Quiz, 5 questions

# ✓ Congratulations! You passed!

Next Item



1.

The function

$$eta(\mathbf{x},\mathbf{y}) = \mathbf{x}^Tegin{bmatrix} 2 & -1 \ -1 & 1 \end{bmatrix}\mathbf{y}$$

is



positive definite

# Correct

Yes, the matrix has only positive eigenvalues and  $\beta(\mathbf{x},\mathbf{x})>0$  for all  $\mathbf{x}\neq\mathbf{0}$  and  $\beta(\mathbf{x},\mathbf{x})=0\iff \mathbf{x}=\mathbf{0}$ 



bilinear

# Correct

Yes:

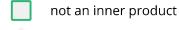
- $\beta$  is symmetric. Therefore, we only need to show linearity in one argument.
- For any  $\lambda \in \mathbb{R}$  it holds that  $\beta(\mathbf{x} + \lambda \mathbf{z}, \mathbf{y}) = \beta(\mathbf{x}, \mathbf{y}) + \lambda \beta(\mathbf{z}, \mathbf{y})$ . This holds because of the rules for vector-matrix multiplication and addition.



symmetric

## Correct

Yes: 
$$\beta(\mathbf{x}, \mathbf{y}) = \beta(\mathbf{y}, \mathbf{x})$$



Quiz, 5 questions



an inner product



It's symmetric, bilinear and positive definite. Therefore, it is a valid inner product.



not bilinear

## **Un-selected is correct**



not symmetric

## **Un-selected** is correct



not positive definite





1/1 point

2.

The function

$$eta(\mathbf{x},\mathbf{y}) = \mathbf{x}^Tegin{bmatrix} 1 & -1 \ -1 & 1 \end{bmatrix}\mathbf{y}$$

is



an inner product

## **Un-selected is correct**



not an inner product

#### Correct

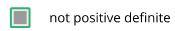
Correct: Since  $\beta$  is not positive definite, it cannot be an inner product.

Quiz, 5 questions

# Correct

Correct:

- ullet eta is symmetric. Therefore, we only need to show linearity in one argument.
- $\beta(\mathbf{x} + \lambda \mathbf{z}, \mathbf{y}) = \beta(\mathbf{x}, \mathbf{y}) + \lambda \beta(\mathbf{z}, \mathbf{y})$ . This holds because of the rules for vector-matrix multiplication and addition.



#### Correct

With  $x = [1,1]^T$  we get  $\beta(\mathbf{x},\mathbf{x}) = 0$ . Therefore  $\beta$  is not positive definite.

positive definite

**Un-selected is correct** 

not symmetric

**Un-selected** is correct

not bilinear

**Un-selected is correct** 

symmetric

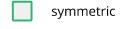
Correct

Correct:  $eta(\mathbf{x},\mathbf{y}) = eta(\mathbf{y},\mathbf{x})$ 

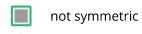


Quiz, 5 questions 
$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is



**Un-selected is correct** 



#### Correct

Correct: If we take  $\mathbf{x} = [1,1]^T$  and  $\mathbf{y} = [2,-1]^T$  then  $\beta(\mathbf{x},\mathbf{y}) = 0$  but  $\beta(\mathbf{y},\mathbf{x}) = 6$ . Therefore,  $\beta$  is not symmetric.

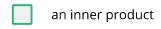


## Correct

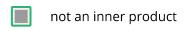
Correct.

not bilinear

**Un-selected** is correct



**Un-selected is correct** 



## Correct

Correct: Symmetry is violated.

Quiz, 5 questions  $\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{y}$ 

is

not positive definite

**Un-selected** is correct

positive definite

#### Correct

It is the dot product, which we know already. Therefore, it is positive definite.

not symmetric

**Un-selected is correct** 

symmetric

## Correct

It is the dot product, which we know already. Therefore, it is symmetric.

not an inner product

**Un-selected is correct** 

an inner product

### Correct

It is the dot product, which we know already. Therefore, it is also an inner product.

not bilinear

**Un-selected** is correct



Properties of inner products

Quiz, 5 questions dot product, which we know already. Therefore, it is positive bilinear.



1/1 point

5.

For any two vectors  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$  write a short piece of code that defines a valid inner product.

```
import numpy as np
 1
 2
 3
   def dot(a, b):
      """Compute dot product between a and b.
 4
 5
      Args:
        a, b: (2,) ndarray as R^2 vectors
 6
 7
 8
      Returns:
      a number which is the dot product between a, b """
9
10
11
      dot_product = np.dot(a, b)
12
13
     return dot_product
14
15
16 # Test your code before you submit.
                                                                                                    Run
17 a = np.array([1,0])
18 b = np.array([0,1])
                                                                                                   Reset
19 print(dot(a,b))
```

# **Correct Response**

Good job!



