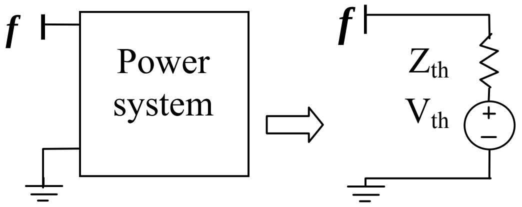
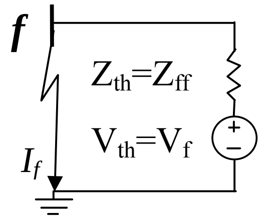
Tutorial 6: Symmetrical Faults

1. Thevenin Equivalent of a power system seen from bus f:



- **Pre-fault voltage at bus *f* : $V_f = V_{th}$**
- **Thevenin impedance at bus *f*: $Z_{ff} = Z_{th}$**
- **Fault current from bus *f* to ground: $I_f = V_f / Z_{ff}$**
- **Fault Level or <u>Short circuit MVA</u>: $S_f = \sqrt{3}\, V_f I_f^*$, $S_{fpu} = V_{fpu} I_{fpu}^*$**
- **The relationship: $Z_{th} = V_f^2 / S_f$ (true for both real and pu value)**

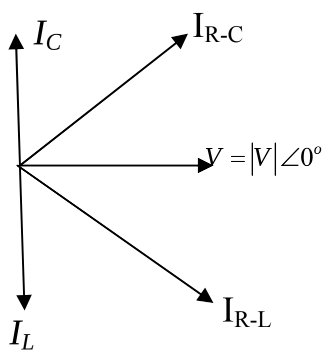


2. Y_{bus} Method:

Y_{bus} for a 4-bus system:

$$\mathbf{Y_{bus}} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}; \quad Z_{\mathbf{bus}} = Y_{bus}^{-1} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix}$$

- **Y_{kk}= sum of all admittances connected to bus *k***
- **Y_{ml}= −(sum of admittances connected between buses *m* and *l*)**
- **Z_{ff} is the diagonal element at *f* row and *f* column of Z_{bus}.**
- **Z_{nf} is the element at *n* row and *f* column of Z_{bus}.**
- $$\boldsymbol{V_n = V_f\left(1 - \frac{Z_{nf}}{Z_{ff}}\right)}$$



3. Direct method (Thevenin impedance)

4. Power factor correction

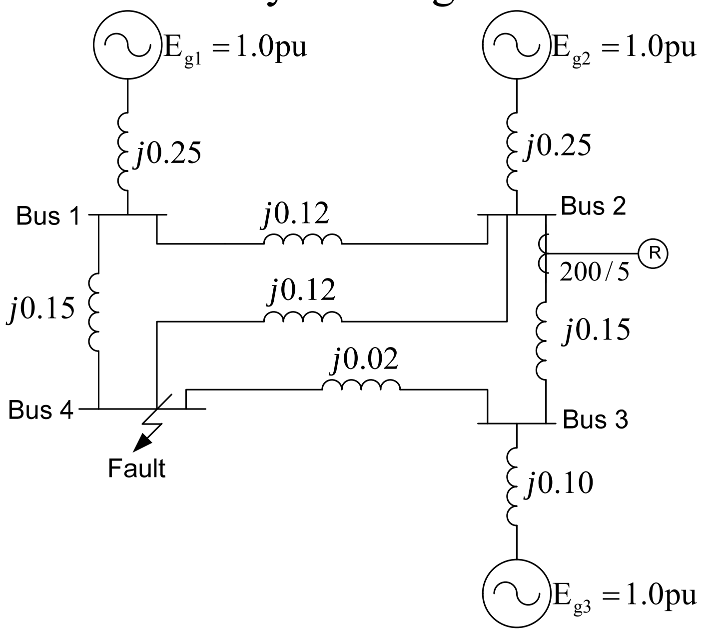
$$I_C = V/\left(-j1/\omega C\right) = V/\left(-jX_C\right) = j\frac{|V|}{X_C} = \frac{|V|}{X_C}\angle 90^o$$

$$I_{R-C} = V/\left(R - j1/\omega C\right) = V/\left(R - jX_C\right) = \frac{|V|}{\sqrt{R^2 + X_C^2}}\angle\varphi_C(possitive)$$

$$I_L = V/\left(j\omega L\right) = V/\left(jX_L\right) = -j\frac{|V|}{X_L} = \frac{|V|}{X_L}\angle -90^o$$

$$I_{R-L} = V/\left(R + j\omega L\right) = V/\left(R + jX_L\right) = \frac{|V|}{\sqrt{R^2 + X_L^2}}\angle\varphi_L(negative)$$

