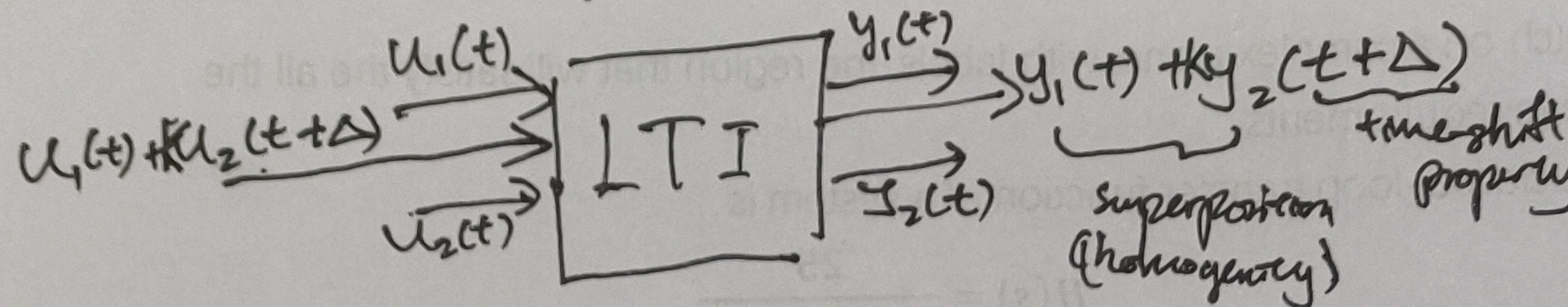
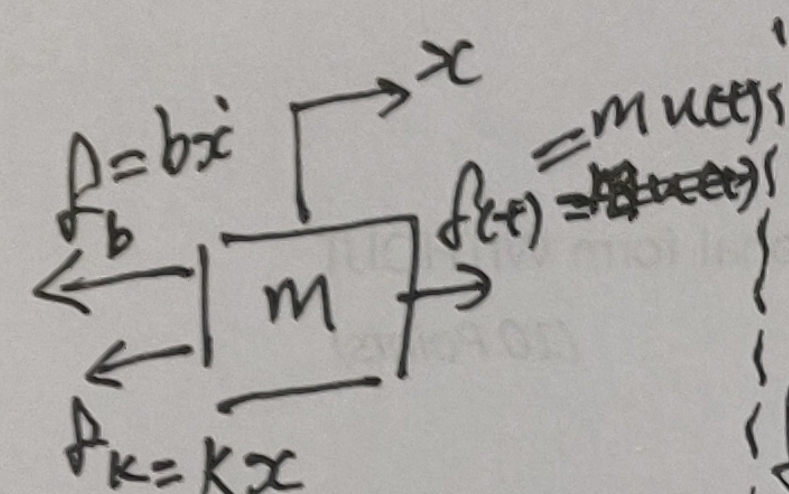
Q1

a) Linear-time-invariant systems satisfy superposition principle and time-shift property (1) (1)



b)



(3)

$ma = f - f_b - f_k$

$m\ddot{x} = f(t) - b\dot{x} - kx$

$f(t) = m\ddot{x} + b\dot{x} + kx$

$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{f(t)}{m}$

$\ddot{x} + \frac{15}{3}\dot{x} + \frac{12}{3}x = \frac{f(t)}{3}$

$\ddot{x}(t) + 5\dot{x}(t) + 4x(t) = u(t)$ (2)

c) Let $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x \\ \dot{x} \end{pmatrix} \Rightarrow$ $\dot{x}_1 = x_2$, $\dot{x}_2 = -5x_2 - 4x_1 + u(t)$

State-eq^n: $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$ (3)

Output eq^n: $y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ (1)

d) Laplace transform $\ddot{x}(t) + 5\dot{x}(t) + 4x(t) = u(t)$ (2)

