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A multi-task optimization algorithm via reinforcement learning for multimodal multi-objective optimization

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ABSTRACT

Solving multimodal multi-objective optimization problems (MMOPs) via evolutionary algorithms has recently garnered increasing attention. Maintaining diversity in both decision and objective spaces is crucial for effectively handling MMOPs. However, most traditional multimodal multi-objective evolutionary algorithms (MMEAs) prioritize convergence in the objective space, often eliminating poorly converged solutions which could enhance diversity in the decision space. To address this issue, this paper proposes a novel MMEA, named QLMTMMEA. Specifically, a multi-task optimization framework comprises a main task and three auxiliary tasks based on different strategies for MMOPs is designed. Then, Q-Learning (QL) is utilized to the adaptively selects optimal auxiliary tasks in the evolution process. In addition, a new diversity enhancement technique is proposed for objective space and decision space by dynamically adjusting the relaxation factor to maintain high quality solutions. Seven state-of-the-art MMEAs are adopted to make comparisons for demonstrating the performance of QLMTMMEA, experimental results show that QLMTMMEA is competitive compared to others MMEAs on 34 complex MMOPs.

1. Introduction

In the real world, there exists various problems with two or more conflict objectives that need to be minimized, these problems are referred to as multi-objective optimization problems (MOPs) (Deb, 2005, 2020). The MOPs can be defined as follows:

$$\text{Minimize}: F(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_m(\mathbf{x})\} \tag{1}$$

$$\text{s.t.}: \mathbf{x} = (x_1, x_2, \cdots, x_n) \in \Omega$$

where Ω denotes the search space, m is the number of objectives, and x is a decision vector consisting of n decision variables x_i.

Over the past few decades, researchers have developed many multi-objective optimization algorithms (MOEAs) and applied them to real-world problems(Liu, Zhang, Zhu, Tong, & Yuan, 2024). The primary goal of a MOEA is to obtain a Pareto set (PS) mapped to the Pareto front (PF)(Coello, 2006). However, in many practical problems, a single PF often corresponds to multiple PSs (Tanabe and Ishibuchi, 2020a). For instance, the feature selection optimization problem, the shop scheduling problem, and the multi-objective knapsack problem are all part of the multi-objective optimization problem with multiple PSs (Jaszkiewicz, 2002; Yacong, Aimin, & Yan, 2020). In such practical problems, MMOPs arise when several feasible configurations or designs meet different optimization criteria, leading to a set of solutions that are optimal in different ways but similarly effective overall. Understanding and efficiently solving MMOPs are crucial for decision makers who aim to explore a range of trade-offs and alternatives rather than settling for a single optimal solution (Li, Zhang, Wang, Huang, & Liang, 2023).

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Therefore, with multiple PS problems, the algorithm that locates only one PS cannot meet the different requirements of decision makers. This kind of MOPs with multiple PS is called MMOPs (Liang, Yue, & Qu, 2016). Current MOEAs struggle to solve MMOPs because they have difficulty locating multiple PSs. Traditional MOEAs are mainly concerned with locating the Pareto front in objective space. However, when dealing with MMOPs, it is crucial that the population maintain diversity in the decision space, and algorithms without this mechanism perform poorly when dealing with MMOPs with multiple PS (Cao, Qi, Chen, & Zhang, 2024; Lin, et al., 2021).

The primary approach for solving MMOPs involves the use of niching techniques, such as fitness sharing (Goldberg & Richardson, 1987), crowding (Thomsen, 2004), and clustering (Wang, et al., 2020). Liu et al. introduced the DNEA-L (Liu, Ishibuchi, Nojima, Masuyama, & Han, 2019). Yue et al. proposed MO_Ring_PSO_SCD, which employs a ring topology to promote stable niching within populations (Yue, Qu, & Liang, 2018). After that, Lin et al. proposed MMOEA/DC, which uses dual clustering in both decision and objective spaces (Lin, et al., 2021). This algorithm targets global PSs while maintaining a balance between global and local PSs during the evolutionary process. Li et al. introduced a MMEA based on a hierarchical ranking method to capture both global and local PSs (Li, Yao, Zhang, Wang, & Wang, 2023).

Recent advancements have driven the development of designing specialized MMEAs for addressing MMOPs (Liang, Lin, Yue, Suganthan, & Wang, 2024). Techniques such as adaptive diversity preservation and multi-population strategies have been proposed to enhance the ability of algorithms to concurrently discover and maintain multiple PSs (Tanabe & Ishibuchi, 2019; Tian, He, Cheng, & Zhang, 2021). Examples include the weighted indicator-based evolutionary algorithm (Li, Zhang, Wang, & Ishibuchi, 2021), the balancing convergence and diversity in objective and decision spaces algorithm(Ming, Gong, Wang, & Gao, 2023), and the coevolutionary framework for dealing with MMOPs (Li, Yao, Li, et al., 2023; Liu, et al., 2024). These algorithms aim to strike a balance between convergence towards PF and exploration of diverse PSs in the decision space.

MMOPs has multiple global optimal solutions, and these solutions may be spread across different search spaces. This makes it necessary for algorithms to explore several potential solution spaces at the same time in order to find as many global optima as possible. Multi-task optimization can simultaneously address multiple related tasks, allowing for a more effective exploration and utilization of the shared information between these tasks (Qiao, Liang, Yu, Ban, et al., 2024). In MMOPs, sharing information helps the algorithm balance multiple objectives and optimize them together. By addressing multiple objectives simultaneously, the algorithm can more easily find solutions that satisfy all of them (Qiao, et al., 2024). Reinforcement learning learns through trial and error and adjusts its strategy based on environmental feedback. This enables it to maximize long-term rewards. Meanwhile, reinforcement learning excels at balancing exploration and exploitation. This balance is crucial in MMOPs because the algorithm must explore various solution spaces while utilizing known information to accelerate the optimization process. For MMOPs, Reinforcement learning can adjust its search strategy to better fit different search objectives (Liang, et al., 2023). This makes it easier for the algorithm to find better solutions when the optimization problem is complex. By designing the reward function well and updating strategies effectively, reinforcement learning can balance exploration and exploitation, improving its efficiency in solving MMOPs.

In summary, multi-task optimization and reinforcement learning offer distinct advantages in addressing MMOPs problems. Multi-task learning enhances algorithm performance by sharing information and optimizing tasks simultaneously, while reinforcement learning adaptively selects auxiliary tasks and guides search strategies throughout the optimization process. By combining these two methods, it is an effective way to solve MMOPs.

One of the primary challenges in handling MMOPs is maintaining sufficient diversity in both objective and decision spaces (Tanabe and Ishibuchi, 2020a). Traditional MOEAs, which are adapt at achieving convergence towards a single PF, often struggle to preserve diversity across multiple PSs. This limitation can lead to premature convergence towards locally optimal subsets of solutions, resulting in missed opportunities for valuable and diverse alternatives (Li, Zhang, et al., 2023). By applying evolutionary multi-task optimization (EMTO) to MMOPs, algorithms can improve diversity in both objective and decision spaces through population diversity and knowledge sharing (Qiao, Liang, et al., 2023). Additionally, introducing reinforcement learning (RL) into EMTO can enhance the effectiveness of the search process (Li, Zhang, & Wang, 2021).

To address this problem, this paper proposed an adaptive auxiliary task selection method based on QL techniques for multimodal multi-objective optimization (QLMTMMEA). Specifically, the main contributions are concluded as follows:

(1) A general multi-task optimization framework is proposed to transform the original MMOPs into a multitasking optimization problem. It includes a main task for the sufficient diversity in decision space and three auxiliary tasks for diversity in objective space, convergence in decision space and convergence in objective space.

(2) The RL techniques are implemented in the framework to choose the most suitable auxiliary task adaptively. As instantiations, the QL-based auxiliary task selection methods are designed. With the help of QL techniques, selecting the most suitable auxiliary task considers not only the historical experience of evolution but also the influence of the strategy in the future state.

(3) The diversity enhancement technique involves using a dynamically adjusted relaxation factor. This factor allows for the preservation of some potential solutions that are dominated in the objective space but exhibit good diversity in the decision space. By relaxing the convergence criteria, this technique aims to improve the detection of all segments of the PS, thereby promoting diversity in both the objective space and the decision space.

The rest of the paper is organized as follows: Section 2 presents the related work of algorithm design and motivation. Section 3 clarifies the details of algorithm. Section 4 presents the experimental results and discussions. Finally, the paper is summarized in Section 5.

2. Related work and motivation

2.1. Convergence-First MMEAs

Convergence-first approaches in MMEAs focus on achieving convergence towards the Pareto front in complex objective spaces with multiple optima. Liu et al. introduced a dual-archive strategy, where one archive focuses on solutions with superior performance indicators to enhance convergence, while the other promotes diversity through non-dominated sorting, effectively balancing exploration and refinement (Liu, Yen, & Gong, 2019). Tanabe and Ishibuchi proposed dividing the objective space into sub-regions and selecting the best-converged solution from each, thereby enhancing exploration and guiding the search toward promising regions of the PF (Tanabe and Ishibuchi, 2020b). Peng and Ishibuchi extended this concept with a diversity-enhanced framework that filters out dominated solutions and employs subset selection to foster both convergence and diversity (Peng & Ishibuchi, 2022). Tanabe et al. utilized a niching-based indicator preserving Pareto dominance for diversity and convergence towards the optimal front (Tanabe & Ishibuchi, 2019). Fan and Yan proposed zoning search, dividing decision space into subspaces and applying niching to maintain diversity (Fan & Yan, 2021). But these algorithms still have some limitations.

Therefore, some researchers consider using ε-Pareto dominance to deal with convergence-first. Different from Pareto dominance, the ε-Pareto dominance is defined as follows (Hernández-Díaz, Santana-Quintero, Coello, & Molina, 2007).

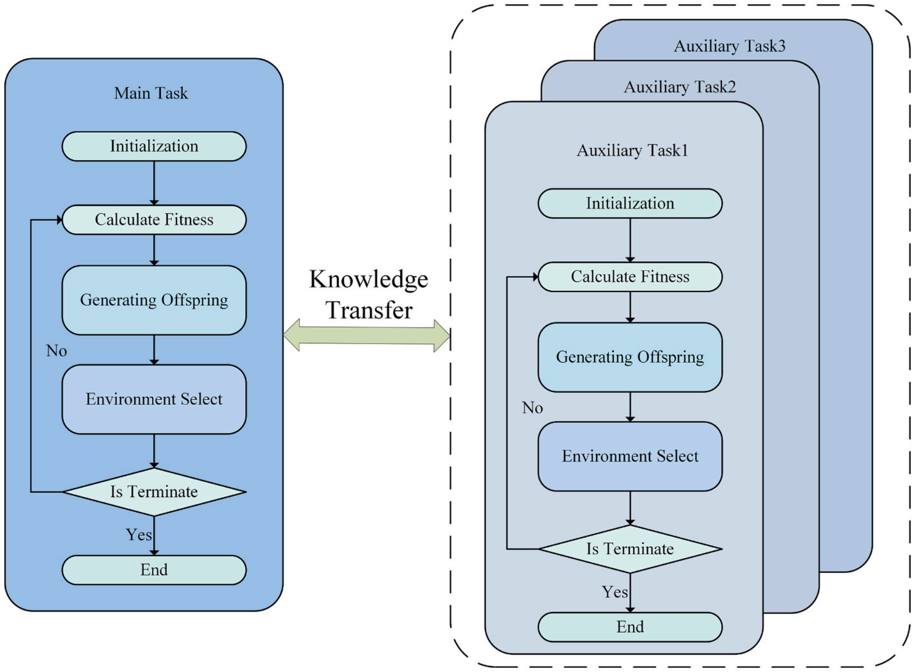

Fig. 1. EMTO mechanism.

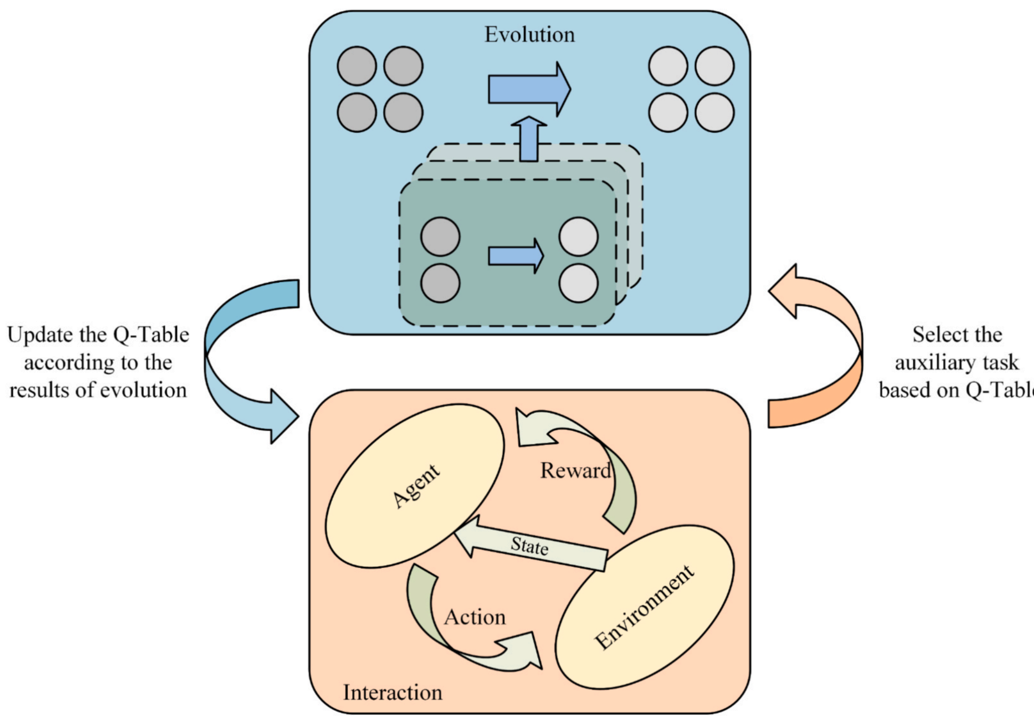

Fig. 2. Evolution process based on QL.

ε-Pareto dominance: for two solutions $x_1, x_2 \in S$. x_1 is said to ε-Pareto dominate x_2 (denoted as $x_1 \prec_\varepsilon x_2$), if the following two conditions are met:

$$\forall i \in \{1, \cdots, m\}, f_i(x_2) - f_i(x_1) \geq \varepsilon \tag{2}$$

$$\exists j \in \{1, \cdots, m\}, f_j(x_2) - f_j(x_1) > \varepsilon$$

It is noteworthy that if $\varepsilon = 0$, then $\prec_\varepsilon$ plays the same role as Pareto dominance. In the context of ε-Pareto dominance, it can be understood that if the difference in objective function values between solutions falls within ε, then a dominance relation does not exist (i.e., they are non-dominated with respect to each other).

2.2. Evolutionary multi-task optimization

EMTO is a method for simultaneously optimizing multiple tasks using evolutionary algorithms, sharing information between tasks to improve overall optimization efficiency (Wang, Kang, & Zhou, 2022). As show in Fig. 1, the core advantage of EMTO is resource sharing and knowledge transfer. By sharing computing resources and data between

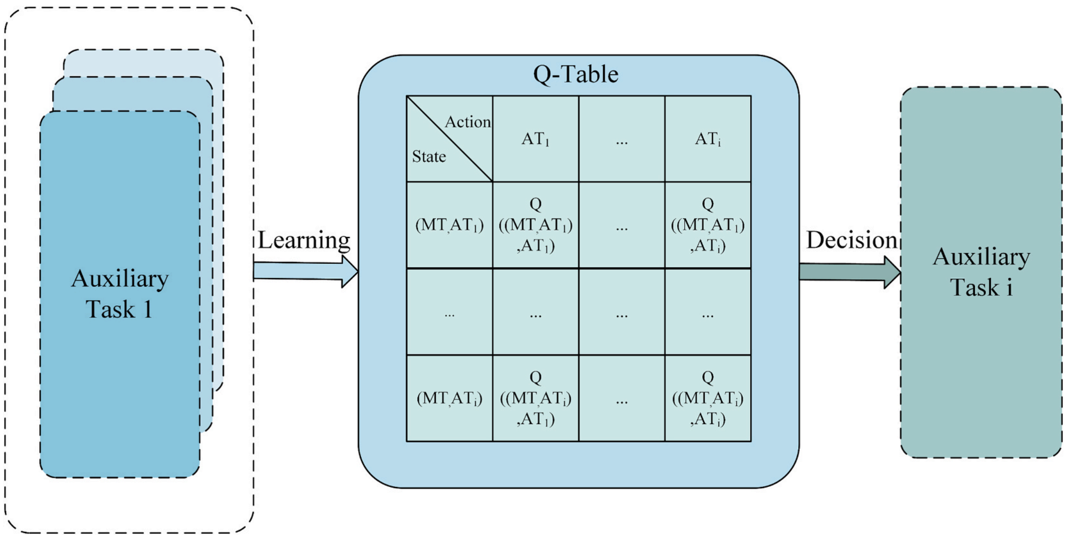

Fig. 3. Q-Table of the proposed model.

Table 1
IGDX results of the compared algorithms on MMOPs.

