NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2016/17 Semester 1 MH2500 Probability and Introduction to Statistics Tutorial 4

For the tutorial on 8 September, let us discuss

- Ex. 2.5.32, 34, 42, 44, 46, 48, 49, 51.

Ex.2.5.32. Suppose that in a city, the number of suicides can be approximated by a Poisson process with $\lambda = 0.33$ per month.

a. Find the probability of k suicides in a year for $k = 0, 1, 2, \ldots$. What is the most probable number of suicides?
b. What is the probability of two suicides in one week?

Ex.2.5.34. Let $f(x) = (1+\alpha x)/2$ for $-1 \leq x \leq 1$ and $f(x) = 0$ otherwise, where $-1 \leq \alpha \leq 1$. Show that f is a density, and find the corresponding cdf. Find the quartiles and the median of the distribution in terms of α.

Ex.2.5.42. Find the probability density for the distance from an event to its nearest neighbor for a Poisson process in the plane.

Ex.2.5.44. Let T be an exponential random with parameter λ. Let X be a discrete random variable defined as $X = k$ if $k \leq T < k + 1$, $k = 0, 1, \ldots$. Find the frequency function of X.

Ex.2.5.46. Recall the gamma function is defined as

$$\Gamma(x) = \int_0^\infty u^{x-1} e^{-u} du, \qquad x > 0$$

and the gamma density function is given by

$$g(t) = \frac{\lambda^\alpha}{\Gamma(\alpha)} t^{\alpha-1} e^{-\lambda t}, \qquad t > 0,$$

where α and λ are two positive parameters. Show that the gamma density integrates to 1.

Ex.2.5.48. T is an exponential random variable, and $P(T < 1) = 0.05$. What is λ?

Ex. 2.5.51. Show that the normal density integrates to 1. (Hint: First make a change of variables to reduce the integral to that for the standard normal. The problem is then to show that $\int_{-\infty}^{\infty} \exp -x^2/2 dx = \sqrt{2\pi}$. Square both sides and reexpress the problem as that of showing

$$\left(\int_{-\infty}^{\infty} \exp(-x^2/2)dx\right)\left(\int_{-\infty}^{\infty} \exp(-y^2/2)dy\right) = 2\pi.$$

Write the product of integrals as a double integral and change to polar coordinates. You might not have learnt these yet, so just assume the following are true.

$$\begin{aligned}\left(\int_{-\infty}^{\infty} \exp(-x^2/2)dx\right)\left(\int_{-\infty}^{\infty} \exp(-y^2/2)dy\right) &= \iint\limits_{R^2} \exp(-(x^2+y^2)/2)dxdy \\ &= \int_0^{\infty}\int_0^{2\pi} e^{-r^2/2} r d\theta dr \\ &= \int_0^{\infty}\int_0^{2\pi} e^{-r^2/2} r \ d\theta dr.\end{aligned}$$

Integrate to show that it is equal to 2π.)