

EE3015: Power Systems & Protection

Wang Peng

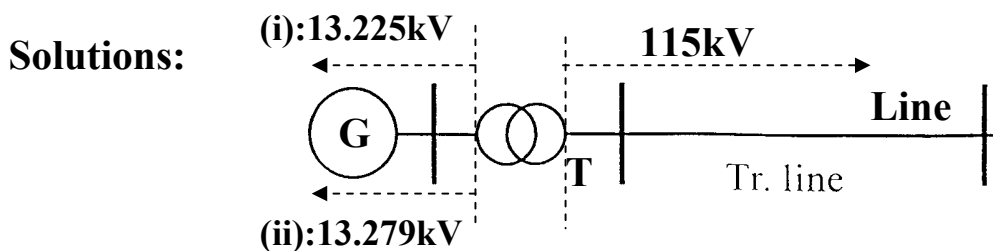
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Tutorial 1: Per Unit System I

1.1 A 3-phase generator rated 100MVA, 13.2kV, $X=20\%$ is connected through a Δ/Y transformer to a transmission line whose series reactance is 40 ohms per phase. Assume that the base values in the line circuit are 200MVA and 115kV.

- (a) Find the pu reactance of the transmission line.
 (b) Find the generator and transformer reactances in pu for the following cases:
 (i) The transformer is a 3-phase unit rated 100MVA, 13.8 Δ /120Y kV, $X = 8\%$
 (ii) The transformer is composed of three single-phase units, each rated 35MVA, 13.8/69 kV, $X=8\%$.



Base values:

$S_{BN}=200\text{MVA}$ (Only one new base power through the calculation)

Two base voltages: (because of one transformer)

Base voltage of the line: (Given) $V_{LBN}=115\text{kV}$ (line to line)

Base voltage of the generator: (Calculate) $V_{GBN} = V_{LBN} T_{ratio}$.

(a) pu reactance of the line: (Given: real impedance in Ω)

$$\text{Base impedance of line: } Z_{LBN} = \frac{V_{LBN}^2}{S_{BN}} = \frac{115^2}{200} = 66.125 \Omega$$

$$\text{pu reactance of line: } X_{Lpu} = \frac{X_L}{Z_{LBN}} = \frac{40}{66.125} = 0.6049\text{pu}$$

(b) pu reactance of the generator and the transformer:

(i) The transformer in a 3-phase unit (Type 1).

Givens (rated): 100MVA (3-Φ); 13.8Δ/120Y kV

$$T_{ratio} = \frac{V_{T\Delta}(\text{rated line-to-line})}{V_{TY}(\text{rated line-to-line})} = \frac{13.8kV}{120kV} \quad (\text{no calculation!!})$$

$$\text{Base voltage of generator: } V_{GBN} = V_{LBN} T_{ratio} = 115 \frac{13.8}{120} = 13.225kV$$

Givens old bases: $S_{TBO}=100\text{MVA (3-}\Phi\text{)}$; $V_{TBO}: 13.8\Delta/120Y \text{ kV}$

New bases: $S_{TBN}=200\text{MVA (3-}\Phi\text{)}$; $V_{TBN}: 13.225\Delta/115Y \text{ kV}$

New pu impedances:

$$\text{Transformer: } X_{TNpu} = X_{TOpu} \frac{V_{TBO}^2}{V_{TBN}^2} \frac{S_{BN}}{S_{TBO}} = 0.08 \frac{120^2}{115^2} \frac{200}{100} = 0.1742pu$$

$$\text{Generator: } X_{GNpu} = X_{GOpu} \frac{V_{GBO}^2}{V_{GBN}^2} \frac{S_{BN}}{S_{GBO}} = 0.2 \frac{13.2^2}{13.225^2} \frac{200}{100} = 0.3985pu$$

(ii) The transformer consists of three single phase units (Type 2).

Givens (rated): $S=35\text{MVA}$ and $V=13.8/69 \text{ kV}$ for each unit.

Three phase transformer (Ratio must be calculated!!!)

Y connection (side): $V_{TY} = \sqrt{3} \times 69 = 119.5kV$ (line-line)

Δ connection (side): $V_{T\Delta} = 13.8kV$ (line-line)

$$T_{ratio} = \frac{V_{T\Delta}}{V_{TY}} = \frac{13.8}{119.5}$$

$S_{BTO}=3 \times 35\text{MVA}=105 \text{ MVA}$ $V_{TBO}: 13.8\Delta/119.5Y \text{ kV}$

New bases: $S_{TBN}=200\text{MVA (3-}\Phi\text{)}$; $V_{TBN}: 13.275\Delta/115Y \text{ kV}$

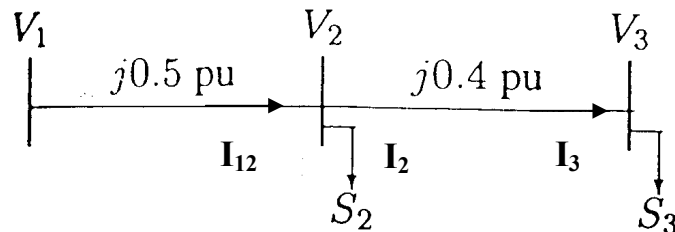
$$\text{Base voltage at generator side: } V_{GBN} = V_{LBN} T_{ratio} = 115 \frac{13.8}{119.5} = 13.279kV$$

New pu impedances:

$$X_{TNpu} = X_{TOpu} \frac{V_{TBO}^2}{V_{TBN}^2} \frac{S_{BN}}{S_{TBO}} = 0.08 \frac{119.5^2}{115^2} \frac{200}{105} = 0.1646pu$$

$$X_{GNpu} = X_{GOpu} \frac{V_{GBO}^2}{V_{GBN}^2} \frac{S_{BN}}{S_{GBO}} = 0.2 \frac{13.2^2}{13.279^2} \frac{200}{100} = 0.3953pu$$

1.2 The one-line diagram of a three-phase power system is shown below. The impedances marked are in per unit on a 100-MVA, 400-kV base. Given that loads $S_2 = (15.93-j33.4)$ MVA, and $S_3 = (77+j14)$ MVA, and that the voltage at bus 3 is maintained at 400kV, determine the magnitude of the voltage at bus 1.



