Tutorial 5

Optimisation

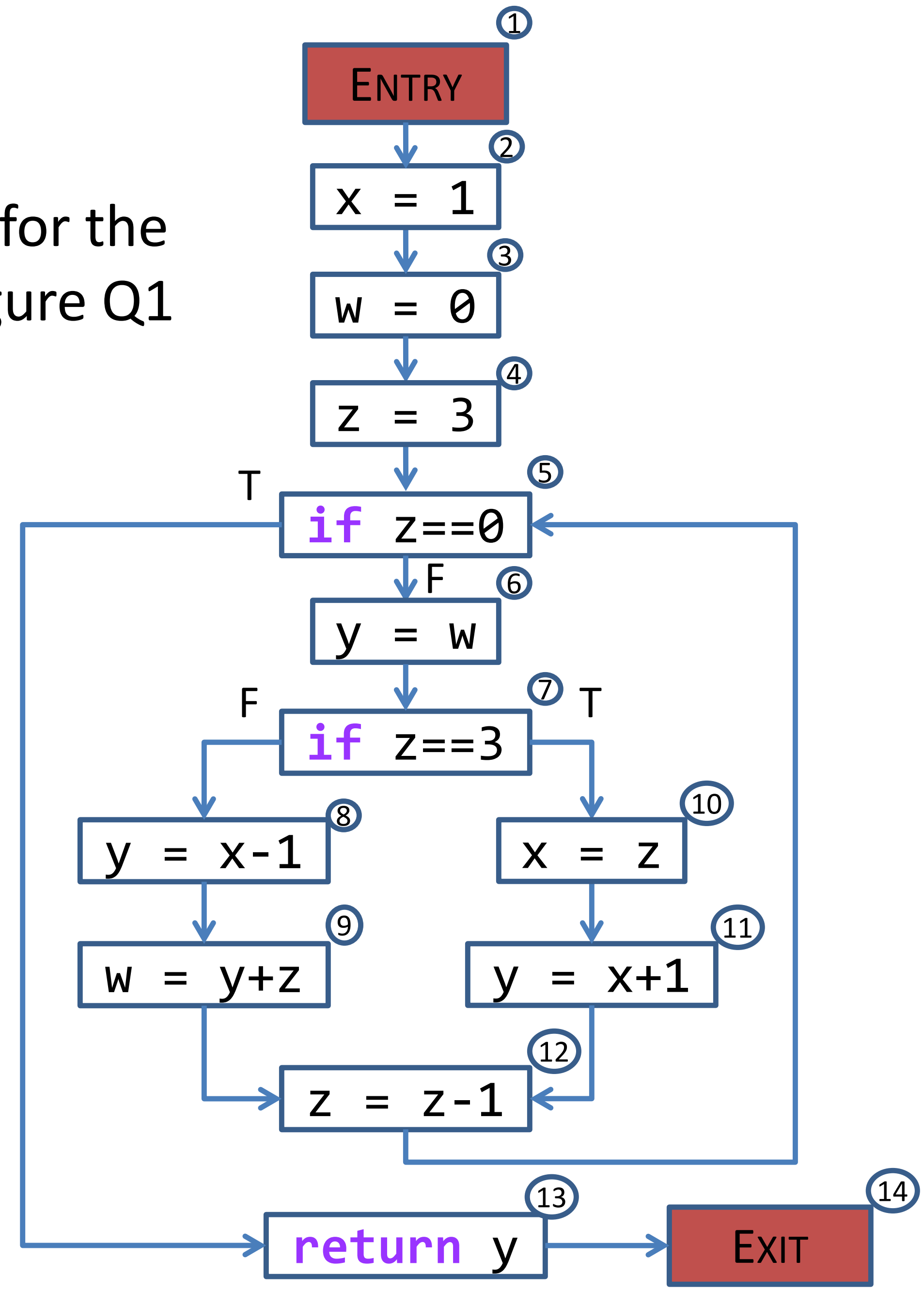
(to be covered in 1.5 tutorials)

Question 1

- Draw the Control Flow Graph for the Jimple-like code snippet in Figure Q1

```
    x = 1;
    w = 0;
    z = 3;
l0: if z==0 goto l4;
l1: y = w;
    if z==3 goto l2;
    y = x-1;
    w = y+z;
    goto l3;
l2: x = z;
    y = x+1;
l3: z = z-1;
    goto l0;
l4: return y;
```

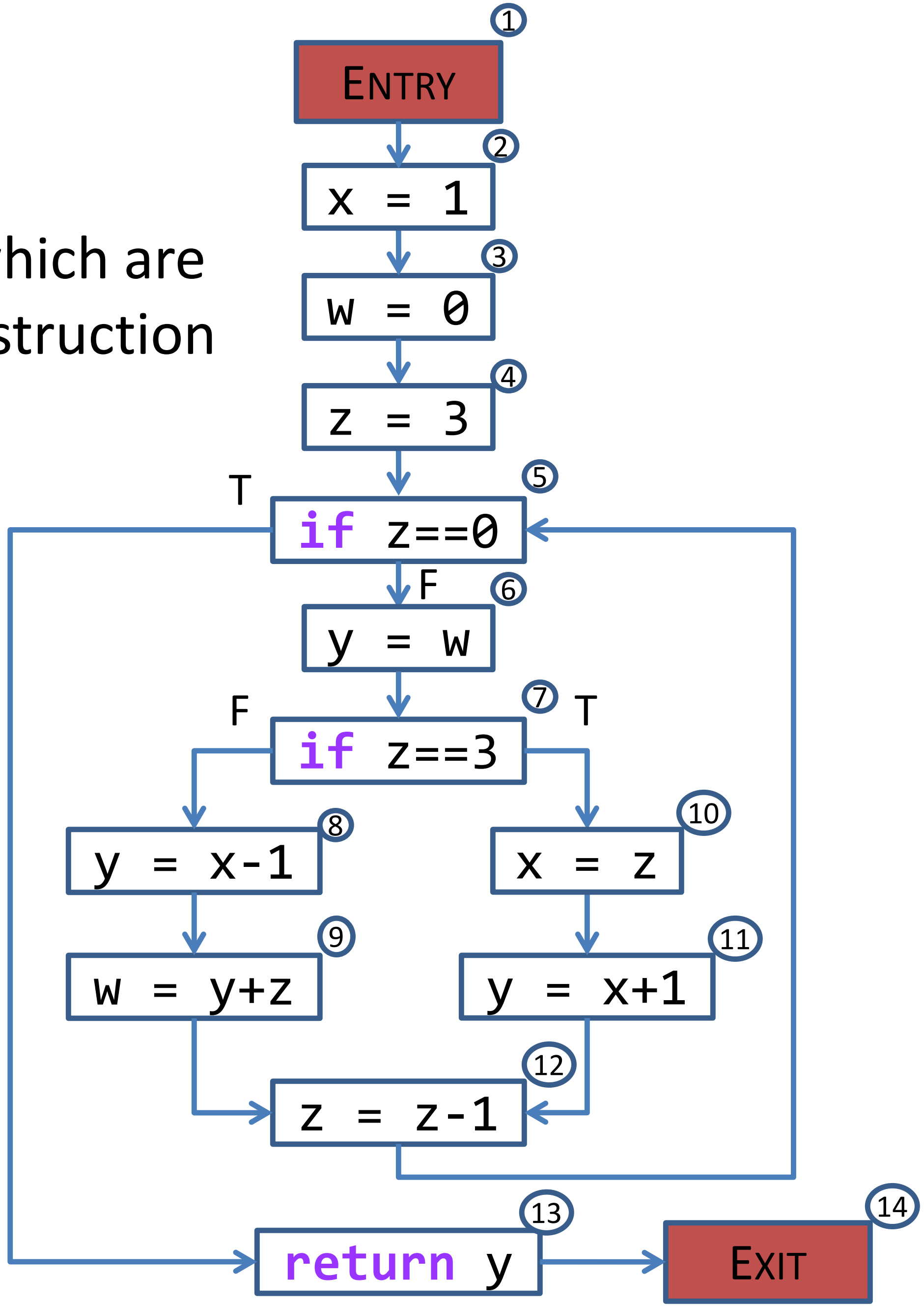
Figure Q1



Question 1

- Which variables are *live* and which are *dead after* the node for the instruction at label `l1` (node 6)

