Part III – Knowledge and Reasoning

- **6 Agents that Reason Logically**
  - Knowledge-based Agents. – Representations.
  - Propositional Logic. – The Wumpus World.
- **7 First-Order Logic**
  - Syntax and Semantics. – Using First-Order Logic.
  - Logical Agents. – Representing Changes.
  - Deducing Properties of the World.
  - Goal-based Agents.
- **8 Building a Knowledge Base**
  - Knowledge Engineering. – General Ontology.

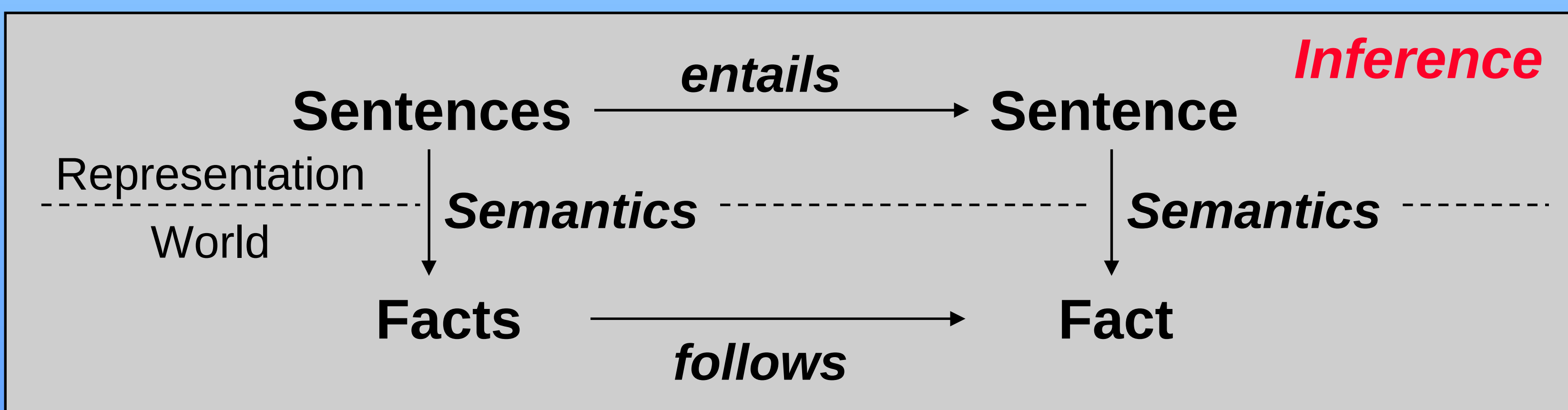
Knowledge Representations

- **Knowledge representation (KR)**
  - KB: set of sentences –> need to
  - Express knowledge in a (computer-) tractable form
- **Knowledge representation language**
  - Syntax – implementation level
    - Possible configurations that constitute sentences
  - Semantics – knowledge level
    - Facts of the world the sentences refer to
    - e.g. language of arithmetics: x, y numbers
      sentence: “x ≥ y”, semantics: “greater or equal”

Logic

Reasoning and Logic

- **Logic**
  - Representation + Inference = Logic
    - Where representation = syntax + semantics
- **Reasoning**
  - Construction of new sentences from existing ones
- **Entailment as logical inference**



Deduction and Induction

- **Mechanical reasoning**
  - Example
    - If a chord sequence is tonal, then it can be generated by a context-sensitive grammar.
    - The twelve-bar blues has a chord sequence that is tonal. |-
    - The twelve-bar blues has a chord sequence that can be generated by a context-sensitive grammar.
- **Deductive inference**
  - KB: Monday $\Rightarrow$ Work, Monday |- Work *sound (MP)*
- **Inductive inference**
  - KB: Monday $\Rightarrow$ Work, Work |- Monday *unsound!*
  - Generalization e.g., *"all swans are white …"*

Entailment and Inference

- **Entailment**
  - Generate sentences that are necessarily true, given that the existing sentences are true
  - Notation: KB $\models \alpha$
    - e.g. Wumpus world:
      { "¬S(1,1)", "¬B(1,1)" } $\models$ "OK(2,1)"
      Arithmetics:
      { "$x \geq y$", "$y \geq z$" } $\models$ "$x \geq z$"
- **Inference**
  - **Tell**, given KB: (KB $\models \alpha$) !
  - **Ask**, given KB and α: (KB $\models \alpha$) ?

