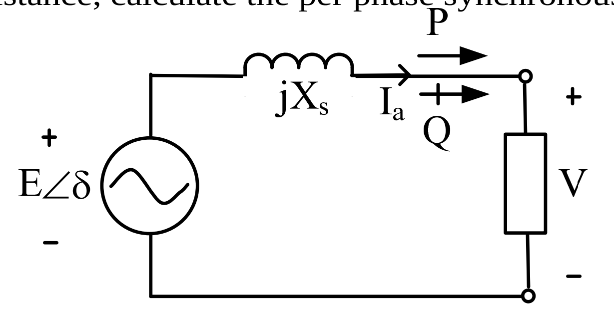
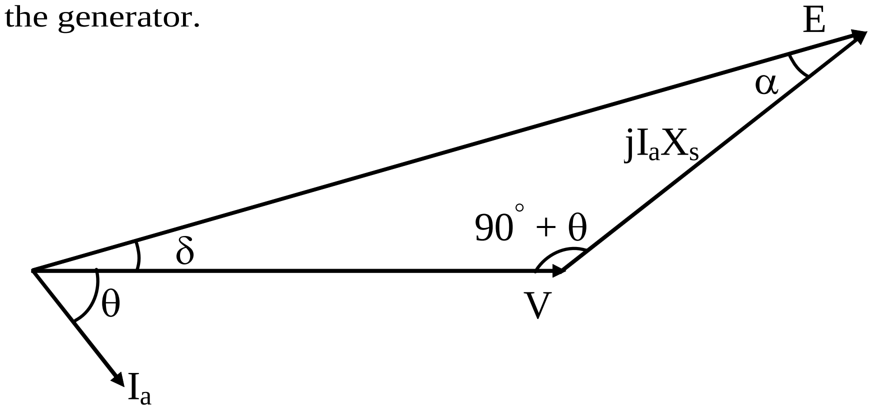
Exercise 10.2

A three-phase synchronous generator has a no-load line-to-line output voltage of 2400 V at a field current of 12 A dc. The generator is connected to a 2300-V infinite bus, and its mechanical drive is adjusted such that the generator output is 48 kW + j12 kVAr, with the same field current of 12 A dc. Assume that the magnetic circuit is unsaturated. Neglecting the armature resistance, calculate the per phase synchronous reactance of the generator.

15.16 Ω





Given: $|V|$, $|E|$, P and Q, i.e. S because $S = P + jQ$

Find: X_s

Solutions: After setting the ref angle, we know V, I_a (both magnitude and angle) and θ

We know I_a and θ because $I_a = (S/V)^*$ and θ can be derived from S.

First Method (via Sine formula):

$$\frac{V}{\text{Sin}\alpha} = \frac{E}{\text{Sin}(90^\circ + \theta)} \quad \rightarrow \quad \alpha = \text{Sin}^{-1}\left[\frac{V\text{Sin}(90^\circ + \theta)}{E}\right]$$

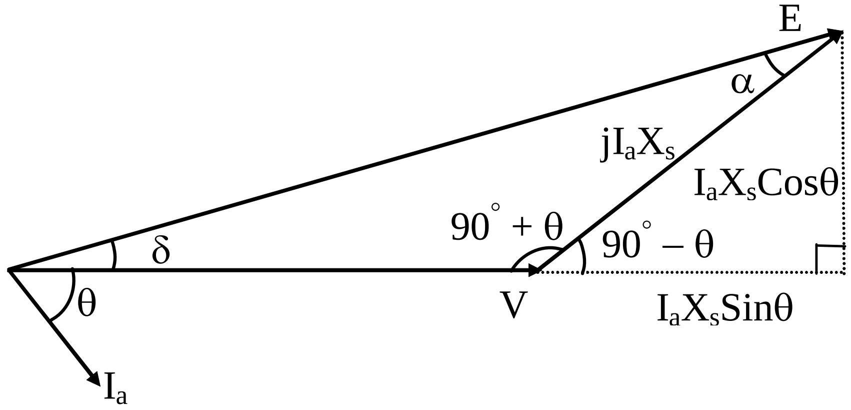
$$\delta = 180 - (90^\circ + \theta) - \alpha \quad \rightarrow \quad \frac{I_a X_s}{\text{Sin}\delta} = \frac{V}{\text{Sin}\alpha}$$

$$X_s = \frac{V\text{Sin}\delta}{I_a \text{Sin}\alpha}$$

Second Method (after obtaining δ)

$$I_a X_s \operatorname{Cos}\theta = E\operatorname{Sin}\delta \quad \rightarrow \quad X_s = \frac{E\operatorname{Sin}\delta}{I_a \operatorname{Cos}\theta}$$

$$\text{or } P = \frac{EV}{X_s}\operatorname{Sin}\delta \quad \rightarrow \quad X_s = \frac{EV\operatorname{Sin}\delta}{P}$$



Third Method (via Cosine formula):

$$E^2 = V^2 + (I_a X_s)^2 - 2V(I_a X_s)\operatorname{Cos}(90^\circ + \theta)$$

$$(I_a X_s)^2 - 2V(I_a X_s)\operatorname{Cos}(90^\circ + \theta) + (V^2 - E^2) = 0$$

$$a X_s{}^2 + bX_s + c = 0 \quad \text{where } a = I_a{}^2,\ b = -2VI_a\operatorname{Cos}(90^\circ + \theta),\ c = V^2 - E^2$$

$$X_s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \text{ ignore negative } X_s$$

Fourth Method (via KVL equation)

$$E = V + jI_a X_s = (V + I_a X_s \operatorname{Sin}\theta) + jI_a X_s \operatorname{Cos}\theta$$

$$E^2 = (V + I_a X_s \operatorname{Sin}\theta)^2 + (I_a X_s \operatorname{Cos}\theta)^2$$

$$E^2 = V^2 + 2VI_a X_s \operatorname{Sin}\theta + (I_a X_s \operatorname{Sin}\theta)^2 + (I_a X_s \operatorname{Cos}\theta)^2$$

$$E^2 = V^2 + 2VI_a X_s \operatorname{Sin}\theta + (I_a X_s)^2$$