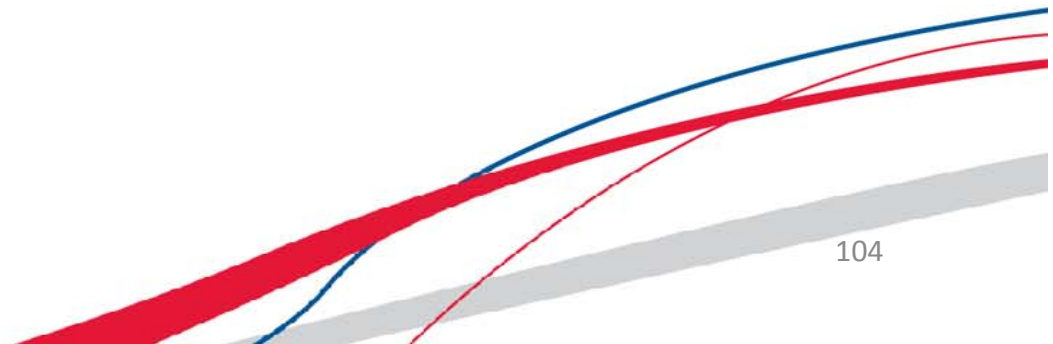
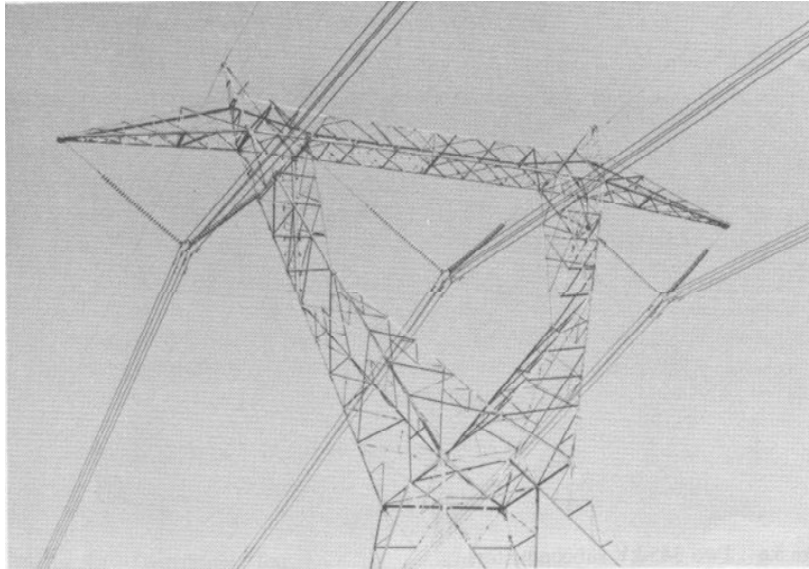


## ELECTRIC POWER TRANSMISSION

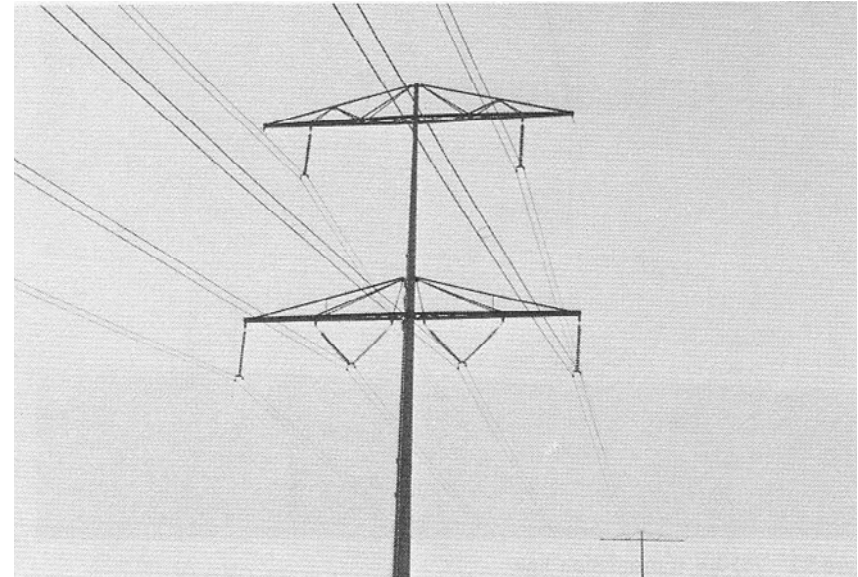
- Power system transmission lines are the electrical connections between the generating stations and the load centres.
- Basically, overhead lines or cables are used for this purpose.
- The need for power transmission arises from the fact that bulk power generation is done at large electric power plants remotely located. Transmission offers the following advantages :
  1. Use of remote energy sources
  2. Reduction of the power reserve of gen. stations
  3. Utilization of the difference between various time zones when the peak demands are not coincident
  4. Improved reliability of the power network!
- The design of power line depends on :
  1. Amount of real power it has to transmit
  2. Distance over which power must be carried
  3. Cost of the power line
  4. Esthetic considerations, urban congestion, ease of installation, expected load growth

- The basic characteristics of power lines are :
  1. To carry real power (& reactive power)
  2. As far as possible, voltage should remain constant over the line length
  3. Line losses must be small, so that operating efficiency is high
  4.  $I^2R$  losses must not overheat the conductors
- Overhead lines are made up of metal conductors suspended on insulators from a tower which is made of metal, wood, or reinforced concrete.
  - Most modern overhead lines are constructed using ACSR (aluminium conductor steel reinforced) which consists of a central steel core on which aluminium wires are wound. (The steel core increases the mech. strength)
  - Multiwire conductors (strands) are preferred over single-wire conductors
  - Towers or poles carrying six trans. lines are called double-circuit lines
  - Steel ground wires are placed atop towers/poles for protection against direct lightning strikes

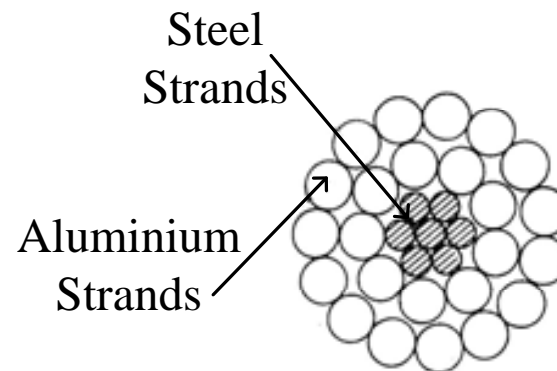




765-kV, 4-conductors/phase, 3- $\phi$  lines on tower



345-kV, double-circuit, 3- $\phi$  lines



Cross-section of ACSR conductors 24/7

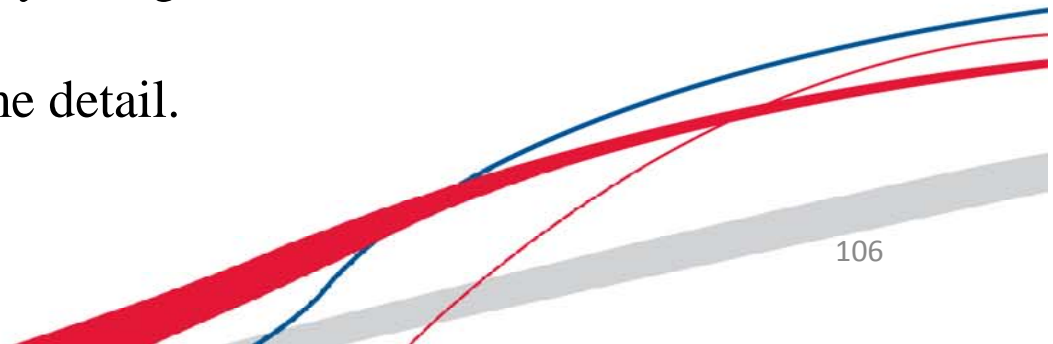
- Cables have conductors insulated from one another and enclosed in protective sheaths. [Please refer to Appen. C for details.]
  - Cables laid directly in the soil, or in a bed of sand, or within special cable ducts.
- By and large, overhead lines favored due to lower initial costs, ease/simplicity of repair & maintenance. Cables, however, utilized in urban areas (eg. Singapore has 100% cable network)
- Power transmitted

$$\propto V^2$$

$$\propto \frac{1}{\text{Length of line}}$$

⇒ Trans. system voltages are usually v. high

- We will study overhead lines w some detail.



