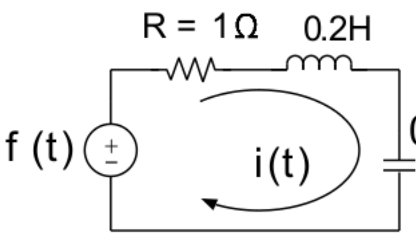
Zhejiang University – University of Illinois at Urbana-Champaign Institute

ECE-210 Analog Signal Processing Spring 2022
Homework #7: Submission Deadline 6th April (10:00 PM)

1. Consider the circuit drawn below, where the frequency response of the circuit is: $H(\omega) = \frac{I}{F}$.



(a) $2\pi f L = \frac{1}{2\pi f C} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$

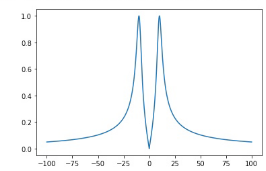
$\therefore f = \frac{5}{\pi}$

(b) $|H(\omega)| = \left|\frac{I}{F}\right|$

$I = \frac{F}{Z} = \frac{F}{R + \frac{1}{j\omega C} + j\omega L} \Rightarrow \frac{I}{F} = \frac{1}{Z} = \frac{1}{R + \frac{1}{j\omega C} + j\omega L}$

(b)

```
[3]: import numpy as np
     import matplotlib.pyplot as plt
     def HwFunc(w,j):
         return abs(1/(1+20/(j*w)+j*w/5))
     list1 = np.linspace(-100,100,10000)
     plt.plot(list1,HwFunc(list1,complex(0,1)))
[3]: [<matplotlib.lines.Line2D at 0x7f7f26970400>]
```



(e) $f = \frac{1}{2\pi\sqrt{LC}}$

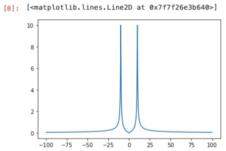
$\Rightarrow$ f remains same for 10Ω & 0.1Ω

$f = \frac{5}{\pi}$

However, for the plot:

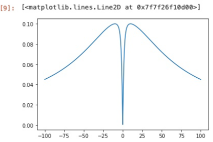
0.1Ω case.

```
[8]: import numpy as np
     import matplotlib.pyplot as plt
     def HwFunc(w,j):
         return (abs(1/(0.1+20/(j*w)+j*w/5)))
     list1 = np.linspace(-100,100,10000)
     plt.plot(list1,HwFunc(list1,complex(0,1)))
[8]: [<matplotlib.lines.Line2D at 0x7f7f26e3b640>]
```



10Ω case.

```
[9]: import numpy as np
     import matplotlib.pyplot as plt
     def HwFunc(w,j):
         return (abs(1/(10+20/(j*w)+j*w/5)))
     list1 = np.linspace(-100,100,10000)
     plt.plot(list1,HwFunc(list1,complex(0,1)))
[9]: [<matplotlib.lines.Line2D at 0x7f7f26f10d00>]
```



(a) What is the resonant frequency of this circuit?

(b) Plot $|H(\omega)|$, and label the resonant frequency on this plot. You may use Matlab, Mathematica, etc.

(c) Plot $Re\{H(\omega)\}$ and $Im\{H(\omega)\}$. You may use Matlab, Mathematica, etc.

(d) Explain why this circuit might be called a "bandpass" filter.

(e) Repeat (a) and (b) for resistor values of $10\,\Omega$ and $0.1\,\Omega$.

(f) Based on (d), how does the resistor value relate to the passband of the filter (e.g., does a larger value for the resistor give a narrower or wider passband)?

(d) Because we can see that: to plot image, the extreme case $w \to \infty$ $w \to 0$ the passing rate of the signal is really small; only the medium case, there is a "band" of signals could pass the circuit.

(f) as the ploting image, $\boxed{\text{larger value resistor}}$ will result in a $\boxed{\text{wider passband.}}$ iff the criteria of signal dection is respectively comparing from frequency to frequency.

iff criteria is to compare the signal to a $H(w) \geq k$, where k is fixed number

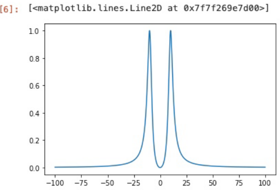
then $\boxed{\text{larger resistor}}$ will result in a $\boxed{\text{smaller pass band}}$

Image.

