9 – INFERENCE IN FIRST-ORDER LOGIC

“In which we define inference mechanisms that can efficiently answer questions posed in first-order logic.”

Part III – Knowledge and Reasoning

- **9 Inference in First-Order Logic**
  - Inference Rules. – Generalised Modus Ponens.
  - Forward and Backward Chaining. – Resolution.
- **10 Logical Reasoning Systems**
  - Indexing, Retrieval and Unification. – Logic Programming / Prolog. – Production Systems.
  - Frames and Semantic / Conceptual Networks.
  - Managing Retractions, Assumptions, and Explanations.

Inferences Rules for FOL

- **Inference rules from Propositional Logic**
  - *Modus Ponens*
    - $$\frac{\alpha \Rightarrow \beta,\ \alpha}{\beta}$$
  - Double-Negation-Elimination
    - $$\frac{\neg\neg\alpha}{\alpha}$$
  - And-Elimination
    - $$\frac{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}{\alpha_i}$$
  - And-Introduction
    - $$\frac{\alpha_1, \alpha_2, \ldots, \alpha_n}{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}$$
  - Or-Introduction
    - $$\frac{\alpha_i}{\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_n}$$
  - Resolution
    - $$\frac{\alpha \vee \beta,\ \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

Inferences Rules with Quantifiers

- **Substitutions**
  - SUBST(θ , α): binding list θ applied to a sentence α
    - e.g.: SUBST({x / John, y / Richard}, Brother(x, y)) = Brother(John, Richard)

- **Inference rules**
  - Universal Elimination
    - $$\frac{\forall\, x\ \ \alpha}{\text{SUBST}(\ \{x/g\},\ \alpha)}$$

$\forall$ x Dog(x) $\Rightarrow$ Friendly(x)

|– Dog(Snoopy) $\Rightarrow$ Friendly(Snoopy)

  - Existential Introduction
    - $$\frac{\alpha}{\exists\ x\ \text{SUBST}(\ \{g/v\},\ \alpha)}$$

  - Existential Elimination
    - $$\frac{\exists\ \ x\ \ \alpha}{\text{SUBST}(\ \{x/\ K\},\ \alpha)}$$

(Skolemization)

$\exists$x Dog (x) Λ Owns(John, x)

|– Dog (Lassie), Owns(John,Lassie)

An Example of Logical Proof

- **Proof procedure**
  - Analysis of the problem description (Natural Language)
  - Translation from NL to first-order logic
  - Application of inference rules (proof)

- **Problem statement**
  - *"It is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold by Col. West, who is American."*

Translation in First-Order-Logic

- "It is a crime for an American to sell weapons to hostile nations …"
  (1) $\forall x,y,z\ American(x) \wedge Weapon(y) \wedge Nation(z) \wedge Hostile(z) \wedge Sells(x,z,y) \Rightarrow Criminal(x)$
- "The country Nono […] has some missiles, …"
  (2) $\exists x\ Owns(Nono, x) \wedge Missile(x)$
- "… all of its missiles were sold by Col. West, …"
  (3) $\forall x\ Owns(Nono,x) \wedge Missile(x) \Rightarrow Sells(West,Nono,x)$
- A missile is a weapon.
  (4) $\forall x\ Missile(x) \Rightarrow Weapon(x)$
- An enemy of America is hostile.
  (5) $\forall x\ Enemy(x,America) \Rightarrow Hostile(x)$
- "… West, who is American."
  (6) $American(West)$
- "The country Nono …"
  (7) $Nation(Nono)$
- "Nono, an enemy of America …"
  (8) $Enemy(Nono,America)$
  (9) $Nation(America)$

