Equalization techniques

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Motivation

Intersymbol Interference

- A digital communication system requires transmit and receive filters.

$$x[k] = x\left(\frac{kT}{M} + \tau\right) = \sum_{i=-\infty}^{\infty} s[i]\, h\left(\frac{kT}{M} + \tau - iT\right) + v[k]$$

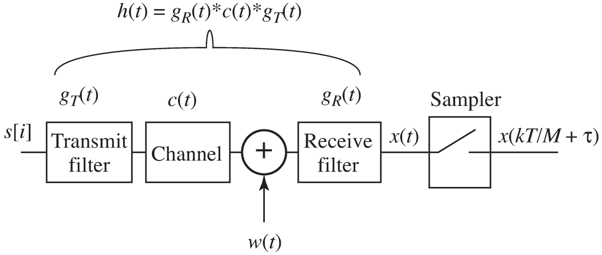

Figure 1: Digital communication system model[a].

[a] Source http://cnx.org/content/m46053/latest/

Intersymbol Interference

- Transmit filter shapes transmitted signal to meet spectral requirements.
- Receive filter accomplishes two roles:
  - Recover the symbol sent (by shaping the signal before sampling).
  - Limit the noise effect (its bandwidth imposes the *noise bandwidth*).
- This is achieved when $h_T(t) = g_T(t) * c(t)$ and $g_R(t)$ constitute a matched filter pair.

$$g_R(t) = h_T(T - t)$$

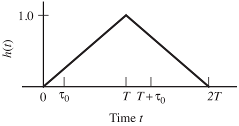

Figure 2: Overall response to a $g_T(t) * c(t)$ with rectangular form and $g_R(t)$ its matched filter.

Intersymbol Interference

- When a system is designed according to the matched filter criterion, there is no *intersymbol interference* (ISI).
  - When sampling at the correct instant, with period T, we get back $s[i]$.
  - At these instants, the $h(t)$ response left from the other symbols is zero.
- Problems:
  - A situation like the one mentioned before ($h_T(t)$ rectangular) requires infinite bandwidth.
  - $c(t)$ will always be a *bandlimited channel*.
  - The channel can be time varying (e.g. wireless channels).
- All this could lead to intersymbol interference.
  - Tails from other symbols' responses overlap at the sampling instants.
  - System gets *memory*.
  - The performance of the system (BER) degrades.
  - System gets more sensitive to synchronization and timing errors.

Intersymbol Interference

- An *eye diagram* is a periodic depiction of a digital waveform.
  - It helps to visualize the behavior of the system, presence of ISI, etc.

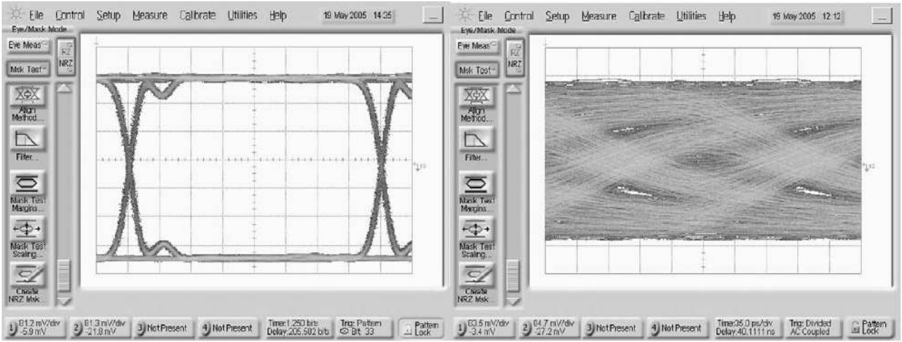

Figure 3: Left: eye diagram of the signal sent. Right: received signal with high ISI (source: Agilent Technologies).

Intersymbol Interference

- The eye diagram provides information about working margins.
  - Synchronization.
  - Noise resistance.

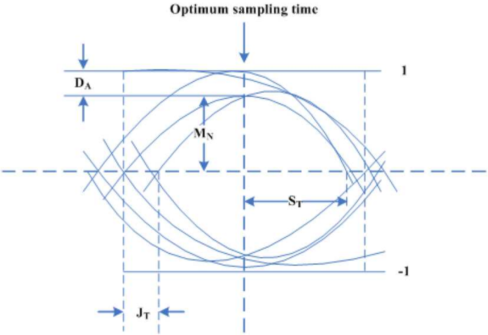

Figure 4: Eye diagram structure (source: http://cnx.org/content).

Intersymbol Interference

- In channels where ISI cannot be avoided:
  - $g_T(t)$ and $g_R(t)$ are designed as matched pairs.
  - The effect of $c(t)$ is counteracted by means of an *equalizer*.

