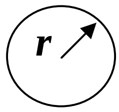
Tutorial 5: Transmission lines

1. Basic line parameters:

a. Resistance $R_{DC} = \rho \dfrac{l}{A}(\Omega)$

$R_{AC} = (1.05 to 1.1) \times R_{DC}$



Line with single conductor

b. Inductance and capacitance

➢ **Single conductor (each of phase *a*, *b* or *c*)**

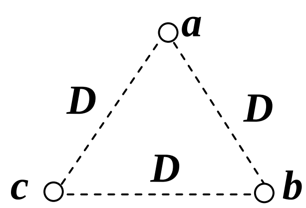
Inductance: $L_p = 2\times 10^{-7}\,\ell n \dfrac{GMD}{GMR}(H/m)$

Capacitance: $C_n = \dfrac{2\pi\varepsilon}{\ell n \dfrac{GMD}{r}}(F/m)$

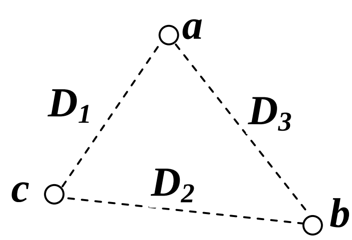
GMR for single conductor is usually <u>given</u>

Equal space between two phases: $GMD=D$

Unequal space: $GMD=\sqrt[3]{D_1 D_2 D_3}$

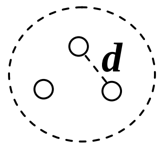


Equal space *D*



Unequal space D_1, D_2 and D_3

➢ **Bundled conductors (each of phase *a*, *b* or *c*)**



Line with Bundled conductors

Inductance: $L_p = 2\times 10^{-7}\,\ell n \dfrac{GMD}{GMR_b}(H/m)$

Capacitance: $C_n = \dfrac{2\pi\varepsilon}{\ell n \dfrac{GMD}{r_b}}(F/m)$

$GMR_b = \sqrt{GMR\times d}$ **for 2 conductors**

$GMR_b = \sqrt[3]{GMR\times d^2}$ **for 3 conductors**

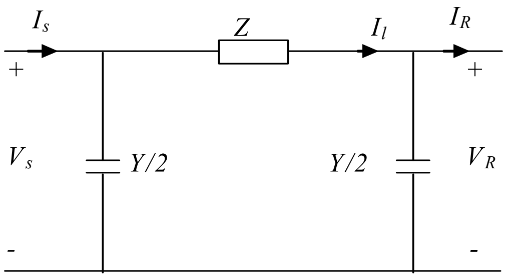
$GMR_b = 1.09\sqrt[4]{GMR\times d^3}$ **for 4 conductors**

$r_b = \sqrt{r\times d}$ **for 2 conductors**

$r_b = \sqrt[3]{r\times d^2}$ **for 3 conductors**

$r_b = 1.09\sqrt[4]{r\times d^3}$ **for 4 conductors**

2. Line equivalent circuit and ABCD parameters



Admittance: Y=j*ωC*
Impedance: Z=*R*+ j*ωL*

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \Rightarrow \begin{pmatrix} V_S = AV_R + BI_R \\ I_S = CV_R + DI_R \end{pmatrix}$$

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

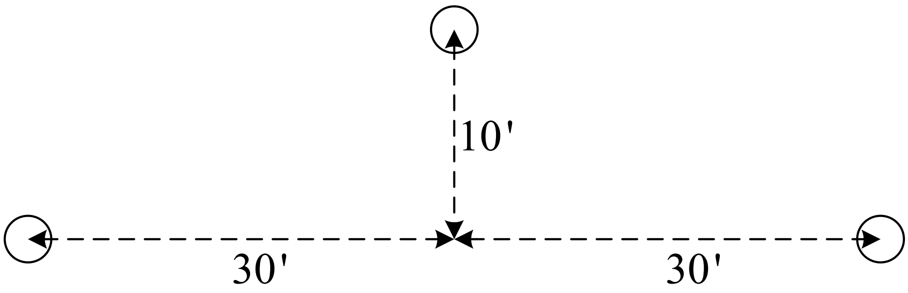
$$A = (1 + \frac{ZY}{2}) \qquad B = Z$$

$$C = Y(1 + \frac{ZY}{4}) \qquad D = (1 + \frac{ZY}{2})$$

Given V_S and I_S, calculate V_R, I_R and S_R
Given V_R and I_R, calculate V_S, I_S and S_S

