NANYANG TECHNOLOGICAL UNIVERSITY
School of Electrical & Electronic Engineering

EE/IM4152 Digital Communications

Tutorial No. 12 (Sem 1, AY2016-2017)

1. For linear block codes, we adopt the maximum-likelihood (ML) strategy for decoding. We decide the codeword $\mathbf{c}_i$ if $\Pr(\mathbf{r}\,|\,\mathbf{c}_i)$ has the largest value among all codewords, where $\mathbf{r}$ is the received word. Suppose the Hamming distance between $\mathbf{r}$ and $\mathbf{c}_i$ is d_i. That is, the channel noise causes errors in d_i positions and the Hamming weight of $w(\mathbf{r}\oplus\mathbf{c}_i)=d_i$.
   (a) Determine $\Pr(\mathbf{r}\,|\,\mathbf{c}_i)$ if the channel bit-error probability is ε over a binary symmetric channel (BSC), where $\varepsilon<0.5$.
   (b) Taking the logarithm of $\Pr(\mathbf{r}\,|\,\mathbf{c}_i)$ will not affect the comparisons. Show that $\max_i \Pr(\mathbf{r}\,|\,\mathbf{c}_i)$ is equivalent to $\min_i [d_i]$.
   (c) Show that the result in part (b) is equivalent to choosing the error pattern with the minimum Hamming weight.

2. For a t-error-correcting (n,k) linear block code, the received word is in error if more that t errors occur in n coded bits (i.e., the worst case).
   (a) Suppose the probability of channel bit error is q_c. Determine the probability of word error, P_{ew}, in terms n, t and q_c.
   (b) Given that a received word is wrong, show that the average probability of bit error is $P_{ec}=\sum_{j=t+1}^{n}\binom{n-1}{j-1}(1-q_c)^{n-j}q_c^{j}$.

3. For the convolutional encoder with (n, k, N) = (2, 1, 3), as shown in Fig. 1, design a trellis diagram to generate the encoded sequence corresponding to the information sequence $\mathbf{x}=[1101011]$. As observed from Fig. 1, the output bits are not only determined by the present $k=1$ information bit but also by the previous information bits. This dependence on the previous information bits causes the encoder to be a finite state machine. Here we use the contents of (d_{k-2},d_{k-1}) to define the state of the encoder: a = (0, 0), b = (1, 0), c = (0, 1) and d = (1, 1). Remember to append $N-1=2$ zero bits to end the transmission of the current block $\mathbf{x}=[1101011]$, which is different from the truncation approach used in the lecture notes. In this way, the trellis path will be forced to return to state a = (0, 0) due to those appended zeros.

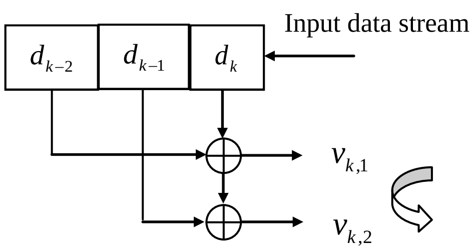

Figure 1: Convolutional encoder with (n, k, N) = (2, 1, 3)

Assuming hard-decision decoding, the received sequence is $\mathbf{y}=[01\,10\,11\,11\,01\,00\,01]$. Find the maximum-likelihood information sequence and the corresponding number of errors contained in $\mathbf{y}$.