ECE 486 Control Systems

Lecture 11: Dynamic Compensation

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**The syllabus and much of the lecture content originated from UIUC ECE 486 course material

Checklist

Wk	Topic	Ref.
1	✓ Introduction to feedback control	Ch. 1
	✓ State-space models of systems; linearization	Sections 1.1, 1.2, 2.1–2.4, 7.2, 9.2.1
2	✓ Linear systems and their dynamic response	Section 3.1, Appendix A
	✓ Transient and steady-state dynamic response with arbitrary initial conditions	Section 3.1, Appendix A
3	✓ National Holiday Week	
4	✓ System modeling diagrams; prototype second-order system	Sections 3.1, 3.2, lab manual
	✓ Transient response specifications	Sections 3.3, 3.14, lab manual
5	✓ Effect of zeros and extra poles; Routh-Hurwitz stability criterion	Sections 3.5, 3.6
	✓ Basic properties and benefits of feedback control; Introduction to Proportional-Integral-Derivative (PID) control	Section 4.1-4.3, lab manual
6	✓ Review A	
	✓ Term Test A	
7	✓ Introduction to Root Locus design method	Ch. 5
	✓ Root Locus continued; introduction to dynamic compensation	Root Locus
8	☐ **Lead and lag dynamic compensation**	Ch. 5
	✓ Introduction to frequency-response design method	Sections 5.1-5.4, 6.1

Modeling

Analysis

Modeling Analysis Design

Root Locus

Frequency Response

State-Space

Wk	Topic	Ref.
9	Bode plots for three types of transfer functions	Section 6.1
	Stability from frequency response; gain and phase margins	Section 6.1
10	Control design using frequency response	Ch. 6
	Control design using frequency response continued; PI and lag, PID and lead-lag	Frequency Response
11	Nyquist stability criterion	Ch. 6
	Nyquist stability criterion continued; gain and phase margins from Nyquist plots	Ch. 6
12	Review B	
	Term Test B	
13	Introduction to state-space design	Ch. 7
	Controllability, stability, and pole-zero cancellations; similarity transformation; conversion of controllable systems to Controller Canonical Form	Ch. 7
14	Pole placement by full state feedback	Ch. 7
	Observer design for state estimation	State-Space
15	Joint observer and controller design by dynamic output feedback; separation principle	Ch. 7
	In-class review	Ch. 7
16	END OF LECTURES: Revision Week	
	Final	

Lecture Overview

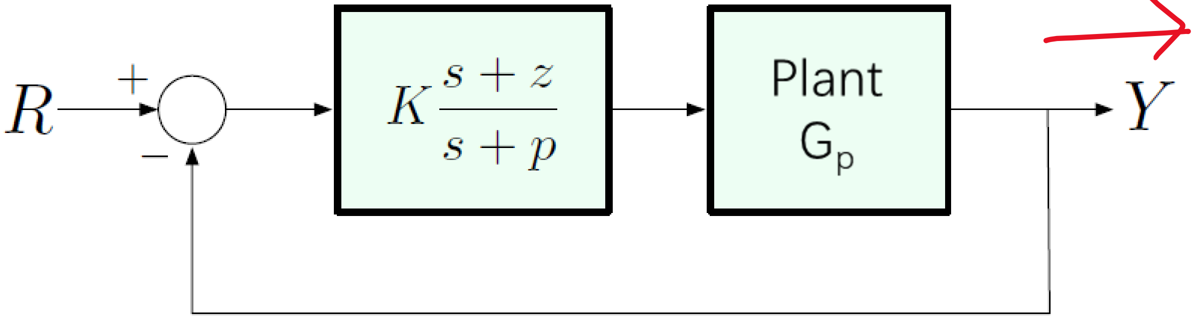
- Review: Root Locus & Dynamic Compensation
- Goal: Introduce the use of lead and lag dynamic compensators for approximate implementation of PD and PI control

PID Control

Reading: FPE, Chapter 5

What is Dynamic Compensation?

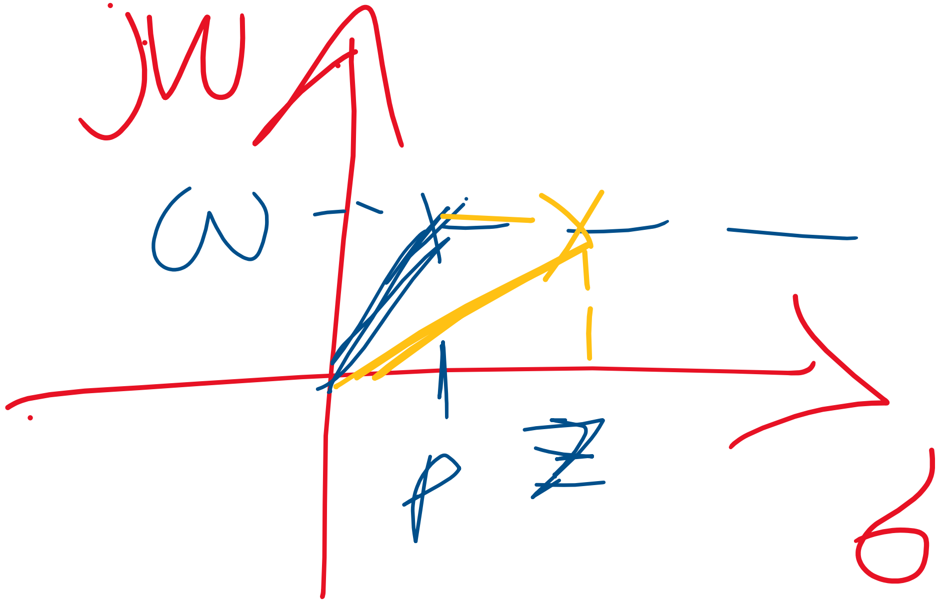
Objectives: stabilize the system and satisfy given time response specs using a *stable*, *causal* controller.