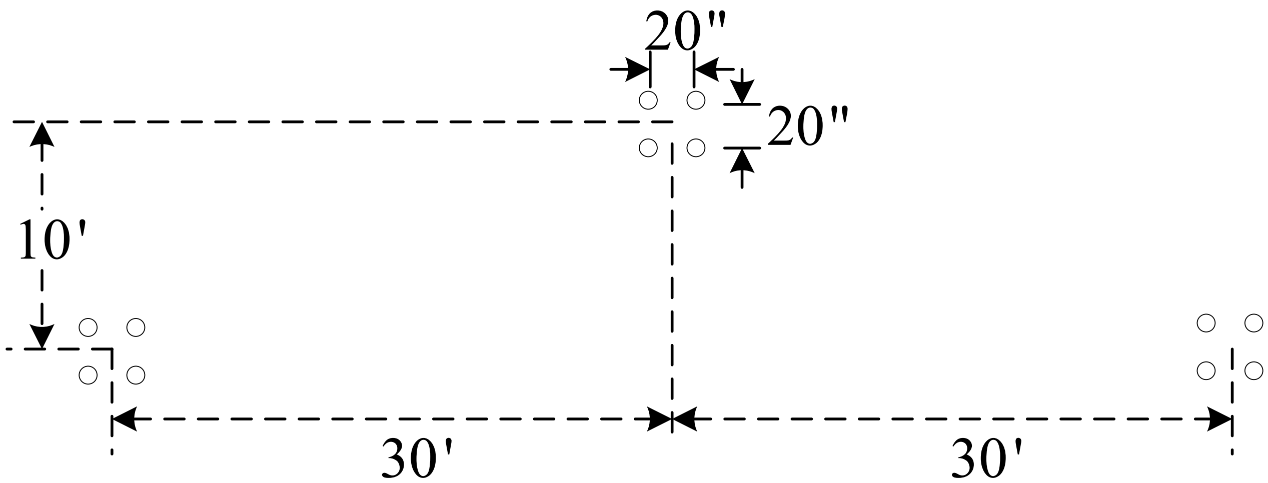
Voltage regulation: $V\,\text{Regulation} = \dfrac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100 \quad (\%)$

5.1(a) A three-phase transmission line consisting of stranded ACSR conductors is shown below. Given that the system frequency is 50 Hz, R/mile=0.1204ohms, GMR=0.0403ft, conductor diameter =1.196inches, calculate the resistance, inductance, series reactance, capacitance, and shunt admittance per km of the line.



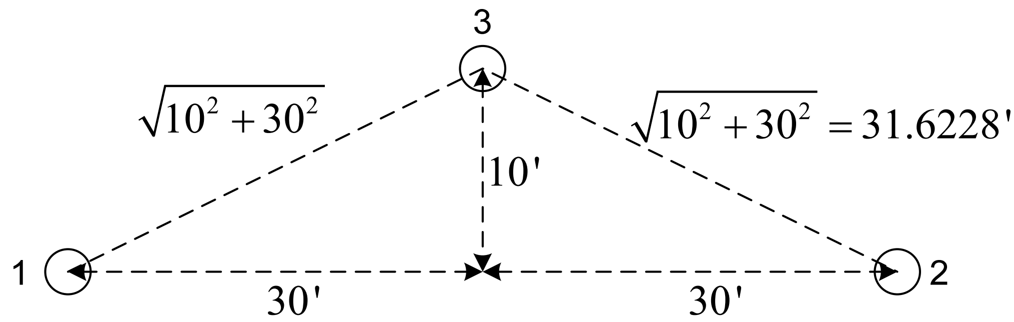
(b) Each phase of the line in part (a) above consists of a bundle of 4 stranded conductors as shown below. Find the new R, L, C, X and Y for the line.

