ECE 486: Control Systems Homework 2

Question 1 (6 points)

Given a transfer function $H(s) = \frac{s+3}{(s^2+3s+2)}$

a) Draw a block with integrators and gains describing the system.
b) Find $\mathcal{L}^{-1}(H(s))$.
c) Sketch the time response

Question 2 (4 points)

Simplify the block diagram shown in Figure 1, using block diagram reduction technique to a single block with input *U(s)* and *Y(s)*. Write down the transfer function relating the input *U(s)* with *Y(s)* in terms of H_1, H_2, G_1, G_2.

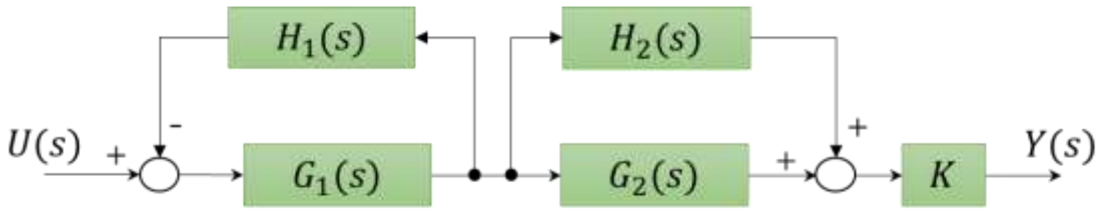


Figure 1

Question 3 (7 points)

Final-Value Theorem (FVT) states that the steady state value $y_{ss} = \lim_{s\to 0}\{sY(s)\}$, if all poles of *sY(s)* lie in the open left half-plane i.e. Re*(s)* < 0 (strictly stable).

a) Given $Y(s) = \frac{2s+1}{s(s^2+4s+5)}$, find the steady state value y_{ss} using FVT.

b) Obtain an expression for $y(t)$ using the following transformations (# 20 and 21 of the Laplace table in the course textbook):

$\boxed{L\{e^{-at}\sin bt\} = \frac{b}{(s+a)^2+b^2}}$ and $\boxed{L\{1-e^{-at}(\cos bt + \frac{a}{b}\sin bt)\} = \frac{a^2+b^2}{s[(s+a)^2+b^2]}}$

c) Inspect the limit $\lim_{t\to\infty} y(t)$ to validate your results in (a)

Question 4 (3 points)

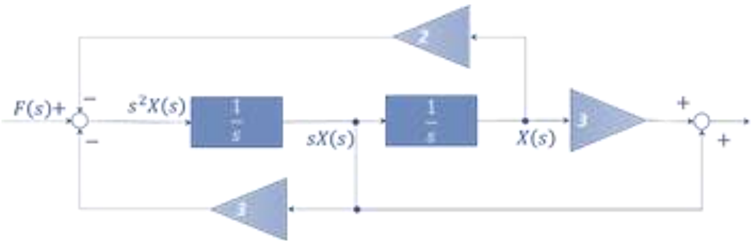
For a transfer function

$$X(s) = \frac{8s}{4s^2+1}$$

Show that FVT is not applicable and briefly explain why.

Solution

1 a)



b)

$$H(s)=\frac{s+3}{(s+1)(s+2)}=\boxed{\frac{A}{s+1}}+\boxed{\frac{B}{s+2}}=\frac{A(s+2)+B(s+1)}{(s+1)(s+2)}=\frac{(A+B)s+2A+B}{(s+1)(s+2)}$$

The denominators of the original and the final fractions are identical (by design), so we force their respective numerators to be identical, that is,

$$s+3\equiv(A+B)s+2A+B$$

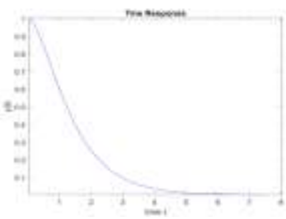
However, this identity holds only if the coefficients of like powers of s on both sides are the same. So, we have

$$\begin{array}{lll} \text{Coefficient of } s: & 1=A+B & \overset{\text{Solve}}{\Rightarrow} & A=2 \\ \text{Constant term}: & 3=2A+B & & B=-1 \end{array}$$

Insert the two residues into the partial fractions, and perform term-by-term inverse Laplace transformation to obtain

$$H(s)=\frac{2}{s+1}-\frac{1}{s+2}\quad\overset{\mathcal{L}^{-1}}{\Rightarrow}\quad h(t)=2\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}-\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}=2e^{-t}-e^{-2t}$$

c)



2)

$$\frac{Y(s)}{U(s)}=\frac{KG_1(s)(H_2(s)+G_2(s))}{1+G_1(s)H_1(s)}$$

3a)

The poles of *X(s)* are at 0 and $-2\pm j$. The complex conjugate pair lies in the left half plane , and 0 is a simple pole (at the origin), all allowed by FVT.

Therefore,

$$x_{ss}=\lim_{s\to 0}(sX(s))=\lim_{s\to 0}\left(\frac{2s+1}{s^2+4s+5}\right)=\frac{1}{5}$$

b)

$$Y(s) = \frac{2s+1}{s(s^2+4s+5)} = \frac{2}{s^2+4s+5} + \frac{1}{s(s^2+4s+5)}$$
$$= \frac{2(1)}{((s+2)^2+1)} + \frac{1}{5}\frac{1^2+2^2}{s((s+2)^2+1)}$$
$$y(t) = \mathcal{L}^{-1}(Y(s)) = 2e^{-2t}\sin t + \frac{1}{5} + \frac{1}{5}e^{-2t}(\cos t + 2\sin t)$$
$$= \frac{1}{5} + \frac{8}{5}e^{-2t}\sin t - \frac{1}{5}e^{-2t}\cos t$$

c)

$$\lim_{s\to\infty} y(t) = \lim_{s\to 0}\left(\frac{1}{5} + \frac{8}{5}e^{-2t}\sin t - \frac{1}{5}e^{-2t}\cos t\right) = \frac{1}{5}$$

4)

Let $X(s) = \frac{2s}{s^2+\frac{1}{4}}$, so that its poles are at $\pm\frac{1}{2}j$, on the imaginary axis, not permitted by the FVT. Therefore, FVT is not applicable and should not be applied. If it were to be applied, it would yield

$$x_{ss} = \lim_{s\to 0}\{sX(s)\} = \lim_{s\to 0}\left\{\frac{2s^2}{s^2+\frac{1}{4}}\right\} = 0$$

which is obviously false. To explain this, we first find $x(t) = \mathcal{L}^{-1}\{X(s)\} = 2\cos(\frac{1}{2}t)$. Then, it is clear that $\lim_{t\to\infty} x(t)$ does not exist, since $x(t)$ is oscillatory and there is no steady-state value.