## Transmission (Overhead) Lines

- Have 4 parameters : Resistance, inductance, capacitance, and conductance.
  - Series inductance & shunt capacitance represent the effects of the magnetic & electric fields respectively around the line conductors.
  - Conductance exists betn. conductors or betn. conductors & the ground. It accounts for the leakage current at the insulators (thru cable insulation for cables)  
⇒ Usually negligible, hence conductance effect is neglected.

### Line Resistance (R)

$$R_{DC} = \rho \frac{l}{A} \Omega$$

where  $\rho$  – Conductor resistivity,  $\Omega\text{-m}$

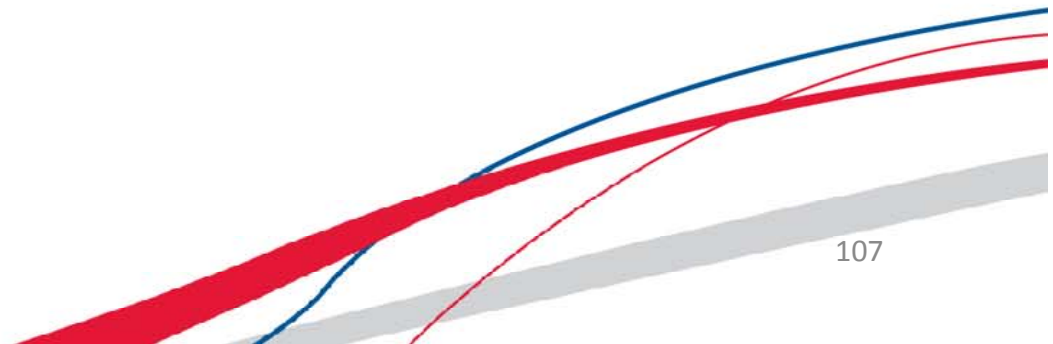
$l$  = Conductor length, m

$A$  = Cross-sectional conductor area,  $\text{m}^2$

=  $\pi r^2$  for circular conductors

$r$  = Conductor radius, m

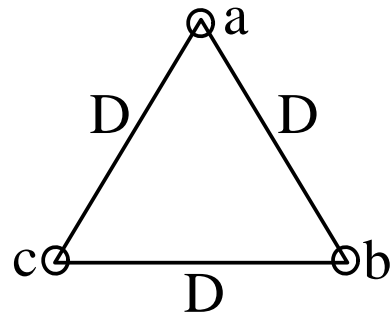
$$R_{AC} \simeq (1.05 \text{ to } 1.10) \times R_{DC}$$



Due to stranding, temp. effects & skin effect (varying freq. causes I to segregate along outer edge or “skin” of conductor; non-uniformly distributed I)

## Line Inductance (L)

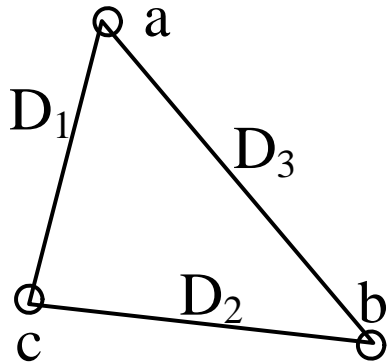
### (a) Equal spacing between the 3 phases



$$L_{\text{phase}} = 2 \times 10^{-7} \times \ell \ln \left( \frac{\text{GMD}}{\text{GMR}} \right) \text{H/m}$$

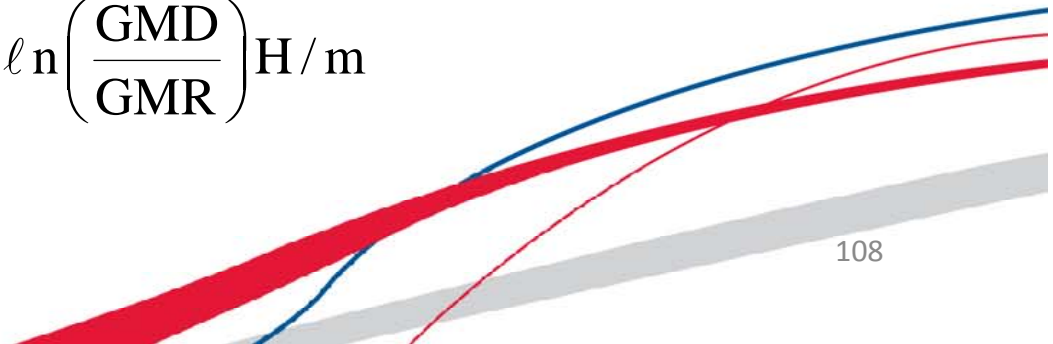
where : GMD – Geometric Mean Distance – D  
GMR = Geometric Mean Radius of conductor

### (b) Unequal spacing between phases



$$\text{GMD} = \sqrt[3]{D_1 D_2 D_3}$$

$$\& L_{\text{phase}} = 2 \times 10^{-7} \times \ell \ln \left( \frac{\text{GMD}}{\text{GMR}} \right) \text{H/m}$$



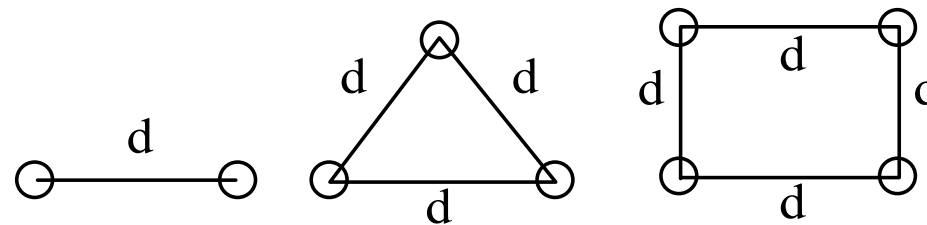


Line reactance per phase

$$X_L = \omega L = 2\pi fL(\Omega/\text{m})$$

(c) Use of bundled conductors

- To reduce corona effect (extra-high voltages on lines ionize the air & produce a discharge resulting in power loss, high-freq. noise, electro-magnetic interference with TV/Radio sets)
- To reduce line inductance & resistance
- 3 ways :



Each phase has 2, 3 or 4 conductors arranged as shown.

$$\begin{aligned} \text{2-conductor case : } GMR_b &= GMR_{\text{bundle}} \\ &= \sqrt{GMR_{\text{cond}} \times d} \end{aligned}$$

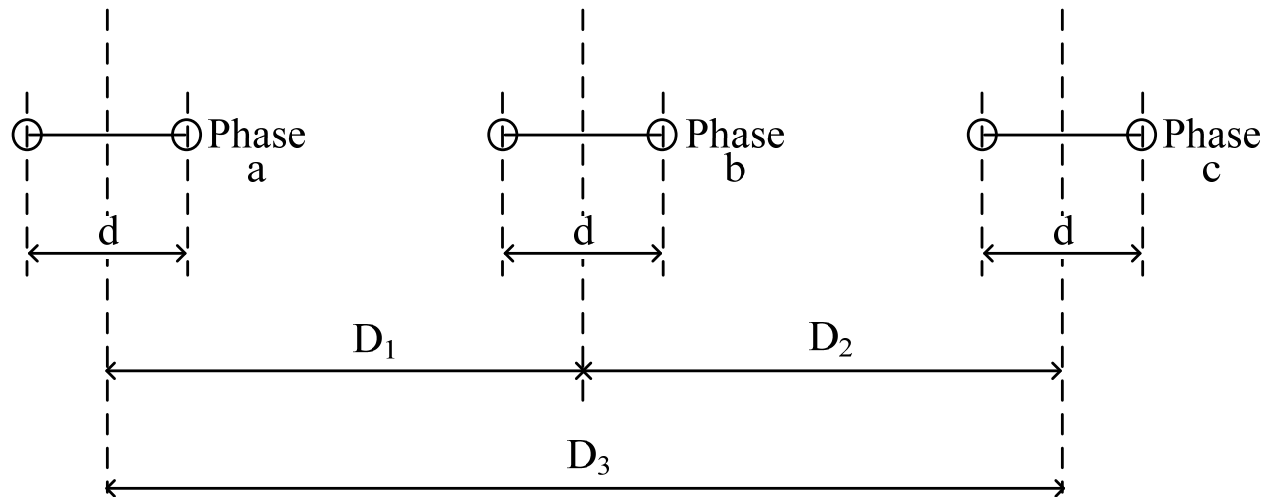
$$\text{3-conductor case : } GMR_b = \sqrt[3]{GMR_{\text{cond}} \times d^2}$$

$$\text{4-conductor case : } GMR_b = 1.09 \times \sqrt[4]{GMR_{\text{cond}} \times d^3}$$

$$\therefore L_{\text{phase}} = 2 \times 10^{-7} \times \ell \ln \left( \frac{\text{GMD}_b}{\text{GMR}_b} \right) \text{ H / m}$$

