EJERCICIO 5

Hallar las derivadas parciales de primer y segundo orden de las siguientes funciones:
Mientras no se diga lo contrario se supone que se puede aplicar el teorema de Swartz. Es decir, se supone que las derivadas parciales mixtas respecto a las mismas variables son iguales, con independencia del orden en que se efectue la derivación.

A continuación en color azul se presentan las expresiones de las distintas derivadas parciales, utilizarlas para comprobar los resultados de vuestros cálculos.

a)
$z := x^4 + y^4 - 4 \cdot x^2 \cdot y^2$

$$z := x^4 - 4\,x^2\,y^2 + y^4 \qquad (1)$$

Derivadas Parciales de primer orden:

$\frac{\partial}{\partial x} z$

$$4\,x^3 - 8\,x\,y^2 \qquad (2)$$

$\frac{\partial}{\partial y} z$

$$-8\,x^2\,y + 4\,y^3 \qquad (3)$$

Derivadas parciales de segundo orden:

$Diff(z, x, x) = diff(z, x, x); Diff(z, y, x) = diff(z, y, x); Diff(z, y, y) = diff(z, y, y)$

$$\frac{\partial^2}{\partial x^2}\left(x^4 - 4\,x^2\,y^2 + y^4\right) = 12\,x^2 - 8\,y^2$$

$$\frac{\partial^2}{\partial y\,\partial x}\left(x^4 - 4\,x^2\,y^2 + y^4\right) = -16\,x\,y$$

$$\frac{\partial^2}{\partial y^2}\left(x^4 - 4\,x^2\,y^2 + y^4\right) = -8\,x^2 + 12\,y^2 \qquad (4)$$

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b)

$z := \ln(x^2 + y^2)$

$$z := \ln(x^2 + y^2) \qquad (5)$$

Derivadas Parciales de primer orden:

$\frac{\partial}{\partial x} z$

$$\frac{2\,x}{x^2+y^2} \tag{6}$$

$\frac{\partial}{\partial y} z$

$$\frac{2\,y}{x^2+y^2} \tag{7}$$

Derivadas parciales de segundo orden:

$Diff(z, x, x) = diff(z, x, x); Diff(z, y, x) = diff(z, y, x); Diff(z, y, y) = diff(z, y, y)$

$$\frac{d^2}{dx^2} \ln(x^2+y^2) = \frac{2}{x^2+y^2} - \frac{4\,x^2}{(x^2+y^2)^2}$$
$$\frac{\partial^2}{\partial y\partial x} \ln(x^2+y^2) = -\frac{4\,y\,x}{(x^2+y^2)^2}$$
$$\frac{d^2}{dy^2} \ln(x^2+y^2) = \frac{2}{x^2+y^2} - \frac{4\,y^2}{(x^2+y^2)^2} \tag{8}$$

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c)

$z := x \cdot y + \frac{x}{y}$

$$z := x\,y + \frac{x}{y} \tag{9}$$

Derivadas Parciales de primer orden:

$\frac{\partial}{\partial x} z$

$$y + \frac{1}{y} \tag{10}$$

$\frac{\partial}{\partial y} z$

$$x - \frac{x}{y^2} \tag{11}$$

Derivadas parciales de segundo orden:

$Diff(z, x, x) = diff(z, x, x); Diff(z, y, x) = diff(z, y, x); Diff(z, y, y) = diff(z, y, y)$

$$\frac{\partial^2}{\partial x^2} \left(x\,y + \frac{x}{y}\right) = 0$$

$$\frac{\partial^2}{\partial y \partial x}\left(x\,y+\frac{x}{y}\right)=1-\frac{1}{y^2}$$
$$\frac{\partial^2}{\partial y^2}\left(x\,y+\frac{x}{y}\right)=\frac{2\,x}{y^3} \tag{12}$$

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d)

$z := \arctan\left(\frac{x}{y}\right)$

$$z := \arctan\left(\frac{x}{y}\right) \tag{13}$$

Derivadas Parciales de primer orden:

$\frac{\partial}{\partial x} z : simplify(\%)$

$$\frac{y}{x^2+y^2} \tag{14}$$

$\frac{\partial}{\partial y} z : simplify(\%)$

$$-\frac{x}{x^2+y^2} \tag{15}$$

Derivadas parciales de segundo orden:

$Diff(z, x, x) = simplify(diff(z, x, x)); Diff(z, y, x) = simplify(diff(z, y, x)); Diff(z, y, y) = simplify(diff(z, y, y))$

$$\frac{d^2}{dx^2}\arctan\left(\frac{x}{y}\right)=-\frac{2\,y\,x}{(x^2+y^2)^2}$$
$$\frac{\partial^2}{\partial y \partial x}\arctan\left(\frac{x}{y}\right)=\frac{x^2-y^2}{(x^2+y^2)^2}$$
$$\frac{d^2}{dy^2}\arctan\left(\frac{x}{y}\right)=\frac{2\,y\,x}{(x^2+y^2)^2} \tag{16}$$

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e)

$z := x \cdot \sin(x+y)$

$$z := x\sin(x+y) \tag{17}$$

Derivadas Parciales de primer orden:

$$\frac{\partial}{\partial x} z$$

$$\sin(x+y) + x\cos(x+y) \qquad (18)$$

$$\frac{\partial}{\partial y} z$$

$$x\cos(x+y) \qquad (19)$$

Derivadas parciales de segundo orden:

$Diff(z, x, x) = diff(z, x, x); Diff(z, y, x) = diff(z, y, x); Diff(z, y, y) = diff(z, y, y)$

$$\frac{\partial^2}{\partial x^2} (x\sin(x+y)) = 2\cos(x+y) - x\sin(x+y)$$

$$\frac{\partial^2}{\partial y \partial x} (x\sin(x+y)) = \cos(x+y) - x\sin(x+y)$$

$$\frac{\partial^2}{\partial y^2} (x\sin(x+y)) = -x\sin(x+y) \qquad (20)$$

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f)

$z := (x^2 + y^2) \cdot \exp(x+y)$

$$z := (x^2 + y^2)\, e^{x+y} \qquad (21)$$

Derivadas Parciales de primer orden:

$$\frac{\partial}{\partial x} z$$

$$2\, x\, e^{x+y} + (x^2 + y^2)\, e^{x+y} \qquad (22)$$

$$\frac{\partial}{\partial y} z$$

$$2\, y\, e^{x+y} + (x^2 + y^2)\, e^{x+y} \qquad (23)$$

Derivadas parciales de segundo orden:

$Diff(z, x, x) = diff(z, x, x); Diff(z, y, x) = diff(z, y, x); Diff(z, y, y) = diff(z, y, y)$

$$\frac{\partial^2}{\partial x^2} ((x^2 + y^2)\, e^{x+y}) = 2\, e^{x+y} + 4\, x\, e^{x+y} + (x^2 + y^2)\, e^{x+y}$$

$$\frac{\partial^2}{\partial y \partial x} ((x^2 + y^2)\, e^{x+y}) = 2\, y\, e^{x+y} + 2\, x\, e^{x+y} + (x^2 + y^2)\, e^{x+y}$$

$$\frac{\partial^2}{\partial y^2}\left((x^2+y^2)\,e^{x+y}\right) = 2\,e^{x+y} + 4\,y\,e^{x+y} + (x^2+y^2)\,e^{x+y} \tag{24}$$

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g) Difícil. Consultar la tabla de derivadas de la sección 4.2 de los resúmenes de teoría (Tema 4).

