Unit 2.1: Understanding Parallelism

Video lesson 3: Speed–up and efficiency

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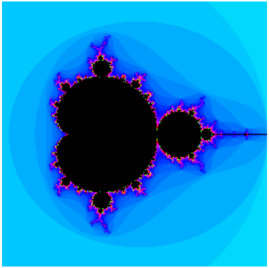
Motivation: Mandelbrot set



- The Mandelbrot set is the set of complex numbers p in a delimited two-dimensional space for which the sequence $z_{n+1} = z_n^2 + p$ (starting with $z_0 = 0$):
$$p,\ p^2 + p,\ (p^2 + p)^2 + p,\ ((p^2 + p)^2 + p)^2 + p,\ \ldots$$
fulfils $|z_\infty| < 2$

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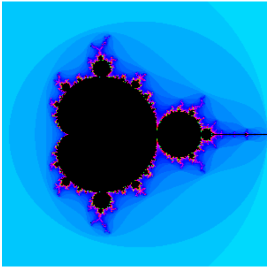
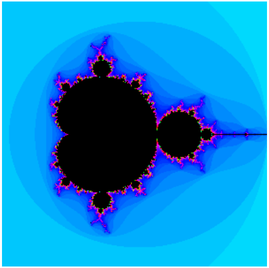


```
n = 0; z.real = z.imag = 0;
do  {
    temp = z.real*z.real - z.imag*z.imag + p.real;
    z.imag = 2*z.real*z.imag + p.imag;
    z.real = temp;
    norm_sq = z.real*z.real + z.imag*z.imag;
} while (norm_sq < (2*2) && ++n < max);
```

limiting the exploration of the sequence up to a maximum number of steps ($n < max$)

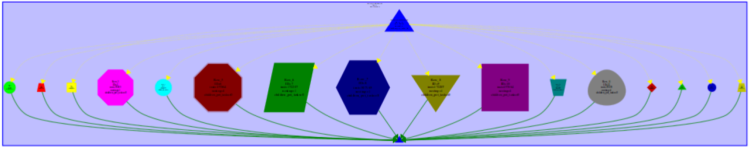
Motivation: Mandelbrot set



- The plot of the Mandelbrot set is created by coloring each point p in the complex plane according to the number of steps n (dark in the plot above if ($n = max$)

- Assume a task corresponds with the computation of the previous recurrence for a set of consecutive rows of the two-dimensional space
- Embarrassingly parallel decomposition of the problem in tasks

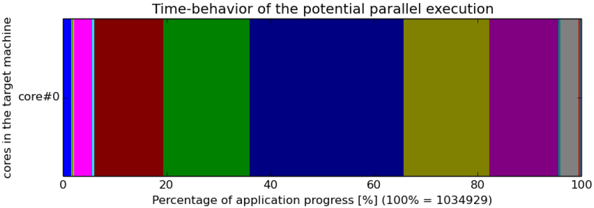
...



... but heavily unbalanced in terms of computational load

Motivation: Mandelbrot set with Tareador

- If we execute the tasks generated in a machine with a single processor ...



the execution time is $T_1 = 1034929$, which we will take as reference time for the sequential execution

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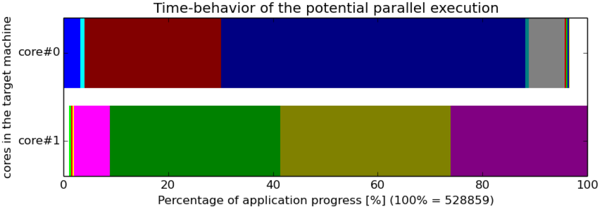
Motivation: Mandelbrot set with Tareador

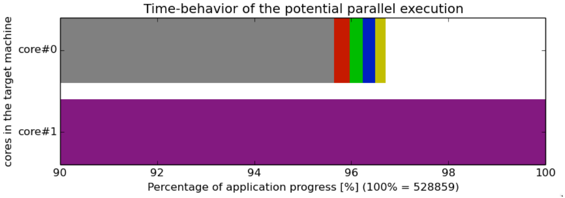
- What if we execute with 2 processors?



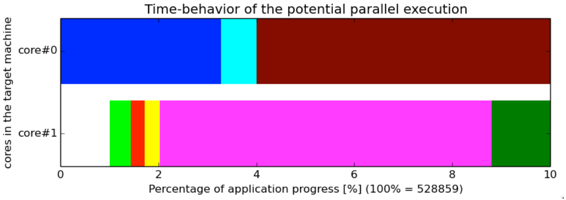
resulting in an execution time of $T_2 = 528859$ time units, 1.95 times faster than the sequential execution

