Lecture 2

Linear Regression

Sung Kim <hunkim+ml@gmail.com>

Acknowledgement

- Andrew Ng's ML class
  - https://class.coursera.org/ml-003/lecture
  - http://www.holehouse.org/mlclass/ (note)
- Convolutional Neural Networks for Visual Recognition.
  - http://cs231n.github.io/
- Tensorflow
  - https://www.tensorflow.org
  - https://github.com/aymericdamien/TensorFlow-Examples

Predicting exam score: regression

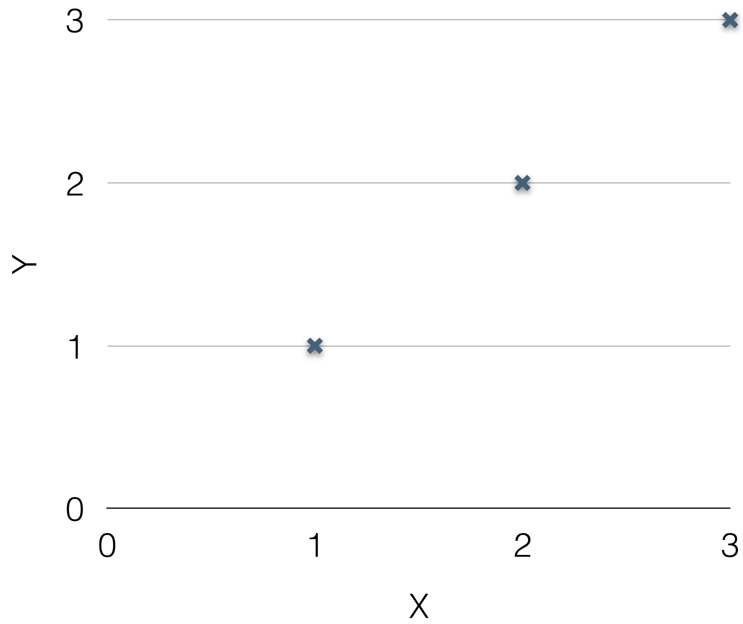
x (hours)	y (score)
10	90
9	80
3	50
2	30

Regression (data)

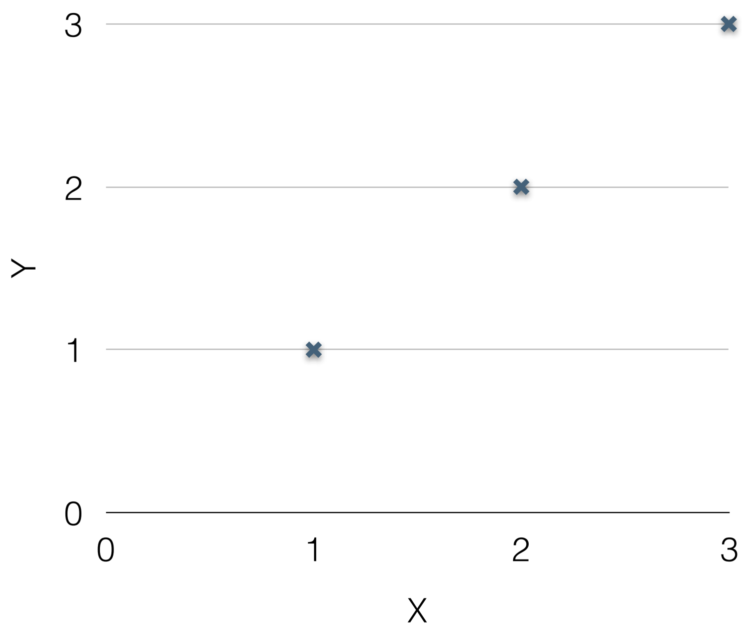
x	y
1	1
2	2
3	3

Regression (presentation)