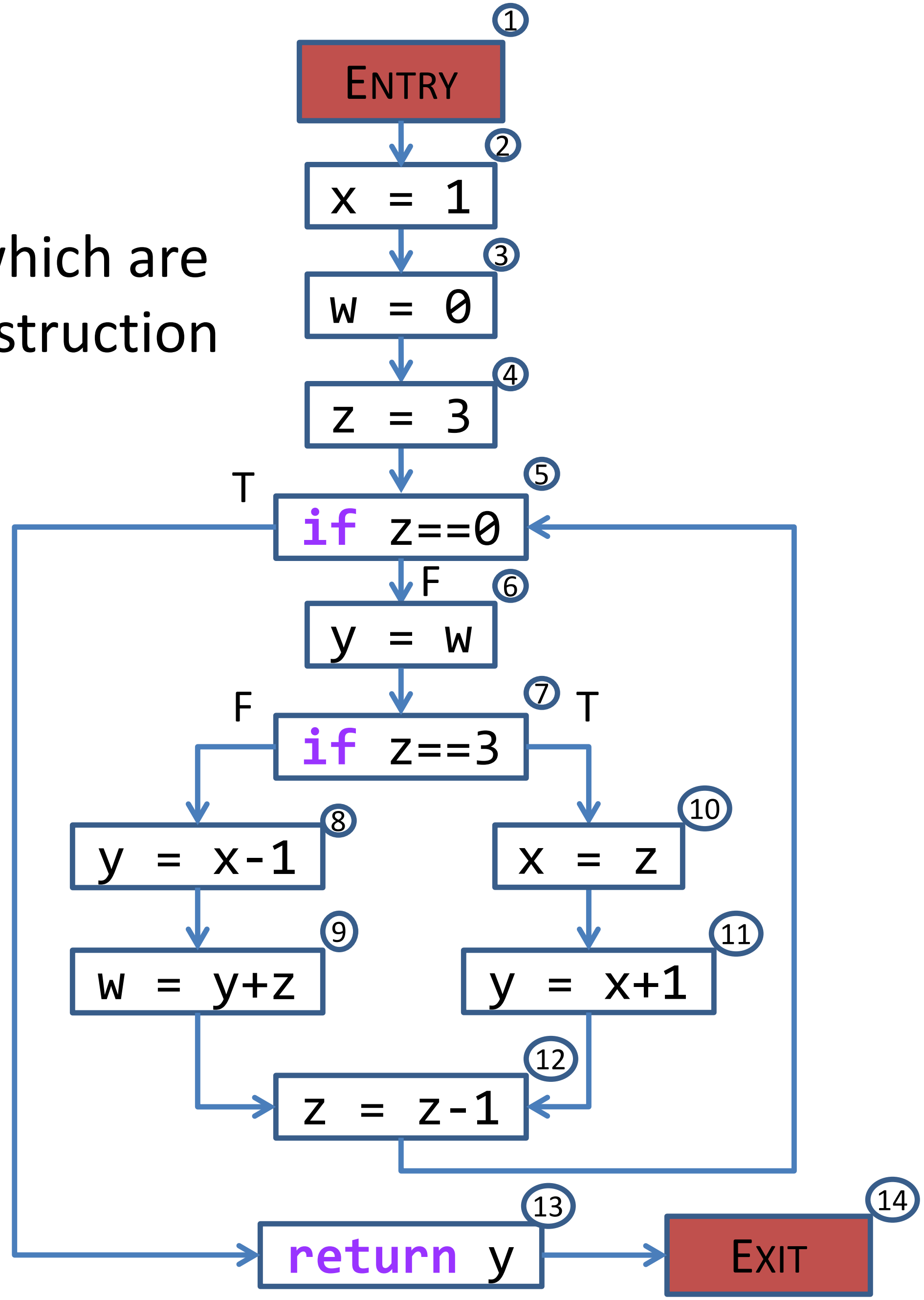
- Variable x is **live** because there is a branch where it is read by node 8 and this node is not preceded by any other node that writes x
- It does not matter that x is written by node 10 on the other branch



Question 1

- Which variables are *live* and which are *dead after* the node for the instruction at label `l1` (node 6)

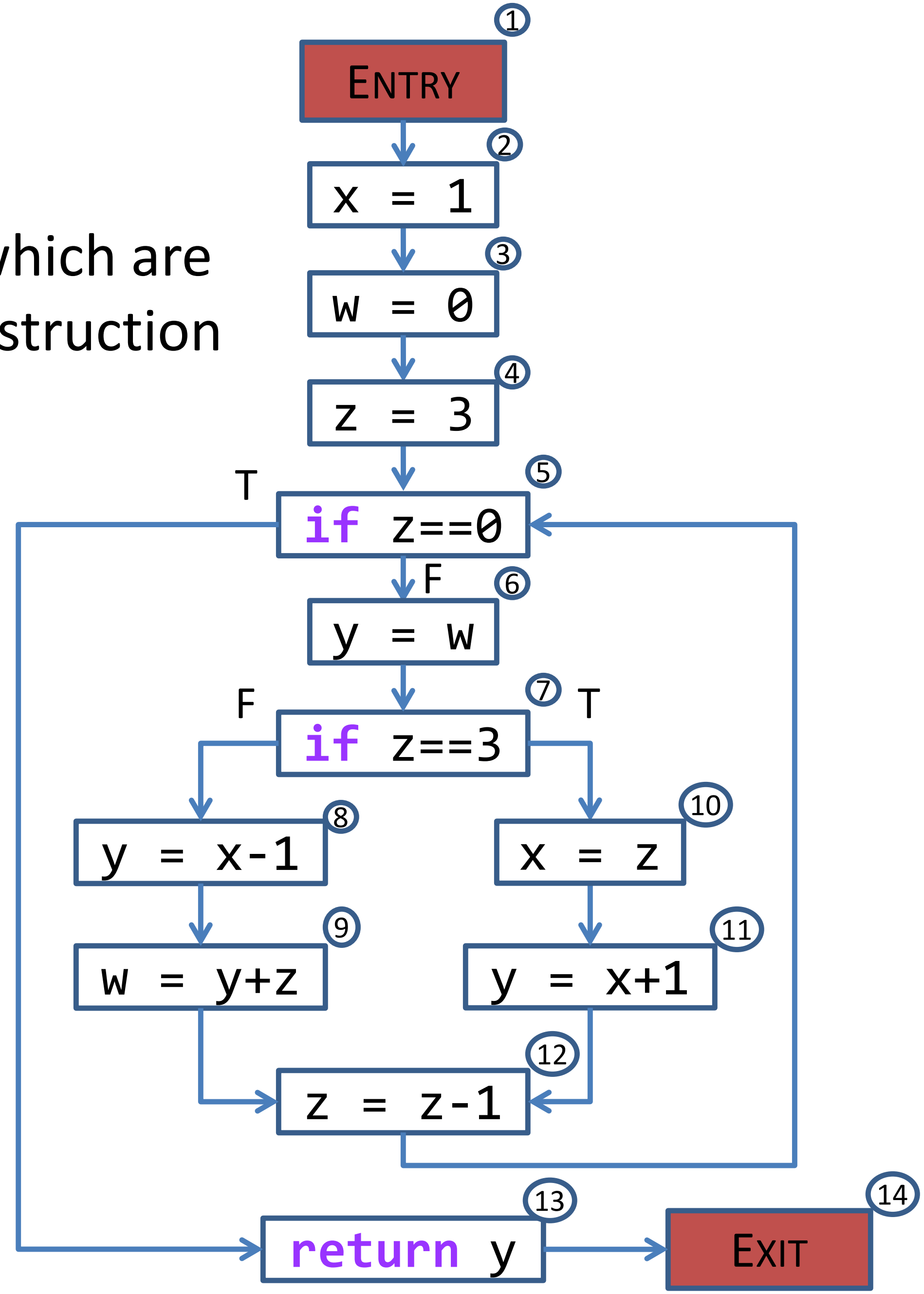
- Variable y is **dead** because it is written without being read on both branches by node 8 and node 11 respectively
- Variable z is **live** because it is read in the conditional jump in the next node, node 7



Question 1

- Which variables are *live* and which are *dead after* the node for the instruction at label `l1` (node 6)

- Although variable w is written by node 9 in one branch, the other branch neither reads nor writes w
- Variable w is **live** because it is read by node 6 in the next iteration of the loop and it is not previously written by any other node on the path



Question 2

- The transfer equations for *liveness analysis* for a Control Flow Graph with 12 nodes are as shown in Figure Q2
- For simplicity, the $out_L(n)$ expressions have been substituted away
- Solve the equations *iteratively* by filling in successive columns of Table Q2 until there are no further changes

