EE3015 POWER SYSTEMs and PROTECTION

Part II-Fault Analysis and Power System Protection

BY

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INTRODUCTION

- **A power system is a <u>three-phase</u> system because of three-phase voltage sources produced by three-phase generators.**
- **A power system usually operates in normal (<u>three phases are balanced</u>) conditions, a <u>single-phase diagram</u> is used in power system analysis and calculations.**
- **A power system sometime operates in abnormal (<u>three phases are unbalanced</u>) condition when a short circuit occurs. In this case, system must be protected and a <u>single-phase diagram cannot be used for power system analysis</u>.**
- **The main objectives of this part of the lecture:**
  - **To study methods for analyzing power systems under <u>abnormal</u> (unbalanced) conditions.**
  - **To study techniques for protecting power system equipment and network when a abnormal condition occurs.**

COURSE OUTLINE

1. Review of Basic Concepts of Power System Analysis
2. Fault Analysis (voltages and currents after a fault)
3. Power System Protection (Techniques)

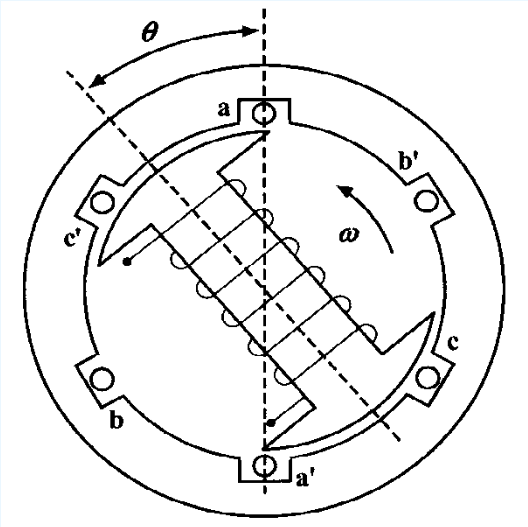
References:

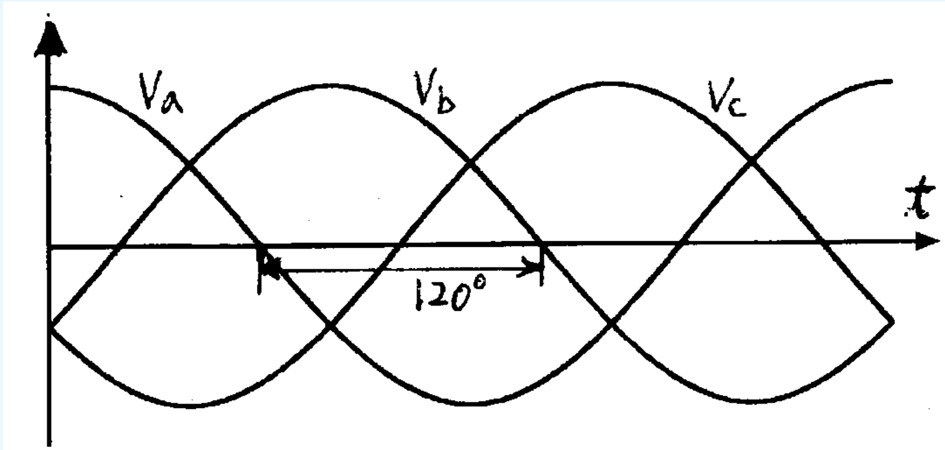
1. Power System Analysis, John J. Grainger and William D. Stevenson, JR., McGraw-Hill, 1994.
2. Power system analysis, Arthur R. Bergen, Vijay Vittal, Prentice Hall, Inc. , 2000.
3. J. Lewis Blackburn, "*Protective Relaying Principle and Application*," Marcel Dekker, Inc. New York, 1997.
4. GEC Measurements, "*Protective Relays Application Guide*," The General Electric Company, p.l.c., of England, London & Wisbech, 1987.
5. P. M. Anderson, "*Power System Protection*," McGraw-Hill, IEEE Press, New York, 1999.

1. Basic Concepts of Power System Operation and Analysis

1.1 Three Phase Generators(three balanced voltage sources)

Three voltage sources in a power system are always balanced. There is 120 degree phase difference between two sources.





Time domain voltage equations:

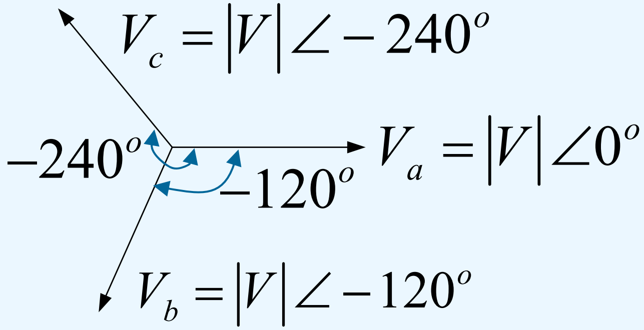
$$v_a(t) = |V|\cos(\omega t)$$

$$v_b(t) = |V|\cos(\omega t - 120^o)$$

$$v_c(t) = |V|\cos(\omega t - 240^o)$$

$$v_a(t) + v_b(t) + v_c(t) = 0$$

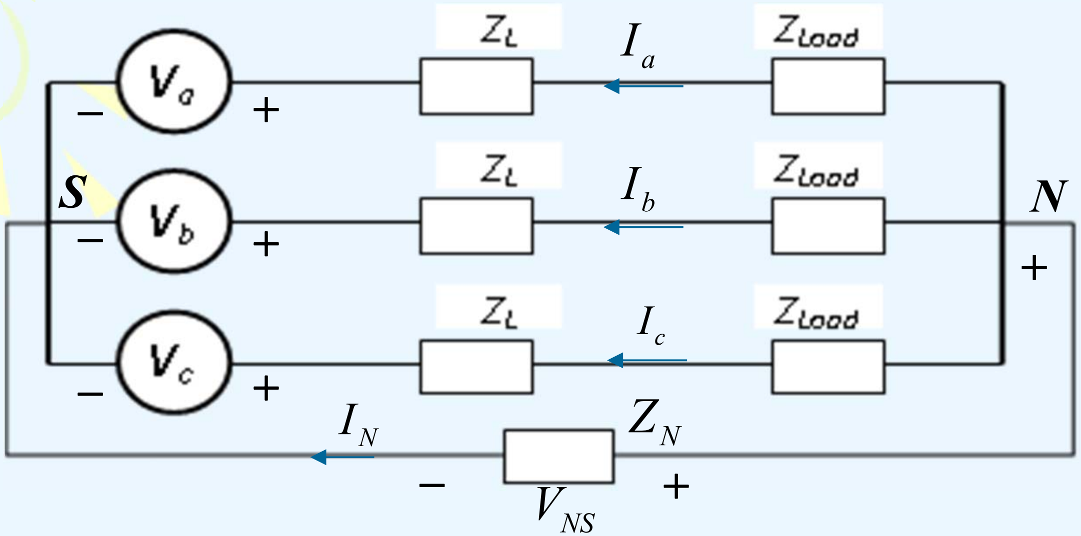
Phasor representation:



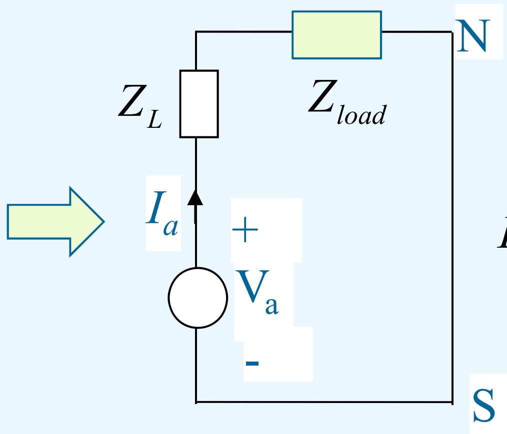
$$V_a + V_b + V_c$$
$$= |V|\angle 0 + |V|\angle -120 + |V|\angle -240 = 0$$

1.2 Three-phase balanced power system (normal operation)

When a three-phase generator supplies three identical loads through three identical lines, the three-phase system is balanced.



Three phase diagram (Y connection)



$$I_a = \frac{V_a}{Z}$$

Single phase diagram

KCL: $I_N + I_a + I_b + I_c = 0 \Rightarrow \frac{V_{NS}}{Z_N} + \frac{V_{NS} - V_a}{Z_L + Z_{load}} + \frac{V_{NS} - V_b}{Z_L + Z_{load}} + \frac{V_{NS} - V_c}{Z_L + Z_{load}} = 0$

$$V_{NS}\left(\frac{1}{Z_N} + \frac{1}{Z_L + Z_{load}} + \frac{1}{Z_L + Z_{load}} + \frac{1}{Z_L + Z_{load}}\right) = \frac{1}{Z_L + Z_{load}}\left(V_a + V_b + V_c\right) \Rightarrow \underline{V_{NS} = 0}$$

$\Rightarrow$ **N and S can be connected together. A three phase power system can be solved by a single phase diagram.**

Example 1.1: A balanced Y-connected load of (2+j3)Ω per phase and a balanced delta-connected load of (6+j6)Ω per phase are connected in parallel to a three-phase, 400V, 50Hz supply. Calculate the current in each supply line, the total power supplied and the overall power factor.

Solutions:

$$Z_{Y2} = Z_{\Delta 2}/3 = \frac{6+j6}{3} = 2.828427\angle 45^o$$

$$Z_{Y1} = 2 + j3 = 3.60555\angle 56.31^o$$

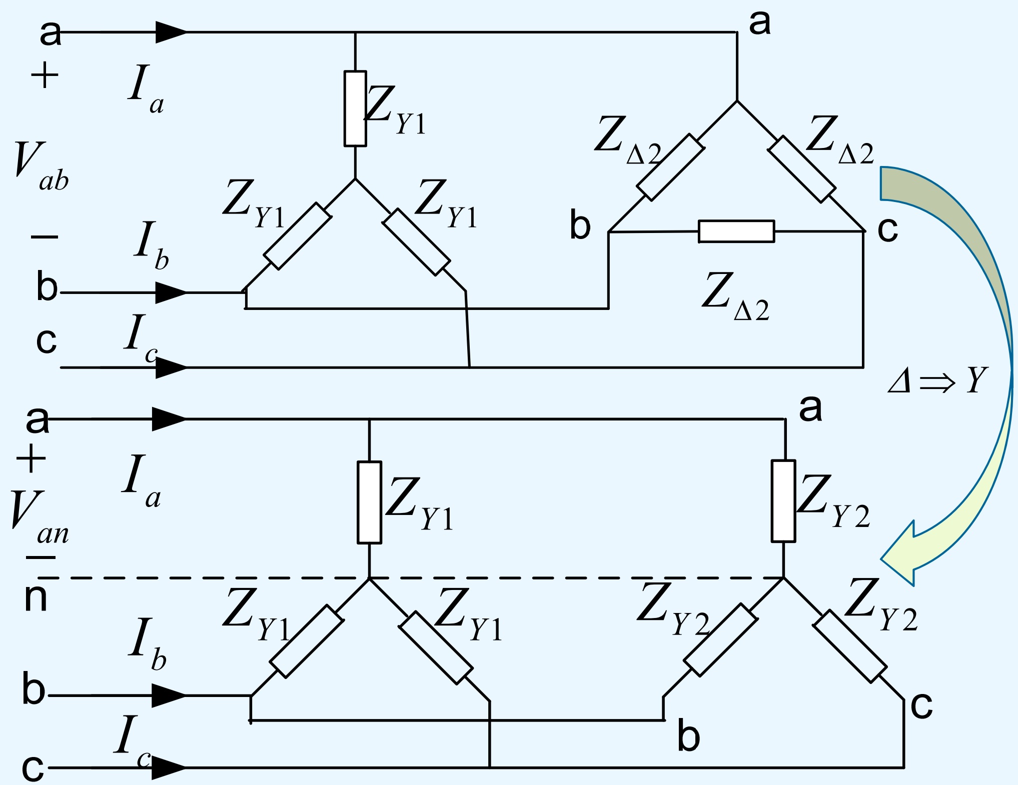
$$Z_{ta} = Z_{Y1}//Z_{Y2} = \frac{Z_{Y1}Z_{Y2}}{Z_{Y1}+Z_{Y2}} = 1.592666\angle 49.97^o$$

$$V_{ab} = 400V, \quad V_{an} = \frac{400V}{\sqrt{3}}\angle 0° = 230.940$$

$$I_a = \frac{V_{an}}{Z_{ta}} = 145.002\angle -49.97^o$$

$$p.f. = \cos(49.97°) = 0.6432$$

$$S = 3V_{an}I_a^* = 100.46\angle 49.97^o\, kVA$$



***All delta connected loads and components must be converted to Y connection for single phase analysis.**

1.3 Unbalance three-phase systems (abnormal operation)

When a short circuit fault occurs cross phase-*a* load in the power system as shown in the figure, the system becomes unbalanced (<u>abnormal</u> operation). The system <u>cannot</u> be solved by <u>a single phase circuit</u> and can be solved by <u>KCL and KVL equations</u>.

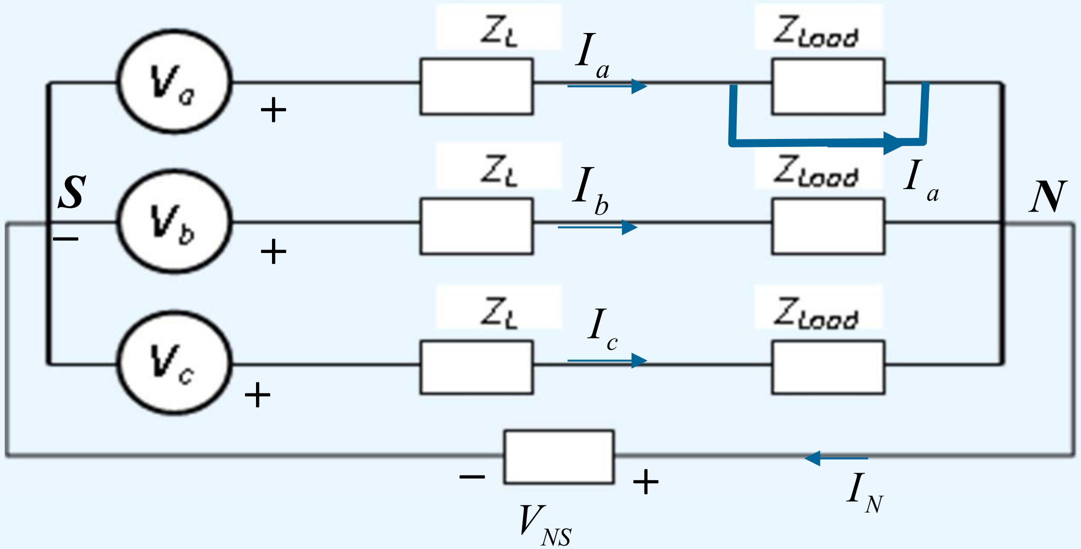
KCL at node N:

$$I_N = I_a + I_b + I_c$$

$$I_a = \frac{V_a - V_{NS}}{Z_L}$$

$$I_b = \frac{V_b - V_{NS}}{Z_L + Z_{load}}$$

$$I_c = \frac{V_c - V_{NS}}{Z_L + Z_{load}}$$

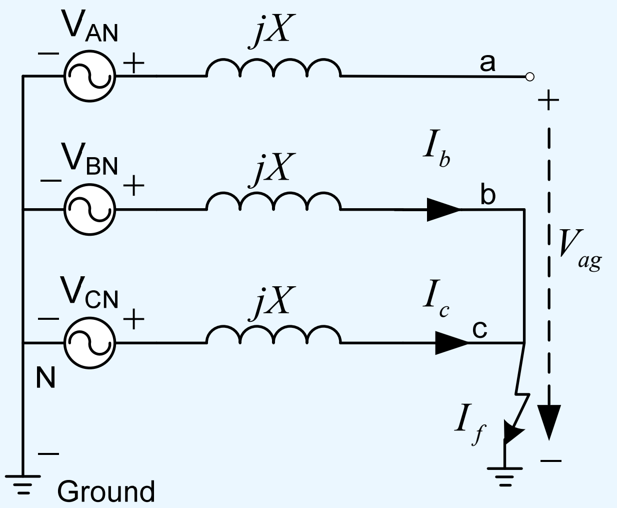


$$I_N = I_a + I_b + I_c \Rightarrow \frac{V_{NS}}{Z_N} = \frac{V_a - V_{NS}}{Z_L} + \frac{V_b - V_{NS}}{Z_L + Z_{load}} + \frac{V_c - V_{NS}}{Z_L + Z_{load}}$$

$$V_{NS} = \left(\frac{V_a}{Z_L} + \frac{V_b}{Z_L + Z_{load}} + \frac{V_c}{Z_L + Z_{load}}\right)\left(\frac{1}{Z_N} + \frac{1}{Z_L} + \frac{1}{Z_L + Z_{load}} + \frac{1}{Z_L + Z_{load}}\right)^{-1} \neq 0$$

Example 1.2 A double line-to-ground short circuit occurs on phases *b* and *c* of a three-phase power system as shown in the figure below. Calculate the voltage from generator terminal to ground and the current I_f, where $V_{AN} = V_{rms}\angle 0^0, V_{BN} = V_{rms}\angle -120^0, V_{CN} = V_{rms}\angle +120^0$

Solutions:



$$V_{ag} = V_{AN} = V_{rms}\angle 0^0$$

$$I_b = \frac{V_{BN}}{jX}$$

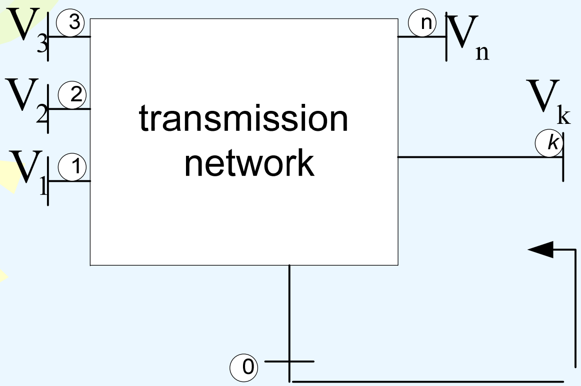
$$I_c = \frac{V_{CN}}{jX}$$

KCL: $I_f = I_b + I_c = \frac{V_{BN}}{jX} + \frac{V_{CN}}{jX}$

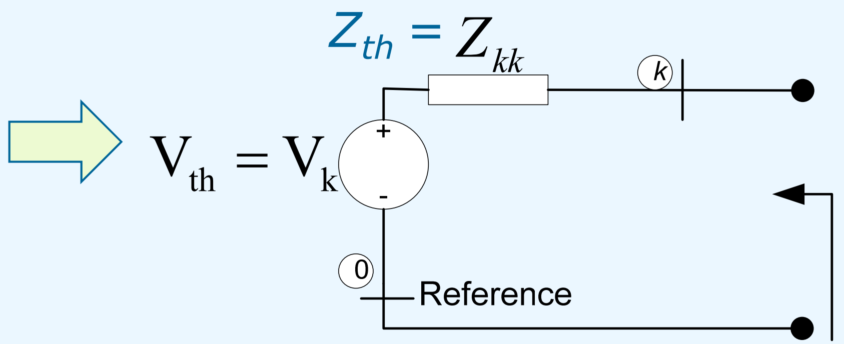
$$= \frac{V_{rms}}{X}\left(\angle -120^o + \angle -240^o\right)\angle -90^o$$

$$= \frac{V_{rms}}{X}\left(\angle -210^o + \angle -330^o\right)$$

1.4 Thevenin Equivalent Viewed from Bus k (Review)



A N-bus Power System

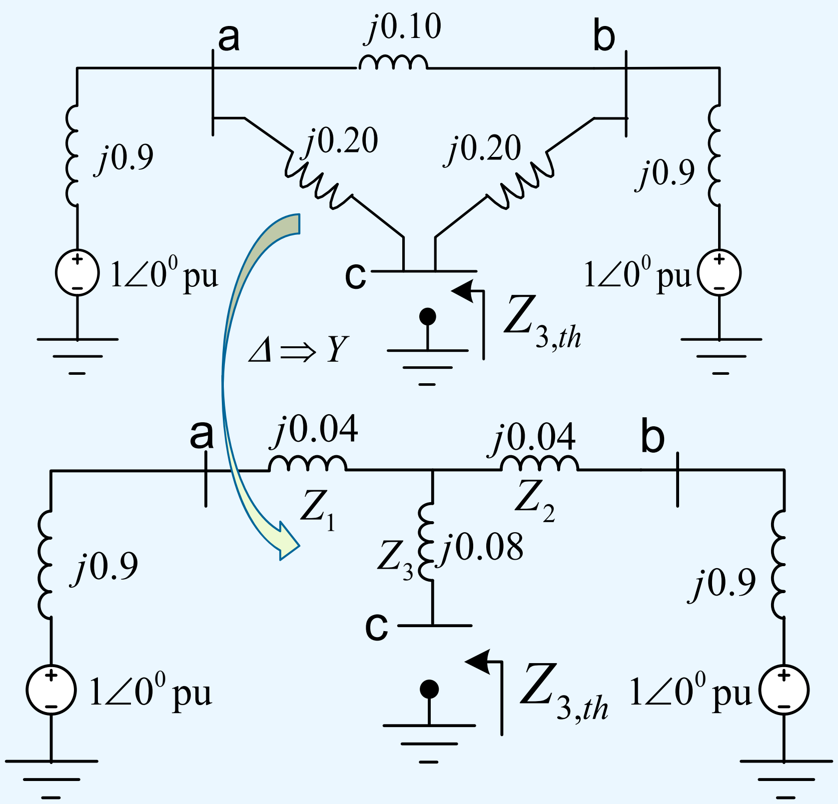


Thevenin Equivalent from Bus k

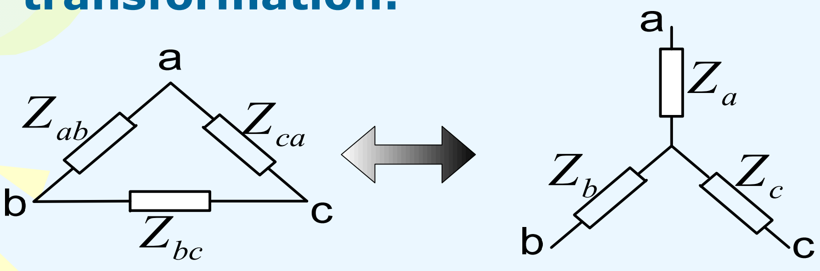
<u>Direct method</u>: Thevenin impedance Z_{th} is the total impedance seen from bus k when kill all sources in the network. V_{th} is open circuit voltage when there is no impedance from bus k to ground.

V_{th} is the pre-fault voltage at bus k, denoted by V_f in the fault analysis.

Example 1. 3: Find the Thevenin Equivalent seen from Bus 3 of the the following power system.



Δ-Y transformation:

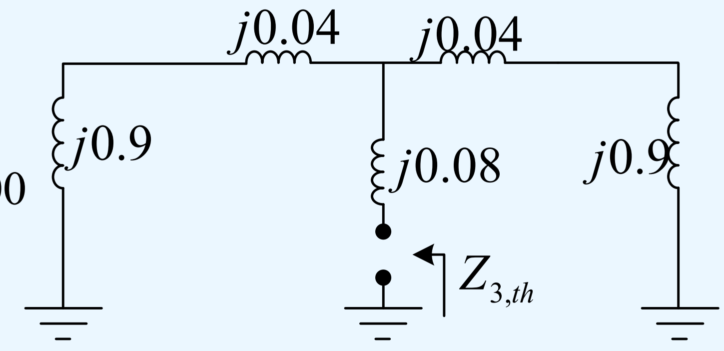


$$Z_a = \frac{Z_{ab} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}} \qquad Z_b = \frac{Z_{ab} Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

$$Z_C = ?$$

$$Z_a = Z_b = \frac{j0.20 \times j0.10}{j0.20 + j0.20 + j0.10} = j0.04$$

$$Z_c = \frac{j0.20 \times j0.20}{j0.20 + j0.20 + j0.10} = j0.08$$



Thevenin impedance from bus 3:

$$Z_{3,th} = j0.08 + (j0.04 + j0.9) \,||\, (j0.04 + j0.9) = j0.5500$$

Thevenin voltage: $V_{3,th} = 1\angle 0^0$ **because there is no current in circuit and no voltage drop.**

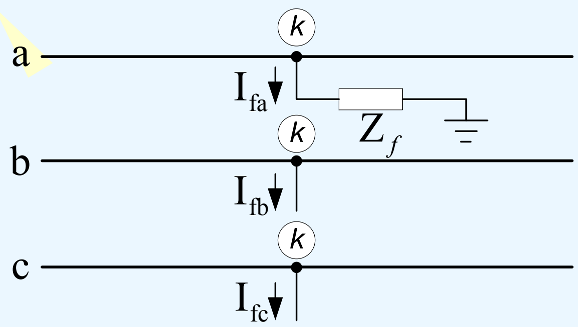
1.5 Important concepts (to add more??)

1) Three phase diagram for a unbalance power system with transformers, generators and transmission lines
2) Single phase pu impedance diagram for a balance power system
3) Δ-Y transformation to simplify complex system configuration into series and parallel system
4) Thevenin equivalent circuit (direct method)
5) The methods for circuit analysis (KCL, KVL, current division, etc)
6) Three phase power formula?
7) Phasor diagram
8) Fault level (fault MVA)

2. Fault Analysis

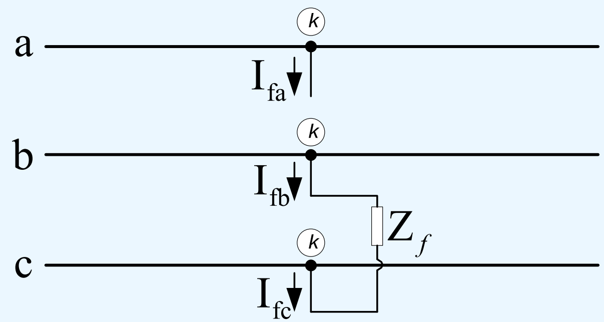
2.1 Faults types

A power system fault is an event which causes abnormal system operation. Faults can be classified into open or short circuit faults. The short circuit faults can be classified into symmetric (type 4) and unsymmetrical faults (types 1, 2 and 3).



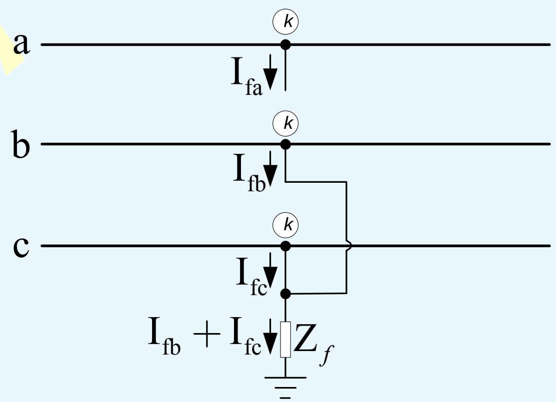
Type 1

Single line-to-ground fault



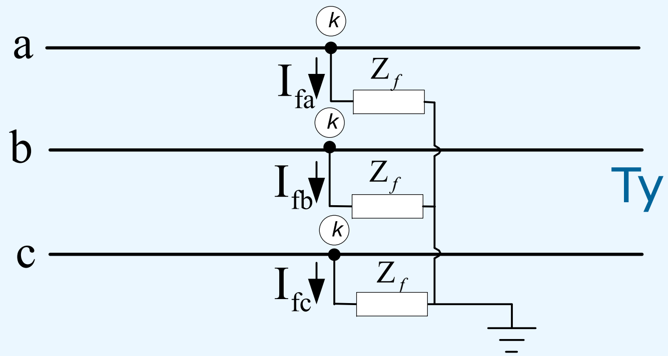
Type 2

Line-to-line fault or double phase fault



Type 3

Double line-to-ground fault

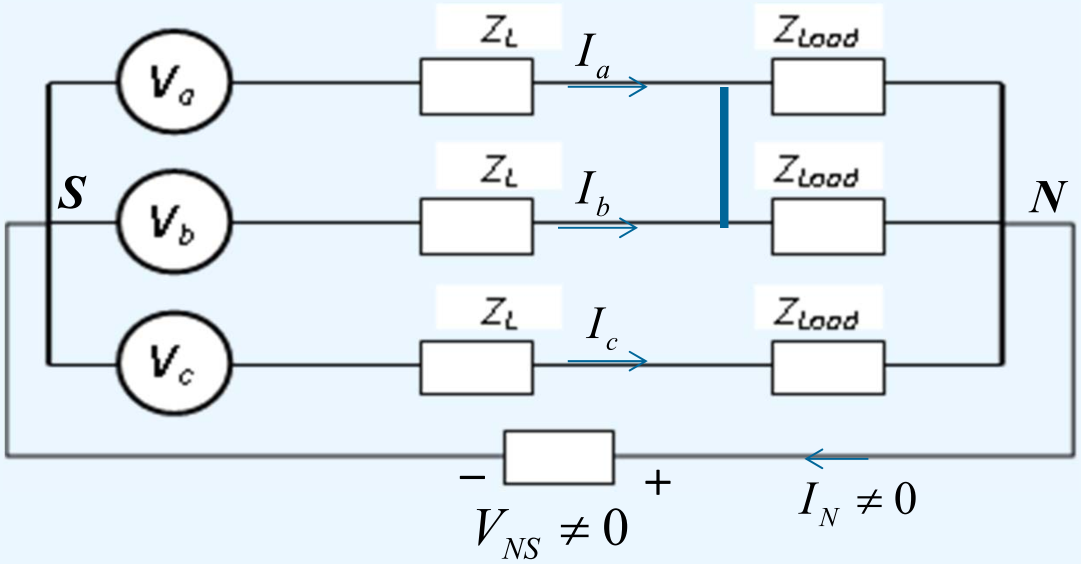


Type 4

Three-phase short-circuit fault

2.2 How to analyze a unbalanced power system?

when a short circuit fault occurs between phase *a* and *b* in the power system as shown in the following figure, the system becomes unbalanced.



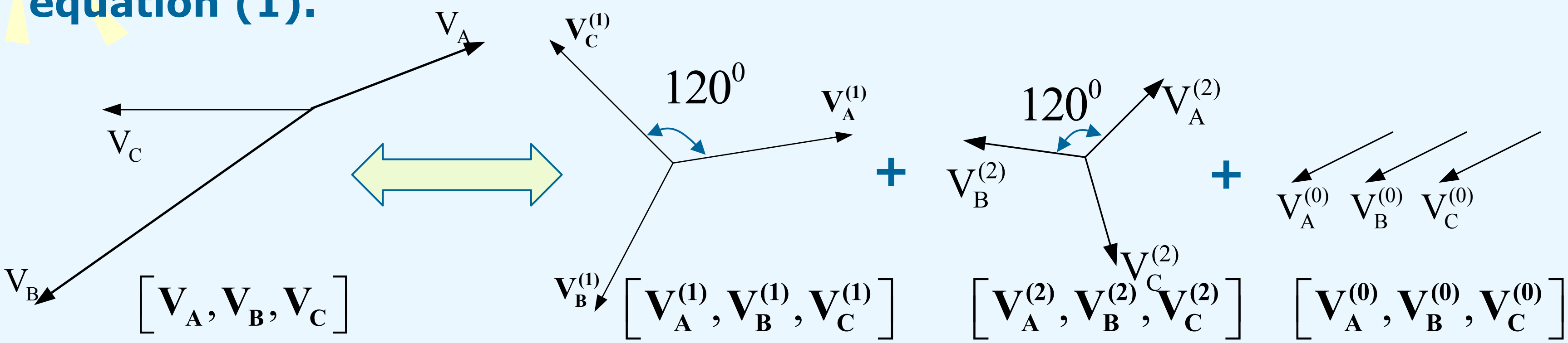
As discussed in Chapter 1, the single-phase diagram can not be used to calculate voltages and currents of a unbalanced system .

