Chapter 8: Classification

1. Naive Bayes Classifier
2. Linear Discriminant Analysis
3. Support Vector Machine
4. Nearest Neighbor Classifier
5. Learning Vector Quantization
6. Decision Trees

Prof. Dr. Thomas A. Runkler

input → continuous

Classification

output → discrete

(True/False, Healthy/Sick, ...)

- data set

input

(input) continuous

$$Z = (X, y) = \{(x_1, y_1), \ldots, (x_n, y_n)\} \subset \mathrm{IR}^p \times \{1, \ldots, c\}$$

output

discrete (output)

- classifier

$$f : \mathrm{IR}^p \to \{1, \ldots, c\}$$

- assessment
  1. true positive (TP): $y = i,\ f(x) = i$
     (a sick patient is classified as sick)
  2. true negative (TN): $y \neq i,\ f(x) \neq i$
     (a healthy patient is classified as healthy)
  3. false positive (FP): $y \neq i,\ f(x) = i$
     (a healthy patient is classified as sick) } it does not harm (false alarm)
  4. false negative (FN): $y = i,\ f(x) \neq i$
     (a sick patient is classified as healthy) } big problem (worse)

errors (3, 4)

Classification Performance

- correct classifications T=TP+TN
  (number of correctly classified patients)
- false classifications F=FP+FN
  (number of incorrectly classified patients)
- relevance R=TP+FN (number of sick patients)
- irrelevance I=FP+TN (number of healthy patients)
- positivity P=TP+FP
  (number of patients that were classified as sick)
- negativity N=TN+FN
  (number of patients that were classified as healthy)
- correct classification rate T/n
  (probability that a patient is correctly classified)
- false classification rate F/n
  (probability that a patient is incorrectly classified)