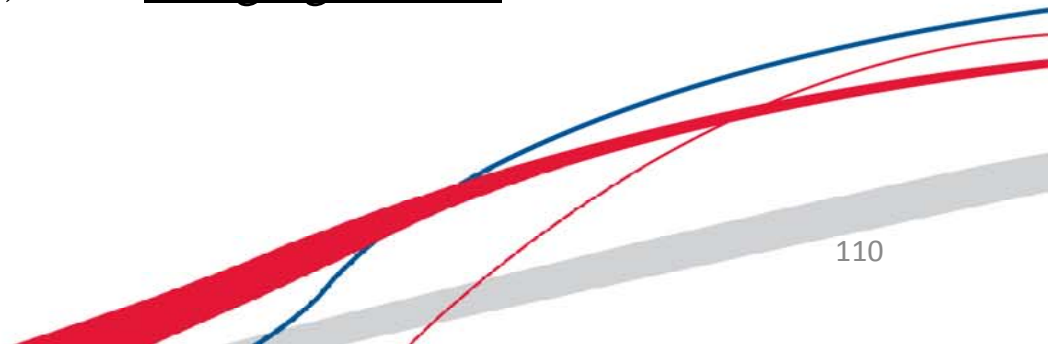
where :  $\text{GMD}_b = \sqrt[3]{D_1 D_2 D_3}$

For example, *flat horizontal line with 2 conductors per phase.*



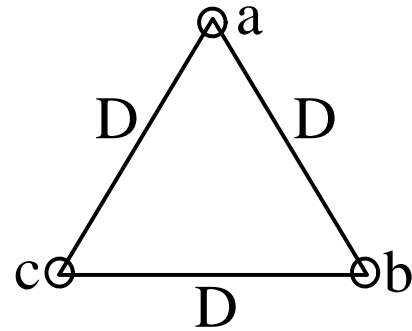
## Line Capacitance (C)

- It is a shunt between conductors; thus charging current flows in the line even when line is not loaded (is open)





(a) Equal conductor spacing

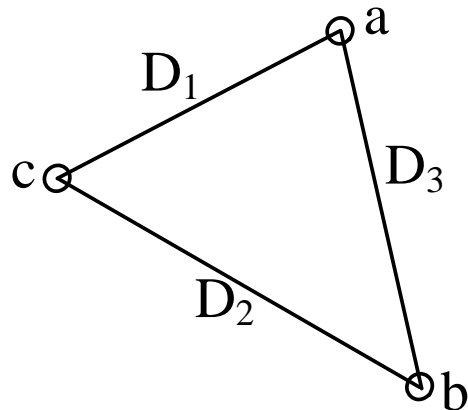


$$C_n = \frac{2\pi\epsilon}{\ln\left(\frac{\text{GMD}}{r}\right)} \text{ F/m to neutral}$$

where :  $\epsilon$  = Permittivity of free space  
 $= 8.85 \times 10^{-12} \text{ F/m}$

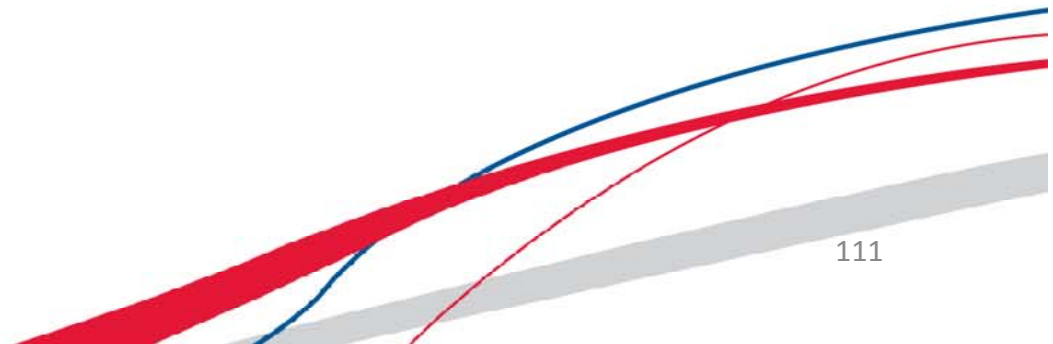
$r$  = radius of conductor &  
 $\text{GMD} = D$

(b) Unequal conductor spacing



$$C_n = \frac{2\pi\epsilon}{\ln\left(\frac{\text{GMD}}{r}\right)} \text{ F/m to neutral}$$

where :  $\text{GMD} = \sqrt[3]{D_1 D_2 D_3}$



## Using bundled conductors

$$C_n = \frac{2\pi\epsilon}{\ell \ln\left(\frac{\text{GMD}_b}{r_b}\right)} \text{ F / m to neutral}$$

where :  $\text{GMD}_b = \sqrt[3]{D_1 D_2 D_3}$ , and

$$r_b = \sqrt{r d} \text{ For 2-conductor case}$$

$$= \sqrt[3]{r d^2} \text{ For 3-conductor case}$$

$$= 1.09 \sqrt[4]{r d^3} \text{ For 4-conductor case}$$

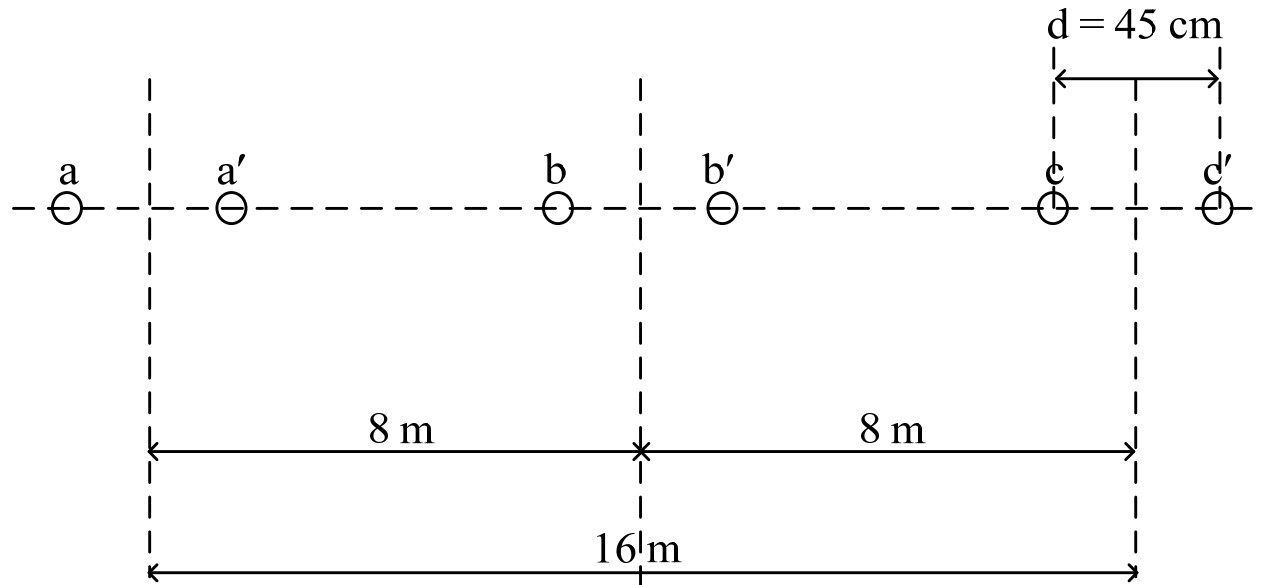
Capacitive reactance per phase

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C_n} \text{ } (\Omega - \text{m})$$

