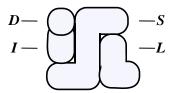
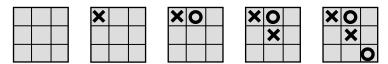
4.1 The logo of the Intelligent Systems Laboratory comprises four design elements: the letters *I*, *S*, *L*, and the circular dot *D*. The graphic designer now wants to color each element red, green, or blue, such that adjacent elements sport different colors.



You are asked to use a *Depth-First Search* algorithm *with Forward Checking* to generate all possible color combinations for the logo. In both cases below, draw the complete search space, calculate the average branching factor and the total number of nodes, and comment on the efficiency of the approach.

- (a) Assume the algorithm arbitrarily colors the elements in *alphabetical order*.
- (b) Assume the algorithm colors the elements in the *order indicated by the best heuristics* that can be used for Constraint Satisfaction Problems. Explain why it is the best in this case and how useful the others are (or not).
- 4.2 The game of Tic-Tac-Toe is played between two players on a board composed of 3x3 squares. Each player takes turn placing an X or O in one empty square, as illustrated by the sample game below (first 4 moves). The X or O player wins the game by aligning 3 X's or O's, respectively, on the same column, row, or diagonal.



A *heuristics* for this game is proposed as follows. Let  $X_n$  be the number of rows, columns, or diagonals with exactly n X's and no O's. Similarly, let  $O_n$  be the number rows, columns, or diagonals with n O's and no X's. The heuristics gives any non-terminal position a value equal to:  $3 X_2 + X_1 - 3 O_2 - O_1$ . Finally, the heuristics assigns +1 to positions with  $X_3$ =1 (a win for X), -1 to positions with  $O_3$ =1 (a loss for X), and 0 to all other terminal positions (a draw).

- (a) Show the *game tree* up to depth 2 i.e., with one X and one O on the board. Make sure to take *symmetry* into account to minimise the size of the tree.
- (b) Indicate on the game tree the *estimated values* of all positions at depth 2. Use the *MiniMax* algorithm to derive the backed-up values for positions at depths 1 and 0. Determine then the best starting move for the X player.