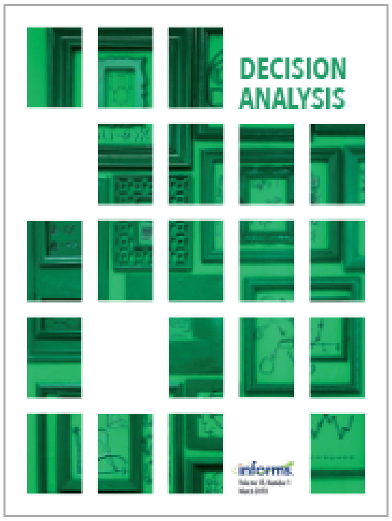

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Modeling and Validating Public–Private Partnerships in Disaster Management

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Abstract. This paper applies game theory and expected utility theory models to study the optimal public–private partnerships (PPPs) in disaster preparedness considering the uncertain consequences of the disasters, the investment costs, and the private sector's potential risk attitudes, including risk-seeking behavior, risk aversion, and risk neutrality. We study the private sector's best response and the subgame perfect Nash equilibrium solutions. Our results show that the public sector provides the least amount of subsidy to the risk-averse private sector and the highest amount of subsidy to the risk-seeking private sector. On the contrary, the risk-averse private sector invests the most, and the risk-seeking private sector invests the least. Finally, we validate the model results using a two-stage experiment with 91 participants. This paper provides insights into how to construct win–win PPPs in disaster preparedness with considerations of the private sector's risk behavior.

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Keywords: public–private partnerships (PPPs) • subsidy • investment • sequential games • risk preferences • model validation

1. Introduction

Public–private partnerships (PPPs) are defined as "the arrangements between government and private sector entities for the purpose of providing public infrastructure, community facilities and related services" (British Columbia Ministry of Municipal Affairs 1999, p. 5). As both natural disasters (Webster et al. 2008) and terrorism activities (Global Terrorism Database 2015) have impacted more and more people over the world in recent decades, PPPs provide a good solution for disaster preparedness and relief through enhancing the resilience (which is defined as the ability to withstand or recover from disasters) of communities. The National Research Council (2011) shows the necessity and advantages of building community disaster resilience through the collaborations between public and private sectors. In a PPP, the private sector is defined as any nongovernment organization or individual (e.g., private business companies, nonprofit organizations, individual citizens, and other nongovernment agencies), and the public sector is defined as any level of government agency (e.g., federal, state, and local governments). The potential benefits of PPPs include cost savings, risk sharing, improved levels of service or maintaining existing levels of service, enhancement of revenues, more efficient implementation, and economic benefits (British Columbia Ministry of Municipal Affairs 1999). The development of win–win PPPs for both private and public sectors has become a hot topic in disaster preparedness.

According to the U.S. Department of Homeland Security (DHS) (U.S. Government Accountability Office 2015, p. 1), "the private sector owns approximately 85 percent of the nation's critical infrastructure." The "infrastructure repair and plans to create funding through public-private partnerships" were addressed by the then president Barrack Obama in his remarks during his visit to the Port of Miami (American Society of Civil Engineers 2013). One recent international example of a PPP is the Zagreb–Macelj toll road in Croatia, where "the government provided in-kind support in the form of land and contingent debt drawn down whenever revenues were insufficient to cover debt service" (World Bank Group 2016). The support extended by the public sector to the private sector could also be in the form of subsidies. For example, the Colombian government provided "subsidies for the very poor as social assistance" (Microinsurance Network 2011, p. 2) in buying products and services from private insurance companies. Thus, every party enjoyed gains: the people were able to access insurance products at affordable rates, the private sector was able to transfer part of the risk to the public sector, and the public sector was able to serve the public. More examples about PPPs are discussed in Pearl (2013).

Specifically in disaster management, the private sector plays an important role in complementing the government's efforts through supplying foods, services, and technical expertise (Federal Emergency Management Agency 2015). For example, according to the major emergency needs for earthquake preparedness and relief, the private sector (research, engineering, architecture, construction, rescue and search, pharmacy, telecommunication, transportation, retail, hotel, education, finance, and nonprofit companies) could allocate their special resources and mobilize their professional staffs in disaster management (California Debt and Investment Advisory Commission 2008). Busch and Givens (2012) provide the following disaster management example in which the public and private sectors worked together: The 2010 Deepwater Horizon oil rig explosion and spill was responded to by both government and nongovernment actors, including British Petroleum, which was a responsible party for the spill, as well as veterans of the Exxon Valdez oil spill. That paper also discusses the potential challenges that underline the need to incentivize the private sector to participate in PPPs. For example, without financial incentives, a private company may not be encouraged to form a PPP with the government if its competitor company decides to not form PPPs.

To be better prepared for future disaster scenarios, securing the private sector's support through contracts and incentive mechanisms is a potential strategy for the government/public sector. For example, the government may identify private companies that have extended support during past disasters and contract with them so that they get paid for supporting again if the need arises in the future. In general, the government could form partnerships with various private companies to improve disaster response and recovery, in which the former provides incentives or subsidies to the latter. Such strategies are investigated in this paper by studying PPPs in disaster management, in particular, the public sector's role in encouraging the private sector to directly invest in disaster management by providing subsidies, considering the private sector's risk preferences. While currently there do not exist any known incentive mechanisms that promote the private sector's participation in PPPs in disaster management, we model and validate the usefulness of generic subsidies, a potential incentive mechanism.

Similar to the decentralized model in Guan and Zhuang (2015), a sequential game model is used to model the interaction between the public and private sectors in this paper. The public sector moves first by declaring the subsidy and the private sector follows by making an investment decision, and both players' moves happen prior to when the disaster strikes. The subsidies may be tax reductions or provisions such as food, water, training, and drills, depending on the particular disaster context. The private sector's investment may be purchasing property/housing insurance, reinforcing buildings, participating in training exercises and drills, aiding relocation to less hazardous areas, or increasing the medical treatment facilities in hospitals. This paper contributes to the literature by providing insights to help us better understand how public–private partnerships can play a critical role in ensuring that private incentives are aligned with public policies, considering the private sector's risk preferences. Moreover, we validate the model using a behavioral experiment with 91 participants. This is another key contribution of this paper—there are very few papers that validate prescriptive game theoretic models using results from behavioral experiments.

The rest of this paper is organized as follows: Section 2 presents the literature review. Section 3 introduces the notation, assumptions, and function definition. Section 4 proposes the optimization models for both the public and private sectors, with considerations of the private sector's risk preferences, and provides the numerical solutions to the models as well as the sensitivity analysis of the model results. Section 5 describes an experiment that was conducted to validate the model results, and Section 6 concludes this paper. The appendices provide proofs of the propositions and some descriptive statistics of the experiment results.

2. Literature Review

Guan and Zhuang (2015) explore optimization models to show how to develop win–win PPPs for both the private and public sectors through an optimal resource allocation problem. However, by the nature of disasters, both the public and private sectors have to face many uncertain factors (Tierney et al. 2001) when they make decisions in disaster management. This paper studies PPPs considering the risk preferences of the private sectors, which are not considered in Guan and Zhuang (2015). In addition, this paper also uses a behavioral experiment to validate the best response of the private sector, as prescribed by the model.

Expected utility theory and game theory are used in this paper, as in Guan and Zhuang (2015). Expected utility theory (Von Neumann and Morgenstern 1947) was originally developed to help people make decisions under uncertainty. It has a normative interpretation and is used to apply in all situations to rational agents. Arrow and Lind (1970) and Deodatis et al. (2014) show that the government is risk neutral, which evaluates the investments through the expected present value (Little and Mirrlees 1974). Expected utility theory is a traditional decision-making approach used in disaster planning. Brookshire et al. (1985) demonstrate that expected utility theory is a reasonable behavior description for the decision makers in disaster planning. Game theory has also been used in the literature, along with expected utility theory, in the context of disaster management. For example, Zhuang and Bier (2007) incorporate expected utility with game theory to identify the defensive strategy equilibrium by maximizing the decision makers' utilities in counterterrorism and natural disaster management.

