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MATH 174 – B2L

48/50

Exercise 1: Basic Polynomial Interpolation

This exercise focuses on constructing an appropriate model for the closing price of ICT stocks in the Philippines from January to September 2022. MATLAB was used to formulate the models and construct the figures. The data on closing prices were exported from https://ph.investing.com/equities/intl-container-historical-data.

(2pts) 1. The closing prices of the ICT stock on the corresponding specified days are tabulated as follows:

Table 1. caption.

Calendar Date (2022)	Closing Price of ICT Stock (in pesos)
January 3	195.00
February 2	200.20
March 1	218.00
April 1	226.60
May 2	215.60
June 1	210.20
July 1	191.00
August 1	183.90
September 1	182.90

Defining the time variable t as the ordinal day number of each date, we have the translated table below.

Table 2. caption

day number t	Closing Price of ICT Stock (in pesos)
3	195.00
33	200.20
60	218.00
91	226.60
122	215.60
152	210.20
182	191.00
213	183.90
244	182.90

You are instructed to express the coefficients in 4 SF

(6pts) 2. Using MATLAB, we obtain the algebraic interpolating polynomial (AIP) $P(t)$ for the closing price of ICT stock as a function of the ordinal day number t, relative to the given 9 data points. $P(t)$ was constructed into its Lagrange form, hence the concept of Lagrange basis polynomial was used in the process. The result from the program using the software is an 8^{th} degree polynomial which is given as follows:

unnecessary

```
The AIP relative to the data points is P(t) =
                                  8                                 7                                   6                                 5                                 4
     711993528820271289 4273 t          42745182838645053003041 9 t          1339940997626182790472609719 t         12507917685863704834314747061 t          5741945317708123750199574435407 t
- ---------------------------------- + ---------------------------------- - ---------------------------------- + ---------------------------------- - ----------------------------------
  349605034432026536359598686476192000   218503146520016585224749179047620 00   174802517216013266179799343238096000   79455689643642394627181519653680 00   3178227585745695785087260786147200 0

                                      3                                   2
    99408640418283966489674294604 7049 t   1043010617776400988399669502229 5681 t   275080306510065451970741681 24257 t   749071806528233770241308 1
  + ---------------------------------- - ---------------------------------- + ---------------------------------- + -------------------------
    87401258609006634089899671619048000   29133752869335544696633223873016000   5685744119698584054768388734000   408402649959946873285 50

or P(t) =
- 2.037e-14*t^8 + 1.956e-11*t^7 - 7.665e-9*t^6 + 1.574e-6*t^5 - 0.0001807*t^4 + 0.01137*t^3 - 0.358*t^2 + 4.838*t + 183.4
```

The graph of P together with the scatter plot of the data points from the table is illustrated below.

(4pts)

