

Fault Analysis of Symmetrical Faults

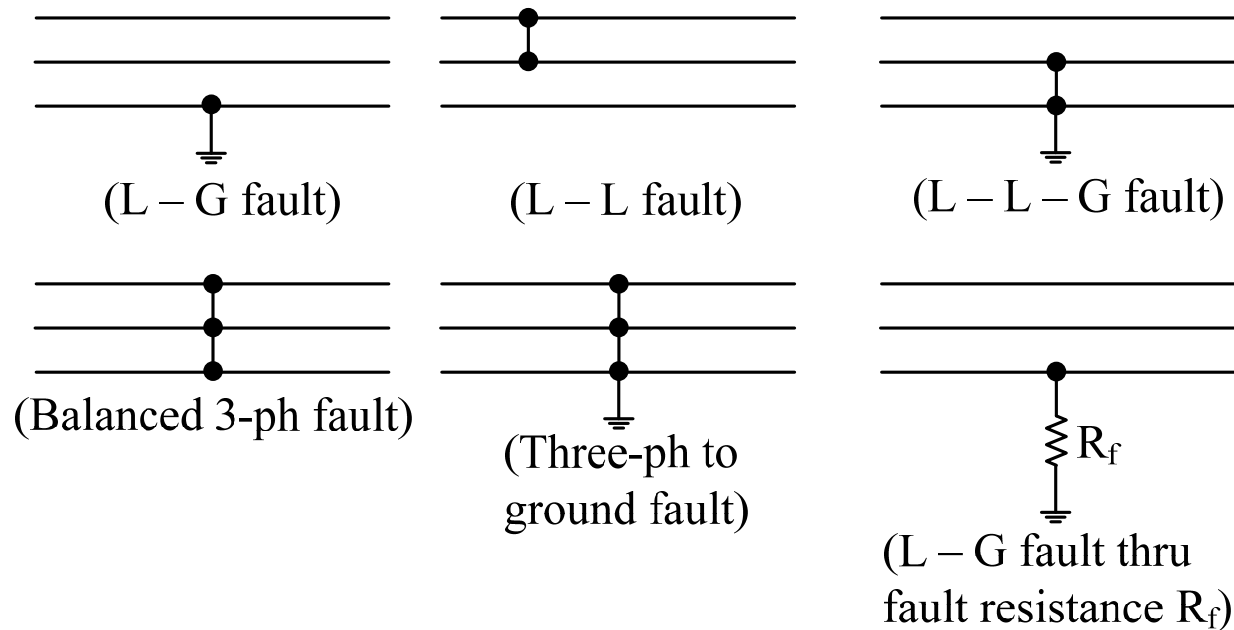
- Under normal conditions, a power system operates as a balanced 3- ϕ system.
- A significant departure from this condition is often caused by a fault (abnormal condition), which may occur due to
 - * lightning strikes
 - * high winds
 - * snow
 - * ice, frost, etc

They cause

- * overload
 - * short-circuit
 - * over-voltage
 - * insulation failure
 - * undervoltage
- Faults give rise to abnormal conditions in the form of excessive currents & voltages at certain points on the system.
 - Protective equipment is used on the system to guard against such faults. e.g. magnitudes of fault currents determine the interrupting capacity of the circuit breakers and settings of protective relays.

- Fault may occur within a generator or at the terminals of a transformer, or on transmission lines/cables.
 - Broadly speaking, two types of faults
 - 1) Three-phase symmetrical faults (network symmetry is maintained under fault conditions; single line representation very useful)
 - least common but most severe fault (which we shall study)
 - 2) Unsymmetrical faults
 - line-to-ground fault (L-G) ← *most common*
 - line-to-line fault (L-L)
 - line-to-line-to-ground fault (L-L-G)
- * *Change in network symmetry \Rightarrow single line representation not enough*





Practical Examples of Faults

- Long term heating & mech. stress on insulation in generators, motors & transformers may result in insulation deterioration, or eventually in breakdown of insulation!
- Insulator string on a T. line tower may fail mechanically due to ice/wind, and the conductor may touch the tower!
- Underwater power cable can get damaged by ships!
- Industrial motor bearings get overheated and jammed, causing excessive current flow & damage to motor windings

Symmetrical Fault Analysis – Methods

1. Z_{Bus} matrix method
2. Thevenin's method

Main Assumptions :

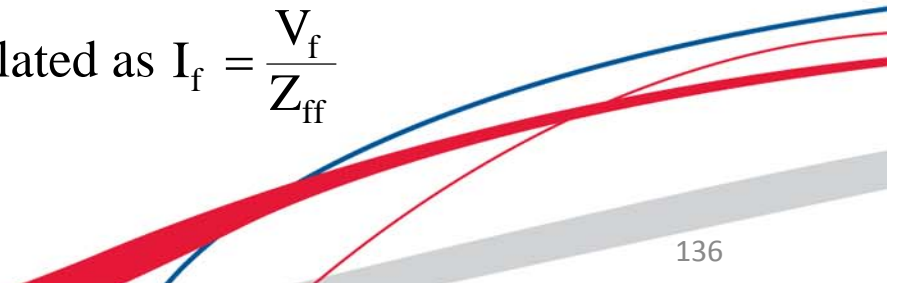
- Magnetizing currents & core losses are neglected for transformers.
- Shunt admittances neglected for lines.
- Component resistances are neglected.
- All pre-fault currents are neglected. Thus, all internal EMFs of sources are 1.0 per unit.

