2D Taylor series

Practice Quiz, 5 questions

5/5 points (100%)

Congratulations! You passed!

Next Item

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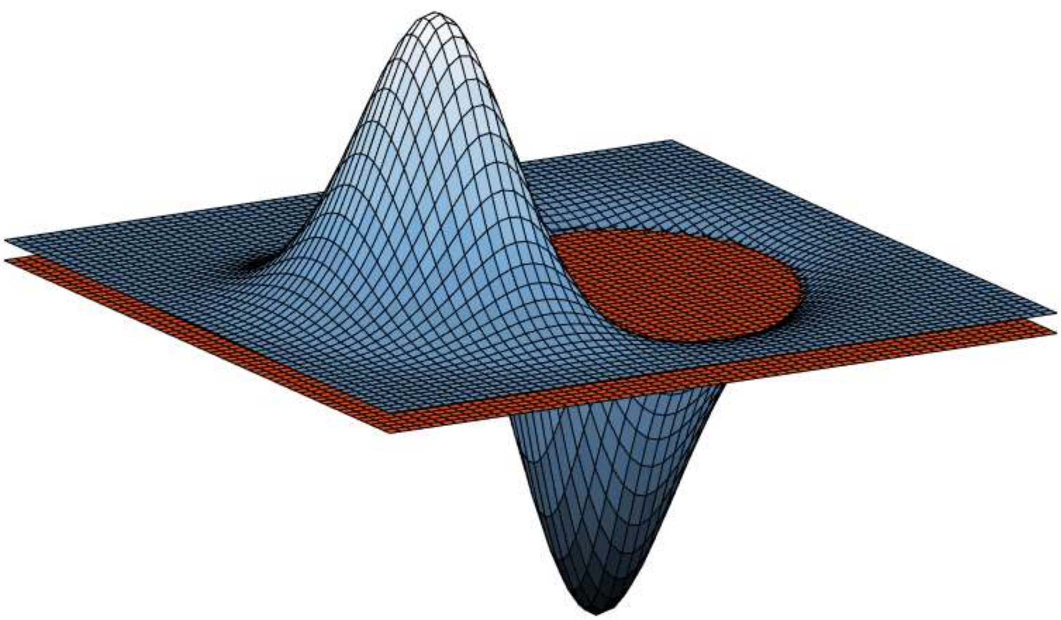
point

1.

Now you have seen how to calculate the multivariate Taylor series, and what zeroth, first and second order approximations look like for a function of 2 variables. In this course we won't be considering anything higher than second order for functions of more than one variable.

In the following questions you will practise recognising these approximations by thinking about how they behave with different x and y, then you will calculate some terms in the multivariate Taylor series yourself.

The following plot features a surface and its Taylor series approximation in red, around a point given by a red circle. What order is the Taylor series approximation?



- Zeroth order

Correct
The red surface is constant everywhere and so has no terms in $\mathbf{\Delta x}$ or $\mathbf{\Delta x}^2$

- First order
- Second order

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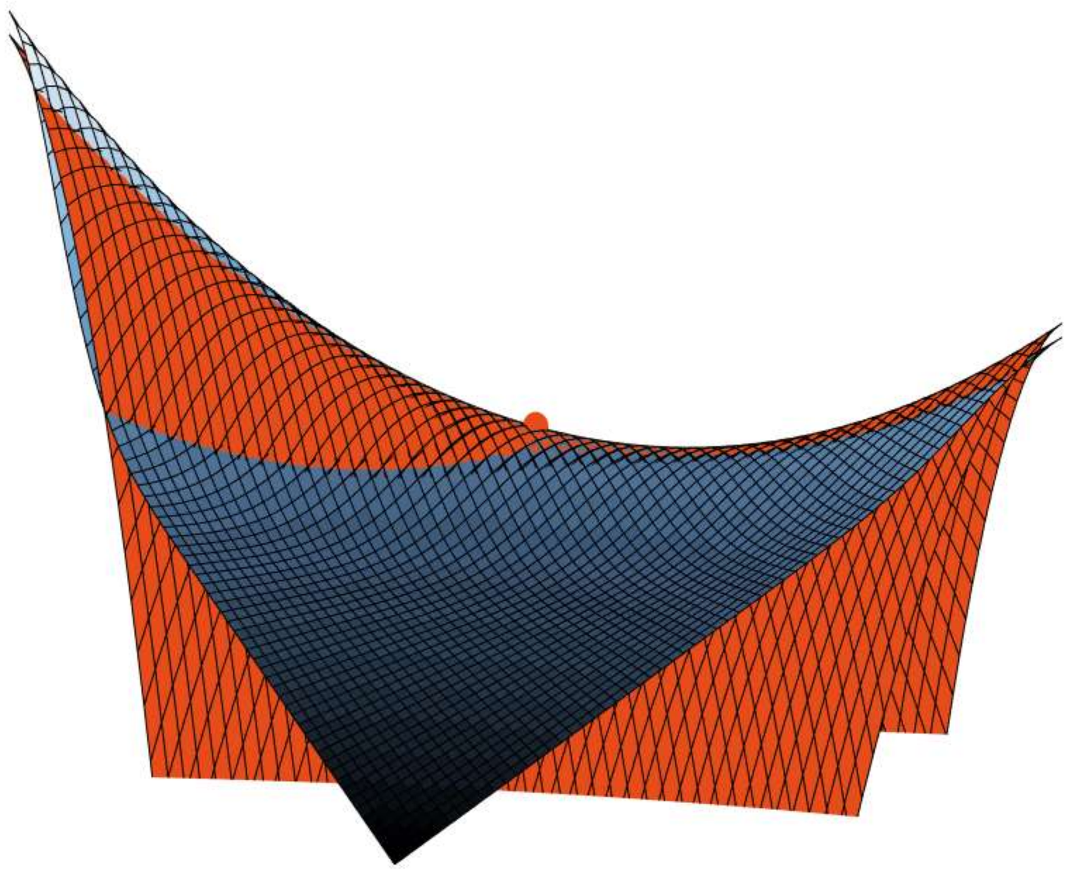
point