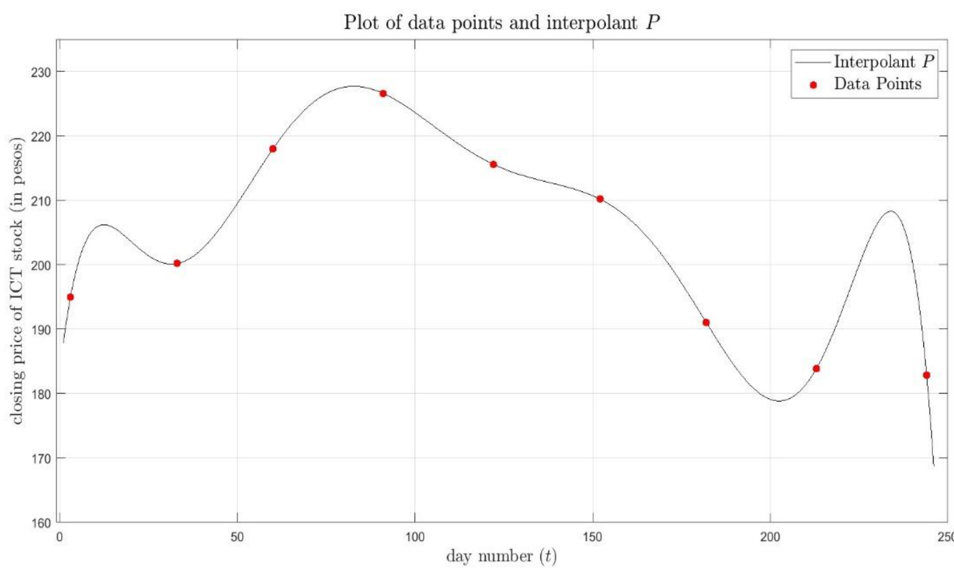

Figure 1. Caption

As observed, the obtained AIP $P(t)$ successfully interpolates the given 9 data points. Hence, the process of constructing P was correct.

(3pts) 3. Using the constructed polynomial $P(t)$, we can estimate the ICT stock's closing price on May 17, i.e., at the ordinal day number $t = 137$. Substituting t in Q using MATLAB, it is obtained that

```
the estimated closing price (in pesos) of ICT stock on May 17 is
  212.8600
```

(3pts) On the other hand, the actual closing price on May 17 was 203.00 pesos. Computing for the relative error of our estimate, we have

$$\frac{|203.00 - 212.86|}{|203.00|} = 0.04857 = 4.857\%$$

From this, it can be deduced that the closing price value $P(137) = 212.86$ is a relatively accurate estimate for the actual value of closing price at $t = 137$. However, this does not guarantee that the polynomial $P(t)$ is an accurate model for all data points.

(6pts) 4. Using MATLAB, the actual historical data points, particularly the closing prices from Jan 3 to Sep 15, was plotted together with the constructed AIP $P(t)$. The figure for this is illustrated as follows:

(4pts)

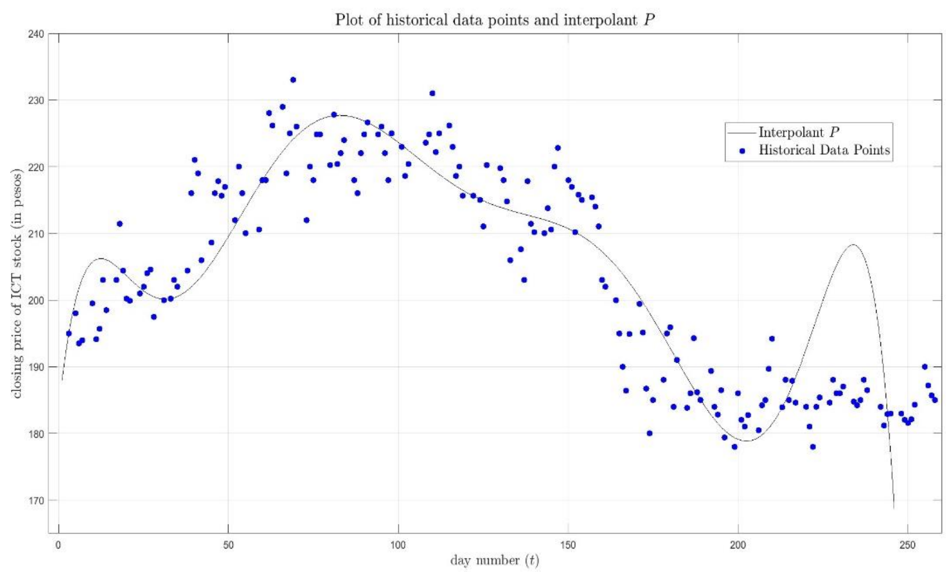

Figure 2. Caption

As observed from the figure, the obtained algebraic interpolating polynomial P satisfies the trend of the data points since the points are positioned relatively close to the graph of P, on average. The function value of $P(t)$ also increases on cases that the actual closing price rise as t increases. Similarly, $P(t)$ decreases on cases that the actual closing price decreases. Some data points apart from the data points used in constructing P intersect with the curve of P, and some having minimal errors. This further shows the goodness of fit of P with the trend of the historical data points.

(6pts) 5. When the number of data points that will be used in constructing the AIP is increased, it is expected that the accuracy of the constructed polynomial will also increase and thus, it's graph will be a better fit for the actual historical data. This, in theory, is because more data values can generate a polynomial with a higher degree and thus, more degrees of freedom. This generated new polynomial will then interpolate the data points better.

