ECE 486 Control Systems

Lecture 20: Pole placement by full state feedback

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**The syllabus and much of the lecture content originated from UIUC ECE 486 course material

Checklist

Modeling Analysis Design

Root Locus
Frequency Response
State-Space

Wk	Topic	Ref.
1	✓ Introduction to feedback control	Ch. 1
	✓ State-space models of systems; linearization	Sections 1.1, 1.2, 2.1–2.4, 7.2, 9.2.1
2	✓ Linear systems and their dynamic response	Section 3.1, Appendix A
	✓ Transient and steady-state dynamic response with arbitrary initial conditions	Section 3.1, Appendix A
3	✓ National Holiday Week	
4	✓ System modeling diagrams; prototype second-order system	Sections 3.1, 3.2, lab manual
	✓ Transient response specifications	Sections 3.3, 3.14, lab manual
5	✓ Effect of zeros and extra poles; Routh-Hurwitz stability criterion	Sections 3.5, 3.6
	✓ Basic properties and benefits of feedback control; Introduction to Proportional-Integral-Derivative (PID) control	Section 4.1-4.3, lab manual
6	✓ Review A	
	✓ Term Test A	
7	✓ Introduction to Root Locus design method	Ch. 5
	✓ Root Locus continued; introduction to dynamic compensation	Root Locus
8	✓ Lead and lag dynamic compensation	Ch. 5
	✓ Introduction to frequency-response design method	Sections 5.1-5.4, 6.1

Modeling
Analysis

Wk	Topic	Ref.
9	**Bode plots for three types of transfer functions**	Section 6.1
	Stability from frequency response; gain and phase margins	Section 6.1
10	**Control design using frequency response: PD and Lead**	Ch. 6
	Control design using frequency response continued; PI and lag, PID and lead-lag	Frequency Response
11	**Nyquist stability criterion**	Ch. 6
	Nyquist stability; gain and phase margins from Nyquist plots	Ch. 6
12	**Review B**	
	Term Test B	
13	**Introduction to state-space design**	Ch. 7
	Controllability, stability, and pole-zero cancellations; similarity transformation; conversion of controllable systems to Controller Canonical Form	Ch. 7
14	**Pole placement by full state feedback**	Ch. 7
	Observer design for state estimation	Ch. 7
15	Joint observer and controller design by dynamic output feedback; separation principle	State-Space
	In-class review	Ch. 7
16	END OF LECTURES: Revision Week	
	Final	

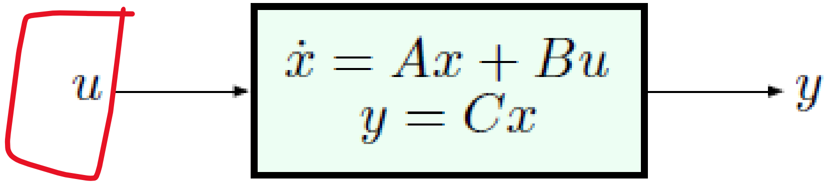
Lecture Overview

- **Review**: coordinate transformations; conversion of any controllable system to CCF.
- **Today's topic:** pole placement by (full) state feedback
- **Learning Goal**: learn how to assign arbitrary closed-loop poles of a controllable system x_ = Ax + Bu by means of state feedback

