

Validity of: $P \Rightarrow Q \Leftrightarrow \neg P \vee Q$

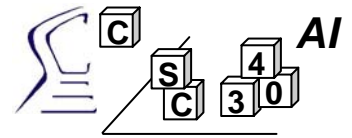
truth table:

P	Q	$\neg P$	$\neg P \vee Q$	$P \Rightarrow Q$	$P \Rightarrow Q \Leftrightarrow \neg P \vee Q$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Validity of: $P \Leftrightarrow Q \Leftrightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P)$

truth table:

P	Q	$P \Leftrightarrow Q$	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$	$P \Leftrightarrow Q \Leftrightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P)$
T	T	T	T	T	T	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T



Validity of: $P \Leftrightarrow Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

no truth table \rightarrow rewriting rules / equivalences

using (ii):

$$P \Leftrightarrow Q \Leftrightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

then using (i):

$$P \Leftrightarrow Q \Leftrightarrow (\neg P \vee Q) \wedge (\neg Q \vee P)$$

using distributivity of \wedge over \vee :

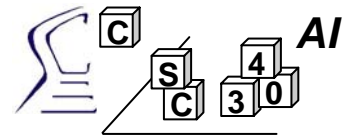
$$P \Leftrightarrow Q \Leftrightarrow (\neg P \wedge \neg Q) \vee (\neg P \wedge P) \vee (Q \wedge \neg Q) \vee (Q \wedge P)$$

simplifying:

$$P \Leftrightarrow Q \Leftrightarrow (\neg P \wedge \neg Q) \vee (Q \wedge P)$$

finally, using commutativity of \vee :

$$P \Leftrightarrow Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$$



Propositional logic and Modus Ponens:

“Amy, Bob, Cal, Don, and Eve were invited to a party last night.” \rightarrow defines what we are talking about

constants: $A \therefore$ “Amy went to the party”, B, C, D, E

knowledge base:

“Cal will always go if Amy and Bob go.”

$$(1) \quad A \wedge B \Rightarrow C \quad (\neg A \vee \neg B \vee C)$$

“Cal will not go if Don goes, and conversely.”

$$(2) \quad D \Rightarrow \neg C \quad (\neg D \vee \neg C)$$

$$(2b) \quad C \Rightarrow \neg D$$

“Amy went to the party with Eve.” $A \wedge E$

$$(3) \quad A \quad (4) \quad E$$

“Bob goes to every party that Eve goes to.”

$$(5) \quad E \Rightarrow B \quad (\neg E \vee B)$$

$$\text{proof: } (4)+(5) \quad E, E \Rightarrow B \quad \vdash \quad B \quad (6)$$

$$(3)+(6)+(1) \quad A, B, A \wedge B \Rightarrow C \quad \vdash \quad C \quad (7)$$

$$(7)+(2b) \quad C, C \Rightarrow \neg D \quad \vdash \quad \neg D$$

Don did not go to the party

The unicorn mystery:

constants: properties of the unicorn

Mythical, Magical, Horned, and

Mammal, Mortal

(Immortal $\Leftrightarrow \neg$ Mortal)

knowledge base:

Mythical $\Rightarrow \neg$ Mortal

\neg Mortal \Rightarrow Horned

\neg Mythical \Rightarrow Mortal

Mammal \Rightarrow Horned

\neg Mythical \Rightarrow Mammal

Horned \Rightarrow Magical

problem: only rules and *no facts* (!)

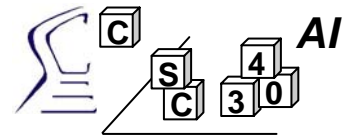
Modus Ponens: $P, P \Rightarrow Q \vdash Q$, but if *no P*?

→ nothing can be inferred directly from the *KB*

→ need some fact(s) i.e.,

1) assume Mythical, then infer (what?)

2) assume \neg Mythical, then ...



Using the Modus Ponens rule of inference:

if the unicorn is mythical:

Mythical, Mythical $\Rightarrow \neg$ Mortal $\vdash \neg$ Mortal

\neg Mortal, \neg Mortal \Rightarrow Horned \vdash Horned

Horned, Horned \Rightarrow Magical \vdash Magical

if the unicorn is not mythical:

\neg Mythical, \neg Mythical \Rightarrow Mammal \vdash Mammal

Mammal, Mammal \Rightarrow Horned \vdash Horned

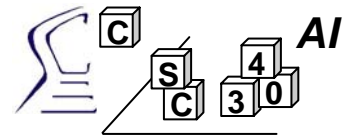
Horned, Horned \Rightarrow Magical \vdash Magical

conclusion: the unicorn is both horned and magical

(true in all cases – Mythical or \neg Mythical)

still no conclusion about the unicorn being mythical

(note: in general this is not a workable approach...)



Using the resolution rule of inference:

binary resolution: $P \vee Q, \neg Q \vee R \vdash P \vee R$

knowledge base (CNF):

1. $\neg \text{Mythical} \vee \neg \text{Mortal}$
2. $\text{Mythical} \vee \text{Mortal}$
3. $\text{Mythical} \vee \text{Mammal}$
4. $\text{Mortal} \vee \text{Horned}$
5. $\neg \text{Mammal} \vee \text{Horned}$
6. $\neg \text{Horned} \vee \text{Magical}$

proof:

7. from 1 and 4: $\neg \text{Mythical} \vee \text{Horned}$
8. from 3 and 5: $\text{Mythical} \vee \text{Horned}$
9. from 7 and 8: Horned
10. from 9 and 6: Magical

conclusion: the unicorn is both horned and magical

still no conclusion about the unicorn being mythical