Name: _____ Tutorial group: T1

Matriculation number: | | | | | | | | | |

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I 2015/16

MH2500– Probability and Introduction to Statistics

20 October 2015 Test 3 40 minutes

INSTRUCTIONS

1. Do not turn over the pages until you are told to do so.
2. Write down your name, tutorial group, and matriculation number.
3. This test paper contains **FOUR (4)** questions and comprises **FIVE (5)** printed pages.
4. Answer **all** questions. The marks for each question are indicated at the beginning of each question.
5. You are allowed three double-sided A4 size cheat sheet.

For graders only	Question	1	2	3	4	Bonus	Total
	Marks						

QUESTION 1. **(12 marks)**

Suppose the joint cumulative distribution function of continuous random variables X and Y is

$$F_{X,Y}(x,y) = \frac{1}{10}(3x^3y + xy^2), \qquad 0 \le x \le 1 \text{ and } 0 \le y \le 2.$$

(a) Find $f_{X,Y}\left(\frac{1}{3}, \frac{1}{2}\right)$, where $f_{X,Y}$ is the joint density of X and Y.

(b) Find $F_X(x)$ and $F_Y(y)$, the marginal cumulative distribution functions for X and Y, respectively.

(c) Find $f_{Y|X}(y|x)$, the conditional density of Y given X.

QUESTION 2. **(8 marks)**

Let X and Y have the joint density function

$$f(x,y) = k(2x+y), \qquad 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 2,$$

and 0 elsewhere.

(a) Find k. Leave your answer as a fraction.

(b) Find $P(Y < X+1)$. Leave your answer as a fraction or to three significant figures.

QUESTION 3. **(6 marks)**

A number, N, is chosen randomly from the set $\{1, 2, 4\}$. A fair coin is then flipped N time. Let H denote the number heads obtained. Find the conditional distribution of N given $H = 2$.

QUESTION 4. **(8 marks)**

Let X and Y have the joint density function $f(x, y)$ and let $Z = 2X + Y$. Show that the density function of Z is

$$f_Z(z) = \int_{-\infty}^{\infty} f\left(x, z - 2x\right) dx$$

by completing the proof below.

Proof:

$$F_Z(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{z-2x} f\left(x, y\right) dydx.$$

Making a change of variable $y = ...$