| *Term* div *Factor*
| *Factor*
Factor → number
| id
| lparen *Expr* rparen

"|" is used to group multiple rules for the same nonterminal.

Factor → number
Factor → id
Factor →lparen *Expr* rparen

back

Notation Adopted

Names Beginning With	Examples	Represent Symbols in
Upper case	A, B, C, Expr, Stmt	N
Lower case and punctuation	a, b, c, if, then, plus	T
X, Y	X_1, X_2	$N \cup T$
Other Greek letters	α, β, γ	$(N \cup T)^*$

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Deriving Sentences

- The language defined by a grammar is the set of all sentences that can be derived from its start symbol.
- Example of one sentence derived using the rules on slide 6:

Expr ⇒ *Expr* plus *Term*
⇒ *Term* plus *Term*
⇒ *Factor* plus *Term*
⇒ *id* plus *Term*
⇒ *id* plus *Term* *mul* *Factor*
⇒ *id* plus *Factor* *mul* *Factor*
⇒ *id* plus *id* *mul* *Factor*
⇒ *id* plus *id* *mul* *id*

- Since there may be multiple rules for a nonterminal, derivation is non-deterministic: we may derive many different sentences from the same initial phrase. (try to derive 3 more sentences for the Expr language)

Deriving Sentences

Slide 6

- If $A \rightarrow \beta$ is a rule of G and $\alpha, \gamma \in (N \cup T)^*$ are phrases of G, then $\alpha A \gamma \Rightarrow \alpha\beta\gamma$ is a **one-step derivation**

  E.g. ***Expr*** plus ***Term*** ⇒ ***Expr*** minus ***Term*** plus ***Term***

- $\alpha \Rightarrow {+}\beta$ means that β is derived in ***one or more*** steps from α,

  $\alpha \Rightarrow {*}\beta$ in ***zero or more*** steps

- The set of terminal strings derivable from *S* are the **set of all sentences** of the language, denoted ***L(G)***

Derivation Sequences

- If there are multiple nonterminals in a phrase, there is a choice as to which nonterminal should be expanded next.

  Term ⇒ ***Term*** mul ***Term***

- In a ***leftmost*** derivation, we always expand the first nonterminal, in a ***rightmost*** one the last nonterminal.

- Ultimately, the derivation order does not matter: we can derive any sentence in *L*(*G*) using any strategy.

- What is important, however, is which rule is applied at each nonterminal occurrence.

Term	→	*Term* mul *Factor*
	|	*Term* div *Factor*
	|	*Factor*

Parse Trees (this is not the AST)

- A ***parse tree*** **represents a derivation**, but abstracts away from the order of derivations.

- Every node in a parse tree is labelled with a symbol:
  - the **root** node is labelled with the **start** symbol;
  - **leaf** nodes are labelled with **terminal** symbols or λ;
  - **inner nodes** are labelled with **nonterminal** symbols.

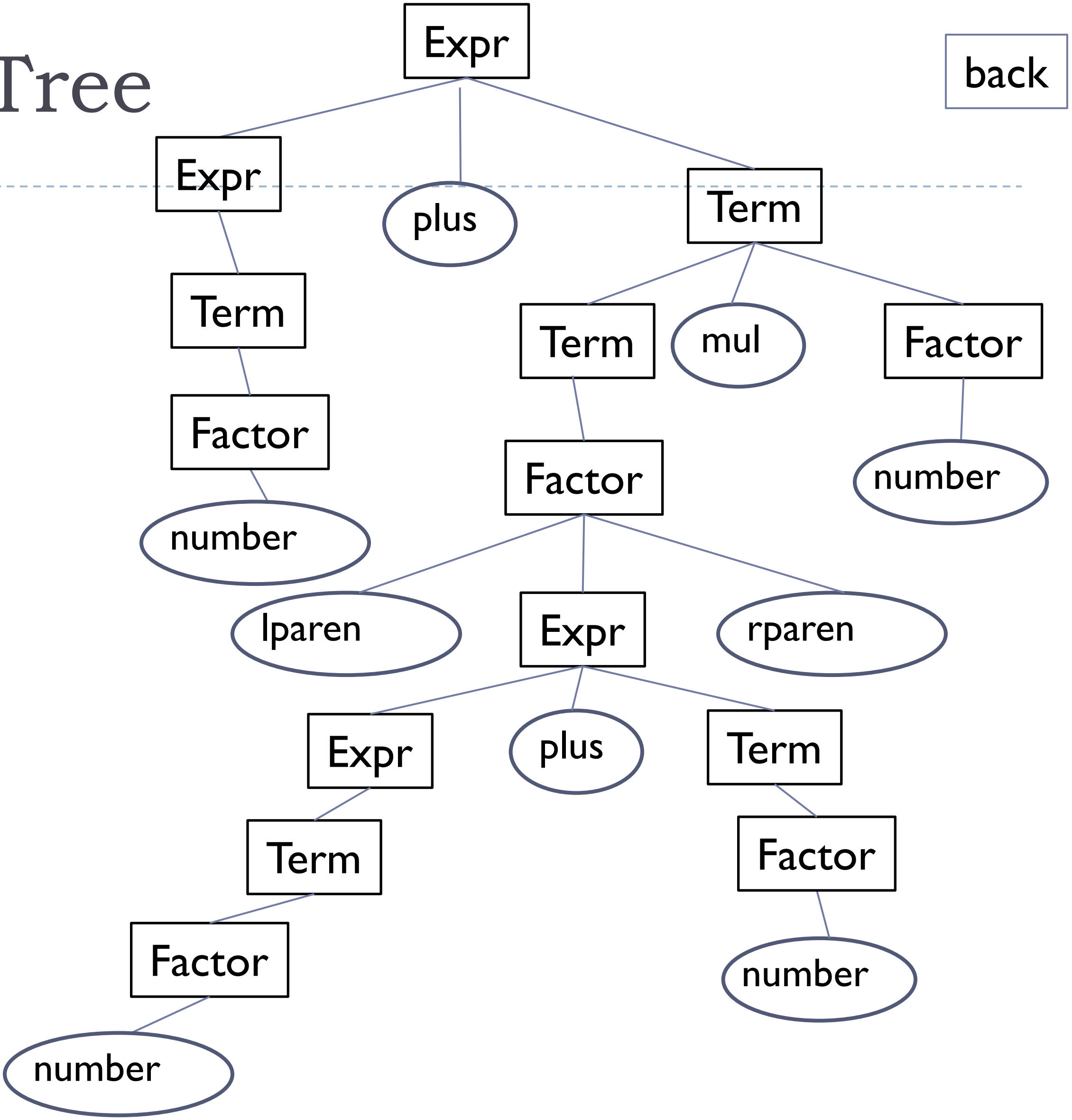
- The labelling obeys the following requirement:

  A node labelled with A which has children labelled $s_1 \dots s_n$, if and only if there is a rule $A \rightarrow s_1 \dots s_n$

Example Parse Tree

back

Expr → *Expr* plus *Term*
| *Expr* minus *Term*
| *Term*
Term → *Term* mul *Factor*
| *Term* div *Factor*
| *Factor*
Factor → number
| lparen *Expr* rparen



Sentence derived:

number plus lparen number plus number rparen mul number

Ambiguous Grammars

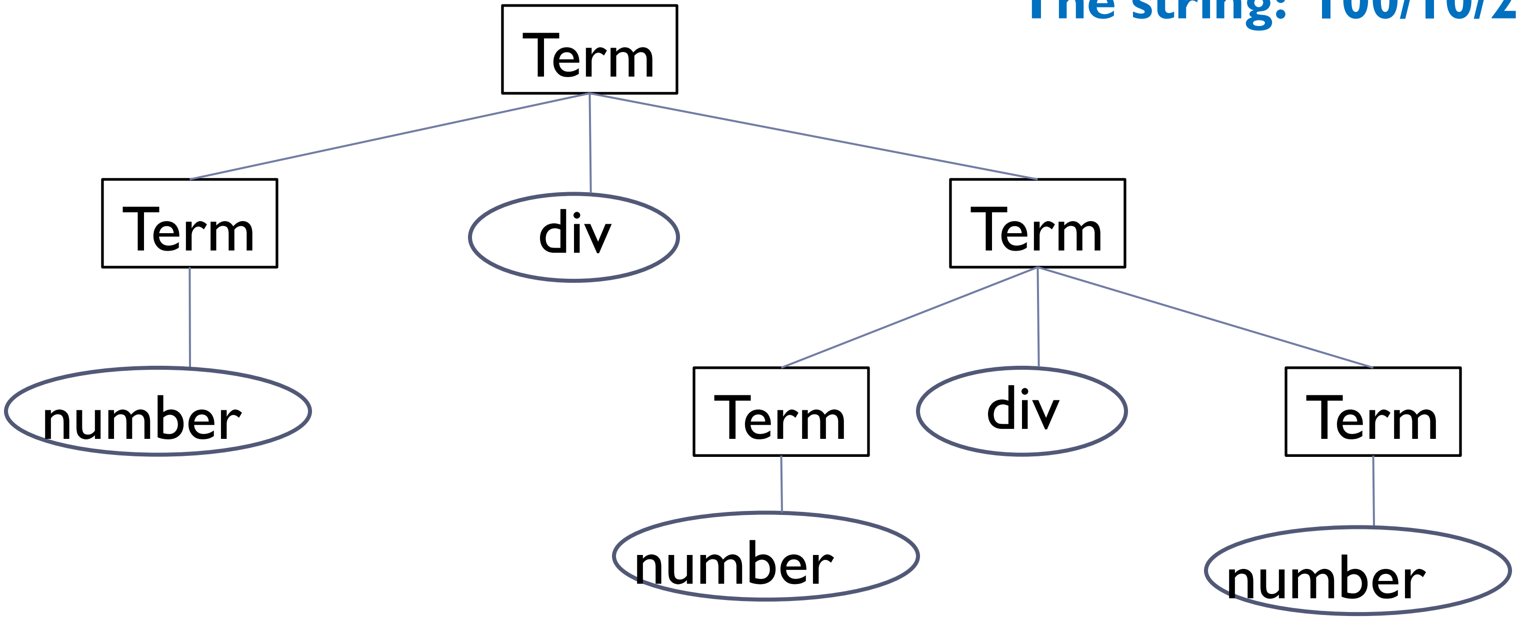
- Consider the grammar *G* defined by the following two rules:

Term → Term div Term // rule 1

| number // rule 2

A parse tree for number div number div number

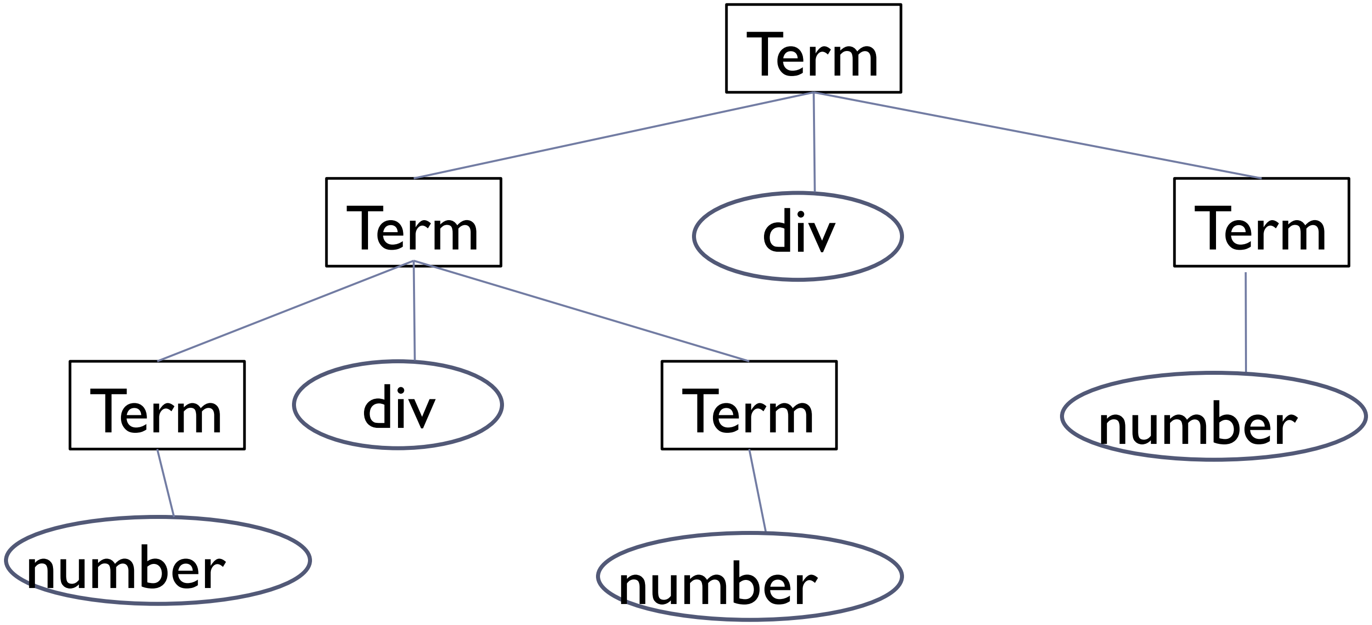
The string: 100/10/2



Ambiguous Grammars

Another parse tree for
number div number div number

The string: 100/10/2



Ambiguous Grammars

- **A grammar that produces more than one leftmost derivation or more than one rightmost derivation for the same sentence are called ambiguous.**
- Ambiguous grammars are not useful for compilers.
- There is no algorithm that can check an arbitrary context free grammar for ambiguity.

The Parsing Problem

Formal Statement

Given a context-free grammar $G = (T, N, S, P)$ and a sentence $w \in T^*$, decide whether or not $w \in L(G)$.

Slide 2

- ***Top-down parsing***: generates a parse tree by starting at the **root** of the tree (the **start** symbol ***S***), expanding the tree by applying rules in a depth-first manner.

Slide 12

- ***Bottom-up parsing***: generates a parse tree by starting at the tree's **leaves** (and *w*) and working towards its root. A node is inserted into the tree only after its children have been inserted.

back

Top-down Parsing

back

- This parsing technique is known by a few names:

1. **Top-down**, because it begins with the grammar's start symbol and grows a parse tree from its root to its leaves.
2. **Predictive**, because it predicts at each step which grammar rule is to be used.

Slide 12

3. **LL(k)**, because it scans the input from left to right, producing a leftmost derivation, using k symbols of lookahead. We will consider only **LL(1)**.
4. **Recursive descent**, because it can be implemented by a collection of mutually recursive procedures.

Recursive Descent Parsing

back

- In a recursive descent parser, for every nonterminal A there is a corresponding method parseA that can parse sentences derived from A.

The grammar:

$S \rightarrow A\ C$

$C \rightarrow c$

$\quad |\ \lambda$

$A \rightarrow a\ B\ C\ d$

$\quad |\ B\ Q$

$B \rightarrow b\ B$

$\quad |\ \lambda$

$Q \rightarrow q$

$\quad |\ \lambda$

The corresponding methods:

parseS {…}

parseC {…}

parseA {…}

parseB {…}

parseQ {…}

Recursive Descent Parsing

- If there is more than one rule for A, parseA inspects the next input token(s) and choose a production rule among the rules for A to apply.

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- The code for applying a production rule is obtained by processing the RHS of the rule, symbol by symbol

  $A \rightarrow X_1X_2\ldots X_m$

Slide 21

- If the next symbol X_i is a terminal *t*, confirms the next input token is *t*.
- If it is a nonterminal B, call the parsing function parseB.
- The code for applying a production rule A → λ will do nothing and simply return.

back

Recursive Descent Parsing

back

- The parsing of the whole program starts from the parse method for the start symbol.
- Recursive descent parsers use one token of lookahead to determine which rule to use.
- Lookahead has to be unambiguous: there cannot be more than one rule (for the same nonterminal) whose RHS starts with the same token.

