

**NANYANG TECHNOLOGICAL UNIVERSITY**  
**School of Electrical & Electronic Engineering**

**EE/IM4152 Digital Communications**

**Tutorial No. 8 (Sem 1, AY2016-2017)**

1. In the lecture, we have derived the transfer function of the matched filter as

$$H(f) = P(-f) e^{-j2\pi f T_o}$$

where  $p(t) \leftrightarrow P(f)$  is a Fourier transform pair and  $T_o$  is the pulse duration. Take the inverse Fourier transfer and verify its impulse response to be  $h(t) = p(T_o - t)$ .

2. It is important to know that the corrector output and the matched filter output are the same **only** at  $t = T_o$ , where  $T_o$  is the pulse duration. Let us ignore the input noise and assume the input signal to be a sine wave as

$$p(t) = \sin 2\pi f_c t, \quad 0 \leq t \leq T_o$$

where  $f_c$  is the carrier frequency. Determine and plot the output of the correlation receiver

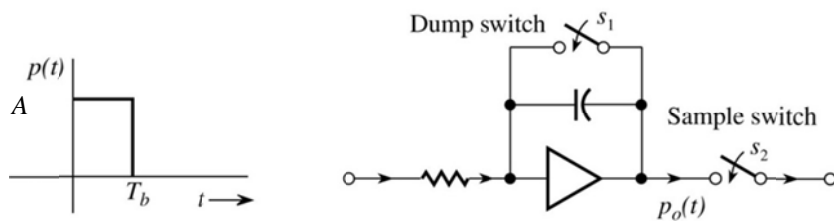
$$z_c(t) = \int_0^t p^2(x) dx$$

and the output of the matched filter

$$z_M(t) = \int_0^t p(x) p(T_o - t + x) dx, \quad 0 \leq t \leq T_o.$$

[Hint: you may ignore the double-frequency terms when computing the integration,]

3. The so-called integrate-and-dump filter is shown in Figure 1. The feedback amplifier is an ideal integrator. The switch  $s_1$  close momentarily and then opens at the instant  $t = T_b$ , thus dumping all the charge in the capacitor and causing the output to go to zero. The switch  $s_2$  samples the output immediately before the dumping action.
- (a) Determine the output  $p_o(t)$  when the rectangular pulse  $p(t)$  with amplitude  $A$  is applied to the input of the filter.
- (b) Find the noise power  $\sigma_n^2$  due to additive white Gaussian noise (AWGN) at the output.
- (c) Compute  $\rho^2 = p_o^2(T_b) / \sigma_n^2$  for the integrate-and-dump filter and compare it with that of the matched filter [see (12) of your lecture notes].



**Figure 1**

4. Consider the following set of signals

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_s}{T}} \cos(2\pi f_c t + i\pi/2), & 0 \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$

where  $i = 1, 2, 3, 4$ ,  $T$  is the pulse duration and  $f_c T = k$  for some integer  $k$ . Choose the basis functions,  $\phi_1(t) = \sqrt{2/T} \cos 2\pi f_c t$  and  $\phi_2(t) = \sqrt{2/T} \sin 2\pi f_c t$ , over the same  $T$ -second interval and plot the locations of  $s_i(t)$ ,  $i = 1, 2, 3, 4$  in the 2-dimensional signal space formed by  $\phi_1(t)$  and  $\phi_2(t)$ .