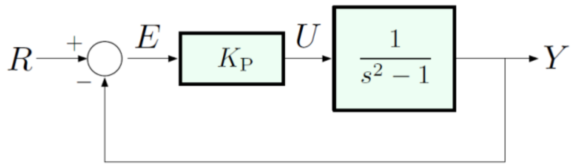
Proportional Feedback. (P-control).



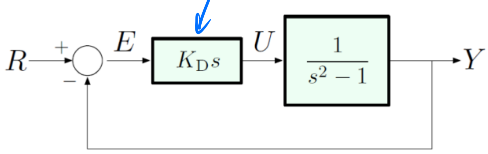
K_P – "proportional gain" (P-gain) $\qquad U = K_P E$

$$\frac{Y}{R} = \frac{\frac{k_P}{s^2-1}}{1+\frac{k_P}{s^2-1}} = \frac{k_P}{s^2-1+k_P}$$ has the form $\frac{a_1}{s^2+a_0 s+a_1}$ with $a_0=0$,

$\Rightarrow$ unstable regardless of k_P.

Derivative Feedback. (D- control).

multiplication by s is not a causal element.



Note: $\dot{e} = \frac{e(t+\delta)-e(t)}{\delta}$.

if $\delta>0$: $e(t+\delta)$ is in the future of $e(t)$.

1. Lack of Causality.

① $\begin{cases} \dot{x} = Ax+Bu. \\ y = Cx. \end{cases} \Leftrightarrow \begin{cases} sX = AX+BU. \\ Y = CX. \end{cases} \Rightarrow \begin{cases} (s-A)X = BU. \\ \frac{Y}{U} = \frac{C-B}{s-A} = \frac{q(s)}{p(s)} \end{cases}$

deg (q) < deg (p) : strictly proper transfer function

② $\begin{cases} \dot{x} = Ax+Bu. \\ y = Cx+Du. \end{cases} \Leftrightarrow \begin{cases} sX = AX+BU. \\ Y = CX+DU. \end{cases} \Rightarrow$ $(s-A)X = BU.$

$$\frac{Y}{U} = \frac{CB}{s-A} + D = \frac{CB+D(s-A)}{s-A}.$$

deg (q) = deg (p) : proper transfer function.

Causal system have proper transfer functions.

In D control:

if $u = k\dot{e}$ i.e. $U = ksE \Rightarrow \frac{U}{E} = ks = \frac{q}{p}$. $\leftarrow$ not implementable.

deg(q) > deg (p). : improper system. (lack of causality).

1. Use Approximation: $\frac{k_D as}{a+s} \to k_D s$ as $a \to \infty$.
2. Use finite differences: $\dot{e}(t) \approx \frac{e(t+\delta)-e(t)}{\delta}$ (have to tolerate delays).

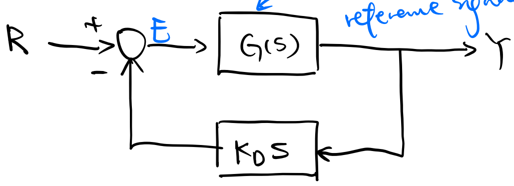
signal for t is generated at $t+\delta$.

2. Noise Amplification. (differentiators amplify noise).

rapid changes in the reference.

To avoid noise:

Smooth out the reference signal.



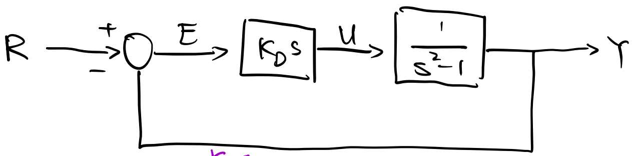
original: $\frac{Y}{R} = \frac{K_D s G(s)}{1+K_D s G(s)}$

now: $\frac{Y}{R} = \frac{G(s)}{1+K_D s G(s)}$

different zeros.

Poles: $1+K_D s G(s) = 0$.

