ECE 486: Control Systems Homework 7

Question 1

Determine (from the observability matrix) whether or not the following systems are observable.

Consider the system given by $\dot{\boldsymbol{x}} = \begin{pmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{0} & \mathbf{3} \end{pmatrix} \boldsymbol{x}$ and output $\boldsymbol{y} = \boldsymbol{x_2}$

i) $\dot{\boldsymbol{x}} = \begin{pmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{0} & \mathbf{3} \end{pmatrix} \boldsymbol{x}$; $\boldsymbol{y} = \boldsymbol{x_2}$ (4 Points)

ii) $\dot{\boldsymbol{x}} = \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{-5} \\ \mathbf{3} & \mathbf{3} & \mathbf{-2} \end{pmatrix} \boldsymbol{x}$; $\boldsymbol{y} = (\mathbf{1} \quad \mathbf{1} \quad \mathbf{1})\boldsymbol{x}$ (6 Points)

Question 2

Consider the control system:

$$\dot{\boldsymbol{x}} = \begin{pmatrix} \mathbf{0} & \mathbf{-1} & \mathbf{2/3} \\ \mathbf{-1} & \mathbf{-2} & \mathbf{1} \\ \mathbf{0} & \mathbf{-3} & \mathbf{1} \end{pmatrix} \boldsymbol{x} + \begin{pmatrix} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{pmatrix} \boldsymbol{u}; \quad \boldsymbol{y} = \boldsymbol{x_2}$$

Design an observer with observer poles (A-LC) placed at -20 and -20±2j (10 Points)

Solution

Question 1

$$\dot{x} = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}, \quad y = x_2 = \begin{pmatrix} 0 & 1 \end{pmatrix} x$$

$$\mathcal{O}(C, A) = \begin{pmatrix} C \\ CA \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 3 \end{pmatrix}, \quad \det(\mathcal{O}(C, A)) = 0$$

$$\implies \text{system unobservable}$$

$$\dot{x} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & -5 \\ 3 & 3 & -2 \end{pmatrix} x, \quad y = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} x$$

$$\mathcal{O}(C, A) = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 6 & -6 \\ -14 & -4 & -14 \end{pmatrix}, \quad \det(\mathcal{O}(C, A)) = 100$$

$$\implies \text{system observable}$$

Solution

Question 2

$$\dot{x} = \begin{pmatrix} 0 & -1 & 2/3 \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} x, \quad y = x_2 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} x$$

$$\mathcal{O}(C, A) = \begin{pmatrix} 0 & 1 & 0 \\ -1 & -2 & 1 \\ 2 & 2 & -5/3 \end{pmatrix}, \quad \det \mathcal{O}(C, A) \neq 0 \implies \text{system observable}$$

Auxiliary system ("dual" system)

$$F = A^T, \; G = C^T, \; \dot{\bar{x}} = F\bar{x} + Gu, \; \mathcal{C}(F, G) = \mathcal{O}^T(C, A)$$

The desired characteristic equation for

$$F - GK = (s+20)(s+20+2j)(s+20-2j) = s^3 + 60s^2 + 1204s + 8080$$

CCF for above system:

$$\dot{x}_1 = \bar{F}x_1 + \bar{G}u$$

where

$$\bar{F} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix}, \quad \bar{G} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$T_{F \to \bar{F}} = \mathcal{C}(\bar{F}, \bar{G})\mathcal{C}^{-1}(F, G) = \begin{pmatrix} 3 & 0 & 3 \\ 2 & 0 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$\bar{F} - \bar{G}\bar{K} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 - \bar{k}_1 & -\bar{k}_2 & -1 - \bar{k}_3 \end{pmatrix} \overset{(*)}{\Longrightarrow} \begin{cases} \bar{k}_1 = 8081 \\ \bar{k}_2 = 1204 \\ \bar{k}_3 = 59 \end{cases}$$

$$\bar{K} = \begin{pmatrix} 8081 & 1204 & 59 \end{pmatrix},$$
$$K = \bar{K}T = \begin{pmatrix} 26769 & 59 & 28032 \end{pmatrix}$$

and

$$\mathcal{L} = K^T = \begin{pmatrix} 26769 \\ 59 \\ 28032 \end{pmatrix}$$