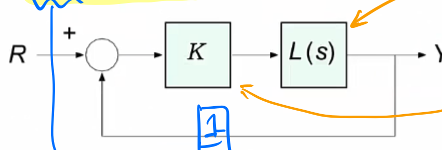
Design methods: RL / Freq Response / State-space.

Consider this unity feedback configuration:

Evan's Form



not necessarily plant

not necessarily controller.

feedback gain = 1.

where

- K is a constant gain
- $L(s) = \dfrac{b(s)}{a(s)}$, where $a(s)$ and $b(s)$ are some polynomials

$$\frac{Y}{R} = \frac{KL}{1+KL}, \quad L = \frac{b(s)}{a(s)}.$$

Poles: $1+KL=0 \Rightarrow L = -\frac{1}{K}$.

$$\Updownarrow$$

$$1+\frac{Kb}{a}=0$$

$$\Updownarrow$$

Characteristic Eq. $a(s)+Kb(s)=0$.

Root Locus

The root locus for $\boxed{1+KL(s)=0}$ is the set of all closed-loop poles, i.e., the roots of $1+KL(s)=0$ as K varies from 0 to ∞

The Phase Condition.

$L(s) = -\frac{1}{K} \in \mathbb{R}$.

The phase condition: The root locus of $1+KL(s)$ is the set of all $s \in \mathbb{C}$, such that $\angle L(s) = 180°$, i.e., $L(s)$ is real and negative.

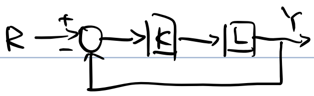
Rules for Sketching Root Loci. $L(s) = \frac{b(s)}{a(s)}$

A. # of Branches. #(branches) = deg(a(s)).

B. starting point: $K=0$, i.e. OL poles. $(a(s)=0)$

open-loop pole

Characteristic eq: $1+KL(s)=0 \Rightarrow L = -\frac{1}{K} = \frac{b(s)}{a(s)} \Big|_{K\to 0} \Rightarrow a(s)=0$.



open-loop gain: KL. (no feedback).

closed-loop gain: $\frac{KL}{1+KL}$.

C. End point. $K\to\infty$, i.e. OL zeros. $(b(s)=0)$.

☆ if # of branches > # of zeros: remaining → infinity.

D. Real locus. $1+KL(s)=0 \longleftrightarrow \angle L(s) = 180°$,

★ n = # of poles

m = # of zeros.

$\angle L(s) = \angle \frac{b(s)}{a(s)}$

zeros poles.

$= \sum_{i=1}^{m} \angle(s-z_i) - \sum_{j=1}^{n} \angle(s-p_j) = \pm\pi$. for $\forall s$ on RL.

Rule D: If s is *real*, then it is on the RL of $1+KL$ if and only if there are an odd number of *real open-loop poles and zeros* to the right of s.

奇数个. → 以维持相位, $(2l\pm1)\pi$.

e.g. s z p x

$\Sigma\phi = -2\pi = 0$.

E. Asymptotes If $\angle s^{n-m} = 180°$, then $\angle s = \frac{180° + l\cdot 360°}{n-m}$ where $l = 0, 1, \cdots, n-m-1$.

Rule E: Branches near ∞ have phase

$$\angle s \simeq \frac{180° + \ell \cdot 360°}{n-m}$$
$$= \frac{(2\ell+1)\cdot 180°}{n-m}, \qquad \ell = 0, 1, \ldots, n-m-1$$

Note: if $m = n$, then there are no branches at ∞.

F. jω-crossings transitions from stability to instability.

① solve for critical value of K.

② plug $K_{critical}$ in and solve the characteristic equation

Double Integrator with PD-Control

$\boxed{L(s) = \frac{s+1}{s^2}}$

Characteristic equation: $1 + K \cdot \frac{s+1}{s^2} = 0$ $\Rightarrow 1+KL = \frac{s^2+Ks+K}{s^2}$.

Here we can still write out the roots explicitly:

$$s^2 + Ks + K = 0 \qquad \Longrightarrow \qquad s = \frac{-K \pm \sqrt{K^2 - 4K}}{2}$$

$n = 2$

$m = 1$.

But let's actually draw the RL using the rules:

Rule A: 2 branches $\deg(s^2) = 2$.

Rule B: both start at $s = 0$ poles.

Rule C: one ends at $z_1 = -1$, the other at ∞

Rule D: one branch will go off to $-\infty$

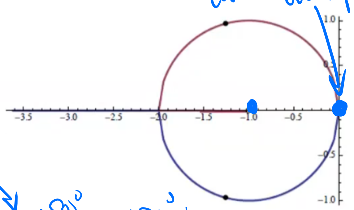
Rule E: asymptote angles at 180° $\frac{180°}{2-1} = 180°$.

Rule F: no $j\omega$-crossings except for $s = p_1 = p_2 = 0$

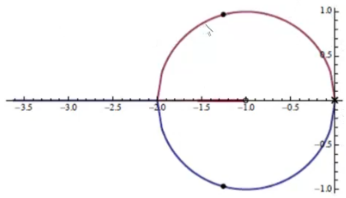
↓

$K=0. \Rightarrow$ no soln for characteristic eqn.

two zeros overlaped.



Characteristic equation: $1 + K \cdot \frac{s+1}{s^2} = 0$



What can we conclude from this root locus about stabilization?

- all closed-loop poles are in LHP (we already knew this from Routh, but now can visualize)
- nice damping, so can meet reasonable specs

So, the effect of D-gain was to introduce an *open-loop zero* into LHP, and this zero "pulled" the root locus into LHP, thus stabilizing the system.