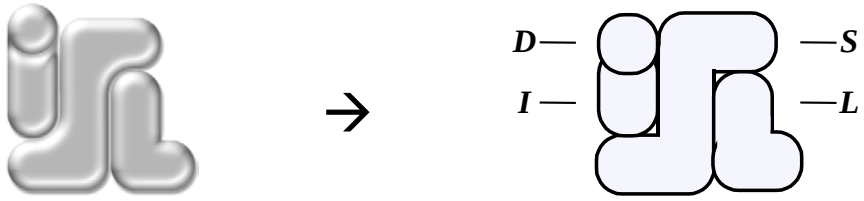
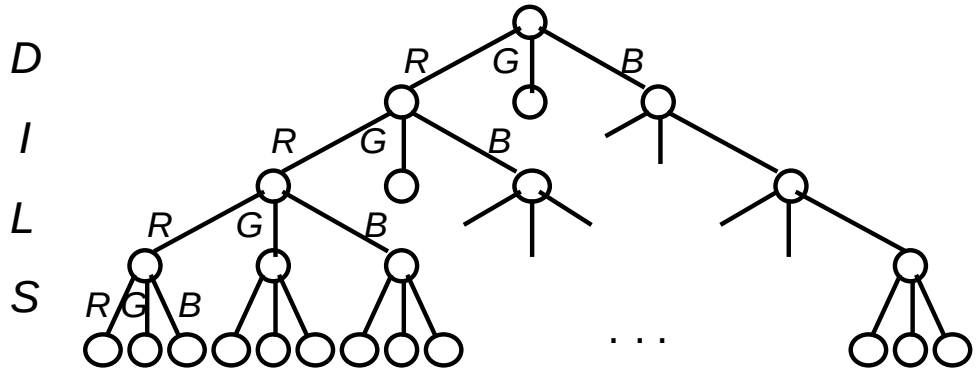
ISL logo coloring problem:



Depth-First Search (no forward checking):

search space:



uniform depth of 4
branching factor = 3

number of nodes: exactly
$1 + 3 + 3^2 + 3^3 + 3^4 = 121$

DFS a good choice?

yes → depth-limited search space, all solutions at maximum depth, many possible solutions

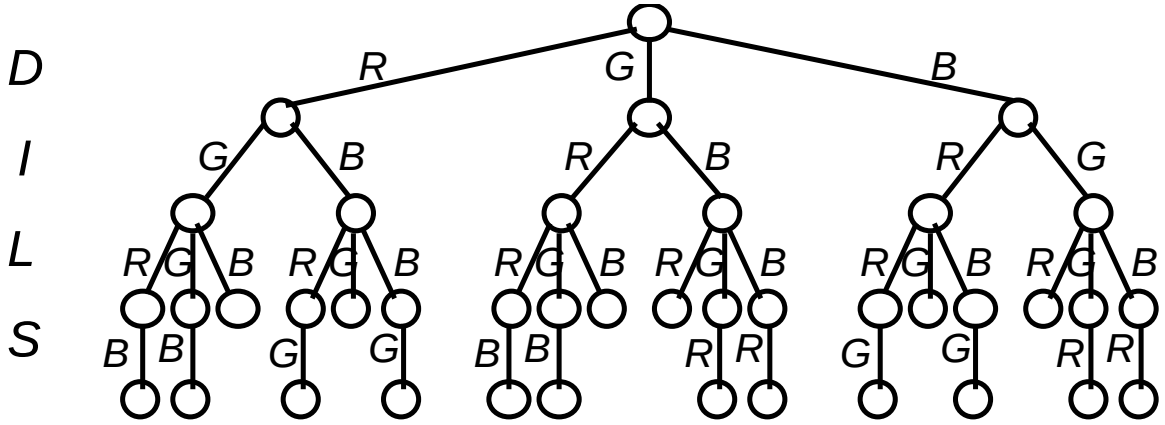
no → need to generate all solutions (not just one), blind search generates invalid colorings, backtracks unnecessarily