Government could allocate its resources to critical infrastructure protection (Zhuang 2010, Golalikhani and Zhuang 2011), or by providing subsidies to private sectors. To our best knowledge, literature studies government incentives to the various private sectors, but not from a disaster management perspective. For example, there are works on government incentives for computer security (August and Tunca 2006; e.g., free software to computer users), education (Dearden et al. 2004, Schultz 2004; e.g., scholarships, loans, and subsidies to students), climate change (Nushiwat 2007; e.g., subsidies and foreign aid to companies or developing countries to reduce greenhouse-gas emissions), healthcare (Gruber and Levitt 2011; e.g., subsidized insurance to patients), and transportation (Dodgson 1986; e.g., subsidies to support urban public transportation). This paper fills this gap by not only studying an arrangement where a government is giving a subsidy to a private sector in disaster management, but also by incorporating risk preferences of the private sector. The behavioral experiment-based validation is another key aspect that distinguishes this paper from other related literature, as there are very few papers that validate prescriptive game theoretic models using results from behavioral experiments.

3. Notation, Assumptions, and Function Definitions

3.1. Notation and Assumptions

The notation used throughout this paper is defined in Table 1, including two decision variables, nine model parameters, and five functions.

Both public and private sectors are assumed to be rational and want to protect the same asset from a disaster. The public and private sectors value the asset differently (V and v, respectively). The public sector is assumed to move first by providing the subsidy s to protect the public asset that it values as V. Its investment cost is αs, where α is the cost coefficient. As the

Table 1. Notation Used in This Paper

Notation	Explanation
s	Public subsidy
p	Private investment
v	The private asset valuation
V	The public asset valuation
θ	Cost coefficient of private investment
α	Cost coefficient of public subsidy
γ	Investment effectiveness coefficient
P_d	Probability that the disaster occurs
A	Defense inherent level
a	Private sector's risk preference indicator
W	Private sector's initial wealth
$P(s,p)$	The damage level function, $0 \le P(s,p) \le 1$
$L_G(s,p)$	Expected loss and cost of the public sector
$U_P(s,p)$	Expected utility of the private sector
$\hat{p}(s)$	Private sector's best response function
(s^*,p^*)	Subgame perfect Nash equilibria for a sequential game

second mover, the private sector invests p amount of (private) resource with an associated unit investment cost of θ to protect the asset that it values as v after observing the public sector's decision. It is assumed that both these moves happen before the disaster strikes. Both the public subsidy and the private investment contribute in reducing the expected damage level $P(s,p)V$ of the public asset once the disaster happens with probability P_d. In this paper, the goal of the public sector (which is risk neutral, as explained in Section 1) is to minimize its expected loss $P_d P(s,p)V$ and investment cost (αs): $L_G(s,p) = P_d P(s,p)V + \alpha s$. The objective of the private sector is to maximize its expected utility $U_P(s,p)$, which equals $(W - (P(s,p)v + \theta p))^a$ when the disaster happens with probability P_d, and $(W - \theta p)^a$ when the disaster does not happen with probability $(1 - P_d)$. In particular, W is the private sector's initial wealth, and a is the risk preference indicator of the private sector's assumed power utility function. The private sector can be considered risk seeking if $a > 1$, risk averse if $0 < a < 1$, and risk neutral if $a = 1$.

The probability that a disaster occurs, P_d, is assumed to be not impacted by either public or private investments. For example, whether an earthquake happens or not would not depend on how much the public or private sectors invested in disaster preparedness. On the other side, we assume that the damage function $P(s,p)$ is jointly impacted by the public and private investments. The damage level is assumed to be twice differentiable, continuous, and with diminishing marginal returns with respect to the public subsidy s and the private investment p. This means that the higher the public and private sectors' investments in disaster preparedness, the less damage to the public and private assets. Thus, the public subsidy s and private investment p are assumed to jointly reduce the damage level $P(s,p)$:

$$\begin{gathered}\frac{\partial P}{\partial s} \le 0, \quad \frac{\partial P}{\partial p} \le 0, \\ \frac{\partial^2 P}{\partial s^2} \ge 0, \quad \frac{\partial^2 P}{\partial p^2} \ge 0, \quad \frac{\partial^2 P}{\partial p \partial s} \ge 0.\end{gathered} \tag{1}$$

3.2. Definition of Investment Cost Functions and Damage Level Functions

Following Bier et al. (2008), Hao et al. (2009), and Guan and Zhuang (2015), we model the damage level function $P(s,p)$ as exponentially decreasing in public subsidy s and in private investment p; that is,

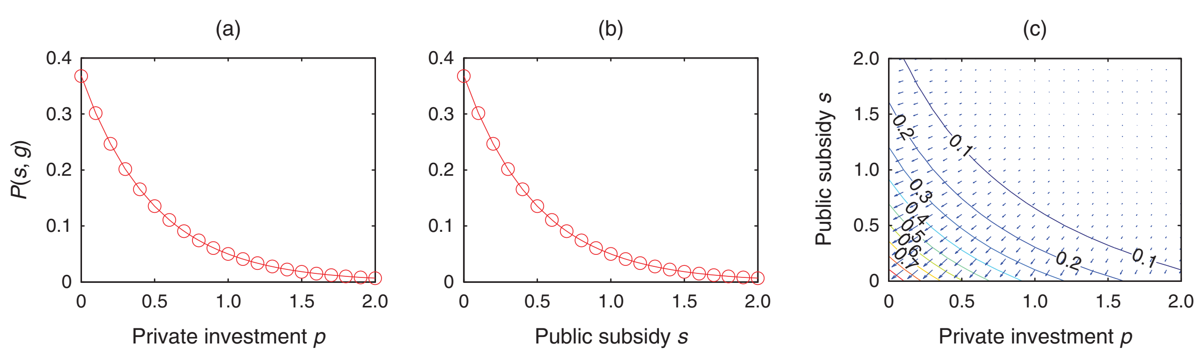
$$P(s,p) = \exp[-\gamma(\underbrace{s+p}_{\text{Single effects}} + \underbrace{sp}_{\text{Joint effects}})], \tag{2}$$

where γ is the investment effectiveness coefficient.

We use sp to model the joint effects between the private investment and the public subsidy. The public and private sectors can enjoy the joint effects only if both of them invest. For example, if the public sector provides positive subsidy $s > 0$, to receive the extra benefits from the joint effects of partnerships, the private sector has to invest positively. The more the private sector invests, the greater additional partnership benefits it will receive, which increases in p with a slope of s. Otherwise if $s = 0$, then the private sector does not receive any additional benefits from the partnerships.

Figure 1 illustrates the exponential damage rate function as a function of the private and public investments. The baseline values of the model parameters are

Figure 1. (Color online) Illustrations for Exponential Damage Rate Function $P(s,g)$



Note. The $P(s,g)$ values are along the y-axis in (a) and (b), and along the contour lines in (c).

$\gamma = 1$, $s = 1$, and $p = 1$. Figure 1 shows that the probability of damage $P(s, g)$ decreases exponentially in the private investment p and the public subsidy s.

4. The Model

The goal of the private sector is to maximize its expected utility over two scenarios, the disaster scenario and the nondisaster scenario, which occur with probabilities of P_d and $1 - P_d$, respectively. The private sector may invest its resources (p) to reduce the damage level $P(s,p)$ of the private asset with the investment cost of θp. In particular, the exponential form damage function $P(s,p)$ is defined in Equation (2). Following Machina (1987) and Raschky and Weck-Hannemann (2007), the utility function of the private sector is defined as follows:

$$\max_{p\geq 0} U_P(s,p) = P_d\underbrace{(W-(P(s,p)v+\theta p))^a}_{\text{Disaster happens}} + (1-P_d)\underbrace{(W-\theta p)^a}_{\substack{\text{Disaster}\\\text{does not}\\\text{happen}}}, \tag{3}$$

where W is the private sector's initial wealth, and a is the risk preference indicator of the private sector. The private sector has two possible outcomes. If the disaster happens with a probability of P_d, the outcome of the private sector is equal to its initial wealth W minus the expected loss and investment cost $(P(s,p)v + \theta p)$. If no disaster happens with a probability of $1 - P_d$, the outcome of the private sector is equal to its initial wealth W subtracting the investment costs θp.

As suggested by Arrow and Lind (1970), the government (public sector) would evaluate its investment strategy under uncertainty through the expected value. Therefore, the goal of the public sector is assumed to be to minimize its expected damage $(P(s,p)P_d V)$ and investment cost αS:

$$\underset{s\geq 0}{\text{minimize}}\ L_G(s,p) = P(s,p)P_d V + \alpha s,$$

where L_G is the objective of the public sector, and the subsidy s is the public sector's decision variable.