$g := (x, y, z) \rightarrow x^{\frac{y}{z}} : g(x, y, z)$

$$x^{\frac{y}{z}} \tag{25}$$

Derivadas Parciales de primer orden:

$\frac{\partial}{\partial x} g(x, y, z)$

$$\frac{x^{\frac{y}{z}}\,y}{z\,x} \tag{26}$$

$\frac{\partial}{\partial y} g(x, y, z)$

$$\frac{x^{\frac{y}{z}}\,\ln(x)}{z} \tag{27}$$

$\frac{\partial}{\partial z} g(x, y, z)$

$$-\frac{x^{\frac{y}{z}}\,y\ln(x)}{z^2} \tag{28}$$

Derivadas parciales de segundo orden:

$Diff(g(x, y, z), x, x) = diff(g(x, y, z), x, x); Diff(g(x, y, z), y, x) = diff(g(x, y, z), y, x);$
$Diff(g(x, y, z), z, x) = diff(g(x, y, z), z, x)$

$$\frac{\partial^2}{\partial x^2}\left(x^{\frac{y}{z}}\right) = \frac{x^{\frac{y}{z}}\,y^2}{z^2\,x^2} - \frac{x^{\frac{y}{z}}\,y}{z\,x^2}$$

$$\frac{\partial^2}{\partial y\,\partial x}\left(x^{\frac{y}{z}}\right) = \frac{x^{\frac{y}{z}}\,y\ln(x)}{z^2\,x} + \frac{x^{\frac{y}{z}}}{z\,x}$$

$$\frac{\partial^2}{\partial z \partial x}\left(x^{\frac{y}{z}}\right) = -\frac{x^{\frac{y}{z}}\, y^2 \ln(x)}{z^3\, x} - \frac{x^{\frac{y}{z}}\, y}{z^2\, x} \tag{29}$$

$Diff(g(x, y, z), y, y) = diff(g(x, y, z), y, y); Diff(g(x, y, z), y, z) = diff(g(x, y, z), y, z)$

$$\frac{\partial^2}{\partial y^2}\left(x^{\frac{y}{z}}\right) = \frac{x^{\frac{y}{z}} \ln(x)^2}{z^2}$$

$$\frac{\partial^2}{\partial y \partial z}\left(x^{\frac{y}{z}}\right) = -\frac{x^{\frac{y}{z}}\, y \ln(x)^2}{z^3} - \frac{x^{\frac{y}{z}} \ln(x)}{z^2} \tag{30}$$

$Diff(g(x, y, z), z, z) = diff(g(x, y, z), z, z)$

$$\frac{\partial^2}{\partial z^2}\left(x^{\frac{y}{z}}\right) = \frac{x^{\frac{y}{z}}\, y^2 \ln(x)^2}{z^4} + \frac{2\, x^{\frac{y}{z}}\, y \ln(x)}{z^3} \tag{31}$$

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h) Aplicar repetidamente la regla de derivación del producto de funciones. Las expresiones se pueden simplificar sacando factor común.

$h := (x, y, z) \rightarrow x \cdot y \cdot z \cdot \exp(x + y + z) : h(x, y, z)$

$$x\, y\, z\, e^{x+y+z} \tag{32}$$

Derivadas Parciales de primer orden:

$\frac{\partial}{\partial x} h(x, y, z)$

$$y\, z\, e^{x+y+z} + x\, y\, z\, e^{x+y+z} \tag{33}$$

$\frac{\partial}{\partial y} h(x, y, z)$

$$x\, z\, e^{x+y+z} + x\, y\, z\, e^{x+y+z} \tag{34}$$

$\frac{\partial}{\partial z} h(x, y, z)$

$$x\, y\, e^{x+y+z} + x\, y\, z\, e^{x+y+z} \tag{35}$$

Derivadas parciales de segundo orden:

$Diff(h(x, y, z), x, x) = diff(h(x, y, z), x, x); Diff(h(x, y, z), y, x) = diff(h(x, y, z), y, x);$
$Diff(h(x, y, z), z, x) = diff(h(x, y, z), z, x)$

$$\frac{\partial^2}{\partial x^2}\left(x\,y\,z\,\mathrm{e}^{x+y+z}\right)=2\,y\,z\,\mathrm{e}^{x+y+z}+x\,y\,z\,\mathrm{e}^{x+y+z}$$