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- A grammar that fulfills this condition is an ***LL*(1) *grammar***.
- A language for which there exists an *LL*(1) grammar is an ***LL*(1) *language***.

Example

Slide 17 back

```
parseS(ts)
{  // ts is the input token stream
   if (ts.peek() ∈ predict($p_1$))
         parseA(ts);  parseC(ts);
    else /* syntax error */        }

parseA(ts)
{  if (ts.peek() ∈ predict($p_4$))
         match(a);   parseB(ts);
         parseC(ts);   match(d);
   else if (ts.peek() ∈ predict($p_5$))
         parseB(ts);  parseQ(ts);
   else /* syntax error */        }
```

/* peek() examines the next input token
without advancing the input */

$S \rightarrow A\ C$	p_1
$C \rightarrow c$	p_2
$\quad \mid\ \lambda$	p_3
$A \rightarrow \mathbf{a\ B\ C\ d}$	p_4
$\quad \mid\ \mathbf{B\ Q}$	p_5
$B \rightarrow b\ B$	p_6
$\quad \mid\ \lambda$	p_7
$Q \rightarrow q$	p_8
$\quad \mid\ \lambda$	p_9

/* match()
confirms a token */

Computing predict(p)

back

Consider a production rule p: $X \rightarrow X_1X_2\ldots X_m$, $m \geq 0$.

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- The set of tokens that **predicts** rule p includes
  - The set of possible tokens that are first produced in some derivation from $X_1X_2\ldots X_m$
    - The set of first tokens in X_1
    - If X_1 may be empty, the set of first tokens in X_2, and so on.
  - Those tokens that can follow X in some derivation from $X_1X_2\ldots X_m$
    - If $X_1X_2\ldots X_m$ may be empty, the first tokens that may follow X

S → A C	p_1
C → c	p_2
| λ	p_3
A → **a B C d**	p_4
| **B Q**	p_5
B → b B	p_6
| λ	p_7
Q → q	p_8
| λ	p_9

Computing predict(p)

Example: p: **A → B Q**

- The set of possible tokens that are first produced in some derivation from B Q for rule A → B Q is { b, q}
- Those tokens that can follow A in some derivation from B Q is {c}

S	→	A C	p_1
C	→	c	p_2
	|	λ	p_3
A	→	**a B C d**	p_4
	|	**B Q**	p_5
B	→	b B	p_6
	|	λ	p_7
Q	→	q	p_8
	|	λ	p_9

The set of tokens that predicts rule p:

{b, q, c}

Computing predict(p)

- To compute the set of tokens that predict rule p, we need to know whether or not
  - a nonterminal can derive empty
  - The RHS of a rule can derive empty
- Two boolean arrays are used:
  - symbolDerivesEmpty[X] for $X \in N$
  - ruleDerivesEmpty[p] for $p \in P$

$S \rightarrow A\ C$	p_1
$C \rightarrow c$	p_2
$\quad\mid\ \lambda$	p_3
$A \rightarrow$ **a B C d**	p_4
$\quad\mid$ **B Q**	p_5
$B \rightarrow b\ B$	p_6
$\quad\mid\ \lambda$	p_7
$Q \rightarrow q$	p_8
$\quad\mid\ \lambda$	p_9

Computing predict(p)

- For the grammar on the right
  - symbolDerivesEmpty[X] for *X* ∈ *N*

S	C	A	B	Q
T	T	T	T	T

  - ruleDerivesEmpty[p] for p ∈ *P*:

p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
T	F	T	F	T	F	T	F	T

$S \rightarrow A\ C$	p_1
$C \rightarrow c$	p_2
$\quad \mid\ \lambda$	p_3
$A \rightarrow$ **a B C d**	p_4
$\quad \mid$ **B Q**	p_5
$B \rightarrow b\ B$	p_6
$\quad \mid\ \lambda$	p_7
$Q \rightarrow q$	p_8
$\quad \mid\ \lambda$	p_9

back

Computing predict(p)

```
predict(p: $X \to X_1X_2\ldots X_m$) // returns a set of tokens     Slide 21
{    ans = first($X_1X_2\ldots X_m$);
     if ruleDerivesEmpty[p] then  // when $X_1X_2\ldots X_m$ may be empty
          ans = ans ∪ follow(X);                                     slide 22
     return ans;
}
```

back

- **first**($X_1X_2\ldots X_m$) returns a set of tokens each of which is the **first token in a sentence** derived from $X_1X_2\ldots X_m$. Formally:

$$\text{first}(X_1X_2\ldots X_m) = \{t \in T \mid \exists w \in T^*, [X_1X_2\ldots X_m \Rightarrow^* tw] \}$$

Computing first ($X_1X_2\ldots X_m$)

```
first ($X_1X_2\ldots X_m$) // returns a set of tokens
{   for each nonterminal X in the language
        visitedFirst[X] = false;
    ans = internalFirst($X_1X_2\ldots X_m$);
    return ans;
}
```

Computing first ($X_1X_2 \ldots X_m$)

The main ideas for computing **internalFirst**($X_lX_2 \ldots X_{m:}$):

1. If $X_lX_2 \ldots X_m = \lambda$, there is no first token. Return empty set.

   internalFirst(λ) returns ∅

2. If X_l is a terminal symbol, the first token is this symbol. Return $\{X_l\}$.

   internalFirst(b B) returns b

3. If X_l is a nonterminal
   1. Look at every rule for X_l and find the first tokens of X_l.
   2. If X_l may derive empty, find the first tokens for $X_2 \ldots X_m$.

What does internalFirst(A C) do?

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internalFirst($X_1X_2\ldots X_m$) //returns a set of tokens

```
{   if ($m$ == 0)  return $\emptyset$;                 /* 1 */
    if ($X_1$ is a terminal symbol)  return {$X_1$}    /* 2 */
    /* $X_1$ is a nonterminal */
    $ans$ = $\emptyset$;
    if not visitedFirst[$X_1$]
        visitedFirst[$X_1$] = true;
        for the $RHS$ of each rule for $X_1$           /* 3.i */
            $ans$ = $ans \cup$ internalFirst($RHS$);
    if symbolDerivesEmpty[$X_1$]                        /* 3.ii */
        $ans$ = $ans \cup$ internalFirst($X_2\ldots X_m$);

    return $ans$;          }
```

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S → A C
C → c
  | λ
A → a B C d
  | B Q
B → b B
  | λ
Q → q
  | λ

internalFirst(B Q) returns {b, q}

Example first(*B*)

$$\begin{array}{lcl} A & \rightarrow & B \\ & | & a \\ B & \rightarrow & A \\ & | & b \end{array}$$

```
first ($X_1X_2 \ldots X_m$) // returns a set of tokens
{    for each nonterminal X in the language
          visitedFirst[X] = false;
     ans = internalFirst($X_1X_2 \ldots X_m$);
     return ans;
}
```

	A	*B*
visitedFirst	F	F

internalFirst(*B*)

```
/* $X_1$ = B i.e. a nonterminal */
ans = ∅;
if not visitedFirst[$X_1$]
     visitedFirst[$X_1$] = true;
     for the RHS of each rule for $X_1$
          ans = ans ∪ internalFirst(RHS);
```

internalFirst(*B*) ➡ {*a*,*b*}

	A	*B*
visitedFirst	F	T

internalFirst(*A*) ∪
internalFirst(*b*)

$A \rightarrow B$
$\quad | \; a$
$B \rightarrow A$
$\quad | \; b$

internalFirst(A)

```
/* $X_1$ = $A$ i.e. a nonterminal */
$ans = \varnothing$;
if not visitedFirst[$X_1$]
    visitedFirst[$X_1$] = true;
    for the $RHS$ of each rule for $X_1$
        $ans = ans \cup$ **internalFirst**($RHS$);
if symbolDerivesEmpty[$X_1$]
    $ans = ans \cup$ **internalFirst**($X_2 \ldots X_m$);

return $ans$;
```

visitedFirst:

A	B
T	T

internalFirst(B) $\cup$
internalFirst(a)

symbolsDerivesEmpty:

A	B
F	F

internalFirst(A) → **{a}**

$A \rightarrow B$
$\quad\;\, \mid\; a$
$B \rightarrow A$
$\quad\;\, \mid\; b$

internalFirst(*B*)

```
/* $X_1$ = $B$ i.e. a nonterminal */
$ans$ = ∅;
if not visitedFirst[$X_1$]
    visitedFirst[$X_1$] = true;
    for the $RHS$ of each rule for $X_1$
        $ans$ = $ans$ ∪ internalFirst($RHS$);
if symbolDerivesEmpty[$X_1$]
    $ans$ = $ans$ ∪ internalFirst($X_2 \ldots X_m$);

return $ans$;  ⇨  ∅
```

visitedFirst:

A	*B*
T	T

symbolsDerivesEmpty:

A	*B*
F	F

internalFirst(*B*) ➡ ∅

Computing follow(X)

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- **follow**(X) returns a set of tokens that can appear right behind the nonterminal X in a phrase derived from the start symbol *S*. Formally,

  $\text{follow}(X) = \{t \in T \mid \exists \alpha,\beta \in (N \cup T)^*, [S \Rightarrow^* \alpha X t \beta]\}$

```
follow (X) // returns a set of tokens that may follow X
{    for each nonterminal Y in the language
          visitedFollow[Y] = false;
     ans = internalFollow(X);
     return ans;
}
```

back

Main ideas of internalFollow(*X*)

How do we find what tokens may follow *X*?

1. Find each occurrence of X in all RHS, E.g. what may follow B:

   A → a B C d
   A → B Q
   B → b B

2. For each such occurrence, find the **first tokens** of the string after X. If this string derives empty, call internalFollow(LHS) to find what tokens follow the LHS nonterminal. E.g. in A → B Q.

S → A C
C → c
  | λ
A → a B C d
  | B Q
B → b B
  | λ
Q → q
  | λ

back

Computing follow(X)

```
internalFollow(Y) // Y is a nonterminal
{   ans = $\emptyset$;
    if not visitedFollow[Y]
        visitedFollow[Y] = true;
        for each occurrence of Y in the RHS of all rules
            tail = stream of symbols that appear after Y ;
            ans = ans $\cup$ first(tail);
            if allDeriveEmpty(tail)
                target = LHS of the rule;
                ans = ans $\cup$ internalFollow(target);
    return ans;
}
```

E.g. internalFollow(B)

Occurrence of B in
‘A → a B C d’

tail = ‘C d’
first(tail) returns {c, d}
ans = ∅ ∪ {c, d}

Occurrence of B in
‘A → B Q’

tail = ‘Q’
first(Q) returns {q}
ans = {c, d} ∪ {q}
target = ‘A’
internalFollow(A)
returns {c}
ans = {c, d, q} ∪ {c}

Occurrence of B in
‘B → b B’

…
ans = {c, d, q} ∪ ∅

Computing follow(A)

```
allDeriveEmpty(β) // β is a stream of symbols
{
    for each symbol X in β
        if X is a terminal or not symbolDerivesEmpty[X]
            return false;
    return true;
}
```

back

E.g. allDerivesEmpty(tail) is called when tail = “C d”. It will return false.

	S	C	A	B	Q
symbolDerivesEmpty	T	T	T	T	T

Obtaining LL(1) Grammars

- LL(1) requires a unique combination of a nonterminal and a lookahead symbol to decide which rule to use.

slide21

- Two common categories of production rules make a grammar not LL(1): *common prefixes* and *left recursion*.

Common Prefixes

- If the RHSs of two rules for the same nonterminal start with the same lookahead symbol, the grammar is not LL(1).

Example

Expr → number plus *Expr*
  | number

Obtaining LL(1) Grammars

Example

Expr → number plus *Expr* | *Factor*
Factor → number

- One way is to eliminate common prefixes is by introducing new nonterminals (left factoring a grammar):

Example

Expr → number *Expr'*
Expr' → plus Expr | λ

This grammar accepts the same language as the one on the previous slide, and this language is LL(1).

Obtaining LL(1) Grammars

Left Recursion

- If the RHS of a rule starts with the LHS nonterminal, the grammar is not LL(1):

Example

StmtList	→	*StmtList* semicolon *Stmt*
	|	*Stmt*
…		

The method that parses *StmtList*, parseStmtList() will call itself repeatedly 'forever'.

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Obtaining LL(1) Grammars

1. Change left recursion to right recursion:

$$\begin{array}{lll} \textit{StmtList} & \rightarrow & \textit{Stmt} \text{ semicolon } \textit{StmtList} \\ & | & \textit{Stmt} \end{array}$$

2. Remove the common prefix:

$$\begin{array}{lll} \textit{StmtList} & \rightarrow & \textit{Stmt} \;\; \textit{StmtListTail} \\ \textit{StmtListTail} & \rightarrow & \text{semicolon } \textit{StmtList} \\ & | & \lambda \end{array}$$

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3. May want to remove mutual recursion:

$$\begin{array}{lll} \textit{StmtList} & \rightarrow & \textit{Stmt} \;\; \textit{StmtListTail} \\ \textit{StmtListTail} & \rightarrow & \text{semicolon } \textit{Stmt} \;\; \textit{StmtListTail} \\ & | & \lambda \end{array}$$

Syntactic Error Recovery

- A compiler should produce a useful set of diagnostic messages when presented with a faulty program.
- Thus after an error is detected it is desirable to recover from it and continue the syntax analysis.
- Semantic analysis and code generation will be disabled.
- In a simple form of error recovery, the parser skips input tokens until it finds a delimiter (e.g. a semicolon) to end the parsing of the current nonterminal.
- The method for parsing a nonterminal is augmented with an extra parameter that is a set of delimiters.

Example

d should follow B

```
parseA(ts, termset)
{  // ts is the input token stream
   if (ts.peek() ∈ {a})
         match(a);   parseB(ts, {d} ∪ termset);
         match(d);   match(e);
   else if (ts.peek() ∈ {b})
         parseB(ts, {q} ∪ termset);  …
   else
         error("expected an a or b");
         skip input till a symbol in termset is found
}
```

q should follow B

$A \to a\ B\ d\ e$

$\quad\mid\ B\ Q\ e$

$B \to b$

$Q \to q$

End-of-file symbol is in the *termset* of every parsing method

Bottom-up Parsing

- Recall: Slide 16

- Bottom-up parsers are commonly used in the syntax analysis phase of a compiler because of its power, efficiency and ease of construction.

- Grammar features like common prefixes and left recursion need to be addressed before top-down parsing can be used. But they can be accommodated without issue in bottom-up parsing.

Bottom-up Parsing

- This parsing technique is known by a few names:

1. **Bottom-up**, because it works its way from the terminal symbols to the grammar's start symbol.
2. **Shift-reduce**, because the two prevalent actions taken by the parser are to *shift* symbols onto the parse stack and to *reduce* a string of such symbols at the top-of-stack to one of the grammar's nonterminals.
3. **LR(k)**, because it scans the input from left to right, producing a rightmost derivation in reverse, using k symbols of lookahead.