Characteristic equation:

$$1 + K \cdot \frac{s+z}{s+p} \cdot G_p = 1 + KL(s) = 0$$

What is Lead & Lag Compensation



Consider a general controller of the form

$$K\frac{s+z}{s+p} \qquad \text{— } K, z, p > 0 \text{ are design parameters}$$

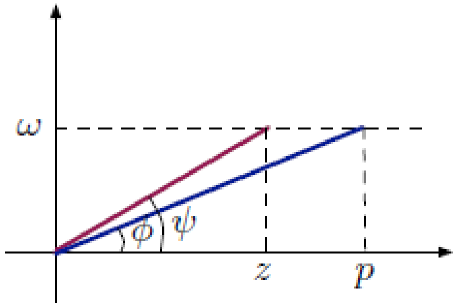
Depending on the relative values of z and p, we call it:

- a lead compensator when $z < p$
- a lag compensator when $z > p$

Why the name "lead/lag?" — think frequency response

$$\angle\frac{j\omega+z}{j\omega+p} = \angle(j\omega+z) - \angle(j\omega+p) = \psi - \phi$$

- if $z < p$, then $\psi - \phi > 0$ (phase lead)
- if $z > p$, then $\psi - \phi < 0$ (phase lag)



Recall: from last week

Using RL to Select Parameter Values

$K \to 0$ to ∞

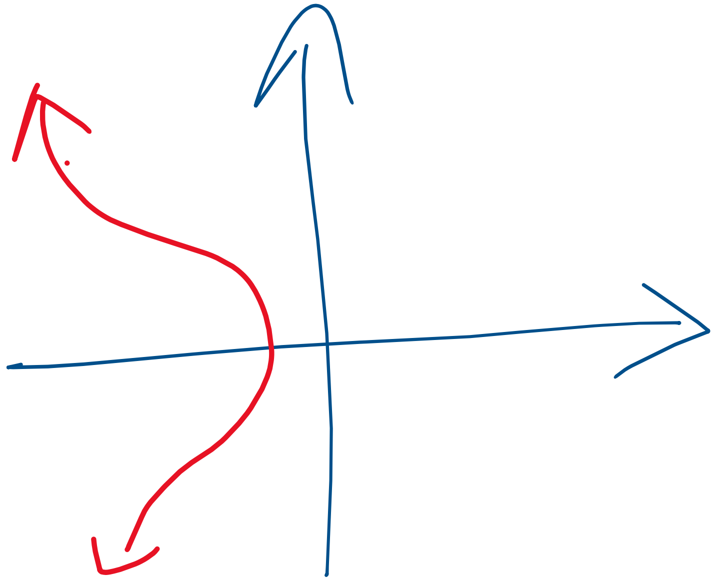
Selecting a value of gain K that corresponds to a desired pole on the root locus

Here is one way of doing it:

$$L(s) = -\frac{1}{K} \qquad \text{— negative real number}$$

$$\Updownarrow$$

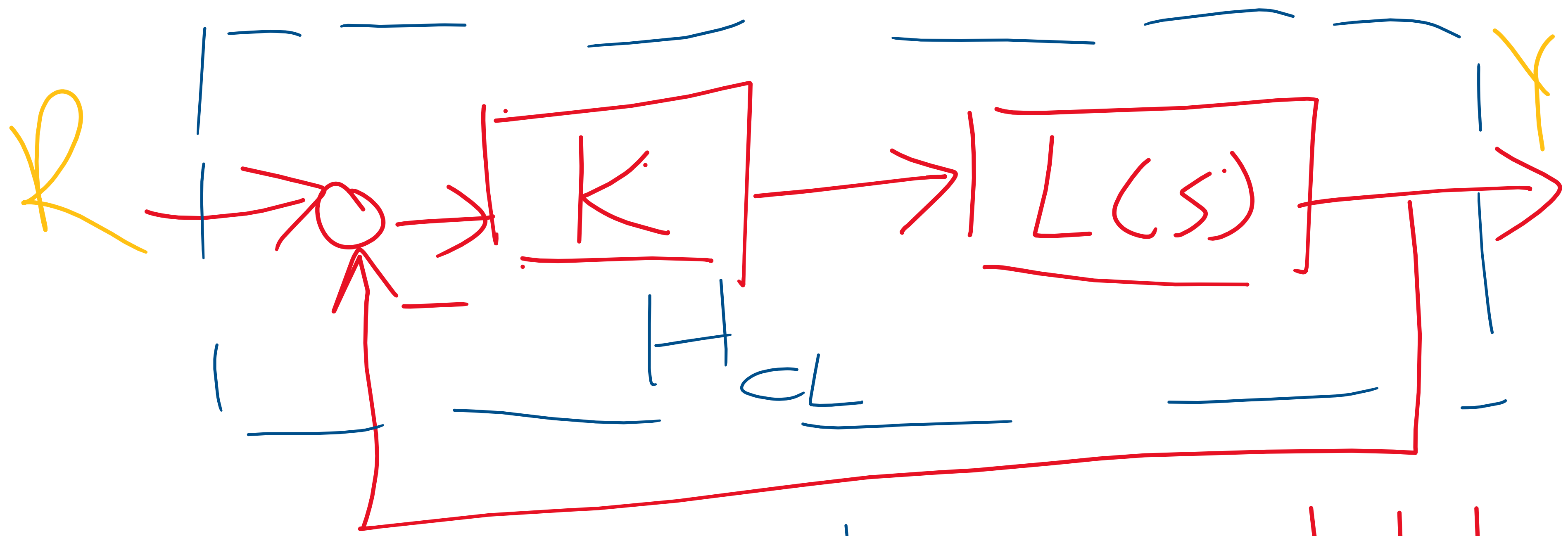
$$K = -\frac{1}{L(s)} = \frac{1}{|L(s)|}$$



$$L(s) = \frac{(s-z_1)\dots(s-z_m)}{(s-p_1)\dots(s-p_n)}$$

$$\Longrightarrow \quad K = \frac{1}{|L(s)|} = \frac{|s-p_1|\dots|s-p_n|}{|s-z_1|\dots|s-z_m|}$$