In fault analysis, <u>a simple single phase method </u>is introduced in this section to calculate the voltages and currents of a power system under <u>unbalanced conditions</u>.

2.3 Sequence components of 3 unbalanced phasors

It has been proven mathematically that <u>an unbalanced set</u> of three-phase voltages or currents could be broken down into the <u>three balanced sequence sets</u> each with three-phase sequence components.

Or: three unbalanced voltages are the sum of the respective balanced sequence components as shown in the following phasor diagram or in equation (1).



<u>An unbalanced set</u> **<u>Positive sequence set</u>** **<u>Negative sequence set</u>** **<u>Zero sequence set</u>**

$$\begin{aligned}
\mathbf{V_A} &= \mathbf{V}_A^{(0)} + \mathbf{V}_A^{(1)} + \mathbf{V}_A^{(2)} \\
\mathbf{V_B} &= \mathbf{V}_B^{(0)} + \mathbf{V}_B^{(1)} + \mathbf{V}_B^{(2)} \\
\mathbf{V_C} &= \mathbf{V}_C^{(0)} + \mathbf{V}_C^{(1)} + \mathbf{V}_C^{(2)}
\end{aligned} \qquad \textbf{(1)}$$

Introducing phasor constant $a = 1\angle 120^0$ **,**

The properties of *a* are as follows:

$$a^2 = a \times a = 1\angle 120^0 \times 1\angle 120^0 = 1\angle 240^0$$

$$a^3 = 1; \quad a^4 = a; \quad 1 + a + a^2 = 0$$

<u>Positive-sequence components</u>: A set of three balance positive-sequence components have the same magnitude, displaced from each other by 120⁰ with the sequence ABC. ($a = 1\angle 120^0 \quad a^2 = 1\angle 240^0$)

$$\mathbf{V_A^{(1)}} = |V|\angle 0^o$$

$$\mathbf{V_B^{(1)}} = \mathbf{V_A^{(1)}} \times 1\angle -120^0 = \mathbf{V_A^{(1)}} \times 1\angle 240^0 = a^2\mathbf{V_A^{(1)}} = |V|\angle -120^o$$

$$\mathbf{V_C^{(1)}} = \mathbf{V_A^{(1)}} \times 1\angle -240^0 = \mathbf{V_A^{(1)}} \times 1\angle 120^0 = a\mathbf{V_A^{(1)}} = |V|\angle -240^o$$

Note: $V_{\mathrm{A}}^{(1)} + V_{\mathrm{B}}^{(1)} + V_C^{(1)} = 0$

<u>Negative-sequence components</u>: A set of three balance negative-sequence components have the same magnitude, displaced from each other by 120⁰ with the sequence ACB.

$$\mathbf{V_A^{(2)}} = |V|\angle 0^o$$

$$\mathbf{V_B^{(2)}} = \mathbf{V_A^{(2)}} \times 1\angle 120^0 = \mathbf{V_A^{(2)}} \times 1\angle -240^0 = a\mathbf{V_A^{(2)}} = |V|\angle 120^o$$

$$\mathbf{V_C^{(2)}} = \mathbf{V_A^{(2)}} \times 1\angle -120^0 = \mathbf{V_A^{(2)}} \times 1\angle 240^0 = a^2\mathbf{V_A^{(2)}} = |V|\angle 240^o$$

Note: $V_{\mathrm{A}}^{(2)} + V_{\mathrm{B}}^{(2)} + V_C^{(2)} = 0$

<u>Zero-sequence components</u>: A set of three balance zero-sequence components consisting of three phasors equal in magnitude and phase.

$$\mathbf{V_A^{(0)}} = \mathbf{V_B^{(0)}} = \mathbf{V_C^{(0)}} \qquad \text{Note: } V_{\mathrm{A}}^{(0)} + V_{\mathrm{B}}^{(0)} + V_C^{(0)} = 3V_{\mathrm{A}}^{(0)}$$

(Replace sequence components of phase B and C by sequence elements of phase A in (1):

$$\begin{aligned}
V_A &= V_A^{(0)} + V_A^{(1)} + V_A^{(2)} = V_A^{(0)} + V_A^{(1)} + V_A^{(2)} \\
V_B &= V_B^{(0)} + V_B^{(1)} + V_B^{(2)} = V_A^{(0)} + a^2 V_A^{(1)} + a V_A^{(2)} \\
V_C &= V_C^{(0)} + V_C^{(1)} + V_C^{(2)} = V_A^{(0)} + a V_A^{(1)} + a^2 V_A^{(2)}
\end{aligned}$$

Matrix form of (1): (find phase components from sequence components)

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix} = \mathbf{A} \begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix} \quad (2)$$

where $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$

Multiply both side of (2) by A^{-1}: (find sequence components from phase components)

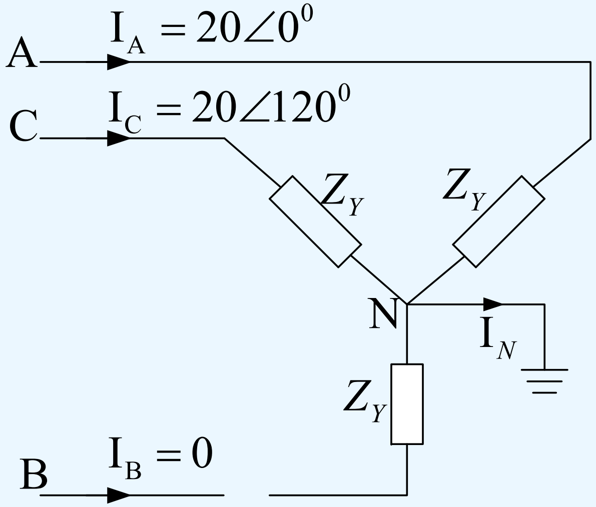
$$\begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix} = A^{-1} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} \quad (3)$$

where $\mathbf{A}^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$

Or $V_A^{(0)} = \frac{1}{3}(V_A + V_B + V_C)$ $\quad V_A^{(1)} = \frac{1}{3}(V_A + a V_B + a^2 V_C)$ $\quad V_A^{(2)} = \frac{1}{3}(V_A + a^2 V_B + a V_C)$

Example 2.1: A three-phase line feeding a balanced Y connected load has phase *b* opened. The load neutral is grounded, and the unbalanced line currents are shown in the figure.

Calculate
(a) the sequence currents in phase a;
(b) sequence currents in phase b and I_B
(c) the neutral current I_N.



Solution:

$$\begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \mathbf{A}^{-1}\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

a)

$$I_A^{(0)} = \frac{1}{3}(I_A + I_B + I_C) = \frac{1}{3}(20\angle 0^0 + 0 + 20\angle 120^0) = 6.666\angle 60^0 (A)$$

$$I_A^{(1)} = \frac{1}{3}(I_A + aI_B + a^2 I_C) = \frac{1}{3}\left[20\angle 0^0 + 0 + 20\angle(120^0 + 240^0)\right] = 13.333\angle 0^0 (A)$$

$$I_A^{(2)} = \frac{1}{3}(I_A + a^2 I_B + aI_C) = \frac{1}{3}\left[20\angle 0^0 + 0 + 20\angle(120^0 + 120^0)\right] = 6.666\angle -60^0 (A)$$

b) The sequence components in phase b:

$$\mathrm{I}_B^{(0)} = \mathrm{I}_A^{(0)} = 6.666\angle 60^0 \qquad (\mathrm{A})$$

$$\mathrm{I}_B^{(1)} = a^2\mathrm{I}_A^{(1)} = 13.333\angle 240^0 \qquad (\mathrm{A})$$

$$\mathrm{I}_B^{(2)} = a\mathrm{I}_A^{(2)} = 6.666\angle(-60^0 + 120^0) = 6.666\angle 60^0 \qquad (\mathrm{A})$$

$$\mathrm{I}_B = \mathrm{I}_B^{(0)} + \mathrm{I}_B^{(1)} + \mathrm{I}_B^{(2)} = 6.666\angle 60^0 + 13.333\angle 240^0 + 6.666\angle 60^0 = 0$$

c) $$\mathrm{I}_N = \mathrm{I}_A + \mathrm{I}_B + \mathrm{I}_C = 20\angle 0^0 + 0 + 20\angle 120^0 = 20\angle 60^0 = 3\mathrm{I}_A^{(0)}$$

This result means that <u>only zero sequence currents</u> can flow from neutral to ground.

There are no positive current and negative current from the neutral to ground because

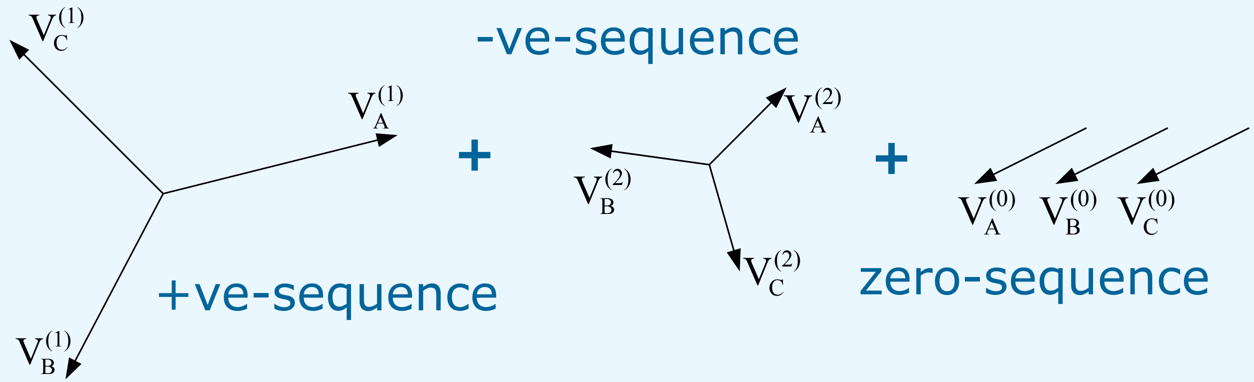
$$I_A^{(1)} + I_B^{(1)} + I_C^{(1)} = 0$$

$$I_A^{(2)} + I_B^{(2)} + I_C^{(2)} = 0$$

Summary

- A unsymmetrical set can be decomposed into three symmetrical sets:

$$V_A = V_A^{(0)} + V_A^{(1)} + V_A^{(2)}$$
$$V_B = V_B^{(0)} + V_B^{(1)} + V_B^{(2)}$$
$$V_C = V_C^{(0)} + V_C^{(1)} + V_C^{(2)}$$



- Transform phase components to sequence components:

$$\underline{\underline{\begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \mathbf{A}^{-1}\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}}}$$

$$\begin{aligned} V_B^{(0)} &= V_A^{(0)} & V_C^{(0)} &= V_A^{(0)} \\ V_B^{(1)} &= a^2 V_A^{(1)} & V_C^{(1)} &= a V_A^{(1)} \\ V_B^{(2)} &= a V_A^{(2)} & V_C^{(2)} &= a^2 V_A^{(2)} \end{aligned}$$

- Transform sequence components to phase components:

$$\underline{\underline{\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix} = \mathbf{A}\begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix}}}$$

2.4 Sequence circuits of power system components

During normal operation of a power system, three voltage sources are balanced and operate in positive sequence. Voltage and current calculation is based on positive sequence circuits of system components such as generators, transformers and transmission lines.

When system operates in unbalanced conditions, voltages and currents are not balanced. Those unbalanced voltages and currents can be represented by positive, negative and zero sequence voltages and currents. Therefore positive, negative and zero sequence circuits for system components are required for power system analysis under abnormal conditions.

In this section, single phase positive, negative and zero sequence circuits for system components will be introduced to analyze system sequence voltages and currents during abnormal operation.

The main objective is to find the relationship of sequence voltage and sequence current and the associate sequence circuit for system components.

2.4.1 Sequence circuits of a 3-phase Y-connected load with neutral impedance Z_N

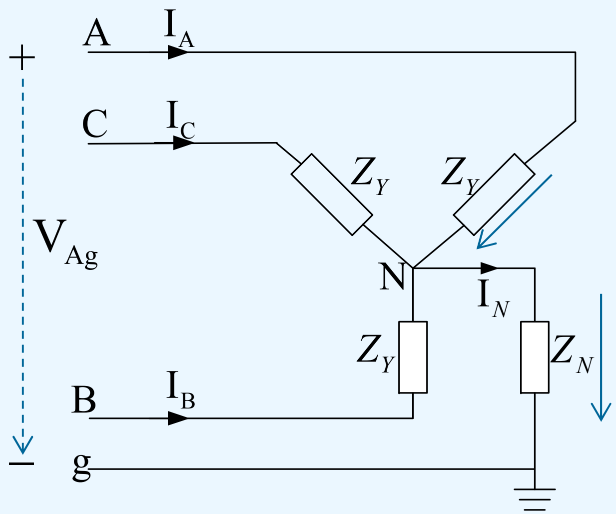
a) The relationships of unbalanced phase voltages and currents:

KVL: $$\mathrm{V_{Ag}} = Z_Y\mathrm{I_A} + Z_N\mathrm{I_N} = Z_Y\mathrm{I_A} + Z_N(\mathrm{I_A}+\mathrm{I_B}+\mathrm{I_C})$$

$$= (Z_Y + Z_N)\mathrm{I_A} + Z_N\mathrm{I_B} + Z_N\mathrm{I_C} \quad (1)$$

$$\mathrm{V_{Bg}} = Z_N\mathrm{I_A} + (Z_N + Z_Y)\mathrm{I_B} + Z_N\mathrm{I_C} \quad (2)$$

$$\mathrm{V_{Cg}} = Z_N\mathrm{I_A} + Z_N\mathrm{I_B} + (Z_Y + Z_N)\mathrm{I_C} \quad (3)$$



Three phase circuit

Matrix form of KVL equations (1)-(3):

$$\begin{bmatrix}\mathrm{V_{Ag}}\\ \mathrm{V_{Bg}}\\ \mathrm{V_{Cg}}\end{bmatrix} = \begin{bmatrix} Z_Y+Z_N & Z_N & Z_N \\ Z_N & Z_Y+Z_N & Z_N \\ Z_N & Z_N & Z_Y+Z_N \end{bmatrix}\begin{bmatrix}\mathrm{I_A}\\ \mathrm{I_B}\\ \mathrm{I_C}\end{bmatrix} = Z\begin{bmatrix}\mathrm{I_A}\\ \mathrm{I_B}\\ \mathrm{I_C}\end{bmatrix} \quad (4)$$

Replacing phase voltages and currents of (4) by the sequence voltages (5) and currents (6) and multiplying both sides of (4) by $\mathbf{A}^{-1}$, gives (7)

$$\begin{bmatrix}\mathrm{V_{Ag}}\\ \mathrm{V_{Bg}}\\ \mathrm{V_{Cg}}\end{bmatrix} = \mathbf{A}\begin{bmatrix}\mathrm{V_{Ag}^{(0)}}\\ \mathrm{V_{Ag}^{(1)}}\\ \mathrm{V_{Ag}^{(2)}}\end{bmatrix} \quad (5) \qquad \begin{bmatrix}\mathrm{I_A}\\ \mathrm{I_B}\\ \mathrm{I_C}\end{bmatrix} = \mathbf{A}\begin{bmatrix}\mathrm{I_A^{(0)}}\\ \mathrm{I_A^{(1)}}\\ \mathrm{I_A^{(2)}}\end{bmatrix} \quad (6) \qquad \begin{bmatrix}\mathrm{V_{Ag}^{(0)}}\\ \mathrm{V_{Ag}^{(1)}}\\ \mathrm{V_{Ag}^{(2)}}\end{bmatrix} = \mathbf{A}^{-1}Z\mathbf{A}\begin{bmatrix}\mathrm{I_A^{(0)}}\\ \mathrm{I_A^{(1)}}\\ \mathrm{I_A^{(2)}}\end{bmatrix} \quad \mathbf{(7)}$$

b) Multiplying $\mathbf{A}^{-1}\mathbf{ZA}$ in (7), the relationship of sequence voltages, currents and impedances are as:

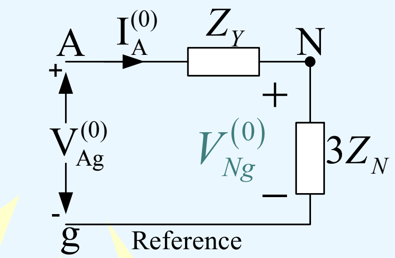
$$\begin{bmatrix} V_{Ag}^{(0)} \\ V_{Ag}^{(1)} \\ V_{Ag}^{(2)} \end{bmatrix} = \begin{bmatrix} Z_Y + 3Z_N & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix} \begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix}$$

or

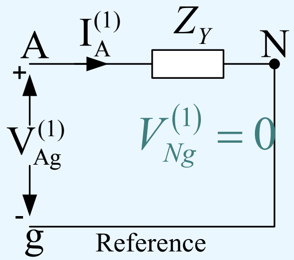
$$\begin{aligned} V_{Ag}^{(0)} &= (Z_Y + 3Z_N) I_A^{(0)} = Z_0 I_A^{(0)} \\ V_{Ag}^{(1)} &= Z_Y I_A^{(1)} = Z_1 I_A^{(1)} \qquad (8) \\ V_{Ag}^{(2)} &= Z_Y I_A^{(2)} = Z_2 I_A^{(2)} \end{aligned}$$

where $Z_0=Z_Y+3Z_N$, $Z_1=Z_Y$ and $Z_2=Z_Y$ are positive, negative and zero sequence impedance respectively.

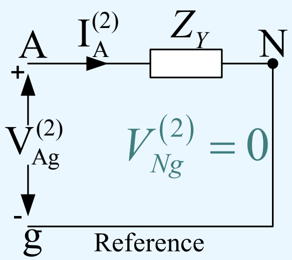
c) From (8), phase-A sequence circuits of the load:



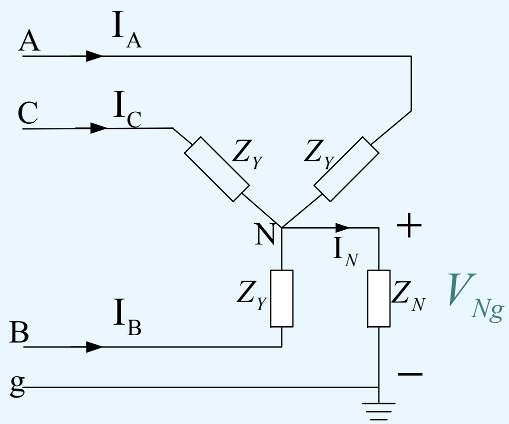
(a) Zero-sequence circuit



(b) Positive-sequence circuit



(c) Negative-sequence circuit



Y connected load

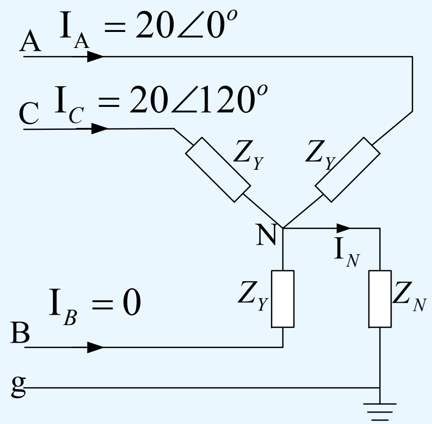
The sequence circuits of a complicated three phase load circuit for phase-*A* is very simple. Positive and negative impedances is the same with the load impedance. Zero sequence impedance is $Z_0 = Z_Y + 3Z_N$.

From above circuits: $V_{Ng} = I_N Z_N = V_{Ng}^{(0)} + 0 + 0 = 3I_A^{(0)} Z_N \Rightarrow I_N = 3I_A^{(0)}$

* $I_N = 3I_A^{(0)}$ **shows that only three zero sequence currents through Z_N.**

Example 2.2: A three phase Y connected load with neutral impedance Z_N =j1Ω. Z_Y =j1Ω. If three phase unbalanced currents are supplied to the three phase loads,

a) Determine three sequence currents.
b) Determine three sequence impedances.
c) Calculate sequence voltages for phase-a.
d) Calculate voltage from neutral to ground.



Solutions:

a) $$I_A^{(0)} = \frac{1}{3}(I_A + I_B + I_C) = \frac{1}{3}(20\angle 0^0 + 0 + 20\angle 120^0) = 6.666\angle 60^0 (A)$$

$$I_A^{(1)} = \frac{1}{3}(I_A + aI_B + a^2 I_C) = \frac{1}{3}\left[20\angle 0^0 + 0 + 20\angle(120^0 + 240^0)\right] = 13.333\angle 0^0 (A)$$

$$I_A^{(2)} = \frac{1}{3}(I_A + a^2 I_B + aI_C) = \frac{1}{3}\left[20\angle 0^0 + 0 + 20\angle(120^0 + 120^0)\right] = 6.666\angle -60^0 (A)$$

b) Sequence impedances: $Z_1 = Z_2 = Z_Y = j1\,\Omega;\ \ Z_0 = Z_Y + 3Z_N = j4\,\Omega$

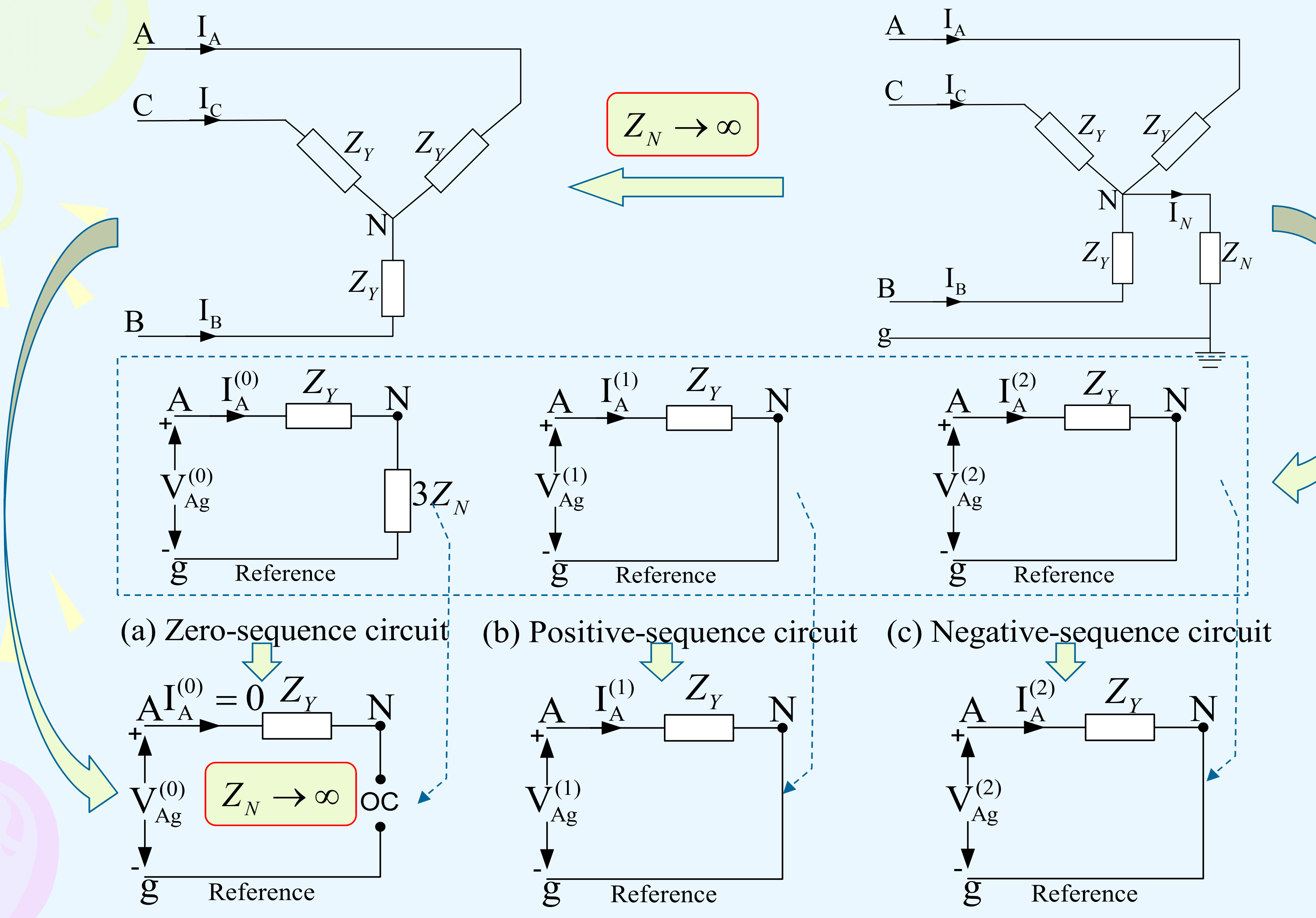
c) Sequence voltages: $V_{Ag}^{(1)} = Z_1 I_A^{(1)} = j1 \times 13.333 = j13.333\ \ (V)$

$$V_{Ag}^{(2)} = Z_1 I_A^{(2)} = j1 \times 6.666\angle -60^o = 6.666\angle 30^o\ \ (V)$$

$$V_{Ag}^{(0)} = Z_0 I_A^{(0)} = j4 \times 6.666\angle 60^o = 26.664\angle 150^o\ \ (V)$$

d) $V_N = I_N Z_N = 3I_A^{(0)} Z_N \left(\because I_N = 3I_A^{(0)}\right) = j3 \times 6.666\angle 60^o = 19.998\angle 150^o\ \ (V)$

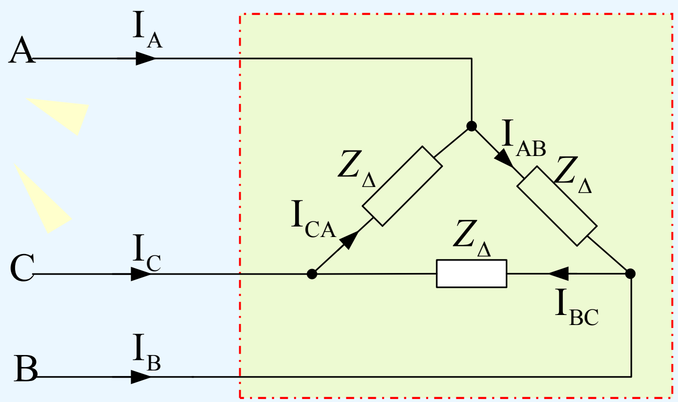
2.4.2 a Y-connected load without neutral impedance



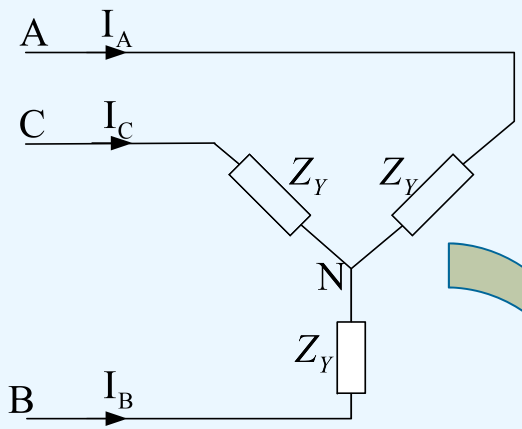
(a) Zero-sequence circuit (b) Positive-sequence circuit (c) Negative-sequence circuit

2.4.3 a 3-phase Δ-connected balanced load

Convert Δ-connected loads into Y-connected:

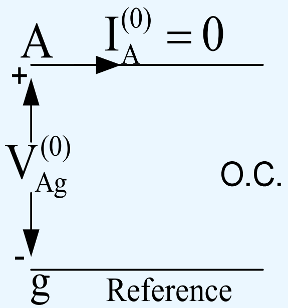


$$Z_Y = \frac{1}{3} Z_\Delta$$

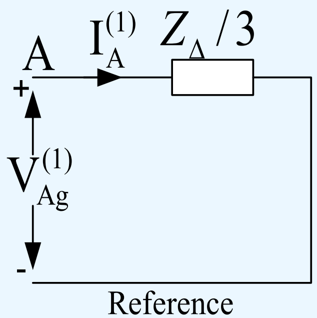


$Z_N = \infty$

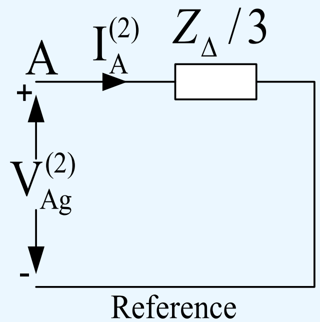
Sequence circuits:



(a) Zero-sequence circuit



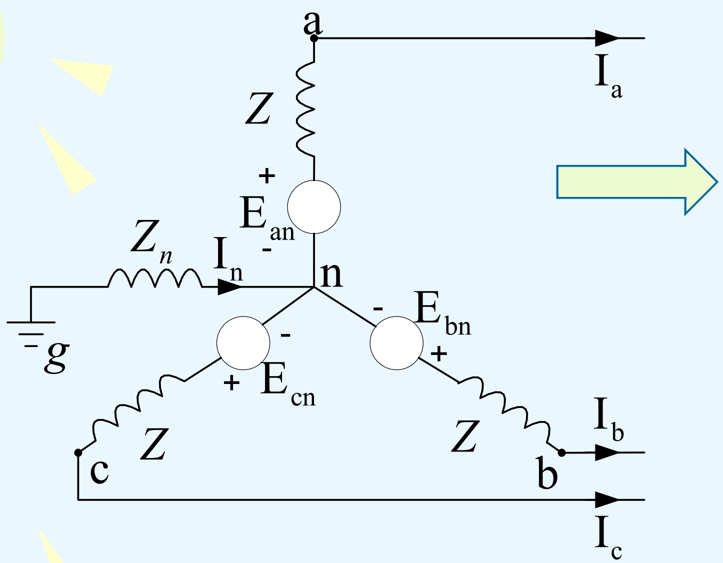
(b) Positive-sequence circuit

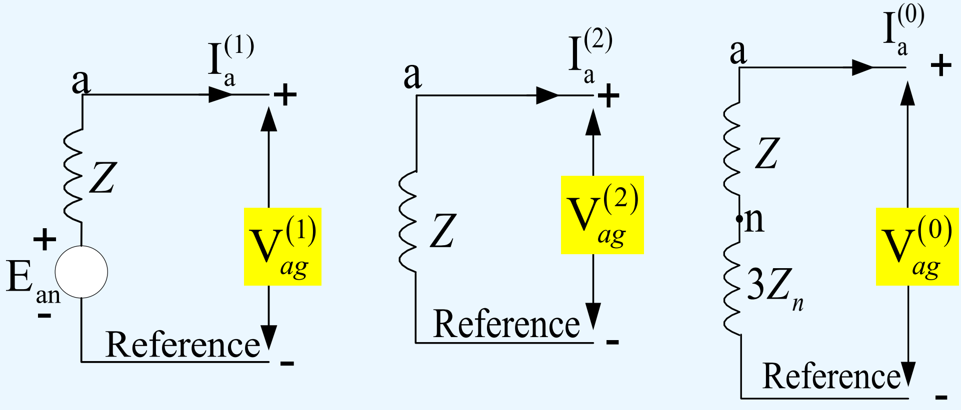


(c) Negative-sequence circuit

2.4.4 Sequence circuits of a synchronous generator

Using the similar method as 2.4.1, the sequence circuits are as: (See appendix 1)





Positive, negative and zero sequence circuits

Sequence circuit equations:

$$\begin{cases} V_{ag}^{(1)} = E_{an} - ZI_a^{(1)} = E_{an} - Z_1 I_a^{(1)} \\ V_{ag}^{(2)} = -ZI_a^{(2)} = -Z_2 I_a^{(2)} \\ V_{ag}^{(0)} = -\left(Z + 3Z_n\right) I_a^{(0)} = -Z_0 I_a^{(0)} \end{cases}$$

***Only zero sequence current through Z_n.**

***For neutral solidly grounded $Z_n = 0$ and for neutral opened $Z_n = \infty$.**

2.4.5 Sequence circuits of a synchronous motor

When a fault occurs, synchronous motors temporarily act as generators. The sequence networks are the same with generator.