4.1. Private Sector's Best Response and the Subgame Perfect Nash Equilibrium Solutions

Since the private sector is assumed to be the second mover in the sequential game, to find the equilibrium solution for both the public and the private sectors, we first obtain the best response function $\hat{p}(s)$ of the private sector. Given a certain level of public subsidy s, the private sector's best response $\hat{p}(s)$ is its best strategy that maximizes its utility $U_P(s,p)$, which is defined in Equation (4):

$$\hat{p}(s) \equiv \arg\max_{p\geq 0} U_P(s,p). \tag{4}$$

Using backward induction, we substitute the private sector's best response function in Equation (4) into the public sector's optimization model and solve for the subgame perfect Nash equilibrium (s^*, p^*), which is defined as follows:

Definition 1. In a sequential game, we call a collection of strategies (s^*, p^*) a subgame perfect Nash equilibrium (SPNE) if and only if both Equations (5) and (6) are satisfied:

$$s^* = \arg\min_{s\geq 0} L_G(s, \hat{p}(s)); \tag{5}$$

$$p^* = \hat{p}(s^*) = \arg\max_{p\geq 0} U_P(s^*, p). \tag{6}$$

According to the public and private investment statuses—investing or not investing—we have four possible cases of equilibrium solutions:

$$\begin{cases} \text{Case 1} & s^* > 0, p^* > 0; \\ \text{Case 2} & s^* = 0, p^* > 0; \\ \text{Case 3} & s^* > 0, p^* = 0; \\ \text{Case 4} & s^* = 0, p^* = 0. \end{cases} \tag{7}$$

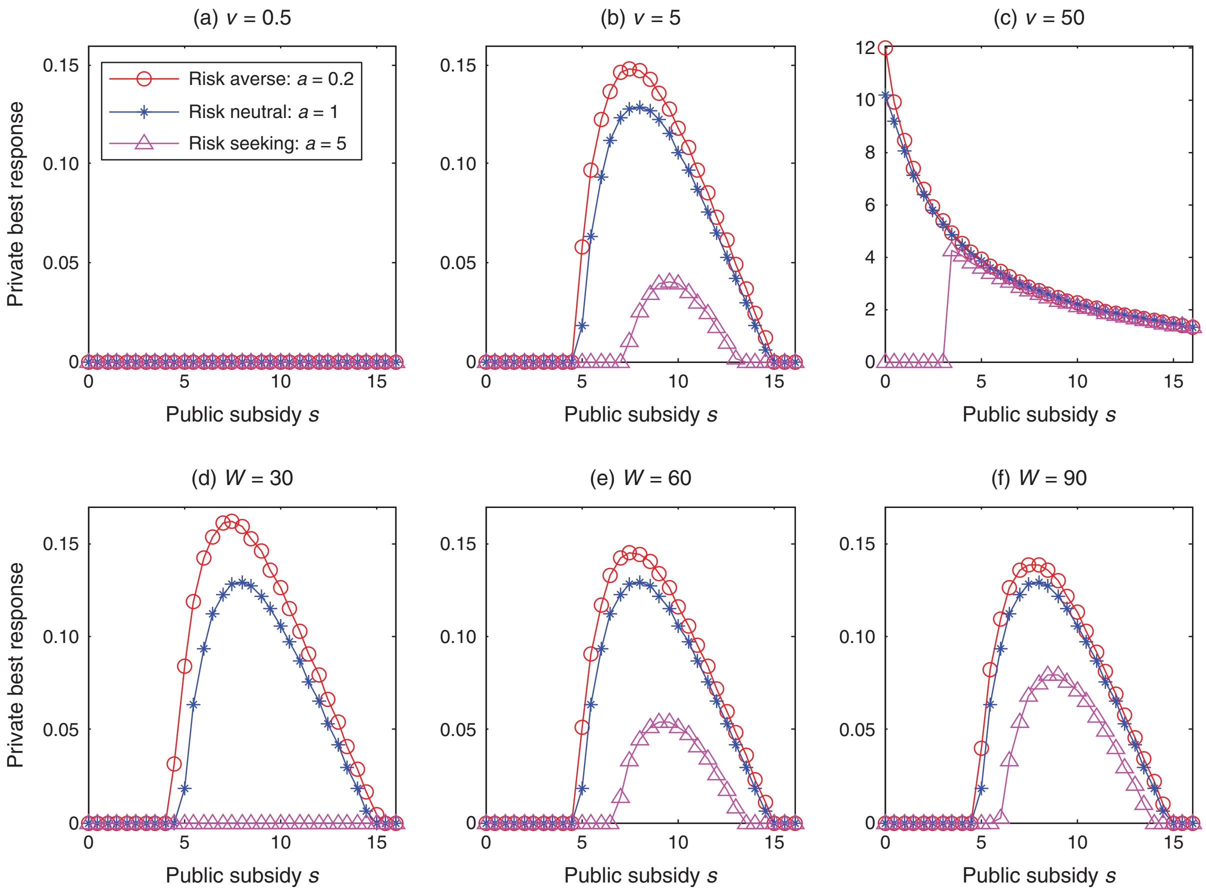
4.2. Private Sector's Best Response and Equilibrium Solutions

Since the private sector is the second mover in the sequential game, to solve the equilibrium solutions, we first study its best response function.

Proposition 1. *For the risk-neutral private sector* ($a = 1$), *the private best response function is as follows*:

$$\hat{p}(s) = \begin{cases} \dfrac{\ln\{P_d v\gamma(1+s)/\theta\} - \gamma s}{\gamma(1+s)}, & \textit{if } P_d v > \dfrac{\theta e^{\gamma s}}{\gamma(1+s)}, \\ 0, & \textit{otherwise}. \end{cases} \tag{8}$$

Figure 2. (Color online) Comparison of the Private Sector's Best Responses with Three Different Levels of Target Valuations and Private Initial Wealth Under Different Risk Preferences



Remark 1. We note that $\hat{p}(s)$ can be either strategic complements or strategic substitutes to the public subsidy s; i.e., $\hat{p}(s)$ could either increase in s (when $0 \le s < (\theta e^{1-\gamma} - P_d v\gamma)/(P_d v\gamma)$) or decrease in s (when $s > (\theta e^{1-\gamma} - P_d v\gamma)/(P_d v\gamma)$), and it does not depend on the initial wealth W.

To avoid intractable solution, instead of obtaining analytical solutions, we study the private sector's best responses numerically for the risk-seeking and the risk-averse cases. In Figure 2(a)–(c), we compare the best responses of the private sector with different risk preferences: risk seeking ($a = 5$), risk averse ($a = 0.2$), and risk neutral ($a = 1$) in three different asset valuation settings (low-valued asset $v = 0.5$, moderate-valued asset $v = 5$, and high-valued asset $v = 50$). The baseline values of other model parameters are $\gamma = 0.1$, $\theta = 0.9$, $P_d = 0.5$, and $W = 50$.

Figure 2(a) shows that the best responses of the risk-seeking, the risk-averse, and the risk-neutral private sectors are to not invest given any subsidy level between 0 and 16 ($s \in [0, 16]$) if the private asset valuation is low ($v = 0.5$). From Figure 2(a), it could be interpreted that the private asset valuation is too low, such that that neither the public sector nor the private sector is interested in investing. When the private asset valuation increases to a moderate value, as shown in Figure 2(b), where the moderate-valued asset $v = 5$, the risk-averse private sector will invest first, then the risk-neutral private sector, and the risk-seeking private sector will invest last. In addition, the private sector's best response first increases and then decreases in subsidy s for all three cases. This is to say a low public subsidy will not motivate the private sector to invest; when the public subsidy increases to certain amount, the private sector will invest. However, when the public subsidy continues to increase, the private sector investment will decrease to zero since the public subsidy is large and there is no motivation for the private sector to invest. For the high-valued asset $v = 50$, the best responses of the risk-seeking and the risk-neutral private sectors decrease in subsidy s, which is shown in Figure 2(c). From Figure 2, (b) and (c), we note that the risk-averse private sector's best response is the highest, the risk-seeking private sector's best response is the lowest, and the risk-neutral private sector's best response is between the former two, given the same level of the subsidy s.