(Assume that transmission line is completely transposed)



Solution:

(a)



Given $R = 0.1204\Omega/\text{mile} = \dfrac{0.1204}{1.609} = 0.0748\Omega/\text{km}$

GMR=0.0403'

Conductor diameter = 1.196" $\Longrightarrow$ radius r=0.598"

D_2=60', D_1= D_3=31.6228'

$GMD = \sqrt[3]{D_1 D_2 D_3} = 39.1487' = 469.784''$

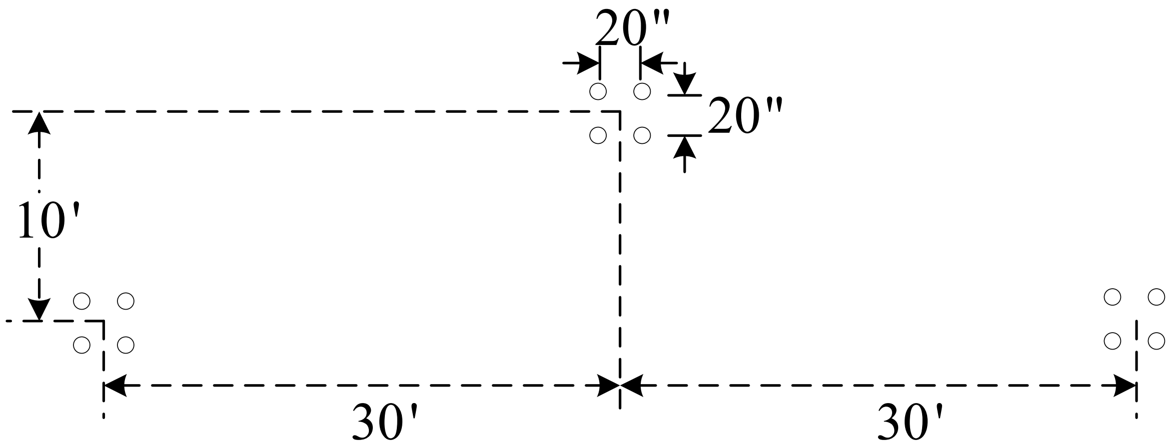
$$L_p = 2\times10^{-7}\,\ell n\frac{GMD}{GMR} = 2\times10^{-7}\,\ell n\frac{39.1487'}{0.0403'} = 13.7575\times10^{-7}\,(H/m)$$

$$\therefore\ jX_L = j\omega L = j2\pi fL = j2\times3.14\times50\times13.7575\times10^{-4} = j0.432\Omega/\text{km}$$

$$C_n = \frac{2\pi\varepsilon}{\ell n\dfrac{GMD}{r}} = \frac{2\pi\times8.85\times10^{-12}}{\ell n\dfrac{469.784''}{0.598''}} = 8.3412\times10^{-12}\,(F/m)$$

$$\therefore \mathrm{Y} = j\omega C_n = j2\pi\times50\times8.3412\times10^{-9} = j2.62\mu\text{S/km}$$

(b)



Per phase resistance $R = \dfrac{0.0748}{4} = 0.0187\Omega/\text{km}$

d=20"; GMR=0.0403'=0.4836"

$$\boldsymbol{GMR_b = 1.09\sqrt[4]{GMR \times d^3} = 1.09\sqrt[4]{0.4836 \times 20^3} = 8.596''}$$