1. Z_{Bus} Matrix Method : [Details are provided in Appendix D.]

Step 1 : Form the bus admittance (Y_{Bus}) matrix.

Step 2 : $[Z_{\text{Bus}}] = [Y_{\text{Bus}}]^{-1}$

Step 3 : Fault current (I_f) at bus f can be calculated as $I_f = \frac{V_f}{Z_{ff}}$



where : V_f = pre-fault voltage at bus f (usually 1.0 pu)

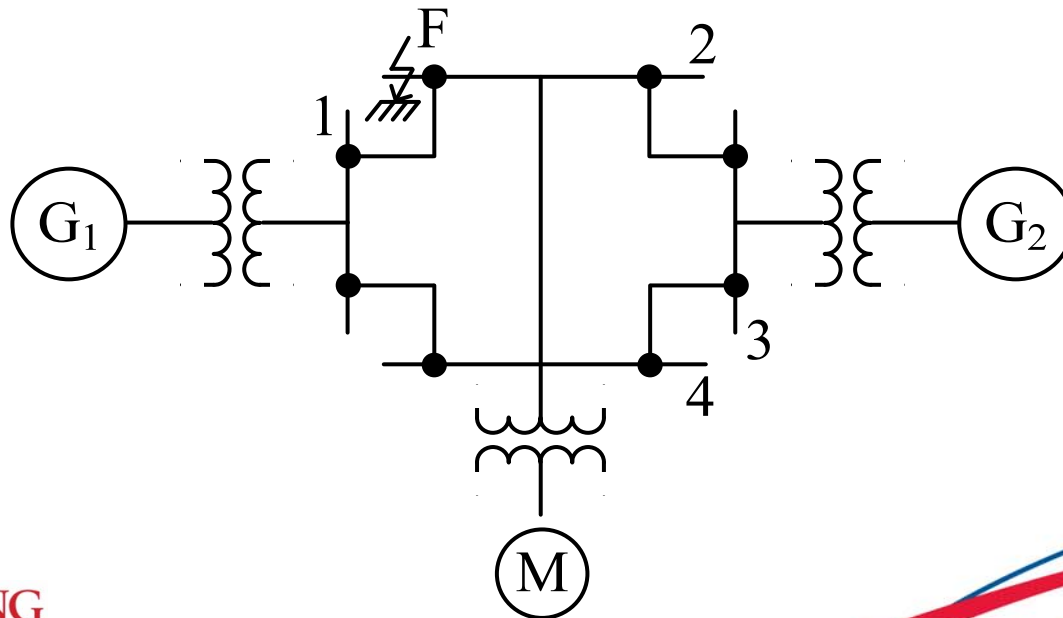
& Z_{ff} = driving-point impedance of the fault bus f (from Z_{Bus})

Step 4 : Post-fault voltages V_n ($n = 1, 2, \dots$ no. of buses) can be evaluated.

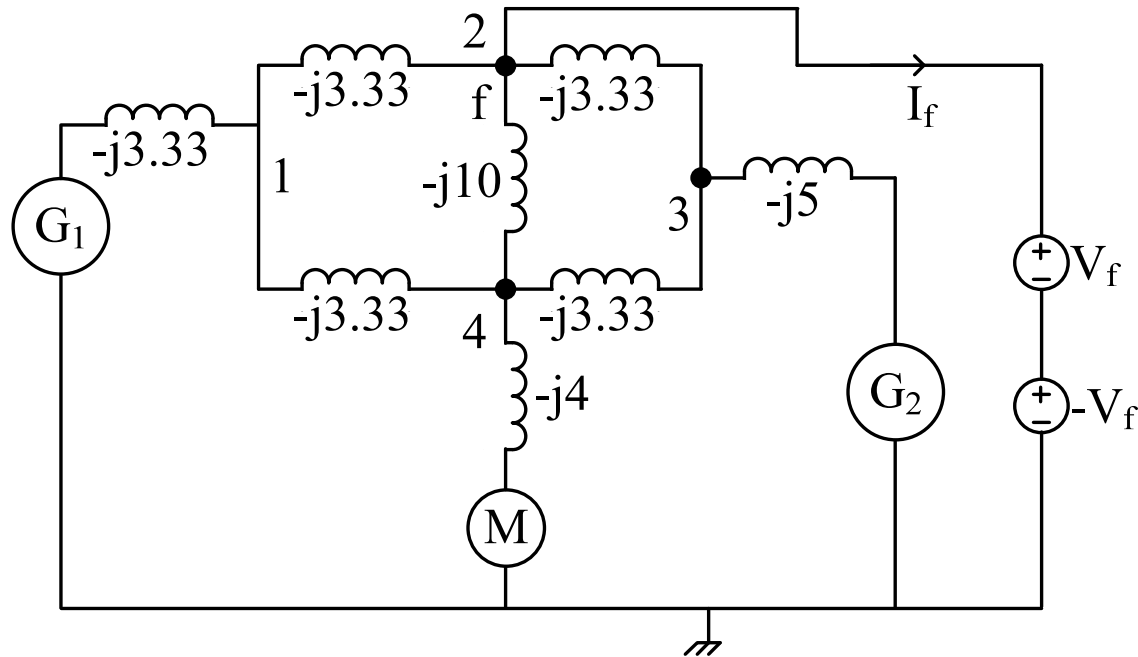
$$V_n = V_f \left[1 - \frac{Z_{nf}}{Z_{ff}} \right] \text{ where } f \text{ is the faulted bus}$$

