NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2016/17 Semester 1 MH2500 Probability and Introduction to Statistics Tutorial 6

For the tutorial on 22 September, let us discuss

- Ex. 3.8.3, 7, 9, 13, 22, 23

Ex. 3.8.3. Three players play 10 independent rounds of a game, and each player has probability $\frac{1}{3}$ of winning each round. Find the joint distribution of the number of games won by each of the three players.

Ex. 3.8.7. Find the joint and marginal densities corresponding to the cdf

$$F(x, y) = (1 - e^{-\alpha x})(1 - e^{-\beta y}), \qquad x \geq 0, \qquad y \geq 0, \qquad \alpha > 0, \qquad \beta > 0.$$

Ex. 3.8.9. Suppose that (X, Y) is uniformly distributed over the region defined by $0 \leq y \leq 1 - x^2$ and $-1 \leq x \leq 1$.

a Find the marginal densities of X and Y.

b Find the two conditional densities.

Ex. 3.8.13. A fair coin is thrown once; if it lands heads up, it is thrown a second time. Find the frequency function of the total number of heads.

Ex. 3.8.22. Consider a Poisson process on the real line and denote by $N(t_1, t_2)$ the number of events in the interval (t_1, t_2). If $t_0 < t_1 < t_2$, find the conditional distribution of $N(t_0, t_1)$ given that $N(t_0, t_2) = n$. (Hint: Use the fact that the numbers of events in disjoint subsets are independent.)

Ex. 3.8.23. Suppose that, conditional on N, X has a binomial distribution with N trials and probability p of success, and that N is a binomial random variable with m trials and probability r of success. Find the unconditional distribution of X.