Problem	DN-NSGA-II	MO_Ring_PSO_SCD	TriMOEA-TA&R	MMEA-WI	HREA	CMMO	CoMMEA	QLMTMMEA
IDMPM2T1	5.3868e-1 (2.74e-1) −	8.3418e-2 (2.00e-1) −	6.2982e-1 (1.65e-1) −	8.1677e-4 (4.38e-5) −	6.1014e-4 (2.40e-4) −	4.5416e-2 (1.71e-1) −	5.5935e-4 (2.13e-5) −	**4.7079e-4 (6.67e-5)**
IDMPM2T2	4.2669e-1 (3.30e-1) −	3.8945e-3 (1.80e-3) −	4.2694e-1 (3.29e-1) −	1.0359e-3 (5.54e-5) −	8.2363e-4 (4.20e-5) −	7.2022e-4 (6.23e-5) =	7.7573e-4 (6.51e-5) −	**7.0409e-4 (9.96e-5)**
IDMPM2T3	6.3940e-2 (1.66e-1) −	2.3132e-3 (1.84e-4) −	1.8244e-1 (2.76e-1) −	1.4769e-3 (1.03e-4) =	1.3856e-3 (2.93e-4) =	1.3017e-3 (2.18e-4) =	**1.0793e-3 (6.51e-5)** +	1.4531e-3 (4.70e-4)
IDMPM2T4	6.0610e-1 (2.05e-1) −	7.1526e-3 (8.69e-3) −	5.8361e-1 (2.32e-1) −	2.3317e-2 (1.23e-1) −	7.3972e-4 (9.80e-4) =	9.0289e-2 (2.33e-1) =	5.5394e-4 (1.53e-5) =	**7.5483e-4 (9.22e-4)**
IDMPM3T1	5.5700e-1 (2.85e-1) −	1.8957e-1 (1.75e-1) −	7.1437e-1 (2.57e-1) −	9.4365e-3 (2.26e-4) −	8.1130e-3 (1.02e-4) −	8.2024e-3 (1.73e-4) −	8.9570e-3 (1.43e-4) −	**7.6512e-3 (4.39e-4)**
IDMPM3T2	5.1401e-1 (2.31e-1) −	9.5370e-2 (1.12e-1) −	5.7452e-1 (2.29e-1) −	1.7674e-2 (4.43e-2) −	**8.2212e-3 (1.23e-4)** =	4.1288e-2 (8.55e-2) =	9.0079e-3 (1.67e-4) −	8.9574e-3 (2.87e-3)
IDMPM3T3	4.2729e-1 (3.28e-1) −	1.5225e-2 (2.46e-3) +	4.6380e-1 (2.88e-1) −	1.0071e-2 (2.31e-4) +	**9.0708e-3 (6.90e-4)** =	9.7701e-3 (2.91e-3) =	9.7348e-3 (1.59e-4) +	1.7586e-2 (4.58e-2)
IDMPM3T4	7.4230e-1 (2.65e-1) −	1.2832e-1 (1.51e-1) −	7.5369e-1 (2.01e-1) −	1.3161e-1 (1.24e-1) −	**8.0877e-3 (1.17e-4)** =	1.0266e-1 (1.49e-1) −	2.5087e-2 (6.18e-2) =	3.4604e-2 (7.15e-2)
IDMPM4T1	1.1480e+0 (1.23e-1) −	9.1238e-1 (3.40e-1) −	1.1778e+0 (7.89e-2) −	4.5862e-2 (9.07e-2) =	3.0541e-1 (3.06e-1) −	5.2114e-1 (3.57e-1) −	4.5523e-2 (9.11e-2) −	**1.5538e-2 (6.37e-3)**
IDMPM4T2	1.1120e+0 (1.40e-1) −	3.3193e-1 (2.49e-1) −	1.0173e+0 (2.21e-1) −	2.5728e-1 (1.83e-1) =	4.3796e-1 (2.69e-1) −	3.4546e-1 (2.63e-1) −	2.3578e-1 (1.84e-1) =	**1.0265e-1 (1.15e-1)**
IDMPM4T3	8.6557e-1 (3.14e-1) −	**5.5233e-2 (7.76e-2)** =	9.4512e-1 (2.32e-1) −	2.7840e-1 (2.13e-1) −	6.7373e-1 (3.62e-1) −	2.4794e-1 (2.20e-1) −	8.1051e-2 (1.17e-1) +	9.5025e-2 (1.12e-1)
IDMPM4T4	1.0395e+0 (2.05e-1) −	2.6885e-1 (2.16e-1) −	1.1135e+0 (1.55e-1) −	3.8129e-1 (1.66e-1) −	6.4599e-1 (3.75e-1) −	2.0494e-1 (2.23e-1) =	2.9158e-1 (2.56e-1) =	**1.4109e-1 (1.33e-1)**
MMF1	4.3416e-2 (2.24e-3) −	6.2752e-2 (8.51e-3) −	5.9986e-2 (6.18e-3) −	4.1399e-2 (2.44e-3) −	3.3850e-2 (1.04e-3) −	3.3320e-2 (1.02e-3) −	3.4035e-2 (8.75e-4) −	**2.7824e-2 (8.47e-4)**
MMF2	5.1855e-2 (2.32e-2) −	7.5404e-2 (4.05e-2) −	8.4114e-2 (5.83e-2) −	2.0109e-2 (4.77e-3) −	2.4989e-2 (8.89e-3) −	1.9112e-2 (4.00e-3) −	1.7609e-2 (3.09e-3) −	**1.3055e-2 (2.34e-3)**
MMF3	4.4269e-2 (2.27e-2) −	6.0600e-2 (2.95e-2) −	5.2659e-2 (2.79e-2) −	1.7304e-2 (4.92e-3) −	1.8117e-2 (4.52e-3) −	1.6104e-2 (3.72e-3) −	1.4119e-2 (2.54e-3) −	**1.2201e-2 (2.23e-3)**
MMF4	2.7325e-2 (3.04e-3) −	4.7347e-2 (1.30e-2) −	6.9321e-2 (1.18e-1) −	2.3523e-2 (1.25e-3) −	1.8472e-2 (5.77e-4) −	1.9081e-2 (9.13e-4) −	2.2402e-2 (5.28e-4) −	**1.6524e-2 (7.26e-4)**
MMF5	8.3705e-2 (4.80e-3) −	1.2259e-1 (2.23e-2) −	9.9107e-2 (1.23e-2) −	6.9872e-2 (3.65e-3) −	5.7855e-2 (1.38e-3) −	6.0276e-2 (1.82e-3) −	5.7461e-2 (1.38e-3) −	**4.4251e-2 (1.69e-3)**
MMF6	7.3141e-2 (3.70e-3) −	9.2163e-2 (1.12e-2) −	8.0144e-2 (5.59e-3) −	6.1891e-2 (1.85e-3) −	5.3925e-2 (1.48e-3) −	5.3497e-2 (1.26e-3) −	5.3979e-2 (1.01e-3) −	**4.0379e-2 (1.54e-3)**
MMF7	2.2887e-2 (2.06e-3) −	4.0815e-2 (9.18e-3) −	4.1515e-2 (2.44e-2) −	2.4825e-2 (1.92e-3) −	1.9198e-2 (6.99e-4) −	1.9323e-2 (8.86e-4) −	2.1671e-2 (7.76e-4) −	**1.6157e-2 (9.61e-4)**
MMF8	9.4430e-2 (3.75e-2) −	1.3032e-1 (4.84e-2) −	3.9238e-1 (1.10e-1) −	6.0551e-2 (1.14e-2) −	4.0744e-2 (2.87e-3) −	4.4797e-2 (6.57e-3) −	4.8174e-2 (1.38e-3) −	**3.5486e-2 (2.13e-3)**
+/-/=	0/20/0	1/18/1	0/20/0	1/16/3	0/15/5	0/14/6	3/13/4	

different tasks, EMTO reduces the computational overhead per task and improves the overall optimization efficiency. In addition, the knowledge and strategies accumulated during optimization can be transferred between different tasks, speeding up the convergence process for each task, even when the tasks exhibit some differences (Qiao, et al., 2022). EMTO also preserves the population diversity by swapping individuals among multiple tasks, avoiding premature convergence to a local optimal solution. This diversity maintenance mechanism enhances the discovery of

Table 2
IGD results of the compared algorithms on MMOPs.

Problem	DN-NSGA-II	MO_Ring_PSO_SCD	TriMOEA-TA&R	MMEA-WI	HREA	CMMO	CoMMEA	QLMTMMEA
IDMPM2T1	5.0956e-4 (2.61e-5) −	1.6536e-3 (1.21e-4) −	6.6886e-4 (1.70e-5) −	6.5237e-4 (2.51e-5) −	5.1267e-4 (3.93e-5) −	5.0733e-4 (5.30e-5) −	4.5297e-4 (5.82e-5) =	**4.5260e-4 (4.27e-5)**
IDMPM2T2	4.9991e-4 (2.67e-5) −	8.3031e-4 (3.28e-5) −	6.5225e-4 (4.44e-5) −	6.4464e-4 (3.00e-5) −	5.1866e-4 (1.57e-5) −	4.8650e-4 (1.63e-5) −	4.1607e-4 (1.27e-5) =	**4.1972e-4 (3.06e-5)**
IDMPM2T3	4.5686e-4 (2.52e-5) −	8.9218e-4 (4.36e-5) −	1.2607e-2 (2.58e-2) −	6.8342e-4 (2.79e-5) −	5.5757e-4 (1.59e-5) −	5.1209e-4 (1.78e-5) −	4.8100e-4 (1.68e-5) −	**4.2779e-4 (2.87e-5)**
IDMPM2T4	4.8494e-4 (1.85e-5) −	1.0014e-3 (6.38e-5) −	6.6808e-4 (3.50e-5) −	6.9063e-4 (4.47e-5) −	4.7726e-4 (2.49e-5) −	4.7075e-4 (5.11e-5) −	4.1677e-4 (4.91e-5) =	**4.2053e-4 (2.45e-5)**
IDMPM3T1	6.3591e-3 (1.66e-4) −	9.9529e-3 (5.71e-4) −	1.1296e-2 (7.40e-4) −	6.2925e-3 (1.73e-4) −	5.8170e-3 (1.68e-4) −	5.7577e-3 (1.99e-4) −	5.8669e-3 (1.68e-4) −	**5.4478e-3 (3.24e-4)**
IDMPM3T2	6.3266e-3 (2.41e-4) −	7.8087e-3 (3.58e-4) −	1.0951e-2 (8.06e-4) −	6.1277e-3 (2.12e-4) −	5.6507e-3 (1.15e-4) −	5.5536e-3 (1.40e-4) −	5.6795e-3 (1.70e-4) −	**5.1895e-3 (3.37e-4)**
IDMPM3T3	6.2933e-3 (1.89e-4) −	8.0973e-3 (4.42e-4) −	1.0752e-2 (8.83e-4) −	6.2643e-3 (1.64e-4) −	5.7628e-3 (1.41e-4) −	5.6206e-3 (1.31e-4) −	6.0627e-3 (1.58e-4) −	**5.2829e-3 (3.33e-4)**
IDMPM3T4	6.2602e-3 (2.88e-4) −	9.5724e-3 (8.02e-4) −	1.1467e-2 (7.59e-4) −	6.1496e-3 (2.03e-4) −	5.6598e-3 (1.52e-4) −	5.5237e-3 (1.74e-4) −	5.6638e-3 (1.62e-4) −	**5.2332e-3 (3.86e-4)**
IDMPM4T1	9.2609e-3 (2.83e-4) −	2.6602e-2 (2.03e-3) −	2.7074e-2 (1.75e-3) −	9.9250e-3 (6.49e-4) −	9.0515e-3 (1.23e-3) −	8.6109e-3 (1.08e-3) =	9.6964e-3 (7.70e-4) −	**8.1555e-3 (8.37e-4)**
IDMPM4T2	8.5909e-3 (2.10e-4) −	1.7665e-2 (1.64e-3) −	2.7229e-2 (1.96e-3) −	8.9702e-3 (3.43e-4) −	7.9957e-3 (6.91e-4) =	8.1899e-3 (5.35e-4) −	8.1675e-3 (6.72e-4) −	**7.7093e-3 (7.66e-4)**
IDMPM4T3	8.8118e-3 (3.97e-4) −	1.8431e-2 (1.52e-3) −	2.9380e-2 (1.54e-2) −	9.2273e-3 (3.89e-4) −	7.9619e-3 (1.02e-3) =	8.3025e-3 (4.13e-4) −	9.0123e-3 (3.60e-4) −	**7.9505e-3 (6.38e-4)**
IDMPM4T4	8.5441e-3 (2.22e-4) −	2.4097e-2 (3.29e-3) −	2.7583e-2 (1.96e-3) −	9.1517e-3 (9.19e-4) −	8.5179e-3 (1.52e-3) =	8.4727e-3 (5.20e-4) −	8.1922e-3 (7.10e-4) =	**7.9943e-3 (7.22e-4)**
MMF1	**1.2583e + 0 (1.04e-4)** +	1.2602e + 0 (1.65e-3) −	1.2635e + 0 (7.24e-3) =	1.2584e + 0 (3.36e-4) +	1.2586e + 0 (4.95e-4) −	1.2584e + 0 (7.31e-5) =	1.2584e + 0 (8.72e-5) =	1.2584e + 0 (7.30e-5)
MMF2	6.1076e-1 (1.21e-2) =	**4.8781e-1 (4.14e-2)** +	5.9308e-1 (4.26e-2) =	6.1395e-1 (1.64e-3) =	6.1422e-1 (3.39e-3) =	6.1395e-1 (1.90e-3) =	6.1323e-1 (3.27e-3) =	6.1344e-1 (2.59e-3)
MMF3	4.6493e-1 (3.56e-3) =	**4.4594e-1 (5.27e-3)** +	4.5843e-1 (7.23e-3) +	4.6676e-1 (1.13e-3) =	4.6556e-1 (3.59e-3) =	4.6699e-1 (1.20e-3) −	4.6511e-1 (3.09e-3) =	4.6612e-1 (1.57e-3)
MMF4	6.1640e-1 (3.85e-2) −	5.6474e-1 (5.30e-2) =	6.2553e-1 (2.81e-2) −	6.0887e-1 (3.73e-2) =	5.7391e-1 (6.16e-2) =	5.9581e-1 (5.83e-2) =	5.9792e-1 (4.54e-2) =	**5.8138e-1 (5.54e-2)**
MMF5	1.6900e + 0 (3.55e-2) −	**1.6390e + 0 (5.56e-2)** +	1.6637e + 0 (6.09e-2) =	1.6783e + 0 (4.23e-2) =	1.6710e + 0 (5.14e-2) =	1.6602e + 0 (5.29e-2) =	1.6589e + 0 (5.29e-2) =	1.6653e + 0 (5.68e-2)
MMF6	1.3835e + 0 (7.05e-3) −	1.3747e + 0 (6.86e-3) =	1.3806e + 0 (6.92e-3) =	1.3808e + 0 (6.61e-3) =	1.3804e + 0 (6.86e-3) =	1.3786e + 0 (7.73e-3) =	1.3788e + 0 (7.53e-3) =	**1.3778e + 0 (7.74e-3)**
MMF7	1.0510e + 0 (3.56e-5) =	1.0521e + 0 (9.33e-4) −	**1.0508e + 0 (3.83e-4)** +	1.0510e + 0 (9.84e-5) +	1.0511e + 0 (8.91e-5) =	1.0511e + 0 (1.63e-4) =	1.0511e + 0 (2.58e-4) =	1.0512e + 0 (6.26e-4)
MMF8	3.7215e + 0 (2.66e-4) −	**2.6227e + 0 (6.74e-1)** +	3.7222e + 0 (9.13e-5) −	3.7213e + 0 (2.37e-4) =	3.7213e + 0 (2.66e-4) =	3.7216e + 0 (2.50e-4) −	3.7203e + 0 (6.36e-4) +	3.7213e + 0 (2.63e-4)
+/-/=	1/16/3	4/14/2	2/14/4	2/12/6	0/10/10	0/13/7	1/8/11	

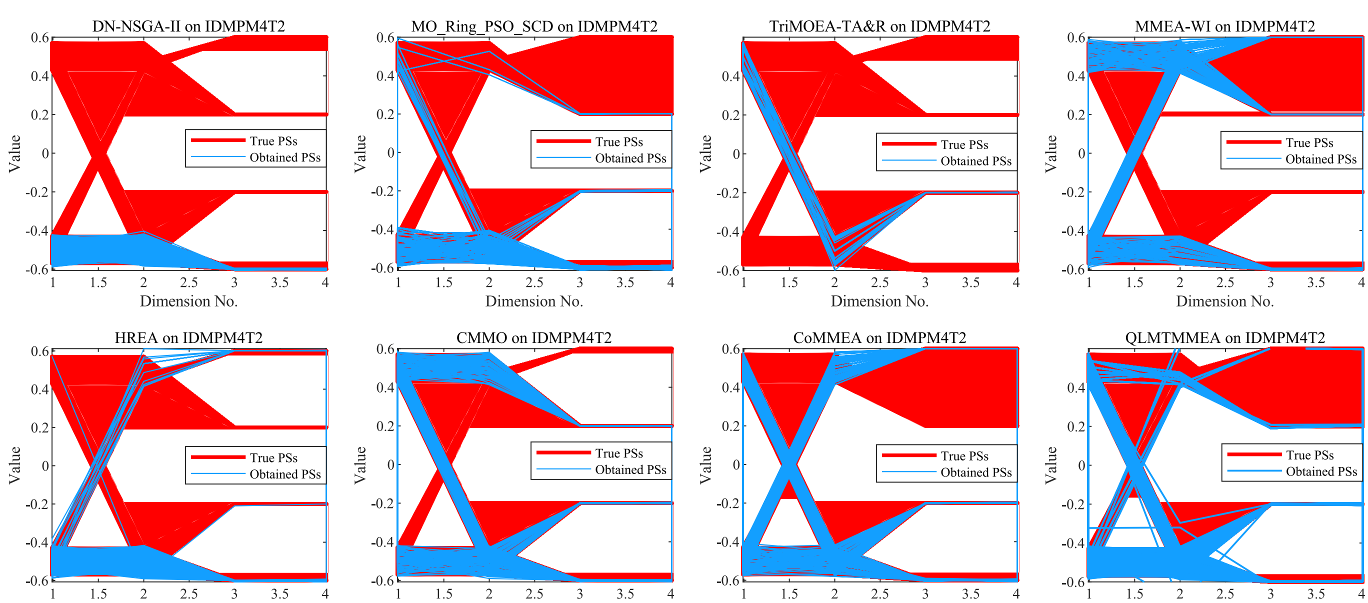

Fig. 4. Distribution of solutions in the decision space on IDMPM4T2 obtained by different algorithms.