$$u = -Kx$$

Reading: FPE, Chapter 7

State-Space Realization



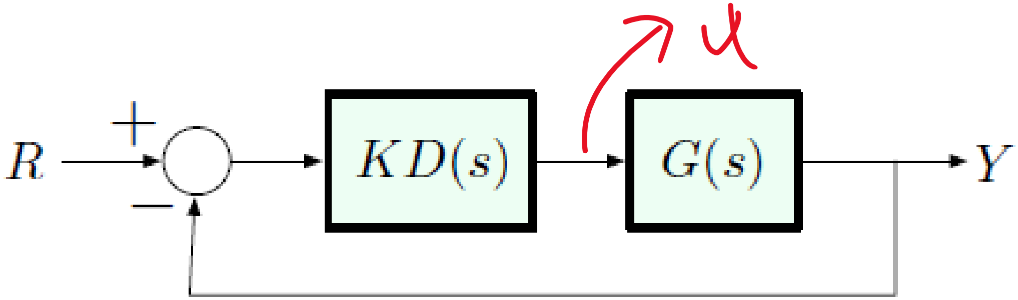
$$\downarrow$$

$$G(s) = C(Is - A)^{-1}B$$

Open-loop poles are the eigenvalues of A:

$$\det(Is - A) = 0$$

Then we add a controller to move the poles to desired locations:



Goal: Pole Placement by State Feedback

Consider a single-input system in state-space form:

$$u \longrightarrow \boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}} \longrightarrow y$$

Today, our goal is to establish the following fact:

If the above system is *controllable*, then we can assign arbitrary closed-loop poles by means of a state feedback law

$$\begin{aligned} u = -Kx &= -\begin{pmatrix} k_1 & k_2 & \dots & k_n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \\ &= -(k_1 x_1 + \dots + k_n x_n), \end{aligned}$$

where K is a $1 \times n$ matrix of feedback gains.

Review: Controllability

Consider a single-input system ($u \in \mathbb{R}$):

$$\dot{x} = Ax + Bu, \qquad y = Cx \qquad\qquad x \in \mathbb{R}^n$$

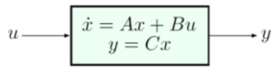
The Controllability Matrix is defined as

$$\mathcal{C}(A, B) = [B \,|\, AB \,|\, A^2B \,|\, \dots \,|\, A^{n-1}B]$$

We say that the above system is controllable if its controllability matrix $\mathcal{C}(A, B)$ is *invertible*.

- As we will see today, if the system is controllable, then we may assign arbitrary closed-loop poles by *state feedback* of the form $u = -Kx$.
- Whether or not the system is controllable depends on its state-space realization.

State-Space Realizations



- a given transfer function $G(s)$ can be realized using infinitely many state-space models
- certain properties make some realizations preferable to others
- one such property is *controllability*

Controller Canonical Form

A single-input state-space model

$$\dot{x} = Ax + Bu, \qquad y = Cx$$

is said to be in Controller Canonical Form (CCF) is the matrices A, B are of the form

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ * & * & * & \dots & * & * \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

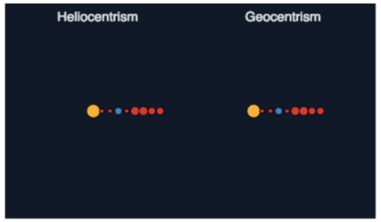
A system in CCF is *always controllable*!!

(The proof of this for $n > 2$ uses the Jordan canonical form, we will not worry about this.)

Coord. Transform and State-Space Models

- We will see that state feedback design is particularly easy when the system is in CCF.
- Hence, we need a way of constructing a CCF state-space realization of a given controllable system.
- We will do this by suitably changing the coordinate system for the state vector.

Same system from a different coord. system



Coord. Transform and State-Space Models

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned} \quad \xrightarrow{T} \quad \begin{aligned}\dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u \\ y &= \bar{C}\bar{x}\end{aligned}$$

where $\bar{A} = TAT^{-1}, \quad \bar{B} = TB, \quad \bar{C} = CT^{-1}$

- The transfer function does not change.
- The controllability matrix is transformed:

$$\mathcal{C}(\bar{A}, \bar{B}) = T\mathcal{C}(A, B).$$

- The transformed system is controllable if and only if the original one is.
- If the original system is controllable, then

$$T = \mathcal{C}(\bar{A}, \bar{B}) \left[\mathcal{C}(A, B)\right]^{-1}.$$

This gives us a way of systematically passing to CCF.