In summary, to any given level of public subsidy s, the best responses of the private sector from the highest to the lowest are those of the risk-averse private sector, the risk-neutral private sector, and the risk-seeking private sector.

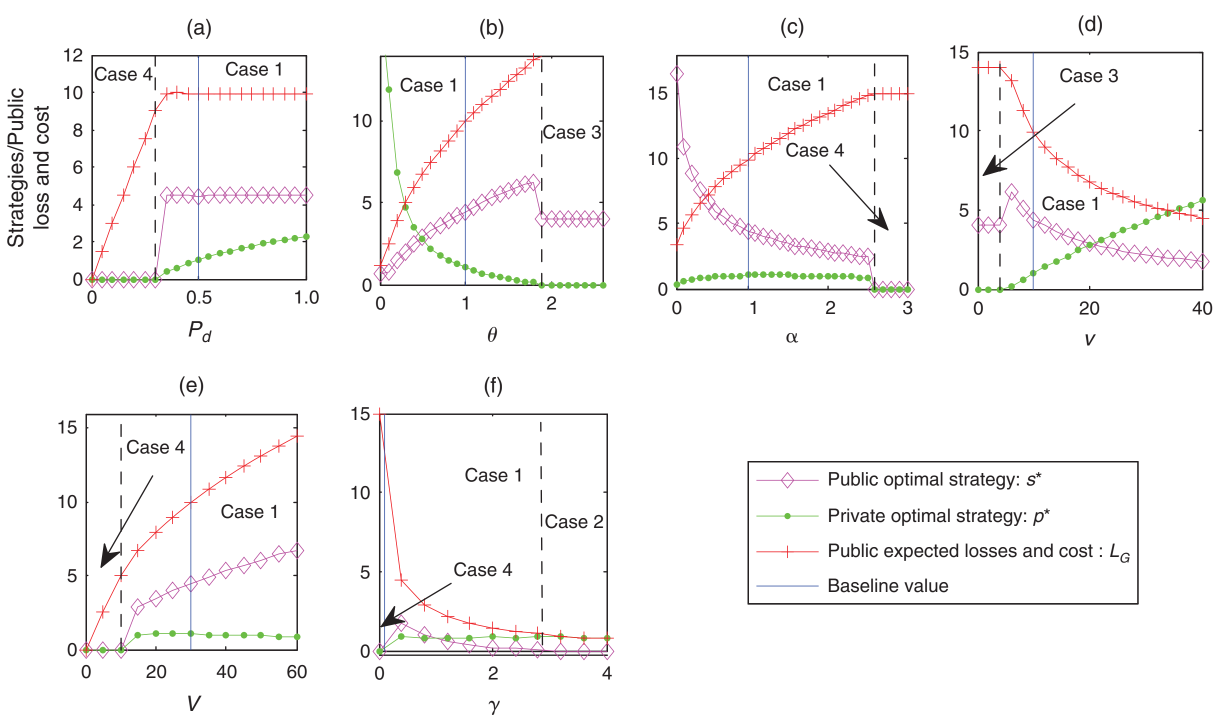
With the same setting of the model parameter baseline values, Figure 2(d)–(f), shows the comparison of the private best responses with three kinds of risk preferences under three initial wealth levels ($W = 30, 60,$ and 90 in Figure 2(d)–(f), respectively), with the private asset valuation setting as $v = 5$. From Figure 2(d)–(f), as the initial wealth W increases, we note that the risk-averse private sector decreases the investment as its best response, while the risk-seeking private sector increases the investment as its best response. The risk-neutral private sector's best response is not sensitive to the change of its initial wealth, which is consistent with Proposition 1.

4.3. Sensitivity Analyses of Equilibrium Solutions

With the definition of the equilibrium solution defined in Definition 1, we study how the equilibrium solutions are sensitive to changes in the values of model parameters. We change the following parameters one at a time: the probability of disaster occurrence, P_d; the private investment cost coefficient, θ; the public investment cost coefficient, α; the private asset valuation, v; the public asset valuation, V; and the investment effectiveness coefficient, γ.

The baseline values of the model parameters are $v = 10$, $V = 30$, $\theta = \alpha = 1$, $\gamma = 0.1$, $P_d = 0.5$, and $a = 1$ (risk neutral). Figure 3(a) shows that the private and public sectors do not invest if the probability of the disaster P_d is small ($P_d < 0.3$). The private sector increases its

Figure 3. (Color online) Comparison of the Public and Private Strategies, Public Expected Loss and Cost, and Private Utility



investment as the probability of disaster P_d increases when P_d is sufficiently large ($P_d \geq 0.3$). Figure 3(b) shows that as the private investment cost coefficient θ increases, the private best investment p^* decreases to zero, the public optimal subsidy s^* first increases and then decreases, and the public expected loss and cost first increase and then trend to constant when p^* equals zero. In particular, Figure 3(b) shows that the best private investment is $p^* = 0$ when $\theta \geq 1.9$. Figure 3(c) shows that as the public investment cost coefficient α increases, the optimal private investment p^* first increases and then decreases, the public subsidy s^* decreases to zero, and the public expected loss and cost L_G increases and remains constant when it is up to some level. The public sector stops subsidizing the private sector when $\alpha > 2.6$. Figure 3(d) shows that the private sector will not invest if its asset valuation v is small ($v < 4$). The private investment increases in v when the private asset valuation v is sufficiently large ($v \geq 4$). The provision of the optimal public subsidy is the same when there is no private sector involvement ($p^* = 0$), and then the public sector first increases and then decreases the optimal subsidy in the private asset valuation. The public sector's expected loss and cost decrease in v. Figure 3(e) shows that if the public asset valuation V is small ($V < 10$), both the public and the private sectors will not invest. Otherwise, as V increases, the private sector first increases and then decreases its investment, and public sector increases the provision of the subsidy. The public sector's expected loss and cost increase in V. Figure 3(f) shows that both the public and the private sectors will not invest if the investment is not efficient at all ($\gamma = 0$), leading to the highest public expected loss and cost. As the investment effectiveness coefficient γ increases, both the public and the private sectors' optimal strategies first increase and then decrease, and the public expected loss and cost decrease.

Figure 4, (a) and (c), shows that the public sector increases the subsidy to the risk-averse private sector, but decreases the subsidy to the risk-seeking private sector, as the initial wealth W increases. From Figure 4(b), we note that the private investment p^* and the public subsidy s^* to the risk-neutral private sector are independent of W. Figure 4(a) shows that the investment of the risk-averse private sector decreases in W. Figure 4(c) shows that the investment of the risk-seeking private sector increases in W. By comparing panels (a)–(c) in Figure 4, given the same amount of initial wealth W, we find that the risk-averse private sector invests the most but receives the least amount of the subsidy from the public sector.

Figure 5 shows the analysis of the risk preference indicator a: the public sector provides the highest subsidy to the most risk-seeking private sector, while the most risk-seeking private sector invests the least.

Figure 4. (Color online) Comparison of the Public and Private Strategies, and Public Expected Loss and Cost Under Different Risk Preferences as a Function of Initial Wealth W

