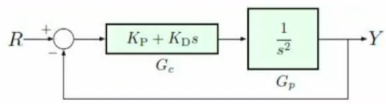
Using RL to Select Parameters.

$$L(s) = -\frac{1}{K} \Rightarrow K = -\frac{1}{L} = \frac{1}{|L(s)|}$$

$$= \frac{|s-p_1| \cdot \cdots \cdot |s-p_n|}{|s-z_1| \cdot \cdots \cdot |s-z_m|}.$$

Double Integrator with PD-Control



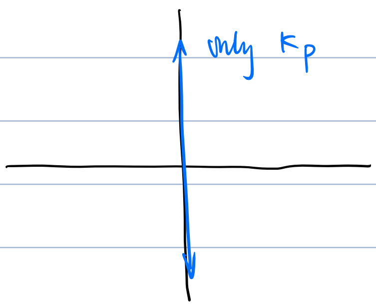
$$\text{Characteristic equation:} \quad 1 + \underbrace{(K_P + K_D s)}_{G_c(s)} \cdot \underbrace{\frac{1}{s^2}}_{G_p(s)} = 0$$

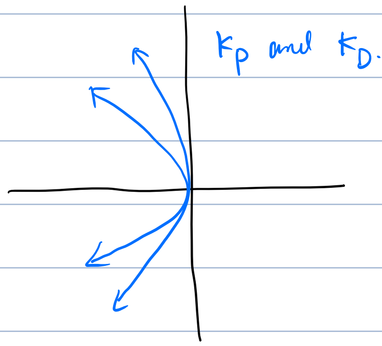
$$s^2 + K_D s + K_P = 0$$

To use the RL method, we need to convert it into the Evans form $1 + KL(s) = 0$, where $L(s) = \frac{b(s)}{a(s)} = \frac{s^m + b_1 s^{m-1} + \dots}{s^n + a_1 s^{n-1} + \dots}$

$$1 + (K_P + K_D s)\frac{1}{s^2} = 1 + K_D \cdot \frac{s + K_P/K_D}{s^2}$$

$$\Longrightarrow K = K_D, \; L(s) = \frac{s + K_P/K_D}{s^2} \qquad \text{(assume } K_P/K_D \text{ fixed, } = 1)$$



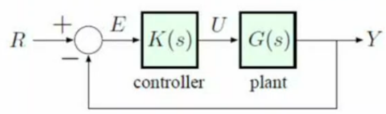


Dynamic Compensation.

★ PD control lacks causality, thus not physically implementable.

Approx. PD using dynamic compensation

$$\dot{z} = Az + Be$$
$$u = Cz + De$$



$$K_D \frac{ps}{s+p} \to K_D s \quad (p \to \infty).$$

$$\frac{ps}{s+p} = \frac{s}{\frac{s}{p}+1} = \left(\lim_{p\to\infty}\right) \frac{s}{1} = s.$$

$$\Rightarrow K_P + K_D s \approx K_P + K_D \boxed{\frac{ps}{s+p}}$$

Poles : $1 + \left(K_P + K_D \frac{ps}{s+p}\right) G(s) = 0.$

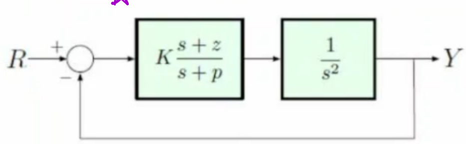
$$\Rightarrow K_P + K_D \frac{ps}{s+p} = (K_P + pK_D) \cdot \frac{s + \frac{pK_P}{K_P + pK_D}}{s+p}.$$ Evans's Canonical Form.

$\frac{pK_P}{K_P + pK_D} \to z.$

$$\Rightarrow \boxed{\begin{cases} K = K_P + pK_D \\ \text{OL zero: } -z = -\frac{pK_P}{K} \end{cases}}$$

Objectives: stabilize the system and satisfy given time response specs using a *stable* *causal* controller.

★



Characteristic equation:

$$1 + K \boxed{\frac{s+z}{s+p}} \frac{1}{s^2} = 1 + KL(s) = 0$$

Lead and Lag Compensation. $K \cdot \boxed{\frac{s+z}{s+p}}$

Lead compensator: $z < p$.

lag : $z > p$.

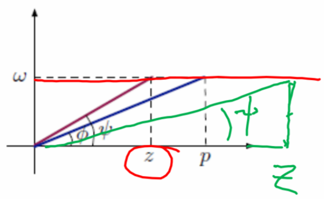
Why the name "lead/lag?" — think frequency response

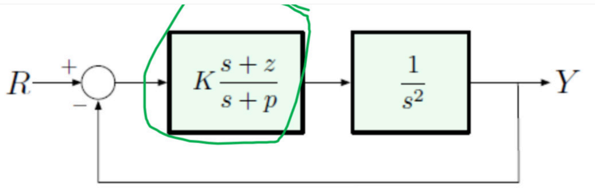
$$\angle \frac{j\omega + z}{j\omega + p} = \angle(j\omega + z) - \angle(j\omega + p) = \psi - \phi$$

output leading

- if $z < p$, then $\psi - \phi > 0$ (phase lead)
- if $z > p$, then $\psi - \phi < 0$ (phase lag)

output lagging





note: large p → similar to PD → noise amplification.

Controller transfer function is $K\frac{s+z}{s+p}$, where:

$$K = K_P + pK_D, \qquad z = \frac{pK_P}{K_P + pK_D} \xrightarrow{p\to\infty} \frac{K_P}{K_D}$$

so, as $p \to \infty$, z tends to a constant, so we get a lead controller. $z<p$.

We use lead controllers as dynamic compensators for approximate PD control.

Large p: good damping, bad noise suppression.

Small : better noise supp., RL close to $j\omega$-axis; no break-in for small p.

intermediate: transition btw. two types of RL; break-in & break-away.

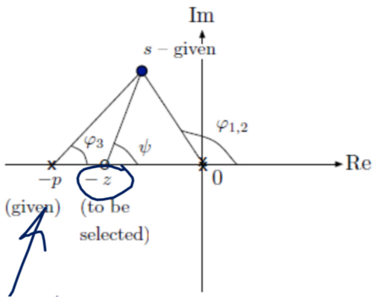
Find z with given p.

Back to our example: double integrator with lead compensation

$$KL(s) = K\frac{s+z}{s+p} \cdot \frac{1}{s^2} \quad \leftarrow \text{OL transfer function.}$$

Problem: given p and a desired closed-loop pole s, find the value of z that will guarantee this (if possible).

Solution: use the phase condition



Must have

$$\underbrace{\psi}_{\substack{\text{angle from}\\ s \text{ to zero}}} - \sum_i \underbrace{\varphi_i}_{\substack{\text{angles from}\\ s \text{ to poles}}} = 180^\circ$$

So, we want $\psi = 180^\circ + \sum_i \varphi_i$