Proof

Knowledge Base	Inferences
(1) $\forall$x,y,z American(x) Λ Weapon(y) Λ Nation (z) Λ Hostile(z) Λ Sells(x,z,y) $\Rightarrow$ Criminal(x)	From (2) and Existential-Elimination:
(2) $\exists$x Owns(Nono, x) Λ Missile(x)	**(10)** Owns(Nono, M1) Λ Missile(M1)
(3) $\forall$x Owns(Nono,x) Λ Missile(x) $\Rightarrow$ Sells(West,Nono,x)	From (10) and And-Elimination:
(4) $\forall$x Missile(x) $\Rightarrow$ Weapon(x)	**(11)** Owns(Nono, M1)
(5) $\forall$x Enemy(x,America) $\Rightarrow$ Hostile(x)	**(12)** Missile(M1)
(6) American(West)	From (4) and Universal-Elimination:
(7) Nation(Nono)	**(13)** Missile(M1) $\Rightarrow$ Weapon(M1)
(8) Enemy(Nono,America)	From (12,13) and Modus Ponens:
(9) Nation(America)	**(14)** Weapon(M1)

Proof (2)

Knowledge Base	Inferences
(1) $\forall$x,y,z American(x) Λ Weapon(y) Λ Nation (z) Λ Hostile(z) Λ Sells(x,z,y) $\Rightarrow$ Criminal(x) … **(3)** $\forall$x Owns(Nono,x) Λ Missile(x) $\Rightarrow$ Sells(West,Nono,x) … **(6)** American(West) … **(10)** Owns(Nono, M1) Λ Missile(M1) … **(14)** Weapon(M1)	From (3) and Universal-Elimination: **(15)** Owns(Nono, M1) Λ Missile(M1) $\Rightarrow$ Sells(West,Nono,M1) From (15,10) and Modus Ponens: **(16)** Sells(West,Nono,M1) From (1) and Universal-Elimination (three times): **(17)** American(West) Λ Weapon(M1) Λ Nation (Nono) Λ Hostile(Nono) Λ Sells(West,Nono,M1) $\Rightarrow$ Criminal(West)

Proof (3)

Knowledge Base	Inferences
…	From (5) and Universal-Elimination:
(5) $\forall$x Enemy(x,America) $\Rightarrow$ Hostile(x)	**(18)** Enemy(Nono,America) $\Rightarrow$ Hostile(Nono)
(6) American(West)	From (8,18) and Modus Ponens:
(7) Nation(Nono)	**(19)** Hostile(Nono)
(8) Enemy(Nono,America)	From (6,7,14,16,19) and And-Intro.:
…	**(20)** American(West) $\wedge$ Weapon(M1) $\wedge$ Nation (Nono) $\wedge$ Hostile(Nono) $\wedge$ Sells(West,Nono,M1)
(14) Weapon(M1)	From (17,20) and Modus Ponens:
(16) Sells(West,Nono,M1)	**(21)** **Criminal(West)**
(17) American(West) $\wedge$ Weapon(M1) $\wedge$ Nation (Nono) $\wedge$ Hostile(Nono) $\wedge$ Sells(West,Nono,M1) $\Rightarrow$ Criminal(West)	

Proof as a Search Problem

- **Proof procedure**
  - Sequence of inference rules applied to the KB
- **Search problem formulation**
  - Initial state: KB (sentences 1 to 9)
  - Operators: applicable inference rules
  - Goal state: KB containing Criminal(West)
- **Characteristics**
  - Solution depth: 14
  - Branching factor increases as the KB grows, very large for some operators (e.g. Universal Elimination)
  - Common inference patterns (using U.E., A.I., M.P.)

Part III – Knowledge and Reasoning

- **9 Inference in First-Order Logic**
  - – Inference Rules. – Generalised Modus Ponens.
  - – Forward and Backward Chaining. – Resolution.
- **10 Logical Reasoning Systems**
  - – Indexing, Retrieval and Unification. – Logic Programming / Prolog. – Production Systems.
  - – Frames and Semantic / Conceptual Networks.
  - – Managing Retractions, Assumptions, and Explanations.

