1. <u>INTRODUCTION TO POWER SYSTEMS</u>

The structure of the electric power or energy system is very large & complex. Nevertheless, it can be divided into five basic stages/components/subsystems.

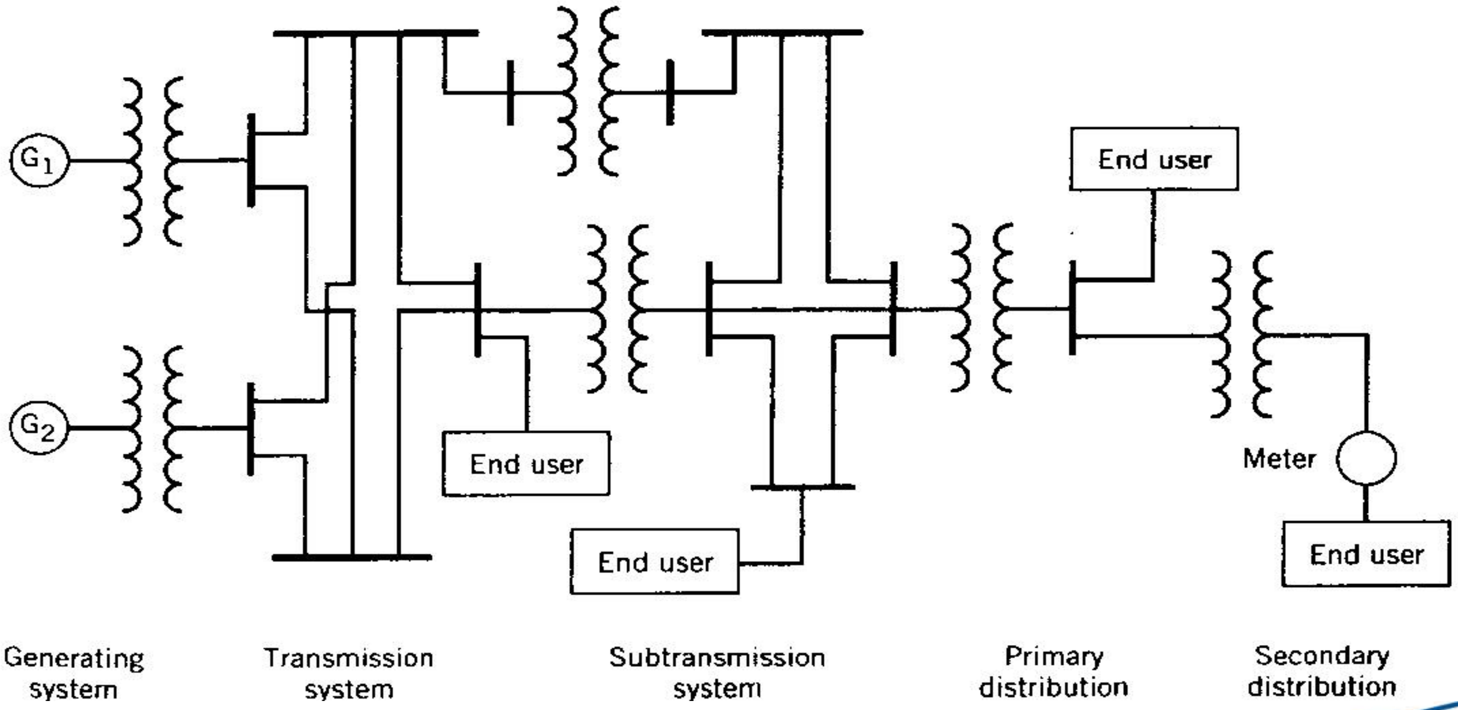
Energy source (Fuel) → **Energy converter (Generator)** → **Transmission system** → **Distribution system** → **Load (Energy sink)**

- Energy source may be
  - coal, gas or oil (fossil fuel)
  - fissionable material (nuclear)
  - water in a dam (hydro)
  - renewable sources e.g. solar, wind, tidal, biofuels, geothermal

- Generator that transforms non-electrical energy to electrical energy; usually rotating-machinery type; power output from few kilowatts to few thousand MW; voltage levels 440 V to 25 kV.

- Transmission system transports generated energy from generating stations to major load centres; voltage levels 115 kV to 765 kV (less than 138 kV usually referred to as sub-transmission system); overhead lines & underground cables.

- Transformers used to change voltage levels (to high over transmission system & low over distribution system, etc)
- Distribution system transports transmitted energy from transmission system to users; voltage levels typically 1 kV to 33 kV.
- Loads : industrial, commercial, residential, farm, etc.



(11-30 kV) (138-765 kV) (33-138 kV) (33-11 kV) (<11 kV)

Basic Structure of Power System

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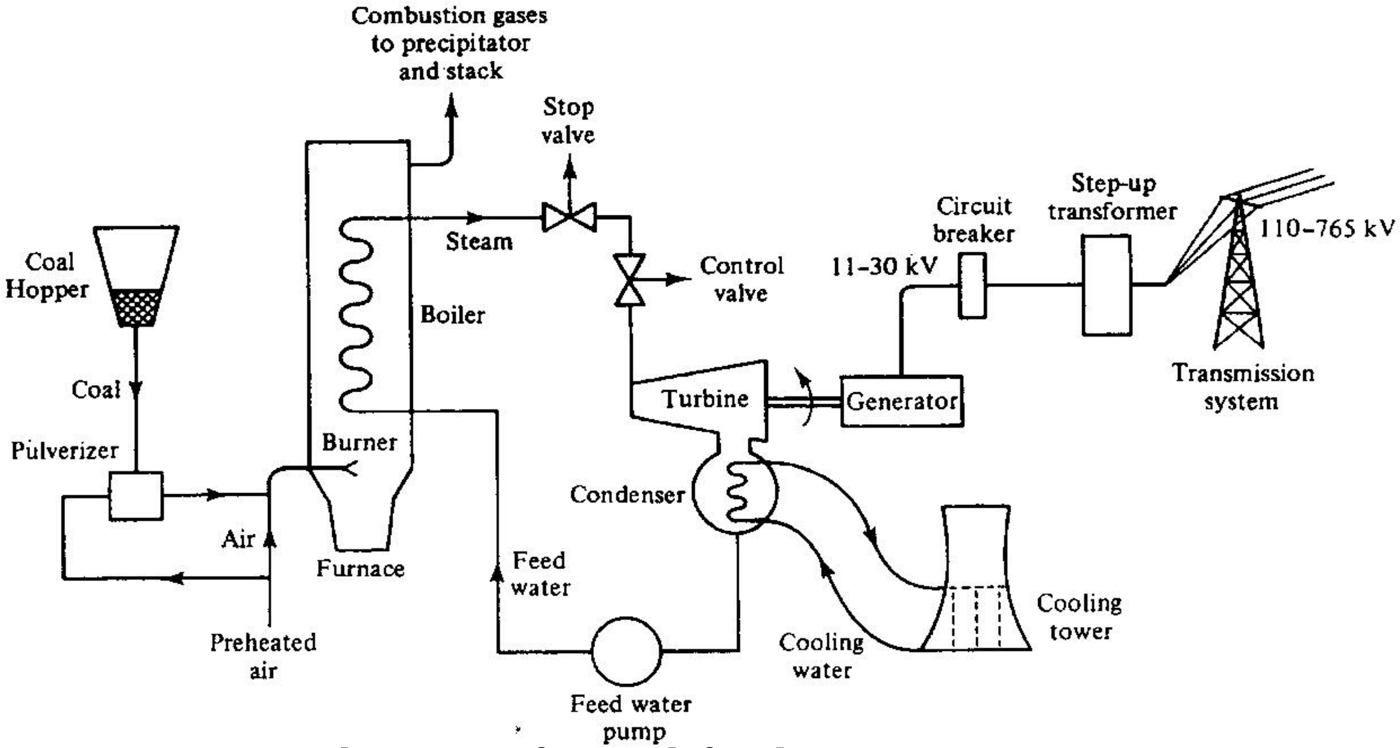

Schematic of a Coal-fired Generating Station

2. REVIEW OF SINGLE-PHASE & THREE-PHASE CIRCUITS

Before we embark on a detailed study of the generators & lines, it would help to quickly review the basic concepts of :

- 3-phase systems (as most power systems operate as 3-ϕ systems)
- Single-phase equivalent of 3-ϕ systems (since for balanced 3-ϕ systems, we can use one phase of the star(Y)-equivalent)
- Per unit (p.u.) systems

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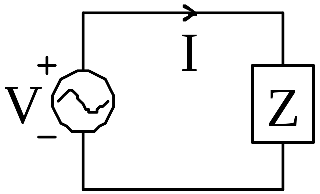
⇒ <u>Recall</u> that all the above have been covered elsewhere (EE2005/EE3010), but since we will be using them extensively, it is worth taking a look back in time!

⇒ First, a review of the <u>one-phase system</u>.



Load represented by impedance Z

Let $V = |V|\angle 0^o$ (reference) & $Z = |Z|\angle\theta$, where θ is <u>power factor angle</u>.

$\therefore Z = R + jX$ (assuming inductive load)

$\Rightarrow R = |Z| \cos\theta$ & $X = |Z| \sin\theta$

$$\Rightarrow I = \frac{V}{Z} = \frac{|V|\angle 0^o}{|Z|\angle\theta} = \frac{|V|}{|Z|}\angle -\theta = |I|\angle -\theta$$

Apparent power S (VA, kVA, MVA and $|S| = |V|\,|I|$)

$$\begin{aligned} \Rightarrow S \quad &= VI^* = (|V|\angle 0^o)(|I|\angle\theta) = |V|\,|I|\angle\theta = |S|\angle\theta \\ &= |V|\,|I| \cos\theta + j|V|\,|I| \sin\theta \\ &= P + jQ \end{aligned}$$

where P : Real (active) power (W, kW, MW)

Q: Imaginary (reactive) power (var, kvar, Mvar)

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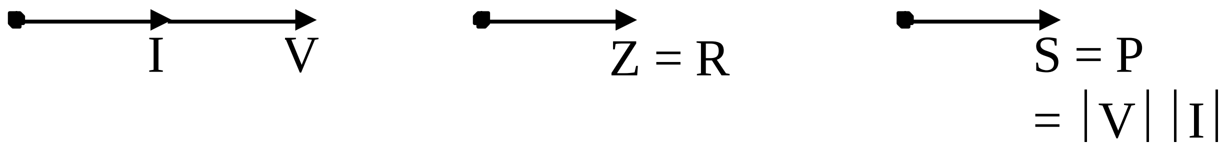
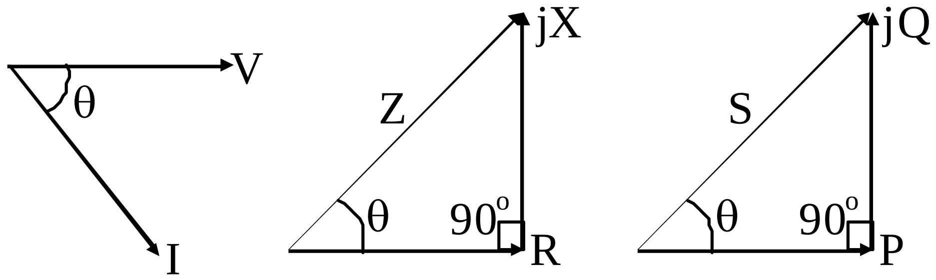
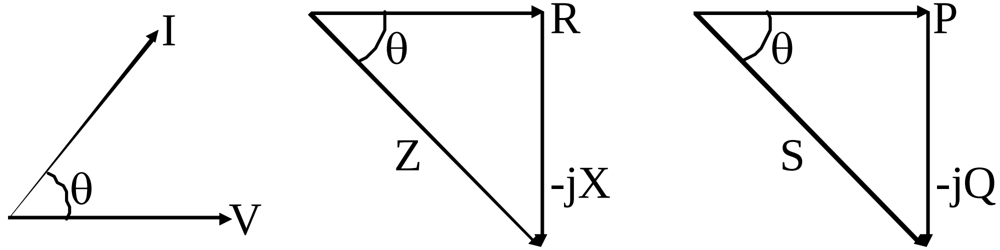
⇒ **For $\theta = 0^o$ (unity power factor load)**



⇒ **For lagging P.F. loads,**



⇒ **For leading P.F. loads,**



What are Real & Reactive Powers?

