NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2016/17 Semester 1 MH2500 Probability and Introduction to Statistics Tutorial 8

For the tutorial on 13 October, let us discuss

- Ex. 3.8.43, 48, 51, 67, 70, 74

Ex. 3.8.43. Let U_1 and U_2 be independent and uniform on [0,1]. Find and sketch the density function of $S = U_1 + U_2$.

Ex. 3.8.48. Let T_1 and T_2 be independent exponentials with parameters λ_1 and λ_2. Find the density function of $T_1 + T_2$.

Ex. 3.8.51. Let X and Y have the joint density function $f(x, y)$ and let $Z = XY$. Show that the density function of Z is

$$f_Z(z) = \int_{-\infty}^{\infty} f\left(y, \frac{z}{y}\right) \frac{1}{|y|} dy.$$

Ex. 3.8.67. A card contains n chips and has an error-correcting mechanism such that the card still functions if a single chip fails but does not function if two or more chips fail. If each chip has a lifetime that is an independent exponential with parameter λ, find the density function of the card's lifetime.

Ex. 3.8.70. If five numbers are chosen at random in the interval $[0, 1]$, what is the probability that they all lie in the middle half of the interval?

Ex. 3.8.74. Let U_1, U_2, and U_3 be independent uniform random variables.

a. Find the joint density of $U_{(1)}, U_{(2)}$, and $U_{(3)}$.
b. The locations of three gas stations are independently and randomly placed along a mile of highway. What is the probability that no two gas stations are less than $\frac{1}{3}$ mile apart?