6. The additional data points together with the previous data points in Table 1 can be tabulated as follows:

Calendar Date (2022)	**day number t**	**Closing Price of ICT Stock (in pesos)**
January 3	3	195.00
January 14	14	198.50
February 2	33	200.20
February 15	46	216.00
March 1	60	218.00
March 15	74	220.00
April 1	91	226.60
April 13	103	220.40
May 2	122	215.60
May 16	136	207.60
June 1	152	210.20
June 15	166	190.00
July 1	182	191.00
July 15	196	179.40
August 1	213	183.90
August 15	227	184.60
September 1	244	182.90
September 15	258	185.00

Running a similar algorithm in MATLAB using the new data points, we generate the new algebraic interpolating polynomial Q which is of degree 17.

The AIP relative to the 18 data points is Q(t) =

$$
\begin{aligned}
&- \frac{1111859061282782529655253670878567980420046876202480127\, t^{17}}{124284864593736393924319532834513741129289380968315548430661717571376407993384960000} + \frac{2016981734782025226211425863846234768685913752372027288521\, t^{16}}{103570720494780328270266277362094784274407817473596290358884764642813673327820800000} \\
&- \frac{175272309539395197998559906537492644682618437638984580850637\, t^{15}}{9006149608241767675675328466269111676035462389008373074685631708070754202419200000} + \frac{306470147615193916989419517668995014091938011391578127730549871\, t^{14}}{25892680123695082067566569340523696068601954368399072589721191160703418331955200000} \\
&- \frac{1014298707805670179858347590122013819327660965407294071657940847891\, t^{13}}{207141440989560656540532554724189568548815634947192580717769529285627346655641600000} + \frac{13735995293286045386751460165475931000300544448550064900440315251287\, t^{12}}{941552004498002984275147976019043493403707431578148094171679678571033393892800000} \\
&- \frac{20086547544622055898893643304171927879175362472955386066908065424782979\,1\, t^{11}}{621424322968681969621597664172568705646446904841577742153308587856882039966924800000} + \frac{1477927342262291296583108187553277408338548324958029608602821984166645\,61\, t^{10}}{272554527617842969132279677268670484932652151246306027260223064849509666521600000} \\
&- \frac{270444143630438148603510640955437102963452410095237328364223390743804468\,1\, t^{9}}{38936361088263281304611382466952926418950307320900861037174723549929952378880000} + \frac{177124445996587165741246669522539656975441573139873118602933688644679872\,43\, t^{8}}{2604897396750008256294423474901780288591745912313789998965914603692496814080000} \\
&- \frac{654702519958819980725096165256471257073987290108033091365140305205491285001883\, t^{7}}{1294634006184754103378328467026184803430097718419953629486059558035170916597760000} + \frac{4879261166710367900841583206246914430657779603581238921721692673847327927274\,3\, t^{6}}{1730794125915446662270492602976182892286226896283360467227352350314399621120000} \\
&- \frac{14077621575368982375798996768312665277693789704686039268717039028451532167553\, t^{5}}{122262929176192197833429410613551078808387787100364559491613214869156249600000} + \frac{8964420241415582413379512785965866115270755788794879494412986153116485718743299\,01\, t^{4}}{269715417955157104870485097297121834047937024670823672809595741257327274291200000} \\
&- \frac{616583901233920134625155472884971935182743770511023667449427552114507536047662281\, t^{3}}{9632693498398468031088753474897208358854893738243702600342705044904545510400000} + \frac{1204506098541201718542734017990764270466228657766592425358492341323924529067287\, t^{2}}{1615139754929320595420649476005568135287540868250117806898508558837113600000} \\
&- \frac{2702665931842300454740600780757094893985288299664132435693979631996388941221\, t}{621665596539063244182421029558045676150443455881680784813264960520400000} + \frac{141531712889252337334670347424550175403662334689993568039647}{17668027598583989183792445134270420997416123031357780000}
\end{aligned}
$$

```
or Q(t) =
- 8.946e-30*t^17 + 1.947e-26*t^16 - 1.946e-23*t^15 + 1.184e-20*t^14 - 4.897e-18*t^13 +

 1.459e-15*t^12 - 3.232e-13*t^11 + 5.423e-11*t^10 - 6.946e-9*t^9 + 6.8e-7*t^8 - 5.057e-5*t^7 +

 0.002819*t^6 - 0.1151*t^5 + 3.324*t^4 - 64.01*t^3 + 745.8*t^2 - 4347.0*t + 8011.0
```

