NANYANG TECHNOLOGICAL UNIVERSITY
School of Electrical & Electronic Engineering

EE/IM4152 Digital Communications

Tutorial No. 9 (Sem 1, AY2016-2017)

1. Show that the following set of waveforms can be used as the basis signals of a 3-dimensional signal space:

$$\begin{cases} \varphi_1(t) = \dfrac{1}{\sqrt{T_0}}, & 0 \le t \le T_0 \\ \varphi_2(t) = \sqrt{\dfrac{2}{T_0}} \sin\left(\dfrac{2\pi}{T_0} t\right), & 0 \le t \le T_0 \\ \varphi_3(t) = \sqrt{\dfrac{2}{T_0}} \cos\left(\dfrac{2\pi}{T_0} t\right), & 0 \le t \le T_0. \end{cases}$$

With respect to above basis functions, find the energy of

$$x(t) = \frac{2}{\sqrt{T_0}} + 3 \times \sqrt{\frac{2}{T_0}} \sin\left(\frac{2\pi}{T_0} t\right) + 5 \times \sqrt{\frac{2}{T_0}} \cos\left(\frac{2\pi}{T_0} t\right).$$

2. Using the Schwarz inequality, we have obtained the transfer function of the matched filter (MF) $H(f) = P(-f)e^{-j2\pi f T_b}$, where $P(f) = \mathrm{F}[p(t)]$ is the Fourier transform of the input signal $p(t)$ and T_b is the duration of $p(t)$. The impulse response of the MF is $h(t) = p(T_b - t)$. It is observed that both $p(t)$ and $h(t)$ have the same duration T_b. Hence, the convolution of them, $p_0(t) = h(t) \otimes p(t)$, has a width of $2T_b$ seconds, with its peak occurring at $t = T_b$.
   (a) Suppose that a real signal $g(t)$ is an even signal, i.e., $g(t) = g(-t)$. Show that $G(f) = \mathrm{F}[g(t)]$ is real. Now, if $g(t)$ is time-shifted by τ, determine its spectrum $\mathrm{F}[g(t-\tau)]$ by using the properties of Fourier transform..
   (b) Find the spectrum $P_0(f) = \mathrm{F}[p_0(t)]$ and compare its form with the results obtained in part (a). Show that $p_0(t)$ is symmetrical about $t = T_b$.

3. Binary data are transmitted by using $s_0(t) = p(t)$ for binary 0 and $s_1(t) = 3p(t)$ for binary 1, where the duration of $p(t)$ is T_b seconds. The binary signal waveform is transmitted over an additive white Gaussian noise (AWGN) channel with two-sided noise power spectral density function $N_0/2$. The matched-filter receiver is shown in Fig. 1.

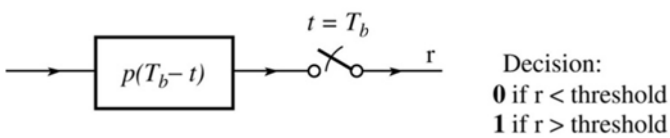

Figure 1

   (a) Determine the output a_0 due to the input signal $s_0(t)$ in terms of

$$E_p = \int_0^{T_b} p^2(t)\,dt .$$

   (b) Determine the output a_1 due to the input signal $s_1(t)$ in terms of E_p.
   (c) Find the mean and variance of the noise component at the output of the receiver.
   (d) If the threshold is chosen to be $\gamma = (a_0 + a_1)/2$, derive the error probability.