Example 14

Consider a 3- ϕ symmetrical fault at bus 2 of the power system shown below. Find
(i) Y-matrix (ii) fault currents in all segments.



Solution



(Admittance values given in p.u.)

Y-matrix Eqn.

$$\Rightarrow [Z] = [Y]^{-1} =$$

①	②	③	④		①	②	③	④	
①	$\begin{bmatrix} 0 \\ -I_f \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} ① \\ ② \\ ③ \\ ④ \end{bmatrix}$	$\begin{bmatrix} -10 & 3.33 & 0 & 3.33 \\ 3.33 & -16.67 & 3.33 & 10 \\ 0 & 3.33 & -11.67 & 3.33 \\ 3.33 & 10 & 3.33 & -20.67 \end{bmatrix}$	$\begin{bmatrix} \Delta V_1 \\ -V_f \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix}$	j	①	②	③	④
						$\begin{bmatrix} 0.1502 & 0.0807 & 0.0431 & 0.0702 \\ 0.0807 & 0.1471 & 0.0692 & 0.0953 \\ 0.0431 & 0.0692 & 0.1226 & 0.0602 \\ 0.0702 & 0.0953 & 0.0602 & 0.1155 \end{bmatrix}$			

[Z] will be provided to you for more than 2-bus systems

$$Y_{11} = y_{11} + y_{12} + y_{13} + \dots + y_{1n} = -j3.33 - j3.33 + 0 + (-j3.33) = -j10$$

$$Y_{12} = -y_{12} = -(-j3.33) = +j3.33$$

$$Y_{13} = -y_{13} = 0$$

$$Y_{14} = -y_{14} = -(-j3.33) = +j3.33$$

$$Y_{21} = Y_{12}$$

$$Y_{31} = Y_{13}$$

$$Y_{41} = Y_{14}$$

$$Y_{22} = y_{22} + y_{21} + y_{23} + y_{24} = 0 + (-j3.33) + (-j3.33) + (-j10) = -j16.6667$$

$$Y_{23} = -y_{23} = -(-j3.33) = +j3.33 = Y_{32}$$

$$Y_{24} = -y_{24} = -(-j10) = +j10 = Y_{42}$$

$$Y_{33} = y_{31} + y_{32} + y_{33} + y_{34} = 0 - j3.33 - j5 - j3.33 = -j11.6667$$

$$Y_{43} = Y_{34} = -(y_{34}) = -(-j3.33) = +j3.33$$

$$Y_{44} = y_{41} + y_{42} + y_{43} + y_{44} = -j3.33 + (-j10) + (-j3.33) + (-j4) = -j20.6667$$

$$I_f \text{ (Bus 2)} = \frac{V_f}{Z_{ff}} = \frac{1.0}{j0.1471} = -j6.80 \text{ p.u.}$$

(assuming pre-fault voltage of 1 p.u.)

$$\Rightarrow V_n = V_f \left[1 - \frac{Z_{nf}}{Z_{ff}} \right]$$

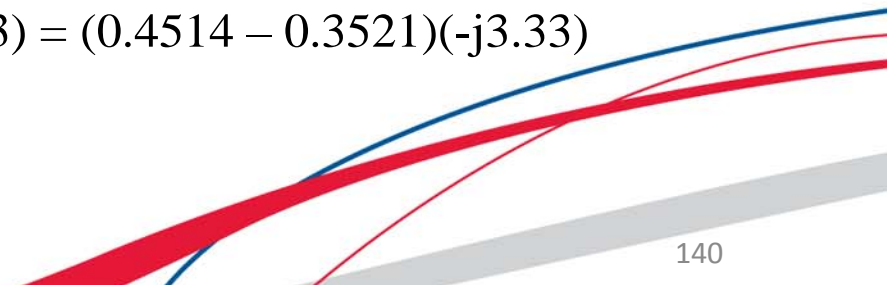
$$\therefore V_1 = 1.0 \left[1 - \frac{Z_{12}}{Z_{22}} \right] = 1.0 \left[1 - \frac{j0.0807}{j0.1471} \right] = 0.4514 \text{ p.u.}$$

$$V_3 = 1.0 \left[1 - \frac{Z_{32}}{Z_{22}} \right] = 1.0 \left[1 - \frac{j0.0692}{j0.1471} \right] = 0.5296 \text{ p.u.}$$

$$V_4 = 1.0 \left[1 - \frac{Z_{42}}{Z_{22}} \right] = 1.0 \left[1 - \frac{j0.0953}{j0.1471} \right] = 0.3521 \text{ p.u.}$$

