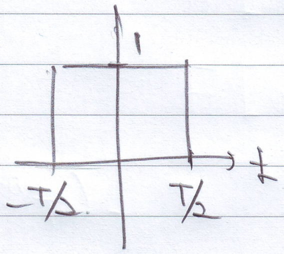
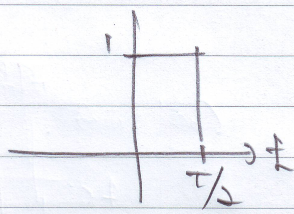
1.

a). $x(t) = \mathrm{rect}\left(\frac{t-\tau/4}{\tau/2}\right) - \mathrm{rect}\left(\frac{t+\tau/4}{\tau/2}\right)$

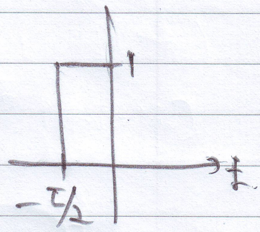
$\mathrm{rect}(t/T)$



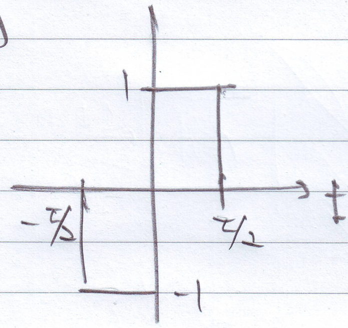
$\mathrm{rect}\left(\frac{t-\tau/4}{\tau/2}\right)$



$\mathrm{rect}\left(\frac{t+\tau/4}{\tau/2}\right)$



$x(t)$



$$\mathcal{F}[x(t)] = \mathcal{F}\left[\mathrm{rect}\left(\frac{t-\tau/4}{\tau/2}\right) - \mathrm{rect}\left(\frac{t+\tau/4}{\tau/2}\right)\right]$$

$$= \mathcal{F}\left[\mathrm{rect}\left(\frac{t-\tau/4}{\tau/2}\right)\right] - \mathcal{F}\left[\mathrm{rect}\left(\frac{t+\tau/4}{\tau/2}\right)\right]$$

$$= \frac{\tau}{2}\,\mathrm{sinc}(f\tau/2)\,e^{-j2\pi f\tau/4} - \frac{\tau}{2}\,\mathrm{sinc}(f\tau/2)\,e^{+j2\pi f\tau/4}$$

$$= \frac{1}{32B}\left\{\mathrm{sinc}(f/32B)\,e^{-j2\pi f\frac{1}{64B}} - \mathrm{sinc}(f/32B)\,e^{j2\pi f\, 1/64B}\right\}$$

$$= \frac{1}{32B}\,\mathrm{sinc}(f/32B)\left(e^{-j2\pi f\, 1/64B} - e^{j2\pi f\, 1/64B}\right)$$

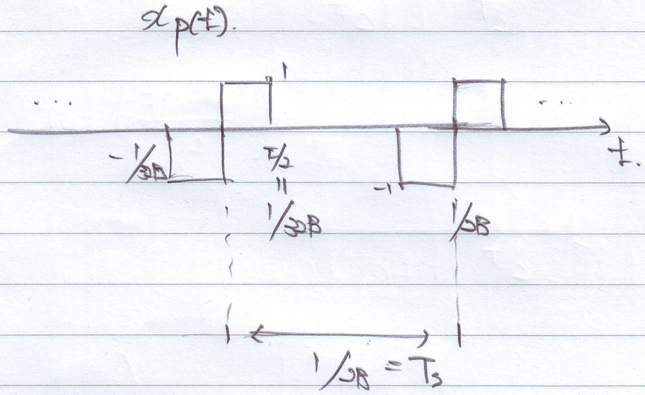
$$= X(f)$$

b).

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(t - nT_s) \qquad \left(\begin{array}{l} T_s = 1/2B \\ f_s = 2B. \end{array} \right.$$

$$= \sum_{n=-\infty}^{\infty} C_n \, e^{j2\pi n f t}$$

$$\hookrightarrow C_n = \frac{1}{T_s} \int_{T_s} x_p(t) \, e^{-j2\pi f_s n t} \, dt.$$



$$C_0 = 2B \int_0^{1/2B} x_p(t) \, e^{-0} \, dt$$

$$= 2B \int_0^{1/2B} x_p(t) \, dt$$

$$= 0.$$

$$C_1 = 2B \int_0^{1/2B} x_p(t) \, e^{-j2\pi f_s t} \, dt.$$

$$= 2B \int_{-1/2B}^{0} (-1) e^{-j2\pi f_s t} \, dt + 2B \int_0^{1/2B} (+1) e^{-j2\pi f_s t} \, dt$$