Example: Convert a Controllable Sys. to CCF

$y = Cx$

$$A = \begin{pmatrix} -15 & 8 \\ -15 & 7 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \boxed{(C \text{ is immaterial})}$$

Step 1: check for controllability.

$B \quad AB$

$$\mathcal{C} = \begin{pmatrix} 1 & -7 \\ 1 & -8 \end{pmatrix} \qquad \det \mathcal{C} = -1 \qquad \text{– controllable}$$

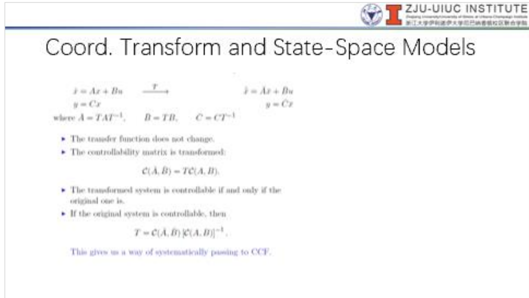
Step 2: Determine desired $\mathcal{C}(\bar{A}, \bar{B})$.

$$\mathcal{C}(\bar{A}, \bar{B}) = [\bar{B} \,|\, \bar{A}\bar{B}] = \begin{pmatrix} 0 & 1 \\ 1 & -8 \end{pmatrix}$$

Step 3: Compute T.

$$T = \mathcal{C}(\bar{A}, \bar{B}) \cdot [\mathcal{C}(A, B)]^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & -8 \end{pmatrix} \begin{pmatrix} 8 & -7 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$T\mathcal{C}(A,B) = \mathcal{C}(\bar{A},\bar{B})$



Finally, Pole Placement via State Feedback

Consider a state-space model

$$\dot{x} = Ax + Bu, \qquad x \in \mathbb{R}^n, u \in \mathbb{R}$$
$$y = x$$

Let's introduce a *state feedback law*

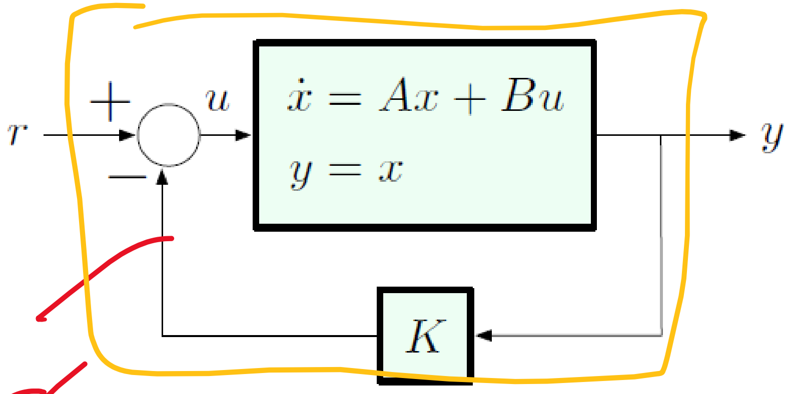
$$u = -Ky \equiv -Kx$$
$$= -\begin{pmatrix} k_1 & k_2 & \dots & k_n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = -(k_1 x_1 + \dots + k_n x_n)$$

Closed-loop system:

$$\dot{x} = Ax - BKx = (A - BK)x$$
$$y = x$$

Pole Placement via State Feedback

Let's also add a reference input:



$-Kx$

$$\dot{x} = Ax + B(-Kx + r) = (A - BK)x + Br, \qquad y = x$$

Take the Laplace transform:

$$sX(s) = (A - BK)X(s) + BR(s), \;\; Y(s) = X(s)$$

$$Y(s) = \underbrace{(Is - A + BK)^{-1}B}_{G}\,R(s)$$

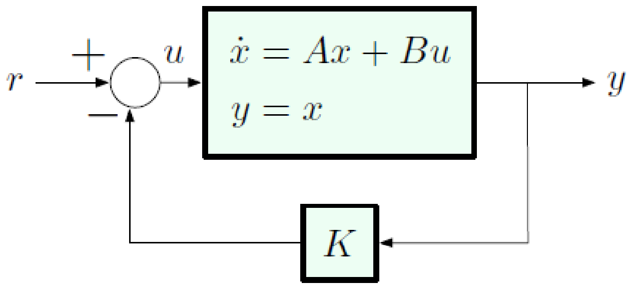
$sX - (A - BK)X = BR$

$(Is - A + BK)Y = BR$

$Y = (Is - A + BK)^{-1}BR$

Closed-loop poles are the eigenvalues of $A - BK$!!

Pole Placement Via State Feedback



assigning closed-loop poles = assigning eigenvalues of $A - BK$

Now we will see that this is particularly straightforward if the (A, B) system is in CCF.

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_2 & -a_1 \end{pmatrix}, \quad \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

The Beauty of CCF

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_2 & -a_1 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

Claim.

$$\det(Is - A) = s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n$$

— the last row of the A matrix in CCF consists of the coefficients of the characteristic polynomial, in reverse order, with "–" signs.

Pole Placement

Proof of the Claim

A nice way is via Laplace transforms:

$$\dot{x} = Ax + Bu$$

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_2 & -a_1 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

Represent this as a system of ODEs:

$$\dot{x}_1 = x_2 \qquad\qquad X_2 = sX_1$$

$$\dot{x}_2 = x_3 \qquad\qquad X_3 = sX_2 = s^2X_1$$

$$\vdots \qquad\qquad \vdots$$

$$\dot{x}_n = -\sum_{i=1}^{n} a_{n-i+1}x_i + u \qquad\qquad \underbrace{(s^n + a_1 s^{n-1} + \dots + a_n)}_{\text{char. poly.}} X_1 = U$$

... And, Back to Pole Placement

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_2 & -a_1 \end{pmatrix}$$

$$BK = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} k_1 & k_2 & \dots & k_n \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 \\ k_1 & k_2 & k_3 & \dots & k_{n-1} & k_n \end{pmatrix}$$

$$(A - BK) = -\begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ a_n + k_1 & a_{n-1} + k_2 & a_{n-2} + k_3 & \dots & a_2 + k_{n-1} & a_1 + k_n \end{pmatrix}$$

— still in CCF!!

Pole Placement in CCF

$$\dot{x} = (A - BK)x + Br, \qquad y = Cx$$

$$A - BK = -\begin{pmatrix} 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 1 \\ a_n + k_1 & a_{n-1} + k_2 & \dots & a_2 + k_{n-1} & a_1 + k_n \end{pmatrix}$$

Closed-loop poles are the roots of the characteristic polynomial

$$\det(Is - A + BK)$$
$$= s^n + (a_1 + k_n)s^{n-1} + \ldots + (a_{n-1} + k_2)s + (a_n + k_1)$$

Key observation: When the system is in CCF, each control gain affects only *one* of the coefficients of the characteristic polynomial, and these coefficients can be assigned arbitrarily by a suitable choice of $k_1, \ldots, k_n$.

Hence the name Controller Canonical Form — convenient for control design.

Pole Placement by State Feedback

0. Check for controllability

General procedure for any *controllable* system:

1. Convert to CCF using a suitable invertible coordinate transformation T (such a transformation exists by controllability).
2. Solve the pole placement problem in the new coordinates.
3. Convert back to original coordinates.

Example

$$\mathcal{C}(A,B) = \begin{bmatrix} 1 & -7 \\ 1 & -6 \end{bmatrix}$$

Given $\dot{x} = Ax + Bu$

$$A = \begin{pmatrix} -15 & 8 \\ -7 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Goal: apply state feedback to place closed-loop poles at $\boxed{-10 \pm j}$.