Solutions: (Convert all parameters into pu values)

Base values: $S_B=100\text{MVA}$ $V_B=400\text{ kV}$

pu voltage at bus 3 (as reference): $V_{3\text{pu}}=400/400=1 \angle 0^\circ$

pu power of loads: (Load are given in the form of power $S=P+jQ$)

$$S_2=(15.93-j33.4)=37.0044 \angle -64.501^\circ \text{ (MVA)}$$

$$S_{2\text{pu}}=S_2/S_B=0.370044 \angle -64.501^\circ$$

$$S_3=(77+j14)=78.262 \angle 10.3^\circ \text{ (MVA)}$$

$$S_{3\text{pu}}=S_3/S_B=0.78262 \angle 10.3^\circ$$

pu voltages at bus 2 and bus 1:

$$I_{3\text{pu}}=(S_{3\text{pu}}/V_{3\text{pu}})^* = 0.78262 \angle -10.3^\circ$$

$$\text{KVL: } V_{2\text{pu}}=V_{3\text{pu}}+I_{3\text{pu}} \times j0.4=1.1 \angle 16.26^\circ$$

$$I_{2\text{pu}}=(S_{2\text{pu}}/V_{2\text{pu}})^* = 0.3364 \angle 80.78^\circ$$

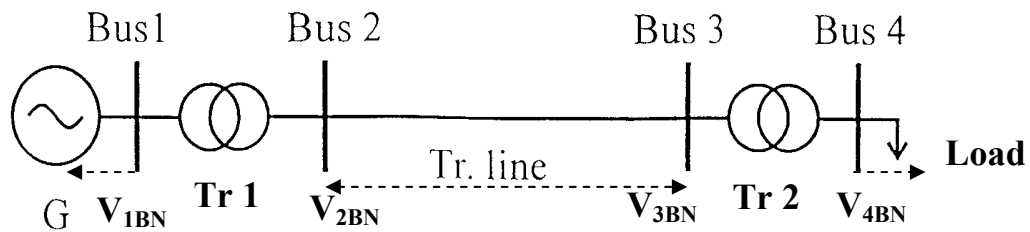
$$\text{KCL at bus 2: } I_{12\text{pu}}=I_{2\text{pu}}+I_{3\text{pu}}=0.846 \angle 13.12^\circ$$

$$\text{KVL: } V_{1\text{pu}}=V_{2\text{pu}}+I_{12\text{pu}} \times j0.5=1.2 \angle 36.87^\circ$$

$$\text{Absolute voltages: } |V_1|=|V_{1\text{pu}}| \times V_B = 480\text{kV}$$

$$|V_2|=440\text{kV}$$

1.3 Obtain the pu impedance diagram for the system without modeling the generator. Assume a base of 200MVA, 11kV in the generator circuit. The transmission line $X=50$ ohms and $R=2.5$ ohms; transformer ratings for Tr 1 are 50MVA, 11/132 kV, $X=12\%$, $R=2\%$, and for Tr 2 are 40MVA, 132/33 kV, $X = 10\%$, $R = 1\%$. The load connected at bus 4 is 40MW at 0.8 pf lag, at a voltage of 30kV.



Solutions:

Base Power (given): $S_{BN}=200$ MVA,

Base voltage at bus 1 (given): $V_{1BN}=11$ kV,

Base voltage at bus 2: $V_{2BN} = V_{1BN} T_{1ratio} = 11 \frac{132}{11} = 132$ kV $= V_{3BN}$

Base voltage at bus 4: $V_{4BN} = V_{3BN} T_{2ratio} = 132 \frac{33}{132} = 33$ kV

Line pu impedance (given real impedance):

$$\text{Base impedance: } Z_{BLN} = \frac{V_{BL}^2}{S_{BN}} = \frac{132^2}{200} = 87.12 \Omega$$

$$\text{pu impedance: } Z_{Lpu} = \frac{Z_L}{Z_{BL}} = \frac{2.5 + j50}{81.12} = 0.0287 + j0.5739 \text{ pu}$$

T1 pu impedance (given old pu impedance):

$$Z_{T1pu} = Z_{T1Opu} \frac{V_{2BO}^2}{V_{2BN}^2} \frac{S_{BN}}{S_{BT1O}} = (0.02 + j0.12) \frac{200}{50} = 0.08 + j0.48 \text{ pu}$$

T2 pu impedance (given old pu impedance):

$$Z_{T2pu} = Z_{T2Opu} \frac{S_{BN}}{S_{BT2O}} = (0.01 + j0.1) \frac{200}{40} = 0.05 + j0.50 \text{ pu}$$

