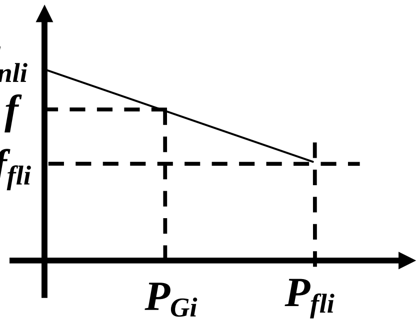
Tutorial 4: Synchronous Generators II

1. Relationship among *f*, *n*, and *p*: $f = \dfrac{np}{120}$

2. For a single machine *i*:

Slope of droop: $S_{Pi} = \dfrac{P_{fli}}{f_{nli} - f_{fli}}$

***P*-*f* or droop curve:** $P_{Gi} = S_{pi}(f_{nli} - f)$



3. Real power sharing for parallel machines:

All machines must operate at the <u>same frequency f_s.</u>

Load is shared by all machines: $P_{load} = P_{G1} + P_{G2} + \ldots + P_{GN}$.

$P_{G1} = S_{p1}(f_{nl1} - f_s)$

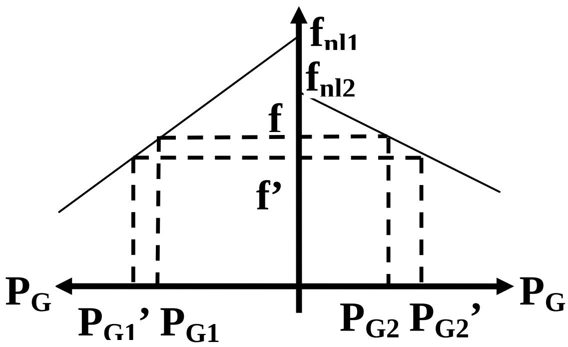
$P_{G2} = S_{p2}(f_{nl2} - f_s)$

Two machines example:

Case 1:

$P_{load} = P_{G1} + P_{G2}$ ⇑ $P_{load}' = P_{G1}' + P_{G2}'$

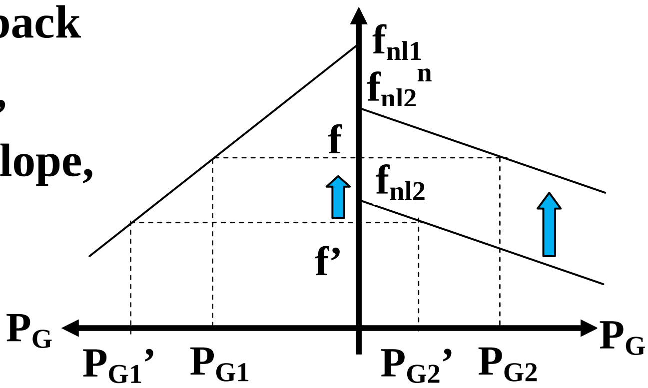
Frequency f ⇓ f'.



Case 2: To restore frequency back from f' to f with the same load, shift f_{nl2} to f_{nl2}^{n} with the same slope,

$P_{G1}' + P_{G2}' = P_{load} \Rightarrow f'$

$P_{G1} + P_{G2} = P_{load} \Rightarrow f$

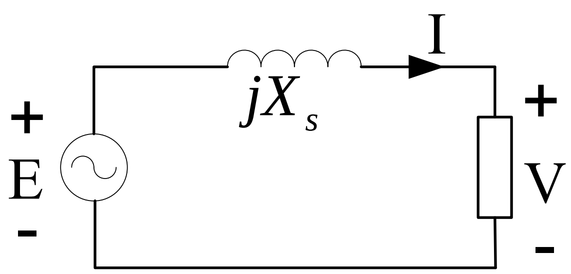


4.1 A 24MVA, 17.32kV, 60Hz, Y-connected, 3-phase synchronous generator has a synchronous reactance of 5ohms per phase, and negligible armature resistance.

(i) At a certain excitation, the generator delivers rated MVA at 0.8 power factor lagging, to an infinite bus operating at 17.32kV. Determine the magnitude of the internal emf *per phase* and the power angle of the generator.