Step 1: convert to CCF — already did this

$$T = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \quad \longrightarrow \quad \bar{A} = \begin{pmatrix} 0 & 1 \\ -15 & -8 \end{pmatrix}, \ \bar{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Step 2: find $u = -\bar{K}\bar{x}$ to place closed-loop poles at $-10 \pm j$.

Desired characteristic polynomial:

$$(s + 10 + j)(s + 10 - j) = (s + 10)^2 + 1 = s^2 + 20s + 101$$

Thus, the closed-loop system matrix should be

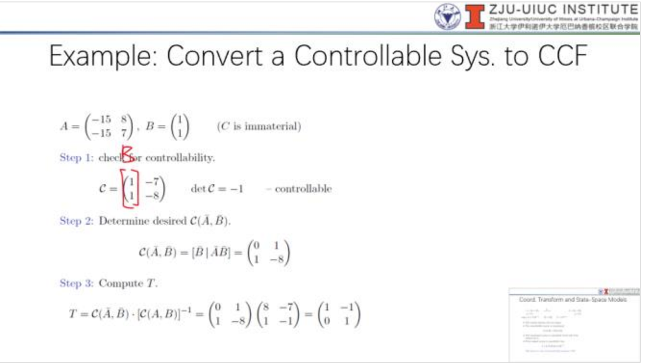
$$\bar{A} - \bar{B}\bar{K} = \begin{pmatrix} 0 & 1 \\ -101 & -20 \end{pmatrix}$$

On the other hand, we know

$$\bar{A} - \bar{B}\bar{K} = \begin{pmatrix} 0 & 1 \\ -(15 + \bar{k}_1) & -(8 + \bar{k}_2) \end{pmatrix} \quad \Longrightarrow \quad \bar{k}_1 = 86, \ \bar{k}_2 = 12$$

This gives the control law

$$u = -\bar{K}\bar{x} = -\begin{pmatrix} 86 & 12 \end{pmatrix} \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix}$$



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Example: Convert a Controllable Sys. to CCF

$$A = \begin{pmatrix} -15 & 8 \\ -15 & 7 \end{pmatrix}, \ B = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad (C \text{ is immaterial})$$

Step 1: check for controllability.

$$\mathcal{C} = \begin{pmatrix} 1 & -7 \\ 1 & -8 \end{pmatrix} \qquad \det \mathcal{C} = -1 \qquad \text{controllable}$$

Step 2: Determine desired $\mathcal{C}(\bar{A}, \bar{B})$.

$$\mathcal{C}(\bar{A}, \bar{B}) = [\bar{B} \mid \bar{A}\bar{B}] = \begin{pmatrix} 0 & 1 \\ 1 & -8 \end{pmatrix}$$

Step 3: Compute T.

$$T = \mathcal{C}(\bar{A}, \bar{B}) \cdot [\mathcal{C}(A, B)]^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & -8 \end{pmatrix} \begin{pmatrix} 8 & -7 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

Coord. Transform and State-Space Models

Example

Step 3: convert back to the old coordinates.

$$\begin{aligned} u &= -\bar{K}\bar{x} \\ &= -\underbrace{\bar{K}T}_{K}x \end{aligned}$$

— therefore,

$$\begin{aligned} K &= \bar{K}\boxed{T} \\ &= \begin{pmatrix} 86 & 12 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 86 & -74 \end{pmatrix} \end{aligned}$$

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$-10 \pm j$$

The desired state feedback law is

$$u = \begin{pmatrix} -86 & 74 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

State-space control method (so far…)

