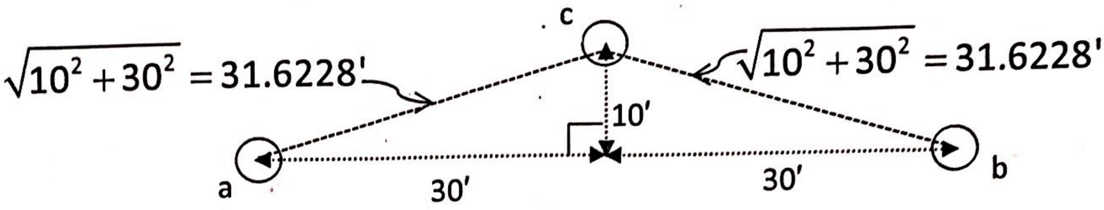
EE3015 Tutorial #5

5.1(a) Given f = 50 Hz, R = 0.1204 Ω/mile
1 mile = 5280 ft x 0.3048 m/ft = 1609.344 m = 1.609344 km

R = 0.1204/1.609344 Ω/km = 0.0748 Ω/km



GMR = 0.0403', r = diameter/2 = 1.196"/2 = 0.598"

$$GMD = \sqrt[3]{D_{ab}D_{bc}D_{ca}} = \sqrt[3]{60 \text{ x} 31.6228^2} = 39.1487' = 469.7841''$$

$$L_{ph} = 2 \text{ x } 10^{-7}\ln\frac{GMD}{GMR} = 2 \text{ x } 10^{-7}\ln\frac{39.1437'}{0.0403'}$$

$$= 1.37575 \text{ x } 10^{-6} \text{ H/m}$$

$$= 1.37575 \text{ x } 10^{-3} \text{ H/km}$$

$$X_L = \omega L_{ph} = (2\pi f)L_{ph} = 2 \text{ x } 3.14159 \text{ x } 50 \text{ x } 1.37575 \text{ x } 10^{-3}$$

$$= 0.432\ \Omega/\text{km}$$

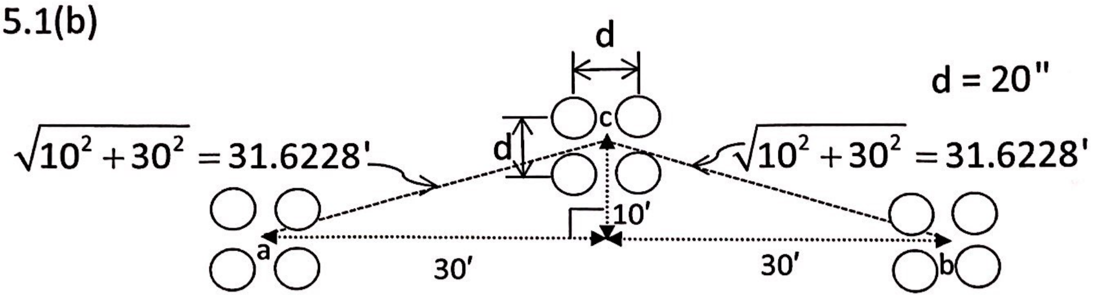
$$C_n = \frac{2\pi\varepsilon}{\ln\frac{GMD}{r}} = \frac{2\pi \text{ x } 8.85 \text{ x } 10^{-12}}{\ln\frac{469.78412''}{0.598''}} = 8.3412 \text{ x } 10^{-12}\text{ F/m to neutral}$$

$$= 8.3412 \text{ x } 10^{-9} \text{ F/km to neutral}$$

$$Y = j\omega C_n = j(2\pi \text{ x} 50) \text{ x } 8.3412 \text{ x } 10^{-9}$$

$$= j2.62 \text{ x } 10^{-6}\text{ S/km}$$

5.1(b)



Centre of each bundle conductor coincides with the vertex of the square of length d.

R = 0.0748/4 = 0.0187 Ω/km four conductors in parallel connection

GMR = 0.0403' = 0.4836", r = 1.196"/2 = 0.598" and $\mathrm{GMD_b} = \mathrm{GMD}$ $= 39.1487' = 469.7841"$; All are the same as before

$$\mathrm{GMR_b} = 1.09 \text{ x } \sqrt[4]{\mathrm{GMR} \text{ x } d^3} = 1.09 \text{ x } \sqrt[4]{0.4836 \text{ x } 20^3} = 8.596"$$

$$L_{b,ph} = 2 \text{ x } 10^{-7} \ln\frac{\mathrm{GMD_b}}{\mathrm{GMR_b}} = 2 \text{ x } 10^{-7} \ln\frac{469.7841"}{8.596"}$$
$$= 0.8002 \text{ x } 10^{-3} \text{ H/km}$$

