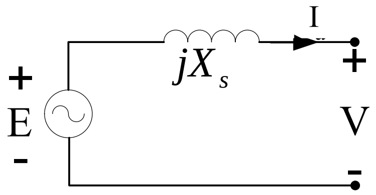
Tutorial 3: Synchronous Generators 1

Equivalent circuit and basic concepts:



E=V+I×jX$_s$ = $|E|\angle\delta$

V: **the terminal voltage**

X$_s$: **synchronous reactance**

- **The magnitude of <u>generated voltage</u> (or emf) $|E|$ is produced by the field or <u>excitation</u> current <u>I_f</u>.**
- **<u>Real power</u> output *P* is converted from <u>mechanical power</u> supplied by the <u>prime mover</u>.**

<u>An infinite bus</u> is a bus with constant voltage (<u>cannot be changed</u>)

Complex power output from generator (pu):

- **given** $I = |I|\angle\theta$ **and** $V = |V|\angle 0^0$**:**

  $\boldsymbol{S = VI^* = |V||I|\angle(-\theta) = |V||I|\cos\theta - j|V||I|\sin\theta}$

  $\Rightarrow$ **real** $P = |V||I|\cos\theta$ **; reactive** $Q = |V||I|\sin\theta$

  θ **is the current angle.**

  Power factor: $pf = \cos\theta$

- **given** $V = |V|\angle 0^0$ **and** $\boldsymbol{E} = |E|\angle\delta$**, (*δ* is called power angle)**

  $$I = \frac{E - V}{jX_S} \qquad S = VI^* = V\frac{E^* - V^*}{-jX_S}$$

  $$\Rightarrow \boldsymbol{P = \frac{|E||V|}{X_s}\sin\delta} \quad \textbf{and} \quad \boldsymbol{Q = \frac{|V|}{X_s}\left(|E|\cos\delta - |V|\right)}$$

- **Maximum real power through *X$_s$*:**

  $$\boldsymbol{P_{max} = \frac{|E||V|}{X_s}\sin 90^o = \frac{|E||V|}{X_s}}$$

3.1 A three-phase, 125-MVA, 0.8 pf (lag), 13.2-kV, 1800-rpm synchronous generator has a synchronous reactance of 1.7 Ω per phase and negligible stator resistance.

(a) If the machine is connected to **an infinite bus** operating at 13.2 kV, and it delivers 100MW at **unity power factor**, find the magnitude of the generated voltage, the power angle, and the reactive power.

(b) Assume that the **prime mover input** to the generator of part (a) is **reduced by 50% without changing the excitation**. Find the new operating conditions.

(c) Starting at the condition of part (a), assume that the **excitation is adjusted** to provide power 100MW at 0.8 pf leading. Evaluate the new operating conditions.

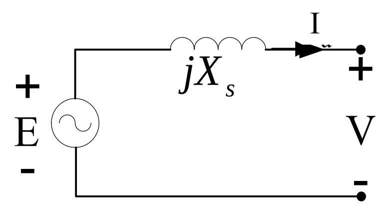
(d) Sketch a single **phasor diagram** to depict the three operating conditions in parts (a)-(c).

Solutions:

Select: $\mathbf{V_B=13.2kV}$

$\mathbf{S_B=125MVA}$

$\mathbf{Z_B=V_B^2/S_B=1.3939\Omega}$



Given:

$\mathbf{X_{spu}=X_s/Z_B=1.2196}$

$\mathbf{V_{pu}=V/V_B=13.2/13.2=1\angle 0^o}$ **(reference, constant for infinite bus)**

(a) Find $E_a=|E_a|\angle\delta_a$ **and** $\mathbf{Q_a}$

Given: $\mathbf{P_a=100}$ **and** $\mathbf{pf_a=1}$

$\mathbf{S_{apu}=S_a/S_B=P_a/S_B=100/125=0.8\angle 0^o}$

Reactive power: $\mathbf{Q_{apu}=0}$

$\mathbf{I_{apu}=(S_{apu}/V_{pu})^*=0.8\angle 0^o}$

$\mathbf{E_{apu}=V_{pu}+I_{apu}\times jX_{spu}=1.3971\angle 44.295^o}$

Generated voltage: $\mathbf{|E_a|=1.3971\times V_B=18.442\quad kV}$

Power angle: $\mathbf{\delta_a=44.295^o}$

(b) Find δ_b , I_b and Q_b

Given: $P_b=0.5\times P_a=50MW$, **(**prime mover input **reduced)**

$P_{bpu}=50/125=0.4$

Given: $|E_{bpu}|=|E_{apu}|$ **(same excitation** with (a)**)**

$$\mathbf{P_{bpu}=\frac{|V_{pu}||E_{bpu}|}{X_{spu}}\sin\delta_b=\frac{1\times 1.3971}{1.2196}\sin\delta_b=0.4}$$