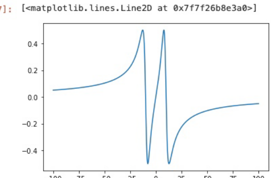
Q1 (C)

Real

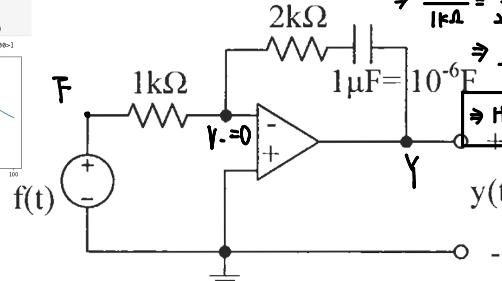
```
[6]: import numpy as np
     import matplotlib.pyplot as plt
     def HwFunc(w,j):
         return ((1/(1+20/(j*w)+j*w/5)).real)
     list1 = np.linspace(-100,100,10000)
     plt.plot(list1,HwFunc(list1,complex(0,1)))
[6]: [<matplotlib.lines.Line2D at 0x7f7f269e7d00>]
```



```
[7]: import numpy as np
     import matplotlib.pyplot as plt
     def HwFunc(w,j):
         return ((1/(1+20/(j*w)+j*w/5)).imag)
     list1 = np.linspace(-100,100,10000)
     plt.plot(list1,HwFunc(list1,complex(0,1)))
[7]: [<matplotlib.lines.Line2D at 0x7f7f26b8e3a0>]
```



2. Determine the frequency response $H(\omega)$ of the following circuit.



$\Rightarrow \frac{F-0}{1k\Omega} = \frac{Y}{2k\Omega + \frac{1}{j\omega C}}$

$\Rightarrow \frac{Y}{F} = 2 + \frac{1}{j\omega C \cdot 1k\Omega}$

$\boxed{\Rightarrow H(\omega) = \frac{Y}{F} = 2 - \frac{1000}{\omega} j}$

3. A linear system with input $f(t)$ and output $y(t)$ is described by the ODE

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y(t) = \frac{df}{dt} + 2\frac{d^2f}{dt^2}.$$

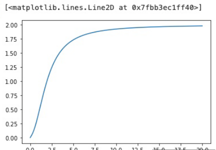
(a) $(j\omega)^2 Y + 4(j\omega)Y + 4Y = (j\omega)F + 2(j\omega)^2 F$

$(-\omega^2 + 4j\omega + 4)Y = (j\omega - 2\omega^2)F$

$\Rightarrow H(\omega) = \frac{Y}{F} = \frac{j\omega - 2\omega^2}{(4-\omega^2) + 4j\omega}$

(b)

```
import numpy as np
import matplotlib.pyplot as plt
def HwFunc(w,j):
    return (abs((j*w-2*w**2)/(4-w**2+4*j*w)))
list1 = np.linspace(0,20,1000)
plt.plot(list1,HwFunc(list1,complex(0,1)))
[<matplotlib.lines.Line2D at 0x7fbb3ec1f40>]
```



$\rightarrow |H(\omega)| = \frac{\omega\sqrt{4\omega^2+1}}{\omega^2+4}$

(a) Determine the frequency response $H(\omega)$ of the system.

(b) Determine and plot the magnitude response $|H(\omega)|$ for $0 < \omega < 20$ rad/s. You may use Matlab, Mathematica, etc.

(c) Determine if this filter is lowpass, bandpass, highpass, or none of these; and indicate why.

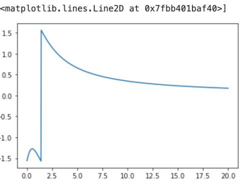
(C) High pass filter as the plot figure indicates that higher frequency will pass more completely in a rate of two

(d) Determine and plot the phase response $\angle H(\omega)$ for $0 < \omega < 20$ rad/s. You may use Matlab, Mathematica, etc.

$\rightarrow \angle H(\omega) = \arctan\left(\frac{(7\omega^2+4)}{2(\omega^2-2)\omega}\right)$

3(d)

```
import numpy as np
import matplotlib.pyplot as plt
def HwFunc(w,j):
    return (np.arctan((7*w**2+4)/(2*w*(w**2-2))))
list1 = np.linspace(0.01,20,1000)
plt.plot(list1,HwFunc(list1,complex(0,1)))
<matplotlib.lines.Line2D at 0x7fbb401baf40>
```



4. A linear system with input $f(t)$ and output $y(t)$ is described by the ODE

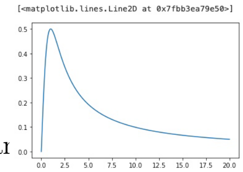
$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y(t) = \frac{df}{dt}.$$

(a) $(j\omega)^2 Y + 2j\omega Y + Y = (j\omega)F$

$H(\omega) = \frac{Y}{F} = \frac{j\omega}{1-\omega^2+2j\omega} = \frac{(\omega-\omega^3)j + 2\omega^2}{(1-\omega^2)^2+4\omega^2}$

(b)

```
import numpy as np
import matplotlib.pyplot as plt
def HwFunc(w,j):
    return (abs((j*w)/(1-w**2+2*j*w)))
list1 = np.linspace(0,20,1000)
plt.plot(list1,HwFunc(list1,complex(0,1)))
[<matplotlib.lines.Line2D at 0x7fbb3ea79e50>]
```



(a) Determine the frequency response $H(\omega)$ of the system.

