Examples of Implication: P⇒Q

PREV

If one has very high level of blood glucose, then he got diabetes

P (high glucose level)	Q (diagnosed as diabetes)	P⇒Q
Yes	Yes	True
Yes	No	False
No	Yes	True
No	No	True

Solution 6.1 a(i)

Validity of: $P \Rightarrow Q \Leftrightarrow \neg P \vee Q$

P	Q	$\neg P$	$\neg P \vee Q$	$P \Rightarrow Q$	$P \Rightarrow Q \Leftrightarrow \neg P \vee Q$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Solution 6.1 a(ii)

Validity of: $P \Leftrightarrow Q \quad \Leftrightarrow \quad (P \Rightarrow Q) \wedge (Q \Rightarrow P)$

P	Q	$P \Leftrightarrow Q$	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$	$P \Leftrightarrow Q \Leftrightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P)$
T	T	T	T	T	T	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T

Solution 6.1 b(iii)

Validity of: $P \Leftrightarrow Q \quad \Leftrightarrow \quad (P \wedge Q) \vee (\neg P \wedge \neg Q)$

(no truth table, rewriting rules / equivalences)

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
\neg(\neg \alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg \beta \Rightarrow \neg \alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg \alpha \vee \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg \alpha \vee \neg \beta) && \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg \alpha \wedge \neg \beta) && \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Solution 6.1 b(iii)

Validity of: $P \Leftrightarrow Q \quad \Leftrightarrow \quad (P \wedge Q) \vee (\neg P \wedge \neg Q)$

(no truth table, rewriting rules / equivalences)

using (ii):

$$P \Leftrightarrow Q \Leftrightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

then using (i):

$$P \Leftrightarrow Q \Leftrightarrow \underline{(\neg P \vee Q)} \wedge (\neg Q \vee P)$$

using distributivity of $\wedge$ over $\vee$:

$$P \Leftrightarrow Q \Leftrightarrow ((\neg P \vee Q) \wedge \neg Q) \vee ((\neg P \vee Q) \wedge P)$$

$$P \Leftrightarrow Q \Leftrightarrow (\neg P \wedge \neg Q) \vee (Q \wedge \neg Q) \vee (\neg P \wedge P) \vee (Q \wedge P)$$

False

Simplifying:

$$P \Leftrightarrow Q \Leftrightarrow (\neg P \wedge \neg Q) \vee (Q \wedge P)$$

finally, using commutativity of $\vee$:

$$P \Leftrightarrow Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$$

Solution 6.2

Propositional logic and Modus Ponens:

Amy, Bob, Cal, Don, and Eve were invited to a party last night."
➔ defines what we are talking about

Constants: A $\therefore$ *"Amy went to the party"*, B, C, D, E

Knowledge base:

"Cal will always go if Amy and Bob go."

(1) $A \wedge B \Rightarrow C$ $\quad$ $(\neg A \vee \neg B \vee C)$

"Cal will not go if Don goes, and conversely."

(2a) $D \Rightarrow \neg C$ $\quad$ $(\neg D \vee \neg C)$

(2b) $C \Rightarrow \neg D$

"Amy went to the party with Eve." $\quad$ $A \wedge E$

(3) A $\quad$ (4) E

"Bob goes to every party that Eve goes to."

(5) $E \Rightarrow B$ $\quad$ $(\neg E \vee B)$

Proof:

(4)+(5)	$E, E \Rightarrow B$	$\vdash B$	(6)
(3)+(6)+(1)	$A, B, A \wedge B \Rightarrow C$	$\vdash C$	(7)
(7)+(2b)	$C, C \Rightarrow \neg D$	$\vdash \neg D$	

Conclusion: **Don did not go to the party**

Solution 6.3

Using the Modus Ponens rule of inference:

If the unicorn is mythical:

Mythical, Mythical $\Rightarrow \neg$ Mortal	$\vdash \neg$ Mortal
$\neg$ Mortal, $\neg$ Mortal $\Rightarrow$ Horned	$\vdash$ Horned
Horned, Horned $\Rightarrow$ Magical	$\vdash$ Magical

If the unicorn is not mythical:

$\neg$ Mythical, $\neg$ Mythical $\Rightarrow$ Mammal	$\vdash$ Mammal
Mammal, Mammal $\Rightarrow$ Horned	$\vdash$ Horned
Horned, Horned $\Rightarrow$ Magical	$\vdash$ Magical

KB:

Mythical $\Rightarrow \neg$ Mortal

$\neg$ Mythical $\Rightarrow$ Mortal

$\neg$ Mythical $\Rightarrow$ Mammal

$\neg$ Mortal $\Rightarrow$ Horned

Mammal $\Rightarrow$ Horned

Horned $\Rightarrow$ Magical

Conclusion: the unicorn is both horned and magical
(true in all cases: Mythical or $\neg$ Mythical)

still no conclusion about the unicorn being mythical

(note: in general this is not a workable approach...)

Solution 6.3

Using the resolution rule of inference:

Binary resolution: $P \vee Q, \neg Q \vee R \quad |- \quad P \vee R$

Knowledge base (**CNF**):

1. ¬Mythical ∨ ¬ Mortal
2. Mythical ∨ Mortal
3. Mythical ∨ Mammal
4. Mortal ∨ Horned
5. ¬Mammal ∨ Horned
6. ¬Horned ∨ Magical

KB (INF):

Mythical ⇒ ¬ Mortal

¬ Mythical ⇒ Mortal

¬ Mythical ⇒ Mammal

¬ Mortal ⇒ Horned

Mammal ⇒ Horned

Horned ⇒ Magical

Proof:

7. from 1 and 4: ¬ Mythical ∨ Horned ⇐ ((¬Mythical ∨ ¬ Mortal) ∧ (Mortal ∨ Horned))
8. from 3 and 5: Mythical ∨ Horned
9. from 7 and 8: Horned
10. from 9 and 6: Magical

Conclusion: the unicorn is both horned and magical

still no conclusion about the unicorn being mythical

Solution 6.3

The unicorn mystery:

Constants: properties of the unicorn

Mythical, Magical, Horned, Mammal, Mortal

(Immortal $\Leftrightarrow \neg$ Mortal)

Knowledge base:

Mythical $\Rightarrow \neg$ Mortal　　$\neg$ Mythical $\Rightarrow$ Mortal　　$\neg$ Mythical $\Rightarrow$ Mammal

$\neg$ Mortal $\Rightarrow$ Horned　　Mammal $\Rightarrow$ Horned

Horned $\Rightarrow$ Magical

Problem: only rules and *no facts* (!)

ModusPonens: P, P $\Rightarrow$Q　　|– Q, but if *no P?*

- ➔ nothing can be inferred directly from the *KB*
- ➔ need some fact(s) i.e.,
  1) assume Mythical, then infer (what?)
  2) assume $\neg$ Mythical, then ...