NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2016/17 Semester 1 MH2500 Probability and Introduction to Statistics Tutorial 10

For the tutorial on 27 October, let us discuss

- Ex. 4.7.42, 45, 49, 50, 54, 60.

Ex. 4.7.42. Let X be an exponential random variable with standard deviation σ. Find $P(|X - E(X)| > k\sigma)$ for $k = 2, 3, 4$, and compare the results to the bounds from Chebyshev's inequality.

Ex. 4.7.45. Find the covariance and correlation of N_i and N_j, where $N_1, N_2, \ldots, N_r$ are multinomial random variables. (Hint: Express them as sums.)

Ex. 4.7.49. Two independent measurements, X and Y, are taken of a quantity μ. Suppose $E(X) = E(Y) = \mu$ but σ_X and σ_Y are unequal. The two measurements are combined by means of a weighted average to give

$$Z = \alpha X + (1 - \alpha)Y$$

where α is a scalar and $0 \leq \alpha \leq 1$.

a. Show that $E(Z) = \mu$.
b. Find α in terms of σ_X and σ_Y to minimize $\mathrm{Var}(Z)$.
c. Under what circumstances is it better to use the average $(X + Y)/2$ than either X or Y alone?

Ex. 4.7.50. Suppose that X_i, where $i = 1, \ldots, n$ are independent random variables with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2$. Let $\overline{X} = n^{-1} \sum_{i=1}^{n} X_i$. Show that $E(\overline{X}) = \mu$ and $Var(\overline{X}) = \sigma^2/n$.

Ex. 4.7.54. Let X, Y, and Z be uncorrelated random variables with variances σ_X^2, σ_Y^2, and σ_Z^2, respectively. Let

$$U = Z + X$$
$$V = Z + Y.$$

Find $\mathrm{Cov}(U, V)$ and ρ_{UV}.

Ex. 4.7.60. Let Y have a density that is symmetric about zero and let $X = SY$, where S is an independent random variable taking on the values $+1$ and -1 with probability $\frac{1}{2}$ each. Show that $\mathrm{Cov}(X, Y) = 0$, but that X and Y are not independent.