Instructions

1. Do not start writing until you are instructed to do so.
2. Do not continue to write when you are told to stop.
3. You are not allowed to communicate with one another during the quiz.
4. The quiz is closed-book, closed-notes. You may bring two sheets of notes (each double-sided) with any necessary formulas. A calculator will NOT be necessary NOR helpful.
5. Answer in the answer-sheet and submit both question- and answer-sheet before the end of the quiz.
6. Write your name and student number clearly in the all sheets.
7. There are 2 questions (40 points in total) with sub-questions

Question 1

a) A system can be represented by the block diagram shown in Figure 4.

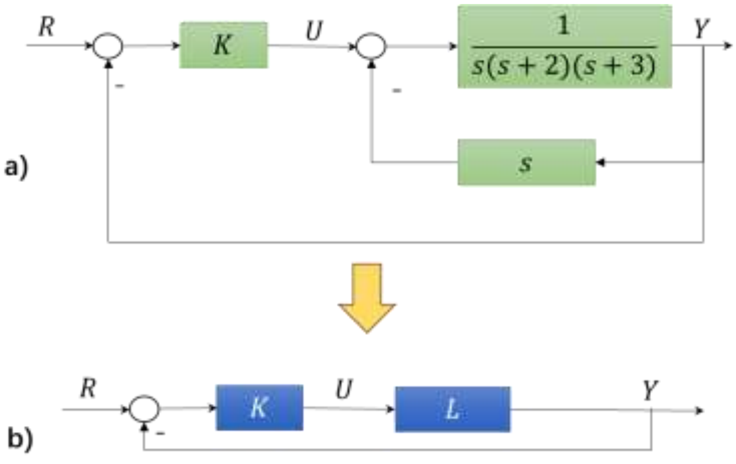


Figure 4

i. Obtain the expression for L in the block diagram (b) reduced from (a). *(2 Points)*

ii. Write down the closed-loop transfer function of the system. *(2 Points)*

iii. Write down the characteristic equation. *(1 Points)*

b) Figure 5 is a plot of the root locus.

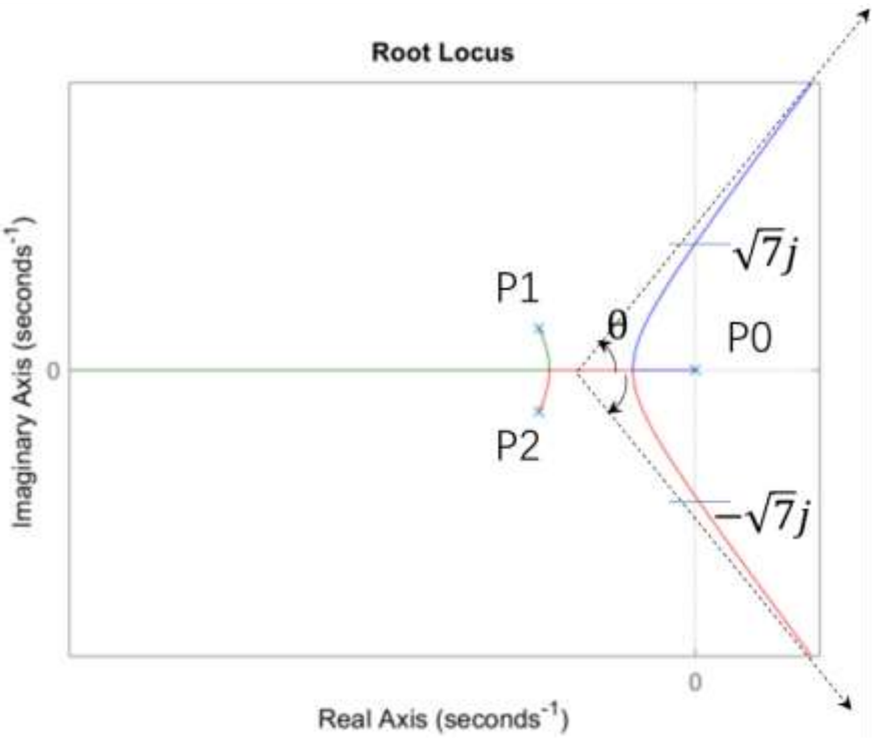


Figure 5

i. Obtain the range of values of K satisfying the Routh-Hurwitz Criteria. *(5 Points)*

ii. Obtain the value for P1 and P2 as indicated in the root locus plot. *(4 Points)*

iii. Obtain the value of θ. *(4 Points)*

iv. Validate using the Routh-Hurwitz Criteria that the $j\omega$-crossing is $\pm\sqrt{7}j$ *(2 Points)*

Question 2

a) A plate attached to a spring and damper with insignificant mass with zero-initial conditions is subjected to a force as shown.