2.4.6 Sequence circuits of transformers

A transformer is connected between two buses. Three windings on each side can be connected into Y or Δ. Therefore, there are six type transformers based on connections.

Type 1: Three-phase Y-Y transformer with the two grounded neutrals. Z is the total impedance of transformer refer to primary side.

Based on dot convention: I_A and I_a are in phase. V_{An} and V_{an} are in phase.

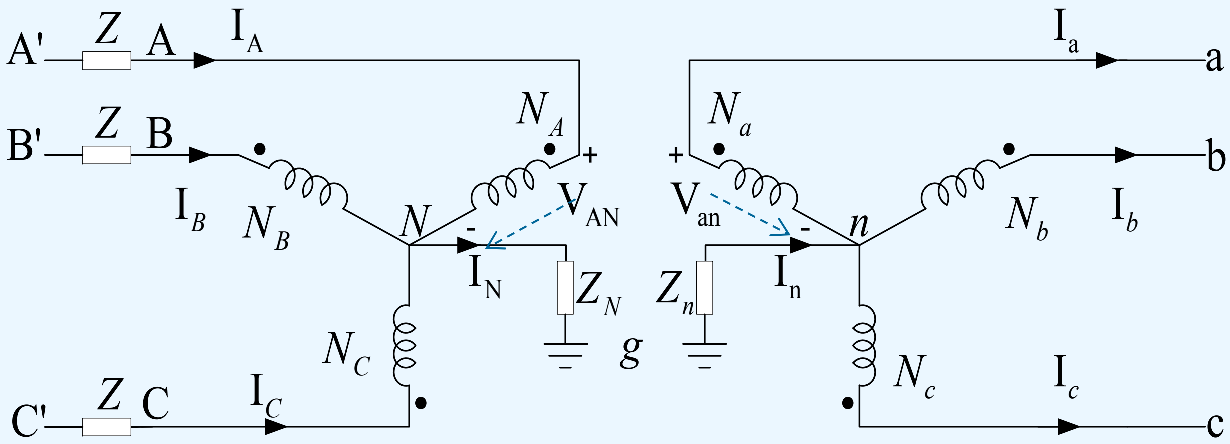
Phasor diagram:

Voltage equation:

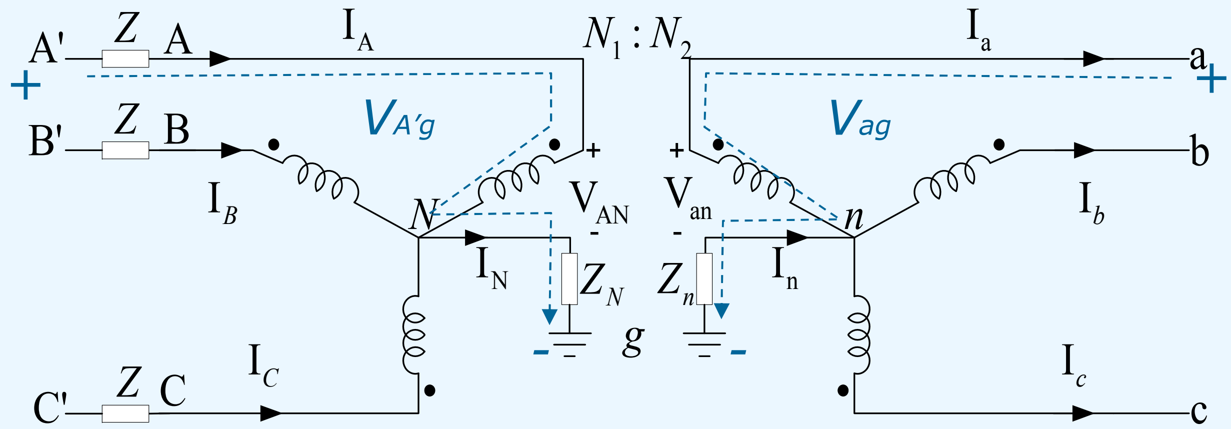
$$\frac{V_{AN}}{V_{an}} = \frac{N_A}{N_a}$$

Current equation:

$$\frac{I_A}{I_a} = \frac{N_a}{N_A}$$



a) The relationship of phase currents, voltages and impedance of the Y-Y transformer under per unit system:



Under per unit system:

$$V_{AN} = V_{an}$$

$$I_A = I_a$$

KVL equations:

Primary side: $V_{A'}\left(V_{A'g}\right) = V_{AN} + ZI_A + Z_N I_N = V_{AN} + \left(Z + Z_N\right) I_A + Z_N I_B + Z_N I_C$

Secondary sides: $V_a\left(V_{ag}\right) = V_{an} - Z_n I_n = V_{an} - Z_n I_a - Z_n I_b - Z_n I_c$

Voltage cross two buses A′ and a:

$$V_{A'} - V_a = \left(Z + Z_N + Z_n\right) I_A + \left(Z_N + Z_n\right) I_B + \left(Z_N + Z_n\right) I_C$$
$$= Z_S I_A + Z_m I_B + Z_m I_C \quad \text{where } \left(Z_S = Z + Z_N + Z_n; Z_m = Z_N + Z_n\right) \qquad (1)$$

Voltage cross B′ and b: $V_{B'} - V_b = Z_m I_A + Z_s I_B + Z_m I_C$ (2)

Voltage cross C′ and c: $V_{C'} - V_c = Z_m I_A + Z_m I_B + Z_s I_C$ (3)

Matrix form of equations (1)-(3):

$$\begin{bmatrix} V_{A'} - V_a \\ V_{B'} - V_b \\ V_{C'} - V_c \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \qquad (4)$$

Replacing phase voltages and currents in (4) by the sequence voltages (5) and currents (6), equation (4) becomes:

$$\begin{bmatrix} V_{A'}-V_a \\ V_{B'}-V_b \\ V_{C'}-V_c \end{bmatrix} = A \begin{bmatrix} V_{A'}^{(0)}-V_a^{(0)} \\ V_{A'}^{(1)}-V_a^{(1)} \\ V_{A'}^{(2)}-V_a^{(2)} \end{bmatrix} \quad (5) \qquad \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = A \begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} \quad (6)$$

$$A \begin{bmatrix} V_{A'}^{(0)}-V_a^{(0)} \\ V_{A'}^{(1)}-V_a^{(1)} \\ V_{A'}^{(2)}-V_a^{(2)} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} A \begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} \quad (7)$$

Multiplying both sides of (7) by A^{-1}, gives:

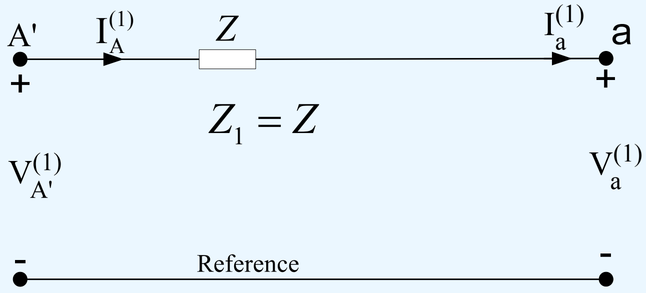
$$A^{-1} A \begin{bmatrix} V_{A'}^{(0)}-V_a^{(0)} \\ V_{A'}^{(1)}-V_a^{(1)} \\ V_{A'}^{(2)}-V_a^{(2)} \end{bmatrix} = \underline{A^{-1} \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} A} \begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} \quad (8)$$

b) Equations of Sequence voltages and currents becomes:

$$\begin{bmatrix} V_{A'}^{(0)}-V_a^{(0)} \\ V_{A'}^{(1)}-V_a^{(1)} \\ V_{A'}^{(2)}-V_a^{(2)} \end{bmatrix} = \begin{bmatrix} Z+3Z_N+3Z_n & & \\ & Z & \\ & & Z \end{bmatrix} \begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} = \begin{bmatrix} Z_0 & & \\ & Z_1 & \\ & & Z_2 \end{bmatrix} \begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} \quad (9)$$

c) From (9), sequence circuits and equations for two neutrals-grounded Y-Y transformer are as follows:

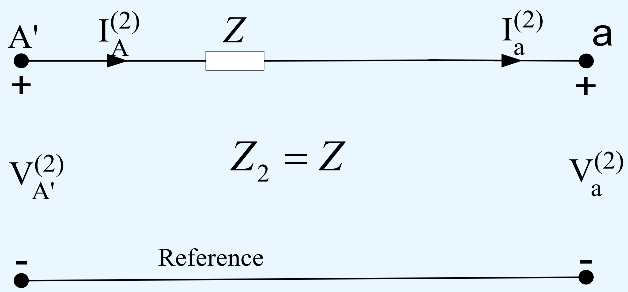
Positive-sequence



$$V_{A'}^{(1)} - V_a^{(1)} = Z_1 I_A^{(1)}$$

$$V_{A'}^{(1)} = V_a^{(1)} + Z_1 I_A^{(1)}$$

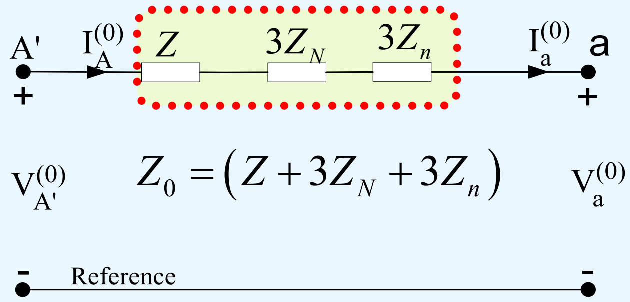
Negative-sequence



$$V_{A'}^{(2)} - V_a^{(2)} = Z_2 I_A^{(2)}$$

$$V_{A'}^{(2)} = V_a^{(2)} + Z_2 I_A^{(2)}$$

Zero-sequence

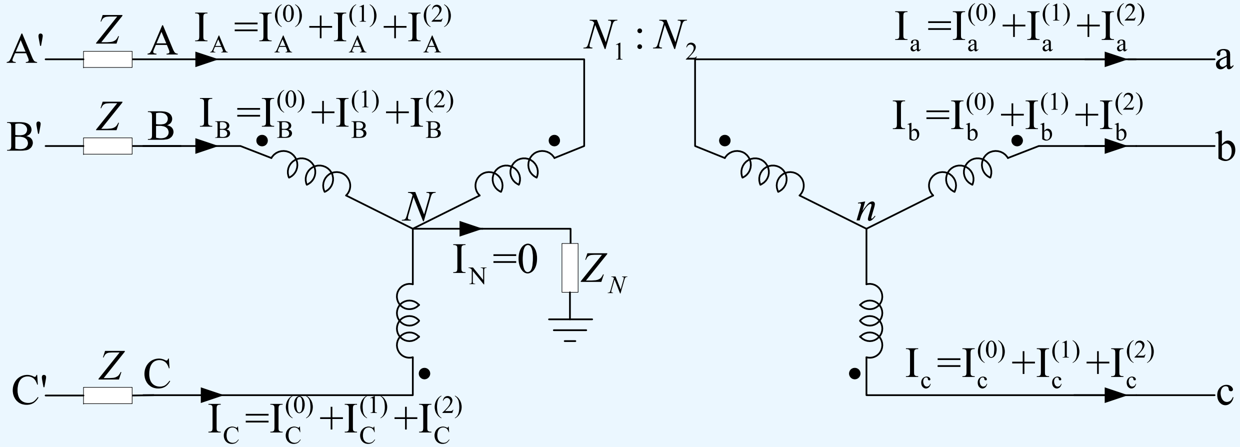


$$V_{A'}^{(0)} - V_a^{(0)} = Z_0 I_A^{(0)}$$

$$V_{A'}^{(0)} = V_a^{(0)} + Z_0 I_A^{(0)}$$

Type 2: Y-Y transformer with one neutral-grounded

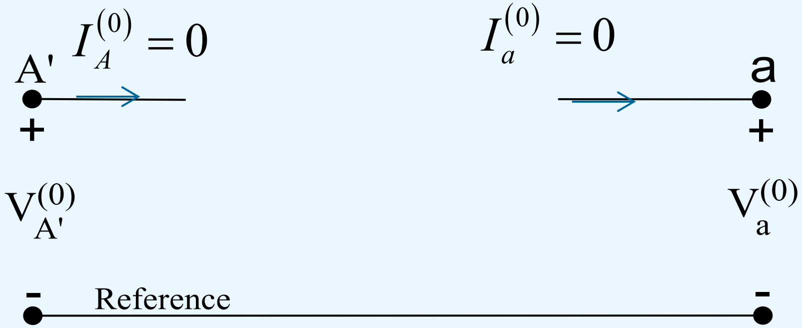
The positive- and negative-sequence circuits of type 2 are the same as those of Type 1 because they are only related to transformer winding impedance Z.



There is no zero-sequence currents can not exist in the ungrounded Y side based on KCL:

$$I_a^{(0)} = \frac{1}{3}\left(I_a + I_b + I_c\right) = 0$$

***Due to the magnetic coupling** $I_A^{(0)} = \frac{N_2}{N_1} I_a^{(0)} = 0$ **.**



Zero-sequence circuit

Type 3: Δ-Δ connected transformer

The positive and negative sequence circuits for all the transformer are the same because they are only related to winding impedance *Z*.

However, the zero sequence circuit depends on the routes of zero sequence currents and magnetic couplings.

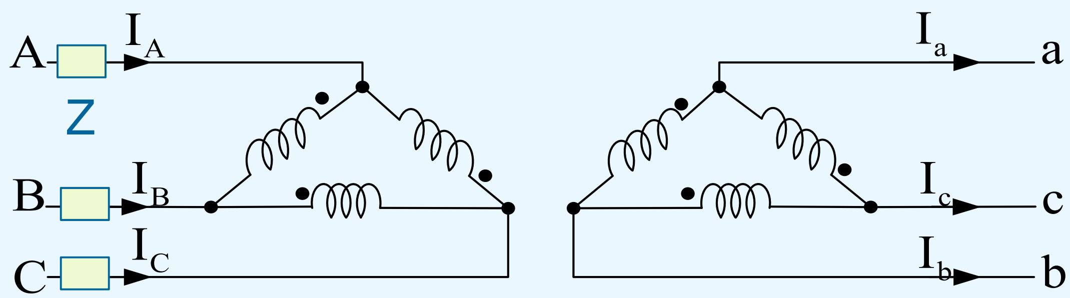
KCL at primary side:

$$I_A + I_B + I_C = 0$$

$$I_A^{(0)} = \frac{1}{3}(I_A + I_B + I_C) = 0$$

The same for secondary side:

$$I_a^{(0)} = 0$$



Therefore there are no zero sequence currents at the terminal of the transformer.

There are not zero sequence currents in the Δ loops because of no zero sequence energy source in the Δ loops.

Zero-sequence circuit

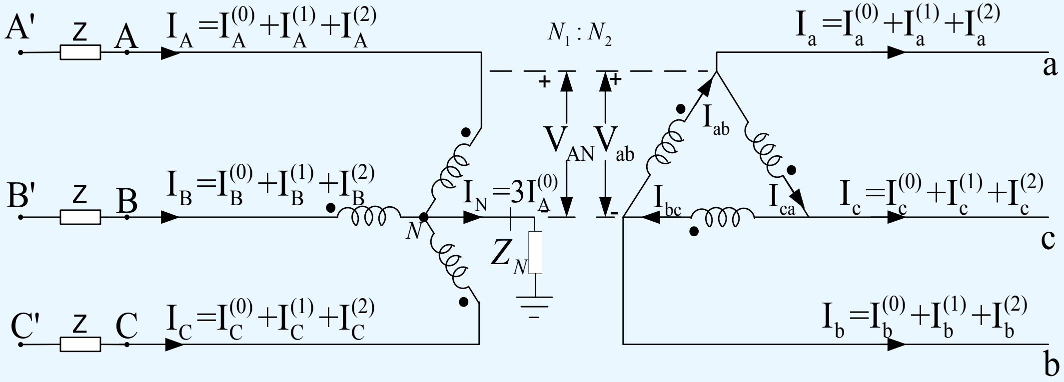
Type 4: Y-Δ transformer with grounded Y

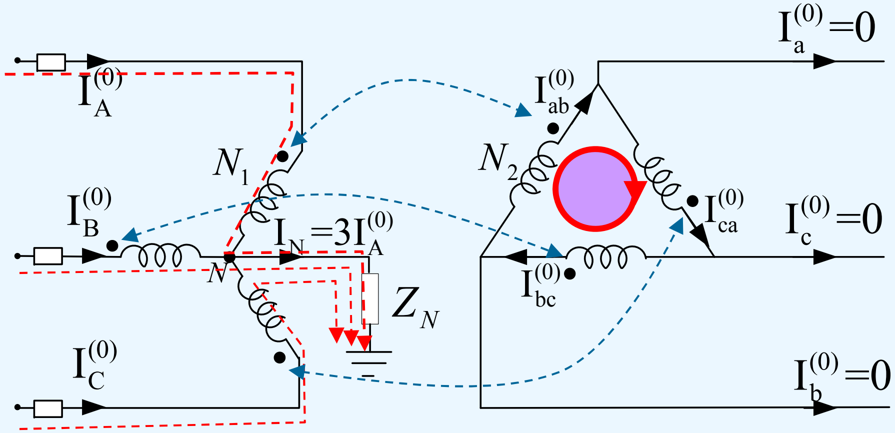
Magnetic coupling:

$$N_1 I_A^{(0)} = N_2 I_{ab}^{(0)}$$

$$N_1 I_B^{(0)} = N_2 I_{bc}^{(0)}$$

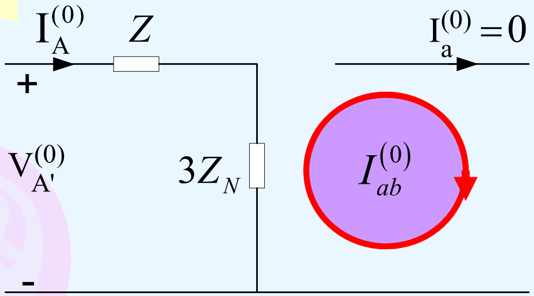
$$N_1 I_C^{(0)} = N_2 I_{ca}^{(0)}$$





When there are zero-sequence currents in Y windings, there will be zero sequence currents in the Δ loop also because they are equal and can circulate inside.

Zero sequence circuit:



$$V_{A'}^{(0)} = \left(Z + 3Z_N\right) I_{A'}^{(0)} \qquad I_a^{(0)} = 0$$

Zero-sequence current circulates within Δ-loop of the three-phase windings because $I_{ab}^{(0)} = I_{bc}^{(0)} = I_{ca}^{(0)}$

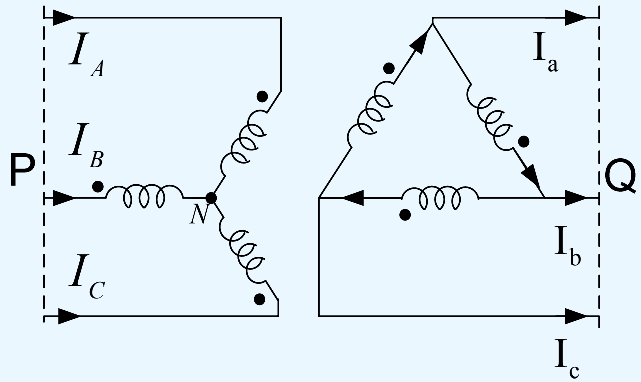
No zero sequence currents at the secondary terminal because: $I_a^{(0)} = I_{ab}^{(0)} - I_{ca}^{(0)} = 0$

Type 5: Y-Δ with ungrounded Y transformer

There are no zero-sequence currents in Y side because of KCL.

$$I_A + I_B + I_C = 0$$

$$I_A^{(0)} = \frac{1}{3}\left(I_A + I_B + I_C\right) = 0$$



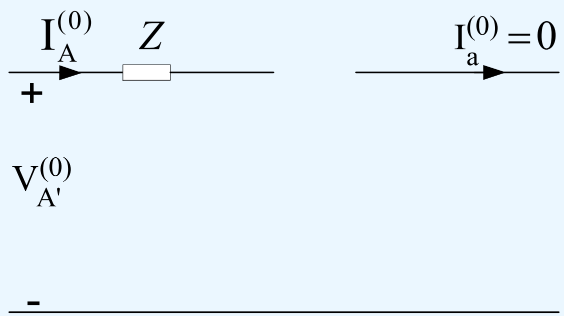
Similarly, there are no zero-sequence currents in Δ side terminal because of KCL.

$$I_a + I_b + I_c = 0$$

$$I_a^{(0)} = \frac{1}{3}\left(I_a + I_b + I_c\right) = 0$$

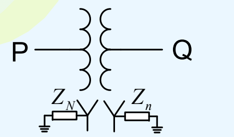
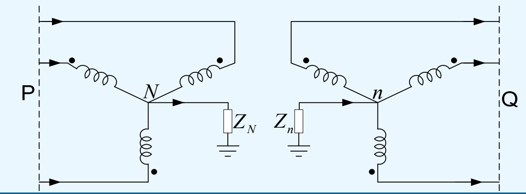
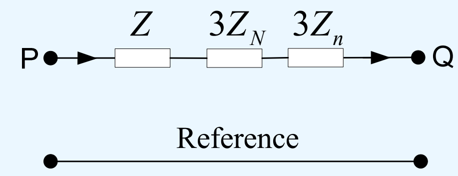
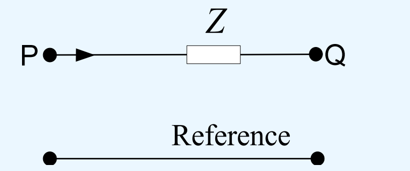
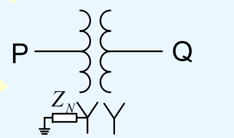
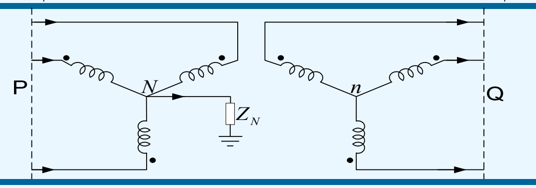
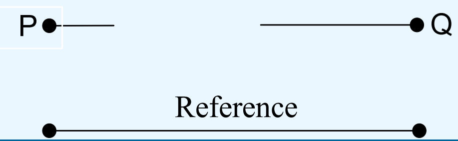
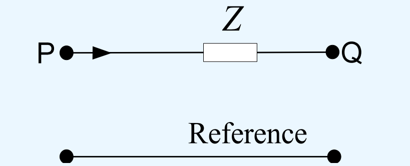
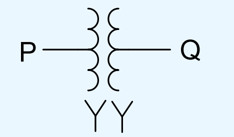
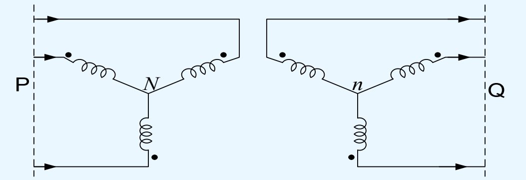
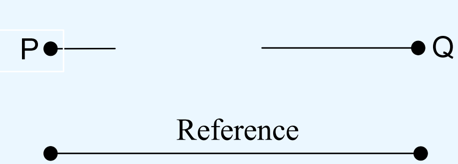
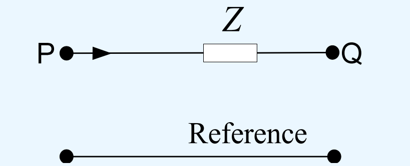
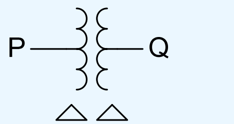
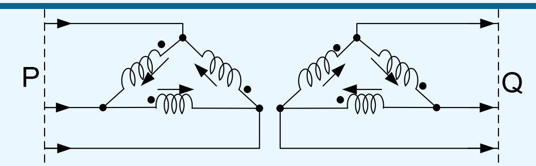
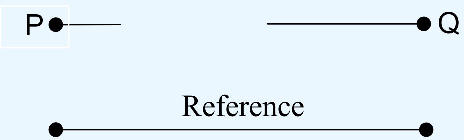
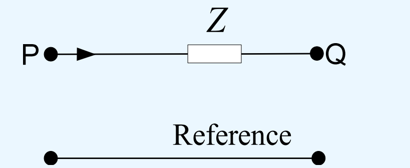
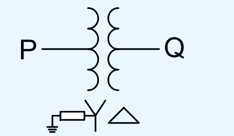
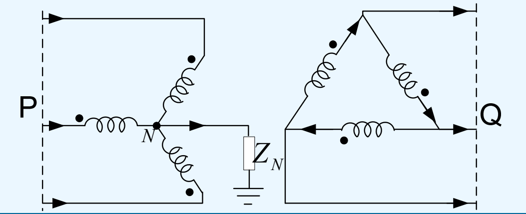
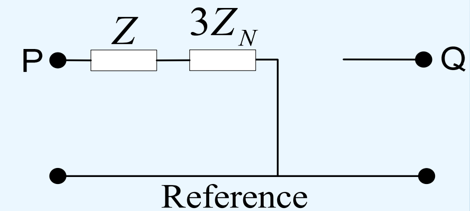
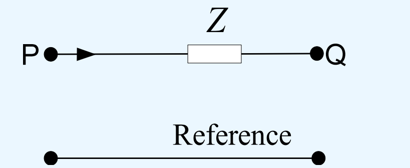
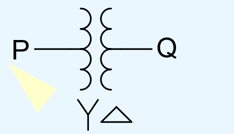
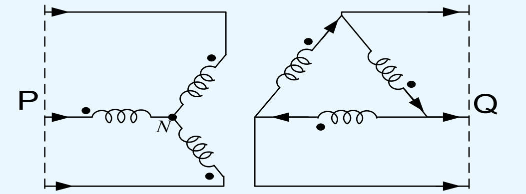
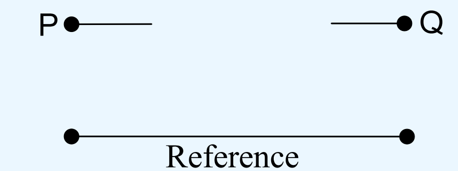
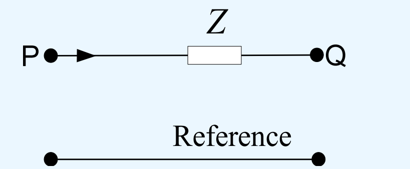
There are no zero-sequence currents in the Δ loop because of no sources in the loop. Therefore:

$$I_A^{(0)} = I_a^{(0)} = I_{ab}^{(0)} = 0$$



Zero sequence circuit:

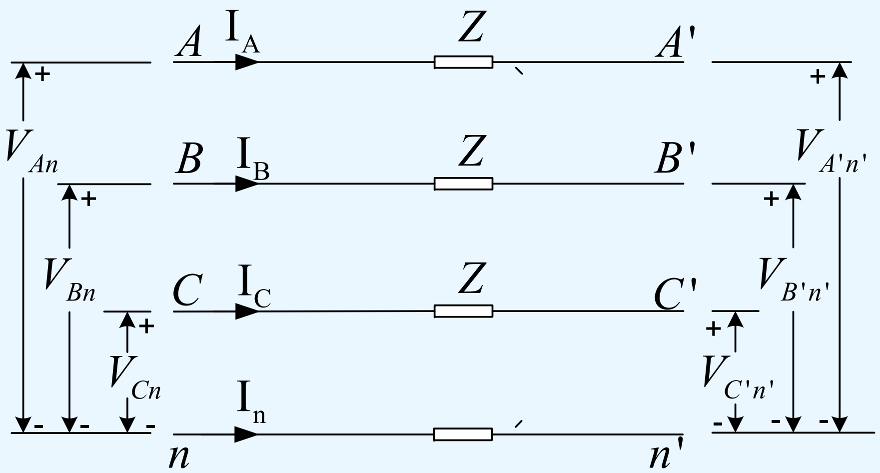
Summary of sequence circuits of three-phase transformers:

Type	Symbol	Connection diagram	Zero-sequence ,	+-tive circuits
1				
2				
3				
4				
5				
6				

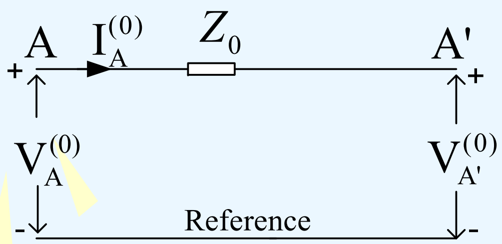
2.4.7 Sequence circuits of transmission lines (Appendix 2)

Transmission line is connected between two buses A and A′. The relationship of the sequence voltages, currents and impedances is derived in detail in Appendix.

