

Most general unifier (MGU):

matching logical sentences is the first step in MP

e.g., $\text{Cat}(x) \Rightarrow \text{Mammal}(x), \text{Cat}(\text{Felix}) \vdash \text{Mammal}(\text{Felix})$

UNIFY[$P(A, B, y, z), P(x, y, z, B)$]

same predicate $P()$, same arity of 4 arguments

unifier: $A \equiv x, B \equiv y, y \equiv z, z \equiv B$

$$\theta = \{ x / A, y / B, z / B \}$$

UNIFY[$\text{Knows}(x, x), \text{Knows}(\text{Father}(y), y)$]

same predicate $\text{Knows}()$, same arity of 2 arguments

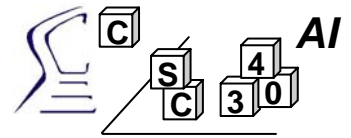
unifier: $x \equiv \text{Father}(y), x \equiv y$

$$\theta = \{ x / \text{Father}(y), x / y \}$$

implies $\text{Father}(y) = y$?!/? $F(y) = y$?

→ impossible if assuming the most obvious semantics i.e., “fatherhood”; then $\theta = \{ \}$

→ possible if assuming another semantics e.g., “provider” etc. (F not defined!)



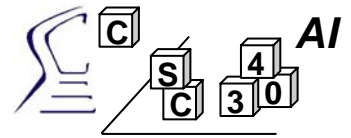
Backward chaining inference:

knowledge base (Horn clauses):

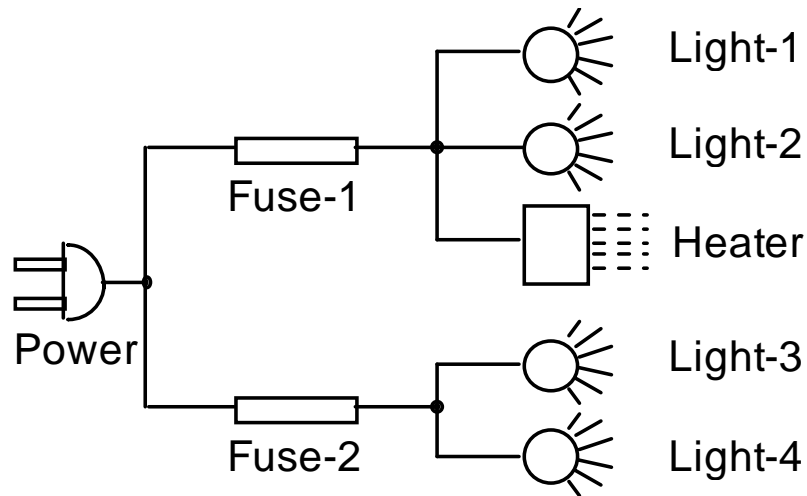
- (1) Sick(John, 2006)
- (2) Sick(Mary, y)
- (3) SitsNextTo(Ginger, Mary, 2006)
- (4) $\text{SitsNextTo}(a, b, y) \wedge \text{Sick}(b, y) \Rightarrow \text{Sick}(a, y)$
- (5) $\text{Parent}(a, b) \wedge \text{HasCold}(b, y) \Rightarrow \text{HasCold}(a, y)$
- (6) $\text{HasCold}(a, y) \Rightarrow \text{Sick}(a, y)$

backward chaining MP: ? Sick(x, 2006)

- (1) under x=John |– Sick(John, 2006)
- (2) under x=Mary, y=2006 |– Sick(Mary, 2006)
- (3) *fails*
- (4) prove Sick(a, y) under x=a, y=2006
 → SitsNextTo(a, b, 2006) and Sick(b, 2006)
 → (3) under a=Ginger, b=Mary
 (2) b=Mary |– Sick(Ginger, 2006)
- (5) *fails*
- (6) prove Sick(a, y) under x=a, y=2006
 → HasCold(a, 2006)
 → (5) Parent(a, b) \wedge HasCold(b, 2006)
 fails (no matching clause...)



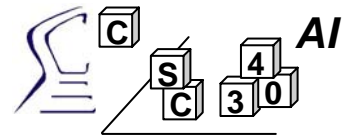
Expert System for an electric network



constants: P for Power; F1, F2 for Fuse-1, Fuse-2
L1, L2 etc. for Light-1 ... , H for Heater

predicates:

On(d)	“device d is turned on”
Working(d)	“device d is working”
Broken(d)	“device f is broken”
Connected(e, f)	“devices e and f are connected”
Intact(f)	“device f is intact”
Fuse(f)	“device f is a fuse”
Hot(d)	“device d is hot”
Device(d) ?	→ not needed (everything is...)



Knowledge base rules in FOL:

“All devices are on if there is power and off otherwise.”

- (1) $\forall d \text{ On(Power)} \Rightarrow \text{On(d)}$
- (2) $\forall d \neg \text{On(Power)} \Rightarrow \neg \text{On(d)}$
or $\forall d \text{ On(d)} \Rightarrow \text{On(Power)}$

“If the room is hot, the heater is working.”