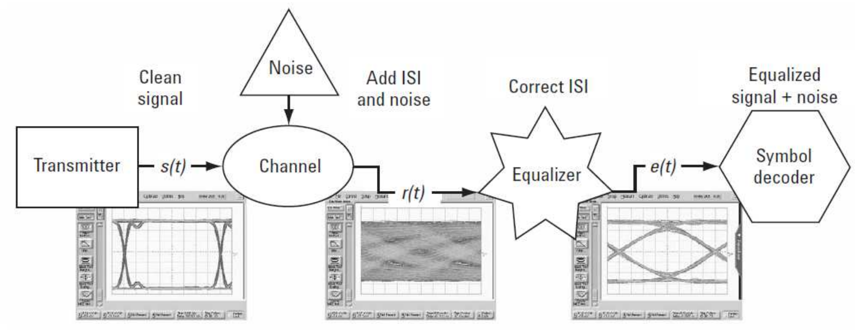

Figure 5: Communication model with equalization (source: Agilent Technologies).

Intersymbol Interference

- Condition for no ISI:

$$\sum_{i=-\infty}^{\infty} H\left(f + \frac{i}{T}\right) = \text{constant}$$

- The ideal case with the smallest bandwidth possible is the *Nyquist channel*.
  - It is not feasible.
  - Real transmission requires higher bandwidth.

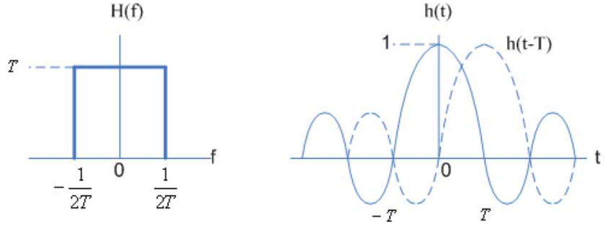

Figure 6: Nyquist channel (source: http://cnx.org/content).

Intersymbol Interference

- Regardless of ISI, the role of transmit filter is important to fit the transmitted spectrum into the appropriate spectral mask.
  - This requires the usage of longer duration waveforms.
  - The spectral occupancy reduction helps in reducing ISI.
  - It allows transmission with controlled co-channel interference.

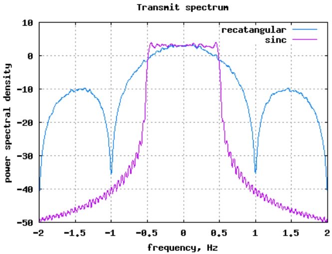

Figure 7: Effect of transmit pulse shaping (source: http://www.dsplog.com).

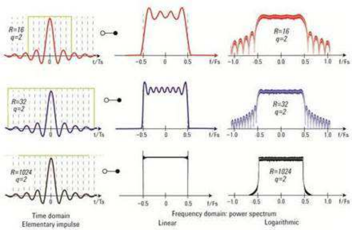

Figure 8: Different pulse shapes (source: http://www.lightwaveonline.com).

Intersymbol Interference

- $g_T(t)$ and $g_R(t)$ are often implemented as *root-raised cosine* (RRC) pairs.
    - They provide feasible waveforms (unlike rectangular ones).
    - The bandwidth required is managed by means of a design parameter.
    - If $c(t) = a \cdot \delta(t - \tau_0)$ ($a \in \mathbb{C}$) within the transmision bandwidth $\rightarrow$ no ISI, optimal TX/RX pair.

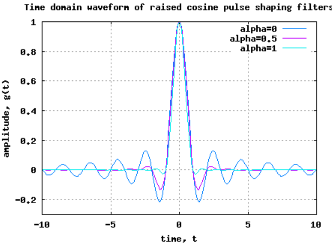

Figure 9: Raised cosine time waveforms (source: http://www.dsplog.com).

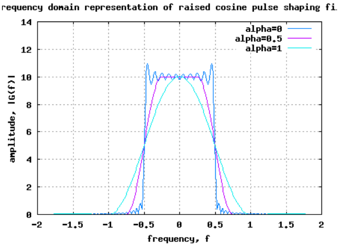

Figure 10: Raised cosine spectra (source: http://www.dsplog.com).

Intersymbol Interference

- Nevertheless, do not forget channels and systems are not ideal!
  - Some amount of ISI will be present, along with other undesirable effects.
  - In wideband high-speed modern digital communications, some degree of equalization is mandatory to get appropriate performances.

