

## Exercises for Chapter 3

3.1 Consider the time series (920, 980, 1.03, 950, 990).

- a) Which stochastic and deterministic errors does this time series seem to contain? *Stochastic errors: noise || deterministic errors: outlier 1.03*
- b) What may be reasons for these errors? *stochastic errors → measurement noise  
Deterministic errors → wrong format 1.03 ↔ 1030*
- c) Compute the output of an asymmetric median filter with window length 3 for this time series. *(920, 950, 950) Median of 3 values*
- d) Which effect does this filter have on the observed errors? *Noise is reduced, outlier is removed*

3.2 Which of these filters are FIR, IIR, or none of these?

- a)  $x_k + x_{k-1} + y_k = 0$  *FIR, because it is linear but does not consider previous values of y*
- b)  $x_k + x_{k-1} + x_{k-2} = 0$  *none, because it does not contain y*
- c)  $x_k + y_{k-1} + y_k = 0$  *IIR, because it is linear and does consider previous values of y*

3.3 Consider the IIR filter  $y_k = 2y_{k-1} - y_{k-2} + x_k + x_{k-1}$ ,  $k = 3, 4, \dots$ ,  $y_1 = y_2 = 0$ .

- a) What is the filter output sequence  $y$  for the input sequence  $x = (0, 0, 1, 0, 0, 0, 0, 0)$ ?
- b) What is the filter output sequence  $y$  for the input sequence  $x = (0, 0, 1, a, b, 0, 0, 0)$ ,  $a, b \in \mathbb{R}$ ?
- c) Give a formula for the filter output  $y_k$ ,  $k = 8, 9, 10, \dots$ , for  $x = (0, 0, 1, a, b, 0, 0, 0, \dots)$ ,  $a, b \in \mathbb{R}$ !
- d) For which finite values of  $a$  and  $b$  will the filter be unstable?
- e) For which finite values of  $a$  and  $b$  will the filter converge to  $\lim_{k \rightarrow \infty} y_k = 0$ ?

3.3 Consider the IIR filter  $y_k = 2y_{k-1} - y_{k-2} + x_k + x_{k-1}$ ,  $k = 3, 4, \dots$ ,  $y_1 = y_2 = 0$ .

- a) What is the filter output sequence  $y$  for the input sequence  $x = (0, 0, 1, 0, 0, 0, 0)$ ?

$$y_1 = y_2 = 0$$

$$y_3 = 0 - 0 + 1 + 0 = 1 \quad || \quad y_4 = 3 \quad || \quad y_5 = 5 \quad || \quad y_6 = 7 \quad || \quad y_7 = 9 \quad || \quad y_8 = 11$$

$$y = (0, 0, 1, 3, 5, 7, 9, 11)$$

- b) What is the filter output sequence  $y$  for the input sequence  $x = (0, 0, 1, a, b, 0, 0, 0)$ ,  $a, b \in \mathbb{R}$ ?

$$y_1 = y_2 = 0$$

$$y_3 = 0 - 0 + 1 + 0 = 1 \quad || \quad y_4 = 2 - 0 + a + 1 = 3 + a$$

$$y = (0, 0, 1, 3+a, 5+3a+b, 7+5a+3b, 9+7a+5b, 11+9a+7b)$$

- c) Give a formula for the filter output  $y_k$ ,  $k = 8, 9, 10, \dots$ , for  $x = (0, 0, 1, a, b, 0, 0, 0, \dots)$ ,  $a, b \in \mathbb{R}$ !

$$y_k = 2k - 5 + (2k - 7)a + (2k - 9)b$$

- d) For which finite values of  $a$  and  $b$  will the filter be unstable?

- e) For which finite values of  $a$  and  $b$  will the filter converge to  $\lim_{k \rightarrow \infty} y_k = 0$ ?

$$d) \quad y_k = 2k - 5 + 2ka - 7a + 2kb - 9b$$

$$y_k = 2k(1+a+b) - 5 - 7a - 9b \quad \text{not finite for } k \rightarrow \infty \\ \text{if } 1+a+b \neq 0$$

e)

$$\text{for } 1+a+b = 0 \Leftrightarrow b = -1-a : \lim_{k \rightarrow \infty} y_k = -5 - 7a - 9b =$$

$$4 + 2a = 0 \quad \text{for } a = -2, \quad b = -1 + 2 = 1$$