1. (a)$75^\circ = 75 \cdot \frac{\pi}{180}\text{rad} = \frac{75\pi}{180}\text{rad} = \frac{15\pi}{36}\text{rad}$
1. (b)$\frac{2\pi}{3}\text{rad} = \frac{2\pi}{3} \cdot \frac{180^\circ}{\pi} = \frac{360^\circ}{3} = 120^\circ$
2. Given that $0 < \theta < 180$ and $\sin\theta = \frac{24}{25}$, find:
(a) $\cos\theta$

$$
\begin{aligned}
\cos\theta &= \pm\sqrt{1-\sin^2\theta} \\
&\pm\sqrt{1-\left(\frac{24}{25}\right)^2} \\
&\pm\sqrt{1-\left(\frac{576}{625}\right)} \\
&\pm\sqrt{1-0.9216} \\
&\pm\sqrt{0.0784} \\
&\pm 0.28
\end{aligned}
$$

(b) $\sin 2\theta$

$$
\begin{aligned}
\sin 2\theta &= 2\sin\theta\cos\theta \\
&\pm 2\left(\frac{24}{25}\right)(0.28) \\
&\pm 0.5376
\end{aligned}
$$

(c) $\cos 2\theta$

$$
\begin{aligned}
\cos 2\theta &= \cos^2\theta - \sin^2\theta \\
&\pm\left(0.28^2 - \left(\frac{24}{25}\right)^2\right) \\
&\pm(0.0784 - 0.9216) \\
&\pm 0.8432
\end{aligned}
$$

(d) $\sin\frac{\theta}{2}$

$$
\begin{aligned}
\sin\frac{\theta}{2} &= \pm\sqrt{\frac{1-\cos\theta}{2}} \\
&\pm\sqrt{\frac{1-0.28}{2}} \\
&\pm\sqrt{\frac{0.72}{2}} \\
&\pm\sqrt{0.36} \\
&\pm 0.6
\end{aligned}
$$

(e) $\cos\frac{\theta}{2}$

$$\begin{aligned}
\cos\frac{\theta}{2} &= \pm\sqrt{\frac{1+\cos\theta}{2}} \\
&\pm\sqrt{\frac{1+0.28}{2}} \\
&\pm\sqrt{\frac{1.28}{2}} \\
&\pm\sqrt{0.64} \\
&\pm 0.8
\end{aligned}$$

3. A triangle has sides of length 5, 7, and 8. Use the law of cosines to find the angle opposite the side of length 7 to the nearest degree. (ref. p 349)

$a = 5$, $b = 7$,$c = 8$

Solve for the largest angle first.

Angle C:

$$\begin{aligned}
\cos C &= \frac{a^2+b^2-c^2}{2ab} \\
&\frac{5^2+7^2-8^2}{2\cdot 5\cdot 7} \\
&\frac{1}{7} \\
C &= \arccos\left(\frac{1}{7}\right) \\
&1.42745 \\
&1.42745\cdot\left(\frac{180}{\pi}\right) \\
&81.787^\circ
\end{aligned}$$

Solve for the smallest angle next.

Angle A:

$$\begin{aligned}
\cos A &= \frac{b^2+c^2-a^2}{2bc} \\
&\frac{7^2+8^2-5^2}{2\cdot 7\cdot 8} \\
&\frac{11}{14} \\
A &= \arccos\frac{11}{17} \\
&0.66695
\end{aligned}$$

$$\begin{aligned}&0.66695 \cdot \frac{180}{\pi}\\&38.213^\circ\end{aligned}$$

Solve for angle B.

$$\begin{aligned}B &= 180^\circ - A - C\\&\quad 180^\circ - 81.787^\circ - 38.213^\circ\\&\quad 60^\circ\end{aligned}$$

4. Find A, θ such that $7\sin\theta + 24\cos\theta = A\sin(\theta+\alpha)$.
Pick θ such that $\sin\theta = \cos\theta$, or $\theta = \frac{\pi}{4}$. Now:

$$\begin{aligned}7\sin\theta + 24\cos\theta &= A\sin(\theta+\alpha)\\ \sin\theta + 3\cos\theta &= \frac{A\sin\theta}{7}\\ \sin\theta - \frac{A\sin\theta}{7} + 3\cos\theta &= 0\end{aligned}$$

When $\theta = \frac{\pi}{4}$, then $\sin\theta = \cos\theta = \frac{\sqrt{2}}{2}$:

$$\begin{aligned}\frac{\sqrt{2}}{2} - \frac{A\sqrt{2}}{14} + \frac{3\sqrt{2}}{2} &= 0\\ \frac{\sqrt{2}}{2} - \frac{A\sqrt{2}}{14} &= -\frac{3\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\\ -\frac{A\sqrt{2}}{14} &= -\frac{2\sqrt{2}}{2}\\ \frac{A\sqrt{2}}{14} &= \sqrt{2}\\ A\sqrt{2} &= 14\sqrt{2}\\ A &= 14\end{aligned}$$

5. Points A and B are 400 miles apart. At noon, one train starts from A heading toward B, while another train starts from B heading for A. The faster train is 20 MPH faster than the slower train. The two trains meet at 4 PM. How far has each train travelled?

This is a vector problem in one dimension. At the start:

A is at $x = 0$, and B is at $x = 400$.

Either train can be the slower train. Let B be the slower train. If A travels m mph then B travels $m - 20$ mph.

At time t:

A will be at $x = mt$.

B will be at $x = 400 - (m - 20)t$
At time $t = 4$ the trains meet, therefore:
$mt = 400 - (m - 20)t$, when $t = 4$.
Substitute $t = 4$ and solve for m:

$$\begin{aligned} 4m &= 400 - 4(m - 20) \\ 4m &= 400 - 4m - 80 \\ 8m &= 320 \\ m &= 40 \end{aligned}$$

Now, find x when $t = 4$ for A and B:
A:

$$\begin{aligned} x &= mt \\ x &= 40 \cdot 5 \\ x &= 200 \end{aligned}$$

B:

$$\begin{aligned} x &= 400 - (m - 20)t \\ x &= 400 - 4(40 - 20) \\ x &= 320 \end{aligned}$$

Because B started at $x = 400$,B has traveled $400 - 320 = 80$ miles and A has traveled 200 miles at $t = 4$.

6. Jane can do a certain job in 10 minutes by herself. Sally would need 15 minutes to do the same job by herself. If the girls work together, how much time is needed for the job?

This is a pre-calculus summation problem.

In one minute, Jane can do $\frac{1}{10}$ of the job, and in one minute Sally can do $\frac{1}{15}$ of the job. If the job is done in m minutes, then:

$$\begin{aligned} \frac{m}{10} + \frac{m}{15} &= 1 \\ 5m &= 30 \\ m &= 6 \end{aligned}$$