$in_L(1) = in_L(2)$

$in_L(2) = in_L(3) \setminus \{z\}$

$in_L(3) = in_L(4) \setminus \{x\}$

$in_L(4) = in_L(11) \cup in_L(5) \cup \{x, z\}$

$in_L(5) = in_L(6) \setminus \{r\} \cup \{x\}$

$in_L(6) = in_L(7) \setminus \{t\} \cup \{z\}$

$in_L(7) = in_L(8) \cup in_L(9) \cup \{t, z\}$

$in_L(8) = in_L(10) \setminus \{x\} \cup \{r, t\}$

$in_L(9) = in_L(10) \setminus \{x\} \cup \{r, z\}$

$in_L(10) = in_L(4) \cup \{z\}$

$in_L(11) = in_L(12) \cup \{r\}$

$in_L(12) = \emptyset$

Figure Q2

Question 2

$in_L(1) = in_L(2)$

$in_L(2) = in_L(3) \setminus \{z\}$

$in_L(3) = in_L(4) \setminus \{x\}$

$in_L(4) = in_L(11) \cup in_L(5) \cup \{x, z\}$

$in_L(5) = in_L(6) \setminus \{r\} \cup \{x\}$

$in_L(6) = in_L(7) \setminus \{t\} \cup \{z\}$

$in_L(7) = in_L(8) \cup in_L(9) \cup \{t, z\}$

$in_L(8) = in_L(10) \setminus \{x\} \cup \{r, t\}$

$in_L(9) = in_L(10) \setminus \{x\} \cup \{r, z\}$

$in_L(10) = in_L(4) \cup \{z\}$

$in_L(11) = in_L(12) \cup \{r\}$

$in_L(12) = \varnothing$

Table Q2

	0	1	2	3	4	5	6
$in_L(1)$	$\varnothing$						
$in_L(2)$	$\varnothing$						
$in_L(3)$	$\varnothing$						
$in_L(4)$	$\varnothing$						
$in_L(5)$	$\varnothing$						
$in_L(6)$	$\varnothing$						
$in_L(7)$	$\varnothing$						
$in_L(8)$	$\varnothing$						
$in_L(9)$	$\varnothing$						
$in_L(10)$	$\varnothing$						
$in_L(11)$	$\varnothing$						
$in_L(12)$	$\varnothing$						

Question 2

$$in_L(1) = in_L(2)$$
$$in_L(2) = in_L(3) \setminus \{z\}$$
$$in_L(3) = in_L(4) \setminus \{x\}$$
$$in_L(4) = in_L(11) \cup in_L(5) \cup \{x, z\}$$
$$in_L(5) = in_L(6) \setminus \{r\} \cup \{x\}$$
$$in_L(6) = in_L(7) \setminus \{t\} \cup \{z\}$$
$$in_L(7) = in_L(8) \cup in_L(9) \cup \{t, z\}$$
$$in_L(8) = in_L(10) \setminus \{x\} \cup \{r, t\}$$
$$in_L(9) = in_L(10) \setminus \{x\} \cup \{r, z\}$$
$$in_L(10) = in_L(4) \cup \{z\}$$
$$in_L(11) = in_L(12) \cup \{r\}$$
$$in_L(12) = \varnothing$$

Table Q2

	0	1	2	3	4	5	6
$in_L(1)$	$\varnothing$	$\varnothing$					
$in_L(2)$	$\varnothing$	$\varnothing$					
$in_L(3)$	$\varnothing$	$\varnothing$					
$in_L(4)$	$\varnothing$	x,z					
$in_L(5)$	$\varnothing$	x					
$in_L(6)$	$\varnothing$	z					
$in_L(7)$	$\varnothing$	t,z					
$in_L(8)$	$\varnothing$	r,t					
$in_L(9)$	$\varnothing$	r,z					
$in_L(10)$	$\varnothing$	z					
$in_L(11)$	$\varnothing$	r					
$in_L(12)$	$\varnothing$	$\varnothing$					

Question 2

$in_L(1) = in_L(2)$

$in_L(2) = in_L(3) \setminus \{z\}$

$in_L(3) = in_L(4) \setminus \{x\}$

$in_L(4) = in_L(11) \cup in_L(5) \cup \{x, z\}$

$in_L(5) = in_L(6) \setminus \{r\} \cup \{x\}$

$in_L(6) = in_L(7) \setminus \{t\} \cup \{z\}$

$in_L(7) = in_L(8) \cup in_L(9) \cup \{t, z\}$

$in_L(8) = in_L(10) \setminus \{x\} \cup \{r, t\}$

$in_L(9) = in_L(10) \setminus \{x\} \cup \{r, z\}$

$in_L(10) = in_L(4) \cup \{z\}$

$in_L(11) = in_L(12) \cup \{r\}$

$in_L(12) = \varnothing$

Table Q2

	0	1	2	3	4	5	6
$in_L(1)$	$\varnothing$	$\varnothing$	$\varnothing$				
$in_L(2)$	$\varnothing$	$\varnothing$	$\varnothing$				
$in_L(3)$	$\varnothing$	$\varnothing$	z				
$in_L(4)$	$\varnothing$	x,z	r,x,z				
$in_L(5)$	$\varnothing$	x	x,z				
$in_L(6)$	$\varnothing$	z	z				
$in_L(7)$	$\varnothing$	t,z	r,t,z				
$in_L(8)$	$\varnothing$	r,t	r,t,z				
$in_L(9)$	$\varnothing$	r,z	r,z				
$in_L(10)$	$\varnothing$	z	x,z				
$in_L(11)$	$\varnothing$	r	r				
$in_L(12)$	$\varnothing$	$\varnothing$	$\varnothing$				

Question 2

$in_L(1) = in_L(2)$

$in_L(2) = in_L(3) \setminus \{z\}$

$in_L(3) = in_L(4) \setminus \{x\}$

$in_L(4) = in_L(11) \cup in_L(5) \cup \{x, z\}$

$in_L(5) = in_L(6) \setminus \{r\} \cup \{x\}$

$in_L(6) = in_L(7) \setminus \{t\} \cup \{z\}$

$in_L(7) = in_L(8) \cup in_L(9) \cup \{t, z\}$

$in_L(8) = in_L(10) \setminus \{x\} \cup \{r, t\}$

$in_L(9) = in_L(10) \setminus \{x\} \cup \{r, z\}$

$in_L(10) = in_L(4) \cup \{z\}$

$in_L(11) = in_L(12) \cup \{r\}$

$in_L(12) = \varnothing$

Table Q2

	0	1	2	3	4	5	6
$in_L(1)$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$			
$in_L(2)$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$			
$in_L(3)$	$\varnothing$	$\varnothing$	z	r,z			
$in_L(4)$	$\varnothing$	x,z	r,x,z	r,x,z			
$in_L(5)$	$\varnothing$	x	x,z	x,z			
$in_L(6)$	$\varnothing$	z	z	r,z			
$in_L(7)$	$\varnothing$	t,z	r,t,z	r,t,z			
$in_L(8)$	$\varnothing$	r,t	r,t,z	r,t,z			
$in_L(9)$	$\varnothing$	r,z	r,z	r,z			
$in_L(10)$	$\varnothing$	z	x,z	r,x,z			
$in_L(11)$	$\varnothing$	r	r	r			
$in_L(12)$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$			

Question 2

$in_L(1) = in_L(2)$

$in_L(2) = in_L(3) \setminus \{z\}$

$in_L(3) = in_L(4) \setminus \{x\}$

$in_L(4) = in_L(11) \cup in_L(5) \cup \{x, z\}$

$in_L(5) = in_L(6) \setminus \{r\} \cup \{x\}$

$in_L(6) = in_L(7) \setminus \{t\} \cup \{z\}$

$in_L(7) = in_L(8) \cup in_L(9) \cup \{t, z\}$

$in_L(8) = in_L(10) \setminus \{x\} \cup \{r, t\}$

$in_L(9) = in_L(10) \setminus \{x\} \cup \{r, z\}$

$in_L(10) = in_L(4) \cup \{z\}$

$in_L(11) = in_L(12) \cup \{r\}$

$in_L(12) = \varnothing$

Table Q2

	0	1	2	3	4	5	6
$in_L(1)$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$		
$in_L(2)$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	r		
$in_L(3)$	$\varnothing$	$\varnothing$	z	r,z	r,z		
$in_L(4)$	$\varnothing$	x,z	r,x,z	r,x,z	r,x,z		
$in_L(5)$	$\varnothing$	x	x,z	x,z	x,z		
$in_L(6)$	$\varnothing$	z	z	r,z	r,z		
$in_L(7)$	$\varnothing$	t,z	r,t,z	r,t,z	r,t,z		
$in_L(8)$	$\varnothing$	r,t	r,t,z	r,t,z	r,t,z		
$in_L(9)$	$\varnothing$	r,z	r,z	r,z	r,z		
$in_L(10)$	$\varnothing$	z	x,z	r,x,z	r,x,z		
$in_L(11)$	$\varnothing$	r	r	r	r		
$in_L(12)$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$		

