NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2016/17 Semester 1 MH2500 Probability and Introduction to Statistics Tutorial 7

For the tutorial on 6 October, let us discuss

- Ex. 3.8.5, 6, 11, 18, 19, 30

Ex. 3.8.5. (Buffon's Needle Problem) A needle of length L is dropped randomly on a plane ruled with parallel lines that are distance D apart, where $D \geq L$. Show that the probability that the needle comes to rest crossing a line is $2L/(\pi D)$. Explain how this gives a mechanical means of estimating the value of π.

Ex. 3.8.6. A point is chosen randomly in the interior of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Find the marginal densities of the x and y coordinates of the point.

Ex. 3.8.11. Let U_1, U_2, and U_3 be independent random variables uniform on [0,1]. Find the probability that the roots of the quadratic $U_1x^2 + U_2x + U_3$ are real.

Ex. 3.8.18. Let X and Y have the joint density function

$$f(x, y) = k(x - y), \qquad 0 \leq y \leq x \leq 1,$$

and 0 elsewhere.

a. Sketch the region over which the density is positive and use it in determining limits of integration to answer the following questions.
b. Find k.
c. Find the marginal densities of X and Y.
d. Find the conditional densities of Y given X and X given Y.

Ex. 3.8.19. Suppose that two components have independent exponentially distributed lifetimes, T_1 and T_2, with parameters α and β, respectively. Find (a) $P(T_1 > T_2)$ and (b) $P(T_1 > 2T_2)$.

Ex. 3.8.30. For $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$, show that $C(u, v) = \min(u^{1-\alpha}v, uv^{1-\beta})$ is a copula (the Marshall-Olkin copula) (valid for $0 \leq u, v \leq 1$. Somehow the author expects us to know that u and v cannot be negative or else $C(u, v)$ is not defined, and u, v cannot exceed 1, or else $C(u, v)$ may exceed 1.) What is the joint density?