APPENDIX D

Fault Analysis Using Z_{Bus} Matrix

Assumptions

1) Magnetizing currents & core losses neglected for transformers.

2) Shunt capacitances neglected for T. line models

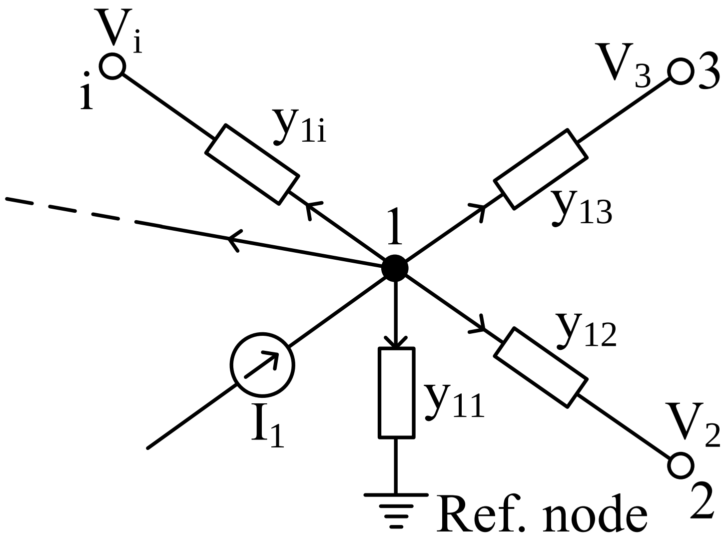
3) Transformers set at nominal tap positions.

4) All internal voltage sources are set equal to $1.0\angle 0°$ $\Rightarrow$ pre-fault load currents are neglected.

5) Resistances of all components assumed negligible; not included therefore.

6) Impedance at the fault is taken to be zero, i.e. solid fault considered.

Recall Generalized KCL Eqns



V_i = Node voltage at node i, wrt ref.
I_1 = Sum of all currents (sources) directly connected to node 1
y_{11} = Admittance betn. node 1 & ref. node
y_{1i} = Admittance betn. node 1 & node i
n = Total no. of nodes (excluding ref.)

Then,

$$I_1 = y_{11}V_1 + (V_1 - V_2)\,y_{12} + (V_1 - V_3)\,y_{13} + \dots + (V_1 - V_n)\,y_{1n}$$
$$= Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + \dots + Y_{1n}V_n$$

where: $Y_{1i} = -y_{1i}$ (for $i \neq 1$) $\Rightarrow Y_{ni} = -y_{ni}$ ($i \neq n$)
$Y_{11} = y_{11} + y_{12} + y_{13} + \dots + y_{1n}$

e.g. $Y_{12} = -y_{12}$; $Y_{13} = -y_{13}$

Matrix Form:

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_j \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1i} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2i} & \cdots & Y_{2n} \\ \vdots & & & & & \\ Y_{j1} & Y_{j2} & \cdots & & \cdots & Y_{jn} \\ \vdots & & & & & \\ Y_{n1} & Y_{n2} & \cdots & & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_j \\ \vdots \\ V_n \end{bmatrix}$$

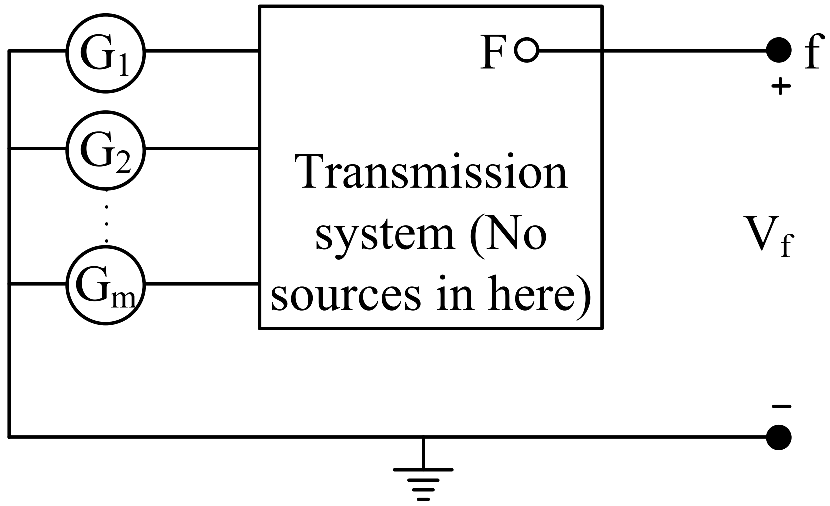
$$\Rightarrow [I] = [Y][V]$$

Note that:

I_j : Sum of currents entering node j from sources directly connected.

