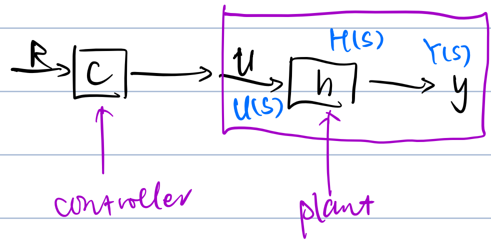
$y(t) = y_{tr}(t) + y_{ss}(t)$.



e.g. $u = m\ddot{x} + b\dot{x} + k$ ← mass-spring system.

Libraries

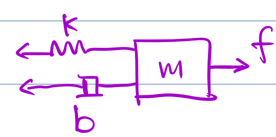
Integrator: $\dot{y} \rightarrow \boxed{\frac{1}{s}} \rightarrow y$ ($\dot{y} \leftrightarrow sY$, $y \leftrightarrow Y$)

★ non-zero initial condition:

$$\mathcal{L}\{\ddot{y}\} = s^2Y(s) + y(0)sY(s) + \dot{y}(0)$$

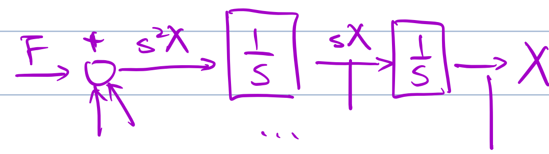
Summing Junction: $u_1 \xrightarrow{+} \bigcirc \rightarrow u = u_1 - u_2$, with u_2 entering at $-$.

Constant gain: $u \rightarrow \boxed{a} \rightarrow y = au$.

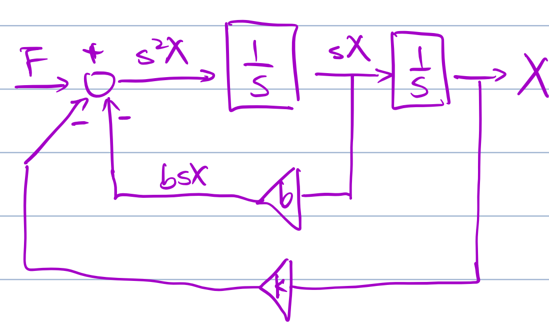


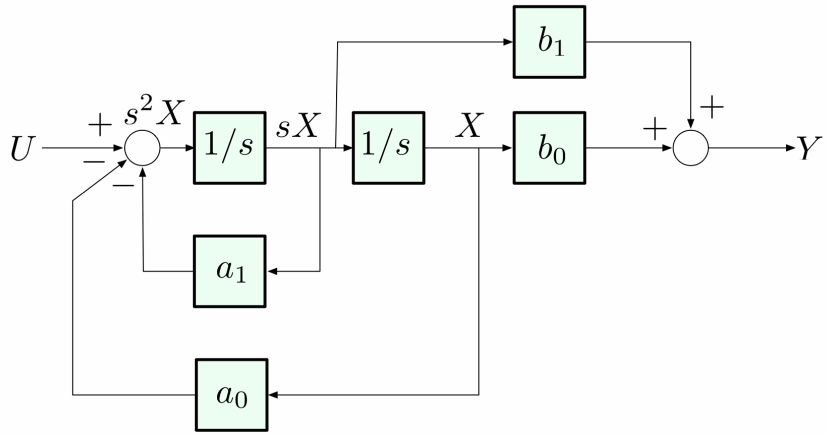
$f = m\ddot{x} + b\dot{x} + k$.

$\leftrightarrow \quad F = s^2X + sbX + kX$.



★ $s^2X = F - bsX - kX$.





State-space model:

$$s^2X = U - a_1 sX - a_0 X \qquad Y = b_1 sX + b_0 X$$
$$\ddot{x} = -a_1\dot{x} - a_0 x + u \qquad y = b_1\dot{x} + b_0 x$$

$x_1 \equiv x, \quad x_2 \equiv \dot{x}$:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -a_0 & -a_1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

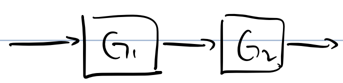
$$y = (b_0 \ \ b_1)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

State-variable form.

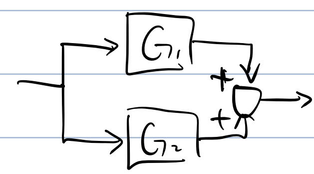
Let $x_i = x^{(i-1)}$ $(x_1 = x,\ x_2 = \dot{x},\ \cdots)$.

state $s = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix}$.

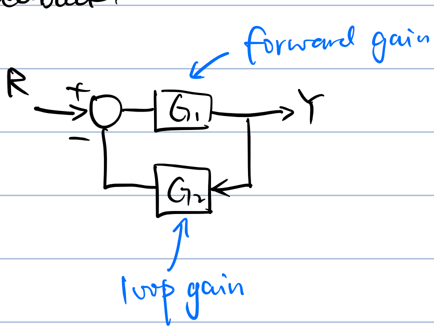
Series



Parallel



Negative feedback.



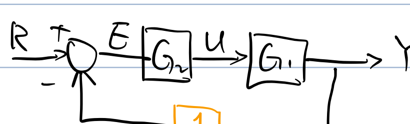
$U = R - W$

$$Y = G_1 \cdot U = \cdots = \frac{G_1}{1 + G_1 G_2} R$$

Gain: $\dfrac{\text{fwd gain}}{1 + \text{loop gain}}$.

i.e. everything.

Unity Feedback



$$\text{gain} = \frac{G_1 G_2}{1 + G_1 G_2}.$$