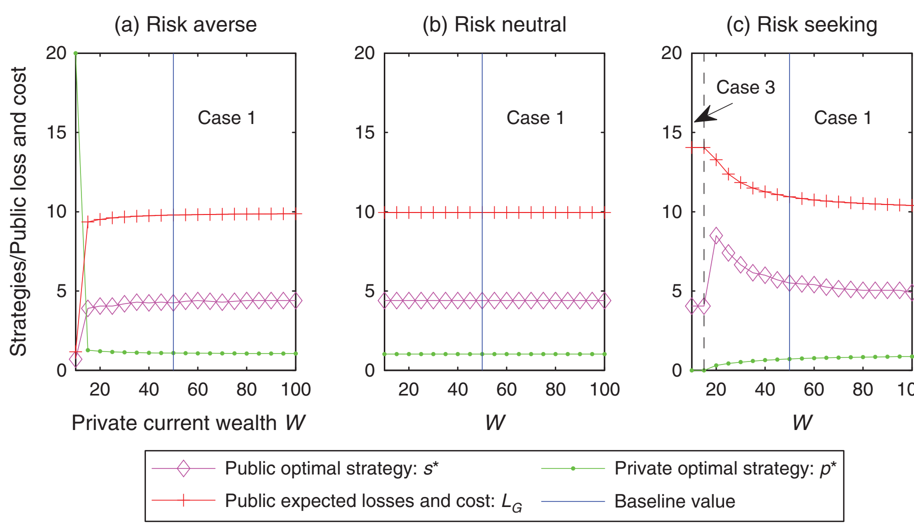
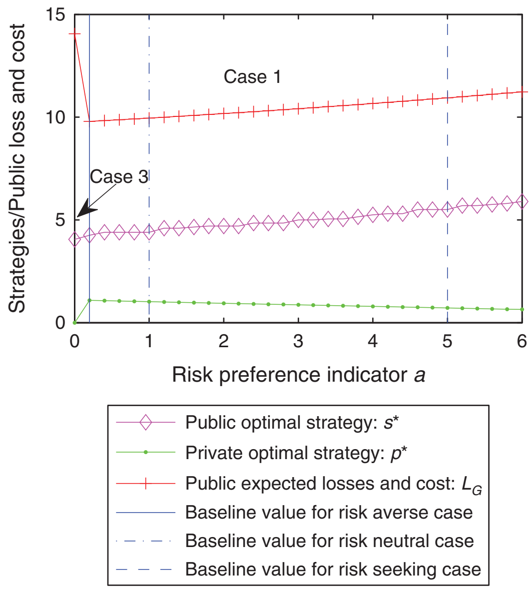


Figure 5. (Color online) Comparison of the Public and Private Strategies, and Public Expected Loss and Cost Under Different Risk Preferences as a Function of the Private Risk Preference Indicator *a*



5. Experiment and Model Validation

In this section, we describe a behavioral experiment that we designed and conducted in a computer-simulated environment to validate our model results. We first state the objective, then present the design, and finally describe the analysis and results of the experiment.

5.1. Experiment Objective

The objective of this experiment is to validate the theoretical model presented in Section 4. In particular, we want to look at how the private sector's best response function as proposed by our model performs when compared with the experimental results. This is because for the public sector (first mover of the game), as a decision maker, it is necessary to understand the private sector's (second mover of the game) response function before making optimal policy decisions on subsidies.

5.2. Theory

Section 4 used numerical illustrations to explain the private sector's best response, as solving for analytical solutions was impossible. We want to compare those numerical results with the experimental data. In particular, we perform comparisons and statistical analyses for four different cases, namely, the expected loss values of 1, 50, 250, and 450, respectively.

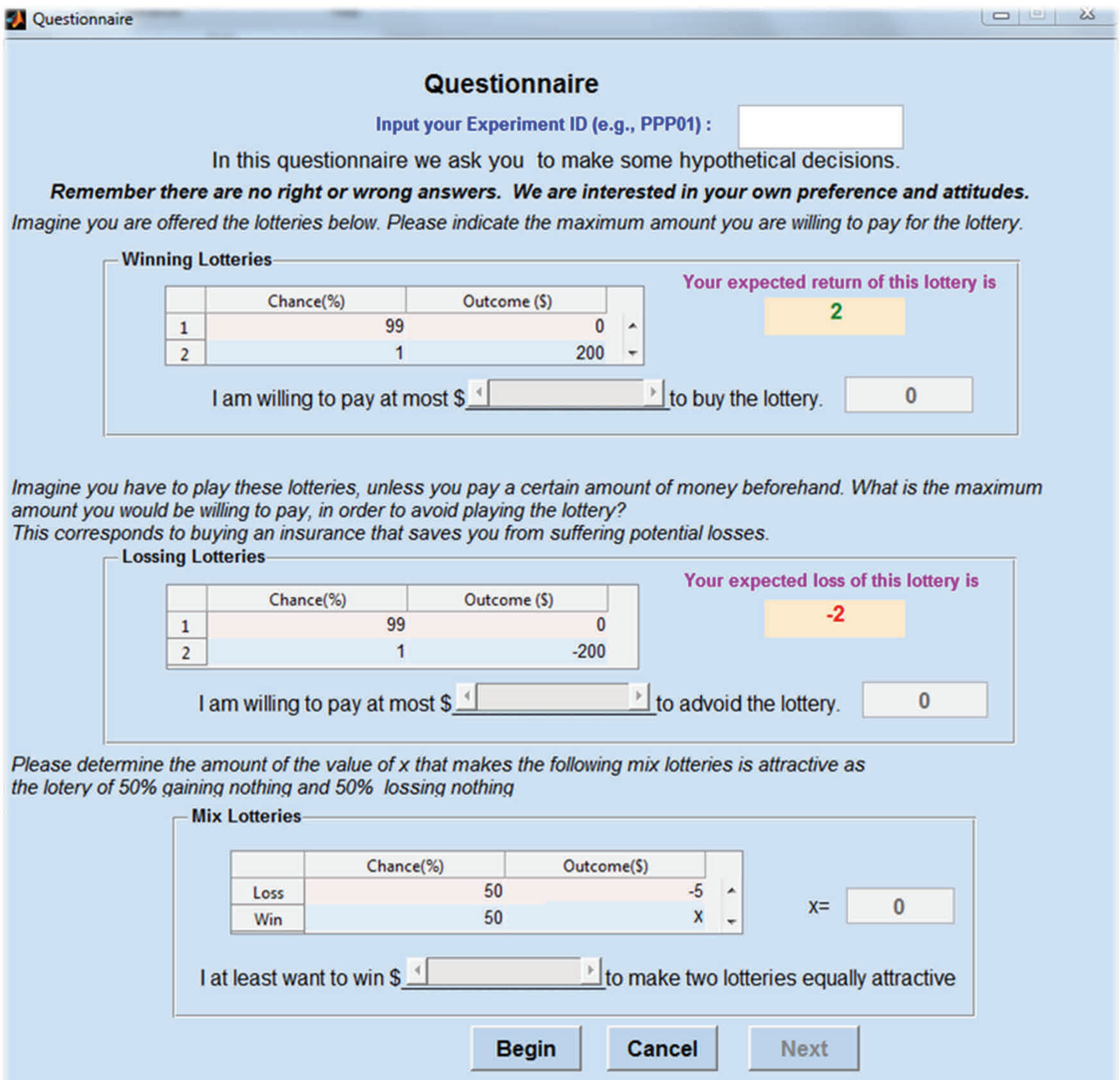
5.3. Experiment Procedure

We recruited 91 students from the University at Buffalo to participate in the experiment. We believe that 91 is a reasonably good number of participants, when compared with other similar works on experimental validation (e.g., 25 participants in Tversky and Kahneman 1992). In total, there were 12 sections of the experiment with 5 to 15 students in each section, and all sections were conducted in a computer lab. All participants were asked to complete the experiment in about two hours, and each participant was paid $10 per hour for participation. To make sure that the participants really understood the instructions and contents of the experiment, before each section of the experiment started, we had an introduction session that lasted for 15–20 minutes. This introduction session gave participants the opportunity to attempt sample questions to familiarize themselves with the experiment setup and ask questions if necessary. Each experiment section included two parts. In the first part, the goal was to study the private sector's risk behavior under uncertainty, and the participants were asked to answer a questionnaire with 28 sets of questions. In the second part, each participant was asked to play a role of the private sector, and was asked to make their disaster preparedness investment decisions based on the given information using his or her own preferences and judgments. We collected the participants' responses for certain disaster scenarios explained in Section 5.4. Then, we processed the collected data to check whether the participants' decisions in the second part of the experiment were consistent with the private sector's best response in the model prediction results based on their risk preferences obtained from the first part of the experiment. A graphical user interface (GUI) developed in MATLAB was used in the experiment for data collection.

5.4. Experiment Design

Following Tversky and Kahneman (1992) and Rieger et al. (2014), in the first part of the experiment, we provided a questionnaire with 28 sets of lottery games to the participants and asked them to answer the questions based on their best knowledge. Each participant was told that there was no correct or incorrect answer, and that we were only interested in measuring their risk preferences. Each set of lottery games included three questions: a winning lottery, a losing lottery, and

Figure 6. (Color online) The First Part of the Experiment: Risk Preference Analysis



a 50% chance mixed lottery. Figure 6 shows one set of those lottery game questions.