(ii) The internal emf and terminal voltage are maintained constant at 13.4kV/phase and 10kV/phase respectively. What is the maximum three-phase real power this generator can develop before it pulls out of synchronism? What are the armature current and reactive power under this condition?

Solution:



Let $S_b = 24\text{MVA}$ $\quad V_b = 17.32\text{kV}$

$$\Rightarrow Z_b = \frac{V_b^2}{S_b} = 12.5\Omega$$

$$\Rightarrow X_{spu} = \frac{5}{Z_b} = 0.4 \quad \& \quad I_b = \frac{S_b \times 10^3}{\sqrt{3}V_b} = \frac{24\times10^3\text{kVA}}{\sqrt{3}\times17.32\text{kV}} = 800.023\text{A}$$

(i) Internal emf *per phase:* $\mathrm{E}=?,\quad \delta=?$

Givens: S; pf ; $\mathrm{V}_{pu}=1\angle 0^0$

$\mathrm{S_{pu}}=1\angle\cos^{-1}0.8=1\angle 36.87^0;$

$$\mathrm{I}_{pu}=\left(\frac{S_{pu}}{V_{pu}}\right)^*=1\angle-36.87^0$$

$$\mathrm{E}=\mathrm{V}+\mathrm{I}(jX_s)=1\angle 0^0+(1\angle-36.87^0)(0.4\angle 90^0)=1.2806\angle 14.47^0\,\mathrm{pu}$$

$$|E|=1.2806\times V_b=22.18\mathrm{kV}$$

$$|E_p|{=}22.18/\sqrt{3}=12.806\mathrm{kV};\delta{=}14.47^0$$

(ii) **Maximum real power (**at δ=90^0)

$$|\,\mathrm{E}\,|=\frac{13.4\sqrt{3}}{17.32}=1.34\mathrm{pu},\qquad |\,\mathrm{V}\,|=\frac{10\sqrt{3}}{17.32}=1.00\mathrm{pu}$$

$$P_{\max}=\frac{|V||E|}{X_s}=\frac{1\times 1.34}{0.4}=3.35\mathrm{pu}=3.35\times S_b{=}80.4\mathrm{MW}\ (\text{at } \delta{=}90^0)$$

$$Q_{\delta=90}=\frac{|V||E|}{X_s}\cos\delta-\frac{|V|^2}{X_s}=-\frac{|V|^2}{X_s}=-\frac{1^2}{0.4}=-2.5\mathrm{pu}=-60\mathrm{MVAr}$$

$$\mathrm{S}=3.35-j2.5=4.18\angle-36.73^0\,\mathrm{pu}$$

$$\mathrm{I}=\left(\frac{\mathrm{S}}{\mathrm{V}}\right)^*=\left(\frac{4.18\angle-36.73^0}{1\angle 0^0}\right)^*=4.18\angle 36.73^0\,\mathrm{pu}$$

Armature current: $|I_{pu}|=4.18\times I_b=3344.1\mathrm{A}$

4.2 Two generators, rated 3MW each, are operating in parallel to supply a total load of 4MW at 0.8 pf (lag). The generators have no-load frequencies of 52Hz and 51Hz respectively. The frequencies of both generators fall to 49Hz at full load.

(a) What is the system frequency and how much power is supplied by each generator?

(b) Find the **pf** of the second generator if the first generator is operating at 0.85 (lag)

(c) Determine the **no-load frequency setting** of the second generator to bring the system back to 50Hz at this load.

Solution:

Givens: $P_{LOAD} = 4\text{MW}$; pf= 0.8 , lag;

f_{nl1}=52Hz; $f_{nl2} = 51\text{Hz}$; $f_{fl1} = f_{fl2} = 49\text{Hz}$

$$s_{p1} = \frac{3\text{MW}}{52-49} = 1\text{MW/Hz} \qquad s_{p2} = \frac{3\text{MW}}{51-49} = 1.5\text{MW/Hz}$$

$$P_{G1} = s_{p1}(f_{nl1} - f_{sys}) = 1(52 - f_{sys}); \; P_{G2} = s_{p2}(f_{nl2} - f_{sys}) = 1.5(51 - f_{sys})$$

(a): f_{sys}? P_{G1}? P_{G2}?