$$\frac{\partial^2}{\partial y\partial x}\left(x\,y\,z\,\mathrm{e}^{x+y+z}\right)=z\,\mathrm{e}^{x+y+z}+x\,z\,\mathrm{e}^{x+y+z}+y\,z\,\mathrm{e}^{x+y+z}+x\,y\,z\,\mathrm{e}^{x+y+z}$$

$$\frac{\partial^2}{\partial z\partial x}\left(x\,y\,z\,\mathrm{e}^{x+y+z}\right)=y\,\mathrm{e}^{x+y+z}+x\,y\,\mathrm{e}^{x+y+z}+y\,z\,\mathrm{e}^{x+y+z}+x\,y\,z\,\mathrm{e}^{x+y+z} \tag{36}$$

$Diff(h(x, y, z), y, y) = diff(h(x, y, z), y, y); Diff(h(x, y, z), y, z) = diff(h(x, y, z), y, z)$

$$\frac{\partial^2}{\partial y^2}\left(x\,y\,z\,\mathrm{e}^{x+y+z}\right)=2\,x\,z\,\mathrm{e}^{x+y+z}+x\,y\,z\,\mathrm{e}^{x+y+z}$$

$$\frac{\partial^2}{\partial y\partial z}\left(x\,y\,z\,\mathrm{e}^{x+y+z}\right)=x\,\mathrm{e}^{x+y+z}+x\,z\,\mathrm{e}^{x+y+z}+x\,y\,\mathrm{e}^{x+y+z}+x\,y\,z\,\mathrm{e}^{x+y+z} \tag{37}$$

$Diff(h(x, y, z), z, z) = diff(h(x, y, z), z, z)$

$$\frac{\partial^2}{\partial z^2}\left(x\,y\,z\,\mathrm{e}^{x+y+z}\right)=2\,x\,y\,\mathrm{e}^{x+y+z}+x\,y\,z\,\mathrm{e}^{x+y+z} \tag{38}$$

Mathematics 2

Taylor's theorem and local extrema (Chapter 9), Problem 6

Exercise 6

Find a degree-2 Taylor polynomial for function $f(x,y) = xy^2 + \sin xy$ at point $\left(1, \frac{\pi}{2}\right)$.

SOLUTION:

Let us compute the partial derivatives to compute the Taylor polynomial

$$\frac{\partial f}{\partial x}(x,y) = y^2 + y\cos(xy); \qquad \frac{\partial f}{\partial y}(x,y) = 2xy + x\cos(xy); \qquad \frac{\partial^2 f}{\partial x^2}(x,y) = -y^2\sin(xy);$$
$$\frac{\partial^2 f}{\partial xy}(x,y) = 2y + \cos(xy) - yx\sin(xy); \qquad \frac{\partial^2 f}{\partial y^2}(x,y) = 2x - x^2\sin(xy).$$

Hence we have

$$f\left(1,\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\right)^2 + 1, \qquad \frac{\partial f}{\partial x}\left(1,\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\right)^2, \qquad \frac{\partial f}{\partial y}\left(1,\frac{\pi}{2}\right) = \pi,$$
$$\frac{\partial^2 f}{\partial x^2}\left(1,\frac{\pi}{2}\right) = -\left(\frac{\pi}{2}\right)^2, \qquad \frac{\partial^2 f}{\partial xy}\left(1,\frac{\pi}{2}\right) = \frac{\pi}{2}; \qquad \frac{\partial^2 f}{\partial y^2}\left(1,\frac{\pi}{2}\right) = 1.$$