Question 2

$in_L(1) = in_L(2)$

$in_L(2) = in_L(3) \setminus \{z\}$

$in_L(3) = in_L(4) \setminus \{x\}$

$in_L(4) = in_L(11) \cup in_L(5) \cup \{x, z\}$

$in_L(5) = in_L(6) \setminus \{r\} \cup \{x\}$

$in_L(6) = in_L(7) \setminus \{t\} \cup \{z\}$

$in_L(7) = in_L(8) \cup in_L(9) \cup \{t, z\}$

$in_L(8) = in_L(10) \setminus \{x\} \cup \{r, t\}$

$in_L(9) = in_L(10) \setminus \{x\} \cup \{r, z\}$

$in_L(10) = in_L(4) \cup \{z\}$

$in_L(11) = in_L(12) \cup \{r\}$

$in_L(12) = \varnothing$

Table Q2

	0	1	2	3	4	5	6
$in_L(1)$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	r	
$in_L(2)$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	r	r	
$in_L(3)$	$\varnothing$	$\varnothing$	z	r,z	r,z	r,z	
$in_L(4)$	$\varnothing$	x,z	r,x,z	r,x,z	r,x,z	r,x,z	
$in_L(5)$	$\varnothing$	x	x,z	x,z	x,z	x,z	
$in_L(6)$	$\varnothing$	z	z	r,z	r,z	r,z	
$in_L(7)$	$\varnothing$	t,z	r,t,z	r,t,z	r,t,z	r,t,z	
$in_L(8)$	$\varnothing$	r,t	r,t,z	r,t,z	r,t,z	r,t,z	
$in_L(9)$	$\varnothing$	r,z	r,z	r,z	r,z	r,z	
$in_L(10)$	$\varnothing$	z	x,z	r,x,z	r,x,z	r,x,z	
$in_L(11)$	$\varnothing$	r	r	r	r	r	
$in_L(12)$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	

Question 2

No further change after iteration 6

$in_L(1) = in_L(2)$

$in_L(2) = in_L(3) \setminus \{z\}$

$in_L(3) = in_L(4) \setminus \{x\}$

$in_L(4) = in_L(11) \cup in_L(5) \cup \{x, z\}$

$in_L(5) = in_L(6) \setminus \{r\} \cup \{x\}$

$in_L(6) = in_L(7) \setminus \{t\} \cup \{z\}$

$in_L(7) = in_L(8) \cup in_L(9) \cup \{t, z\}$

$in_L(8) = in_L(10) \setminus \{x\} \cup \{r, t\}$

$in_L(9) = in_L(10) \setminus \{x\} \cup \{r, z\}$

$in_L(10) = in_L(4) \cup \{z\}$

$in_L(11) = in_L(12) \cup \{r\}$

$in_L(12) = \varnothing$

Table Q2

	0	1	2	3	4	5	6
$in_L(1)$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	r	r
$in_L(2)$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	r	r	r
$in_L(3)$	$\varnothing$	$\varnothing$	z	r,z	r,z	r,z	r,z
$in_L(4)$	$\varnothing$	x,z	r,x,z	r,x,z	r,x,z	r,x,z	r,x,z
$in_L(5)$	$\varnothing$	x	x,z	x,z	x,z	x,z	x,z
$in_L(6)$	$\varnothing$	z	z	r,z	r,z	r,z	r,z
$in_L(7)$	$\varnothing$	t,z	r,t,z	r,t,z	r,t,z	r,t,z	r,t,z
$in_L(8)$	$\varnothing$	r,t	r,t,z	r,t,z	r,t,z	r,t,z	r,t,z
$in_L(9)$	$\varnothing$	r,z	r,z	r,z	r,z	r,z	r,z
$in_L(10)$	$\varnothing$	z	x,z	r,x,z	r,x,z	r,x,z	r,x,z
$in_L(11)$	$\varnothing$	r	r	r	r	r	r
$in_L(12)$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

Question 3

- Consider the Control Flow Graph (CFG) shown in Figure Q3
- Derive the set of transfer functions for available expressions analysis, i.e. the functions for $in_A(n)$ and $out_A(n)$ for each node n in the CFG

Set of all (arithmetic and logical) expressions that occur in the method: U = {w*2, x+y, z-1}

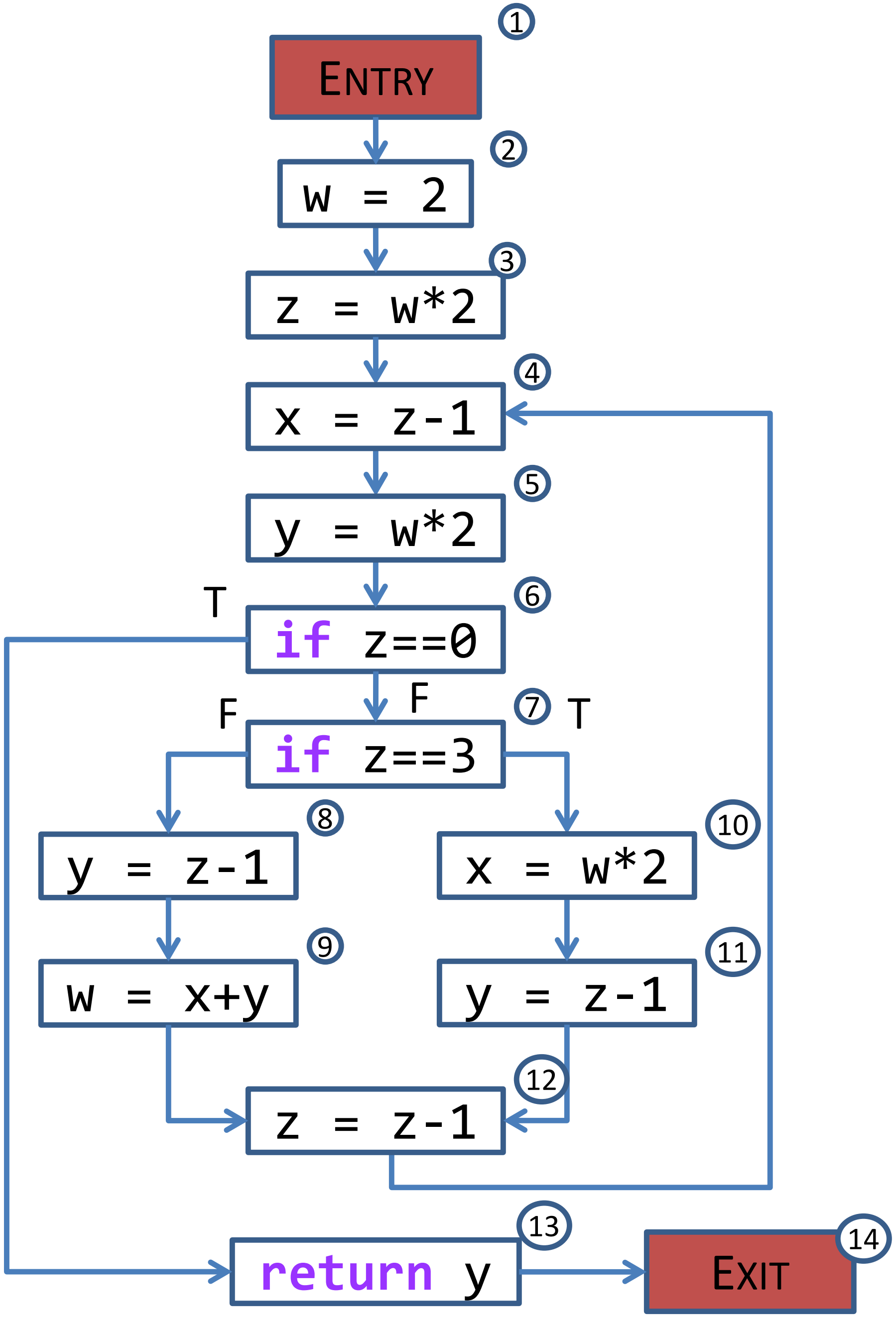


Figure Q3

Question 3

Transfer Functions

$$\mathrm{out}_A(n) = \mathrm{in}_A(n) \setminus \{e \mid \mathrm{vars}(e) \cap \mathrm{def}(n) \neq \emptyset\} \cup \mathrm{comp}(n)$$