0-IC $\Rightarrow$ $s^2X(s) + 5sX(s) + 4X(s) = U(s)$

$H(s)$: $\frac{X(s)}{U(s)} = \frac{1}{s^2+5s+4}$ (2)

e) poles at $s^2 + 5s + 4 = 0$

$(s+4)(s+1) = 0$

$\therefore p_{1,2} = -4$ or -1 (2)

f) since $H(s) = \frac{1}{(s+4)(s+1)} = \frac{1}{s+4} \cdot \frac{1}{s+1}$

$\Rightarrow$ (i) 4 , (ii) 1 (2)

g) $\left[U\left(\frac{1}{s+1}\right) - U\left(\frac{1}{s+4}\right)\right] \times \frac{1}{3} = X$

$\frac{X}{U} = \frac{1}{3}\left(\frac{1}{s+1} - \frac{1}{s+4}\right) = \frac{1}{3}\left(\frac{s+4-s-1}{(s+1)(s+4)}\right)$

$= \frac{1}{(s+4)(s+1)} = \frac{1}{s^2+5s+4}$ (verified) (2)

h) $x(t) = ?$ ie $x(t) = 1(t) * h(t)$

$X(s) = \frac{1}{s}H(s)$ (1)

$= \frac{1}{s}\frac{1}{s^2+5s+4}$

$= \frac{1}{s}\frac{1}{3}\left(\frac{1}{s+1} - \frac{1}{s+4}\right)$ (1)

$= \frac{1}{3}\left[\frac{1}{(s+0)(s+1)} - \frac{1}{(s+0)(s+4)}\right]$

$\mathcal{L}^{-1}\{X(s)\}$

$x(t) = \frac{1}{3}\left[\frac{1}{1-0}\left(e^{0} - e^{-t}\right) - \frac{1}{4-0}\left(e^{0} - e^{-4t}\right)\right]$

$x(t) = \frac{1}{3}\left[\left(1 - e^{-t}\right) - \frac{1}{4}\left(1 - e^{-4t}\right)\right]$ (1)

$= \frac{1}{3}\left[\frac{3}{4} - e^{-t} + \frac{1}{4}e^{-4t}\right]$

$x(t) = \frac{1}{4} - \frac{1}{3}e^{-t} + \frac{1}{12}e^{-4t}$ (1)

Q2

a) Advantage: Reject disturbance, Track reference
Disadvantage: Introduce errors, may amplify noise, costy to implement ②

b) To reduce steady-state error ①

c) D-term or derivative term can be noise prone and adversely affect noise suppression ①

d) i) $H_p = \frac{1}{s^2+3s+2}$ ①, $H_c = K + \frac{K_I}{s}$, $H_{CL} = \frac{H_p H_c}{1+H_p H_c} = \frac{H_c}{\frac{1}{H_p}+H_c}$

$$\therefore H_{CL} = \frac{K+\frac{K_I}{s}}{(s^2+3s+2)+(K+\frac{K_I}{s})} = \frac{Ks+K_I}{s^3+3s^2+2s+Ks+K_I}$$

$$= \frac{Ks+K_I}{s^3+3s^2+(K+2)s+K_I} \quad ③$$

ii) Characteristic eq^n: $s^3+3s^2+(K+2)s+K_I = 0$

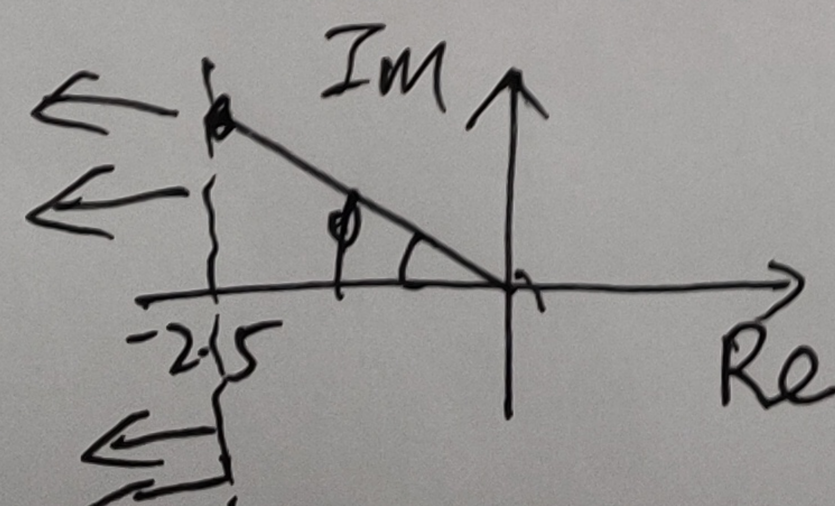
$$\begin{array}{c|cc|l} s^3 & 1 & K+2 & ① \\ s^2 & 3 & K_I & ① \\ s^1 & \frac{-[K_I-3(K+2)]}{3} & 0 & ② \\ s^0 & K_I & & ② \end{array} \quad ⑥$$

