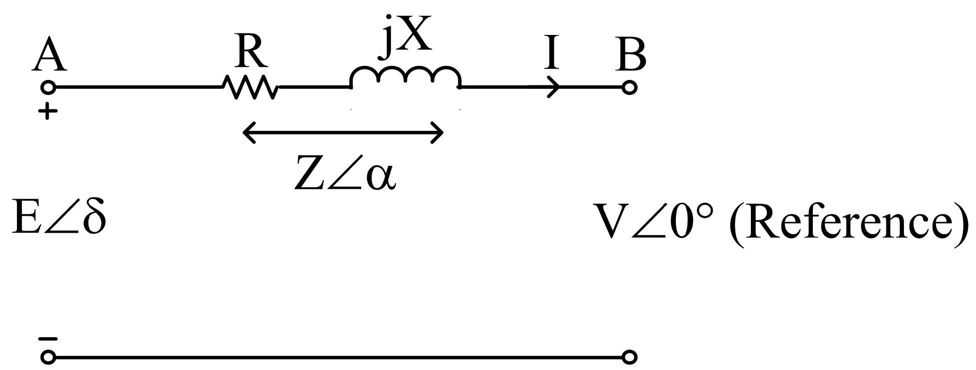
APPENDIX A

<u>Power Transfer & Reactive Power</u>



Consider the transfer of power from a source bus (A) to a load bus (B), through a line of impedance Z = R + jX = $|Z|\angle\alpha$.

Then,

$$I = \frac{E\angle\delta - V\angle 0°}{R + jX} = \frac{E\angle\delta - V\angle 0°}{Z\angle\alpha}$$

$$= \frac{E}{Z}\angle(\delta - \alpha) - \frac{V}{Z}\angle - \alpha$$

$\therefore$ Power delivered to B

$$S_B = V_B I_B^* = [V\angle 0°]\left[\frac{E}{Z}\angle(\alpha - \delta) - \frac{V}{Z}\angle\alpha\right]$$

$$= \frac{VE}{Z}\angle(\alpha - \delta) - \frac{V^2}{Z}\angle\alpha$$

Since $S_B = P_B + jQ_B$

$\Rightarrow$ $P_B = S_B \cos\theta$
$Q_B = S_B \sin\theta$

$\Rightarrow$ Real power delivered

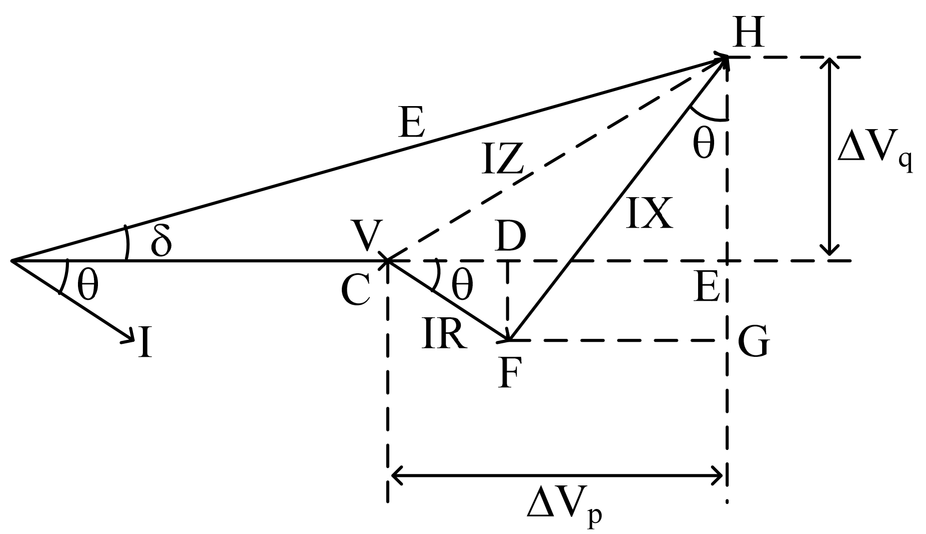
$$P_B = \frac{VE}{Z}\cos(\alpha-\delta) - \frac{V^2}{Z}\cos\alpha \qquad (1)$$

& Reactive power delivered

$$Q_B = \frac{VE}{Z}\sin(\alpha-\delta) - \frac{V^2}{Z}\sin\alpha \qquad (1)$$

Let us understand these relations from the vector diagram.

<u>Note</u> : θ is power factor angle at B i.e. I lags V by θ (assumed).



$$\sin\,\delta = \frac{\Delta V_q}{E} \Rightarrow \Delta V_q = E \sin\delta$$

$$\cos\,\delta = \frac{V + \Delta V_p}{E} \Rightarrow E\,\cos\,\delta = V + \Delta V_p$$

$$\Rightarrow \Delta V_p = E \cos\delta - V \tag{1a}$$

$$\begin{aligned} \text{But } \Delta V_p &= CD + DE = CD + FG \\ &= IR\cos\theta + IX\sin\theta \\ &= \frac{1}{V}\left[(VI\cos\theta)\,R + (VI\sin\theta)\,X\right] \end{aligned}$$

$$\Rightarrow \Delta V_p = \frac{1}{V}(PR + QX) \tag{2}$$

$$\begin{aligned} \text{Also, } \Delta V_q &= HG - EG = HG - DF \\ &= IX\cos\theta - IR\sin\theta \\ &= \frac{1}{V}\left[(VI\cos\theta)\,X - (VI\sin\theta)\,R\right] \end{aligned}$$

$$\Rightarrow \Delta V_q = \frac{1}{V}(PX - QR) \tag{3}$$

Let $R << X$, then $Z \simeq X$ & $\alpha \simeq 90°$ ($R \simeq 0$)

$\Rightarrow$ From (1)

$$P_B = \frac{VE}{X} \sin \delta$$
$$\& \; Q_B = \frac{VE}{X} \cos \delta - \frac{V^2}{X} \qquad (4)$$

$\Rightarrow$ From (2)

$$\Delta V_p = \frac{PR + QX}{V} = \frac{QX}{V} \qquad (5)$$

$\Rightarrow$ From (3)

$$\Delta V_q = \frac{PX - QR}{V} = \frac{PX}{V} \qquad (6)$$

Since in physical systems E & V do not vary much, and X is a constant, we conclude that :

- Real power P depends only on

  (i) sin δ [see eqn. 4]
  (ii) ΔV_q (which is a slightly different measure of δ) : [see eqn. 6]

δ is called the ***power angle*** *or torque angle.*

ΔV_q $\rightarrow$ *quadrature component of voltage difference betn. V & E.*

$\Rightarrow$ P flows primarily due to phase angle difference between E & V; i.e. due to angle δ.

Hence, if :

$\angle E > \angle V$ Real power will flow from A to B
$\angle E < \angle V$ Real power will flow from B to A
$\angle E = \angle V$ No Real power will flow

- Reactive power Q depends only on

  (i) ΔV_p [see eqn. 5]
  (ii) when $\delta \simeq$ small, $\Delta V_p \simeq E - V$ [see eqn. 1a]

  Q primarily depends on the magnitude difference between E & V measured in terms of ΔV_p (the in-phase component of voltage difference)

Hence, if :

$|V| = |E|$ & $\delta \simeq$ small,

Then Q = 0 [see eqn. set 1]

If $|E| > |V|$ Q flows from A to B
If $|E| < |V|$ Q flows from B to A

Summary :

1) P flows from a bus with greater voltage phase angle relative to a bus with smaller phase angle of voltage.
2) Q flows from a bus at a higher voltage magnitude to a bus with a lower voltage magnitude.