$1 + KL = 0$



Evan's form

$$H_{CL} = \frac{K L(s)}{(1 + K L(s))} = 0$$

$$1 + K L(s) = 0$$

$$L(s) = -\frac{1}{K}$$

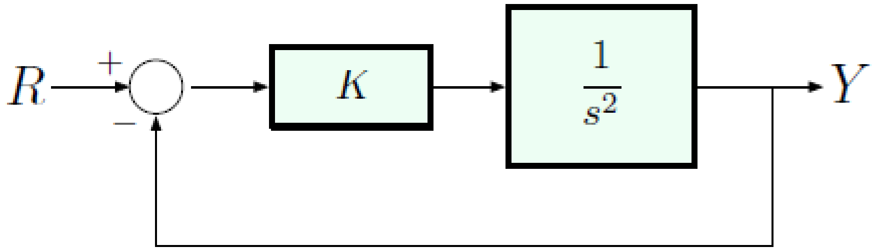
Recall: from last week

Control Design Using Root Locus

Case study: double integrator, transfer function $G(s) = \dfrac{1}{s^2}$

Control objective: ensure stability; meet time response specs.

First, let's try a simple P-gain:

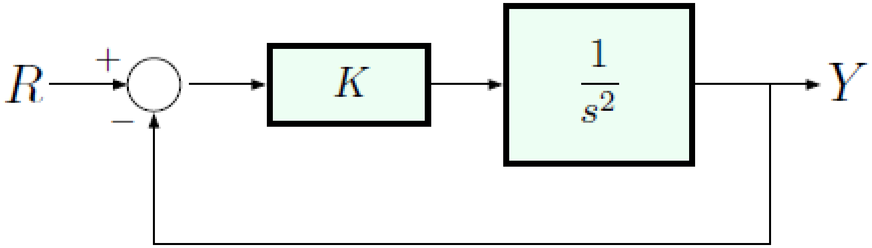


Closed-loop transfer function:

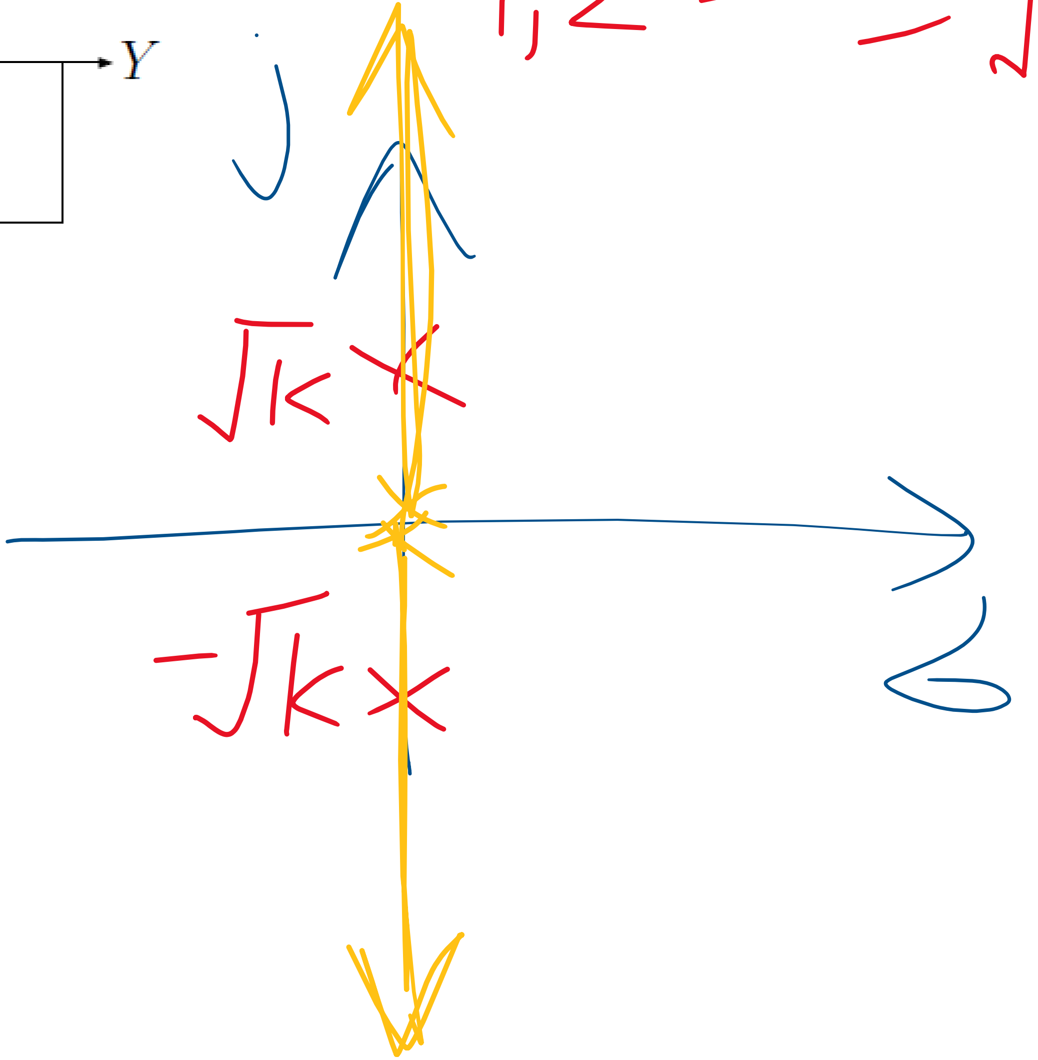
$$\frac{\frac{K}{s^2}}{1+\frac{K}{s^2}} = \frac{K}{(s^2+K)} = 0$$

Recall: from last week

Double Integrator with P-Gain



$s_{1,2} = \pm\sqrt{K}\,j$



Closed-loop transfer function:

$$\frac{\frac{K}{s^2}}{1+\frac{K}{s^2}} = \frac{K}{s^2+K}$$

Characteristic equation:

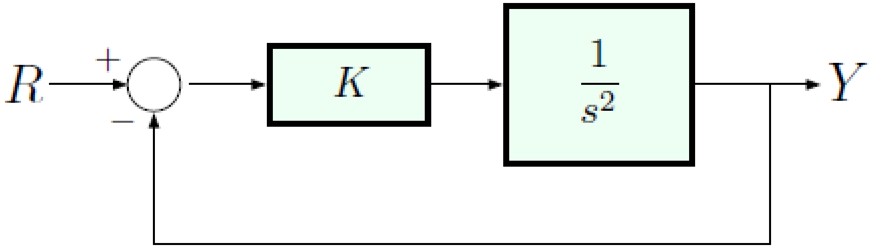
$$s^2 + K = 0$$

Closed-loop poles:

Recall: from last week

Double Integrator with P-Gain

$$\frac{180+360(l-1)}{n-m}$$



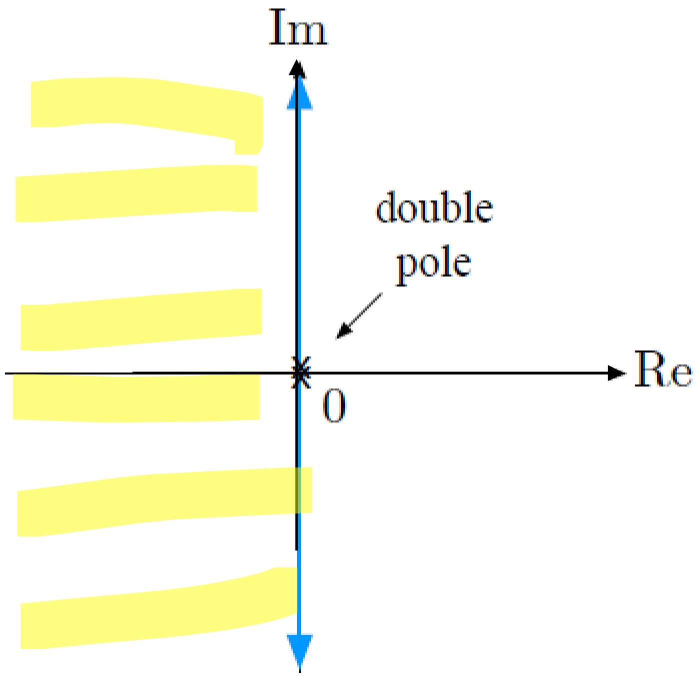
Closed-loop transfer function:

$$\frac{\frac{K}{s^2}}{1+\frac{K}{s^2}} = \frac{K}{s^2+K}$$

Characteristic equation:

$$s^2 + K = 0$$

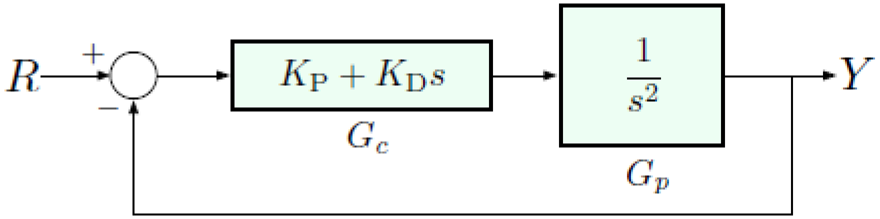
Closed-loop poles: $s = \pm\sqrt{K}j$



This confirms what we already knew: P-gain alone does not deliver stability.

Recall: from last week

Double Integrator with PD-Control



$$\text{Characteristic equation:} \quad 1 + \underbrace{(K_\mathrm{P} + K_\mathrm{D} s)}_{G_c(s)} \cdot \underbrace{\frac{1}{s^2}}_{G_p(s)} = 0$$

$$s^2 + K_\mathrm{D} s + K_\mathrm{P} = 0$$

To use the RL method, we need to convert it into the Evans form $1 + KL(s) = 0$, where $L(s) = \dfrac{b(s)}{a(s)} = \dfrac{s^m + b_1 s^{m-1} + \dots}{s^n + a_1 s^{n-1} + \dots}$

$$1 + (K_\mathrm{P} + K_\mathrm{D} s)\frac{1}{s^2} = 1 + K_\mathrm{D} \cdot \frac{s + K_\mathrm{P}/K_\mathrm{D}}{s^2}$$

$$\Longrightarrow K = K_\mathrm{D}, \; L(s) = \frac{s + K_\mathrm{P}/K_\mathrm{D}}{s^2} \qquad (\text{assume } K_\mathrm{P}/K_\mathrm{D} \text{ fixed}, = 1)$$

Recall: from last week

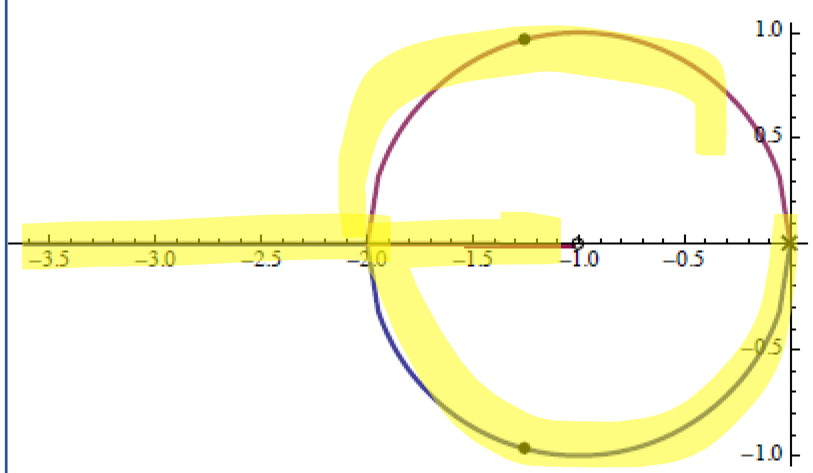
Double Integrator with PD-Control

Characteristic equation: $1 + K \cdot \dfrac{s+1}{s^2} = 0$

Here we can still write out the roots explicitly:

$$s^2 + Ks + K = 0 \qquad \Longrightarrow \qquad s = \frac{-K \pm \sqrt{K^2 - 4K}}{2}$$