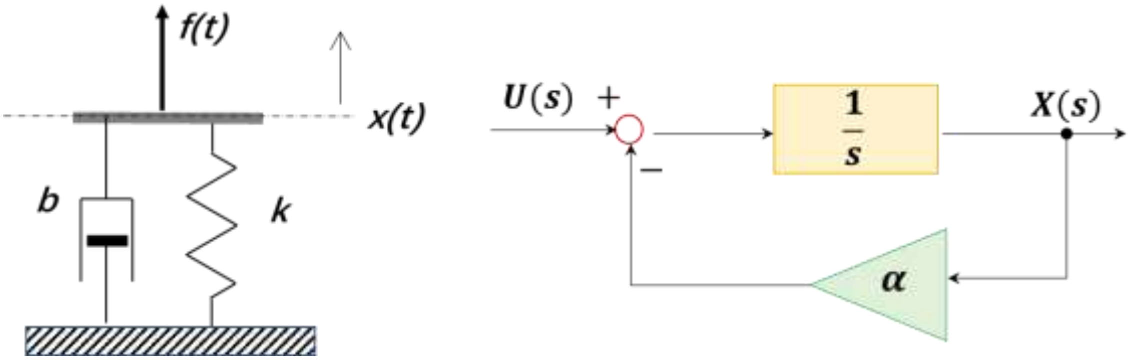


Figure 3b

i) Show that the system can be represented with the given block diagram and provide the expressions of $U(s)$ and α *(2 Points)*

ii) Write down the frequency response function $G(j\omega)$ (2 points)

iii) Express $G(j\omega)$ in terms of its magnitude and phase given *k=b=1*. (2 points)

iv) Sketch the Bode diagrams representing gain $G(j\omega)$ (4 points)

v) Assuming significant plate mass *m=1, b=6, k=5*, rewrite the new transfer function of the plant $G_p(s)$ (1 Points)

A feedback control system is implemented as represented by the shown block diagram.

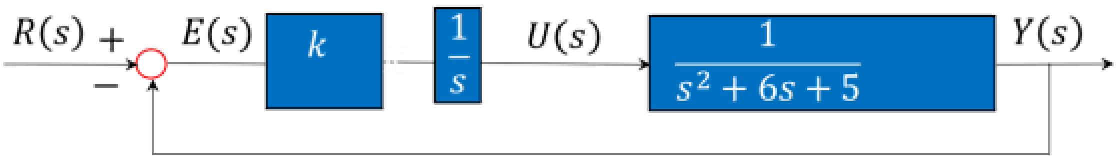


Figure 3c

vi) When K = 10, the bode plot is given by **Figure 3d**. Indicate the frequency values where there are changes in the magnitude slope. (4 Points)

vii) Given the Gain Margin (GM)=+8 dB, Phase Margin (PM)=+21°, on the bode plot on Figure 2, label the Gain Margin and Phase Margin. *(2 Points)*

viii) Comment on how changing the value of *K affect stability using the Bode plot. (3 points)*

Sample for Term Test II

Question 1

a) A system can be represented by the block diagram shown in Figure 4.

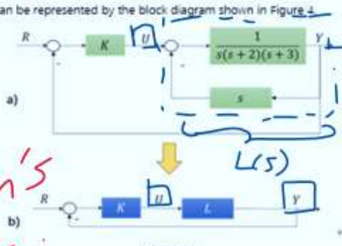


Figure 4

Eran's form

$L(s) = \frac{Y(s)}{U(s)}$

$L(s) = \frac{H_1}{[1 + H_1 H_2]} = \frac{\frac{1}{s(s+2)(s+3)}}{1 + \frac{s}{s(s+2)(s+3)}}$

$= \frac{1}{s(s+2)(s+3)+s}$

$L(s) = \frac{1}{s^3+5s^2+7s}$

$= \frac{1}{[s(s^2+5s+7)]}$

Characteristic eq^n: $[s(s^2+5s+7)] = 0$

i. Obtain the expression for L in the block diagram (b) reduced from (a). (2 Points)
ii. Write down the closed-loop transfer function of the system. (2 Points)
iii. Write down the characteristic equation. (1 Points)

$H_{CL} = \frac{KL}{(1+KL)}$ $= 0$ characteristic equation

$H_{CL} = \frac{K}{s^3+5s^2+7s+K}$

$s^3+5s^2+7s+K=0$

b) Figure 5 is a plot of the root locus.

