Diagonalisation and applications

Practice Quiz, 7 questions

7/7 points (100%)

Congratulations! You passed!

Next Item

1 / 1
point

1.
In this quiz you will diagonalise some matrices and apply this to simplify calculations.

Given the matrix $T = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$ and change of basis matrix $C = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ (whose columns are eigenvectors of T), calculate the diagonal matrix $D = C^{-1}TC$.

- [x] $\begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$

Correct
Well done!

- [ ] $\begin{bmatrix} 6 & 0 \\ 0 & 3 \end{bmatrix}$
- [ ] $\begin{bmatrix} 9 & 0 \\ 0 & 20 \end{bmatrix}$
- [ ] $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

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1 / 1
point

2.

Given the matrix $T = \begin{bmatrix} 2 & 7 \\ 0 & -1 \end{bmatrix}$ and change of basis matrix $C = \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix}$ (whose columns are eigenvectors of T), calculate the diagonal matrix $D = C^{-1}TC$.

- ( ) $\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$
- ( ) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$
- ( ) $\begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix}$
- (x) $\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$

Correct
Well done!

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1 / 1
point

3.
Given the matrix $T = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ and change of basis matrix $C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ (whose columns are eigenvectors of T), calculate the diagonal matrix $D = C^{-1}TC$.

- ( ) $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$
- ( ) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- (x) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Correct
Well done!

- ( ) $\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$

4.

Given a diagonal matrix $D = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$, and a change of basis matrix $C = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ with inverse $C = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$, calculate $T = CDC^{-1}$.

- $\begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix}$
- $\begin{bmatrix} -a & 0 \\ 0 & -a \end{bmatrix}$
- $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

Correct
Well done! As it turns out, because D is a special type of diagonal matrix, where all entries on the diagonal are the same, this matrix is just a scalar multiple of the identity matrix. Hence, given any change of co-ordinates, this matrix remains the same.

- $\begin{bmatrix} -a & 0 \\ 0 & a \end{bmatrix}$

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1 / 1
point

5.

Given that $T = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$, calculate T^3.

- $\begin{bmatrix} 122 & 186 \\ -61 & 3 \end{bmatrix}$
- $\begin{bmatrix} -61 & 3 \\ 122 & 186 \end{bmatrix}$
- $\begin{bmatrix} 186 & -61 \\ 122 & 3 \end{bmatrix}$

Correct
Well done!

$\begin{bmatrix} 3 & 122 \\ 186 & -61 \end{bmatrix}$

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1 / 1
point

6.

Given that $T = \begin{bmatrix} 2 & 7 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & -1/3 \\ 1 & 7/3 \end{bmatrix}$, calculate T^3.

- (selected) $\begin{bmatrix} 8 & 21 \\ 0 & -1 \end{bmatrix}$

Correct
Well done!

- $\begin{bmatrix} 21 & 8 \\ 0 & -1 \end{bmatrix}$
- $\begin{bmatrix} 0 & -1 \\ 21 & 8 \end{bmatrix}$
- $\begin{bmatrix} -1 & 21 \\ 8 & 0 \end{bmatrix}$

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1 / 1
point

7.

Given that $T = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, calculate T^5.

- (selected) $\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

Correct
Well done!

- $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

- $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

- $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$