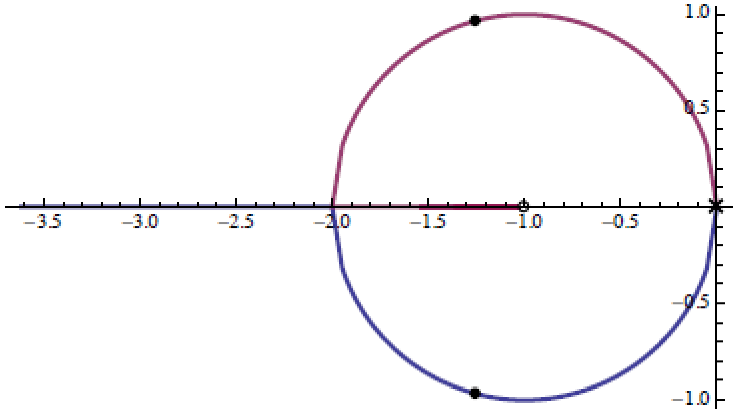
But let's actually draw the RL using the rules:



Recall: from last week

Double Integrator with PD-Control

$$\text{Characteristic equation:} \quad 1 + K \cdot \frac{s+1}{s^2} = 0$$



What can we conclude from this root locus about stabilization?

- all closed-loop poles are in LHP (we already knew this from Routh, but now can visualize)
- nice damping, so can meet reasonable specs

So, the effect of D-gain was to introduce an *open-loop zero* into LHP, and this zero "pulled" the root locus into LHP, thus stabilizing the system.

Dynamic Compensation

We can use RL to *visualize* the effect of adding D-gain: add a LHP zero, pull the closed-loop poles into LHP — stabilization!!

However: we already know that PD control is not physically realizable (lack of causality).

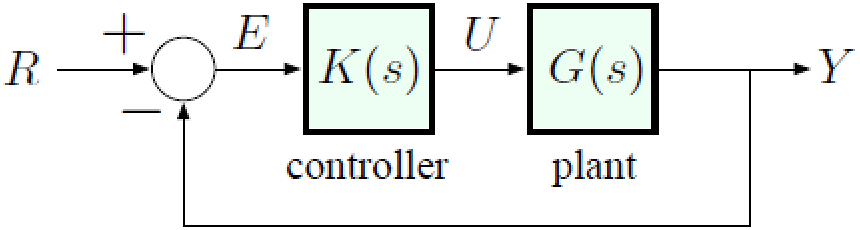
$s = \frac{s^1}{s^0}$

Dynamic compensation (or dynamic control): consider controllers more general than just P-gain, but implementable by *causal systems* of the form

$m > n$

$$\dot{z} = Az + Be$$
$$u = Cz + De$$

$\boxed{n \geq m}$



— so, any proper transfer function is admissible

Recall: from last week

Approximate PD Using Dynamic Compensation

Reminder: we can approximate the D-controller $K_D s$ by

$$K_D \boxed{\frac{ps}{s+p}} \longrightarrow K_D s \text{ as } p \to \infty$$

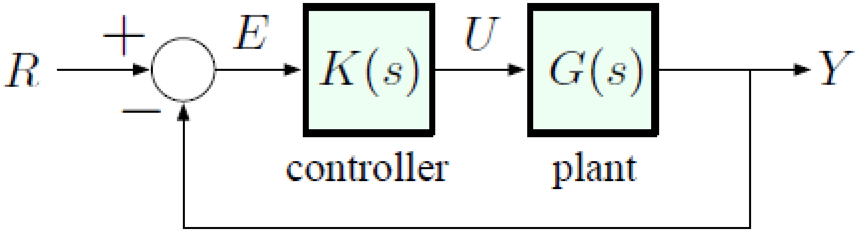
approx: s

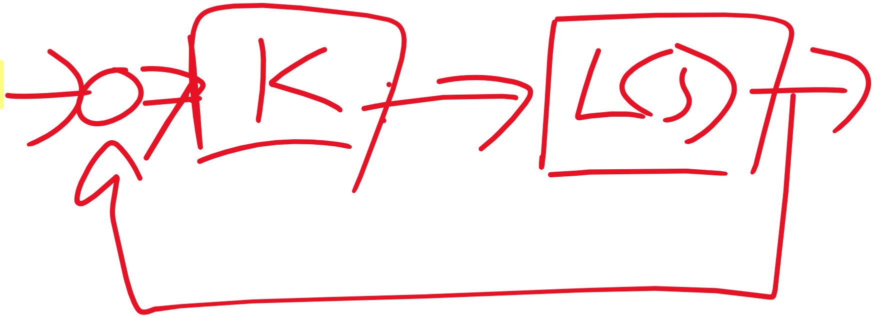
$$\frac{ps/p}{\frac{s}{p}+p/p} = \frac{s}{\frac{s}{p}+1}$$

— here, $-p$ is the *pole* of the controller.