By convention,

- Lagging vars $\Rightarrow$ Inductive load $\Rightarrow$ Q (+VE)
- Leading vars $\Rightarrow$ Capacitive load $\Rightarrow$ Q (–VE)

P = Real or active power, consumed/dissipated in resistance R

$= |V||I| \cos\theta = (|I||Z|)|I| \cos\theta = |I|^2|Z| \cos\theta$

$= |I|^2R$

Q = Reactive power absorbed by inductive reactance X (or generated by capacitive reactance)

$= |V||I| \sin\theta = |I||Z||I| \sin\theta = |I|^2|Z| \sin\theta$

$= |I|^2X$

Represents the energy exchange between source & reactor/capacitor i.e. represents stored energy.

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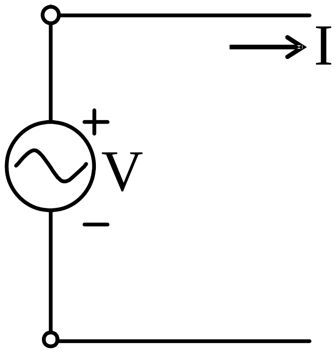
Convention for Real & Reactive Powers

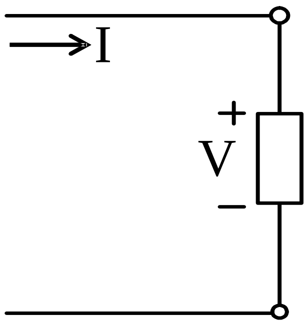
1) Generators



$$S = VI^* = P + jQ$$

- If P is positive, real power is delivered/supplied
- If P is negative, real power is absorbed by source
- If Q is positive, reactive power is supplied/delivered
- If Q is negative, reactive power is absorbed by source

2) Loads



$$S = VI^* = P + jQ$$

- If P is positive, real power is absorbed by the load
- If P is negative, real power is supplied by the load
- If Q is positive, reactive power is absorbed by the load (lagging PF load $\Rightarrow$ inductive load)
- If Q is negative, reactive power is supplied by the load (leading PF load $\Rightarrow$ capacitive load)

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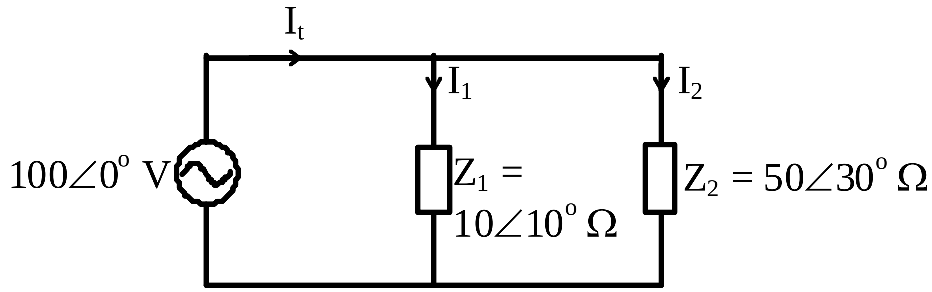
⇒ Inductive load <u>absorbs</u> reactive power.

⇒ Capacitive load <u>generates</u> reactive power, or <u>absorbs negative reactive power</u>.

<u>Review Exercises (1 & 2)</u>

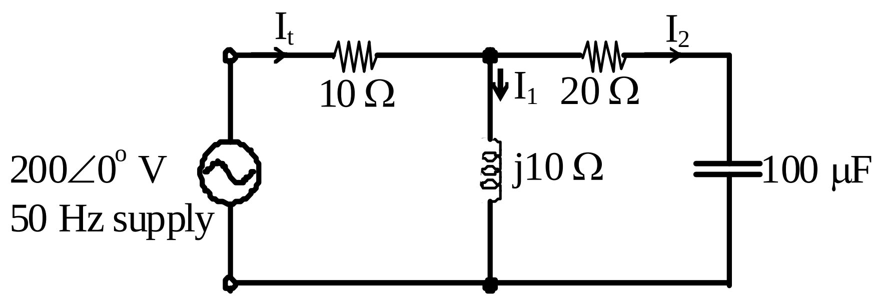
(1)



Find the

a) Real power P (<u>1158.014 W</u>)
b) Reactive power Q (<u>273.657 VAr lag</u>)
c) Apparent power S (<u>1189.91 VA</u>)
d) Power factor of the Network (<u>0.973 lag</u>)

(2)



Find the Network

a) Power factor (<u>0.7 lag</u>)
b) Real power (<u>1601.16</u> W)
c) Reactive power (<u>1628.2 VAr</u>)
d) Apparent power (<u>2283.6 VA</u>)

Review of Balanced 3-phase Systems

- Mesh or Delta (Δ) connection
- Star or Wye (Y) connection

Usually (By default if not specified), 3-ϕ systems specified in terms of

→ total 3-ϕ S, P, Q
→ line-to-line voltages
→ line currents

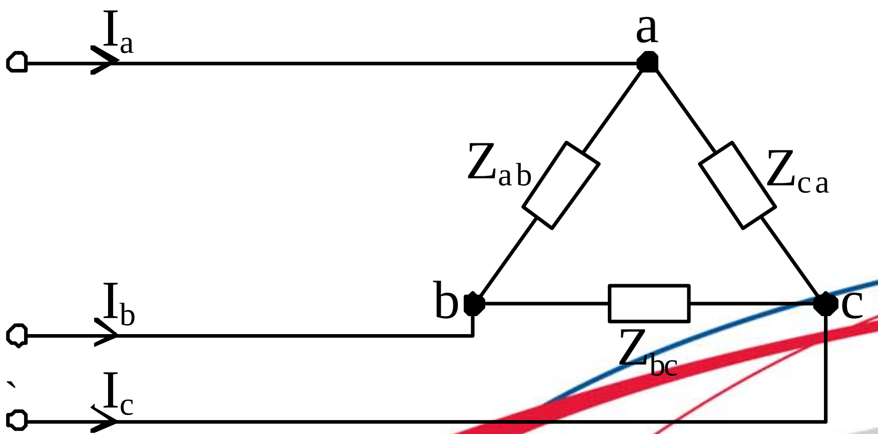
Remember : cos θ = power factor
θ = power factor angle, where θ is always between phase voltage & phase current of the same phase.

Delta Connection

$$Z_{ab} = Z_{ca} = Z_{bc} = |Z| \angle\theta$$

$$I_{ab} = \frac{V_{ab}}{Z_{ab}}, \quad I_{bc} = \frac{V_{bc}}{Z_{bc}} \quad \&$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}}$$



If V_{ab} = Reference = $|V_{ab}|\angle 0^o$ V

Then $V_{bc} = |V_{ab}|\angle -120^o$ V

& $V_{ca} = |V_{ab}|\angle 120^o$ V

Line currents $I_a = I_{ab} - I_{ca}$

$I_b = I_{bc} - I_{ab}$ & $I_c = I_{ca} - I_{bc}$

$$\Rightarrow \quad \begin{aligned} I_a &= \sqrt{3}|I_{ab}|\angle -\theta-30^o \\ I_b &= \sqrt{3}|I_{bc}|\angle -120-\theta-30^o \Rightarrow \\ I_c &= \sqrt{3}|I_{ca}|\angle 120-\theta-30^o \end{aligned} \qquad \begin{aligned} |I_{LINE}| &= \sqrt{3}|I_{PHASE}| \\ \\ |V_{LINE}| &= |V_{PHASE}| \end{aligned}$$

$$\begin{aligned} \text{Apparent power } S &= 3\, V_{PHASE}\, I^*_{PHASE} \\ &= 3\, |V_{PHASE}|\,|I_{PHASE}|\angle\theta \\ &= 3\, |V_{LINE}|\frac{1}{\sqrt{3}}|I_{LINE}|\angle\theta \\ &= \sqrt{3}\, |V_{LINE}|\,|I_{LINE}|\angle\theta \text{ VA} \end{aligned}$$

$$\begin{aligned} \Rightarrow S &= \sqrt{3}|V_{LINE}|\,|I_{LINE}|\cos\theta + j\sqrt{3}|V_{LINE}|\,|I_{LINE}|\sin\theta \\ &= P + jQ \end{aligned}$$

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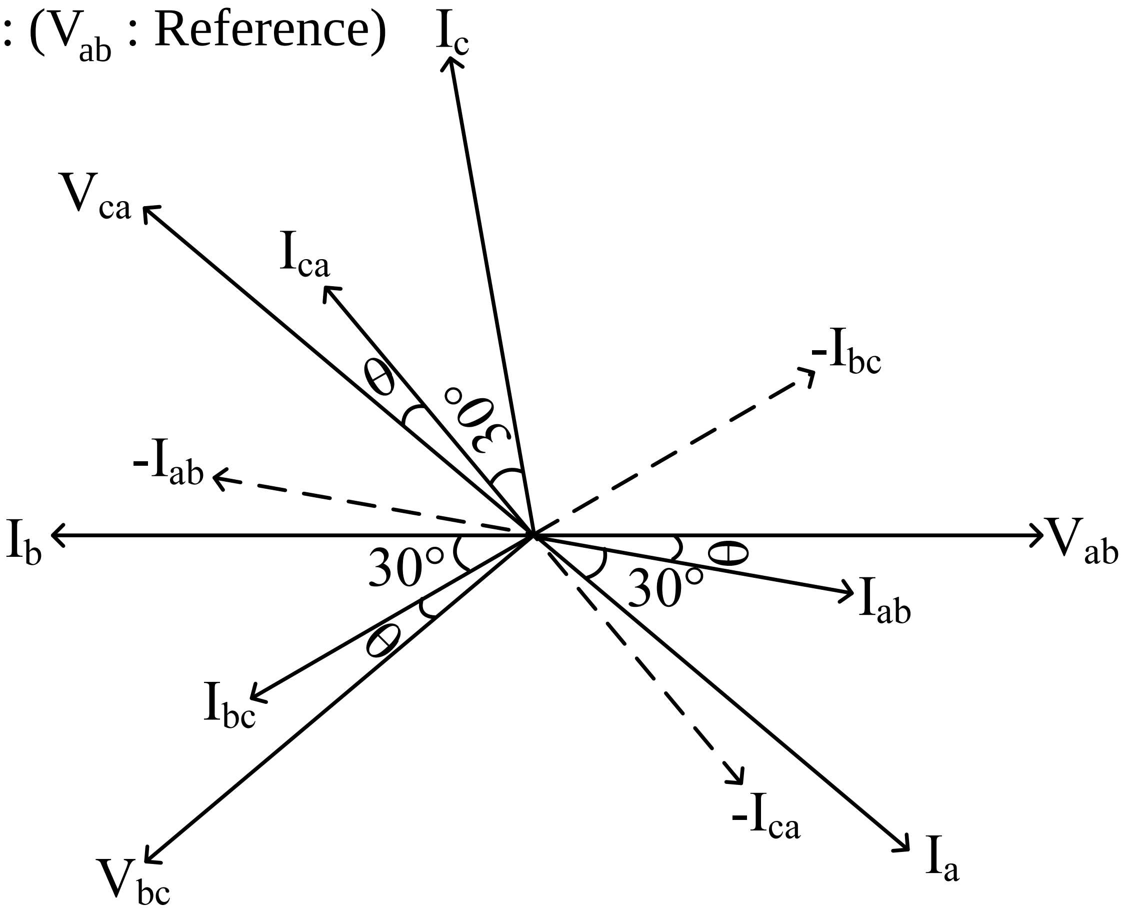
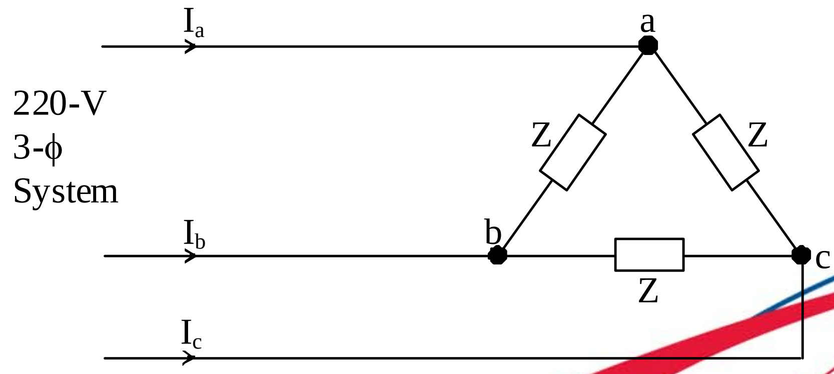
Phasor Diagram : (V_{ab} : Reference)