2.
What order Taylor series approximation, expanded around the red circle, is the red surface in the following plot?



Zeroth order

First order

Second order

Correct
The gradient of the surface is not constant, so we must have a term of higher order than $\Delta\mathbf{x}$.

None of the above

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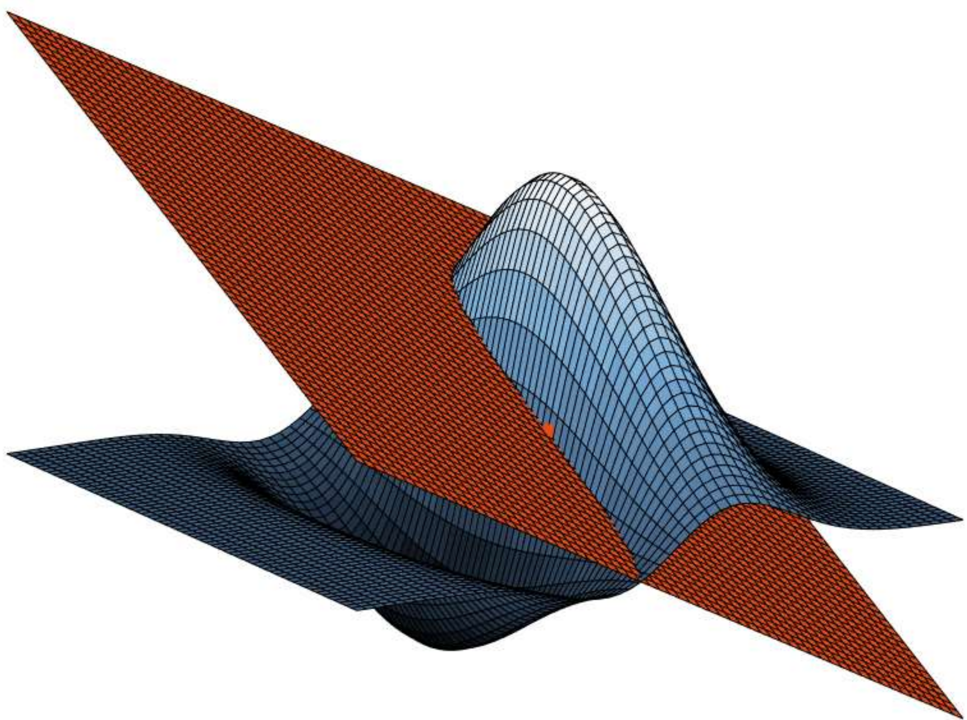
1 / 1
point

