Exercises for Chapter 3

3.1 Consider the time series $(920, 980, 1.03, 950, 990)$.

a) Which stochastic and deterministic errors does this time series seem to contain?

stochastic errors: noise, deterministic errors: outlier 1.03

b) What may be reasons for these errors?

stochastic errors: measurement noise, deterministic errors: wrong format $1.030 \leftrightarrow 1030$

c) Compute the output of an asymmetric median filter with window length 3 for this time series.

$(920, 950, 950)$

d) Which effect does this filter have on the observed errors?

noise is reduced, outlier is removed

3.2 Which of these filters are FIR, IIR, or none of these?

a) $x_k + x_{k-1} + y_k = 0$

FIR, because it is linear but does not consider previous values of y

b) $x_k + x_{k-1} + x_{k-2} = 0$

none, because it does not contain y

c) $x_k + y_{k-1} + y_k = 0$

IIR, because it is linear and does consider previous values of y

3.3 Consider the IIR filter $y_k = 2y_{k-1} - y_{k-2} + x_k + x_{k-1}$, $k = 3, 4, \ldots$, $y_1 = y_2 = 0$.

a) What is the filter output sequence y for the input sequence $x = (0, 0, 1, 0, 0, 0, 0, 0)$?

$y_3 = 2y_2 - y_1 + x_3 + x_2 = 0 - 0 + 1 - 0 = 1$
$y_4 = 2y_3 - y_2 + x_4 + x_3 = 2 - 0 + 0 + 1 = 3$
$y_5 = 2y_4 - y_3 + x_5 + x_4 = 6 - 1 + 0 + 0 = 5$
$y_6 = 2y_5 - y_4 + x_6 + x_5 = 10 - 3 + 0 + 0 = 7$
$y_7 = 2y_6 - y_5 + x_7 + x_6 = 14 - 5 + 0 + 0 = 9$
$y_8 = 2y_7 - y_6 + x_8 + x_7 = 18 - 7 + 0 + 0 = 11$
$y = (0, 0, 1, 3, 5, 7, 9, 11)$

b) What is the filter output sequence y for the input sequence $x = (0, 0, 1, a, b, 0, 0, 0)$, $a, b \in \mathbb{R}$?

$y_3 = 2y_2 - y_1 + x_3 + x_2 = 0 - 0 + 1 - 0 = 1$
$y_4 = 2y_3 - y_2 + x_4 + x_3 = 2 - 0 + a + 1 = 3 + a$
$y_5 = 2y_4 - y_3 + x_5 + x_4 = 6 + 2a - 1 + b + a = 5 + 3a + b$
$y_6 = 2y_5 - y_4 + x_6 + x_5 = 10 + 6a + 2b - 3 - a + 0 + b = 7 + 5a + 3b$
$y_7 = 2y_6 - y_5 + x_7 + x_6 = 14 + 10a + 6b - 5 - 3a - b + 0 + 0 = 9 + 7a + 5b$
$y_8 = 2y_7 - y_6 + x_8 + x_7 = 18 + 14a + 10b - 7 - 5a - 3b + 0 + 0 = 11 + 9a + 7b$
$y = (0, 0, 1, 3 + a, 5 + 3a + b, 7 + 5a + 3b, 9 + 7a + 5b, 11 + 9a + 7b)$

c) Give a formula for the filter output y_k, $k = 8, 9, 10, \ldots$, for $x = (0, 0, 1, a, b, 0, 0, 0, \ldots)$, $a, b \in \mathbb{R}$!

$y_k = 2k - 5 + (2k - 7)a + (2k - 9)b$

d) For which finite values of a and b will the filter be unstable?

$y_k = 2k(1 + a + b) - 5 - 7a - 9b$ not finite for $k \to \infty$ if $1 + a + b \neq 0$

e) For which finite values of a and b will the filter converge to $\lim_{k\to\infty} y_k = 0$?

for $1 + a + b = 0 \Leftrightarrow b = -1 - a$: $\lim_{k\to\infty} y_k = -5 - 7a - 9b = 4 + 2a = 0$
for $a = -2$, $b = -1 + 2 = 1$