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Lab 4.

Recap: • random variable (Lab 1 & 2)

• random process (Lab 3)

In this Lab:

- standardized random variable
- Parameter estimation
  - ML
  - MAP
- Confidence intervals

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Standardized random variables

Standard means:

- mean = 0
- variance = 1

Give any random variable Y with mean μ and variance σ^2, its standardized random variable X is

$$X = \frac{Y - \mu}{\sigma}$$

Check:
- $\mathbb{E}[X] = 0$
- $Var(X) = 1$

Purpose: sometimes we do not know the distribution of Y, but we know the distribution of X. The $dist^n$ of Y can be derived from the $dist^n$ of X.

Parameter estimation

Motivation:

We have a distribution with an unknown parameter. But we have some samples generated by the distribution. Goal: estimate the unknown parameter from the samples.

Maximum likelihood (ML) estimation

Idea: our estimated value of the parameter is the value that maximizes the probability/likelihood of observing the sample(s).

e.g. $X \sim Poi(\lambda)$ λ is unknown.

Observation : $X = 4$.

prob. / likelihood of observing $X = 4$:

$$\Pr(X=4) = p_{X,\lambda}(4)$$

$$= \frac{\lambda^4}{4!} e^{-\lambda}$$

What is the value of λ that maximizes the above?

$$\frac{d}{d\lambda}\left(\frac{\lambda^4}{4!} e^{-\lambda}\right) = \frac{1}{4!}\left(4\lambda^3 e^{-\lambda} + \lambda^4 e^{-\lambda}(-1)\right)$$

$$= \frac{1}{4!}\lambda^3 e^{-\lambda}(4-\lambda) = 0$$

$$\Rightarrow \quad \lambda^* = 4.$$

$$\hat{\lambda}_{ML} = 4.$$

In general.

$\Pr(X=k)$, $X \sim$ some dist$^n(\theta)$

$$\hat{\theta}_{ML}(k) = \arg\max_{\theta} p_{X,\theta}(k)$$

Maximum A Posteriori Probability (MAP) Estimation

Motivation:

In ML Estimation, we make no assumption on the value of the unknown parameter before we make observations. That is equivalent to assuming that all possible values of the unknown parameter are equally likely.

But what if, even before we observe, we know some values are more likely, while some other values are less likely?

The MAP Estimator "refines" the ML Estimator by incorporating this extra information about the unkown parameter.

$$\hat{\theta}_{MAP}(k) = \arg\max_{\theta} \; p_{X,\theta}(k) \; p_{\Theta}(\theta)$$

$Pr(X=k)$

$X \sim$ some dist$^n(\theta)$

$Pr(\Theta=\theta)$

given the dist^n of Θ

Confidence interval

Motivation:

There is a population parameter that we want to know. But we can only select a sample of the population and get a sample estimate of the true parameter. How good is our estimate?

eg true population parameter p.

$$P\left\{ p \in \left(\hat{p} - \frac{a}{2\sqrt{n}}, \hat{p} + \frac{a}{2\sqrt{n}}\right) \right\} \geq 1 - \frac{1}{a^2}$$

$\left(\hat{p} - \frac{a}{2\sqrt{n}}, \hat{p} + \frac{a}{2\sqrt{n}}\right)$: confidence interval

$1 - \frac{1}{a^2}$: confidence level.

$\hat{p}$ is 47% with ± 2% accuray and 95% confidence means,

before the estimate is taken, we are 95% confident that the true p will lies in $(\hat{p} - 2\%, \hat{p} + 2\%)$

Usualy:

- we choose a to control the confidence level

Then choose n to control the desired accuray (confidence interval width)