Lab Report

Lab #3: Digital Simulation of a Closed-Loop System

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Exercise 1

	Calculated Value in Prelab			**Experimental Data**		
Controller	***Mp***	**t_r[s]**	**t_s[s]**	***Mp***	**t_r[s]**	**t_s[s]**
#1	49.8%	0.052	0.400	49.8%	0.046	0.393
#2	2.84%	0.180	0.400	2.84%	0.268	0.315
#3	2.84%	0.052	0.097	3.20%	0.067	0.087

Table 1: Comparison of system characteristics between calculated value in prelab and experimental data

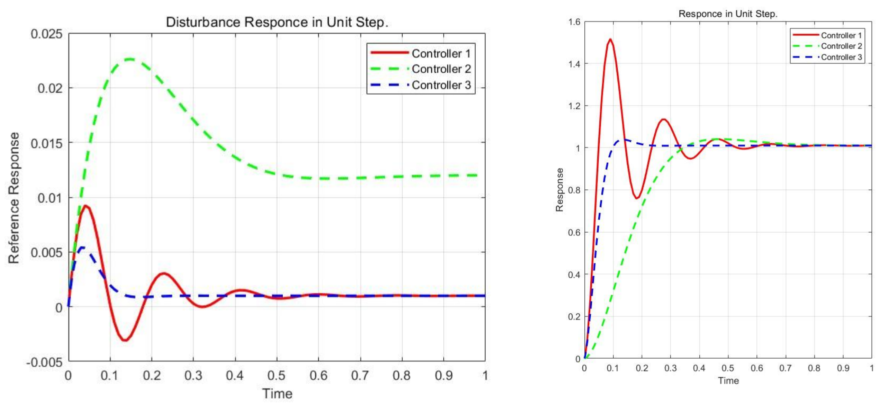

Figure 1: Disturbance Response

We can find that Controller #3 met the specifications. It responds quickly to step inputs and is not affected by disturbances.

For Controller #1, the dumping coefficient ζ is about 0.217 which is much less than 0.75. For Controller #2, steady-state tape speed ω_{ss} is 0.12 rad/s which is larger than 0.11 rad/s.

Exercise 2

The system can be represented in s-domain,

$$\Omega(s) = [(\Omega_r(s)K_r - \Omega(s))KH_1(s) + T_d(s)]\, H_2(s)$$

$$\Omega(\Omega_r, T_d) = \frac{\Omega_r K_r H_1 H_2 + T_d H_2}{1 + KH_1 H_2}$$

Then, the steady-state error could be represented as,

$$\begin{aligned} E_{ss}(\Omega_r, T_d) &= \Omega_r - \frac{\Omega_r K_r H_1 H_2 + T_d H_2}{1 + KH_1 H_2} \\ &= \frac{\Omega_r(1 + KH_1 H_2 - K_r K H_1 H_2) + T_d H_2}{1 + KH_1 H_2} \\ &= \frac{\Omega_r(s^2 + 15s + 60K - 60KK_r + 36) + T_d \cdot 4(s+3)}{s^2 + 15s + 60K + 36} \end{aligned}$$

So the s-domain steady-state error is,

$$\begin{aligned} e_{ss} &= \lim_{s\to 0} E_{ss}(\Omega_r, T_d) \\ &= \frac{(60K - 60KK_r + 36)\Omega_r + 12T_d}{60K + 36} = \boxed{\frac{(5K - 5KK_r + 3)\Omega_r + T_d}{5K + 3}} \end{aligned}$$

Exercise 3

We have the polynomials that,

$$\begin{aligned} 2\zeta\omega_n &= 60KK_d + 15 \\ \omega_n^2 &= 60K + 36 \end{aligned}$$

The result is solved as,

$$\begin{aligned} \omega_n &= \boxed{2\sqrt{15K + 9}}\,[\text{rad/s}] \\ \zeta &= \boxed{\frac{60KK_d + 15}{4\sqrt{15K + 9}}} \end{aligned}$$

According to the specifications of Controller 3, we have poles,

$$\begin{aligned} s_1 &= \frac{-(60KK_d + 15) + \sqrt{(60KK_d + 15)^2 - 4(60K + 36)}}{2} \\ s_2 &= \frac{-(60KK_d + 15) - \sqrt{(60KK_d + 15)^2 - 4(60K + 36)}}{2} \end{aligned}$$

The negative value shows that there is no RHP poles and we can fine the real and imaginary part of $s1$, $s2$,

$$\Re(s) = -\frac{1}{2}(60KK_d + 15)$$
$$\Im(s) = \pm\frac{1}{2}\sqrt{4(60K + 36) - (60KK_d + 15)^2}$$

And the relationship can be described as,

$$\Re(s)^2 + \Im(s)^2 = 36 - 60K$$

With larger K_d, real part and absolute value of imaginary part will both increase. The dominant pole will converge to zero. And the figure will then get farther away from the origin of the poles. This can be seen in the figure

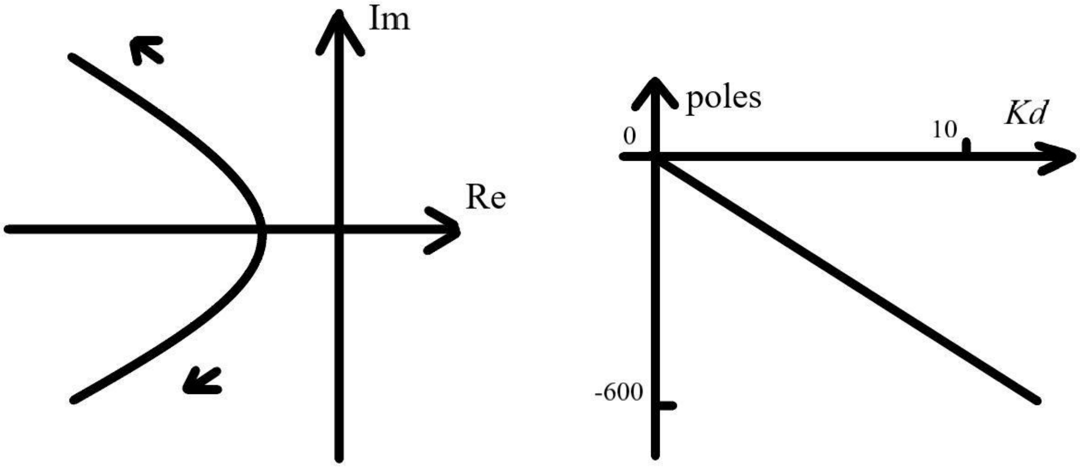

Figure 2: The track of poles