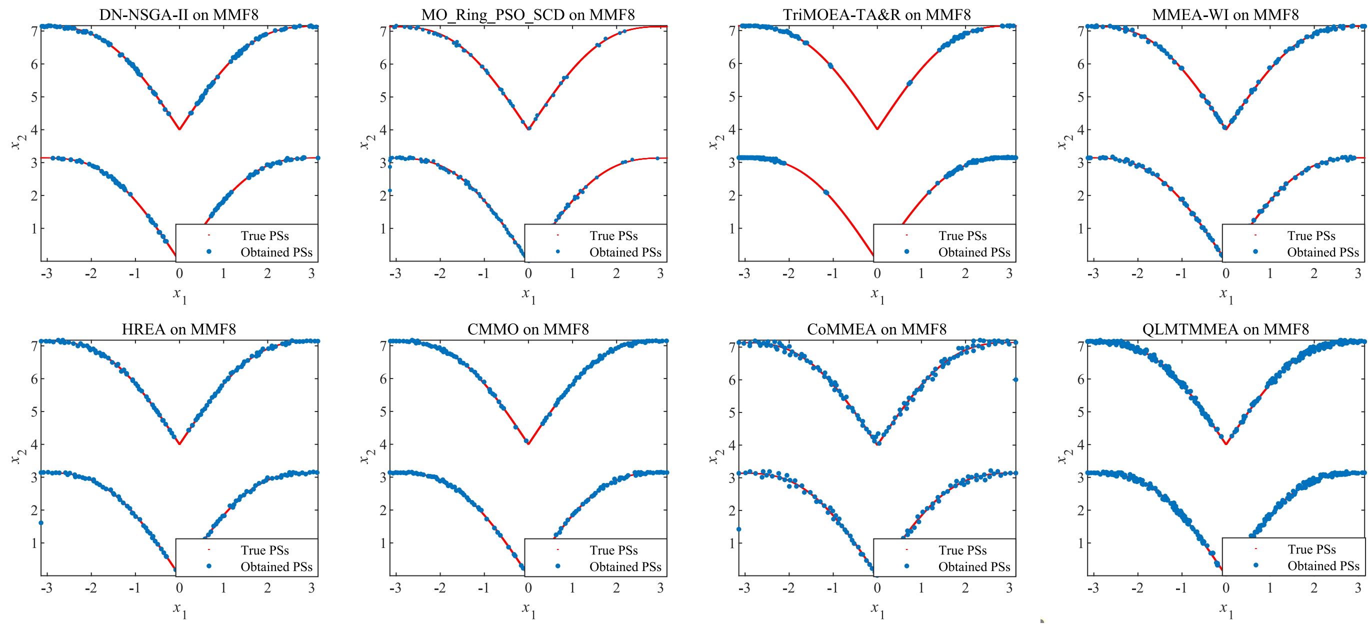

Fig. 5. Distribution of solutions in the decision space on MMF8 obtained by different algorithms.

Table 3
IGDX results of the compared algorithms on MMOPLs.

Problem	DN-NSGA-II	MO_Ring_PSO_SCD	TriMOEA-TA&R	MMEA-WI	HREA	CMMO	CoMMEA	QLMTMMEA
IDMPM2T1_ee	6.7320e-1 (2.60e-5) –	6.7351e-1 (4.75e-4) –	6.7327e-1 (1.93e-5) –	6.7325e-1 (5.85e-5) –	6.7317e-1 (4.93e-5) –	6.7317e-1 (3.73e-5) –	6.7212e-1 (2.22e-3) –	**4.1612e-4 (9.75e-5)**
IDMPM2T2_ee	6.7331e-1 (1.05e-4) –	6.7346e-1 (4.14e-4) –	6.7326e-1 (3.45e-5) –	6.7329e-1 (1.88e-4) –	9.2601e-4 (5.22e-5) –	6.7325e-1 (2.47e-4) –	8.3963e-4 (4.36e-5) –	**4.7738e-4 (6.39e-5)**
IDMPM2T3_ee	5.0200e-1 (1.77e-1) –	2.4445e-1 (1.69e-1) –	4.8002e-1 (1.83e-1) –	3.0094e-1 (3.01e-4) –	1.1685e-3 (3.31e-5) –	3.0058e-1 (1.75e-4) –	1.2825e-3 (6.83e-5) –	**9.6557e-4 (1.58e-4)**
IDMPM2T4_ee	1.0080e + 0 (8.47e-5) –	1.0079e + 0 (1.17e-3) –	1.0109e + 0 (1.93e-2) –	1.0081e + 0 (4.23e-4) –	**1.0204e-2 (1.42e-2)** +	1.0080e + 0 (3.77e-5) –	2.2808e-2 (2.55e-2) +	4.9229e-2 (4.26e-2)
IDMPM3T1_ee	7.4059e-1 (1.76e-1) –	6.6572e-1 (9.46e-2) –	7.7760e-1 (1.77e-1) –	6.2546e-1 (8.93e-4) –	8.5585e-3 (1.40e-4) –	6.2463e-1 (6.70e-4) –	6.2460e-1 (9.80e-4) –	**6.6182e-3 (4.11e-4)**
IDMPM3T2_ee	7.9643e-1 (1.68e-1) –	5.3020e-1 (9.65e-2) –	8.5877e-1 (1.26e-1) –	4.9672e-1 (8.76e-4) –	2.5526e-1 (9.70e-4) –	5.0927e-1 (7.13e-2) –	4.9554e-1 (8.90e-4) –	**6.4724e-3 (8.86e-4)**
IDMPM3T3_ee	7.9597e-1 (2.56e-1) –	5.0237e-1 (3.21e-3) –	8.1035e-1 (2.44e-1) –	4.9915e-1 (1.10e-3) –	2.5470e-1 (1.72e-3) –	4.9954e-1 (1.98e-3) –	4.9796e-1 (9.51e-4) –	**8.2952e-3 (1.63e-3)**
IDMPM3T4_ee	9.5244e-1 (1.38e-1) –	8.2228e-1 (8.73e-2) –	9.2590e-1 (1.34e-1) –	9.2282e-1 (1.50e-1) –	5.0405e-1 (9.21e-4) –	8.4872e-1 (9.57e-4) –	8.7159e-1 (9.14e-2) –	**2.7739e-1 (1.82e-1)**
MMF10	1.9765e-1 (7.13e-3) –	1.7063e-1 (9.46e-3) –	2.0144e-1 (5.25e-5) –	1.9306e-1 (1.61e-2) –	8.0754e-3 (4.45e-4) –	2.0132e-1 (3.52e-5) –	1.5685e-1 (1.23e-4) –	**7.0986e-3 (7.85e-4)**
MMF11	2.4903e-1 (2.57e-4) –	2.3209e-1 (2.27e-2) –	2.5244e-1 (7.20e-5) –	2.4889e-1 (2.90e-4) –	1.0641e-2 (2.14e-3) –	2.5026e-1 (6.00e-4) –	8.0629e-3 (5.43e-4) –	**4.4065e-3 (3.73e-4)**
MMF12	2.4521e-1 (3.53e-4) –	2.2082e-1 (3.65e-2) –	2.4798e-1 (3.69e-4) –	2.4444e-1 (3.55e-4) –	**2.8037e-3 (1.33e-4)** +	2.4540e-1 (2.37e-4) –	4.0682e-2 (1.15e-4) –	4.7579e-3 (1.99e-3)
MMF13	2.5874e-1 (4.57e-3) –	2.5444e-1 (1.11e-2) –	2.7027e-1 (6.46e-3) –	2.5371e-1 (5.80e-4) –	6.3124e-2 (2.88e-3) –	2.6007e-1 (7.96e-4) –	6.5608e-2 (1.63e-3) –	**4.7975e-2 (2.61e-3)**
MMF15	2.6092e-1 (2.81e-3) –	2.0885e-1 (2.07e-2) –	2.7181e-1 (3.05e-4) –	2.6135e-1 (1.92e-3) –	5.3862e-2 (2.12e-3) –	2.5631e-1 (9.16e-3) –	5.2132e-2 (6.74e-4) –	**3.9323e-2 (1.28e-3)**
MMF15_a	2.1380e-1 (5.32e-3) –	2.0015e-1 (1.21e-2) –	2.2190e-1 (2.24e-3) –	2.1408e-1 (2.04e-3) –	6.8229e-2 (3.39e-3) –	2.1230e-1 (3.49e-3) –	6.4237e-2 (1.04e-3) –	**4.8810e-2 (2.87e-3)**
+/-/=	0/14/0	0/14/0	0/14/0	0/14/0	2/12/0	0/14/0	1/13/0	

superior solutions in high-dimensional and complex optimization problems. EMTO can decompose complex problems into multiple interconnected subtasks to optimize more efficiently (Qiao, Yu, et al., 2023). By integrating conditions and experiences from individual subtasks, EMTO not only enhances algorithm robustness but also increases stability across diverse problem types.

EMTO has expanded its applications across diverse fields such as machine learning, engineering optimization, and bioinformatics. It effectively tackles multi-task optimization problems and shows promising potential for collaborative optimization in complex systems (Qiao, Yu, et al., 2024). Overall, EMTO significantly improves efficiency and effectiveness in multi-task optimization through mechanisms such as resource sharing, knowledge transfer, and diversity maintenance, making it a robust tool for solving complex optimization challenges (Liang, et al., 2024; Qiao, Liang, Yu, Yue, et al., 2024).

2.3. Reinforcement learning

RL is a branch of machine learning where an agent learns to make decisions by interacting with an environment. Through a trial-and-error process, the agent receives feedback in the form of rewards or penalties based on its actions, enabling it to learn optimal strategies to achieve

Table 4
IGD results of the compared algorithms on MMOPLs.

Problem	DN-NSGA-II	MO_Ring_PSO_SCD	TriMOEA-TA&R	MMEA-WI	HREA	CMMO	CoMMEA	QLMTMMEA
IDMPM2T1_ee	7.2359e-3 (1.23e-5) –	**6.9359e-3 (7.68e-5)** +	7.4137e-3 (8.18e-6) –	7.2236e-3 (2.44e-5) =	7.1931e-3 (3.21e-5) =	7.2259e-3 (2.04e-5) =	7.1338e-3 (7.54e-5) +	7.2042e-3 (4.32e-5)
IDMPM2T2_ee	7.2665e-3 (1.42e-5) –	7.1485e-3 (4.71e-5) –	7.4187e-3 (6.10e-6) –	7.2860e-3 (3.27e-5) –	8.7974e-4 (3.12e-5) –	7.1977e-3 (1.20e-5) –	7.9541e-4 (1.64e-5) –	**4.8413e-4 (7.91e-5)**
IDMPM2T3_ee	5.0056e-3 (1.66e-5) –	6.5143e-3 (6.01e-4) –	2.2900e-2 (2.26e-2) –	5.1368e-3 (3.07e-5) –	9.8889e-4 (3.30e-5) –	5.0122e-3 (1.68e-5) –	1.0033e-3 (7.35e-5) –	**6.6144e-4 (6.32e-5)**
IDMPM2T4_ee	1.6146e-2 (5.26e-5) –	1.5109e-2 (2.90e-4) –	1.5944e-2 (1.79e-3) –	1.6106e-2 (8.49e-5) –	1.5327e-3 (7.91e-5) +	1.6179e-2 (2.67e-5) –	**1.2506e-3 (9.87e-5)** +	1.9713e-3 (8.65e-4)
IDMPM3T1_ee	2.5921e-2 (2.99e-4) –	2.4180e-2 (3.27e-4) –	2.9091e-2 (3.81e-4) –	2.5496e-2 (1.77e-4) –	8.1691e-3 (2.45e-4) –	2.5117e-2 (1.85e-4) –	2.4359e-2 (1.72e-4) –	**5.9761e-3 (4.89e-4)**
IDMPM3T2_ee	3.7968e-2 (2.68e-4) –	3.6788e-2 (7.89e-4) –	4.1245e-2 (2.74e-4) –	3.7222e-2 (2.64e-4) –	1.6991e-2 (2.04e-4) –	3.7170e-2 (1.83e-4) –	3.6134e-2 (2.32e-4) –	**6.2490e-3 (6.57e-4)**
IDMPM3T3_ee	3.7809e-2 (2.88e-4) –	3.6205e-2 (4.91e-4) –	4.0876e-2 (7.01e-4) –	3.6987e-2 (2.32e-4) –	1.6760e-2 (1.44e-4) –	3.6995e-2 (2.17e-4) –	3.5744e-2 (2.15e-4) –	**7.5648e-3 (1.39e-3)**
IDMPM3T4_ee	5.7845e-2 (6.08e-3) –	5.3350e-2 (3.62e-3) –	5.8903e-2 (6.84e-3) –	5.8805e-2 (4.11e-4) –	2.6268e-2 (2.99e-4) –	5.9030e-2 (3.64e-4) –	5.6897e-2 (2.20e-3) –	**2.0583e-2 (3.21e-3)**
MMF10	1.9070e-1 (1.50e-2) –	1.5688e-1 (9.42e-3) –	2.2771e-1 (6.12e-3) –	1.9092e-1 (1.63e-2) –	2.5715e-2 (1.71e-3) –	1.9435e-1 (1.05e-4) –	1.3614e-1 (3.05e-4) –	**9.0267e-3 (9.32e-4)**
MMF11	9.8142e-2 (9.73e-4) –	8.8110e-2 (7.50e-3) –	1.6393e-1 (8.62e-3) –	9.7884e-2 (2.23e-3) –	3.8449e-2 (6.03e-3) –	9.3649e-2 (2.35e-4) –	2.1876e-2 (9.56e-4) –	**1.0177e-2 (1.12e-3)**
MMF12	8.3196e-2 (1.20e-4) –	6.7138e-2 (1.30e-2) –	8.5577e-2 (8.59e-4) –	8.3899e-2 (5.13e-4) –	**6.4499e-3 (4.50e-4)** +	8.3041e-2 (2.65e-4) –	5.8002e-2 (1.24e-3) –	9.3984e-3 (3.99e-3)
MMF13	1.5557e-1 (4.38e-3) –	1.0624e-1 (2.61e-2) –	2.4903e-1 (7.85e-3) –	1.5848e-1 (8.52e-3) –	4.6954e-2 (6.13e-3) –	1.5079e-1 (1.42e-3) –	3.6541e-2 (3.34e-3) –	**2.0797e-2 (1.81e-2)**
MMF15	1.9372e-1 (4.53e-3) –	1.9279e-1 (4.20e-3) –	2.0960e-1 (6.88e-4) –	1.9482e-1 (2.57e-3) –	1.2708e-1 (5.19e-3) –	1.8577e-1 (2.89e-3) –	1.1099e-1 (1.16e-3) –	**8.0113e-2 (2.46e-3)**
MMF15_a	1.9365e-1 (3.76e-3) –	1.8948e-1 (4.03e-3) –	1.9876e-1 (3.06e-3) –	1.8979e-1 (3.09e-3) –	1.4555e-1 (7.14e-3) –	1.8201e-1 (2.21e-3) –	1.1854e-1 (1.33e-3) –	**8.4560e-2 (4.09e-3)**
+/-/=	0/14/0	0/14/0	0/14/0	0/14/0	2/12/0	0/14/0	1/13/0	

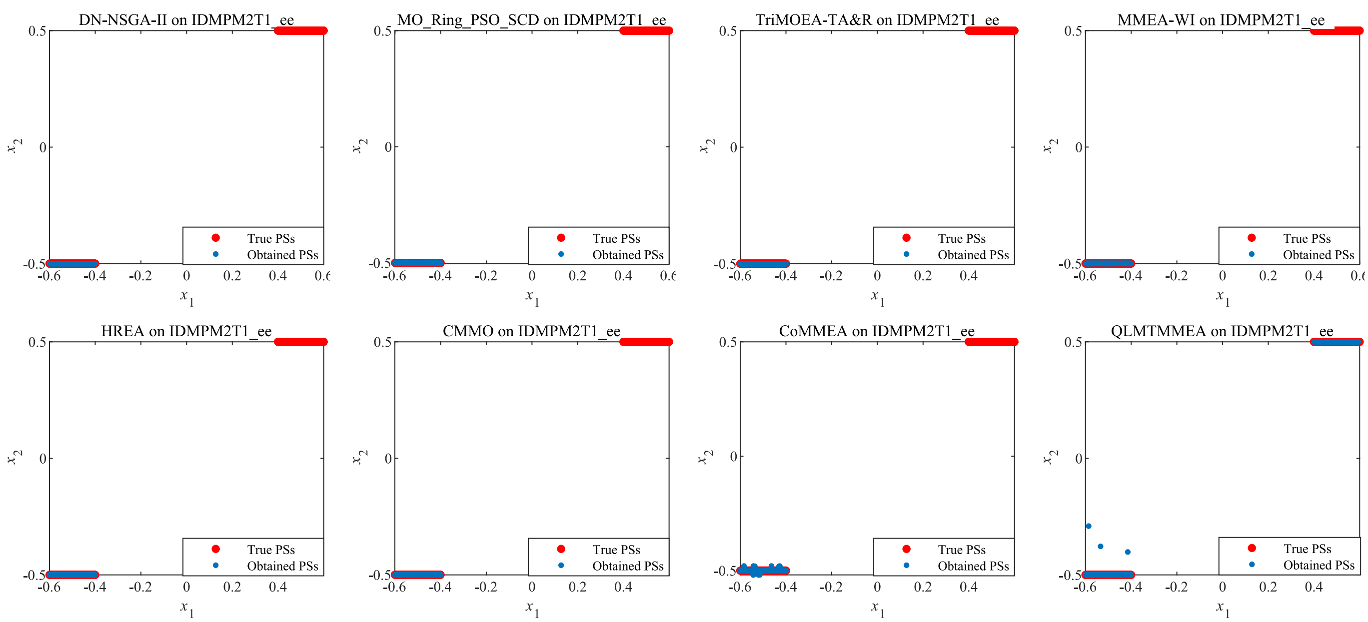

Fig. 6. Distribution of solutions in the decision space on IDMPM2T1_ee obtained by different algorithms.

long-term goals(Luong, et al., 2019). RL has shown remarkable success in various domains such as robotics, game playing, finance, and healthcare(Arulkumaran, Deisenroth, Brundage, & Bharath, 2017). A fundamental concept in RL is the policy (π), a strategy that the agent follows to choose actions based on the current state. The goal is to learn an optimal policy π^* that maximizes the expected cumulative reward, known as the return G_t:

$$G_t = \sum_{k=0}^{\infty} \gamma^k r_{t+k+1} \tag{3}$$

where $\gamma(0 \le \gamma < 1)$ is the discount factor that prioritizes immediate rewards over future rewards.

