

“Not all students take both History and Biology.”

constants: History, Biology

predicates: Student(x) “x is a student”
 Takes(x, c) “x takes course c”

sentence:

$$\neg (\forall x \text{ Student}(x) \Rightarrow \text{Takes}(x, \text{History}) \wedge \text{Takes}(x, \text{Biology}))$$

equivalent to:

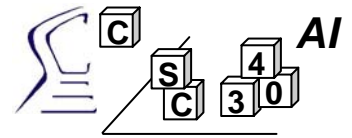
$$\exists x \text{ Student}(x) \wedge (\neg \text{Takes}(x, H) \vee \neg \text{Takes}(x, B))$$

“There is a barber in town who shaves all men who do not shave themselves.”

predicates: Barber(x) “x is a barber”
 InTown(x) “x live in town”
 Man(x) “x is a man”
 Shaves(x, y) “x shaves y”

sentence:

$$\exists x \text{ Barber}(x) \wedge \text{InTown}(x) \wedge (\forall y \text{ Man}(y) \wedge \neg \text{Shaves}(y, y) \Rightarrow \text{Shaves}(x, y))$$



“Politicians

***can fool all of the people some of the time, they
can even fool some of the people all of the time,
but they can't fool all of the people all of the time.”***

predicates:

Politician(x)	“x is a politician”
Person(x)	“x is a person”
Time(t)	“t is a time”
Fools(x, y, t)	“x fools y at t”

sentence:

$\forall x \text{ Politician}(x) \Rightarrow$

$(\forall y \text{ Person}(y) \Rightarrow \exists t \text{ Time}(t) \wedge \text{Fools}(x, y, t)) \wedge$

$(\exists y \text{ Person}(y) \wedge (\forall t \text{ Time}(t) \Rightarrow \text{Fools}(x, y, t))) \wedge$

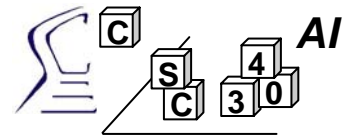
$\neg (\forall y \forall t \text{ Person}(y) \wedge \text{Time}(t) \Rightarrow \text{Fools}(x, y, t))$

or 3 sentences:

$\forall x \text{ Pol}(x) \Rightarrow (\forall y \text{ Per}(y) \Rightarrow \exists t \text{ T}(t) \wedge \text{F}(x, y, t))$

$\forall x \text{ Pol}(x) \Rightarrow (\exists y \forall t \text{ Per}(y) \wedge \text{T}(t) \Rightarrow \text{F}(x, y, t))$

$\forall x \text{ P}(x) \Rightarrow \neg (\forall y, t \text{ Per}(y) \wedge \text{T}(t) \Rightarrow \text{F}(x, y, t))$



Logical sentence interpretation:

$$(i) \quad \forall x (\text{Boy}(x) \Rightarrow \exists y (\text{Girl}(y) \wedge \text{Likes}(x, y)))$$

“every boy likes a girl”

$$(ii) \quad \exists y (\text{Girl}(y) \wedge \forall x (\text{Boy}(x) \Rightarrow \text{Likes}(x, y)))$$

“there is a girl that all boys like”

Equivalence and entailment:

equivalent? *no*

which entails which?

$(i) \Rightarrow (ii) ?$

$(ii) \Rightarrow (i) ?$

(i) does *not* entail (ii)

every boy likes a girl
but not nec. the same

(ii) entails (i)

every boy likes *that* girl
so they all like one

How to prove it logically:

proof: *counter-example!*

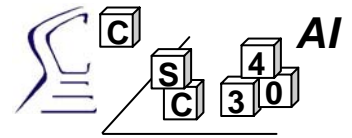
case where P true and Q
is not? nec. $P \Rightarrow Q$ is false

e.g., 2 boys, Al and Ben, and
2 girls, Cloe and Dora,
 $\text{Likes}(\text{Al}, \text{Cloe}), \text{Likes}(\text{Ben}, \text{Dora})$
 \rightarrow (i) is true but (ii) is not

proof: *by contradiction*

$P \Rightarrow Q$? suppose not i.e.
 $\neg(P \Rightarrow Q)$, that is: $P \wedge \neg Q$

then show ...
the hypothesis ...
yields a ...
contradiction

**A working proof:**hypothesis: (ii) $\wedge \neg(i)$

$$(\exists y \text{ Girl}(y) \wedge \forall x (\text{Boy}(x) \Rightarrow \text{Likes}(x, y))) \wedge \\ \neg (\forall z \text{ Boy}(z) \Rightarrow \exists u (\text{Girl}(u) \wedge \text{Likes}(z, u)))$$

converting to CNF:

$$\begin{aligned} & \underline{\text{Girl}(G)} \wedge \forall x (\underline{\neg \text{Boy}(x)} \vee \underline{\text{Likes}(x, G)}) \wedge \\ & \neg (\forall z \neg \text{Boy}(z) \vee \exists u (\text{Girl}(u) \wedge \text{Likes}(z, u))) \\ & \quad \exists z \text{ Boy}(z) \wedge \neg \exists u (\text{Girl}(u) \wedge \text{Likes}(z, u)) \\ & \quad \underline{\text{Boy}(B)} \wedge \forall u (\underline{\neg \text{Girl}(u)} \vee \underline{\neg \text{Likes}(B, u)}) \end{aligned}$$

KB of 4 CNF sentences:

- | | |
|---------------------|---|
| 1. $\text{Girl}(G)$ | 2. $\neg \text{Boy}(x) \vee \text{Likes}(x, G)$ |
| 3. $\text{Boy}(B)$ | 4. $\neg \text{Girl}(u) \vee \neg \text{Likes}(B, u)$ |

resolving 1+4 with $\{u/G\}$: resolving 2+3 with $\{x/B\}$:

- | | |
|------------------------------|-------------------------|
| 5. $\neg \text{Likes}(B, G)$ | 6. $\text{Likes}(B, G)$ |
|------------------------------|-------------------------|

resolving 5 and 6 yields a contradiction: \emptyset hence the *hypothesis is false* and (ii) \Rightarrow (i)