$$\mathrm{out}_A(1) = \mathrm{in}_A(1)$$

$$\mathrm{out}_A(2) = \mathrm{in}_A(2) \setminus \{\, w*2 \,\}$$

$$\mathrm{out}_A(3) = \mathrm{in}_A(3) \setminus \{\, z-1 \,\} \cup \{\, w*2 \,\}$$

$$\mathrm{out}_A(4) = \mathrm{in}_A(4) \setminus \{\, x+y \,\} \cup \{\, z-1 \,\}$$

$$\mathrm{out}_A(5) = \mathrm{in}_A(5) \setminus \{\, x+y \,\} \cup \{\, w*2 \,\}$$

$$\mathrm{out}_A(6) = \mathrm{in}_A(6)$$

$$\mathrm{out}_A(7) = \mathrm{in}_A(7)$$

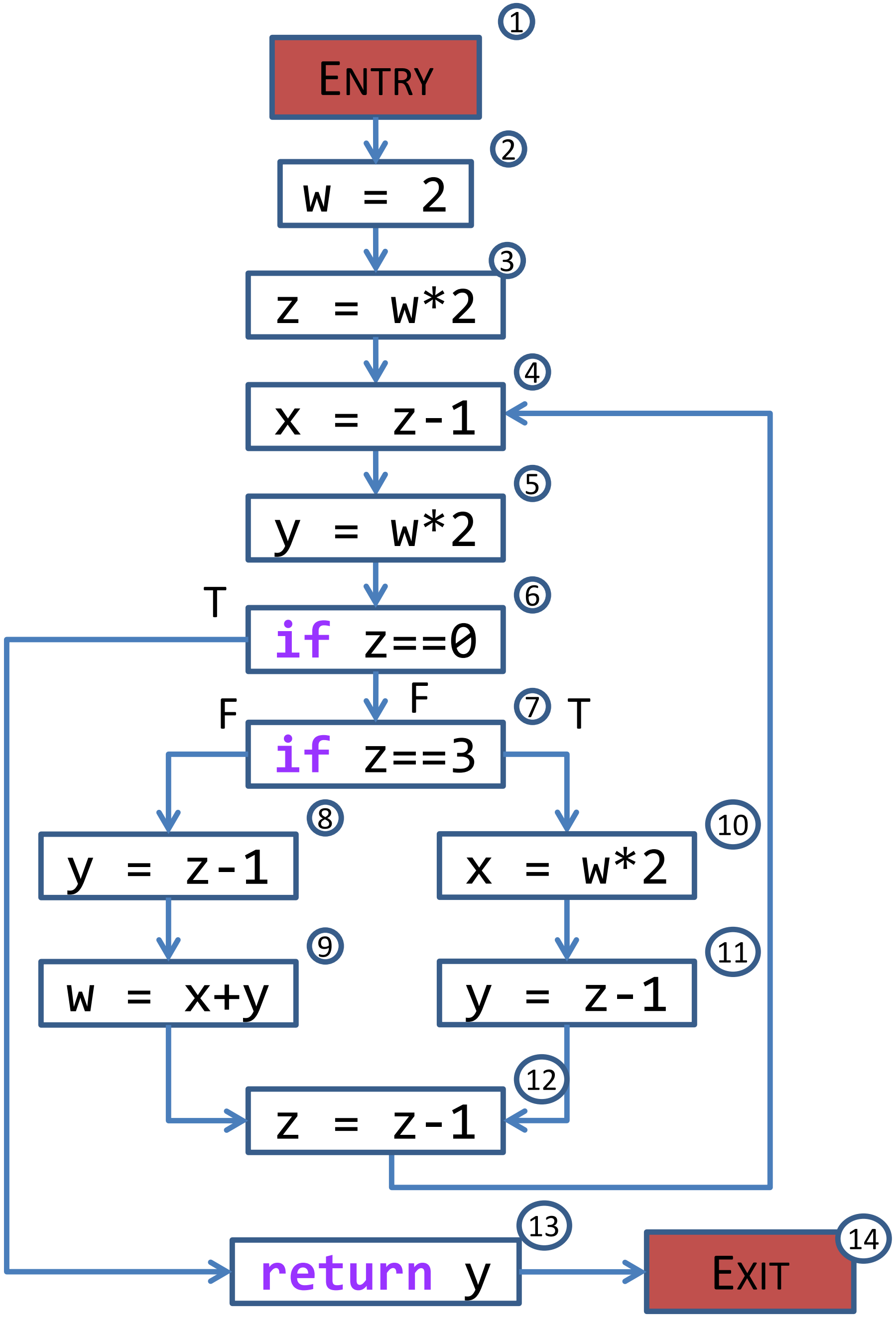


Figure Q3

Question 3

Transfer Functions

$$out_A(n) = in_A(n) \setminus \{e \mid vars(e) \cap def(n) \neq \emptyset\} \cup comp(n)$$

$out_A(8) = in_A(8) \setminus \{ x+y \} \cup \{ z-1 \}$

$out_A(9) = in_A(9) \setminus \{ w*2 \} \cup \{ x+y \}$

$out_A(10) = in_A(10) \setminus \{ x+y \} \cup \{ w*2 \}$

$out_A(11) = in_A(11) \setminus \{ x+y \} \cup \{ z-1 \}$

$out_A(12) = in_A(12) \setminus \{ z-1 \}$

$out_A(13) = in_A(13)$

$out_A(14) = in_A(14)$

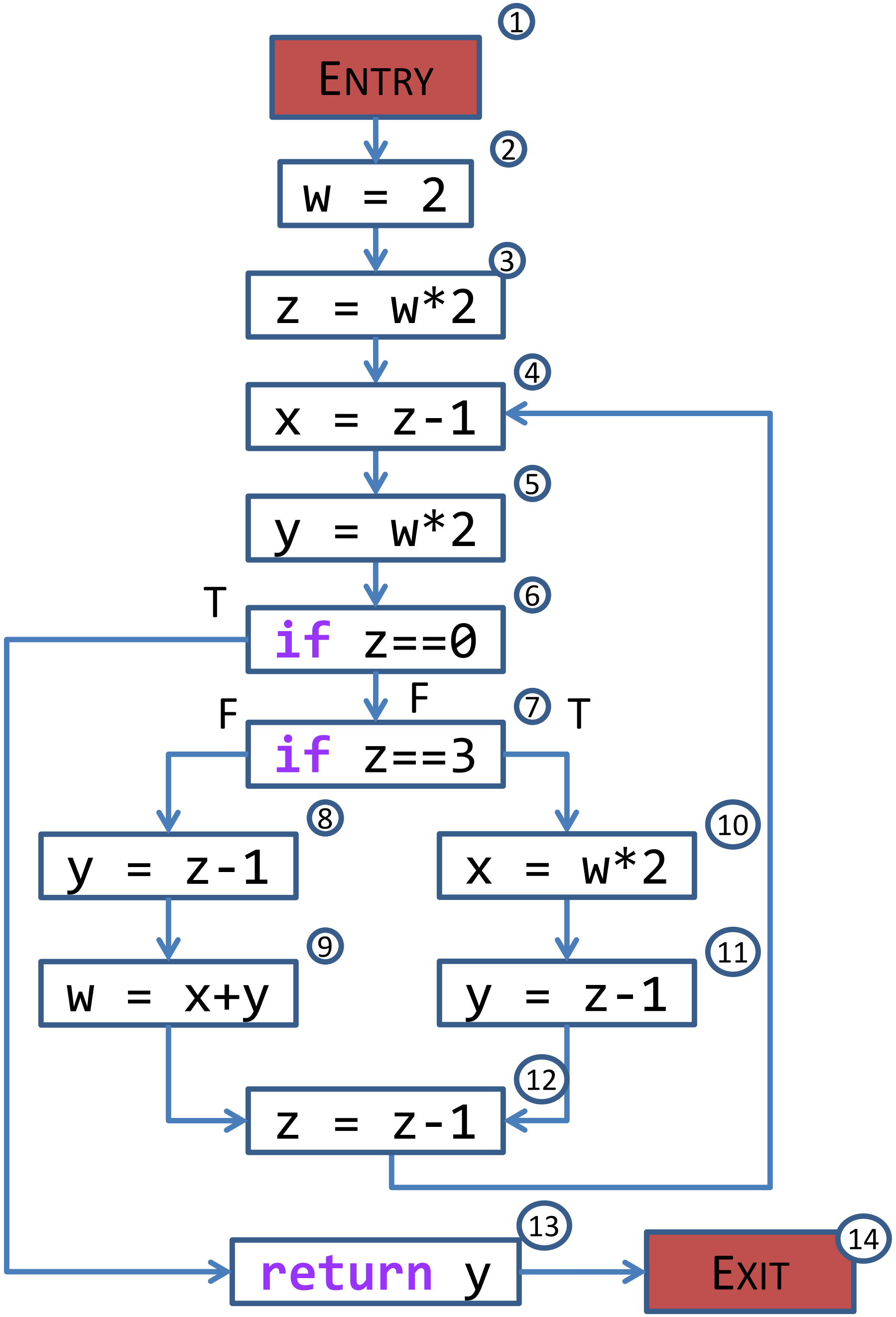


Figure Q3

Question 3

Transfer Functions

$$in_A(n) = \cap \{ out_A(m) \mid m \in pred(n) \}$$
$$in_A(\text{ENTRY}) = \emptyset$$

