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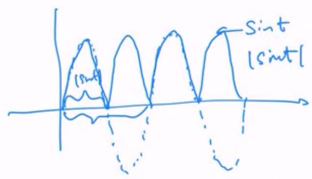
Find Fourier Series of $f(t) = |\sin t|$



$\sin t \Longrightarrow \omega_0 = 1$

$T = \frac{2\pi}{\omega_0} = 2\pi$

$|\sin t| \quad T = \frac{2\pi}{2} = \pi$

period of $|\sin t|$

F_n integrate 0 to π

$\underline{0<t<\pi} \quad |\sin t| = \sin t$

$f(t) = \sin t$

$$F_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega_0 t}\, dt$$

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$$F_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_0^T \sin t\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_0^T \frac{e^{jt}-e^{-jt}}{2j}\, e^{-jn\omega_0 t}\, dt \qquad \omega_0 = 2$$

$$= \frac{1}{2\pi j}\int_0^\pi \left[e^{j(1-2n)t} - e^{-j(1+2n)t}\right] dt$$

$$= \frac{1}{2\pi j}\left[\frac{e^{j(1-2n)t}}{j(1-2n)} + \frac{e^{-j(1+2n)t}}{j(1+2n)}\right]\Bigg|_0^\pi \qquad -1$$

$$= -\frac{1}{2\pi}\left[\frac{e^{j(1-2n)\pi} - e^0}{1-2n} + \frac{e^{-j(1+2n)\pi} - e^0}{1+2n}\right] \Rightarrow \frac{1}{\pi}\left[\frac{1}{1-2n} + \frac{1}{1+2n}\right]$$

$e^{-j2n\pi} \rightarrow$ integer

$e^{-j\pi} \cdot e^{-j2n\pi} = e^{-j\pi} = -1$

$$e^{-j(1+2n)\pi} = e^{-j\pi}\cdot \underbrace{e^{-j2n\pi}}_{1} = e^{-j\pi} = -1$$

$$e^{-jn\pi} = \underbrace{\cos(-2n\pi)}_{1} + j\underbrace{\sin(-2n\pi)}_{0}$$

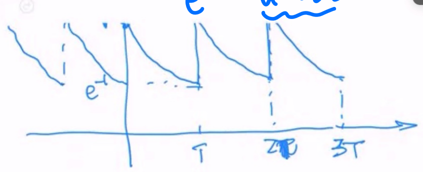
$$= \frac{1}{\cancel{2}\pi}\left(\frac{\cancel{-2}}{1-2n} + \frac{\cancel{-2}}{1+2n}\right)$$

$$= \frac{1}{\pi}\left(\frac{1}{1-2n} + \frac{1}{1+2n}\right) = \frac{2}{\pi}\frac{1}{1-4n^2} = F_n$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{2}{\pi}\frac{1}{1-4n^2}e^{j2nt} \leftarrow \text{exponential form}$$

e^{-at} $\quad a = 0.5\,s^{-1}$

for $f(t)$ $\quad$ a and T



$$F_n = \frac{1}{T}\int_0^T e^{-at}\cdot e^{-jn\omega_0 t}\,dt$$

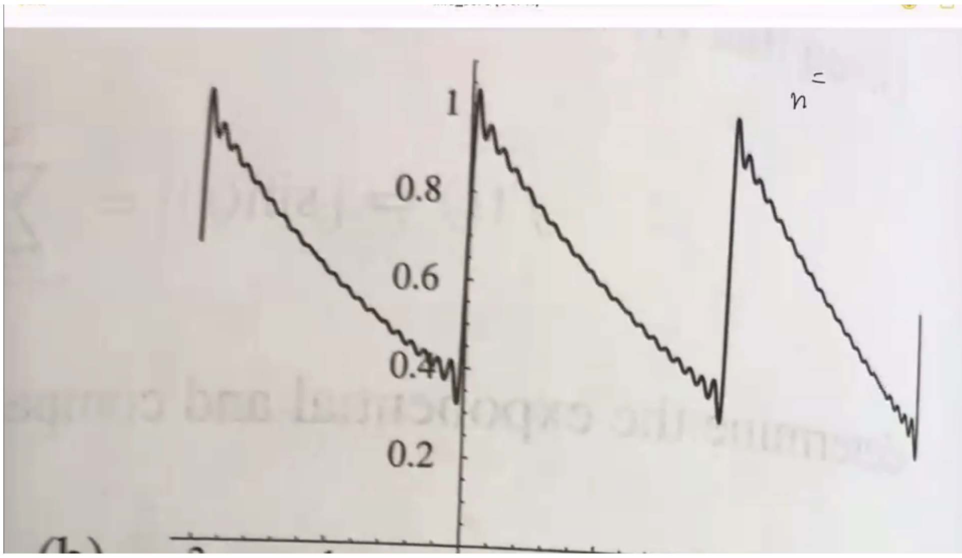
$$= \frac{1}{T}\int_0^T e^{-(a+jn\omega_0)t}\,dt$$