Review Exercise (3)

(3)



$Z = 30\angle{-15^o}\ \Omega$

Find phase & line currents. Sketch phasor diagram. ($I_a = 12.7\angle{-15^o}$ A)

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Y-connection

$$I_a = I_{an} = \frac{V_{an}}{Z}$$

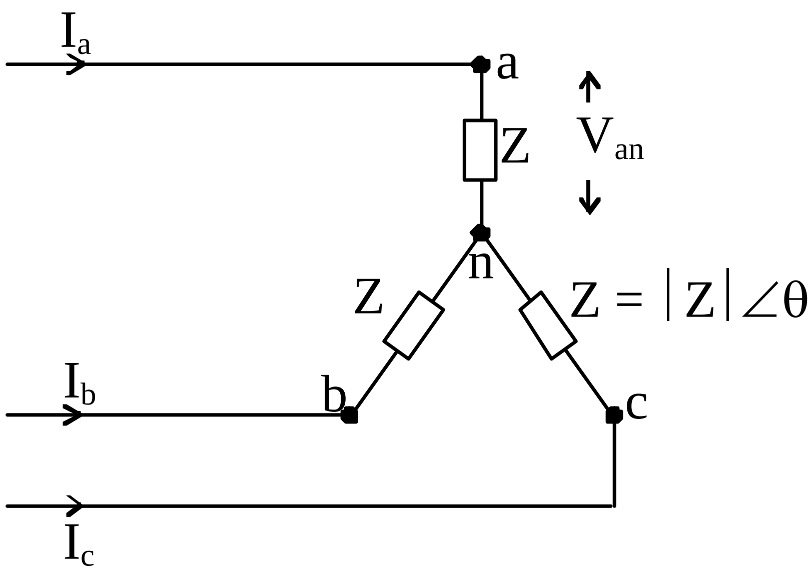
$$= \frac{|V_{an}|}{|Z|}\angle -\theta$$

$$\text{Reference} = V_{an} = |V_{an}|\angle 0$$

$$V_{ab} = V_{an} + V_{nb}$$

$$V_{bc} = V_{bn} + V_{nc}$$

$$V_{ca} = V_{cn} + V_{na}$$



Here $|I_{LINE}| = |I_{PHASE}|$

& $|V_{LINE}| = \sqrt{3}|V_{PHASE}|$

$$\begin{aligned}
S &= 3\, V_{PHASE}\, I^*_{PHASE} \\
&= 3|V_{PHASE}|\angle 0°\left[|I_{PHASE}|\angle\theta\right] \\
&= 3\frac{1}{\sqrt{3}}\, |V_{LINE}|\,|I_{LINE}|\angle\theta \\
&= \sqrt{3}|V_{LINE}|\,|I_{LINE}|\angle\theta \\
&= \sqrt{3}|V_{LINE}|\,|I_{LINE}|\cos\theta + j\sqrt{3}\,|V_{LINE}|\,|I_{LINE}|\sin\theta \\
&= P + jQ
\end{aligned}$$

Review Exercise (4)

(4) $V_{AB} = 208 \angle 30^{o}$ V
Load $Z = (4 + j3)\ \Omega$ in each phase.

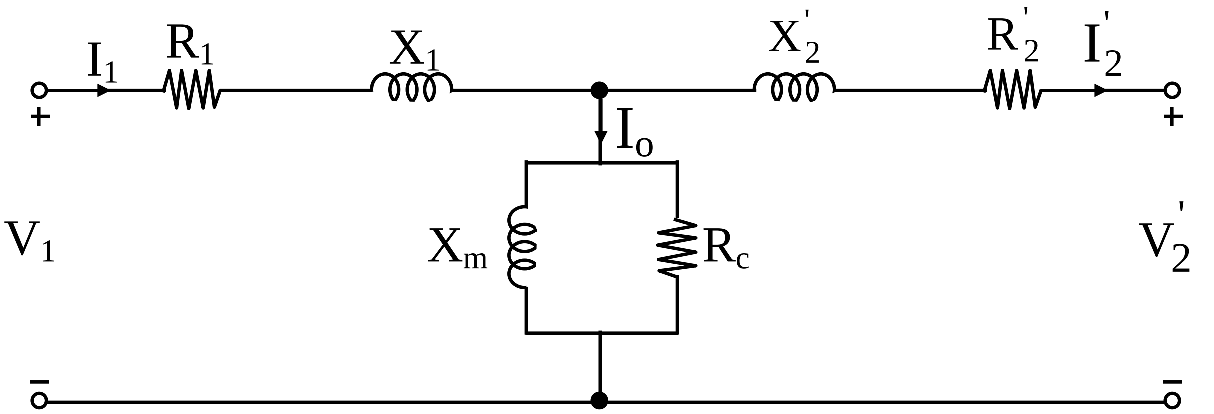
- Find the line currents & power dissipated in the loads (total power).
- Also find Q & S.

[Y-connected system, phase seq. A-B-C]

$I_a = 24.02 \angle -36.87^{o}$ A, P = 6923 W, Q = 5192.17 VAr lag, S = 8653.7VA

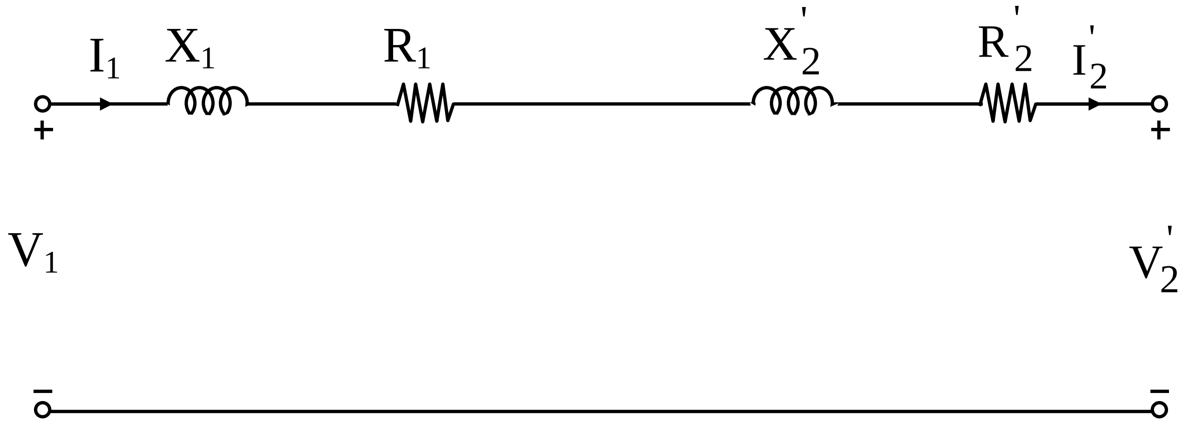
Review of Transformer's Equivalent CKT.

⇒ Equivalent circuit (per phase)

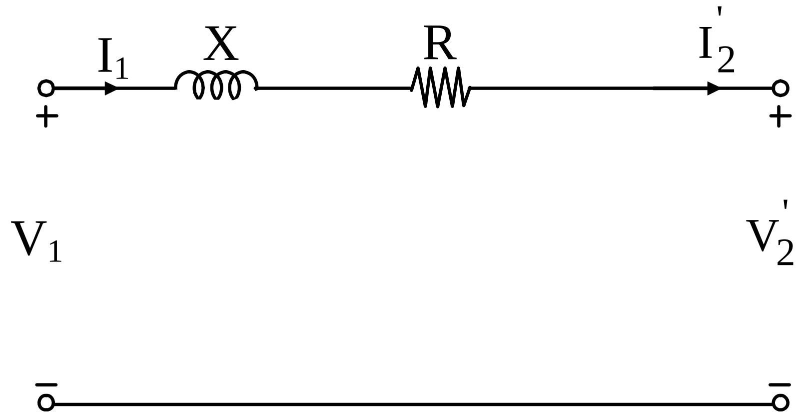


(All quantities referred to the primary)

⇒ Approximate equivalent circuit (neglecting magnetizing current)



which reduces to



where $R = R_1 + R'_2$ = total equivalent resistance & $X = X_1 + X'_2$ = total equivalent reactance.

$\Rightarrow$ Frequently R is neglected, in which case

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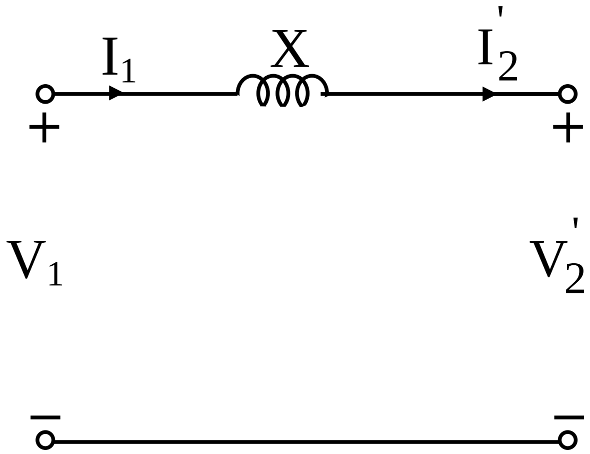

In other words, the transformer can be replaced by an equivalent impedance, or just an equivalent reactance.

3. PER-UNIT (P.U.) SYSTEM

- Power transmission lines are operated at very high voltage levels (kilovolts). Due to the large amount of power transmitted, megawatts & megavoltamps are commonly-used terms!
- It therefore would be more meaningful to scale down all physical values of Ω, A, kV, MVA, MW using scaling factors called based values. For example, if a base voltage of 100 kV is selected, then physical system voltages of 80 kV, 110 kV & 100 kV become $\frac{80}{100}$, $\frac{110}{100}$ & $\frac{100}{100}$ per unit respectively!