The main methods of RL include QL, Policy Gradient methods, and Deep Q Networks, which have enabled RL to tackle complex tasks with high-dimensional state and action spaces. In recent years, various studies have applied RL techniques in handling MOPs(Drugan, 2019; K. Li, et al., 2021). Nevertheless, the application of RL in MMOPs is just in its infant stage.

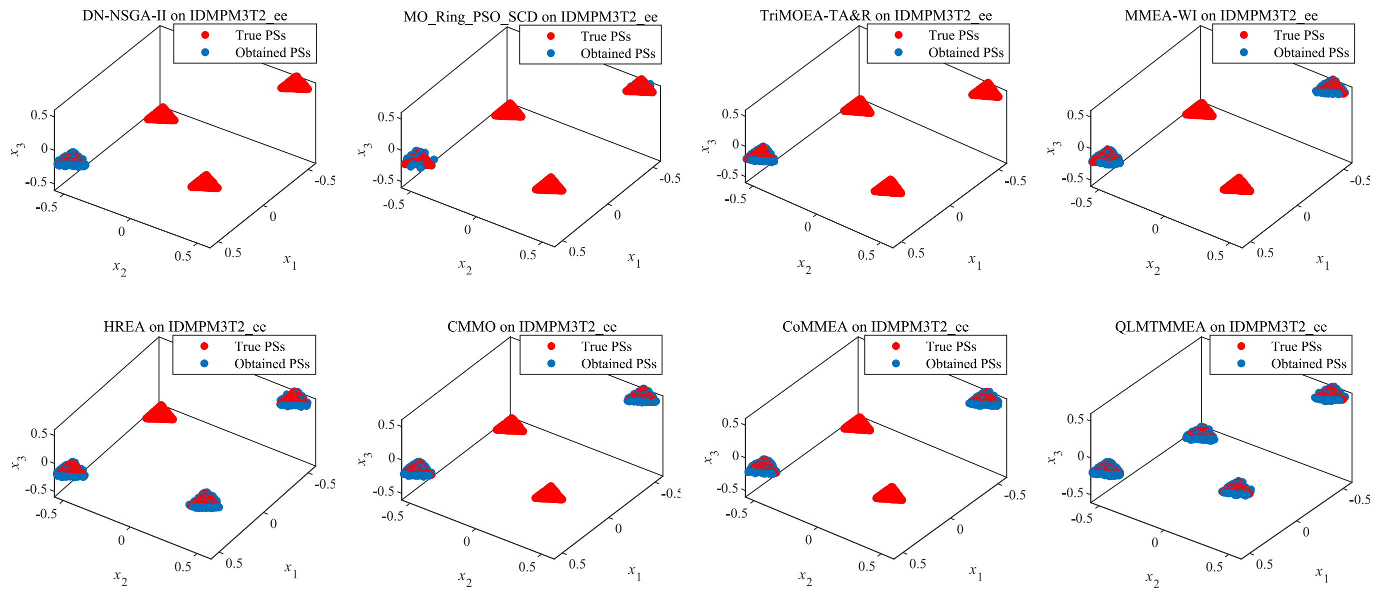

Fig. 7. Distribution of solutions in the decision space on IDMPM3T2_ee obtained by different algorithms.

Table 5
Average rank of IGDX and IGD over 34 test problems with different values of α.

α	10 %	20 %	30 %	40 %	50 %	60 %	70 %	80 %	90 %	100 %
IGDX	7.38	6.94	1.91	7.35	2.09	7.00	6.47	5.97	5.71	4.18
IGD	5.18	5.44	4.22	6.21	3.49	6.07	6.13	6.54	6.75	4.97

2.4. Motivation

It is evident from the previously mentioned literature that most existing methods adopt a convergence-first principle. However, their preference for non-dominated solutions often leads to the algorithms easily becoming trapped in local PSs. Therefore, further insights and new visions are required in the development of MMEAs to address the challenge of balancing convergence and diversity in MMOPs.

In recent years, EMTO has garnered considerable attention due to its unique advantages of sharing information and resources among multiple tasks. Through knowledge transfer and resource sharing, optimization efficiency is significantly improved, and the algorithm's robustness is enhanced. In the field of MOPs, EMTO has demonstrated excellent performance in effectively handling complex optimization problems. By applying EMTO in MMOPs, more comprehensive and effective solutions can be achieved through the maintenance of population diversity and facilitation of knowledge sharing. This approach can enhance algorithmic diversity in both the target and decision spaces and mitigate the issue of falling into local optimal solutions. In addition, RL has exhibited competitive performance in solving complex decision problems in recent years. Through the interaction between the agent and the environment, RL gradually improves its strategy and possesses adaptive and dynamic learning abilities. Integrating reinforcement learning into EMTO can further enhance its capability to address complex optimization problems, the policy improvement mechanism of RL helps EMTO explore and utilize the search space more efficiently, thereby enhancing the overall performance of multi-task optimization.

In summary, combining the advantages of EMTO in multi-objective optimization with the powerful capabilities of reinforcement learning in dynamic decision making and policy improvement may provide efficient solutions to MMOPs. Given the advantages of both in dealing with MOPs, a multi-task optimization algorithm via RL for MMOPs is designed in this paper.

3. Proposed method

3.1. Framework of QLMTMMEA

Generally, the algorithm comprises two stages: learning and evolving. During the learning stage, all auxiliary tasks evolve by generating offspring and undergoing environmental selection to achieve two objectives. Firstly, they converge according to their algorithmic strategies to prevent their populations from straying too far from the PF, ensuring effective transfer. Secondly, they evolve to enhance Q-learning exploration. In contrast, during the evolving stage, only the selected task evolves. This stage focuses function evaluations on the most suitable auxiliary task to leverage learned policies for exploitation.

The learning stage initiates by generating offspring sets for all auxiliary tasks. Subsequently, based on the Q-table, one auxiliary task is chosen. Knowledge from this task is transferred to the main task *MT*, and environmental selection selects population for the next generation. Following this, the Q-table is updated based on interaction feedback, after which all auxiliary tasks undergo environmental selection and evolve.

During the evolving stage, an auxiliary task AT_j is selected based on the Q-table. Offspring sets of AT_j are generated, and beneficial solutions are transferred to *MT* to assist in its environmental selection. Then AT_j is evolved, and the Q-table is updated accordingly. Upon algorithm termination, population of main task *MT* is output as the final solution set. The pseudo-code for the proposed QLMTMMEA is outlined in Algorithm 1.

Algorithm 1: Framework of QLMTMMEA

Input: N (population size) Gmax (termination condition) i (number of auxiliary tasks)
Output: P (final solution set)
1. Initialize population for the main task MT;
2. Initialize populations for auxiliary tasks AT_1 to AT_i;
3. Initialize the Q-Table as zeros;
4. **while** g < Gmax **do**

(*continued on next page*)

(*continued*)

Algorithm 1: Framework of QLMTMMEA

5. **if** g < α * Gmax **then** /* α=50% */
6. Generate offspring set for main task MT;
7. Generate offspring sets for auxiliary tasks AT_1 to AT_i;
8. Select an action according to Q-learning;
9. P =*Transferandevolution* (MT, AT_i); /*Transfer solutions from selected auxiliary task AT_i to main task MT and evolve MT, Algorithm 2*/
10. Q = *UpdateQ −Table* (Q, MT, AT_i); /*Update Q-Table based on the success of the transfer, Algorithm 3*/
11. Evolve auxiliary tasks AT_1 to AT_i;
12. **else**
13. Generate offspring set for main task MT;
14. Select an action according to Q-learning;
15. Generate offspring set for selected auxiliary task AT_j;
16. P =*TransferAndEvolve* (MT, AT_j); /*Transfer solutions from selected auxiliary task T_i to main task T and evolve T, Algorithm 2*/
17. Q = *UpdateQTable* (Q, MT, AT_j); /*Update Q-Table based on the success of the transfer, Algorithm 3*/
18. Evolve selected auxiliary task AT_j;
19. **end if**
20. g = g + 1;
21. **end while**
22. **return** P

This algorithm outlines a multitask evolutionary approach that integrates QL. Initially, it establishes populations for the main task and auxiliary tasks, along with initializing a Q-table set to zero. In the initial phase, offspring are produced for both the main task and auxiliary tasks, using QL to determine optimal transfer actions. In the subsequent phase, offspring generation focuses on the main task and a selected auxiliary task, continuously updating the Q-table based on transfer effectiveness. The algorithm iterates until it reaches the maximum number of generations, ultimately producing the final solution set.

3.2. Transfer and evolution

In QL, the states, actions, and rewards should be set so the agent can learn the Q-Table and make decisions accordingly. Based on this model, the proposed QL-based evolution can be illustrated in Fig. 2. The evolutionary process contains two parts: (1) Evolution: In the evolution process, the useful knowledge(solutions) in the selected auxiliary task is transferred to the main task to assist in the evolution of the main task (line 2). (2) Interaction: In the feedback loop process, the agent performs the action (transfer knowledge from the selected auxiliary task) and determines the reward and the next state (line 6–7).

Algorithm 2: Transfer and evolution

Input: MT (main task), AT_i (auxiliary task i)
Output: MT, AT_i
1.**While** g ≤ Gmax **do**
2. Use reinforcement learning to select auxiliary tasks;
3. Calculating the fitness;
4. Generating mating pools based on tournament selection method;
5. Using GA to generate OffA and OffB based on mating pools;
6. [MT] ← *EnvironmentalSelectionMT* (P,OffA,OffB,N);/* Algorithm 4*/
7. [AT_i] ← *EnvironmentalSelectionATi* (P,OffA,OffB,N);/* Algorithm 5*/
8.**end while**
9.**reture** MT, AT_i

Table 6
IGD results of the compared algorithms on MMOPs with different values of α.

Problem	10 %	20 %	30 %	40 %	50 %	60 %	70 %	80 %	90 %	100 %
IDMPM2T1	4.6830e-4 (8.05e-5)	4.7352e-4 (7.02e-5)	4.7000e-4 (4.33e-5)	4.6976e-4 (5.73e-5)	4.5260e-4 (4.27e-5)	4.4638e-4 (4.79e-5)	4.4170e-4 (4.19e-5)	**4.3701e-4 (5.25e-5)**	4.3917e-4 (4.11e-5)	4.4309e-4 (4.53e-5)
IDMPM2T2	4.3921e-4 (3.33e-5)	4.3385e-4 (3.74e-5)	4.3193e-4 (2.57e-5)	4.3112e-4 (5.42e-5)	**4.1972e-4 (3.06e-5)**	5.2335e-4 (1.12e-4)	5.0098e-4 (1.03e-4)	5.4944e-4 (1.36e-4)	4.8020e-4 (7.65e-5)	4.7748e-4 (6.09e-5)
IDMPM2T3	5.0873e-4 (1.04e-4)	5.1273e-4 (1.19e-4)	4.4352e-4 (2.30e-5)	5.0937e-4 (8.92e-5)	**4.2779e-4 (2.87e-5)**	5.3398e-4 (8.48e-5)	5.4712e-4 (1.36e-4)	5.2818e-4 (1.19e-4)	6.4804e-4 (3.07e-4)	5.2160e-4 (1.01e-4)
IDMPM2T4	5.6566e-4 (4.23e-4)	4.4493e-4 (7.11e-5)	4.4183e-4 (6.28e-5)	4.9567e-4 (1.84e-4)	**4.2053e-4 (2.45e-5)**	5.4088e-4 (2.53e-4)	5.2997e-4 (1.71e-4)	5.9296e-4 (3.03e-4)	5.9473e-4 (2.37e-4)	5.7142e-4 (2.11e-4)
IDMPM3T1	6.0497e-3 (7.39e-4)	6.0151e-3 (6.20e-4)	5.5771e-3 (2.17e-4)	6.0927e-3 (6.42e-4)	**5.4478e-3 (3.24e-4)**	6.1578e-3 (9.76e-4)	5.9202e-3 (6.55e-4)	6.3809e-3 (9.21e-4)	6.4186e-3 (1.31e-3)	5.9381e-3 (6.08e-4)
IDMPM3T2	5.3593e-3 (4.48e-4)	5.4225e-3 (6.20e-4)	5.3335e-3 (2.39e-4)	5.4895e-3 (5.92e-4)	**5.1895e-3 (3.37e-4)**	5.4549e-3 (4.77e-4)	5.5827e-3 (7.60e-4)	5.4868e-3 (5.07e-4)	5.6632e-3 (6.12e-4)	5.2753e-3 (4.64e-4)
IDMPM3T3	5.6504e-3 (6.22e-4)	5.4680e-3 (4.74e-4)	5.3658e-3 (2.64e-4)	5.5763e-3 (5.19e-4)	**5.2829e-3 (3.33e-4)**	6.0427e-3 (8.75e-4)	5.5095e-3 (5.11e-4)	5.9187e-3 (7.72e-4)	5.7881e-3 (5.37e-4)	5.8148e-3 (9.33e-4)
IDMPM3T4	5.2991e-3 (7.87e-4)	5.2349e-3 (6.18e-4)	5.3404e-3 (3.15e-4)	5.2667e-3 (6.78e-4)	**5.2332e-3 (3.86e-4)**	5.3911e-3 (6.46e-4)	5.4956e-3 (8.72e-4)	5.8498e-3 (8.63e-4)	5.6302e-3 (9.70e-4)	5.9728e-3 (1.56e-3)
IDMPM4T1	7.8829e-3 (7.61e-4)	**7.8369e-3 (9.11e-4)**	8.6903e-3 (8.66e-4)	8.1607e-3 (8.83e-4)	8.1555e-3 (8.37e-4)	9.0013e-3 (1.71e-3)	9.5180e-3 (1.15e-3)	9.3247e-3 (1.02e-3)	1.0543e-2 (1.64e-3)	1.1924e-2 (2.20e-3)
IDMPM4T2	**7.3842e-3 (8.40e-4)**	7.5561e-3 (7.35e-4)	8.0569e-3 (5.33e-4)	7.4872e-3 (1.12e-3)	7.7093e-3 (7.66e-4)	7.8469e-3 (1.16e-3)	8.4121e-3 (1.36e-3)	8.5780e-3 (1.48e-3)	1.1183e-2 (3.58e-3)	1.2763e-2 (5.12e-3)
IDMPM4T3	8.1403e-3 (1.35e-3)	8.4202e-3 (1.55e-3)	8.1754e-3 (6.98e-4)	9.1569e-3 (1.77e-3)	**7.9505e-3 (6.38e-4)**	9.3052e-3 (1.79e-3)	1.0260e-2 (2.52e-3)	1.2245e-2 (7.15e-3)	1.2548e-2 (4.23e-3)	1.8300e-2 (8.54e-3)
IDMPM4T4	**7.2997e-3 (6.13e-4)**	7.4744e-3 (5.06e-4)	8.5444e-3 (8.35e-4)	8.7054e-3 (1.74e-3)	7.9943e-3 (7.22e-4)	9.8535e-3 (3.53e-3)	1.1460e-2 (3.51e-3)	1.3132e-2 (4.97e-3)	1.5570e-2 (7.12e-3)	1.7811e-2 (9.55e-3)
MMF1	1.2613e + 0 (5.55e-3)	1.2624e + 0 (8.80e-3)	1.2585e + 0 (4.32e-4)	1.2650e + 0 (9.21e-3)	**1.2584e + 0 (7.30e-5)**	1.2597e + 0 (3.10e-3)	1.2611e + 0 (6.50e-3)	1.2606e + 0 (4.90e-3)	1.2624e + 0 (8.85e-3)	1.2594e + 0 (2.33e-3)
MMF2	6.0613e-1 (1.41e-2)	6.0993e-1 (1.20e-2)	6.1373e-1 (1.78e-3)	6.0220e-1 (3.24e-2)	6.1344e-1 (2.59e-3)	5.9654e-1 (3.04e-2)	6.0481e-1 (1.54e-2)	5.9863e-1 (2.82e-2)	6.0521e-1 (1.71e-2)	**5.9568e-1 (2.28e-2)**
MMF3	4.5983e-1 (1.15e-2)	4.6243e-1 (7.32e-3)	4.6618e-1 (1.40e-3)	4.6150e-1 (7.13e-3)	4.6612e-1 (1.57e-3)	**4.5890e-1 (1.06e-2)**	4.6202e-1 (6.72e-3)	4.6024e-1 (9.16e-3)	4.6068e-1 (8.99e-3)	4.6070e-1 (5.98e-3)
MMF4	5.7434e-1 (5.68e-2)	**5.4789e-1 (5.93e-2)**	5.9296e-1 (4.70e-2)	5.7999e-1 (5.58e-2)	5.8138e-1 (5.54e-2)	5.6772e-1 (6.00e-2)	5.6080e-1 (5.83e-2)	5.6772e-1 (6.12e-2)	5.5017e-1 (6.15e-2)	5.6517e-1 (6.12e-2)
MMF5	1.6633e + 0 (6.03e-2)	1.6613e + 0 (6.21e-2)	1.6617e + 0 (5.13e-2)	1.6514e + 0 (5.91e-2)	1.6653e + 0 (5.68e-2)	1.6536e + 0 (5.36e-2)	1.6583e + 0 (5.91e-2)	1.6599e + 0 (5.70e-2)	**1.6332e + 0 (5.71e-2)**	1.6451e + 0 (6.04e-2)
MMF6	1.3811e + 0 (9.45e-3)	1.3820e + 0 (1.32e-2)	1.3792e + 0 (6.89e-3)	1.3848e + 0 (1.31e-2)	**1.3778e + 0 (7.74e-3)**	1.3779e + 0 (8.63e-3)	1.3832e + 0 (8.84e-3)	1.3790e + 0 (7.57e-3)	1.3802e + 0 (6.69e-3)	1.3795e + 0 (7.71e-3)
MMF7	1.0521e + 0 (2.91e-3)	1.0523e + 0 (2.31e-3)	1.0511e + 0 (3.16e-4)	1.0529e + 0 (4.61e-3)	1.0512e + 0 (6.26e-4)	1.0516e + 0 (1.69e-3)	1.0523e + 0 (3.09e-3)	1.0513e + 0 (1.15e-3)	1.0526e + 0 (2.62e-3)	**1.0510e + 0 (1.61e-4)**
MMF8	**3.6040e + 0 (4.26e-1)**	3.6596e + 0 (3.15e-1)	3.7213e + 0 (2.07e-4)	3.7209e + 0 (5.26e-4)	3.7213e + 0 (2.63e-4)	3.7203e + 0 (2.98e-3)	3.6998e + 0 (1.08e-1)	3.7208e + 0 (4.52e-4)	3.7134e + 0 (3.88e-2)	3.7212e + 0 (2.02e-4)

Table 7
IGDX results of the compared algorithms on MMOPs with different values of α.