Using the constructed polynomial $Q(t)$, we can again estimate the ICT stock's closing price on May 17, i.e., at the ordinal day number $t = 137$. It is obtained using MATLAB, that

(3pts)

```
The estimated closing price (in pesos) of ICT stock on May 17 is
  207.6200
```

which then have the corresponding relative error

(3pts)

$$\frac{|203.00 - 207.62|}{|203.00|} = 0.02276 = 2.276\%$$

This shows that the estimated closing price obtained using $Q(t)$ is a better estimate than the one obtained using the polynomial $P(t)$.

7. Plotting the graph of the algebraic interpolating polynomial Q with the scatter plot of the historical data from Jan 3 to Sep 15 using MATLAB, we obtain the figure below:

(4pts)

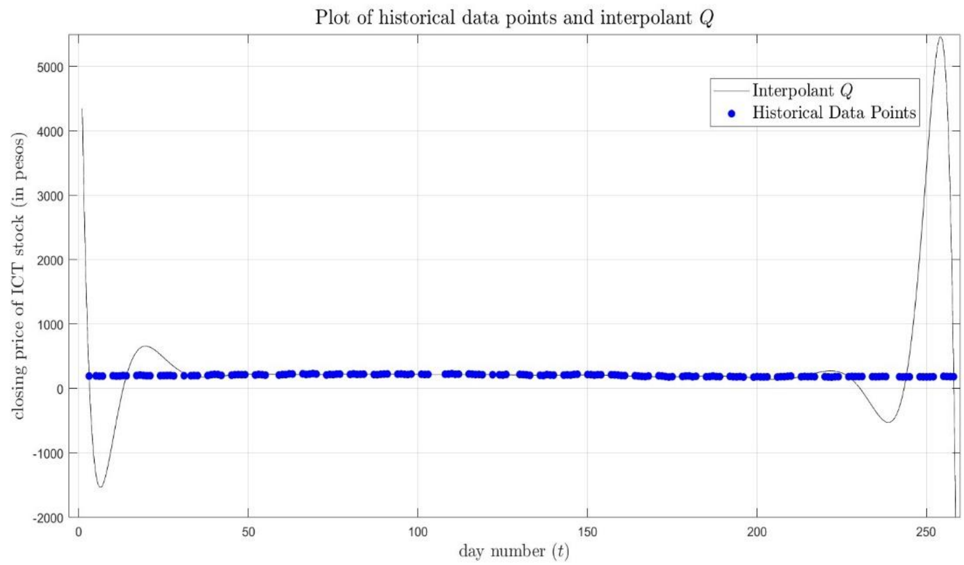

Figure 3. Caption

From the figure, we can infer that Q seems to estimate most of the historical data points better. However, it can be noted that the range intervals of the plot are high. So, zooming in on the graph, we obtain the figure below.