$$= \frac{-1}{2\pi} \left(\sin(\pi/8) + j(1 - \cos \pi/8) \right) + \frac{1}{2\pi} \left(\sin(\pi/8) + j(\cos \pi/8 - 1) \right)$$

$$= \frac{1}{2\pi} \left(j(2\cos \pi/8 - 2) \right) = \frac{1}{\pi} \cdot j(\cos \pi/8 - 1).$$

$$e^{jx} = \cos(x) + j\sin x$$
$$e^{-jx} = \cos x - j\sin x$$
Euler's formula.

$$\int_{-1/2B}^{0} (-1)\, e^{-j(2\pi f_s t)}\, dt$$

$$2\pi f_s t = x = 4\pi B t.$$
$$2\pi(2B) = dx/dt.$$
$$4\pi B = dx/dt.$$

$$= -\int_{-\frac{\pi}{8}}^{0} e^{-jx}\, dx \cdot \frac{1}{4\pi B}$$

$$= -\frac{1}{4\pi B}\int_{-\pi/8}^{0} \cos x - j\sin x\, dx$$

$$= -\frac{1}{4\pi B}\Big[\sin x + j\cos x\Big]_{-\pi/8}^{0}$$

$$= -\frac{1}{4\pi B}\Big[j - (\sin(-\pi/8) + j\cos(-\pi/8)\Big]$$

$$= -\frac{1}{4\pi B}\left(\sin(\pi/8) + j(1 - \cos\pi/8)\right)$$

$$\int_{0}^{1/2B} (+1)\, e^{-j2\pi f_s t}\, dt$$

$$= \frac{1}{4\pi B}\int_{0}^{\pi/8} \cos x - j\sin x\, dx = \Big[\sin x + j\cos x\Big]_{0}^{\pi/8}\frac{1}{4\pi B}$$

$$= \frac{1}{4\pi B}\Big[\sin(\pi/8) + j\cos(\pi/8) - j\Big] = \frac{1}{4\pi B}\left(\sin(\pi/8) + j(\cos\pi/8 - 1)\right)$$

$$C_2 = 2B\int_0^{1/2B} x_p(t)\, e^{-j2\pi f_s 2t}\, dt.$$

$$= 2B\int_{-1/2B}^{0} (-1)\, e^{-j2\pi f_s 2t}\, dt + 2B\int_0^{1/2B} (+1)\, e^{-j2\pi f_s 2t}\, dt.$$

$$= -\frac{1}{4\pi}\left(\frac{1}{\sqrt{2}} + j\left(1-\frac{1}{\sqrt{2}}\right)\right) + \frac{1}{4\pi}\left(\frac{1}{\sqrt{2}} + j\left(\frac{1}{\sqrt{2}}-1\right)\right)$$

$$= \frac{1}{4\pi}\left\{0 + j\left(\frac{1}{\sqrt{2}} - 1 - 1 + \frac{1}{\sqrt{2}}\right)\right\}$$

$$= \frac{1}{4\pi}\, j\left(\frac{2}{\sqrt{2}} - 2\right)$$

$$= \frac{1}{2\pi}\, j\left(\frac{1}{\sqrt{2}} - 1\right)$$

$$2B\int_{-1/2B}^{0}(-1)e^{-j2\pi f_s 2t}\,dt$$

$$f_s = 2B$$
$$j2\pi f_s 2t = j8\pi Bt$$
$$j8\pi Bt = x$$
$$dt = \frac{1}{8\pi B}dx$$

$$2B\int_{-\frac{\pi}{4}}^{0} -e^{-jx}\,dx \cdot \frac{1}{8\pi B}$$

$$= \frac{-1}{4\pi}\int_{-\pi/4}^{0} e^{-jx}\,dx$$

$$= -\frac{1}{4\pi}\int_{-\pi/4}^{0} \cos x - j\sin x\,dx$$

$$= -\frac{1}{4\pi}\Big[\sin x + j\cos x\Big]_{-\pi/4}^{0}$$

$$= -\frac{1}{4\pi}\Big\{(0+j) - (\sin(-\pi/4) + j\cos(-\pi/4)\Big\}$$

$$= -\frac{1}{4\pi}\left(j + \sin(\pi/4) - j\cos(\pi/4)\right)$$

$$= -\frac{1}{4\pi}\left(\frac{1}{\sqrt{2}} + j\left(1 - \frac{1}{\sqrt{2}}\right)\right)$$

