NANYANG TECHNOLOGICAL UNIVERSITY
School of Electrical & Electronic Engineering

EE/IM4152 Digital Communications

Tutorial No. 2 (Sem 1, AY2016-2017)

1. A signal $g(t)$ band-limited to B Hz is sampled using a triangular pulse train $x_p(t) = \sum_{n=-\infty}^{\infty} \Delta\left(\frac{t - nT_s}{\tau}\right)$, where $\Delta(t/\tau)$ is a triangular pulse spreading over 2τ seconds (from $-\tau$ to τ). Note that $\tau = 1/(16B)$ and the sampling period $T_s = 1/(2B)$.

   (a) For the triangular pulse train, the pulse at the centre is $x(t) = \Delta(t/\tau)$. Find its Fourier transform $X(f)$.

   (b) Determine the Fourier coefficients $\{C_n\}$ of the periodic sampling function $x_p(t)$.

   (c) Show that the sampled-data signal $\bar{g}(t)$ is given by

   $$\bar{g}(t) = \sum_{n=-\infty}^{\infty} \frac{1}{8} \mathrm{sinc}^2\left(\frac{n}{8}\right) g(t) e^{j2\pi n f_s t}$$

   where $f_s = 2B$. Can the signal $g(t)$ be recovered by passing $\bar{g}(t)$ through an ideal low-pass filter (LPF) of bandwidth B Hz and gain 8?

2. The signal $s(t) = \cos 2\pi t$ is sampled every 0.4 sec and sent over the channel using natural-sampled PAM with pulse width of 0.1 sec. The transmission channel can be modelled as an ideal low-pass filter (LPF) with cut-off frequency at 6.25 Hz. Determine the received waveform. How can we perfectly recover the original signal $s(t)$ from the received waveform?

3. A signal $g(t) = x(t) \otimes x(t)$ is sampled using uniformly spaced impulses at 40 samples/s, where $x(t) = \mathrm{sinc}(50t)$ and the symbol $\otimes$ denotes the convolution operation. The sampled-data function $\bar{g}(t)$ is fed to an ideal low-pass filter (LPF) with cutoff frequency 25 Hz.

   (a) Obtain the frequency-domain and time-domain representations of $g(t)$. Determine the bandwidth of $g(t)$ and its Nyquist sampling rate.

   (b) Sketch the spectrum of the sampled-data function $\bar{g}(t)$.

   (c) Sketch the spectrum of the output signal $y(t)$ of the LPF. Determine the frequency-domain and time-domain representations of $y(t)$.