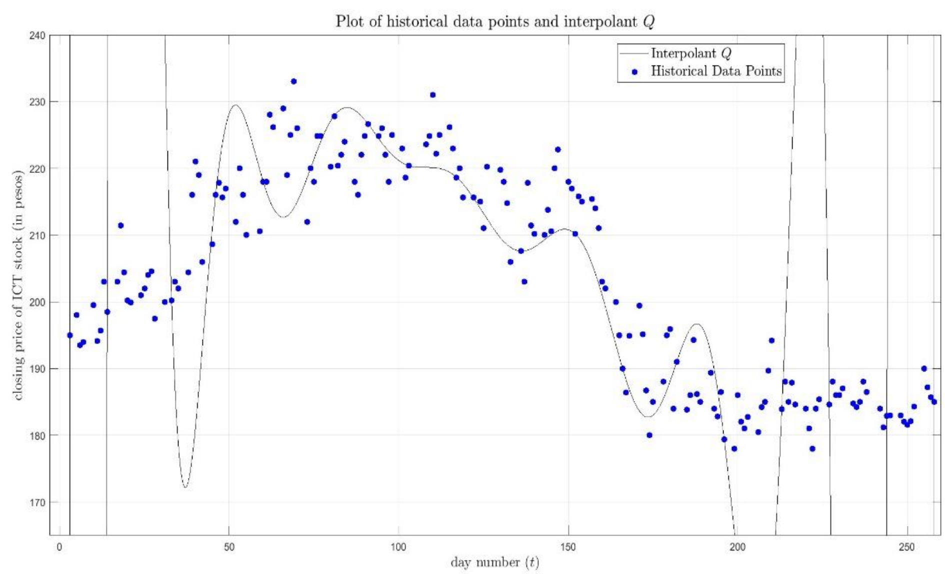

Figure 4. Caption

We can observe from this figure that more data points are relatively “closer” to the graph of Q than with that of the graph of P. This indicates that the respective relative errors of the estimates using Q are smaller and thus, Q is a better model for the historical data. Moreover, we can observe that the graph of Q oscillates

(2pts)

↳ Note that this is only applicable when you zoom in. The question refers to the general behavior of Q.

more than our previous graph for P and the amplitude of oscillation is higher. We can infer from this that constructing an algebraic interpolating polynomial using more data points would result in a more accurate model for the data since it interpolates more points. But it would not be a good forecasting model due to the increase in oscillations which would result in higher relative errors.

References:

International Container Terminal Services Inc (ICT) Historical Prices - Investing.com PH. (2022, September 19). Investing.com Philippines. Retrieved September 19, 2022, from https://ph.investing.com/equities/intl-container-historical-data

(2pts)