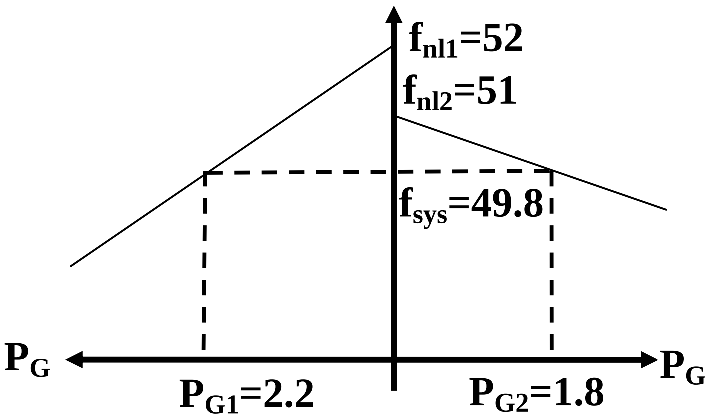
$$P_{LOAD} = P_{G1} + P_{G2} = 4$$

$$1(52 - f_{sys}) + 1.5(51 - f_{sys}) = 4$$

$$\Rightarrow f_{sys} = 49.8\text{Hz}$$

$$P_{G1} = 1(52 - 49.8) = 2.2\text{MW}$$

$$P_{G2} = 1.5(51 - 49.8) = 1.8\text{MW}$$



(b): pf of G2?

Givens: P_{G1}, pf_{G1}=0.85 lag; P_{LOAD}=4, pf_{LOAD}=0.8 lag

$$\left|S_{LOAD}\right| = \frac{P_{LOAD}}{0.8} = 5 \quad \Rightarrow \quad S_{LOAD} = 5\angle\cos^{-1}0.8 = 5\angle 36.87^0\,\mathrm{MVA}$$

$$\left|S_{G1}\right| = \frac{P_{G1}}{0.85} = \frac{2.2}{0.85} = 2.5882\mathrm{MVA}$$

$$S_{G1} = 2.5882\angle\cos^{-1}0.85 = 2.5882\angle 31.788^0\,\mathrm{MVA}$$

$$S_{LOAD} = S_{G1} + S_{G2}$$

$$S_{G2} = S_{LOAD} - S_{G1} = 5\angle 36.87^0 - 2.5882\angle 31.788^0 = 2.433\angle 42.28^0\,\mathrm{MVA}$$

$$\Rightarrow \text{p.f. (G2)} = \cos 42.28^0 = 0.74(\text{lag})$$

(c): Keeping f_{nl1} the same to supply the same load,

how to restore f_{sys}=50 Hz?? (from 49.8HZ)?

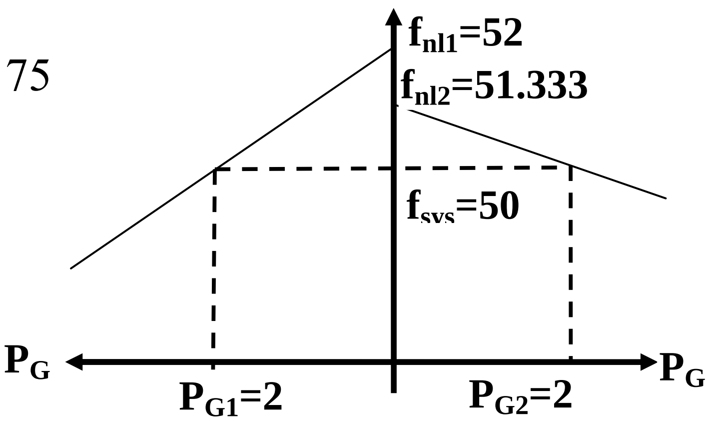
Solution: Shift f_{nl2} from the original 51 HZ

