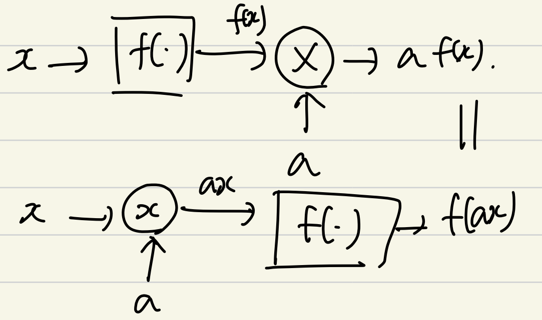
Linearity : $f(x)$, operation,

1) super position : $f(x_1+x_2) = f(x_1) + f(x_2)$

2) homogeniety : $f(ax) = af(x)$:



$$f(a_1x_1 + a_2x_2) = a_1f(x_1) + a_2f(x_2)$$

$$f\left(\sum_{k=1}^{n} a_k x_k\right) = \sum_{k=1}^{n} a_k f(x_k)$$

· Vector (Column notation)

$$v = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \quad w = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \quad A = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$AX = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = x \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} + y \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$$

· Linear Combination of Column vectors ⇒ AX

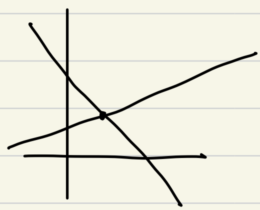
$$\left(\begin{array}{l} x+2y=3 \\ 4x+5y=6 \end{array} \right. \quad \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

· Singular Case. → No solution

→ Infinite Solutions

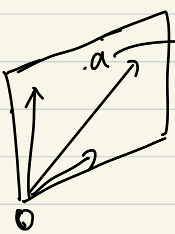
(1) Row form ⇒ parallel

⇒ overlap



(2) Column form : $x\begin{bmatrix} \ \end{bmatrix} + y\begin{bmatrix} \ \end{bmatrix} \Rightarrow \begin{bmatrix} \ \end{bmatrix}$

.a → No sol



→ infinite sol

Back-substitution.

1.3 Gauss Elimination.

$$\begin{cases} 2u + v + w = 5 \\ 4u - 6v = -2 \\ -2u + 7v + 2w = 9 \end{cases} \Rightarrow \begin{aligned} 2u + v + w &= 5 \\ -8v - 2w &= -12 \quad ② - ① \times 2 \\ 8v + 3w &= 14 \quad ③ + ① \end{aligned} \Rightarrow \begin{aligned} 2u + v + w &= 5 \\ -8v - 2w &= -12 \\ 1w &= 2 \quad ③ + ② \end{aligned}$$

대입

∵ all pivots are Non-zero

⇒ G.E has unique solution.

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 4 & -6 & 0 & -2 \\ -2 & 7 & 2 & 9 \end{array}\right] \quad \left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 0 & -8 & -2 & -12 \\ 0 & 8 & 3 & 14 \end{array}\right] \quad \left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 0 & -8 & -2 & -12 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Upper Triangular Matrix.

- Break Down. ⇒ when a zero appears in a pivot position.
  - ⇒ G.E has to be stopped.
  - ⇒ The order of eqns has to be changed
    - ↳ pivoting.

Ex 1)

$$\begin{aligned} u + v + w &= a \\ 2u + 2v + 5w &= b \\ 4u + 6v + 8w &= c \end{aligned} \Rightarrow \begin{aligned} u + v + w &= a \\ 3w &= b - 2a \quad ② - ① \times 2 \\ 2v + 4w &= c - 4a \quad ③ - ① \times 4 \end{aligned}$$

$\boxed{\text{Pivoting}}$

$$\Rightarrow \begin{aligned} u + v + w &= a \\ 2v + 4w &= c - 4a \\ 3w &= b - 2a \end{aligned}$$

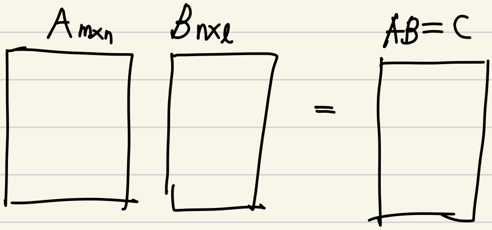
Ex 2)

$$\begin{aligned} u + v + w &= a \\ 2u + 2v + 5w &= b \\ 4u + 4v + 8w &= c \end{aligned} \overset{GE}{\Rightarrow} \begin{aligned} u + v + w &= a \\ 3w &= b - 2a \\ 4w &= c - 4a \end{aligned}$$

$w = \frac{b-2a}{3} = \frac{c-4a}{4}$: 무수히 많은 해

$w = \frac{b-2a}{3} \neq \frac{c-4a}{4} = w$: 해가 없음.

1.4 Matrix multiplication.



$$(AB)_{ij} = \sum_{k=1}^{l} a_{ik} b_{kj}$$

$$AB \neq BA \qquad AB = A\begin{bmatrix} b_1 & b_2 & \cdots & b_l \end{bmatrix} = \begin{bmatrix} Ab_1 & Ab_2 & \cdots & Ab_l \end{bmatrix}$$