0: Check for controllability

- Pole placement via state feedback

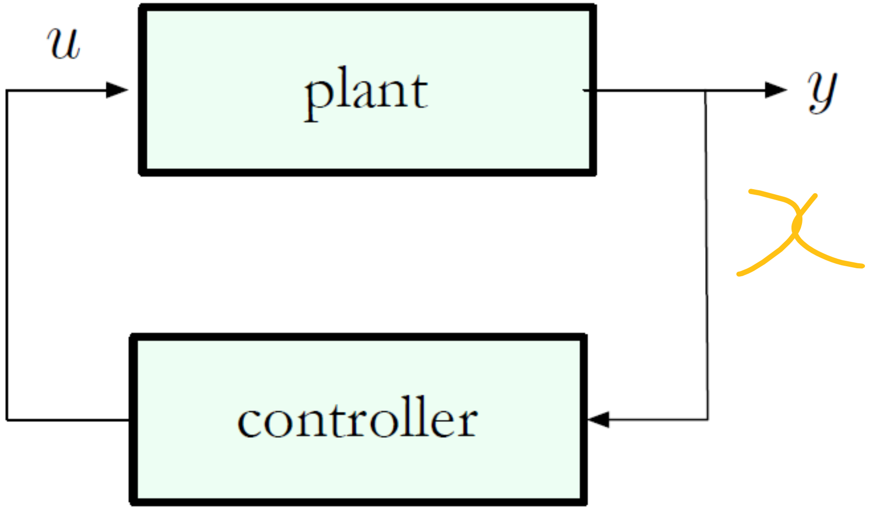
General Procedure for any controllable system

1. Convert to CCF using a suitable invertible coordinate transformation T (such a transformation exists by controllability).
2. Solve the pole placement problem in the new coordinates.
3. Convert back to original coordinates.

Next Lecture

In a typical system, measurements are provided by sensors:

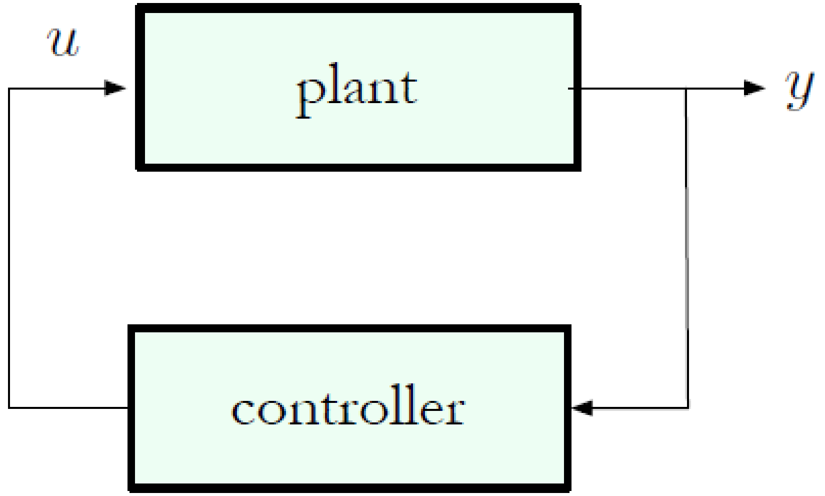
$u = -kx$



Full state feedback $u = -Kx$ is *not implementable*!!

Is Full State Feedback always available?

In a typical system, measurements are provided by sensors:



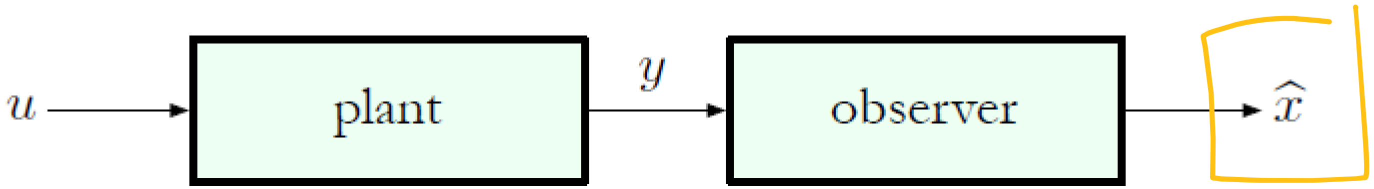
We need an observer!!