6.1 The figure below shows a 4-bus power system where all the *pu* values are on 33kV, 100MVA bases.
(a) Form the bus admittance matrix for the system.
(b) Find the voltages at all the buses and the fault currents in all the lines, for a 3-phase fault at bus 4 (as shown). Assume that the voltage throughout the network is 1.0pu before the fault. The impedance matrix for the system is given below.



$$\mathbf{Z_{bus}} = j\begin{bmatrix} 0.0929 & 0.0532 & 0.0416 & 0.0483 \\ 0.0532 & 0.08 & 0.0467 & 0.0517 \\ 0.0416 & 0.0467 & 0.0647 & 0.06 \\ 0.0483 & 0.0517 & 0.06 & 0.0731 \end{bmatrix}$$

Solution: (Y matrix method)

Base values: $V_b = 33\text{kV}$ $\quad S_b = 100\text{MVA}$

(a) $$Y_{11} = \frac{1}{j0.25} + \frac{1}{j0.12} + \frac{1}{j0.15} = -j19$$

$$Y_{22} = \frac{1}{j0.25} + \frac{1}{j0.12} + \frac{1}{j0.12} + \frac{1}{j0.15} = -j27.333$$

$$Y_{33} = \frac{1}{j0.1} + \frac{1}{j0.15} + \frac{1}{j0.02} = -j66.67$$

$$Y_{44} = \frac{1}{j0.02} + \frac{1}{j0.12} + \frac{1}{j0.15} = -j65$$

$$Y_{12} = Y_{21} = -\left(\frac{1}{j0.12}\right) = j8.33, \qquad Y_{14} = Y_{41} = -\left(\frac{1}{j0.15}\right) = j6.67$$

$$Y_{23} = Y_{32} = -\left(\frac{1}{j0.15}\right) = j6.67, \qquad Y_{24} = Y_{42} = -\left(\frac{1}{j0.12}\right) = j8.33$$

$$Y_{34} = Y_{43} = -\left(\frac{1}{j0.02}\right) = j50, \qquad Y_{13} = Y_{31} = 0$$

$$\therefore \mathbf{Y} = \begin{bmatrix} -j19 & j8.33 & 0 & j6.67 \\ j8.33 & -j27.33 & j6.67 & j8.33 \\ 0 & j6.67 & -j66.67 & j50 \\ j6.67 & j8.33 & j50 & -j65 \end{bmatrix} \quad \text{pu}$$

(b) Given $\mathbf{Z_{bus}} = \mathbf{Y^{-1}} = j\begin{bmatrix} 0.0929 & 0.0532 & 0.0416 & 0.0483 \\ 0.0532 & 0.08 & 0.0467 & 0.0517 \\ 0.0416 & 0.0467 & 0.0647 & 0.06 \\ 0.0483 & 0.0517 & 0.06 & 0.0731 \end{bmatrix}$

For the fault at bus 4:

$$\mathrm{I_f} = \frac{\mathrm{V_f}}{Z_{ff}} \Rightarrow \left|\mathrm{I_4}\right| = I_f = \left|\frac{1.0}{Z_{44}}\right| = \left|\frac{1.0}{j0.0731}\right| = 13.6799\text{pu}$$

$$\therefore \left|\mathrm{I_f}\right| = I_{f\,pu} \times I_b = 23.9336\text{kA} \quad \left(I_b = \frac{S_b \times 10^3}{\sqrt{3} \times V_b} = 1749.5463\text{A}\right)$$

Voltages at other buses:

$$\Rightarrow V_1 = 1\left[1 - \frac{Z_{14}}{Z_{44}}\right] = 1 - \frac{j0.0483}{j0.0731} = 0.3393\text{pu} \rightarrow 11.196\text{kV}$$

$$V_2 = 1\left[1 - \frac{Z_{24}}{Z_{44}}\right] = 1 - \frac{j0.0517}{j0.0731} = 0.2927\text{pu} \rightarrow 9.661\text{kV}$$

$$V_3 = 1\left[1 - \frac{Z_{34}}{Z_{44}}\right] = 1 - \frac{j0.06}{j0.0731} = 0.1792\text{pu} \rightarrow 5.914\text{kV}$$

$$V_4 = 1\left[1 - \frac{Z_{44}}{Z_{44}}\right] = 0$$

Currents in the lines:

$$I_{12} = \frac{V_1 - V_2}{j0.12} = (V_1 - V_2)(-j8.333) = -j0.3883\text{pu} \rightarrow 679.4\text{A}$$