- The per-unit value of any quantity is defined as the ratio of the actual quantity to an "arbitrarily" chosen value (base or reference) of the same dimensions.

$$\therefore \text{ Quantity in per unit} = \frac{\text{physical quanity}}{\text{base value}}$$

$$\& \text{ Quantity in percent} = \frac{\text{physical quantity}}{\text{base value}} \times 100$$

- Percent system should be used with caution (due to mult. factor of 100)
- Per-unit system is preferred in power system calculations as it offers the following advantages :

Advantages of Per-unit System

- Analysis is greatly simplified, e.g. all impedances of a given equivalent CKT can be directly added without considering system voltages.
- Use of "√3" is eliminated! The base values account for these easily.
- Manufacturers of electrical equipment usually specify the impedance in per unit or percent of nameplate ratings.

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- Electrical machines & transformers have widely varying internal impedances with size & rating. However, it turns out that in the p.u. system, these impedances fall within a fairly narrow range. Hence, if the actual impedance of a machine is not known, its per unit value can be easily assigned!
- Circuit analyst is relieved of the worry of referring quantities to one side or other side of transformer, especially in large networks containing many transformers of different turns ratios.

⇒Eliminates the possible cause of making serious calculation mistakes!

- P.U. values are more convenient in simulating machine systems on digital computers.

Significant advantage : Per unit impedance of transformer is the same on both sides of the transformer!

Per Unit (p.u.) Quantities in 3-phase Power System

- There are 4 base values
  1. Power base S_b, usually in MVA.
  2. Impedance base Z_b, usually in OHMS.
  3. Current base I_b, usually in A.
  4. Voltage base V_b, usually in kV.

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- In 3-phase systems, usually
  - → S, P, Q are <u>three-phase powers</u> in MVA, MW & MVAr respectively
  - → Voltages considered are <u>line-to-line</u> values
  - → Currents considered are <u>line</u> values
  - → Impedances considered are <u>phase</u> values of <u>equivalent star</u> (Y) configuration (This means that all impedances in Ω must be converted to equivalent Y value in Ω, before converting to its p.u. value)

- The 4 base values (S_b, Z_b, I_b & V_b) are related as follows :

$$S_b = \sqrt{3}\frac{V_b I_b}{1000} \text{ MVA} \qquad (1)$$

where V_b is line-to-line base voltage <u>in kV</u> & I_b is the line current <u>in AMPS.</u>

$$\Rightarrow \quad I_b = \frac{S_b \times 1000}{\sqrt{3} \times V_b} \qquad (2)$$

$$\text{Next, } Z_b = \frac{(V_b / \sqrt{3})}{I_b} \times 1000 \quad \Omega \qquad (3)$$

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<u>Note</u> : $\frac{V_b}{\sqrt{3}}$ is the phase voltage of the equivalent Y system.

Substituting (2) in (3), we get :

$$Z_b = \frac{(V_b/\sqrt{3})}{\left(\frac{S_b \times 1000}{\sqrt{3} \times V_b}\right)} \times 1000$$

$$\Rightarrow \quad Z_b = \frac{{V_b}^2}{S_b}\ \Omega \qquad (4)$$

- From the above, it is clear that we <u>need to select only 2 base values</u>, instead of all 4. For example, if S_b & V_b are selected, then I_b & Z_b can be calculated using equations (2) & (4) respectively!
- V_b (and consequently Z_b & I_b too) changes from <u>transformer</u> primary to secondary as follows :

$$\frac{V_b \text{ (primary)}}{V_b \text{ (secondary)}} = \frac{\text{No. of turns (primary)}}{\text{No. of turns (secondary)}}$$

$$= \frac{N_1}{N_2} \text{(line} - \text{to} - \text{line turns ratio, a)}$$

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- Per unit values of Z, R, X & I are the same on either side of the transformer, but the actual values are not!

⇒ Per unit quantities can now be evaluated as :

$$Z_{p.u.} = \frac{Z(\Omega)}{Z_b(\Omega)}; \; I_{p.u.} = \frac{I(AMPS)}{I_b(AMPS)};$$

$$V_{p.u.} = \frac{V(kV)}{V_b(kV)}; \; P_{p.u.} = \frac{P(MW)}{S_b(MVA)}$$

⇒ If necessary, actual voltages/currents/ohms etc. can also be obtained as :

Actual value = Base value × p.u. value

General Guidelines for Obtaining P.U. Values

Objective : To reduce the number of computations by selecting suitable values for V_b & S_b.

- Base MVA (S_b) is the same for all parts of the system. Normally S_b in MVA is used.
- Base kV (V_b) is selected in one part of the system; for other parts base kV is obtained according to the line-to-line voltage ratios of transformers.

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- Base impedances will be different in different parts of the system.
- In general, the p.u. (or %) impedances of electrical equipment are specified in terms of their own MVA & kV ratings. These values need to be converted to the system bases selected in (1) & (2) above. This conversion is done using the following formula :

$$Z_{p.u.\ NEW} = \frac{Z_{ACTUAL}}{Z_{b\ NEW}}$$

$$Z_{p.u.\ OLD} = \frac{Z_{ACTUAL}}{\underset{\substack{\uparrow \\ \text{Manufacturer's base (Equipment base)}}}{Z_{b\ OLD}}}$$

where : $Z_{b\ NEW}$ = Base impedance selected in that part of the system where this component is placed

$$= \frac{(V_{b\ NEW})^2}{S_{b\ NEW}} \Leftarrow \text{these are base values in that section}$$

$$\&\ Z_{b\ OLD} = \frac{(V_{b\ OLD})^2}{S_{b\ OLD}} \Leftarrow \text{these are component bases/ratings}$$

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$$\therefore \frac{Z_{\text{p.u. NEW}}}{Z_{\text{p.u. OLD}}} = \frac{Z_{\text{b OLD}}}{Z_{\text{b NEW}}} = \frac{(V_{\text{b OLD}})^2 / S_{\text{b OLD}}}{(V_{\text{b NEW}})^2 / S_{\text{b NEW}}}$$

$$\Rightarrow Z_{\text{p.u. NEW}} = Z_{\text{p.u. OLD}} \times \left[\frac{V_{\text{b OLD}}}{V_{\text{b NEW}}}\right]^2 \times \left[\frac{S_{\text{b NEW}}}{S_{\text{b OLD}}}\right] \qquad (*)$$

<u>Example 1</u> : A component rated for 13.2 kV, 30 MVA & with Z = 0.2 p.u. (on its own ratings) is placed in a power system portion where V_b = 13.8 kV & S_b = 50 MVA. What is the new p.u. Z of the component?

Here,

$S_{\text{b NEW}} = 50$ $\quad$ $S_{\text{b OLD}} = 30$
$V_{\text{b NEW}} = 13.8$ $\quad$ $V_{\text{b OLD}} = 13.2$
$Z_{\text{p.u. OLD}} = 0.2$ $\quad$ $Z_{\text{p.u. NEW}} = ?$

$$\therefore Z_{\text{p.u. NEW}} = 0.2 \times \left[\frac{13.2}{13.8}\right]^2 \times \left[\frac{50}{30}\right] = 0.306 \text{ p.u.}$$

- If a transformer impedance is given in p.u., then this value is based on the MVA rating of the transformer.

$\Rightarrow$ Use equation in (*) above to convert this given p.u. to the new p.u. on the system base kVA, Sb.

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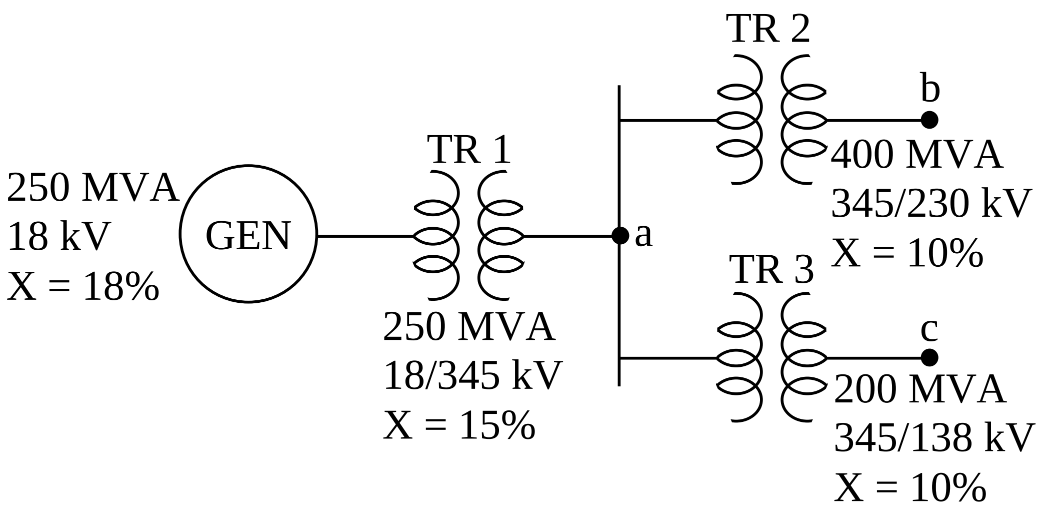
- If the Sb (base MVA) is not specified (and that is usually the case), the system component that has the largest MVA rating is chosen to give us the base MVA. Occasionally, a nice round number such as 100 MVA is selected as the base MVA!

Example 2 : For the power system shown below,

(i) Find appropriate voltage bases by selecting V_b = 18 kV at the generator terminals.

(ii) Find all impedances in p.u. Use S_b = 100 MVA.



Solution : Given $V_{b,\,GEN}$ = 18 kV (generator circuit)

Base voltage at point a = $V_{b,\,a}$

$$V_{b,\,GEN} \times \frac{345}{18} = 345 \text{ kV}$$

Base voltage at point b = $V_{b,\,b} = V_{b,\,a} \times \dfrac{230}{345} = 230 \text{ kV}$

Finally, base voltage at point c $= V_{b,\,c} = V_{b,\,a} \times \dfrac{138}{345} = 138 \text{ kV}$

Next, assuming a power base of S_b = 100 MVA, let us find all the impedances in p.u.

$$Z_{\text{GEN,NEW p.u.}} = Z_{\text{GEN,OLD p.u.}} \times \left[\frac{V_{\text{b OLD,GEN}}}{V_{\text{b,GEN}}}\right]^2 \times \frac{S_{\text{b NEW}}}{S_{\text{b OLD,GEN}}}$$

$$= 0.18 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{100}{250}\right) = 0.072 \text{ p.u.}$$

$$Z_{\text{TR1,NEW p.u.}} = Z_{\text{TR1,OLD p.u.}} \times \left[\frac{V_{\text{b OLD,TR1}}}{V_{\text{b,GEN}}}\right]^2 \times \left[\frac{S_{\text{b NEW}}}{S_{\text{b OLD,TR1}}}\right]$$

↑

Calculated on the primary side

$$= 0.15 \times \left[\frac{18}{18}\right]^2 \times \left[\frac{100}{250}\right]$$

$$= 0.06 \text{ p.u.}$$

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$$Z_{TR\,2,NEW\ p.u.} = Z_{TR2,OLD\ p.u.} \times \left[\frac{V_{b\ OLD,TR2}}{V_{b,a}}\right]^2 \times \left[\frac{S_{b\ NEW}}{S_{b\ OLD,TR2}}\right]$$