D_2=60', D_1= D_3=31.6228'

$$\boldsymbol{GMD = \sqrt[3]{D_1 D_2 D_3} = 39.1487' = 469.784''}$$

$$\boldsymbol{L_p = 2 \times 10^{-7}\,\ell n \frac{GMD}{GMR_b} = 2 \times 10^{-7}\,\ell n \frac{469.784''}{8.596''} = 0.8002(mH\ /\ km)}$$

$$\therefore\ jX_L = j(2\pi f)L = j0.2514\Omega/\text{km}$$

$$\boldsymbol{r_b = 1.09\sqrt[4]{r \times d^3} = 1.09\sqrt[4]{0.598 \times 20^3} = 9.0651''}$$

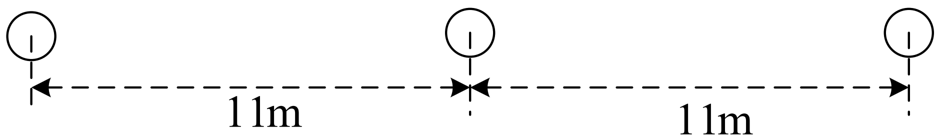
$$\boldsymbol{C_n = \frac{2\pi\varepsilon}{\ell n \frac{GMD}{r_b}} = \frac{2\pi \times 8.85 \times 10^{-12}}{\ell n \frac{469.784''}{9.0651''}} = 14.085 \times 10^{-9}(F\ /\ km)}$$

$$\therefore \text{Y} = j\omega C_n = j(2\pi \times 50) \times 14.085 \times 10^{-9} = j4.43\mu\text{S/km}$$

5.2 A three-phase transposed transmission line is composed of one ACSR conductor per phase with flat horizontal spacing of 11m between adjacent conductors. The conductors have a diameter of 3.625cm and a GMR of 1.439cm each. The line is to be replaced by a 3-conductor bundle of ACSR conductors having a diameter of 2.1793cm and a GMR of 0.8839cm each. The replaced line will also have a flat horizontal configuration, but it is to be operated at a higher voltage and therefore the phase spacing is increased to 14m between adjacent bundles. The spacing between the conductors in the bundles is 45cm. ($\varepsilon = 8.85\times10^{-12}$ F/m)
Determine:

(i) the percentage change in the line inductance
(ii) the percentage change in the line capacitance

Solution:



1-conductor case:

GMR=1.439cm

Conductor diameter= 3.625cm; r=1.8125cm

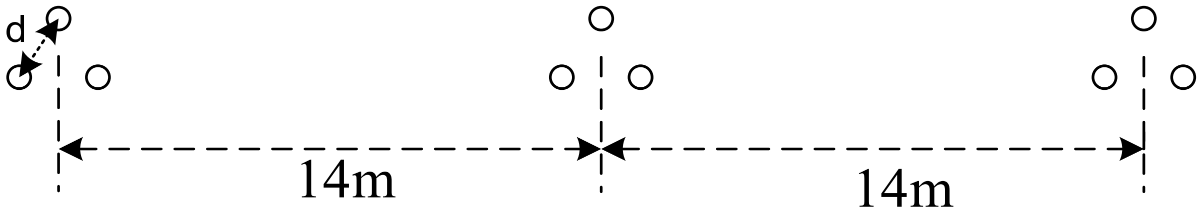
D_2=22m, D_1= D_3=11m

$$\boldsymbol{GMD = \sqrt[3]{D_1 D_2 D_3} = 13.8591m}$$

$$\boldsymbol{L_{old} = L_p = 2\times10^{-7}\,\ell n\frac{GMD}{GMR} = 2\times10^{-7}\,\ell n\frac{13.8591}{0.01439} = 13.74\times10^{-7}\left(H/m\right)}$$

$$\boldsymbol{C_{old} = C_n = \frac{2\pi\varepsilon}{\ell n\dfrac{GMD}{r}} = \frac{2\pi\times8.85\times10^{-12}}{\ell n\dfrac{13.8591}{0.018125}} = 8.3751\times10^{-12}\left(F/m\right)}$$