Also, [V] = [Z] [I], where [Z] = bus impedance matrix (Z_{Bus})

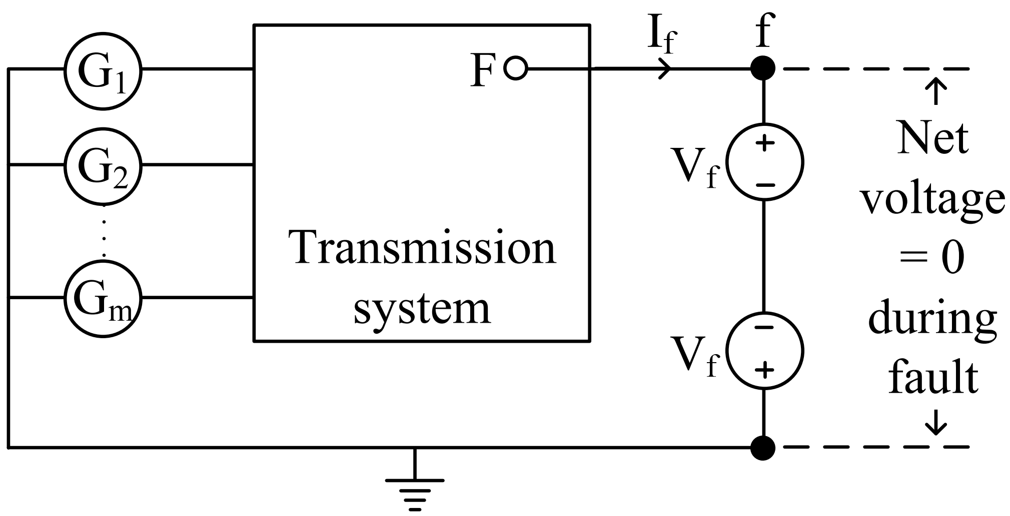
Now, let the power system be represented as follows (Thevenin's theorem):



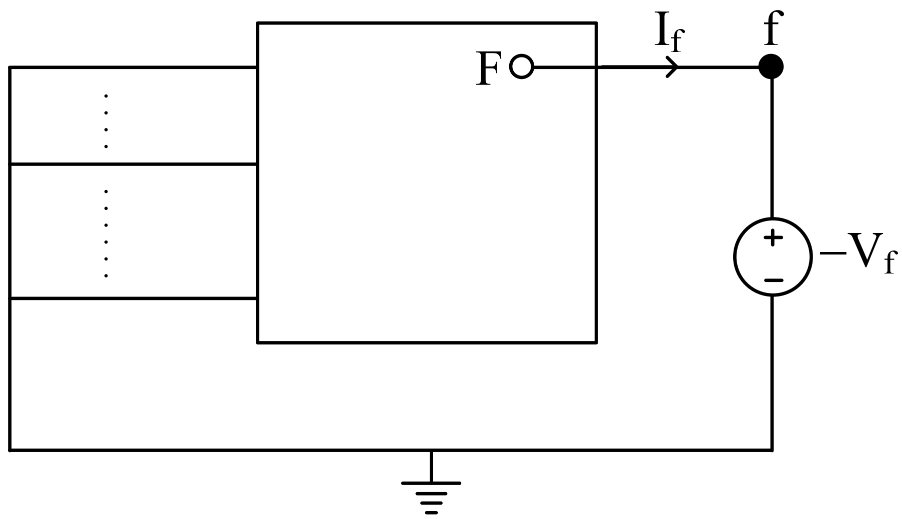
V_f: Voltage betn. node F & ground before the fault occurs (*actual p.u. voltage at point f before fault occurs*)

→ When a 3-phase-to-ground fault occurs at node F, voltage there drops to zero.

→ This is equivalent to applying an equal and opposite voltage V_f at node F as shown:



→ Effects of the fault can be analyzed using superposition theorem, by considering the changes caused by ($-V_f$) as follows:



→ Matrix eqn. (with $-V_f$ as the only source)

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_f \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & \cdots & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & \cdots & \cdots & Y_{2n} \\ \vdots & & & & & \\ Y_{f1} & \cdots & \cdots & Y_{ff} & \cdots & Y_{fn} \\ \vdots & & & & & \\ Y_{n1} & \cdots & \cdots & Y_{nf} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_f \\ \vdots \\ \Delta V_n \end{bmatrix}$$

ΔV_f : $0 - V_f = -V_f$

ΔV : Indicates <u>changes</u> in the voltages at the nodes due to the current ($-I_f$) injected into bus f by the fault.

$(-I_f)$: Signifies current entering the node F (from external sources)

$$\Rightarrow \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_f \\ \vdots \\ \Delta V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & \cdots & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & \cdots & \cdots & Z_{2n} \\ \vdots & & & & & \\ Z_{f1} & Z_{f2} & \cdots & Z_{ff} & \cdots & Z_{fn} \\ \vdots & & & & & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nf} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_f \\ \vdots \\ 0 \end{bmatrix}$$

$$\Rightarrow \Delta V_f = -V_f = Z_{ff}(-I_f)$$

$$\Rightarrow I_f = \frac{V_f}{Z_{ff}}$$

$$\Delta V_n = Z_{nf}\,(-I_f)$$

$$= Z_{nf}\left[\frac{-V_f}{Z_{ff}}\right]$$

Assuming that pre-fault voltage is the same at all nodes $= V_f$, then voltage at node n when fault current is flowing is given by:

$$V_n = V_f + \Delta V_n = V_f - \frac{Z_{nf}\ V_f}{Z_{ff}}$$

$$\Rightarrow V_n = V_f \left[1 - \frac{Z_{nf}}{Z_{ff}} \right]$$

V_n : Total voltage at node n after the fault.

$\Rightarrow$ In this manner, voltages at all nodes (*after-fault values*) can be obtained.

$\Rightarrow$ Ohm's law can be used to find all currents flowing between the various nodes.

$\Rightarrow$ Voltages at <u>all</u> buses can be evaluated using the <u>pre-fault voltage V_f</u> of the fault bus <u>and</u> the elements in the <u>f^{th}</u> column of the Z_{Bus} matrix!