DFS with forward checking, arbitrary ordering:

logo colouring = Constraint Satisfaction Problem

forward checking: *take constraints into account to significantly prune the search space*

search space:



number of nodes: 1 + 3 + 3∗2 + 6∗3 + 6∗2 = 40

avg branching factor: (40 – 1) / (1 + 3 + 6 + 6∗2) = 1.77

efficiency: significant improvement i.e.,
search space reduced from 121 to 40 nodes,
branching factor decreased from 3 to 1.77,
no invalid colorings, little backtracking

however elements are assigned a color in arbitrary (alphabetical) order → *not* optimal

General CSP heuristics, and their usefulness:

Most Constraining Variable (Degree)

select among yet unassigned variables the one involved in the largest number of constraints

i.e., logo element with the largest nb of neighbors

→ useful to optimize the order of variable assigments

Most Constrained Variable (Minimum-Remaining Value)

select the variable with fewest possible values

i.e., logo element with fewest colors available *(all 3!)*

→ useful to complement the above in case of a tie

Least Constraining Value

select a value that leaves the largest choice of values for other constraint-related variables

i.e., color that less constrains other logo elements

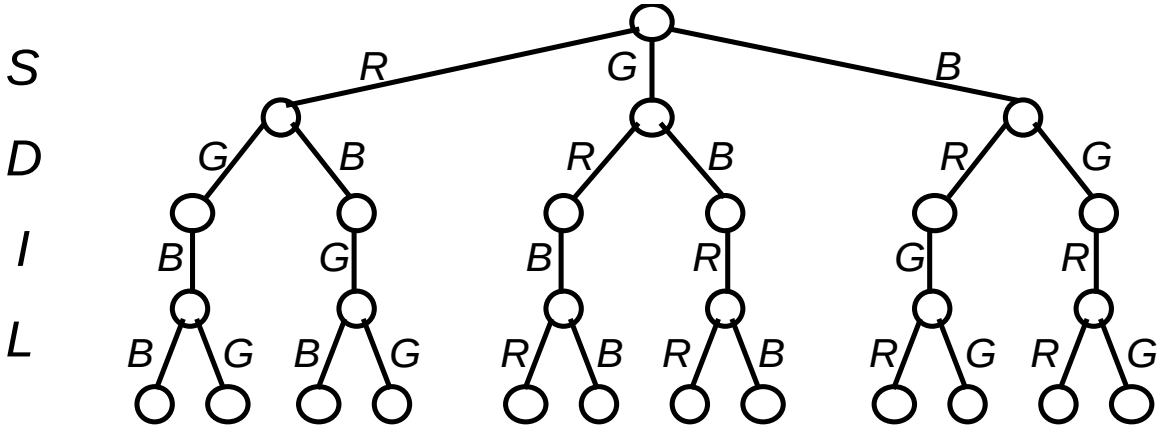
→ useful to prevent deadlocks, reduce backtracking

best heuristics: *Most Constraining Variable*

DFS with forward checking, heuristic ordering:

most constraining logo element: *S* with 3 neighbors,
followed by *D* and *I* with 2 neighbors each,
then *L* with only 1 neighbor

search space:



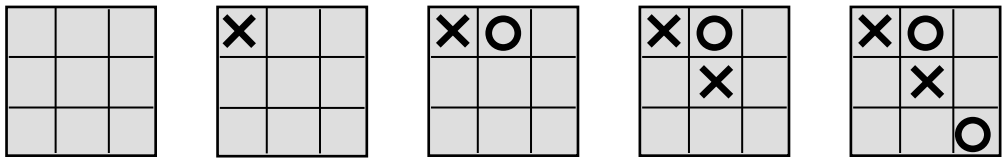
number of nodes: 1 + 3 + 3∗2 + 6∗1 + 6∗2 = 28

avg branching factor: (28 – 1) / (1 + 3 + 6 + 6∗1) = 1.69

efficiency: further good improvement i.e.,
search space reduced from 40 to 28 nodes,
branching factor decreased from 1.77 to 1.69, no invalid colorings, no backtracking

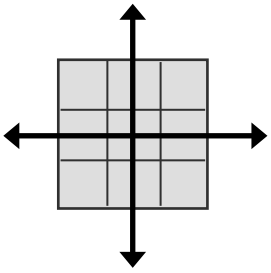
elements are assigned a color in *optimal* order