### **Example 11**

Each conductor of the bundled-conductor trans. line shown below has a GMR of 0.0466', and outside diameter of 1.382". Find :

- (1) Line inductance per phase per metre
- (2) Inductive reactance per phase in  $\Omega/\text{mile}$
- (3) Line capacitance in F/metre to neutral
- (4) Capacitive reactance to neutral in  $\Omega\text{-km}$  per phase (given  $f = 60 \text{ Hz}$ )



**Solution :**

$$\begin{aligned} \text{GMR}_{\text{conductor}} &= 0.0466' = 0.0466 \times 0.3048 \\ &= 0.0142036 \text{ m} \end{aligned}$$

$$\Rightarrow \text{GMR}_b (\text{for 2-cond. bundle}) = \sqrt{\text{GMR}_{\text{conductor}} \times d} = \sqrt{0.0142036 \times 0.45} = 0.08 \text{ m}$$

$$\text{GMD}_b = \sqrt[3]{D_1 D_2 D_3} = \sqrt[3]{8 \times 8 \times 16} = 10.0794 \text{ m}$$

(1)  $\therefore$  Line inductance

$$\begin{aligned} L_{\text{phase}} &= 2 \times 10^{-7} \times \ell \ln \left( \frac{\text{GMD}_b}{\text{GMR}_b} \right) \text{ H/m} \\ &= 9.6724 \times 10^{-7} \text{ H/m} \end{aligned}$$

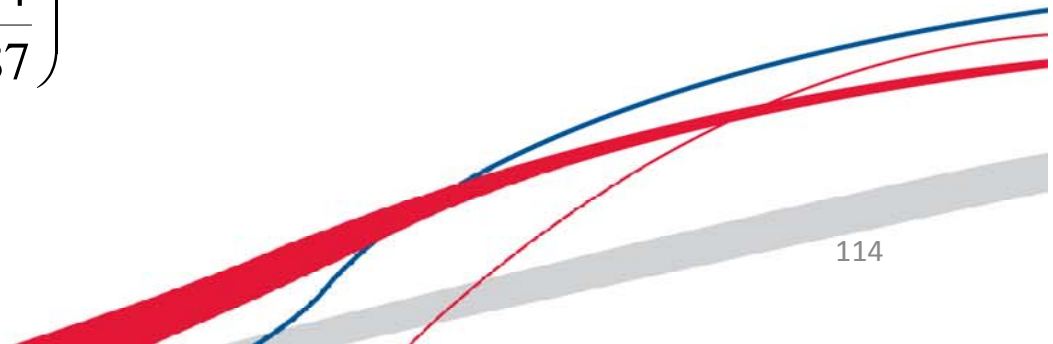
(2) Line reactance

$$\begin{aligned} X_L &= 2\pi fL = 2\pi \times 60 \times 9.672 \times 10^{-7} \\ &= 3646.43 \times 10^{-7} \Omega/\text{m} \\ &= 0.587 \Omega/\text{mile} \end{aligned}$$

$$(3) r = \frac{1.382''}{2} = 0.691'' = 0.0175514 \text{ m}$$

$$\begin{aligned} \therefore r_b (\text{for 2-cond. bundle}) &= \sqrt{r d} = \sqrt{0.0175514 \times 0.45} \\ &= 0.08887 \text{ m} \end{aligned}$$

$$\begin{aligned} \therefore C_n &= \frac{2\pi\epsilon}{\ell \ln \left( \frac{\text{GMD}_b}{r_b} \right)} = \frac{2\pi \times 8.85 \times 10^{-12}}{\ell \ln \left( \frac{10.0794}{0.08887} \right)} \\ &= 11.7534 \times 10^{-12} \text{ F/m to neutral} \end{aligned}$$

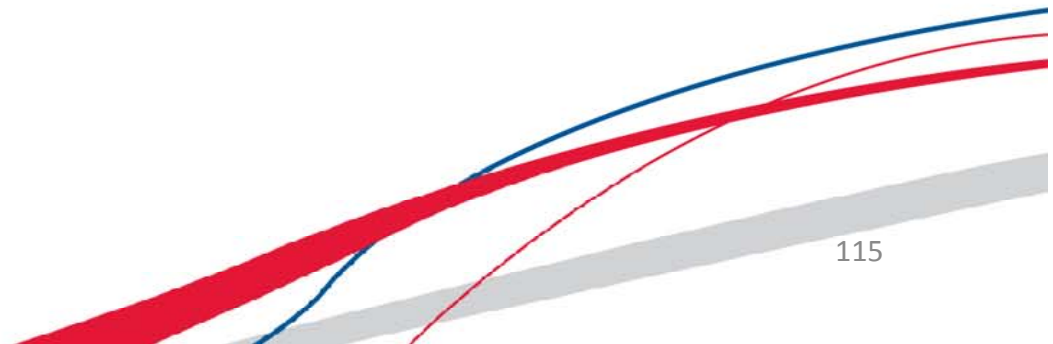


#### (4) Capacitive reactance

$$\begin{aligned}X_c &= \frac{1}{2\pi fC} = \frac{1}{2\pi \times 60 \times 11.7534 \times 10^{-12}} \\&= 2.257 \times 10^8 \, \Omega\text{-m} \\&= 0.2257 \times 10^6 \, \Omega\text{-km per phase}\end{aligned}$$

#### Exercise 16

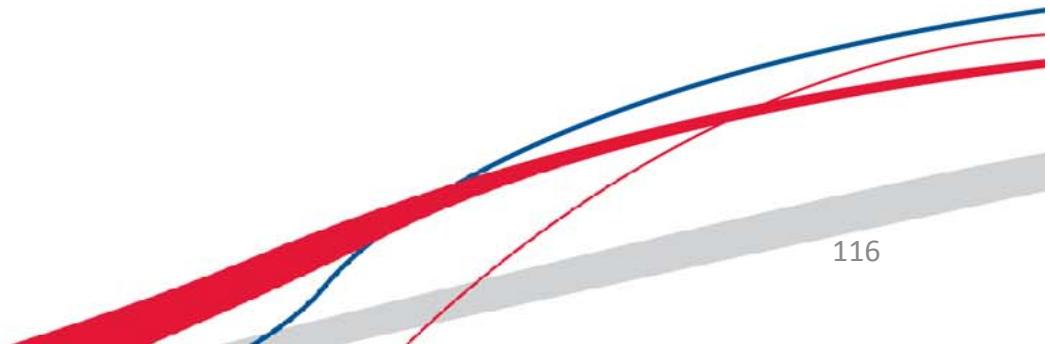
- (i) A three-phase transposed line is composed of one ACSR (aluminium conductors steel-reinforced) conductor per phase, with a flat horizontal spacing of 8 m between adjacent conductors. The geometric mean radius (GMR) of each conductor is 1.515 cm. Determine the inductance per phase per km of the line.
- (i)  $13 \times 10^{-4} \text{ H/km}$
- (ii) The line in part (i) is to be replaced by a two-conductor bundle with 8 m spacing between the centres of adjacent bundles. The spacing between the conductors in the bundle is 40 cm. If the new line inductance per phase per km is to be 77% of the inductance in part (i), what would be the GMR of each new conductor in the bundle?
- 1.1416 cm



# ELECTRICAL NETWORK MODELS OF TRANSMISSION LINES

- We have examined the parameters (R, L, C) of a transmission line, and are ready to consider the line as an element of a power system.
- Analysis of power system requires a mathematical model of transmission lines in order to calculate voltages, currents, and power flows.
- T. line models must account for the series resistance & inductance, and shunt capacitance of each phase. These parameters are distributed along the entire line length.
  1. Experience reveals that a simple series impedance with negligible capacitive shunting is acceptable for lines up to 80 km long. These lines are called short lines.
  2. When T. line lengths are between 80 & 240 km, the capacitive effect becomes sufficiently important to be included in the analysis.