This implies that $P_2 = P_2\left(f, x, y, 1, \frac{\pi}{2}\right)$

$$\begin{aligned}
P_2 &= f\left(1,\frac{\pi}{2}\right) + \frac{\partial f}{\partial x}\left(1,\frac{\pi}{2}\right)(x-1) + \frac{\partial f}{\partial y}\left(1,\frac{\pi}{2}\right)\left(y-\frac{\pi}{2}\right) + \\
&\quad + \frac{\partial^2 f}{\partial x^2}\left(1,\frac{\pi}{2}\right)\frac{(x-1)^2}{2!} + 2\frac{\partial^2 f}{\partial xy}\left(1,\frac{\pi}{2}\right)\frac{(x-1)\left(y-\frac{\pi}{2}\right)}{2!} + \frac{\partial^2 f}{\partial y^2}\left(1,\frac{\pi}{2}\right)\frac{\left(y-\frac{\pi}{2}\right)^2}{2!} \\
&= \left(\frac{\pi}{2}\right)^2 + 1 + \frac{\pi^2}{4}(x-1) + \pi\left(y-\frac{\pi}{2}\right) - \frac{\pi^2}{8}(x-1)^2 + \frac{\pi}{2}(x-1)\left(y-\frac{\pi}{2}\right) + \frac{1}{2}\left(y-\frac{\pi}{2}\right)^2 \\
&= 1 - \frac{\pi^2}{4} + \frac{\pi^2}{4}x - \frac{\pi^2}{8}x^2 + \frac{\pi}{2}xy + \frac{1}{2}y^2.
\end{aligned}$$

Taller M2: FÓRMULA DE TAYLOR
2019-20, M2, FIB

TALLER 9.2

Problema 8

8 Fent ús de polinomis de Taylor de segon grau per a funcions de dues variables, calculeu aproximadament:

a) $\sqrt{1.03 + 2.98}$; b) $\sqrt[3]{0.98 \times 1.02}$; c) $0.95^{2.01}$.

Apartat a)

Si considerem la funció

$f := (x, y) \to \sqrt{x + y}$

$$f := (x, y) \mapsto \sqrt{x + y} \tag{1.1.1.1}$$

aleshores el valor que volem aproximar és $f(1.03, 2.98)$.

El punt $(1.03, 2.98)$ podem considerar que és proper al punt $(1, 3)$, un punt "fàcil" per a la funció ja que $f(1, 3) = \sqrt{4} = 2$

Decidim fer, doncs, el polinomi de **Taylor de grau 2 de f en el punt $(1, 3)$**

$f(1, 3)$

$$2 \tag{1.1.1.2}$$

$\frac{\partial}{\partial x} f(x, y)$

$$\frac{1}{2\sqrt{x + y}} \tag{1.1.1.3}$$

$\frac{\partial}{\partial x} f(x, y)(1, 3) = \frac{1}{4}$

$\frac{\partial}{\partial y} f(x, y)$

$$\frac{1}{2\sqrt{x + y}} \tag{1.1.1.4}$$

$\frac{\partial}{\partial y} f(x, y)(1, 3) = \frac{1}{4}$

$$\frac{\partial^2}{\partial x \partial x} f(x, y)$$

$$-\frac{1}{4\,(x+y)^{3/2}} \tag{1.1.1.5}$$

$$\frac{\partial^2}{\partial x \partial x} f(x, y)(1, 3) = -\frac{1}{32}$$

$$\frac{\partial^2}{\partial x \partial y} f(x, y)$$

$$-\frac{1}{4\,(x+y)^{3/2}} \tag{1.1.1.6}$$

$$\frac{\partial^2}{\partial x \partial y} f(x, y)(1, 3) = -\frac{1}{32}$$

$$\frac{\partial^2}{\partial y \partial y} f(x, y)$$

$$-\frac{1}{4\,(x+y)^{3/2}} \tag{1.1.1.7}$$

$$\frac{\partial^2}{\partial y \partial y} f(x, y)(1, 3) = -\frac{1}{32}$$

Polinomi de Taylor

$$P(x, y) = 2 + \frac{1}{4}(x-1) + \frac{1}{4}(y-3) + \frac{1}{2}\left(-\frac{1}{32}(x-1)^2 - \frac{2\cdot 1}{32}(x-1)(y-3) - \frac{1}{32}(y-3)^2\right)$$

$$\boxed{P(x, y) = 2 + \frac{1}{4}(x-1) + \frac{1}{4}(y-3) - \frac{1}{64}(x-1)^2 - \frac{1}{32}(x-1)\cdot(y-3) - \frac{1}{64}(y-3)^2}$$