Problem	10 %	20 %	30 %	40 %	50 %	60 %	70 %	80 %	90 %	100 %
IDMPM2T1	5.2982e-4 (1.07e-4)	5.0665e-4 (1.21e-4)	5.5111e-4 (2.61e-4)	5.0011e-4 (8.37e-5)	4.7079e-4 (6.67e-5)	5.0513e-4 (1.12e-4)	4.9909e-4 (8.22e-5)	**4.6023e-4 (6.85e-5)**	4.9668e-4 (1.01e-4)	4.7339e-4 (7.67e-5)
IDMPM2T2	7.9058e-4 (1.71e-4)	7.3684e-4 (1.09e-4)	7.2495e-4 (8.89e-5)	1.0843e-3 (1.09e-3)	**7.0409e-4 (9.96e-5)**	4.1334e-3 (7.01e-3)	3.8604e-3 (4.12e-3)	2.7147e-3 (1.65e-3)	3.7994e-3 (4.28e-3)	3.8083e-3 (3.41e-3)
IDMPM2T3	4.1171e-3 (4.51e-3)	3.1441e-3 (1.76e-3)	**1.2365e-3 (2.05e-4)**	4.0463e-3 (2.28e-3)	1.4531e-3 (4.70e-4)	3.6897e-3 (2.74e-3)	3.0869e-3 (1.12e-3)	4.1731e-3 (2.51e-3)	3.8479e-3 (2.56e-3)	3.3946e-3 (1.69e-3)
IDMPM2T4	4.5578e-2 (1.49e-1)	9.3534e-2 (2.09e-1)	**5.7237e-4 (2.63e-4)**	9.2943e-2 (2.11e-1)	7.5483e-4 (9.22e-4)	3.3118e-2 (1.16e-1)	3.0286e-2 (1.13e-1)	7.0174e-2 (1.76e-1)	3.0821e-2 (1.09e-1)	2.9781e-2 (1.09e-1)
IDMPM3T1	1.9928e-2 (7.44e-3)	1.3991e-2 (6.65e-3)	7.9296e-3 (7.52e-4)	4.1742e-2 (7.44e-2)	**7.6512e-3 (4.39e-4)**	1.8478e-2 (8.65e-3)	1.8451e-2 (8.28e-3)	3.6784e-2 (6.20e-2)	1.9682e-2 (7.34e-3)	2.4666e-2 (4.35e-2)
IDMPM3T2	6.6489e-2 (9.19e-2)	7.1725e-2 (9.37e-2)	**8.2351e-3 (8.18e-4)**	5.8486e-2 (9.25e-2)	8.9574e-3 (2.87e-3)	5.0293e-2 (7.23e-2)	6.0170e-2 (8.12e-2)	3.0752e-2 (3.99e-2)	4.6494e-2 (6.74e-2)	4.2262e-2 (6.41e-2)
IDMPM3T3	6.8816e-2 (9.56e-2)	5.1034e-2 (6.91e-2)	**9.2517e-3 (8.24e-4)**	5.0198e-2 (7.45e-2)	1.7586e-2 (4.58e-2)	5.9531e-2 (8.16e-2)	6.1386e-2 (8.44e-2)	5.6741e-2 (7.45e-2)	5.7400e-2 (7.86e-2)	4.2053e-2 (5.97e-2)
IDMPM3T4	1.9212e-1 (1.71e-1)	1.3978e-1 (1.19e-1)	**2.4083e-2 (5.94e-2)**	1.2878e-1 (1.17e-1)	3.4604e-2 (7.15e-2)	1.2216e-1 (1.61e-1)	1.3396e-1 (1.47e-1)	1.8571e-1 (1.78e-1)	1.1504e-1 (1.08e-1)	1.1946e-1 (1.38e-1)
IDMPM4T1	2.6114e-1 (2.45e-1)	2.2007e-1 (1.69e-1)	2.1718e-2 (4.71e-2)	2.5296e-1 (2.43e-1)	**1.5538e-2 (6.37e-3)**	1.4624e-1 (1.20e-1)	2.2290e-1 (2.14e-1)	2.2749e-1 (2.16e-1)	2.0105e-1 (1.65e-1)	2.4795e-1 (2.28e-1)
IDMPM4T2	2.9176e-1 (2.24e-1)	3.8662e-1 (2.08e-1)	**4.7708e-2 (7.64e-2)**	2.7954e-1 (2.46e-1)	1.0265e-1 (1.15e-1)	3.0083e-1 (2.47e-1)	3.2090e-1 (1.90e-1)	3.2162e-1 (2.18e-1)	3.0016e-1 (2.43e-1)	2.7768e-1 (2.59e-1)
IDMPM4T3	1.8659e-1 (1.21e-1)	1.3509e-1 (1.25e-1)	**7.1646e-2 (1.03e-1)**	2.6064e-1 (2.28e-1)	9.5025e-2 (1.12e-1)	1.8708e-1 (1.90e-1)	2.0392e-1 (1.80e-1)	1.6291e-1 (1.74e-1)	1.7914e-1 (1.61e-1)	1.0696e-1 (4.56e-2)
IDMPM4T4	4.3218e-1 (2.49e-1)	3.2541e-1 (2.75e-1)	**9.5689e-2 (1.10e-1)**	3.3933e-1 (2.68e-1)	1.4109e-1 (1.33e-1)	3.4749e-1 (2.75e-1)	3.4098e-1 (3.02e-1)	2.5932e-1 (1.88e-1)	2.1115e-1 (1.80e-1)	2.2113e-1 (1.66e-1)
MMF1	6.0348e-2 (1.74e-2)	5.9241e-2 (1.83e-2)	**2.7321e-2 (1.11e-3)**	5.5065e-2 (1.44e-2)	2.7824e-2 (8.47e-4)	4.9840e-2 (1.55e-2)	4.9581e-2 (1.01e-2)	4.9316e-2 (1.09e-2)	5.0205e-2 (8.81e-3)	4.0939e-2 (1.77e-3)
MMF2	4.2527e-2 (1.62e-2)	4.1286e-2 (1.31e-2)	**1.2412e-2 (2.13e-3)**	4.1381e-2 (1.19e-2)	1.3055e-2 (2.34e-3)	4.8088e-2 (1.79e-2)	4.6262e-2 (1.20e-2)	4.3783e-2 (9.87e-3)	4.5908e-2 (9.11e-3)	4.5856e-2 (1.04e-2)
MMF3	3.3168e-2 (1.01e-2)	3.4266e-2 (1.28e-2)	**1.1650e-2 (1.47e-3)**	3.4694e-2 (9.75e-3)	1.2201e-2 (2.23e-3)	3.4777e-2 (7.39e-3)	3.5834e-2 (9.38e-3)	3.5470e-2 (4.74e-3)	3.8196e-2 (8.84e-3)	3.7242e-2 (6.59e-3)
MMF4	3.0428e-2 (1.05e-2)	2.7712e-2 (8.12e-3)	1.6903e-2 (8.17e-4)	3.2384e-2 (1.10e-2)	**1.6524e-2 (7.26e-4)**	2.7478e-2 (8.00e-3)	2.6927e-2 (7.36e-3)	2.6351e-2 (7.52e-3)	2.7377e-2 (7.22e-3)	1.9480e-2 (7.71e-4)
MMF5	8.2984e-2 (2.67e-2)	8.8110e-2 (2.08e-2)	4.4479e-2 (1.90e-3)	8.5002e-2 (2.38e-2)	**4.4251e-2 (1.69e-3)**	8.1757e-2 (1.98e-2)	8.6676e-2 (1.97e-2)	7.3375e-2 (1.54e-2)	7.5371e-2 (1.55e-2)	6.1666e-2 (3.48e-3)
MMF6	7.9360e-2 (2.13e-2)	6.9663e-2 (1.91e-2)	**3.9385e-2 (1.62e-3)**	7.2228e-2 (2.36e-2)	4.0379e-2 (1.54e-3)	7.2119e-2 (1.71e-2)	6.7461e-2 (1.65e-2)	6.8289e-2 (1.46e-2)	5.9296e-2 (1.47e-2)	5.1152e-2 (2.21e-3)
MMF7	2.7294e-2 (1.14e-2)	2.9233e-2 (1.01e-2)	**1.6128e-2 (7.68e-4)**	2.8561e-2 (9.07e-3)	1.6157e-2 (9.61e-4)	2.4260e-2 (5.64e-3)	2.5312e-2 (6.48e-3)	2.4182e-2 (5.37e-3)	2.3583e-2 (3.83e-3)	1.9727e-2 (1.86e-3)
MMF8	9.8656e-2 (3.94e-2)	9.2706e-2 (2.78e-2)	**3.3623e-2 (2.27e-3)**	9.8933e-2 (4.04e-2)	3.5486e-2 (2.13e-3)	8.9561e-2 (2.79e-2)	9.1588e-2 (2.76e-2)	8.6021e-2 (3.11e-2)	9.1902e-2 (2.77e-2)	6.1431e-2 (7.60e-3)

Table 8
IGD results of the compared algorithms on MMOPLs with different values of α.

Problem	10 %	20 %	30 %	40 %	50 %	60 %	70 %	80 %	90 %	100 %
IDMPM2T1_ee	7.2355e-3 (7.59e-5)	7.2553e-3 (9.25e-5)	7.1724e-3 (7.20e-5)	7.1964e-3 (5.75e-5)	7.2042e-3 (4.32e-5)	7.2155e-3 (1.02e-4)	7.2126e-3 (1.30e-4)	7.1620e-3 (8.88e-5)	7.1303e-3 (8.17e-5)	**7.1107e-3 (7.81e-5)**
IDMPM2T2_ee	4.2558e-3 (2.07e-3)	3.3955e-3 (1.97e-3)	**4.6488e-4 (4.04e-5)**	3.8911e-3 (2.17e-3)	4.8413e-4 (7.91e-5)	4.2167e-3 (1.96e-3)	3.8909e-3 (2.10e-3)	4.2438e-3 (1.97e-3)	4.6710e-3 (2.14e-3)	4.5245e-3 (1.90e-3)
IDMPM2T3_ee	1.7470e-3 (7.68e-4)	1.7354e-3 (5.68e-4)	**6.4599e-4 (5.28e-5)**	2.0661e-3 (8.59e-4)	6.6144e-4 (6.32e-5)	2.1614e-3 (1.21e-3)	2.1743e-3 (8.74e-4)	2.5099e-3 (1.35e-3)	2.8799e-3 (1.85e-3)	1.8325e-3 (5.94e-4)
IDMPM2T4_ee	3.0337e-3 (1.10e-3)	3.1615e-3 (1.45e-3)	2.1324e-3 (9.51e-4)	3.3917e-3 (1.43e-3)	**1.9713e-3 (8.65e-4)**	3.0849e-3 (1.11e-3)	3.0043e-3 (1.39e-3)	3.2319e-3 (1.08e-3)	3.2366e-3 (1.14e-3)	2.3688e-3 (9.25e-4)
IDMPM3T1_ee	9.4908e-3 (2.40e-3)	1.0111e-2 (2.06e-3)	6.0686e-3 (4.44e-4)	9.9745e-3 (2.97e-3)	**5.9761e-3 (4.89e-4)**	1.0934e-2 (2.73e-3)	1.0394e-2 (2.50e-3)	1.1248e-2 (2.74e-3)	1.0495e-2 (2.67e-3)	9.7443e-3 (1.60e-3)
IDMPM3T2_ee	1.3250e-2 (3.34e-3)	1.3690e-2 (5.08e-3)	**6.2331e-3 (6.93e-4)**	1.5904e-2 (4.41e-3)	6.2490e-3 (6.57e-4)	1.4735e-2 (3.70e-3)	1.6842e-2 (5.18e-3)	1.5439e-2 (4.54e-3)	1.5921e-2 (4.31e-3)	1.7014e-2 (7.55e-3)
IDMPM3T3_ee	1.5639e-2 (3.11e-3)	1.5868e-2 (3.91e-3)	7.6795e-3 (1.07e-3)	1.7606e-2 (3.94e-3)	7.5648e-3 (1.39e-3)	1.7441e-2 (3.88e-3)	1.7134e-2 (3.55e-3)	1.6960e-2 (3.19e-3)	1.6910e-2 (3.10e-3)	1.7292e-2 (5.28e-3)
IDMPM3T4_ee	**1.7289e-2 (5.69e-3)**	1.9753e-2 (4.93e-3)	2.0132e-2 (4.02e-3)	1.8548e-2 (5.93e-3)	2.0583e-2 (3.21e-3)	1.8062e-2 (5.18e-3)	2.1848e-2 (6.04e-3)	1.9847e-2 (7.34e-3)	1.9175e-2 (5.19e-3)	1.7465e-2 (4.26e-3)
MMF10	5.2244e-2 (3.26e-2)	6.0223e-2 (4.27e-2)	9.1263e-3 (1.65e-3)	5.2477e-2 (3.56e-2)	**9.0267e-3 (9.32e-4)**	5.5591e-2 (3.35e-2)	4.6511e-2 (2.17e-2)	4.9570e-2 (2.71e-2)	5.3918e-2 (2.71e-2)	4.6347e-2 (2.04e-2)
MMF11	2.4174e-2 (1.42e-2)	2.7034e-2 (1.55e-2)	1.0231e-2 (1.07e-3)	2.4577e-2 (1.49e-2)	**1.0177e-2 (1.12e-3)**	2.5259e-2 (1.13e-2)	2.2774e-2 (1.03e-2)	2.4257e-2 (1.04e-2)	2.2305e-2 (1.06e-2)	1.1650e-2 (7.61e-4)
MMF12	7.6044e-3 (1.87e-3)	7.5462e-3 (2.34e-3)	1.1416e-2 (5.16e-3)	7.8043e-3 (1.45e-3)	9.3984e-3 (3.99e-3)	7.5633e-3 (2.16e-3)	6.9766e-3 (1.72e-3)	7.6132e-3 (2.20e-3)	7.9148e-3 (1.95e-3)	**5.8482e-3 (1.37e-3)**
MMF13	3.1567e-2 (1.97e-2)	4.0409e-2 (2.11e-2)	2.1170e-2 (2.28e-2)	2.7649e-2 (1.75e-2)	2.0797e-2 (1.81e-2)	2.8445e-2 (1.21e-2)	2.5966e-2 (1.29e-2)	2.9668e-2 (1.13e-2)	2.5087e-2 (1.00e-2)	**1.5131e-2 (2.46e-3)**
MMF15	1.0736e-1 (2.49e-2)	1.0237e-1 (2.42e-2)	7.9400e-2 (2.32e-3)	1.0535e-1 (2.21e-2)	8.0113e-2 (2.46e-3)	1.0179e-1 (1.96e-2)	1.0308e-1 (2.21e-2)	9.8690e-2 (1.98e-2)	9.2913e-2 (1.88e-2)	**7.7115e-2 (2.88e-3)**
MMF15_a	1.2974e-1 (3.63e-2)	1.1399e-1 (3.24e-2)	8.4485e-2 (4.31e-3)	1.2124e-1 (3.18e-2)	8.4560e-2 (4.09e-3)	1.2476e-1 (3.21e-2)	1.1812e-1 (2.96e-2)	1.2203e-1 (2.65e-2)	1.1167e-1 (2.76e-2)	**8.3340e-2 (3.97e-3)**

Table 9
IGDX results of the compared algorithms on MMOPLs with different values of α.