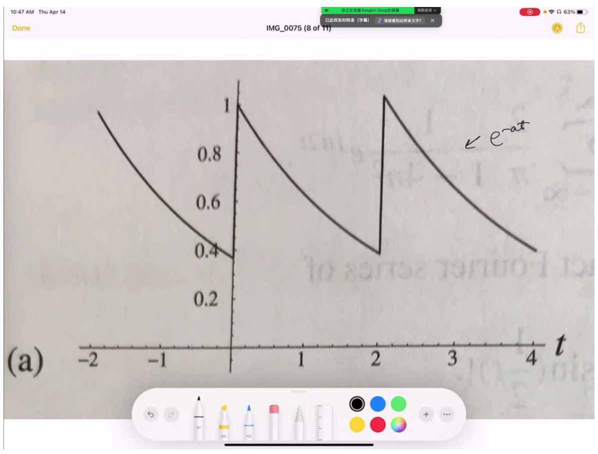
$$= \frac{1}{T}\left.\frac{e^{-(a+jn\omega_0)t}}{-(a+jn\omega_0)}\right|_0^T = \frac{1}{T}\,\frac{1-e^{-(a+jn\omega_0)T}}{a+j\omega_0 n}$$

$$= \frac{1-e^{-aT}\cdot e^{-jn2\pi}}{aT+jn2\pi} = \frac{1-e^{-aT}}{aT+jn2\pi} = \frac{1-e^{aT}}{\sqrt{a^2T^2+(2n\pi)^2}}\cdot e^{-j\tan\frac{2n\pi}{aT}}$$

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1-e^{-aT}}{\sqrt{a^2T^2+(2\pi n)^2}}\, e^{-j\tan\frac{2\pi n}{aT}}\cdot e^{jn\omega_0 t}.$$


1
0.8
0.6
0.4
0.2
n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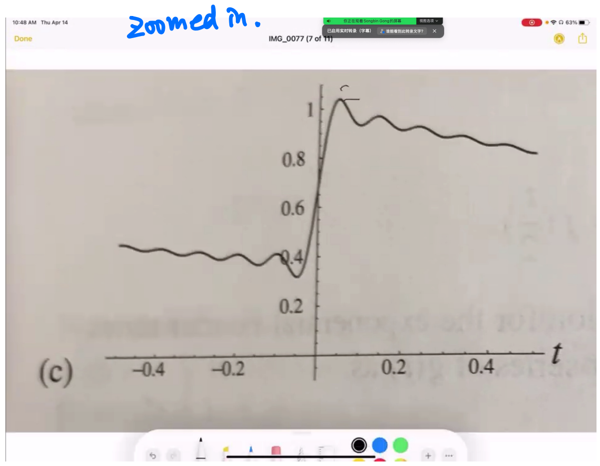
1
0.8
0.6
0.4
0.2
(a)
-2
-1
1
2
3
4
t
e^{-at}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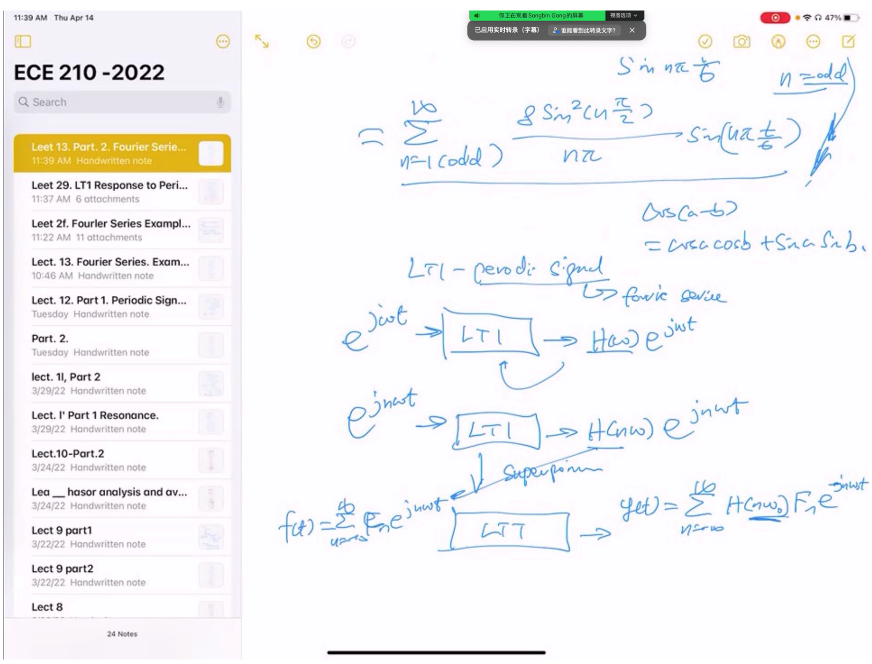
1
0.8
0.6
0.4
0.2
(c)
-0.4
-0.2
0.2
0.4
t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Search
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Lect 9 part2
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Lect 8
24 Notes
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$\sin n\pi \frac{t}{6}$
$n = \text{odd}$
$= \sum_{n=1\,(\text{odd})}^{\infty} \frac{8\sin^2(n\frac{\pi}{2})}{n\pi} \sin(n\pi\frac{t}{6})$
$\cos(a-b)$
$= \cos a \cos b + \sin a \sin b.$
LTI - perodic signal
fouric serice
$e^{j\omega t} \rightarrow \boxed{LTI} \rightarrow H(\omega) e^{j\omega t}$
$e^{jn\omega t} \rightarrow \boxed{LTI} \rightarrow H(n\omega) e^{jn\omega t}$
superposition
$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega t}$ $\boxed{LTI}$ $\rightarrow$ $y(t) = \sum_{n=-\infty}^{\infty} H(n\omega_0) F_n e^{jn\omega t}$

