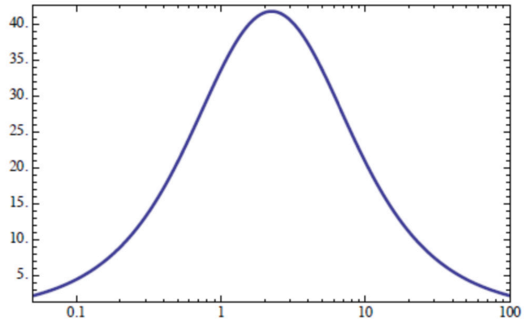
Unstable Zeros/Poles. (RHP).

different exists in phasor plots c.f. stable o/p.

all LHP:

$$G_1(j\omega) = \frac{j\omega + 1}{j\omega + 5} = \frac{1}{5}\frac{j\omega + 1}{\frac{j\omega}{5} + 1}$$

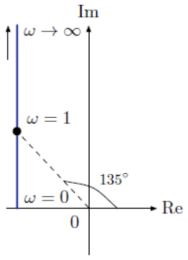
- Low-frequency term: $\frac{1}{5}(j\omega)^0$— $n = 0$, so phase starts at
- Break-points at $\omega_n = 1$ (phase goes up by 90°) and at $\omega_n = 5$ (phase goes down by 90°)



RHP:

$$G_2(j\omega) = \frac{j\omega - 1}{j\omega + 5} = \frac{1}{5}\frac{j\omega - 1}{\frac{j\omega}{5} + 1}$$

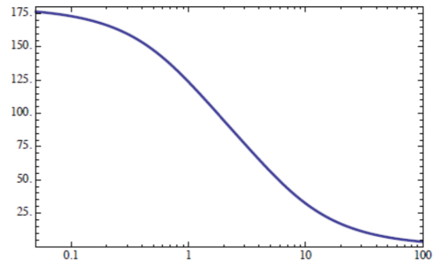
Let's do a Nyquist plot for $j\omega - 1$:



New type of behavior —

- $\omega \approx 0$: $\phi \approx 180°$ (real and negative)
- $\omega \gg 1$: $\phi \approx 90°$ (Re $= -1$, Im $= \omega \gg 1$)
- $\omega \approx 1$: $\phi \approx 135°$

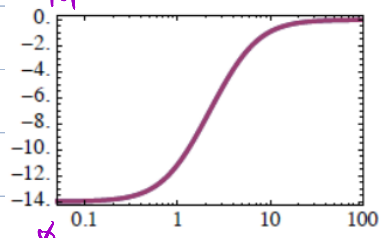
For a RHP zero, the phase starts out at 180° and goes down by 90° through the break-point (135° at break-point).



For a RHP zero, the phase plot is similar to what we had for a LHP pole: goes down by 90° ... However, it starts at 180°, and not at 0°.

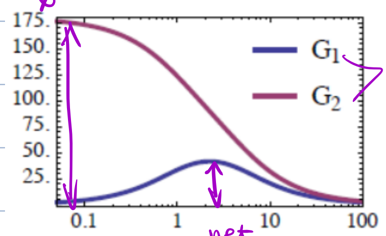
Minimum-/ Non-Minimum-Phase Zeros

M



Among all transfer functions with the same magnitude plot, the one with only LHP zeros has the minimal net phase change as ω goes from 0 to ∞ — hence the term *minimum-phase* for LHP zeros.

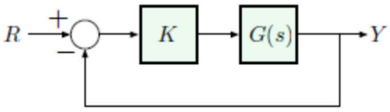
ϕ



see above.

net phase change.