$\uparrow$

Calculated on the primary side

$$= 0.10 \times \left[\frac{345}{345}\right]^2 \times \left[\frac{100}{400}\right]$$

$$= 0.025 \text{ p.u.}$$

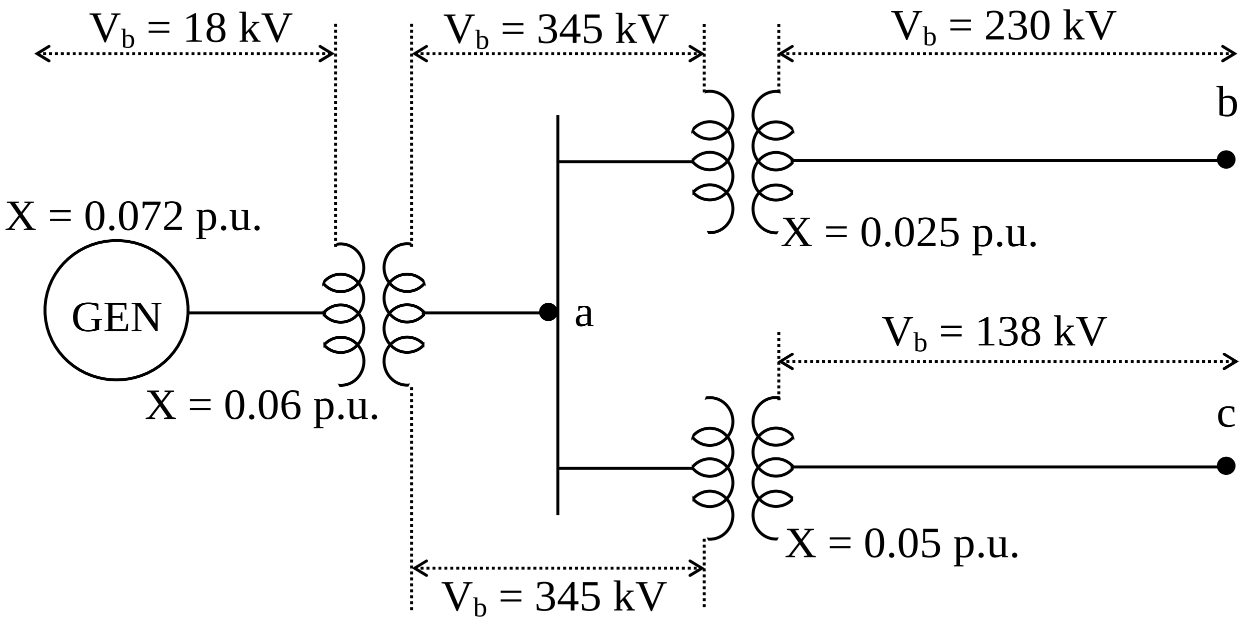
$$Z_{TR\,3,NEW\ p.u.} = Z_{TR3,OLD\ p.u.} \times \left[\frac{V_{b\ OLD,TR3}}{V_{b,a}}\right]^2 \times \left[\frac{100}{200}\right]$$

$\uparrow$

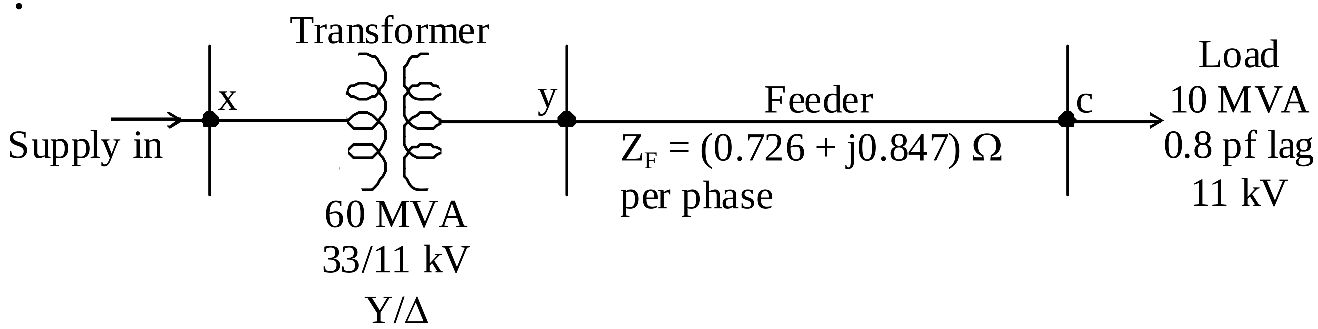
Calculated on the primary side

$$= 0.10 \times \left[\frac{345}{345}\right]^2 \times \left[\frac{1}{2}\right]$$

$$= 0.05 \text{ p.u.}$$



Example 3 :



$Z_T = (0.2723 + j0.726)\ \Omega$ per phase (Referred to H.V. side)

Calculate the input & output line voltages of the 3-ϕ transformer i.e. voltages at x and y.

Solution : Let base MVA = S_b = 60 MVA (constant throughout the system)`

Transformer H.V. side

Base kV, $V_{b\ x}$ = 33 kV (assumed)

$$Z_{b\ \text{primary of trans.}} = Z_{b\ x} = \frac{V_{b\ x}^{\ 2}}{S_b} = \frac{33^2}{60} = 18.15\ \Omega = Z_{b\ pri}$$

$$\therefore Z_{T\ p.u.} = \frac{Z_{T\ \text{actual pri}}}{Z_{b\ pri}} = \frac{0.2723 + j0.726}{18.15} = 0.015 + j0.04 = 0.0427\angle 69.44°\ \text{p.u.}$$

Transformer L.V. side

$$\text{Base kV} = V_{b\ y} = V_{b\ x}\frac{11}{33} = 11\ \text{kV}$$

$$\therefore Z_{b\ y} = \frac{(V_{b\ y})^2}{S_b} = \frac{11^2}{60} = 2.017\ \Omega$$

$$\therefore Z_{F\ p.u.} = \frac{Z_F(\Omega)}{Z_{b\ y}} = \frac{0.726 + j0.847}{2.017} = 0.36 + j0.42 = 0.5532\ \angle 49.4°\ \text{p.u.}$$

$\therefore$ Total impedance

$Z_{TOT\ p.u.} = Z_{T\ p.u.} + Z_{F\ p.u.} = 0.5935\angle 50.81°$ p.u.

S_{Load} = 10 MVA, 0.8 pf lag = $10\angle 36.87^o$ MVA

$$\therefore S_{\text{Load p.u.}} = \frac{S_{\text{Load}}}{S_b} = \frac{10\angle 36.87^\circ}{60} = 0.1667\angle 36.87^\circ \text{ p.u.}$$

$$V_{\text{Load}} = 11\text{ kV} \Rightarrow V_{\text{Load p.u.}} = \frac{V_{\text{Load}}}{V_{b\ c}} = \frac{11}{11} = 1.0\angle 0^{o} \text{ p.u.}$$

$\Rightarrow$ Load current

$$I^{*}_{\text{Load p.u.}} = \frac{S_{\text{Load p.u.}}}{V_{\text{Load p.u.}}} = \frac{0.1667\angle 36.87^\circ}{1\angle 0^\circ} = 0.1667\angle 36.87^\circ \text{ p.u.}$$

$$\Rightarrow I_{\text{Load p.u.}} = 0.1667\ \angle\text{-}36.87^{o} \text{ p.u.}$$

$\therefore$ Voltage at the L.V. side of transformer

$$V_{\text{y p.u.}} = (I_{\text{Load p.u.}})(Z_{\text{F p.u.}}) + V_{\text{Load p.u.}}$$

$$= (0.1667\angle\text{-}36.87^\circ)(0.5532\angle 49.4^\circ) + 1\angle 0^\circ = 1.09\angle 1.05^\circ \text{ p.u.}$$

Actual voltage $|V_y| = V_{\text{y p.u.}}$ x Base voltage at y = 1.09 x 11 kV $\simeq$ $\underline{\underline{12 \text{ kV}}}$

Finally, voltage at H.V. side of transformer

$$V_{\text{x p.u.}} = (Z_{\text{T p.u.}})(I_{\text{Load p.u.}}) + V_{\text{y p.u.}}$$

$$= (0.0427\angle 69.44^\circ)(0.1667\angle\text{-}36.87^\circ) + 1.09\angle 1.05^\circ = 1.0963\angle 1.24^\circ \text{ p.u.}$$

Actual voltage, $|V_x|$ = $V_{\text{x p.u.}}$ x Base voltage at x

= 1.0963 x 33 = $\underline{\underline{36.18 \text{ kV}}}$

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Single-line Diagram (SLD) & Impedance (Reactance) Diagram

- You have already learnt the circuit models for transformers. Very soon, you will learn circuit models for synchronous machines & transmission lines & loads.

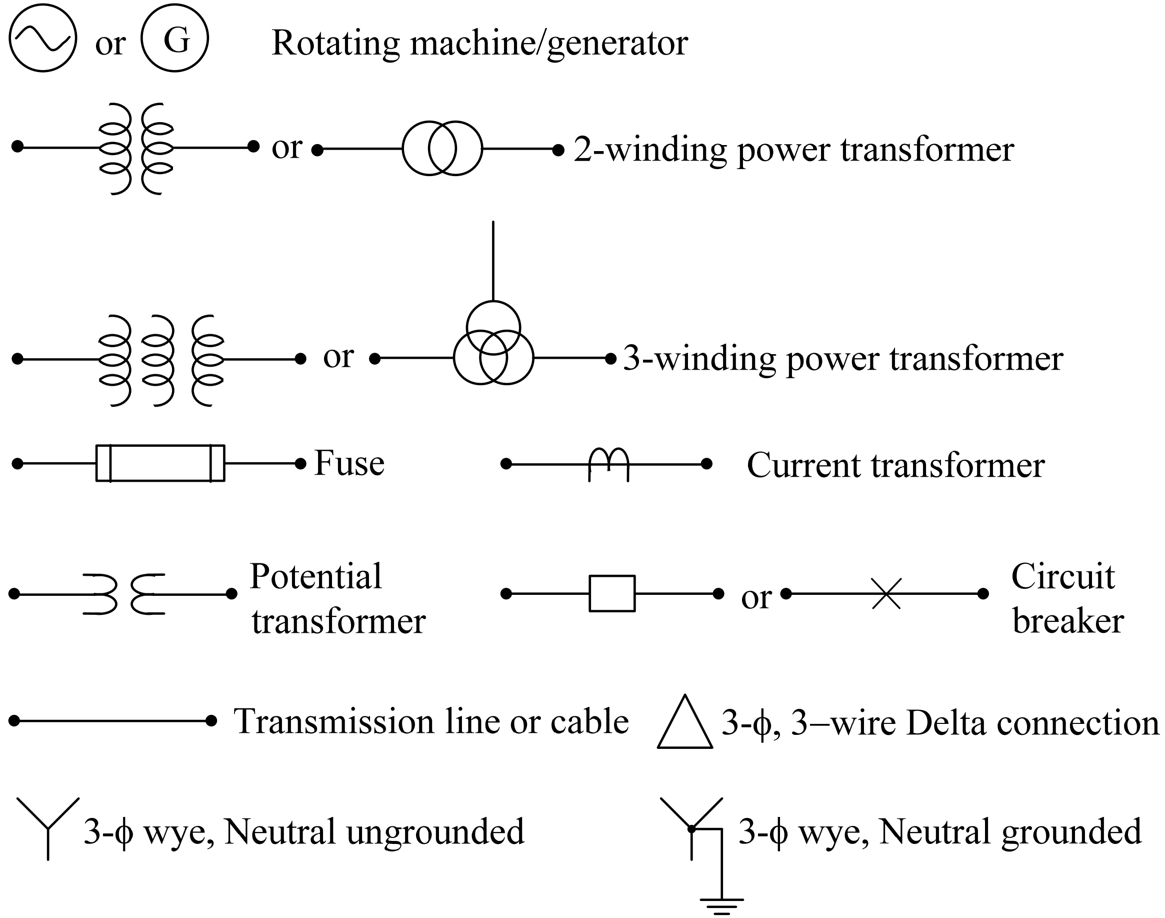
- Our present interest is in "how to portray the assemblage of these components to model a power system in its entirety".