Rightmost Derivation in Reverse

Rule	Derivation
1	Start ⇒ S $
2	⇒ A C $
3	⇒ A c $
5	⇒ a B C d c $
4	⇒ a B d c $
7	⇒ a b B d c $
7	⇒ a b b B d c $
8	⇒ a b b d c $

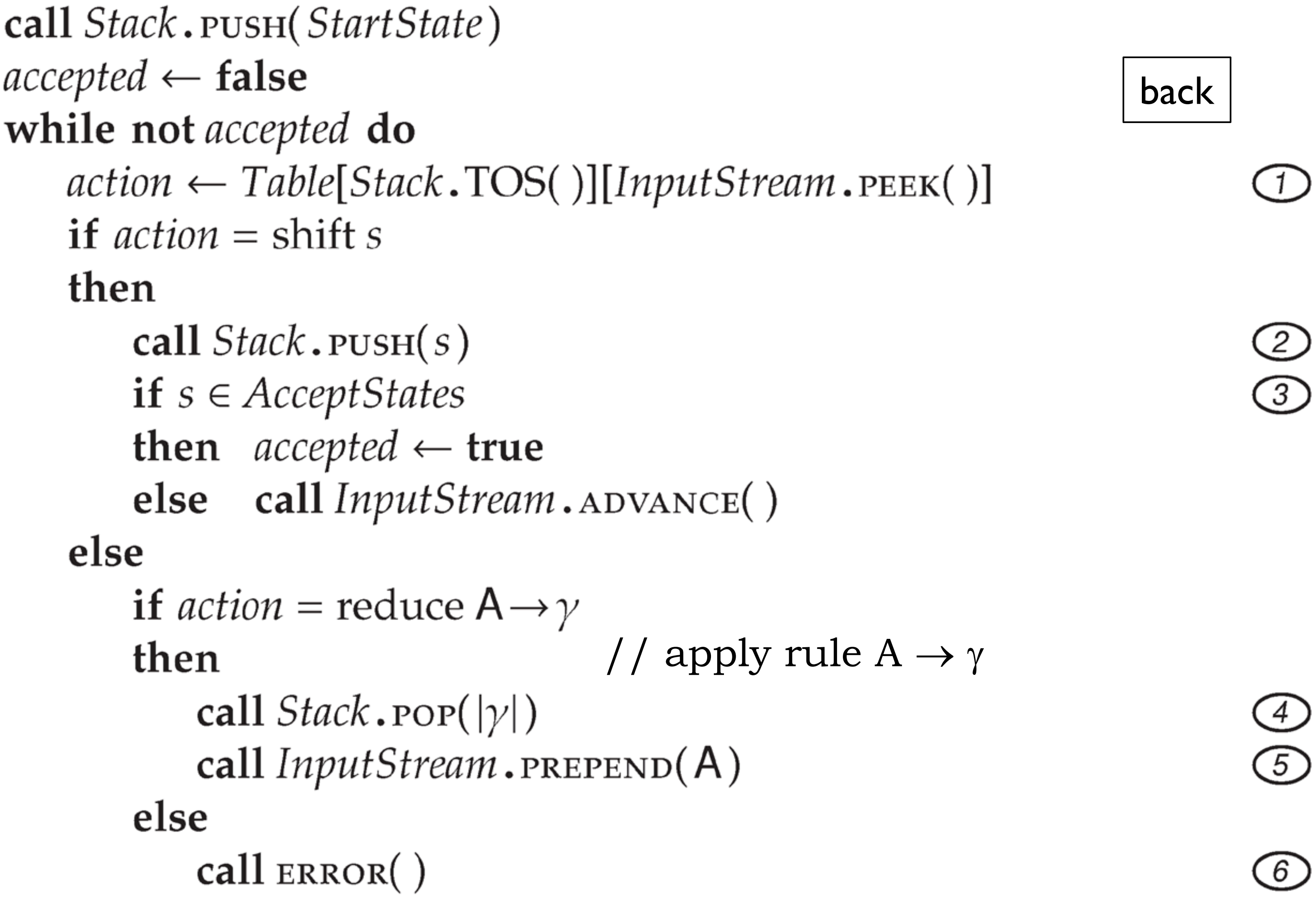
1. Start → S $
2. S → A C
3. C → c
4. | λ
5. A → a B C d
6. | B Q
7. B → b B
8. | λ
9. Q → q
10. | λ

back

LR Parsing Engine

- The parsing engine is driven by a table.
- The table is indexed using the parser's *current state* and the *next input symbol*.
- At each step, the engine looks up the table based on the current state and the next input symbol for an action.
- The table entry indicates the action to perform (either a shift or a reduce, till the final action which is accept).

back



```
call Stack.PUSH(StartState)
accepted ← false
while not accepted do
    action ← Table[Stack.TOS( )][InputStream.PEEK( )]                  ①
    if action = shift s
    then
        call Stack.PUSH(s)                                             ②
        if s ∈ AcceptStates                                            ③
        then   accepted ← true
        else   call InputStream.ADVANCE( )
    else
        if action = reduce A→γ
        then                          // apply rule A → γ
            call Stack.POP(|γ|)                                        ④
            call InputStream.PREPEND(A)                                ⑤
        else
            call ERROR( )                                              ⑥
```

Figure 6.3: Driver for a bottom-up parser.

Slide 23, chap 2

Slide 47

shift state 3

Reduce by rule 8

input: a b b d c $

0

input: b b d c $

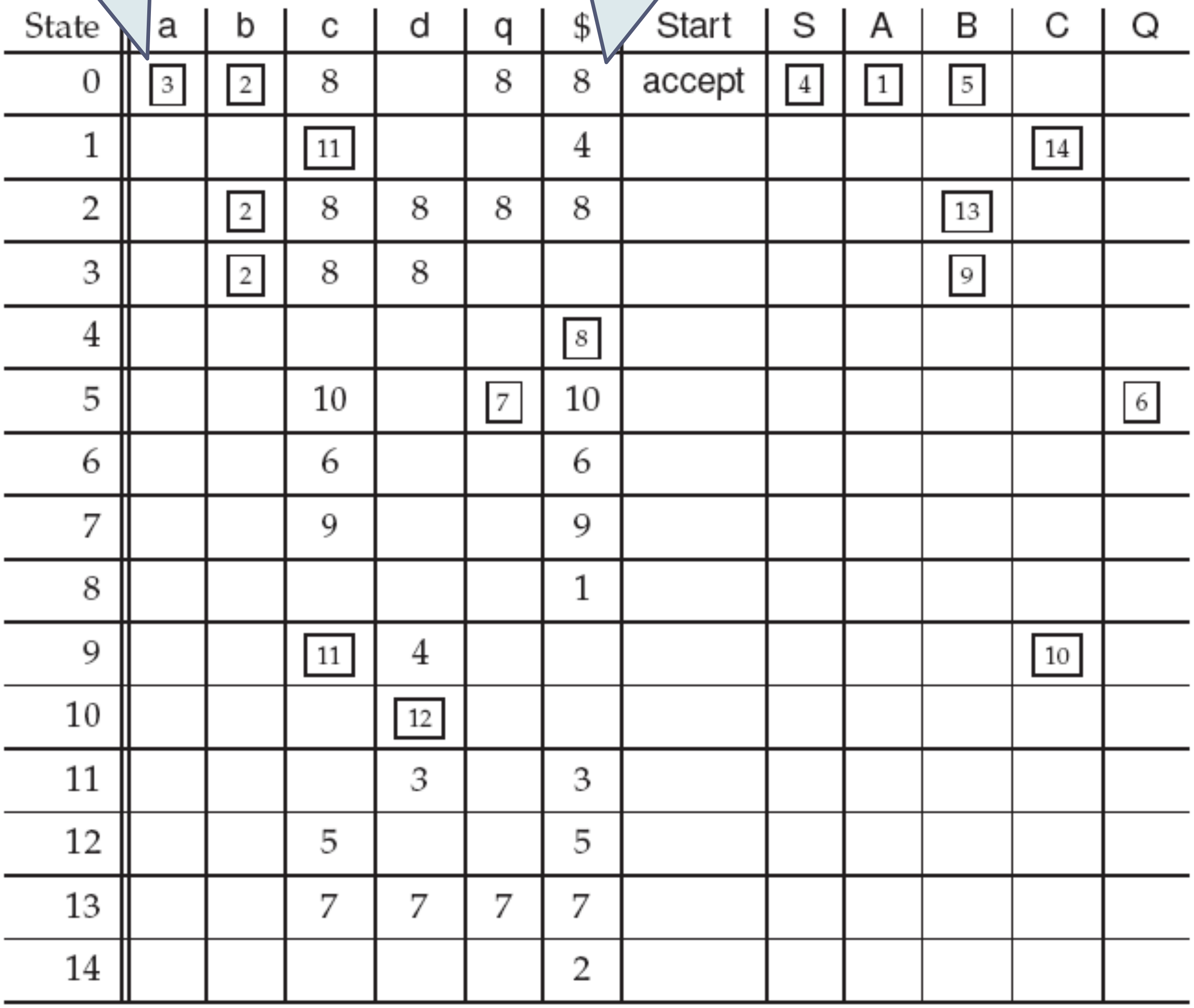
0	3
	a

input: b d c $

0	3	2
	a	b

input: d c $

0	3	2	2
	a	b	b



State	a	b	c	d	q	$	Start	S	A	B	C	Q
0	$\boxed{3}$	$\boxed{2}$	8		8	8	accept	$\boxed{4}$	$\boxed{1}$	$\boxed{5}$		
1			$\boxed{11}$			4					$\boxed{14}$	
2		$\boxed{2}$	8	8	8	8				$\boxed{13}$		
3		$\boxed{2}$	8	8						$\boxed{9}$		
4						$\boxed{8}$						
5			10		$\boxed{7}$	10						$\boxed{6}$
6			6			6						
7			9			9						
8						1						
9			$\boxed{11}$	4							$\boxed{10}$	
10				$\boxed{12}$								
11				3		3						
12			5			5						
13			7	7	7	7						
14						2						

Figure 6.5: Parse table for the grammar shown in Slide 45.

back

Slide 47

8. B → λ

input: B d c $

0	3	2	2
	a	b	b

input: d c $

0	3	2	2	13
	a	b	b	B

7. B → b B

input: B d c $

0	3	2
	a	b

input: d c $

0	3	2	13
	a	b	B

shift state 3

Reduce by rule 8

State	a	b	c	d	q	$	Start	S	A	B	C	Q
0	$\boxed{3}$	$\boxed{2}$	8		8	8	accept	$\boxed{4}$	$\boxed{1}$	$\boxed{5}$		
1			$\boxed{11}$			4					$\boxed{14}$	
2		$\boxed{2}$	8	8	8	8				$\boxed{13}$		
3		$\boxed{2}$	8	8						$\boxed{9}$		
4						$\boxed{8}$						
5			10		$\boxed{7}$	10						$\boxed{6}$
6			6			6						
7			9			9						
8						1						
9			$\boxed{11}$	4							$\boxed{10}$	
10				$\boxed{12}$								
11				3		3						
12			5			5						
13			7	7	7	7						
14						2						

Figure 6.5: Parse table for the grammar shown in **Slide 45**

back

shift state 3

Reduce by rule 8

7. B → b B

input: B d c $

0	3
	a

input: d c $

0	3	9
	a	B

4. C → λ

input: C d c $

0	3	9
	a	B

input: d c $

0	3	9	10
	a	B	C

State	a	b	c	d	q	$	Start	S	A	B	C	Q
0	$\boxed{3}$	$\boxed{2}$	8		8	8	accept	$\boxed{4}$	$\boxed{1}$	$\boxed{5}$		
1			$\boxed{11}$			4					$\boxed{14}$	
2		$\boxed{2}$	8	8	8	8				$\boxed{13}$		
3		$\boxed{2}$	8	8						$\boxed{9}$		
4						$\boxed{8}$						
5			10		$\boxed{7}$	10						$\boxed{6}$
6			6			6						
7			9			9						
8						1						
9			$\boxed{11}$	4							$\boxed{10}$	
10				$\boxed{12}$								
11				3		3						
12			5			5						
13			7	7	7	7						
14						2						

Figure 6.5: Parse table for the grammar shown in **Slide 45**

Slide 47

input: c $

0	3	9	10	12
	a	B	C	d

5. A → a B C d

input: A c $

0

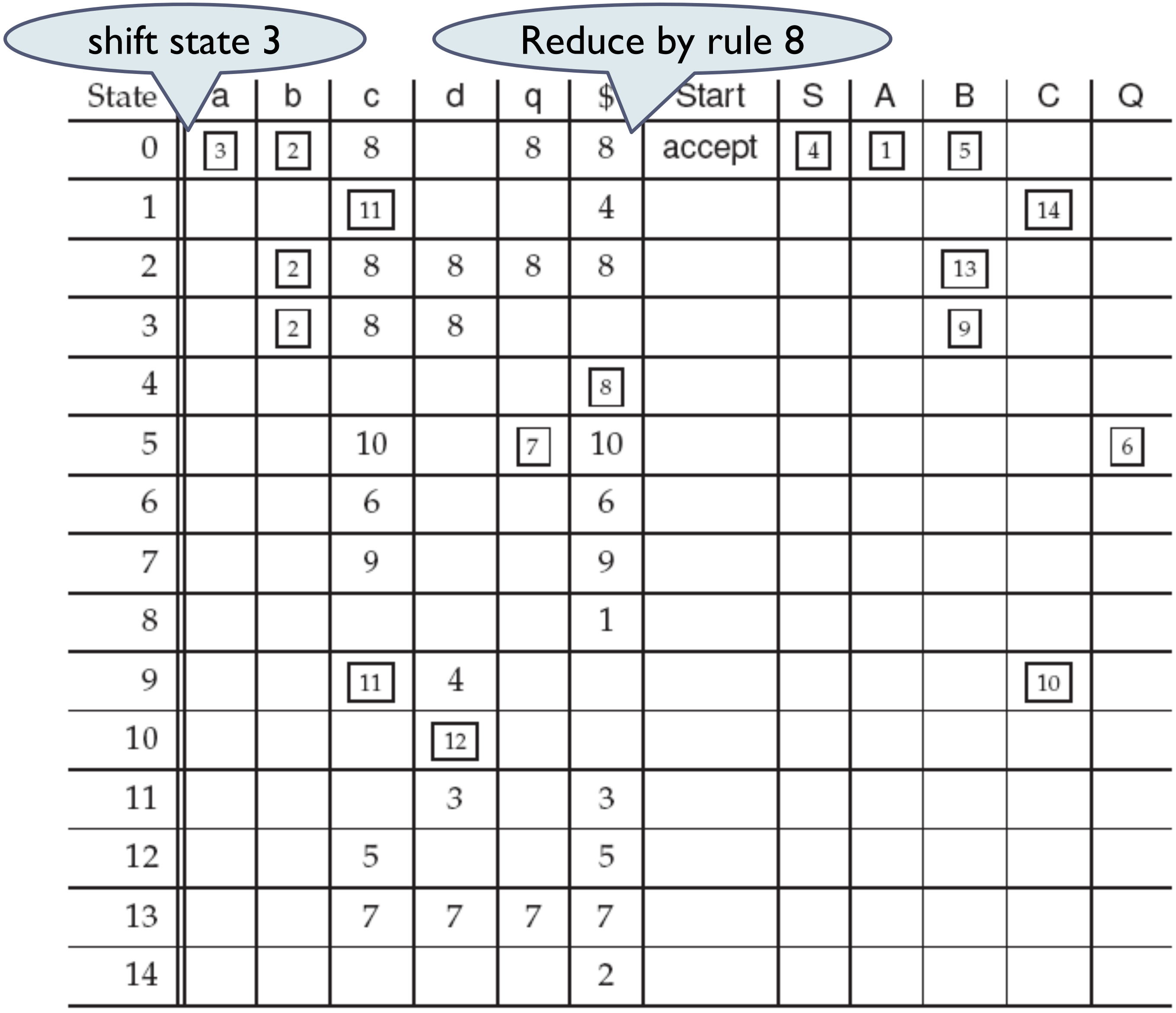
input: c $

0	1
	A

input: $

0	1	11
	A	c

back



shift state 3

Reduce by rule 8

State	a	b	c	d	q	$	Start	S	A	B	C	Q
0	$\boxed{3}$	$\boxed{2}$	8		8	8	accept	$\boxed{4}$	$\boxed{1}$	$\boxed{5}$		
1			$\boxed{11}$			4					$\boxed{14}$	
2		$\boxed{2}$	8	8	8	8				$\boxed{13}$		
3		$\boxed{2}$	8	8						$\boxed{9}$		
4						$\boxed{8}$						
5			10		$\boxed{7}$	10						$\boxed{6}$
6			6			6						
7			9			9						
8						1						
9			$\boxed{11}$	4							$\boxed{10}$	
10				$\boxed{12}$								
11				3		3						
12			5			5						
13			7	7	7	7						
14						2						