Fault Currents

$$I_{14} = \text{current from node 1 to 4} = (V_1 - V_4)(-j3.33) = (0.4514 - 0.3521)(-j3.33) \\ = -j0.3307 \text{ p.u.}$$



$$\begin{aligned} I_{12} &= (V_1 - V_2)(-j3.33) = (0.4514 - 0)(-j3.33) \\ &= -j1.5032 \text{ p.u.} \end{aligned}$$

etc. etc.

Current fed by G_1

$$\begin{aligned} &= (E_{G1} - V_1)(-j3.33) = (1.0 - 0.4514)(-j3.33) \\ &= -j1.828 \text{ p.u.} \end{aligned}$$

etc. etc.

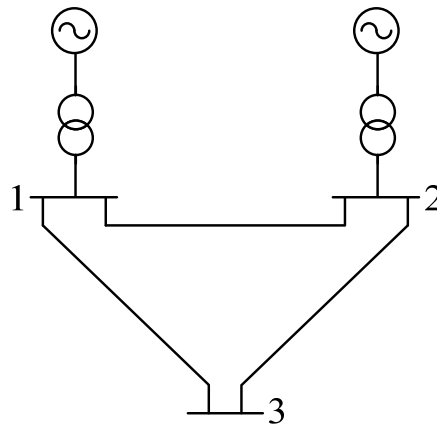
Fault MVA (Fault Level)

$$\begin{aligned} S_f &= V_f I_f^* \Rightarrow S_f = I_f^* \text{ in per unit} \\ &= j6.80 \text{ p.u.} \end{aligned}$$

Exercise 22

The three bus system shown in the figure below has 100 MVA and 200 MVA generators feeding power, via step-up transformers, into buses 1 and 2 respectively. The three transmission lines are assumed to be of equal impedance, $j0.20$ pu on a 100 MVA base. Generator reactance plus transformer reactance equals $j0.30$ pu based upon their respective ratings. All three pre-fault bus voltages are assumed to have 1 pu magnitude.





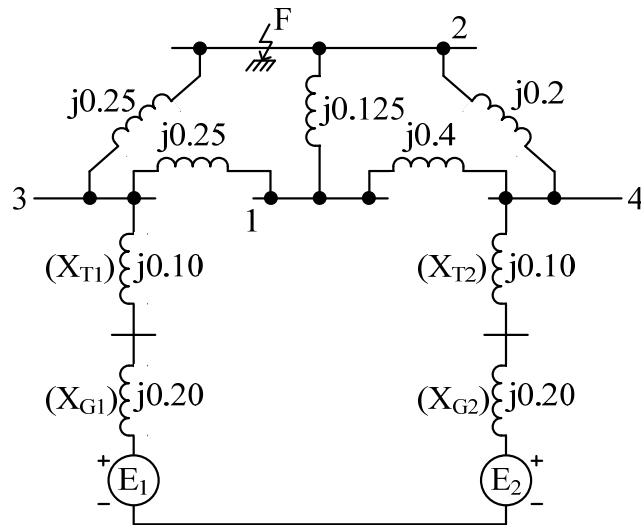
- (a) Find the bus admittance matrix assuming a 50 MVA base.
- (b) Calculate the short-circuit line currents and bus voltages following a solid symmetrical fault at bus 3. Given the Z_{bus} matrix is :

$$j \begin{bmatrix} 0.0730 & 0.0386 & 0.0558 \\ 0.0386 & 0.0558 & 0.0472 \\ 0.0558 & 0.0472 & 0.1014 \end{bmatrix}$$

- (a) $Y_{11} = -j26.67, Y_{22} = -j33.33, Y_{33} = -j20$, off diagonal elements = $j10$
- (b) $I_{12} = 0.85, I_{23} = 5.35, I_{31} = 4.50$
 $V_1 = 0.45, V_2 = 0.535, V_3 = 0$



Exercise 23



Reactance diagram (All values of impedances in p.u.)

$$Z_{\text{Bus}} = j \begin{bmatrix} 0.2436 & 0.1938 & 0.1544 & 0.1456 \\ 0.1938 & 0.2295 & 0.1494 & 0.1506 \\ 0.1544 & 0.1494 & 0.1954 & 0.1046 \\ 0.1456 & 0.1506 & 0.1046 & 0.1954 \end{bmatrix}$$