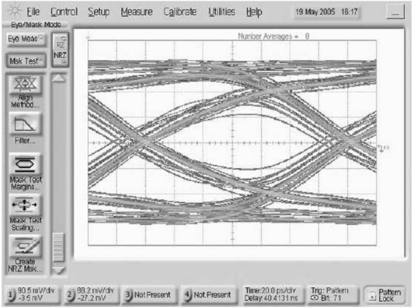

Figure 11: An average system with residual ISI (source: Agilent Technologies).

Intersymbol Interference

- Equalization is as well about compensating other effects that distort the received data constellation.
  - Observe the rotation and warping in the figure.
  - In many cases, de-rotating and compensating linear and nonlinear effects require knowledge about $c(t)$.
  - In bandpass transmission, eye diagrams and compensation should heed I and Q channels.

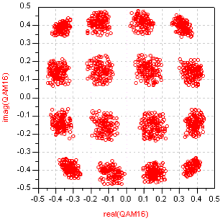

Figure 12: 16-QAM distorted constellation (source: Agilent Technologies).

Equalization strategies

Equalization strategies

- System model for equalization.
  - It is valid both for baseband and bandpass transmission.
  - In bandpass systems, equalization is performed over I and Q channels.

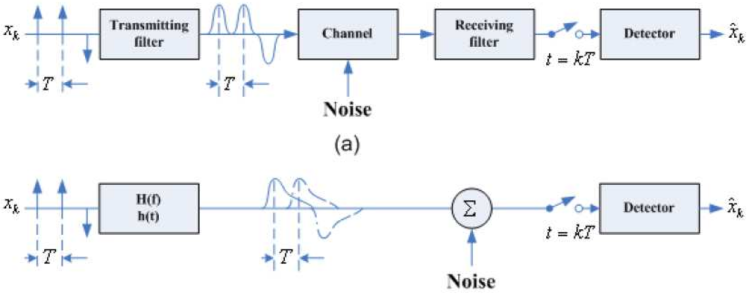

Figure 13: System model (source: http://cnx.org/content).

Equalization strategies

- The natural way to compensate $H(f)$ would be to invert it: $H^{-1}(f)$.
- This strategy has some drawbacks:
  - Enhances noise where signal is greatly attenuated (noise has to be taken into account!).
  - Could lead to instabilities (implementation is an important issue!).

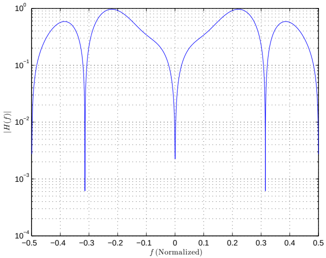

Figure 14: Channel example.

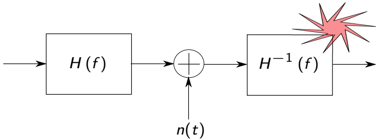

Figure 15: Equalization scheme.

Equalization strategies

- Equalization should take care of:
  - Removing ISI.
  - While ensuring a maximum SNR.
  - And allowing a feasible implementation.
- Remember: $H(f) = G_R(f)\, C(f)\, G_R(f)$.
  - $G_T(f)$ and $G_R(f)$ are matched filter pairs (e.g. RRC filters).
  - Equalization can be done by estimating the properties of $H(f)$ (**non-blind equalization**: channel identification), or just by resorting to the properties of the data carrying signals (**blind equalization**: matching TX/RX data).

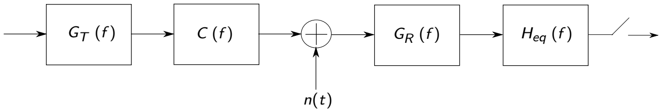

Figure 16: A system with a linear equalizer.

Equalization strategies

- Apart form the blind / non-blind classification, equalizers can be classified into:
  - Linear equalizers ($H_{eq}(f)$).
    - Zero forcing equalizer (ZFE).
    - Minimum mean square error (MMSE) equalizer.
  - Non-linear equalizers.
    - Decision-feedback equalizer (DFE).
    - Maximum likelihood sequence estimation (MLSE).
- Depending on the treatment of the received symbols, the equalizers can be classified into:
  - Symbol-by-symbol (SBS): each symbol is equalized separately and then used for decision (linear equalizers and DFE are of this kind).
    - Easier to implement, but not optimal to remove all the memory effects.
  - Sequence estimators (SE): a sequence of symbols is equalized as a block (MLSE is of this kind).
    - Optimal when considering all the memory in the channel, but complex, and even unfeasible as memory grows.