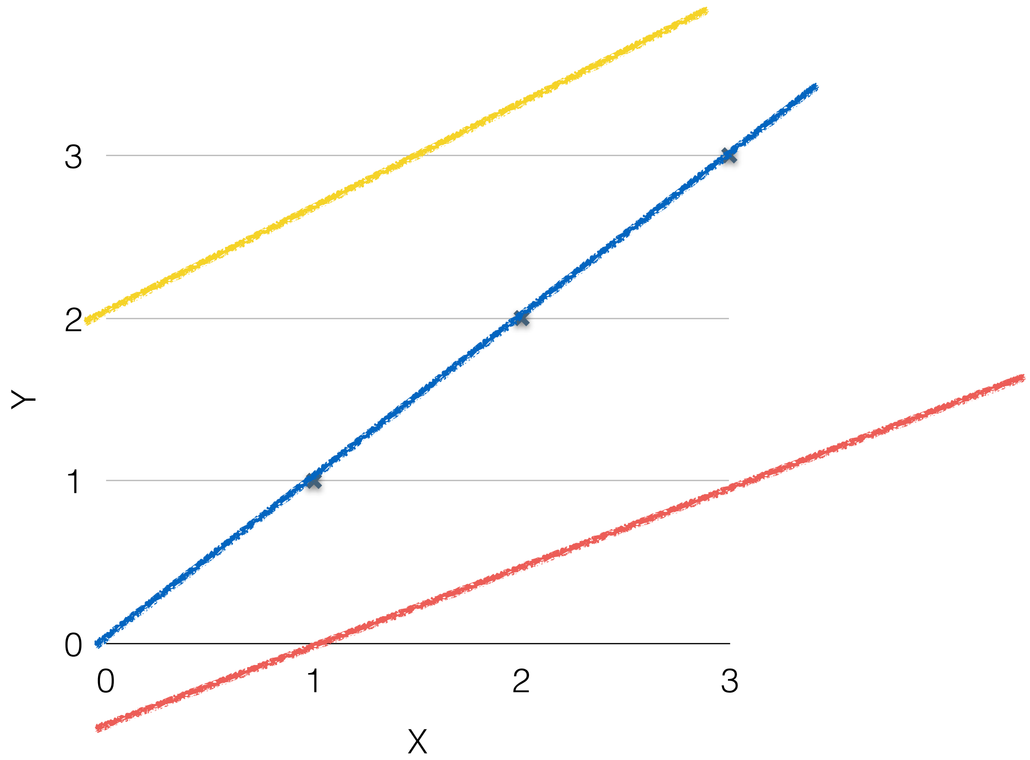
X	Y
1	1
2	2
3	3



(Linear) Hypothesis

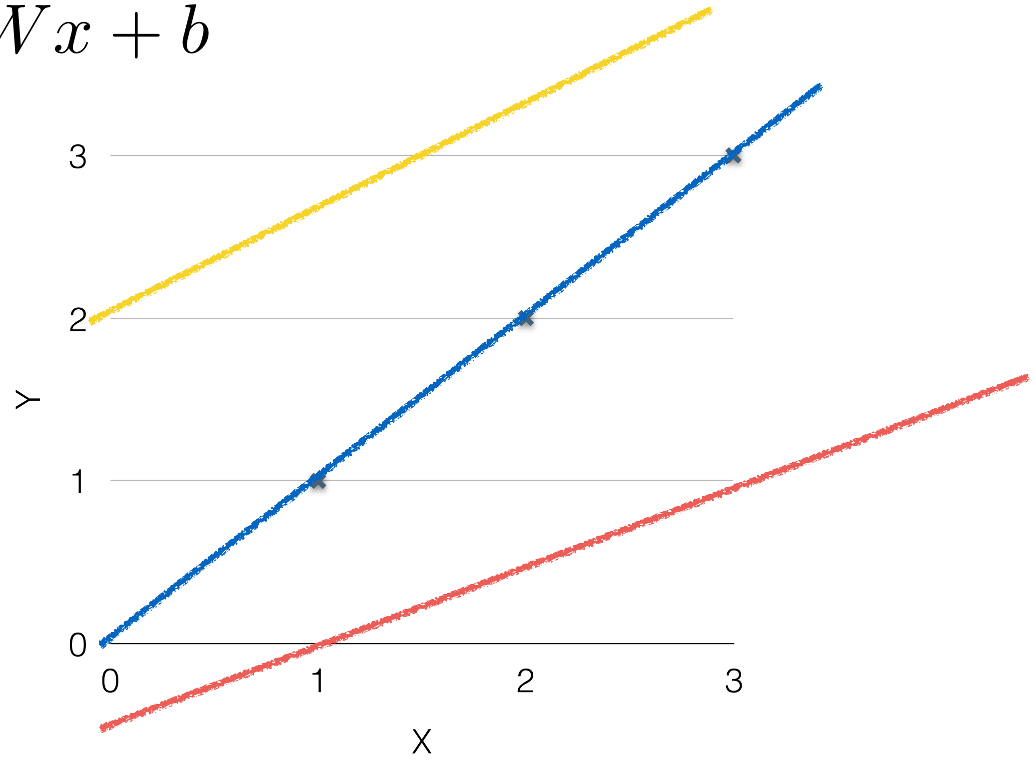


(Linear) Hypothesis

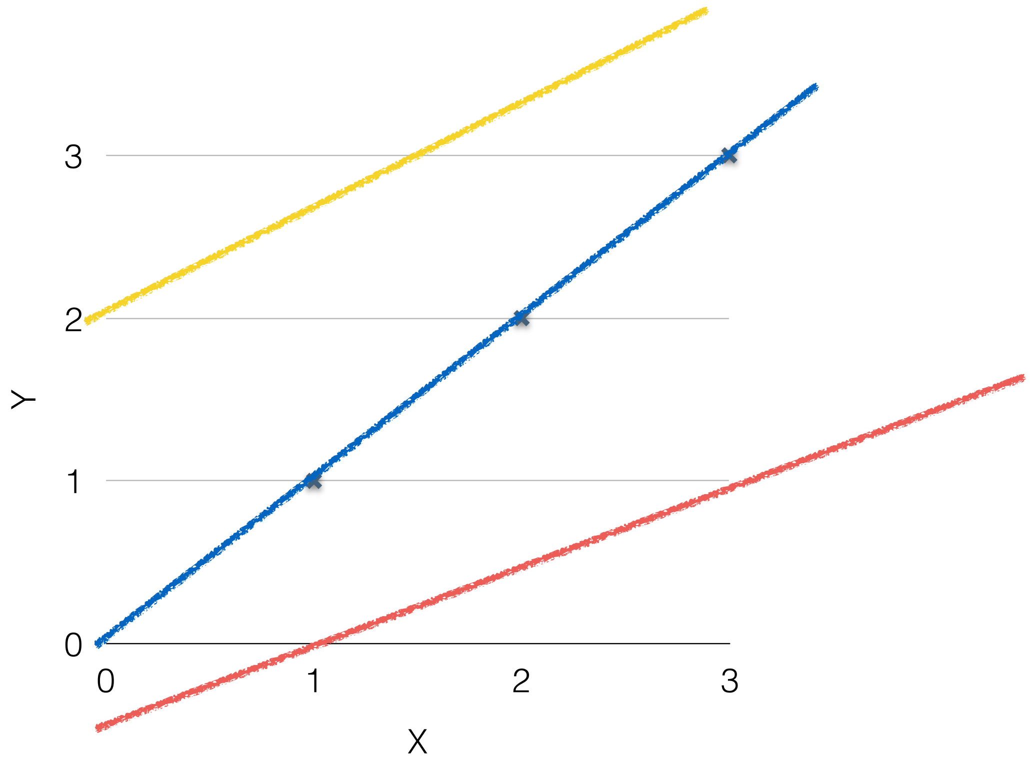


(Linear) Hypothesis

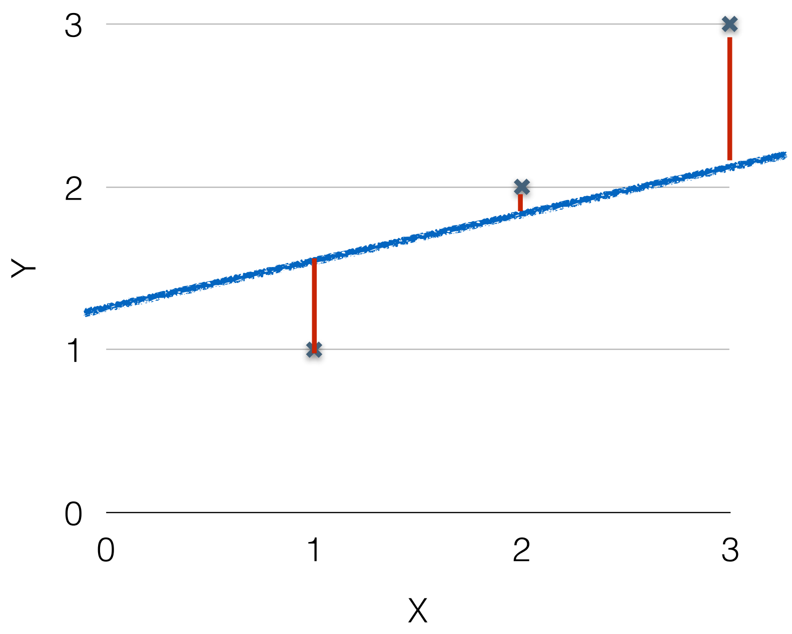
$$H(x) = Wx + b$$



Which hypothesis is better?



Which hypothesis is better?

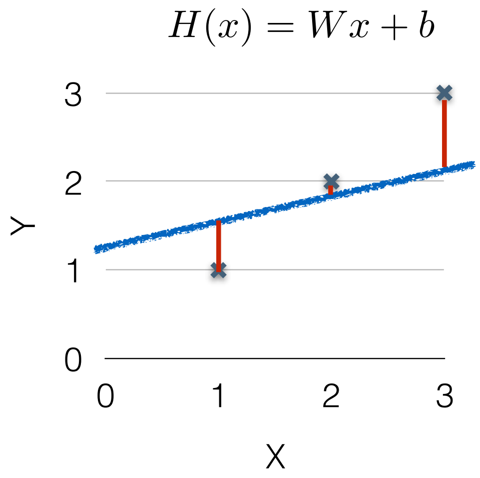


Cost function =loss function

- How fit the line to our (training) data

$$H(x) - y$$

값이 0이 될 수 있음->
절대값 or 제곱을 취함

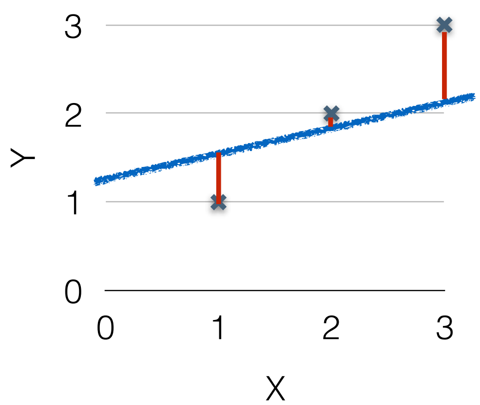


Cost function

- How fit the line to our (training) data

$$H(x) = Wx + b$$

$$\frac{(H(x^{(1)}) - y^{(1)})^2 + (H(x^{(2)}) - y^{(2)})^2 + (H(x^{(3)}) - y^{(3)})^2}{3}$$



$$cost = \frac{1}{m} \sum_{i=1}^{m} (H(x^{(i)}) - y^{(i)})^2$$

Cost function

$$cost = \frac{1}{m}\sum_{i=1}^{m}(H(x^{(i)}) - y^{(i)})^2$$

$$H(x) = Wx + b$$

$$cost(W, b) = \frac{1}{m}\sum_{i=1}^{m}(H(x^{(i)}) - y^{(i)})^2$$

Goal: Minimize cost

$$\underset{W,b}{\text{minimize}}\, cost(W, b)$$

Lecture 3

How to minimize cost

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  - https://github.com/aymericdamien/TensorFlow-Examples

Hypothesis and Cost

$$H(x) = Wx + b$$

$$cost(W, b) = \frac{1}{m} \sum_{i=1}^{m} (H(x^{(i)}) - y^{(i)})^2$$

Simplified hypothesis

$$H(x) = Wx$$

$$cost(W) = \frac{1}{m}\sum_{i=1}^{m}(Wx^{(i)} - y^{(i)})^2$$

What *cost(W)* looks like?

$$cost(W) = \frac{1}{m}\sum_{i=1}^{m}(Wx^{(i)} - y^{(i)})^2$$

X	Y
1	1
2	2
3	3

- *W*=1, *cost(W)*=?

What *cost(W)* looks like?