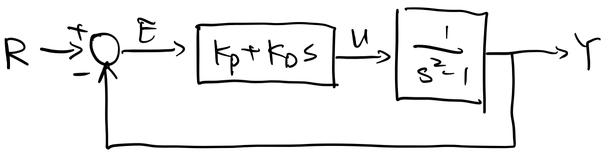
A typical derivative feedback:



$$H_{cl} = \frac{Y}{R} = \frac{\frac{K_D s}{s^2-1}}{1+\frac{K_D s}{s^2-1}} = \frac{K_D s}{s^2+K_D s-1}.$$

negative => unstable.

Proportional-Derivative (PD) Control.



$$\frac{Y}{R} = \frac{\frac{K_p+K_D s}{s^2-1}}{1+\frac{K_p+K_D s}{s^2-1}} = \frac{K_p+K_D s}{s^2+K_D s+(K_p-1)}.$$

stable: $\begin{cases} K_D > 0 \\ K_p > 1. \end{cases}$

Pole: different K_D, K_p => arbitrary pole placement

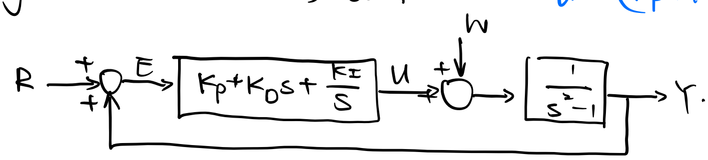
Zero: $K_D s + K_p = 0$ => $s = -\frac{K_p}{K_D}$ at LHP.

DC gain: $= \frac{Y}{R}\Big|_{s=0} = \frac{K_p}{K_p-1} \neq 1$. => can't have perfect tracking of const reference.

Proportional-Integral-Derivative (PID) Control.

$u = \left(K_p + K_D s + \frac{K_I}{s}\right)E$.

(class 3-term controller).



Goal: tracking a const reference, r.

rejecting a const disturbance w.

$Y = \frac{1}{s^2-1}(U+W)$, $\quad U = \left(K_P + K_D s + \frac{K_I}{s}\right)(R-Y)$.

$$\Rightarrow Y = \frac{K_P + K_D s + \frac{K_I}{s}}{s^2-1}(R-Y) + \frac{1}{s^2-1}W.$$

simplify: $\boxed{Y = \frac{K_D s^2 + K_P s + K_I}{s^3 + K_D s^2 + (K_P-1)s + K_I} R + \frac{s}{s^3 + K_D s^2 + (K_P-1)s + K_I} W}$

stability: ① $K_D, K_I > 0$; $K_P > 1$. (necessary).

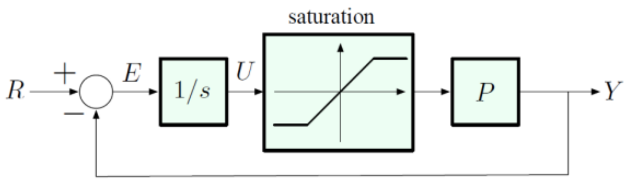
$K_D(K_P-1) > K_I$. (R-H Criterion).

② arbitrary poles. ← choosing K_D, K_P, K_I.

DC gain $(R \to Y) = 1$. → perfect tracking.

$(W \to Y) = 0$ → complete attenuation of const disturbance.

Wind-up Phenomenon.



actuator saturates → error still integrated.

→ large overshoot.

Summary.

P-gain simplest to implement, but not always sufficient for stabilization

D-gain helps achieve stability, improves time response (more control over pole locations)

- arbitrary pole placement only valid for 2nd-order response; in general, we still have control over two *dominant poles*
- cannot be implemented directly, so need approximate implementation; D-gain also amplifies noise

I-gain essential for perfect steady-state tracking of constant reference and rejection of constant disturbance

- but $1/s$ is not a stable element by itself, so one must be careful: it can destabilize the system if the feedback loop is broken (integrator wind-up)