⇒ Medium lines



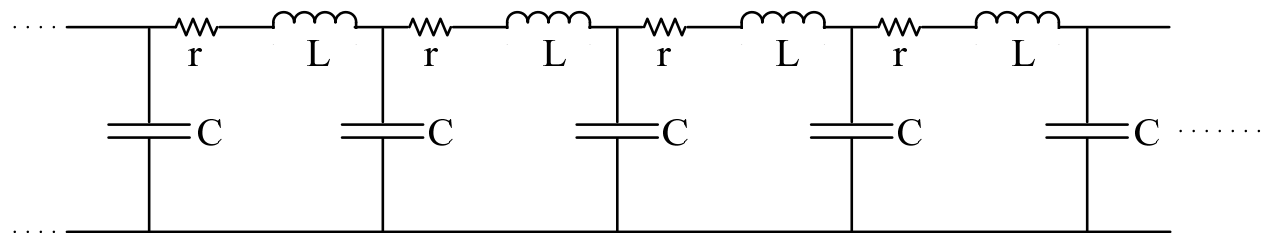


3. Finally, for line lengths exceeding 240 km, it becomes necessary to treat it by a continuous distribution of elemental parameters (rather than lumped parameters, as in 1 & 2)

⇒ Long lines

- We shall study the short & medium lines only!

We shall adopt the following nomenclature :



(Accurate line representation)

$Z = \text{Total series impedance per phase} = (r + j\omega L) \times l$

where :  $r = \text{resistance } \Omega/\text{m}$

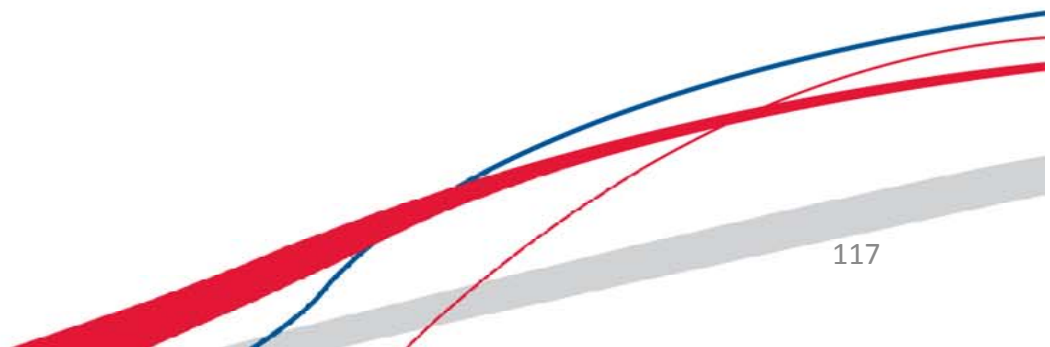
$$\omega = 2\pi f$$

$L = \text{inductance H/m}$

$l = \text{line length}$

$$R = rl$$

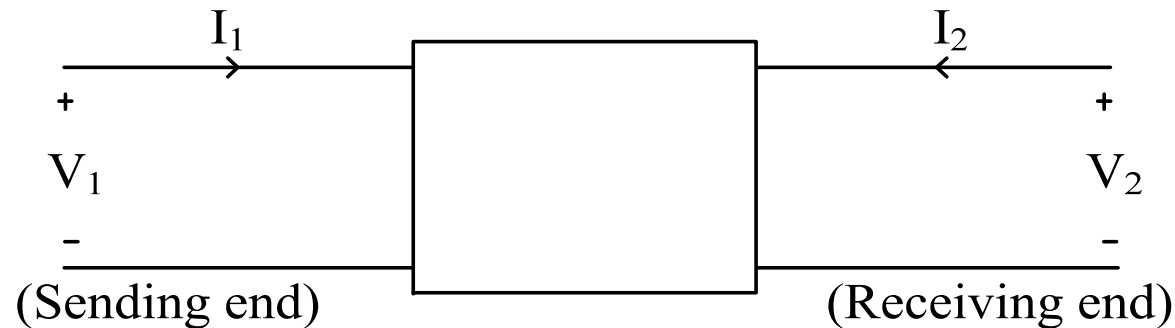
$$jX = j\omega Ll$$



$$\begin{aligned}
 Y &= \text{Total shunt admittance per phase to neutral} \\
 &= 1/(-jX_c) \\
 &= (j\omega C) \times l
 \end{aligned}$$

where :  $C$  = capacitance F/m

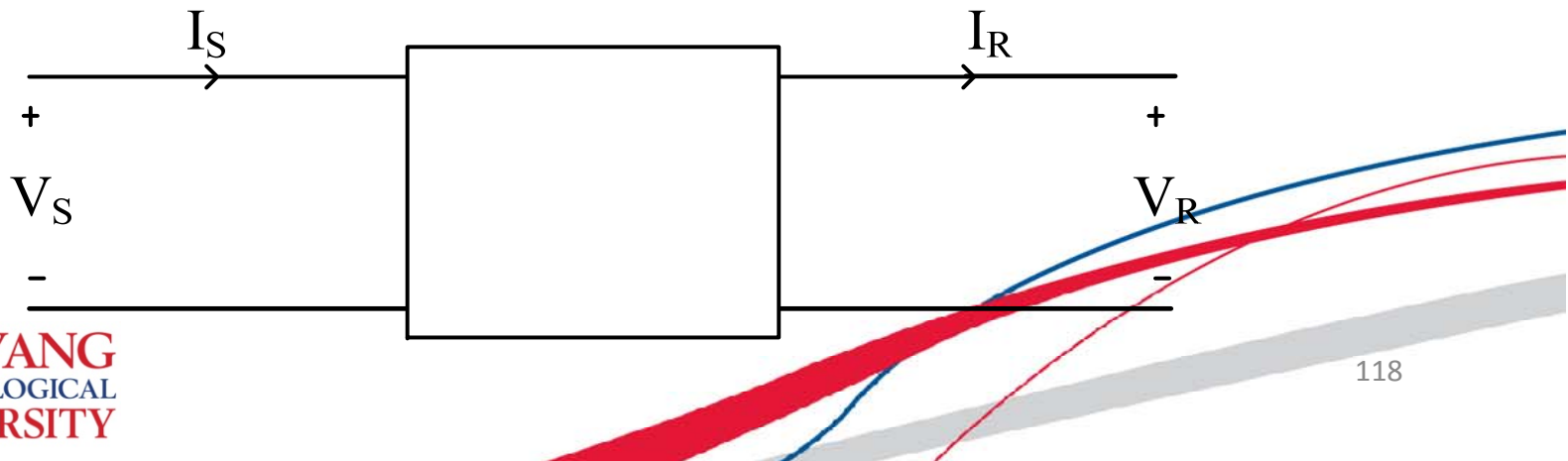
### Review of ABCD-parameters of a 2-port network



Recall

$$\begin{aligned}
 V_1 &= AV_2 - BI_2 \text{ (-ve sign for } I_2\text{)} \\
 I_1 &= CV_2 - DI_2
 \end{aligned}$$

Now

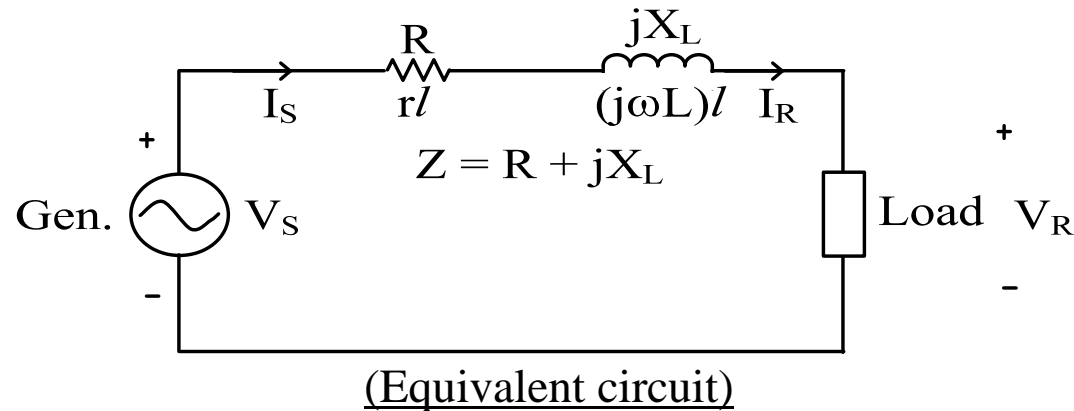


Then  $I_R = -I_2$

$$\begin{aligned} V_S &= AV_R + BI_R \\ \therefore I_S &= CV_R + DI_R \end{aligned} \Rightarrow \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Reciprocal  $\Rightarrow AD - BC = 1$ ; Symmetric  $\Rightarrow A = D$ .

