Linear system: / State-space model.

$x_1 \equiv x,\ x_2 \equiv \dot{x},\ x_3 \equiv \ddot{x},\ \cdots$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} ? \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

$$\begin{cases} \dot{x} = \underset{\text{dynamics}}{A}x + \underset{\text{control}}{B}\underset{\text{input}}{u} \\ \underset{\text{measured output}}{y} = \underset{\text{sensor}}{C}\underset{\text{state}}{x} + Du. \end{cases}$$

if $u_1 \to y_1$, $u_2 \to y_2$ $\Rightarrow u_1 + u_2 = y_1 + y_2$.

Laplace: $F(s) = \int_{\mathbb{R}} f(\tau) e^{-s\tau} d\tau.$

$s = e^{j\omega} = \cos\omega + j\sin\omega.$

Impulse response. $\int_{-a}^{a} \delta(t)\,dt = 1.$

$\int_{\mathbb{R}} \delta(t-\tau) f(t)\,dt = f(\tau)$

Causal: output not affected by future time.

Characteristic Polynomial: $P(\lambda) = \det(A - \lambda I)$, λ = eigen value for $P(\lambda) = 0$.

Vector division: $\vec{z}_1 = |z_1| e^{j\phi_1}$, $\vec{z}_2 = |z_2| e^{j\phi_2}$

$\Rightarrow \frac{z_1}{z_2} = \frac{|z_1|}{|z_2|} e^{j(\phi_1 - \phi_2)}.$

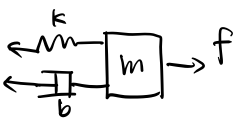
Transient response: ① vanishes at $t \to \infty$. ② $\mathcal{L}$ gives transient rsps.

DC gain: $= y(t \to \infty)$. where $u(t) = 1(t)$.

FVT: (all) Poles of $sY(s)$ lies in OLHP $\Leftrightarrow$ $Re(s) < 0$: $y(t \to \infty) = \lim_{s \to 0} sY(s)$.

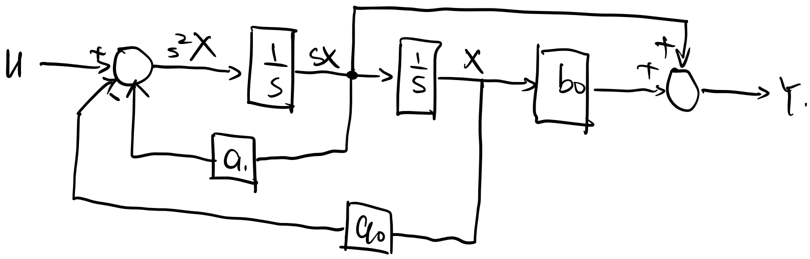
or satisfies R-H Criterion.

Mass-Damping:



$f = m\ddot{x} + b\dot{x} + k.$ $\Leftrightarrow F = s^2X + sbX + kX.$

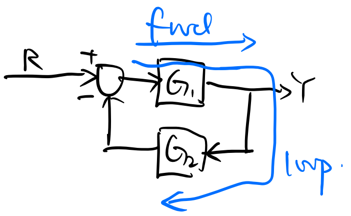
2nd-Order Damping System.



$$\begin{cases} s^2X = U - a_1 sX - a_0 X \\ Y = b_1 sX + b_0 X. \end{cases}$$

i.e. $$\begin{cases} \frac{U}{X} = s^2 + a_1 s + a_0 \\ \frac{Y}{X} = b_1 X + b_0 \end{cases} \Rightarrow \frac{Y}{U} = \frac{Y}{X} \cdot \frac{X}{U} = \frac{b_1 X + b_0}{s^2 + a_1 s + a_0}.$$

For Feedback System:



$$\text{Gain} = \frac{\text{fwd gain}}{1 + \text{loop gain}} = \frac{G_1}{1 + G_1 G_2}$$

rise: $10\% \sim 90\%$.

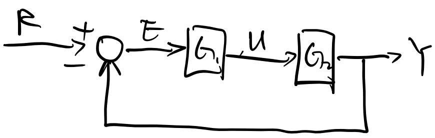
$t_r \doteq \frac{1.8}{\omega_n}$, exact at $\zeta = 0.5$

$t_p = \frac{\pi}{\omega_d}$

$M_p = e^{-\pi\zeta / \sqrt{1-\zeta^2}}$

$t_s \doteq \frac{3}{\sigma}.$

Unity Feedback



$$\text{gain} = \frac{G_1 G_2}{1 + G_1 G_2}$$

A damping system: $H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ poles $s = -\omega_n(\zeta \pm \sqrt{\zeta^2 - 1})$,

$= \frac{\omega_n^2}{(s+\sigma)^2 + \omega_d^2}$. $y(t) = 1 - e^{-\sigma t}\left[\cos(\omega_d t) + \frac{\sigma}{\omega_d}\sin(\omega_d t)\right]$

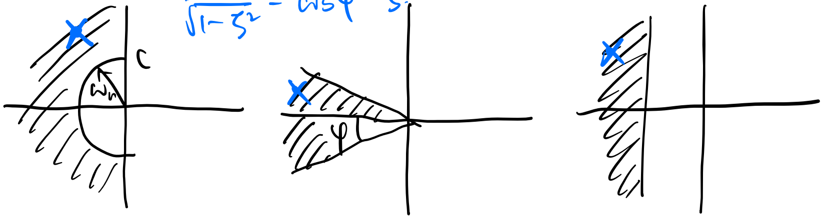
ζ: damping coef. / damping ratio

ω_n: natural freq.