$$cost(W) = \frac{1}{m} \sum_{i=1}^{m} (Wx^{(i)} - y^{(i)})^2$$

X	Y
1	1
2	2
3	3

- W=1, cost(W)=0

  $$\frac{1}{3}((1 * 1 - 1)^2 + (1 * 2 - 2)^2 + (1 * 3 - 3)^2)$$

- W=0, cost(W)=4.67

  $$\frac{1}{3}((0 * 1 - 1)^2 + (0 * 2 - 2)^2 + (0 * 3 - 3)^2)$$

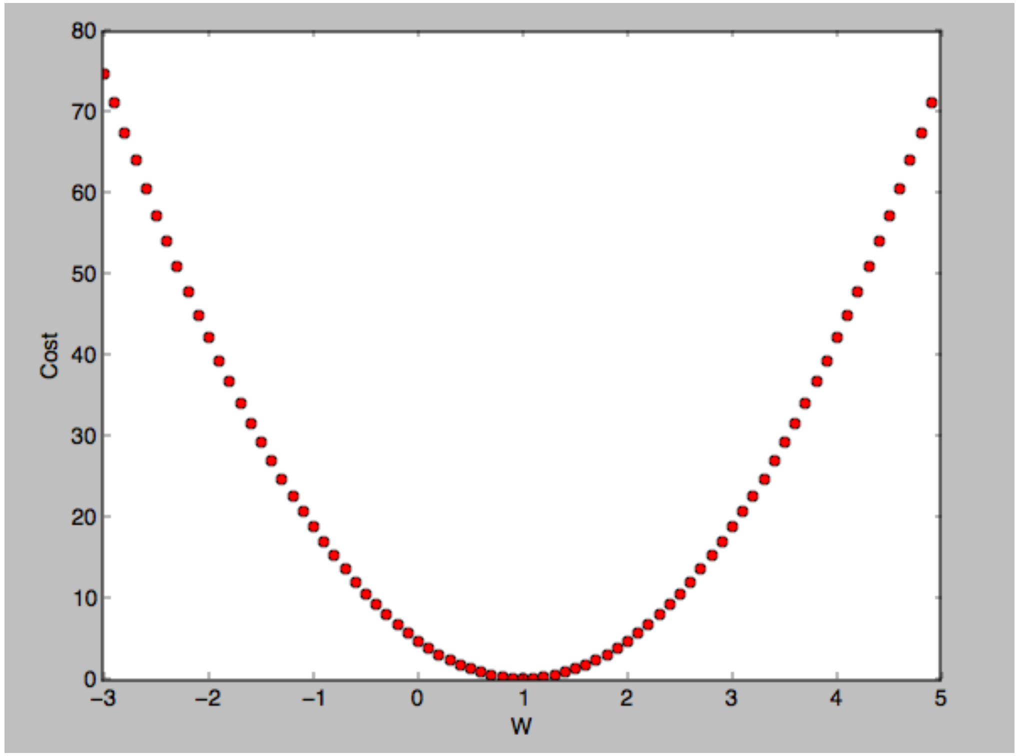
- W=2, cost(W)=?

What *cost(W)* looks like?

- W=1, cost(W)=0
- W=0, cost(W)=4.67
- W=2, cost(W)=4.67

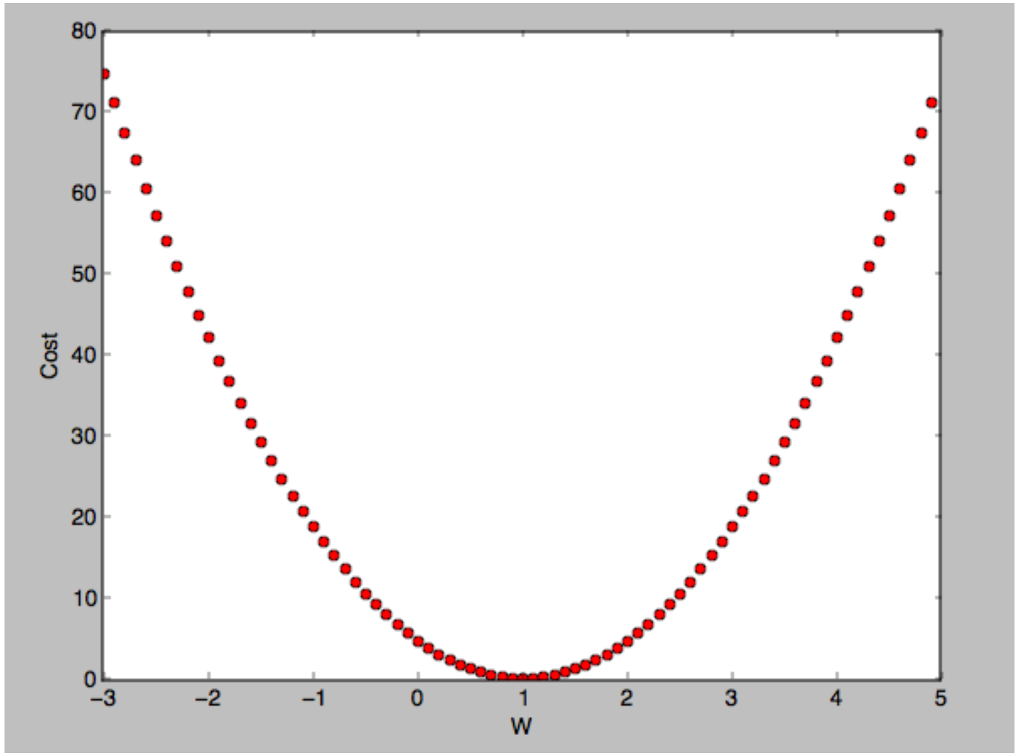
What *cost(W)* looks like?

$$cost(W) = \frac{1}{m} \sum_{i=1}^{m} (Wx^{(i)} - y^{(i)})^2$$



How to minimize cost?