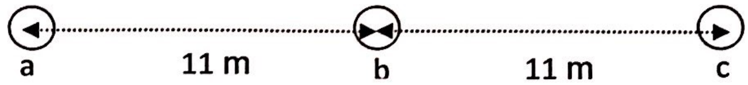
$$X_{b,L} = \omega L_{b,ph} = (2\pi f)L_{b,ph} = 2 \text{ x } 3.14159 \text{ x } 50 \text{ x } 0.8002 \text{ x } 10^{-3}$$
$$= 0.2514 \ \Omega/\text{km}$$

$$r_b = 1.09 \text{ x } \sqrt[4]{r \text{ x } d^3} = 1.09 \text{ x } \sqrt[4]{0.598 \text{ x } 20^3} = 9.0651"$$

$$C_{b,n} = \frac{2\pi\varepsilon}{\ln\frac{\mathrm{GMD_b}}{r_b}} = \frac{2\pi \text{ x } 8.85 \text{ x } 10^{-12}}{\ln\frac{469.78412"}{9.0651"}} = 14.085 \text{ x } 10^{-12} \text{ F/m to neutral}$$
$$= 1.4085 \text{ x } 10^{-8} \text{ F/km to neutral}$$

$$Y_b = j\omega C_{b,n} = j(2\pi \text{ x } 50) \text{ x } 1.4085 \text{ x } 10^{-8}$$
$$= j4.43 \text{ x } 10^{-6} \text{ S/km}$$

5.2 Given r = 3.625/2 = 1.8125 cm ad GMR = 1.439 cm

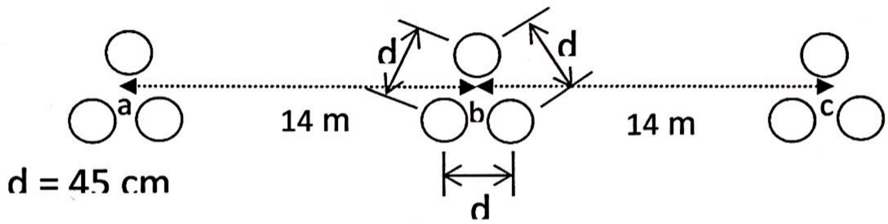


$$GMD = \sqrt[3]{D_{ab}D_{bc}D_{ca}} = \sqrt[3]{11 \text{ x } 11 \text{ x } 22} = 13.8591 \text{ m}$$

$$L_{ph} = 2 \text{ x } 10^{-7} \ln\frac{GMD}{GMR} = 2 \text{ x } 10^{-7} \ln\frac{13.8591 \text{ m}}{0.01439 \text{ m}} = 1.37403 \text{ x } 10^{-6} \text{ H/m}$$

$$C_n = \frac{2\pi\varepsilon}{\ln\frac{GMD}{r}} = \frac{2\pi \text{ x } 8.85 \text{ x } 10^{-12}}{\ln\frac{13.8591 \text{ m}}{0.018125 \text{ m}}} = 8.3751 \text{ x } 10^{-12} \text{ F/m to neutral}$$

<u>3-conductor bundle</u> Given r = 2.1793/2 = 1.08965 cm and GMR = 0.8839 cm



Centre of each bundle conductor coincides with the vertex of the equilateral triangle of length d.

$$GMD_b = \sqrt[3]{D_{ab}D_{bc}D_{ca}} = \sqrt[3]{14 \text{ x } 14 \text{ x } 28} = 17.6388 \text{ m}$$

$$GMR_b = \sqrt[3]{GMR \text{ x } d^2} = \sqrt[3]{0.8839 \text{ x } 45^2} = 12.1413 \text{ cm}$$

$$r_b = \sqrt[3]{r \text{ x } d^2} = \sqrt[3]{1.08965 \text{ x } 45^2} = 13.01879 \text{ cm}$$

$$L_{b,ph} = 2 \text{ x } 10^{-7} \ln\frac{GMD_b}{GMR_b} = 2 \text{ x } 10^{-7} \ln\frac{17.6388 \text{ m}}{0.121413 \text{ m}} = 9.9573 \text{ x } 10^{-7} \text{ H/m}$$

$$C_{b,n} = \frac{2\pi\varepsilon}{\ln\frac{GMD_b}{r_b}} = \frac{2\pi \text{ x } 8.85 \text{ x } 10^{-12}}{\ln\frac{17.6388 \text{ m}}{0.1301879 \text{ m}}} = 11.3277 \text{ x } 10^{-12} \text{ F/m to neutral}$$

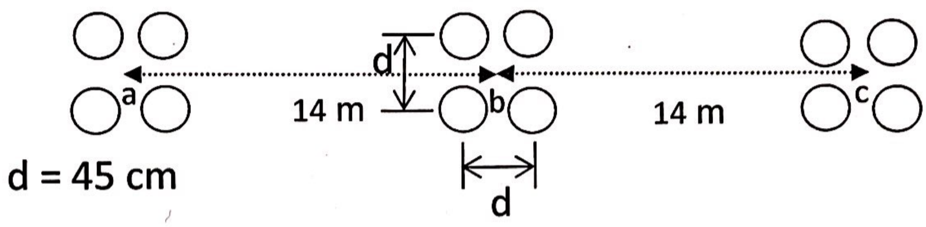
(i) Percentage change in L $= \dfrac{L_{b,ph} - L_{ph}}{L_{ph}}$ x 100% $= \dfrac{9.9573 - 13.7403}{13.7403}$

x 100% = –27.53% or 27.53% decrease.

