

10/10 Excellent!

Problem Set 2.1

- a. One technique in deriving numerical differentiation formula is by constructing the algebraic interpolating polynomial relative to a given set of data points and simply obtaining its derivative. The other method uses concept of Taylor's theorem to construct formulas for function values evaluated at each of the given data points. The formulas are then manipulated and combined in order to isolate an approximating formula for the required derivative where the order of convergence is also known.
- b. Due to the limited number of digits that can be read and calculated by machines, really small values of increments (step size) causes non-distinction of consecutive data points and thus results with cancellation errors due to machine precision. With the dependence of numerical differentiation formulas to values of step size, machine-introduced error is unavoidable which causes inaccuracies in the calculations in addition to the hypothetical error of the formula.
- c. Newton-Cotes and Gauss-Legendre are types of quadrature rules which differ in how their weights and the abscissas are chosen. For Newton-Cotes, discrete abscissas are used, equally-spaced or not, to construct the algebraic interpolating polynomial which is then integrated to find the approximating formula. On the other hand, Gauss-Legendre uses the roots of the Legendre polynomial of the correct degree as abscissas and calculates the weights using these points which would result to a quadrature rule of high degree of precision.
- d. Increasing the number of nodes in the aim of improving quadrature rule estimates is not an efficient way as such method become more subject to the Runge phenomenon. Instead, similar with the method done with splines, the intervals are subdivided and *low order* interpolation is done on each sub-interval. By integrating the polynomials constructed on each sub-interval and with some analytical steps, an accumulated formula for composite quadrature rules is obtained.
- e. MATH 174 has personally been the most enjoyable course this semester. I find the topics to be very relevant and the setup of the discussion to be very effective in having a profound understanding of the topics involved. Assessments are really challenging which pushed me to think critically and explore things on my own. I'm glad to finally have learned to use MATLAB to my advantage. Overall, I highly appreciate the value of *Numerical Methods* as a mathematical field that I now consider also to study more concepts on it independently.