Figure 6.5: Parse table for the grammar shown in **Slide 45**

Slide 47

shift state 3

Reduce by rule 8

3. C → c

input: C $

0	1
	A

input: $

0	1	14
	A	C

2. S → A C

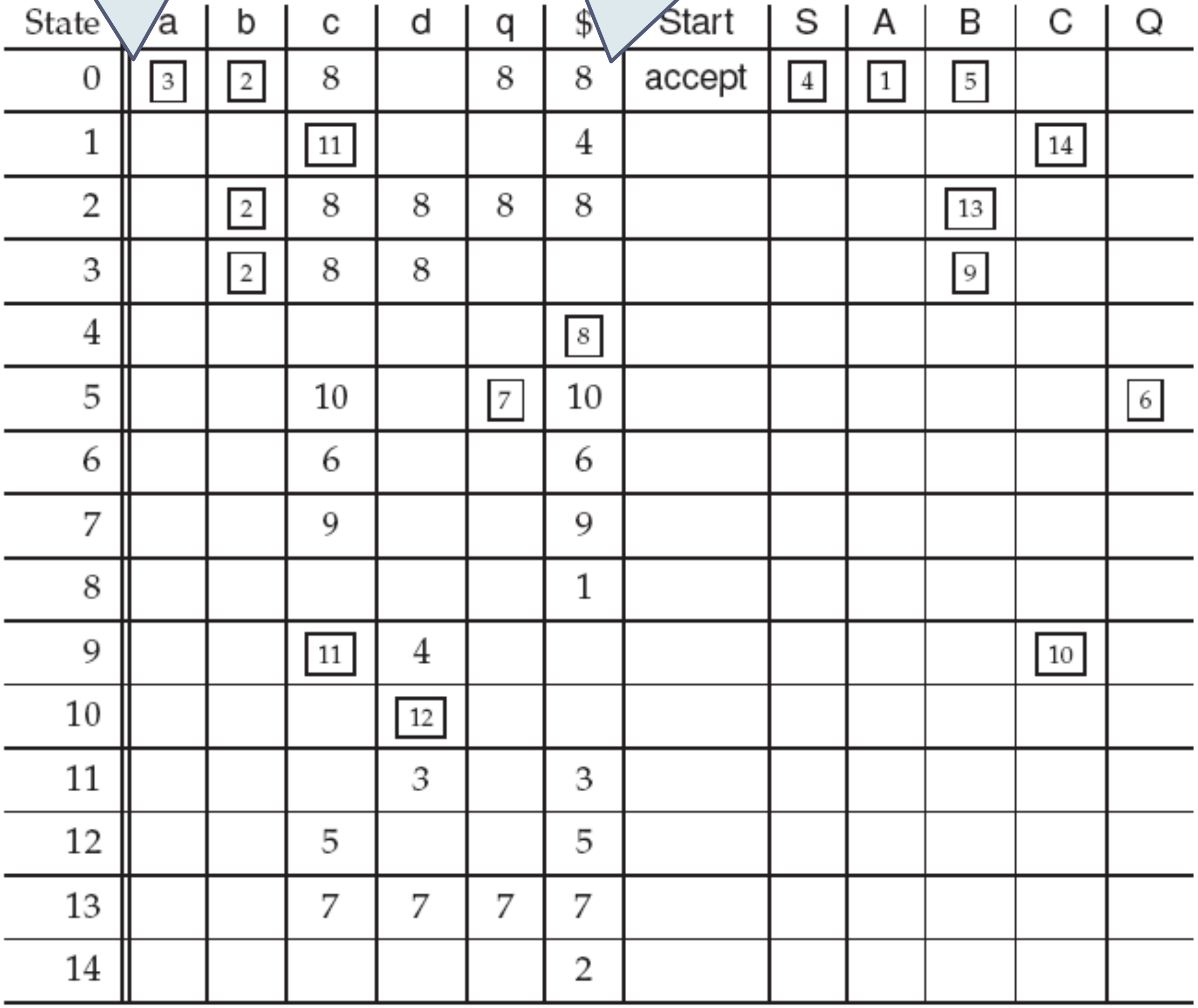
input: S $

0

input: $

0	4
	S

back



State	a	b	c	d	q	$	Start	S	A	B	C	Q
0	$\boxed{3}$	$\boxed{2}$	8		8	8	accept	$\boxed{4}$	$\boxed{1}$	$\boxed{5}$		
1			$\boxed{11}$			4					$\boxed{14}$	
2		$\boxed{2}$	8	8	8	8				$\boxed{13}$		
3		$\boxed{2}$	8	8						$\boxed{9}$		
4						$\boxed{8}$						
5			10		$\boxed{7}$	10						$\boxed{6}$
6			6			6						
7			9			9						
8						1						
9			$\boxed{11}$	4							$\boxed{10}$	
10				$\boxed{12}$								
11				3		3						
12			5			5						
13			7	7	7	7						
14						2						

Figure 6.5: Parse table for the grammar shown in **Slide 45**

Slide 47

shift state 3

Reduce by rule 8

input: $

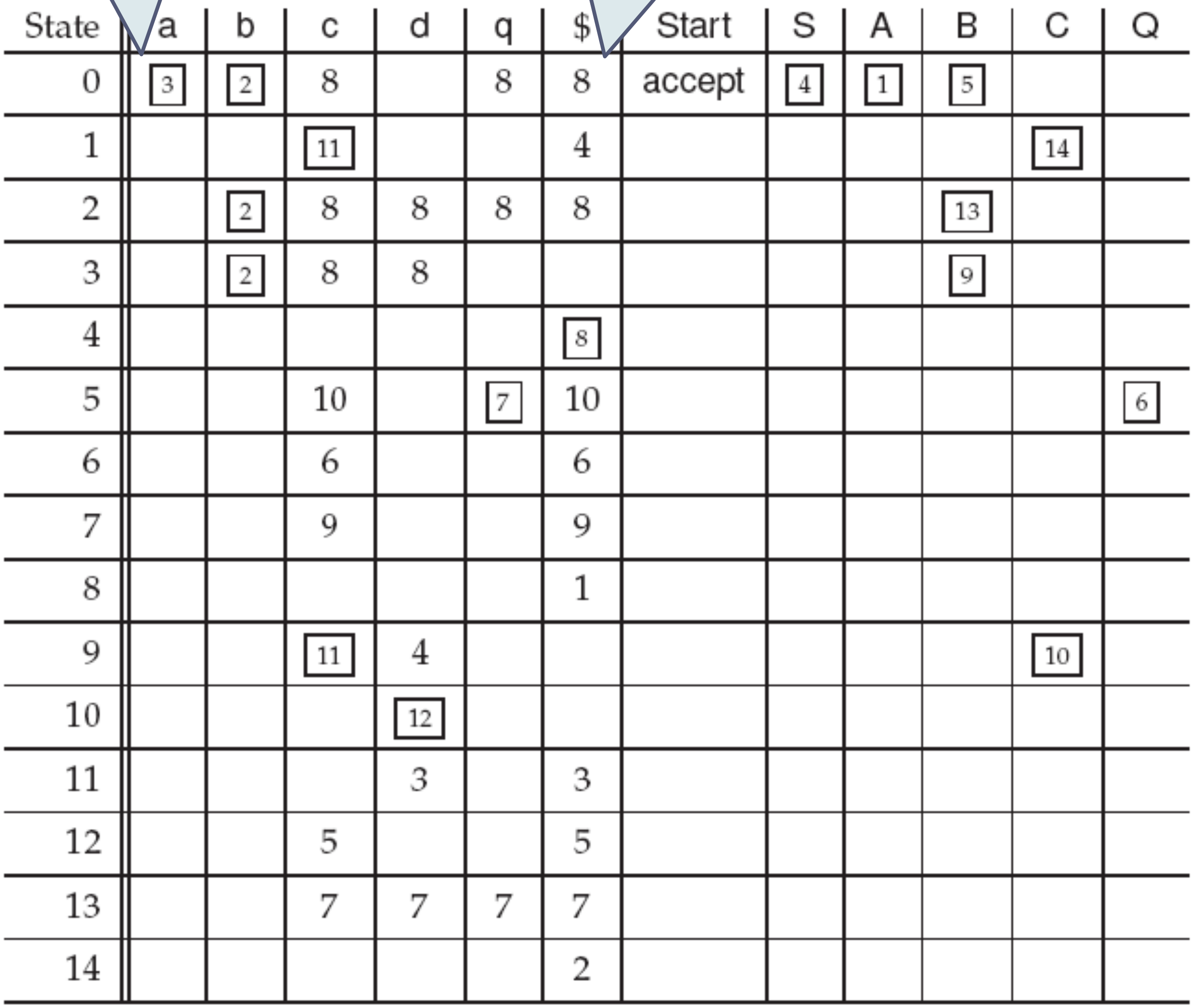
0	4	8
	S	$

1. Start → S $

input: Start $

0

accept



State	a	b	c	d	q	$	Start	S	A	B	C	Q
0	$\boxed{3}$	$\boxed{2}$	8		8	8	accept	$\boxed{4}$	$\boxed{1}$	$\boxed{5}$		
1			$\boxed{11}$			4					$\boxed{14}$	
2		$\boxed{2}$	8	8	8	8				$\boxed{13}$		
3		$\boxed{2}$	8	8						$\boxed{9}$		
4						$\boxed{8}$						
5			10		$\boxed{7}$	10						$\boxed{6}$
6			6			6						
7			9			9						
8						1						
9			$\boxed{11}$	4							$\boxed{10}$	
10				$\boxed{12}$								
11				3		3						
12			5			5						
13			7	7	7	7						
14						2						

Figure 6.5: Parse table for the grammar shown in **Slide 45**

Slide 47

Classes of Bottom-up Parsers

- In practice, bottom-up parsers only depend on a single token of lookahead.
- They can also be described as finite automata (though of a more complicated kind than DFAs)
- There are several methods for constructing parse tables:
  - LR(0): simplest, fails for many practical grammars;
  - LR(1): quite general, can handle almost all practically interesting grammars;
  - LALR(1): faster, slightly weaker variant of LR(1), used by most bottom-up parser generators.

LR(0) Automata and Table Construction

- The table construction process analyzes the grammar.
- Each state corresponds to a row of the parser table.
- Each symbol in the terminal and nonterminal sets corresponds to a column of the table.

Slide 52

- During parsing, we want to keep track of where we are in the grammar.
- To do this, we use **LR(0) items**: an LR(0) item is **a grammar rule with a marker "•"** showing the current progress of the parser in recognizing the RHS of the rule.

LR(0) Automata and Table Construction

- Symbols before the • have already been seen. The first symbol after • is what we expect next. For examples, E → • plus E E, E → plus • E E, E → plus E • E

- For an item A→α • β, the item is called ***initial*** if α →λ, and ***final*** if β →λ. A final item for the start symbol is called accepting.

E. g.

Start → • S $ // an initial item

S → A C • // a final item

LR(0) Automata and Table Construction

- An LR(0) state is **a set of LR(0) items**, which is ***closed*** in the sense that if the state contains an item with a nonterminal A immediately following the marker, we add the initial items for A, i.e., items for all rules of A with the marker at the beginning of the RHS. This is called taking ***the closure of the item***.

E.g., For an item S → • A,

we add

A → • a B d, A→ • B Q,

B → • b

i.e. closure = {S → • A, A → • a B d,

A→ • B Q, B → • b}

S → A

A → a B d

| B Q

B → b

Q → q

LR(0) Automata and Table Construction

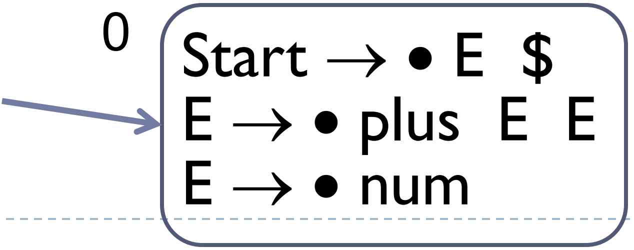
- We describe the parsing process as a finite automaton.
- The start state is the closure of the initial items of the start symbol:

For the grammar on the right, there is only one initial item for the Start symbol:

Start → • E $

Start → E $
E → plus E E
  | num

Taking the closure, we get the set of items in state 0:



0

Start → • E $
E → • plus E E
E → • num

In slides 48, 51, 52, state 0 is on TOS

LR(0) Automata and Table Construction

- For each symbol γ that appears to the right of the marker of an item (or items), we can **shift** over it and transition to a new state:
  - replace an item of the form $A \to \alpha \bullet \gamma \beta$ with $A \to \alpha \gamma \bullet \beta$
  - throw away all other items;
  - take closure

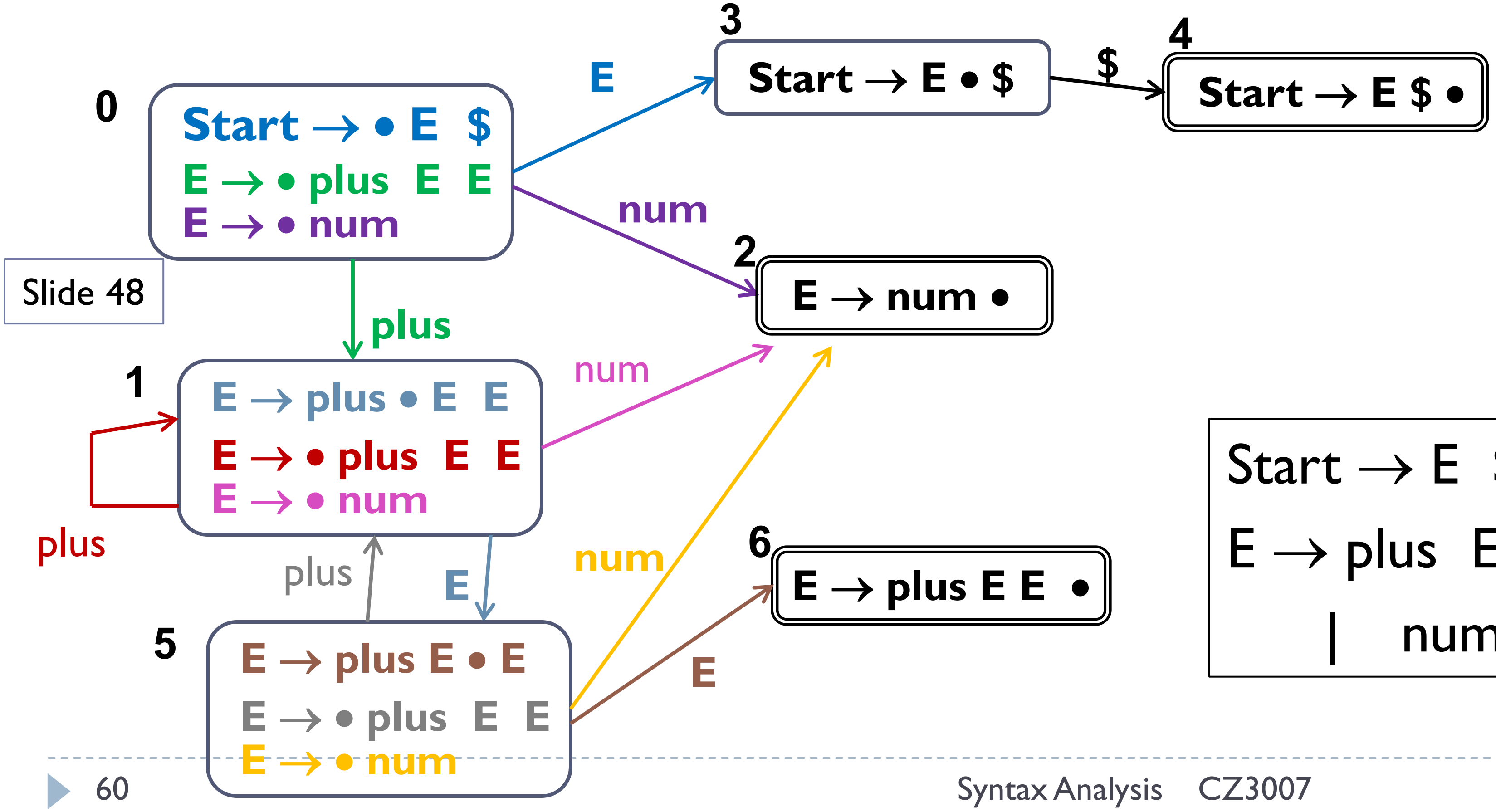
- Transitions are labelled by γ which is either a terminal or a nonterminal.