Aproximació

$$f(1.03, 2.98) \approx P(1.03, 2.98) = 2 + \frac{1}{4}0.03 - \frac{1}{4}0.02 - \frac{1}{64}0.03^2 + \frac{1}{32}0.03\cdot 0.02 - \frac{1}{64}0.02^2$$

$f(1.03, 2.98) \approx P(1.03, 2.98) = 2 + \frac{3}{400} - \frac{2}{400} - \frac{9}{640000} + \frac{6}{320000} - \frac{4}{64000}$

$= 2 + \frac{1}{400} - \frac{1}{640000}$

$2 + \frac{1}{400} - \frac{1}{640000}$

$$\frac{1281599}{640000} \tag{1.1.1.8}$$

$evalf(\%)$

$$2.002498438 \tag{1.1.1.9}$$

Resposta: $\sqrt{1.03 + 2.98} \approx 2.002498438$

Observació: podeu fer el mateix amb el polinomi de Taylor de $g(x) = \sqrt{x}$ en el punt 4

$P(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$

$P(4.01) = 2 + \frac{1}{4}0.01 - \frac{1}{64}0.01^2 = 2 + \frac{1}{400} - \frac{1}{640000}$

Apartat b)

Si considerem la funció

$f := (x, y) \rightarrow \sqrt[3]{x \cdot y}$

$$f := (x, y) \mapsto (y\,x)^{1/3} \tag{1.1.2.1}$$

aleshores el valor que volem aproximar és $f(0.98, 1.02)$.

El punt $(0.98, 1.02)$ podem considerar que és proper al punt $(1, 1)$, un punt "fàcil" per a la funció ja que $f(1, 1) = \sqrt[3]{1} = 1$

Decidim fer, doncs, el polinomi de **Taylor de grau 2 de f en el punt $(1, 1)$**

$f(1, 1)$

$$1 \tag{1.1.2.2}$$

$\frac{\partial}{\partial x} f(x, y)$

$$\frac{y}{3\,(x\,y)^{2/3}} \tag{1.1.2.3}$$

$$\frac{\partial}{\partial x} f(x, y)(1, 3) = \frac{1}{3}$$

$$\frac{\partial}{\partial y} f(x, y)$$

$$\frac{x}{3\,(x\,y)^{2/3}} \qquad \textbf{(1.1.2.4)}$$

$$\frac{\partial}{\partial y} f(x, y)(1, 3) = \frac{1}{3}$$

$$\frac{\partial^2}{\partial x\,\partial x} f(x, y)$$

$$-\frac{2\,y^2}{9\,(x\,y)^{5/3}} \qquad \textbf{(1.1.2.5)}$$

$$\frac{\partial^2}{\partial x\,\partial x} f(x, y)(1, 3) = -\frac{2}{9}$$

$$\mathit{simplify}\left(\frac{\partial^2}{\partial x\,\partial y} f(x, y)\right)$$

$$\frac{1}{9\,(x\,y)^{2/3}} \qquad \textbf{(1.1.2.6)}$$

$$\frac{\partial^2}{\partial x\,\partial y} f(x, y)(1, 3) = \frac{1}{9}$$

$$\frac{\partial^2}{\partial y\,\partial y} f(x, y)$$

$$-\frac{2\,x^2}{9\,(x\,y)^{5/3}} \qquad \textbf{(1.1.2.7)}$$

$$\frac{\partial^2}{\partial y \partial y} f(x, y)(1, 3) = -\frac{2}{9}$$

Polinomi de Taylor

$$P(x, y) = 1 + \frac{1}{3}(x-1) + \frac{1}{4}(y-1) + \frac{1}{2}\left(-\frac{2}{9}(x-1)^2 + \frac{2 \cdot 1}{9}(x-1)(y-1) - \frac{2}{9}(y-1)^2\right)$$

$$P(x,y) = 1 + \frac{1}{3}(x-1) + \frac{1}{3}(y-1) - \frac{1}{9}(x-1)^2 + \frac{1}{9}(x-1) \cdot (y-1) - \frac{1}{9}(y-1)^2$$