So, we replace the PD controller $K_P + K_D s$ by

$$K(s) = K_P + K_D \frac{ps}{s+p}$$

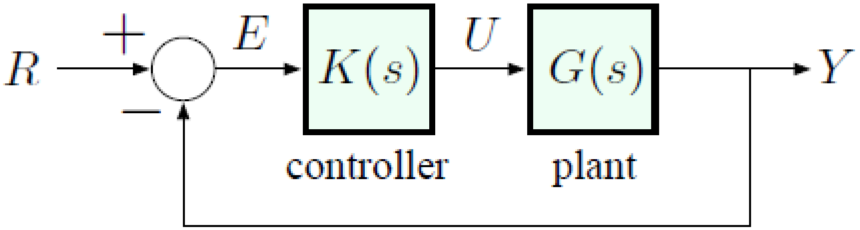




Closed-loop poles: $1 + \left(K_P + K_D \frac{ps}{s+p}\right) G(s) = 0$

Recall: from last week

Approximate PD Using Dynamic Compensation



Closed-loop poles: $1 + \left(K_{\mathrm{P}} + K_{\mathrm{D}} \frac{ps}{s+p} \right) G(s) = 0$

Transform into Evans' canonical form:

$$K_{\mathrm{P}} + K_{\mathrm{D}} \frac{ps}{s+p} = \frac{(K_{\mathrm{P}} + pK_{\mathrm{D}})s + pK_{\mathrm{P}}}{s+p}$$

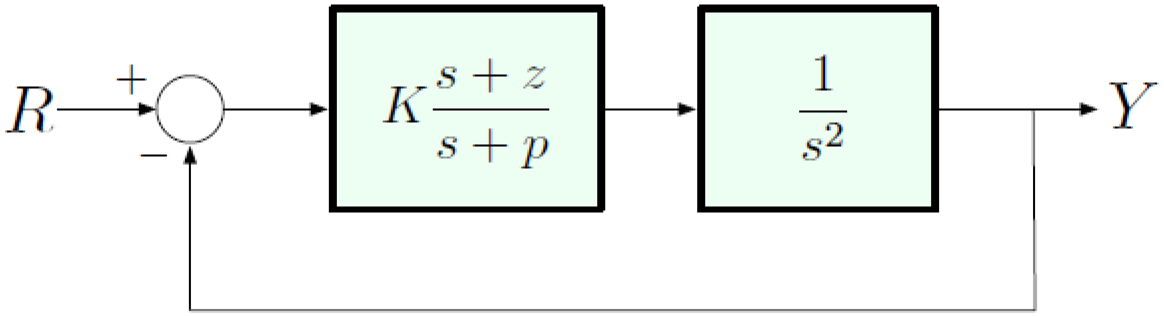
$$= (K_{\mathrm{P}} + pK_{\mathrm{D}}) \cdot \frac{s + \frac{pK_{\mathrm{P}}}{K_{\mathrm{P}}+pK_{\mathrm{D}}}}{s+p}$$

Thus, we can write the controller as $K \cdot \frac{s+z}{s+p}$, where:

- the parameter $K = K_{\mathrm{P}} + pK_{\mathrm{D}}$ is a combination of P-gain, D-gain, and p
- the controller has an open-loop zero at $-z = -\frac{pK_{\mathrm{P}}}{K}$

Approximate PD Using Dynamic Compensation

Double integrator:



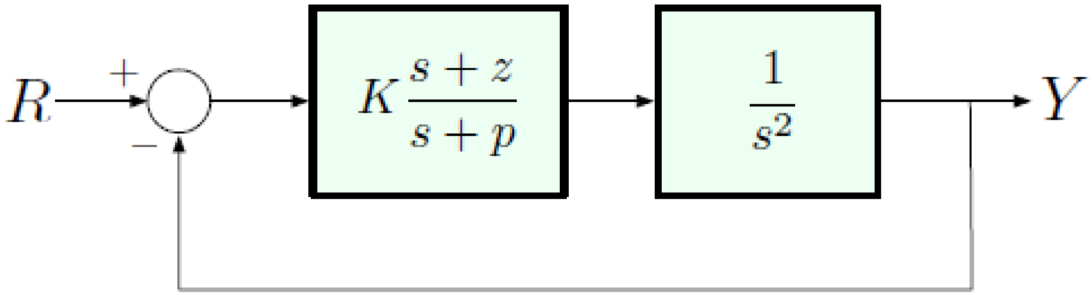
Characteristic equation:

$$1 + K \cdot \frac{s+z}{s+p} \cdot \frac{1}{s^2} = 1 + KL(s) = 0$$

Note: $L(s)$ is *not* the open-loop transfer function; it comes from the forward gain shaped by the controller acting on the plant.

Dynamic Compensation

Objectives: stabilize the system and satisfy given time response specs using a *stable, causal* controller.



Characteristic equation:

$$1 + K \cdot \frac{s+z}{s+p} \cdot \frac{1}{s^2} = 1 + KL(s) = 0$$

Dynamic Compensation to Approx. PD

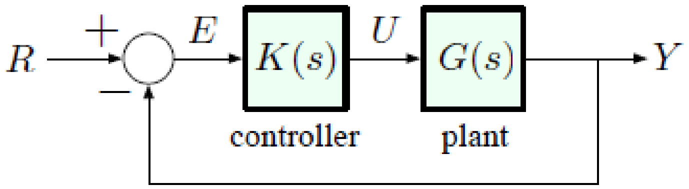
Reminder: we can approximate the D-controller $K_{\mathrm{D}}s$ by

$$K_{\mathrm{D}}\frac{ps}{s+p} \longrightarrow K_{\mathrm{D}}s \text{ as } p \to \infty$$

— here, $-p$ is the *pole* of the controller.

So, we replace the PD controller $K_{\mathrm{P}} + K_{\mathrm{D}}s$ by

$$K(s) = K_{\mathrm{P}} + K_{\mathrm{D}}\frac{ps}{s+p}$$



Closed-loop poles: $1 + \left(K_{\mathrm{P}} + K_{\mathrm{D}}\frac{ps}{s+p}\right)G(s) = 0$

Lead & Lag Compensation

$$\frac{e^a}{e^b} = e^{(a-b)}$$

Consider a general controller of the form

$$K\frac{s+z}{s+p} \qquad \text{— } K, z, p > 0 \text{ are design parameters}$$

Depending on the relative values of z and p, we call it:

- a lead compensator when $z < p$
- a lag compensator when $z > p$

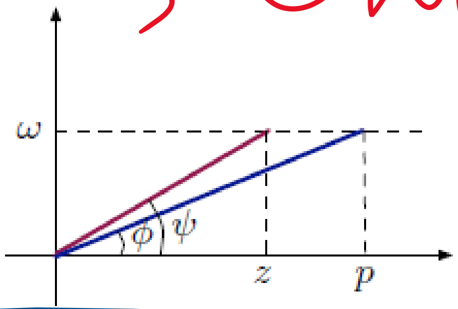
Why the name "lead/lag?" — think frequency response

$$\angle\frac{j\omega+z}{j\omega+p} = \angle(j\omega+z) - \angle(j\omega+p) = \psi - \phi$$

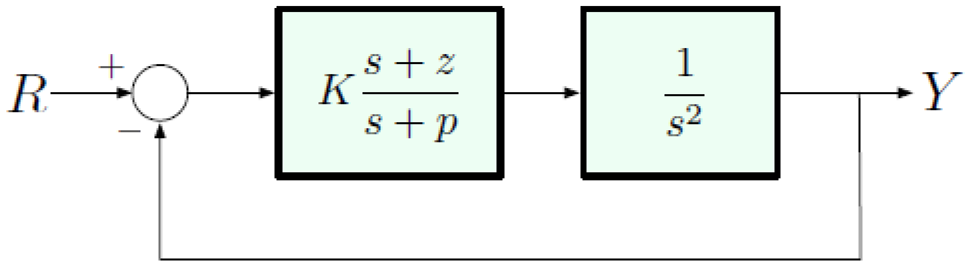
output leads input

// lag //

- if $z < p$, then $\psi - \phi > 0$ (phase lead)
- if $z > p$, then $\psi - \phi < 0$ (phase lag)



Back to Double Integrator



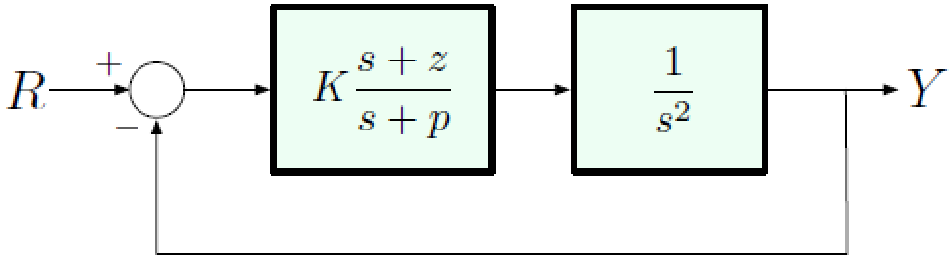
Controller transfer function is $K\dfrac{s+z}{s+p}$, where:

$$K = K_P + pK_D, \qquad z = \frac{pK_P}{K_P + pK_D} \xrightarrow{p\to\infty} \frac{K_P}{K_D}$$

so, as $p \to \infty$, z tends to a constant, so we get a lead controller.

We use lead controllers as dynamic compensators for approximate PD control.

Double integrator & Lead Compensator



To keep things simple, let's set $K_P = K_D$. Then:

$$K = K_P + pK_D = (1+p)K_D$$

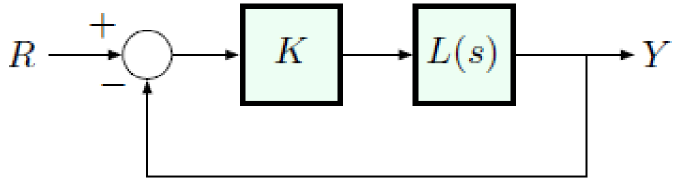
$$z = \frac{pK_P}{K_P + pK_D} = \frac{pK_D}{(1+p)K_D} = \frac{p}{1+p} \xrightarrow{p\to\infty} 1$$

Since we can choose p and z directly, let's take

$$z = 1 \qquad \text{and} \qquad p \text{ large.}$$

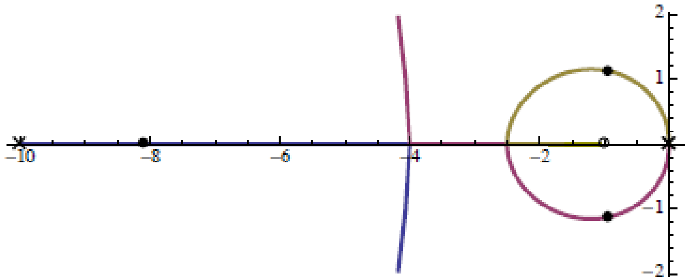
We expect to get behavior similar to PD control.

Double integrator & Lead Compensator



$$L(s) = \frac{s+z}{s+p} \cdot \frac{1}{s^2} \overset{z=1}{=} \frac{s+1}{s^2(s+p)}$$

Let's try a few values of p. Here's $p = 10$:



Close to $j\omega$-axis, this root locus looks similar to the PD root locus. However, the pole at $s = -10$ makes the locus look different for s far into LHP.

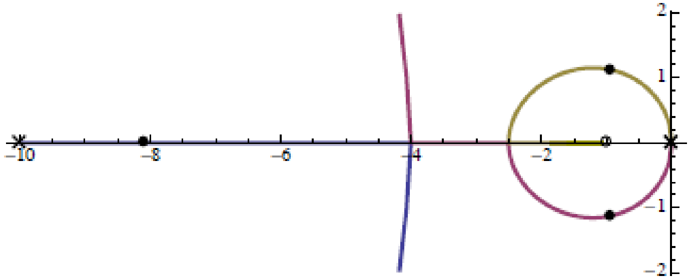
Double integrator & Lead Compensator

$$s^2 + 2\boxed{\zeta}\omega_n s + \omega_n^2$$

Damping ratio

$$L(s) = \frac{s+1}{s^2(s+p)}$$

Root locus for $p = 10$:



The design seems to look good: nice damping, can meet reasonable specs.