Problem	10 %	20 %	30 %	40 %	50 %	60 %	70 %	80 %	90 %	100 %
IDMPM2T1_ee	2.5073e-2	4.5419e-3	4.5875e-4	2.5582e-2	**4.1612e-4**	2.5039e-2	4.6337e-2	4.2386e-3	2.5192e-2	4.3845e-3
	(1.20e-1)	(3.32e-3)	(4.15e-4)	(1.17e-1)	**(9.75e-5)**	(1.19e-1)	(1.61e-1)	(2.17e-3)	(1.16e-1)	(3.45e-3)
IDMPM2T2_ee	4.6042e-3	4.4169e-3	4.9816e-4	4.1275e-3	**4.7738e-4**	5.0875e-3	4.8959e-3	5.3183e-3	7.6612e-3	4.6854e-3
	(3.58e-3)	(9.60e-3)	(8.10e-5)	(3.95e-3)	**(6.39e-5)**	(5.47e-3)	(6.55e-3)	(8.35e-3)	(1.07e-2)	(4.13e-3)
IDMPM2T3_ee	2.7565e-3	2.7090e-3	**8.9823e-4**	3.8849e-3	9.6557e-4	5.4937e-3	3.7561e-3	1.5523e-2	1.6297e-2	2.9216e-3
	(1.51e-3)	(1.23e-3)	**(5.41e-5)**	(2.87e-3)	(1.58e-4)	(6.81e-3)	(3.08e-3)	(6.03e-2)	(6.05e-2)	(1.66e-3)
IDMPM2T4_ee	1.1629e-1	1.1779e-1	5.1264e-2	9.7794e-2	**4.9229e-2**	1.1697e-1	1.1057e-1	1.0051e-1	1.2118e-1	9.8301e-2
	(6.12e-2)	(6.24e-2)	(5.48e-2)	(5.94e-2)	**(4.26e-2)**	(7.49e-2)	(6.14e-2)	(6.07e-2)	(7.65e-2)	(5.05e-2)
IDMPM3T1_ee	1.3189e-2	1.5384e-2	**6.5099e-3**	1.3563e-2	6.6182e-3	1.4474e-2	1.4198e-2	1.3836e-2	1.3616e-2	1.2460e-2
	(6.26e-3)	(8.28e-3)	**(3.28e-4)**	(4.21e-3)	(4.11e-4)	(4.57e-3)	(5.60e-3)	(4.57e-3)	(3.87e-3)	(2.77e-3)
IDMPM3T2_ee	5.1344e-2	5.1805e-2	6.5163e-3	7.0337e-2	**6.4724e-3**	6.4837e-2	4.1692e-2	3.9255e-2	3.7909e-2	4.9302e-2
	(8.25e-2)	(8.64e-2)	(8.15e-4)	(9.55e-2)	**(8.86e-4)**	(9.30e-2)	(6.03e-2)	(6.08e-2)	(5.66e-2)	(7.71e-2)
IDMPM3T3_ee	7.9919e-2	8.0362e-2	**8.0106e-3**	1.0890e-1	8.2952e-3	7.8951e-2	7.9201e-2	7.3989e-2	4.2198e-2	9.3573e-2
	(1.01e-1)	(1.05e-1)	**(7.74e-4)**	(1.40e-1)	(1.63e-3)	(9.33e-2)	(1.00e-1)	(9.69e-2)	(6.39e-2)	(1.06e-1)
IDMPM3T4_ee	2.7101e-1	2.7671e-1	2.5335e-1	2.9380e-1	2.7739e-1	2.8681e-1	2.8718e-1	2.5439e-1	2.1395e-1	**2.0023e-1**
	(1.14e-1)	(1.04e-1)	(1.96e-1)	(1.21e-1)	(1.82e-1)	(1.38e-1)	(1.47e-1)	(1.22e-1)	(1.10e-1)	**(1.21e-1)**
MMF10	2.0631e-2	2.4400e-2	7.1952e-3	1.9247e-2	**7.0986e-3**	2.1481e-2	1.7566e-2	1.8190e-2	1.8784e-2	1.5425e-2
	(1.09e-2)	(1.66e-2)	(9.10e-4)	(1.08e-2)	**(7.85e-4)**	(1.43e-2)	(9.25e-3)	(1.12e-2)	(9.98e-3)	(6.63e-3)
MMF11	8.9052e-3	9.5460e-3	**4.3904e-3**	9.0771e-3	4.4065e-3	9.1004e-3	8.4595e-3	9.0521e-3	8.6141e-3	5.3933e-3
	(4.40e-3)	(4.74e-3)	**(3.92e-4)**	(5.08e-3)	(3.73e-4)	(3.71e-3)	(3.18e-3)	(3.24e-3)	(3.59e-3)	(3.89e-4)
MMF12	3.8916e-3	3.9852e-3	9.0839e-3	3.9973e-3	4.7579e-3	3.8363e-3	3.7493e-3	4.0400e-3	3.9837e-3	**3.4786e-3**
	(6.95e-4)	(9.74e-4)	(1.15e-2)	(7.36e-4)	(1.99e-3)	(6.08e-4)	(7.47e-4)	(8.35e-4)	(7.50e-4)	**(8.32e-4)**
MMF13	6.8362e-2	7.7662e-2	**4.7031e-2**	6.2599e-2	4.7975e-2	6.2958e-2	6.0634e-2	6.5357e-2	5.9995e-2	4.9725e-2
	(1.96e-2)	(2.11e-2)	**(2.43e-3)**	(1.58e-2)	(2.61e-3)	(1.06e-2)	(1.28e-2)	(1.12e-2)	(1.15e-2)	(2.38e-3)
MMF15	5.0801e-2	4.9351e-2	**3.9059e-2**	5.0125e-2	3.9323e-2	4.9001e-2	4.9764e-2	4.7720e-2	4.5295e-2	3.9975e-2
	(8.96e-3)	(9.22e-3)	**(8.96e-4)**	(8.22e-3)	(1.28e-3)	(7.43e-3)	(9.62e-3)	(6.92e-3)	(6.94e-3)	(1.88e-3)
MMF15_a	7.2582e-2	6.4090e-2	4.8656e-2	6.7753e-2	4.8810e-2	7.0279e-2	6.5275e-2	6.7522e-2	6.2642e-2	**4.8011e-2**
	(1.96e-2)	(1.75e-2)	(2.61e-3)	(1.73e-2)	(2.87e-3)	(1.76e-2)	(1.55e-2)	(1.39e-2)	(1.45e-2)	**(2.84e-3)**

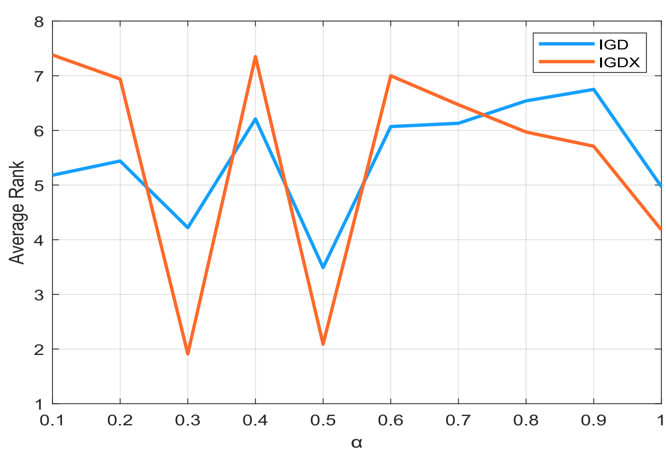

Fig. 8. Average rank of IGDX and IGD over 34 test problems with different values of α.

3.3. Update Q-table

Fig. 3 shows the Q-Table of the proposed model. The Learning process involves updating the Q-Table based on the outcomes of QL, while the Decision process entails selecting the auxiliary task according to the Q-Table. This Q-Table can estimate the Q values of performing any auxiliary task under any state.

This work defines these definitions as follows. The actions are performing one of the auxiliary tasks. They include:

$$A = \{a | a \in \{AT_1, AT_2, \cdots, AT_i\}\} \tag{4}$$

$$S = \{s | s \in \{(MT, AT_1), (MT, AT_2), \cdots, (MT, AT_i)\}\} \tag{5}$$

$$r_t = \frac{|P_j^t|}{|P_j|} \tag{6}$$

where AT_1 to AT_i represents performing the corresponding task, and i is the number of auxiliary tasks. where P_j is the population of AT_j; and P_j^t is the set of solutions transferred to MT (i.e., selected into MT for the next generation). For example, (MT, AT_1) means the algorithm currently adopts AT_1 as the auxiliary task. The reward of performing the j-th task AT_j is defined by the transfer ratio from AT_j to MT. Equation (6) defines the reward r_t obtained by performing an auxiliary task AT_j, which is calculated by dividing the size of the solution set P_j^t transferred from the population P_j of auxiliary task j to the MT solution set (i.e., the number of solutions transferred to MT) by the size of P_j (i.e., the total number of solutions in auxiliary task j). The transfer to MT is based on predefined criteria for each auxiliary task, where the first auxiliary task is based on diversity, the second on fitness, and the third on convergence. The algorithm selects outstanding solutions from the population P_j of AT_j to form P_j^t through a selection mechanism, and then calculates the transfer ratio r_t using Equation (6). This ratio serves as an immediate reward, which is used to update the Q-Table, thereby guiding the algorithm to choose more beneficial auxiliary tasks to optimize the MT. Then, the Q-Table is updated using the Bellman function:

$$Q(s, a) = Q(s, a) + \alpha[r + \gamma \max_{a'} Q(s'a') - Q(s, a)] \tag{7}$$

where α is the learning rate, and γ is the discount factor. As the core idea of QL, the Bellman function considers both the real-time reward of performing action a under state s and the expected future reward of taking the next action.

Algorithm 3: Update Q-Table

Input: (MT, AT_f) (current state), A (action set), i (number of auxiliary tasks), Q (trained Q-Table)
Output: T_j (selected action)
1. k ← Generate a random number in [0,1];
2. **if** k < 0.5 **then**
3. $AT_j = argmax_{a \in A} Q((MT, AT_f), a)$;
4. **else**
5. j ← Generate a random number in {1, 2, …, i};
6. **end if**
7. **return** T_j.

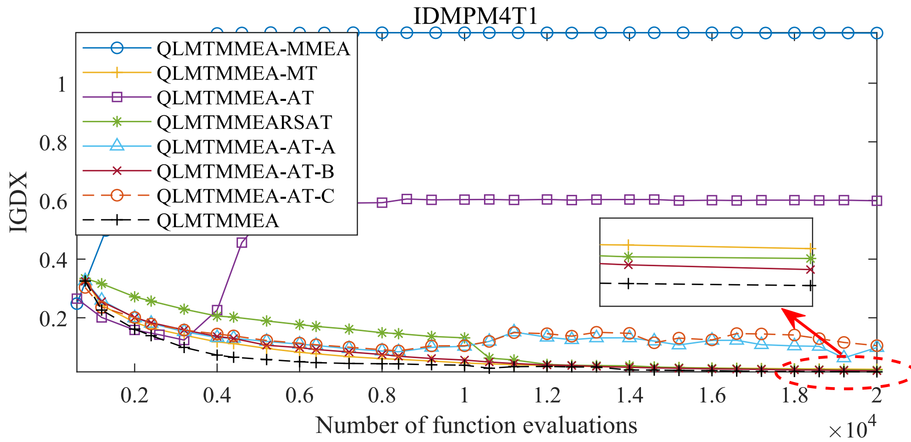

Fig. 9. The convergence curves of the MMOPs.

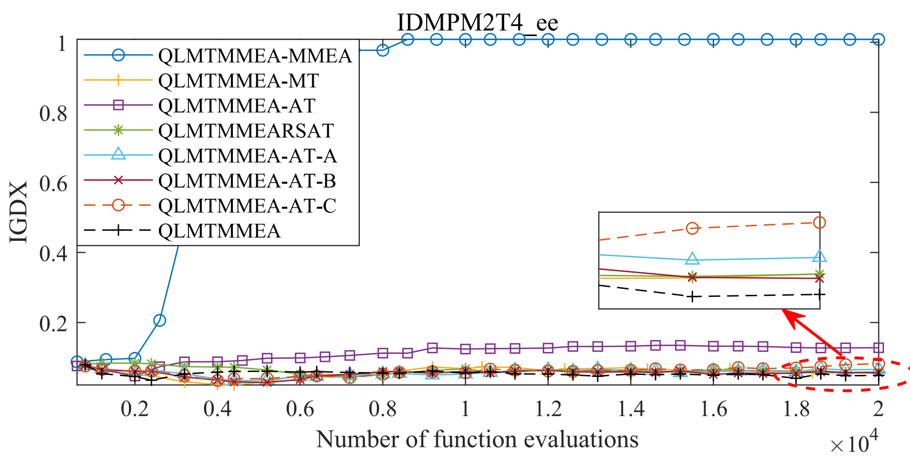

Fig. 10. The convergence curves of the MMOPLs.

3.4. Evolution of the main task

To preserve all PSs, the mating pool is established using a niching method. The selection operator is also modified accordingly. In this approach, only solutions within the same niche compete against each other. This paper adopts the crowding method, which proceeds as follows: (1) Randomly select a solution from the population. (2) Randomly select a certain number of solutions from the population. (3) Calculate the Euclidean distance between the current solution and each of the solutions, and choose the solution closest to the current solution. (4) Add the better of the two solutions the current solution and the nearest solution to the mating pool.

These steps are repeated until the mating pool is filled. Finally, environmental selection is performed based on non-dominated sorting. The environmental selection is as follows:

Algorithm 4: EnvironmentalSelectionMT

Input: P, OffA, OffB, N.
Output: P.
1.P ← ∅;
2.[FNo, MaxF] ← NDsort (P);/*Obtain the FNo (Pareto rank) of the P and the MaxF (max Pareto rank) */
3.Calculate fitness;
4.**while** Num<N **do**
5. From a new population according to the Pareto rank and fitness;
6.**end while**
7.**return** P.

3.5. Environmental selection of auxiliary tasks

Auxiliary task based on the ε-Pareto dominance, namely using the relaxation factor of Pareto dominance changes to ensure that diversity-first. In this paper, a novel formulation of the update strategy of ε is proposed, which is as follows:

$$\varepsilon = \log_2\left(\frac{g}{G}\right) \tag{8}$$

where g represents the current generation; G denotes the maximum generation.

The specific descriptions of the three auxiliary tasks are as follows: Auxiliary Task 1, Diversity-First strategy based on ε-Pareto dominance. The core of this task lies in the use of an improved ε-Pareto dominance, which is an extension of the standard Pareto dominance. By introducing a relaxation factor ε, the dominance relationship is adjusted to prioritize diversity. The update strategy for ε is shown in Equation (8). This formula ensures that as the evolution proceeds, the algorithm gradually shifts from emphasizing diversity (larger ε values) to focusing more on convergence (smaller ε values). By adjusting ε, the algorithm is able to retain more diverse solutions in the early stages, thereby increasing population diversity. This helps the algorithm discover more potential useful solutions during the exploration phase, providing a rich knowledge base for the subsequent convergence process. Auxiliary Task 2: Use of the strength Pareto evolutionary algorithm 2 (SPEA2) (Zitzler, Laumanns, & Thiele, 2001). Auxiliary Task 3: Pareto dominance. In this task, ε is set to zero, meaning the algorithm reverts to the standard Pareto dominance relationship, without considering the diversity-first

Table 10
IGD results of the compared algorithms on MMOPs.

Problem	QLMTMMEA-MMEA	QLMTMMEA-MT	QLMTMMEA-AT	QLMTMMEARSAT	QLMTMMEA-AT-A	QLMTMMEA-AT-B	QLMTMMEA-AT-C	QLMTMMEA
IDMPM2T1	5.0427e-4 (3.38e-5) −	6.4709e-4 (1.08e-4) −	**4.0753e-4 (4.48e-5)** +	7.2498e-4 (2.36e-4) −	5.5956e-4 (2.07e-4) −	6.3018e-4 (1.34e-4) −	5.7418e-4 (1.53e-4) −	4.5260e-4 (4.27e-5)
IDMPM2T2	5.0509e-4 (2.92e-5) −	5.0927e-4 (5.82e-5) −	4.0784e-4 (3.30e-5) =	5.4570e-4 (1.91e-4) −	4.8541e-4 (1.31e-4) −	4.8538e-4 (3.57e-5) −	4.7817e-4 (1.10e-4) −	**4.1972e-4 (3.06e-5)**
IDMPM2T3	4.7102e-4 (3.79e-5) −	5.1376e-4 (4.83e-5) −	4.2384e-4 (1.52e-5) =	5.8264e-4 (2.05e-4) −	5.1120e-4 (1.36e-4) −	5.2089e-4 (1.04e-4) −	5.3550e-4 (1.66e-4) −	**4.2779e-4 (2.87e-5)**
IDMPM2T4	4.9259e-4 (1.22e-5) −	5.3139e-4 (9.87e-5) −	3.8334e-4 (2.21e-5) =	6.8979e-4 (4.00e-4) −	5.3051e-4 (2.09e-4) −	5.4601e-4 (1.43e-4) −	4.5454e-4 (6.13e-5) −	**4.2053e-4 (2.45e-5)**
IDMPM3T1	6.3770e-3 (1.94e-4) −	6.4546e-3 (4.77e-4) −	5.1346e-3 (2.49e-4) =	6.9806e-3 (1.73e-3) −	5.7609e-3 (9.50e-4) =	6.3994e-3 (7.93e-4) −	5.7668e-3 (1.12e-3) =	**5.4478e-3 (3.24e-4)**
IDMPM3T2	6.3534e-3 (2.19e-4) −	5.9086e-3 (3.69e-4) −	4.9760e-3 (2.05e-4) =	6.3669e-3 (1.80e-3) −	5.4236e-3 (1.22e-3) =	5.9089e-3 (7.44e-4) −	5.5305e-3 (8.71e-4) =	**5.1895e-3 (3.37e-4)**
IDMPM3T3	6.3149e-3 (2.31e-4) −	5.7636e-3 (3.03e-4) −	5.1206e-3 (2.03e-4) =	6.5192e-3 (1.58e-3) −	5.4296e-3 (9.03e-4) =	5.8696e-3 (9.51e-4) −	5.2783e-3 (4.68e-4) =	**5.2829e-3 (3.33e-4)**
IDMPM3T4	6.3093e-3 (2.59e-4) −	5.8038e-3 (6.46e-4) −	4.8286e-3 (1.96e-4) =	6.6800e-3 (1.85e-3) −	5.6012e-3 (1.24e-3) =	5.6660e-3 (4.94e-4) −	5.5545e-3 (7.39e-4) =	**5.2332e-3 (3.86e-4)**
IDMPM4T1	9.3530e-3 (3.21e-4) −	9.5121e-3 (1.33e-3) −	**6.6570e-3 (1.70e-4)** +	1.0907e-2 (2.01e-3) −	9.0859e-3 (2.01e-3) =	1.0391e-2 (1.54e-3) −	8.7284e-3 (1.35e-3) =	8.1555e-3 (8.37e-4)
IDMPM4T2	8.6400e-3 (2.40e-4) −	8.3020e-3 (8.97e-4) −	**6.6183e-3 (2.63e-4)** +	8.7891e-3 (1.01e-3) −	7.6464e-3 (1.07e-3) =	9.0665e-3 (1.59e-3) −	7.9411e-3 (8.25e-4) =	7.7093e-3 (7.66e-4)
IDMPM4T3	8.7849e-3 (3.77e-4) −	8.9759e-3 (1.55e-3) −	**7.1488e-3 (5.12e-4)** +	8.9699e-3 (1.29e-3) −	8.9237e-3 (1.20e-3) −	8.9392e-3 (1.23e-3) −	8.6125e-3 (1.26e-3) −	7.9505e-3 (6.38e-4)
IDMPM4T4	8.5350e-3 (2.77e-4) −	9.0134e-3 (1.90e-3) =	**6.7107e-3 (4.27e-4)** +	9.0944e-3 (1.46e-3) −	8.7089e-3 (2.16e-3) =	9.1302e-3 (2.15e-3) −	8.0337e-3 (8.73e-4) =	7.9943e-3 (7.22e-4)
MMF1	1.2583e + 0 (1.47e-4) =	1.2590e + 0 (1.84e-3) =	1.2584e + 0 (3.01e-4) −	1.2586e + 0 (7.71e-4) −	**1.2583e + 0 (2.18e-4)** +	1.2584e + 0 (2.30e-4) −	1.2590e + 0 (1.35e-3) =	1.2584e + 0 (7.30e-5)
MMF2	6.1319e-1 (3.16e-3) =	6.1415e-1 (2.73e-3) =	6.1378e-1 (1.97e-3) −	6.1253e-1 (3.31e-3) =	6.0983e-1 (1.27e-2) =	6.1311e-1 (2.97e-3) =	6.0938e-1 (1.49e-2) =	**6.1344e-1 (2.59e-3)**
MMF3	4.6532e-1 (4.02e-3) =	4.6562e-1 (3.66e-3) =	4.6612e-1 (2.20e-3) −	4.6525e-1 (2.32e-3) =	4.6568e-1 (2.77e-3) =	4.6568e-1 (2.13e-3) =	**4.6235e-1 (6.41e-3)** +	4.6612e-1 (1.57e-3)
MMF4	6.1766e-1 (3.78e-2) −	5.5457e-1 (5.73e-2) =	6.0090e-1 (4.24e-2) −	5.8802e-1 (5.25e-2) =	5.6990e-1 (5.91e-2) =	5.5649e-1 (6.41e-2) =	5.7033e-1 (5.65e-2) =	**5.8138e-1 (5.54e-2)**
MMF5	1.6953e + 0 (3.69e-2) −	1.6482e + 0 (5.39e-2) =	1.6473e + 0 (5.05e-2) =	1.6464e + 0 (5.59e-2) =	1.6425e + 0 (5.61e-2) =	1.6453e + 0 (5.44e-2) =	1.6445e + 0 (4.92e-2) +	**1.6653e + 0 (5.68e-2)**
MMF6	1.3822e + 0 (7.19e-3) −	1.3762e + 0 (8.23e-3) =	1.3766e + 0 (8.09e-3) =	**1.3736e + 0 (7.27e-3)** +	1.3783e + 0 (7.13e-3) =	1.3760e + 0 (7.51e-3) =	1.3765e + 0 (8.22e-3) =	1.3778e + 0 (7.74e-3)
MMF7	1.0510e + 0 (3.64e-5) =	1.0512e + 0 (6.45e-4) +	1.0510e + 0 (8.17e-5) −	1.0511e + 0 (2.57e-4) +	**1.0510e + 0 (5.81e-5)** +	1.0511e + 0 (3.29e-4) =	1.0517e + 0 (2.26e-3) −	1.0512e + 0 (6.26e-4)
MMF8	3.7215e + 0 (2.90e-4) −	3.7207e + 0 (2.59e-3) +	3.7213e + 0 (2.05e-4) =	3.7211e + 0 (3.55e-4) =	3.7211e + 0 (2.99e-4) +	3.7210e + 0 (3.32e-4) +	**3.6871e + 0 (1.86e-1)** +	3.7213e + 0 (2.63e-4)
+/-/=	0/16/4	2/11/7	5/5/10	2/13/5	3/5/12	1/13/6	3/6/11	