$$\Rightarrow\delta_b=20.437^o;\quad E_{bpu}=|E_{bpu}|\angle\delta_b=1.3971\angle 20.437^o.$$

$$\mathbf{Q_{bpu}\ \frac{|V_{pu}||E_{bpu}|}{X_{spu}}\cos\delta_b-\frac{|V_{pu}|^2}{X_{spu}}=0.2535,\ Q_b=31.687MVAr}$$

$$I_{bpu}=\frac{E_{bpu}-V_{pu}}{jX_{spu}}=0.4735\angle-32.37^o$$

(c) Find $|E_c|,\delta_c$, I_c and Q_c

Given: $P_c=P_a=100$ and $pf_c=\cos\theta_c=0.8$

$$|S_c|=\frac{P_c}{\cos\theta_c}=\frac{100}{0.8}=125\quad\Rightarrow\quad|S_{cpu}|=\frac{|S_c|}{S_B}=1$$

$\theta_c=\cos^{-1}pf_c=-36.83$ because of leading pf

$S_{cpu}=|S_{cpu}|\angle\theta_c=1\angle-36.87^o=P_c+jQ_c=0.8-j0.6$

$Q_c=-0.6\times125=-75$ MVAr

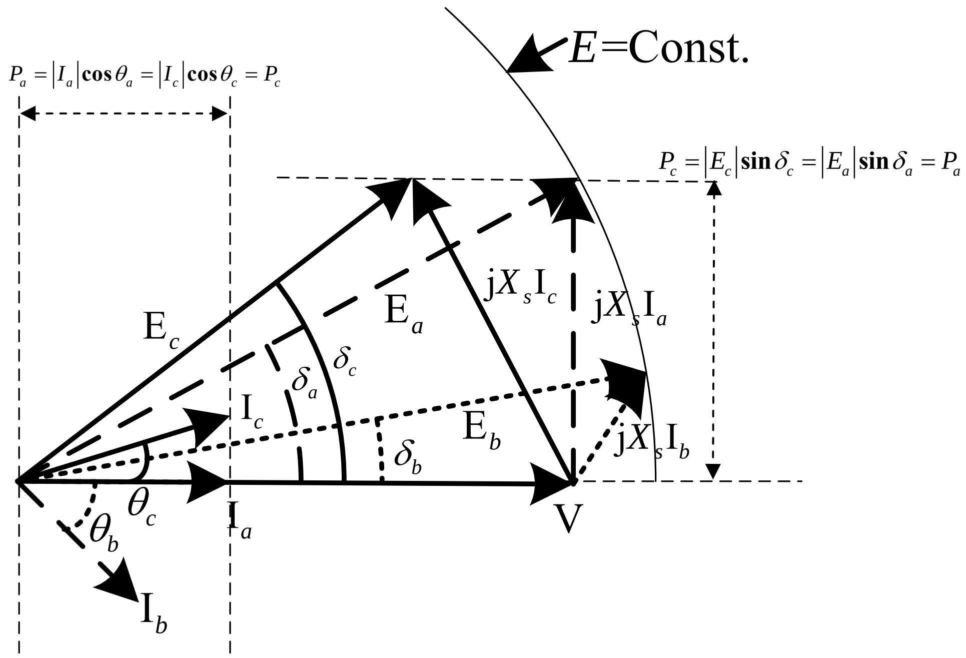
$I_{cpu}=(S_{cpu}/V_{pu})^*=1\angle 36.87^o$

$E_{cpu}=V_{pu}+I_{cpu}\times jX_{spu}=1.0119\angle 74.63^o$,

$$|E_c|=13.3568\quad kV$$

$\delta_c=74.63^o$

(d) Phasor diagram (relationship of E, V and I)



$V_{pu} = 1\angle 0^o$ as reference

(a): $E_{apu}=V_{pu}+I_{apu}\times jX_{spu}$; V_{pu} and I_{apu} are in phase

(b): $|E_{bpu}| = |E_{apu}|$ (same excitation)

(c): $P_c=P_a$=100MW (same real power)

$$P_c = \frac{|V_{pu}||E_{cpu}|}{X_{spu}}\sin\delta_c = \frac{|V_{pu}||E_{apu}|}{X_{spu}}\sin\delta_a = P_a$$

$$\Rightarrow |E_c|\,sin\,\delta_c = |E_a|\,sin\,\delta_a$$

Or P_a= $P_a = |V||I_a|\cos\theta_a = |V||I_c|\cos\theta_c = P_c$

$$\Rightarrow |I_a|\cos\theta_a = |I_c|\cos\theta_c$$

3.2 A three-phase Y-connected synchronous generator has a synchronous reactance of 10 ohms and negligible armature resistance. (1) At **unity power factor** the generator supplies 188 A to a 13.2-kV **infinite bus**. (2) When the generator **excitation is increased by 25%, without changing the mechanical power** input, determine the generator line current and power factor. Use the per unit system for your calculations. Sketch a phasor diagram to show the two operating conditions.