****Load pu impedance (given real power, power factor and voltage):**

pu voltage: $V_{4pu} = 30/33 = 0.909 \angle 0^\circ$

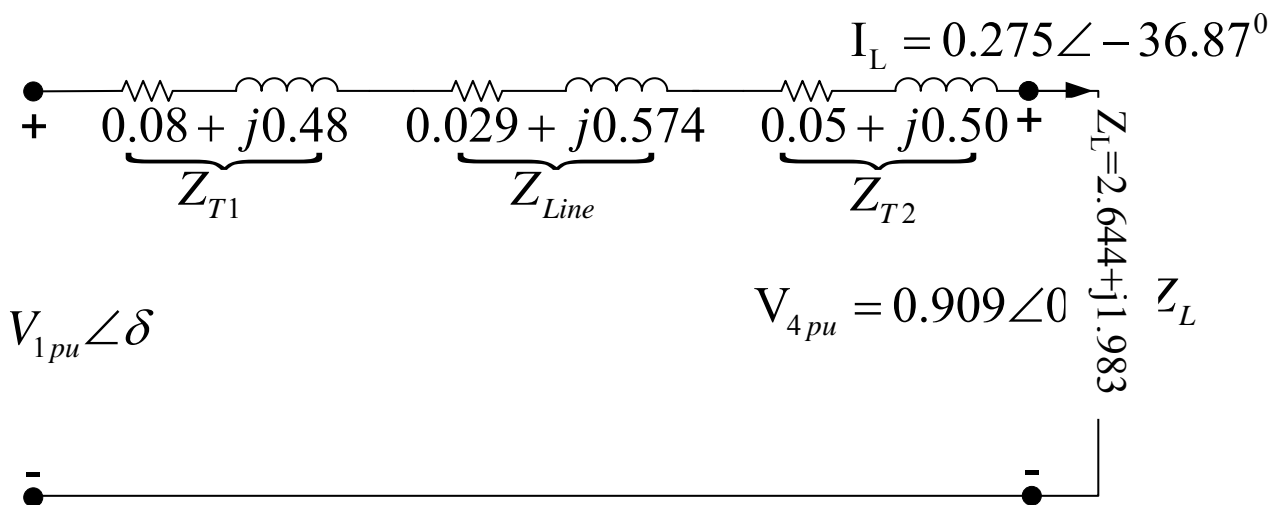
Load: $S_L = 40/0.8 \angle \cos^{-1} 0.8 = 50 \angle 36.87^\circ \text{ MVA}$ (*power triangle)

pu load: $S_{Lpu} = 50/200 \angle 36.87^\circ = 0.25 \angle 36.87^\circ \text{ pu}$

pu current: $I_{Lpu} = (S_{Lpu} / V_{4pu})^* = 0.275 \angle -36.87^\circ$ ($S_{pu} = V_{pu} I_{pu}^*$)

pu load impedance: $Z_{Lpu} = V_{4pu} / I_{Lpu} = 3.305 \angle 36.87^\circ = 2.644 + j1.983$

pu impedance diagram:



Tutorial 2: PU System II

2.1 The system below has the following component ratings:

G1: 12 kV, 50 MVA, $X = 8\%$;

G2: 12 kV, 40 MVA, $X = 8\%$

T1: 3 single-phase units, each rated 13/80kV, 15MVA, $X=9\%$

T2: 3 single-phase units, each rated 6.9/39.8kV, 25MVA, $X=8\%$

T3: 3 single-phase units, each rated 70/80kV, 25MVA, $X=13\%$

T4: 12/138 kV, 80 MVA, $X = 12\%$

L1: $X=30\Omega$ /phase;

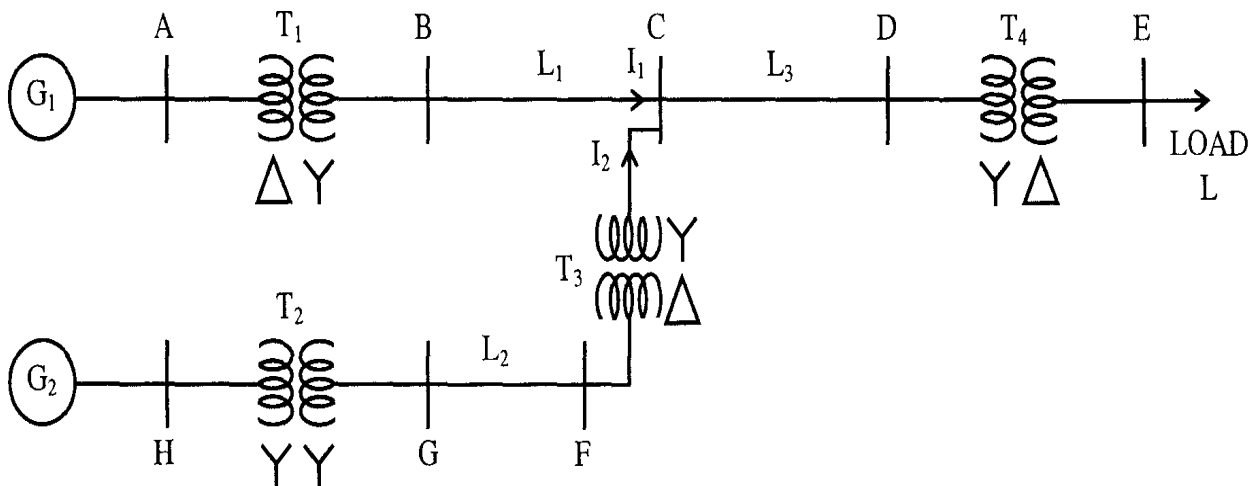
L2: $X=30\Omega$ /phase;

L3: $X=50\Omega$ /phase

Load: 60MW, 0.9pf lag at 11kV

a) Selecting bases of 100MVA and 140kV in L3 draw the per unit impedance diagram.

b) Assume that current phasors I_1 and I_2 are equal under the load. Calculate the magnitude of the voltage at the terminals of each generator.



Solutions:

a)

Given bases: $S_{BN}=100\text{MVA}$, $V_{BN}=140\text{ kV}$ (base voltage at L3)

Step 1: Determine other base voltages(*Transformer types!)