Appendix A
Program Used in MATLAB

```
clearvars -except data  % to clear all existing variables except imported data
(historical data)
close all

t = sym('t');  % declares t as a symbolic variable

%% Item 1: Constructing AIP (P)
% Initializes abscissas and function values

T = [3 33 60 91 122 152 182 213 244];  % time variable t
S = [195 200.20 218 226.6 215.6 210.2 191 183.9 182.9];  % stock prices S

numPts = size(T, 2);
interval = [1, 246];

% Compute for AIP

% since P is a sum of terms, we initialize P as
P = 0;
for k = 1:numPts
    % since the Lagrange basis polynomial is a product of terms
    % we initialize lk as
    lk = 1;

    % computation of lk
    for i = 1:numPts
        if i ~= k
            numer = t - T(i);
            denom = T(k) - T(i);
            lk = lk * (numer/denom);
        end
    end

    P = P + S(k) * lk;
end

%% Item 2: Displaying the polynomial P and plot of results

% polynomial P
P = simplify(P);
disp('The AIP relative to the data points is P(t) = ')
pretty(P)  % displays P decently

disp('or P(t) = ')
disp(vpa(expand(P), 4))  %  displays P having coeff in sci notation w 4 sfs


% plot of 9 data points and interpolant P
figure
fplot(P, interval, 'k') % creates a 2-D plot of polynomial P over interval [1, 246]
hold on  % to include scatter plot with graph of P
scatter(T, S, 'filled', 'r')  % scatter plot of data points
```

```
grid  % adds grid to figure

% specify axis limits
xlim([-1 250])
ylim([160 235])

% plot labels
title('Plot of data points and interpolant $P$', ...
       'Interpreter', 'Latex', ...
       'FontSize', 16)
lgnd = legend('Interpolant $P$', 'Data Points');
xl = xlabel('day number $(t)$');
yl = ylabel('closing price of ICT stock (in pesos)');

for label = [lgnd xl yl]
    set(label,'Interpreter', 'Latex', 'Fontsize', 14)
end

%% Item 3: Estimate on a Date

t_may17 = 137;  % time variable t at May 17
price_may17 = subs(P, t_may17);  % substitute t_may17 to P
price_may17 = double(price_may17);
price_may17 = round(price_may17, 2);  % rounds price to 2 decimal places

disp(' ')
disp('the estimated closing price (in pesos) of ICT stock on May 17 is')
disp(price_may17)
disp(' ')

% computing for relative error of estimate
rel = abs(203 - price_may17) / abs(203);

disp('Hence, the relative error of the estimate is')
disp(round(rel, 4, 'significant'))  % displays rel error w 4 sf
disp('which shows that the estimate is relatively accurate.')

%% Item 4: Historical Data Points and AIP

% the historical data is imported from an excel file
% and was defined as the variable 'data'

data = table2array(data);  % to convert imported table into a matrix
T1 = data(:,1);  % time variable of historical data
S1 = data(:,2);  % stock prices of historical data

% plot of historical data points and interpolant P
figure
fplot(P, interval, 'k')
hold on
scatter(T1, S1, 'filled', 'b')
grid

% specify limits
xlim([-3 260])
```

```
ylim([165 240])

% plot labels
title('Plot of historical data points and interpolant $P$', ...
       'Interpreter', 'Latex', ...
       'FontSize', 16)
lgnd = legend('Interpolant $P$', 'Historical Data Points');
xl = xlabel('day number $(t)$');
yl = ylabel('closing price of ICT stock (in pesos)');

for label = [lgnd xl yl]
    set(label,'Interpreter', 'Latex', 'Fontsize', 14)
end

%% Item 6: Constructing new AIP (Q)

% following similar algorithm in item 1
T2 = [3 14 33 46 60 74 91 103 122 136 152 166 182 196 213 227 244 258];
S2 = [195 198.5 200.2 216 218 220 226.6 220.4 215.6 207.6 210.2 190 191 179.4 183.9
184.6 182.9 185];

numPts1 = size(T2, 2);
interval1 = [1 260];

Q = 0;
for k1 = 1:numPts1
    lk1 = 1;
    for i1 = 1:numPts1
        if i1 ~= k1
            numer1 = t - T2(i1);
            denom1 = T2(k1) - T2(i1);
            lk1 = lk1 * (numer1/denom1);
        end
    end
    Q = Q + S2(k1) * lk1;
end

% plotting new AIP (Q)
Q = simplify(Q);
disp('The AIP relative to the 18 data points is Q(t) = ')
pretty(Q)

disp('or Q(t) = ')
disp(vpa(expand(Q), 4))

% estimated value of Q at May 17
newprice_may17 = subs(Q, t_may17);
newprice_may17 = double(newprice_may17);
newprice_may17 = round(newprice_may17, 2);

disp(' ')
disp('The estimated closing price (in pesos) of ICT stock on May 17 is')
disp(newprice_may17)
disp(' ')
```

```
% computing for relative error of estimate
rel1 = abs(203 - newprice_may17) / abs(203);

disp('Thus, the relative error of the estimate is')
disp(round(rel1, 4, 'significant'))
disp('which shows that the estimate is more accurate')
disp('than the estimate obtained using the polynomial P.')

%% Item 7

% plot of polynomial Q and historical data
figure
fplot(Q, interval1, 'k')
hold on
scatter(T1, S1, 'filled', 'b')
grid

% specify limits
xlim([-3 260])
ylim([-2000 5500])

% plot labels
title('Plot of historical data points and interpolant $Q$', ...
       'Interpreter', 'Latex', ...
       'FontSize', 16)
lgnd = legend('Interpolant $Q$', 'Historical Data Points');
xl = xlabel('day number $(t)$');
yl = ylabel('closing price of ICT stock (in pesos)');

for label = [lgnd xl yl]
    set(label,'Interpreter', 'Latex', 'Fontsize', 14)
end

%% End of Program
```

