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Lab 9

In this lab:

- Binary hypothesis testing
- sequential hypothesis testing
- gambler's ruin.

Review: Binary Hypothesis testing, one-shot

		0	1	2	3
H_0 :	$p_0 =$	0.2	0.2	0.4	0.2
H_1 :	$p_1 =$	0.6	0.3	0.2	0.1

Given observation $k \in \{0, 1, 2, 3\}$.

likelihood ratio $\Lambda(k) \triangleq \dfrac{p_1(k)}{p_0(k)}$

likelihood ratio test

$$\begin{cases} \Lambda(k) > 1 \Rightarrow \text{declare } H_1 \\ \Lambda(k) < 1 \Rightarrow \text{declare } H_0 \end{cases}$$

Two kinds of errors: false alarm, miss

$p_{fa} = Pr(\text{declare } H_1 / H_0)$

$p_m = Pr(\text{declare } H_0 / H_1)$

Binary Hypothesis Testing, multiple shots (Observations)

Idea: take multiple observations to increase the accuracy (reduce errors) of the test.

Given observations $k_0, k_1, \ldots, k_{N-1}$

$$\Lambda(k_0, \ldots, k_{N-1}) = \frac{p_1(k_0)}{p_0(k_0)} \cdot \frac{p_1(k_1)}{p_1(k_1)} \cdots \frac{p_1(k_{N-1})}{p_0(k_{N-1})}$$

Still:

$$\begin{cases} \Lambda > 1 \Rightarrow \text{declare } H_1 \\ \Lambda < 1 \Rightarrow \text{declare } H_0 \end{cases}$$

Expect: smaller p_{fa} and p_m.

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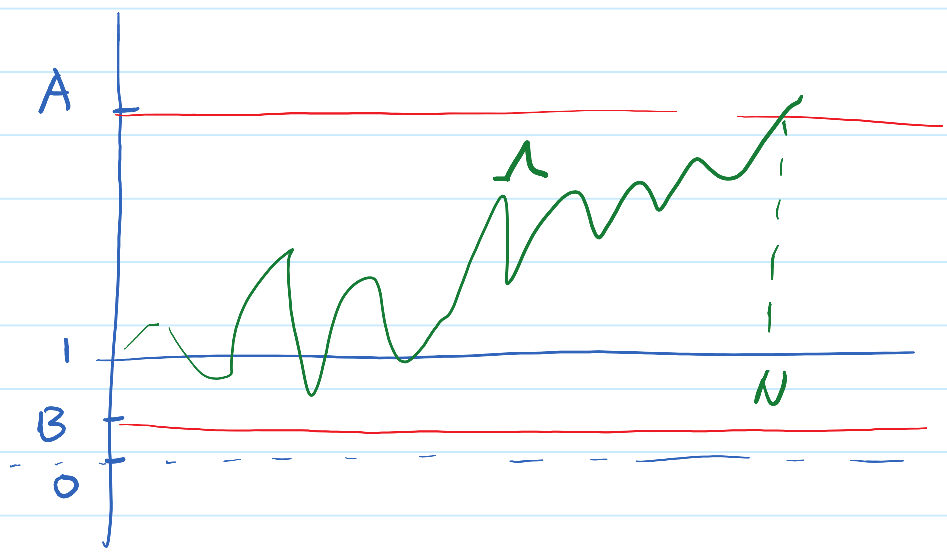
Sequential Hypothesis testing

Idea: N is a variable (different for each particular test) instead of a fixed number.

The test is designed so that $\mathbb{E}N$ is reduced while the targeted error probabilities are still met.



$$\Lambda(k_0, k_1, \cdots, k_i) = \frac{p_1(k_0)}{p_0(k_0)} \cdots \frac{p_1(k_i)}{p_0(k_i)}$$

- continue sampling if $B < \Lambda < A$
- stop when $\Lambda < B$ or $\Lambda > A$

- values of A and B are set based on the desired level of p_{fa} and p_m.

(if $p_{fa} \le \alpha$, $p_m \le \beta$, then $A = \frac{1}{\alpha}$, $B = \beta$

or $A = \frac{1-\beta}{\alpha}$, $B = \frac{\beta}{1-\alpha}$)

- The graph can be drawn on a log scale

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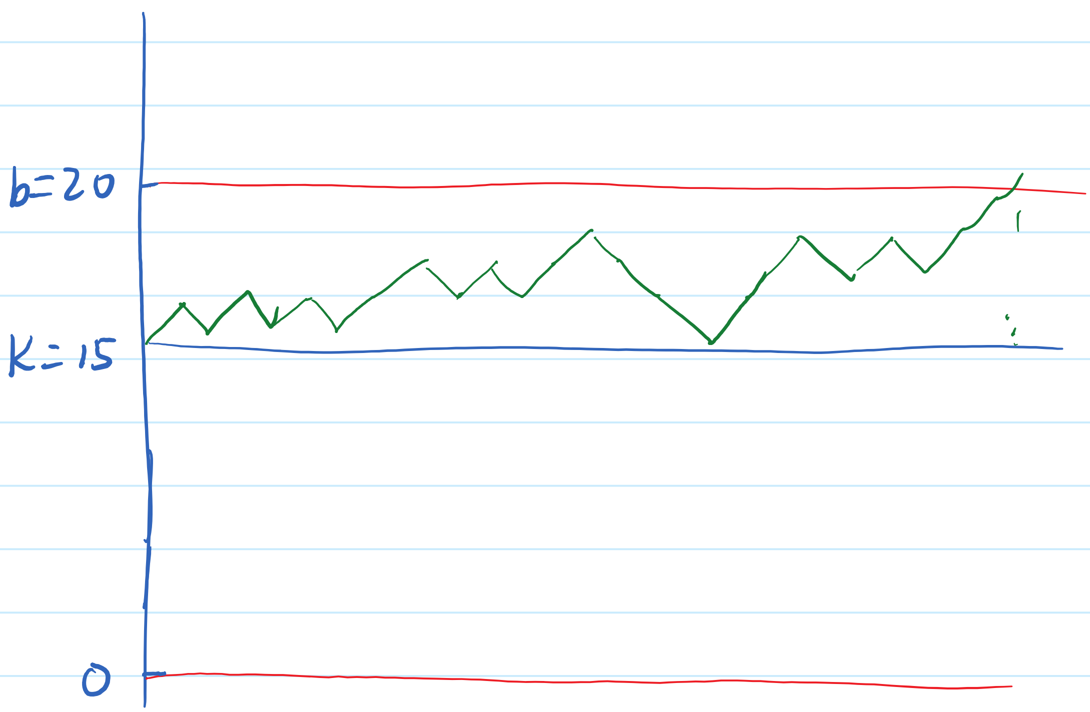
Gambler's ruin

scenario:

- k: initial amount of money
- b: satisfaction level $(b > k)$
- p: probability of winning
- if win, +1 \$; if loss, −1 \$,
- stopping criteria: money $= b$ or $= 0$.

↓ ruined.



Question: find p_{ruin}:

Question:

What is the connection between sequential hypothesis testing and gambler's ruin?