Base voltage at buses B, C and D: $V_{BN}^B = V_{BN}^C = V_{BN}^D = 140\text{kV}$

Base voltage at bus A (T1: type 2 transformer):

$$T_{1ratio} = \frac{13}{\sqrt{3} \times 80} = \frac{13}{138.564}$$

$$V_{BN}^A = V_{BN}^C \times T_{1ratio} = 140 \times \frac{13}{\sqrt{3} \times 80} = 13.1347\text{kV}$$

Base voltage at bus E:

$$V_{BN}^E = V_{BN}^D \times T_{4ratio} = 140 \times \frac{12}{138} = 12.1739\text{kV}$$

Base voltage at buses F and G (T2 and T3: type 2):

$$V_{BN}^F = V_{BN}^G = V_{BN}^C \times T_{3ratio} = 140 \times \frac{70}{\sqrt{3} \times 80} = 70.7254\text{kV}$$

Base voltage at bus H:

$$V_{BN}^H = V_{BN}^G \times T_{2ratio} = 70.7254 \times \frac{\sqrt{3} \times 6.9}{\sqrt{3} \times 39.8} = 12.2614\text{kV}$$

Step 2: determine pu impedance

Generator pu impedances: (given old pu impedance)

$$X_{G1Npu} = Z_{G1Opu} \frac{S_{BN}}{S_{G1BO}} \left(\frac{V_{G1BO}}{V_{BN}^A} \right)^2 = 0.08 \frac{100}{50} \left(\frac{12}{13.1347} \right)^2 = 0.1335\text{pu}$$

$$X_{G2Npu} = Z_{G2Opu} \frac{S_{BN}}{S_{G2BO}} \left(\frac{V_{G2BO}}{V_{BN}^H} \right)^2 = 0.08 \frac{100}{40} \left(\frac{12}{12.2614} \right)^2 = 0.1916\text{pu}$$

Transformer pu impedances: (given old pu impedance)

$$X_{T1Npu} = Z_{T1Opu} \frac{S_{BN}}{S_{T1BO}} \left(\frac{V_{T1BO}^B}{V_{BN}^B} \right)^2 = 0.09 \frac{100}{45} \left(\frac{\sqrt{3} \times 80}{140} \right)^2 = 0.1959 \text{pu}$$

$$X_{T2Npu} = Z_{T2Opu} \frac{S_{BN}}{S_{T2BO}} \left(\frac{V_{T2BO}^H}{V_{BN}^H} \right)^2 = 0.08 \frac{100}{75} \left(\frac{\sqrt{3} \times 6.9}{12.2614} \right)^2 = 0.1013 \text{pu}$$

$$X_{T3Npu} = Z_{T3Opu} \frac{S_{BN}}{S_{T3BO}} \left(\frac{V_{T3BO}^C}{V_{BN}^C} \right)^2 = 0.13 \frac{100}{75} \left(\frac{\sqrt{3} \times 80}{140} \right)^2 = 0.1698 \text{pu}$$

$$X_{T4Npu} = Z_{T4Opu} \frac{S_{BN}}{S_{T4BO}} \left(\frac{V_{T4BO}^C}{V_{BN}^C} \right)^2 = 0.12 \frac{100}{80} \left(\frac{138}{140} \right)^2 = 0.1457 \text{pu}$$

Line pu impedances: (given real impedance in Ω)

$$\text{Base impedance: } Z_{L1B} = Z_{L3B} = \frac{(V_{BN}^B)^2}{S_{BN}} = \frac{140^2}{100} = 196 \Omega$$

$$X_{L1pu} = \frac{X_{L1}}{Z_{L1B}} = \frac{30}{196} = 0.1531 \text{pu}$$

$$X_{L3pu} = \frac{X_{L3}}{Z_{L3B}} = \frac{50}{196} = 0.2551 \text{pu}$$

$$\text{Base impedance: } Z_{L2B} = \frac{(V_{BN}^F)^2}{S_{BN}} = \frac{70.7254^2}{100} = 50.021 \Omega$$

$$X_{L2pu} = \frac{X_{L2}}{Z_{L2B}} = \frac{30}{50.021} = 0.5998 \text{pu}$$

Load pu impedance: (Given P, power factor and voltage)

$$V_{Lpu} = 11/V_{BN}^E = 11/12.1739 = 0.9036pu$$

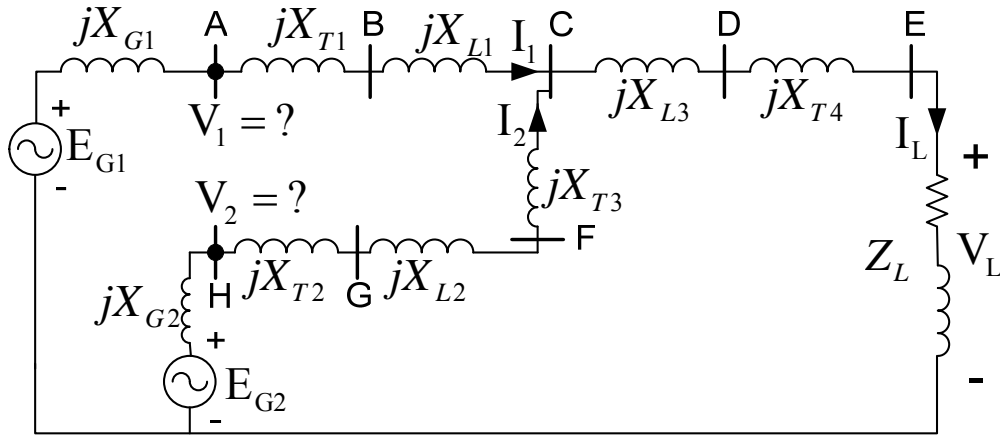
$$S_L = \frac{P}{pf} \angle \theta = \frac{60}{0.9} \angle \cos^{-1} 0.9 = 66.6667 \angle 25.8419^\circ \text{ MVA}$$

$$S_{Lpu} = 0.666667 \angle 25.8419^\circ pu$$

$$I_{Lpu} = \left(\frac{S_{Lpu}}{V_{Lpu}} \right)^* = 0.7378 \angle -25.8419^\circ pu$$

$$Z_{Lpu} = \frac{V_{Lpu}}{I_{Lpu}} = 1.2247 \angle 25.8419^\circ = 1.1022 + j0.5338pu$$