Properties of Inference

  - *Can be described by the sentences it derives, KB |=* α_I
- **Soundness**
  - Generate only entailed sentences
  - Proof: sequence of operations of a sound inference
    - Record of operations that generate a specific entailed sentence

      e.g. "Smoke ⇒ Fire" and "Smoke" |= "Fire"

      "Fire ⇒ Call_911" and "Fire" |= "Call_911"
- **Completeness**
  - A proof can be found for any entailed sentence
- **Proof theory**
  - Specify the reasoning operations that are sound

An Example of Sound Inference

– Sentence: x

• Semantics: an expression; can be a single symbol or number, the concatenation of 2 expressions, etc.

– Sentence: x y

• Semantics: an expression which refers to a quantity that is the <u>product</u> of the quantities referred to by each of the expressions

– Sentence: x = y

• Semantics: the 2 expressions on each side of "**=**" refer to the same quantity

– A sound inference: from $E = mc^2$, $T_1 \geq T_2$ $\models$ $E\ T_1 \geq mc^2\ T_2$

Knowledge Representation Languages

- **Formal (programming) languages**
  - Good at describing algorithms and data structures
    - e.g. the Wumpus world as a 4x4 array, World[2,2] $\leftarrow$ Pit
  - Poor at representing incomplete / uncertain information
    - e.g. “there is a pit in [2,2] or [3,1]”, or “…a wumpus *somewhere*”
  - > *not expressive enough*
- **Natural languages**
  - Very expressive (too much, thus very complex)
  - More appropriate for communication than representation
  - Suffer from ambiguity
    - e.g. “It’s hot!”
    - e.g. “small cats and dogs” compared to “– x + y”.

Properties of Representations

– *KR languages should combine the advantages of both programming and natural languages.*

- **Desired properties**
  - Expressive
    - Can represent everything we need to.
  - Concise
  - Unambiguous
    - Sentences have a unique interpretation.
  - Context independent
    - Interpretation of sentences depends on semantics only.
  - Effective
    - An inference procedure allows to create new sentences.

Properties of Semantics

- **Interpretation (meaning)**
  - *Correspondence between sentences and facts*
  - Arbitrary meaning, fixed by the writer of the sentence
    - e.g. Natural languages: meaning fixed by usage (cf. dictionary)
      exceptions: encrypted messages, codes (e.g. Morse)
      chat
  - Systematic relationship: compositional languages
    - *The meaning of a sentence is a function of the meaning of its parts*.
  - Truth value
    - A sentence make a claim about the world –> TRUE or FALSE
    - *Depends on the interpretation and the state of the world*
      - e.g. Wumpus world: S(1,2) true if means "Stench at [1,2]" and the world has a wumpus at either [1,3] or [2,2].

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Properties of Inference

- **Definition**
  - *Inference (reasoning) is the process by which conclusions are reached*
  - Logical inference (deduction) is the process that implements entailment between sentences
- **Useful properties**
  - Valid sentence (tautology)
    - iff TRUE under all possible interpretations in all possible worlds.
      - e.g. “S or ¬ S” is valid, “S(2,1) or ¬ S(2,1)”, etc.
  - Satisfiable sentence
    - iff there is some interpretation in some world for which it is TRUE
      - e.g. “S and ¬ S” is unsatisfiable

Inference and Agent Programs

- **Inference in computers**
  - Does not know the interpretation the agent is using for the sentences in the KB
  - Does not know about the world (actual facts)
  - Knows only what appears in the KB (sentences)
    - e.g. Wumpus world: doesn't know the meaning of "OK", what a wumpus or a pit is, etc. – can only see: KB |= "[2,2] is OK"
  - > *Cannot reason informally*
    - does not matter, however, if KB |= "[2,2] is OK" is a valid sentence
- **Formal inference**
  - Can handle arbitrarily complex sentences, KB |= P

Different Logics

- **Formal logic**
  - Syntax
    - A set of rules for writing sentences
  - Semantics
    - A set of rules (constraints) for relating sentences to facts
  - Proof theory / inference procedure
    - A set of rules for deducing entailments of sentences
- **Propositional logic**
  - Symbols, representing propositions (facts)
  - Boolean connectives, combining symbols
    - e.g. “Hot” or “Hot and Humid”

Different Logics

- **First-order logic**
  - Objects and predicates, representing properties of and relations between objects
  - Variables, Boolean connectives and quantifiers
    - e.g. “Hot(x)”, “Hot(Air)” or ”Hot(Air) and Humid(Air)”
- **Temporal logic**
  - World ordered by a set of time points (intervals)
- **Probabilistic and fuzzy logic**
  - Degrees of belief and truth in sentences
    - e.g. “Washington is a state” with belief degree 0.4, “a city” 0.6, ”Washington is a large city” with truth degree 0.6

Different Degrees of Truth

- *Q: Is there a tuna sandwich in the refrigerator?*
- *A: 0.5 !*

- **Probabilities**
  - There *is* or there *isn’t* (50% chance either way).
- **Measures**
  - There is *half* a tuna sandwich there.
- **Fuzzy answer**
  - There is *something* there, but it *isn't really* a tuna sandwich. Perhaps it is some other kind of sandwich, or a tuna salad with no bread...

