$\frac{g\pi}{l} \to$ unit step

SAMPLE MIDTERM EXAM 1

---

1. The pendulum dynamics derived in class are

$$\ddot{\theta} = -\frac{g}{\ell}\sin\theta + \frac{1}{m\ell^2}T_e$$

$\ddot{\theta} = -\frac{g}{l}(\pi-\theta) + \frac{1}{ml^2}Te.$

$\ddot{\theta} = \frac{g}{l}\theta + \frac{1}{ml^2}Te - \frac{g\pi}{l}.$

where θ is the angle between the pendulum and the downward vertical direction, g is the gravitational constant, ℓ is the length of the pendulum, m is the tip mass, and T_e is the external torque.

Linearize the above pendulum equation around the upward equilibrium $\theta = \pi$. Write your answer in state space form $\dot{x} = Ax + Bu$ where x is an appropriate vector of state variables and A, B are matrices/vectors of appropriate dimensions.

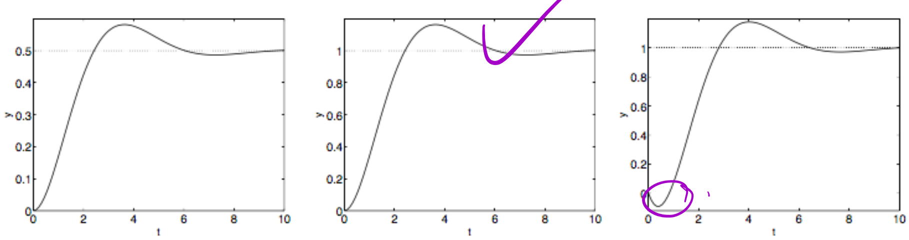
$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \theta_2 \\ \frac{g}{l}\theta_1 + \frac{1}{ml^2}Te - \frac{g\pi}{l} \end{bmatrix} = \cdots$

2. Consider the transfer function $H(s) = \dfrac{1}{s^2+s+1}$.

$\zeta = \frac{1}{2}.$
$\omega_n = 1$

1. Which of the following is the corresponding step response? Explain your choice and why you rejected the other two possibilities.

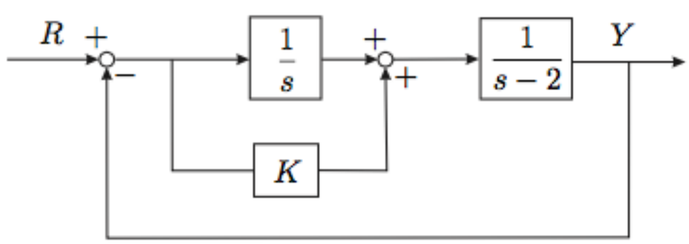
$M_p = e^{-\frac{\frac{1}{2}\pi}{\sqrt{1-\frac{1}{4}}}} = 16\%.$



pole: $s = \frac{-1\pm\sqrt{1-4}}{2} = \frac{-1\pm\sqrt{3}j}{2}$. LHP.

2. Is the transfer function stable? Justify your answer.

3. Consider the system given by the block diagram below:



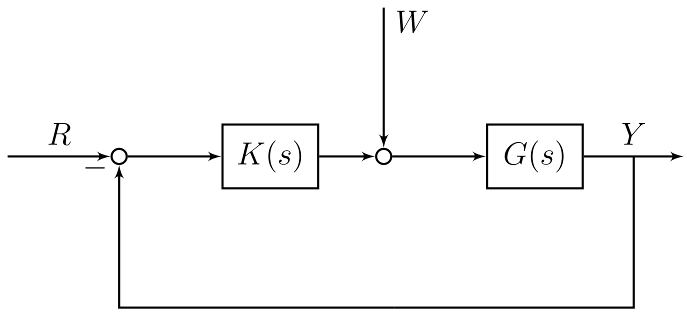
a) Compute the transfer function from the reference R to the output Y.

b) Determine the range of values of K for which the system is stable.

c) Suppose that the reference is a step: $r(t) = 1(t)$. Does the system achieve perfect steady-state tracking of this reference? If yes, justify. If not, characterize the steady-state tracking error.

d) Suppose that the reference is a ramp: $r(t) = t1(t)$. Answer the same questions as in part c).

4. Consider a system with transfer function $G(s) = \frac{1}{s+a}$ $(a > 0)$ in unity negative feedback loop with controller transfer function $K(s)$ and disturbance signal $w(t)$.



1. Assume the controller is of P-type. What is the closed-loop transfer function and what is the DC gain of the system.

2. Assuming the controller is of PD-type, can you choose the parameters of the controller so that the DC gain is exactly one? Justify your answer.

3. Assuming the controller is of PI-type, can you choose a value of the controller parameters so as to have perfect disturbance rejection? Justify your answer.