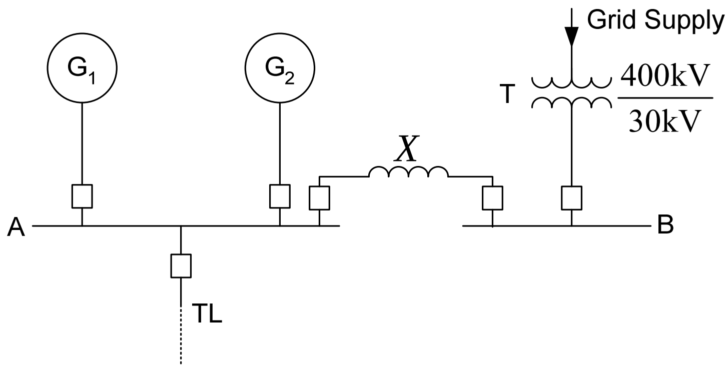
$$I_{14} = \frac{V_1 - V_4}{j0.15} = (V_1 - V_4)(-j6.667) = -j2.262\text{pu} \rightarrow 3957.47\text{A}$$

$$I_{23} = \frac{V_2 - V_3}{j0.15} = (V_2 - V_3)(-j6.667) = -j0.7567\text{pu} \rightarrow 1323.88\text{A}$$

$$I_{24} = \frac{V_2 - V_4}{j0.12} = (V_2 - V_4)(-j8.333) = -j2.4392\text{pu} \rightarrow 4267.43\text{A}$$

$$I_{34} = \frac{V_3 - V_4}{j0.02} = (V_3 - V_4)(-j50) = -j8.96\text{pu} \rightarrow 15.676\text{kA}$$

6.2 Consider the three-phase power system shown below. Generators G1 and G2 are connected in parallel to a 30-kV busbar A. The ratings of G1 and G2 are 100MVA, 30kV, X=20% and 50MVA, 30kV, X=15% respectively. Busbar A feeds a transmission line TL. A grid supply is connected to the station busbar B through a transformer T rated 500MVA, 400/30kV, X=20%. Determine the reactance (in ohms) of a current limiting reactor X to be connected between bus B and bus A such that the short-circuit MVA at bus A does not exceed 1250MVA.



Solutions:

pu impedance:

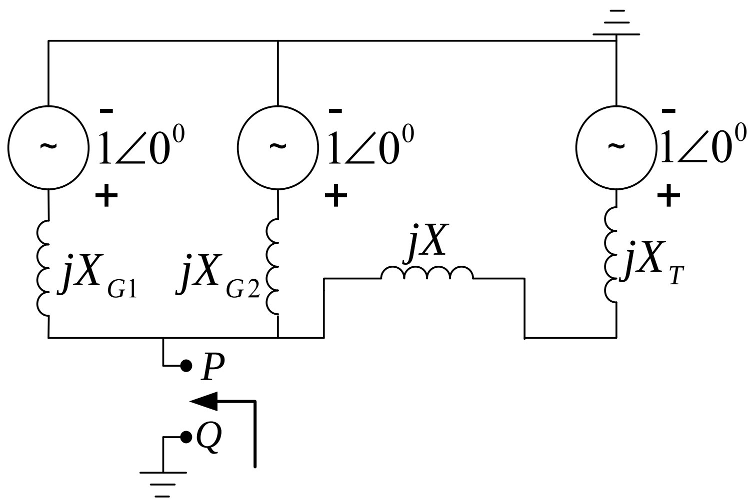
$S_b = 500\text{MVA(largest)}$ & $V_b = 30\text{kV}$ at Busbar A.

$$\Rightarrow I_b = \frac{500\times10^3}{\sqrt{3}\times30} = 9622.5045\text{A} \qquad \& \qquad Z_b = \frac{V_b^2}{S_b} = 1.8\Omega$$