Equalization strategies

- The time varying characteristics of the channel lead to:
  - Non-adaptive equalizers, ideal when the channel is static.
    - This does not mean that the system does not follow a previous *acquisition* step (with the help of known sequences or channel identification).
  - Adaptive equalizers, ideal when the channel is time varying.
    - Apart from the *acquisition* initial setup, the system performs *tracking*, continuously updating the equalizer parameters.
    - The DFE is inherently an adaptive equalizer.
    - Linear equalizers are easily modified as adaptive variants: LMS (least mean squares) filter, RLS (recursive least squares) filter.

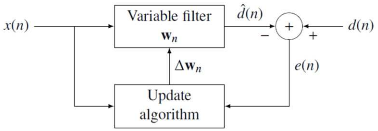

Figure 17: Adaptive equalization based on linear equalizers (source: Wikipedia).

Linear equalizers

Linear equalizers

- The linear equalizers are simple to implement and:
  - Rely on the principle of inverting $H(f)$.
  - Cancel ISI at the cost of possibly enhancing noise (ZFE), or provide a tradeoff between noise enhancement and ISI removal (MMSE).
- In non-blind mode, $H(f)$ is estimated by feeding an impulse.
  - Equalization is performed digitally, so $h[n]$ is what usually matters.
- In blind mode, the system uses known training sequences.
- $h_{eq}[n]$ is implemented as a FIR (finite impulse response) filter.

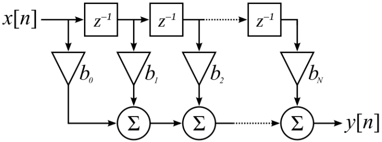

Figure 18: FIR transversal filter (source: Wikipedia).

Linear equalizers

- The ZFE tries to force the non-ISI condition:

$$\sum_{i=-\infty}^{\infty} H_0\left(f+\frac{i}{T}\right)=\mathrm{constant}$$

- Where $H_0(f)=H(f)\,H_{eq}(f)$.
- In the discrete-time domain, this translates into:

$$h_0(nT)=h_0[n]=\delta[n]$$

- We *force* an overall response with *zeros* when $n\neq 0$.
- When the filter length $N+1$ is large enough, $h_0(t)\approx\delta(t)$ and ISI is perfectly compensated.
- In practice, the number of FIR coefficients are limited to a given target channel response length, and some residual ISI remains.
- The time-domain implementation helps to counteract unstability problems of frequency-based domain.

Linear equalizers

- ZFE $\{b_i\}_{i=0}^{N}$ coefficients are calculated with very simple matrix algebra.
- Once we know the impulse response $h[n]$, for $n = -N, \cdots, N$, N even, the non-ISI condition results in:

$$\sum_{i=0}^{N} b_i h\left[m - \left(i - \frac{N}{2}\right)\right] = \delta[m], m = -\frac{N}{2}, \cdots, \frac{N}{2}$$

- In matrix form:

$$\mathbf{v} = \mathbf{H}\mathbf{b} \rightarrow \mathbf{b} = \mathbf{H}^{-1}\mathbf{v}$$

- Where $\mathbf{v}$ and $\mathbf{b}$ are length $N+1$ vectors:

$$\mathbf{v} = (0 \cdots 010 \cdots 0)^T$$
$$\mathbf{b} = (b_0 \cdots b_N)^T$$

- $\mathbf{H}$ and $\mathbf{b}$ contain generally complex numbers.

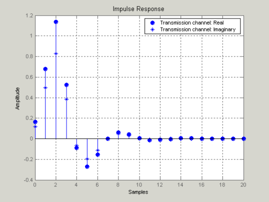

Figure 19: Channel impulse response.

Linear equalizers

- $\mathbf{H}$ $((N+1)\times(N+1))$ is built from the impulse response as:

$$\mathbf{H} = \begin{pmatrix} h\left[0\right] & \cdots & h\left[-\frac{N}{2}\right] & \cdots & h\left[-N\right] \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ h\left[\frac{N}{2}\right] & \cdots & h\left[0\right] & \cdots & h\left[-\frac{N}{2}\right] \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ h\left[N\right] & \cdots & h\left[\frac{N}{2}\right] & \cdots & h\left[0\right] \end{pmatrix}$$

- First of all, $\mathbf{H}^{-1}$ is calculated numerically.
- The vector of FIR coefficientes corresponding to the zero-forcing solution, $\mathbf{b}_{zf} \rightarrow h_{eq}\left[n\right]$, would be given by the central column of $\mathbf{H}^{-1}$.
- The non-ISI condition is met for output values up to $\frac{N}{2}$ samples at both sides of the center of the impulse response.
- The ZFE filter works reasonably well if the impulse response coefficients for $n < -\frac{N}{2}$ and $n > \frac{N}{2}$ are very low.