Before each section of the experiment, each participant was assigned a unique experiment ID (PPP01 to PPP91). The participant input their experiment ID to start the experiment after the introduction session. As shown in the example in Figure 6, the first question was a winning lottery game, which asked the participant to answer the following question:

- *Winning lottery*: If there is a game in which you have a 99% chance of getting or losing nothing and a 1% chance of winning $200, how much at most do you want to pay to play the game?

The expected (average) return ($2) was also provided to the participant for their reference. The participant was asked to move the slider at the bottom of the winning lottery table to make their decision.

The second question was a losing lottery game, which asked the participant the following question:

- *Losing lottery*: If you are forced to play a game in which you have a 99% chance of getting or losing nothing and a 1% chance of losing $200, how much at most do you want to pay to avoid playing the game?

The expected (average) loss (−\$2) was also provided to the participant for their reference. The participant was asked to move the slider at the bottom of the losing lottery table to determine their decision.

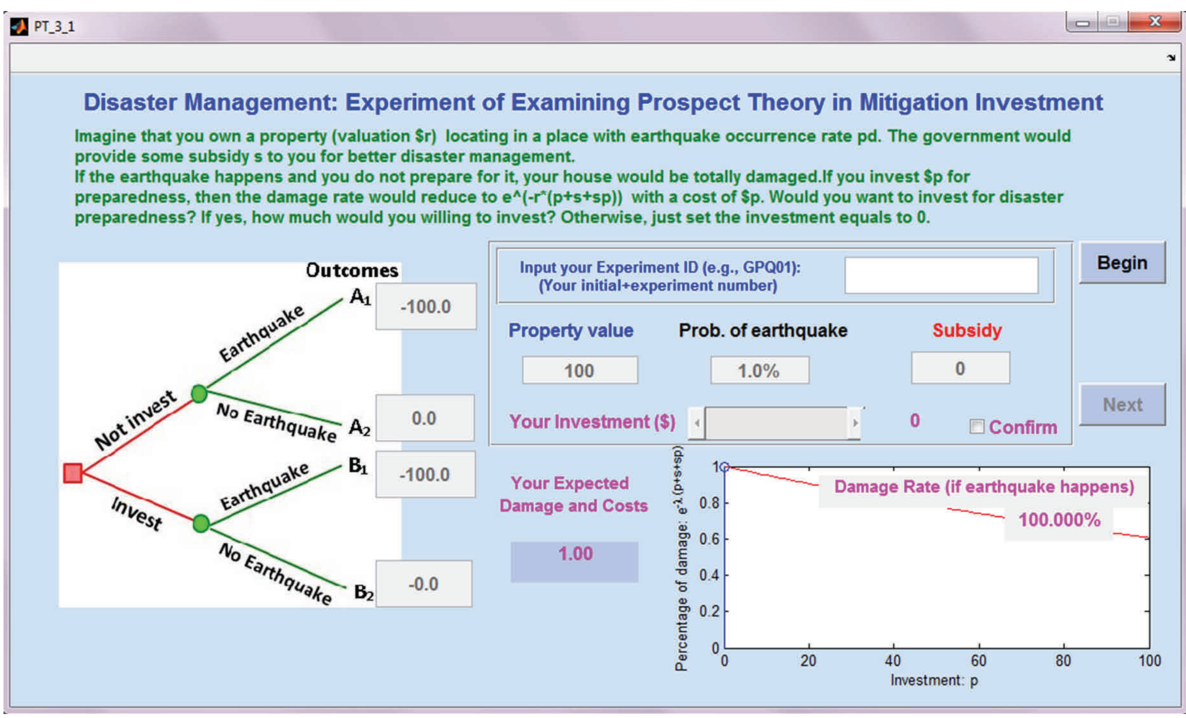
The last question was the mixed lottery game, which asked the participant the following question:

• *Mixed lottery*: If you are offered a game in which you have a 50% chance of losing \$5 and a 50% chance of gaining \$$X$, what is the minimum of X that makes this game attractive to you?

The participant was asked to move the slider at the bottom of the mixed lottery table to determine their decision. After the completion of the first part of the experiment, the second part of the experiment was introduced to the participant. They were asked to decide how much of their resources they planned to invest for disaster preparedness under different scenarios (given the information on available resources, property value, public subsidy level, the probability of disaster happening, all the possible outcomes in a decision tree, and the expected damage to property once the disaster happens). The goal of the second part of the experiment was to validate the predicted results of the private sector's response to the government's incentive level using the model inputs (private sector's utility function) that were obtained in the first part of the experiment. There were 36 scenarios total in the second part of the experiment. Figure 7 shows an example scenario, which describes the following to the participant:

• Imagine you own a \$100-valued property in a place with a 1% chance that an earthquake will happen, and the government does not provide any subsidy (\$0) to you for disaster management. You are asked to make your investment decision based on the given information. The given information includes all your possible outcomes from the decision tree on the left: A_1(−\$100), your outcome if you do not invest and an earthquake happens, which has a 1% chance possibility; A_2(\$0), your outcome if you do not invest and an earthquake does not happen, which has a 99% chance possibility; B_1(−\$100), your outcome if you invest and an earthquake happens, which has a 1% chance possibility; B_2(\$0), your outcome if you invest and an earthquake happens, which has a 99% chance possibility (since the investments in B_1 and B_2 equal to zero in this example, so the investing case is the same as the not

Figure 7. (Color online) The Second Stage of the Experiment: Private Best Response Validation



investing case); the expected (average) damage of the property and your investment costs ($1 for this example) from the text box in the middle at the bottom; and the damage level of your property (100% for this example, because both the private sector and the government do not invest in disaster preparedness) from the graph on the bottom left.

After inputting their experiment ID, a participant could start the experiment. They could move the slider, which was to the right of the text "Your Investment ($)" to choose their investment level. When a participant moved the investment slider, changing the number (investment level) in text box next to the slider, all the information of the four possible outcomes in the decision tree, the expected damage and costs, and the property damage level were updated automatically. After the participant carefully considered the problem and decided to make their decision, they were asked to confirm their answer and move to the next scenario.

5.5. Validation and Results

For validation, we compare the model result, which is calculated by using the private sector's risk behavior input from the first part of the experiment, with the experiment results of private sector's real decision under different scenarios. In the experiment, we construct the scenarios by setting different public subsidy levels ($s = \$0, \$1, \$2, \$3, \$5, \$10, \$20, \$30, \$100$), different probabilities of disaster ($P_d = 1\%, 10\%, 50\%, 90\%$), and different asset valuations (V = \$100, \$500). With the combinations of different probabilities of disaster and asset valuations, those scenarios can be classified into four cases according to the private expected loss: $1 (low), $50 (moderate), $250 (high), and $450 (high). We study the private sector's best responses to different levels of public subsidy in each case.

The idea here is to show that the private sector's decision making is well represented by the PPP model presented in Section 4. To prove this claim, we present descriptive statistical analysis, as explained in the remaining part of this section. We first evaluate the participants' utility function by using the data of the winning lotteries in the first part of the experiment. Based on the expected utility function defined in Equation (9), the model parameter a (the risk preference indicator of the private sector) is estimated by using parametric nonlinear regression:

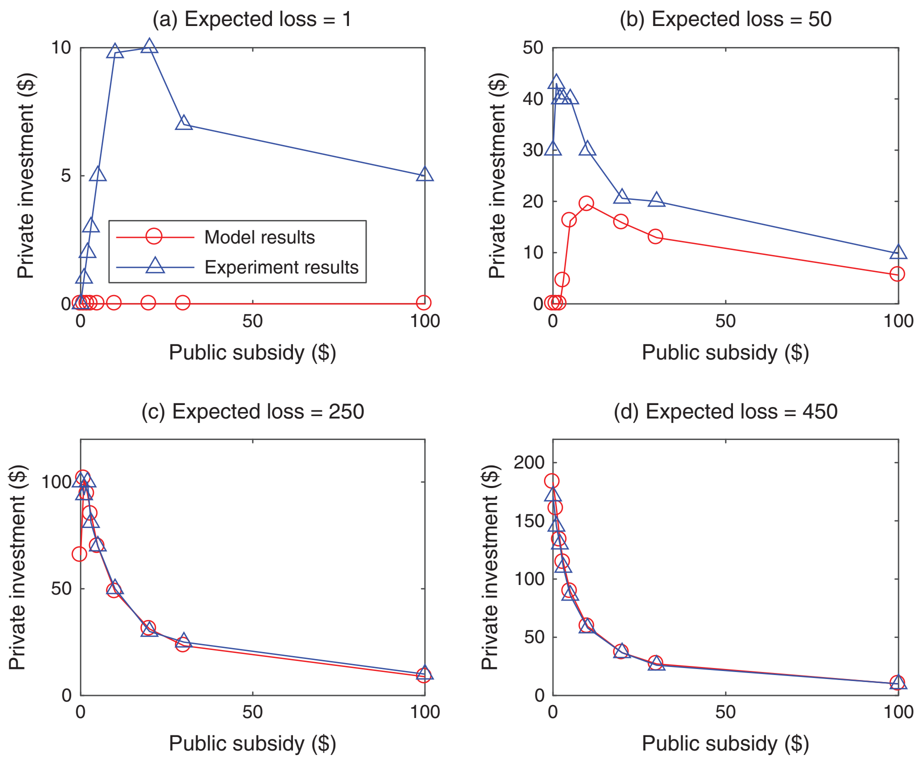
$$EU = \sum_{i=1}^{n} P_d^i (W - x_i)^a. \qquad (9)$$