Appendix B
Table of Historical Data Points

Date	ICT Stock Closing Price (in pesos)
9/15/2022	185
9/14/2022	185.7
9/13/2022	187.2
9/12/2022	190
9/9/2022	184.3
9/8/2022	182.1
9/7/2022	181.6
9/6/2022	182
9/5/2022	183
9/2/2022	183
9/1/2022	182.9
8/31/2022	181.2
8/30/2022	184
8/26/2022	186.5
8/25/2022	188
8/24/2022	185
8/23/2022	184.2
8/22/2022	184.8
8/19/2022	187
8/18/2022	186
8/17/2022	186
8/16/2022	188
8/15/2022	184.6
8/12/2022	185.4
8/11/2022	184
8/10/2022	178
8/9/2022	181
8/8/2022	184
8/5/2022	184.6
8/4/2022	187.9
8/3/2022	185
8/2/2022	188
8/1/2022	183.9
7/29/2022	194.2
7/28/2022	189.7
7/27/2022	185
7/26/2022	184.2
7/25/2022	180.5
7/22/2022	182.7
7/21/2022	181
7/20/2022	182
7/19/2022	186
7/18/2022	178

7/15/2022	179.4
7/14/2022	186.5
7/13/2022	182.8
7/12/2022	184
7/11/2022	189.4
7/8/2022	185
7/7/2022	186.2
7/6/2022	194.3
7/5/2022	186
7/4/2022	183.8
7/1/2022	191
6/30/2022	184
6/29/2022	195.9
6/28/2022	195
6/27/2022	188
6/24/2022	185
6/23/2022	180
6/22/2022	186.7
6/21/2022	195.1
6/20/2022	199.4
6/17/2022	194.9
6/16/2022	186.4
6/15/2022	190
6/14/2022	195
6/13/2022	200
6/10/2022	202
6/9/2022	203
6/8/2022	211
6/7/2022	214
6/6/2022	215.4
6/3/2022	215
6/2/2022	215.8
6/1/2022	210.2
5/31/2022	217
5/30/2022	218
5/27/2022	222.8
5/26/2022	220
5/25/2022	210.6
5/24/2022	213.8
5/23/2022	210
5/20/2022	210.2
5/19/2022	211.4
5/18/2022	217.8
5/17/2022	203
5/16/2022	207.6
5/13/2022	206

5/12/2022	214.8
5/11/2022	218
5/10/2022	219.8
5/6/2022	220.2
5/5/2022	211
5/4/2022	215
5/2/2022	215.6
4/29/2022	215.6
4/28/2022	220
4/27/2022	218.6
4/26/2022	223
4/25/2022	226.2
4/22/2022	225
4/21/2022	222.2
4/20/2022	231
4/19/2022	224.8
4/18/2022	223.6
4/13/2022	220.4
4/12/2022	218.6
4/11/2022	223
4/8/2022	225
4/7/2022	218
4/6/2022	222
4/5/2022	226
4/4/2022	224.8
4/1/2022	226.6
3/31/2022	224.8
3/30/2022	222
3/29/2022	216
3/28/2022	218
3/25/2022	224
3/24/2022	222
3/23/2022	220.4
3/22/2022	227.8
3/21/2022	220.2
3/18/2022	224.8
3/17/2022	224.8
3/16/2022	218
3/15/2022	220
3/14/2022	212
3/11/2022	226
3/10/2022	233
3/9/2022	225
3/8/2022	219
3/7/2022	229
3/4/2022	226.2

3/3/2022	228
3/2/2022	218
3/1/2022	218
2/28/2022	210.6
2/24/2022	210
2/23/2022	216
2/22/2022	220
2/21/2022	212
2/18/2022	217
2/17/2022	215.6
2/16/2022	217.8
2/15/2022	216
2/14/2022	208.6
2/11/2022	206
2/10/2022	219
2/9/2022	221
2/8/2022	216
2/7/2022	204.4
2/4/2022	202
2/3/2022	203
2/2/2022	200.2
1/31/2022	200
1/28/2022	197.5
1/27/2022	204.6
1/26/2022	204
1/25/2022	202
1/24/2022	201
1/21/2022	199.9
1/20/2022	200.2
1/19/2022	204.4
1/18/2022	211.4
1/17/2022	203
1/14/2022	198.5
1/13/2022	203
1/12/2022	195.7
1/11/2022	194.1
1/10/2022	199.5
1/7/2022	194
1/6/2022	193.5
1/5/2022	198
1/3/2022	195