$$cost(W) = \frac{1}{m} \sum_{i=1}^{m} (Wx^{(i)} - y^{(i)})^2$$

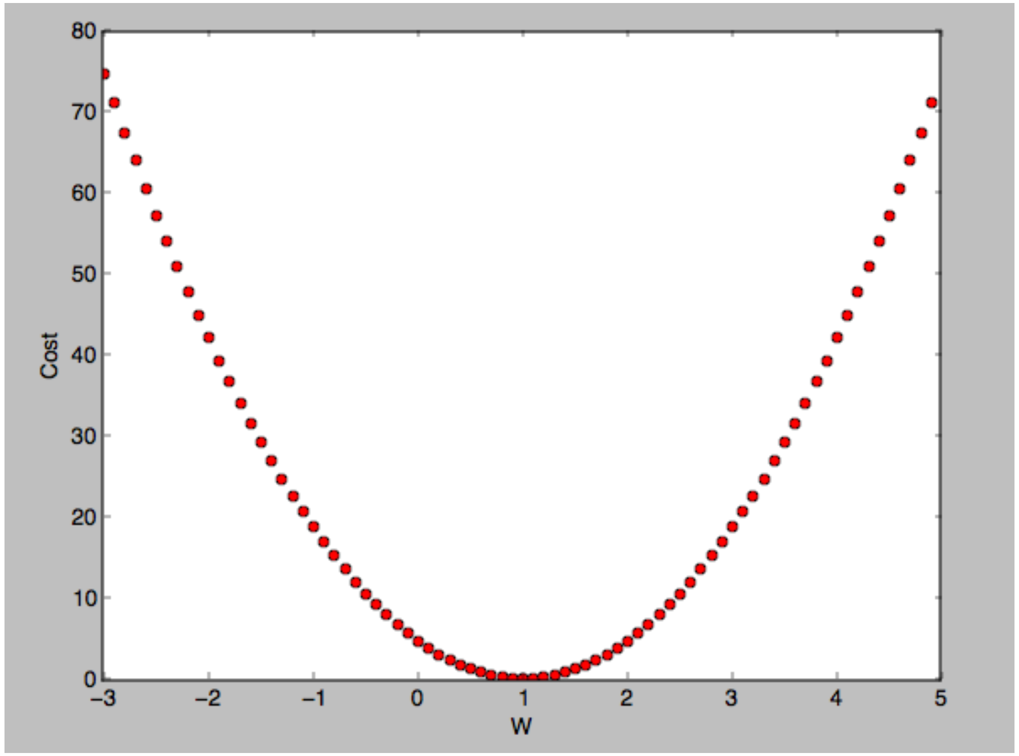


Gradient descent algorithm

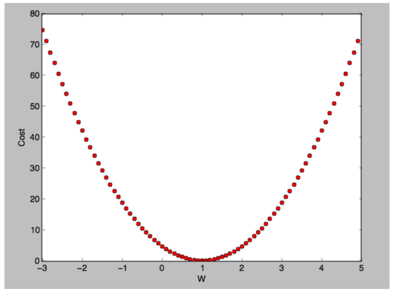
- Minimize cost function
- Gradient descent is used many minimization problems
- For a given cost function, *cost (W, b)*, it will find *W, b* to minimize cost
- It can be applied to more general function: *cost (w1, w2, …)*

How it works?

How would you find the lowest point?



How it works?



- Start with initial guesses
  - Start at 0,0 (or any other value)
  - Keeping changing W and b a little bit to try and reduce cost(W, b)
- Each time you change the parameters, you select the gradient which reduces cost(W, b) the most possible
- Repeat
- Do so until you converge to a local minimum
- Has an interesting property
  - Where you start can determine which minimum you end up

http://www.holehouse.org/mlclass/01_02_Introduction_regression_analysis_and_gr.html

Formal definition

$$cost(W) = \frac{1}{m}\sum_{i=1}^{m}(Wx^{(i)} - y^{(i)})^2$$

$$\Downarrow$$

$$cost(W) = \frac{1}{2m}\sum_{i=1}^{m}(Wx^{(i)} - y^{(i)})^2$$

Formal definition

$$cost(W) = \frac{1}{2m} \sum_{i=1}^{m} (Wx^{(i)} - y^{(i)})^2$$

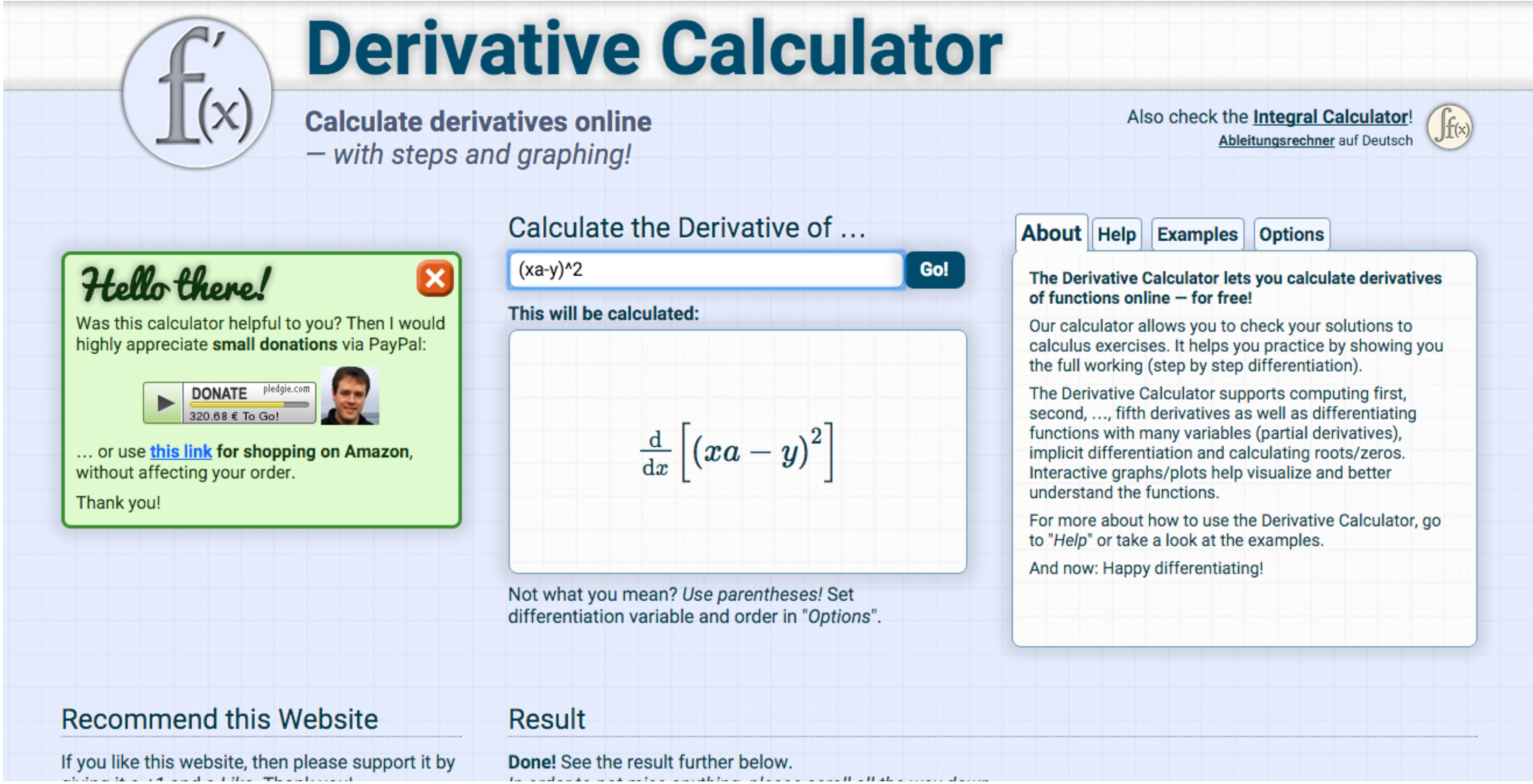
$$W := W - \alpha \frac{\partial}{\partial W} cost(W)$$