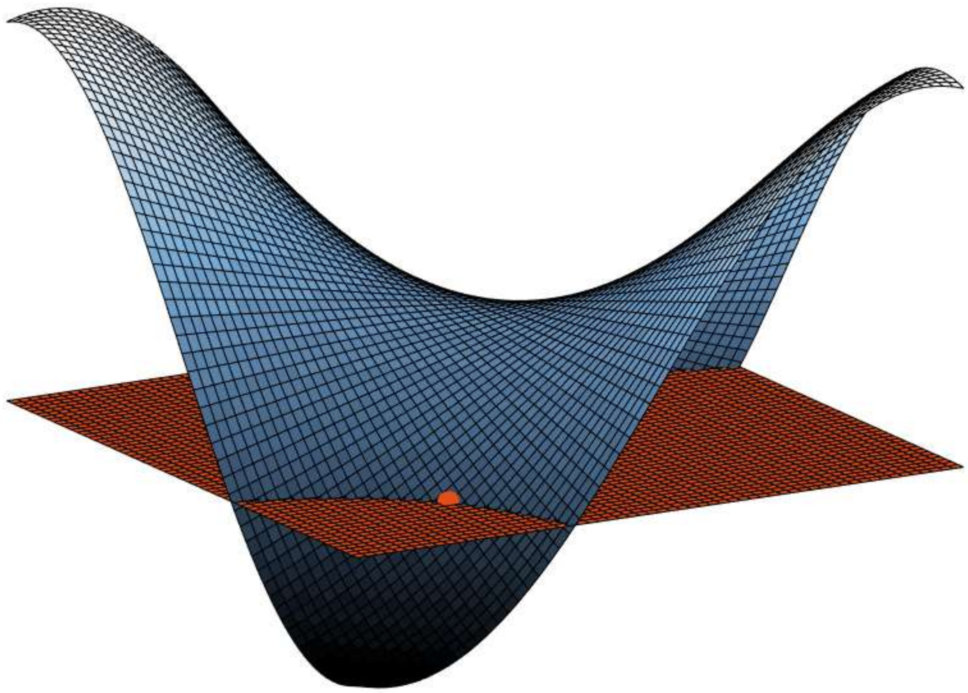
3.

Which red surface in the following images is a first order Taylor series approximation of the blue surface? The original functions are given, but you don't need to do any calculations.

