$$\begin{cases} 2u+v+w=5 & ① \\ 4u-6v=-2 & ② \\ -2u+7v+2w=9 & ③ \end{cases}$$

$$\begin{aligned} 2u+v+w&=5 \\ -8v-2w&=-12 \Leftarrow ② - \boxed{2}\times ① \\ -2u+7v+2w&=9 \Leftarrow ③ + ① \end{aligned}$$

$$\overset{E_{21}}{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{A}{\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ -2 & 7 & 2 \end{bmatrix}$$

$$\overset{A}{\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}$$

$$\begin{aligned} 2u+v+w&=5 \\ -8v-2w&=-12 \\ 8v+3w&=14 \end{aligned}$$

$$\overset{E_{31}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}} \overset{A}{\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}} = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ 0 & 8 & 3 \end{bmatrix}$$

$$\therefore E_{31} E_{21} A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 8 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} 2u+v+w&=5 \\ -8v-2w&=-12 \\ w&=2 \quad ③ - \boxed{-1}\times ② \end{aligned}$$

$$\underset{E_{32}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}} \begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 8 & 3 \end{bmatrix} = \underset{U}{\begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\therefore E_{32} E_{31} E_{21} A = U$$

- Elementary Matrix in G.E

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -l_{21} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} : ② - l_{21}\times ① \to ②' \Rightarrow ② = l_{21}\times ① + ②'$$

$$E_{21} \cdot A = ②' \Rightarrow \underbrace{E_{21}^{-1}}_{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} ②' = A$$

$$\therefore A = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} U$$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_{31} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & l_{32} & 1 \end{bmatrix} U = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}}_{\text{: Lower triangular Matrix}} U$$

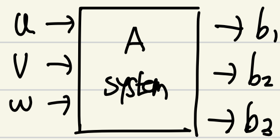
1.5 Triangular Factors.

$Ax = b$

$A = LU$

$\Rightarrow$ LU factorization (decomposition)

$\Rightarrow$ 연산이 용이함.



$2u + v + w = 5$

$\boxed{A} \to \boxed{L} \times \boxed{U}$

$$Ax = b$$

$$\underline{L^{-1}A}x = L^{-1}b = c$$

$$Ux = c$$

$$A = LU \quad . \quad Lc = b$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \qquad \text{ex)} \; L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 4 & 1 \end{bmatrix}$$

$$\text{ex)} \; A = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix} \Rightarrow U = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$$

$$\det(A) = 2 \times 1 \qquad L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

EX4)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = L\underbrace{I}_{=U}$$

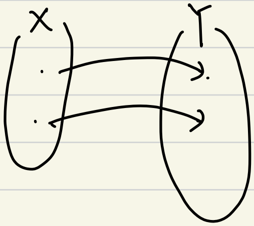
$$U = \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & d_3 & \\ 0 & & \ddots & d_n \end{bmatrix} \Rightarrow \begin{bmatrix} d_1 & & 0 \\ & d_2 & \\ & & \ddots & \\ 0 & & & d_n \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{u_{12}}{d_1} & \frac{u_{13}}{d_1} & \cdots \\ & 1 & \frac{u_{23}}{d_2} & \cdots \\ & & 1 & \\ 0 & & & 1 & \\ & & & & \ddots \end{bmatrix}$$

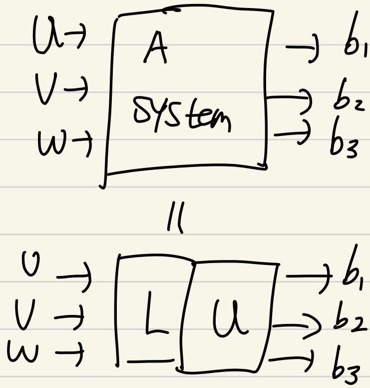
: 대각행렬. Diagonal Matrix.

$$= DU \qquad D^n = \begin{bmatrix} d_1^n & & 0 \\ & d_2^n & \\ 0 & & \ddots & d_n^n \end{bmatrix}$$

$$\therefore A = LU = LDU$$

⊙ LU factorization is Unique!!





⊙ Row Exchange (pivoting)

⇒ permutation . P

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{= P_{21}} \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -6 & 0 \\ 2 & 1 & 1 \\ -2 & 7 & 2 \end{bmatrix}$$

- permutation Matrix has the same rows with I
  → There is a single '1' in every row and Column

$$P_{31} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad P_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P_{32} P_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$P_{21} P_{32} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \neq P_{32} P_{21}$ $\therefore$ 교환법칙 X.

$A = LU$ or $PA = LU$

$P^{-1} = P^T$

$A = P^T L U$