ω_d: damped freq.

Specifications.

$\frac{\zeta}{\sqrt{1-\zeta^2}} = \cos\varphi \doteq \xi$.



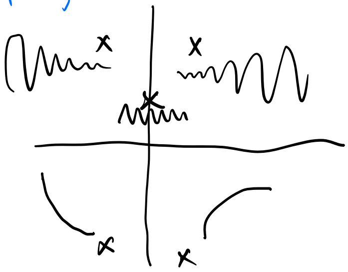
Rise. Mp ts

× : pole (σ, ω_d).

$\begin{cases} \sigma = \zeta\omega_n \\ \omega_d = \omega_n \cdot \sqrt{1-\zeta^2} \end{cases}$

$\omega_n = \sigma^2 + \omega_d^2$.

$\zeta^2 \omega_n^2 \quad 1 - \zeta^2$.



Effects of zeros.

LHP: ① increased overshoot. ② little influence on settling time

③ $a \to \infty$ yields less significant effect.

RHP: ① delays the response.

② creates an undershoot. (when a is small enough.)

Extra Poles: ☆ extra LHP poles, → real parts 5× that of dominant LHP poles.

$$y = \Sigma C_k e^{-\lambda_k t}. \qquad Re(\text{pole}) = \lambda_k.$$

Pole locations: ① RHS → unstable. ② LHS → stable.

③ Im axis $\begin{cases} \text{impulse / step : unstable if } \omega \neq 0. \\ \omega = 0 : \begin{cases} \text{impulse}: Y = \frac{1}{s}, \ y = 1(t) \to \text{stable} \\ \text{step}: Y = \frac{1}{s^2}, \ y = t. \to \text{unit ramp.} \end{cases} \end{cases}$

Stability. $\frac{\cdots}{a_0 s^n + a_1 s^{n-1} + \cdots + a_n}$

necessary: $a_0, a_1, \cdots, a_n > 0$.

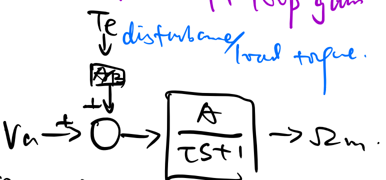
R-H. For lower order

2nd: $s^2 + a_1 s + a_2$ is stable iff $a_1, a_2 > 0$

3rd: $s^3 + a_1 s^2 + a_2 s + a_3$ is stable iff $\begin{cases} a_1, a_2, a_3 > 0 \\ a_1 a_2 > a_3. \end{cases}$

$H = \frac{Y}{R} = \frac{\text{Fwd gain}}{1 + \text{loop gain}} = \frac{\cdots}{\text{polynomial}}$ ← check if it's stable.

DC Motor:



$$\Omega_m = \frac{A}{\tau s+1} V_a + \frac{B}{\tau s+1} T_e.$$

Disturbance Rejection. see coefficient of disturbance.

open-loop motor: $\omega_m(\infty) = \omega_{ref} + B T_e$ poor rejection.

closed : $\omega_m(\infty) = \frac{AK_{cl}}{1+AK_{cl}} \omega_{ref} + \frac{B}{1+AK_{cl}} T_e$.

As K_{cl} large, this item → 0. → good rejection.

Sensitivity: $S = \frac{\partial T/T}{\partial A/A}$. $S_{ol} = 1$, $S_{cl} = \frac{1}{1+AK_{cl}}$.

PID.

P: $\frac{Y}{R} = \frac{K_p}{s^2 - 1 + K_p}$, unstable. D: $\frac{Y}{R} = \frac{K_D s}{s^2 + K_D s - 1}$. unstable.

PD: $\frac{Y}{R} = \frac{K_p + K_D s}{s^2 + K_D s + (K_p - 1)}$. stable when $\begin{cases} K_D > 0 \\ K_p > 1. \end{cases}$ no perfect tracking, DC gain $= \frac{K_p}{K_p - 1} \neq 1$.

PID: $\frac{K_D s^2 + K_p s + K_I}{s^3 + K_D s^2 + (K_p - 1)s + K_I} R + \frac{s}{s^3 + K_D s^2 + (K_p - 1)s + K_I} W$.

↖ DC gain = 1. perfect tracking. ↖ DC gain = 0. perfect rejection.