Step 3: pu impedance diagram



b): System voltage calculations:

$$V_{pu}^C = V_{Lpu} + I_{Lpu} (jX_{T4pu} + jX_{L3pu}) = 1.0662 \angle 14.45^\circ$$

$$I_{1pu} = I_{2pu} = 0.5 I_{Lpu} = 0.3689 \angle -25.8419^\circ$$

$$V_{1pu} = V_{pu}^A = V_{pu}^C + I_{1pu} \times j(X_{T1pu} + X_{L1pu}) = 1.1536 \angle 19.335^\circ pu$$

$$V_{2pu} = V_{pu}^H = V_{pu}^C + I_{2pu} \times j(X_{T2pu} + X_{T3pu} + X_{L2pu}) = 1.2973 \angle 25.34^\circ pu$$

$$|V_1| = |V_{1pu}| \times V_{BN}^A = 15.1527kV$$

$$|V_2| = |V_{2pu}| \times V_B^H = 15.9066kV$$

2.2 The system shown has the following component ratings:

Line: $X = 7.2 \Omega/\text{phase}$; $R=1.5 \Omega/\text{phase}$

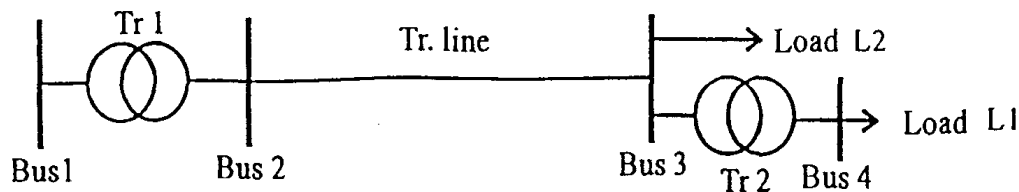
Load L1: Y-connected $R=32 \Omega/\text{phase}$

Load L2: 4 MW, 0.8pf lag, 22kV

T1: 10MVA, 230/22kV, $Z=(2 + j8)\%$

T2: 5MVA, 22/12 kV, $Z=(1.2 + j6)\%$

- Sketch the pu impedance diagram. Assume 10MVA and 22kV as base values in the transmission line circuit.**
- If the voltage at bus 3 is maintained at 22kV, calculate the voltages at Buses 2 and 1.**



Solutions:

a) Given base values: $S_{BN}=10\text{MVA}$, $V_{2BN}=22\text{kV}$ at line circuit

Step1: determine other base voltages

Base voltage at buses 2 and 3: $V_{2BN} = V_{3BN} = 22\text{kV}$

Base voltage at bus 1: $V_{1BN} = V_{2BN} \frac{230}{22} = 230\text{kV}$

Base voltage at bus 4: $V_{4BN} = V_{3BN} \frac{12}{22} = 12\text{kV}$

Step2: determine new pu impedances

Transformer impedances (given old pu impedance):

T1: $Z_{T1Npu} = 0.02 + j0.08\text{pu}$ (new bases are the same with the old)

T2: $Z_{T2Npu} = Z_{T2Opu} \frac{S_{BN}}{S_{T2BO}} = (0.012 + j0.06) \frac{10}{5} = 0.024 + j0.12\text{pu}$

Line impedances (real impedance is given):

$$Z_{liB} = \frac{V_{2BN}^2}{S_{BN}} = \frac{22^2}{10} = 48.4 \Omega$$

$$Z_{llpu} = \frac{Z_l}{Z_{liB}} = \frac{1.5 + j7.2}{48.4} = 0.03099 + j0.1487 \text{ pu}$$

Load impedances:

Load L1: (given real power, pf and voltage)

$$V_{3pu} = 22/V_{3BN} = 1 \text{ pu}$$

$$S_{L2} = \frac{4}{0.8} \angle \cos^{-1} 0.8 = 5 \angle 36.87^\circ \text{ MVA} = 0.5 \angle 36.87^\circ \text{ pu}$$

$$I_{L2pu} = \left(\frac{S_{L2pu}}{V_{3pu}} \right)^* = 0.5 \angle -36.87^\circ \text{ pu}$$