Motivation: Mandelbrot set with Tareador

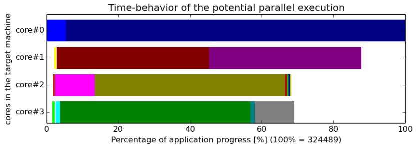
- Load unbalance



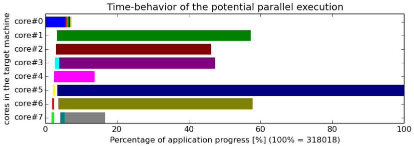
- Task creation overhead



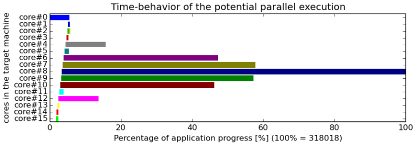
Motivation: Mandelbrot set with Tareador



$T_4 = 324489$, 3.18 times faster



$T_8 = 318018$, 3.25 times faster



$T_{16} = 318018$, 3.25 times faster

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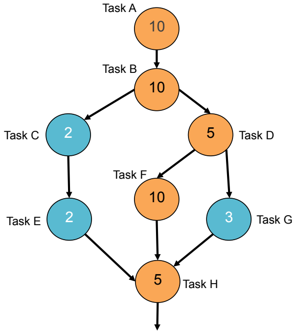
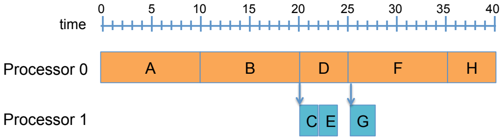
Execution time bounds on P processors



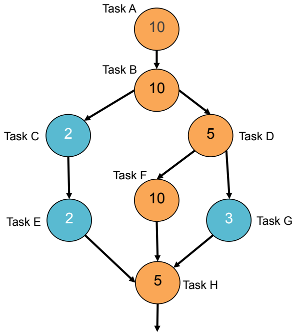
- T_p = execution time on P processors
- **Task scheduling**: how are tasks assigned to processors? For example:



- Lower bounds
  - $T_p \geq T_1/P$
  - $T_p \geq T_\infty$

Speed–up

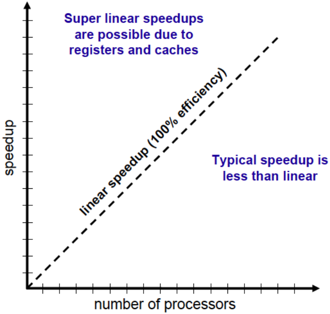
Speedup S_p: relative reduction of the sequential execution time when using P processors



- $S_p = T_1 \div T_p$
- In this example:
  $T_2 = 40, \ S_2 = 47/40 = 1.175$

Scalability and efficiency

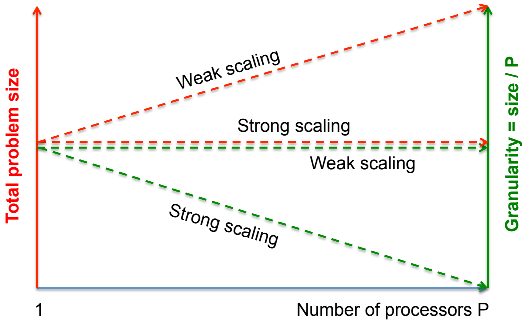
- Scalability: how the speed-up evolves when the number of processors is increased
- Efficiency: $E_p = S_p \div P$



Strong vs. weak scalability

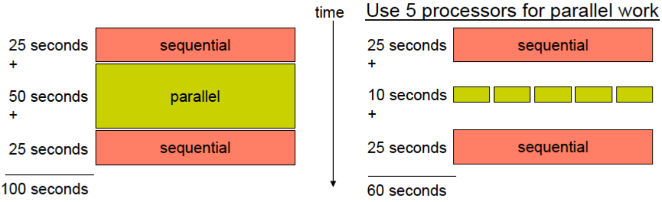
Two usual scenarios to evaluate the scalability of one application:

- Increase the number of processors P with constant problem size (strong scaling $\rightarrow$ reduce the execution time)
- Increase the number of processors P with problem size proportional to P (weak scaling $\rightarrow$ solve larger problem)



Amdahl's law

Performance improvement is limited by the fraction of time the program does not run in fully parallel mode



- Parallel part is 5 times faster: $Speedup_{parallel\_part} = 50/10 = 5$
- Parallel version is just 1.67 times faster: $S_p = 100/60 = 1.67$, $E_p = 1.67/5 = 0.33$

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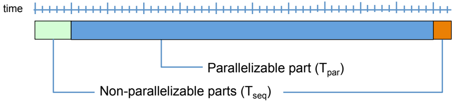
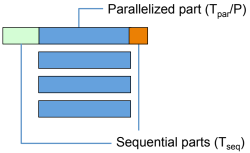
Amdahl's law

Assume the following simplified case, where the parallel fraction φ is the fraction, of total execution time, the program can be parallelized



$$T_1 = T_{seq} + T_{par}$$
$$\varphi = T_{par}/T_1$$
$$T_{seq} = (1-\varphi) \times T_1$$
$$T_{par} = \varphi \times T_1$$

$$T_1 = (1-\varphi) \times T_1 + \varphi \times T_1$$



$$T_P = T_{seq} + T_{par}/P$$
$$T_P = (1-\varphi) \times T_1 + (\varphi \times T_1/P)$$