Linear equalizers

- Example for baseband transmission: provide ZFE with 5 taps.

$$\mathbf{h} = \begin{pmatrix} 0.01 & -0.02 & 0.05 & -0.1 & 0.2 & 1.0 & 0.15 & -0.15 & 0.05 & -0.02 & 0.005 \end{pmatrix}^T$$

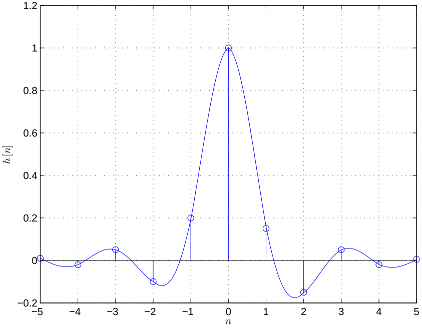

Figure 20: Measured impulse response.

Linear equalizers

- Build $\mathbf{H}$ matrix:

$$\mathbf{H}=\begin{pmatrix} 1.0 & 0.2 & -0.1 & 0.05 & -0.02 \\ 0.15 & 1.0 & 0.2 & -0.1 & 0.05 \\ -0.15 & 0.15 & 1.0 & 0.2 & -0.1 \\ 0.05 & -0.15 & 0.15 & 1.0 & 0.2 \\ -0.02 & 0.05 & -0.15 & 0.15 & 1.0 \end{pmatrix}$$

- Get inverse:

$$\mathbf{H}^{-1}=\begin{pmatrix} 1.0774 & -0.2682 & 0.1932 & -0.1314 & 0.0806 \\ -0.2266 & 1.1272 & -0.2983 & 0.2034 & -0.1314 \\ 0.2326 & -0.2737 & 1.1517 & -0.2983 & 0.1932 \\ -0.1405 & 0.2516 & -0.2737 & 1.1272 & -0.2682 \\ 0.0888 & -0.1405 & 0.2326 & -0.2266 & 1.0774 \end{pmatrix}$$

- Filter coefficients for $h_{eq}[n]$:

$$\mathbf{b}_{zf}=\begin{pmatrix} 0.1932 & -0.2983 & 1.1517 & -0.2737 & 0.2326 \end{pmatrix}^T$$

Linear equalizers

- The overall impulse response $h_0[n] = h[n] * h_{eq}[n]$ is:

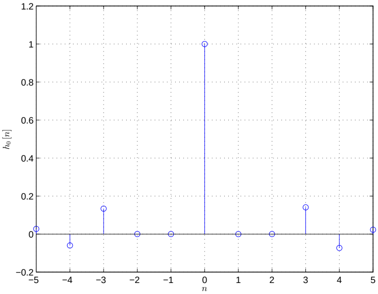

Figure 21: Overall impulse response.

- Observe how not all the ISI is compensated: the non-zero tails for $n > 2$, $n < -2$ take still significant large values.

Linear equalizers

- Results in spectrum:

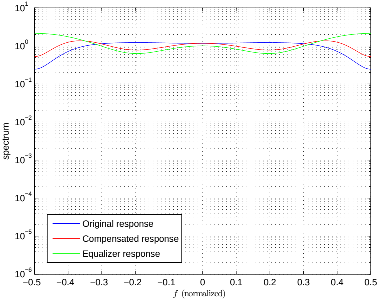

Figure 22: Overall impulse response.

- Observe how, for a usual lowpass impulse response $h[n]$, high frequency noise is actually enhanced.

Linear equalizers

- ZFE strategy suffers from the noise-enhancing issue.
  - The strategy relies on a perfect estimation of $h[n]$.
  - The noise has not been taken into account at all.
- A ZFE is not very useful in wireless channel.
  - Wireless channels suffer from frequency-selective fades.
  - They pose a challenge to keep up pace with channel identification.
- MMSE tries to reach a tradeoff between noise and ISI effects.

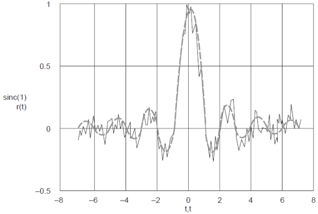

Figure 23: Noise effects in channel impulse response (source: Texas Instruments).

Linear equalizers

- A transveral FIR filter can equalize the worst-case ISI only when the peak distortion is small[1].
  - In presence of noise, the peak distortion grows.
- The MMSE gives the filter coefficients to keep a minimum mean square error between the output of the equalizer and the desired signal.
  - The MMSE equalizer requires training sequences ($d(t)$).
  - $y(t)$ and $v(t)$ are signals affected by noise.

