Name: _____ Tutorial group: T1

Matriculation number: | | | | | | | | | |

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I 2016/17

MH2500– Probability and Introduction to Statistics

30 August 2016 Test 1 40 minutes

INSTRUCTIONS

1. Do not turn over the pages until you are told to do so.
2. Write down your name, tutorial group, and matriculation number.
3. This test paper contains **FOUR (4)** questions and comprises **FIVE (5)** printed pages.
4. Answer **all** questions. The marks for each question are indicated at the beginning of each question.
5. You are allowed one side of an A4 size paper as cheat sheet.

For graders only	Question	1	2	3	4	Bonus	Total
	Marks						

QUESTION 1. **(8 marks)**

(a) Suppose X is a geometric random variable with $p = 0.3$. Find $P(X = 5)$. Give your answer to three significant figures.

[Answer:]

$$P(X = 5) = (1 - p)^4 p = 0.7^4 \times 0.3 \approx 0.0720.$$

3 marks for up to $0.7^4 \times 0.3$ and 1 mark for the calculation.

(b) Let Y be the uniform random variable on $[3, 5]$. Give the cumulative distribution function of Y.

[Answer:]

For $y \in [3, 5]$,

$$F(y) \;=\; \int_3^y \frac{1}{5-3} dy \;=\; \Big[\frac{y}{2}\Big]_3^y = \frac{y-3}{2}.$$

Therefore,

$$F(y) = \begin{cases} 1, & y > 5; \\ \frac{y-3}{2}, & 3 \le y \le 5; \\ 0, & y < 3. \end{cases}$$

3 marks for $F(y) = \frac{y-3}{2}$ and 1 mark for the rest.

QUESTION 2. **(10 marks)**

(a) A deck of 52 cards is shuffled thoroughly and five cards are drawn. What is the probability that the five cards consist of two spades and three hearts and there are no kings?

5 marks each.

[Answer:]

Let A be the event that the five cards drawn consist of two spades and three hearts and no kings. There are 13 spades and 13 hearts in a deck of cards. Excluding the kings, there are 12 spades and 12 hearts. Thus

$$\begin{aligned} P(A) &= \frac{\binom{12}{3}\binom{12}{2}}{\binom{52}{5}} \\ &= \frac{121}{21658} \\ &\approx 0.00559 \end{aligned}$$

(b) There are n students in a room where n is an integer greater than 1. Write down in terms of n, the probability that at most one of them is born on 30 August. (For simplicity, assume there are 365 days in a year.)

[Answer:]

Let A be the event that at most one of the n students is born on 30 August, let B be the event that none of the students are born on 30 August, and C be the event that exactly one of the n students is born on 30 August. Then A is the disjoint union of B and C. Thus

$$\begin{aligned} P(A) &= P(B) + P(C) \\ &= \frac{364^n}{365^n} + \frac{\binom{n}{1}364^{n-1}}{365^n}. \end{aligned}$$

QUESTION 3. **(8 marks)**

A university conducted a survey on students' overall satisfaction level, and students are to answer whether they are "satisfied", "neutral", or "dissatisfied". Based on the survey results, 70% of the students who did the survey answered "satisfied", 10% of the students answered "neutral", and 20% of the students answered "dissatisfied". We are told that the proportion of Mathematics majors amongst those students who answered "satisfied", "neutral", and "dissatisfied" are 25%, 10%, and 5%, respectively.

(i) Find the probability that a randomly selected student is a Mathematics major.

(ii) Find the probability that a randomly selected student answered "satisfied" if we know that he is a Mathematics major.

Give your answer correct to three significant figures.

4 marks each

[Answer:]

(i) Let S, N, and D denote the event that a randomly selected student answered "satisfied", "neutral", and "dissatisfied", respectively. Let M denote that a randomly selected student is a Mathematics major. By the law of total probability,

$$\begin{aligned} P(M) &= P(M|S)P(S) + P(M|N)P(N) + P(M|D)P(D) \\ &= 0.25 \times 0.7 + 0.1 \times 0.1 + 0.2 \times 0.05 \\ &= 0.195. \end{aligned}$$

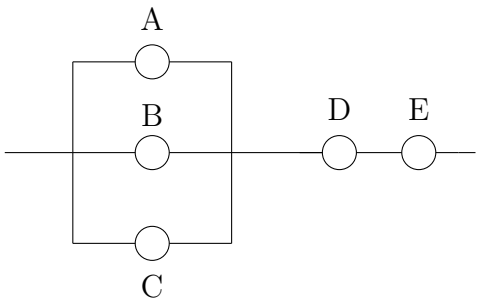
(ii)

$$P(S|M) = \frac{P(S \cap M)}{P(M)} = \frac{0.25 \times 0.7}{0.195} = \frac{35}{39} \approx 0.897.$$

QUESTION 4. **(8 marks)**

A system consists of five components, A, B, C, D, and E, connected in the way illustrated below. Assume units A, B, and C each fails with probability 0.1, and units D, E each fails with probability 0.2 and that all five units fail independently. What is the probability that the system works? Leave your answer correct to 3 significant figures.

Remark: The system works if all three conditions below are satisfied.
(1) at least one of A, B, C works,
(2) D works,
(3) E works.



[Answer:]

$P(\text{at least one of A, B, C works}) = 1 - P(\text{all A, B, C fails}) = 1 - 0.1^3 = 0.999.$

Thus

$$\begin{aligned} P(\text{system works}) &= P(\text{at least one of A, B, C works })P(\text{D works})P(\text{E works}) \\ &= (1 - 0.999)(1 - 0.2)(1 - 0.2) \\ &= 0.639. \end{aligned}$$