

NANYANG TECHNOLOGICAL UNIVERSITY
School of Electrical & Electronic Engineering

EE/IM4152 Digital Communications

Tutorial No. 5 (Sem 1, AY2016-2017)

1. Suppose a discrete memoryless source M emits messages m_1, m_2, m_3 , and m_4 with probabilities p_1, p_2, p_3 , and p_4 , respectively. Show that the entropy $H(M)$ of the source is maximized when all the messages are equiprobable. Determine the corresponding entropy.
2. A zero-memory source emits messages x_1 and x_2 with probabilities 0.9 and 0.1, respectively. Find the optimum binary code for this source as well as for its second- and third-order extensions. Determine the code efficiency in each case.
3. A channel has the following transition matrix:

$$\mathbf{P}(\mathbf{Y} | \mathbf{X}) = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}$$

- (a) Sketch the channel diagram showing all transition probabilities.
 - (b) Determine the channel output probabilities assuming that the input probabilities are equal.
 - (c) Determine the channel input probabilities that result in equally likely channel outputs.
 - (d) Determine the joint probability matrix using the result in part (c) above.
4. Using the definitions of entropy and conditional entropy, derive
$$H(X, Y) = H(X | Y) + H(Y)$$