relaxation strategy. When ε is zero, the algorithm relies entirely on the standard Pareto dominance relationship to select and retain solutions. With diversity no longer prioritized, the algorithm can converge to the PF more rapidly.

Algorithm 5: EnvironmentalSelectionAT$_i$

Input: P, i, OffA, OffB, N.
Output: P
1.P ← ∅;
2.**if** i = 1 **then**
3. Calculate indicator of diversity by eq. (8);
4.**else if** i = 2 **then**
5. Adopt the Environmental Selection of SPEA2;
6.**else**
7. ε is set to zero;
8.**end if**
9.**while** Num<N **do**
10. From a new population according to the Pareto rank and indicators of diversity;
11.**end while**
12.**return** P.

3.6. Computational complexity

Given that QLMTMMEA employs distinct strategies across two stages of evolution, the worst-case time complexity for mating selection and offspring generation is $2 \times O(TN)$ and $2 \times O(TND)$, respectively. Here, T represents the number of tasks, M denotes the number of objectives, D signifies the number of decision variables, and N is the population size. Consequently, the overall complexity per generation is primarily $O(TMN^2)$ for both stages. Therefore, the total computational complexity of the proposed QLMTMMEA is $O(TMN^2)$.

4. Experiment

4.1. Experimental settings

1)Competitive Algorithms: To examine the performance of QLMTMMEA in dealing with MMOPs, seven state-of-the-art MMEAs were chosen as competitor algorithms, including: DN-NSGA-II(J. J. Liang, et al., 2016), MO_Ring_PSO_SCD(Yue, et al., 2018), TriMOEA-TA&R(Yiping Liu, et al., 2019), MMEA-WI(W. Li, et al., 2021), HREA (Li, Yao, Zhang, Wang, & Wang, 2023), CMMO (Ming, et al., 2023), CoMMEA (Li, Yao, Li, et al., 2023). DN-NSGA-II, MO_Ring_PSO_SCD and TriMOEA-TA&R are representative algorithms in the field of multi-objective multimodal optimization. MMEA-WI and HREA are two representative MMEAs known for their strong performance in searching for local PSs. CMMO and CoMMEA are two new yet competitive MMEAs.

2)Parameter setting: To ensure a fair comparison among the algorithms, the following parameter settings are applied consistently across all experiments: the population size N is set to 200, and the maximum number of function evaluations is set to 20000, where D represents the number of decision variables. This scaling ensures that the computational effort is proportional to the problem's dimensionality, a standard practice in many related works. Except for MO_Ring_PSO_SCD, which uses the Particle Swarm Optimization (PSO) operator, all other

Table 11
IGDX results of the compared algorithms on MMOPs.

Problem	QLMTMMEA-MMEA	QLMTMMEA-MT	QLMTMMEA-AT	QLMTMMEARSAT	QLMTMMEA-AT-A	QLMTMMEA-AT-B	QLMTMMEA-AT-C	QLMTMMEA
IDMPM2T1	6.2836e-1 (1.71e-1) −	7.4711e-4 (1.28e-4) −	2.2024e-3 (2.47e-3) −	7.2926e-4 (1.79e-4) −	8.7383e-4 (4.17e-4) −	6.3816e-4 (2.35e-4) −	7.1834e-4 (3.85e-4) −	**4.7079e-4 (6.67e-5)**
IDMPM2T2	3.8184e-1 (3.39e-1) −	9.7081e-4 (1.77e-4) −	2.7089e-3 (4.06e-3) −	9.4526e-4 (2.01e-4) −	1.1710e-3 (4.03e-4) −	7.9949e-4 (9.31e-5) −	1.1135e-3 (6.75e-4) −	**7.0409e-4 (9.96e-5)**
IDMPM2T3	2.6206e-1 (3.19e-1) −	1.9174e-3 (3.78e-4) −	2.2728e-3 (6.44e-4) −	1.7706e-3 (3.14e-4) −	2.0921e-3 (6.09e-4) −	1.6021e-3 (5.06e-4) −	2.2319e-3 (7.18e-4) −	**1.4531e-3 (4.70e-4)**
IDMPM2T4	6.5080e-1 (1.23e-1) −	1.2537e-3 (1.09e-3) −	1.3159e-1 (2.46e-1) −	1.0943e-3 (4.47e-4) −	1.7182e-3 (1.54e-3) −	8.4275e-4 (4.28e-4) −	2.1715e-3 (4.72e-3) −	**7.5483e-4 (9.22e-4)**
IDMPM3T1	6.5291e-1 (2.45e-1) −	8.8289e-3 (7.20e-4) −	2.9077e-2 (6.02e-2) −	9.3172e-3 (1.46e-3) −	9.8837e-3 (2.72e-3) −	7.9980e-3 (1.05e-3) =	1.0281e-2 (3.17e-3) −	**7.6512e-3 (4.39e-4)**
IDMPM3T2	6.0325e-1 (1.70e-1) −	9.6208e-3 (1.48e-3) −	5.8826e-2 (9.23e-2) −	1.0742e-2 (1.87e-3) −	2.2763e-2 (4.47e-2) −	8.8555e-3 (1.60e-3) =	1.3227e-2 (5.59e-3) −	**8.9574e-3 (2.87e-3)**
IDMPM3T3	4.0243e-1 (2.72e-1) −	1.2800e-2 (2.56e-3) +	6.0100e-2 (9.59e-2) −	1.3358e-2 (2.63e-3) +	1.7165e-2 (8.17e-3) +	**1.0923e-2 (1.86e-3)** +	1.5285e-2 (4.68e-3) +	1.7586e-2 (4.58e-2)
IDMPM3T4	7.5942e-1 (2.79e-1) −	1.4711e-2 (6.09e-3) +	2.0585e-1 (2.26e-1) −	**1.3534e-2 (3.77e-3)** +	2.6323e-2 (4.50e-2) +	1.3673e-2 (1.04e-2) =	2.3760e-2 (4.36e-2) +	3.4604e-2 (7.15e-2)
IDMPM4T1	1.1709e + 0 (8.28e-2) −	2.4228e-2 (6.33e-3) −	5.9903e-1 (2.80e-1) −	2.1910e-2 (3.95e-3) −	9.8495e-2 (1.99e-1) −	1.9306e-2 (1.09e-2) −	1.0502e-1 (1.59e-1) −	**1.5538e-2 (6.37e-3)**
IDMPM4T2	1.0815e + 0 (1.85e-1) −	7.6852e-2 (8.93e-2) =	5.1356e-1 (2.88e-1) −	4.8217e-2 (4.97e-2) =	2.4039e-1 (2.54e-1) −	6.5769e-2 (8.67e-2) =	2.0331e-1 (2.08e-1) −	**1.0265e-1 (1.15e-1)**
IDMPM4T3	8.2205e-1 (3.41e-1) −	4.0781e-2 (4.73e-2) =	2.5659e-1 (1.40e-1) −	3.3042e-2 (1.86e-2) =	8.2442e-2 (1.28e-1) =	4.1474e-2 (4.18e-2) =	6.0896e-2 (7.94e-2) =	**9.5025e-2 (1.12e-1)**
IDMPM4T4	1.0733e + 0 (1.72e-1) −	7.7440e-2 (7.80e-2) =	6.3031e-1 (3.06e-1) −	7.4148e-2 (7.45e-2) =	1.4494e-1 (1.82e-1) =	7.8883e-2 (9.55e-2) =	1.8018e-1 (1.90e-1) =	**1.4109e-1 (1.33e-1)**
MMF1	4.5400e-2 (1.45e-2) −	3.7079e-2 (1.47e-3) −	2.8413e-2 (8.38e-4) −	3.2255e-2 (3.09e-3) −	3.0966e-2 (2.08e-3) −	2.9864e-2 (2.41e-3) −	3.1823e-2 (5.73e-3) −	**2.7824e-2 (8.47e-4)**
MMF2	5.5769e-2 (2.99e-2) −	2.4045e-2 (4.87e-3) −	1.6224e-2 (4.55e-3) −	1.8680e-2 (4.76e-3) −	1.9966e-2 (4.91e-3) −	1.9285e-2 (4.71e-3) −	1.9895e-2 (4.42e-3) −	**1.3055e-2 (2.34e-3)**
MMF3	4.2682e-2 (1.71e-2) −	2.2867e-2 (4.15e-3) −	1.2615e-2 (2.37e-3) =	1.5561e-2 (4.27e-3) −	1.8566e-2 (3.56e-3) −	1.8172e-2 (3.95e-3) −	1.8033e-2 (5.29e-3) −	**1.2201e-2 (2.23e-3)**
MMF4	2.7630e-2 (3.21e-3) −	2.2285e-2 (1.05e-3) −	**1.5707e-2 (5.12e-4)** +	1.8967e-2 (2.60e-3) −	1.8754e-2 (2.02e-3) −	1.8766e-2 (1.78e-3) −	2.1069e-2 (5.87e-3) −	1.6524e-2 (7.26e-4)
MMF5	8.2855e-2 (5.24e-3) −	6.4221e-2 (2.20e-3) −	**4.2950e-2 (1.13e-3)** +	5.0301e-2 (6.20e-3) −	4.9124e-2 (4.61e-3) −	4.9597e-2 (5.21e-3) −	5.0946e-2 (1.01e-2) −	4.4251e-2 (1.69e-3)
MMF6	7.3674e-2 (3.66e-3) −	5.4447e-2 (1.93e-3) −	**3.8061e-2 (1.12e-3)** +	4.5062e-2 (3.34e-3) −	4.2961e-2 (2.81e-3) −	4.2462e-2 (3.61e-3) −	4.8966e-2 (1.14e-2) −	4.0379e-2 (1.54e-3)
MMF7	2.2893e-2 (1.88e-3) −	1.9527e-2 (9.45e-4) −	**1.5817e-2 (6.91e-4)** +	1.7790e-2 (1.66e-3) −	1.7732e-2 (1.34e-3) −	1.7766e-2 (1.65e-3) −	1.8549e-2 (4.52e-3) −	1.6157e-2 (9.61e-4)
MMF8	1.0114e-1 (3.34e-2) −	5.6377e-2 (5.30e-3) −	4.6305e-2 (1.14e-2) −	4.3729e-2 (7.76e-3) −	4.4770e-2 (5.26e-3) −	4.3729e-2 (6.15e-3) −	4.9316e-2 (1.37e-2) −	**3.5486e-2 (2.13e-3)**
+/-/=	0/20/0	2/15/3	4/15/1	2/15/3	2/16/2	1/13/6	2/16/2	

algorithms utilize the Simulated Binary Crossover (SBX) and Polynomial Mutation (PM) operators for generating offspring. These operators are widely used in evolutionary algorithms for their effectiveness in exploring and exploiting the search space. The parameters specific to each comparison algorithm are set according to the values reported in their original publications to ensure that each algorithm is evaluated under conditions that reflect its intended operational settings. All experiments are conducted using the PlatEMO platform, which provides a comprehensive environment for implementing and testing multi-objective evolutionary algorithms (Tian, Cheng, Zhang, & Jin, 2017). The experiments are run on a PC equipped with an NVIDIA GTX 1650 processor operating at 2.40 GHz, with 16 GB of RAM. This setup ensures that the computational resources are sufficient to handle the demands of the experiments, providing reliable and reproducible results.

3) Benchmark Problems: To benchmark the performance of QLMTMMEA, the proposed test problems in IDMP_ee are used (Li, Yao, Zhang, Wang, & Wang, 2023). Additionally, benchmarks from previous studies are included, as they feature local PFs. Given that the proposed QLMTMMEA is also intended to solve traditional MMOPs, both Multi-Modal Function (MMF) problems and IDMPs are employed as test problems (Liang, Qu, Gong, & Yue, 2019; Liu, Ishibuchi, Yen, Nojima, & Masuyama, 2020). MMF problems are designed by mirroring the original Pareto optimal solution to create multiple equivalent subsets. IDMPs are used to evaluate the algorithm's performance across problems with varying levels of difficulty in discovering different PSs. The parameters for each benchmark problem are set according to the recommended values from their original papers. Furthermore, the true PF and PS reference points are provided by these original papers.

4)Performance metrics: Typically, the Inverted Generational Distance (IGD) and IGDX serve as performance metrics for assessing how well obtained solution sets approximate the true PF and PSs (Zitzler, Thiele, Laumanns, Fonseca, & Fonseca, 2003). Here's how they're calculated for a solution set:

$$IGD(X) = \frac{1}{|X^*|}\sum_{y\in X^*}\min_{x\in X}\{D(F(x),F(y))\} \tag{9}$$

$$IGDX(X) = \frac{1}{|X^*|}\sum_{y\in X^*}\min_{x\in X}\{D(x,y)\} \tag{10}$$

where $D(F(x),F(y))$ is the Euclidean distance between $F(x)$ and $F(y)$, $D(x,y)$ is the Euclidean distance between x and y, X and X^* denote the obtained solution set and a set of a finite number of Pareto optimal solutions uniformly sampled from the true PS. It is worth noting that a small IGD value implies that the solution set X has reasonable convergence and diversity in the objective space, which indicates that the MMEA is an effective MOEA.

4.2. Experimental results

1)Results of MMOPs: This section shows the performance comparison between QLMTMMEA and competitor algorithms on IDMP and

Table 12
IGD results of the compared algorithms on MMOPLs.