- Since a balanced 3-ϕ system is always solved as a single-phase equivalent circuit composed of one of the three lines & the neutral return, it is seldom necessary to show more than 1-phase & neutral return when drawing a diagram of the system.

- Often the diagram is simplified further by omitting the completed CKT thru the neutral, and by indicating the components by standard symbols rather than by equiv. CKTs.

$\Rightarrow$ Such a simplified diagram is called one-line diagram or single-line diagram.

Standard Symbols Used in SLD



⇒ Most transformer neutrals in transmission systems are solidly grounded.

⇒ Generator neutrals are usually grounded thru fairly high R or L to limit the flow of current to ground during a fault (abnormal condition).

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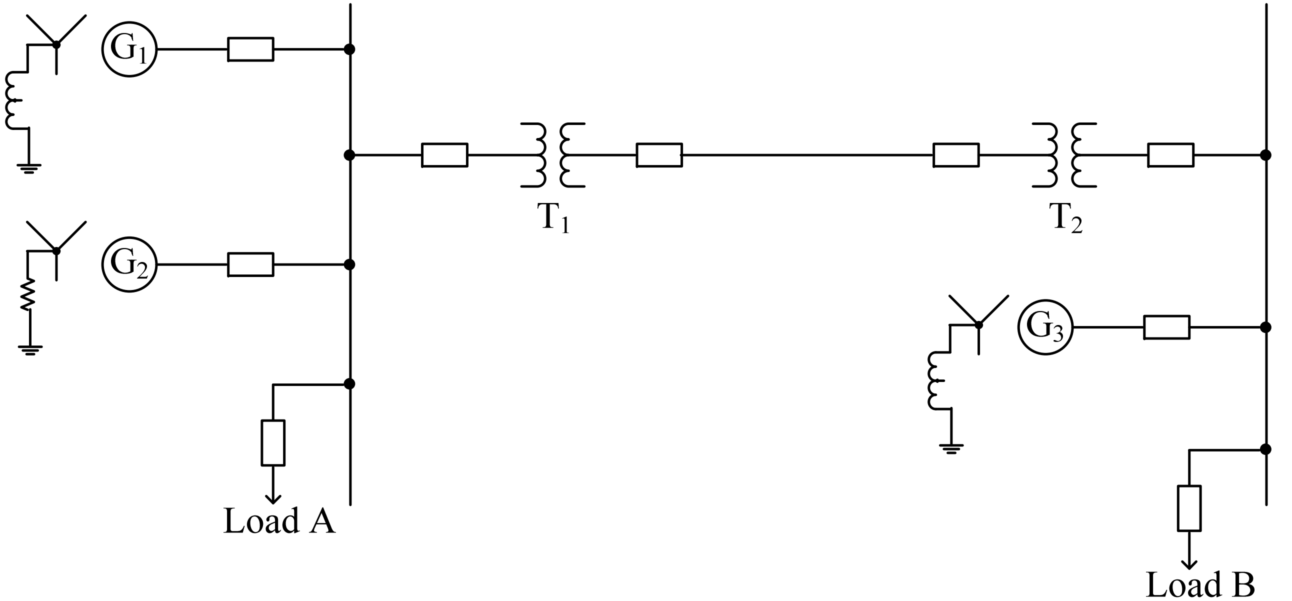
Example of SLD :



⇒ The amount of information presented on the SLD depends on the purpose for which the diagram is intended.

⇒ As stated earlier, p.u. system is used to solve for the unknowns.

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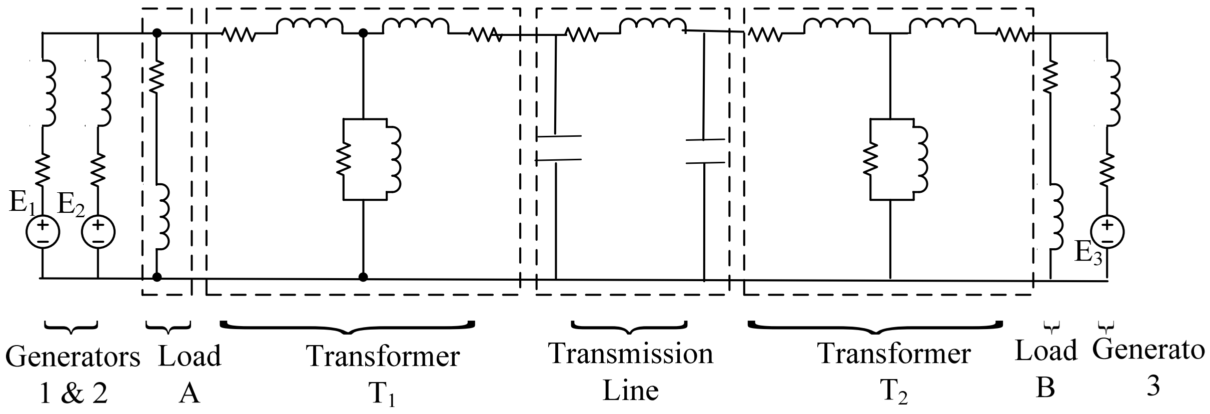
Impedance & Reactance Diagrams

→ When the individual components of a SLD are represented by their equivalent circuits, then the resultant drawing is called the per-phase impedance diagram.

→ The impedance diagram of the system shown on the previous page is as follows :



→ Since the shunt current of a transformer is usually insignificant compared with the full-load current, the shunt admittance is usually omitted from the equiv. CKT of transformer.

→ Resistance is often omitted since X >> R.

→ Transmission-line capacitances may also be omitted.

⇒ Resultant diagram is called <u>reactance diagram.</u>

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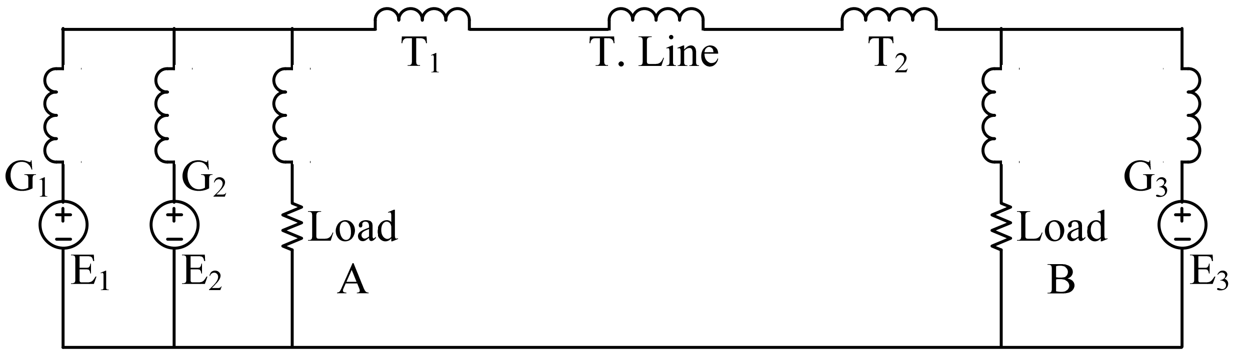
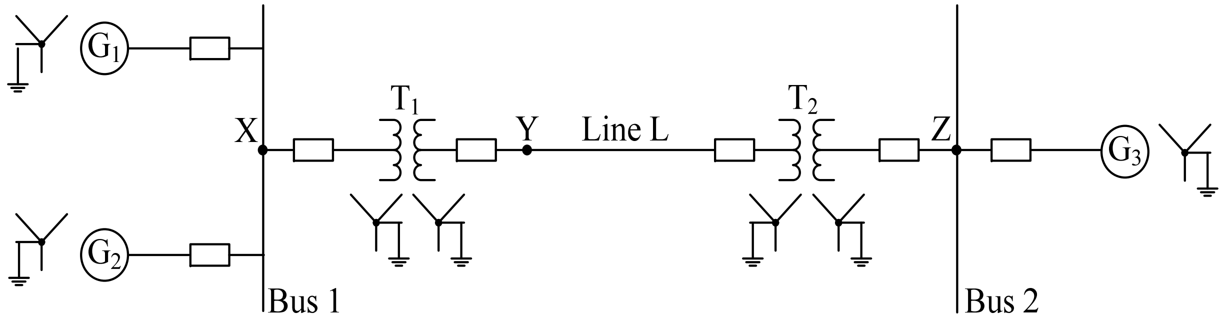


Note : Consistent base kVs must be specified in different sections of the system to analyze using the p.u. system.

Example 4 : Consider the power system shown :



G_1 : 20 MVA, 6.6 kV, X = 0.655 Ω

G_2 : 10 MVA, 6.6 kV, X = 1.31 Ω

G_3 : 30 MVA, 3.81 kV, X = 0.1452 Ω

T_1 : 10 MVA, 6.7/38 kV, X = 14.52 Ω per phase (on 38 kV side)

T_2 : 12 MVA, 38/3.8 kV, X = 14.52 Ω per phase (on 38 kV side)

X_L = 17.4 Ω per phase

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Using a 30 MVA base & a 6.6 kV base in G_1 circuit, obtain the p.u. impedance diagram.

Solution : Base MVA = 30 for all sections.

Base kV in G_1 & G_2 CKT = 6.6 kV = $V_{b\,X}$

$$Z_{b\,X} = Z_{Base\,X} = \frac{(V_{b\,X})^2}{S_b} = \frac{6.6^2}{30} = 1.452\ \Omega$$

$$\therefore X_{G_1\,p.u.} = \frac{X_{G_1}(\Omega)}{Z_{b\,X}} = \frac{0.655}{1.452} = 0.451\ \text{p.u.}$$

$$X_{G_2\,p.u.} = \frac{X_{G_2}(\Omega)}{Z_{b\,X}} = \frac{1.31}{1.452} = 0.9022\ \text{p.u.}$$

<u>Base kV in T_1 Secondary (38 kV side) + Line</u>

$$= V_{b\,Y} = V_{b\,X} \times \frac{38}{6.7} = 6.6 \times \frac{38}{6.7} = 37.4328\ \text{kV}$$

$$\therefore Z_{b\,Y} = \frac{(V_{b\,Y})^2}{S_b} = \frac{(37.4328)^2}{30} = 46.707\ \Omega$$

$$\therefore X_{T_1\,p.u.} = \frac{X_{T_1\,Y}(\Omega)}{Z_{b\,Y}} = \frac{14.52}{46.707} = 0.3109\ \text{p.u.}$$

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$$X_{L\text{ p.u.}} = \frac{X_L(\Omega)}{Z_{b\ Y}} = \frac{17.4}{46.707} = 0.3725 \text{ p.u.}$$

$$\text{Now, } X_{T_2\text{ p.u.}} = \frac{X_{T_2\ Y}(\Omega)}{Z_{b\ Y}} = \frac{14.52}{46.707} = 0.3109 \text{ p.u.}$$