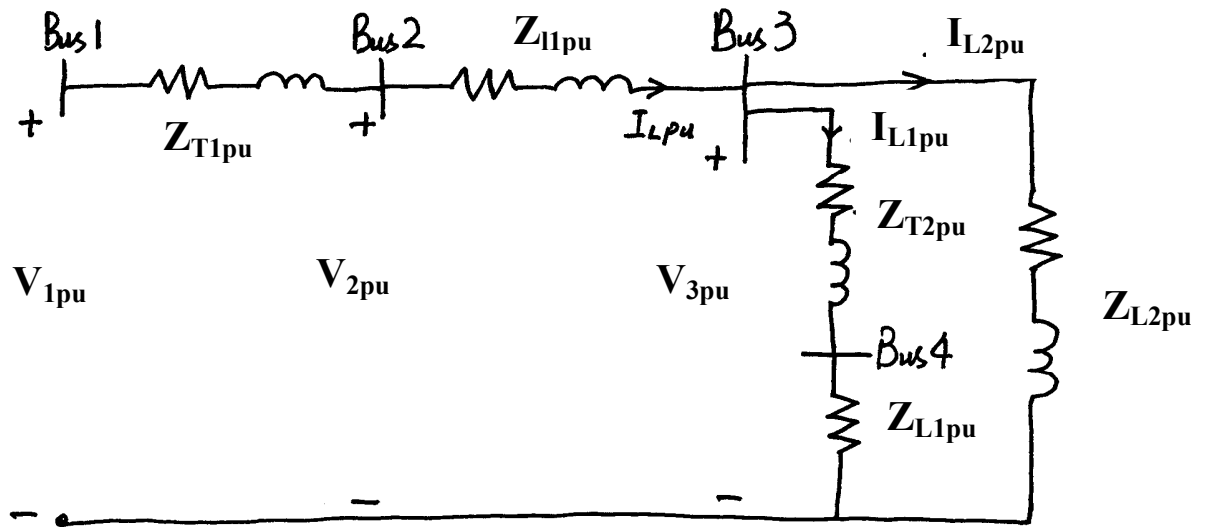
$$Z_{L2pu} = \frac{V_{3pu}}{I_{L2pu}} = 2 \angle 36.87^\circ \text{ pu} = 1.6 + j1.2 \text{ pu}$$

Load L2: (given real impedance)

$$Z_{L1B} = \frac{V_{4BN}^2}{S_{BN}} = \frac{12^2}{10} = 14.4 \Omega$$

$$Z_{L1pu} = \frac{Z_{L1}}{Z_{L1B}} = \frac{32}{14.4} = 2.2222 \text{ pu}$$

Step 3: pu impedance diagram



b) Voltage calculation

$$I_{L1pu} = \frac{V_{3pu}}{Z_{L1pu} + Z_{T1pu}} = 0.4446 \angle -3.058^\circ \text{ pu}$$

$$I_{L2pu} = \left(\frac{S_{L2pu}}{V_{3pu}} \right)^* = 0.5 \angle -36.87^\circ \text{ pu (from part a)}$$

$$I_{Lpu} = I_{L1pu} + I_{L2pu} = 0.904 \angle -20.985^\circ \text{ pu}$$

$$V_{2pu} = V_{3pu} + I_{Lpu} \times Z_{l1pu} = 1.0805 \angle 6.134^\circ \text{ pu}$$

$$V_{1pu} = V_{2pu} + I_{Lpu} \times Z_{T1pu} = 1.13095 \angle 8.98^\circ \text{ pu}$$

$$|V_2| = 23.77 \text{ kV}$$

$$|V_1| = 260.12 \text{ kV}$$

2.3 The component ratings for the system are as follows:

G: 90MVA, 22kV, $X=18\%$

T1: 50MVA, 220/22kV, $X=10\%$

T2: 40MVA, 11/220kV, $X=6\%$

T3: 40MVA, 110/22kV, $X=6.4\%$

T4: 40MVA, 11/110kV, $X=8\%$

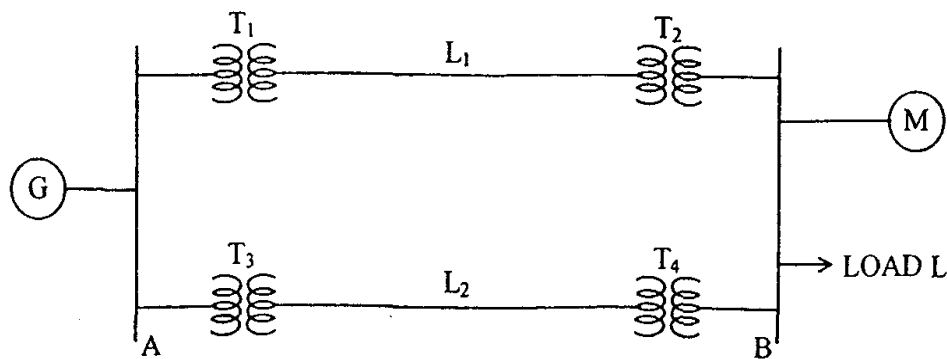
M: 66.5MVA, 10.45kV, $X=18.5\%$

Load L: 57MVA, 0.6pf lag, 10.45kV (R & X in series)

L1: $X=48.4$ ohms per phase

L2: $X=65.43$ ohms per phase

- a) Selecting a base of 100MVA and 22kV in the generator circuit, sketch the per unit impedance diagram of the system.
- b) Assuming that the motor M operates at full-load 0.8 pf leading at a terminal voltage of 10.45kV, find
 - i) the voltage at the generator bus
 - ii) the generator and motor internal emfs.



Solutions:

a) pu impedance diagram

Given Bases: $S_{BN}=100\text{MVA}$, $V_{GBN} = 22\text{kV}$ at generator side

Step1: determine other base voltages

$$\text{Base voltage at line } L_1: V_{L1BN} = V_{GBN} \frac{220}{22} = 220\text{kV}$$

$$\text{Base voltage at line } L_2: V_{L2BN} = V_{GBN} \frac{110}{22} = 110\text{kV}$$

$$\text{Base voltage at motor bus: } V_{MBN} = V_{L1BN} \frac{11}{220} = 11\text{kV}$$

(**Old and New voltage for all transformers are the same)

Step2: determine pu impedance

Transformer pu impedances (given old pu impedance):

$$\text{T1: } Z_{T1Npu} = Z_{T1Opu} \frac{S_{BN}}{S_{T1BO}} = 0.1 \frac{100}{50} = j0.2\text{pu}$$

$$\text{T2: } Z_{T2Npu} = Z_{T2Opu} \frac{S_{BN}}{S_{T2BO}} = 0.06 \frac{100}{40} = j0.15\text{pu}$$

$$\text{T3: } Z_{T3Npu} = Z_{T3Opu} \frac{S_{BN}}{S_{T3BO}} = 0.064 \frac{100}{40} = j0.16\text{pu}$$

$$\text{T4: } Z_{T4Npu} = Z_{T4Opu} \frac{S_{BN}}{S_{T4BO}} = 0.08 \frac{100}{40} = j0.2\text{pu}$$

