Bonus Problem (Moment generating functions)

For the random variable X, the moments can be expressed as

$$E\left(X^n\right) = \frac{2^n}{n+1},\ n = 1, 2, 3, \ldots$$

Find some (in fact, the unique) distribution of X having these moments.
Hint: Study the moment generating function of X and use the fact that

$$e^{tX} = \sum_{n=0}^{\infty} \frac{t^n X^n}{n!}$$