Solutions:

Select: V_b=13.2kV, I_b=188A $\Rightarrow$ $S_b=\sqrt{3}\,V_bI_b$=4.298MVA, $Z_b=V_b^2/S_b$=40.537Ω.

(1) Old operating condition:

Given: I=188A, pf=1, V=13.2kV

$X_{spu}=10/Z_b=0.2467$

$V_{pu}=1\angle 0^o$, $I_{opu}=1\angle 0^o$,

$S_{pu}=1\angle 0^o$, $P_{pu}=1$, $Q_{pu}=0$

$E_{opu}=V_{pu}+I_{pu}\times jX_{spu}$

$=1.03\angle 13.857^o$,

$$\boldsymbol{\delta_o = 13.857^o}$$

(2) New operating condition:

$|\mathbf{E}_{npu}| = 1.25|\mathbf{E}_{opu}|$,

$P_{npu}=P_{opu}$ (same)=1

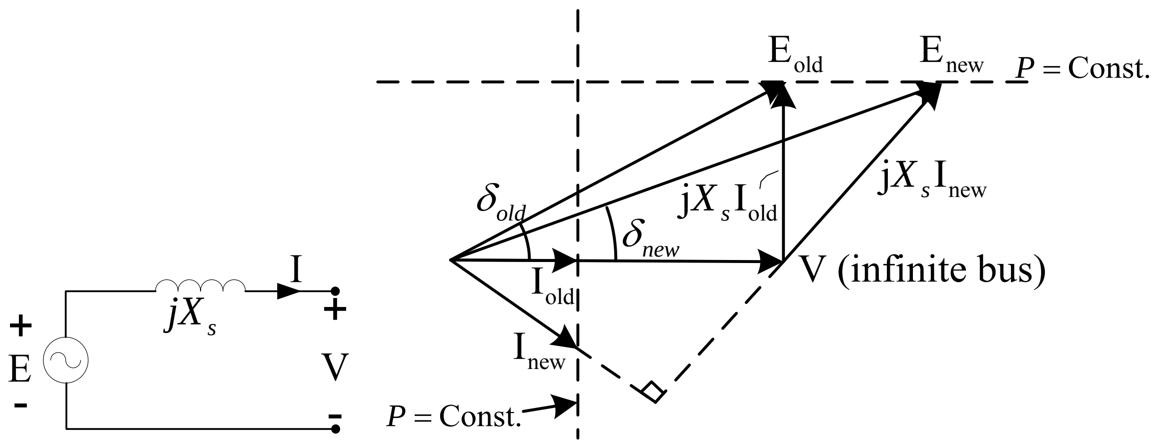
$|\mathbf{E}_{npu}|\,\boldsymbol{sin}\,\delta_n = |\mathbf{E}_{opu}|\,\boldsymbol{sin}\,\delta_o$

$1.25|\mathbf{E}_{opu}|\,\boldsymbol{sin}\,\delta_n = |\mathbf{E}_{opu}|\,\boldsymbol{sin}\,13.857^o$

$\delta_n = \boldsymbol{11.046^o}$

$$\mathbf{I}_{npu} = \frac{\mathbf{E}_{npu} - \mathbf{V}_{pu}}{\mathbf{jX}_{spu}} = 1.4636\angle -46.9^o$$

$\mathbf{pf} = \boldsymbol{cos(}-46.9^o\boldsymbol{)} = 0.683$

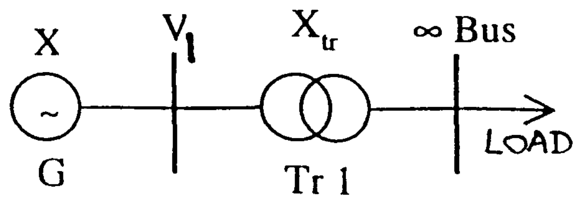


3.3 A 120-MVA, 22-kV synchronous generator has X= 1.l6pu. The generator supplies power to a large power system (infinite bus) through a 150MVA, 22/345kV, X= 0.06pu transformer, as shown below.

(a) Determine the induced emf and the terminal voltage of the generator when the load supplied to the **infinite bus** is 80MW at 0.8pf leading, at the rated voltage of 345kV.