(ii) Percentage change in C $= \dfrac{C_{b,n} - C_n}{C_n}$ x 100% $= \dfrac{11.3277 - 8.3751}{8.3751}$

x 100% = 35.25% or 35.25% increase.

5.3 Given f = 60 Hz, r = 3.625/2 = 1.8125 cm and GMR = 1.439 cm, length of line = 400 km = 4×10^5 m



Centre of each bundle conductor coincides with the vertex of the square of length d.

$$GMR_b = 1.09 \times \sqrt[4]{GMR \times d^3} = 1.09 \times \sqrt[4]{1.439 \times 45^3} = 20.742 \text{ cm}$$

$$L_{b,ph} = 2 \times 10^{-7} \ln \frac{GMD_b}{GMR_b} = 2 \times 10^{-7} \ln \frac{17.6389 \text{ m}}{0.20742 \text{ m}} = 8.8862 \times 10^{-7} \text{ H/m}$$

For the entire line,

$$L_{b,ph} = 8.8862 \times 10^{-7} \text{H/m} \times 4 \times 10^5 \text{ m} = 35.5449 \times 10^{-2} \text{ H}$$

$$r_b = 1.09 \times \sqrt[4]{r \times d^3} = 1.09 \times \sqrt[4]{1.8125 \times 45^3} = 21.9738 \text{ cm}$$

$$C_{b,n} = \frac{2\pi\varepsilon}{\ln \dfrac{GMD_b}{r_b}} = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln \dfrac{17.6389 \text{ m}}{0.219738 \text{ m}}} = 12.67977 \times 10^{-12} \text{F/m}$$

For the entire line,

$C_{b,n} = 12.67977 \times 10^{-7}$ F/m x 4 x 10^5 m $= 50.71908 \times 10^{-7}$ F

$Z = R + j\omega L_{b,ph} = 0 + j2\pi \times 60 \times 35.5449 \times 10^{-2} = j134.0011\ \Omega$

R = 0 due to lossless line

$Y = j\omega C_{b,n} = j(2\pi \times 60) \times 50.71908 \times 10^{-7} = j19120.644 \times 10^{-7}$

V_b = 765 kV, S_b = 2,000 MVA $\rightarrow$ $Z_b = V_b^2/S_b = 292.6125\ \Omega$ and $I_b = S_b/(\sqrt{3}\, V_b) = 2{,}000/(\sqrt{3} \times 765) = 1.509412$ kA

In pu, $Z = j134.0011\ \Omega/Z_b = j0.45795$ pu

In pu, $Y = Y / Y_b = Y \times Z_b = j19120.644 \times 10^{-7} \times 292.6125 = j0.5505$ pu

Using ABCD parameters,

A = D = (ZY)/2 + 1 = (j0.45795 x j0.5595)/2 + 1 = 0.8719 pu

B = Z = = j0.45795 pu

C = Y[(ZY)/4 + 1] = j0.5595[(j0.45795 x j0.5595)/4 + 1] = j0.52366 pu

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.8719 & j0.45795 \\ j0.52366 & 0.8719 \end{bmatrix} \text{pu}$$

(a) $V_S = 765/V_b = 1\angle 0^\circ$ pu, $S_S = (1{,}920 + j600)/S_b = 2011.5666\angle 17.354^\circ/2{,}000 = 1.00578\angle 17.354^\circ$ pu, $I_S = 1.00578\angle -17.354^\circ$ pu

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \frac{1}{AD - BC} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

But $AD - BC = 1$ since the power network is symmetric

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$= \begin{bmatrix} 0.8719 & -j0.45795 \\ -j0.52366 & 0.8719 \end{bmatrix} \begin{bmatrix} 1\angle 0^\circ \\ 1.00578\angle -17.354^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 0.856\angle -30.9^\circ \\ 1.1477\angle -43.172^\circ \end{bmatrix} \text{pu}$$

$S_R = V_R I_R^* = 0.856 \times 1.1477\angle(-30.9+43.172)^\circ = 0.98243\angle 12.272^\circ = 0.96 + j0.2088$ pu.

$V_R = |V_R| \times V_b = 0.856 \times 765 = 654.86$ kV, $I_R = |I_R| \times I_b = 1.1477 \times 1.509412 = 1.73231$ kA, $S_R = |S_R| \times S_b = 0.98243 \times 2{,}000 = 1964.86$ MVA, $P_R = 0.96 \times S_b = 0.96 \times 2{,}000 = 1920$ MW, $Q_R = 0.2088 \times S_b = 0.2088 \times 2{,}000 = 417.6$ Mvar lag because S_R has a positive angle.

(b) $I_R = 0 \rightarrow V_S = AV_R + BI_R = AV_R \rightarrow |V_R| = |V_S|/|A| = 1/0.8719 = 1.14692$ pu $\rightarrow$ Actual $|V_R| = 1.14692 \times 765 = 877.39$ kV