Question 1

1. In class we derived the closed-loop system obtained with dynamic output feedback in $(x, \hat{x})$-coordinates:

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & -BK \\ LC & A - LC - BK \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

and later rewrote it in (x, e)-coordinates. Rewrite the same system in $(\hat{x}, e)$-coordinates.

(6 Points)

$$\dot{\hat{x}} = A\hat{x} - BK\hat{x}$$
$$= A\hat{x} - Ae - BK\hat{x}$$
$$= (A - BK)\hat{x} - Ae.$$

$$\dot{e} = (A - LC)e \Rightarrow \begin{pmatrix} \dot{\hat{x}} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} A-BK & -A \\ 0 & A-LC \end{pmatrix} \begin{pmatrix} \hat{x} \\ e \end{pmatrix}$$

Question 2

Consider the system:

$$\dot{x} = \begin{pmatrix} 0 & -1 & 2/3 \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} u, \qquad y = x_2$$

a) Write down the open-loop characteristic equation. (This involves computing a 3×3 determinant, which you can do either by hand or in MATLAB using a symbolic variable s.) Are all open-loop poles in the LHP?

b) Using the formula given in class, compute the transfer function of this system. (Use the general formula, do *not* take Laplace transform of individual differential equations. Look up the procedure for inverting a matrix by hand, or use the MATLAB command inv.)

c) Find another state-space realization of the same transfer function, in controller canonical form

$$\dot{x} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u, \qquad y = \begin{pmatrix} b_3 & b_2 & b_1 \end{pmatrix} x$$

Hint: you should see that, similarly to the 2×2 case discussed in class, there is a simple relationship between the entries in the above matrices and the coefficients in the transfer function.

(14 Points)

(a) $\dot{x} = \begin{pmatrix} 0 & -1 & \frac{2}{3} \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} u \quad y = x_2$

open loop characteristic Eq:

$$\det(sI - A) = \det\left\{ \begin{pmatrix} s & & \\ & s & \\ & & s \end{pmatrix} - \begin{pmatrix} 0 & -1 & 2/3 \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} \right\}$$

$$= s^3 + s^2 - 1$$

open loop poles $= \{0.75, -0.88 \pm 0.75j\} \Rightarrow$ unstable.

(b) $G(s) = C(sI-A)^{-1}B = (0 \ 1 \ 0)\,(sI-A)^{-1} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$$(sI-A)^{-1} = \begin{pmatrix} s & 1 & -2/3 \\ 1 & s+2 & -1 \\ 0 & 3 & s-1 \end{pmatrix}^{-1} = \frac{1}{s^3+s^2-1} \begin{pmatrix} s^2+s+1 & -s-1 & \frac{1+2s}{3} \\ -s+1 & s^2-s & s-2/3 \\ 3 & -3s & s^2+2s-1 \end{pmatrix}$$

$$\Rightarrow G(s) = \frac{2s^2-1}{s^3+s^2-1} \quad \Rightarrow b_1 = 2 \quad b_2 = 0 \quad b_3 = -1 \quad a_1 = 1$$

(c) $\dot{x} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$

$y = (-1 \ 0 \ 2)\, x$