3-conductor case



D_2=28m, D_1= D_3=14m

$$\boldsymbol{GMD = \sqrt[3]{D_1 D_2 D_3} = 17.6388m}$$

$$\boldsymbol{GMR_b = \sqrt[3]{GMR \times d^2} = \sqrt[3]{0.8839 \times 45^2} = 12.1413cm}$$

$$\boldsymbol{r_b = \sqrt[3]{r \times d^2} = \sqrt[3]{\frac{2.1793}{2} \times 45^2} = 13.01879cm}$$

$$\boldsymbol{L_{new} = L_p = 2 \times 10^{-7} \ell n \frac{GMD}{GMR_b} = 2 \times 10^{-7} \ell n \frac{17.6388}{0.1214} = 9.9573 \times 10^{-7} \left(H / m\right)}$$

$$\boldsymbol{C_{new} = C_n = \frac{2\pi\varepsilon}{\ell n \frac{GMD}{r_b}} = \frac{2\pi \times 8.85 \times 10^{-12}}{\ell n \frac{17.6388}{0.1302}} = 11.3277 \times 10^{-12} \left(F / m\right)}$$

(i) $\%\text{change in } L = \frac{L_{old} - L_{new}}{L_{old}} \times 100\% = \frac{13.74 - 9.957}{13.74} \times 100\% = 27.53\%$

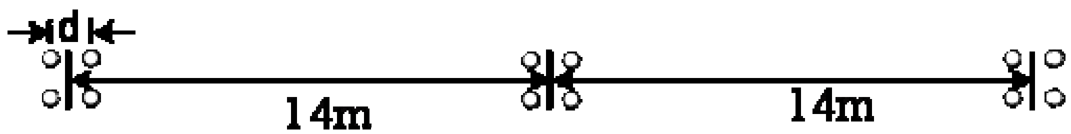
(ii) $\%\text{change in C} = \frac{C_{old} - C_{new}}{C_{old}} \times 100\% = \frac{8.3751 - 11.3277}{8.3751} \times 100\% = -35.25\%$

5.3 A 3-phase, 765kV, 60Hz transmission line is composed of four ACSR conductors per phase with a flat horizontal spacing of 14m between adjacent conductors. The conductors have a diameter of 3.625cm, and a GMR of 1.439cm. The bundle spacing is 45cm, and the line is 400km long. Assume the line to be lossless and model it as a nominal-π equivalent with base values of 765kV and 2000MVA ($\varepsilon = 8.85 \times 10^{-12}\,\text{F/m}$).

(a) Determine the receiving-end voltage, current and complex power when 1920MW and 600MVAr (lag) are being transmitted at 765kV at the sending end.

(b) If the line is energized with 765kV at the sending end when the load at the receiving end is removed, what will be the receiving-end voltage?

Solution:



r=1.8125cm, GMR=1.439cm

d=45cm, $\text{Length} = 400 \times 10^3\,\text{m}$

D_2=28m, $D_1 = D_3$=14m

$$GMD = \sqrt[3]{D_1 D_2 D_3} = 17.6389m$$

$$GMR_b = 1.09\sqrt[4]{GMR \times d^3} = 1.09\sqrt[4]{1.439 \times 45^3} = 20.742cm$$

$$r_b = 1.09\sqrt[4]{r \times d^3} = 1.09\sqrt[4]{1.8125 \times 45^3} = 21.9738cm$$

$$L_p = 2 \times 10^{-7}\,\ell n\frac{GMD}{GMR_b} = 8.8862 \times 10^{-7}\,(H/m)$$

***Total: L=L_p*×400000=0.355448H**

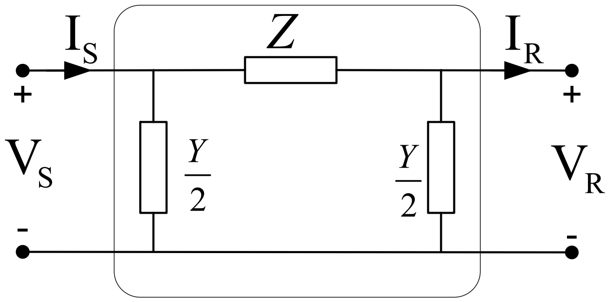
$$C_n = \frac{2\pi\varepsilon}{\ell n\frac{GMD}{r_b}} = 12.6798\times 10^{-12}\left(F\ /\ m\right)$$