Formal definition

$$W := W - \alpha \frac{\partial}{\partial W} \frac{1}{2m} \sum_{i=1}^{m} (Wx^{(i)} - y^{(i)})^2$$

$$W := W - \alpha \frac{1}{2m} \sum_{i=1}^{m} 2(Wx^{(i)} - y^{(i)})x^{(i)}$$

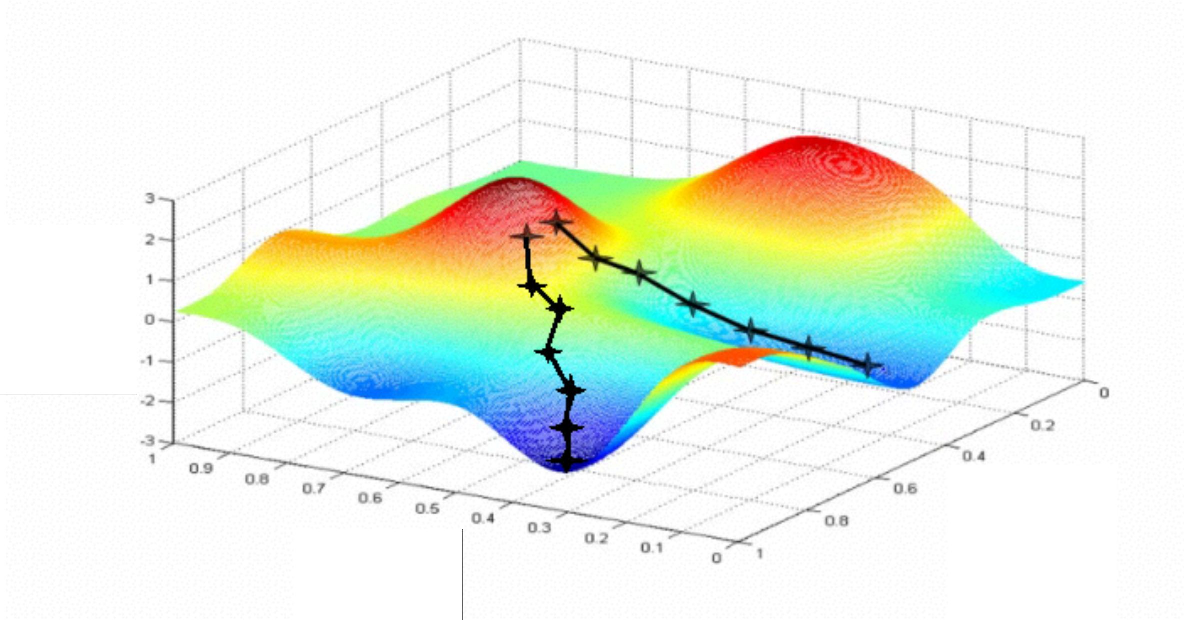
$$W := W - \alpha \frac{1}{m} \sum_{i=1}^{m} (Wx^{(i)} - y^{(i)})x^{(i)}$$


Derivative Calculator
Calculate derivatives online
— with steps and graphing!
Also check the Integral Calculator!
Ableitungsrechner auf Deutsch
Calculate the Derivative of …
(xa-y)^2
Go!
This will be calculated:
$\frac{d}{dx}\left[(xa-y)^2\right]$
Not what you mean? Use parentheses! Set differentiation variable and order in "Options".
Hello there!
Was this calculator helpful to you? Then I would highly appreciate small donations via PayPal:
DONATE pledgie.com
320.68 € To Go!
… or use this link for shopping on Amazon, without affecting your order.
Thank you!
About
Help
Examples
Options
The Derivative Calculator lets you calculate derivatives of functions online — for free!
Our calculator allows you to check your solutions to calculus exercises. It helps you practice by showing you the full working (step by step differentiation).
The Derivative Calculator supports computing first, second, …, fifth derivatives as well as differentiating functions with many variables (partial derivatives), implicit differentiation and calculating roots/zeros. Interactive graphs/plots help visualize and better understand the functions.
For more about how to use the Derivative Calculator, go to "Help" or take a look at the examples.
And now: Happy differentiating!
Recommend this Website
If you like this website, then please support it by
Result
Done! See the result further below.