### The Short Transmission Line ( $\leq 80$ km)



Obviously,  $I_S = I_R = 0V_R + 1I_R$

and  $V_S - V_R + I_R Z$

Compare these with  $I_S = CV_R + DI_R$

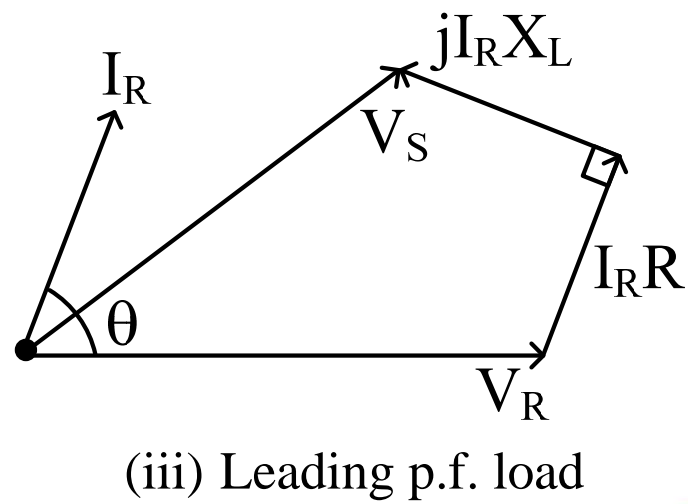
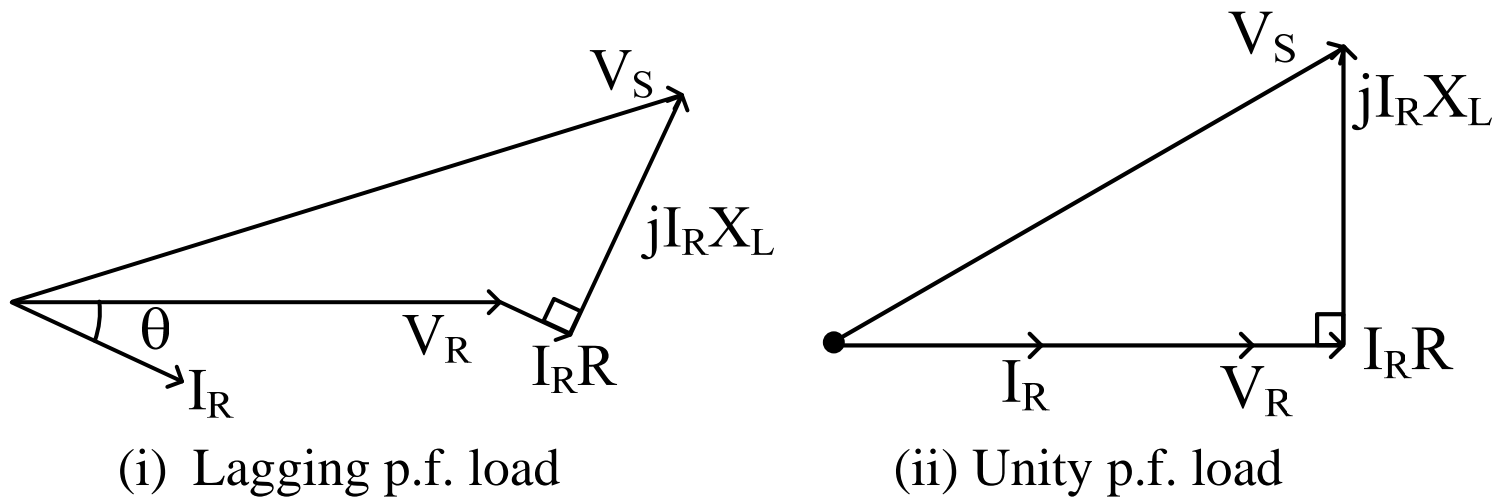
and  $V_S = AV_R + BI_R$

$\Rightarrow C = 0 \quad D = 1$

$A = 1 \quad B = Z$

Note : Often  $R \ll X_L$ . Hence  $R$  is ignored

## Phasor Diagrams



$$\% \text{ Voltage regulation (VR) of a line} = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100$$

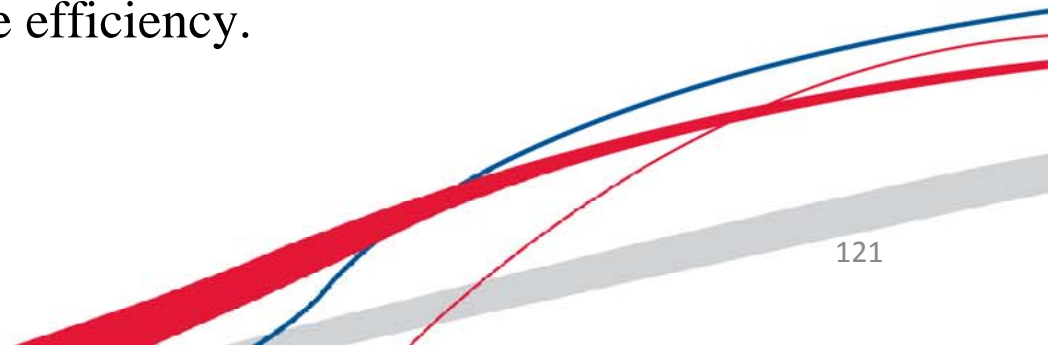
where :  $|V_{R,NL}|$  = Magn. of receiving-end voltage at no load  
 $|V_{R,FL}|$  = Magn. of receiving-end voltage at full load with  $|V_S|$   
 = constant

Note : VR is positive for lagging & UPF loads, but can be negative for leading pf loads

### **Example 12**

A 60-Hz, 3- $\phi$  T. line is 40 miles long & has a series impedance of  $(35 + j140) \Omega$ . It delivers 40 MW at 220 kV, 0.9 pf lag.

- Find the voltage, current & pf at the line sending end.
- Find the voltage regulation & line efficiency.



## Solution

$$\text{Let } V_b = 220 \text{ kV} \ \& \ S_b = \frac{40}{0.9} = 44.44 \text{ MVA}$$

$$\text{Given } S_R = \frac{40 \text{ MW}}{0.9 \text{ pf}} = 44.44 \text{ MVA} \Rightarrow |S_{R \text{ p.u.}}| = 1.0$$

$$\Rightarrow S_{R \text{ pu}} = 1 \angle 25.84^\circ \ \& \ V_{R \text{ pu}} = 1 \angle 0^\circ \text{ (reference)}$$

$$\Rightarrow I_{R \text{ pu}} = 1 \angle -25.84^\circ$$

Next,

$$Z_b = \frac{V_b^2}{S_b} = \frac{220^2}{44.44} = 1089.0 \ \Omega$$

$$\Rightarrow Z_{\text{Line p.u.}} = \frac{(35 + j140) \ \Omega}{1089 \ \Omega} \equiv 0.1325 \angle 75.96^\circ$$

$$V_S = V_R + Z_{\text{Line}} I_R = 1 \angle 0^\circ + (0.1325 \angle 75.96^\circ)(1 \angle -25.84^\circ)$$

$$= 1 + 0.1325 \angle 50.12^\circ = 1.085 + j0.1017 = \underline{\underline{1.09 \angle 5.35^\circ}}$$

$$\therefore |V_{S \text{ Actual}}| = |V_{S \text{ pu}}| \times V_b = \underline{\underline{239.74 \text{ kV (Line-to-line)}}$$



$$|I_S| = |I_R| = I_{R \text{ pu}} \times I_b$$

$$= 1.0 \times \left( \frac{S_b \times 10^3}{\sqrt{3} \times V_b} \right) = \underline{116.6 \text{ A}}$$

$$\text{Angle between } V_{S \text{ pu}} \text{ \& } I_{S \text{ pu}} = 5.35^\circ - (-25.84^\circ) = 31.2^\circ$$

$$\Rightarrow \text{Sending end pf} = \cos 31.2^\circ = \underline{0.855 \text{ lag}}$$