- (3) $\text{Hot(R)} \Rightarrow \text{Working(H)}$

“If a device is on, connected to a fuse and the fuse is intact, but the device is not working, then it is broken.”

- (4) $\forall d, f \text{ On(d)} \wedge \text{Connected(d, f)} \wedge \text{Fuse(f)} \wedge \text{Intact(f)} \wedge \neg \text{Working(d)} \Rightarrow \text{Broken(d)}$

“A fuse is intact if a device connected to it is working.”

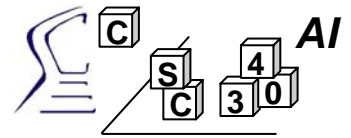
- (5) $\forall d, f \text{ Fuse(f)} \wedge \text{Connected(d, f)} \wedge \text{Working(d)} \Rightarrow \text{Intact(f)}$

“If two different devices connected to the same fuse are on but not working, the fuse is not intact.”

- (6) $\forall d, e, f \text{ Connected(d, f)} \wedge \text{Connected(e, f)} \wedge \neg d = e \wedge \text{Fuse(f)} \wedge \text{On(d)} \wedge \text{On(e)} \wedge \neg \text{Working(d)} \wedge \neg \text{Working(e)} \Rightarrow \neg \text{Intact(f)}$

“A working device is clearly on.”

- (7) $\forall d \text{ Working(d)} \Rightarrow \text{On(d)}$

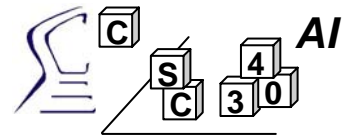


Knowledge base rules in CNF:

all \forall already on the left, no \exists ... need convert all \Rightarrow
 replace $P \Rightarrow Q$ by $\neg P \vee Q$, distribute \neg , etc.

$$P_1 \wedge P_2 \wedge \dots \wedge P_N \Rightarrow Q \quad \Leftrightarrow \quad \neg P_1 \vee \neg P_2 \vee \dots \vee \neg P_N \vee Q$$

- (1) $\neg \text{On}(\text{Power}) \vee \text{On}(d)$
- (2) $\text{On}(\text{Power}) \vee \neg \text{On}(d)$
- (3) $\neg \text{Hot}(R) \vee \text{Working}(H)$
- (4) $\neg \text{On}(d) \vee \neg \text{Connected}(d, f) \vee \neg \text{Fuse}(f) \vee \neg \text{Intact}(f) \vee \text{Working}(d) \vee \text{Broken}(d)$
- (5) $\neg \text{Fuse}(f) \vee \neg \text{Connected}(d, f) \vee \neg \text{Working}(d) \vee \text{Intact}(f)$
- (6) $\neg \text{Connected}(d, f) \vee \neg \text{Connected}(e, f) \vee d = e \vee \neg \text{Fuse}(f) \vee \neg \text{On}(d) \vee \neg \text{On}(e) \vee \text{Working}(d) \vee \text{Working}(e) \vee \neg \text{Intact}(f)$
- (7) $\neg \text{Working}(d) \vee \text{On}(d)$



Knowledge base facts:

electric network information:

- (8) Connected(L1, F1)
- (9) Connected(L2, F1)
- (10) Connected(H, F1)
- (11) Connected(L3, F2)
- (12) Connected(L4, F2)
- (13) Connected(F1, P)
- (14) Connected(F2, P)
- (15) Fuse(F1)
- (16) Fuse(F2)

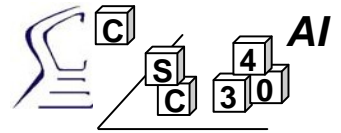
additional information:

“All lights except Light-2 are working ...”

- (17) Working(L1)
- (18) \neg Working(L2)
- (19) Working(L3)
- (20) Working(L4)

“... and the room is hot.”

- (21) Hot(R)



c) Proof by resolution: "Is Light-2 broken?"

resolving (4) with (9), (15), and (18) under $d=L2$, $f=F1$:

$$(22) \quad \neg \text{On}(L2) \vee \neg \text{Intact}(F1) \vee \text{Broken}(L2)$$

resolving (5) with (8), (15) and (17) under $d=L1$, $f=F1$:

$$(23) \quad \text{Intact}(F1)$$

resolving (22) with (23):

$$(24) \quad \neg \text{On}(L2) \vee \text{Broken}(L2)$$

resolving (1) with (24) under $d=L2$:

$$(25) \quad \neg \text{On}(P) \vee \text{Broken}(L2)$$

resolving (3) with (7) and (21) under $d=H$:

$$(26) \quad \text{On}(H)$$

resolving (2) with (26) under $d=H$:

$$(27) \quad \text{On}(P)$$

resolving (25) and (27) :

$$(28) \quad \underline{\text{Broken}(L2)} \quad \rightarrow \text{Light-2 is broken}$$