NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2016/17 Semester 1 MH2500 Probability and Introduction to Statistics Tutorial 11

For the tutorial on 3 November, let us discuss

- Ex. 4.7. 70, 73, 77, 80, 85, 96.

Ex. 4.7.70 If X and Y are independent, show that $E(X|Y = y) = E(X)$.

Ex. 4.7.73. A fair coin is tossed n times, and the number of heads, N, is counted. The coin is then tossed N more times. Find the expected total number of heads generated by this process.

Ex. 4.7.77. Let X and Y have the joint density

$$f(x, y) = e^{-y}, \qquad 0 \le x \le y.$$

a. Find $\mathrm{Cov}(X, Y)$ and the correlation of X and Y.
b. Find $E(X|Y = y)$ and $E(Y|X = x)$.
c. Find the density functions of the random variables $E(X|Y)$ and $E(Y|X)$.

Ex. 4.7.80. Let X be a continuous random variable with density function

$$f(x) = 2x, \qquad 0 \le x \le 1.$$

Find the moment-generating function of X, $M(t)$, and verify that $E(X) = M'(0)$ and that $E(X^2) = M''(0)$.

Ex. 4.7.85. Find the mgf of a geometric random variable, and use it to find the mean and the variance.

Ex. 4.7.96. If X and Y have a joint distribution, their joint moment-generating function is defined as

$$M_{XY}(s, t) = E(e^{sX+tY}),$$

which is a function of two variables, s and t.

Show how to find $E(XY)$ from the joint moment-generating function of X and Y.