(b) Determine and plot the magnitude response $|H(\omega)|$ for $0 < \omega < 20$ rad/s. You can use Matlab, Mathematica, etc.

$\Rightarrow |H(\omega)| = \frac{1}{(1-\omega^2)^2+4\omega^2}\sqrt{\omega^2+\omega^6-2\omega^4+4\omega^4} = \frac{\omega+\omega^3}{(1-\omega^2)^2+4\omega^2} = \frac{\omega}{1+\omega^2}$

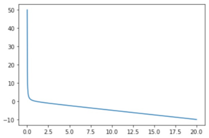
(c) Determine if this filter is lowpass, bandpass, highpass, or none of these; and indicate why.

band pass. as both $w \to 0$ & $w \to \infty$ would result in the truly low passing fraction $H(\omega)$

$\angle H(w) = \frac{w - w^3}{2w^2}$
$= \frac{1 - w^2}{2w}$

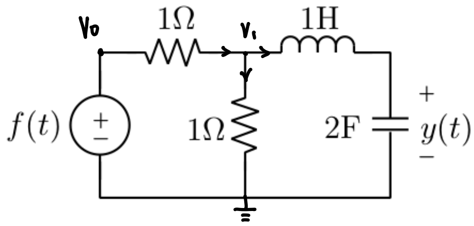
```python
import numpy as np
import matplotlib.pyplot as plt
def HwFunc(w,j):
    return ((1-w**2)/(2*w))
list1 = np.linspace(0.01,20,1000)
plt.plot(list1,HwFunc(list1,complex(0,1)))
```

[<matplotlib.lines.Line2D at 0x7fbb3ff08490>]



(d) Determine and plot the phase response $\angle H(\omega)$ for $0 < \omega < 20$ rad/s. You can use ica, etc.

5. In the following circuit, the input is $f(t) = 4 + \cos(2t)$. Determine the steady-state output $y(t)$ of the circuit.



$$\begin{cases} L\frac{d^2y}{dt^2} + y = V_1 \\ \frac{V_0 - V_1}{1\Omega} = \frac{V_1}{1\Omega} + \frac{dy}{dt} \end{cases}$$

$\Rightarrow \frac{dy}{dt} + 2\left(L\frac{d^2y}{dt^2} + y\right) = f(t)$

$2\frac{d^2y}{dt^2} + \frac{dy}{dt} + 2y = f(t)$

$\Rightarrow 2\frac{d^2y}{dt^2} + \frac{dy}{dt} + 2y = 4 + \cos(2t)$ As for 4: the out put is $n=0$ $\alpha=0$

$2r^2 + r + 2 = 0$

$2(r^2 + \frac{1}{2}r + \frac{1}{16}) + 2 - \frac{1}{8} = 0$

$(r + \frac{1}{4})^2 + \frac{15}{16} = 0$

$r = -\frac{1}{4} \pm \frac{\sqrt{15}}{4}i$

$\Rightarrow \tilde{y} = 2$

for $\cos 2t = Re\{e^{i2t}\}$

assume $\tilde{y} = a e^{i2t}$

$(2i)^2 a \cdot e^{i2t} + 2i a e^{i2t} + 2a e^{i2t} = e^{i2t}$

$\therefore -4a e^{i2t} + 2ia e^{i2t} + 2a e^{i2t} = e^{i2t}$

$\Rightarrow -4a + 2ai + 2a = 1$

$\therefore 2a(-1+i) = 1$ $\therefore a = \frac{1}{2i-2} = \frac{(i+1)}{2(i^2-1)} = -\frac{1}{4}(i+1)$

$\Rightarrow \tilde{y} = Re\{-\frac{1}{4}(i+1)(\cos 2t + i\sin 2t)\}$

$= -\frac{1}{4}(\cos 2t - \sin 2t)$

$\Rightarrow$ steady state $= 2 - \frac{1}{4}(\cos 2t - \sin 2t)$ V

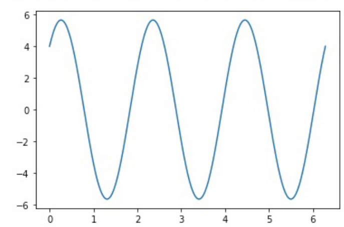
6. Given an input $f(t) = 2e^{-j2t} + (2+j2)e^{-jt} + (2-j2)e^{jt} + 2e^{j2t}$ and $H(\omega) = \frac{1+j\omega}{2+j\omega}$ determine the steady-state response $y(t)$ of the system $H(\omega)$ and express it as a real valued signal.

