Lab Report
Lab #3: Digital Simulation of a Closed-Loop System

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Exercise 1

	Calculated Value in Prelab			Experimental Data		
Controller	***Mp***	t_r**[s]**	t_s**[s]**	***Mp***	t_r**[s]**	t_s**[s]**
#1	49.8%	0.052	0.400	49.8%	0.046	0.393
#2	2.84%	0.180	0.400	2.84%	0.268	0.315
#3	2.84%	0.052	0.149	2.84%	0.067	0.090

Table 1: Comparison of system characteristics between calculated value in prelab and experimental data

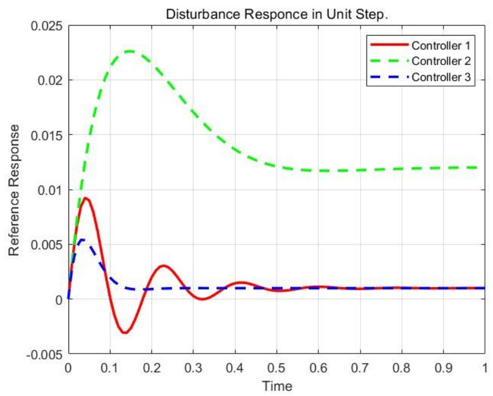

Figure 1: Disturbance Response

We can find that Controller #3 met the specifications.

For Controller #1, the dumping coefficient ζ is about 0.217 which is much less than 0.75. For Controller #2, steady-state tape speed ω_{ss} is 0.12 rad/s which is larger than 0.11 rad/s.

Exercise 2

The system can be represented in s-domain,

$$\Omega(s) = [(\Omega_r(s)K_r - \Omega(s))KH_1(s) + T_d(s)]\,H_2(s)$$

$$\Omega(\Omega_r, T_d) = \frac{\Omega_r K_r H_1 H_2 + T_d H_2}{1 + KH_1 H_2}$$

Then, the steady-state error could be represented as,

$$\begin{aligned}
E_{ss}(\Omega_r, T_d) &= \Omega_r - \frac{\Omega_r K_r H_1 H_2 + T_d H_2}{1 + KH_1 H_2} \\
&= \frac{\Omega_r(1 + KH_1H_2 - K_r KH_1H_2) + T_d H_2}{1 + KH_1H_2} \\
&= \frac{\Omega_r(s^2 + 15s + 60K - 60KK_r + 36) + T_d \cdot 4(s+3)}{s^2 + 15s + 60K + 36}
\end{aligned}$$

So the s-domain steady-state error is,

$$\begin{aligned}
e_{ss} &= \lim_{s\to 0} E_{ss}(\Omega_r, T_d) \\
&= \frac{(60K - 60KK_r + 36)\Omega_r + 12T_d}{60K + 36} = \boxed{\frac{(5K - 5KK_r + 3)\Omega_r + T_d}{5K + 3}}
\end{aligned}$$

Exercise 3

We have the polynomials that,

$$\begin{aligned}
2\zeta\omega_n &= 60KK_d + 15 \\
\omega_n^2 &= 60K + 36
\end{aligned}$$

The result is solved as,

$$\begin{aligned}
\omega_n &= \boxed{2\sqrt{15K + 9}}\,[\text{rad/s}] \\
\zeta &= \boxed{\frac{60KK_d + 15}{4\sqrt{15K + 9}}}
\end{aligned}$$

According to the specifications of Controller 3, we have poles,

$$\begin{aligned}
s_1 &= \frac{-(60KK_d + 15) + \sqrt{(60KK_d + 15)^2 - 4(60K + 36)}}{2} \\
s_2 &= \frac{-(60KK_d + 15) - \sqrt{(60KK_d + 15)^2 - 4(60K + 36)}}{2}
\end{aligned}$$

The negative value shows that there is no RHP poles and we can fine the real and imaginary part of $s1$, $s2$,

$$\Re(s) = -\frac{1}{2}(60KK_d + 15)$$
$$\Im(s) = \pm\frac{1}{2}\sqrt{4(60K + 36) - (60KK_d + 15)^2}$$

And the relationship can be described as,

$$\Re(s)^2 + \Im(s)^2 = 36 - 60K$$

With larger K_d, real part and absolute value of imaginary part will both increase. The figure will then get farther away from the origin of the poles.

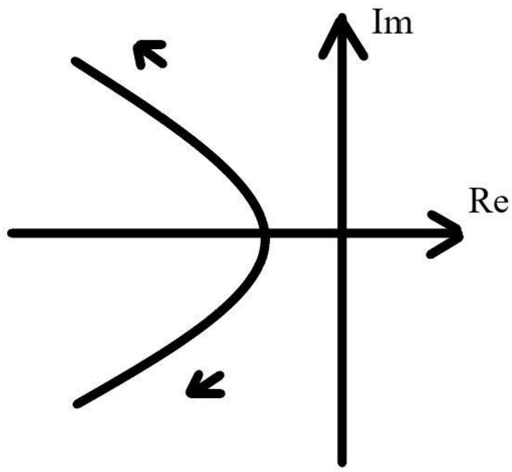

Figure 2: The track of poles