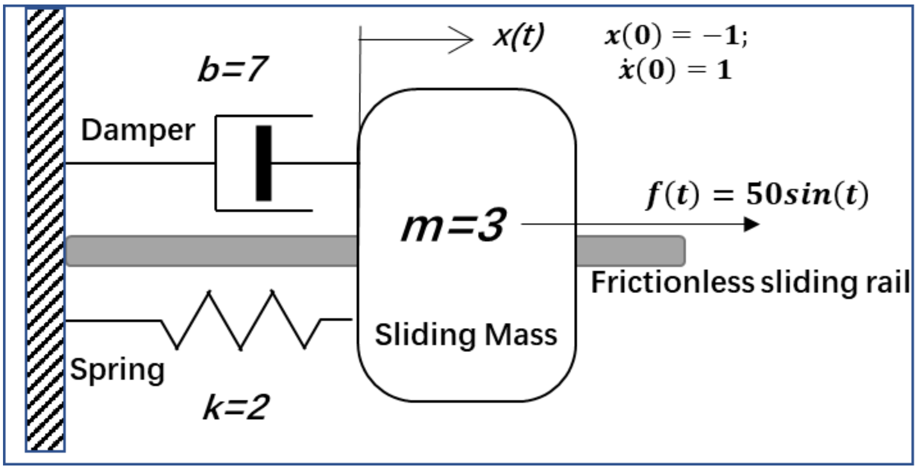
ECE 486: Control Systems — Assignment # 01

Issued: Oct 04 — Due: Oct 11, 2023

Question 1



a) Write down the dynamic equation in the form of a 2^{nd} order differential equation. (2 points)

b) Write down the state-space equation of the system. (2 points)

c) Express the equation in the s-domain. (4 points)

d) Obtain the system response for the input $f(t)=50\sin(t)$. (7 points)

Question 2

a) Obtain the impulse response, $x(t)$, of the system with a transfer function (2 points)

$$X(s) = \frac{8s}{4s^2 + 1}$$

b) Show that FVT is not applicable and briefly explain why. (3 points)

Solution

1. a)

$$m\ddot{x} + b\dot{x} + kx = f(t)$$
$$3\ddot{x} + 7\dot{x} + 2x = f(t)$$
$$3\ddot{x} + 7\dot{x} + 2x = 50sin(t)$$

b)

$$\ddot{x} + \frac{7}{3}\dot{x} + \frac{2}{3}x = \frac{f(t)}{3} = \frac{50}{3}sin(t)$$

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}0 & 1\\ -\frac{2}{3} & -\frac{7}{3}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}\cdot\frac{50}{3}sin(t);\ y = \begin{pmatrix}b_0 & b_1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}$$

c)

$$[s^2X(s) - sx(0) - \dot{x}(0)] + \frac{7}{3}[sX(s) - x(0)] + \frac{2}{3}X(s) = U(s)$$

$$3s^2X + 7sX + 2X - (3sx_0 + 3\dot{x}_0 + 7x_0) = \frac{50}{(s^2+1)}$$

d)

$$(3s^2 + 7s + 2)X = \frac{50}{(s^2+1)} - 3s - 4$$

$$(3s+1)(s+2)X = -3s - 4 + \frac{50}{(s^2+1)}$$

$$X = \frac{-3s-4}{(3s+1)(s+2)} + \frac{50}{(3s+1)(s+2)(s^2+1)}$$

$$X = \frac{-9}{5(3s+1)} + \frac{-2}{5(s+2)} + \frac{27}{(3s+1)} + \frac{-2}{(s+2)} + \frac{-7s}{(s^2+1)} + \frac{-1}{(s^2+1)}$$

$$X = \frac{42}{5\left(s+\frac{1}{3}\right)} - \frac{12}{5(s+2)} - \frac{7s}{(s^2+1)} - \frac{1}{(s^2+1)}$$

Inverse Laplace:

$$x(t) = \frac{42}{5}e^{\frac{-t}{3}} - \frac{12}{5}e^{-2t} - 7cos(t) - sin(t)$$

2. a) $x(t) = 2cos\left(\frac{t}{2}\right)$

b)

Let $X(s) = \frac{2s}{s^2+\frac{1}{4}}$, so that its poles are at $\pm\frac{1}{2}j$, on the imaginary axis, not permitted by the FVT. Therefore, FVT is not applicable and should not be applied. If it were to be applied, it would yield

$$x_{ss} = \lim_{s\to 0}\{sX(s)\} = \lim_{s\to 0}\left\{\frac{2s^2}{s^2+\frac{1}{4}}\right\} = 0$$

which is obviously false. To explain this, we first find $x(t) = \mathcal{L}^{-1}\{X(s)\} = 2\cos(\frac{1}{2}t)$. Then, it is clear that $\lim_{t\to\infty} x(t)$ does not exist, since $x(t)$ is oscillatory and there is no steady-state value.