Gradient descent algorithm

$$W := W - \alpha \frac{1}{m} \sum_{i=1}^{m} (Wx^{(i)} - y^{(i)})x^{(i)}$$

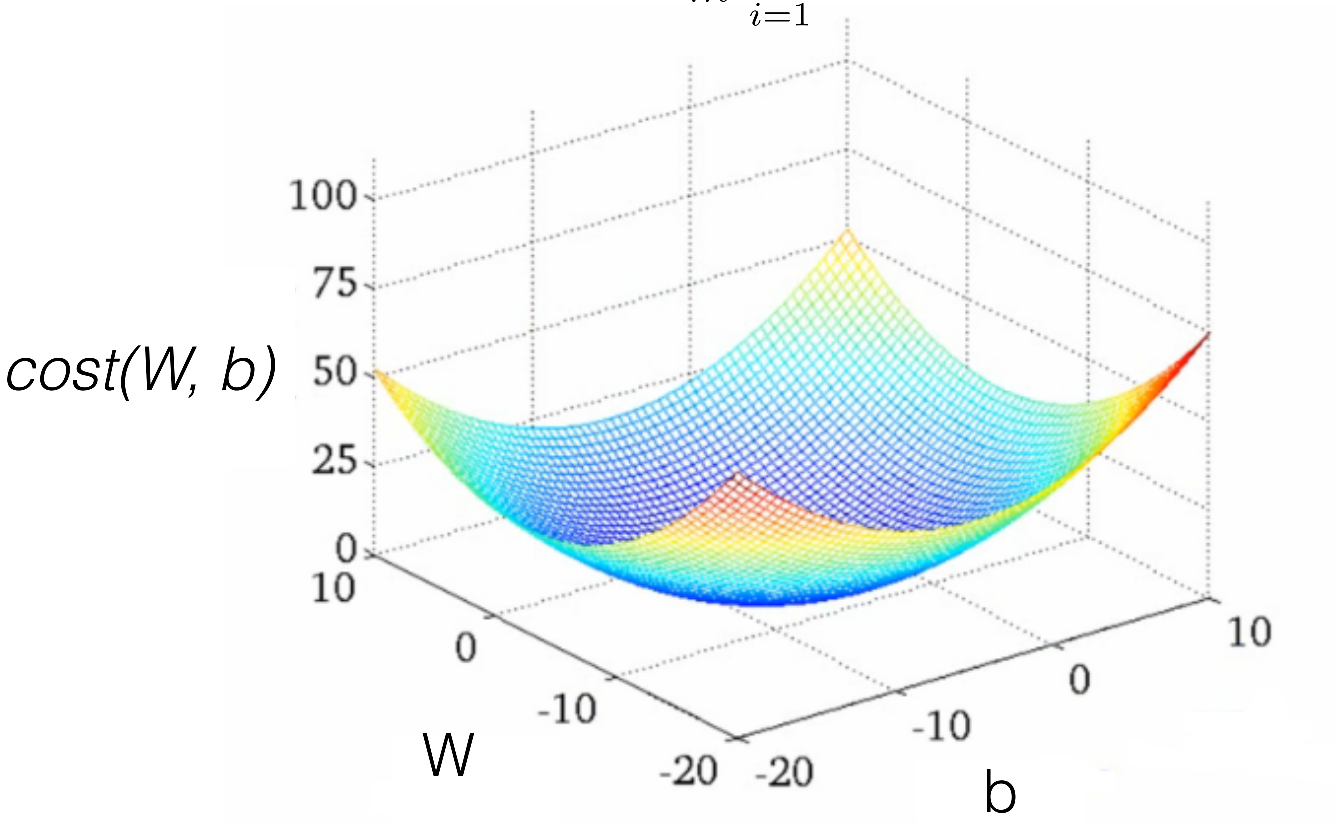
Convex function



www.holehouse.org/mlclass/

Convex function

$$cost(W, b) = \frac{1}{m}\sum_{i=1}^{m}(H(x^{(i)}) - y^{(i)})^2$$



www.holehouse.org/mlclass/


Next
Multivariable logistic
regression

Lecture 4

Multivariable linear regression

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http://hunkim.github.io/ml/

Video (Korean): https://youtu.be/kPxpJY6fRkY

Recap

- Hypothesis
- Cost function
- Gradient descent algorithm

Recap

- Hypothesis

$$H(x) = Wx + b$$

- Cost function

$$cost(W, b) = \frac{1}{m} \sum_{i=1}^{m} (H(x^{(i)}) - y^{(i)})^2$$

- Gradient descent algorithm

Predicting exam score: regression using one input (x)

one-variable
one-feature

x (hours)	y (score)
10	90
9	80
3	50
2	60
11	40

Predicting exam score:
regression using three inputs (x1, x2, x3)

multi-variable/feature

x_1 (quiz 1)	x_2 (quiz 2)	x_3 (midterm 1)	Y (final)
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

Test Scores for General Psychology

Hypothesis

$$H(x) = Wx + b$$

Hypothesis

$$H(x) = Wx + b$$

$$H(x_1, x_2, x_3) = w_1x_1 + w_2x_2 + w_3x_3 + b$$

Cost function

$$H(x_1, x_2, x_3) = w_1x_1 + w_2x_2 + w_3x_3 + b$$

$$cost(W, b) = \frac{1}{m} \sum_{I=1}^{m} (H(x_1^{(i)}, x_2^{(i)}, x_3^{(i)}) - y^{(i)})^2$$

Multi-variable

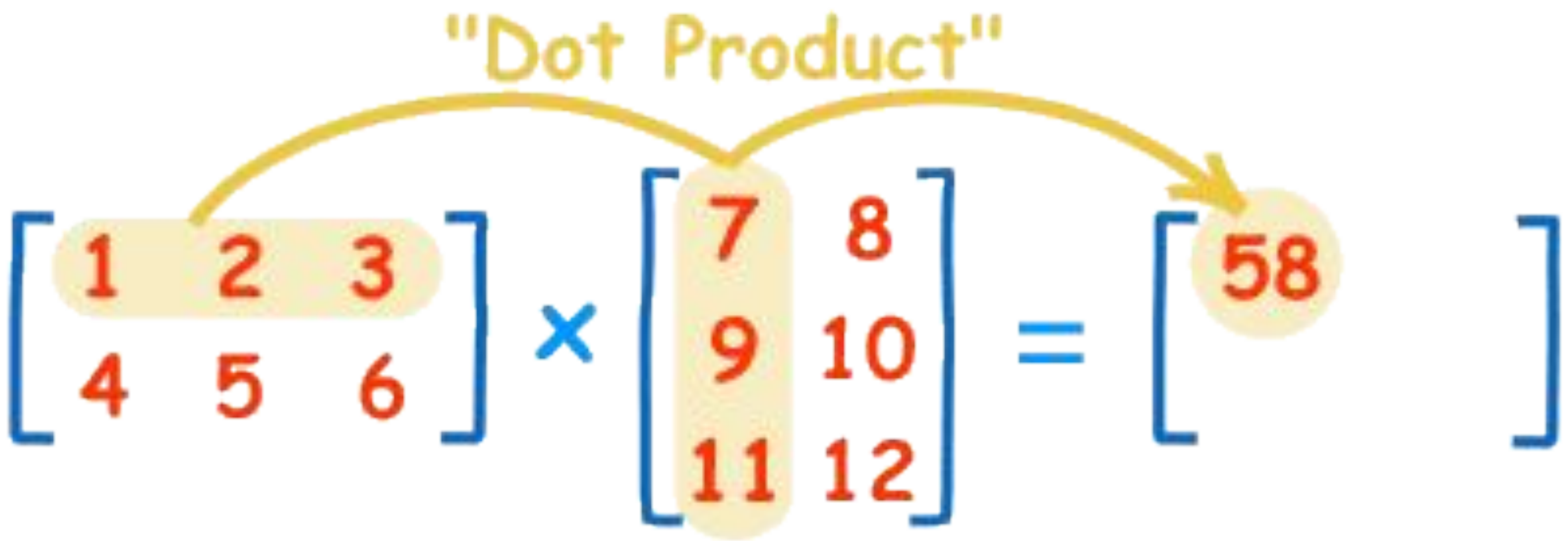
$$H(x_1, x_2, x_3) = w_1x_1 + w_2x_2 + w_3x_3 + b$$