Alt.,

$$S_{S \text{ pu}} = V_{S \text{ pu}} I_{S \text{ pu}}^* = (1.09 \angle 5.35^\circ)(1 \angle 25.84^\circ) = 1.09 \angle 31.2^\circ = 0.932 + j0.565$$

$$= P_{S \text{ pu}} + jQ_{S \text{ pu}}$$

$$\text{Volt. regulation} = \frac{|V_{R \text{ NL}}| - |V_{R \text{ FL}}|}{|V_{R \text{ FL}}|} \times 100 = \frac{1.09 - 1.0}{1.0} \times 100 \simeq 9\%$$

$$\text{Line efficiency, } \eta = \frac{P_R}{P_S} \times 100\%$$

$$\text{where } P_S = \text{Sending-end power} = V_{S \text{ pu}} I_{S \text{ pu}} \cos \theta_S = 1.09 \times 1.0 \times 0.855$$

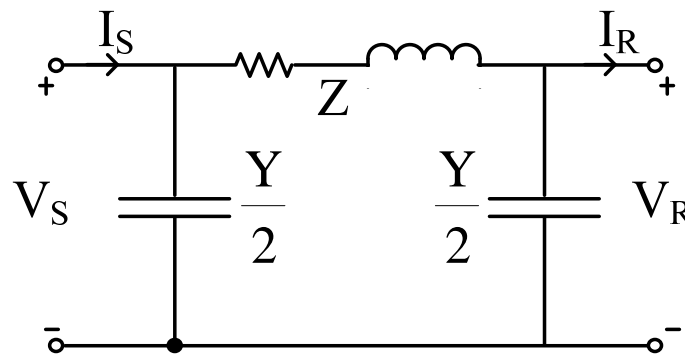
$$= 0.9317 \text{ pu}$$

&  $P_R = \text{Receiving-end power}$   
 $= 40 \text{ MW} = \frac{40}{44.44} = 0.9 \text{ pu}$

$\therefore \eta = \frac{0.9}{0.9317} \times 100 = 96.6\%$

### **Medium Lines ( $\geq 80 \text{ km}$ but $\leq 240 \text{ km}$ )**

- (1) Nominal- $\pi$  model : Here shunt admittance (Y) is divided into 2 equal parts – one is placed at sending end, other at receiving end.



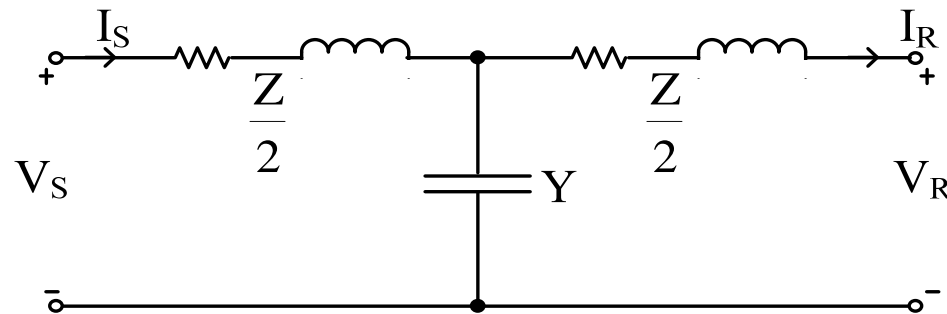
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{ZY}{2} + 1 & Z \\ Y\left(\frac{ZY}{4} + 1\right) & \frac{ZY}{2} + 1 \end{bmatrix}$$

$$V_S = AV_R + BI_R \Rightarrow A = \left. \frac{V_S}{V_R} \right|_{I_R=0} \quad [\text{Rec. end open}]$$

$$\Rightarrow V_{R \text{ NL}} = \frac{|V_S|}{|A|}$$

$$\Rightarrow \text{Voltage regulation} = \frac{V_{R \text{ NL}} - V_{R \text{ FL}}}{V_{R \text{ FL}}} = \frac{\frac{|V_S|}{|A|} - |V_{R \text{ FL}}|}{|V_{R \text{ FL}}|} \times 100\%$$

(2) Nominal-T model: Here series imped. (Z) is split into 2 equal parts & Y is placed at the line mid-point.



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{ZY}{2} + 1 & \left( \frac{ZY}{4} + 1 \right) Z \\ Y & \frac{ZY}{2} + 1 \end{bmatrix}$$

### Example 13

A 3- $\phi$ , 100-mile, 60-Hz T. line has a total series impedance of  $(18.26 + j78.4)$  ohms per phase, and a total capacitive reactance of  $185.5 \times 10^3 \angle -90^\circ$   $\Omega$ -mile per phase. A 200-MVA, 230-kV, unity pf load is connected at the receiving end. Using the nominal- $\pi$  model,

- (a) Find the sending-end voltage, current, real & reactive powers.
- (b) Find the voltage regulation.

### Solution

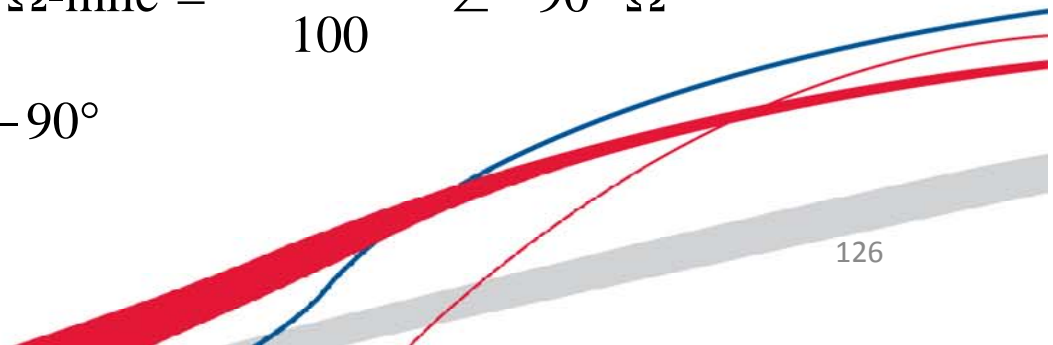
- (a) Let  $S_b = 200$  MVA &  $V_b = 230$  kV

$$\Rightarrow Z_b = \frac{V_b^2}{S_b} = 264.5 \Omega$$

$$\therefore Z_{pu} = \frac{18.26 + j78.4}{264.5} = 0.3043 \angle 76.9^\circ$$

$$\text{Given } -jX_C = 185.5 \times 10^3 \angle -90^\circ \Omega\text{-mile} = \frac{185.5 \times 10^3}{100} \angle -90^\circ \Omega$$

$$\Rightarrow -jX_{C_{pu}} = -j \frac{1855}{264.5} = 7.0132 \angle -90^\circ$$



$$\Rightarrow Y_{pu} = \frac{1}{-jX_{C_{pu}}} = 0.1426 \angle 90^\circ$$

Given  $S_R = 200 \angle 0^\circ$  MVA  $= 1 \angle 0^\circ$  pu

&  $V_R = 230 \angle 0^\circ$  kV  $= 1 \angle 0^\circ$  pu

$\Rightarrow I_R = 1 \angle 0^\circ$  pu

$$D = A = \frac{ZY}{2} + 1 = \frac{(0.3043 \angle 76.9^\circ)(0.1426 \angle 90^\circ)}{2} + 1$$

$$= 0.9789 \angle 0.288^\circ \text{ pu}$$

$B = Z = 0.3043 \angle 76.9^\circ$  pu

$$C = Y \left[ \frac{ZY}{4} + 1 \right] = 0.1411 \angle 90.142^\circ \text{ pu}$$

Now,

$$V_S = AV_R + BI_R$$

$$= (0.9789 \angle 0.288^\circ)(1 \angle 0^\circ) + (0.3043 \angle 76.9^\circ)(1 \angle 0^\circ)$$

$$= 1.0903 \angle 16.04^\circ \text{ pu}$$

$$\Rightarrow |V_S|_{\text{Actual}} = |V_S|_{pu} \times V_b = \underline{250.775 \text{ kV}}$$