(b) If the generator **excitation is increased by 20%** while supplying the constant load of 80MW, calculate:
   (i) the generator terminal voltage
   (ii) the Q output of the generator at its terminals
   (iii) the Q delivered to the infinite bus.

(c) Determine the **maximum real powers** the generator can supply to the infinite bus at the excitation levels in (a) and (b) respectively. In addition, find the power factors of the load at the infinite bus while delivering these maximum powers.



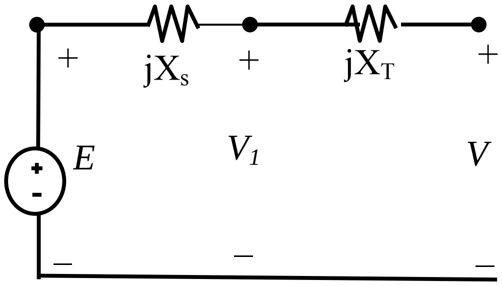
Solutions:

Select S_B=150 MVA, V_{1B}=22 kV ⇒∞bus:V_B=345 kV

Generator: $$X_{sNpu} = X_{sOpu}\frac{S_{BN}}{S_{BO}} = 1.16\frac{150}{120} = 1.45$$

Transformer: $X_{TNpu}=X_{TOpu}=0.06$

pu Voltage at ∞ bus: $$V_{pu}= \frac{345}{345} = 1\angle 0^{o}$$



(a) Load: $S_a = \frac{P}{pf}\angle -\cos 0.8 = 100\angle -36.87^o$ **MVA**

$S_{apu}=S_a/S_B=100/150=0.8\angle -36.87^o$

$I_{apu}=(S_{apu}/V_{pu})^*= 0.667\angle 36.87^o$

$V_{1apu}=V_{pu}+I_{apu}\times jX_{TNpu}=0.97652\angle 1.88^o$

$E_{apu}=V_{1apu}+I_{apu}\times jX_{sNpu}=0.8974\angle 63.815^o$

Or $E_{apu}=V_{pu}+I_{apu}\times(jX_{sNpu}+ jX_{TNpu})=0.8974\angle 63.815^o$

(b) $|E_{bpu}| = 1.2|E_{apu}|=1.077$

$P_{bpu}=P_{apu} \Rightarrow |E_{bpu}|\sin\delta_b = |E_{apu}|\sin\delta_a \Rightarrow \delta_b = 48.4^o$

$E_{bpu}=V_{pu}+I_{bpu}\times j(X_{sNpu}+X_{TNpu})=1.077\angle 48.4^o$

$\Rightarrow I_{bpu}=0.5657\angle 19.49^o$

$V_{1bpu}=V_{pu}+I_{bpu}\times jX_{TNpu}=0.9892\angle 1.854^o$

Generator terminal: $S_{1bpu}=V_{1bpu}I_{bpu}^*=0.5332-j0.1697$

Infinite bus: $S_{bpu}=V_{pu}I_{bpu}^*=0.5332-j0.1887$

(c) When δ=90°, $P_{max} = \frac{|V_{pu}||E_{pu}|}{X_{pu}} sin 90 = \frac{|V_{pu}||E_{pu}|}{X_{pu}}$

******* $X_{pu}=X_{sNpu}+X_{tNpu}=1.51$

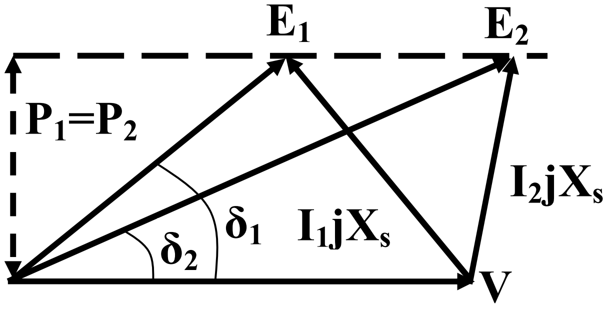
For (a): $E_{apu}=0.8974\llcorner 90^o$ **and** $P_{a\,max} = \frac{|V_{pu}||E_{apu}|}{X_{pu}}=0.5943$

$I_{apu} = \frac{E_{apu}-V_{pu}}{jX_{pu}} = 0.8898\angle 48.09^o$, $pf_a=\cos 48.09^o=0.6678$

For (b): $E_{bpu}=1.077\llcorner 90^o$ **and** $P_{b\,max} = \frac{|V_{pu}||E_{bpu}|}{X_{pu}}=0.71316$

$I_{bpu} = \frac{E_{bpu}-V_{pu}}{jX_{pu}} = 0.9733\angle 42.88^o$, $pf_b=\cos 42.88^o=0.7328$

$$P = \frac{|E||V|}{X_s} \sin\delta$$



$$P = |V||I| \cos\varphi$$