- (i) Find the Y-matrix.
- (ii) Given that

Find the voltages at all buses with a fault at bus 2. Also find all currents.

$$\text{Ans (i) } Y = \begin{bmatrix} -j14.5 & j8 & j4 & j2.5 \\ j8 & -j17 & j4 & j5 \\ j4 & j4 & -j11.33 & 0 \\ j2.5 & j5 & 0 & -j10.83 \end{bmatrix}$$

- (ii) $V_1 = 0.1556 \text{ pu}$, $V_3 = 0.349 \text{ p.u.}$, $V_4 = 0.3438 \text{ pu}$
 $I_{31} = -j0.7736 \text{ p.u. etc.}$
 $I_{12} = -j1.245 \text{ p.u.}$
 $I_{32} = -j1.396 \text{ p.u.}$
 $I_{42} = -j1.719 \text{ p.u.}$



Exercise 24

The single-line diagram of an unloaded three-phase power system is shown in Figure 1. The ratings of the components are as follows :

Generator G_1 : 20 MVA, 18 kV, $X = 20\%$

Generator G_2 : 20 MVA, 18 kV, $X = 20\%$

Generator G_3 : 30 MVA, 13.8 kV, $X = 20\%$

Transformers T_1, T_2, T_3, T_4 : 20 MVA, 138 Y/20 Y kV, $X = 10\%$

Transformers T_5, T_6 : 15 MVA, 138 Y/13.8 Δ kV, $X = 10\%$

Line L_1 : j40 Ω per phase

Line L_2 : j20 Ω per phase

Line L_3 : j20 Ω per phase

- (a) Draw the impedance diagram for the power system. Mark all impedances in per unit. Use a base of 50 MVA, 138 kV in the circuit of line L_1 .

$$\underline{X_{G1}} = j0.405, \underline{X_{G2}} = j0.405, \underline{X_{G3}} = j0.333$$

$$\underline{X_{T1}} = \underline{X_{T2}} = \underline{X_{T3}} = \underline{X_{T4}} = j0.25$$

$$\underline{X_{T5}} = \underline{X_{T6}} = j0.333$$

$$\underline{X_{L1}} = j0.105, \underline{X_{L2}} = \underline{X_{L3}} = j0.0525$$

- (b) Assume that the system is unloaded and that the voltage throughout the system is 1.0 pu on bases selected in part (a) above. Find the magnitude of the short-circuit current if a three-phase symmetrical fault occurs from bus C to ground.

Hint : By symmetry, G_1 & G_2 will supply equal currents, and NO CURRENT will flow between buses A & B, i.e. in Line L_1 hence this line is open-circuited.

- (c) What is the MVA supplied by each generator under the conditions of part (b) above?

$$\underline{I_F} = 10295.87 \text{ A}$$

$$\underline{S_{G1}} = \underline{S_{G2}} = 48.04 \text{ MVA}$$

$$\underline{S_{G3}} = 150 \text{ MVA}$$

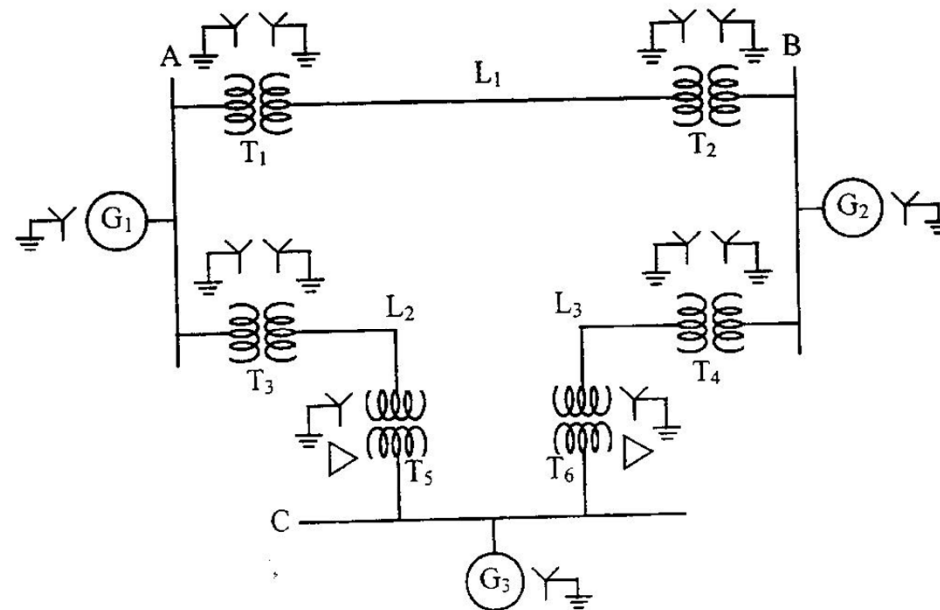


Figure 1

Exercise 25

A three-phase, 765-kV, 60-Hz transposed transmission line is composed of four ACSR conductors per phase with flat horizontal spacing of 14 m. The conductors have a diameter of 3.625 cm and GMR of 1.439 cm. The bundle spacing is 45 cm. The line is 400 km long. Assume that the line is lossless and that it can be modelled by a nominal- π equivalent. Use the per unit system with bases of 765 kV and 2000 MVA. ($\epsilon = 8.85 \times 10^{-12}$ F/m)

The line is energized with 765 kV at the sending end when a three-phase short-circuit occurs at the receiving end. Find the receiving-end and sending-end currents.