The sequence circuits and voltage equations are as follows:

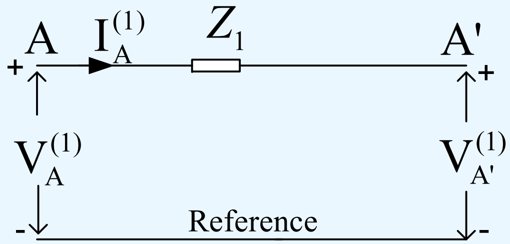


A three-phase transmission line



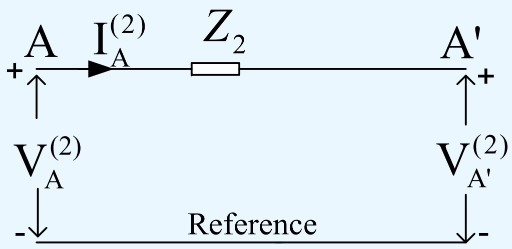
Zero-sequence circuit

$$V_{An}^{(0)} - V_{A'n'}^{(0)} = Z_0 I_A^{(0)}$$



Positive-sequence circuit

$$V_{An}^{(1)} - V_{A'n'}^{(1)} = Z_1 I_A^{(1)}$$

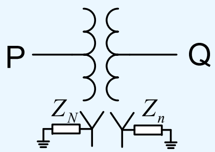
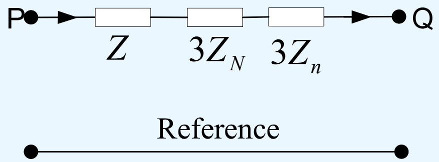
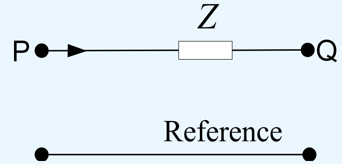
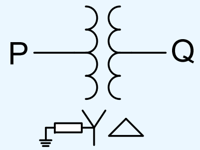
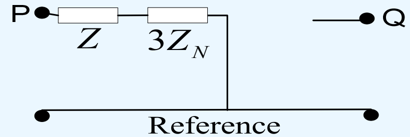
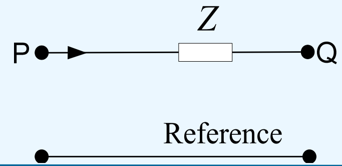
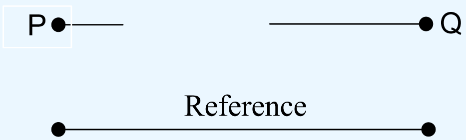
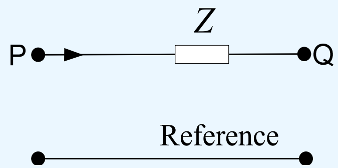
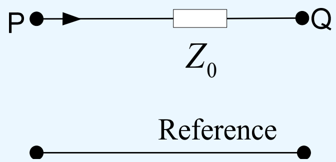
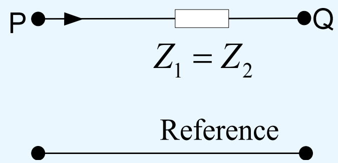
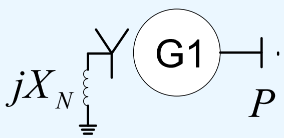
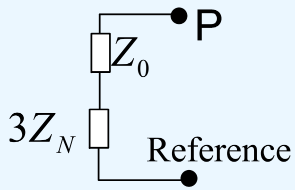
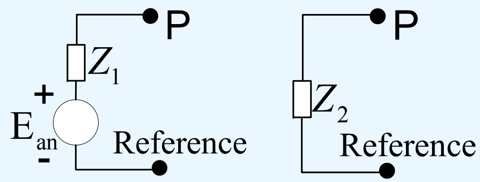
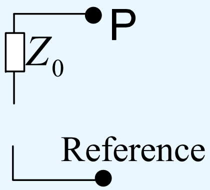
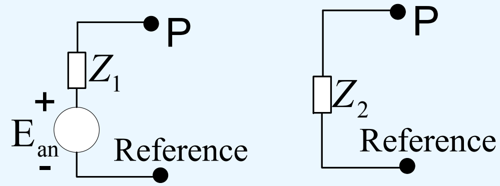


Negative-sequence circuit

$$V_{An}^{(2)} - V_{A'n'}^{(2)} = Z_1 I_A^{(2)}$$

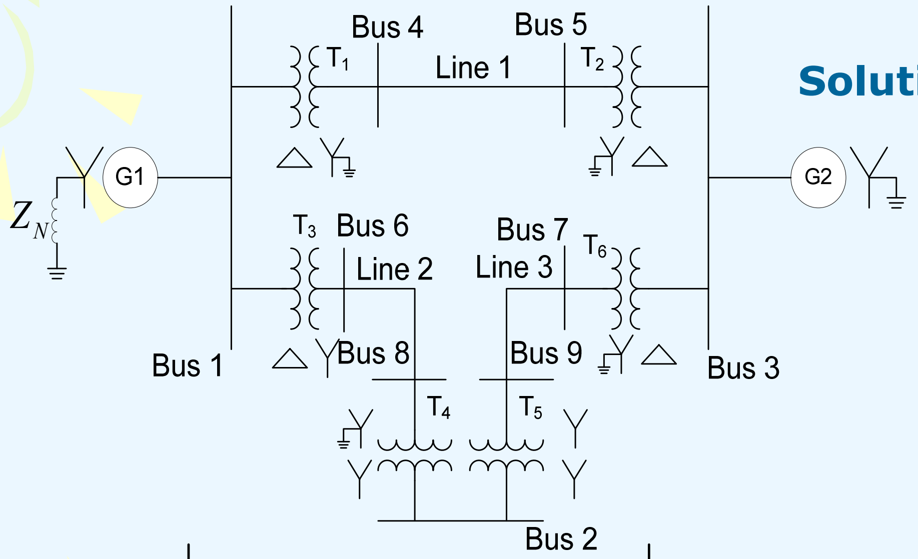
The positive and negative sequence impedances are the same. However, <u>zero sequence impedance is larger</u> than the positive sequence impedance because of different sequence magnetic fields.

The sequence circuits of major components for fault analysis

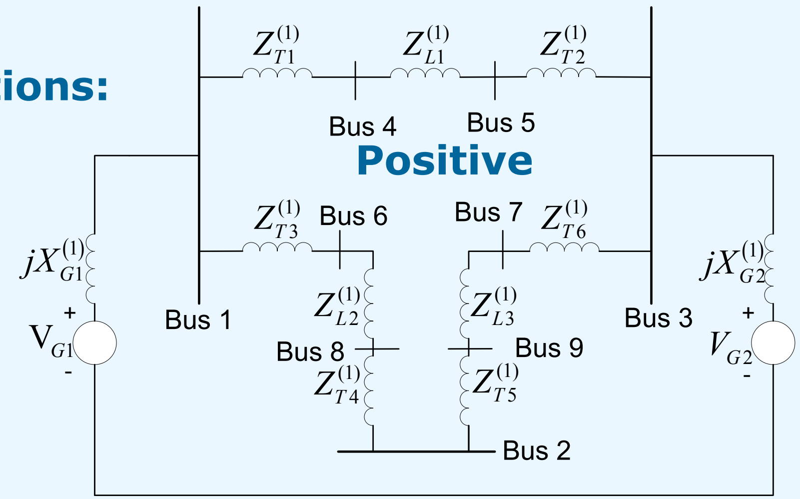
Symbol	Zero-sequence ,	+-tive circuits
		
		
Other connections of transformers		
Transmission line		
		
Other connections of machines		

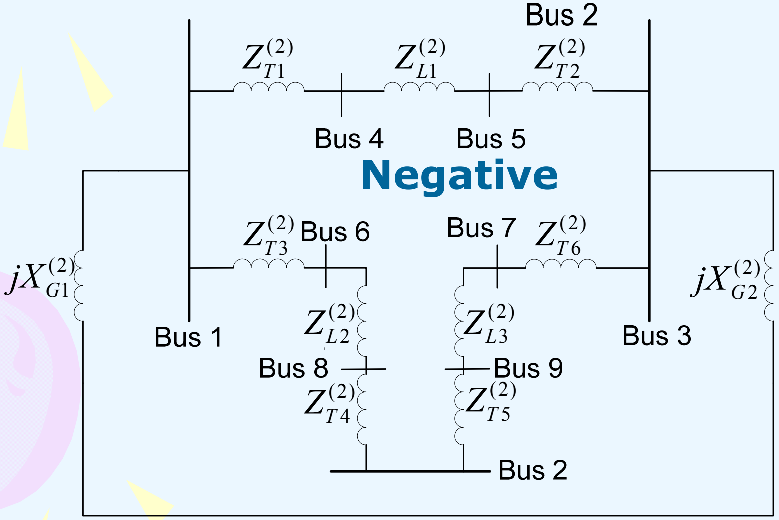
2.4.8 Sequence circuits of a power system

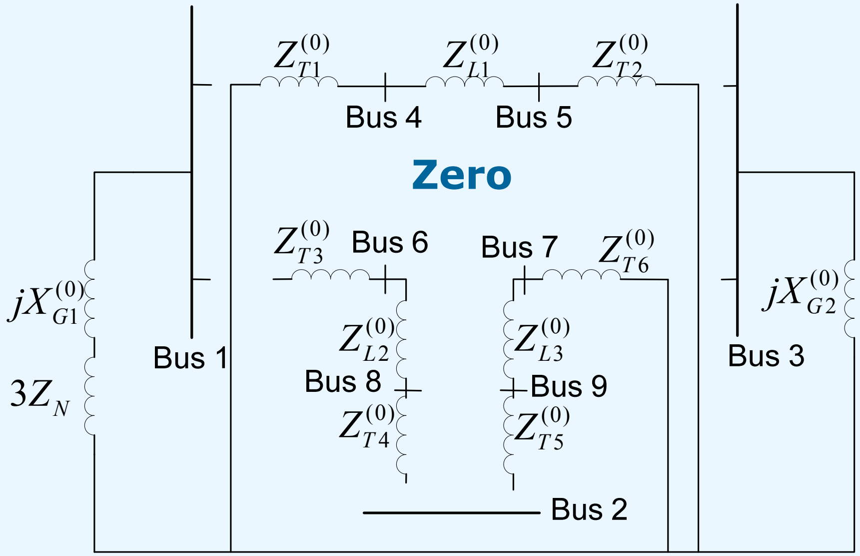
Example 2.3: Sketch the positive-, negative- and zero-sequence equivalent circuits for the following power system. (Connect the sequence impedances of the components to the corresponding buses to form the system sequence circuits)



Solutions:

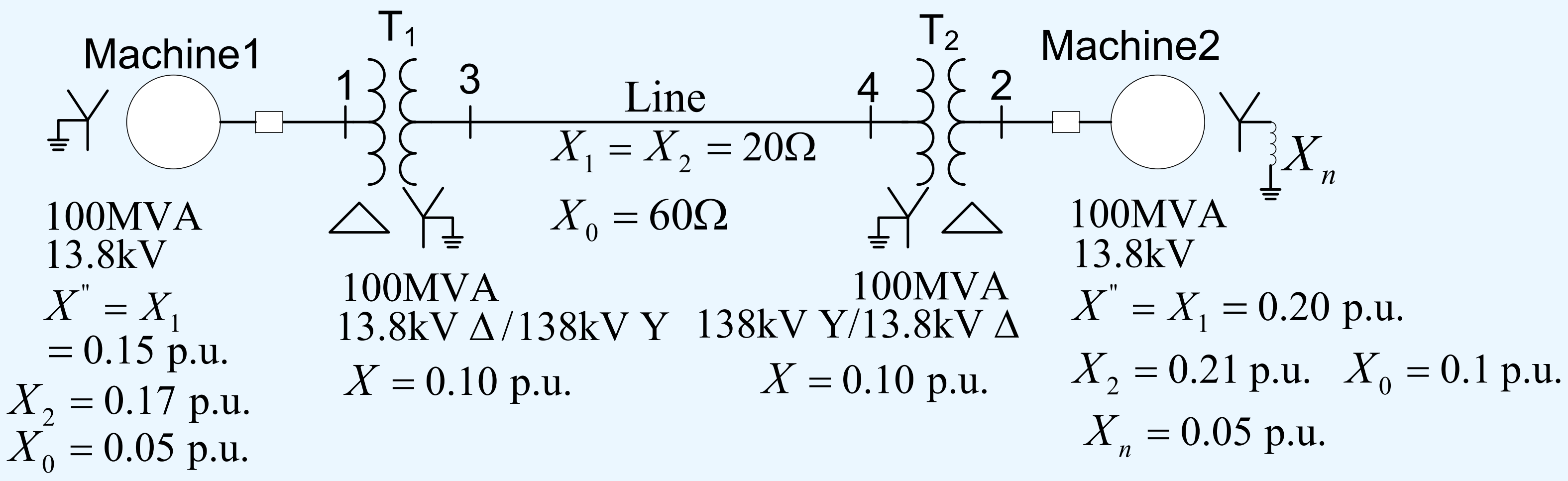






2.4.9 Thevinin equivalent circuits of sequence circuits

Example 2.4: The single-phase diagram of a power system is shown in the following figure, where the reactances are also given. The neutral of the synchronous motor (Machine 2) is grounded through a reactance X_n=0.05 per unit on the motor base. Draw the per-phase per-unit sequence networks and associated Thevinin equivalent circuits seen from bus 2 on 100-MVA, 13.8-kV (machine side) base. Assume that source voltage V_s=1.05pu



Solution:

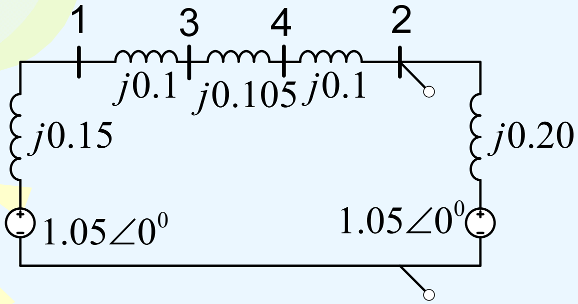
Base impedance for the line: $$\frac{V_{LL}^2}{S_{3\varphi}} = \frac{(138\text{kV})^2}{100\text{MVA}} = 190.4(\Omega)$$

The per unit impedances for the transmission line:

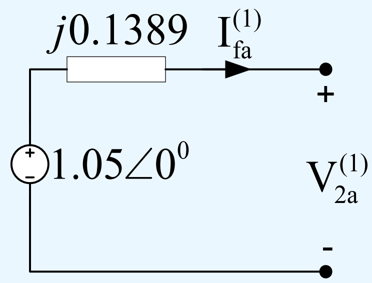
X_1=X_2=20/190.4=0.105 and X_0=60/190.4=0.315pu.

Positive-sequence network:

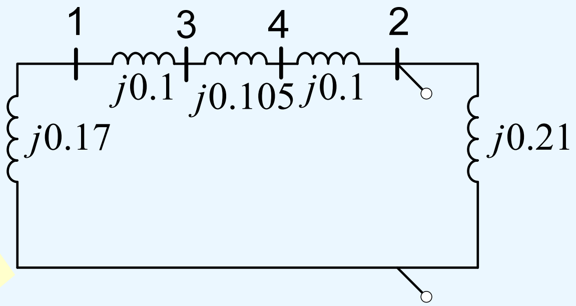
Thevenin equivalent circuits



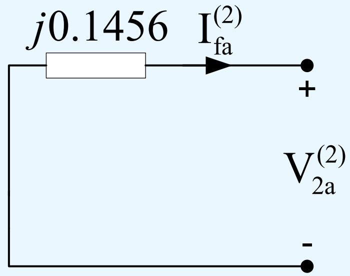
$$
\begin{aligned}
Z_{th}^{(1)} &= \\
&= j0.20 \,\|\, (j0.15 + j0.305) \\
&= j0.1389
\end{aligned}
$$



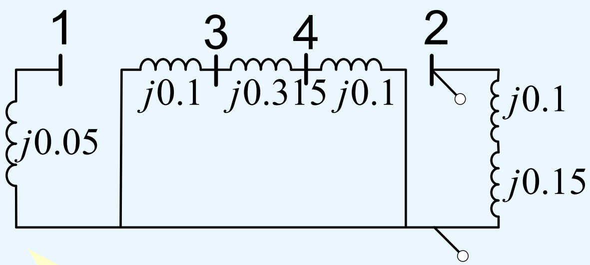
Negative-sequence network:



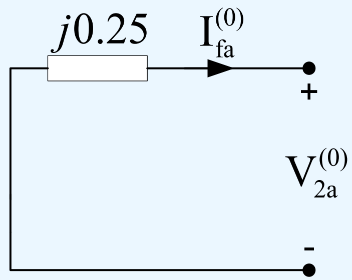
$$
\begin{aligned}
Z_{th}^{(2)} &= \\
&= j0.21 \,\|\, (j0.17 + j0.305) \\
&= j0.1456
\end{aligned}
$$



Zero sequence network:



$$
\begin{aligned}
Z_{th}^{(0)} &= \\
&= (j0.1 + j0.15) \\
&= j0.25
\end{aligned}
$$



Exercise: The one-line diagram for a power system with two synchronous generators is given in the following figure. The ratings and reactances of the generators and transformers are

G1 and G2: 100MVA, 20kV, $X_1=X_2=20\%$, $X_0=6\%$ and $X_N=5\%$.

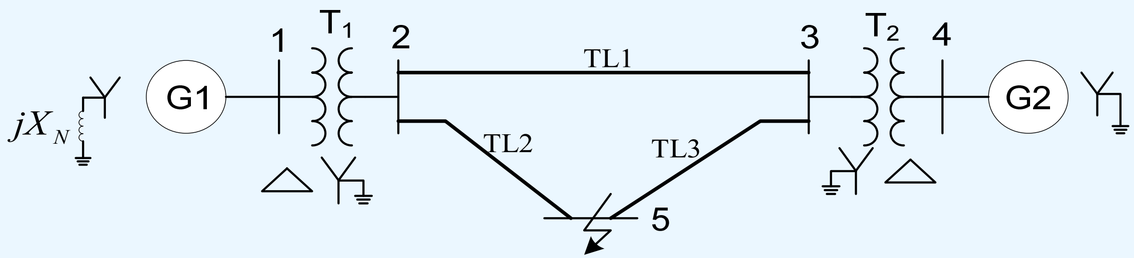
Transformers T1 and T2: 100MVA, $20\Delta / 345Y$ kV and X=10%.

The three transmission lines have the same sequence reactance.

On the base of 100MVA 345kV, the line sequence reactances:

$$X_1 = X_2 = 10\%,\ X_0 = 30\%$$

Determine the per-unit Thevenin impedances from bus 5 for the positive-, negative-, and zero-sequence networks, respectively.



Ans.:

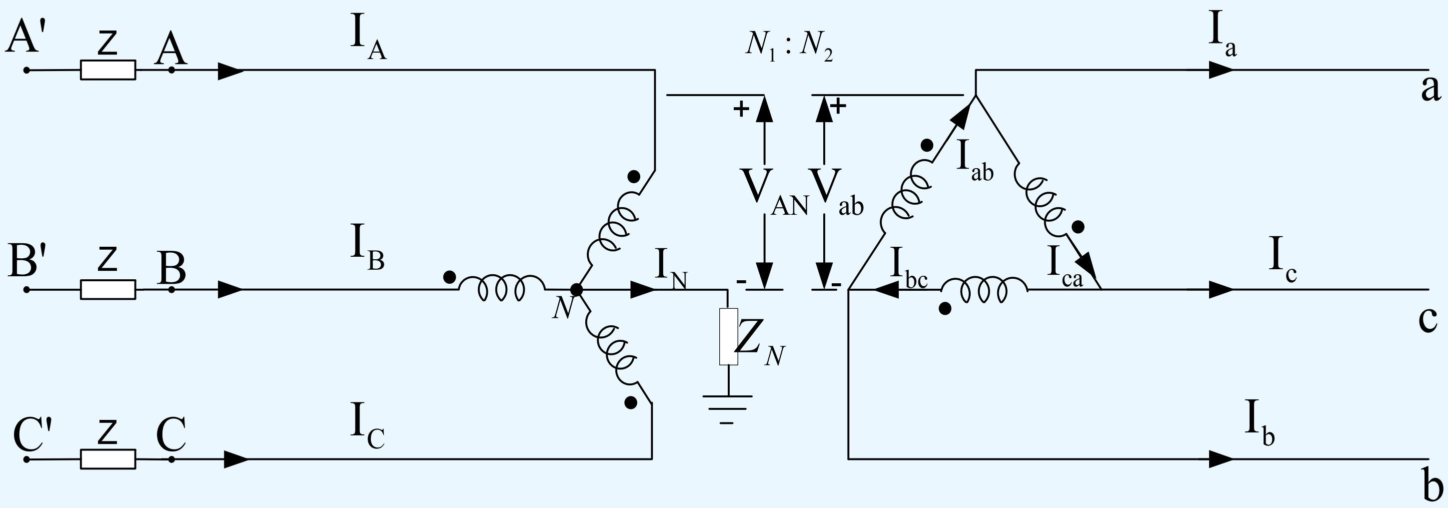
$$Z_{th}^{(1)} = j0.2\text{pu},\ Z_{th}^{(2)} = j0.2\text{pu},\ Z_{th}^{(0)} = j0.2\text{pu}$$

2.4.10 Phase shift of a Y-Δ transformer

1) Dot convention:

The voltages from the dot point to non-dot point of two coupling transformer windings are in phase: V_{AN} and V_{ab} are in phase; V_{BN} and V_{bc} are in phase; V_{CN} and V_{ca} are in phase.

The current entering dot point in one winding is in phase with the current leaving dot point in the coupling winding. I_A and I_{ab} are in phase. I_B and I_{bc} are in phase. I_C and I_{ca} are in phase.



Noted: All Y-Δ transformers used in this lecture are the same connection as shown in this figure.

2) Positive sequence: (phase shift!!!!!! Different current angle)

$I_A^{(1)}$ leads $I_a^{(1)}$ by 30^0 $\Rightarrow I_A^{(1)} = 1\angle 30^0 \times I_a^{(1)}$

$V_{AB}^{(1)}$ leads $V_{ab}^{(1)}$ by 30^0

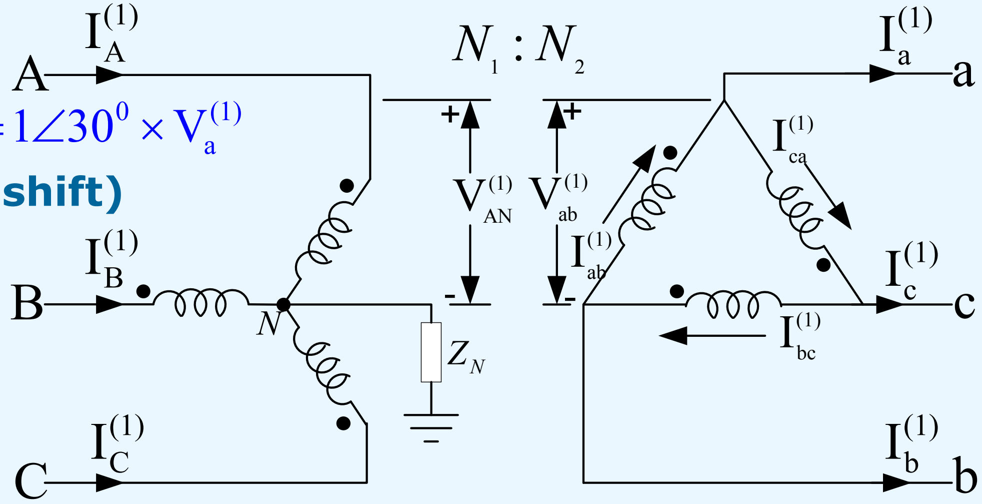
$V_{AN}^{(1)}$ leads $V_{an}^{(1)}$ by $30^0 \Rightarrow V_A^{(1)} = 1\angle 30^0 \times V_a^{(1)}$

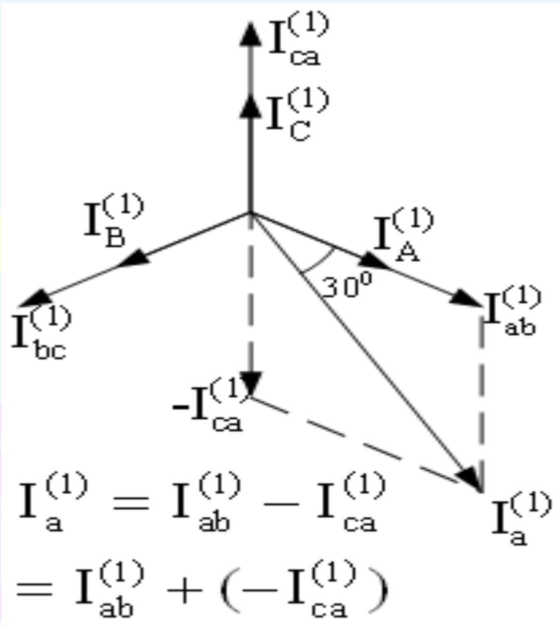
3) Negative sequence: (phase shift)

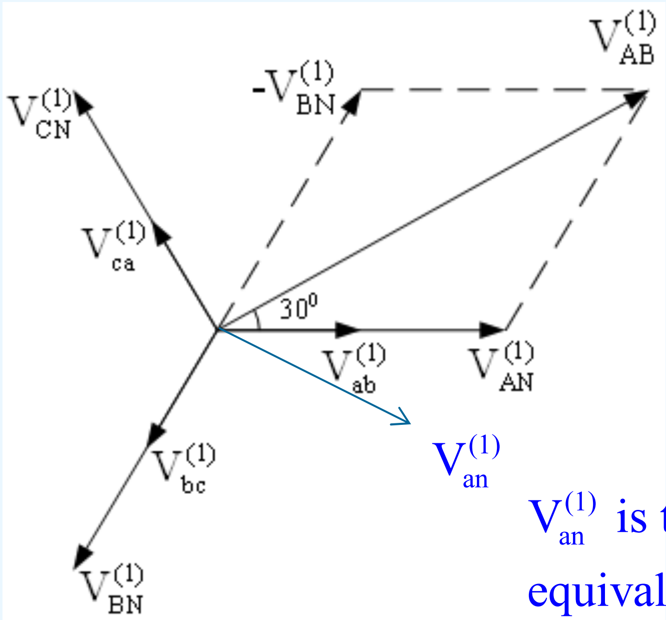
$I_A^{(2)}$ lags $I_a^{(2)}$ by 30^0

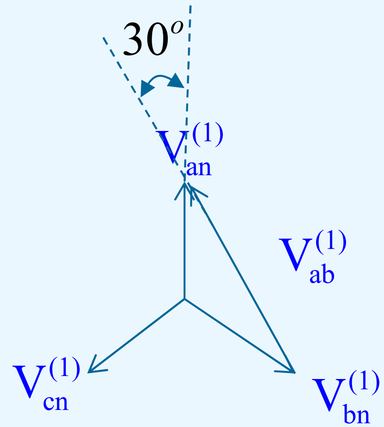
$V_{AB}^{(2)}$ lags $V_{ab}^{(2)}$ by 30^0

$V_{AN}^{(2)}$ lags $V_{an}^{(2)}$ by 30^0









$V_{an}^{(1)}$ is the phase voltage of the equivalent Y of the Δ windings

4) Sequence current calculation of a Y-Δ transformer

The following two circuits have the same impedance seen from terminals 11'.

KVL for Circuit 1:

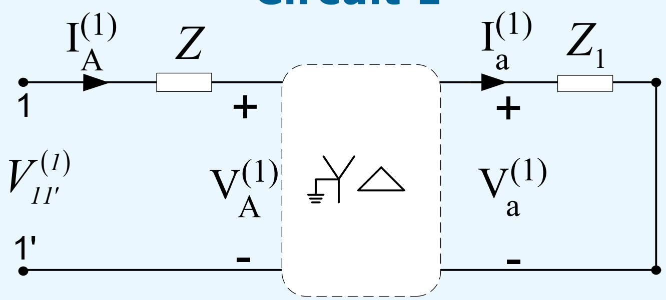
$$V_{11'}^{(1)} = I_A^{(1)}Z + V_A^{(1)} = I_A^{(1)}Z + V_a^{(1)}\angle 30^o \quad \because V_A^{(1)} = 1\angle 30^0 \times V_a^{(1)}$$

$$= I_A^{(1)}Z + I_a^{(1)}Z_1\angle 30^o$$

$$= I_A^{(1)}Z + I_A^{(1)}\angle -30^o Z_1\angle 30^o \quad \because I_A^{(1)} = 1\angle 30^0 \times I_a^{(1)}$$

$$= I_A^{(1)}\left(Z + Z_1\right) \quad \textbf{(1)}$$

Circuit 1

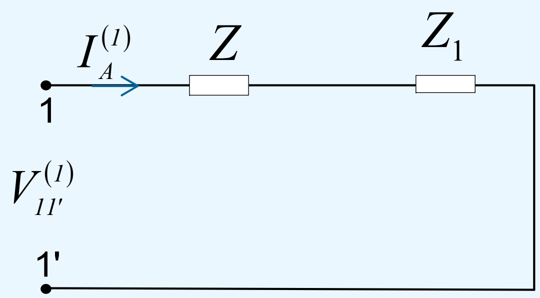


Phase shifter

KVL for Circuit 2: $V_{11'}^{(1)} = I_A^{(1)}\left(Z + Z_1\right)$ **(2)**

$$I_A^{(1)} = \frac{V_{11'}^{(1)}}{Z + Z_1}$$

Circuit 2



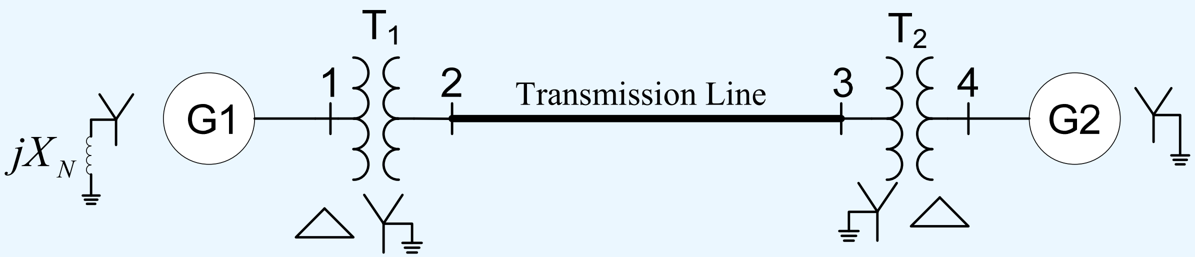
It can be seen that current and voltage phase shift does not affect the calculation of $I_A^{(1)}$ because (1) and (2) from the two circuits are the same. The same for negative sequence circuit.

*****During fault analysis, sequence voltages and currents on one side of a Δ/Y transformer can be calculated without considering the phase shift. The phase shifts can be added to the currents and voltages on other side.**

Example 2.5: A bolted single line-to-ground fault occurs on phase a at bus 2 of the power system. The sequence components of phase-*a* current flowing from Y side of T_1 to bus 2 are:

$$I_a^{(0)} = 1.8492\angle -90^0 \text{pu} \quad I_a^{(2)} = 1.3586\angle -90^0 \text{pu} \quad I_a^{(1)} = 1.3586\angle -90^0 \text{pu}$$

Calculate per-unit phase currents flowing from G1 to bus 1.



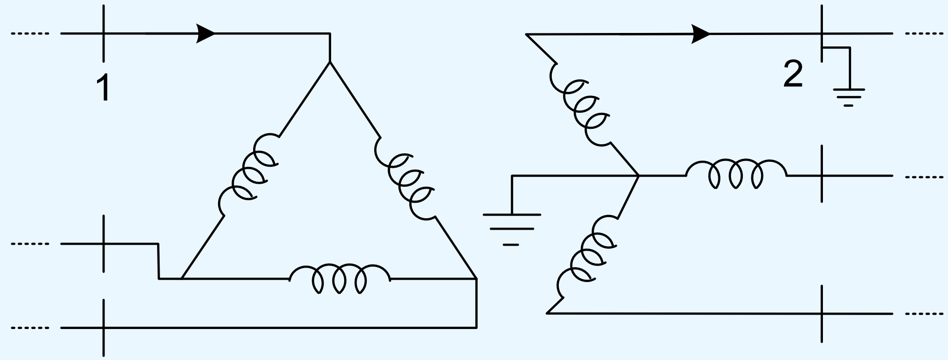
$20\Delta / 345\text{Y kV}$

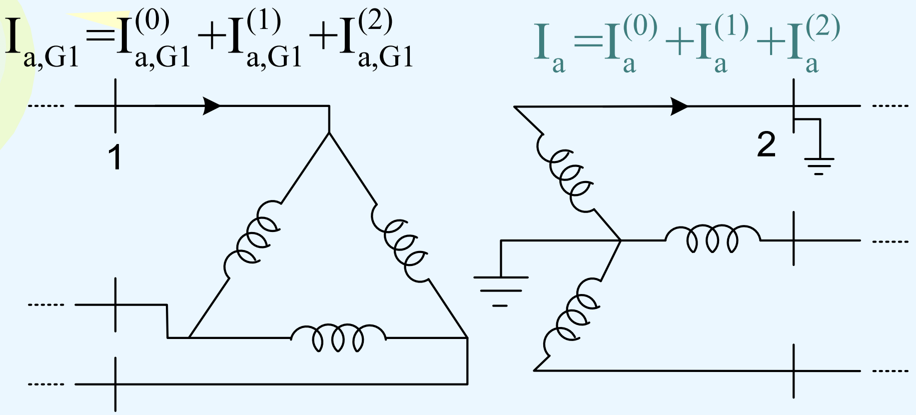
Solutions:

Convert sequence currents form Y side to Δ side.