Stability in FR



Transition:

① $j\omega$-crossings

② some ω s.t. $\begin{cases} M=1 \\ \phi = 180^\circ \end{cases}$

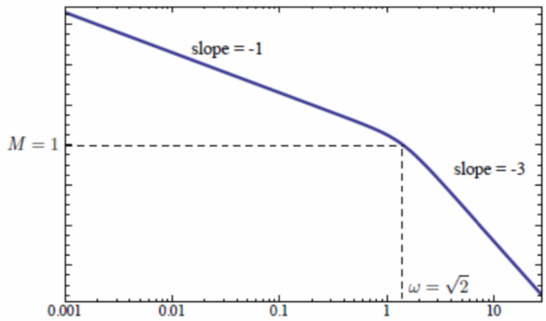
Stability from frequency response. If $s = j\omega$ is on the root locus (for some value of K), then

$$|KG(j\omega)| = 1 \qquad \text{and} \qquad \angle KG(j\omega) = 180^\circ \mod 360^\circ$$

Crossover Frequency and Stability

CF: ω at $M=1$. (ω_c).

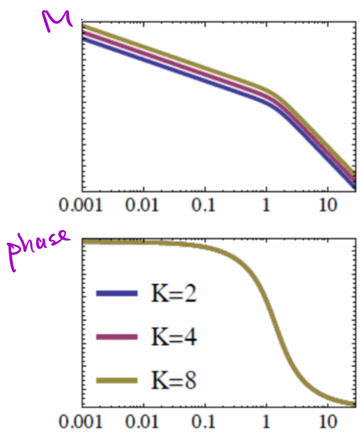
Definition: The frequency at which $M = 1$ is called the *crossover frequency* and denoted by ω_c.



Transition from stability to instability on the Bode plot:

$$\text{for critical } K, \qquad \angle G(j\omega_c) = 180^\circ$$

Effect of Varying K



What happens as we vary K?

- ϕ independent of $K \Longrightarrow$ only the M-plot changes
- If we multiply K by 2:

$$\log(2M) = \log 2 + \log M$$

  – M-plot shifts up by $\log 2$
- If we divide K by 2:

$$\log(\tfrac{1}{2}M) = \log\frac{1}{2} + \log M = -\log 2 + \log M$$

  – M-plot shifts down by $\log 2$

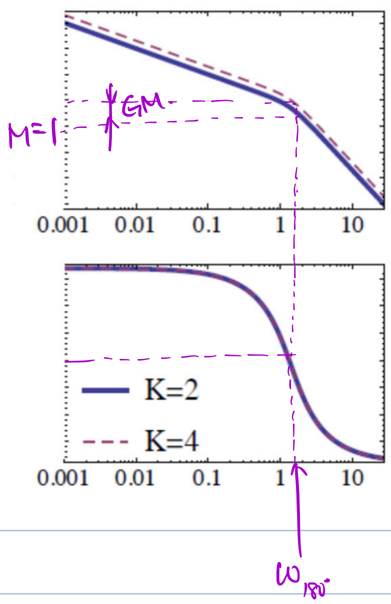
☆ Changing the value of K moves the crossover frequency ω_c!!

but will not change phase.

Gain Margin.

Gain Margin

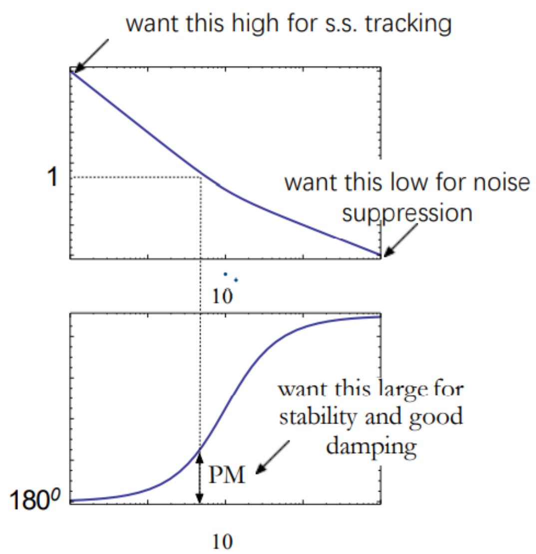
Back to our example: $G(s) = \dfrac{1}{s(s^2+2s+2)}$, $K = 2$ (stable)



Gain margin (GM) is the factor by which K can be multiplied before we get $M = 1$ when $\phi = 180^\circ$

Since varying K doesn't change ω_{180°, to find GM we need to inspect M at $\omega = \omega_{180^\circ}$

☆ GM < 1 (i.e. < 0dB): system unstable.



See L17 for phase margin.