1. (a)$75^\circ = 75 \cdot \frac{\pi}{180}\text{rad} = \frac{75\pi}{180}\text{rad} = \frac{15\pi}{36}\text{rad}$
1. (b)$\frac{2\pi}{3}\text{rad} = \frac{2\pi}{3} \cdot \frac{180^\circ}{\pi} = \frac{360^\circ}{3} = 120^\circ$
2. Given that $0 < \theta < 180$ and $\sin\theta = \frac{24}{25}$, find:
(a) $\cos\theta$

$$\begin{aligned}
\cos\theta &= \pm\sqrt{1-\sin^2\theta} \\
& \pm\sqrt{1-\left(\frac{24}{25}\right)^2} \\
& \pm\sqrt{1-\left(\frac{576}{625}\right)} \\
& \pm\sqrt{1-0.9216} \\
& \pm\sqrt{0.0784} \\
& \pm 0.28
\end{aligned}$$

(b) $\sin 2\theta$

$$\begin{aligned}
\sin 2\theta &= 2\sin\theta\cos\theta \\
& \pm 2\left(\frac{24}{25}\right)(0.28) \\
& \pm 0.5376
\end{aligned}$$

(c) $\cos 2\theta$

$$\begin{aligned}
\cos 2\theta &= \cos^2\theta - \sin^2\theta \\
& \pm\left(0.28^2 - \left(\frac{24}{25}\right)^2\right) \\
& \pm(0.0784 - 0.9216) \\
& \pm 0.8432
\end{aligned}$$

(d) $\sin\frac{\theta}{2}$

$$\begin{aligned}
\sin\frac{\theta}{2} &= \pm\sqrt{\frac{1-\cos\theta}{2}} \\
& \pm\sqrt{\frac{1-0.28}{2}} \\
& \pm\sqrt{\frac{0.72}{2}} \\
& \pm\sqrt{0.36} \\
& \pm 0.6
\end{aligned}$$

(e) $\cos\frac{\theta}{2}$

$$\begin{aligned}
\cos\frac{\theta}{2} &= \pm\sqrt{\frac{1+\cos\theta}{2}} \\
& \pm\sqrt{\frac{1+0.28}{2}} \\
& \pm\sqrt{\frac{1.28}{2}} \\
& \pm\sqrt{0.64} \\
& \pm 0.8
\end{aligned}$$

3. A triangle has sides of length 5, 7, and 8. Use the law of cosines to find the angle opposite the side of length 7 to the nearest degree. (ref. p 349)

$a = 5$, $b = 7$,$c = 8$

Solve for the largest angle first.

Angle C:

$$\begin{aligned}
\cos C &= \frac{a^2+b^2-c^2}{2ab} \\
& \frac{5^2+7^2-8^2}{2\cdot 5\cdot 7} \\
& \frac{1}{7} \\
C &= \arccos\left(\frac{1}{7}\right) \\
& 1.42745 \\
& 1.42745\cdot\left(\frac{180}{\pi}\right) \\
& 81.787^\circ
\end{aligned}$$

Solve for the smallest angle next.

Angle A:

$$\begin{aligned}
\cos A &= \frac{b^2+c^2-a^2}{2bc} \\
& \frac{7^2+8^2-5^2}{2\cdot 7\cdot 8} \\
& \frac{11}{14} \\
A &= \arccos\frac{11}{17} \\
& 0.66695
\end{aligned}$$

$$0.66695 \cdot \frac{180}{\pi}$$
$$38.213^\circ$$

Solve for angle B.

$$\begin{aligned} B &= 180^\circ - A - C \\ & 180^\circ - 81.787^\circ - 38.213^\circ \\ & 60^\circ \end{aligned}$$

4. Find A, θ such that $7\sin\theta + 24\cos\theta = A\sin(\theta+\alpha)$.
Pick θ such that $\sin\theta = \cos\theta$, or $\theta = \frac{\pi}{4}$. Now:

$$\begin{aligned} 7\sin\theta + 24\cos\theta &= A\sin(\theta+\alpha) \\ \sin\theta + 3\cos\theta &= \frac{A\sin\theta}{7} \\ \sin\theta - \frac{A\sin\theta}{7} + 3\cos\theta &= 0 \end{aligned}$$

When $\theta = \frac{\pi}{4}$, then $\sin\theta = \cos\theta = \frac{\sqrt{2}}{2}$:

$$\begin{aligned} \frac{\sqrt{2}}{2} - \frac{A\sqrt{2}}{14} + \frac{3\sqrt{2}}{2} &= 0 \\ \frac{\sqrt{2}}{2} - \frac{A\sqrt{2}}{14} &= -\frac{3\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \\ -\frac{A\sqrt{2}}{14} &= -\frac{2\sqrt{2}}{2} \\ \frac{A\sqrt{2}}{14} &= \sqrt{2} \\ A\sqrt{2} &= 14\sqrt{2} \\ A &= 14 \end{aligned}$$