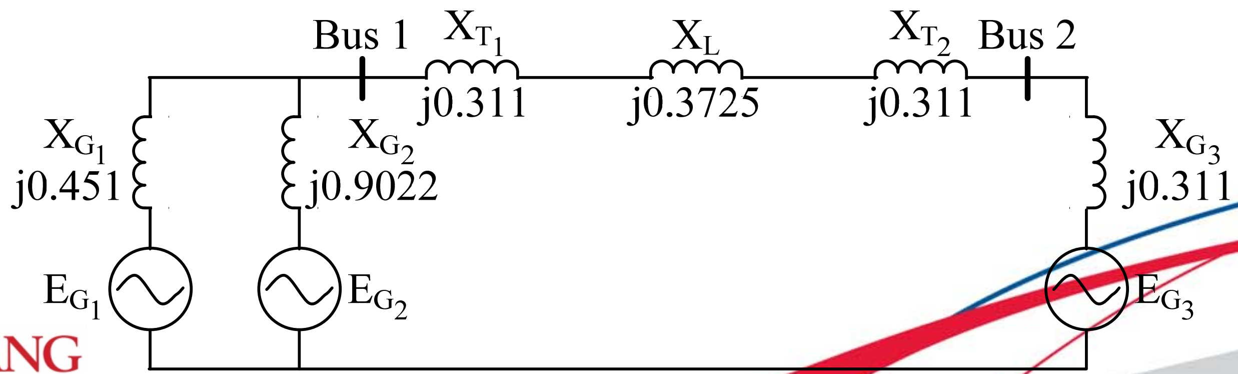
Base kV on Secondary (3.8 kV) side of T_2

$$= V_{b\ Z} = V_{b\ Y} \times \frac{3.8}{38} = 37.4328 \times \frac{3.8}{38} = 3.7433 \text{ kV}$$

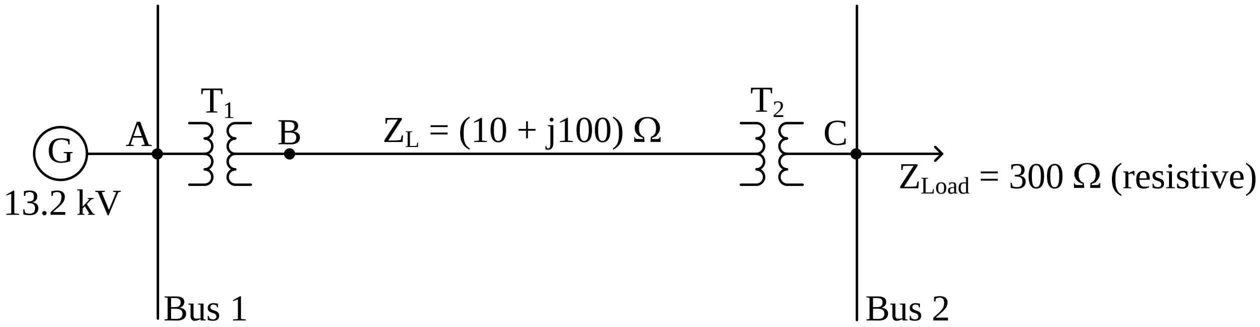
$$\Rightarrow Z_{b\ Z} = \frac{(V_{b\ Z})^2}{S_b} = \frac{3.7433^2}{30} = 0.4671\ \Omega$$

$$\therefore X_{G_3\text{ p.u.}} = \frac{X_{G_3}(\Omega)}{Z_{b\ Z}} = \frac{0.1452}{0.4671} = 0.3109 \text{ p.u.}$$

P.U. Impedance (Reactance) Diagram



Example 5 : (Last example on p.u./reactance diagram)



T_1 : 5 MVA, 13.2 Δ/132 Y kV, X = 10%

T_2 : 10 MVA, 138 Y/69 Δ kV, X = 8%

Assume that generator terminal voltage magnitude is 13.2 kV (L-L). (Ignore the generator reactance).

Find actual values of : Generator current, line current, load current, load voltage & MVA.

Solution : Assume S_b = Base MVA = 10 MVA & $V_{b\ A}$ = 13.2 kV

$$\therefore X_{T_1\ \text{p.u. NEW}} = X_{T_1\ \text{p.u. OLD}} \times \left[\frac{V_{b\ \text{OLD}\ T_1}}{V_{b\ A}}\right]^2 \times \left[\frac{S_{b\ \text{NEW}}}{S_{b\ T_1}}\right] = 0.10 \times \left[\frac{13.2}{13.2}\right]^2 \times \left[\frac{10}{5}\right] = 0.20 \text{ p.u.}$$

↑

(Calculated on the primary side)

$$\underline{\text{Base kV in Section B}} = V_{b\ B} = V_{b\ A} \times \frac{132}{13.2} = 132\ \text{kV}$$

$$\therefore Z_{b\ B} = \frac{132^2}{10} = 1742.4\ \Omega$$

$$\therefore Z_{L\ p.u.} = \frac{Z_L(\Omega)}{Z_{b\ B}} = \frac{10 + j100}{1742.4} = 0.00574 + j0.0574 = 0.0577\angle 84.29^o\ \text{p.u.}$$

Now,

$$X_{T_2\ p.u.\ NEW} = X_{T_2\ p.u.\ OLD} \times \left[\frac{V_{b\ OLD\ T_2}}{V_{b\ B}}\right]^2 \times \left[\frac{S_{b\ NEW}}{S_{b\ T_2}}\right]$$

↑

(Calculated on primary side)

$$= 0.08 \times \left[\frac{138}{132}\right]^2 \times \left[\frac{10}{10}\right] = 0.087438\ \text{p.u.}$$

Next, base kV in section C

$$= V_{b\ C} = V_{b\ B} \times \frac{69}{138} = 132 \times \frac{69}{138} = 66\ \text{kV}$$

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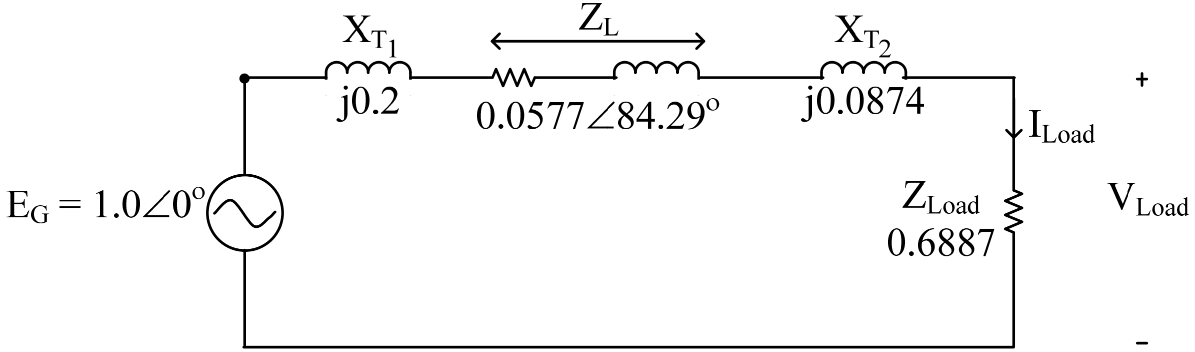
$$\therefore Z_{b\,C} = \frac{(V_{b\,C})^2}{S_b} = \frac{66^2}{10} = 435.6\ \Omega$$

$$\therefore Z_{Load\ p.u.} = \frac{Z_{load}(\Omega)}{Z_{b\,C}} = \frac{300}{435.6} = 0.6887\ \text{p.u.}$$

4. IMPEDANCE DIAGRAM (P.U.)



$$I_{Load\ p.u.} = \frac{1\angle 0^\circ}{j0.2 + (0.00574 + j0.0574) + j0.0874 + 0.6887}$$

$$= \frac{1\angle 0^\circ}{0.694445 + j0.3448} = \frac{1\angle 0^\circ}{0.7753\angle 26.404^o} = 1.2898\angle -26.4^o\ \text{p.u.}$$

<u>Note</u> : $I_{Load\ p.u.}$ represents different <u>actual</u> currents in sections A, B & C.

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Now,

$$V_{\text{Load p.u.}} = I_{\text{Load p.u.}} \times 0.6887\angle 0^{o} = 0.88826\angle -26.4^{\circ} \text{ p.u.}$$

$$S_{\text{Load p.u.}} = (V_{\text{Load p.u.}})(I_{\text{Load p.u.}})^{*} = 1.1457\angle 0^{o}$$

$\Rightarrow$ Load power (actual) = $S_{\text{Load p.u.}}$ x S_b = 11.458 MVA $\Leftarrow$

$|V_{\text{Load}}|$ Actual = $|V_{\text{Load p.u.}}|$ x base kV in section C = 0.88826 x 66 = <u>58.625 kV</u> $\Leftarrow$

<u>Last step</u> : Find base currents in all sections.

$$I_{b\ A} = \frac{S_b}{\sqrt{3}\ V_{b\ A}} = \frac{10 \times 10^3}{\sqrt{3} \times 13.2} = 437.386 \text{ Amps}$$

$\Rightarrow I_{Gen} = I_{b\ A} \times I_{\text{Load p.u.}} = 437.386 \times 1.2898 =$ <u>564.14 Amps</u>

$$I_{b\ B} = \frac{S_b}{\sqrt{3}\ V_{b\ B}} = \frac{10 \times 10^3}{\sqrt{3} \times 132} = 43.7387 \text{ Amps}$$

$\Rightarrow I_{Line} = I_{b\,B} \times I_{Load\ p.u.}$

$= 43.7387 \times 1.2898 = \underline{56.414 \text{ Amps}}$

$$I_{b\,C} = \frac{S_b}{\sqrt{3}\ V_{b\,C}} = \frac{10 \times 10^3}{\sqrt{3} \times 66} = 87.4773 \text{ Amps}$$

$\Rightarrow I_{Load} = I_{b\,C} \times I_{Load\ p.u.}$

$= 87.4773 \times 1.2898 = \underline{112.828 \text{ Amps}}$

Load Representations

→ Actual P & Q demands of load depend on system frequency & voltage.

→ Residential loads behave differently than (say) commercial or industrial loads. They are less affected by frequency and are usually resistive – kW demand sensitive to voltage of supply.

→ Industrial loads e.g. induction motor loads draw reactive power which is sensitive to voltage level (real power demand, kW, does not vary significantly).

→ Accurate estimate of load very difficult. Following two representations quite reasonable/satisfactory :

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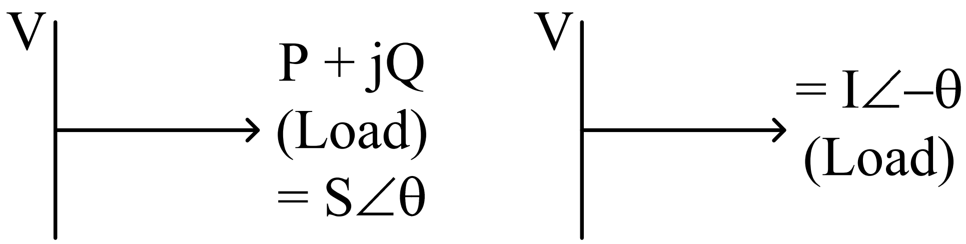
1) <u>Constant Power Model (e.g. Air-conditioning loads)</u>

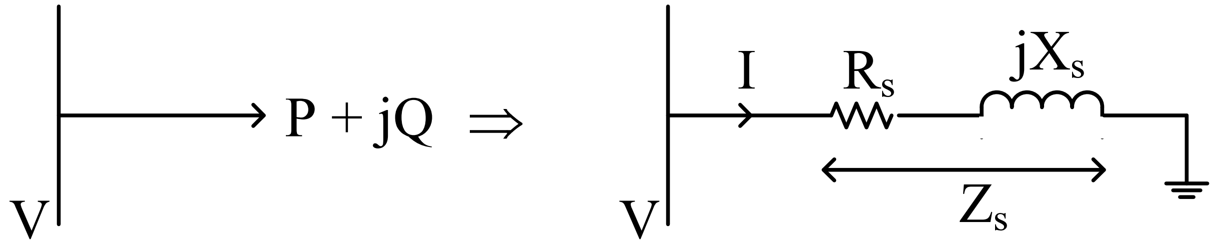


$$S = VI^* \Rightarrow I^* = \frac{S\angle\theta}{V\angle 0^\circ} \Rightarrow I = \frac{S}{V}\angle -\theta = \frac{\sqrt{P^2+Q^2}}{V}\angle -\theta$$

where θ is load power factor angle (assumed lagging)

⇒ Here load power (S) remains constant, although load voltage (V) & current (I) will change; e.g. air-conditioning loads.