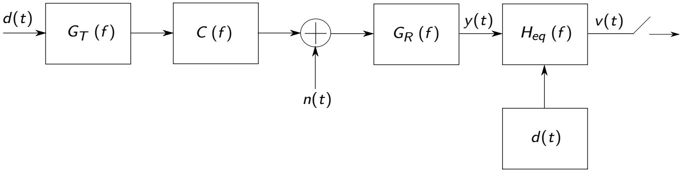

Figure 24: MMSE setup.

[1] Peak distortion: magnitude of the difference between the output of the channel and the desired signal

Linear equalizers

- MMSE criterion can be read as

$$\epsilon = \mathrm{E}\left[\left(v\left(t\right) - d\left(t\right)\right)^2\right]$$

- Where

$$v(t) = y(t) * h_{eq}(t) = \sum_{n=0}^{N} b_n y\left(t - nT\right)$$

- The minimization parameters are the *filter coefficients*, according to

$$\frac{\partial \epsilon}{\partial b_m} = 0 = 2\mathrm{E}\left[\left(v(t) - d(t)\right)\frac{\partial v(t)}{\partial b_m}\right]$$
$$m = 0, \cdots, N, \; N \text{ even}$$

- Where it has been taken into account the linearity of the operators $\mathrm{E}\left[\cdot\right]$ and $\frac{\partial}{\partial x}$, and the fact that the expected value is taken over the noise distribution, independently of the filter coefficients.

Linear equalizers

- The previous expressions lead to the next *orthogonality condition*.
  - The error sequence between the output of the equalizer and the desired signal, and the received data sequence should be *statistically orthogonal*.

$$\mathrm{E}\left[(v(t)-d(t))\, y\,(t-mT)\right] = R_{yv}\,(mT) - R_{yd}\,(mT) = 0$$
$$m = 0, \cdots, N$$
$$R_{yv}\,(mT) = \mathrm{E}\left[y(t)v\,(t+mT)\right]$$
$$R_{yd}\,(mT) = \mathrm{E}\left[y(t)d\,(t+mT)\right]$$

- All random processes involved are considered *widesense stationary* (*wss* processes) and *jointly wss*.
  - A process is *wss* when its mean and covariance do not vary with time.
  - Two random processes A and B are *jointly wss* when they are *wss* and their crosscorrelation only depends on the time difference.

$$R_{AB}\,(t,\tau) = R_{AB}\,(t-\tau)$$

Linear equalizers

- $R_{yv}(mT)$ can be written as

$$R_{yv}(mT) = \mathrm{E}\left[y(t)v(t+mT)\right] =$$
$$= \mathrm{E}\left[y(t)\sum_{n=0}^{N} b_n y\left(t+(m-n)T\right)\right] = \sum_{n=0}^{N} b_n R_y\left((m-n)\,T\right)$$
$$m = 0, \cdots, N$$

- Where $R_y(\tau)$ is the autocorrelation of $y(t)$.
- This set of equations can be written in vector-matrix form as

$$\mathbf{R}_y \mathbf{b}_{\mathrm{MMSE}} = \mathbf{R}_{yd}$$

- They are known as the *Wiener-Hopf equations*.
- The filter coefficients can be finally calculated as

$$\mathbf{b}_{\mathrm{MMSE}} = \mathbf{R}_y^{-1}\mathbf{R}_{yd}$$

Linear equalizers

- $\mathbf{R}_y$ $((N+1)\times(N+1))$ is built from the autocorrelation of $y(t)$ as:

$$\mathbf{R}_y = \begin{pmatrix} R_y(0) & \cdots & R_y\left(-\frac{N}{2}T\right) & \cdots & R_y(-NT) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ R_y\left(-\frac{N}{2}T\right) & \cdots & R_y(0) & \cdots & R_y\left(-\frac{N}{2}T\right) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ R_y(NT) & \cdots & R_y\left(\frac{N}{2}T\right) & \cdots & R_y(0) \end{pmatrix}$$

- In case the signals are real, $R_y(t) = R_y(-t)$ and the matrix is symmetric.
- The vector $\mathbf{R}_{yd}$ is given as:

$$\mathbf{R}_{yd} = \begin{pmatrix} R_{yd}\left(-\frac{N}{2}T\right) & \cdots & R_{yd}(0) & \cdots & R_{yd}\left(\frac{N}{2}T\right) \end{pmatrix}^T$$

