Frequency Response

$$u(t) = A\cos(\omega t) \rightarrow y(t) = \frac{A}{2}\left(H(j\omega)e^{j\omega t} + H(-j\omega)e^{-j\omega t}\right)$$

$$\begin{cases} H(j\omega) = M(\omega)e^{j\varphi(\omega)} \\ H(-j\omega) = M(\omega)e^{-j\varphi(\omega)} \end{cases}$$

$$\Rightarrow y(t) = \frac{A}{2}M(\omega)\left[e^{j(\omega t+\varphi(\omega))} + e^{-j(\omega t+\varphi(\omega))}\right]$$

$$= AM(\omega)\cos(\omega t + \underbrace{\varphi(\omega)}_{\text{delay}}).$$

Dynamic Response: Laplace Transf.

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt, \quad s \text{ is a complex param.}$$

? slide unit step: why $Re(s)>0$.

Transient & Steady-state Response.

e.g. $\dot{y} = -y + u \Rightarrow$ given $u = \cos t$,

$$y = \underbrace{-\frac{1}{2}e^{-t}}_{\text{transient}} + \underbrace{\frac{1}{\sqrt{2}}\cos(t - \frac{\pi}{4})}_{\text{steady-state}}$$

* ① transient response vanishes at $t \to \infty$.
② $\mathcal{L}$ only gives steady-state response.

Differentiation in $\mathcal{L}$:

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0),$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0) = \mathcal{L}\{(f'(t))'\}.$$

DC Gain.

steady-state value of the step response.

$$\text{gain} = y(\infty) = \lim_{t\to\infty} y(t), \quad \text{for } u(t) = 1(t).$$

The Final Value Theorem.

If all poles of $sY(s)$ are *strictly stable* or lie in the *open left half-plane* (OLHP), i.e., have $\mathrm{Re}(s) < 0$, then

$$y(\infty) = \lim_{s\to 0} sY(s).$$