Q1

a) Conservation of mass (assume incompressible flow)

$$\Delta mass_{tank} = Q_{in} - Q_{out}$$

$$A\dot{y}(t) = q_{in} - \frac{g}{R}y(t)$$

$$A\dot{y}(t) + \frac{g}{R}y(t) = q_{in} \quad \text{(shown)}$$

(3)

b) $y(t) = \{q_{in} * h(t)\}(t)$ where $h(t)$ is the impulse response

(1)

c) ~~when~~ If $q_{in} = 0$, $\overset{1}{\cancel{A}}\dot{y}(t) + \frac{g}{R}y(t) = 0$

Free response, $Ce^{-at} = y(t)$

$y(0) = 5$ $\quad y(t=0) = C = 5$

$$\therefore y(t) = 5e^{-\frac{g}{R}t}$$

Alternative

using Laplace transf^m,

$$A\,(sY(s) - y(0)) + \frac{g}{R}Y(s) = 0$$

$$Y(s) = \frac{5}{(As + \frac{g}{R})}$$

inverse Laplace

$$y(t) = 5e^{-\frac{g}{AR}t} = 5e^{-\frac{g}{R}t}$$

(3)

d) at $t = \tau$ $\quad y(\tau) = \frac{5}{e} = 5e^{-1}$

$$\Rightarrow \quad -\frac{gt}{AR} = -1 \quad \therefore t = \frac{AR}{g}$$

(2)

e) Assuming zero-initial condition

$$AsY + \frac{g}{R}Y = Q_{in}$$

$$\therefore H(s) = \frac{Y}{Q_{in}} = \frac{1}{As + \frac{g}{R}}$$

(3)

**Question 2 (6 Points)**

A dynamic system with no input is governed by the equation:

$\ddot{x} = 0.5(x^2 - 1)\dot{x} + 1.5x$

a. Choosing state variables $(x_1\ x_2) = (x\ \dot{x})$, write down its non-linear state-space model for the system. *(2 Points)*

b. Derive the linearized state-space model at the equilibrium point. *(4 Points)*

a) $\dot{x}_1 = x_2$ (Already linear)

$\dot{x}_2 = \ddot{x} = 0.5(\cancel{x_1^2}\, x_1^2 - 1)x_2 + 1.5x_1$ (need to linearize)

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1.5 & 0.5(x^2-1)\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}$$ 2

b) $$\left.\frac{\partial f_2}{\partial x_1}\right|_0 = \frac{\partial}{\partial x_1}\left[0.5(x_1^2 x_2 - x_2) + 1.5x_1\right]\Big|_0$$

$$= 0.5(2x_2x_1 - 0) + 1.5\,\big|_0$$ 2

$$= x_2x_1 + 1.5\,\big|_{x_1=0} = 1.5$$

$$\left.\frac{\partial f_2}{\partial x_2}\right|_0 = 0.5x_2 = 0$$ 2

Q3. a) $Y(s) = \left(\frac{1}{s+4}\right)\left(\frac{1}{s+1}\right) 4 \cdot U(s)$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{4}{(s+4)(s+1)} = \frac{4}{s^2+5s+4}$$

where $\omega_n = 2$ , $\zeta = \frac{5}{2(2)} = 1.25$

b) for $2^{nd}$ order system both neccesary AND ~~and~~ sufficient conditions are met iff coefficients are ~~All~~ positive

Full Mark

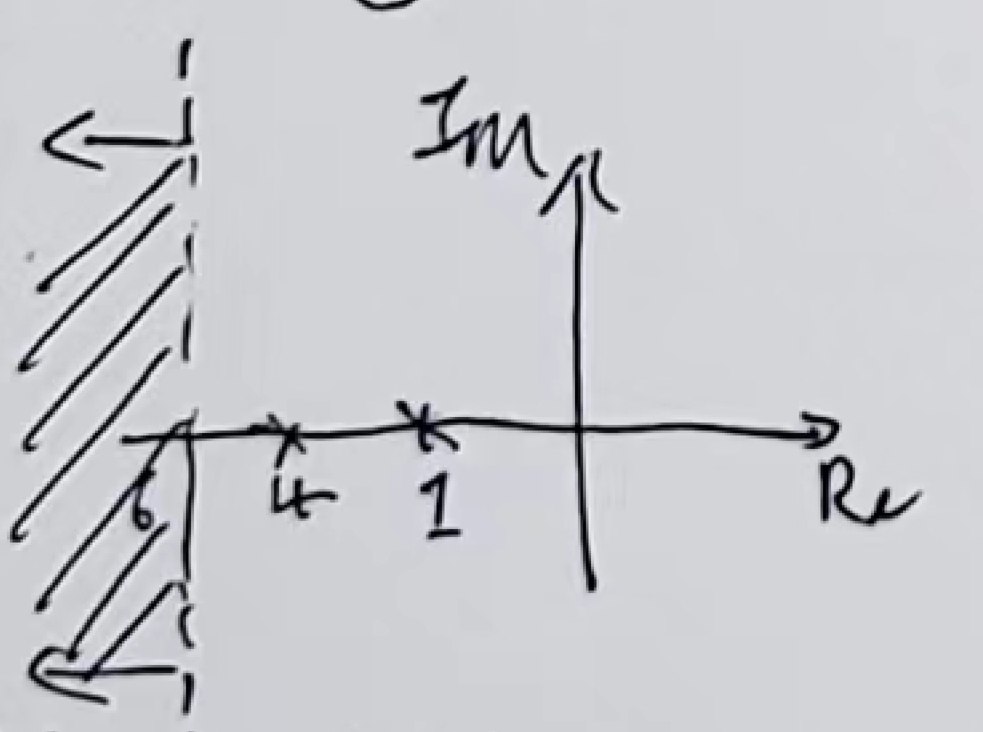
for,

| | | |
|---|---|---|
| $s^2$ : | 1 | 4 |
| $s^1$ : | 5 | 0 |
| $s^0$ : | $\frac{-1}{5}\begin{vmatrix} & 4 \\ 5 & 0\end{vmatrix} = 5$ | |