$in_A(1) = \emptyset$

$in_A(2) = out_A(1)$

$in_A(3) = out_A(2)$

$in_A(4) = out_A(3) \cap out_A(12)$

$in_A(5) = out_A(4)$

$in_A(6) = out_A(5)$

$in_A(7) = out_A(6)$

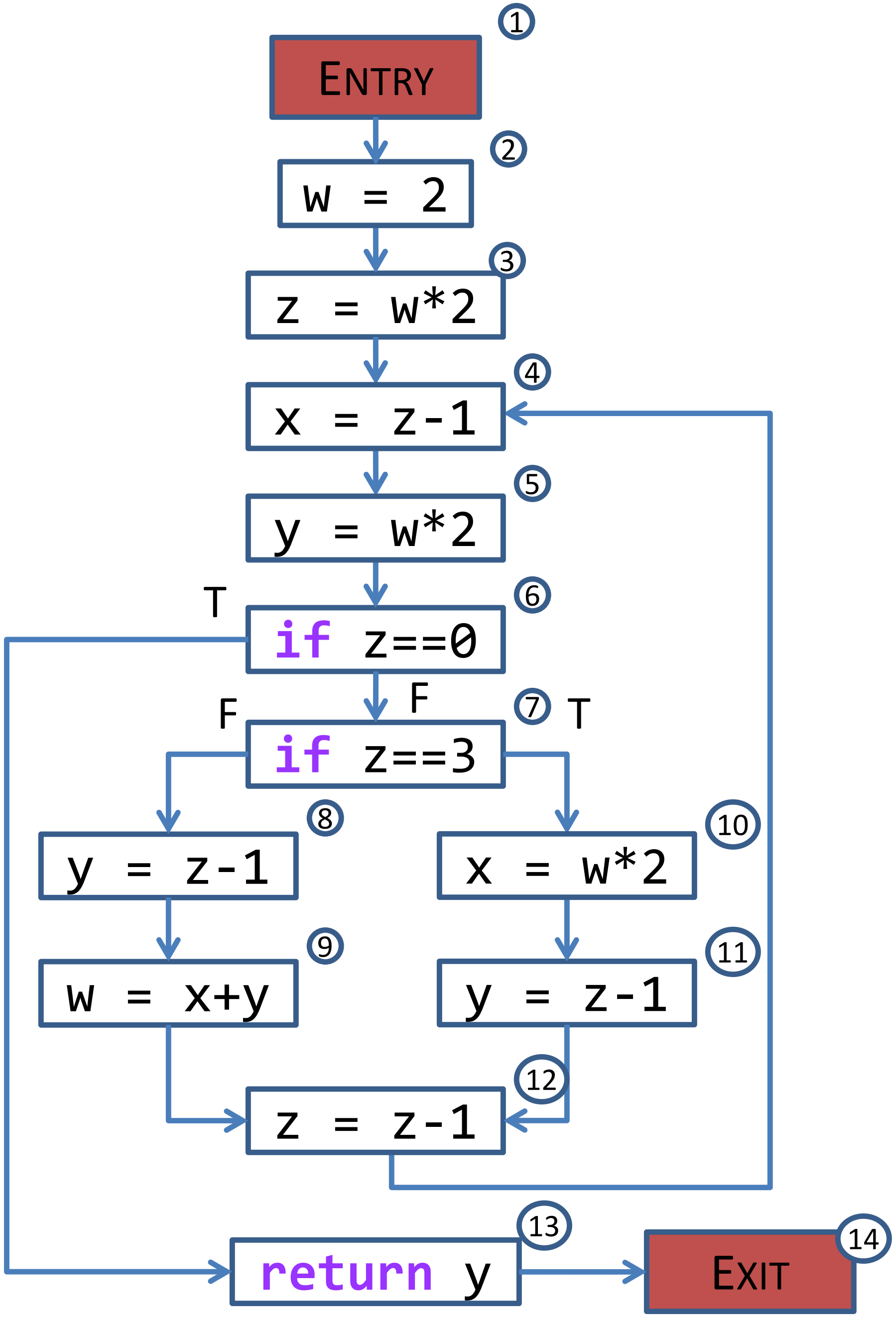


Figure Q3

Question 3

Transfer Functions

$in_A(n) = \cap \{ out_A(m) \mid m \in pred(n) \}$
$in_A(\text{ENTRY}) = \emptyset$

$in_A(8) = out_A(7)$

$in_A(9) = out_A(8)$

$in_A(10) = out_A(7)$

$in_A(11) = out_A(10)$

$in_A(12) = out_A(9) \cap out_A(11)$

$in_A(13) = out_A(6)$

$in_A(14) = out_A(13)$

Question 4

- Use the worklist algorithm shown in Figure Q4 to solve the set of transfer functions derived in Question 3

worklist = [all nodes]
while worklist != empty **do**
    m = removeFirst(worklist)
    recompute $out_A(m)$
    if $out_A(m)$ has changed **then**
        for each successor *n* of *m*
            compute $in_A(n)$
            if $in_A(n)$ has changed **then** put *n* into worklist
(if not already in worklist)

Figure Q4

Question 4

- You may assume that the worklist is initialized to the set of all nodes, in increasing order of node number, as shown in Figure Q3

- We initialize the worklist to contain all nodes in the CFG to ensure that each node is evaluated at least once
- As far as possible the nodes should be ordered so that a node is only evaluated after all its predecessors have updated their values (but this can be difficult when the CFG has cycles)
- Note: for all nodes, $\mathrm{out}_A(n)$ and $\mathrm{in}_A(n)$ are initialized to U, the set of all expressions in the method, except $\mathrm{in}_A(1) = \emptyset$

Question 4

worklist = [all nodes]

while worklist != empty do

    m = removeFirst(worklist)