Prof. Dr. Thomas A. Runkler

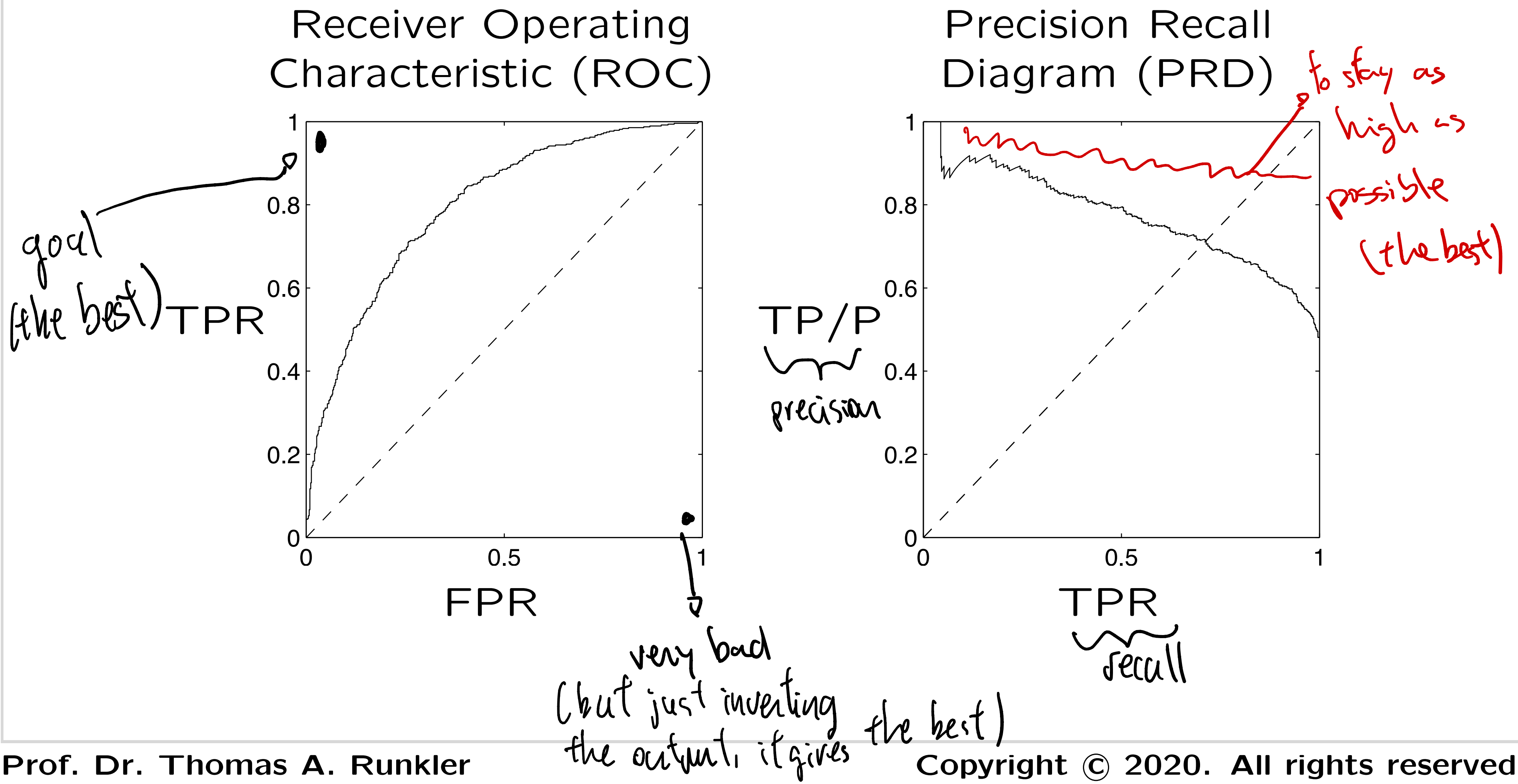
Classification Performance

- true positive rate, sensitivity, recall TPR=TP/R
  (probability that a sick patient is classified as sick)
- true negative rate, specifity TNR=TN/I
  (probability that a healthy patient is classified as healthy)
- false positive rate, false alarm rate FPR=FP/I
  (probability that a healthy patient is classified as sick)
- false negative rate FNR=FN/R
  (probability that a sick patient is classified as healthy)

Classification Performance

- positive prediction, precision TP/P
  (probability that a sick classified patient is sick)
- negative prediction TN/N
  (probability that a healthy classified patient is healthy)
- negative false classification rate FN/N
  (probability that a healthy classified patient is sick)
- positive false classification rate FP/P
  (probability that a sick classified patient is healthy)
- F measure F=2/(P/TP+R/TP)=2TP/(P+R)
  (harmonic mean of precision and recall)

Classifier Diagrams



Prof. Dr. Thomas A. Runkler

probabilistic classifier

Naive Bayes Classifier

- given:
  - class probabilities

$$p(1), \ldots, p(c)$$

  - conditional feature related class probabilities

$$\begin{array}{ccc} p(x^{(1)} \mid 1), & \ldots & p(x^{(1)} \mid c) \\ \vdots & \ddots & \vdots \\ p(x^{(p)} \mid 1), & \ldots & p(x^{(p)} \mid c) \end{array}$$

- wanted: classification probabilities

$$p(1 \mid x), \ldots, p(c \mid x)$$

Naive Bayes Classifier

- naive Bayes classifier:

$$p(i \mid x) = \frac{p(i) \cdot p(x \mid i)}{\sum\limits_{j=1}^{c} p(j) \cdot p(x \mid j)}$$

→ Bayes Rule

naive assumption

(it does not always hold)

$$p(x \mid i) = \prod_{k=1}^{p} p(x^{(k)} \mid i)$$

→ individual features are independent of each other

Example Naive Bayes Classifier

	exam passed	exam failed
went to class	21	4
did not go to class	1	3
studied material	16	2
did not study material	6	5

- given: x : went to class, studied material
- wanted: $p(\text{passed} \mid x)$

Prof. Dr. Thomas A. Runkler

Example Naive Bayes Classifier

$$p(\text{went to class}|\text{passed}) = \frac{21}{21+1} = \frac{21}{22}$$
$$p(\text{studied material}|\text{passed}) = \frac{16}{16+6} = \frac{16}{22}$$
$$\Rightarrow p(x|\text{passed}) = \frac{21\cdot 16}{22\cdot 22} = \frac{84}{121}$$

$$p(\text{went to class}|\text{not passed}) = \frac{4}{4+3} = \frac{4}{7}$$
$$p(\text{studied material}|\text{not passed}) = \frac{2}{2+5} = \frac{2}{7}$$
$$\Rightarrow p(x|\text{not passed}) = \frac{4\cdot 2}{7\cdot 7} = \frac{8}{49}$$

$$p(\text{passed}) = \frac{22}{22+7} = \frac{22}{29}$$
$$p(\text{not passed}) = \frac{7}{22+7} = \frac{7}{29}$$

$$p(\text{passed}) \cdot p(x \mid \text{passed}) = \frac{22}{29} \cdot \frac{84}{121} = \frac{168}{319}$$
$$p(\text{not passed}) \cdot p(x \mid \text{not passed}) = \frac{7}{29} \cdot \frac{8}{49} = \frac{8}{203}$$

$$\Rightarrow p(\text{passed} \mid x) = \frac{\frac{168}{319}}{\frac{168}{319}+\frac{8}{203}} = \frac{168\cdot 203}{168\cdot 203+8\cdot 319} = \frac{147}{158} \approx 93\%$$

Prof. Dr. Thomas A. Runkler

$+/-$ Naive Bayes Classifier

$+$ training data have to be evaluated only once
$+$ missing data can be simply ignored

$-$ features must be independent
$-$ features must be discrete

Prof. Dr. Thomas A. Runkler