- If there is a final item of the form $A \to \alpha \bullet$, we can **reduce**.

- An accepting state is one that contains a final item for start symbol.

LR(0) Automata and Table Construction

- Continue from State 0 of our finite automata in slide 58, each of the three items will lead to a new state. Each new state leads to more new states.



Start → E $
E → plus E E
| num

LR(0) Automata and Table Construction

- The LR(0) automaton needs to be used together with a stack of states; this kind of automaton is known as a ***pushdown automaton.***
- The state of the parser is the state on top of the parser stack.

Slide 51

- Actions taken during bottom-up parsing are of four types:
  - **shift**(i): consume next input symbol, push state i onto state stack;
  - **reduce**(A→γ): reduce by rule A→γ, i.e., pop |γ| states from the stack and consider A as next input symbol;
  - **accept**: report that input was parsed successfully;
  - **error**: report a parse error

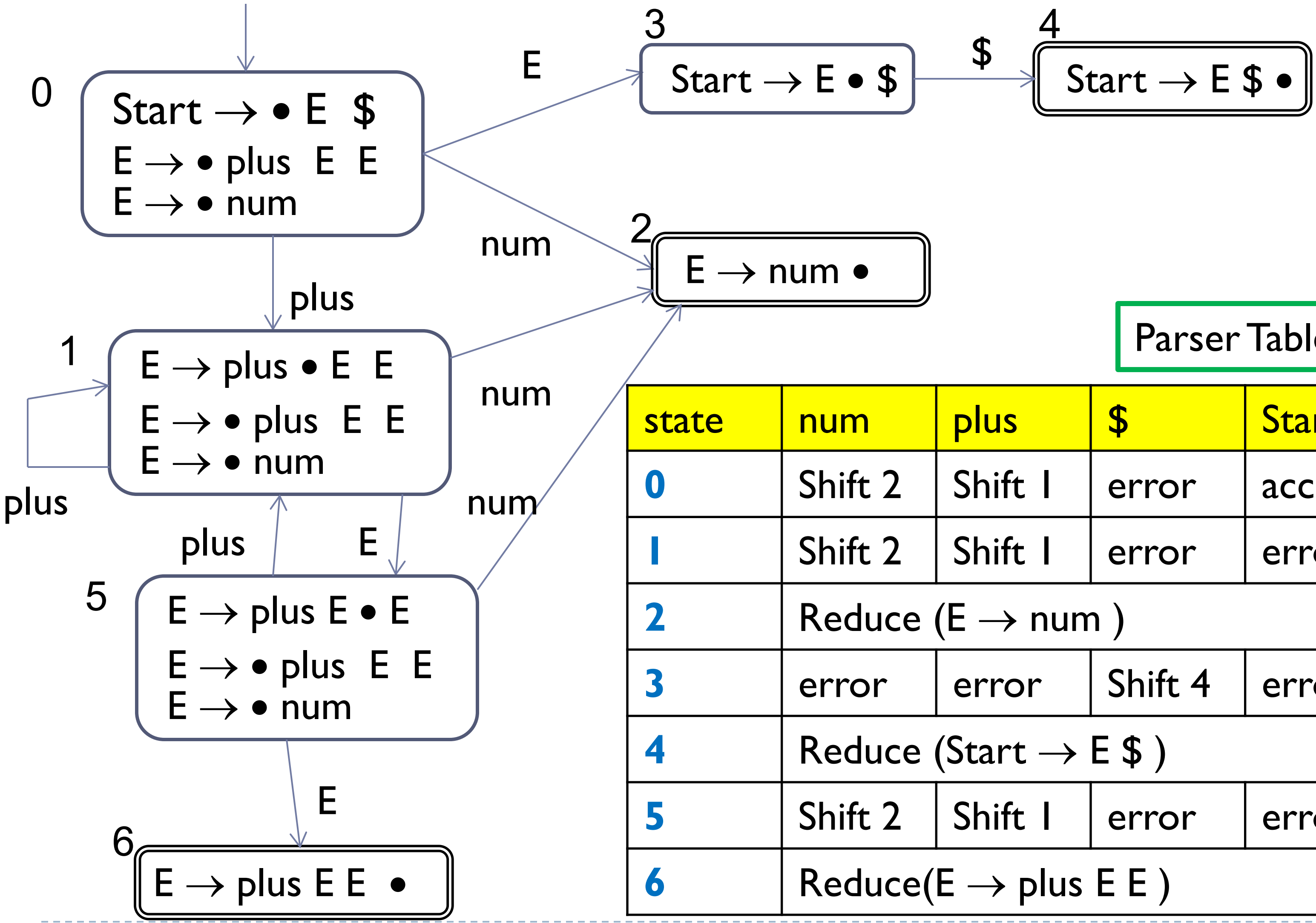
Slide 47

LR(0) Automata and Table Construction

- An LR(0) parse table is a compact representation of an LR(0) automaton
- Rows are indexed by states, columns by symbols; cell in row r , column c contain a single parsing action to take when encountering input symbol c in state r
- Constructing a parse table from an LR(0) automaton is easy:
  - For every transition from state s to state s' labelled with symbol x, enter **shift**(s') into the cell in row s, column x
  - If state s contains a final item $A \rightarrow \beta \bullet$, enter **reduce**($A \rightarrow \beta$) into all cells of row s
  - For cell (0, StartSymbol), enter **accept**
  - Enter **error** into any remaining empty cells

back

Example



back

Parser Table

state	num	plus	$	Start	E
0	Shift 2	Shift 1	error	accept	Shift 3
1	Shift 2	Shift 1	error	error	Shift 5
2	Reduce (E → num)				
3	error	error	Shift 4	error	error
4	Reduce (Start → E $)				
5	Shift 2	Shift 1	error	error	Shift 6
6	Reduce(E → plus E E)				

Conflicts

- Sometimes when trying to construct an LR(0) parse table we end up with two different actions in the same cell; this is known as a conflict.

- There are two kinds of conflicts:
  - **shift-reduce conflict**: the same cell contains both a **shift()** action and a **reduce()** action;
  - **reduce-reduce conflict**: the same cell contains two different **reduce()** actions.

- **Question: Can there be a shift-shift conflict?**

Shift-reduce Conflict

- **shift-reduce conflict**: if a state contains both a non-final item A → β • γ (a **shift()** action) and a final item A → β • (a **reduce()** action)

  E.g. A state has these two items:

  IfStatement → IF Cond THEN StatList • ELSE StatList

  IfStatement → IF Cond THEN StatList •

- Example:

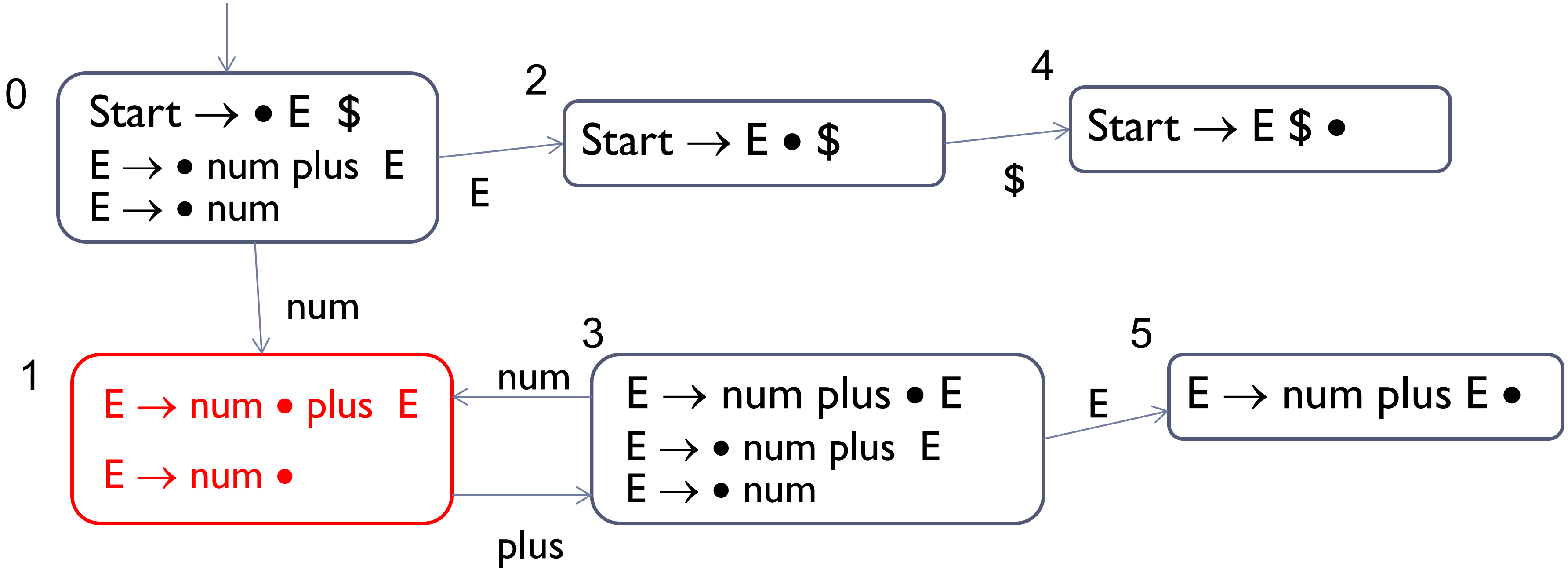
  Start → E $

  E → num plus E

  | num

back

Example: shift-reduce conflict



state	num	plus	$	Start	E
0	Shift 1	error	error	accept	Shift 2
1		Shift/reduce?			
2	…				

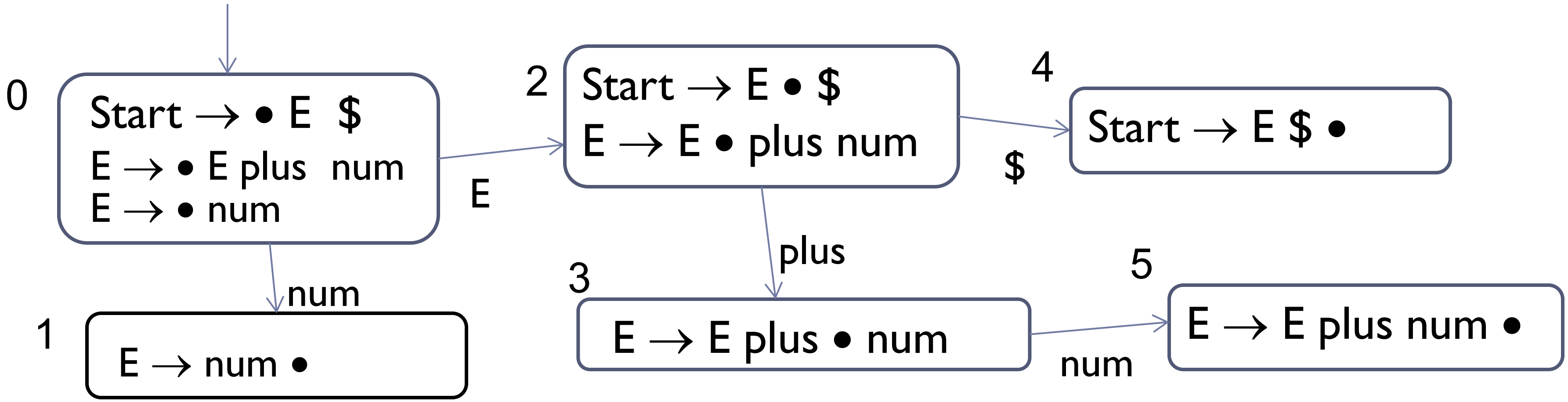
back

Shift-reduce Conflict

- **A shift-reduce conflict** may be eliminated by rewriting the grammar. For example,

  Rewrite to

Start → E $	Start → E $
E → num plus E	E → E plus num
| num	| num



An Ambiguous Grammar

- Ambiguous grammars always lead to conflicts. For example,

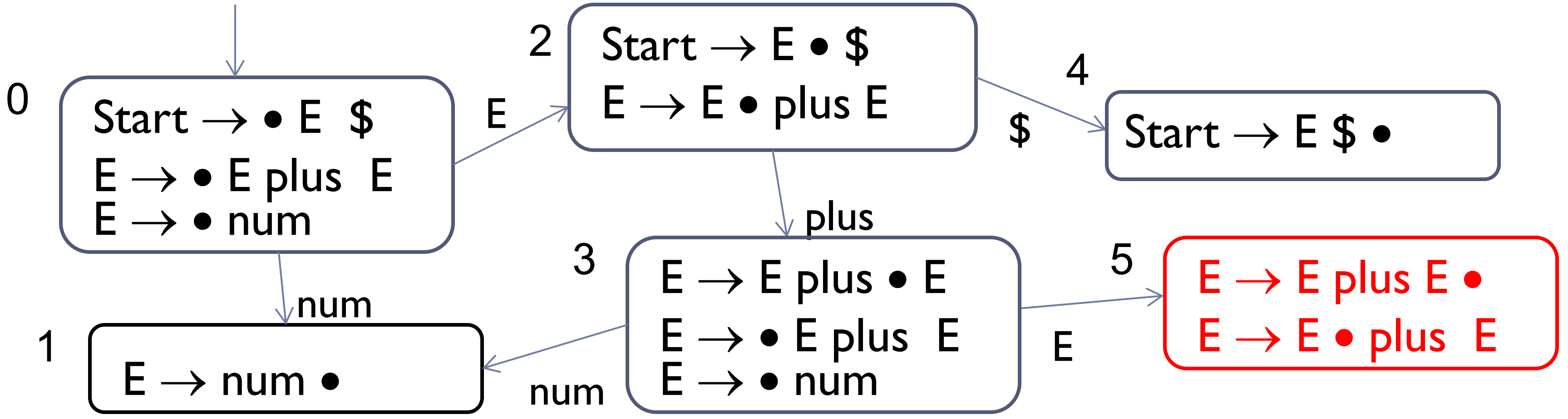
Start → E $

E → E plus E

| num

Rewriting the grammar will solve the problem in this case.