3296 A; 2873.74 A

POWER SYSTEMS OPERATION

We are concerned with two things :

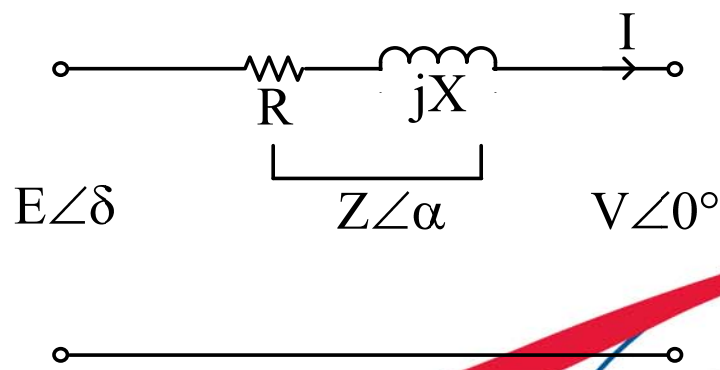
1. Maintaining constant (within tolerable limits) system voltages at different loads.
2. Maintaining constant (within tolerable limits) system frequency – as it affects rotating m/cs, drives & clock-related applications. As stated in synch. m/c studies, variation of freq. is dependent on real power, and we have already looked at this aspect.

⇒ We now concentrate on voltage control.

Recall : Reactive power is less sensitive to changes in freq, and is mainly dependent on changes in voltage magnitude.

Reactive Power & Voltage Control

Recall :



Neglecting R since $X \gg R$, $\Delta V_p = \frac{QX}{V}$

\Rightarrow Control of voltages in a network requires adjusting the MVar flow, and vice versa. If $|E| > |V|$, Q flows from left \rightarrow right & if $|E| < |V|$, Q flows from right \rightarrow left.

- Control of Q flow is complicated by the fact that most power system components may either absorb or generate Q depending on the operating conditions.

(i) Synch. Generator :

- Generates Q when operating at lagging p.f. (overexcited);
- Absorbs Q when operating at leading p.f. (underexcited); and
- Generator is the main source of supplying both positive & negative VARs to the system.

(ii) Overhead T. Lines :

- At low loads, produce Q due to shunt capacitance.



$$\text{i.e. } I^2 X_L < \frac{V^2}{X_C} \quad (X_L = \omega L) \quad (X_C = \frac{1}{\omega C})$$

- At heavy loads, absorb Q due to series reactance,

$$\text{i.e. } I^2 X_L > \frac{V^2}{X_C}$$

(iii) Cables :

- Generate Q due to their high shunt capacitance;
e.g. a 275-kV, 240 MVA, 3- ϕ cable produces 6 to 7 MVAr per km line length.

(iv) Transformers :

- Always absorb Q.

(v) Loads :

- Majority are of lagging p.f. type; hence they absorb Q.
- Some with leading p.f. will generate Q.
- Compensating devices are used to supply or absorb Q, as required.

Methods of Voltage Control

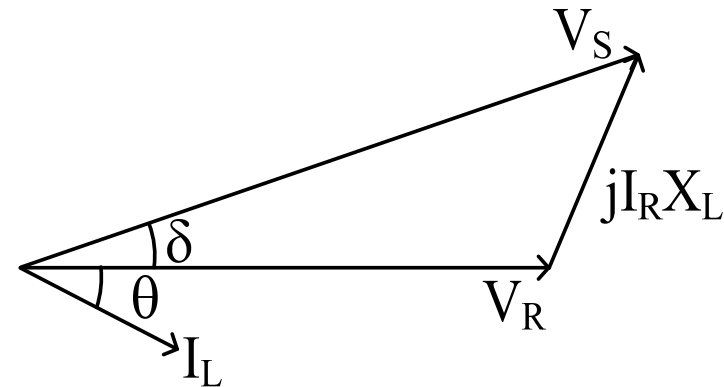
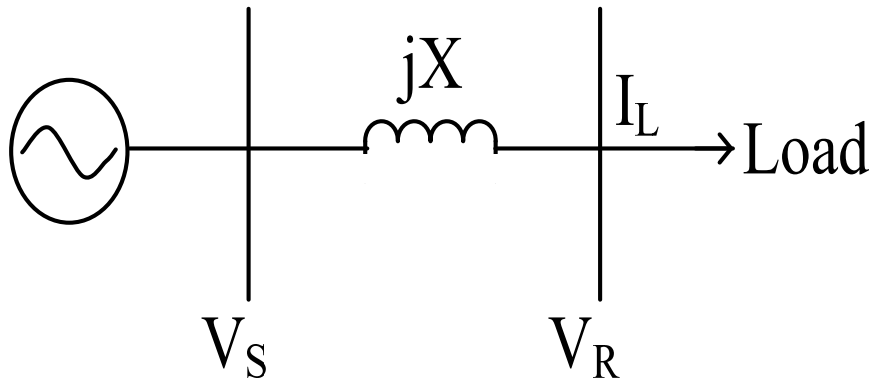
1. Static shunt capacitors & reactors
2. Static series capacitors
3. Synchronous condensers/compensators
4. Static VAr compensators (SVC)
5. Regulating transformers
 - Tap-changing transformers
 - Booster transformers