$$2B \int_0^{1/8B} (+1)\, e^{-j2\pi f_0 2t}\, dt$$

$$= 2B \int_0^{\pi/4} e^{-jx}\, dx \cdot \frac{1}{8\pi B}$$

$$= \frac{1}{4\pi} \int_0^{\pi/4} \cos x - j \sin x \, dx$$

$$= \frac{1}{4\pi} \Big[\sin x + j \cos x \Big]_0^{\pi/4}$$

$$= \frac{1}{4\pi} \left\{ \left(\frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} \right) - (0 + j) \right\}$$

$$= \frac{1}{4\pi} \left(\frac{1}{\sqrt{2}} + j \left(\frac{1}{\sqrt{2}} - 1 \right) \right)$$

c)

$\bar{g}(t) = g(t) \times x_p(t)$.

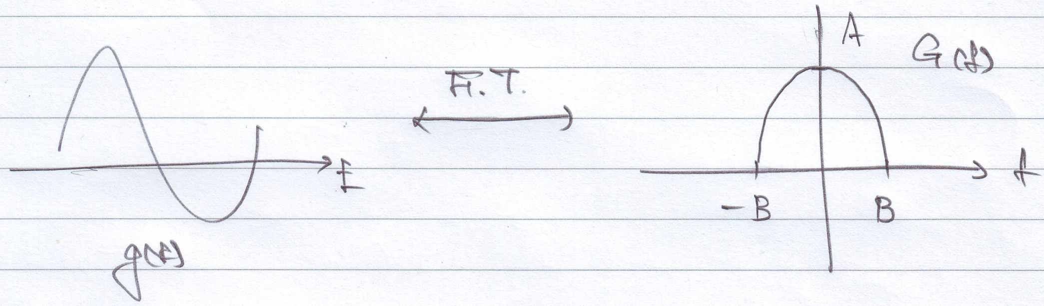
sampling rate $\Rightarrow T_s = 1/2B$

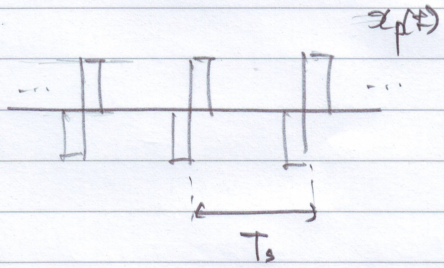
$f_s = 2B$.

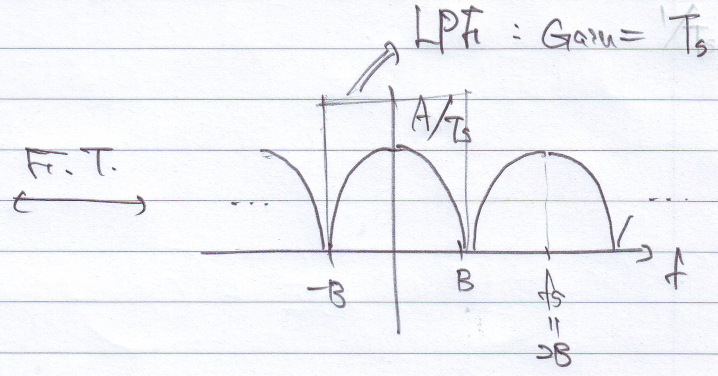
$f_0 = B$.

$f_s \geq 2B$ Hz $\Rightarrow$ Nyquist frequency.

$\therefore$ It is possible to recover $g(t)$ from $\bar{g}(t)$.



