NANYANG TECHNOLOGICAL UNIVERSITY
School of Electrical & Electronic Engineering

EE/IM4152 Digital Communications

Tutorial No. 6 (Sem 1, AY2016-2017)

1. Mutual information $I(X;Y)$ is defined as the decrease in the observer's average uncertainty of the transmitted signal when the output is received. That is,

$$I(X;Y) = H(X) - H(X\,|\,Y).$$

Using the inequality $\ln(x) \le x - 1$, show that $H(X\,|\,Y) - H(X) \le 0$.

2. A channel is described by the transition probability matrix

$$\mathbf{P(Y\,|\,X)} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{10} & \frac{9}{10} \end{bmatrix}.$$

The source probabilities are $p(x_1) = 1/3$ and $p(x_2) = 2/3$. Determine $H(Y)$, $H(Y\,|\,X)$ and $I(X;Y)$.

3. Consider the binary symmetric channel (BSC) shown in Figure 1. Let α be the probability of choosing binary symbol x_1. Accordingly, the probability of selecting binary symbol x_2 is $1-\alpha$. The transition probability of the BSC is denoted by ε, as shown in Figure 1.
   (a) Find the mutual information $I(X;Y)$ between the channel input and channel output. What is the value of α that maximizes $I(X;Y)$? Show that the channel capacity is equal to $C = 1 - \Omega(\varepsilon)$, where $\Omega(\varepsilon) \equiv -\varepsilon \log_2 \varepsilon - (1-\varepsilon)\log_2(1-\varepsilon)$.
   (b) Suppose two BSCs are connected in cascade, as shown in Figure 2. Assuming that both channels have the same transition probability diagram shown in Figure 1. Determine the matrix of transition probabilities of the cascaded system. Is it binary symmetric? Find the overall channel capacity of the cascaded connection.

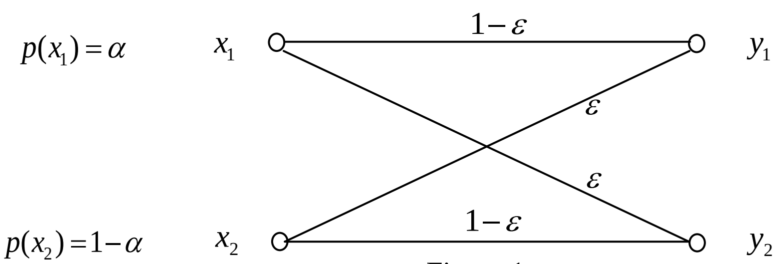

Figure 1

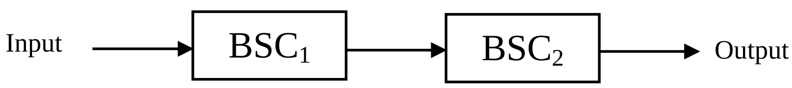

Figure 2