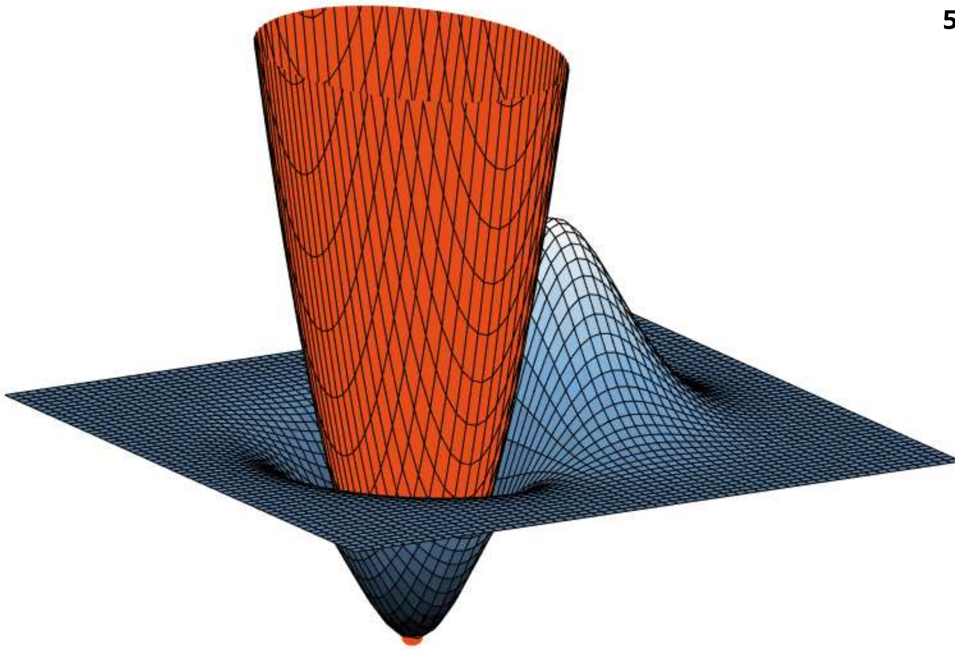
$f(x, y) = (x^2 + 2x)e^{-x^2-y^2/5}$



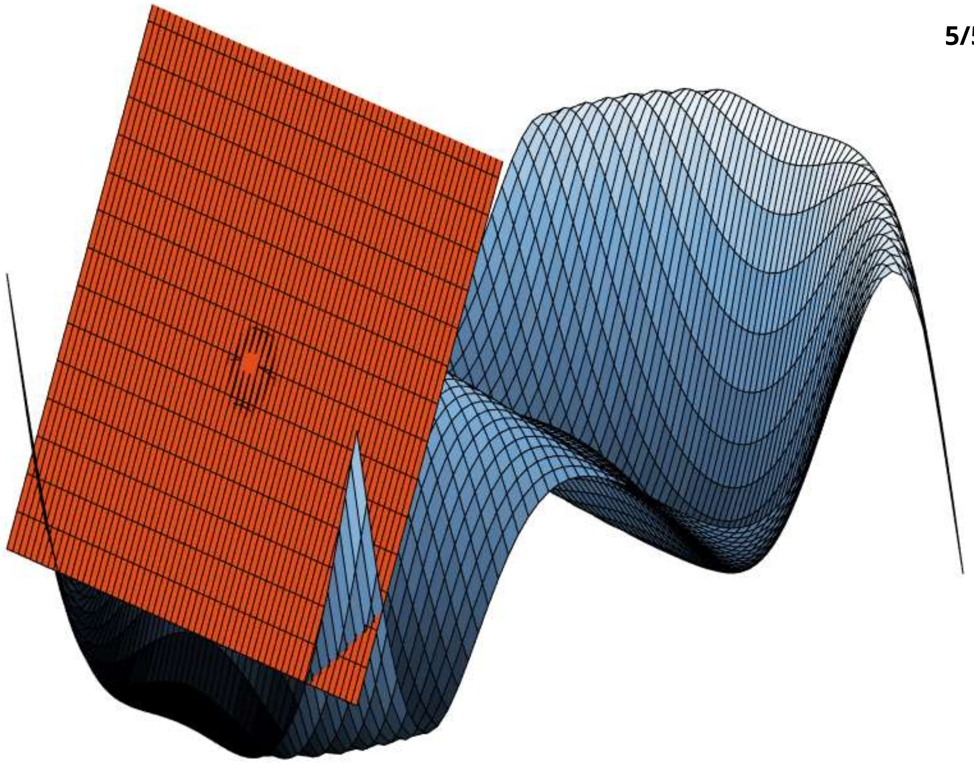
$f(x, y) = \sin(xy/5)$



- [ ] $f(x,y) = xe^{-x^2-y^2}$



$f(x, y) = x \sin(x^2/2 + y^2/4)$



Correct
The gradient of the red surface is non-zero and constant, so the $\mathbf{\Delta x}$ terms are the highest order.

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1 / 1
point

4.
Recall that up to second order the multivariate Taylor series is given by
$f(\mathbf{x}+\mathbf{\Delta x}) = f(\mathbf{x}) + J_f \mathbf{\Delta x} + \frac{1}{2}\mathbf{\Delta x}^T H_f \mathbf{\Delta x} + \dots$

Consider the function of 2 variables, $f(x,y) = xy^2 e^{-x^4-y^2/2}$. Which of the following is the first order Taylor series expansion of f around the point $(-1, 2)$?

- [ ] $f_1(-1+\Delta x, 2+\Delta y) = -4e^{-3} - 4e^{-3}\Delta x + 4e^{-3}\Delta y$
- [ ] $f_1(-1+\Delta x, 2+\Delta y) = -4e^{-3} + 16e^{-3}\Delta x - 8e^{-3}\Delta y$
- [ ] $f_1(-1+\Delta x, 2+\Delta y) = 2e^{-33/2} - 63e^{-33/2}\Delta x - 2e^{-33/2}\Delta y$
- [x] $f_1(-1+\Delta x, 2+\Delta y) = -4e^{-3} - 12e^{-3}\Delta x + 4e^{-3}\Delta y$

1 / 1
point

5.
Now consider the function $f(x, y) = \sin(\pi x - x^2 y)$. What is the Hessian matrix H_f that is associated with the second order term in the Taylor expansion of f around $(1, \pi)$?

- $H_f = \begin{pmatrix} -2\pi & -2 \\ -2 & 1 \end{pmatrix}$
- $H_f = \begin{pmatrix} -\pi^2 & \pi \\ \pi & -1 \end{pmatrix}$
- $H_f = \begin{pmatrix} -2 & -2\pi \\ 2\pi & 1 \end{pmatrix}$
- $H_f = \begin{pmatrix} -2\pi & -2 \\ -2 & 0 \end{pmatrix}$

Correct
Good, you can check your second order derivatives here:

$$\partial_{xx} f(x, y) = -2y\cos(\pi x - x^2 y) - (\pi - 2xy)^2 \sin(\pi x - x^2 y)$$

$$\partial_{xy} f(x, y) = -2x\cos(\pi x - x^2 y) - x^2(\pi - 2xy)\sin(\pi x - x^2 y)$$

$$\partial_{yx} f(x, y) = -2x\cos(\pi x - x^2 y) - x^2(\pi - 2xy)\sin(\pi x - x^2 y)$$

$$\partial_{yy} f(x, y) = -x^4 \sin(\pi x - x^2 y)$$