ECE 486: Control Systems Homework 3

Question 1

If the percentage overshoot of a second order system is to be kept within 4.3%, sketch and indicate the region for the poles of the transfer function on a complex plane to meet this specification. *(3 Points)*

Question 2

a) A system is represented by the following block diagram.

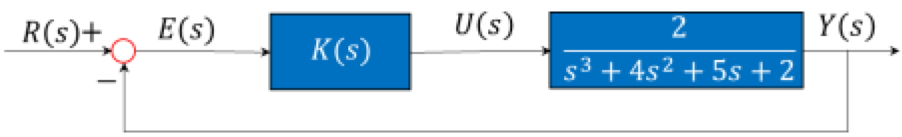


Figure 1

i) Write down the closed-loop transfer function of the block diagram representation. *(1 Points)*

ii) If $\boldsymbol{K(s)} = K$,which is a constant, using Routh-Hurwitz stability criterion, obtain an appropriate range for K. *(2 Points)*

b) An integral term is incorporated such that $\boldsymbol{K(s) = K_P + \frac{K_I}{s}}$.

i) Draw the new block diagram containing the individual blocks of $\boldsymbol{K_P}$, $\boldsymbol{K_I}$ and $\boldsymbol{\frac{1}{s}}$.

(2 Points)

ii) Using Routh-Hurwitz criterion, express the necessary and sufficient conditions for stability in terms of $\boldsymbol{K_I}$ and $\boldsymbol{K_P}$. *(5 Points)*

Question 3

The goal of this exercise is to compare sensitivity of open-loop and closed-loop control with respect to errors in the *control gain*. Consider the DC motor model discussed in class, with no disturbance ($\tau_L = 0$). Let the control gain sensitivity be defined as follows: when the controller gain changes from K to $K + \delta K$ and, as a result, the steady state gain (DC gain) of the overall system changes from T to $T + \delta T$, we define $S_K = \dfrac{\delta T/T}{\delta K/K}$. (The motor gain A remains fixed here.)

a) Compute the sensitivity S_K in the open-loop case, starting from the nominal values $K_{ol} = 1/A$ and $T_{ol} = 1$.

b) Compute the sensitivity S_K for a feedback gain K_{cl}, using the approximate formula $\delta T = \dfrac{dT}{dK}\delta K$ and the fact that the nominal system gain is, as derived in class, $T_{cl} = \dfrac{AK_{cl}}{1 + AK_{cl}}$.

Hint: your final answers in a) and b) should be the same as the ones derived in class for sensitivity to errors in the motor gain A.

(6 Points)

Solution:

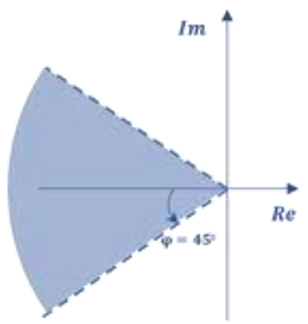
Question 1

Percentage Overshoot M_p<4.3%

$$\exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right) < 0.043$$

$$\frac{\pi\zeta}{\sqrt{1-\zeta^2}} > \ln 0.043$$

$$\boxed{\begin{array}{c} \zeta > 0.707 \\ \boldsymbol{\varphi} < \cos^{-1}(\mathbf{0.707}) \lesssim \mathbf{45^o} \end{array}}$$



Question 2

a)

i)

$$H(s)_{cl} = \frac{KL}{1 + K(L)(1)} = \frac{2K}{s^3 + 4s^2 + 5s + (2K + 2)}$$

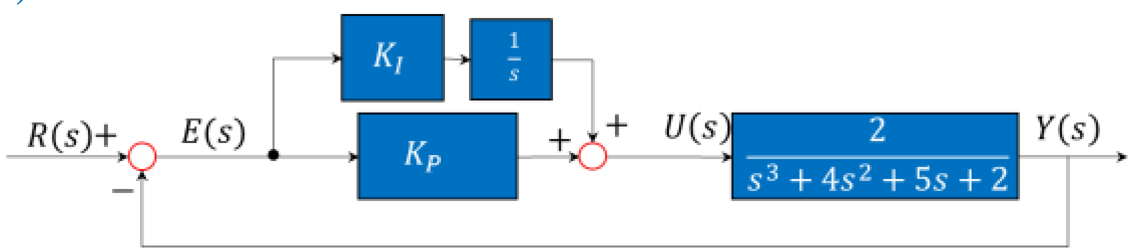
ii)

Routh Array

$$\begin{array}{c|c|c|} s^3 & 1 & 5 \\ s^2 & 4 & 2K+2 \\ s^1 & \dfrac{18-2K}{4} & \\ s^0 & 2K+2 & \end{array}$$

b)

i)



ii)

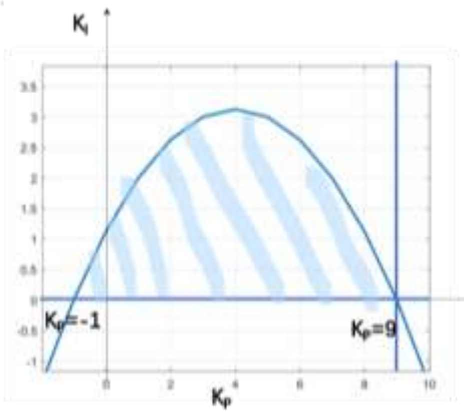
$$H_2(s)_{cl} = \frac{(K_P + \frac{1}{s}K_I)L}{1 + (K_P + \frac{1}{s}K_I)L}$$

Characteristic Equation:

$$1 + \left(K_P + \frac{1}{s}K_I\right)\frac{2}{s^3 + 4s^2 + 5s + 2} = 0$$
$$s^4 + 4s^3 + 5s^2 + (2K_P + 2)s + 2K_I = 0$$

Routh Array

$$\begin{array}{c|c|c|c}
s^3 & 1 & 5 & 2K_I \\
 & 4 & 2K_P + 2 & \\
s^2 & \dfrac{18 - 2K_P}{4} & 2K_I & \\
s^1 & * & & \\
s^0 & 2K_I & &
\end{array}$$



$K_P < 9;$

$K_I > 0$

$(*) \rightarrow \frac{1}{8}(1 + K_P)(9 - K_P) - K_I > 0$

$K_I < \frac{1}{8}(1 + K_P)(9 - K_P)$

Question 3

a)

$$T_{cl} = 1, \quad T_{cl} + \delta T_{cl} = A(K_{cl} + \delta K_{cl}) = A \times \frac{1}{A} + A \times \delta\left(\frac{1}{A}\right) = T_{cl} + A \times \delta\left(\frac{1}{A}\right)$$

$$\Rightarrow \delta T_{cl} = A\delta\left(\frac{1}{A}\right) = A\delta K_{cl}$$

$$\Rightarrow S_k = \frac{\frac{\delta T_{cl}}{T_{cl}}}{\frac{\delta K_{cl}}{K_{cl}}} = \frac{\frac{A\delta K_{cl}}{1}}{\frac{\delta K_{cl}}{1/A}} = 1$$

b)

$$\frac{\delta T_{cl}}{\delta K_{cl}} = \frac{A}{(1 + AK_{cl})^2}$$

$$S_{K_{cl}} = \frac{\delta T_{cl}}{\delta K_{cl}} \times \frac{K_{cl}}{T_{cl}} = \frac{A}{(1 + AK_{cl})^2} \times \frac{K_{cl}}{\frac{AK_{cl}}{1+AK_{cl}}} = \frac{1}{1 + AK_{cl}}$$