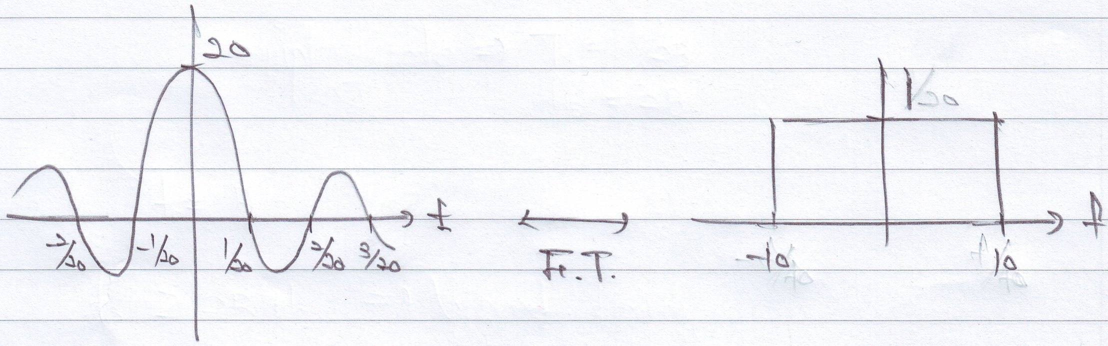




No. 5

2. $g(t) = 20\ \mathrm{sinc}(20t)$

a).



$g(t) = 20\,\mathrm{sinc}(20t)$

$G(f) = 20 \cdot \frac{1}{20}\,\mathrm{rect}(f/20)$

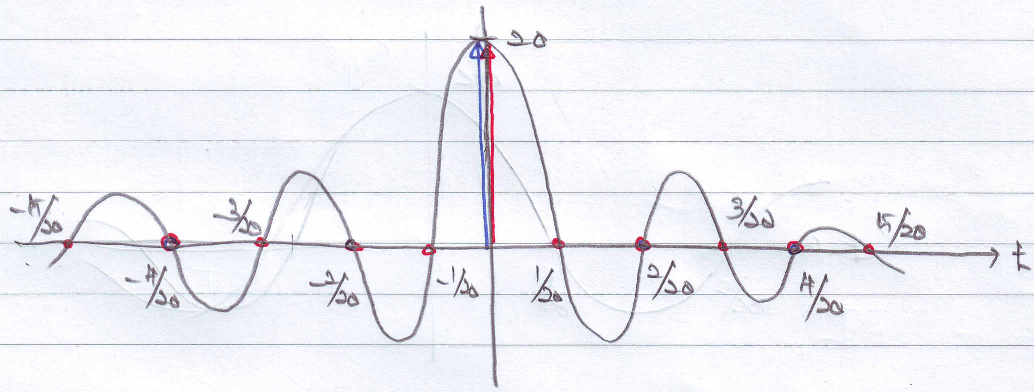
$= \mathrm{rect}(f/20)$

B.W of $g(t) \Rightarrow B = 10$ Hz.

Nyquist Sampling Rate $\geq 2B$

$= 20$ Hz.

b).



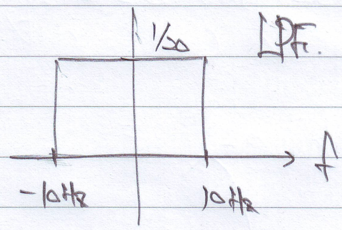
$f_s = 1/T_s$

$T_s = 1/f_s$

i) $T_s = 1/10$ blue

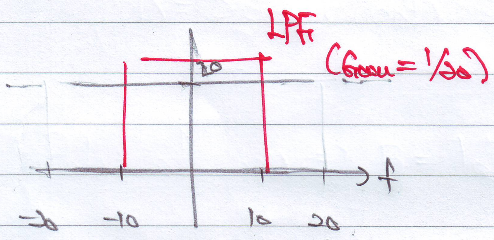
ii) $T_s = 1/20$. red.

c)



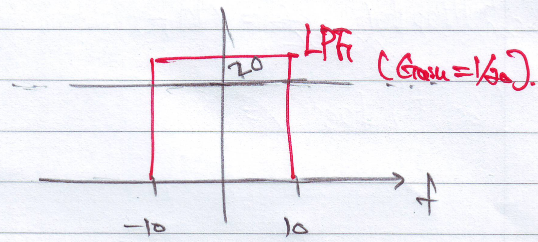
I)

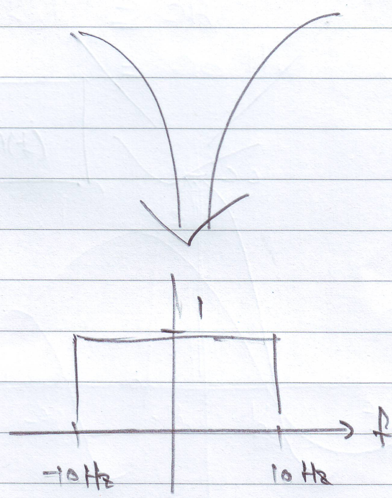
$\bar{G}(f)$



II).

$\bar{G}(f)$





We can recover in both cases.