Calculate phase currents from the sequence currents.

$$I_{a,G1} = I_{a,G1}^{(0)} + I_{a,G1}^{(1)} + I_{a,G1}^{(2)} \qquad I_a = I_a^{(0)} + I_a^{(1)} + I_a^{(2)}$$





$$I_a^{(0)} = 1.8492\angle -90^0 \text{pu}$$

$$I_a^{(1)} = 1.3586\angle -90^0 \text{pu}$$

$$I_a^{(2)} = 1.3586\angle -90^0 \text{pu}$$

Considering phase shifts for positive and negative currents between two sides of the transformer, and also considering the path of zero sequence current, the sequence components of phase-a current from G1 to bus 1:

$$I_{a,G1}^{(0)} = 0$$

$$I_{a,G1}^{(1)} = I_a^{(1)} \times 1\angle -30^0 = 1.3586\angle(-90^0 - 30^0) = 1.3586\angle -120^0$$

$$I_{a,G1}^{(2)} = I_a^{(2)} \times 1\angle 30^0 = 1.3586\angle(-90^0 + 30^0) = 1.3586\angle -60^0$$

$$\begin{bmatrix} I_{a,G1} \\ I_{b,G1} \\ I_{c,G1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a,G1}^{(0)} \\ I_{a,G1}^{(1)} \\ I_{a,G1}^{(2)} \end{bmatrix} = [\mathbf{A}] \begin{bmatrix} 0 \\ 1.3586\angle -120^0 \\ 1.3586\angle -60^0 \end{bmatrix} = \begin{bmatrix} -j2.3532 \\ j2.3532 \\ 0 \end{bmatrix} \text{pu}$$

Summary

a) Transform three unbalance voltages or currents into three sets of balance sequence voltages.

b) Transform the sequence voltages into three unbalance voltages or currents.

c) The sequence circuits of components:

*Zero Sequence circuits for transformers and Machines

d) The positive, negative and zero sequence networks of a power system

e) Thevenin equivalent circuits of the sequence networks seen from any bus

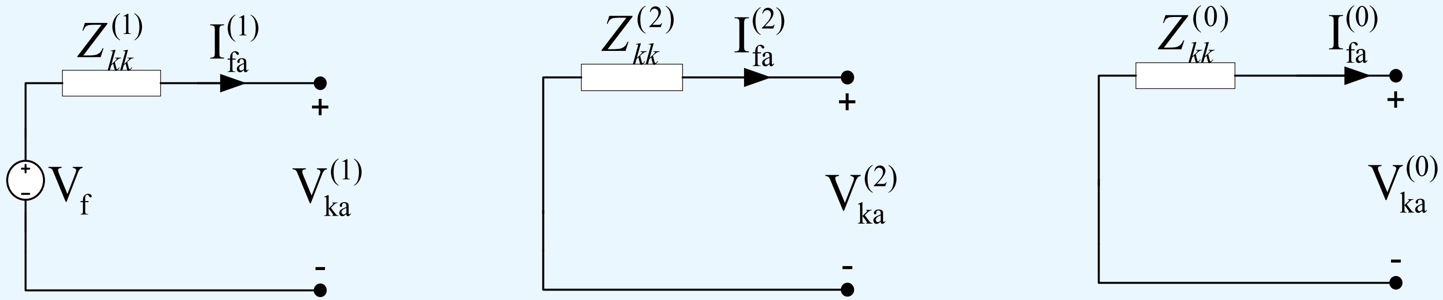
f) Phase shifts of positive and negative sequence voltages and currents cross Δ/Y transformers

2.5 Procedure of fault analysis

The procedure to calculate voltages and currents after a fault includes seven steps.

√Step 1: Draw system single <u>phase-*a*</u> <u>sequence</u> networks based on the sequence circuits of components and network connection (before the fault).

√Step 2: Determine Thevenin equivalent circuits of the <u>phase-*a*</u> <u>sequence</u> networks seen from the fault bus k.



Positive-sequence **Negative-sequence** **Zero-sequence**

$$V_{ka}^{(1)} = V_f - Z_{kk}^{(1)} I_{fa}^{(1)} \qquad V_{ka}^{(2)} = -Z_{kk}^{(2)} I_{fa}^{(2)} \qquad V_{ka}^{(0)} = -Z_{kk}^{(0)} I_{fa}^{(0)}$$

$V_{ka}^{(1)}, V_{ka}^{(2)}$, and $V_{ka}^{(0)}$ **: Sequence components of voltage V_{ka} at the fault bus**

$I_{fa}^{(0)}, I_{fa}^{(1)}, and\ I_{fa}^{(2)}$ **: Sequence components of current I_{fa} at the fault point**

$Z_{kk}^{(1)}, Z_{kk}^{(2)}$, and $Z_{kk}^{(0)}$**: Thevenin sequence impedances**

V_f: pre-fault voltage on phase *a* at fault bus k.

2.5 Procedure of fault analysis (continued)

***Step 3*: Find <u>phase</u> voltage and current circuit equations <u>at the fault location</u> from <u>three phase network</u> for a given fault type.**

***Step 4*: Determine the <u>sequence</u> current and voltage circuit equations based on phase voltages and current equations <u>at the fault location</u>.**

Step 5: Determine the connection of the <u>Thevenin equivalent circuits</u> of the <u>sequence</u> networks <u>at the fault location</u> based on the sequence circuit equations developed in Step 3.

Step 6: Calculate the <u>sequence</u> currents and voltages <u>at the fault location</u> based on the connection of the sequence Thevenin equivalent networks at the fault point.

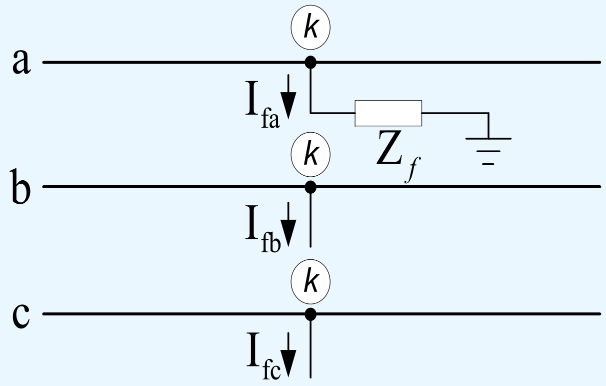
Step 7: Calculate the <u>sequence</u> currents and voltages in the <u>other parts</u> of a power system by <u>adding the sequence currents</u> from step 5 as <u>current sources at the fault bus</u> in the <u>original sequence networks (without considering phase shift</u> caused by Δ/Y transformers).

<u>Add the phase shift</u> for the sequence voltages and currents cross Δ/Y transformers.**

Step 8: Calculate the <u>phase</u> currents and voltages based on the the sequence currents and voltages using A matrix.

2.6 Single line-to-ground fault (Single phase-to-ground fault)

Step 3: Phase voltage and current equations at the fault bus k: $I_{fb} = I_{fc} = 0$ **and** $V_{ka} = Z_f I_{fa}$



(phase-a-to-ground fault)

Step 4: Sequence voltage and current equations

$$\begin{bmatrix} I_{fa}^{(0)} \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{fa} \\ 0 \\ 0 \end{bmatrix}$$

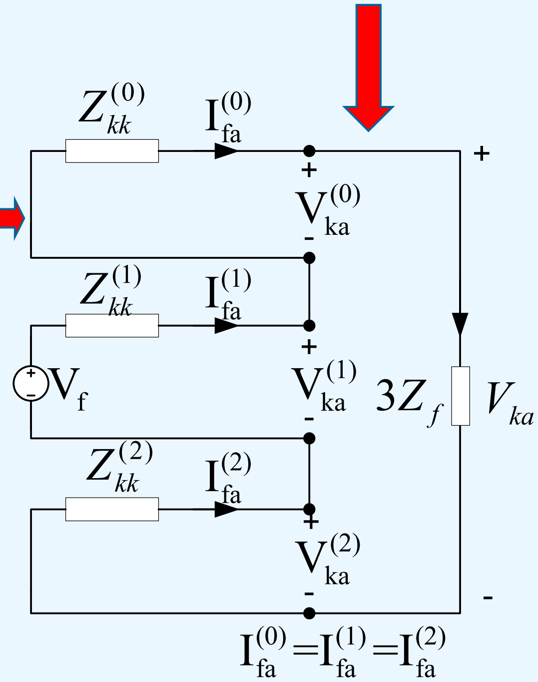
$$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = I_{fa}/3$$ ⇨ **Three sequence networks are connected in series**

$$I_{fa} = 3I_{fa}^{(0)}$$

$$V_{ka} = Z_f I_{fa} = 3Z_f I_{fa}^{(0)}$$ **(Z_f: fault impedance)**

$$V_{ka} = \underline{V_{ka}^{(0)} + V_{ka}^{(1)} + V_{ka}^{(2)}} = \underline{3Z_f I_{fa}^{(0)}}$$ ⇨ **Sequence networks and $3Z_f$ are in the same loop**

Step 5:



Step 6: $$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = \frac{V_f}{Z_{kk}^{(0)} + Z_{kk}^{(1)} + Z_{kk}^{(2)} + 3Z_f}$$

$$V_{ka}^{(1)} = V_f - Z_{kk}^{(1)} I_{fa}^{(1)}$$

⇨ $$V_{ka}^{(2)} = -Z_{kk}^{(2)} I_{fa}^{(2)}$$

$$V_{ka}^{(0)} = -Z_{kk}^{(0)} I_{fa}^{(0)}$$

Step 8: $$\begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} V_{ka}^{(0)} \\ V_{ka}^{(1)} \\ V_{ka}^{(2)} \end{bmatrix}$$

Example 2.6: In Example 2.4, a bolted single line-to-ground fault occurs on phase *a* at bus 2.

a)Determine the sequence currents of phase *a* at the fault point.

b)Calculate the fault currents flowing from bus 2 to ground.

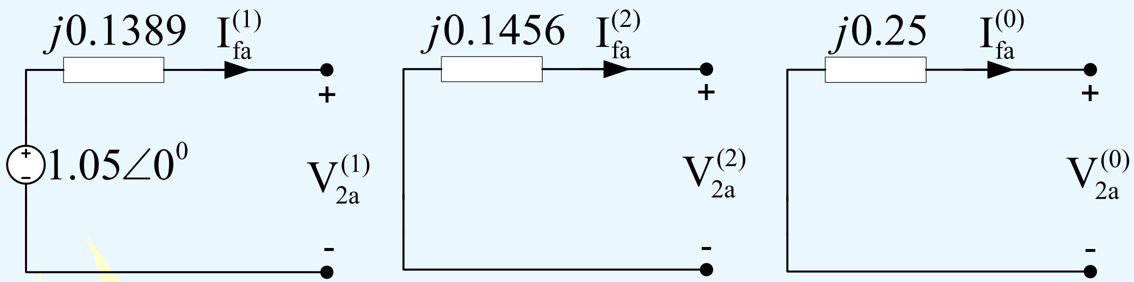
c) Add the sequence currents to the corresponding sequence networks.

d)Calculate phase currents flowing from machine 2 to bus 2.

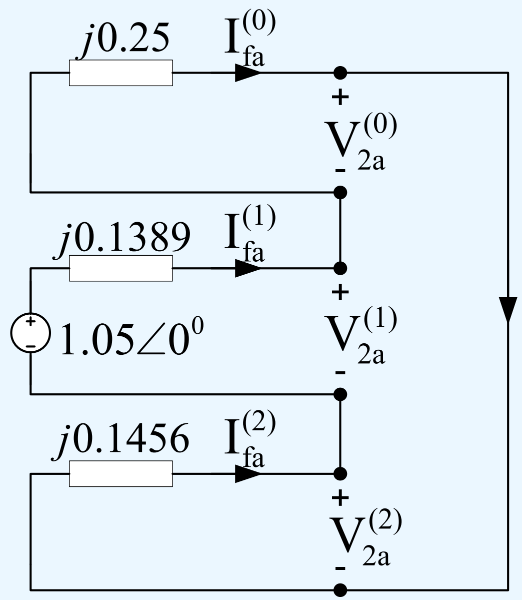
e)Calculate phase currents through transmission line.

Solutions:

From the solutions of example 2.4, the sequence networks of phase a:



Connection (step 5)



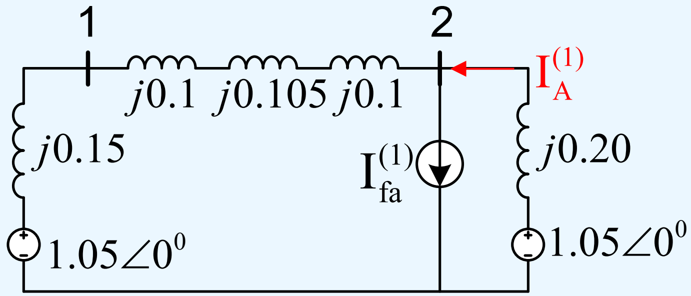
a) $$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = \frac{V_f}{Z_{22}^{(0)} + Z_{22}^{(1)} + Z_{22}^{(2)} + 3\times 0}$$ **(step 6)**

$$= \frac{1.05\angle 0^0}{j0.25 + j0.1389 + j0.1456} = -j1.9645\ pu$$

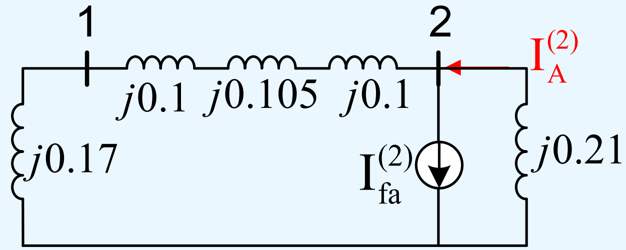
b) $$I_{fa} = 3I_{fa}^{(0)} = 3\times(-j1.9645) = -j5.8934\ pu$$ **(step 8)**

c) Connect the sequence currents to the corresponding system sequence networks at bus 2 (step 7):

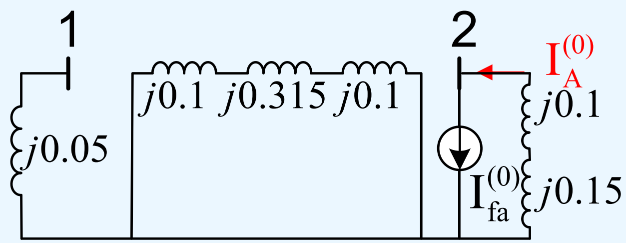
Positive-sequence network with positive-sequence current at the fault location: ⇨



Negative-sequence network with negative-sequence current at the fault location: ⇨



Zero-sequence network with zero-sequence current at the fault location: ⇨



d) Sequence currents from machine 2 to bus 2 (using current divider):

$$\mathbf{I}_A^{(1)} = \mathbf{I}_{fa}^{(1)} \frac{\mathbf{j0.455}}{\mathbf{j0.2 + j0.455}} = \mathbf{0.695 I}_{fa}^{(1)}$$

$$\mathbf{I}_A^{(2)} = \mathbf{I}_{fa}^{(2)} \frac{\mathbf{j0.475}}{\mathbf{j0.21 + j0.475}} = \mathbf{0.693 I}_{fa}^{(2)}$$

$$\mathbf{I}_A^{(0)} = \mathbf{I}_{fa}^{(0)}$$

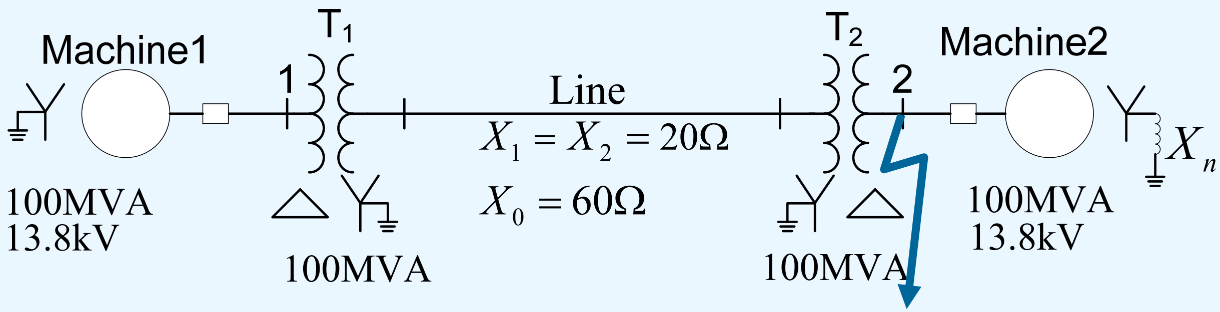
⇨

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} = ??$$

Phase currents from machine 2 to bus 2

e) The sequence currents through transmission line:

Because transmission line is connected to Y side and the fault is at Δ side of the transformer, the phase shifts for positive and negative sequence currents have to be considered.



without considering phase shift:

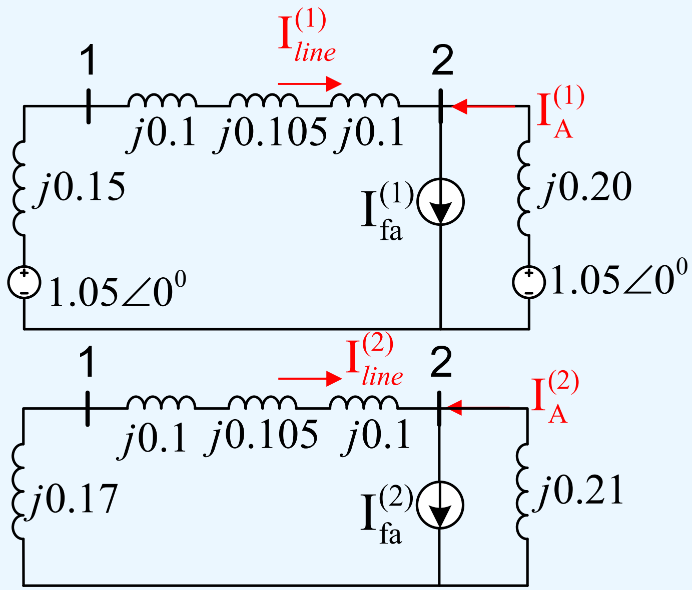
$$\mathrm{I}_{line}^{(1)} = \mathrm{I}_{fa}^{(1)} - \mathrm{I}_{A}^{(1)} \quad \mathrm{I}_{line}^{(2)} = \mathrm{I}_{fa}^{(2)} - \mathrm{I}_{A}^{(2)} \quad \mathrm{I}_{line}^{(0)} = 0$$

with phase shift:

$$\mathrm{I}_{line}^{(1)} = \left(\mathrm{I}_{fa}^{(1)} - \mathrm{I}_{A}^{(1)}\right)\angle 30^{o} \quad \mathrm{I}_{line}^{(2)} = \left(\mathrm{I}_{fa}^{(2)} - \mathrm{I}_{A}^{(2)}\right)\angle -30^{o}$$

The phase currents :

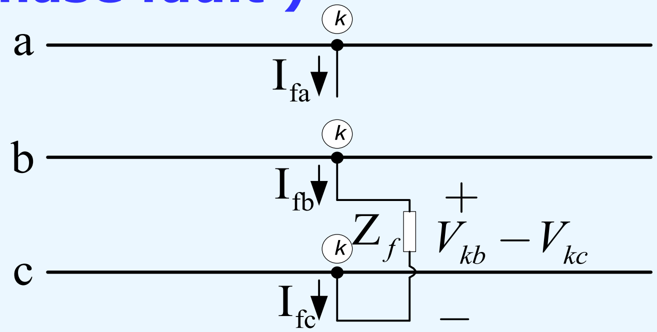
$$\begin{bmatrix} \mathrm{I}_{faline} \\ \mathrm{I}_{fbline} \\ \mathrm{I}_{fcline} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \mathrm{I}_{line}^{(0)} \\ \mathrm{I}_{line}^{(1)} \\ \mathrm{I}_{line}^{(2)} \end{bmatrix} = ?$$



2.7 Line-to-Line fault analysis (double phase fault)

Step 3: Phase current and voltage equations for the fault between phase b and c at bus k:

$$I_{fa}=0 \quad I_{fb}=-I_{fc} \quad V_{kb}-V_{kc}=Z_f I_{fb}$$



(phase-b-to-phase-c fault)

Step 4: The sequence current and voltage equations at bus k:

$$\begin{bmatrix} I_{fa}^{(0)} \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_{fb} \\ -I_{fb} \end{bmatrix} \Rightarrow \begin{cases} \underline{I_{fa}^{(0)}=0} \\ \underline{I_{fa}^{(1)}}=-I_{fa}^{(2)}=\dfrac{1}{3}(a-a^2)I_{fb} \end{cases}$$

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} I_{fa}^{(0)} \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 0 \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{bmatrix} \Rightarrow I_{fb}=a^2 I_{fa}^{(1)}+aI_{fa}^{(2)}=(a^2-a)I_{fa}^{(1)}$$

$$\begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} V_{ka}^{(0)} \\ V_{ka}^{(1)} \\ V_{ka}^{(2)} \end{bmatrix} \Rightarrow \begin{matrix} V_{kb}=V_{ka}^{(0)}+a^2V_{ka}^{(1)}+aV_{ka}^{(2)} \\ V_{kc}=V_{ka}^{(0)}+aV_{ka}^{(1)}+a^2V_{ka}^{(2)} \end{matrix} \Rightarrow V_{kb}-V_{kc}=Z_f I_{fb}$$

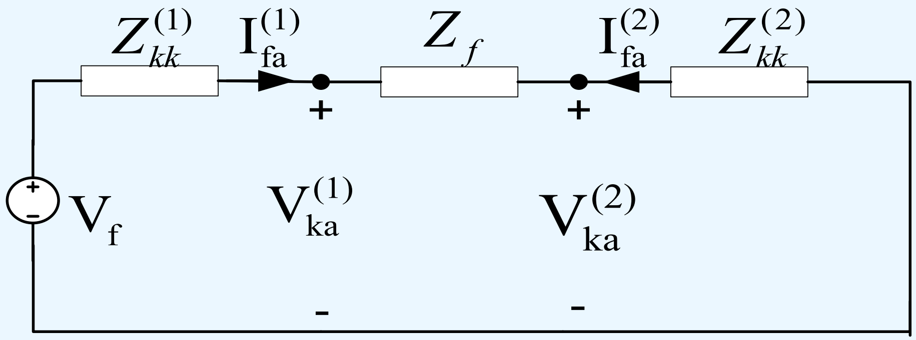
$$\Rightarrow (a^2-a)V_{ka}^{(1)}+(a-a^2)V_{ka}^{(2)}=Z_f(a^2-a)I_{fa}^{(1)} \Rightarrow \underline{V_{ka}^{(1)}-V_{ka}^{(2)}=Z_f I_{fa}^{(1)}}$$

Step 5: Sequence network connection at bus k:

$\mathrm{I}_{\mathrm{fa}}^{(0)} = 0$ **: No zero sequence current for this fault.**

Only positive and negative sequence circuits exist:

$$\left.\begin{aligned} \mathrm{I}_{\mathrm{fa}}^{(1)} &= -\,\mathrm{I}_{\mathrm{fa}}^{(2)} \\ \mathrm{V}_{\mathrm{ka}}^{(1)} - \mathrm{V}_{\mathrm{ka}}^{(2)} &= Z_f \mathrm{I}_{\mathrm{fa}}^{(1)} \end{aligned}\right\} \Rightarrow$$



Step 6: Sequence currents at bus k:

$$\mathrm{I}_{\mathrm{fa}}^{(1)} = -\,\mathrm{I}_{\mathrm{fa}}^{(2)} = \frac{\mathrm{V_f}}{Z_{kk}^{(1)} + Z_{kk}^{(2)} + Z_f}$$

Step 8:

Fault currents at bus k:

$$\begin{bmatrix} \mathrm{I_{fa}} \\ \mathrm{I_{fb}} \\ \mathrm{I_{fc}} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \mathrm{I}_{\mathrm{fa}}^{(0)} \\ \mathrm{I}_{\mathrm{fa}}^{(1)} \\ \mathrm{I}_{\mathrm{fa}}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ (a^2 - a)\mathrm{I}_{\mathrm{fa}}^{(1)} \\ (a - a^2)\mathrm{I}_{\mathrm{fa}}^{(1)} \end{bmatrix}$$

Fault voltages at bus k:

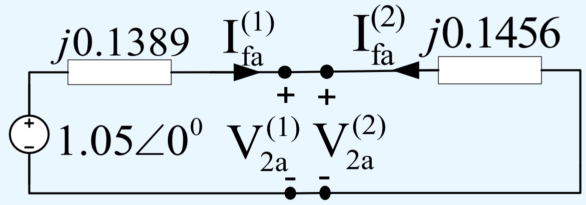
$$\begin{bmatrix} \mathrm{V_{ka}} \\ \mathrm{V_{kb}} \\ \mathrm{V_{kc}} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \mathrm{V}_{\mathrm{ka}}^{(0)} \\ \mathrm{V}_{\mathrm{ka}}^{(1)} \\ \mathrm{V}_{\mathrm{ka}}^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ \mathrm{V}_{\mathrm{ka}}^{(1)} \\ \mathrm{V}_{\mathrm{ka}}^{(2)} \end{bmatrix}$$

Example 2.7: In Example 2.4, a bolted line-to-line fault between phases b and c occurs at bus 2.

a)Determine the sequence fault currents at the fault point (bus 2).

b)Calculate the fault currents on phases a, b and c at the fault point.

c)Calculate the phase voltages at bus 2 after the fault.



Solutions:

For a bolted fault, Z_f =0

a): $I_{fa}^{(0)} = 0 \qquad I_{fa}^{(1)} = -I_{fa}^{(2)} = \dfrac{V_f}{Z_{22}^{(1)} + Z_{22}^{(2)} + 0} = \dfrac{1.05\angle 0^0}{j0.1389 + j0.1456} = -j3.6907\,\text{pu}$

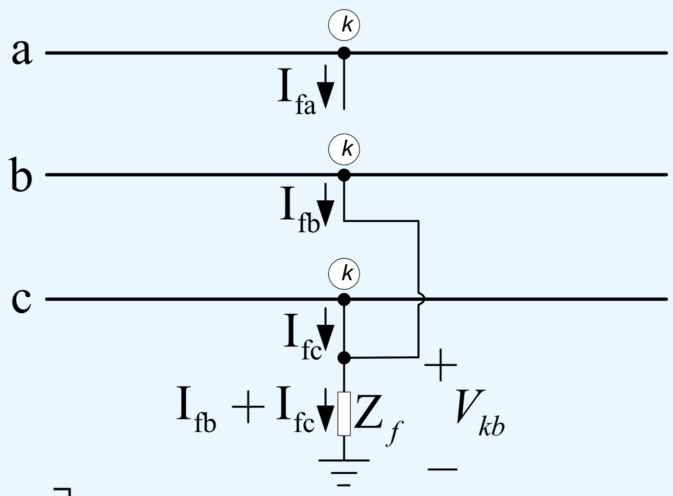
b): $$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{fa}^{(0)} \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j3.6907 \\ j3.6907 \end{bmatrix} = \begin{bmatrix} 0 \\ -6.3925 \\ 6.3925 \end{bmatrix} \text{pu}$$

c): $V_{2a}^{(2)} = V_{2a}^{(1)} = -Z_{22}^{(2)} I_{fa}^{(2)} = -j0.1456 \times j3.6907 = 0.5374\text{pu}$ **and** $V_{2a}^{(0)} = 0$

$$\begin{bmatrix} V_{2a} \\ V_{2b} \\ V_{2c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{2a}^{(0)} \\ V_{2a}^{(1)} \\ V_{2a}^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.5374 \\ 0.5374 \end{bmatrix} = \begin{bmatrix} 1.0748 \\ -0.5374 \\ -0.5374 \end{bmatrix} pu$$

2.8 Double line-to-ground fault analysis (double phase-to-ground fault)

Step 3: For a phase *b* and *c* to ground fault, phase current and voltage equations:



$$I_{fa}=0 \qquad V_{kb}=V_{kc}=(I_{fb}+I_{fc})Z_f$$

Step 4: The sequence currents and voltage equations:

$$I_{fa}^{(0)}=(I_{fb}+I_{fc}+0)1/3 \Rightarrow I_{fb}+I_{fc}=3I_{fa}^{(0)}$$

$$V_{kb}=V_{kc}=\left(I_{fb}+I_{fc}\right)Z_f=3Z_fI_{fa}^{(0)}$$

$$\begin{bmatrix} V_{ka}^{(0)} \\ V_{ka}^{(1)} \\ V_{ka}^{(2)} \end{bmatrix}=\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kc} \end{bmatrix}=\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_{ka} \\ 3Z_fI_{fa}^{(0)} \\ 3Z_fI_{fa}^{(0)} \end{bmatrix} \quad \textbf{(1)}$$

The second and third rows of (1) show that $V_{ka}^{(1)}=V_{ka}^{(2)}$.