Full state feedback $u = -Kx$ is *not implementable*!!

State Estimation using an Observer

$u = -Kx$

- When full state feedback is unavailable, the observer is used to estimated the state *x*.



State Estimation using an Observer

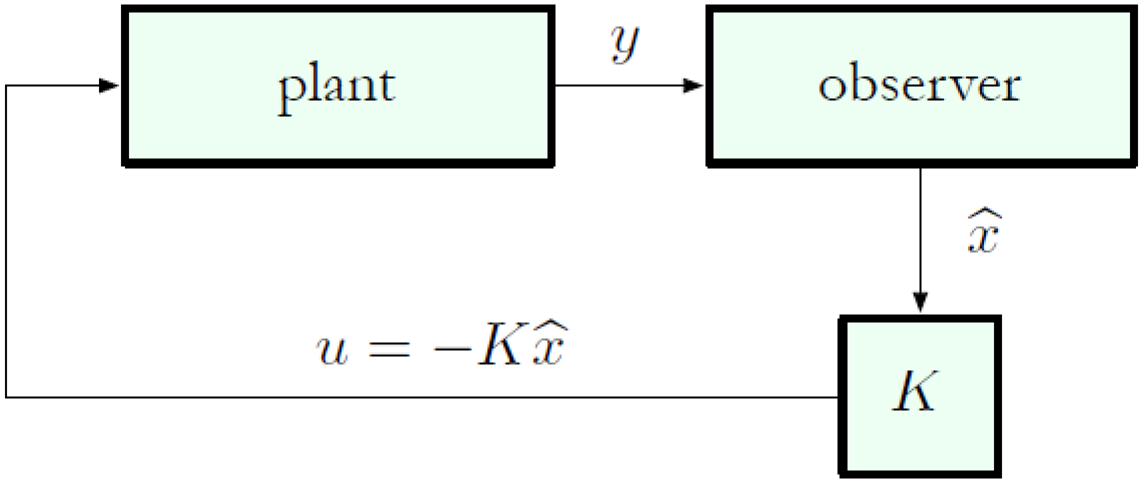
The idea is to design the observer in such a way that the state estimate $\hat{x}$ is *asymptotically accurate*:

$$\|\hat{x}(t) - x(t)\| = \sqrt{\sum_{i=1}^{n} (\hat{x}_i(t) - x_i(t))^2} \xrightarrow{t\to\infty} 0$$

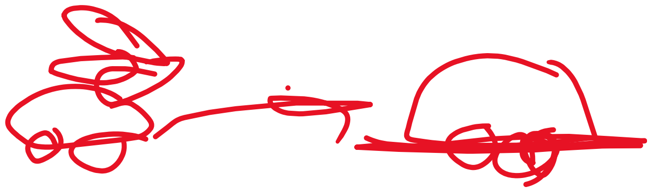
$u = -Kx$

$u = -K\hat{x}$

If we are successful, then we can try estimated state feedback:



A New Concept: Observability



- Before, we saw that closed-loop poles can be assigned arbitrarily by full state feedback when the plant is controllable.
- Now, we will see that asymptotically accurate state estimation will be possible when the system is observable.
- Observability is a system property which is dual to controllability

Observability

$\mathcal{C}(A, B) = [B \mid AB \cdots]$

$\mathcal{O}(A, C)$

Consider a single-output system ($y \in \mathbb{R}$):

$$\dot{x} = Ax + Bu, \qquad y = Cx \qquad\qquad x \in \mathbb{R}^n$$

The Observability Matrix is defined as

$$\mathcal{O}(\boxed{A, C}) = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

— recall that C is $1 \times n$ and A is $n \times n$, so $\mathcal{O}(A, C)$ is $n \times n$;
— the observability matrix only involves A and C, not B

We say that the above system is observable if its observability matrix $\mathcal{O}(A, C)$ is *invertible*.

(This definition is only true for the single-output case; the multiple-output case involves the *rank* of $\mathcal{O}(A, C)$.)