2) <u>Constant-impedance Model (e.g. Water heaters & light bulbs)</u>



$$Z_s = \frac{V}{I} = \frac{V}{\left(\frac{S}{V}\angle -\theta\right)} = \left|\frac{V^2}{S}\right|\angle\theta$$

$$P = I^2 R_S \Rightarrow R_S = \frac{P}{I^2} = \frac{P}{\left(\dfrac{P^2 + Q^2}{V^2}\right)}$$

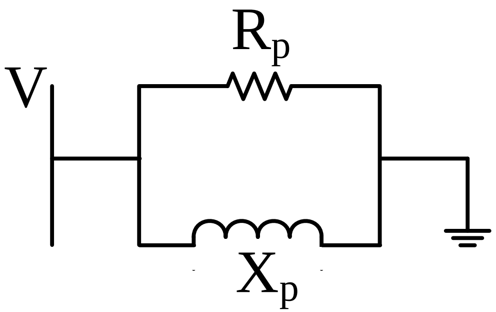
$$\Rightarrow R_S = \frac{P\,V^2}{P^2 + Q^2}\,\Omega$$

$$Q = I^2 X_S \Rightarrow X_S = \frac{Q}{I^2} = \frac{Q}{\left(\dfrac{P^2 + Q^2}{V^2}\right)} = \frac{Q\,V^2}{P^2 + Q^2}\,\Omega$$

<u>In p.u. system</u> : $Z_{S\ p.u.} = R_{S\ p.u.} + jX_{S\ p.u.}$

$\Rightarrow$ For a constant-impedance load, the load impedance is kept constant but the current & power drawn at various voltages will be different.

<u>Note</u> : Constant-impedance load may also be represented by R & X in parallel.



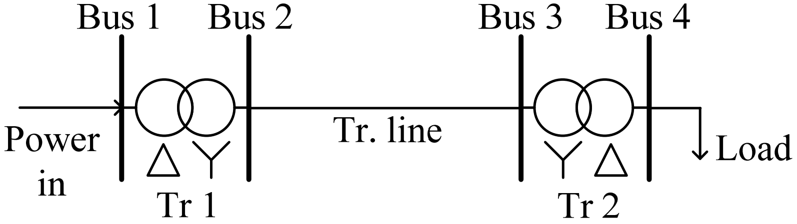
$$\Rightarrow Z_p = \frac{(R_p)(jX_p)}{R_p + jX_p}\,\Omega$$

$$\text{where } R_p = \frac{V^2}{P}\,\Omega$$

$$\& \ X_p = \frac{V^2}{Q}\,\Omega$$

REVIEW EXERCISES (5)

(5) Draw the impedance diagram for the 3-phase system shown in the following figure indicating the pu values of impedances.



Tr line : Z = 12.8 + j64 Ω

Tr 1 : 1 × 3-phase 120 MVA Transformer, Δ/Y, 34.5 kV/345 kV, Z = (1 + j8)%

Tr 2 : 3 × 1-phase 30 MVA Transformer, 200 kV/20 kV, Z = (1 + j7)%

Take the base values at bus 2 as 100 MVA and 345 kV.

(Hint : List the base values for all buses and then convert parameters to the common base)

Tr 1 : (0.0083 + j0.0667) pu, Tr. line : (0.0108 + j0.0538) pu, Tr 2 : (0.0112 + j0.0784) pu

(6) The system in problem (5) delivers a load of 60 MW at bus 4 at the rated voltage of 20 kV. Indicate the load terminal conditions in pu and calculate : (a) the voltage at bus 1, (b) phase angle of voltage at bus 1, for the following cases :

(i) 0.8 (lag), and (ii) 0.8 (lead)

(i) $V_L = 1.004\angle 0^o$ pu, $I_L = 0.7469\angle -36.87^o$ pu, 38.51 kV, 5.41°

(ii) $V_L = 1.004\angle 0^o$ pu, $I_L = 0.7469\angle 36.87^o$ pu, 32.51 kV, 8.08°

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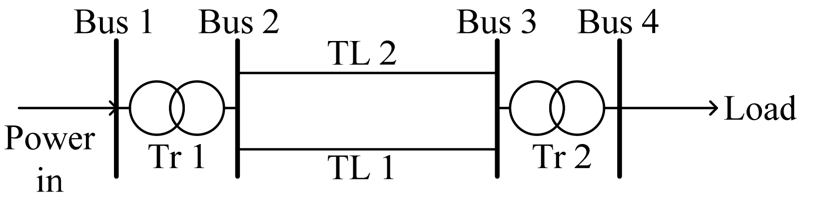
REVIEW EXERCISES (7 & 8)

(7)(a) Draw the impedance diagram for the system shown in the figure indicating the p.u. values of impedance, for the following system data.

Tr. Lines : TL1 and TL2 : R = 16 Ω, X = 120 Ω.
Load : (12 + j9) MVA at 13.2 kV.
Tr 1 : 20 MVA, 11/120 kV, Z = (2 + j10)%
Tr 2 : 20 MVA, 120/13.8 kV, Z = (1 + j8)%



Represent the load by a constant series impedance and take the base values for the transmission line as 60 MVA, and 120 kV.

(b) If the voltage at Bus 1 is maintained at rated value (11 kV), calculate the load terminal voltage.

(a) TL1 & TL2 : 0.067 + j0.50 p.u.; Load impdance : 2.928 + j2.196 p.u.;
Tr 1 : 0.06 + j0.3 p.u.; Tr 2 : 0.03 + j0.24 p.u.
(b) V_L = 11.83 kV

Note : Load voltage for (b) will NOT be 13.2 kV as in (a). The load is fixed but the load voltage can vary!

(8) Consider the power system shown below.

(i) Draw the per unit reactance diagram using a 30 MVA, 13.8 kV base in the generator circuit.
(ii) If motor loads M_1 and M_2 draw 16 MW and 8 MW respectively at unity power factors and 12.5 kV, find the internally generated generator voltage (i.e., voltage before the generator reactance itself).

(i) X_{G1} = j0.15; X_{T1} = j0.078; X_{Line} = j0.166; X_{T2} = j0.0915; X_{M1} = j0.2744; X_{M2} = j0.549
(ii) 1.0391 p.u. (14.339 kV)

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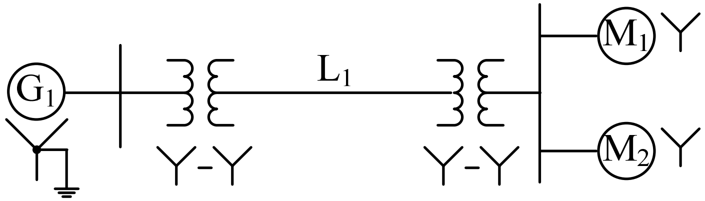

G_1 : 30 MVA, 13.8 kV, 3-ϕ generator, X_{G1} = 15%

M_1 : 20 MVA, 12.5 kV, 3-ϕ motor load, X_{M1} = 20% (synchronous motor)

M_2 : 10 MVA, 12.5 kV, 3-ϕ motor load, X_{M2} = 20% (synchronous motor)

T_1 : 35 MVA, 13.2/115 kV, Transformer, X_{T1} = 10%

T_2 : 30 MVA, 12.5/115 kV, Transformer, X_{T2} = 10%

L_1 : Line X_L = 80 Ω

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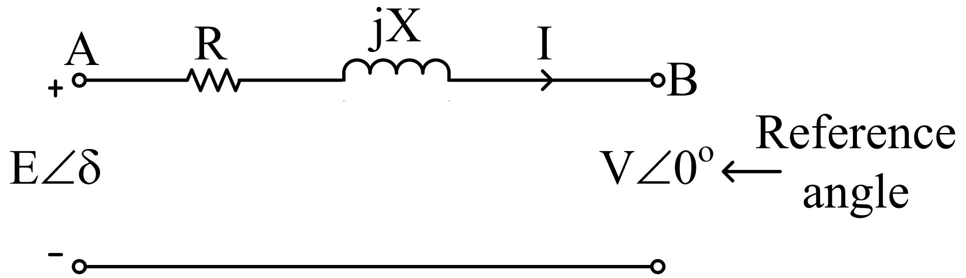
Basic Equations for Real/Reactive Powers (*Please see Appendix A for details*)

- Consider transfer of power from point “A” in a power system to point “B”, thru an impedance $Z = R + jX = |Z|\angle\alpha$



- Voltage at “A” = $E\angle\delta$
- Voltage at “B” = $V\angle 0°$

$$I = \frac{E\angle\delta - V\angle 0}{R + jX} = \frac{E}{Z}\angle(\delta - \alpha) - \frac{V}{Z}\angle -\alpha$$

- Complex power delivered at “B”

S = VI* (in per unit)

$$= (V\angle 0°)\left[\frac{E}{Z}\angle(\alpha - \delta) - \frac{V}{Z}\angle\alpha\right] = \frac{VE}{Z}\angle(\alpha - \delta) - \frac{V^2}{Z}\angle\alpha$$

$$\Rightarrow \text{Real power } P = \frac{VE}{Z}\cos(\alpha-\delta) - \frac{V^2}{Z}\cos\alpha$$

$$\text{Reactive power } Q = \frac{VE}{Z}\sin(\alpha-\delta) - \frac{V^2}{Z}\sin\alpha$$

Special case : $R \simeq 0 \Rightarrow Z = 0 + jX = X\angle 90^o$

$$\therefore S = \frac{VE}{X}\angle(90^\circ - \delta) - \frac{V^2}{X}\angle 90^\circ$$

$$\Rightarrow P = \frac{VE}{X}\sin\delta \;\&\; Q = \frac{VE}{X}\cos\delta - \frac{V^2}{X}$$

- Point “A” is usually

  (1) Internally-generated voltage E for a synchronous generator or
  (2) Sending-end voltage for a transmission line

- Point “B” is usually

  (1) Load voltage V for a generator connected to a load or
  (2) Receiving-end voltage for a transmission line

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- Impedance $Z = R + jX$ (or $Z \simeq jX$) is usually

  (1) Synchronous impedance of a synchronous generator or
  (2) Series impedance of a trans. line

- Note [for $R \simeq 0$]

  $\Rightarrow$ P flows from "A" to "B" for angle δ taking positive values; else "B" to "A"; if $\delta = 0$, $P = 0$ (no real power).
  P depends largely on relative angle difference of voltages at "A" & "B".

  $\Rightarrow$ For small δ : Q flows from "A" to "B" if $|E| > |V|$; else "B" to "A"; if $|E| \simeq |V|$, then $Q \simeq 0$ (no reactive power).
  Q depends largely on voltage magnitudes at "A" or "B".

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