$$H(x_1, x_2, x_3, ..., x_n) = w_1x_1 + w_2x_2 + w_3x_3 + ... + w_nx_n + b$$

Matrix

$$w_1x_1 + w_2x_2 + w_3x_3 + ... + w_nx_n$$

Matrix multiplication



https://www.mathsisfun.com/algebra/matrix-multiplying.html

Hypothesis using matrix

$$w_1x_1 + w_2x_2 + w_3x_3 + ... + w_nx_n$$

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_1w_1 + x_2w_2 + x_3w_3 \end{pmatrix}$$

$$H(X) = XW$$

Hypothesis using matrix

$$H(x_1, x_2, x_3) = x_1w_1 + x_2w_2 + x_3w_3$$

x_1	x_2	x_3	Y
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

Test Scores for General Psychology

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix

$$H(x_1, x_2, x_3) = x_1w_1 + x_2w_2 + x_3w_3$$

x_1	x_2	x_3	Y
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

Test Scores for General Psychology

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_1w_1 + x_2w_2 + x_3w_3 \end{pmatrix}$$

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Many x instances

x_1	x_2	x_3	Y
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

Test Scores for General Psychology

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_1 w_1 + x_2 w_2 + x_3 w_3 \end{pmatrix}$$

http://college.cengage.com/mathematics/brase/understandable_statistics/7e/students/datasets/mlr/frames/frame.html

x_1	x_2	x_3	Y
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

Hypothesis using matrix

$$w_1x_1 + w_2x_2 + w_3x_3 + \ldots + w_nx_n$$

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix

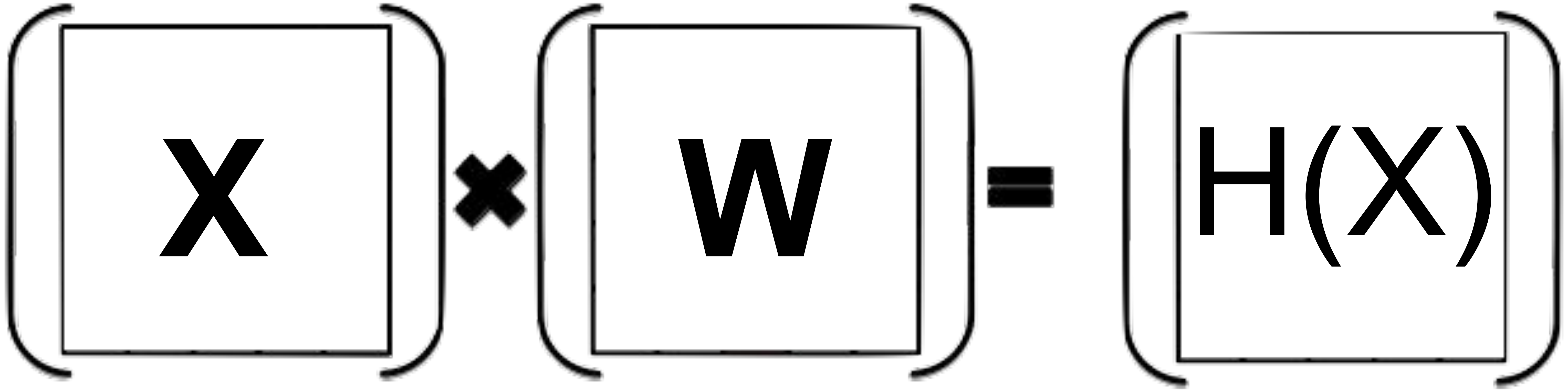
$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

[5, 3] **[3, 1]** [5, 1]

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix



[5, 3] [?, ?] [5, 1]

$$H(X) = XW$$

Hypothesis using matrix

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

`[n, 3]` **`[3, 1]`** `[n, 1]`

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix (n output)

3차원 2차원

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \; ? \; = \begin{pmatrix} x_{11}w_{11}+x_{12}w_{21}+x_{13}w_{31} & x_{11}w_{12}+x_{12}w_{22}+x_{13}w_{32} \\ x_{21}w_{11}+x_{22}w_{21}+x_{23}w_{31} & x_{21}w_{12}+x_{22}w_{22}+x_{23}w_{32} \\ x_{31}w_{11}+x_{32}w_{21}+x_{33}w_{31} & x_{31}w_{12}+x_{32}w_{22}+x_{33}w_{32} \\ x_{41}w_{11}+x_{42}w_{21}+x_{43}w_{31} & x_{41}w_{12}+x_{42}w_{22}+x_{43}w_{32} \\ x_{51}w_{11}+x_{52}w_{21}+x_{53}w_{31} & x_{51}w_{12}+x_{52}w_{22}+x_{53}w_{32} \end{pmatrix}$$

[n, 3] [?, ?] [n, 2]

3*2

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix (n output)

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{pmatrix} = \begin{pmatrix} x_{11}w_{11}+x_{12}w_{21}+x_{13}w_{31} & x_{11}w_{12}+x_{12}w_{22}+x_{13}w_{32} \\ x_{21}w_{11}+x_{22}w_{21}+x_{23}w_{31} & x_{21}w_{12}+x_{22}w_{22}+x_{23}w_{32} \\ x_{31}w_{11}+x_{32}w_{21}+x_{33}w_{31} & x_{31}w_{12}+x_{32}w_{22}+x_{33}w_{32} \\ x_{41}w_{11}+x_{42}w_{21}+x_{43}w_{31} & x_{41}w_{12}+x_{42}w_{22}+x_{43}w_{32} \\ x_{51}w_{11}+x_{52}w_{21}+x_{53}w_{31} & x_{51}w_{12}+x_{52}w_{22}+x_{53}w_{32} \end{pmatrix}$$

`[n, 3]` **`[3, 2]`** `[n, 2]`

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

WX vs **XW**

- Lecture (theory):

$$H(x) = Wx + b$$

- Implementation (TensorFlow)

$$H(X) = XW$$

Next

Logistic Regression

(Classification)