Here, P_d^i ($\sum_{i=1}^{n} P_d^i = 1$) is the probability the outcome x_i happens, and W is the decision maker's initial wealth. Figure B.2 in Appendix B shows the expected utility functions of all the participants, based on the risk preference indicator a estimated from the analysis. In particular, 43 participants were risk seeking, 26 participants were risk neutral, and 22 participants were risk averse. For this classification, we used the assumption risk averse ($a \leq 0.95$), risk neutral ($0.95 \leq a \leq 1.05$), and risk seeking ($a \geq 1.05$). This is because there cannot be a perfect risk-neutral person with $a = 1$. This classification is only intended to show the distribution of participants' risk preferences, and does not in any way affect the calculations or results. Figure B.1 in Appendix B provides some descriptive statistics on the risk preference parameters of participants.

Inputting the private sector's utility function (which is estimated from the first part of the experiment) and the values of other model parameters into the private sector's best response function (Equation (4)), we compare the model predicted results with the experiment results. To eliminate data input error from individual inputs, we analyze the results by using the median value of the participants' inputs (Figure 8). By observation, it is seen that the experiment results are almost identical to the model results for higher expected losses (Figure 8(c), (d)). The resemblance is much less for lower values of expected loss (Figures 8(a), (b)). In fact, the degree of resemblance increases in the expected loss. This could have happened because the participants might have underestimated their "reference points" while making decisions under uncertainty. Hence, the median participant might have invested higher than the model prediction. However, except for the lowest expected loss scenario, the general trend of the private sector's investment $\hat{p(s)}$ first increasing and then decreasing in subsidy s has been preserved (Figure 8(b)–(d)).

Figures 9 and 10 provide more detailed statistical analyses of the results presented in Figure 8. Figure 9 uses a series of vertical box plots to show the distribution of the experiment responses besides the median

Figure 8. (Color online) Comparison of the Experiment and Model Results of the Private Sector's Best Response by Using the Median of 91 Participants' Input in the Expected Utility Model



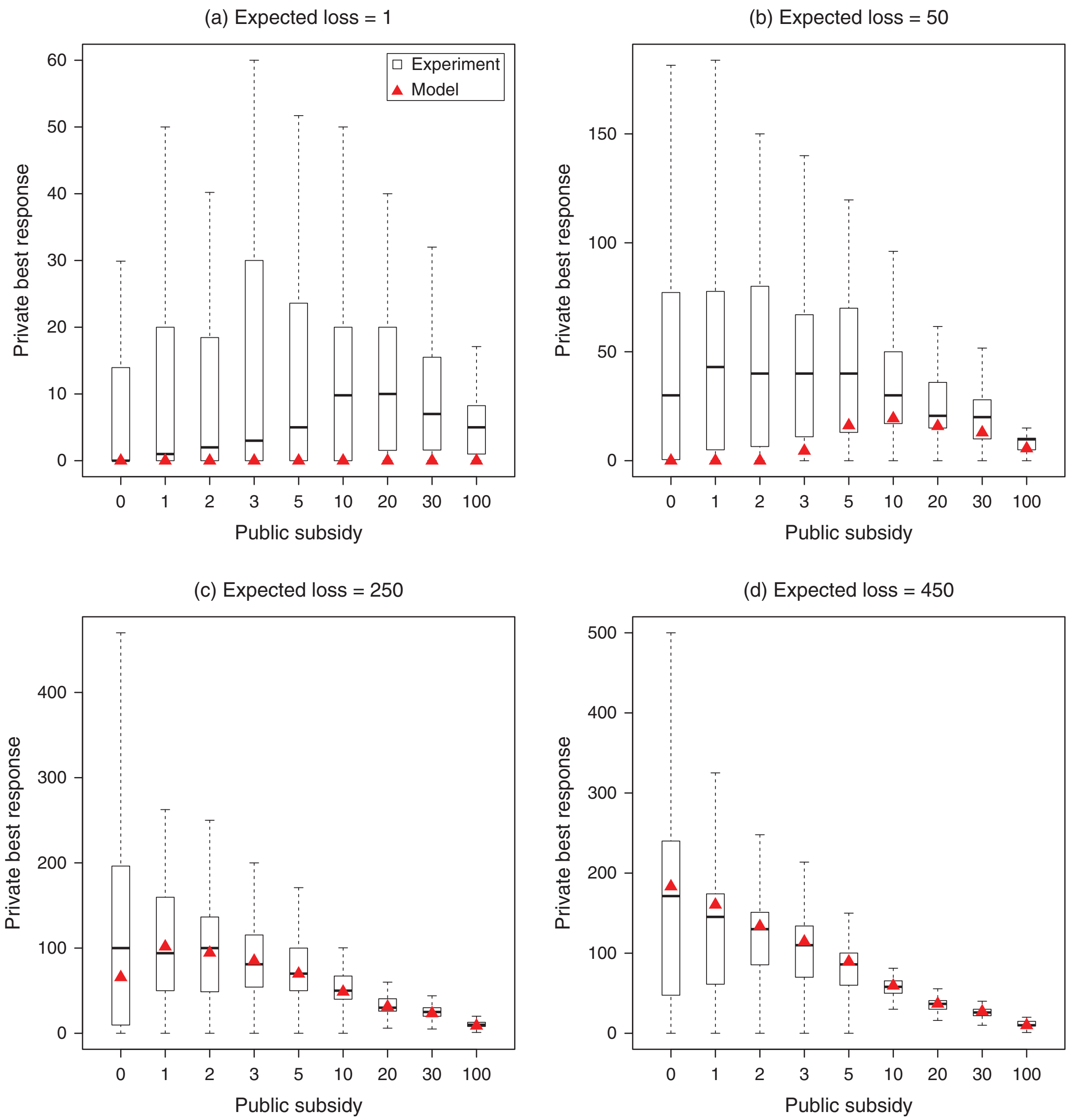
values of the experiment results (the horizontal line inside each box). One new observation here is that there is higher variation in the response values to lower subsidy levels. This suggests that the public sector proposing a low subsidy may consider preparing for a larger variation in the private sectors' responses. Figure 10 reinforces the similarity in the experiment and model results, especially at higher values of expected loss.

Overall, from Figures 8–10, it is possible to conclude, for larger expected loss values, that the private sector's best response investment as a function of subsidy is well represented by our PPP model. To explain the private sector's response at lower expected loss values, minor changes to the experimental setup (in particular, relating to reference points for decision making) may be required, which could be pursued in future research.