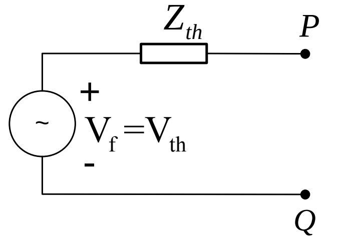
$$X_{G1punew} = \frac{0.2\times30^2/100}{30^2/500} = 1.0, \qquad X_{G2punew} = \frac{0.15\times30^2/50}{30^2/500} = 1.5$$

$$X_{Tpunew} = \frac{0.2\times30^2/500}{30^2/500} = 0.2$$

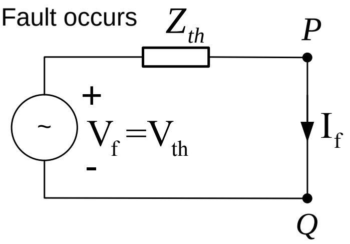
pu impedance diagram:



Thevenin equivalent as viewed from fault point (before fault occurs)



Fault occurs



Thevenin impedance at bus A (from the original circuit):

$$Z_{thpu} = \left[jX_{G1} \,||\, jX_{G2} \right] || \,(jX + jX_T) = (j1 \,||\, j1.5) \,||\, (jX + j0.2)$$

$$= j0.6 \,||\, (jX + j0.2) = \frac{j0.6(jX + j0.2)}{j0.6 + (jX + j0.2)} = \frac{j0.6(jX + j0.2)}{j0.8 + jX}$$

Given short circuit MVA: $S_f = 1250\text{MVA} \Rightarrow \text{S}_{fpu} = \dfrac{1250}{500} = 2.5\text{pu}$

Thevenin impedance (from given $\mathbf{S_{fpu}}$ and $\mathbf{V_{fpu}}$):

$$\text{S}_{\text{fpu}} = \text{V}_{\text{fpu}} \text{I}^*_{\text{fpu}} \Rightarrow | \text{I}_{\text{fpu}} | = | \text{S}_{\text{fpu}} | = 2.5\text{pu} \,,\ \ (\,|\text{V}_{\text{fpu}}| = |\text{V}_{\text{thpu}}| = 1)$$

$$|Z_{thpu}| = \left| \frac{\text{V}^2_{thpu}}{S_{\text{fpu}}} \right| = \frac{1.0}{2.5} = 0.4 \Rightarrow Z_{thpu} = j0.4$$

Both should be the same: $j0.4 = \dfrac{j0.6(jX + j0.2)}{j0.8 + jX}$

$$\Rightarrow X = 1.0\,pu$$

Real impedance: $X(\Omega) = 1.8(\Omega)$

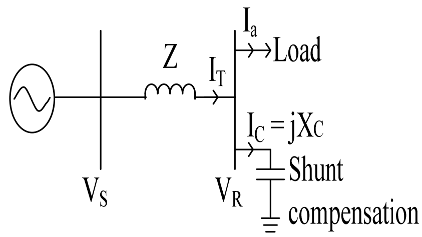
6.3 A feeder from a distribution transformer is 500 feet long. It consists of three wires having an impedance of 0.052 + j0.069 ohms per 1,000 feet and current capacity of 225 amps. The three-phase load is balanced and draws 120 kW at 440 volts with a 60% lagging power factor.

(a) Determine the kVA rating of capacitors capable of correcting the power factor to allow the rated current to flow in the feeder.

(b) Draw the phasor diagram of the feeder current before and after power factor correction and compute the new power factor.

(c) Determine the voltage at the sending end of the feeder.

(d) Determine the voltage regulation of the feeder.



Solutions:

Base values: $S_b = 120\text{x}10^{-3}/0.6 = 0.2$ MVA,

$V_b = 0.44$ kV, $Z_b = V_b^2/S_b = 0.968\ \Omega$,

$I_b = S_b/(\sqrt{3}\times V_b) = 0.26243$ kA

Feeder impedance: $Z_{Tpu} = \dfrac{(0.052 + j0.069)/2}{Z_b} = 0.02686 + j0.03564 \text{ pu}$

(a) pu voltage cross the load: $V_{Rpu} = \dfrac{0.44\angle 0^{\circ}}{V_b} = 1\angle 0^{\circ} \text{ pu}$

