continuity

$A: \mathcal{F} \mapsto \mathcal{G}$ a function

continuity means convergent sequence in $\mathcal{F}$ maps to a convergent sequence in $\mathcal{G}$.

operator norm

$A: \mathcal{F} \mapsto \mathcal{G}$, a linear operator:

$$\|A\| = \sup_{f \in \mathcal{F}} \frac{\|Af\|_{\mathcal{G}}}{\|f\|_{\mathcal{F}}}$$

"maximum scalaring"

$\|A\| < \infty$: bounded operator.

Thm L: linear operator, $(\mathcal{F}, \|\cdot\|_{\mathcal{F}})$, $(\mathcal{G}, \|\cdot\|_{\mathcal{G}})$ normed linear space

① L is bounded

② L is continuous on $\mathcal{F}$

③ L is continuous at 1 point of $\mathcal{F}$

Definition (RKHS)

A Hilbert space $\mathcal{H}$ of functions $f: \underset{\text{non-empty}}{\mathcal{X}} \mapsto \mathbb{R}$ is said to be a RKHS if δ_x is continuous $\forall x \in \mathcal{X}$.

1) $\langle \cdot, \cdot \rangle_{\mathcal{H}}$ is defined
2) complete (没有洞)

$1, 1.4, 1.414, 1.4142, \cdots$
$(\mathbb{Q}, |\cdot|)$ $\boxed{\text{Cauchy sequence}}$

$A: \mathcal{F} \mapsto \mathcal{G}$

convergent sequence in $\mathcal{F}$ is $\underbrace{\text{mapped to}\ \mathcal{G}}_{\text{a convergent sequence in}}$

$\delta_x: \mathcal{H} \mapsto \mathbb{R}$
$f \mapsto f(x)$
evaluation functional.

☆ in SVGD, $\phi(\cdot)$ is not a member of the RKHS $\mathcal{H}$ $\boxed{?}$ √?

$f_1, f_2 \in \mathcal{H}$,
$\|f_1\|_{\mathcal{H}}$, $\|f_2\|_{\mathcal{H}}$ are close, then $\forall x \in \mathcal{X}$, $f_1(x)$ and $f_2(x)$ are close.

$\|f_1 - f_2\|_{\mathcal{H}} < \epsilon \Rightarrow |f_1(x) - f_2(x)| < \delta(\epsilon)$, $\forall x$.

How about "Reproducing Kernel"?

$\mathcal{H}$: Hilbert space of functions $f: \underset{\text{non-empty.}}{\mathcal{X}} \mapsto \mathbb{R}$,

a function $k: \mathcal{X} \times \mathcal{X} \mapsto \mathbb{R}$ is called a "reproducing kernel" of $\mathcal{H}$ if:

1) $\forall x \in \mathcal{X}$, $k(\cdot, x) \in \mathcal{H}$.   e.g. $k(x, x') = \frac{-\|x-x'\|^2}{2\sigma^2}$, $k(\cdot, x) = \frac{-\|\cdot - x\|^2}{2\sigma^2}$.

2) $\forall x \in \mathcal{X}$, $\forall f \in \mathcal{H}$, $\langle f(\cdot), k(\cdot, x) \rangle_{\mathcal{H}} = f(x)$ ("reproducing property")

in particular, for any $x, y \in \mathcal{X}$,

$k(x, y) = \langle k(\cdot, x), k(\cdot, y) \rangle_{\mathcal{H}}$.

because $k(\cdot, x)$, $k(\cdot, y)$ in $\mathcal{H}$. $k(\cdot, x_0)$ is only subset of $\mathcal{H}$.

Thm $\mathcal{H}$ is a RKHS (δ_x is continuous) i.f.f. $\mathcal{H}$ has a r.k.