***Total: C=C_n*×400000=5.07192 (*mF*)**

$Z=R+j\omega L=0+j2\pi fL=j134.0012\Omega$

$Y=j\omega C=j2\pi fC=j1.1912\times 10^{-3}S$

Nominal -π equivalent circuit for the transmission line:



Let $V_b = 765\text{kV}$ & $S_b = 2000\text{MVA} \Rightarrow Z_b = \frac{V_b^2}{S_b} = 292.6125\Omega$

$$I_b = \frac{S_b\times 10^3}{\sqrt{3}\times V_b} = \frac{(2000\times 10^3)\text{kVA}}{\sqrt{3}\times 765\text{kV}} = 1509.4125\text{A}$$

$$\therefore Z_{pu} = j0.45795,\ Y_{pu} = \frac{Y}{Y_b} = YZ_b = j0.5595$$

Line parameters $A = D = \frac{ZY}{2}+1 = \frac{j0.45795\times j0.5595}{2}+1 = 0.8719$

$$B = Z = j0.45795 \qquad C = Y\left[\frac{ZY}{4}+1\right] = j0.5595\left\{\frac{j0.45795\times j0.5595}{4}+1\right\} = j0.52366$$

$$\therefore ABCD = \begin{bmatrix} 0.8719\angle 0^0 & 0.45795\angle 90^0 \\ 0.52366\angle 90^0 & 0.8719\angle 0^0 \end{bmatrix}$$

(a) given $V_s = 1\angle 0^0 \qquad S_s = 1920 + j600 = 2011.5666\angle 17.354^0 \text{(MVA)}$

$$\therefore S_{spu} = 1.00578\angle 17.354^0 \Rightarrow I_{spu} = 1.00578\angle -17.354^0$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \Rightarrow \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} V_s \\ I_s \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \frac{1}{AD - BC} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix} \qquad \text{But } AD - BC = 1$$

$$\therefore V_R = DV_s - BI_s = 0.8719 \times 1 - (0.45795\angle 90^0)(1.00578\angle -17.354^0)$$
$$= 0.8719 - (0.13738 + j0.4396) = 0.856\angle -30.90^0$$

$$\therefore |V_R| = |V_{Rpu}| \times V_b = 654.86\text{kV}$$

$$I_R = -CV_s + AI_s = -j0.52366 + (0.8719)(1.00578\angle -17.354^0)$$
$$= -j0.52366 + 0.837 - j0.2616 = 0.837 - j0.78523 = 1.1477\angle -43.172^0$$
$$\therefore |I_R| = |I_{Rpu}| \times I_b = 1732.31\text{A}$$

$$\therefore S_R = V_R I_R^* = 0.856\angle -30.90^0 \times 1.1477\angle 43.172^0 = 0.98243\angle(-30.90^0 + 43.172^0)$$
$$= 0.98243\angle 12.272^0 = 0.96 + j0.2088$$

$$\therefore |S_R| = 0.98243 \times 2000 = 1964.86\text{MVA}$$
$$|P_R| = 0.96 \times 2000 = 1920\text{MW}$$
$$|Q_R| = 0.2088 \times 2000 = 417.64\text{MVAr (lag)}$$

(b): $\boldsymbol{V_s = AV_R + BI_R \left(\because I_R = 0\right) \Rightarrow V_s = AV_R}$

$$\Rightarrow |V_R| = \left|\frac{V_S}{A}\right| = \left|\frac{1}{0.8719}\right| = 1.14692\text{pu} = 877.39\text{kV}$$