LR(0) automaton:



LALR(1) – Look Ahead LR with one token lookahead

back

- Due to its balance of power and efficiency, LALR(1) is the most popular LR table-building method.
  - For every transition from state s to state s' labelled with symbol x, enter **shift**(s') into the cell in row s, column x
  - **If state s contains a final item A→ β •, enter reduce(A→β) into the cells of row s for each token T ∊ itemFollow(*s*, *item*)**
  - For cell (0, StartSymbol), enter **accept**
  - Enter **error** into any remaining empty cells

Compare: slide 62

- **itemFollow()** keeps track of the symbols that can follow the ***item*** after reduction *from this state*.

Slide 66

LALR Propagation Graph

Slide 63

- Consider the LR(0) table, the pair (s, $A \rightarrow \alpha \bullet \beta$) suffices to identify an item $A \rightarrow \alpha \bullet \beta$ that occurs in state s.
- Each item in an LR(0) table is represented by a vertex in the **LALR propagation graph**.
- We will compute itemFollow() for each item.
- The LALR table is constructed with reference to the itemFollow() computed for the items.
- The propagation graph will not be retained after constructing the LALR table.

Generating the Propagation Graph

A. Setup:

   1. Create the LR(0) finite automaton.
   2. For each (state, item), create a vertex *v* in the graph.
   3. Initialize all itemFollow[*v*] to ∅.
   4. Initialize itemFollow[(0, StartSymbol Productions)] = {$}

B. Build the propagation graph

C. Propagate itemFollow[ ]

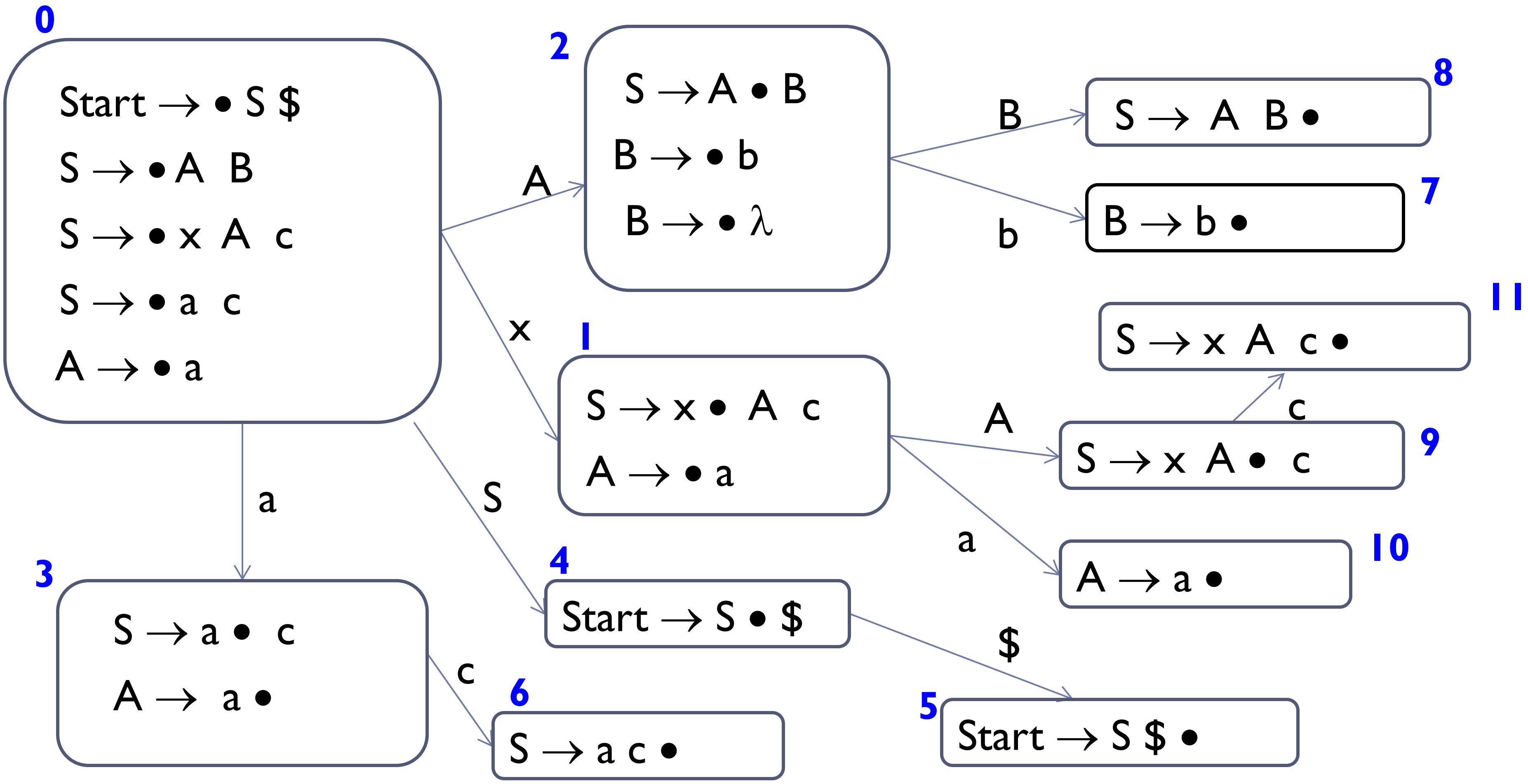
back

Generating the Propagation Graph (Example)

P_1	Start	$\rightarrow$ S \$
P_2	S	$\rightarrow$ A B
P_3		| a c
P_4		| x A c
P_5	A	$\rightarrow$ a
P_6	B	$\rightarrow$ b
P_7		| λ

back

A: Setup (create LR(0) finite automaton)



0

Start → • S $

S → • A B

S → • x A c

S → • a c

A → • a

2

S → A • B

B → • b

B → • λ

8

S → A B •

7

B → b •

1

S → x • A c

A → • a

11

S → x A c •

9

S → x A • c

10

A → a •

3

S → a • c

A → a •

4

Start → S • $

6

S → a c •

5

Start → S $ •

A: Setup (create vertices, initialize itemFollow)

1	0, Start → • S $	{$}
2	0, S → • A B	∅
3	0, S → • x A c	∅
4	0, S → • a c	∅
5	0, A → • a	∅
11	3, S → a • c	∅
12	3, A → a •	∅
8	2, S → A • B	∅
9	2, B → • b	∅
10	2, B → • λ	∅
6	1, S → x • A c	∅
7	1, A → • a	∅
13	4, Start → S • $	∅
15	6, S → a c •	∅
17	8, S → A B •	∅
16	7, B → b •	∅
20	11, S → x A c •	∅
18	9, S → x A • c	∅
19	10, A → a •	∅
14	5, Start → S $ •	∅

B:Building the Propagation Graph (ideas)

- For item (s, $A \to \alpha\bullet B\gamma$), any symbol in First($\gamma$) can follow each closure item (s, $B \to \bullet\delta$) (2)
  - Put First(γ) into the itemFollow set of each initial item of B

Slide 66

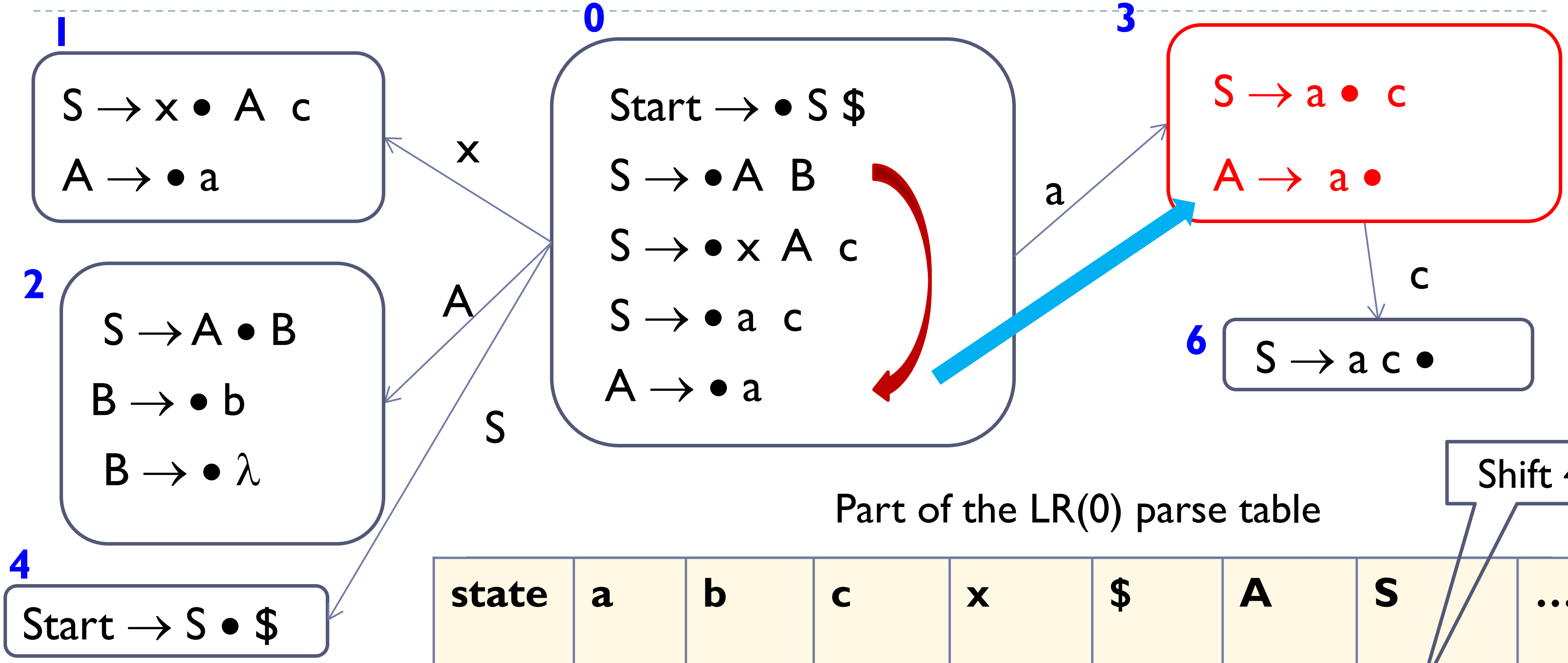
- A propagation edge is placed from an item (s, $A \to \alpha\bullet B\gamma$) to an item (t, $A \to \alpha B\bullet\gamma$) (1)

back

- For item (s, $A \to \alpha\bullet B\gamma$), when $\gamma \Rightarrow^{*} \lambda$, any symbol that can follow A can also follow B (3)
  - Place a propagation edge from (s, $A \to \alpha\bullet B\gamma$) to (s, $B \to \bullet\delta$)

itemFollow() keeps track of the symbols that can follow the ***item*** after reduction *from this state*

itemFollow() keeps track of the symbols that can follow the ***item*** after reduction *from this state*



Part of the LR(0) parse table

state	a	b	c	x	$	A	S	…
0	S3	err	err	S1	err	S2	S4	…
1								
2								
3	err	R5	S6	err	R5	err	err	

Shift 4

Input scenarios:

1) a $
2) a b $
3) a c $

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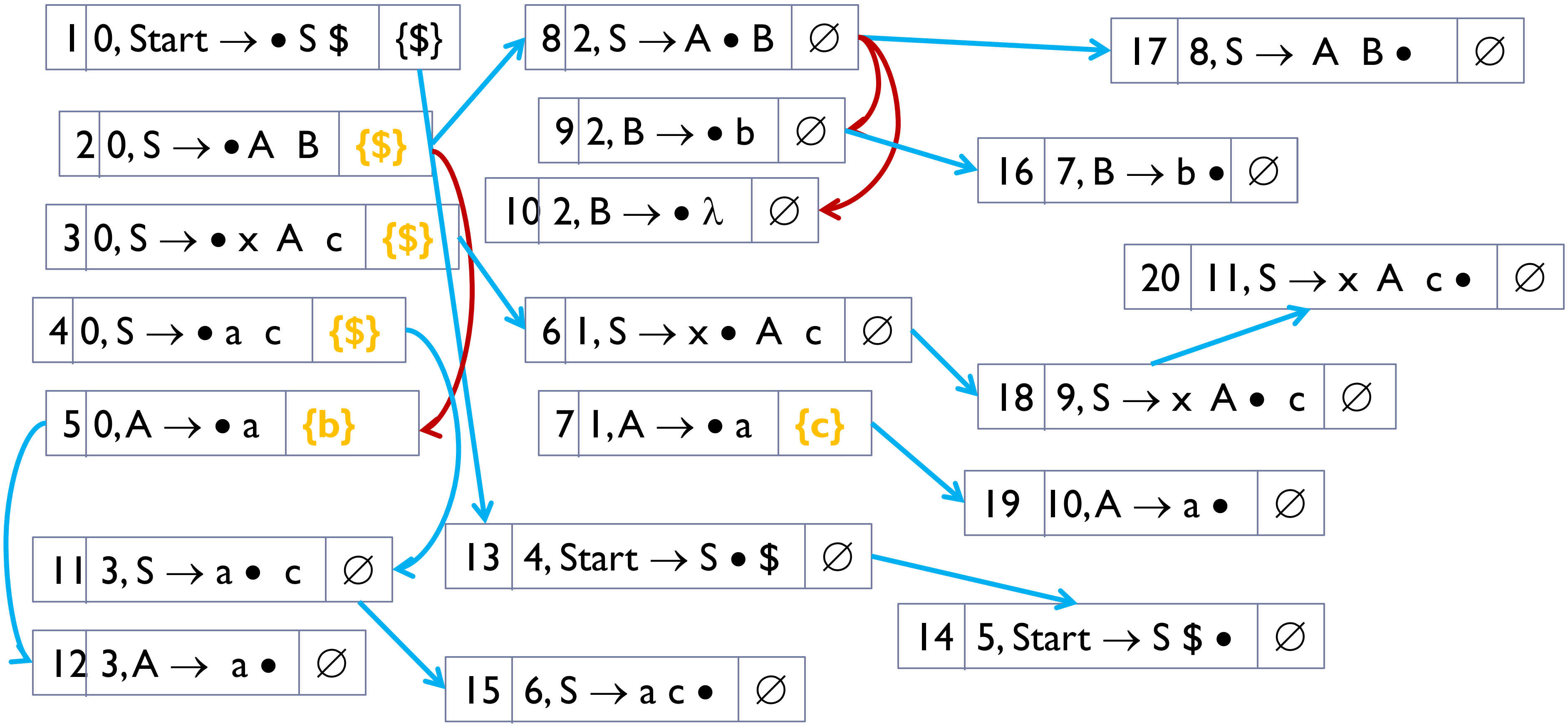
B: Building the Propagation Graph



1 | 0, Start → • S $ | {$}

2 | 0, S → • A B | {$}

3 | 0, S → • x A c | {$}

4 | 0, S → • a c | {$}

5 | 0, A → • a | {b}

11 | 3, S → a • c | ∅

12 | 3, A → a • | ∅

8 | 2, S → A • B | ∅

9 | 2, B → • b | ∅

10 | 2, B → • λ | ∅

6 | 1, S → x • A c | ∅

7 | 1, A → • a | {c}

13 | 4, Start → S • $ | ∅

15 | 6, S → a c • | ∅

17 | 8, S → A B • | ∅

16 | 7, B → b • | ∅

20 | 11, S → x A c • | ∅

18 | 9, S → x A • c | ∅

19 | 10, A → a • | ∅

14 | 5, Start → S $ • | ∅

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B: Building the Propagation Graph

```
For each state s
    For each item of form $A \to \alpha \bullet B \gamma$ in s
        vertex u = (s, $A \to \alpha \bullet B \gamma$)
        vertex v = (s', $A \to \alpha B \bullet \gamma$), successor of u on shift(B)
        Add edge (u, v)

        For each vertex w of form (s, $B \to \bullet \delta$)
            Add first($\gamma$) to itemFollow[w]
            if allDeriveEmpty($\gamma$)
                add edge (u, w)
```

s' = Table[s][B]