Generalised Modus Ponens

- **Inference pattern**
  - Universal-Elimination + And-Introduction + Modus Ponens
    - e.g.: $\forall$x Missile(x) Λ Owns(Nono,x) $\Rightarrow$ Sells(West,Nono,x)
      Missile(M1)
      Owns(Nono,M1) |– Sells(West,Nono,M1)

- **Inference rule**
  - *Generalised Modus Ponens*
    - $$\frac{\chi_1, \chi_2, \ldots, \chi_N, \ (\alpha_1 \Lambda \alpha_2 \Lambda \ldots \Lambda \alpha_N \Rightarrow \beta)}{\text{SUBST}(\theta, \beta)}$$

where $\forall$i SUBST(θ, χ_i) = SUBST(θ, α_i) 30th MARCH-

Example of GMP Application

- **Knowledge base (extract)**
    - $\forall$x Missile(x) Λ Owns(Nono,x) $\Rightarrow$ Sells(West,Nono,x)
    - $\forall$y Owns(y,M1)
    - Missile(M1)
- **Generalized Modus Ponens**
  – Matching
    - $\chi_1 \leftarrow$ Missile(M1)
    - $\chi_2 \leftarrow$ Owns(y,M1)
    - $\theta \leftarrow$ {x/M1, y/Nono)
    - $\alpha_1 \leftarrow$ Missile(x)
    - $\alpha_2 \leftarrow$ Owns(Nono,x)
    - $\beta \leftarrow$ Sells(West,Nono,x)

  – Inference rule
    - $$\frac{\chi_1,\ \chi_2,\ (\alpha_1\ \Lambda\ \alpha_2 \Rightarrow \beta)}{\text{SUBST}(\theta,\ \beta)}$$ $\leftarrow$ Sells(West,Nono,M1)

Using the GMP

- **Characteristics**
  - Combine several inferences into one
  - Use helpful substitutions (rather than random U.E.)
  - Make use of pre-compiled rules in…
- **Canonical form**
  - Matches the premises of the GMP rule
  - Horn sentences (Horn normal forms / clause forms)
    - i.e. $\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n \Rightarrow \beta$
  - Sentences converted when entered in the KB
    - e.g. $\exists x$ Missile(x) $\wedge$ Owns(Nono,x) becomes Missile(M66) and Owns(Nono,M66)

Example on variables

Unification

- **The UNIFY routine**
  - Find a substitution that make 2 atomic sentences alike i.e.

    UNIFY$(\alpha,\beta) = \underbrace{\theta}_{\text{unifier}}$ where SUBST(θ,α) = SUBST(θ,β)

- **Example**
  - Sample rule in canonical form:
    - Knows(John,x) $\Rightarrow$ Hates(John,x)
  - Query: “who does John hate?”
    - ?p, Hates(John,p)
    - Find all the sentences in the KB that unify with Knows(John,x), then apply the unifier to Hates(John,x).

Variable Substitution

- **Renaming**
  - Sentence identical to another, except for variable names
    - e.g. Hates(x,Elizabeth) and Hates(y,Elizabeth)
- **Composition of substitutions**
  - *Substitution with composed unifier identical to the sequence of substitutions with each unifier*
    i.e.
    $$\text{Subst}(\text{Compose}(\theta_1,\theta_2),\alpha) = \text{Subst}(\theta_2 ,\text{Subst}(\theta_1,\alpha))$$
    - e.g. α = Knows(x,y), θ_1 = {x/John}, θ_2 = {y/ Elizabeth}
      Subst(θ_2,Subst(θ_1 ,α)) = Subst(θ_2,Knows(John,y)) =
      Subst({x/John,y/Elizabeth}, Knows(x,y)) = Knows(John,Elizabeth)

