Validity of: $P \Rightarrow Q \Leftrightarrow \neg P \vee Q$

truth table:

P	Q	$\neg$ P	$\neg P \vee Q$	$P \Rightarrow Q$	$P \Rightarrow Q \Leftrightarrow \neg P \vee Q$
T	**T**	F	**T**	**T**	**T**
T	F	F	F	F	**T**
F	**T**	**T**	**T**	**T**	**T**
F	F	**T**	**T**	**T**	**T**

Validity of: $P \Leftrightarrow Q \quad \Leftrightarrow \quad (P \Rightarrow Q) \wedge (Q \Rightarrow P)$

truth table:

P	Q	$P \Leftrightarrow Q$	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$	$P \Leftrightarrow Q \Leftrightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P)$
T	**T**	**T**	**T**	**T**	**T**	**T**
T	F	F	F	**T**	F	**T**
F	**T**	F	**T**	F	F	**T**
F	F	**T**	**T**	**T**	**T**	**T**

Validity of: $P \Leftrightarrow Q \quad \Leftrightarrow \quad (P \land Q) \lor (\neg P \land \neg Q)$

no truth table → rewriting rules / equivalences

using (ii):

$$P \Leftrightarrow Q \quad \Leftrightarrow \quad (P \Rightarrow Q) \land (Q \Rightarrow P)$$

then using (i):

$$P \Leftrightarrow Q \quad \Leftrightarrow \quad (\neg P \lor Q) \land (\neg Q \lor P)$$

using distributivity of $\land$ over $\lor$:

$$P \Leftrightarrow Q \quad \Leftrightarrow \quad (\neg P \land \neg Q) \lor (\neg P \land P) \lor (Q \land \neg Q) \lor (Q \land P)$$

simplifying:

$$P \Leftrightarrow Q \quad \Leftrightarrow \quad (\neg P \land \neg Q) \lor (Q \land P)$$

finally, using commutativity of $\lor$:

$$P \Leftrightarrow Q \quad \Leftrightarrow \quad (P \land Q) \lor (\neg P \land \neg Q)$$

Propositional logic and Modus Ponens:

"Amy, Bob, Cal, Don, and Eve were invited to a party last night." → defines what we are talking about

<u>constants</u>: A ∴ *"Amy went to the party",* B, C, D, E

<u>knowledge base</u>:

"Cal will always go if Amy and Bob go."

(1) $A \wedge B \Rightarrow C$ $(\neg A \vee \neg B \vee C)$

"Cal will not go if Don goes, and conversely."

(2) $D \Rightarrow \neg C$ $(\neg D \vee \neg C)$

(2b) $C \Rightarrow \neg D$

"Amy went to the party with Eve." $A \wedge E$

(3) A (4) E

"Bob goes to every party that Eve goes to."

(5) $E \Rightarrow B$ $(\neg E \vee B)$

<u>proof</u>: (4)+(5) $E,\ E \Rightarrow B$ $\vdash$ B (6)

(3)+(6)+(1) $A, B,\ A \wedge B \Rightarrow C$ $\vdash$ C (7)

(7)+(2b) $C,\ C \Rightarrow \neg D$ $\vdash$ $\neg D$

<u>Don did not go to the party</u>

The unicorn mystery:

constants: properties of the unicorn

Mythical, Magical, Horned, and

Mammal, Mortal (Immortal $\Leftrightarrow \neg$ Mortal)

knowledge base:

Mythical $\Rightarrow \neg$ Mortal	$\neg$ Mortal $\Rightarrow$ Horned
$\neg$ Mythical $\Rightarrow$ Mortal	Mammal $\Rightarrow$ Horned
$\neg$ Mythical $\Rightarrow$ Mammal	Horned $\Rightarrow$ Magical

problem: only rules and *no facts* (!)

Modus Ponens: P, P $\Rightarrow$ Q |– Q, but if *no P?*

→ nothing can be inferred directly from the *KB*

→ need some fact(s) i.e.,

1) assume Mythical, then infer (what?)

2) assume $\neg$ Mythical, then …

Using the Modus Ponens rule of inference:

if the unicorn is mythical:

Mythical, Mythical ⇒ ¬ Mortal |– ¬ Mortal

¬ Mortal, ¬ Mortal ⇒ Horned |– Horned

Horned, Horned ⇒ Magical |– Magical

if the unicorn is <u>not</u> mythical:

¬ Mythical, ¬ Mythical ⇒ Mammal |– Mammal

Mammal, Mammal ⇒ Horned |– Horned

Horned, Horned ⇒ Magical |– Magical

<u>conclusion</u>: the unicorn is <u>both horned and magical</u>

(true in all cases – Mythical or ¬ Mythical)

still no conclusion about the unicorn being mythical

(note: in general this is <u>not</u> a workable approach…)

Using the resolution rule of inference:

binary resolution: $P \vee Q \, , \neg Q \vee R \;\vdash\; P \vee R$

knowledge base (CNF):

1. $\neg$ Mythical $\vee$ $\neg$ Mortal
2. Mythical $\vee$ Mortal
3. Mythical $\vee$ Mammal
4. Mortal $\vee$ Horned
5. $\neg$ Mammal $\vee$ Horned
6. $\neg$ Horned $\vee$ Magical

proof:

7. from 1 and 4: $\neg$ Mythical $\vee$ Horned
8. from 3 and 5: Mythical $\vee$ Horned
9. from 7 and 8: Horned
10. from 9 and 6: Magical

conclusion: the unicorn is both horned and magical

still no conclusion about the unicorn being mythical