DC Gain. steady-state value of the step response.

e.g. $y(t) = \underbrace{\frac{1}{2}1(t)}_{\text{steady-state}} + \underbrace{(2\alpha+\beta-1)e^{-t} + (\frac{1}{2}-\alpha-\beta)e^{-2t}}_{\text{transient}}$.

$\Rightarrow$ gain $= \lim_{t\to\infty} y(t) = \frac{1}{2}1(t) = \frac{1}{2}$.

FVT: Final Value Theorem.

All poles of $\boxed{sY(s)}$ are in OLHS. i.e. have $Re(s)<0$:

$$y(\infty) = \lim_{s\to 0} \boxed{sY(s)}.$$

$\Rightarrow$ if all poles of $sY(s) = H(s)$ are strictly stable, then $y(\infty) = \lim_{s\to\infty} H(s)$.

For high order of [illegible], use Routh-Hurwitz criterion.

State-space Model. high order ODE $\to$ linear.

eigenvalues.

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \quad u = \begin{pmatrix} u_1 \\ \vdots \\ u_m \end{pmatrix}.$$

$$\begin{pmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{pmatrix} = A^{n\times n} \underbrace{\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}}_{\text{state}} + B^{n\times m} \underbrace{\begin{pmatrix} u_1 \\ \vdots \\ u_m \end{pmatrix}}_{\text{input}}. \Rightarrow \dot{x} = Ax + Bu$$

measured output: the one we only cared about.

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

General Procedure of Linearization. ($\dot{x} = f(x,u)$).

Usually require a point at equilibrium: $f(x_0, u_0) = 0$.

$f(x', u') = f(x + x_0, u + u_0) = 0$ with $x' = x + x_0$, $u + u_0$

$\Rightarrow \dot{x}' = \dot{x}_0 = f(x,u). = Ax + Bu$

$\dot{u}' = \dot{u}$.

where $\begin{cases} A_{ij} = \left.\dfrac{\partial f_i}{\partial x_j}\right|_{x=x_0,\, u=u_0} \\ B_{ik} = \left.\dfrac{\partial f_i}{\partial u_k}\right|_{x=x_0,\, u=u_0} \end{cases}$

(comes from: $f(x) = f(x_0) + f'(x_0) f(x - x_0)$.)
Linear approx.