The first row of (1):

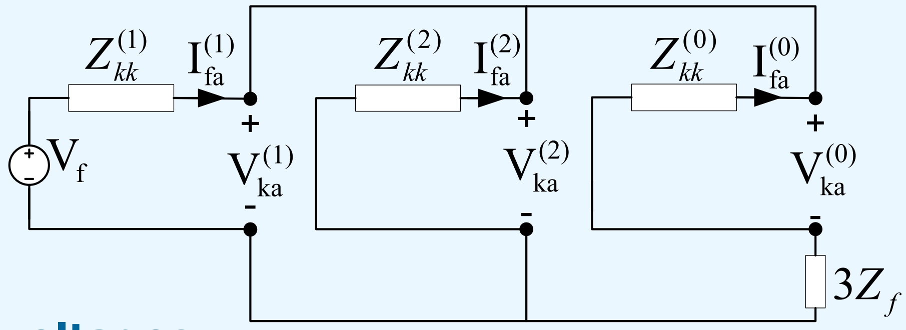
$$3V_{ka}^{(0)}=V_{ka}+2(3Z_fI_{fa}^{(0)})=\overbrace{V_{ka}^{(0)}+V_{ka}^{(1)}+V_{ka}^{(2)}}^{V_{ka}}+2(3Z_fI_{fa}^{(0)})=V_{ka}^{(0)}+2V_{ka}^{(1)}+2(3Z_fI_{fa}^{(0)})$$

$$\Rightarrow \underline{V_{ka}^{(1)}=V_{ka}^{(2)}=V_{ka}^{(0)}-3Z_fI_{fa}^{(0)}}$$

$$I_{fa}=0 \quad \Rightarrow \quad \underline{I_{fa}^{(0)}+I_{fa}^{(1)}+I_{fa}^{(2)}=0}$$

Step 5: network connection:

$$V_{ka}^{(1)} = V_{ka}^{(2)}$$
$$V_{ka}^{(1)} = V_{ka}^{(0)} - 3Z_f I_{fa}^{(0)}$$
$$I_{fa}^{(0)} + I_{fa}^{(1)} + I_{fa}^{(2)} = 0$$



Step 6: sequence currents and voltages:

$$I_{fa}^{(1)} = \frac{V_f}{Z_{kk}^{(1)} + Z_{kk}^{(2)} \,||\, (Z_{kk}^{(0)} + 3Z_f)}$$

$$I_{fa}^{(2)} = -\frac{Z_{kk}^{(0)} + 3Z_f}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f} I_{fa}^{(1)}$$

$$I_{fa}^{(0)} = -\frac{Z_{kk}^{(2)}}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f} I_{fa}^{(1)}$$

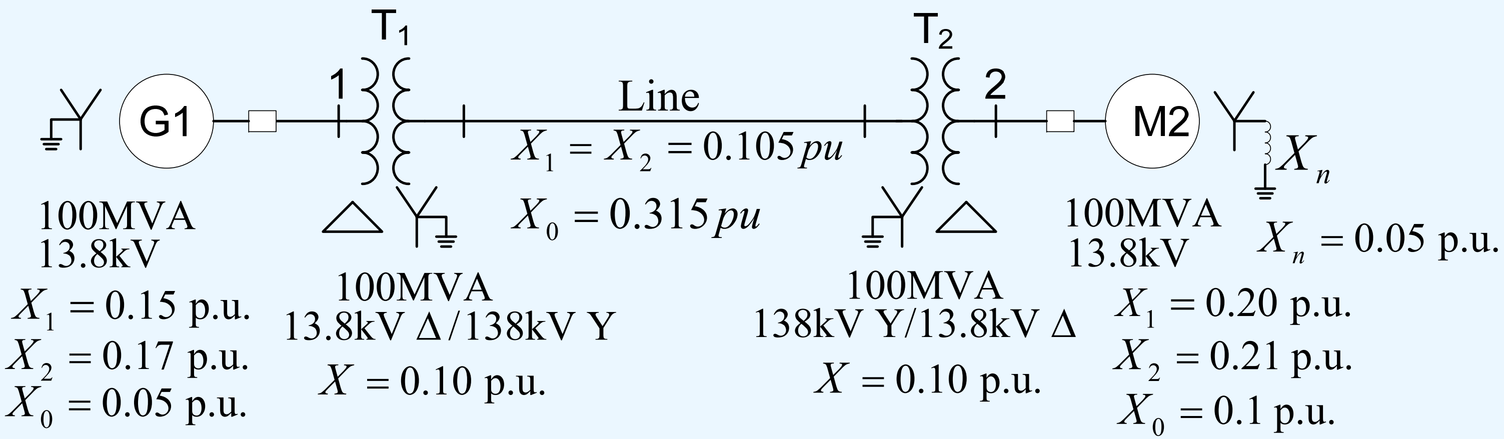
$$V_{ka}^{(1)} = V_f - Z_{kk}^{(1)} I_{fa}^{(1)}$$
$$V_{ka}^{(2)} = -Z_{kk}^{(2)} I_{fa}^{(2)}$$
$$V_{ka}^{(0)} = -Z_{kk}^{(0)} I_{fa}^{(0)}$$

Step 8: Phase voltages and currents:

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{fa}^{(0)} \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{bmatrix} \qquad \begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{ka}^{(0)} \\ V_{ka}^{(1)} \\ V_{ka}^{(2)} \end{bmatrix}$$

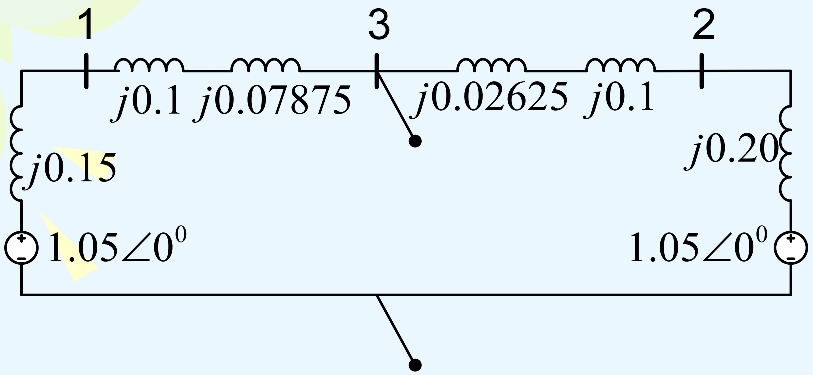
Example 2.8: The sequence reactances of components for a power system are shown in the following figure. The line sequence reactances are on 100MVA and 138kV base. A bolted double phase to ground fault between phase b and phase c occurs at one quarter of the transmission line away from T2. Pre-fault voltage at fault point is $1.05\angle 0^o$ per unit.

a) **Draw the sequence networks for fault analysis.**
b) **Determine sequence components of the fault currents at the fault point.**
c) **Calculate the sequence currents flowing from Y side of T1 to the fault point,**
d) **Calculate the phase currents flowing from the Y side of T1 to the fault point.**

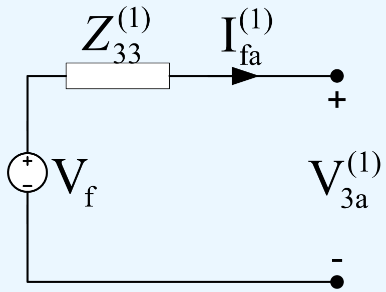


Solutions: add bus 3 at the fault point

a) The positive-sequence network:



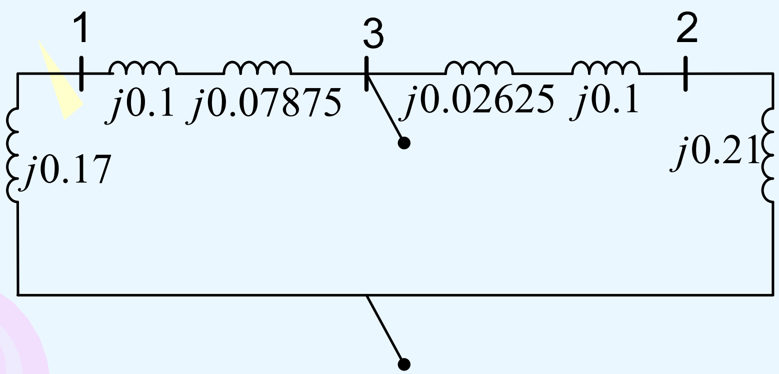
Thevenin equivalent circuit:



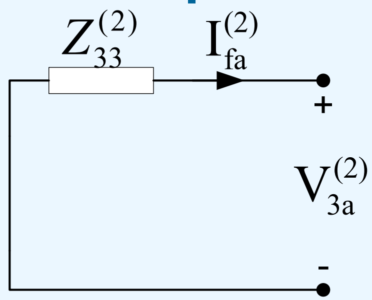
$$V_f = 1.05\angle 0^0 \text{ p.u.}$$

$$Z_{33}^{(1)} = (j0.15 + j0.1 + j0.07875) \,||\, (j0.02625 + j0.1 + j0.20) = j0.16375 \text{p.u.}$$

The negative-sequence network :

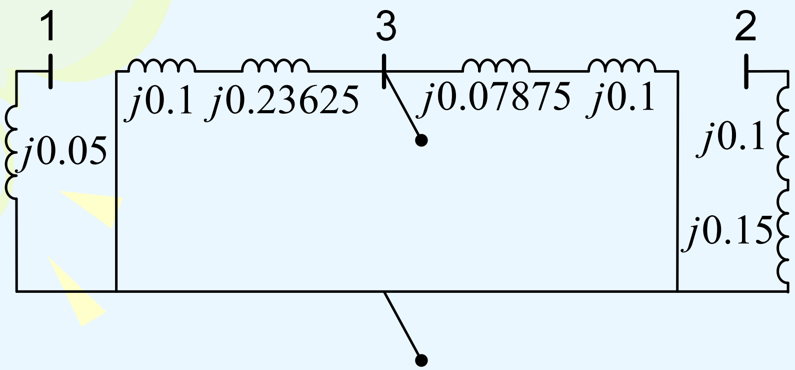


Thevenin equivalent circuit:

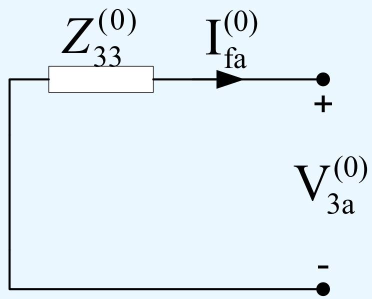


$$Z_{33}^{(2)} = (j0.17 + j0.1 + j0.07875) \,||\, (j0.02625 + j0.1 + j0.21) = j0.17119 \text{p.u.}$$

The zero-sequence network :



Thevenin equivalent circuit:



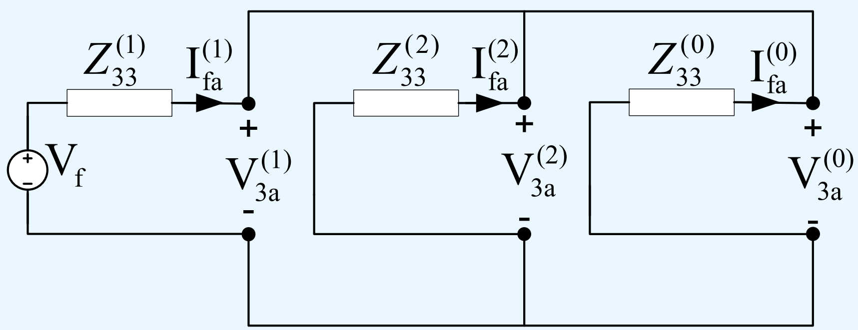
$$Z_{33}^{(0)} = (j0.1 + j0.23625) \parallel (j0.07875 + j0.1) = j0.11671\text{p.u.}$$

b. sequence currents and voltages:

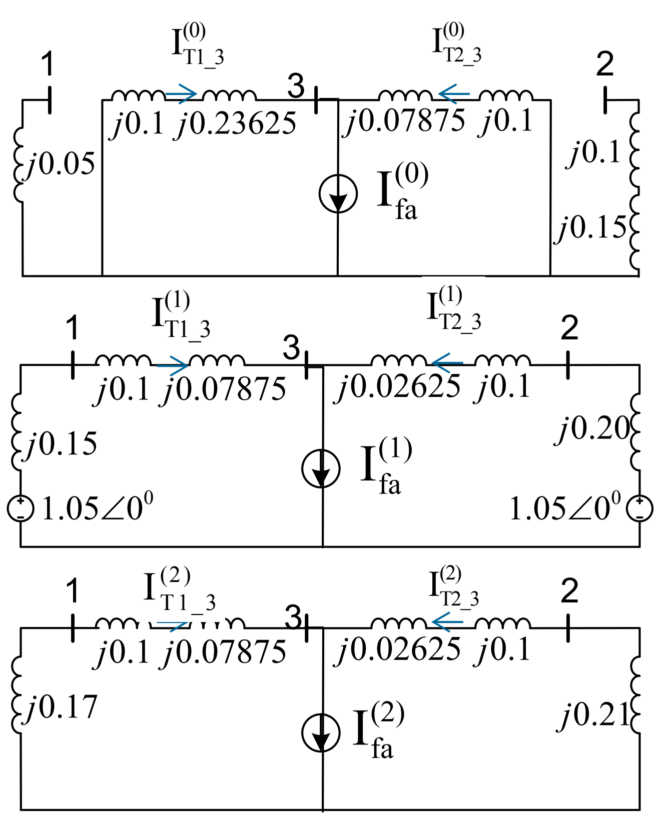
$$I_{fa}^{(1)} = \frac{V_f}{Z_{kk}^{(1)} + Z_{kk}^{(2)} \parallel Z_{kk}^{(0)}} = ???$$

$$I_{fa}^{(2)} = -\frac{Z_{kk}^{(0)} + 3Z_f}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f} I_{fa}^{(1)} = ??$$

$$I_{fa}^{(0)} = -\frac{Z_{kk}^{(2)}}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f} I_{fa}^{(1)} = ??$$



c) and d) Adding sequence currents as current sources to the fault bus in original sequence networks as shown below, the sequence currents flowing from T1 to the fault point can be calculated uing current division as.



The sequence currents flowing from T1 to bus 3:

$$I^{(1)}_{T1\_3} = \frac{j0.02625 + j0.1 + j0.20}{(j0.15 + j0.1 + j0.07875) + (j0.02625 + j0.1 + j0.20)} I^{(1)}_{fa}$$

$$I^{(2)}_{T1\_3} = \frac{j0.02625 + j0.1 + j0.21}{(j0.17 + j0.1 + j0.07875) + (j0.02625 + j0.1 + j0.21)} I^{(2)}_{fa}$$

$$I^{(0)}_{T1\_3} = \frac{j0.07875 + j0.1}{(j0.1 + j0.23625) + (j0.07875 + j0.1)} I^{(0)}_{fa}$$

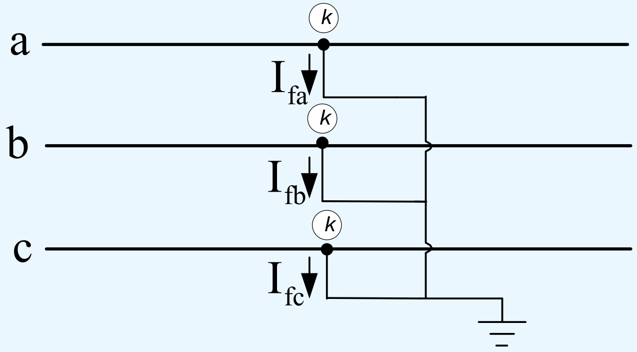
The phase currents flowing from T1 to bus 3 (homework):

$$\begin{bmatrix} I_{aT1\text{-}3} \\ I_{bT1\text{-}3} \\ I_{cT1\text{-}3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I^{(0)}_{T1\text{-}3} \\ I^{(1)}_{T1\text{-}3} \\ I^{(2)}_{T1\text{-}3} \end{bmatrix} = ???$$

2.9 Three phase-to-ground fault

The fault conditions at fault bus k:

$$V_{ka} = V_{kb} = V_{kc} = 0$$



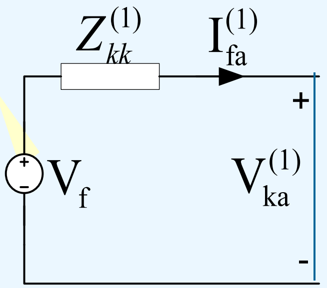
Sequence voltages at bus k:

$$V_{ka}^{(0)} = \frac{1}{3}(V_{ka} + V_{kb} + V_{kc}) = 0$$

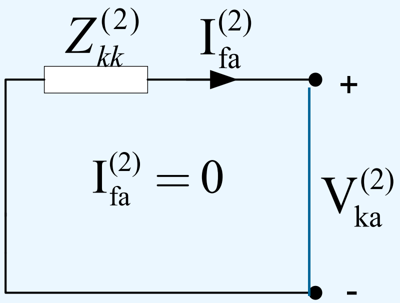
$$V_{ka}^{(1)} = \frac{1}{3}(V_{ka} + aV_{kb} + a^2V_{kc}) = 0$$

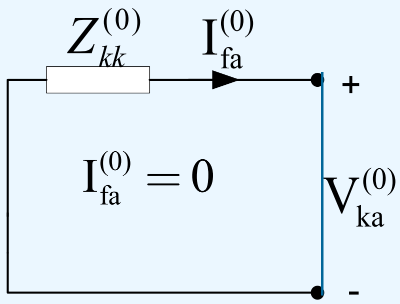
$$V_{ka}^{(2)} = \frac{1}{3}(V_{ka} + a^2V_{kb} + aV_{kc}) = 0$$

Apply the above sequence voltages to the equivalent circuits:



$$I_{fa}^{(1)} = \frac{V_f}{Z_{kk}^{(1)}}$$

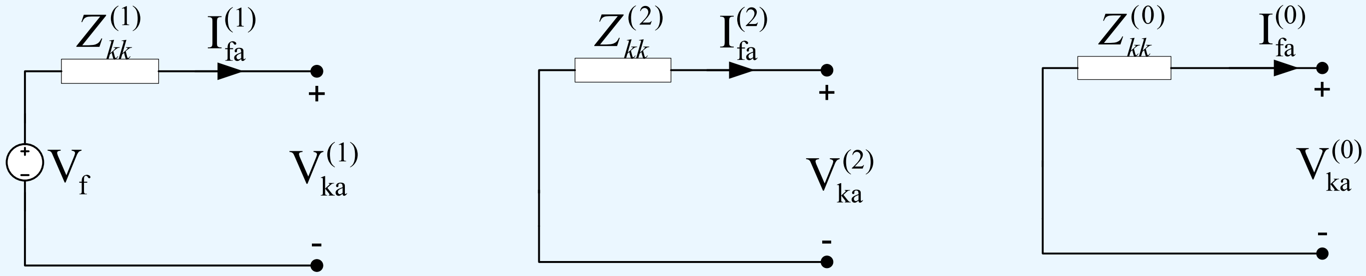




Therefore, three phase fault is the balanced fault and there are only positive fault voltages and currents to be calculated from positive network.

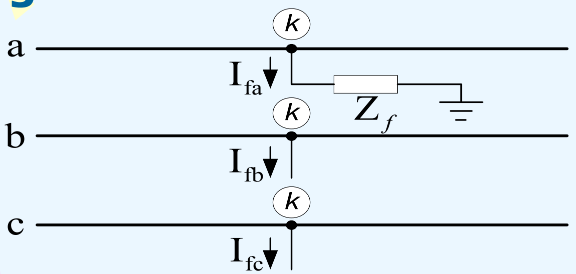
Summary for Chapter 2:

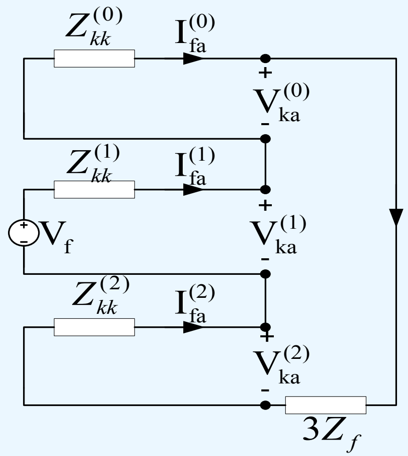
- **Draw the per-phase per-unit positive-, negative-, and zero-sequence networks of a power system.**
- **Develop the Thevenin equivalent circuits as viewed from the fault bus k for the positive-, negative- and zero-sequence networks.**



- **Calculate the sequence voltage and currents at the fault point based on the connection of the sequence networks for the different faults:**

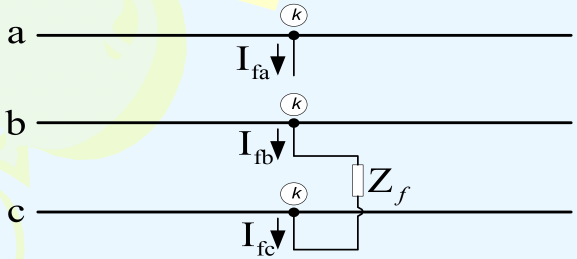
Single Line (or phase) to ground fault:

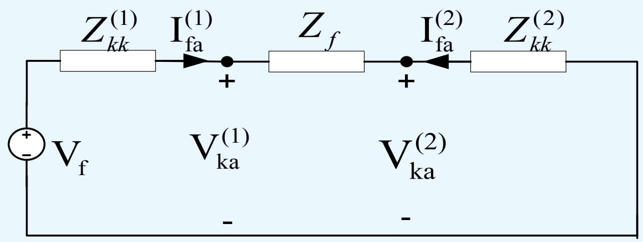




$$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = \frac{V_f}{Z_{kk}^{(0)} + Z_{kk}^{(1)} + Z_{kk}^{(2)} + 3Z_f}$$

Double line (or phase) fault:

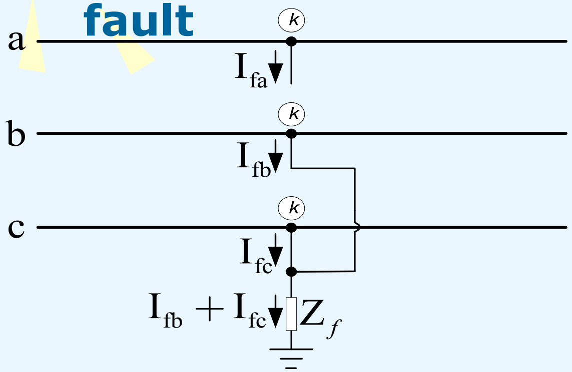


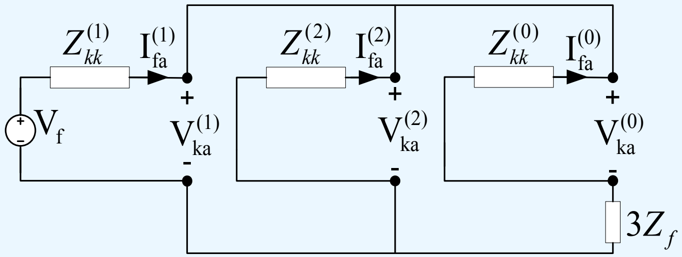


$$I_{fa}^{(1)} = -I_{fa}^{(2)}$$

$$= \frac{V_f}{Z_{kk}^{(1)} + Z_{kk}^{(2)} + Z_f}$$

Double line (or phase)-to-ground fault





$$I_{fa}^{(1)} = \frac{V_f}{Z_{kk}^{(1)} + Z_{kk}^{(2)} \parallel (Z_{kk}^{(0)} + 3Z_f)}$$

$$I_{fa}^{(2)} = -\frac{Z_{kk}^{(0)} + 3Z_f}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f} I_{fa}^{(1)}$$

$$I_{fa}^{(0)} = -\frac{Z_{kk}^{(2)}}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f} I_{fa}^{(1)}$$

- **Add the sequence current to the corresponding sequence network as a current source and calculate the sequence currents and voltage in other parts of the sequence network considering phase shift.**
- **Calculate the phase voltages and currents using matrix A:**

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix} = \mathbf{A} \begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} = \mathbf{A} \begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix}$$

3. Power System Protection

It can be seen from fault analysis in Chapter 2 that there will be large short circuit currents in a power system when a fault occurs. Without controlling the large short circuit currents, power system equipment and load will be damaged. Therefore, protection devices and systems have to be designed and installed in a power system to isolate the fault and protect system equipment.

The objectives of this chapter:

- **Protection systems and requirements**
- **Fundamental principles of various relays**
- **Various protection schemes**

3.1 Basic protection system

3.2 Selection of instrument transformers

3.3 Overcurrent protection

3.4 Differential protection

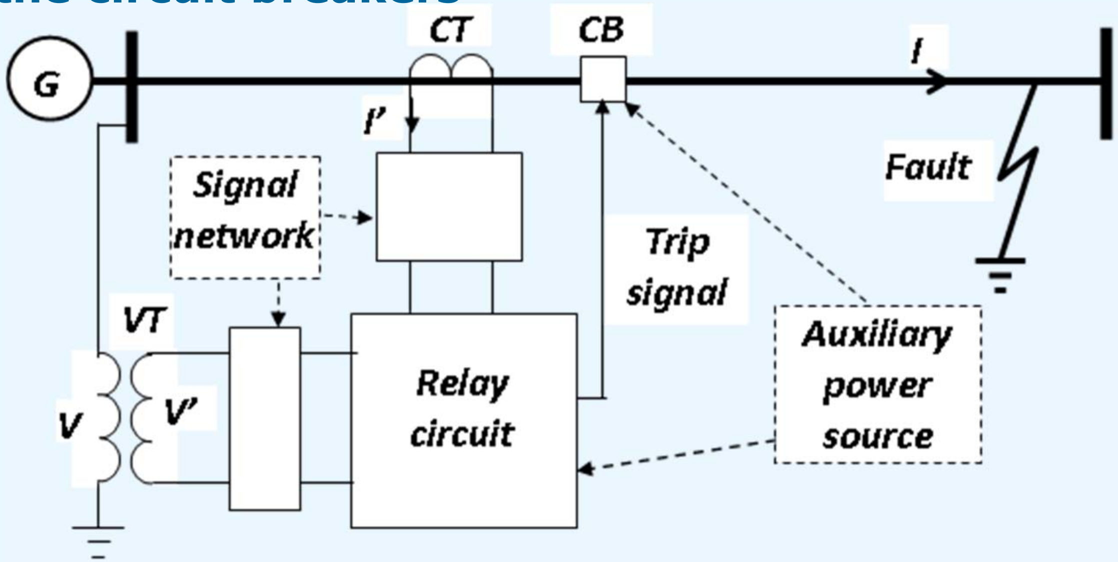
3.1 Basic Protection System

3.1.1 Basic protection system configuration

1) Relay Circuit:

• Detect abnormal conditions through measuring changes of currents, voltages and impedances

• Initiate actions to trip the circuit breakers



2) Sensing devices:

• Current transformer (CT): Provide current signal for relay circuit

• Voltage or potential transformer (VT or PT): Provide voltage signal for relay circuit

3) Signal network:

• Transfer the accurate signals from CTs and VTs to relay circuit

4) Circuit breaker (CB)

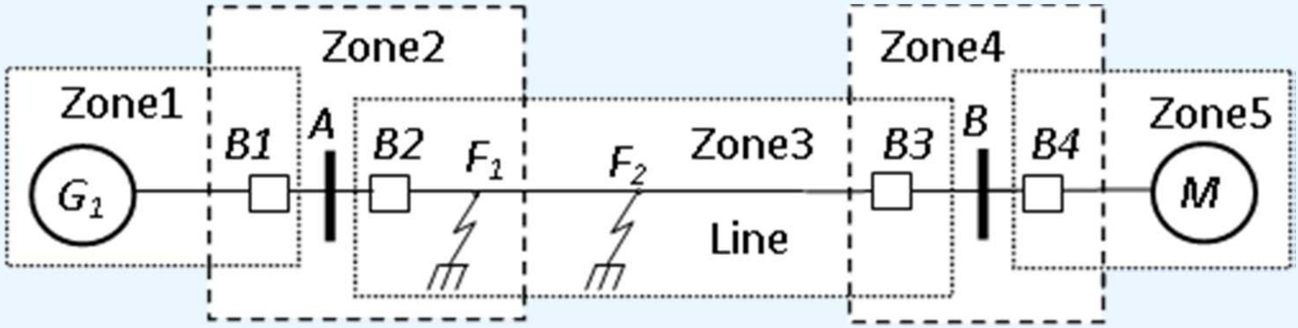
• Isolate the fault through opening circuits (just like a switch)

5) Auxiliary power source: (Battery)

• Provide back-up power for relay and circuit breaker

3.1.2 Protection zones

Power system is the most complicated system in the world. It is not possible to design a single protection system to protect the entire power system. Many protection systems and devices are required. A protection zone is usually defined for one component (sometime two components) and its connection as shown in the following figure. The components in a zone is protected by one or more breakers. If a fault occurs in a zone, *all the breakers* in the zone should operate to isolate the fault, which is the basic design requirement.



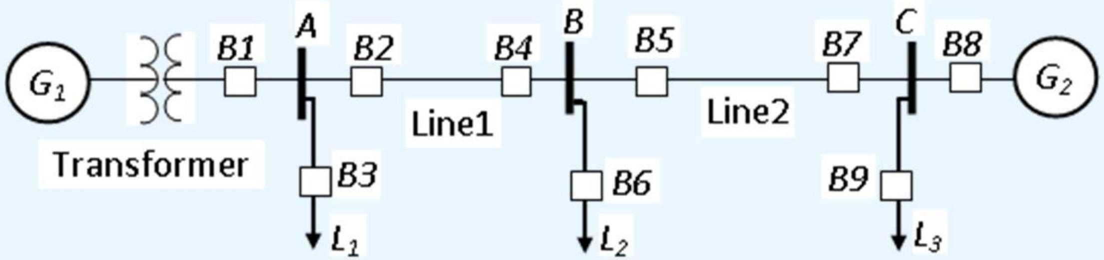
Basic zone requirements:
To protect whole system without leaving any *small area* between two zones unprotected, the *overlapping* of neighboring zones is required. *Breakers* are inserted in overlapping areas. CTs are connected at the border of a Zone.

Problem caused by overlapping:
Both zones will be isolated if a fault occurs in the overlapping area. Therefore, the overlapping area should be as small as possible to reduce the affected area.

***Protection zones are also important to determine the fault location.**

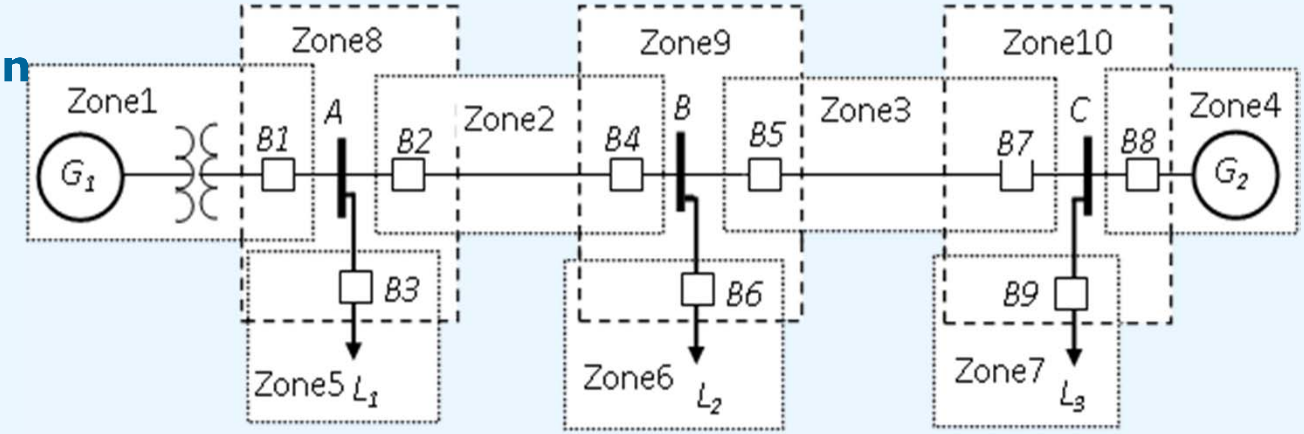
Example 3-1: For the following system, define protection zones based on the locations of the breakers. Assuming that all main protections are 100% reliable, determine the fault location for each of the two cases based on the circuit breakers opened.

Case 1: B2 and B4
Case 2: B1, B2, B3 and B4



Solutions:

The protection zones are as shown in the right figure.



Fault location can be found based on protection zones and the operation of breakers as follows:

Case 1: Zone2 excluding overlapping areas with Zones 8 and 9 (if it is in overlapping area with Zone9, B5 and B6 also opened).
Case 2: The overlapping area of Zone 2 and Zone 8

3.2 CT and VT Selection

Voltage transformer selection is based on voltage level of the protected circuit and the rating of a voltage relay connected to VT secondary side which is 110V. Standard VT ratios are shown in the following table.

