P1

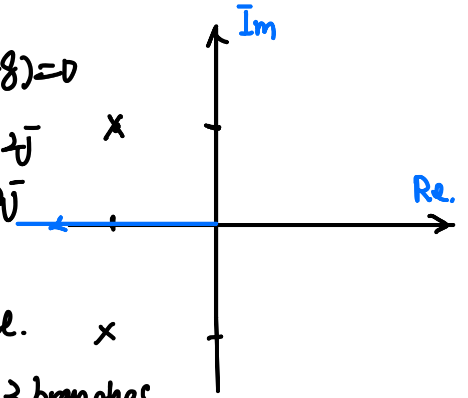
(a) ① poles:

$s(s^2+4s+8)=0$

$\Rightarrow \begin{cases} P_1=-2+2j \\ P_2=-2-2j \\ P_3=0 \end{cases}$

zeros: none.

$\begin{cases} n=3 \\ m=0 \end{cases} \Rightarrow$ 3 branches.



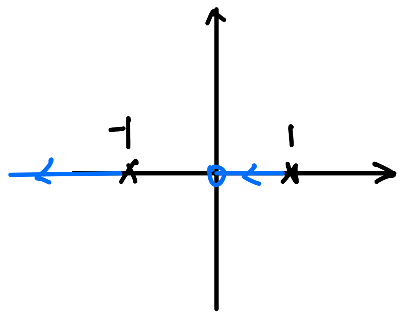
Rule No.2

Branches start from open Loop poles.

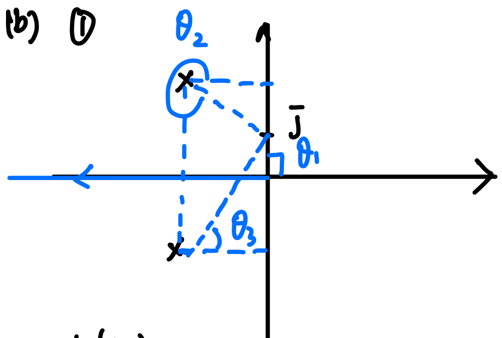
② poles: $\begin{cases} P_1=1 \\ P_2=P_3=P_4=-1 \end{cases}$

zeros: $Z_1=0$

$\begin{cases} n=4 \\ m=1 \end{cases} \Rightarrow$ 4 branches.



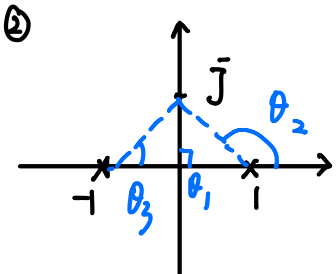
(b) ①



$L(cs) = -\theta_2 - \theta_1 - \theta_3 < 180^\circ$

$\Rightarrow s=j$ not on RL.

②



$\angle L(cs) = -90^\circ - 135^\circ - 3\cdot 45^\circ$

$= -180^\circ$

$\Rightarrow$ on RL.

(c) $\angle S = (2l+1)\times 180^\circ/(n-m) \quad (l \in \mathbb{Z})$

$\Rightarrow l=0 \quad \angle S = 60^\circ$

$l=1 \quad \angle S = 180^\circ \qquad \phi_{dep} = \Sigma\phi_i - \Sigma\phi$

$l=2 \quad \angle S = -60^\circ$

break point:

$KL(s)+1=0$

$\Rightarrow k = \frac{-1}{L(s)}$

$\frac{dk}{ds}=0$

$\Rightarrow 3s^2+8s+8=0$

$br(s) = -\frac{4}{3} + \frac{2}{3}\sqrt{2}j$

$\Rightarrow$ no break point.