Generator pu impedance (given old pu impedance):

$$Z_{GNpu} = Z_{GOpu} \frac{S_{BN}}{S_{GBO}} = 0.18 \frac{100}{90} = j0.2\text{pu}$$

Motor new pu impedance (given old pu impedance):

$$Z_{MNpu} = Z_{MOpu} \frac{V_{MBO}^2}{V_{MBN}^2} \frac{S_{BN}}{S_{MBO}} = 0.185 \frac{10.45^2}{11^2} \frac{100}{66.5} = j0.251pu$$

Line pu impedances (given real impedance):

$$L1: Z_{L1B} = \frac{V_{L1BN}^2}{S_{BN}} = \frac{220^2}{100} = 484 \Omega$$

$$Z_{L1Npu} = \frac{Z_{L1}}{Z_{L1B}} = j \frac{48.4}{484} = j0.1pu$$

$$L2: Z_{L2B} = \frac{V_{L2BN}^2}{S_{BN}} = \frac{110^2}{100} = 121 \Omega$$

$$Z_{L2Npu} = \frac{Z_{L2}}{Z_{L2B}} = j \frac{65.43}{121} = j0.541pu$$

Load pu impedance (given real power, pf and voltage):

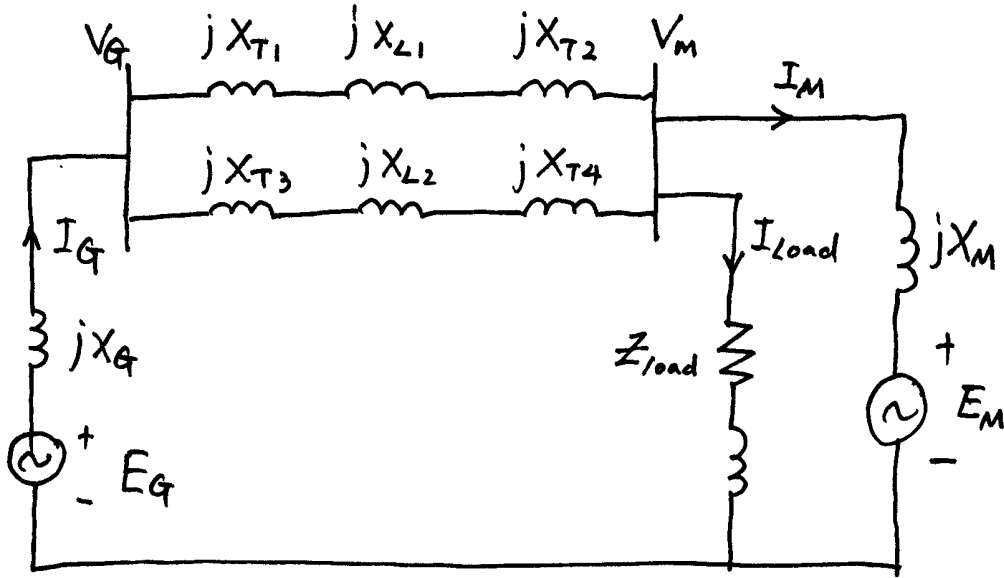
$$S_{loadpu} = \frac{57}{100} \angle \cos^{-1} 0.6 = 0.57 \angle 53.13^\circ pu$$

$$V_{Mpu} = V_{loadpu} = \frac{V_M}{V_{BMN}} = \frac{10.45}{11} = 0.95 \angle 0^\circ pu$$

$$I_{loadpu} = \left(\frac{S_{loadpu}}{V_{Mpu}} \right)^* = 0.6 \angle -53.13^\circ pu$$

$$Z_{loadpu} = \frac{V_{loadpu}}{I_{loadpu}} = 1.583 \angle 53.13^\circ = 0.9498 + j1.2664pu$$

Step 3: pu impedance diagram:



b) Voltage calculations

$$V_{Mpu} = \frac{V_M}{V_{BMN}} = \frac{10.45}{11} = 0.95 \angle 0^\circ \text{ pu}$$

$$S_{Mpu} = \frac{66.5}{100} \angle -\cos^{-1} 0.8 = 0.665 \angle -36.87^\circ \text{ pu}$$

$$I_{Mpu} = \left(\frac{S_{Mpu}}{V_{Mpu}} \right)^* = 0.7 \angle 36.87^\circ \text{ pu}$$

$$I_{Gpu} = I_{Mpu} + I_{loadpu} = 0.9219 \angle -3.73^\circ \text{ pu}$$

$$V_{Gpu} = V_{Mpu} + I_{Gpu} \times j(X_{T1pu} + X_{L1pu} + X_{T2pu}) // j(X_{T1pu} + X_{L1pu} + X_{T2pu}) \\ = 1.0066 \angle 15.917^\circ \text{ pu}$$

$$E_{Gpu} = V_{Gpu} + I_{Gpu} \times jX_{Gpu} = 1.0826 \angle 25.15^\circ \text{ pu}$$

$$E_{Mpu} = V_{Mpu} - I_{Mpu} \times jX_{Mpu} = 1.065 \angle -7.589^\circ \text{ pu}$$

Summary

Basic Values and equations in per unit system:

Base MVA $S_{B3-\phi} = S_B$ (Only select one for an entire system)

Base voltage $V_{BL-L} = V_B$ (different at two sides of a transformer)

Select V_B at one side of transformer.

