

NANYANG TECHNOLOGICAL UNIVERSITY  
SPMS/DIVISION OF MATHEMATICAL SCIENCES

2016/17 Semester 1      MH2500 Probability and Introduction to Statistics      Tutorial 5

For the tutorial on 15 September, let us discuss

- Ex. 2.5.52, 57, 60, 66, 70
- Ex. 3.8.2

**Ex. 2.5.52.** Suppose that in a certain population, individuals' heights are approximately normally distributed with parameters  $\mu = 70$  and  $\sigma = 3$  in.

- a. What proportion of the population is over 6 ft. tall?
- b. What is the distribution of heights if they are expressed in centimeters? In meters?  
(Conversions: 1 inch = 2.54cm and 1 ft = 12 inches.)

**Ex. 2.5.57.** If  $X \sim N(\mu, \sigma^2)$  and  $Y = aX + b$  where  $a < 0$ , show that  $Y \sim N(a\mu + b, a^2\sigma^2)$ .

**Ex. 2.5.60.** Find the density function of  $Y = e^Z$ , where  $Z \sim N(\mu, \sigma^2)$ . This is called the **lognormal density**, since  $\log Y$  is normally distributed.

**Ex. 2.5.66.** Let  $f(x) = \alpha x^{-\alpha-1}$  for  $x \geq 1$  and  $f(x) = 0$  otherwise, where  $\alpha$  is a positive parameter. Show how to generate random variables with this density from a uniform random number generator.

**Ex. 2.5.70.** Let  $U$  be a uniform random variable on  $[0, 1]$ . Find the density function of  $V = U^{-\alpha}$ ,  $\alpha > 0$ . Compare the rates of decrease of the tails of the densities as a function of  $\alpha$ . Does the comparison make sense intuitively?

**Ex. 3.8.2.** An urn contains  $p$  black balls,  $q$  white balls, and  $r$  red balls; and  $n$  balls are chosen without replacement.

- a. Find the joint distribution of the numbers of black, white, and red balls in the sample.
- b. Find the joint distribution of the numbers of black and white balls in the sample.
- c. Find the marginal distribution of the number of white balls in the sample.