

Lab 12

In this Lab:

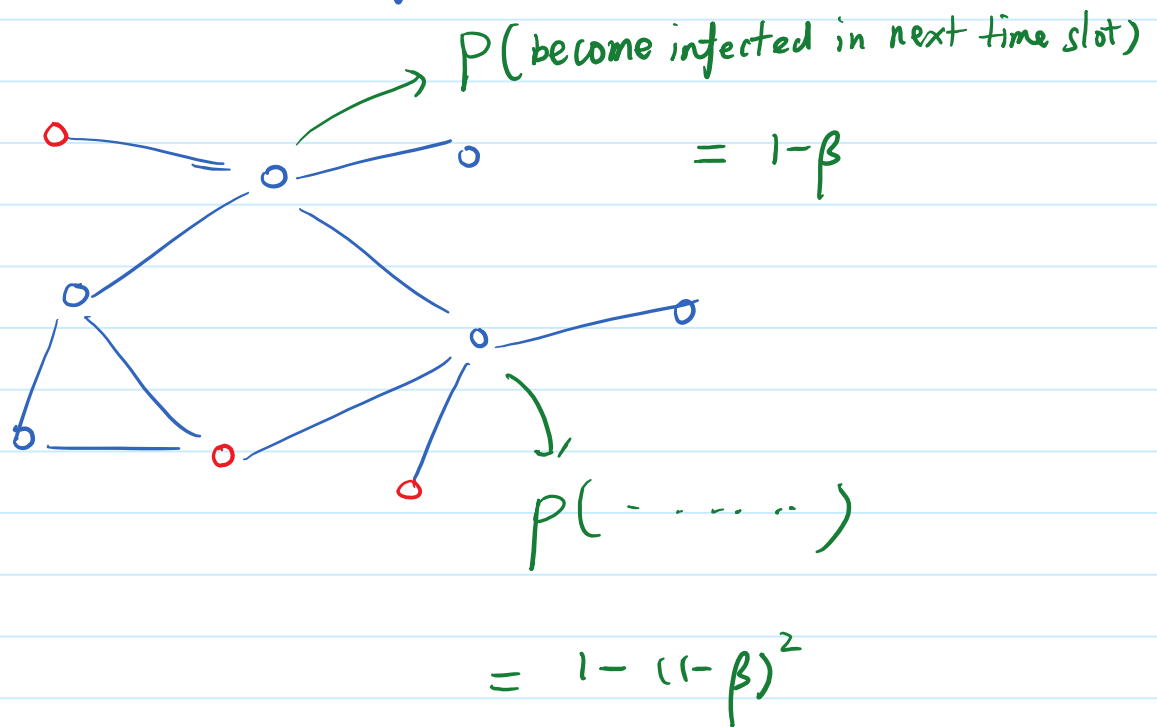
— SIR model for epidemics

SIR model

- A network
- Each node can be in one of three possible states: susceptible, infected, removed
- A susceptible node at time t could become infected at time $t+1$ by an infected neighbor node (at time t) with prob. β
- The prob. of infection from different neighbors are independent
- An infected node becomes removed in the next time slot.

- Once a node is removed, it never gets infected again.

○ : infected
○ : susceptible



Interesting questions:

- How large a fraction will the population gets infected eventually?
- What quickly does the virus spread?

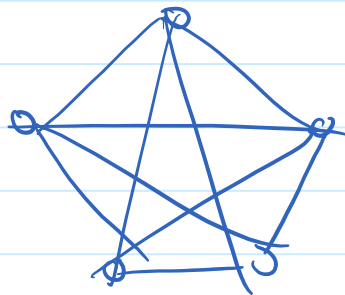
Impact factors:

- # of nodes infected initially.
- β
- The network topology.

The state of the network at time t can be described by a triple (S_t, I_t, R_t)

- $S_t = \#$ of susceptible nodes at time t
- $I_t = \#$ of infected nodes at time t
- $R_t = \#$ of removed nodes at time t

SIR model in a complete graph



(S_t, I_t, R_t) evolves as a Markov process (why?)

$$I_{t+1} \sim \text{Binom}(S_t, 1 - (1 - \beta)^{I_t})$$

$$p = 1 - (1 - \beta)^{I_t} \approx 1 - (1 - \beta I_t) = \beta I_t$$

$$\mathbb{E}[I_{t+1}] \approx \beta I_t S_t$$

Use ODEs to approximate the stochastic evolution :

$$\frac{dS}{dt} = -\beta I S$$

$$\frac{dI}{dt} = \beta I S - I$$

$$\frac{dR}{dt} = I$$

Improved model

include γ : the prob. that an infected node becomes removed in the next time step.

Simulation on a real world network