c) (i) $t_s \approx \frac{3}{\zeta\omega_n} = \cancel{\left(\frac{3}{2.5} \ldots 0.5\right)}$ ~~meet~~

$$t_s \approx \frac{3}{\sigma} < 0.5$$

$$\sigma > 6$$




did Not meet the condition.

**Question 4 (10 Points)**

a) State the key purpose of incorporating integral control in a PID controller *(1 Points)*

b) The sensor of a control system is subject to a lot of noise from the working environment which term in the PID control is likely to worsen the effect of noise. Explain. *(1 Points)*

a) Reduce steady-state error ①, b) Derivitive - term (D--term) ①

c) A control system is implemented as represented by the block diagram.

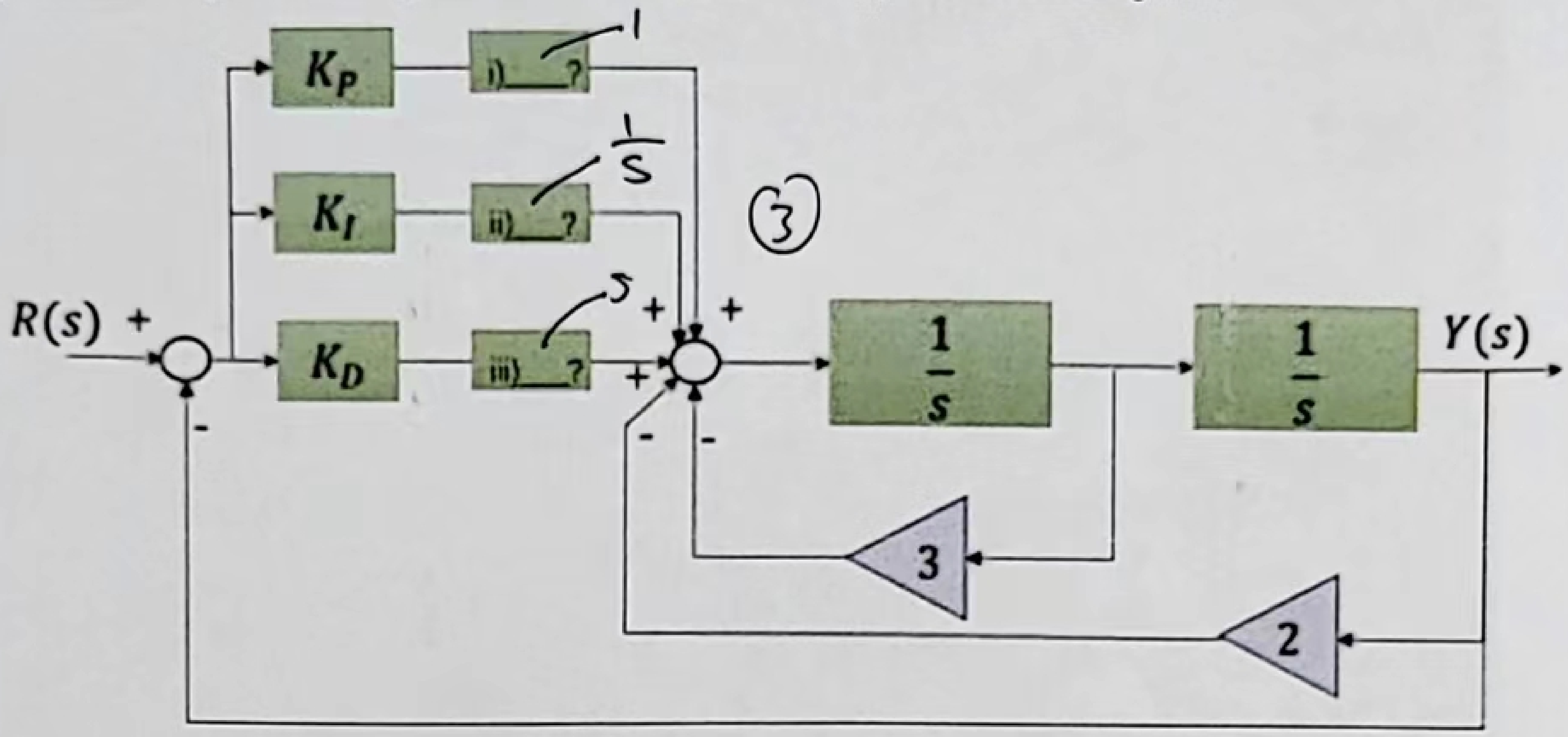



**Figure 3**

Fill in the blocks (i)-(iii) *(3 Points)*

iv) Write down the closed-loop transfer function. *(5 Points)*

③

$$K = K_p + \frac{K_I}{s} + K_D s, \quad G = \frac{1}{s^2+3s+2}$$

$$H_{cL} = \frac{KG}{1+KG} = \frac{K_I + K_p s + K_D s^2}{s(s+2)(s+1)} = \frac{K_D s^2 + K_p s + K_I}{s^3+3s^2+2s}$$

②