NANYANG TECHNOLOGICAL UNIVERSITY
School of Electrical & Electronic Engineering

EE/IM4152 Digital Communications

Tutorial No. 11 (Sem 1, AY2016-2017)

1. (a) Golay codes are powerful block codes with parameters $n = 23, k = 12$ and $t = 3$. Find the number of error patterns whose Hamming distance is *i* from the correct codewords for $i = 1,\ 2,\ 3$. Show that the codes satisfy the Hamming bound exactly.
   (b) Binary codes use two symbols (binary 0 and binary 1) to form codewords and *ternary* codes use three symbols to form codewords. There exists a (11, 6) ternary code that can correct up to 2 errors. Verify that this ternary code is a perfect code.
   (c) There exists a (18, 7) binary code. Discuss its possible error-correcting capability using the maximum-likelihood decoding strategy. Can it correct all single-error patterns? Can it correct all double-error patterns, and so on?

2. A single parity-check code can *detect* single errors but does not have any error-correcting capability. A parity-check bit is appended to form a $(k+1, k)$ block code, which makes the Hamming weight of each codeword even.
   (a) Construct the appropriate generator matrix **G** for this code. Determine the parity-check matrix $\mathbf{H}^T$. Show that $\mathbf{G}\mathbf{H}^T = \mathbf{0}$.
   (b) Generate the codewords for $k = 3$. Discuss its error-detecting capability.

3. Consider the following 3 generator matrices $\mathbf{G}_1$, $\mathbf{G}_2$ and $\mathbf{G}_3$. Identify the generator matrices that can generate systematic linear block codes.

$$\mathbf{G}_1 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{G}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}, \quad \mathbf{G}_3 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

   Construct the codes for these 3 generator matrices and find their $d_{\min}$ values. Discuss their error-correcting capabilities.

4. For a (6, 3) *systematic* block code $\mathbf{c} = [c_1\ c_2\ c_3\ d_1\ d_2\ d_3]$, the parity-check digits are

$$c_1 = d_1 \oplus d_2 \oplus d_3$$
$$c_2 = d_1 \oplus d_2$$
$$c_3 = d_1 \oplus d_3$$

   Note that the message block $\mathbf{d} = [d_1\ d_2\ d_3]$ is placed at the end of the codewords.
   (a) Construct the appropriate generator matrix **G** and the parity-check matrix $\mathbf{H}^T$. Show that they are orthogonal.
   (b) List the codewords generated by the generator matrix **G**.
   (c) Prepare the syndrome decoding table for this block code. Discuss its error-correcting capabilities.
   (d) Decode the received words: 101100, 000110 and 101010.