Name: ______________________________ Tutorial group: T1

Matriculation number: | | | | | | | | | |

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I 2015/16

MH2500– Probability and Introduction to Statistics

1 September 2015 Test 1 40 minutes

<u>INSTRUCTIONS</u>

1. Do not turn over the pages until you are told to do so.
2. Write down your name, tutorial group, and matriculation number.
3. This test paper contains **FOUR (4)** questions and comprises **FIVE (5)** printed pages.
4. Answer **all** questions. The marks for each question are indicated at the beginning of each question.
5. You are allowed one double sided A4 size cheat sheet.

For graders only	Question	1	2	3	4	Bonus	Total
	Marks						

QUESTION 1. **(6 marks)**

An urn contains five red balls and two white balls. Three balls are drawn without replacement from the urn, and the colours are noted in sequence.

Let A be the event that the second ball drawn is white. Find $P(A)$. Leave your answer as a fraction.

[Answer:]

a.

$$\Omega = \{\text{RRR, RRW, RWR, WRR, RWW, WRW, WWR}\}.$$

b. Let A be the event that the second ball drawn is white. Find $P(A)$.

$$P(A) = \frac{5}{7} \times \frac{2}{6} + \frac{2}{7} \times \frac{1}{6} = \frac{2}{7}.$$

Final answer is 2/7, including incomplete working .... 6 points.

Final answer is 2/7 but wrong working ....case by case, maybe 3 points.

Working is correct but final answer is wrong ....5 points.

Some parts are correct ....3 points.

Nothing is correct .... 1 point.

Blank .... 0 points.

QUESTION 2. **(12 marks)**

There are 2 bad batteries in a given lot of 100 batteries.

a An inspector examines 3 batteries which are selected at random and without replacement. Find the probability of at least 1 bad battery among the 3. Leave your answer as a fraction or correct to three significant figures.

b Suppose the inspector examines batteries selected at random but with replacement. What is the minimum number of batteries that he should examine so that the probability of finding at least 1 bad battery is at least $\frac{1}{2}$?

[Answer:]

a. Let A be the event of finding at least 1 bad battery.

$$P(A) \;=\; 1 - P(A^c) \;=\; 1 - \frac{\binom{98}{3}}{\binom{100}{3}} \;=\; 1 - \frac{776}{825} \;=\; \frac{49}{825} \;\approx\; 0.0594.$$

b. Let n be the required number of batteries. Then

$$P(A) \;=\; 1 - P(A^c) \;=\; 1 - \left(\frac{98}{100}\right)^n \;\geq\; \frac{1}{2}.$$

So

$$\left(\frac{98}{100}\right)^n \leq \frac{1}{2}$$

i.e.

$$\left(\frac{100}{98}\right)^n \geq 2$$

Thus,

$$n \geq \frac{\log 2}{\log \frac{100}{98}} \approx 34.3.$$

Therefore, $n = 35$.

QUESTION 3. **(8 marks)**

Let A, R, S, T be events of a certain experiment such that R, S, T are mutually disjoint and $R \cup S \cup T = \Omega$. Suppose $P(A|R) = 0.75$, $P(A|S) = 0.2$, $P(A|T) = 0.5$, $P(R) = 0.60$, $P(S) = 0.25$, and $P(T) = 0.15$.

Find $P(R|A)$. Leave your answer as a fraction or correct to 3 significant figures.

[Answer:]

(a) $P(A \cap S) = P(A|S)P(S) = 0.2 \times 0.25 = 0.05$.

(b) By Bayes Theorem,

$$\begin{aligned} P(R|A) &= \frac{P(A|R)P(R)}{P(A|R)P(R) + P(A|S)P(S) + P(A|T)P(T)} \\ &= \frac{0.75 \times 0.6}{0.75 \times 0.6 + 0.2 \times 0.25 + 0.5 \times 0.15} \\ &= \frac{18}{23} \approx 0.783. \end{aligned}$$

Final answer is $\frac{18}{23}$ or 0.783, including incomplete working . . . . 8 points.

Final answer is $\frac{18}{23}$ but wrong working . . . . case by case basis, maybe 4 points.

Working is correct but final answer is wrong . . . . 7 points.

Some parts are correct . . . . 3 – 6 points.

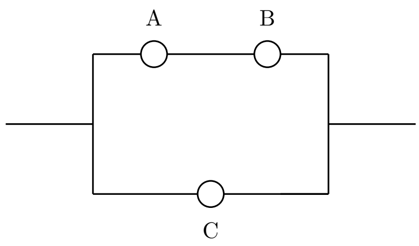
Nothing is correct . . . . 1 point.

Blank . . . . 0 points.

[Remark on Question 4:] There were requests to remove Question 4 from the purpose of grading, as some students do not know how bulbs in parallel and series works. I decided to keep the question but give generous partial credits. My decision was based on (1) there is nothing wrong with the question, (2) it is a textbook styled question (see Ex.1.8.74), (3) we did touch on components connected in series in the tutorial and although we did not really discussed components connected in parallel, there are many examples in the textbook on that. On hindsight, I should have either told you the significance of components connected in parallel, or said nothing at all. Telling you that you may relate the system to a circuit of light bulbs was a bad choice.

QUESTION 4. **(8 marks)**

Three components, A, B, and C, are connected in the way illustrated below. Assume that all three units are independent, units A and B each fails with probability 0.1, and unit C fails with probability 0.2. What is the probability that the system works?



[Intended Solution:]

$$P(\text{A and B work}) = (1 - 0.1)^2 = 0.81.$$

Therefore,

$$P(\text{A or B fails}) = 1 - 0.81 = 0.19$$

and

$$P(\text{A or B fails, and C also fails}) = 0.19 \times 0.2 = 0.038.$$

Therefore,

$$\begin{aligned} P(\text{system works}) &= P(\text{A and B works, or C works}) \\ &= 1 - 0.038 = 0.962. \end{aligned}$$

[Other solutions:] Adding the possible cases so that the system works, e.g.,

$$\begin{aligned} P(\text{system works}) &= P(\text{A and B works, or C works}) \\ &= P(\text{C works}) + P(\text{C fails and both A and B works}) \\ &= 0.8 + 0.2 \times 0.9 \times 0.9 = 0.962. \end{aligned}$$

Most incomplete or wrong solutions earn 4–6 points.

6 points is given for the following solution where at the system works if at least one component works and it disregards the way the components are connected.

$$\begin{aligned} P(\text{at least one works} &= 1 - P(\text{none of them works}) \\ &= 1 - P(\text{A fails})P(\text{B fails})P(\text{C fails}) \\ &= 1 - 0.1 \times 0.1 \times 0.2 = 0.998. \end{aligned}$$