Departure angle:

$\Rightarrow$ ① $p=-2+2j$

$\Rightarrow \phi_1 = -45^\circ$

② $p=-2-2j$

$\Rightarrow \phi_2 = 45^\circ$

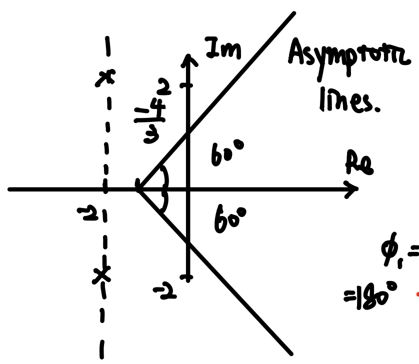
③ $p=0$

$\Rightarrow \phi_3 = 180^\circ$

Departure point

$S = \alpha = \frac{\Sigma p_i - \Sigma z_i}{n-m}$

$= -\frac{4}{3}$



$q\phi_{l,dep} = \Sigma\psi_i - \sum_{i \neq l,dep}\phi_i - 180^\circ$

$\psi_{arrive} = \Sigma\phi_i - \sum_{i \neq arr}\psi_i + 180^\circ$

For $P_1=1$ Departure angle.

$\phi_1 = 180^\circ$

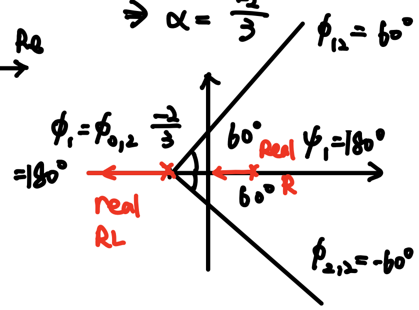
For $P_2=P_3=P_4=-1$ Departure angle

$\Rightarrow 3\phi_{0,2} = 180^\circ, \quad \phi_{0,2} = 180^\circ$

$3\phi_{1,2} = 180^\circ, \quad \phi_{1,2} = 60^\circ$

$3\phi_{2,2} = -180^\circ, \quad \phi_{2,2} = -60^\circ$

$\Rightarrow \alpha = \frac{-2}{3}$

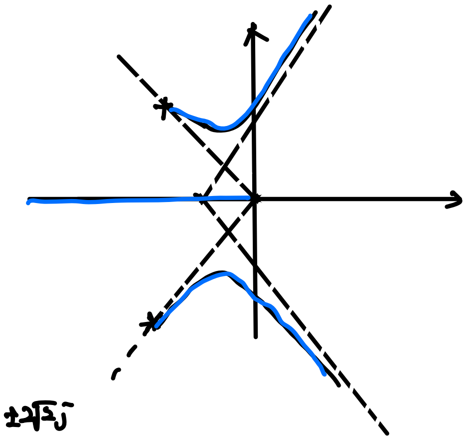


(d) ① $j\omega$ - crossing

$s^3+4s^2+8s+k=0$

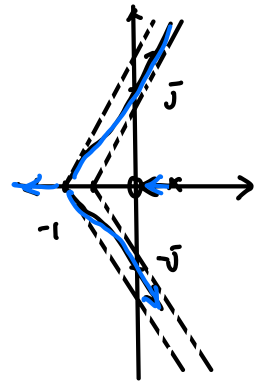
$$\Rightarrow \begin{array}{c|cc} s^3 & 1 & 8 \\ s^2 & 4 & k \\ s^1 & -\frac{1}{4}(k-32) & 0 \\ s^0 & k & \end{array}$$