Linear equalizers

- Note the similarities with the process followed with the ZFE strategy.
  - We calculate a matrix depending on the channel response, and a vector depending on the expected response.
  - The matrix is inverted and we get a solution.
- Note that the matrices and vectors have now *statistical* meaning.
- Note that the algebra is more involved.
- Note that the process is *blind* in nature, since we do not explicitly identify the channel, and solely rely on the properties of the data signals.
- The minimum mean square error we arrive at is given by:

$$\epsilon_{\min} = \mathrm{E}\left[|d(t)|^2\right] - \mathbf{R}_{yd}^T \mathbf{b}_{\mathrm{MMSE}} = \\ = R_d(0) - \mathbf{R}_{yd}^T \mathbf{R}_y^{-1} \mathbf{R}_{yd}$$

Linear equalizers

- Example for baseband transmission: provide an MMSE equalizer with 3 coefficients for a multipath channel with two paths.
- Hypothesis:
  - $y(t) = Ad(t) + bAd(t - T_m) + w(t)$, where $w(t) = n(t) * g_R(t)$ and $d(t - T_m)$ is the multipath trajectory.
  - $n(t)$ is AWGN with power spectral density $N_0/2$.
  - $d(t)$ is a random binary sequence with $R_d(\tau) = \Lambda(\tau/T)$, where $\Lambda(\cdot)$ is the triangle function.
  - $w(t)$ has lowpass spectrum with 3dB cutoff frequency $f_c = 1/T$
    $$S_w(f) = \frac{N_0/2}{1+(f/f_c)^2}.$$
  - In this example, symbol period and delay coincide, $T = T_m$.
  - Results will be given as a function of $E_b/N_0 = A^2T/N_0$ and b.

Linear equalizers

- $R_y(\tau)$ can be calculated as

$$R_y(\tau) = \mathrm{E}\left[y(t)y(t+\tau)\right] =$$
$$= \left(1+b^2\right)A^2 R_d(\tau) + R_w(\tau) + bA^2\left(R_d(\tau - T) + R_d(\tau + T)\right)$$

- Where

$$R_d(\tau) = \mathrm{E}\left[d(t)d(t+\tau)\right] = \Lambda(\tau/T)$$
$$R_w(\tau) = \mathrm{F}^{-1}\left(S_w(f)\right) = \tfrac{\pi N_0}{2T}\exp\left(-\tfrac{2\pi|\tau|}{T}\right)$$

- $\mathbf{R}_{yd}$ is calculated as

$$R_{yd}(\tau) = \mathrm{E}\left[y(t)d(t+\tau)\right] =$$
$$= AR_d(\tau) + bAR_d(T+\tau),\ \tau = 0, \pm T$$

- And then

$$\mathbf{R}_{yd} = \begin{pmatrix} bA & A & 0 \end{pmatrix}^T$$

Linear equalizers

- Matrix $\mathbf{R}_y$ is calculated as

$$\mathbf{R}_y = \begin{pmatrix} (1+b)^2 \frac{E_b}{N_0} + \frac{\pi}{2} & b\frac{E_b}{N_0} + \frac{\pi}{2}\exp\left(-2\pi\right) & \frac{\pi}{2}\exp\left(-4\pi\right) \\ b\frac{E_b}{N_0} + \frac{\pi}{2}\exp\left(-2\pi\right) & (1+b)^2 \frac{E_b}{N_0} + \frac{\pi}{2} & b\frac{E_b}{N_0} + \frac{\pi}{2}\exp\left(-2\pi\right) \\ \frac{\pi}{2}\exp\left(-4\pi\right) & b\frac{E_b}{N_0} + \frac{\pi}{2}\exp\left(-2\pi\right) & (1+b)^2 \frac{E_b}{N_0} + \frac{\pi}{2} \end{pmatrix}$$

- If we consider $A = 1.0$, $b = 0.1$ and $E_b/N_0 = 10\text{dB}$, then

$$\begin{pmatrix} 11.671 & 1.003 & 5.5 \cdot 10^{-6} \\ 1.003 & 11.671 & 1.003 \\ 5.5 \cdot 10^{-6} & 1.003 & 11.671 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 1.0 \\ 0 \end{pmatrix}$$

- Finally, $\mathbf{b}_{\mathrm{MMSE}} = \begin{pmatrix} 0.0116 & 0.8622 & -0.0741 \end{pmatrix}^T$
- Note that the problem has been solved semi-analitically, because we assumed a given channel.
- In real applications, the correlations are calculated numerically, without prior assumptions on the channel shape.

Linear equalizers

- The overall impulse response $h_0[n] = h[n] * h_{eq}[n]$ is:

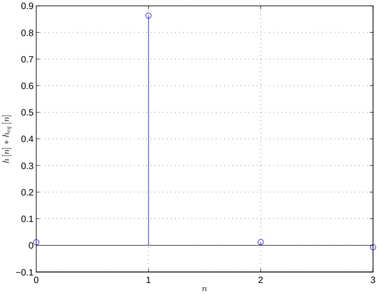

Figure 25: Overall impulse response.

