Exercises for Chapter 7

7.1 We want to predict the sales figures of a startup online shop from the sales of the previous three months: 5000 Euros, 10000 Euros, 15000 Euros.

a) Compute the μ-σ-standardized time series.

b) Construct the (standardized) regression data set for a forecasting model with time horizon 1 and find the optimal linear autoregressive forecasting model with offset ($f(0) \neq 0$) and time horizon 1 for these data.

c) Using this forecasting model compute the (unstandardized) sales forecasts for the next two months.

d) Which value will this linear forecasting model yield if time goes to infinity?

7.2 We construct a *single* layer perceptron (SLP) using the linear forecasting model from Exercise 7.1 followed by a single neuron with hyperbolic tangent activation function. If necessary use the following approximations: $\tanh(3/4) = 5/8$, $\tanh(1) = 3/4$, $\tanh(3/2) = 29/32$, $\tanh(7/4) = 15/16$, $\tanh(2) = 1$.

a) Initialize the SLP with (the standardized equivalent of) 5000 Euros and compute the (unstandardized) sales forecasts for the following 3 months.

b) Which value will this SLP forecasting model yield if time goes to infinity?

7.1 We want to predict the sales figures of a startup online shop from the sales of the previous three months: 5000 Euros, 10000 Euros, 15000 Euros.

a) Compute the μ-σ-standardized time series.

The mean is $\mu = (5000 + 10000 + 15000)/3 = 10\,000$

The standard deviation is

$$\sigma = \sqrt{\frac{1}{3-1}\left((5000-10000)^2 + (10000-10000)^2 + (15\,000-10000)^2\right)}$$

$$= \sqrt{\frac{1}{2}(5000^2 + 0 + 5000^2)} = 5000$$

So, the standardized time series is

$$x = \left(\frac{5000-10000}{5000}, \frac{10\,000-10\,000}{5\,000}, \frac{15\,000-10\,000}{5000}\right) = (-1, 0, 1)$$

b) Construct the (standardized) regression data set for a forecasting model with time horizon 1 and find the optimal linear autoregressive forecasting model with offset ($f(0) \neq 0$) and time horizon 1 for these data.

The forecasting model is $x_t = a x_{t-1} + b$. For the given data this yields

$$\left.\begin{array}{l} x_2 = a x_1 + b \Rightarrow 0 = -a + b \\ x_3 = a x_2 + b \Rightarrow 1 = b \end{array}\right\} a = 1 \quad \Rightarrow x_t = x_{t-1} + 1$$

c) Using this forecasting model compute the (unstandardized) sales forecasts for the next two months.

d) Which value will this linear forecasting model yield if time goes to infinity?

c) We compute the standardized forecasts as $x_4 = x_3 + 1 = 1 + 1 = 2$ and $x_5 = x_4 + 1 = 2 + 1 = 3$, which corresponds to the unstandardized forecasts

$y_4 = 2 \cdot 5000 + 10000 = 20\,000$ and $y_5 = 3 \cdot 5000 + 10000 = 25000$

d) $y_\infty \to \infty$

7.2 We construct a *single* layer perceptron (SLP) using the linear forecasting model from Exercise 7.1 followed by a single neuron with hyperbolic tangent activation function. If necessary use the following approximations: $\tanh(3/4) = 5/8$, $\tanh(1) = 3/4$, $\tanh(3/2) = 29/32$, $\tanh(7/4) = 15/16$, $\tanh(2) = 1$.

a) Initialize the SLP with (the standardized equivalent of) 5000 Euros and compute the (unstandardized) sales forecasts for the following 3 months.

Applying to hyperbolic tangent yields the forecasting model

$$x_t = \tanh(x_{t-1} + 1)$$

The unstandardized initialization $y_1 = 5000$ corresponds to the standardized initialization $x_1 = (5000 - 10000)/5000 = -1$. The standardized forecasts are then computed as

$$x_2 = \tanh(-1 + 1) = 0 \qquad x_4 = \tanh(3/4 + 1) \approx 15/16$$

$$x_3 = \tanh(0 + 1) \approx 3/4$$

This corresponds to the unstandardized forecasts

$$y = \left(-1 \cdot 5000 + 10000,\ 0 \cdot 5000 + 10000,\ 3/4 \cdot 5000 + 10000,\ \tfrac{15}{16} \cdot 5000 + 10000\right)$$

$$= (5000,\ 10000,\ 13750,\ 14687.5)$$

b) Which value will this SLP forecasting model yield if time goes to infinity?

As time goes to infinity we have $x_\infty \approx \tanh(x_\infty + 1)$. With the approximation $\tanh(2) = 1$, we can write $1 \approx$

$$\tanh(1+1) \Rightarrow x_\infty = 1 \Rightarrow y_\infty = 1 \cdot 5000 + 1000 = 15000$$