Calculate others: $V_{B2} = T_{ratio} \times V_{B1}$ $T_{ratio} = V_{B2rated} / V_{B1rated}$ •

$$S_B = \sqrt{3} V_B \times I_B^*$$

Base Current (phase): $I_B^* = S_B / (\sqrt{3} V_B)$

Base impedance $Z_B = \frac{V_{B\phi}}{I_B} = \frac{V_B^2}{S_B}$ (per phase)

Per unit values:

$$S_{pu} = S/S_B; V_{pu} = V/V_B; I_{pu} = I/I_B;$$

$$S_{pu} = V_{pu} I_{pu}^*; Z_{pu} = V_{pu}/I_{pu}$$

Given real impedance (absolute value) $Z(\Omega)$:

$$\text{pu impedance: } Z_{pu} = Z(\Omega)/Z_B(\Omega);$$

Given pu impedance (Z_{Opu}) based on rated S_{BO} and rated voltage V_{BO} :

New pu impedance Z_{Npu} based on new S_{BN} and new V_{BN} :

$$Z_{Npu} = \frac{V_{BO}^2}{V_{BN}^2} \frac{S_{BN}}{S_{BO}} Z_{Opu}$$

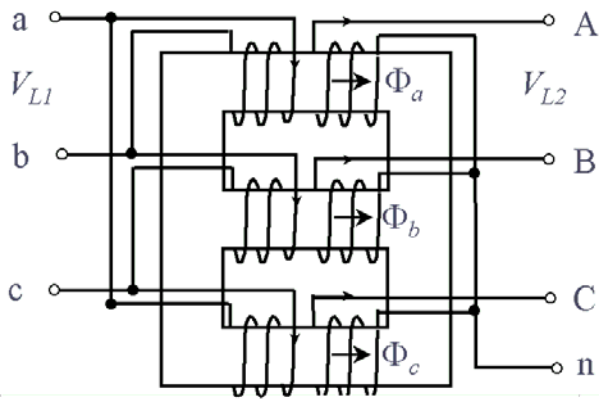
Given power and power factor and voltage?

Find S using power triangle and power factor

$$\text{Find } S_{pu} \Rightarrow \text{Find } V_{pu} \Rightarrow \text{Find } I_{pu} = (S_{pu}/V_{pu}) \Rightarrow \text{Find } Z_{pu} = V_{pu}/I_{pu}$$

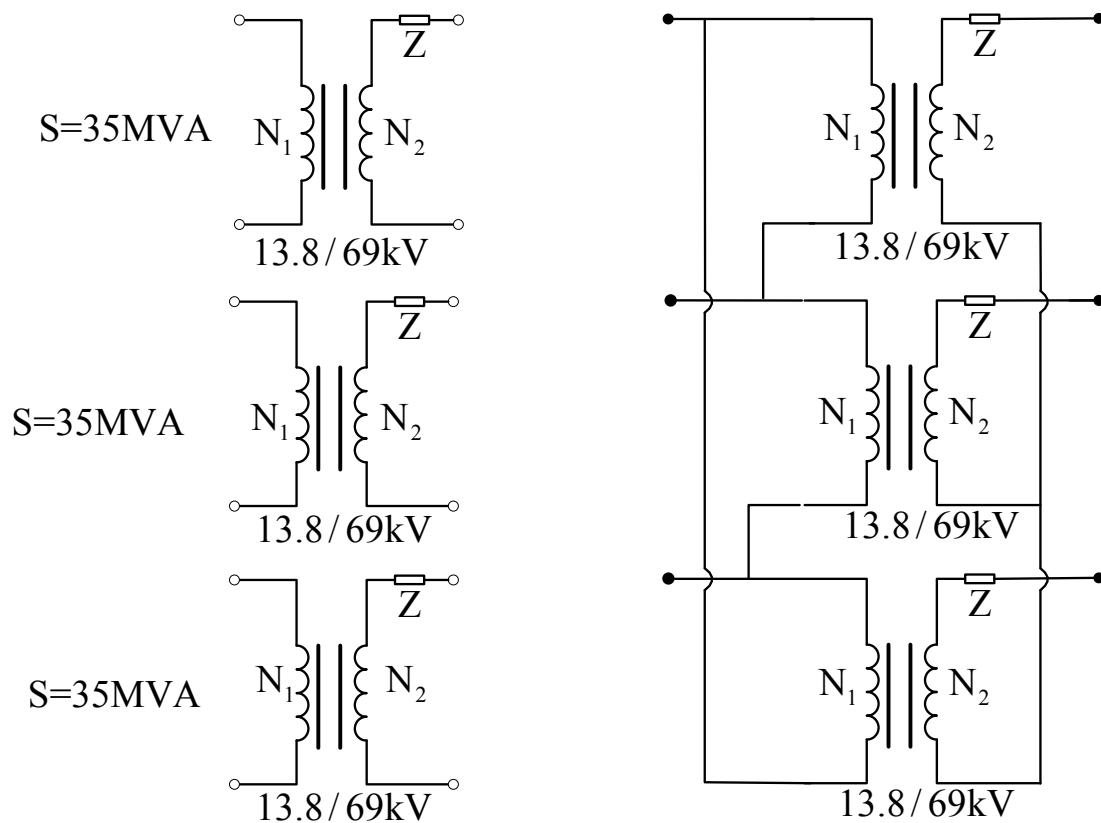
Two types of three-phase transformers:

a) A Δ/Y connected transformer (in a 3-phase unit Type 1)



Line to line voltage ratio are given!!!

b) A transformer (constructed by three single-phase units Type 2)



Three single-phase transformers

Δ/Y connection

Line to line voltage ratio = $13.8\text{kV} / (69\sqrt{3})\text{kV}$

$S = 3 \times 35 = 115\text{MVA}$