1

2

3

Slide 36

C: Propagating itemFollw [ ]

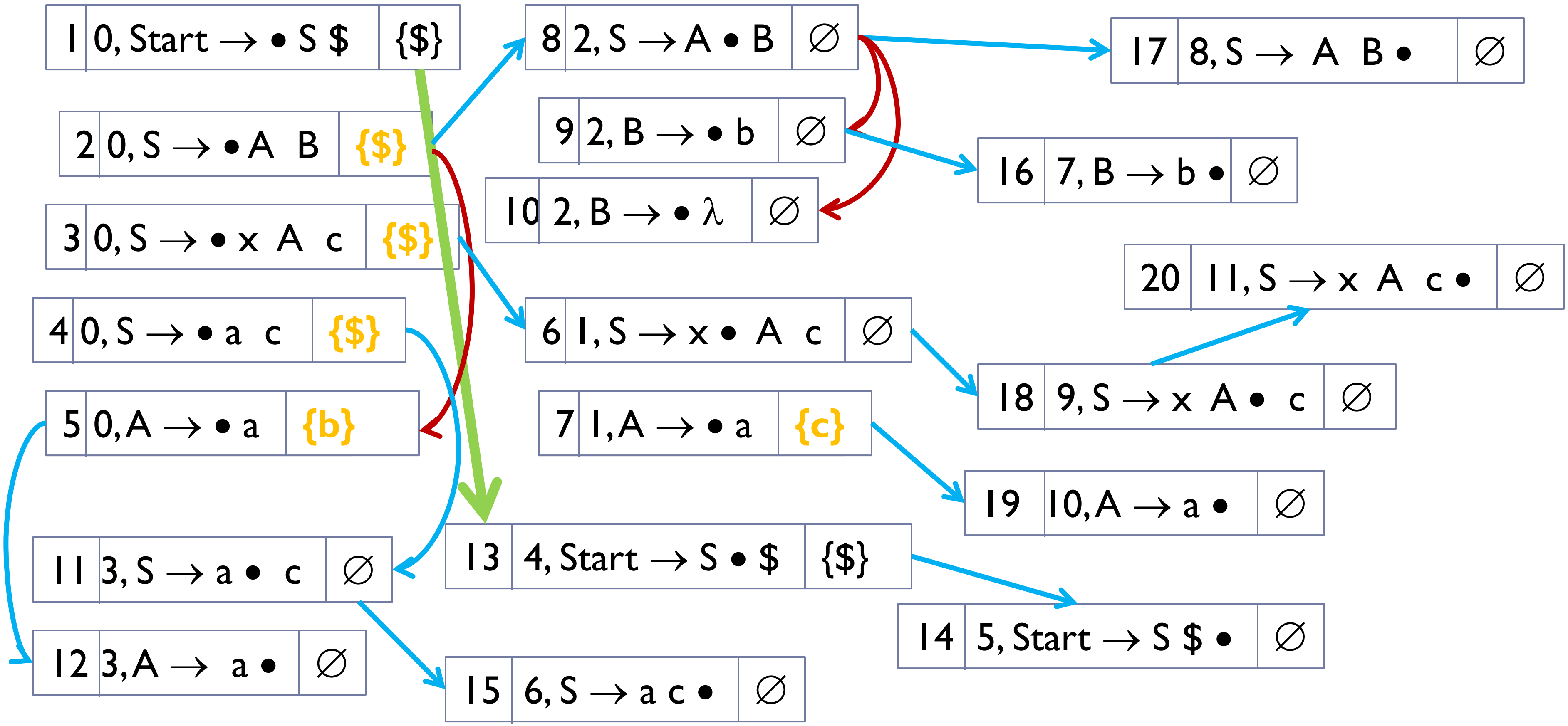
Slide 71

```
While (making progress)
    For each edge (u,v)
        add ItemFollow(u) to ItemFollow(v)
```

4

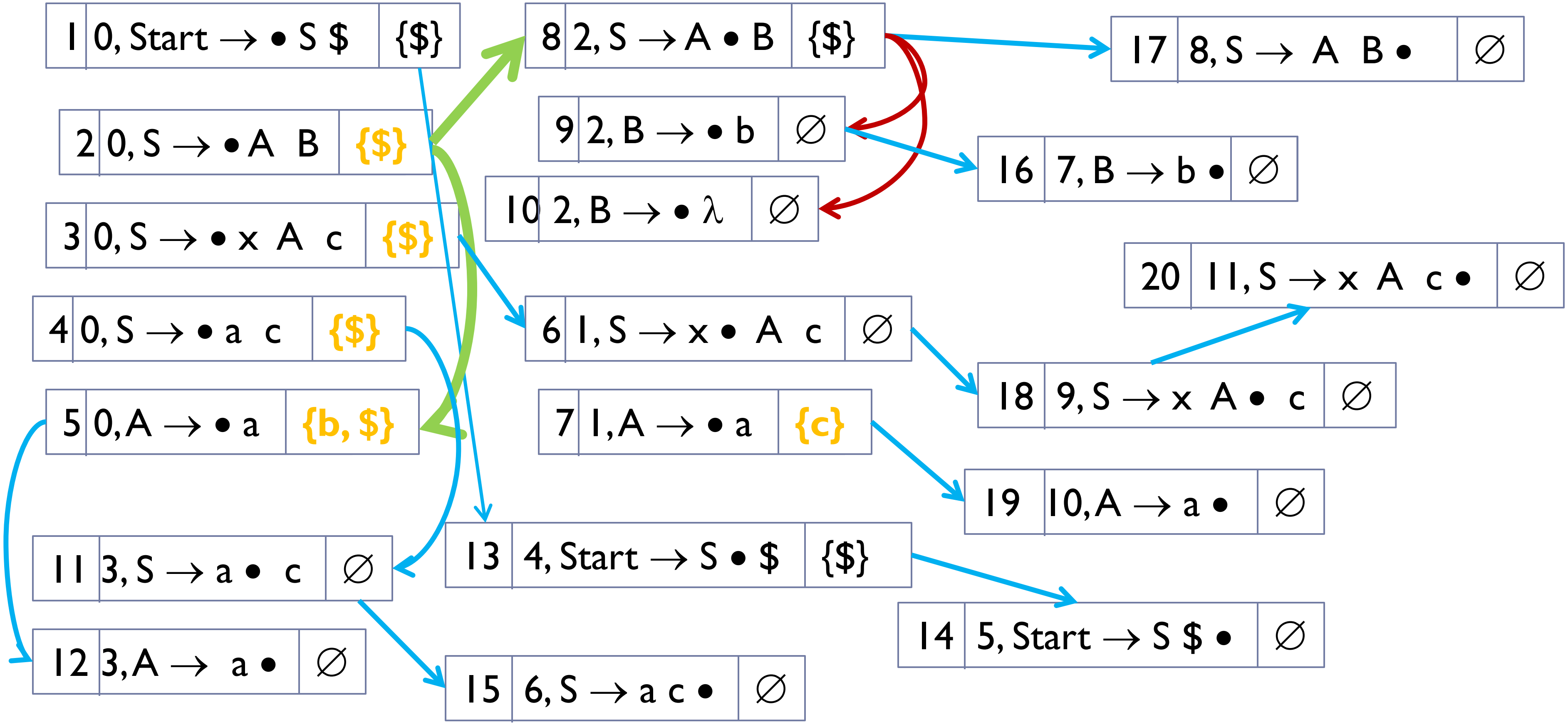
- In general, multiple passes can be required
- In practice, LALR(1) lookahead computations converge quickly, usually in one or two passes
- LALR(1) is a powerful parsing method
- LALR(1) grammars are available for all popular programming languages

C: Propagating itemFollw [ ]



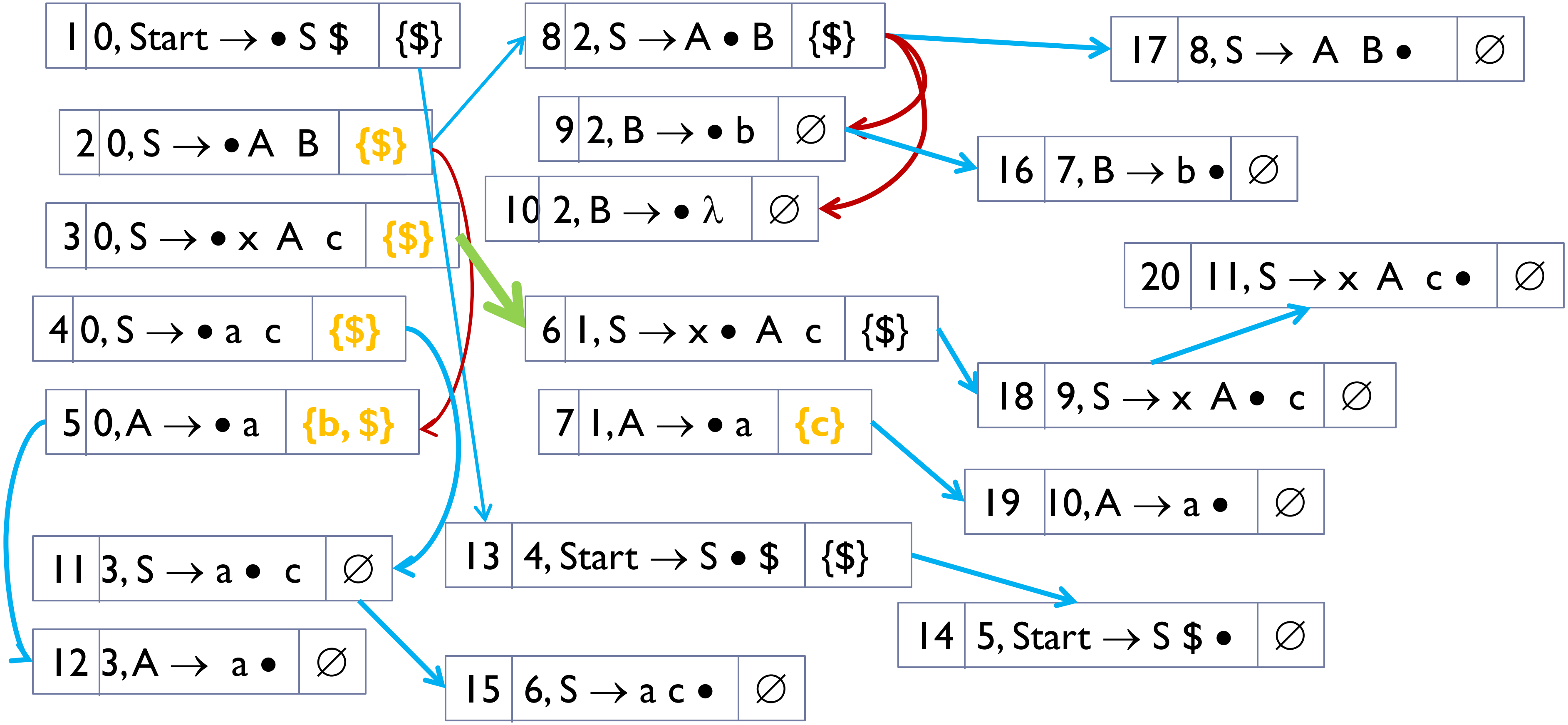
1	0, Start → • S $	{$}
2	0, S → • A B	{$}
3	0, S → • x A c	{$}
4	0, S → • a c	{$}
5	0, A → • a	{b}
6	1, S → x • A c	∅
7	1, A → • a	{c}
8	2, S → A • B	∅
9	2, B → • b	∅
10	2, B → • λ	∅
11	3, S → a • c	∅
12	3, A → a •	∅
13	4, Start → S • $	{$}
14	5, Start → S $ •	∅
15	6, S → a c •	∅
16	7, B → b •	∅
17	8, S → A B •	∅
18	9, S → x A • c	∅
19	10, A → a •	∅
20	11, S → x A c •	∅

C: Propagating itemFollw [ ]



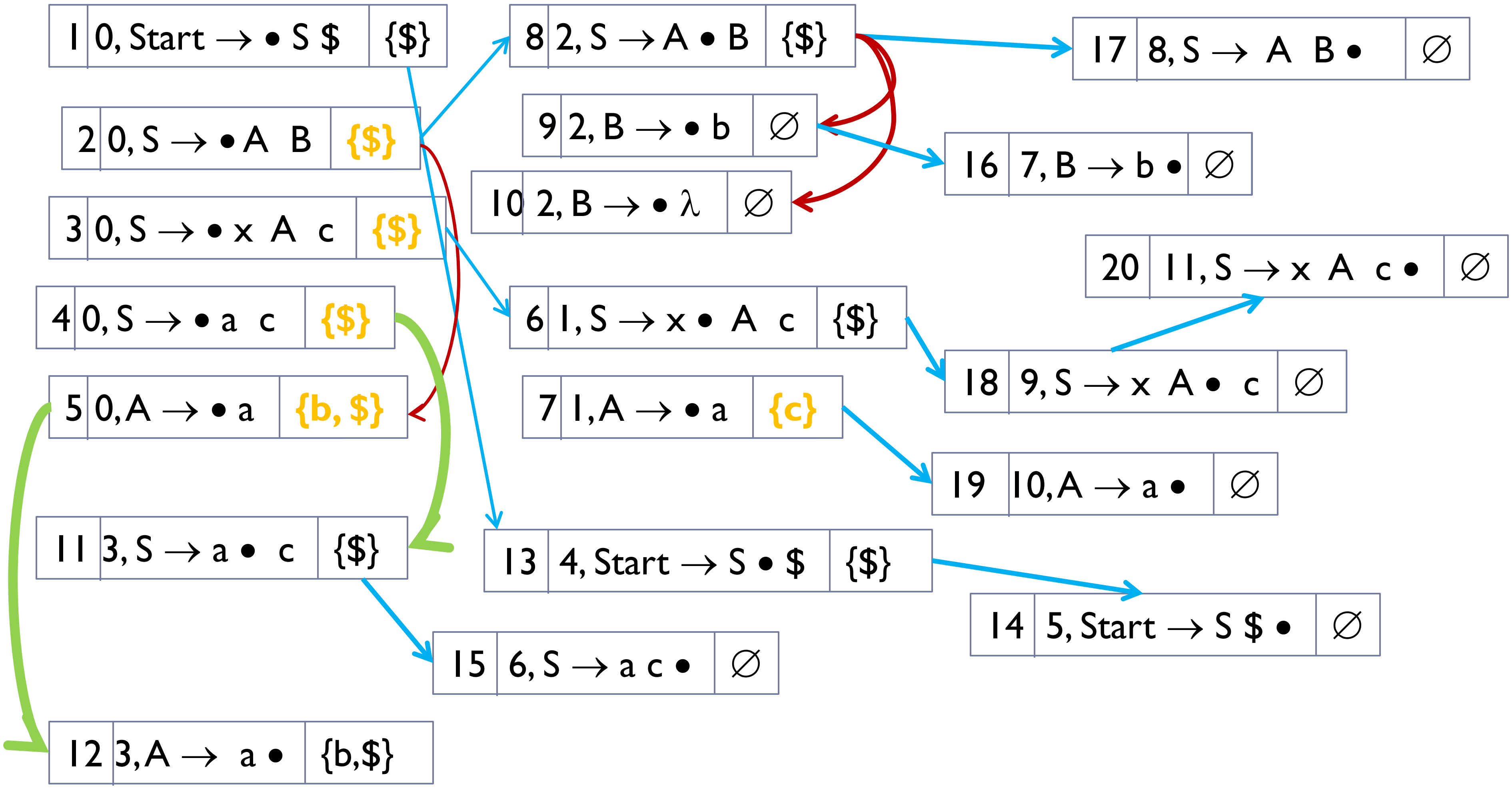
1	0, Start → • S $	{$}
2	0, S → • A B	{$}
3	0, S → • x A c	{$}
4	0, S → • a c	{$}
5	0, A → • a	{b, $}
6	1, S → x • A c	∅
7	1, A → • a	{c}
8	2, S → A • B	{$}
9	2, B → • b	∅
10	2, B → • λ	∅
11	3, S → a • c	∅
12	3, A → a •	∅
13	4, Start → S • $	{$}
14	5, Start → S $ •	∅
15	6, S → a c •	∅
16	7, B → b •	∅
17	8, S → A B •	∅
18	9, S → x A • c	∅
19	10, A → a •	∅
20	11, S → x A c •	∅