$\Rightarrow k<32 \Rightarrow \pm j\omega = \pm 2\sqrt{2}j$



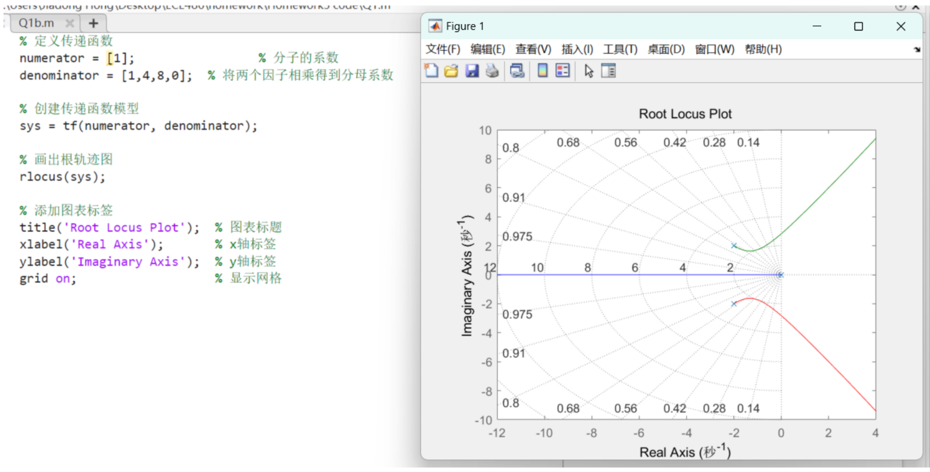
② $s^4+2s^3+(k-2)s-1=0$

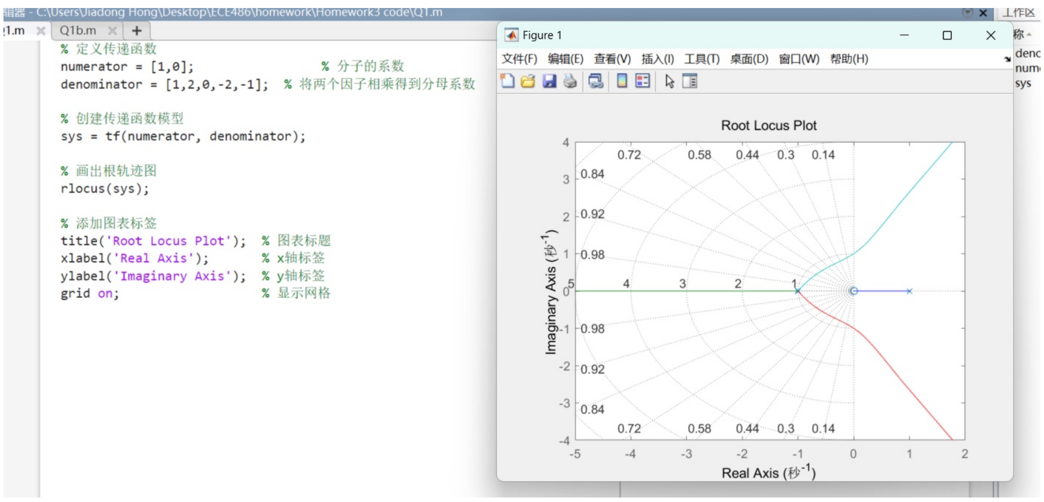
$$\begin{array}{c|ccc} s^4 & 1 & 0 & -1 \\ s^3 & 2 & k-2 & 0 \\ s^2 & 1-\frac{k}{2} & -1 & 0 \\ s^1 & \frac{\frac{k^2}{2}-2k}{\frac{k}{2}-1} & 0 & 0 \\ s^0 & \frac{-(\frac{k}{2}-1)(k^2+4k)}{(-\frac{k^2}{2}+2k)(k-2)} & & \end{array}$$



(e)

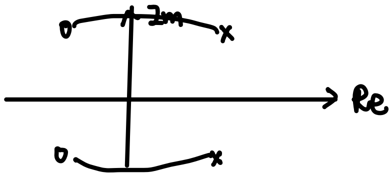
⇒ code:





P2.

(a) $L(s) = \frac{s^2+2s+2}{s^2-2s+2}$ $\Rightarrow$



$\Rightarrow$ poles: $s = 1 \pm j$

Zeros: $s = -1 \pm j$

asymptotes: $\angle s = \frac{360^\circ \ell}{n-m}$ $\Rightarrow$ no branches at $\pm\infty$.

Angles departure from $1+j$

$\angle(s-P_1) = 360^\circ \ell + 0^\circ + 45^\circ - 90^\circ$
$= -45^\circ$

from $1-j$

$\angle(s-P_2) = 45^\circ$

from $-1+j$

$\angle(s-Z_1) = -135^\circ$

from $-1-j$

$\angle(s-Z_2) = 135^\circ$

$1 + KL(s) = 0$

$$\Rightarrow \begin{cases} 1+K>0 \\ 2K-2>0 \\ K+1>0 \\ (1+K)(2K-2)>2(K+1) \end{cases} \Rightarrow \begin{cases} K>-1 \\ K>1 \\ K>2 \end{cases} \Rightarrow K_{critical} = -1$$

(b)