**NANYANG TECHNOLOGICAL UNIVERSITY**

**SPMS/DIVISION OF MATHEMATICAL SCIENCES**

2016/17 Semester 1 MH2500 Probability and Introduction to Statistics Tutorial 3

For the tutorial on 1 September, let us discuss

- Ex. 2.5.6, 11, 13, 16, 22, 26.

**Ex. 2.5.6.** Let $A$ and $B$ be events, and let $I_A$ and $I_B$ be the associated indicator random variables. Show that

$$I_{A\cap B} = I_A I_B = \min\{I_A, I_B\}$$

and

$$I_{A\cup B} = \max\{I_A, I_B\}.$$

**Ex. 2.5.11.** Consider the binomial distribution with $n$ trials and probability $p$ of success on each trial. For what value of $k$ is $P(X = k)$ maximized? This value is called the **mode** of the distribution. (Hint: Consider the ratio of successive terms.)

**Ex. 2.5.13.** A multiple-choice test consists of 20 items, each with four choices. A student is able to eliminate one of the choices on each question as incorrect and chooses randomly from the remaining three choices. A passing grade is 12 items or more correct.

(a) What is the probability that the student passes?

(b) Answer the question in part (a) again, assuming that the student can eliminate two of the choices on each question.

**Ex. 2.5.16.** Show that if $n$ approaches $\infty$ and $r/n$ approaches $p$ and $m$ is fixed, the hypergeometric frequency function tends to the binomial frequency function with parameters $m$ and $p$. Give a heuristic argument for why this is true.

**Ex. 2.5.22.** Three identical fair coins are thrown simultaneously until all three show the same face. What is the probability that they are thrown more than three times?

**Ex. 2.5.26.** The university administration assures a mathematician that he has only 1 chance in 10,000 of being trapped in a much-maligned elevator in the mathematics building. If he goes to work 5 days a week, 52 weeks a year, for 10 years, and always rides the elevator up to his office when h first arrives, what is the probability that he will never be trapped? That he will be trapped once? Twice? Assume that the outcomes on all the days are mutually independent (a dubious assumption in practice).