Linearity (선형성) : $f(x)$ — 조건

① Superposition (중첩) : $f(x_1+x_2) = f(x_1)+f(x_2)$

② Homogeniety . $f(ax) = a f(x)$

Constant

$f(a_1x_1+a_2x_2) = a_1f(x_1) + a_2f(x_2)$

ex) . $y = mx = f(x)$ ∴ 선형성 O

$m(a_1x_1+a_2x_2) = a_1mx_1 + a_2mx_2 = a_1f(x_1) + a_2f(x_2)$

. $y = mx+n$ $(n \neq 0)$ ∴ 선형성 X.

$m(a_1x_1+a_2x_2)+n \neq a_1(mx_1+n) + a_2(mx_2+n)$

. Operation (미분, 적분 ...) about $x_1(t)$, $x_2(t)$ ..

$\frac{d}{dt}(a_1x_1(t)+a_2x_2(t)) = a_1\frac{d}{dt}x_1(t) + a_2\cdot\frac{d}{dt}x_2(t)$

$\int$ 〃 $dt = a_1\int x_1(t)\,dt + a_2\cdot\int x_2(t)\,dt$

. Matrix A × (X_1, X_2)

$[A][x_1][x_2] = [y]$: $A(a_1X_1+a_2X_2) = a_1AX_1 + a_2AX_2$

· Basic Notation of Matrix

vector $V = (a,b,c)^T$ → row vector

$V = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ → Colum vector.

· Transpose (전치) : row ⟺ Column ex) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T \rightarrow \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

· Linear Combination (선형 결합) : $V = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$, $W = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$ ⇒ $\alpha V + \beta W = \alpha\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} + \beta\begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} \alpha a_1+\beta a_2 \\ \alpha b_1+\beta b_2 \\ \alpha c_1+\beta c_2 \end{bmatrix}$

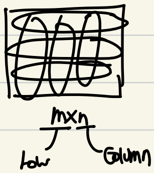
$[V\ W] = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$. $\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha a_1+\beta a_2 \\ \alpha b_1+\beta b_2 \\ \alpha c_1+\beta c_2 \end{bmatrix} = \alpha\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} + \beta\begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$

어떤 행렬과 칼럼 벡터의 곱

〃

칼럼 벡터들이 α, β 계수의 Linear Combination 결과

· Matrix



$A \pm B$

$AB \neq BA$

$AI = IA = A$

항등원 = 단위행렬

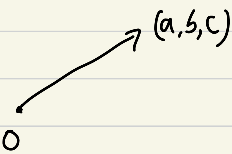
$I = E = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & \ddots \; 1 \end{bmatrix}$

· $AA^{-1} = A^{-1}A = I$

역행렬.

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

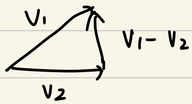
· Vector : $V = (a,b,c)^T = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

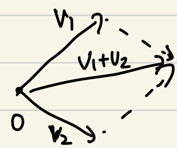


· $V_1 + V_2$

$= (a_1+a_2,\ b_1+b_2,\ C_1+C_2)$

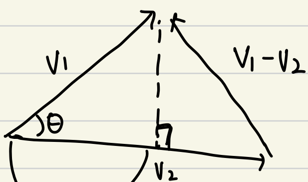
· $V_1 - V_2$.





- Inner product (내적). ≒ Projection (투영)

$V_1 \cdot V_2 = |V_1||V_2| \cdot \cos\theta = (a_1 \cdot a_2 + b_1 \cdot b_2 + C_1 \cdot C_2)$



$|V_1| \cdot \cos\theta$ ⇒ 투영

함수의 내적. $(f_1(t),\ f_2(t))$

… 각각의 element

$f_1(t) = (\cdots\ f_1(t_1),\ f_1(t_2) \cdots)$

$f_2(t) = (\cdots\ ,f_2(t_1),\ f_2(t_2) \cdots)$

$\underbrace{\sum_{k=-\infty}^{\infty} f_1(t_k) \cdot f_2(t_k) \rightarrow \int f_1(t) f_2(t)\, dt}_{\text{Hilbert Space.}}$

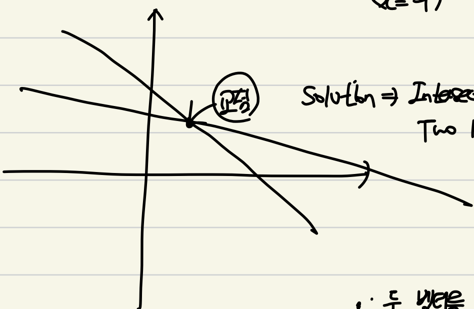
Chapter 1 · Gauss Elimination.

⇒ How to solve Linear system eqn.

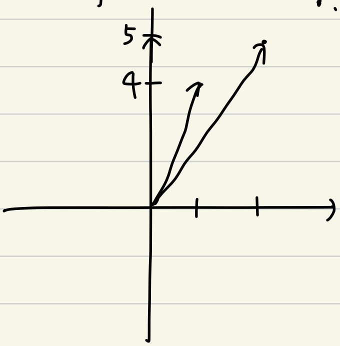
$$\begin{cases} x + 2y = 3 & -\ ① \\ 4x + 5y = 6 & -\ ② \end{cases} \qquad 4 \times ① - ② \Rightarrow 3y = 6 \qquad \begin{pmatrix} y = 2 \\ x = -1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$



Solution ⇒ Intersection of Two lines.



∴ 두 벡터를 이용하여 해당 좌표에 도달.

· For 3-D vector.

$$\begin{cases} 2u + v + w = 5 \\ 4u - 6v = -2 \\ -2u + 7v + 2w = 9 \end{cases}$$

Row form ⇒ Intersection of 3 planes.

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix} \qquad u \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix} + v \begin{bmatrix} 1 \\ -6 \\ 7 \end{bmatrix} + w \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

Column form ⇒ Linear Combination of Column vectors.