C: Propagating itemFollw [ ]



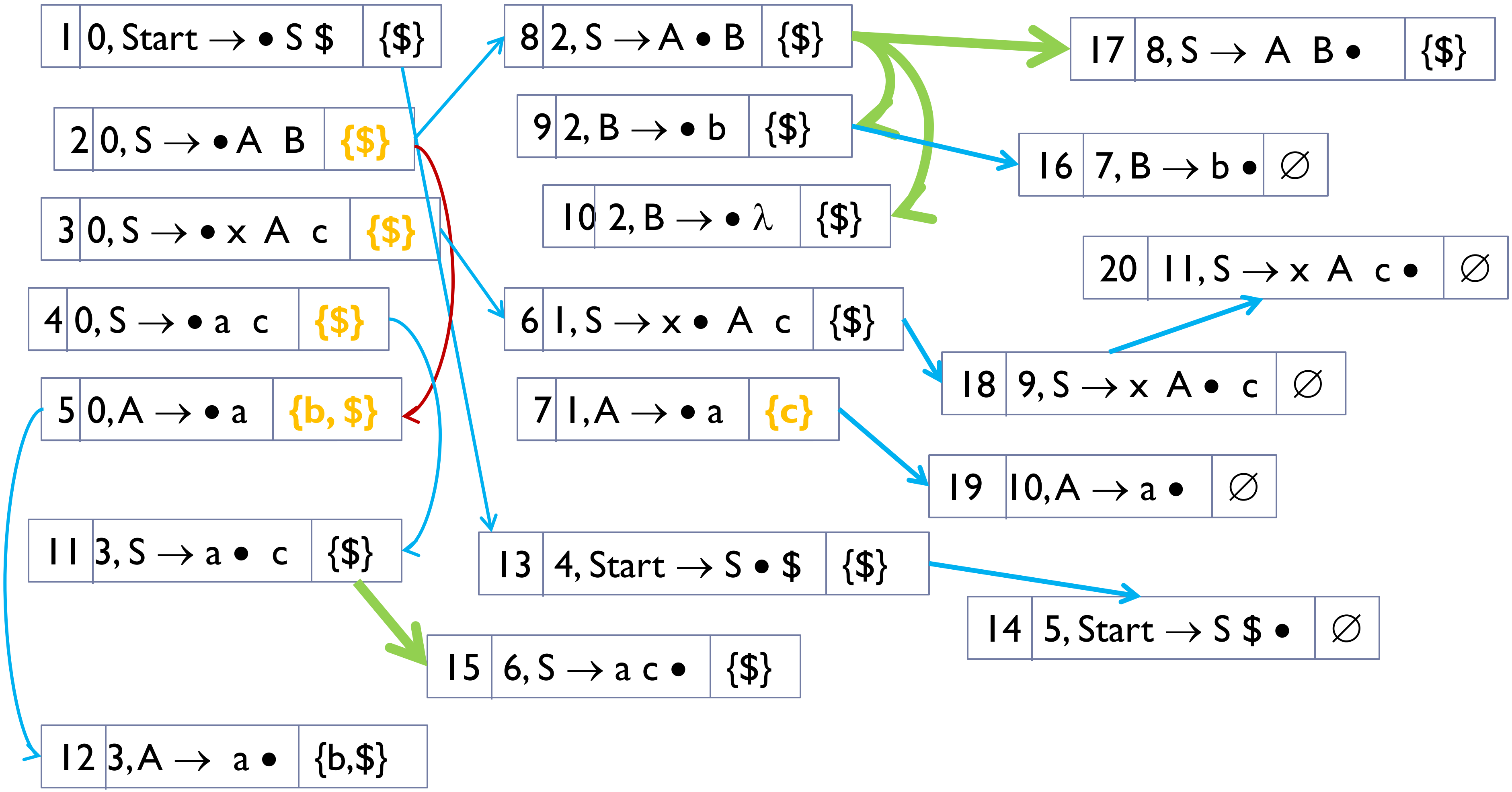
1	0, Start → • S $	{$}
2	0, S → • A B	{$}
3	0, S → • x A c	{$}
4	0, S → • a c	{$}
5	0, A → • a	{b, $}
6	1, S → x • A c	{$}
7	1, A → • a	{c}
8	2, S → A • B	{$}
9	2, B → • b	∅
10	2, B → • λ	∅
11	3, S → a • c	∅
12	3, A → a •	∅
13	4, Start → S • $	{$}
14	5, Start → S $ •	∅
15	6, S → a c •	∅
16	7, B → b •	∅
17	8, S → A B •	∅
18	9, S → x A • c	∅
19	10, A → a •	∅
20	11, S → x A c •	∅

C: Propagating itemFollw [ ]



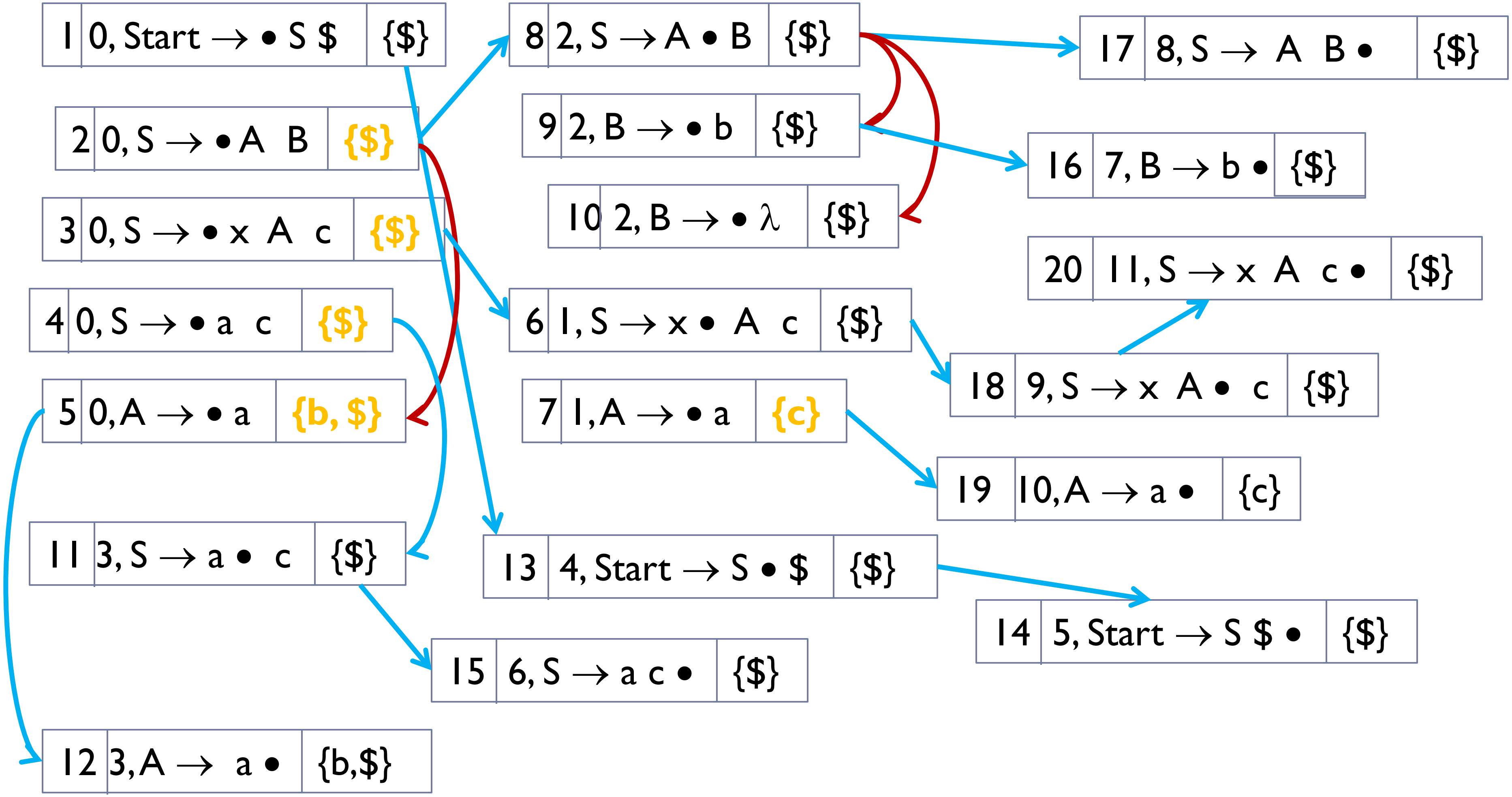
1	0, Start → • S $	{$}
2	0, S → • A B	{$}
3	0, S → • x A c	{$}
4	0, S → • a c	{$}
5	0, A → • a	{b, $}
6	1, S → x • A c	{$}
7	1, A → • a	{c}
8	2, S → A • B	{$}
9	2, B → • b	∅
10	2, B → • λ	∅
11	3, S → a • c	{$}
12	3, A → a •	{b,$}
13	4, Start → S • $	{$}
14	5, Start → S $ •	∅
15	6, S → a c •	∅
16	7, B → b •	∅
17	8, S → A B •	∅
18	9, S → x A • c	∅
19	10, A → a •	∅
20	11, S → x A c •	∅

C: Propagating itemFollw [ ]



1	0, Start → • S $	{$}
2	0, S → • A B	{$}
3	0, S → • x A c	{$}
4	0, S → • a c	{$}
5	0, A → • a	{b, $}
6	1, S → x • A c	{$}
7	1, A → • a	{c}
8	2, S → A • B	{$}
9	2, B → • b	{$}
10	2, B → • λ	{$}
11	3, S → a • c	{$}
12	3, A → a •	{b,$}
13	4, Start → S • $	{$}
14	5, Start → S $ •	∅
15	6, S → a c •	{$}
16	7, B → b •	∅
17	8, S → A B •	{$}
18	9, S → x A • c	∅
19	10, A → a •	∅
20	11, S → x A c •	∅

C: Propagating itemFollw [ ]

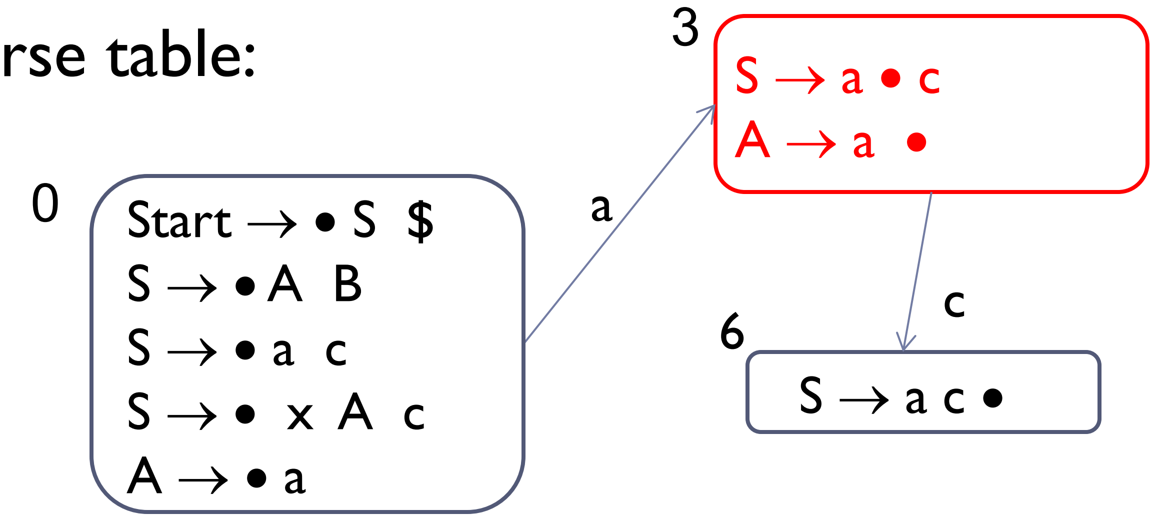


1	0, Start → • S $	{$}
2	0, S → • A B	{$}
3	0, S → • x A c	{$}
4	0, S → • a c	{$}
5	0, A → • a	{b, $}
11	3, S → a • c	{$}
12	3, A → a •	{b,$}
8	2, S → A • B	{$}
9	2, B → • b	{$}
10	2, B → • λ	{$}
6	1, S → x • A c	{$}
7	1, A → • a	{c}
13	4, Start → S • $	{$}
15	6, S → a c •	{$}
17	8, S → A B •	{$}
16	7, B → b •	{$}
20	11, S → x A c •	{$}
18	9, S → x A • c	{$}
19	10, A → a •	{c}
14	5, Start → S $ •	{$}

The Result:

The LALR(1) parse table:

Start → S $
S → A B
| a c
| x A c
A → a
B → b
| λ



0

Start → • S $
S → • A B
S → • a c
S → • x A c
A → • a

a

3

S → a • c
A → a •

c

6

S → a c •

Slide 69

itemFollow(3, A → a •) = {b, $}

state	…	b	c	$	…
…	…				
3		Reduce(A→ a)	Shift 6	Reduce(A→ a)	
…	…				

back

Parser Generator Beaver

- Beaver is a LALR(1) parser generator that generates parsers written in Java.
- Beaver's syntax is very similar to the notation we have been using for context free grammars, except that Beaver uses = where we have used →.

Slide 6

- The rules for a nonterminal must be terminated by a semicolon.
- The directive %**terminals** on the first line declares the set of terminals used by the grammar.
- Beaver implicitly assumes that every name that isn't declared to be a terminal is a nonterminal.

Example of a Beaver specification: grammar for arithmetic expressions

```
%terminals PLUS, MINUS, MUL, DIV, NUMBER, LPAREN, RPAREN;
%goal Expr;
Expr = Expr PLUS Term
        | Expr MINUS Term
        | Term
        ;
Term = Term MUL Factor
        | Term DIV Factor
        | Factor
        ;
Factor = NUMBER
        | LPAREN Expr RPAREN
        ;
```

Some review questions/tasks

1. What does a syntax analyzer do?
2. What are the input and output of a syntax analyzer?
3. What is a context free grammar used for in a compiler?
4. Write the pseudocode for a recursive descent parser for the context free grammar of Expr on slide 6 (reference slide 21). Assume the methods peek(), match() and predict() are provided.
5. Follow the bottom-up parsing engine on slide 47 and use the parse table on slide 48 for the grammar on slide 45 to trace the parsing process for the input "adc$".