The Commitments of Logics

Formal (KR) Language	**Ontological commitment** (what exists in the world)	**Epistemological commitment** (what an agent believes about facts)
Propositional logic	facts	true / false / unknown
First-order logic	facts, objects, relations	true / false / unknown
Temporal logic	facts, objects, rel., times	true / false / unknown
Probability logic	facts	degree of belief 0…1
Fuzzy logic	degrees of truth 0…1	degree of belief 0…1

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Elements of Propositional Logic

- **Symbols**
  - Logical constants: TRUE, FALSE
  - Propositional symbols: P, Q, etc.
  - Logical connectives: $\wedge$, $\vee$, $\Leftrightarrow$, $\Rightarrow$, $\neg$
  - Parentheses: ( )
- **Sentences**
  - Atomic sentences: constants, propositional symbols
  - Combined with connectives, e.g. $P \wedge Q \vee R$
    also wrapped in parentheses, e.g. $(P \wedge Q) \vee R$

Logical Connectives

- Conjunction $\wedge$
  - Binary op., e.g. P $\wedge$ Q, "P and Q", where P, Q are the *conjuncts*
- Disjunction $\vee$
  - Binary op., e.g. P $\vee$ Q, "P or Q", where P, Q are the *disjuncts*
- Implication $\Rightarrow$
  - Binary op., e.g. P $\Rightarrow$ Q, "P implies Q", where P is the *premise* (antecedent) and Q the *conclusion* (consequent)
  - Conditionals, "if-then" statements, or *rules*
- Equivalence $\Leftrightarrow$
  - Binary op., e.g. P $\Leftrightarrow$ Q, "P equivalent to Q"
  - Biconditionals.
- Negation $\neg$
  - Unary op., e.g. $\neg$ P, "not P"

Syntax of Propositional Logic
(Backus-Naur Form)

Sentence	$\rightarrow$	AtomicSentence | ComplexSentence
AtomicSentence	$\rightarrow$	LogicalConstant | PropositionalSymbol
ComplexSentence	$\rightarrow$	(Sentence) | Sentence LogicalConnective Sentence | ¬Sentence
LogicalConstant	$\rightarrow$	TRUE | FALSE
PropositionalSymbol	$\rightarrow$	P | Q | R | …
LogicalConnective	$\rightarrow$	$\wedge$ | $\vee$ | $\Leftrightarrow$ | $\Rightarrow$ | $\neg$

Precedence (from highest to lowest): $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

e.g.: $\neg P \wedge Q \vee R \Rightarrow S$ (not ambiguous), eq. to: $(((\neg P) \wedge Q) \vee R) \Rightarrow S$

Semantics of Propositional Logic

- **Interpretation of symbols**
  - Logical constants have fixed meaning
    - True: always means the fact is the case; valid
    - False: always means the fact is not the case; unsatisfiable
  - Propositional symbols mean "whatever they mean"
    - e.g.: **P** "we are in a pit", etc.
    - Satisfiable, but not valid (true only when the fact is the case)
- **Interpretation of sentences**
  - Meaning derived from the meaning of its parts
    - Sentence as a combination of sentences using connectives
  - Logical connectives as (boolean) functions:

TruthValue ***f*** (TruthValue, TruthValue)

Semantics of Propositional Logic

- **Interpretation of connectives**
  - Truth-table
    - Define a mapping from input to output

P	Q	$\neg$ P	P $\wedge$ Q	P $\vee$ Q	P $\Rightarrow$ Q	P $\Leftrightarrow$ Q
False	False	True	False	False	True	True
False	True	True	False	True	True	False
True	False	False	False	True	False	False
True	True	False	True	True	True	True

  - Interpretation of sentences by decomposition
    - e.g.: $\neg P \wedge Q \vee R \Rightarrow S$, with $P \leftarrow T$, $Q \leftarrow T$, $R \leftarrow F$, $S \leftarrow F$:

$\neg P \leftarrow F$ $\quad$ $((\neg P) \wedge Q) \vee R) \leftarrow F$

$(\neg P) \wedge Q \leftarrow F$ $\quad$ $(((\neg P) \wedge Q) \vee R) \Rightarrow S \leftarrow T$

end