Example: Compute $\mathcal{O}(A,C)$

$$\text{Let} \quad A = \begin{pmatrix} 0 & -6 \\ 1 & -5 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

Here, $n = 2$, $C \in \mathbb{R}^{1\times 2}$, $A \in \mathbb{R}^{2\times 2} \implies \mathcal{O}(A,C) \in \mathbb{R}^{2\times 2}$.

$$\mathcal{O}(A,C) = \begin{bmatrix} C \\ CA \end{bmatrix}$$

where $CA = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -6 \\ 1 & -5 \end{pmatrix} = \begin{pmatrix} 1 & -5 \end{pmatrix}$

$$\therefore\ \mathcal{O}(A,C) = \begin{pmatrix} 0 & 1 \\ 1 & -5 \end{pmatrix}$$

$$\det \mathcal{O}(A,C) = -1 \qquad \implies \qquad \text{the system is observable}$$

— recall: this system is in Observer Canonical Form (OCF) ...

Observer Canonical Form

A single-output state-space model

$$\dot{x} = Ax + Bu, \qquad y = Cx$$

is said to be in Observer Canonical Form (OCF) if the matrices A, C are of the form

$$A = \begin{pmatrix} 0 & 0 & \dots & 0 & 0 & * \\ 1 & 0 & \dots & 0 & 0 & * \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & * \\ 0 & 0 & \dots & 0 & 1 & * \end{pmatrix}, \qquad C = \begin{pmatrix} 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

Fact: A system in OCF is *always observable*!!

(The proof of this for $n > 2$ uses the Jordan canonical form, we will not worry about this.)

Coordinate Transform & Observability

Just like controllability, observability is preserved under invertible coordinate transformations.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad \xrightarrow{T} \quad \begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u \\ y &= \bar{C}\bar{x} \end{aligned}$$

where $\bar{A} = TAT^{-1}, \quad \bar{B} = TB, \quad \bar{C} = CT^{-1}$

$$\mathcal{O}(\bar{A},\bar{C}) = \begin{pmatrix} \bar{C} \\ \bar{C}\bar{A} \\ \vdots \\ \bar{C}\bar{A}^{n-1} \end{pmatrix} = \begin{pmatrix} CT^{-1} \\ CT^{-1}TAT^{-1} \\ \vdots \\ CT^{-1}TA^{n-1}T^{-1} \end{pmatrix}$$

$$= \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{pmatrix} T^{-1} = \mathcal{O}(A,C)T^{-1}$$

If the original system is observable, then

$$T\underbrace{\left[\mathcal{O}(A,C)\right]^{-1}}_{\text{old}} = \underbrace{\left[\mathcal{O}(\bar{A},\bar{C})\right]^{-1}}_{\text{new}}$$

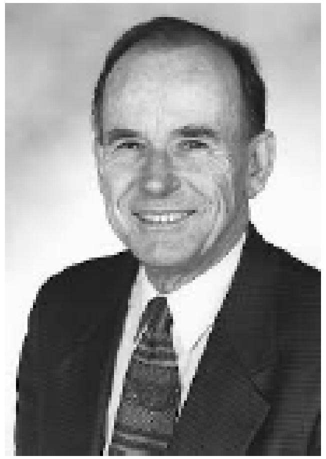
$$\Updownarrow$$

$$T = \underbrace{\left[\mathcal{O}(\bar{A},\bar{C})\right]^{-1}}_{\text{new}} \underbrace{\left[\mathcal{O}(A,C)\right]}_{\text{old}}$$

Next: Observability and State Estimation

As we will show next:

If the system is observable, then there exists an observer (state estimator) that provides an asymptotically convergent estimate $\widehat{x}$ of the state x based on the observed output y.



The particular type of observer we will construct is called the Luenberger observer after David G. Luenberger, who developed this idea in his 1963 Ph.D. dissertation.

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