$$P_{G1} = s_{p1}(f_{nl1} - f_{sys}) = 1(52-50) = 2\mathrm{MW}$$

$$P_{G2} = 4 - 2 = 2\mathrm{MW}$$

$$2 = 1.5(f_{nl2} - f_{sys}) = 1.5f_{nl2} - 75$$

$$f_{nl2} = \frac{77}{1.5} = 51.333\mathrm{Hz}$$



4.3 A 480-V, 200-kW, 2-pole, 3-phase, 50-Hz synchronous generator's prime mover has a no-load speed of 3040 rpm, and a full-load speed of 2975 rpm. It is operating in parallel with a 480-V, 150-kW, 4-pole, 3-phase, 50-Hz synchronous generator whose prime mover has a no-load speed of 1500rpm, and a full-load speed of 1485 rpm. The total load supplied by the two generators is 200kW at 0.85 pf lagging.

(a) Find the **operating frequency** of the power system.

(b) Find the **power** being supplied by each of the two generators.

Solutions:

$$f_{nl} = \frac{n_{nl} p}{120} \qquad \& \qquad f_{fl} = \frac{n_{fl} p}{120}$$

$$f_{nl1} = \frac{n_{nl1} p}{120} = \frac{3040 \times 2}{120} = 50.667\text{Hz}, \quad f_{fl1} = \frac{n_{fl1} p}{120} = \frac{2975 \times 2}{120} = 49.583\text{Hz}$$

$$f_{nl2} = \frac{1500 \times 4}{120} = 50\text{Hz}, \quad f_{fl2} = \frac{1485 \times 4}{120} = 49.5\text{Hz}$$

$$\Rightarrow s_{P1} = \frac{P_1}{f_{nl1} - f_{fl1}} = \frac{200\text{kW}}{50.667 - 49.583} = 0.185\text{MW/Hz}$$

$$s_{P2} = \frac{P_2}{f_{nl2} - f_{fl2}} = \frac{150\text{kW}}{50 - 49.5} = 0.3\text{MW/Hz}$$

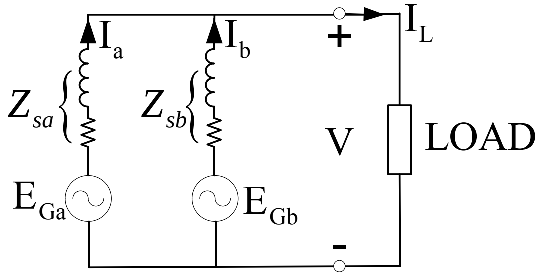
$$\text{(a):} P_{LOAD} = P_1 + P_2 = s_{p1}(f_{nl1} - f_{sys}) + s_{p2}(f_{nl2} - f_{sys})$$

$$\frac{200}{1000} = 0.185(50.667 - f_{sys}) + 0.3(50 - f_{sys}) \Rightarrow f_{sys} = 49.842\text{Hz}$$

$$\text{(b):} P_1 = s_{p1}(f_{nl1} - f_{sys}) = 0.185(50.667 - 49.842) = 153\text{kW}$$

$$P_2 = s_{p2}(f_{nl2} - f_{sys}) = 0.3(50 - 49.842) = 47\text{kW}$$

4.4 Two three-phase, 6.6kV, Y-connected synchronous generators operating in parallel supply a load of 3MW at 6.6 kV and 0.8 pf lagging. The synchronous impedances of the machines A and B are (0.5+j10)Ω and (0.4+j12)Ω respectively. The excitation of machine A is adjusted so that it delivers 150A at a lagging pf, and the governors of the two machines are so set that the load (real power) is shared equally between the machines. Determine the **armature current** of the second machine, and also the **power factor, internal voltage and power angle** of each machine. Sketch a **phasor diagram** showing all currents and voltages.