***Before compensation**: the current in the feeder:

$$S_L = \frac{P}{pf}\angle \cos^{-1} pf = \frac{120}{0.6}\angle 53.13^{\circ} = 0.2\angle 53.13^{\circ} \text{MVA}$$

$$S_{Lpu} = \frac{S_L}{S_b} = \frac{0.2\angle 53.13^{\circ}}{0.2} = 1\angle 53.13^{\circ}$$ pf=cosθ=cos53.13°=0.6 lagging

$$S_{Lpu} = V_{pu} \times I^{*}_{apu} \quad \rightarrow I_{Tpu} = I_{apu} = \left(S_{Lpu}/V_{pu}\right)^{*} = S^{*}_{Lpu} = 1\angle -53.13^{\circ} = 0.6 - j0.8 \text{ pu}$$

After compensation: The required current through the feeder:

$$|I_{Tpu}| = \frac{225}{I_b} = \frac{225}{262.43} = 0.85737 \text{ pu}$$

Since $\dfrac{1}{j\omega C} = -j\dfrac{1}{\omega C} = -jX_C$, the current through capacitor:

$$I_{Cpu} = \frac{V_{Rpu}}{-jX_C} = \frac{1}{-jX_C} = j\frac{1}{X_C} \text{ pu}.$$

KCL: $I'_{Tpu} = I_{apu} + I_{Cpu} = 0.6 - j0.8 + j\dfrac{1}{X_C} = 0.6 + j(\dfrac{1}{X_C} - 0.8) \text{ pu}$

The magnitude of the current should be: $\left|I'_{Tpu}\right| = \sqrt{0.6^2 + (\frac{1}{X_C} - 0.8)^2} = 0.85737\,\text{pu}$

$$\frac{1}{X_C} - 0.8 = \pm\sqrt{0.85737^2 - 0.6^2}$$

$$\frac{1}{X_C} = 0.8 \pm 0.61243 = 0.18757\ \text{pu, or } 1.41243\,\text{pu}$$

$$I'_{Tpu} = I_{apu} + I_{Cpu} = 0.6 - j0.8 + j\frac{1}{X_C} = 0.6\text{-}j0.61243 = 0.8574\underline{\left|-45.587^\circ\right.}\ \text{or???}$$

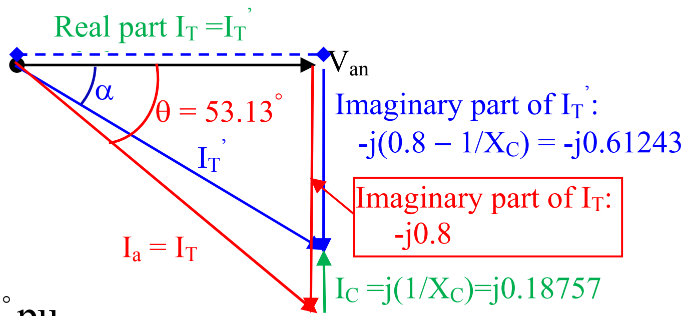
kVA rating of capacitor: $\left|S_{Cpu}\right| = \left|V_{pu}\right| \times \left|I^*_{Cpu}\right| = \frac{1}{X_C} = 0.18757\text{pu}$

S_C= 0.18757 x S_b = 0.18757 x 0.2 x 1000kVA = 37.513 kVA (or 282.468kVA)

(b) Phasor diagram of the feeder current before/after power factor correction, i.e. compensation.

New pf = $\cos\alpha$

= cos 45. 587° = 0.7 lagging



(c) $V_{Spu} = V_{Rpu} + I_{Tpu}Z_{Tpu}$

$= 1.03794 + j0.004934 = 1.03795\angle 0.272^\circ$ pu

Actual $|V_S|$ = 1.038 x V_b = 1.03795 x 440 = 456.7 V

(d) Percent VR $= \dfrac{\left|V_{R,\,NL}\right| - \left|V_{R,\,FL}\right|}{\left|V_{R,\,FL}\right|}$ x 100%

Capacitive reactance = -jXc = -j1/0.18757 = -j5.33134 pu

$$\left|V_{R,\,NL}\right| = \left|\frac{-jX_C}{Z_T - jX_C}V_S\right| = \left|\frac{-j5.33134}{0.02686 + j0.03564 - j5.33134}\right|\left|V_S\right|$$

$$= 1.00672\text{x}1.03795 = 1.0449\ \text{pu}$$

Percent VR $= \dfrac{1.0449 - 1}{1}$ x 100% = 4.49%