    recompute $out_A(m)$

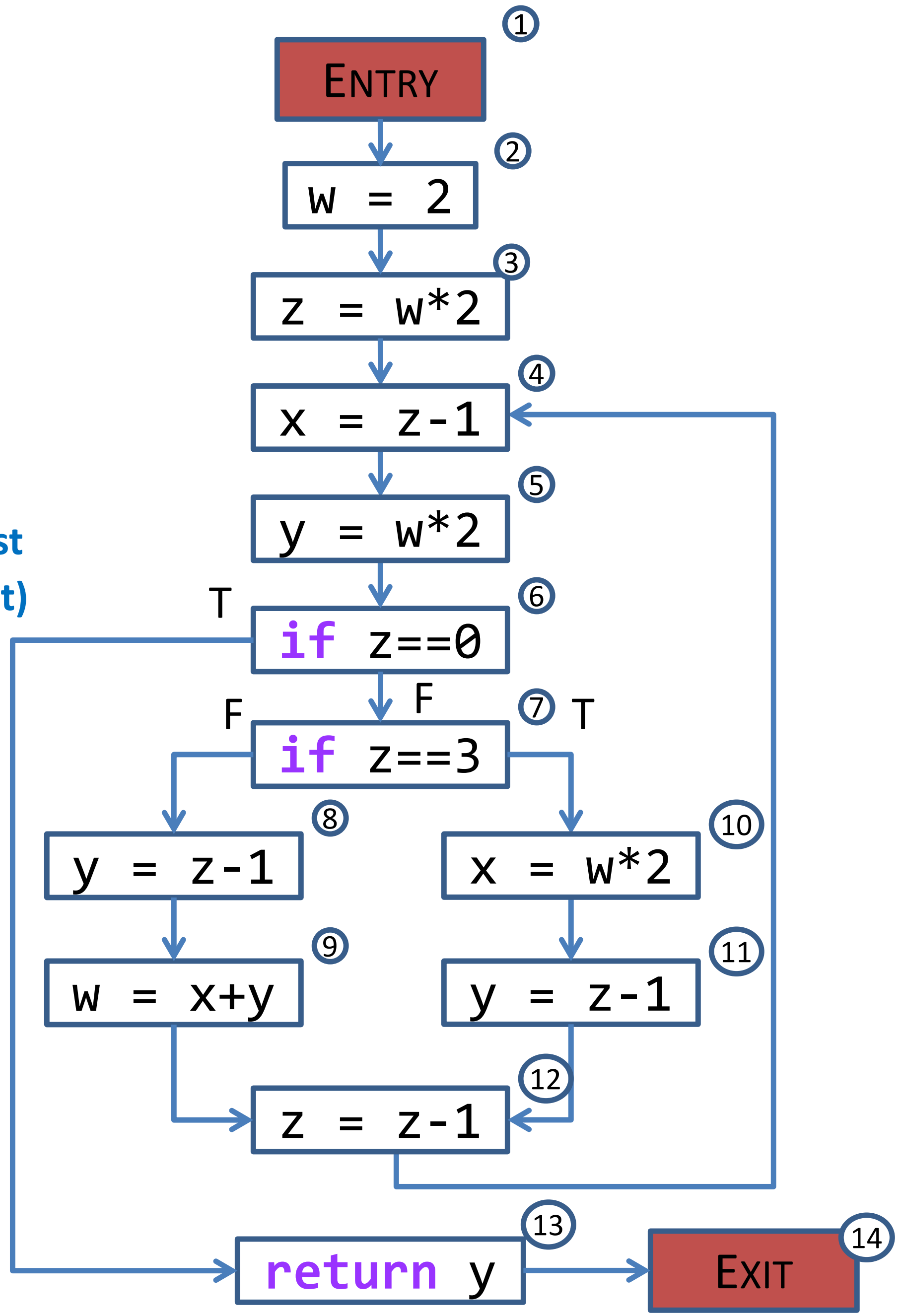
    if $out_A(m)$ has changed then

        for each successor *n* of *m*

            compute $in_A(n)$

            if $in_A(n)$ has changed then put *n* into worklist (if not already in worklist)

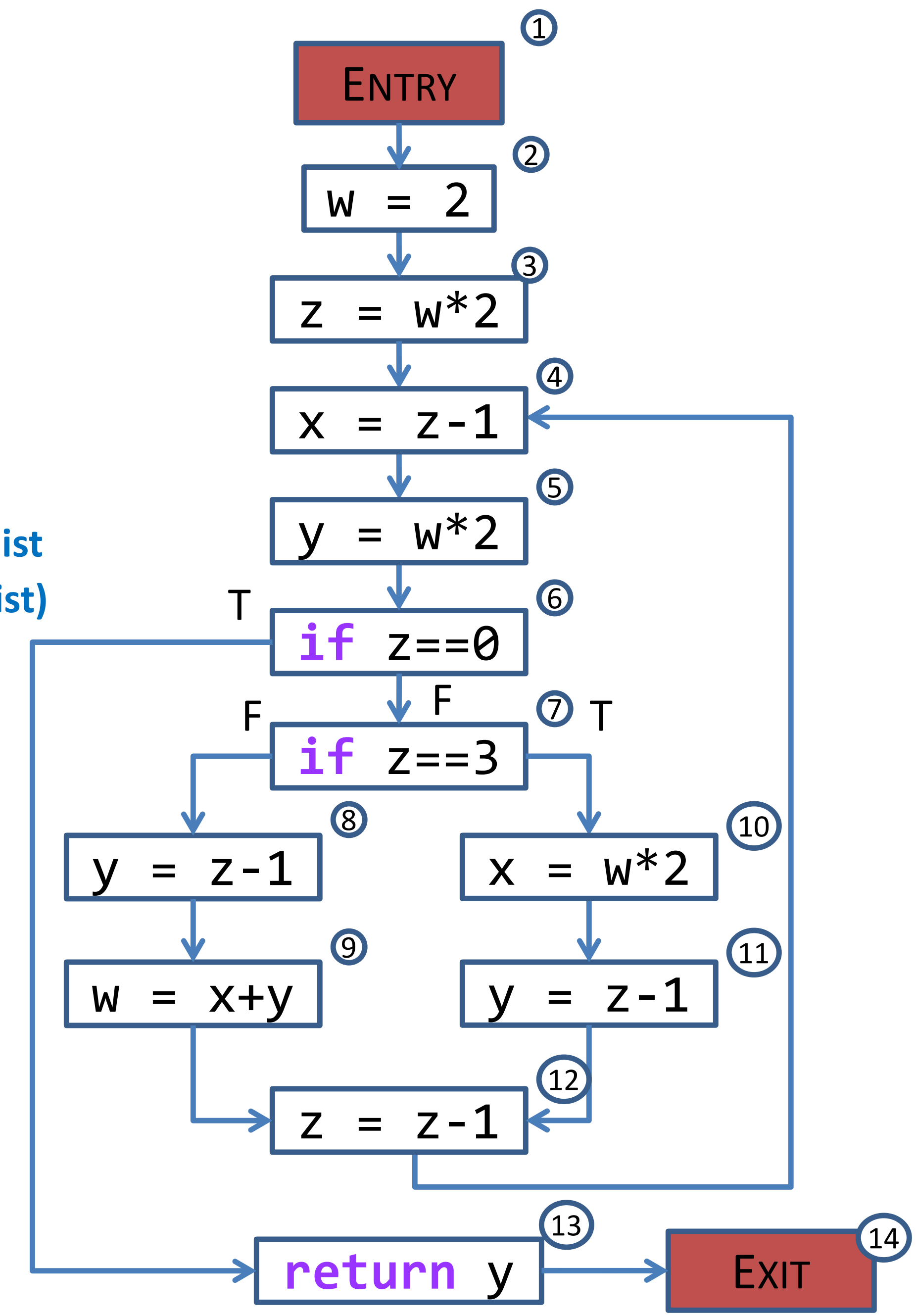
Worklist	**$out_A(m)$** &	**$in_A(n)$**
1, ..., 14	$out_A(1) = \emptyset$	$in_A(2) = \emptyset$
2, ..., 14	$out_A(2) = \emptyset$	$in_A(3) = \emptyset$
3, ..., 14	$out_A(3) = \{ w*2 \}$	
	$in_A(4) = \{ w*2 \}$	
4, ..., 14	$out_A(4) = \{ w*2, z-1 \}$	
	$in_A(5) = \{ w*2, z-1 \}$	



Question 4

worklist = [all nodes]
while worklist != empty do
    m = removeFirst(worklist)
    recompute $out_A(m)$
    if $out_A(m)$ has changed then
        for each successor *n* of *m*
            compute $in_A(n)$
            if $in_A(n)$ has changed then put *n* into worklist
                (if not already in worklist)