Solution:
Select:

$$V_b = 6.6\text{kV};\quad S_b = 3.75\text{MVA}$$

$$Z_b = \frac{V_b^2}{S_b} = \frac{6.6^2}{3.75} = 11.616\Omega$$

$$Z_{sa} = \frac{0.5 + j10}{11.616} = \frac{10.0125}{11.616}\angle 87.138^0 = 0.86196\angle 87.138^0\,\text{pu}$$

$$Z_{sb} = \frac{0.4 + j12}{11.616} = \frac{12.0067}{11.616}\angle 88.091^0 = 1.0336\angle 88.091^0\,\text{pu}$$

$$I_{base} = \frac{S_b \times 10^3}{\sqrt{3}\times V_b} = \frac{(3.75\times 10^3)\text{kVA}}{\sqrt{3}\times 6.6\text{kV}} = 328.0399\text{A}$$

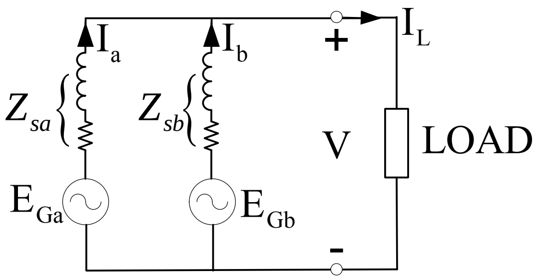
Givens: P_{LOAD}=3MW, at 0.8 pf lag;

$$P_a = 1.5\text{MW} = P_b;\ \ V = 6.6\text{kV};\quad |\text{I}_\text{a}| = 150\text{A}\,(\text{lag})$$

$$\text{S}_{\text{LOAD}} = \frac{3}{0.8}\angle\cos^{-1}0.8 = 3.75\angle 36.87^0\,\text{MVA}=1\angle 36.87^0\,\text{pu}$$

Select voltage cross load as reference: $\text{V} = 1\angle 0^0\,\text{pu}$,

$$\text{I}_\text{L} = \left(\frac{S_{LOAD}}{V}\right)^* = 1\angle -36.87^0\,\text{pu}$$



Machine a:

$$|\text{I}_\text{a}| = 150\text{A(lag)} \Rightarrow |\text{I}_{\text{apu}}| = \frac{150}{I_{base}} = 0.4573\text{pu}$$

$$P_a = P_b = 1.5\text{MW} \Rightarrow P_a = P_b = \frac{1.5}{3.75} = 0.4\text{pu}$$

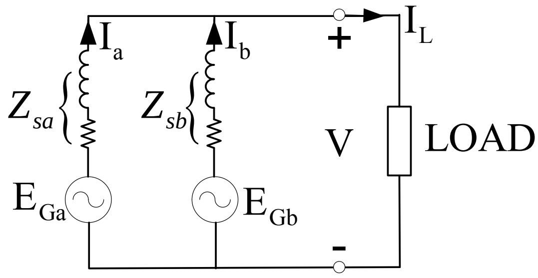
$$P_a = |\text{V}||\text{I}_\text{a}|\cos\theta_a \Rightarrow \cos\theta_a = \frac{P_a}{|\text{V}||\text{I}_\text{a}|}$$

$$\Rightarrow \cos\theta_a = \frac{0.4}{1\times 0.4573} = 0.8747(\text{lag}) \Rightarrow \theta_a = -29^0$$

$$\text{E}_{\text{Ga}} = Z_{sa}\text{I}_\text{a} + \text{V}=(0.86196\angle 87.138^0)(0.4573\angle -29^0) + 1\angle 0^0$$
$$= 1.2081 + j0.3348 = 1.2536\angle 15.49^0\,\text{pu}$$

$$|\text{E}_{\text{Ga}}| = 8.274\text{kV},\ \ \delta_a = 15.49^0$$

Machine b:



KCL:

$$I_b = I_L - I_a$$

$$= 1\angle -36.87^0 - 0.4573\angle -29^0$$

$$= 0.5506\angle -43.41^0\, pu$$

$$|\,I_b\,| = 180.627\text{A};\ (\theta_b = -43.41^0)$$

$$\text{pf}_{G2} = \cos 43.41^0 = 0.7264(lag)$$

$$E_{Gb} = Z_{sb} I_b + V = (1.0336\angle 88.091^0)(0.5506\angle -43.41^0) + 1\angle 0^0$$

$$= 0.5691\angle 44.681^0 + 1\angle 0^0 = 1.4605\angle 15.90^0\,\text{pu}$$

$$|\,E_{Gb}\,| = 9.6393\text{kV},\quad \delta_b = 15.90^0$$