6. Conclusions

In this paper, we develop novel optimization models to study win–win public–private partnerships for both the public and private sectors in a quantitative way with considerations of the risks of the disasters, the investment costs of the disaster preparedness, the interactions between the public and private sectors, and the risk behavior of the decision makers. The primary contribution of this paper is to provide insights to help us better understand how public–private partnerships can play a critical role in ensuring that private incentives are aligned with public policies, considering the private sector's risk preferences. We provide the analytical solution for the risk-neutral case and the numerical solutions of the SPNE strategies for the risk-averse and the risk-seeking cases in the expected utility theory model. We also compare the private sector's best response and the SPNE solutions of both the private and public sectors. We find that to any given level of public subsidy, the best response investments from the highest to the lowest are those of the risk-averse private sector, the risk-neutral private sector, and the risk-seeking private sector. We also find that (i) the best response investment of the risk-averse private sector increases in its initial wealth, (ii) the best response

Figure 9. (Color online) Comparison of the Experiment and Model Results of the Private Sector's Best Response

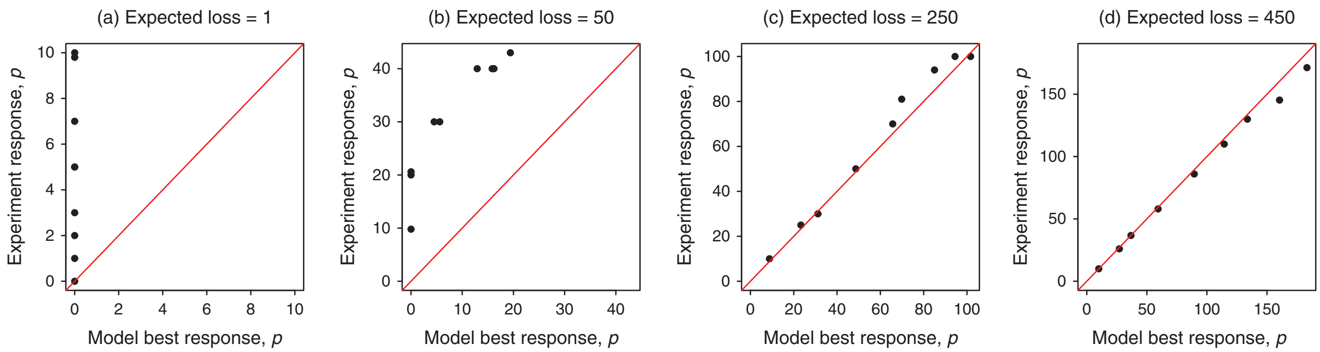


Note. The vertical boxes show the distribution of the responses from 91 participants, with the median shown using a horizontal line across the box.

investment of the risk-seeking private sector decreases in its initial wealth, and (iii) the risk-neutral private sector is not sensitive to changes in initial wealth. From the sensitivity analysis results, we find that the public sector provides the highest amount of subsidy to the risk-seeking private sector and the lowest amount of subsidy to the risk-averse private sector, and subsidy to the risk-neutral private sector is in between the former two. We also find that the risk-seeking private sector invests the least, the risk-averse private sector invests the most, and the investment of the risk-neural sector is in between the former two.

To validate the model results, a GUI was developed in MATLAB to conduct a two-stage experiment in a computer-simulated environment. The responses of the 91 participants are used to (1) elicit the private

Figure 10. (Color online) Comparison of the Experiment and Model Results of the Private Sector's Best Response Using X–Y Plots



Note. The closer the points are to the 45° red line, the better.

sector's utility function and (2) check the consistency of the private sector's best response to the public subsidy level by comparing the model results and the experiment results. We analyze the median data from the first part of the experiment to estimate the participants' utility function based on expected utility theory. The model results (calculated using the data from the first part of the experiment as the model inputs) are validated by comparing with the second part of the experiment results. Overall, for larger expected loss values, the private sector's best response investment as a function of subsidy is well represented by our PPP model. To explain the private sector's response at lower expected loss values, minor changes to the experimental setup (in particular, relative to reference points for decision making) may be required, which could be pursued in future research.

Another potential interesting area for future research is to analyze the different types of incentive mechanisms that the public sector could offer to the private sector. As an example, the public sector could pay the private sector for purchasing tangible assets that are useful for relief operations (e.g., trucks and ferries for transporting goods as well as for evacuating people), and the private company would bear the operational expenses/investments. The private company would benefit from initial cost savings, and they could use the assets for business expansion after disaster recovery.

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Appendix A. Proof of Proposition 1
According to the private sector's optimization model (3), we write the risk-neutral ($a = 1$) private sector's optimization problem as follows:

$$\max_{p\geq 0} U_P(s,p) := P_d(W - (P(s,p)v + \theta p)) + (1-P_d)(W - \theta p). \tag{A.1}$$

The second derivative of $U_P(s,p)$ respect to p is as follows:

$$\frac{\partial^2 U_P(s,p)}{\partial p^2} = -P_d v \frac{\partial^2 P(s,p)}{\partial p^2}. \tag{A.2}$$

Since we have assumptions of $\partial^2 P/\partial p^2 \geq 0$ in Equation (1), we have $-P_d v(\partial^2 P(s,p)/\partial p^2) = \partial^2 U_P(s,p)/\partial p^2 \leq 0$. So, $U_P(s,p)$ is concave in p.

With the definition of the damage rate function $P(s,p)$ in Equation (2), the private sector's optimization model (A.1) becomes

$$\max_{p\geq 0} U_P(s,p) := P_d(W - (e^{-\gamma(s+p+sp)}v + \theta p)) + (1-P_d)(W - \theta p). \tag{A.3}$$

Based on the first order condition of $U_P(s,p)$, if p maximizes $U_P(s,p)$, we have

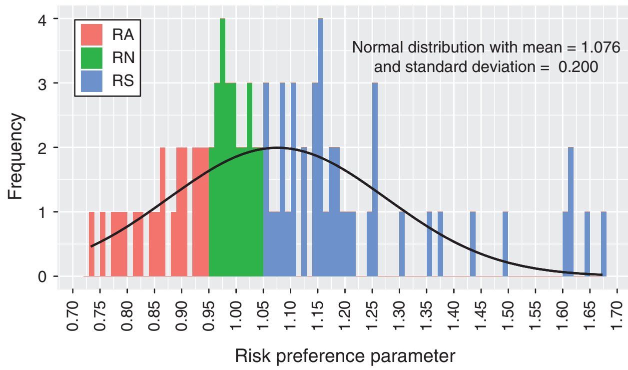
$$\frac{\partial U_P(s,p)}{\partial p} = P_d v\gamma(1+s)e^{-\gamma(s+p+sp)} - \theta = 0. \tag{A.4}$$

After solving Equation (A.4), we have the private sector's best response as follows:

$$\hat{p}(s) = \begin{cases} \dfrac{\ln\{P_d v\gamma(1+s)/\theta\} - \gamma s}{\gamma(1+s)} & \text{if } P_d v > \dfrac{\theta e^{\gamma s}}{\gamma(1+s)}, \\ 0 & \text{otherwise.} \end{cases}$$

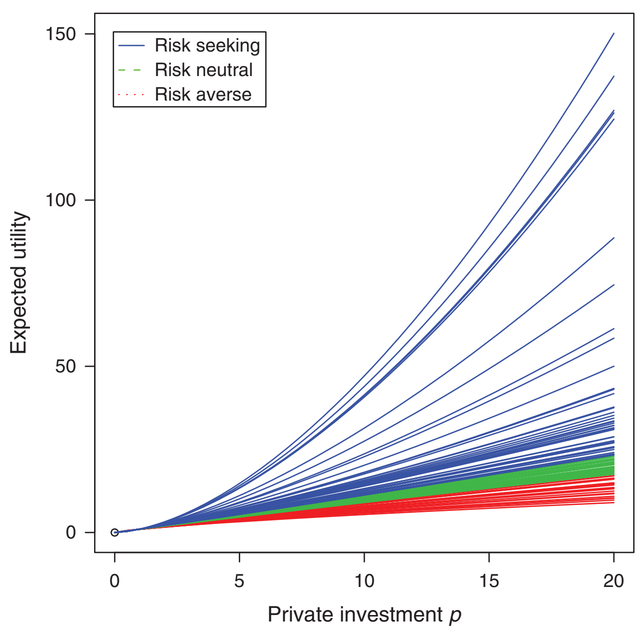
Appendix B. Experiment and Validation Results

Figure B.1. (Color online) Distribution of Risk Preference Parameters of the 91 Participants



Notes. The numbers of risk-seeking (RS), risk-neutral (RN), and risk-averse (RA) participants were 43, 26, and 22 respectively. We assumed that $a \le 0.95 \Rightarrow$ risk averse, $0.95 \le a \le 1.05 \Rightarrow$ risk neutral, and $a \ge 1.05 \Rightarrow$ risk seeking.

Figure B.2. (Color online) Expected Utility Plots for Each of the 91 Participants, Categorized Into Risk Averse, Risk Neutral, and Risk Seeking



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