⇒ We shall study method 1 only!

Shunt Compensation

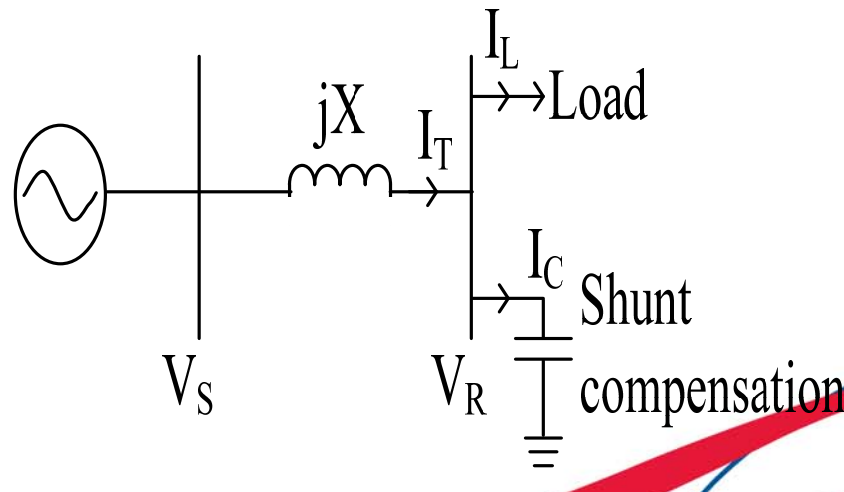
- Shunt capacitors are used for lagging p.f. circuits.
- Reactors are used for leading p.f. circuits (e.g. lightly loaded cables).
- In both cases, the objective is to supply the requisite Q to maintain constant voltage.

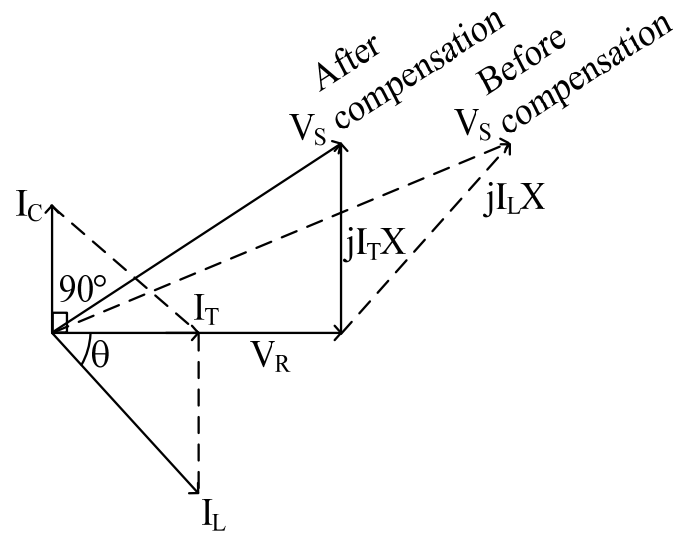
- **For lagging pf loads**



Here $V_S > V_R$ i.e. if $V_R = 1$ pu then $V_S > 1$ pu. And if $V_S = 1$ pu, then $V_R < 1$ pu. Note that for leading p.f., likely $V_S < V_R$.

– Let us add shunt compensation at the load busbars.





- Capacitive compensation reduces the line current from I_L to I_T , and this in turn
 - (i) improves the voltage at receiving end;
 - (ii) reduces Q loss in the T. line from $I_L^2 X$ to $I_T^2 X$; also
 - (iii) reduces P loss in the T. line from $I_L^2 R$ to $I_T^2 R$.
- Compensation is either permanently connected or switched on/off as required.

- Typical problem with this kind of compensation is that voltage control is not smooth.

Example 19

A 275-kV T. line with series impedance of $(0.03 + j0.265) \Omega/\text{km}$ per phase and a length of 140 km supplies a load of 600 MW, 0.9 pf lag. If the sending end voltage is maintained constant at rated value, find the required compensation at the receiving end such that volt. drop along the line is limited to 10%.