Table 3-1 Standard VT ratios

1:1	2:1	2.5:1	4:1	5:1	20:1
40:1	60:1	100:1	200:1	300:1	400:1
600:1	800:1	1000:1	2000:1	3000:1	4500:1

Examples: The ratio n of VT at 400kV circuit should be larger than or =400kV/110V=364. Therefore n=400:1 ratio is selected.

Current transformer selection is based on the maximum load current and the rating of a current relay connected at CT secondary side which is 1A or 5A in Singapore. Standard CT ratios for 5A rating are shown in the following table.

Table 3-2 Standard CT ratios:

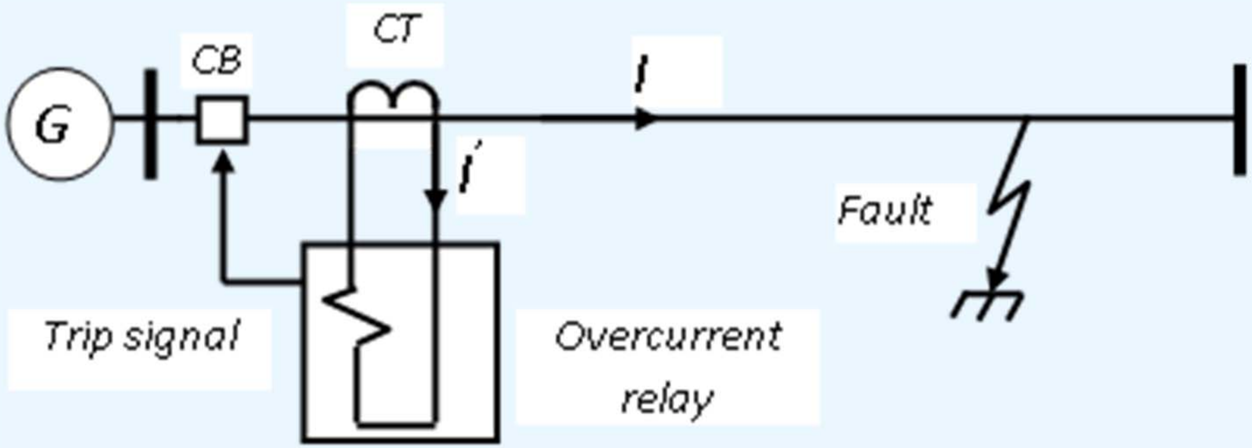
50:5	100:5	150:5	200:5	250:5	300:5	400:5	450:5
500:5	600:5	800:5	900:5	1000:5	1200:5	1500:5	1600:5
2000:5	2400:5	2500:5	3000:5	3200:5	4000:5	5000:5	6000:5

For example, the CT ratio for the maximum load current of 90 A should be larger than or =90/5. Therefore n=100:5 ratio is selected.

3.3 Overcurrent Protection

3.3.1 An overcurrent protection system (overcurrent relay)

An overcurrent protection system is usually designed to protect distribution networks when short circuit faults occur.



Relay signal: Current I' on CT secondary side

Relay setting:

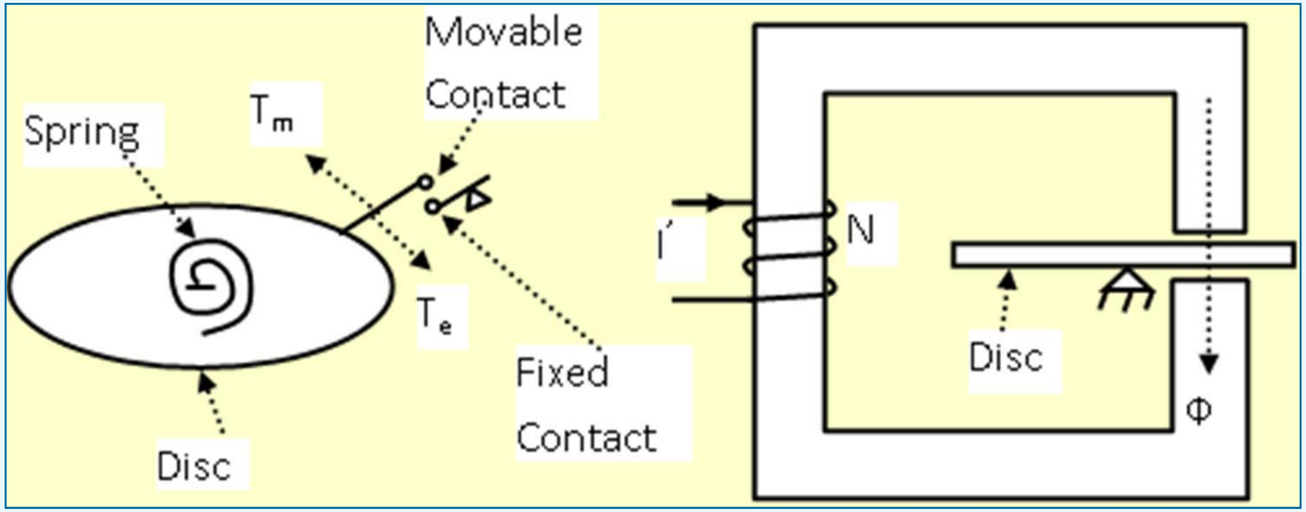
Pick-up or threshold current I_{pickup}

Operation principle:

- **System normal operation: $I' < I_{pickup}$:**
  No trip signal; circuit breaker is closed.
- **Short circuit fault: $I' > I_{pickup}$:**
  Trip signal; the breaker is opened to isolate the line.

3.3.2 An induction disc overcurrent relay

The key component of overcurrent protection is overcurrent relay. An electromagnetic induction disc relay and its components are shown in the figure:



T_m: Mechanical force produced by the spring

T_e : Electromagnetic force created by current I' (*opposite to T_m*)

T_e is proportional to $\Phi=kNI'$, where N is the number of turns of winding and k is coefficient.

Relay operation depends on the balance between T_m and T_e

Normal operation: $I' < I_{pickup}$, $T_e < T_m$, Two contacts are opened; No trip signal.

Fault condition: $I' >= I_{pickup}$, $T_e > T_m$, Two contacts are closed; Trip signal.

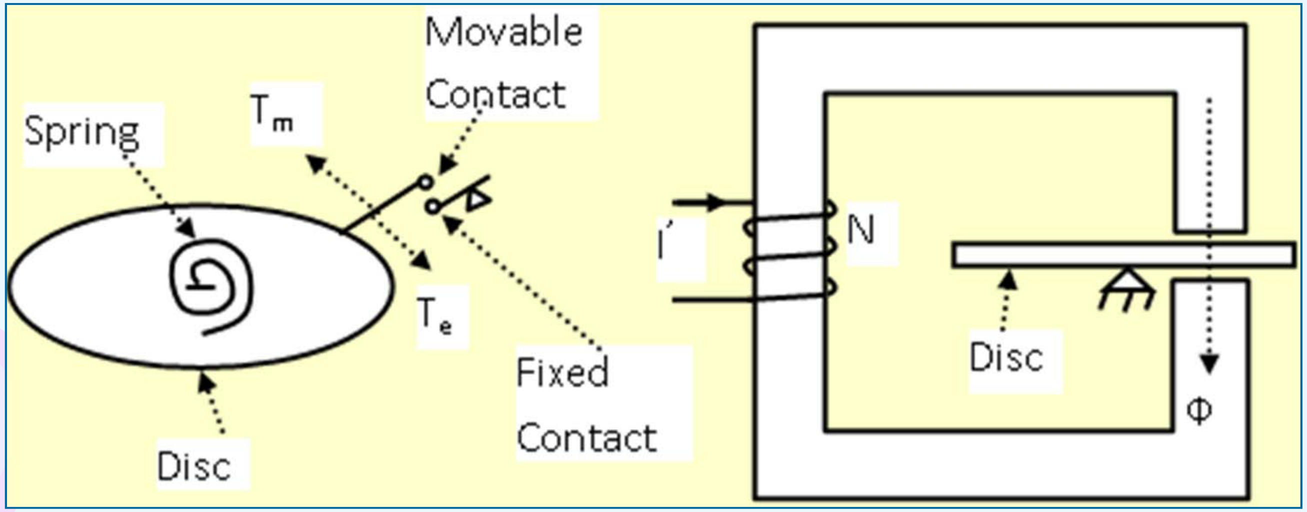
3.3.3 Relay settings

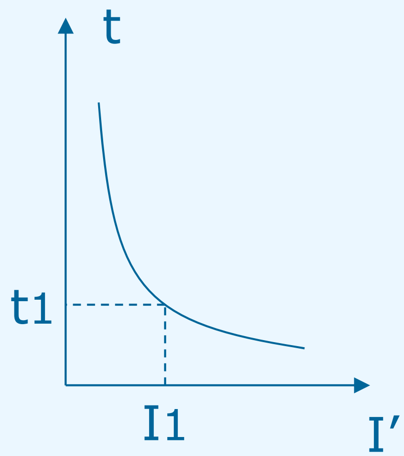
There are two relay parameters to be selected in protection design.

1. Plug Setting (PS): To produce the same T_e=T_m, large I' requires less N and small I' more N *because* T_e =*kNI'*. Therefore pick-up current can be set to different values through changing N designated as PS or tap setting .

2. Time dial Setting (TDS): For the same current and N, operating time of relay can be set to different value designated as TDS through changing the space between the fixed and movable contacts.

3. T-I curve: For a given T_m , N and TDS, relay operating time changes nonlinearly with current, which is represented by a time-current (T-I) curve. For a given current I1, the operating time t1 can be found from T-I curve.





T-I curve

3.3.4 Time-current curves for different overcurrent relays

1. CO-8 relay: The plug settings, the time dial settings and the related I-T <u>curves</u> for a CO-8 relay is shown in the figure.

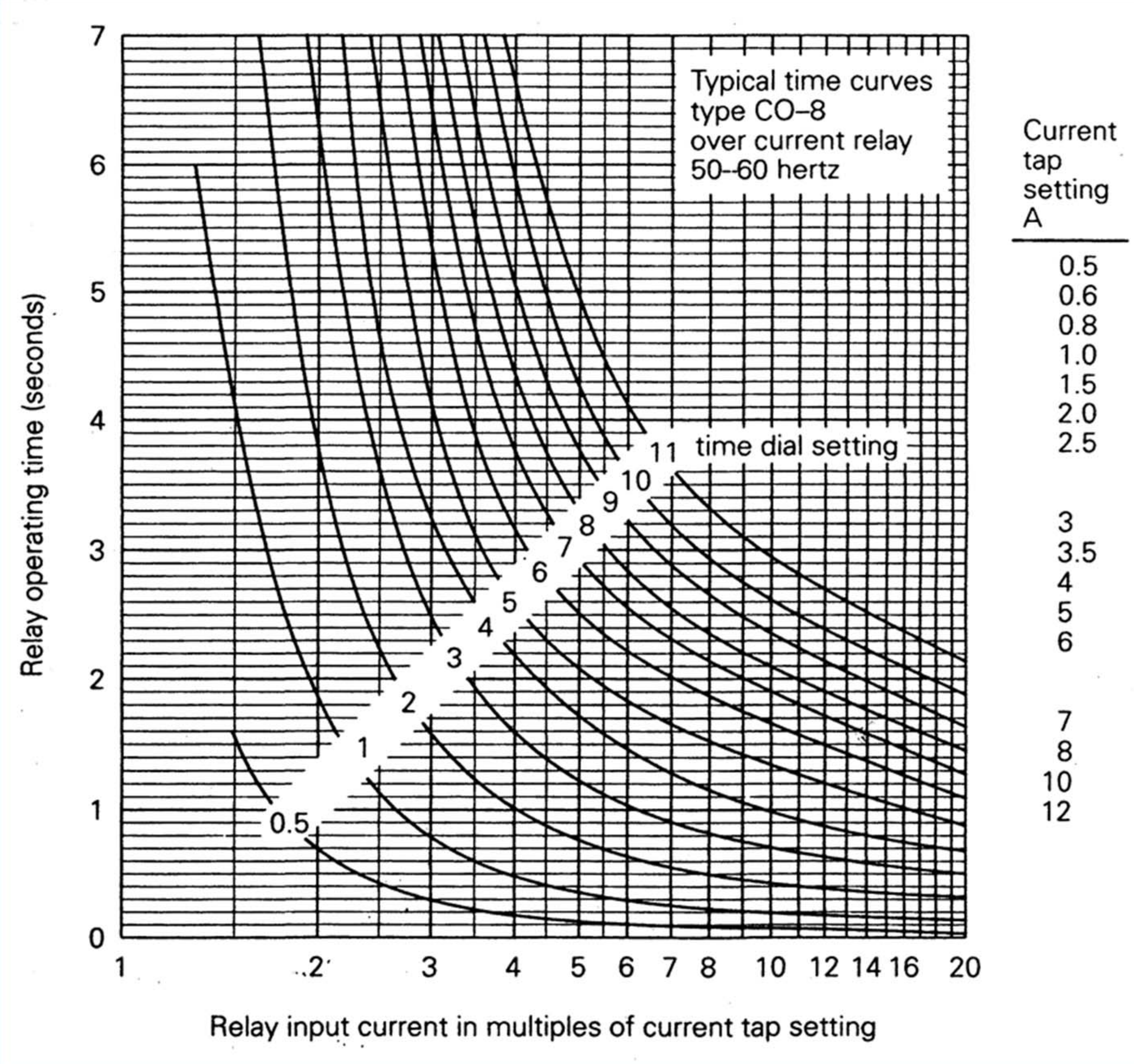
The current axis of the I-T curves is not absolute current (A) but the <u>multiple of pick up current</u>:

$$MPC = I'/I_{pickup}$$

The fastest TDS setting is TDS=0.5.

2. IDMT relay: The I-T <u>curve</u> for the inverse definite minimum time (IDMT) is represented by a <u>equation</u>:

$$t = \frac{0.14}{MPC^{0.02} - 1} \times TDS$$



For the given PS and TDS, the operating time of a relay for a given current can be determined using I-T curves or the equation on page 74.

Example 3-2: A relay is selected to protect a distribution feeder. The fault current through CT primary side is 2000A and CT ratio is 200:5. PS is set to 200% of the rated current 5 A. Determine the operating time of the following two relays if the TDS is set to 2.

- A CO-8 relay with the time current curve
- An IDMT relay with the time-current equation

Solutions:

The current through the relay=2000/40=50A

The pickup current is 200%×5=10A

The MPC is 50/10=5

For the CO-8:

The operating time from the curve of TDS=2 is 0.75s

For the IDMT:

$$t = \frac{0.14}{MPC^{0.02} - 1} \times TDS = \frac{0.14}{5^{0.02} - 1} \times 2 = 8.6s$$

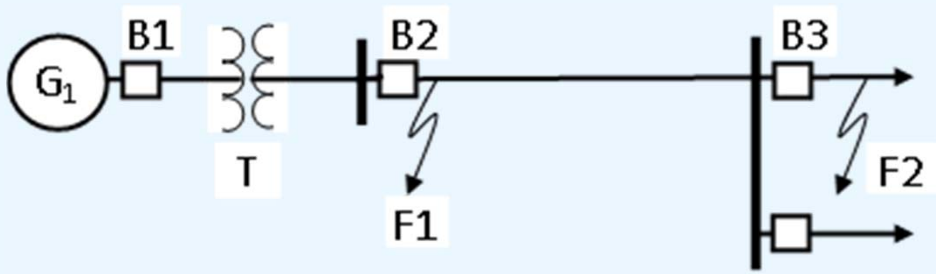
3.3.5 PS and TDS settings for over-current protection

Over-current protection is commonly used to protect system components in a radial distribution system as shown in the figure.

When a fault occurs in a distribution system, circuit breakers in the system have to operate selectively, accurately and reliably to isolate the fault and to limit the affected area to the minimum.

Reliable operation of the protection system for a short circuit fault needs one main protection and one back-up protection. If the main protection fails to operate for a fault in its protecting area, the back-up needs to operate.

For fault F2, B3 is main protection and B2 is the back-up of B3.
For fault F1, B2 is main protection and B1 is the back-up of B2.



The back-up protection has to operate after the main protection with a time delay designated as coordination time interval (CTI), which means that the operating time of the back-up equals to the operating time of the main plus CTI. $t_{req}^{backup} = t_{\min}^{Main} + CTI$

3.3.5 PS and TDS settings for over-current protection (continued)

In order to protect different parts of a distribution system, many relays are required. PSs and TDSs of relays have to be properly selected based on fault currents and operating times required.

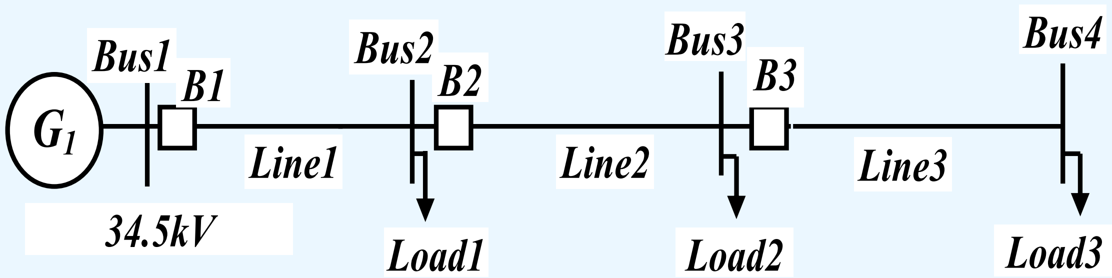
1) Selection of PS (based on the maximum load current):

Step 1: Determine the relay minimum operating current (MOC) seen from the primary side of a CT for the relay to pick up.

In order to avoid malfunction of relays during the peak load periods, Phase relay: MOC is set to 200% of the maximum load current.

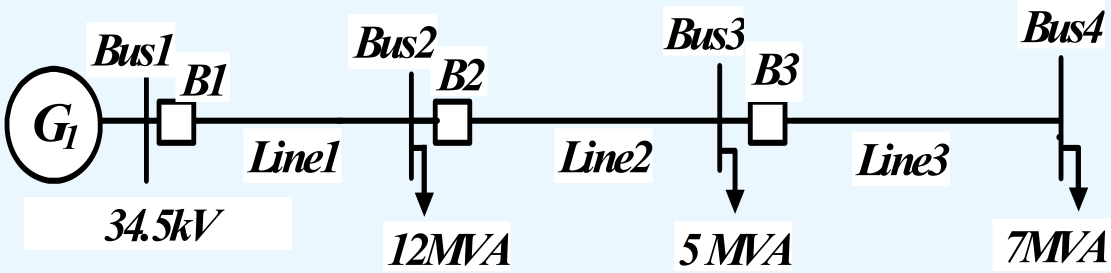
Step 2: Determine the relay plug setting: $I_{pickup}(or\ PS) \geq \dfrac{MOC}{CT\ ratio}$

Example 3.3: A 50-Hz radial distribution system is shown in the figure. The loads at Buses 2, 3 and 4 are 12MVA, 5 MVA and 7MVA respectively with the same power factor. Select PSs for the phase relays.



Solutions:

PS selection: (Ignore line losses and voltage drop when calculate maximum load current)



B1 relay (all loads):

CT Primary load current:

$$|S_{L1}|+|S_{L2}|+|S_{L3}|=12+5+7=24MVA$$

$$|I_{L1}|=\frac{|S_{L1}|+|S_2|+|S_3|}{\sqrt{3}\times|V_1|}=\frac{24\times10^6}{\sqrt{3}\times34.5\times10^3}=401.6A$$

MOC=2×401.6=803.2*A*

CT Secondary current:

$$|I'_{L1}|=\frac{MOC}{CT\ ratio}=\frac{803.2}{400/5}=10.4A$$

Select PS_1 =12A

B2 relay (Load2 anad Load3): $|S_{L2}|+|S_{L3}|=5+7=12MVA$

CT Primary current: $$|I_{L2}|=\frac{|S_2|+|S_3|}{\sqrt{3}\times|V_2|}=\frac{12\times10^6}{\sqrt{3}\times34.5\times10^3}=200.8A$$

MOC=2×200.8=401.6*A*

CT Secondary current: $$|I'_{L2}|=\frac{MOC}{CT\ ratio}=\frac{401.6}{200/5}=10.04A$$

Select PS$_2$ =12A

B3 relay (Load3):

CT Primary current: $$|I_{L3}|=\frac{|S_3|}{\sqrt{3}\times|V_3|}=\frac{7\times10^6}{\sqrt{3}\times34.5\times10^3}=117.1A$$

MOC=2×117.1=234.2*A*

CT Secondary current: $$|I'_{L3}|=\frac{MOC}{CT\ ratio}=\frac{234.2}{200/5}=5.85A$$

Select PS$_3$ =6A

2) Selection of TDSs (based on the maximum fault current):

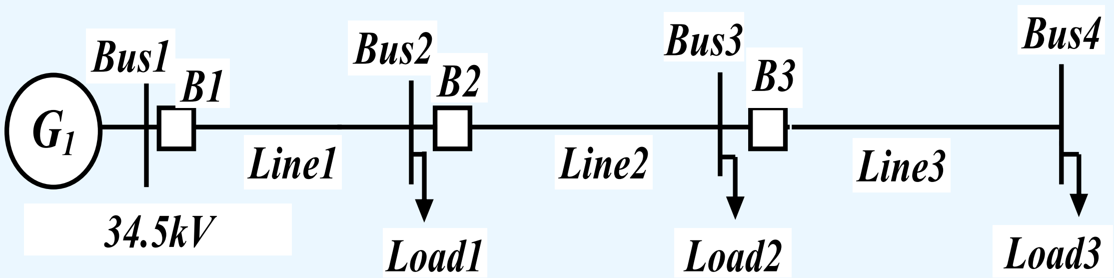
Step1: Select the minimum TDS for the last relays at the end of a radial network.

Step2: Find the pairs of main and back up relays.

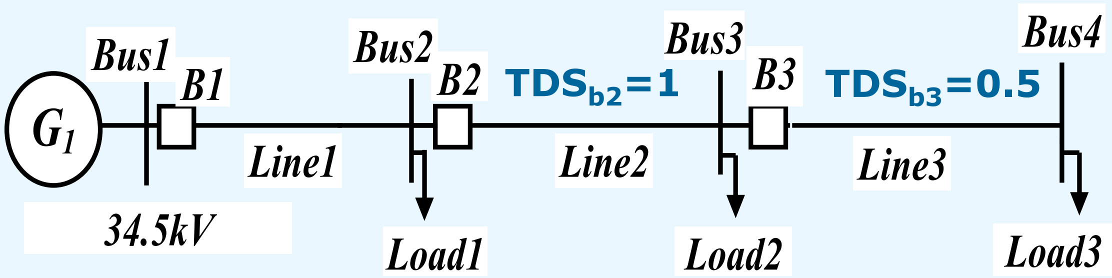
Step3: Calculate the minimum operation time $t_{\min}^{Main}$ of the main relay based on the maximum fault current through the main relay.

Step4: Determine the TDS of the back up relay based on the required operating time $t_{req}^{backup} = t_{\min}^{Main} + CTI$ and MPC under the same fault current.

Example 3.4: For the same system as shown in example 3.3 The maximum fault currents at Buses 2, 3 and 4 are 3500A, 2500A and 1000A respectively. The CO-8 relays are provided for breakers. The ratios of the CTs corresponding to B1, B2 and B3 are 400:5. 200:5 and 200:5 respectively. Set PSs and TDSs for the phase relays. The CTI is 0.5s for two adjacent relays.



B3 is the last relay at the end of network, select TDS_{b3}=0.5 (trips as fast as possible).



<u>For B3 (main) and B2 (back-up) pair:</u>

<u>B3 relay as main protection:</u> (PS_3=6A)

For the faults occurring on line3, B3 relay is the main protection

The max fault current near bus 3 (2500A): $MPC_3 = \dfrac{2500/(200/5)}{6} = 10.4$

The minimum operating time: T3=0.07s from TDS_{b3}=1/2 curve.

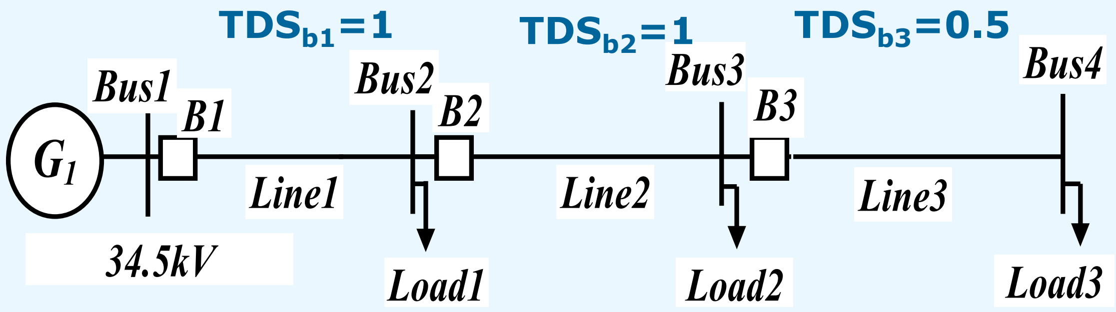
<u>B2 relay as back up of B3:</u> (PS_2=12A)

For the same fault current 2500A, if B3 cannot operate, B2 should operate as the back up for B3.

Required fault clearing time for B2: T2=T3+CTI=0.07+0.5s=0.57s.

For the same current (2500A): $MPC_2 = \dfrac{2500/(200/5)}{12} = 5.2$

Based on the operating point of T2=0.57 and $MPC_2 = 5.2$, select TDS_{b2}=2 from the relay curves.



For B2 (main) and B1 (back-up) pair:

B2 relay as the main protection: (PS_2=12A, TDS=2)
For the faults occurring on line2, B2 relay is the main protection.

The max fault current near bus 2 (3500A): $MPC_2 = \frac{3500/(200/5)}{12} = 7.3$

The minimum operating time: T2=0.53s from TDS_{b2}=2 relay curve.

B1 relay as back up of B2: (PS_2=12A)
For the same fault current 3500A, if B2 cannot operate, B1 should operate as the back up for B2.

Required fault clearing time for B1: T1=T2+CTI=0.53+0.5s=1.03s.

For the same current (3500A): $MPC_1 = \frac{3500/(400/5)}{12} = 3.645$

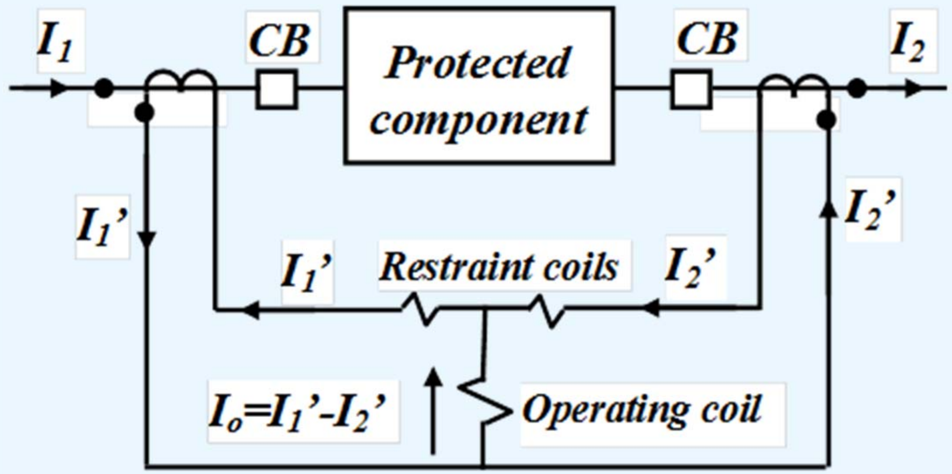
Based on the operating point of T1=1.03 and $MPC_1 = 3.645$, select TDS_{b1}=2 from the relay curves.

3.4 Differential protection

3.4.1 Percentage differential relay

Differential relay is only used to protect only internal faults of a power system element. It will not protect the external faults and overloads. A percentage differential relay is shown in the figure. The relay has two restraint windings and one operating winding. Two CTs provide current signals for the relay. Two circuit breakers are required to isolate each phase of the protected component.

To protect a power system component using differential relay, CTs have to be correctly connected to make sure that two restraint currents must be *in phase* for the external faults and normal load.