Also,

$$\begin{aligned} I_S &= CV_R + DI_R \\ &= (0.1411 \angle 90.14^\circ)(1 \angle 0^\circ) + (0.9789 \angle 0.288^\circ)(1 \angle 0^\circ) = 0.9894 \angle 8.49^\circ \text{ pu} \end{aligned}$$

$$I_b = \frac{S_b \times 10^3}{\sqrt{3} \times V_b} = \frac{200 \times 10^3}{\sqrt{3} \times 230} = 502.044 \text{ A}$$

$$\Rightarrow |I_S|_{\text{Actual}} = |I_S|_{\text{pu}} \times I_b = \underline{496.722 \text{ A}}$$

$$\begin{aligned} \therefore S_{S \text{ pu}} &= V_{S \text{ pu}} I_{S \text{ pu}}^* \\ &= (1.0903 \angle 16.04^\circ)(0.9894 \angle -8.49^\circ) \\ &= 1.0694 + j0.1417 \end{aligned}$$

$$\Rightarrow S_{S \text{ Actual}} = S_{S \text{ pu}} \times S_b = (213.88 + j28.35) \text{ MVA}$$

$$\Rightarrow P_S = 213.88 \text{ MW}$$

$$\& Q_s = 28.35 \text{ MVAr}$$

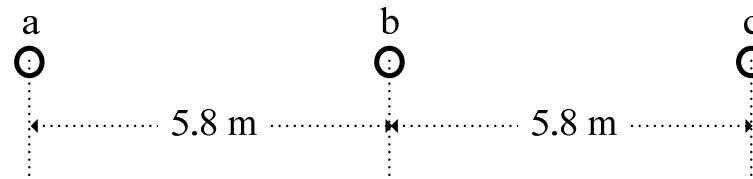


(b)

$$\text{Volt. reg.} = \frac{\frac{|V_S|}{|A|} - |V_R|}{|V_R|} \times 100\% = \frac{\frac{1.0903}{0.9789} - 1}{1} \times 100\% = \underline{11.38\%}$$

### Exercise 17

The configuration of a transposed three-phase, 230-kV, 50-Hz ACSR transmission line is shown below. Each stranded conductor has an outside diameter of 2.814 cm, and a GMR of 1.137 cm.

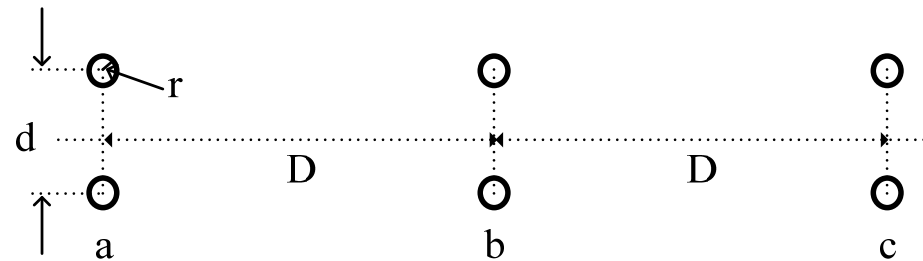


- (a) Calculate the inductance and capacitance per meter of the line.
- (b) If the line length is 240 km, calculate the parameters of the nominal  $\pi$  equivalent circuit. Ignore the resistance of the conductor.  
( $\epsilon = 8.85 * 10^{-12}$  F/m)
- (c) When one end of the line is maintained at the rated voltage with the other end left open, calculate the voltage at the open terminals and the charging current drawn by the line.

- (a)  $12.9314 * 10^{-7}$  H/m;  $8.8933 * 10^{-12}$  F/m
- (b)  $Z = j97.5$  ohms,  $Y/2 = j0.3353$  mS
- (c) 237.77 kV (line to line); 90.456 Amps

### Review Exercise 18

A 3- $\phi$  T. line using bundle conductors is shown below. ( $D = 4$  m,  $d = 20$  cm) (Conductor radius = 0.75 cm) (GMR = 0.5841 cm)



- (i) Find the R, L & C per km of the line. ( $\rho = 1.78 \times 10^{-8} \Omega\text{-m}$ ,  $\epsilon = 8.85 \times 10^{-12} \text{ F/m}$ )
- (ii) Draw the nominal-T representation of the line parameters if the system freq = 50 Hz, and the line length is 200 km.

(i)  $R = 0.0504 \Omega$  per km,  $L = 1$  mH per km,  $C = 0.01142 \mu\text{F}$  per km

(ii)  $\frac{Z}{2} = (5.04 + j31.42) \Omega$

$Y = j0.718 \text{ mS}$

### Exercise 19

A single-circuit, 50-Hz, 3-phase transmission line is 150 km long. The line is connected to supply a load of 30 MVA at 0.85 pf lag and 138 kV. The line constants are :  $R = 0.186 \Omega/\text{km}$ ,  $L = 2.60 \text{ mH}/\text{km}$ , and  $C = 0.012 \mu\text{F}/\text{km}$ . Using nominal  $\pi$  representation calculate :

- (a) the ABCD constants of the line.
- (b) the sending end voltage when supplying (i) no load, and (ii) the above load at the receiving end at the rated voltage of 138 kV.
- (c) If the sending end voltage is fixed at 138 kV, calculate the load bus voltage at (i) no load and (ii) the above load of 30 MVA at 0.85 pf lag.

(a)  $A = D = 0.965 \angle 0.47^\circ$ ,  $B = 125.66 \angle 77.17^\circ \Omega$ ,  $C = 0.556 \times 10^{-3} \angle 90.23^\circ \text{ S}$

(b) 133.2 kV, 153.58 kV (c) 142.95 kV; 117.5 kV

## Exercise 20

A three-phase, 345-kV, 60-Hz, 150-km long transposed line is composed of two ACSR conductors per phase. The conductors are in a flat horizontal configuration, spaced 11 m apart. The conductors have a diameter of 3.195 cm and a GMR of 1.268 cm. The bundle spacing is 45 cm. The resistance of each conductor in the bundle is 0.1076  $\Omega$  per km.

- (a) Determine the line inductance (in Henries) and capacitance (in Farads).
- (b) Determine the element values of the nominal- $\pi$  model.
- (c) Determine the ABCD-constants of the line using the nominal- $\pi$  representation.
- (d) If the line is now delivering 400 MW and 10 MVar at 345 kV at the receiving end, determine the sending-end voltage, current and apparent power. What is the line efficiency under this condition?

(a) 0.15644 H, 1.63659  $\mu$ F

(b)  $\frac{Y}{2} = j0.3085 \times 10^{-3}$ ;  $Z = 59.497 \angle 82.2^\circ \Omega$

(c)  $A = D = 0.98182 \angle 0.145^\circ$   
 $B = 59.497 \angle 82.2^\circ \Omega$

$C = 611.366 \times 10^{-6} \angle 90.072^\circ \text{ S}$

(d) 356.522 kV, 665.784 A, 411.128 MVA; 97.34%

## Exercise 21

A 230-kV, 3-phase transmission line has a per phase series impedance of  $(0.05 + j0.45) \Omega$  per km, and a per phase shunt admittance of  $j3.4 \mu\text{S}$  per km. The line is 80 km long. Using the nominal- $\pi$  model, determine the sending-end voltage and current, sending-end complex power, and the transmission efficiency when the line delivers 200 MVA, 0.8 pf lagging at 220 kV at the receiving end.

242.67 kV, 502.37 A, 163.17 MW + j134 MVar, 98.06%

## Further reading for Lines

1. Weedy & Cory, “Electric Power Systems”, 4<sup>th</sup> Ed., Wiley, 1998.
2. Turan Gonen, “Modern Power System Analysis”, Wiley, 1988.
3. A. R. Bergen & V. Vittal, “Power Systems Analysis”, 2<sup>nd</sup> Ed., Prentice-Hall, 2000.
4. J. J. Grainger & William Stevenson, “Power System Analysis”, McGraw-Hill, 1994.
5. T. R. Bosela, “Introduction to Electrical Power System Technology”, Prentice-Hall, 1997.
6. V. Del Toro, “Electric Power Systems”, Prentice-Hall, 1992.
7. T. Wildi, “Electrical Machines, Drives, and Power Systems”, 3<sup>rd</sup> Ed., Prentice-Hall, 1997.
8. S. A. Nasar, “Electric Energy Systems”, Prentice-Hall, 1996.