Depth-2 game tree for Tic-Tac-Toe:

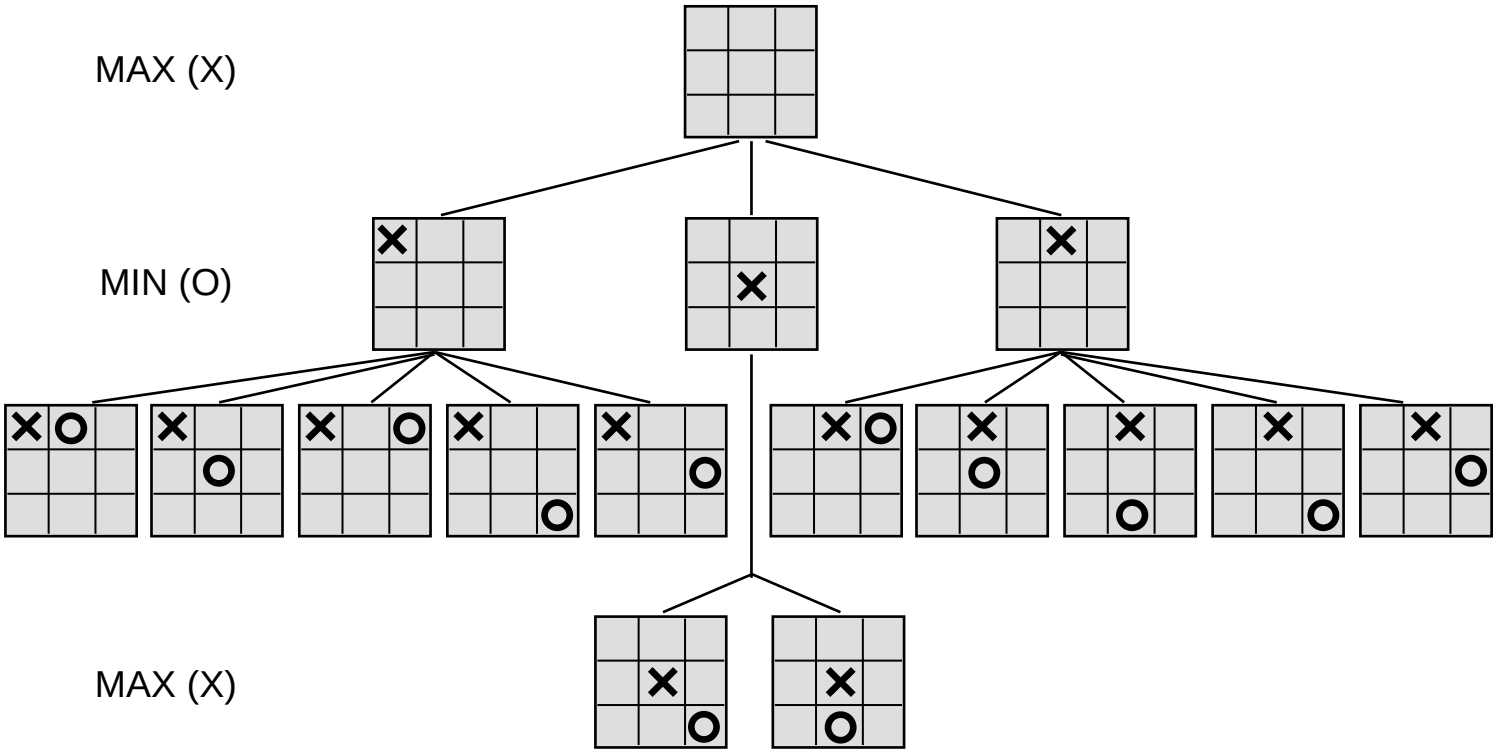


horizontal and vertical symmetry

thus only 3 moves at depth 1 (vs. 9)
and 12 moves at depth 2 (vs. 72)



search tree:

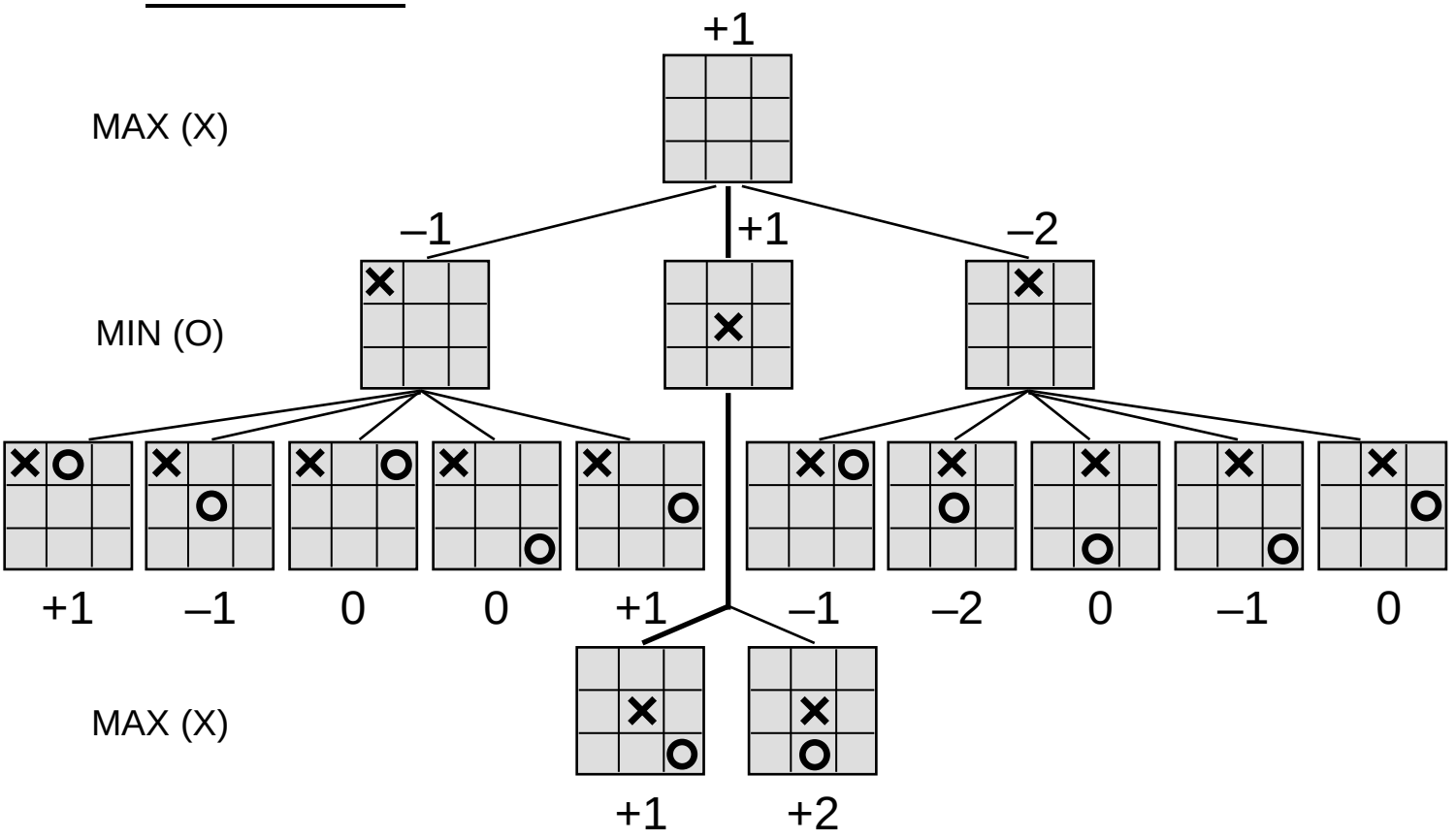


Heuristics for Tic-Tac-Toe:

value of non–terminal nodes: $3\,X_2 + X_1 - 3\,O_2 - O_1$

at depth 2: $X_2 = O_2 = 0$ → $h = X_1 - O_1$

search tree:



best move for MAX: *play in the centre!*

fair game? no, whoever plays first has a strong advantage