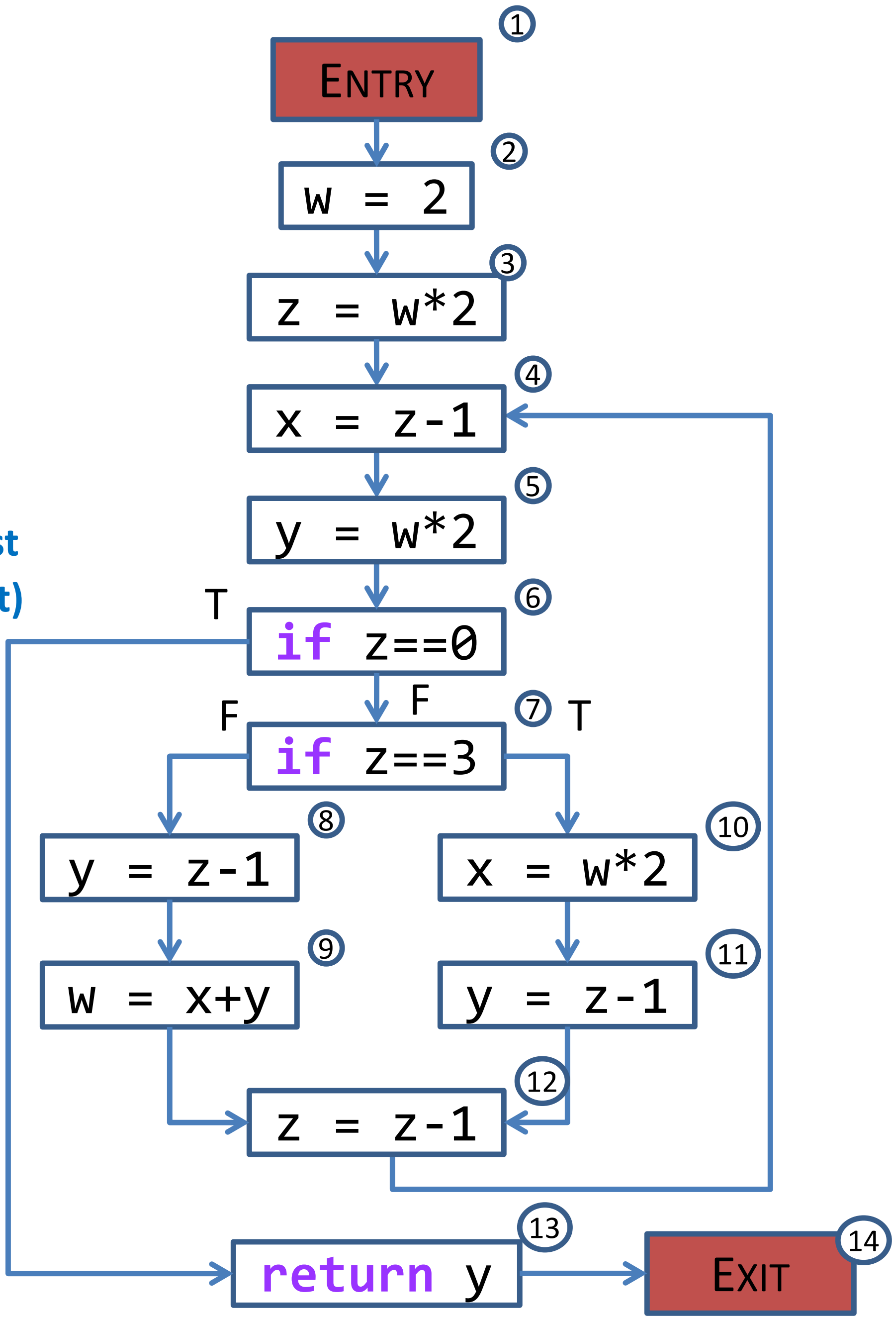
Worklist	**$out_A(m)$ & $in_A(n)$**
5, …, 14	$out_A(5)$ = { w*2, z-1 }
	$in_A(6)$ = { w*2, z-1 }
6, …, 14	$out_A(6)$ = { w*2, z-1 }
	$in_A(7)$ = { w*2 , z-1 }
	$in_A(13)$ = { w*2, z-1 }



Question 4

worklist = [all nodes]
while worklist != empty do
    m = removeFirst(worklist)
    recompute $out_A(m)$
    if $out_A(m)$ has changed then
        for each successor *n* of *m*
            compute $in_A(n)$
            if $in_A(n)$ has changed then put *n* into worklist
                (if not already in worklist)

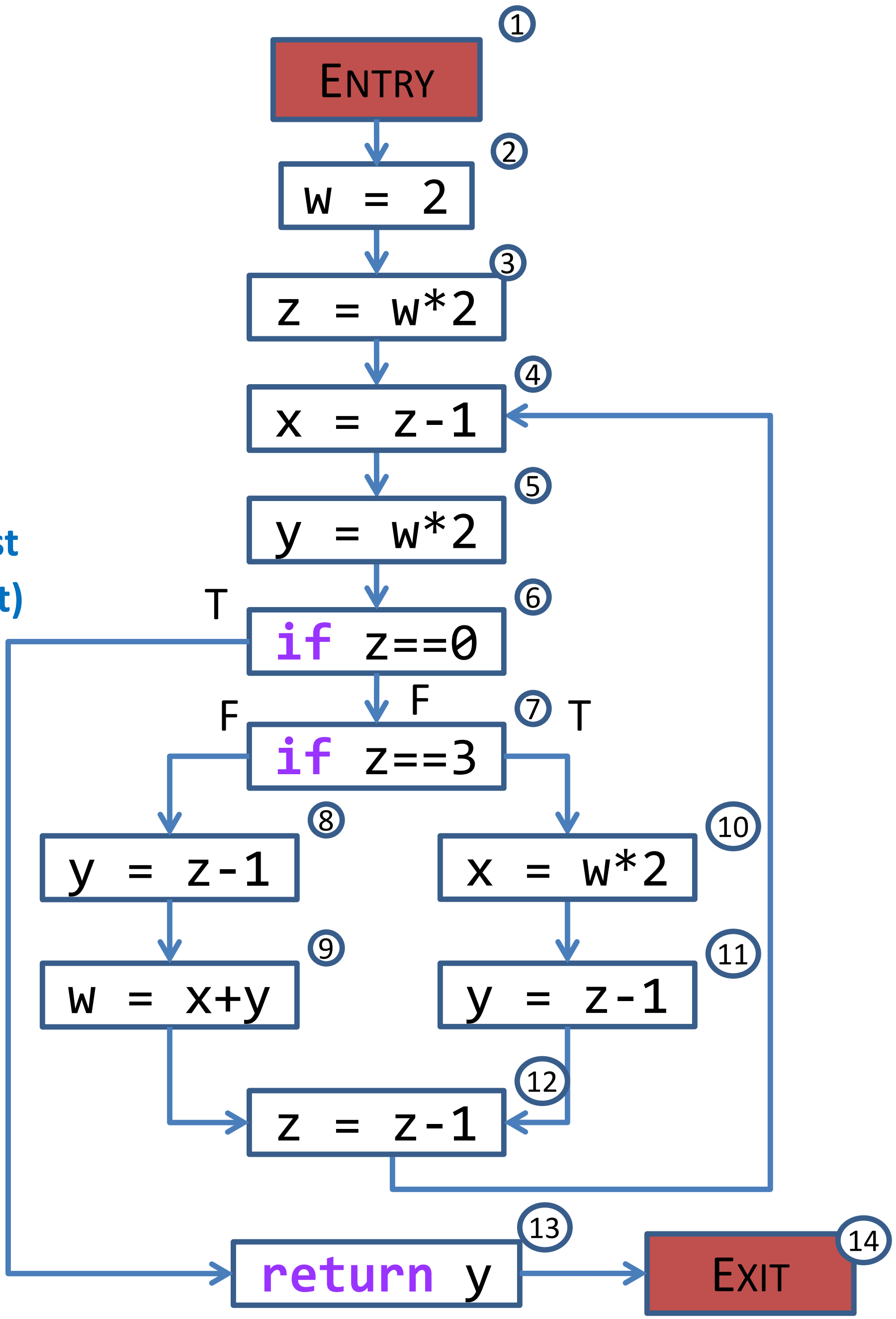
Worklist	$out_A(m)$ & $in_A(n)$
7, ..., 14	$out_A(7)$ = { w*2, z-1 } $in_A(8)$ = { w*2, z-1 } $in_A(10)$ = { w*2, z-1 }
8, ..., 14	$out_A(8)$ = { w*2, z-1 } $in_A(9)$ = { w*2, z-1 }
9, ..., 14	$out_A(9)$ = { z-1, x+y } $in_A(12)$ = { z-1, x+y }



Question 4

worklist = [all nodes]
while worklist != empty do
  m = removeFirst(worklist)
  recompute $out_A(m)$
  if $out_A(m)$ has changed then
    for each successor *n* of *m*
      compute $in_A(n)$
      if $in_A(n)$ has changed then put *n* into worklist
        (if not already in worklist)

Worklist	**$out_A(m)$ & $in_A(n)$**
10, …, 14	$out_A(10) = \{ w*2, z-1 \}$ $in_A(11) = \{ w*2, z-1 \}$
11, …, 14	$out_A(11) = \{ w*2, z-1 \}$ $in_A(12) = \{ z-1 \}$
12, …, 14	$out_A(12) = \varnothing$ $in_A(4) = \varnothing$ (changed)



Question 4

worklist = [all nodes]
while worklist != empty do
    m = removeFirst(worklist)
    recompute $out_A(m)$
    if $out_A(m)$ has changed then
        for each successor *n* of *m*
            compute $in_A(n)$
            if $in_A(n)$ has changed then put *n* into worklist
                (if not already in worklist)

Worklist	**$out_A(m)$ & $in_A(n)$**
13, 14, 4	$out_A(13)$ = { w*2, z-1 } $in_A(14)$ = { w*2, z-1 }
14, 4	$out_A(14)$ = { w*2, z-1 }
4	$out_A(4)$ = { z-1 } (changed) $in_A(5)$ = { z-1 } (changed)
5	$out_A(5)$ = { w*2, z-1 } (unchanged)