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Amdahl's law

From where we can compute the speed–up S_P that can be achieved as

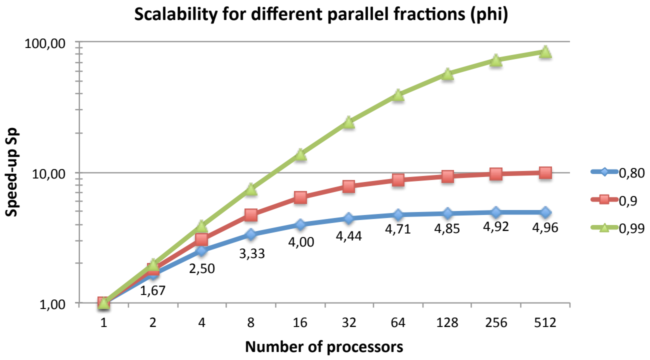
$$S_p = \frac{T_1}{T_p} = \frac{T_1}{(1-\varphi) \times T_1 + (\varphi \times T_1/P)}$$

$$S_p = \frac{1}{((1-\varphi) + \varphi/P)}$$

Two particular cases:

$$\varphi = 0 \rightarrow S_p = 1$$
$$\varphi = 1 \rightarrow S_p = P$$

Amdahl's law



When $P \to \infty$ the expression of the speed–up becomes

$$S_{p\to\infty} = \frac{1}{(1-\varphi)}$$

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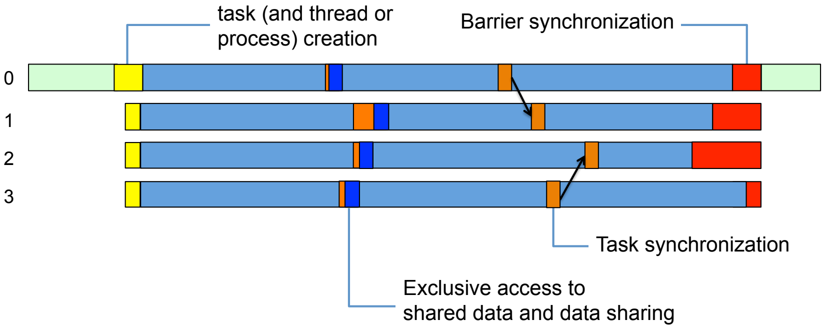
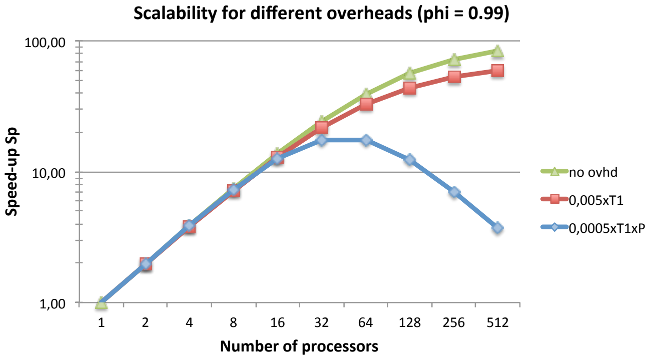
Sources of overhead

Parallel computing is not for free, we should account overheads (i.e. any cost that gets added to a sequential computation so as to enable it to run in parallel)



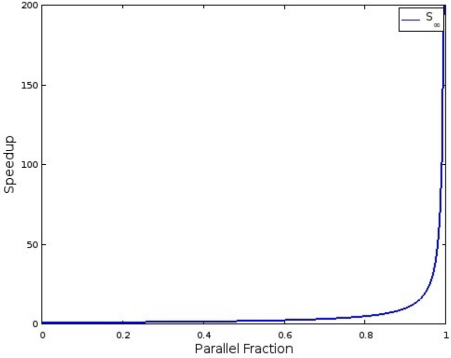
Amdahl's law (with constant and linear overheads)

$$T_p = (1 - \varphi) \times T_1 + \varphi \times T_1/p + overhead(p)$$



Conclusions of Amdahl's Law

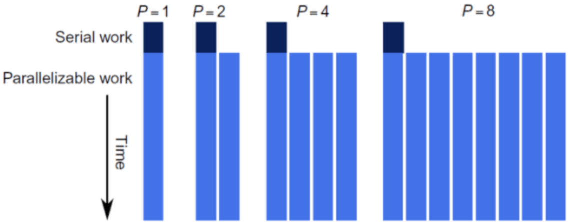
Amdahl's Law can be overly pessimistic:



- Parallel processing might not be worthwhile if there is a large amount of inherently sequential code.

However, often in practice . . .

- The goal of applying parallelism is to increase the accuracy of the solution that can be computed in a fixed amount of time.
  $\rightarrow$ Treat time as constant and let problem size increase with P.
- The serial part grows slowly or remains fixed
  $\rightarrow$ It's proportion gets reduced as the problem size increases.



- Speedup grows as workers are added and the problem size is increased. (Weak Scaling).

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