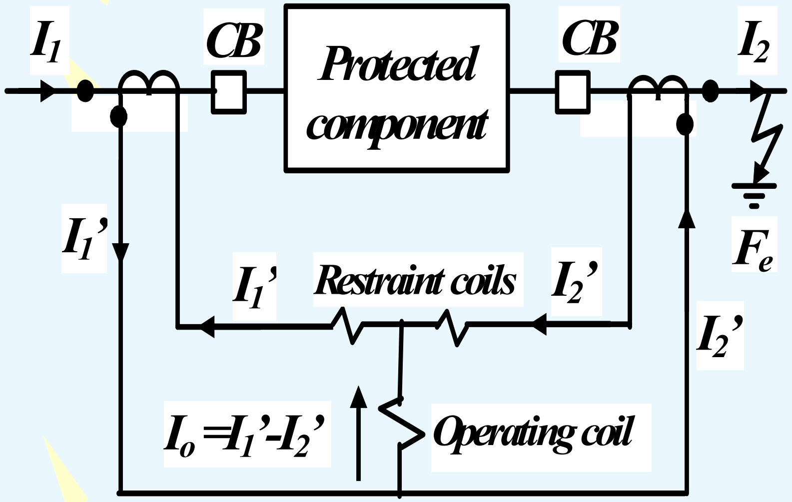


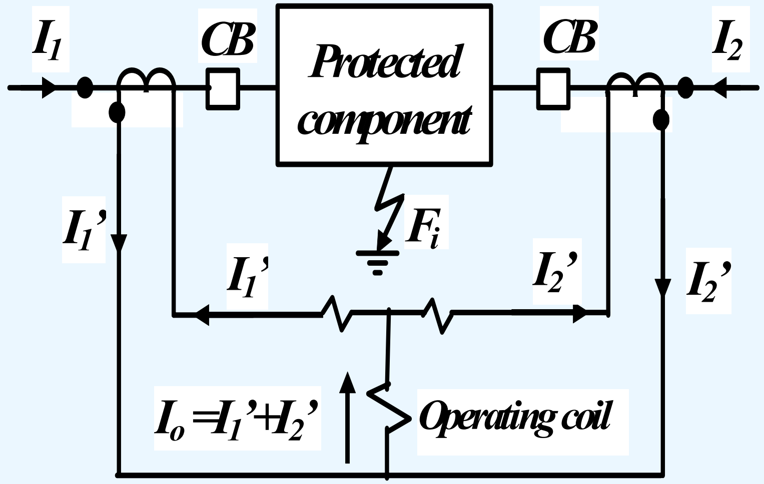
Currents I_1' and I_2' in the restraint windings produce restrain forces to inhibit trip signal. (T_{er})

Current $I_o=I_1'-I_2'$ in the operating winding produces an operating force to create trip signal. (T_{eo})

3.4.2 Relay operation:

External fault: **Relay hardly operates because the operating force is cancelled by the two restrain forces and the operating current $I_o = I_1' - I_2'$ is very small. Relay cannot operate.**

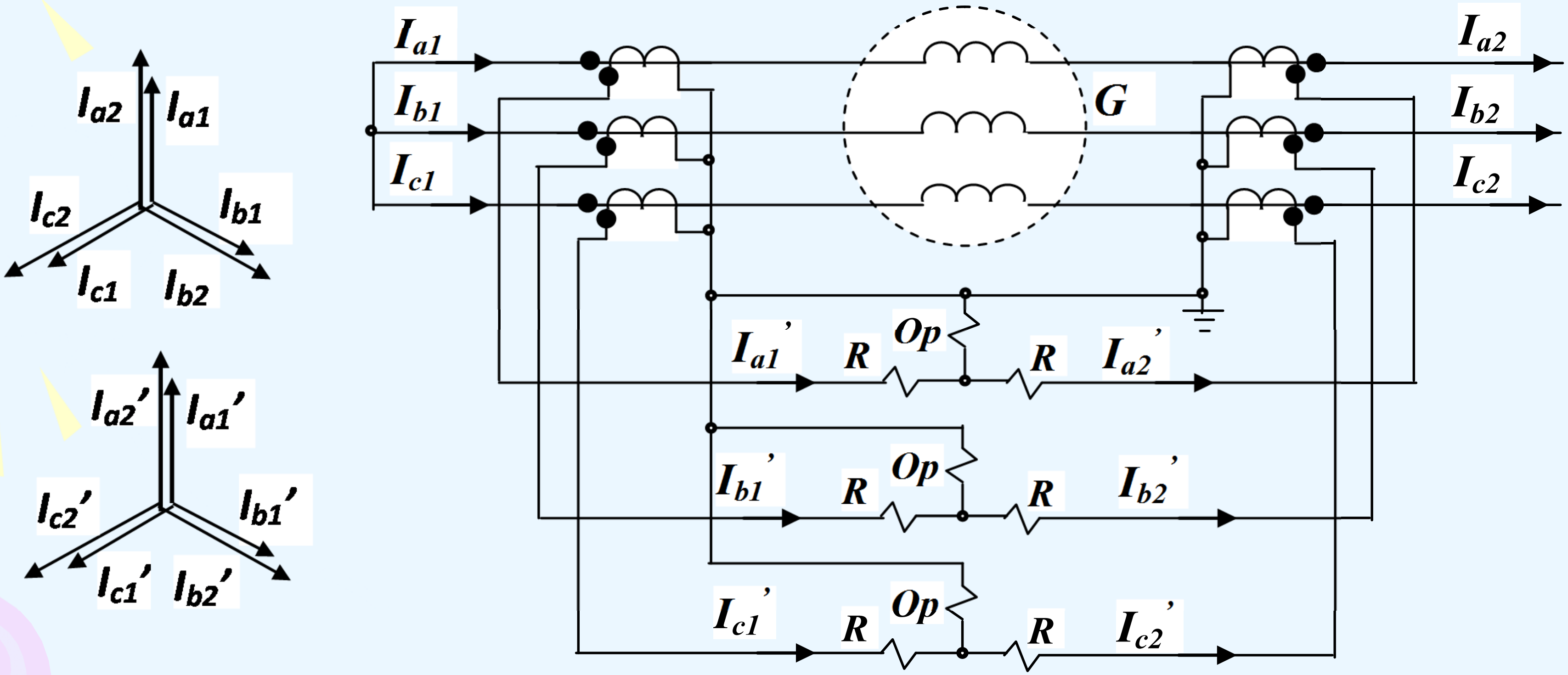




Internal fault: **Smaller operating current is required for relay operation because the two restrain forces are cancelled each other. In this case, relay trip depends on <u>only the pickup current</u> which is usually set from 0.14 A to 3 A depending on application.**

3.4.3 Generator differential protection

Two important aspects of differential protection are CT and relay connections and current calculation. Relay connection must make sure that the currents flow in the two restraint windings are in phase for external faults. Three CTs in both sides of a Y connected generator are connected in Y as shown in the following figure.

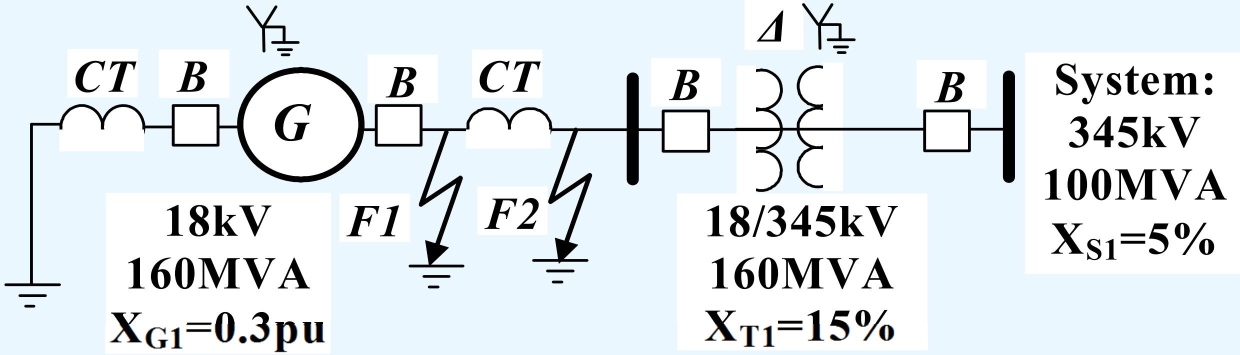


From dot convention, I_{a1} and I_{a1}' (I_{a2} and I_{a2}') are in phase. For external fault I_{a1} and I_{a2} are in phase. I_{a1}' and I_{a2}' are also in phase.

Current calculation: **Example 3.4: A Y-connected generator is connected to a 345 kV system through a Δ-Y 18/345 kV transformer as shown in figure. A differential relay with 1A pick up current is used to protect the generator. The system parameters are also shown in figure. CT ratio is 1100:1. Determine relay operation for three phase faults at *F1* and *F2*.**

S_{BN}=100 MVA

V_{BN}=18 kV at G:

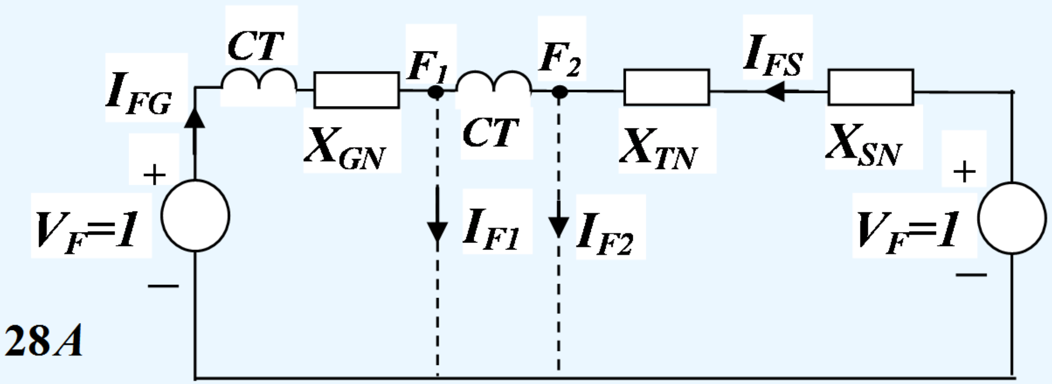


$$I_B = \frac{100000}{\sqrt{3}\times 18} = 3207.5A$$

$$X_{SN1} = 0.05$$

$$X_{GN1} = X_{G1}\frac{S_{BN}}{S_{BO}} = 0.3\frac{100}{160} = 0.1875$$

$$X_{TN1} = 0.15\frac{100}{160} = 0.09375$$



$$I_{FS} = \frac{V_F}{X_{TN1}+X_{SN1}}\times I_B / CTratio = 20.28A$$

$$I_{FG} = \frac{V_F}{X_{GN1}}\times I_B / CTratio = \frac{1}{0.1875}\times 3207.5/1100 = 15.55A$$

For external fault *F2*:

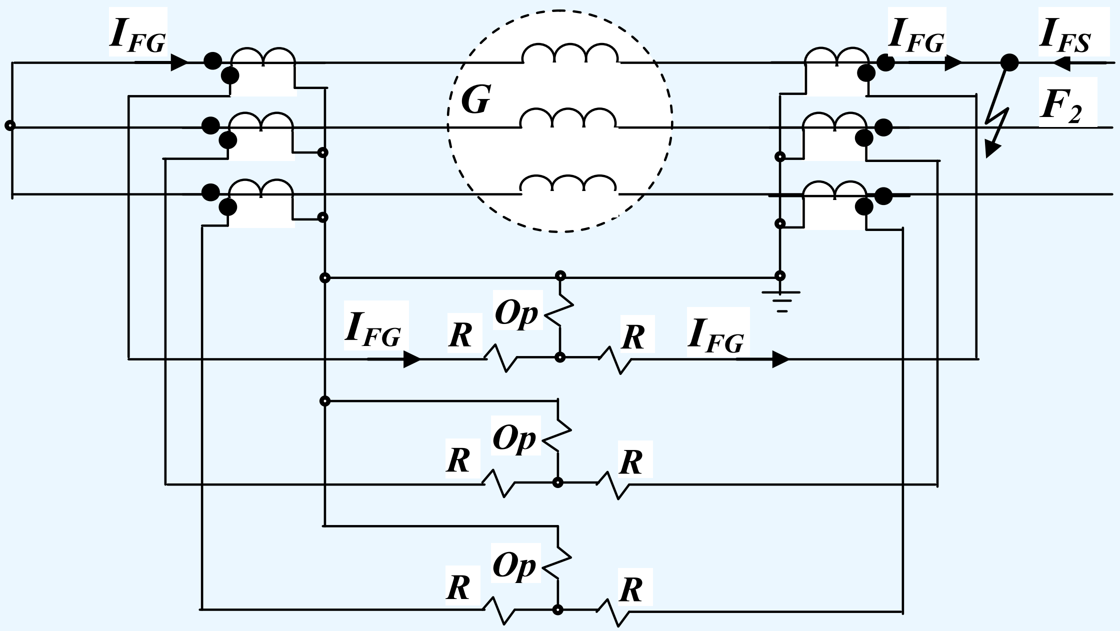
Two restraint currents are the same:

$$I_{FG} = 15.55A$$

Operating current:

$$I_o = I_{FG} - I_{FG} = 0A$$

Relay does not operate



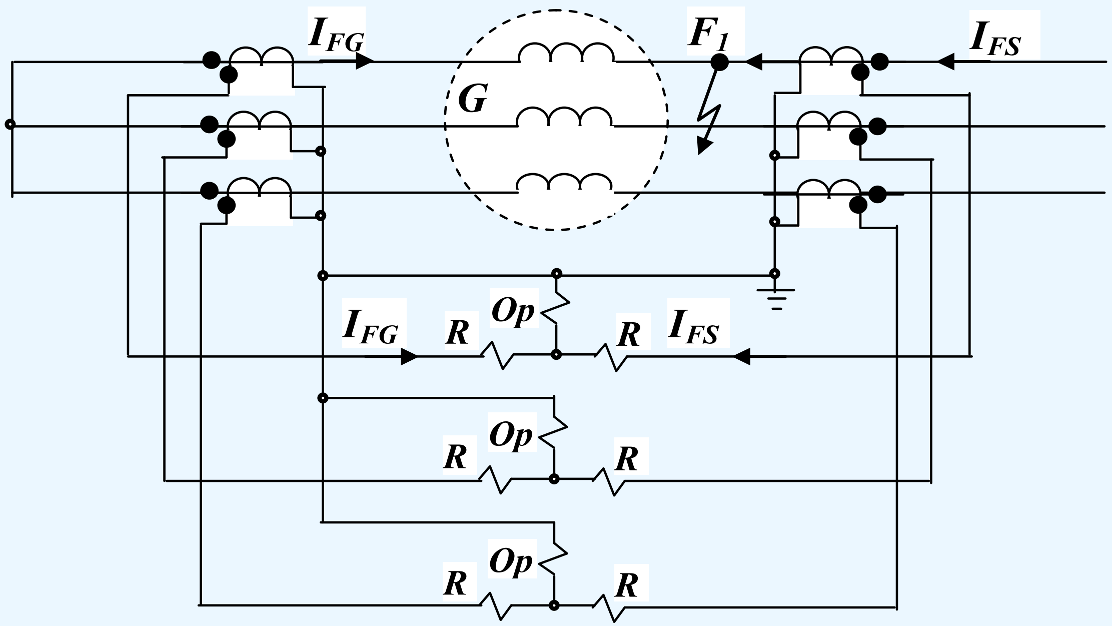
For internal fault *F1*:

Two restraint currents I_{FG} *and* I_{FS} are opposite and different:

Operating current:

$$I_o = I_{FG} + I_{FS} = 35.83A$$

Relay operates because $I_o > 1A$



Summary for protection techniques

a) Protection zones and fault location

b) Maximum load current and maximum fault calculation

c) Select CT and VT ratios

d) Main and back up protection

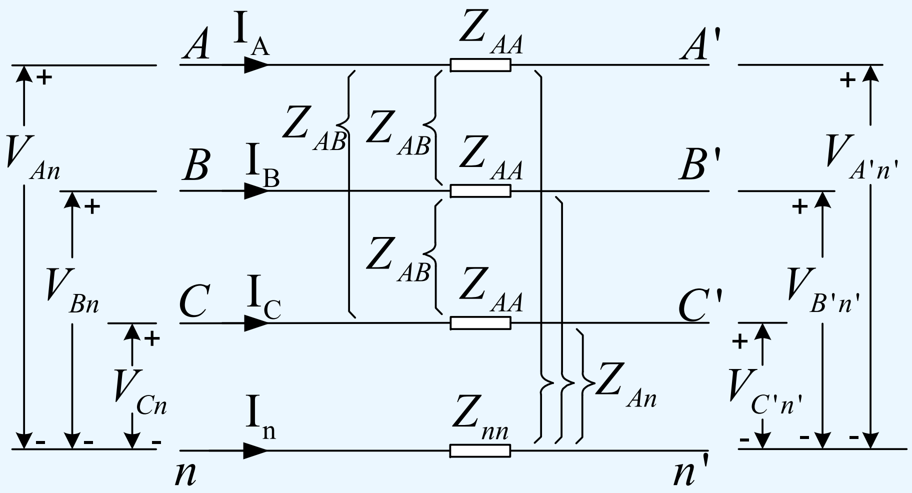
e) Select PS based on maximum load current

f) Select TDS based on CTI between back-up and main protection

g) Configuration of percentage differential relay and current calculation.

2.4.4 Sequence circuits of transmission lines (Appendix 1)

Z_{AA}: self-impedance of a phase line
Z_{nn}: self-impedance of neutral line
Z_{AB}: mutual impedance between any two phase lines
Z_{An}: mutual impedance between any phase line and neutral line



Line circuit model

KVL(the relationship of phase currents and voltages):

$$V_{An} = V_{AA'} + V_{A'n'} + V_{n'n} \qquad V_{An} - V_{A'n'} = V_{AA'} + V_{n'n}$$

$$V_{Bn} = V_{BB'} + V_{B'n'} + V_{n'n} \Rightarrow V_{Bn} - V_{B'n'} = V_{BB'} + V_{n'n}$$

$$V_{Cn} = V_{CC'} + V_{C'n'} + V_{n'n} \qquad V_{Cn} - V_{C'n'} = V_{CC'} + V_{n'n}$$

$$V_{AA'} = Z_{AA}I_A + Z_{AB}I_B + Z_{AB}I_C + Z_{An}I_n = \left(Z_{AA} - Z_{An}\right)I_A + \left(Z_{AB} - Z_{An}\right)I_B + \left(Z_{AB} - Z_{An}\right)I_C$$

$$V_{n'n} = -Z_{An}I_A - Z_{An}I_B - Z_{An}I_C - Z_{nn}I_n = \left(Z_{nn} - Z_{An}\right)I_A + \left(Z_{nn} - Z_{An}\right)I_B + \left(Z_{nn} - Z_{An}\right)I_C$$

$$\begin{aligned} V_{An} - V_{A'n'} &= Z_s I_A + Z_m I_B + Z_m I_C \\ V_{Bn} - V_{B'n'} &= Z_m I_A + Z_s I_B + Z_m I_C \\ V_{Cn} - V_{C'n'} &= Z_m I_A + Z_m I_B + Z_s I_C \end{aligned} \Rightarrow \begin{bmatrix} V_{An} - V_{A'n'} \\ V_{Bn} - V_{B'n'} \\ V_{Cn} - V_{C'n'} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

where:

$$Z_s = \left(Z_{AA} - 2Z_{An} + Z_{nn}\right)$$

$$Z_m = \left(Z_{AB} - 2Z_{An} + Z_{nn}\right)$$

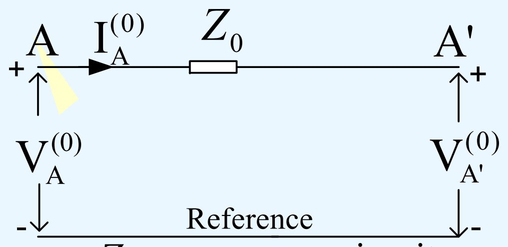
Same method with 2.4.1 to obtain relationship of sequence components:

$$A\begin{bmatrix} V_{An}^{(0)}-V_{A'n'}^{(0)} \\ V_{An}^{(1)}-V_{A'n'}^{(1)} \\ V_{An}^{(2)}-V_{A'n'}^{(2)} \end{bmatrix} = \begin{bmatrix} \mathrm{V}_{An}-V_{A'n'} \\ \mathrm{V}_{Bn}-V_{B'n'} \\ \mathrm{V}_{Cn}-V_{C'n'} \end{bmatrix} \Rightarrow \begin{bmatrix} \mathrm{V}_{An}-V_{A'n'} \\ \mathrm{V}_{Bn}-V_{B'n'} \\ \mathrm{V}_{Cn}-V_{C'n'} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix}\begin{bmatrix} \mathrm{I}_{\mathrm{A}} \\ \mathrm{I}_{\mathrm{B}} \\ \mathrm{I}_{\mathrm{C}} \end{bmatrix} \Leftarrow \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = A\begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix}$$

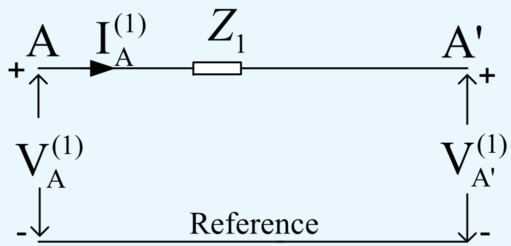
$$\Downarrow$$

$$\begin{bmatrix} V_{An}^{(0)}-V_{A'n'}^{(0)} \\ V_{An}^{(1)}-V_{A'n'}^{(1)} \\ V_{An}^{(2)}-V_{A'n'}^{(2)} \end{bmatrix} = \begin{bmatrix} Z_s+2Z_m & & \\ & Z_s-Z_m & \\ & & Z_s-Z_m \end{bmatrix}\begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} = \begin{bmatrix} Z_0 & & \\ & Z_1 & \\ & & Z_2 \end{bmatrix}\begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix}$$

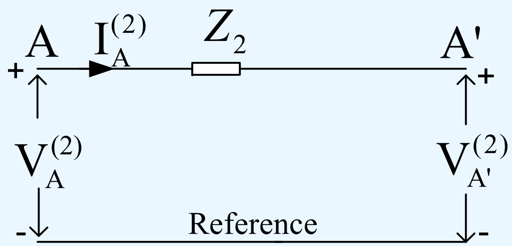
$$V_{An}^{(0)}-V_{A'n'}^{(0)}=Z_0I_A^{(0)} \qquad V_{An}^{(1)}-V_{A'n'}^{(1)}=Z_1I_A^{(1)} \qquad V_{An}^{(2)}-V_{A'n'}^{(2)}=Z_1I_A^{(2)}$$



Zero-sequence circuit



Positive-sequence circuit



Negative-sequence circuit

Where:

$Z_0 = Z_s + 2Z_m$: zero sequence impedance

$Z_1 = Z_s - Z_m$: positive sequence impedance

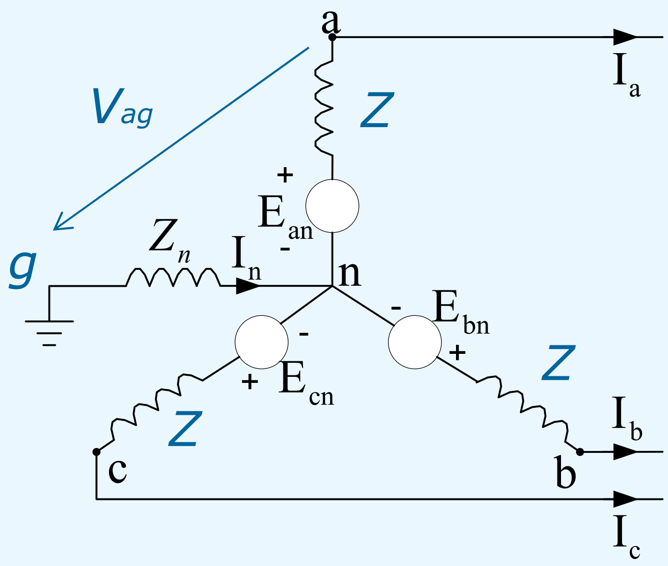
$Z_2 = Z_s - Z_m$: negtive sequence impedance

2.4.5 Sequence circuits of a synchronous generator (Appendix 2)

KVL: $V_{ag} = E_{an} - ZI_a - Z_n I_n = E_{an} - ZI_a - Z_n I_a - Z_n I_b - Z_n I_c = E_{an} - (Z+Z_n)I_a - Z_n I_b - Z_n I_c$

$V_{bg} = E_{bn} - Z_n I_a - (Z+Z_n)I_b - Z_n I_c$

$V_{cg} = E_{cn} - Z_n I_a - Z_n I_b - (Z+Z_n)I_c$



Matrix form:

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} E_{an} \\ E_{bn} \\ E_{cn} \end{bmatrix} - \begin{bmatrix} (Z+Z_n) & Z_n & Z_n \\ Z_n & (Z+Z_n) & Z_n \\ Z_n & Z_n & (Z+Z_n) \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$A\begin{bmatrix} V_{ag}^{(0)} \\ V_{ag}^{(1)} \\ V_{ag}^{(2)} \end{bmatrix} = A\begin{bmatrix} E_{an}^{(0)} \\ E_{an}^{(1)} \\ E_{an}^{(2)} \end{bmatrix} - \begin{bmatrix} (Z+Z_n) & Z_n & Z_n \\ Z_n & (Z+Z_n) & Z_n \\ Z_n & Z_n & (Z+Z_n) \end{bmatrix} A\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

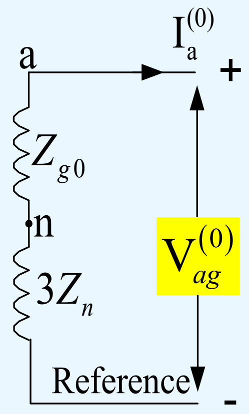
$$\begin{bmatrix} V_{ag}^{(0)} \\ V_{ag}^{(1)} \\ V_{ag}^{(2)} \end{bmatrix} = \begin{bmatrix} E_{an}^{(0)} \\ E_{an}^{(1)} \\ E_{an}^{(2)} \end{bmatrix} - A^{-1}\begin{bmatrix} (Z+Z_n) & Z_n & Z_n \\ Z_n & (Z+Z_n) & Z_n \\ Z_n & Z_n & (Z+Z_n) \end{bmatrix} A\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} V_{ag}^{(0)} \\ V_{ag}^{(1)} \\ V_{ag}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ E_{an} \\ 0 \end{bmatrix} - \begin{bmatrix} (Z+3Z_n) & 0 & 0 \\ 0 & Z & 0 \\ 0 & 0 & Z \end{bmatrix} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} E_{an}^{(0)} \\ E_{an}^{(1)} \\ E_{an}^{(2)} \end{bmatrix} = A^{-1}\begin{bmatrix} E_{an} \\ E_{bn} \\ E_{cn} \end{bmatrix} = A^{-1}\begin{bmatrix} E_{an} \\ a^2 E_{an} \\ aE_{an} \end{bmatrix} = \begin{bmatrix} 0 \\ E_{an} \\ 0 \end{bmatrix}$$

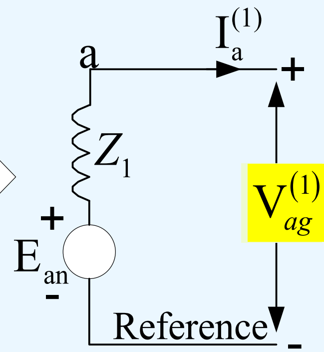
Sequence circuits:

$$V_{ag}^{(0)} = -(Z + 3Z_n)I_a^{(0)}$$ ⇨

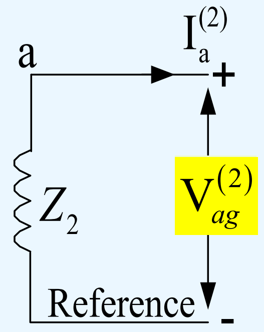


Only zero sequence current through $\underline{Z}_N$

$$V_{ag}^{(1)} = E_{an} - ZI_a^{(1)}$$ ⇨



$$V_{ag}^{(2)} = -ZI_a^{(2)}$$ ⇨



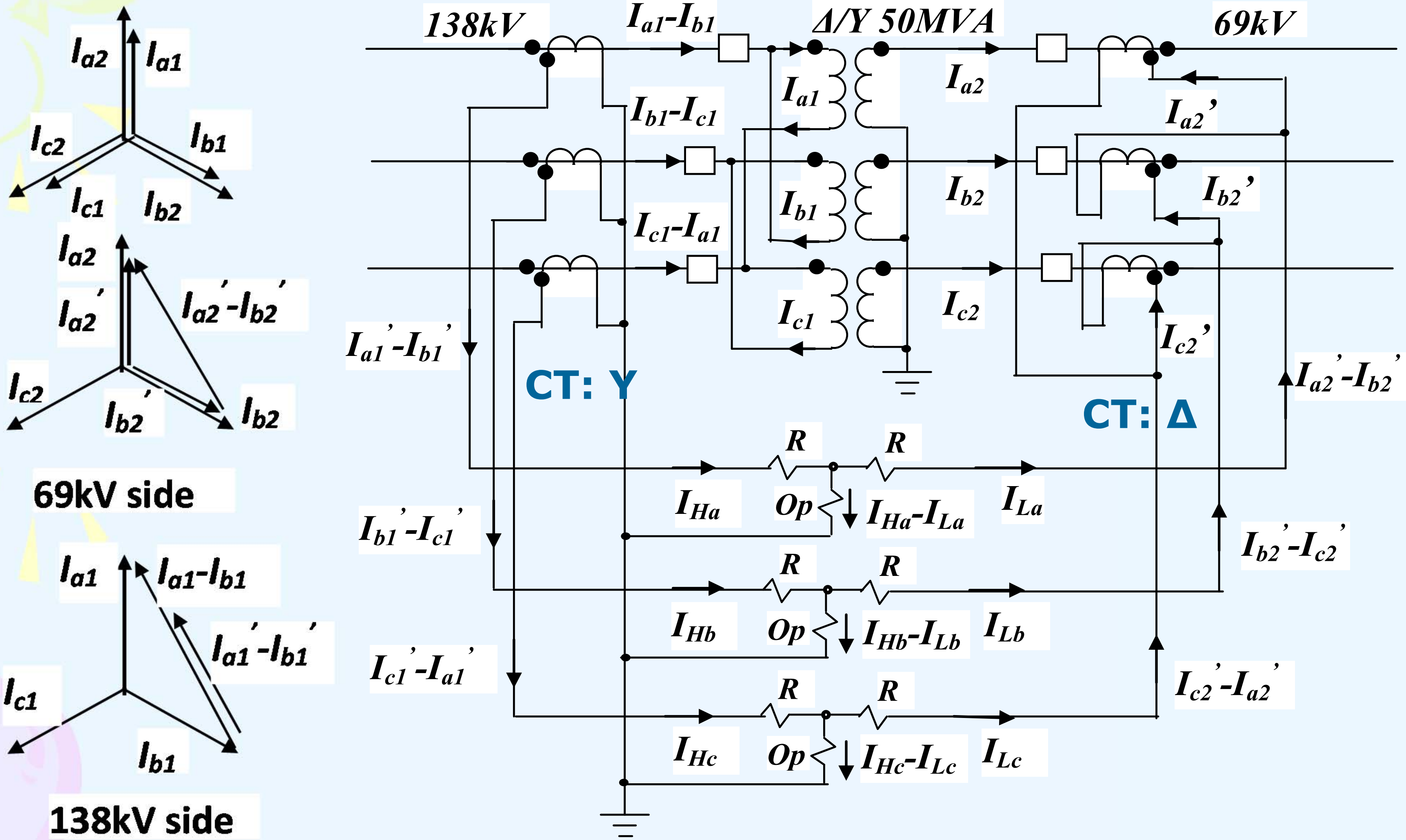
For neutral solidly grounded $Z_n = 0$ and for neutral opened $Z_n = \infty$

2.4.6 Sequence circuits of a synchronous motor

When a fault occurs, synchronous motors temporarily act as generators. The sequence networks are the same with generator.

3.4.4 Transformer differential protection (Appendix 3)

To guarantee two restraint currents I_{Ha} and I_{La} in phase for the normal load, a Δ-Y transformer and CT connections are shown in figure.



I_{Ha} and I_{La} are in phase from the phasor diagram

For normal load and external fault, two restraint currents should as close as possible. When CT ratios are selected based on the maximum load current and the operating current is calculated as follow:

The current at 138 kV (CT primary side): $I_{a1} - I_{b1} = \dfrac{50MVA}{\sqrt{3} \times 138kV} = 209.18A$

Select CT ratio 250:5 =50

The current at left-hand side of the restraint winding:

$$I_{Ha} = I'_{a1} - I'_{b1} = \frac{I_{a1} - I_{b1}}{CTratio} = 4.18A$$

The current at 69 kV (CT primary side): $I_{a2} = \dfrac{50MVA}{\sqrt{3} \times 69kV} = 418.37A$

Select CT ratio 500:5 =100

The current at the secondary side of CT: $I'_{a2} = \dfrac{I_{a2}}{CTratio} = \dfrac{418.37}{100} = 4.18A$

The current at right-hand side of the restraint winding (Δ CTs):

$$I_{La} = I'_{a2} - I'_{b2} = \sqrt{3} I'_{a2} = 7.25A$$

The operating current:

$$I_o = I_{Ha} - I_{La} = 3.07A$$

Example 3.5 For the transformer in Section 3.5.4, the CT ratios at high and low voltage sides of the transformer are 200:5 and 400:5 respectively. The transformer positive impedance is 0.5 pu. Ignore the source impedance. Calculate the currents in two restraint windings for an external three phase to ground fault which occurs at low voltage side of the transformer.

Solution:

The base current at 138 kV: $\left|I_{Bp}\right| = \frac{S_B}{\sqrt{3}V_B} = \frac{50MVA}{\sqrt{3}\times 138kV} = 209A$

The base current at 69 kV: I_B**=418 A**

The fault current=1/0.5=2pu

The fault currents at 138kV and 69kV from CT primary side are 418A and 836A respectively.

The current at left-hand side of the restraint winding:

I_H**=418/40=10.45**

The current at right-hand side of the restraint winding:

I_L**=836/80**$\sqrt{3}$ **=18.1**