7. Consider the function $f(t) = Re\{4e^{j3t} + 4e^{-j3t}\}$. Find its period, T_0, its fundamental frequency, ω_o, and plot it over at least two periods. $f(t) = 4\cos(3t) + 4\sin(3t)$ Period $= \frac{2\pi}{w} = \frac{2\pi}{3} = T_0$

$= 4\sqrt{2}\cos(3t - \frac{\pi}{4})$

```python
list1 = np.linspace(0,2*np.pi,1000)
plt.plot(list1,4*np.sqrt(2)*np.cos(3*list1-np.pi/4))
```

[<matplotlib.lines.Line2D at 0x7fbb3f12c550>]



8. For each one of the following functions of t, indicate whether they are periodic or not. If periodic, indicate its period, and if not periodic, indicate why. Assume n is a positive integer.

(a) $\sin(t) + \sin\left(\frac{t}{2}\right) + \sin\left(\frac{t}{3}\right)$ Yes, $T = 12\pi$.

(b) $\sin(\pi t) + \cos\left(\sqrt{2}\, t\right)$ Not periodic. $\longrightarrow$ if there is a period:

$f(0) = f(T+0)$

$\Rightarrow f(0) = 1$

$f(T) = \sin(\pi T) + \cos(\sqrt{2}T)$

and $\begin{cases} \sin(\pi T) = 0 \text{ ①} \\ \cos(\sqrt{2}T) = 1 \text{ ②} \end{cases}$

$\Rightarrow$ for ①: $T = k$, $k \in \mathbb{Z}$

②: $T = \sqrt{2}k\pi$, $k \in \mathbb{Z}$

only sol. is $k=0$

$\therefore$ There is no period for $\sin(\pi t) + \cos(\sqrt{2}t)$

(c) $\sin\left(\frac{\pi t}{4}\right) + \cos\left(\frac{3\pi t}{2}\right) + \sin\left(\frac{2\pi t}{5}\right)$ Yes, $T = 40$.

(d) $|\sin(nt)|$ Yes, $T = \frac{2\pi}{n}$.

(e) $\cos(\pi t) + \cos\left(\frac{\pi}{n}t\right)$ Yes, $T = 2n$.

(f) $\cos(nt)\sin(nt)$ Yes, $T = \frac{\pi}{n}$

6.

$f(t) = \underbrace{2e^{-j2t}}_{\text{signal 1}} + \underbrace{(2+j2)e^{-jt}}_{\text{signal 2}} + \underbrace{(2-j2)e^{jt}}_{\text{signal 3}} + \underbrace{2e^{j2t}}_{\text{signal 4}}$

$H(w) = \frac{1+jw}{2+jw}$

For signal 1: $F = 2$ $w = -2$

$\Rightarrow Y = H(-2) \cdot F = \frac{1-2j}{2-2j} \cdot 2 = \frac{(1-2j)(1+j)}{(1-j)(1+j)} = \frac{1}{2}(1+j-2j+2) = \frac{1}{2}(3-j) \Rightarrow y_1(t) = \frac{1}{2}(3-j) \cdot e^{i(-2t)}$

For signal 2: $F = (2+j2)$ $w = -1$

$\Rightarrow Y = H(-1) \cdot F = \frac{1-j}{2-j} \cdot (2+2j) = \frac{2 \cdot 2 \cdot (2+j)}{5} = \frac{4}{5}(2+j) \Rightarrow y_2(t) = \frac{4}{5}(2+j)e^{i(-t)}$

For signal 3: $F = (2-j \cdot 2)$ $w = 1$

$\Rightarrow Y = H(1) \cdot F = \frac{1+j}{2+j} \cdot (2-2j) = \frac{4 \cdot (2-j)}{5} \Rightarrow y_3(t) = \frac{4}{5}(2-j)e^{it}$

For signal 4: $F = 2$ $w = 2$

$\Rightarrow Y = H(2) \cdot F = \frac{1+2j}{2+2j} \cdot 2 = \frac{(1+2j)(1-j)}{2} = \frac{1}{2}(1-j+2j+2) = \frac{1}{2}(3+j)$

$\Rightarrow y_4 = \frac{1}{2}(3+j) \cdot e^{j(2t)}$

$$\boxed{\Rightarrow y(t) = \frac{1}{2}(3-j)e^{j(-2t)} + \frac{4}{5}(2+j)e^{j(-t)} + \frac{4}{5}(2-j)e^{jt} + \frac{1}{2}(3+j)e^{j(2t)}}$$