Aproximació

$$f(0.98, 1.02) \approx P(0.98, 1.02) = 1 - \frac{1}{3}0.02 + \frac{1}{3}0.02 - \frac{1}{9}0.02^2 - \frac{1}{9}0.02 \cdot 0.02 - \frac{1}{9}0.02^2$$

$$f(1.03, 2.98) \approx P(1.03, 2.98) = 1 - \frac{4}{30000} = \frac{7499}{7500} = 0.9998666$$

Resposta: $\sqrt[3]{0.98 \cdot 1.02} \approx 0.9998666$

Observació: podeu fer el mateix amb el polinomi de Taylor de $g(x) = \sqrt[3]{x}$ en el punt 1

$$P(x) = 1 + \frac{1}{3}(x-1)$$

$$P(0.98 \times 1.02) = P(0.9996) = 1 - \frac{1}{3}0.0004 = 1 - \frac{4}{3000}$$

Apartat c)

Si considerem la funció

$f := (x, y) \rightarrow x^y$

$$f := (x, y) \mapsto x^y \qquad \textbf{(1.1.3.1)}$$

aleshores el valor que volem aproximar és $f(0.95, 2.01)$.

El punt $(0.95, 2.01)$ podem considerar que és proper al punt $(1, 2)$, un punt "fàcil" per a la funció ja que $f(1, 2) = 1^2 = 1$

Decidim fer, doncs, el polinomi de **Taylor de grau 2 de f en el punt $(1, 2)$**

$f(1, 2)$

$$1 \qquad \textbf{(1.1.3.2)}$$

$\frac{\partial}{\partial x} f(x, y)$

$$\frac{x^y\,y}{x} \quad \textbf{(1.1.3.3)}$$

$\frac{\partial}{\partial x} f(x, y)(1, 2) = 2$

$\frac{\partial}{\partial y} f(x, y)$

$$x^y \ln(x) \quad \textbf{(1.1.3.4)}$$

$\frac{\partial}{\partial y} f(x, y)(1, 2) = 0$

$\frac{\partial^2}{\partial x\,\partial x} f(x, y)$

$$\frac{x^y\,y^2}{x^2} - \frac{x^y\,y}{x^2} \quad \textbf{(1.1.3.5)}$$

$\frac{\partial^2}{\partial x\,\partial x} f(x, y)(1, 2) = 2$

$simplify\left(\frac{\partial^2}{\partial x\,\partial y} f(x, y)\right)$

$$(y \ln(x) + 1)\,x^{y-1} \quad \textbf{(1.1.3.6)}$$

$\frac{\partial^2}{\partial x\,\partial y} f(x, y)(1, 2) = 1$

$\frac{\partial^2}{\partial y\,\partial y} f(x, y)$

$$x^y \ln(x)^2 \quad \textbf{(1.1.3.7)}$$

$\frac{\partial^2}{\partial y\partial y} f(x, y)(1, 2) = 0$

Polinomi de Taylor

$$P(x, y) = 1 + 2(x - 1) + 0(y - 2) + \frac{1}{2}\left(2(x - 1)^2 + 2 \cdot 1(x - 1)(y - 2) - 0(y - 2)^2\right)$$

$$\boxed{P(x, y) = 1 + 2(x - 1) + (x - 1)^2 + (x - 1) \cdot (y - 2)}$$

Aproximació

$f(0.95, 2.01) \approx P(0.95, 2.01) = 1 - 2 \cdot 0.05 + 0.05^2 - 0.05 \cdot 0.01$

$f(0.95, 2.01) \approx P(0.95, 2.01) = 1 - \frac{1}{10} + \frac{25}{10000} - \frac{5}{10000} = 1 - \frac{1}{10} + \frac{1}{500}$

$= \frac{451}{500} = 0.902$

Resposta: $0.95^{2.01} \approx 0.902$

Observació: En aquest cas no es veu cap funció d'una variable amb la qual fer el càlcul.