$$g(t) = \frac{2a}{t^2 + a^2}$$

From the Fourier transform table, we have

$$e^{-a|t|} \quad \leftrightarrow \quad \frac{2a}{a^2 + (2\pi f)^2}$$

Using the duality property of Fourier transform, we have

$$\frac{2a}{a^2 + (2\pi t)^2} \quad \leftrightarrow \quad e^{-a|-f|} = e^{-a|f|}$$

Using the time scaling property, we yield

$$g(t) = \frac{2a}{a^2 + t^2} \quad \leftrightarrow \quad G(f) = |2\pi| e^{-a|2\pi f|} = 2\pi e^{-2\pi a|f|}$$

The energy of the signal is therefore

$$\begin{aligned} E_g &= \int_{-\infty}^{\infty} |G(f)|^2 df \\ &= 4\pi^2 \int_{-\infty}^{\infty} e^{-4\pi a|f|} df \\ &= (4\pi^2)(2)\int_0^{\infty} e^{-4\pi af} df \\ &= \frac{2\pi}{a} \end{aligned}$$

The essential bandwidth B satisfies the following equation:

$$\int_{-B}^{B} |G(f)|^2 df = 0.99 \times E_g$$

or

$$\int_{|f|>B} |G(f)|^2 df = 0.01 \times E_g$$

$$4\pi^2 \times 2 \times \int_B^{\infty} e^{-4\pi af} df = 0.01 \times \frac{2\pi}{a}$$

$$-e^{-4\pi af}\Big|_B^{\infty} = 0.01$$

$$B = \frac{0.3665}{a} \quad \text{(Hz)}$$

Accordingly, the Nyquist sampling rate is given by

$$f_s = 2B = 0.733/a$$

-- end --