an LTI system $H(\omega)$ converts the Fourier Coefficients of the input (F_n) to the coefficients of the LTI response to the input

$$\boxed{\underline{Y_n} = H(n\omega_0) F_n}$$

Example

a linear system

$$H(\omega) = \frac{2+j\omega}{3+j\omega}$$

$$f(t) = \sum_{n=-\infty}^{\infty} \underbrace{\frac{n}{1+n^2}}_{F_n} e^{jn\underbrace{4}_{\omega_0}t}$$

$$H(4n) = \frac{2+j4n}{3+j4n}$$

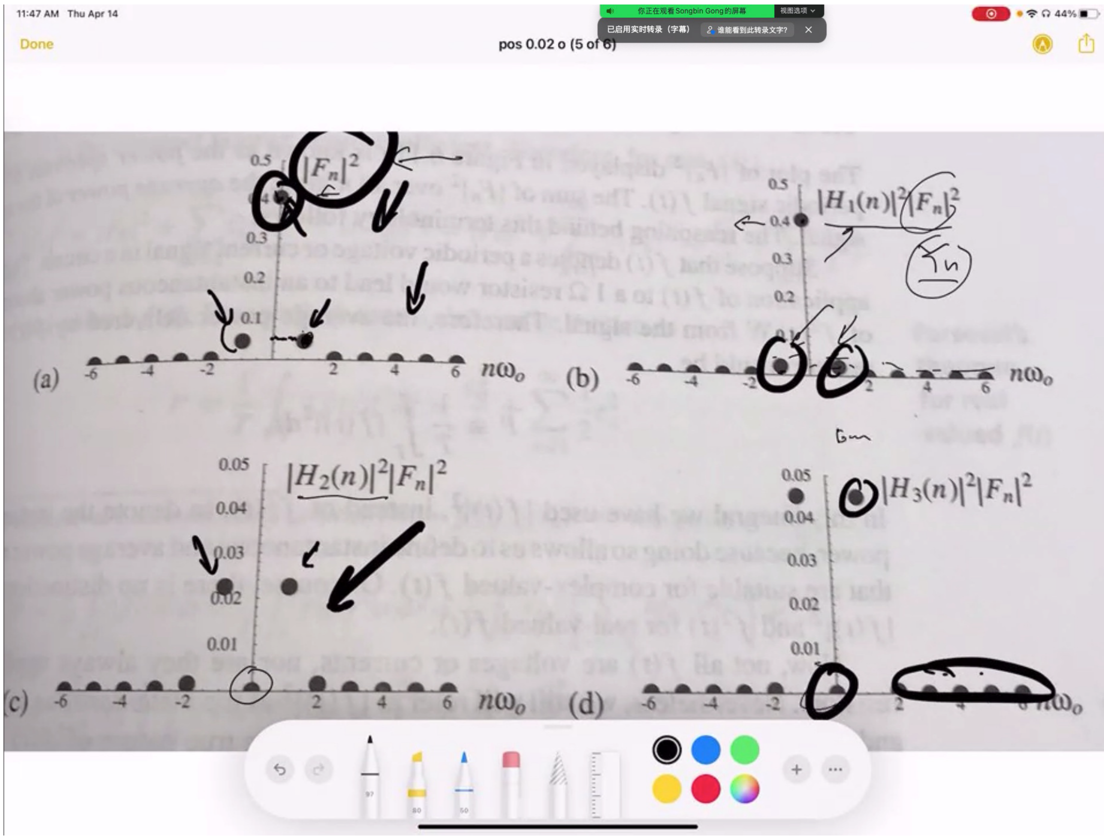
$\underline{Y_n} \to \underline{y(t)}$

$$Y_n = H(n\omega_0) F_n$$

$$= H(4n) F_n$$

$$= \frac{2+j4n}{3+j4n} \cdot \frac{n}{1+n^2}$$

$$y(t) = \sum_{n=-\infty}^{\infty} \frac{2+j4n}{3+j4n} \cdot \frac{n}{1+n^2} \cdot e^{jn4t}$$



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$|F_n|^2$
(a)
$n\omega_o$
$|H_1(n)|^2|F_n|^2$
(b)
$n\omega_o$
$|H_2(n)|^2|F_n|^2$
(c)
$n\omega_o$
$|H_3(n)|^2|F_n|^2$
(d)
$n\omega_o$