Standardising Sentences

- **Example**
  - Knowledge base:
    - Knows(John,Jane)
    - Knows(z,Mother(z))
    - Knows(y,Leonid)
    - Knows(x,Elizabeth)
  - Unifying with Knows(John,x):
    - UNIFY(Knows(John,x), Knows(John,Jane)) = {x/Jane}
    - UNIFY(Knows(John,x), Knows(y,Leonid)) = {x/Leonid, y/John}
    - UNIFY(Knows(John,x), Knows(z,Mother(z))) = {z/John, x/Mother(John)}
    - UNIFY(Knows(John,x), Knows(x,Elizabeth)) = {} *?*
- **Standardise sentences apart**
  - Renaming variables to avoid clashes, e.g. Knows(z,Elizabeth)

Most General Unifier

- **Example**
  - Unifying yields an infinite number of substitutions
    - UNIFY(Knows(John,x), Knows(z,Elizabeth)) = {x/Elizabeth, z/John}
      or {x/Elizabeth, z/John, w/Richard},
      or {x/Elizabeth, y/Elizabeth, z/John},
      or …
- **Most General Unifier (MGU)**
  - *Unifier that makes the least commitments about the bindings of the variables*
  - UNIFY always returns the MGU

Sample Proof Revisited

Knowledge Base	KB in Horn Normal Form
(1) $\forall x,y,z$ American(x) Λ Weapon(y) Λ Nation (z) Λ Hostile(z) Λ Sells(x,z,y) $\Rightarrow$ Criminal(x)	**(1)** American(x) Λ Weapon(y) Λ Nation (z) Λ Hostile(z) Λ Sells(x,z,y) $\Rightarrow$ Criminal(x)
(2) $\exists x$ Owns(Nono, x) Λ Missile(x)	**(2a)** Owns(Nono, M1) **(2b)** Missile(M1)
(3) $\forall x$ Owns(Nono,x) Λ Missile(x) $\Rightarrow$ Sells(West,Nono,x)	**(3)** Owns(Nono,x) Λ Missile(x) $\Rightarrow$ Sells(West,Nono,x)
(4) $\forall x$ Missile(x) $\Rightarrow$ Weapon(x)	**(4)** Missile(x) $\Rightarrow$ Weapon(x)
(5) $\forall x$ Enemy(x,America) $\Rightarrow$ Hostile(x)	**(5)** Enemy(x,America) $\Rightarrow$ Hostile(x)
(6) American(West)	**(6)** American(West)
(7) Nation(Nono)	**(7)** Nation(Nono)
(8) Enemy(Nono,America)	**(8)** Enemy(Nono,America)
(9) Nation(America)	**(9)** Nation(America)

Sample Proof (2)

Knowledge Base (HNF)	Inferences
(1) American(x) Λ Weapon(y) Λ Nation (z) Λ Hostile(z) Λ Sells(x,z,y) $\Rightarrow$ Criminal(x) **(2a)** Owns(Nono, M1) **(2b)** Missile(M1) **(3)** Owns(Nono,x) Λ Missile(x) $\Rightarrow$ Sells(West,Nono,x) **(4)** Missile(x) $\Rightarrow$ Weapon(x) **(5)** Enemy(x,America) $\Rightarrow$ Hostile(x) **(6)** American(West) **(7)** Nation(Nono) **(8)** Enemy(Nono,America) **(9)** Nation(America)	From (2b, 4) and Modus Ponens: **(10)** Weapon(M1) From (8, 5) and Modus Ponens: **(11)** Hostile(Nono) From (2a, 2b, 3) and Modus Ponens: **(12)** Sells(West,Nono,M1) From (6, 10, 7, 11, 12, 1) and Modus Ponens: **(13)** **Criminal(West)**

Semantic Interpretation

- The symbolic representation using 1st order logic is independent of the semantics.
- Each satisfiable interpretation allows a different set of semantics to be defined which represent different worlds.
- The missile world in the example provides a possible interpretation based on the specified semantics.
- Another example is the technology world of network browser descnetscape example

– NEXT

end