- Observe how now we do not force zeros.

Linear equalizers

- Results in spectrum:

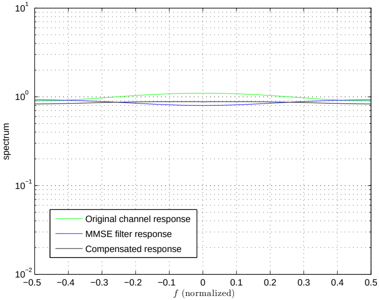

Figure 26: Overall impulse response.

- Observe how now high frequency noise is not enhanced as in ZFE.

Nonlinear equalizers

Nonlinear equalizers

- Linear equalizers are simple to implement, but they have severe limitations in wireless channels.
  - Linear equalizers are not good in compensating for the appearance of spectral zeros.
- The *decision feedback equalizer* (DFE) can help in counteracting these effects.

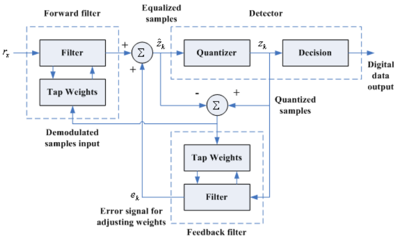

Figure 27: Structure of a DFE (source: http://cnx.org/content).

Nonlinear equalizers

- The DFE works by first estimating the ISI indirectly, in the feedback path.
- The ISI affected signal is reconstructed by using previously decided symbols, and the result is subtracted from the output of the feedforward part of the equalizer.
  - This feedforward part is responsible for compensating the remaining ISI.
- The estimated error is used to calculate the forward filter and the feedback filter coefficients.
  - They can be jointly estimated with a minimum mean square error strategy.
- Advantages
  - The system works better in presence of spectral nulls, because the channel is not inverted.
  - It is inherently adaptive.
- Drawbacks
  - When a symbol is incorrectly decided, this error propagates during some symbol periods, depending on the memory of the system.

Nonlinear equalizers

- A *maximum likelihood sequence estimation* (MLSE) equalizer works over different principles.
  - The channel is considered as a system with memory described by a *trellis*.
  - The state of the system is built by considering the number of successive symbols matching the length of the channel response.
  - The symbols at the input of the channel drive the transitions.

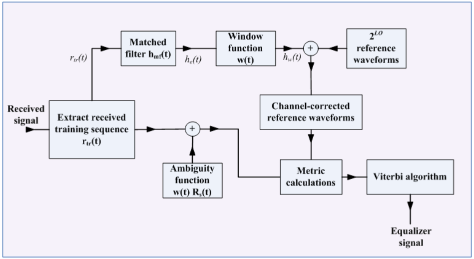

Figure 28: MLSE equalizer structure for GSM (source: http://cnx.org/content).

Nonlinear equalizers

- The MLSE equalizer requires training sequences, as in the case of the DFE or the MMSE.
- It has to build metrics that indicate the likelihood of a possible transition over the trellis.
- The optimal algorithm to implement the MLSE criterion is the *Viterbi algorithm*.
- Advantages
  - It is optimal from the sequence estimation point of view.
  - It can provide the lowest frame error rate.
- Disadvantages
  - It is difficult to implement.
  - It becomes quickly unfeasible when the channel memory grows.
  - The sequence has to be buffered in blocks, adding delay to the operation of the system.

Noninear equalizers

- Equalizers comparative example: BER results drawn with the help of the MATLAB(R) *eqberdemo.m* demo.

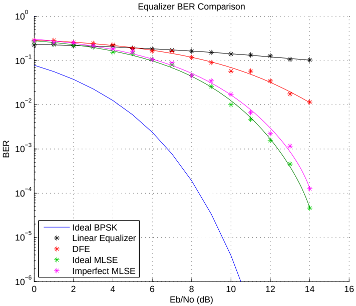

Figure 29: BER comparison for different linear and nonlinear equalizers.

- Observe the important differences in performance.

Conclusions

Conclusions

- Equalization is a process implemented at the RX that is mandatory for almost any modern digital communication system.
- There is a variety of equalization strategies and possible implementations.
  - Adaptive, blind, linear, and so on.
- It is not a closed field, and it is subject to ongoing research and improvements.
- There is not an all-powerful equalization technique, it all depends on the kind of channel, HW availability, target performance, tolerable delay, and so on.
- Equalizers are not left alone: FEC sytems can also contribute to the compensation of residual ISI effects.