$K_I > 0$
$-K_I+K+2>0$
$K+2 > K_I$
optional

2 e) (i) $\varphi = 45^\circ \; (=\cos^{-1}\frac{1}{\sqrt{2}})$

(ii) $t_{s5\%} = \frac{3}{\sigma} \Rightarrow \sigma > \frac{3}{1\cdot 2}$

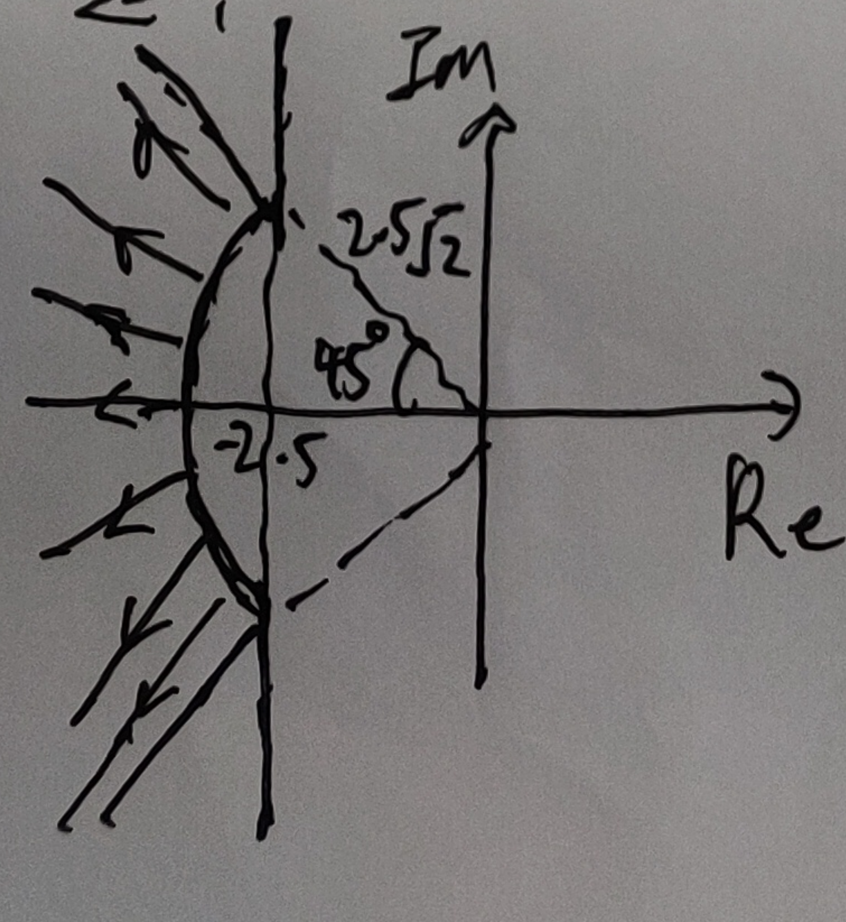
$\therefore \sigma = 2.5$



(iii) $\omega_n \zeta = \sigma$
$\omega_n > 2.5\sqrt{2}$

$$P_{1,2} = \frac{-6 \pm \sqrt{36-100}}{2} = -3 \pm \sqrt{16(-1)}$$
$$P_{1,2} = -3 \pm 4j$$

iv)



(v) $H(s) = \frac{25}{s^2+6s+25} = \frac{25}{(s+3+4j)(s+3-4j)}$

P_1 and P_2 are out of the satisfactory region.