"Not all students take both History and Biology."

constants: History, Biology

predicates: Student(x) "x is a student"
Takes(x, c) "x takes course c"

sentence:

$$\neg (\forall x \; Student(x) \Rightarrow Takes(x, History) \land Takes(x, Biology))$$

equivalent to:

$$\exists x \; Student(x) \land (\neg Takes(x, H) \lor \neg Takes(x, B))$$

"There is a barber in town who shaves all men who do not shave themselves."

predicates: Barber(x) "x is a barber"
InTown(x) "x live in town"
Man(x) "x is a man"
Shaves(x, y) "x shaves y"

sentence:

$$\exists x \; Barber(x) \land InTown(x) \land (\forall y \; Man(y) \land \neg Shaves(y, y) \Rightarrow Shaves(x, y))$$

“Politicians

can fool all of the people some of the time, they can even fool some of the people all of the time, but they can't fool all of the people all of the time.”

predicates:	Politician(x)	“x is a politician”
	Person(x)	“x is a person”
	Time(t)	“t is a time”
	Fools(x, y, t)	“x fools y at t”

sentence:

$$\forall x \text{ Politician}(x) \Rightarrow$$

$$(\forall y \text{ Person}(y) \Rightarrow \exists t \text{ Time}(t) \land \text{Fools}(x, y, t)) \land$$

$$(\exists y \text{ Person}(y) \land (\forall t \text{ Time}(t) \Rightarrow \text{Fools}(x, y, t))) \land$$

$$\neg (\forall y \forall t \text{ Person}(y) \land \text{Time}(t) \Rightarrow \text{Fools}(x, y, t))$$

or 3 sentences:

$$\forall x \text{ Pol}(x) \Rightarrow (\forall y \text{ Per}(y) \Rightarrow \exists t \text{ T}(t) \land \text{F}(x, y, t))$$

$$\forall x \text{ Pol}(x) \Rightarrow (\exists y \forall t \text{ Per}(y) \land \text{T}(t) \Rightarrow \text{F}(x, y, t))$$

$$\forall x \text{ P}(x) \Rightarrow \neg (\forall y,t \text{ Per}(y) \land \text{T}(t) \Rightarrow \text{F}(x, y, t))$$

Logical sentence interpretation:

(i) $\forall x \,(\, Boy(x) \Rightarrow \exists y \,(\, Girl(y) \wedge Likes(x, y)\,)\,)$

"every boy likes a girl"

(ii) $\exists y \,(\, Girl(y) \wedge \forall x \,(\, Boy(x) \Rightarrow Likes(x, y)\,)\,)$

"there is a girl that all boys like"

Equivalence and entailment:

equivalent? *no*

which entails which?

(i) ⇒ (ii) ?

(ii) ⇒ (i) ?

(i) does *not* entail (ii)
every boy likes *a* girl
but not nec. the same

(ii) entails (i)
every boy likes *that* girl
so they all like one

How to prove it logically:

proof: *counter-example!*

case where P true and Q
is not? nec. P⇒Q is false

e.g., 2 boys, Al and Ben, and
2 girls, Cloe and Dora,
Likes(Al,Cloe), Likes(Ben,Dora)
→ (i) is true but (ii) is not

proof: *by contradiction*

P ⇒ Q? suppose not i.e.
¬(P⇒Q), that is: P ∧ ¬Q

then show …
the hypothesis …
yields a …
contradiction

A working proof:

hypothesis: (ii) $\wedge$ ¬(i)

$$(\exists y\ Girl(y) \wedge \forall x\ (Boy(x) \Rightarrow Likes(x, y))) \wedge$$
$$\neg (\forall z\ Boy(z) \Rightarrow \exists u\ (Girl(u) \wedge Likes(z, u)))$$

converting to CNF:

$$\underline{Girl(G)} \wedge \forall x\ (\underline{\neg Boy(x) \vee Likes(x, G)}) \wedge$$

$$\neg (\forall z\ \neg Boy(z) \vee \exists u\ (Girl(u) \wedge Likes(z, u)))$$

$$\exists z\ Boy(z) \wedge \neg \exists u\ (Girl(u) \wedge Likes(z, u))$$

$$\underline{Boy(B)} \wedge \forall u\ (\underline{\neg Girl(u) \vee \neg Likes(B, u)})$$

KB of 4 CNF sentences:

1. $Girl(G)$
2. $\neg Boy(x) \vee Likes(x, G)$
3. $Boy(B)$
4. $\neg Girl(u) \vee \neg Likes(B, u)$

resolving 1+4 with *{ u / G }*: resolving 2+3 with *{ x / B }*:

5. $\neg Likes(B, G)$
6. $Likes(B, G)$

resolving 5 and 6 yields a contradiction: $\varnothing$

hence the *hypothesis is false* and (ii) $\Rightarrow$ (i)