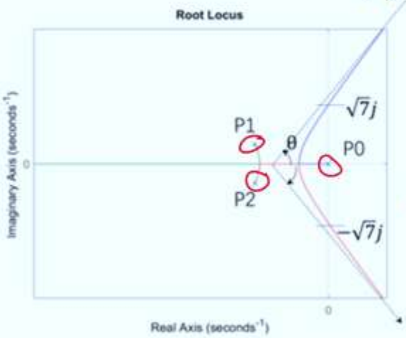


Figure 5

i. Obtain the range of values of K satisfying the Routh-Hurwitz Criteria. (5 Points)
ii. Obtain the value for P1 and P2 as indicated in the root locus plot. (4 Points)
iii. Obtain the value of θ. (4 Points)
iv. Validate using the Routh-Hurwitz Criteria that the $j\omega$-crossing is $\boxed{\pm\sqrt{7}j}$ (2 Points)

i). $H_{CL} = \frac{KL}{1+KL} = \frac{K}{(s^3+5s^2+7s+K)} = 0$

s^3	1	7	
s^2	5	K	$K < 35$
s^1	35-K		$K > 0$
s^0	K		$0 < K < 35$

ii) $s(s^2+5s+7)=0$

$P_0 = S_0 = 0$, $P_{1,2} = S_{1,2} = -\frac{5}{2} \pm \frac{\sqrt{3}}{2}j$

iii) Rule E: $\frac{(2l+1).180°}{m - \cancel{n} \to 0} = 60°, 180°, 300° (= -60°)$
$m = 3$

$\theta = 60°$

v) $K_{critical} = 35$

$s^3+5s^2+7s+35=0$

substitute $s = \pm\sqrt{7}j$ to $LHS = RHS = 0$

Question 2

a) i)

$$f_{external} = f_{damper} + f_{spring}$$
$$f(t) = b\dot{x}(t) + kx(t)$$
$$\frac{f(t)}{b} = \dot{x}(t) + \frac{k}{b}x(t)$$

Letting $\frac{f(t)}{b} = u(t)$, $\frac{k}{b} = \alpha$,

$$u(t) = \dot{x}(t) + \alpha x(t)$$

With zero initial conditions and taking Laplace transform

$$U(s) = sX(s) + \alpha X(s)$$

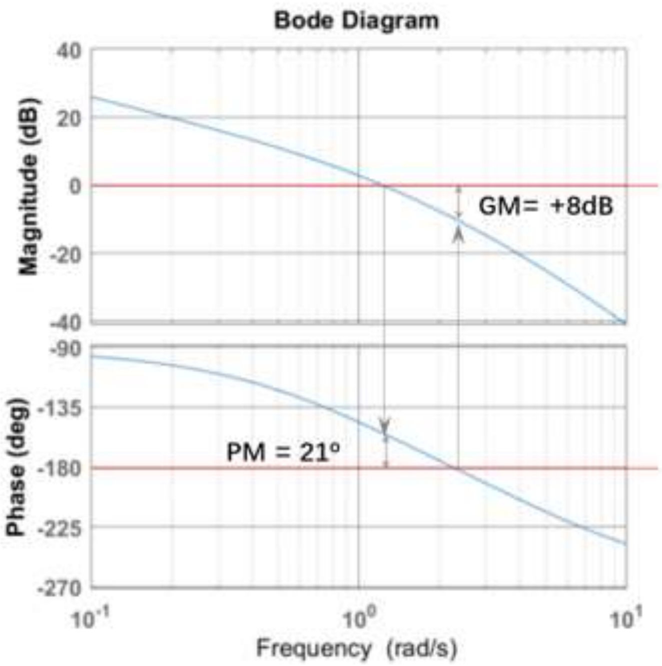
ii)

$$G(j\omega) = \frac{1}{j\omega + \alpha}$$

iii) $|G(j\omega)| = \frac{1}{\sqrt{\omega^2+1}}$ (iv) $\angle G(j\omega) = -\angle(\omega j + 1)$

d) vi) $\omega = 1, 5$

v)



viii) since increasing K shift the magnitude plot downwards but does not change the phase plot, the gain margin will be reduced and eventually become negative and unstable.