Problem	QLMTMMEA-MMEA	QLMTMMEA-MT	QLMTMMEA-AT	QLMTMMEARSAT	QLMTMMEA-AT-A	QLMTMMEA-AT-B	QLMTMMEA-AT-C	QLMTMMEA
IDMPM2T1_ee	7.2448e-3 (1.59e-5) −	7.2013e-3 (5.67e-5) =	7.2382e-3 (2.04e-5) −	7.1835e-3 (1.63e-4) =	7.1666e-3 (5.24e-5) +	**7.1542e-3 (9.44e-5)** +	7.3943e-3 (1.26e-3) −	7.2042e-3 (4.32e-5)
IDMPM2T2_ee	7.2692e-3 (1.52e-5) −	1.4323e-3 (2.50e-4) −	3.7205e-3 (2.17e-3) −	8.4274e-4 (3.76e-4) −	1.8339e-3 (1.07e-3) −	7.8607e-4 (2.01e-4) −	1.2420e-3 (9.29e-4) −	**4.8413e-4 (7.91e-5)**
IDMPM2T3_ee	5.1546e-3 (8.45e-4) −	1.4717e-3 (1.47e-4) −	7.2058e-4 (1.79e-4) =	1.1799e-3 (4.01e-4) −	1.0447e-3 (2.31e-4) −	9.5560e-4 (2.82e-4) −	1.1506e-3 (5.58e-4) −	**6.6144e-4 (6.32e-5)**
IDMPM2T4_ee	1.6150e-2 (2.12e-5) −	2.0226e-3 (5.03e-4) =	2.4870e-3 (1.16e-3) −	1.7608e-3 (5.66e-4) =	1.9210e-3 (7.59e-4) =	2.0553e-3 (8.34e-4) =	2.6331e-3 (8.19e-4) −	**1.9713e-3 (8.65e-4)**
IDMPM3T1_ee	2.5933e-2 (5.98e-4) −	9.0311e-3 (6.92e-4) −	7.7925e-3 (3.53e-3) −	8.4149e-3 (1.67e-3) −	8.2392e-3 (1.38e-3) −	7.5743e-3 (1.33e-3) −	7.2171e-3 (1.66e-3) −	**5.9761e-3 (4.89e-4)**
IDMPM3T2_ee	3.7883e-2 (2.94e-4) −	1.1676e-2 (1.41e-3) −	1.0371e-2 (4.31e-3) −	9.8995e-3 (2.69e-3) −	1.0478e-2 (3.18e-3) −	9.1771e-3 (2.15e-3) −	8.7479e-3 (2.48e-3) −	**6.2490e-3 (6.57e-4)**
IDMPM3T3_ee	3.7751e-2 (3.05e-4) −	1.3191e-2 (1.20e-3) −	1.5754e-2 (5.56e-3) −	1.1975e-2 (3.42e-3) −	1.2400e-2 (2.20e-3) −	1.0734e-2 (2.98e-3) −	1.1178e-2 (3.00e-3) −	**7.5648e-3 (1.39e-3)**
IDMPM3T4_ee	5.9857e-2 (5.55e-4) −	1.6725e-2 (3.92e-3) +	1.6556e-2 (8.15e-3) +	1.7215e-2 (3.96e-3) +	**1.5323e-2 (5.55e-3)** +	1.7568e-2 (4.67e-3) +	1.6299e-2 (5.23e-3) +	2.0583e-2 (3.21e-3)
MMF10	1.9191e-1 (1.36e-2) −	1.5775e-2 (1.60e-3) −	**8.3212e-3 (3.17e-4)** +	1.2474e-2 (3.39e-3) −	1.0891e-2 (1.97e-3) −	1.2735e-2 (3.31e-3) −	1.9330e-2 (1.26e-2) −	9.0267e-3 (9.32e-4)
MMF11	9.8008e-2 (1.02e-3) −	1.8136e-2 (1.34e-3) −	1.0463e-2 (4.16e-4) =	1.1838e-2 (1.35e-3) −	1.1807e-2 (1.56e-3) −	1.2369e-2 (2.07e-3) −	1.6724e-2 (8.90e-3) −	**1.0177e-2 (1.12e-3)**
MMF12	8.3223e-2 (1.60e-4) −	**5.2452e-3 (4.56e-4)** +	8.6802e-3 (2.08e-3) =	8.3931e-3 (2.43e-3) =	7.4295e-3 (4.20e-3) +	6.5630e-3 (3.71e-3) +	1.0465e-2 (3.69e-3) =	9.3984e-3 (3.99e-3)
MMF13	1.5482e-1 (3.75e-3) −	2.4875e-2 (3.06e-3) −	**1.4546e-2 (1.34e-2)** +	2.0175e-2 (1.28e-2) =	1.8397e-2 (9.55e-3) =	1.9962e-2 (4.69e-3) +	1.8152e-2 (7.59e-3) =	2.0797e-2 (1.81e-2)
MMF15	1.9595e-1 (4.14e-3) −	8.8518e-2 (1.45e-3) −	7.8888e-2 (2.27e-3) =	8.2088e-2 (4.20e-3) =	8.1157e-2 (3.69e-3) =	8.0793e-2 (4.99e-3) =	9.4692e-2 (1.84e-2) −	**8.0113e-2 (2.46e-3)**
MMF15_a	1.9482e-1 (3.76e-3) −	1.0954e-1 (4.34e-3) −	**8.0008e-2 (3.02e-3)** +	9.3401e-2 (8.83e-3) −	8.9337e-2 (8.79e-3) =	9.1281e-2 (7.27e-3) −	9.7115e-2 (1.94e-2) −	8.4560e-2 (4.09e-3)
+/-/=	0/14/0	2/10/2	4/6/4	1/8/5	3/7/4	4/8/2	1/11/2	

Table 13
IGDX results of the compared algorithms on MMOPLs.

Problem	QLMTMMEA-MMEA	QLMTMMEA-MT	QLMTMMEA-AT	QLMTMMEARSAT	QLMTMMEA-AT-A	QLMTMMEA-AT-B	QLMTMMEA-AT-C	QLMTMMEA
IDMPM2T1_ee	6.7320e-1 (2.42e-5) −	1.3060e-3 (3.37e-4) −	9.3666e-2 (2.19e-1) −	1.4830e-3 (1.05e-3) −	7.0925e-2 (1.97e-1) −	1.0791e-3 (9.50e-4) −	2.6826e-2 (1.19e-1) −	**4.1612e-4 (9.75e-5)**
IDMPM2T2_ee	6.7330e-1 (1.05e-4) −	1.1291e-3 (1.49e-4) −	4.0377e-3 (5.13e-3) −	7.9927e-4 (2.81e-4) −	1.3191e-3 (6.28e-4) −	7.2824e-4 (1.32e-4) −	9.8257e-4 (6.39e-4) −	**4.7738e-4 (6.39e-5)**
IDMPM2T3_ee	4.5966e-1 (1.77e-1) −	1.8651e-3 (2.80e-4) −	2.3988e-3 (3.72e-3) −	1.4243e-3 (4.07e-4) −	1.6024e-3 (5.12e-4) −	1.1567e-3 (2.83e-4) −	1.6795e-3 (1.06e-3) −	**9.6557e-4 (1.58e-4)**
IDMPM2T4_ee	1.0080e + 0 (5.61e-5) −	5.8686e-2 (3.38e-2) −	1.2803e-1 (7.63e-2) −	5.8637e-2 (3.36e-2) =	6.6322e-2 (3.70e-2) −	5.6635e-2 (3.72e-2) =	8.2504e-2 (4.61e-2) −	**4.9229e-2 (4.26e-2)**
IDMPM3T1_ee	6.9638e-1 (1.44e-1) −	1.0686e-2 (1.15e-3) −	1.0258e-2 (4.26e-3) −	8.6861e-3 (1.75e-3) −	9.4138e-3 (1.48e-3) −	7.7949e-3 (1.50e-3) −	8.8775e-3 (2.51e-3) −	**6.6182e-3 (4.11e-4)**
IDMPM3T2_ee	7.8787e-1 (1.80e-1) −	1.3141e-2 (2.60e-3) −	8.6717e-2 (1.37e-1) −	1.0200e-2 (2.22e-3) −	1.3574e-2 (3.51e-3) −	8.9923e-3 (2.24e-3) −	2.0837e-2 (4.46e-2) −	**6.4724e-3 (8.86e-4)**
IDMPM3T3_ee	7.2239e-1 (2.54e-1) −	1.5815e-2 (2.33e-3) −	1.0281e-1 (1.17e-1) −	1.3602e-2 (3.11e-3) −	1.5218e-2 (4.39e-3) −	1.2596e-2 (4.94e-3) −	2.4797e-2 (4.61e-2) −	**8.2952e-3 (1.63e-3)**
IDMPM3T4_ee	1.0408e + 0 (1.41e-1) −	**8.6802e-2 (8.68e-2)** +	3.4097e-1 (1.26e-1) =	1.0922e-1 (1.13e-1) +	1.6566e-1 (1.72e-1) +	1.4865e-1 (1.61e-1) +	1.9061e-1 (1.77e-1) +	2.7739e-1 (1.82e-1)
MMF10	1.9856e-1 (5.82e-3) −	7.9542e-3 (7.90e-4) −	**5.2472e-3 (3.01e-4)** +	7.7862e-3 (2.02e-3) =	6.0934e-3 (6.84e-4) +	7.2082e-3 (1.63e-3) =	9.8043e-3 (5.37e-3) =	7.0986e-3 (7.85e-4)
MMF11	2.4910e-1 (3.30e-4) −	6.5580e-3 (4.76e-4) −	4.4984e-3 (1.80e-4) =	4.9361e-3 (4.00e-4) −	4.8904e-3 (4.67e-4) −	5.3189e-3 (5.70e-4) −	6.6546e-3 (3.09e-3) −	**4.4065e-3 (3.73e-4)**
MMF12	2.4513e-1 (3.83e-4) −	**3.1740e-3 (2.74e-4)** +	6.0008e-3 (5.47e-3) =	4.2797e-3 (1.17e-3) =	4.4999e-3 (2.35e-3) =	3.6418e-3 (1.36e-3) +	6.2795e-3 (5.77e-3) =	4.7579e-3 (1.99e-3)
MMF13	2.5794e-1 (1.30e-3) −	5.7066e-2 (1.78e-3) −	**4.5835e-2 (1.98e-3)** +	4.9592e-2 (3.84e-3) =	5.0627e-2 (3.40e-3) −	5.0750e-2 (4.26e-3) −	5.1016e-2 (5.79e-3) −	4.7975e-2 (2.61e-3)
MMF15	2.6105e-1 (3.85e-3) −	4.2719e-2 (1.15e-3) −	4.0440e-2 (1.08e-3) −	4.0803e-2 (1.64e-3) −	4.0626e-2 (1.68e-3) −	3.9815e-2 (1.90e-3) =	4.5687e-2 (6.61e-3) −	**3.9323e-2 (1.28e-3)**
MMF15_a	2.1395e-1 (5.77e-3) −	6.0077e-2 (2.47e-3) −	**4.7231e-2 (2.88e-3)** +	5.2752e-2 (4.64e-3) −	5.1324e-2 (4.07e-3) −	5.1522e-2 (3.96e-3) −	5.4723e-2 (8.15e-3) −	4.8810e-2 (2.87e-3)
+/-/=	0/14/0	2/12/0	3/8/3	1/9/4	2/11/1	2/9/3	1/11/2	

MMF test suites. The IDMP test problems represent MMOPs with imbalanced PSs, which are more representative in some aspects. The results, summarized in Table 1, display the mean IGDX values over 30 independent runs. The Wilcoxon rank-sum test ($p < 0.05$) is utilized to compare QLMTMMEA with each competitor algorithm. In the final column of each table, symbols “+” and “−“ denote the number of test problems where the compared algorithm demonstrates significantly better or worse performance, respectively, compared to QLMTMMEA. Additionally, “=” indicates the number of test problems where no significant difference exists between QLMTMMEA and the compared algorithms.

Table 1 illustrates the IGDX comparison results, indicating that QLMTMMEA outperforms other state-of-the-art algorithms across the chosen test problems. Specifically, QLMTMMEA emerges victorious in fifteen instances out of 20 test problems. The table highlights QLMTMMEA, HREA, and CoMMEA as the top three algorithms for addressing these problems. However, DN-NSGA-II and TriMOEA-TA&R, despite being designed for MMOPs, exhibit poor performance, failing to win on any test instance. While DN-NSGA-II represents significant early works in MMOP research, their performance suffers on these benchmark problems due to the imbalance difficulties inherent in the IDMP test suites. These primitive MMEAs struggle to locate all PSs in such scenarios, leading to subpar IGDX performance. Conversely, for traditional MMOPs like the MMF test suite, differences in algorithm performance are negligible. In contrast, for IDMPs, QLMTMMEA, HREA and CoMMEA demonstrate significant superiority over other algorithms.

Table 2 presents the IGD comparison results for these algorithms, revealing that QLMTMMEA, HREA and MO_Ring_PSO_SCD outshine other alternatives. Analysis of the table indicates that QLMTMMEA, HREA, and CoMMEA can successfully capture all distinct PSs. Particularly, QLMTMMEA demonstrates adeptness in balancing both convergence and diversity of solutions. In summary, QLMTMMEA proves to be competitive when measured against other state-of-the-art methods across the selected benchmark problems.

Fig. 4 depicts the distribution of obtained solutions and true solutions in decision spaces. Here, one global PF corresponds to two distinct PSs. Notably, QLMTMMEA successfully identifies all different PSs with commendable convergence and distribution. In contrast, the remaining algorithms are limited to finding a part of PSs. Fig. 5 shows the distribution of the solution sets of the algorithms on MMF8 in the decision space. It can be seen that only QLMTMMEA, MMEA-WI, HREA, and CoMMEA obtained relatively complete PSs.

2) Results of MMOPLs: This section presents the performance of QLMTMMEA and competing algorithms on MMOPLs. Specifically, eight MMOPLs based on the IDMP test suite are used. Additionally, MMF10, MMF11, MMF12, MMF13, MMF15, and MMF15a are identified as MMOPLs. All experiments were conducted 30 times. The Wilcoxon rank-sum test ($p < 0.05$) is utilized to compare QLMTMMEA with each competitor algorithm. In the final column of each table, symbols "+" and "−" denote the number of test problems where the compared algorithm demonstrates significantly better or worse performance, respectively, compared to QLMTMMEA. Additionally, "=" indicates the number of test problems where no significant difference exists between QLMTMMEA and the compared algorithms.

Table 3 illustrates the IGDX comparison results, indicating that QLMTMMEA outperforms other state-of-the-art algorithms across the chosen test problems. Specifically, QLMTMMEA emerges victorious in twelve instances out of 14 test problems. The table highlights QLMTMMEA and HREA as the top two algorithms for addressing these problems.

Table 4 presents the IGD comparison results for these algorithms, highlighting that QLMTMMEA outperforms the other alternatives. The analysis indicates that both QLMTMMEA and HREA successfully capture all distinct PSs. Notably, QLMTMMEA excels in balancing convergence and diversity of solutions. In summary, QLMTMMEA proves to be highly competitive when compared to other state-of-the-art methods across the selected benchmark problems.Fig. 6 shows the PSs obtained by all algorithms on IDMPM2T1_ee. It can be seen from the figure that only QLMTMMEA captures all PSs. Additionally, Fig. 7 illustrates the PSs obtained by all algorithms on IDMPM3T2_ee, which has three decision variables. It is evident that only QLMTMMEA obtains all PSs, while HREA captures three sets of PSs, and MMEA-WI, CMMO, and CoMMEA capture two sets of PSs. DN-NSGA-II, MO_Ring_PSO_SCD, and TriMOEA-TA&R only capture one set of PSs, likely because these three algorithms were developed earlier and did not account for MMOPs with local optima. Overall, QLMTMMEA demonstrates a decisive advantage on MMOPLs.

4.3. Effectiveness analysis of parameter

In Section 3.1, a parameter α to select computing resources is introduced. To analyze the effect of parameter α, QLMTMMEA is examined on 34 MMOPs problems with different $p \in (0,1]$. Then, for each test problem, the average IGDX and IGD values over 30 runs are used to calculate the rank. The smaller the average rank is, the better the algorithm. From the Tables 5–9 and Fig. 8, it is clearly that there is little difference in the average rank of IGDX and IGD as the value of parameter α changes, which means that the performance of QLMTMMEA is not sensitive to α. As a tradeoff, we set $\alpha = 50$ % as the default value. Fig. 8Table 5

4.4. Effectiveness analysis of strategies

To validate the effectiveness of the proposed strategies in this study, seven variants of the algorithm were designed. Among them, QLMTMMEA-MMEA is a variant of the proposed algorithm that does not incorporate a multitasking optimization mechanism. This means that the algorithm operates in a traditional, single-task manner, focusing solely on optimizing the primary task without leveraging the potential benefits of multitasking. QLMTMMEA-MT integrates a multi-task optimization mechanism in the algorithm, but does not include reinforcement learning for adaptive selection of auxiliary tasks, which is to optimize auxiliary tasks directly with the main task. QLMTMMEA-AT employs auxiliary tasks but does not include diversity enhancement techniques for these tasks. This variant test the impact of diversity enhancement on the performance of the algorithm. Random selection of auxiliary tasks (QLMTMMEA-RSAT) variant randomly picks one of the three available auxiliary tasks to assist in the evolution process. When the choice task was only two, there were the following three variants: the first was to use auxiliary task 1 and auxiliary task 2 (QLMTMMEA-AT-A), the second was to use auxiliary task 1 and auxiliary task 3 (QLMTMMEA-AT-B), and the third was to use auxiliary task 2 and auxiliary task 3 (QLMTMMEA-AT-C).

According to the convergence curves of the two test questions in Fig. 9 and Fig. 10, the indispensable role of the proposed strategy in improving convergence and population diversity is verified. Table 10 and Table 12 list the IGD results of the compared algorithms, while Table 11 and Table 13 show the IGDX results of the compared algorithms. Figs. 9 and 10.

5. Conclusion

This paper presented a novel Multi-Task MMEA designed to address the challenge of maintaining diversity in both decision and objective spaces for MMOPs. QLMTMMEA introduced a multi-task optimization framework that included a main task and three auxiliary tasks, each employing different strategies tailored for MMOPs. By utilizing QL techniques, this framework adaptively selected the most optimal auxiliary tasks, ensuring a balanced focus on both convergence and diversity. Additionally, QLMTMMEA featured a new diversity enhancement technique that dynamically adjusted the relaxation factor to maintain a set of promising solutions, thereby improving the diversity of the objective space and decision space. The effectiveness of the proposed strategy was demonstrated through comparative experiments.

QLMTMMEA also has some limitations, such as increased computational complexity and potential overfitting to specific problem types. Future work will focus on reducing computational costs, enhancing scalability, and validating the algorithm on a broader range of real-world problems. Further exploration of adaptive mechanisms for task selection and diversity enhancement will also be pursued.

CRediT authorship contribution statement

Jie Cao: Conceptualization, Project administration, Funding acquisition, Supervision. **Yuze Yang:** Conceptualization, Methodology, Data curation, Writing – original draft. **Jianlin Zhang:** Writing – review & editing, Funding acquisition, Supervision. **Zuohan Chen:** Visualization, Validation, Writing – review & editing, Funding acquisition. **Zongli Liu:** Conceptualization, Project administration, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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