Lab 11

In this Lab:

- solving ODEs with python (tool)

failure rate function (practice)

- evolutionary games (meat)

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Integration of ODEs

$$\dot{y} = \frac{dy}{dt}$$

$$\dot{y} = f(y)$$

e.g. $\dot{y} = 2y$, $\dot{y} = \cos y$

2D e.g. $\begin{pmatrix} \dot{y}_0 \\ \dot{y}_1 \end{pmatrix} = \begin{pmatrix} 2y_1 - 0.1y_0 \\ -y_0 \end{pmatrix}$

$$\dot{y}_0 = 2y_1 - 0.1y_0$$

$$\dot{y}_1 = -y_0$$

- closed-form solution is hard to find in general
- numerically solve it with python.

Find the CDF from the failure rate function

$$h(t) = \lim_{\varepsilon \to \infty} \frac{P(t < T \le t+\varepsilon \mid T > t)}{\varepsilon}$$

interpretation: $h(t)$ is the rate of failure prob. at time t, given that the object is still working at time t.

or, given that the machine is still working at time t, the prob. that it will fail in the next ε time unit is $h(t)\varepsilon + o(\varepsilon)$.

In class notes,

$$h(t) = \frac{f_T(t)}{1 - F_T(t)}$$

$\Rightarrow f_T(t) = (1 - F_T(t))\, h(t)$

let $F_T(t) = y$, then $f_T(t) = \dot{y}$

$\dot{y} = (1-y)\, h(t) \quad \Rightarrow$ solve for y.

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Evolutionary games

Cricket game

	small	large
small	5	1
large	8	3

round	#(small)	#(large)	
0	4	4	
1	14	4	Small meets small
2	15	12	Small meets large
3	15	18	large meets large
4			
5			
⋮	⋮		

Suppose at time t, we have n_0 small crickets and n_1 large crickets.

$$P(\text{small meets small}) = \frac{\binom{n_0}{2}}{\binom{n_0+n_1}{2}} = \frac{n_0(n_0-1)}{(n_0+n_1)(n_0+n_1-1)}$$

$$P(\text{small meets large}) = \frac{n_0 n_1}{\binom{n_0+n_1}{2}}$$

$$P(\text{large meets large}) = \frac{\binom{n_1}{2}}{\binom{n_0+n_1}{2}}$$

A more realistic model:

- At each round, all the crickets go around and meet other crickets. Not just one pair meets.
- Exponential growth of population.

We can simulate the evolution of the population of the crickets. Each simulation generates a sample path of the evolution. (random)

But we can also model the evolution by an ODE , which is deterministic

— Compare the ODE prediction with the simulation results.

Dove - Hawk game

	dove	hawk
dove	3	1
hawk	5	0