Solution

Let $S_b = 600 \text{ MVA}$; $V_b = 275 \text{ kV}$

$$\Rightarrow Z_b = \frac{V_b^2}{S_b} = \frac{275^2}{600} = \underline{126.042 \Omega}$$

$$\begin{aligned} Z_{\text{Line p.u.}} &= (0.03 + j0.265) \Omega/\text{km} \\ &= (4.2 + j37.1) \Omega \end{aligned}$$

$$\begin{aligned} \therefore Z_{\text{Line p.u.}} &= \frac{Z_{\text{Line}} (\Omega)}{Z_b} = \frac{4.2 + j37.1}{126.042} \\ &= \frac{37.3337 \angle 83.54^\circ}{126.042} = 0.296 \angle 83.54^\circ \end{aligned}$$

Given $|V_S| = \text{rated value} = 275 \text{ kV} = 1 \text{ pu}$

Since load is lagging pf, $|V_S| > |V_R|$

For a 10% drop in T. line, $|V_R| = 0.9 \text{ pu}$

$$\begin{aligned}\text{Reference } V_R &= |V_R| \angle 0^\circ \\ &= 0.9 \angle 0^\circ \text{ pu} \\ \Rightarrow V_S &= |V_S| \angle \delta = 1.0 \angle \delta \text{ pu}\end{aligned}$$

$$\begin{aligned}P_{\text{Load pu}} &= \frac{600 \text{ MW}}{S_b} \\ &= 1.0\end{aligned}$$

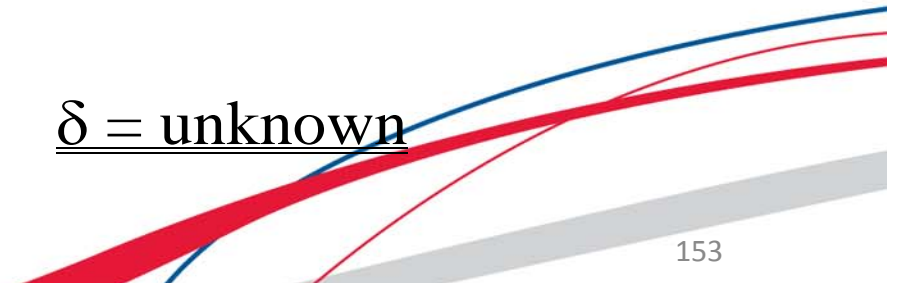
$$\begin{aligned}S_{\text{Load pu}} &= \frac{600}{0.9 \times 600} \angle 25.842^\circ \\ &= 1.111 \angle 25.84^\circ = (1.0 + j0.4843)\end{aligned}$$

$P_{\text{Load}} = 1 \text{ pu}$ (old & new)

$Q_{\text{Load}} = 0.4843 \text{ pu}$ (old)

Recall : Power delivered

$$V = V_R, \quad E = V_S, \quad \alpha = 83.54^\circ \quad \underline{\underline{\delta = \text{unknown}}}$$



$$P = \frac{VE}{Z} \cos(\alpha - \delta) - \frac{V^2}{Z} \cos \alpha$$

$$Q = \frac{VE}{Z} \sin(\alpha - \delta) - \frac{V^2}{Z} \sin \alpha$$

$$\therefore 1.0 = \frac{0.9 \times 1}{0.296} \cos (83.54^\circ - \delta) - \frac{0.9^2}{0.296} \cos 83.54^\circ$$

$$1.0 = 3.04054 \cos (83.54^\circ - \delta) - 0.3079$$

$$\Rightarrow \cos (83.54^\circ - \delta) = 0.4301 \Rightarrow 83.54^\circ - \delta = 64.523^\circ$$

$$\Rightarrow \underline{\delta = 19.02^\circ}$$

$$\therefore Q_{\text{new}} = \frac{0.9 \times 1}{0.296} \sin (83.54^\circ - 19.02) - \frac{0.9^2}{0.296} \sin 83.54^\circ$$

$$= 2.7449 - 2.71911 = 0.025764 \text{ pu}$$

$$= 15.46 \text{ MVAr}$$

$$\therefore Q_{\text{sh}} = (Q_{\text{new}} - Q_{\text{old}}) = 0.025764 - 0.4843 = -0.459 \text{ pu}$$

$$= -275 \text{ MVAr}$$

\therefore Capacitor bank rated for 275 MVAr is required at the receiving end.

Further reading on Faults and Power System Operation

1. Grainger & Stevenson, "Power System Analysis", 2nd Ed., McGraw-Hill, 1994.
2. A. R. Bergen & V. Vittal, "Power Systems Analysis", 2nd Ed., Prentice-Hall, 2000.
3. Weedy & Cory, "Electric Power Systems", 5th Ed., Wiley, 2012.
4. V. Del Toro, "Electric Power Systems", Prentice-Hall, 1992.
5. T. Gonen, "Modern Power System Analysis", Wiley, 1988.

