Name: For the 6 students who had timetable clash Tutorial group: T1

Matriculation number: | | | | | | | | | |

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I 2015/16

MH2500– Probability and Introduction to Statistics

22 September 2015 Test 2 40 minutes

INSTRUCTIONS

1. Do not turn over the pages until you are told to do so.
2. Write down your name, tutorial group, and matriculation number.
3. This test paper contains **FOUR (4)** questions and comprises **FIVE (5)** printed pages.
4. Answer **all** questions. The marks for each question are indicated at the beginning of each question.
5. You are allowed two double-sided A4 size cheat sheet.

For graders only	Question	1	2	3	4	Bonus	Total
	Marks						

QUESTION 1. **(6 marks)**

An urn contains 20 red balls and 10 white balls. Two balls are drawn simultaneously from the urn and then put back into the urn. This process is repeated until the two balls drawn are of the same colour. Find the probability that the process stopped at the fifth draw.

[Answer:] Probability of success is

$$p = \frac{\binom{20}{2} + \binom{10}{2}}{\binom{30}{2}} = \frac{47}{87}.$$

Let X be the number of draws until two balls of the same color are drawn.

$$P(X = 5) = (1 - p)^4 p = \approx 0.241.$$

QUESTION 2. **(12 marks)**

Suppose an office receives telephone calls as a Poisson process with $\lambda = 0.4$ per min.

(a) Find the probability that there are 10 calls in a one hour interval. Give your answer to three significant figures.

(b) Let T denote the time taken (in minutes) between one phone call and three more phone calls, i.e., time between the first and the fourth phone calls. Find the probability density function of T.

[Answer:]

a. Process is Poisson with $\lambda = 2.4 \times 10 = 24$.

$$P(X = 10) \;=\; \frac{24^{10}}{10!} e^{-24} = 0.000660.$$

b. Note that the number of phone calls per t minutes is Poisson with parameter $0.3t$. Suppose at time 0, there is a phone call. Let T denote the time taken (in minutes) to receive two more phone calls. Then

$$\begin{aligned} P(T \le t) &= 1 - P(T > t) \\ &= 1 - P(\text{no phone calls in time } (0,t]) - P(\text{one phone call in time } (0,t]) \\ &\quad - P(\text{two phone call in time } (0,t]) \\ &= 1 - \frac{(0.4t)^0}{0!} e^{-0.4t} - \frac{(0.4t)^1}{1!} e^{-0.4t} - \frac{(0.4t)^2}{2!} e^{-0.4t} \\ &= 1 - (1 + 0.4t + 0.08t^2) e^{-0.4t}. \end{aligned}$$

Hence the density is

$$f_T(t) = -(0.4 + 0.16t) e^{-0.4t} + 0.4(1 + 0.4t + 0.08t^2) e^{-0.4t} = 0.032 t^2 e^{0.4t}.$$

QUESTION 3. **(8 marks)**

Suppose X is a random variable with density

$$f_X(x) = \begin{cases} \frac{1}{8}x, & 0 < x < 4; \\ 0, & \text{otherwise.} \end{cases}$$

and let Y be a random variable defined by $Y = \dfrac{4}{4-X}$. Find the density function of Y.

[Answer:]

$$\begin{aligned} P(Y \le y) &= P\left(\frac{4}{4-X} \le y\right) \\ &= P\left(\frac{4}{y} \le 4 - X\right) \\ &= P\left(X \le 4\left(1 - \frac{1}{y}\right)\right) \\ &= \frac{1}{16} \cdot 4^2 \left(1 - \frac{1}{y^2}^2\right) \\ &= 1 - \frac{2}{y} + \frac{1}{y^2}, \qquad (1 \le y < \infty). \end{aligned}$$

Hence

$$f_Y(y) = \frac{d}{dy}\left(1 - \frac{2}{y} + \frac{1}{y^2}\right) = \frac{2}{y^2} - \frac{2}{y^3} \qquad (1 \le y < \infty).$$

QUESTION 4. **(8 marks)**

There are 10 apples, 5 bananas, and 5 oranges in a basket. Alan chooses 6 fruits from the basket.

(a) Find the joint distribution of the number of apples, bananas, and oranges chosen by Alan.

(b) Find the marginal distribution of the number of apples chosen by Alan.

[Answer]

(a) Let A, B, C be the number of apples, bananas, and oranges chosen by Alan, respectively. Then for $0 \leq a \leq 6$ and $0 \leq b, c \leq 5$ with $a + b + c = 6$,

$$P(A = a, B = b, C = c) = \frac{\binom{10}{a}\binom{5}{b}\binom{5}{c}}{\binom{20}{6}}.$$

Otherwise, $P(A = a, B = b, C = c) = 0$.

(b) For $0 \leq a \leq 6$,

$$P(A = a) = \frac{\binom{10}{a}\binom{10}{6-a}}{\binom{20}{6}}.$$