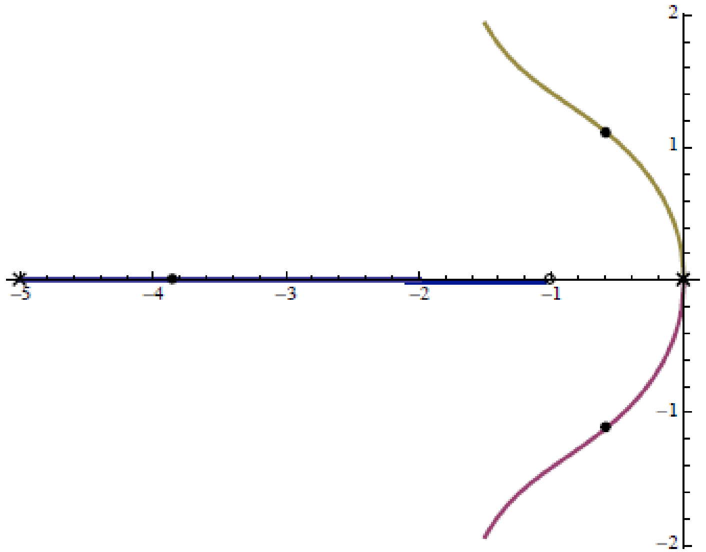
Any concerns with large values of p?

When p is large, we are very close to PD control, so we run into the same issue: noise amplification.
(This is just intuition for now — we will confirm it later using frequency-domain methods.)

Double integrator & Lead Compensator

$$L(s) = \frac{s+1}{s^2(s+p)}$$

Let's try $p = 5$:

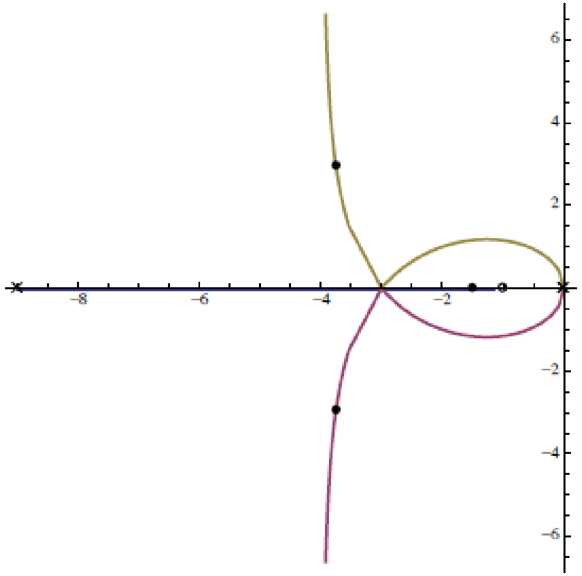


— for this value of p, the root locus is different, not nearly as nicely damped as for $p = 10$.

Double integrator & Lead Compensator

$$L(s) = \frac{s+1}{s^2(s+p)}$$

Let's try p in between $p = 5$ and $p = 10$, say $p = 9$:

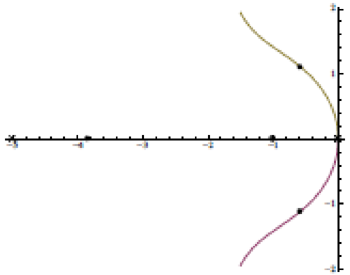


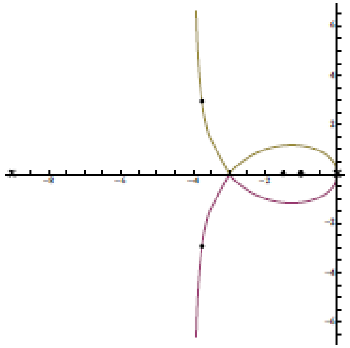
— for this value of p, the branches meet (*break in*) and separate (*break away*) at the same point on the real axis.

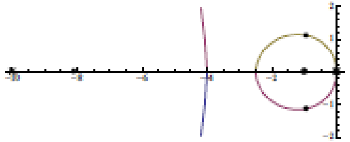
Summary of Design Trade-offs

From what we have seen so far:

- p large — good damping, but bad noise suppression (too close to PD); the branches first break in (meet at the real axis), then break away.
- p small — noise suppression is better, but RL is too close to $j\omega$-axis, which is not good; no break-in for small values of p.
- intermediate values of p — transition between two types of RL; break-in and break-away points are the same.







Lead Controller Design

With a lead controller in place, we have

$$KL(s) = K\frac{s+z}{s+p} \cdot G_p(s)$$

where the lead zero parameter z and lead pole parameter p are constrained to satisfy $z < p$.

In our example with $G_p(s) = 1/s^2$, we have set $z = 1$ to approximate PD control. Then $p > 1$ is our design parameter (and, of course, K is the gain parameter in the root locus).

Alternatively, we can assume that p is given (say, from noise suppression considerations), and we look for z that will give us a desired pole on the RL.

Is there a systematic procedure for doing this?

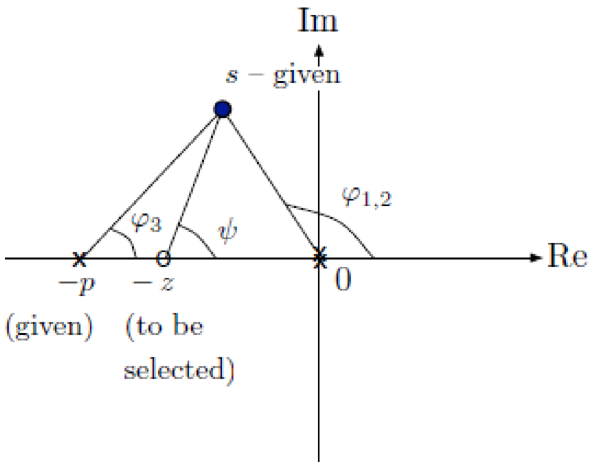
Pole Placement using RL

Back to our example: double integrator with lead compensation

$$KL(s) = K\frac{s+z}{s+p} \cdot \frac{1}{s^2}$$

Problem: given p and a desired closed-loop pole s, find the value of z that will guarantee this (if possible).

Solution: use the phase condition

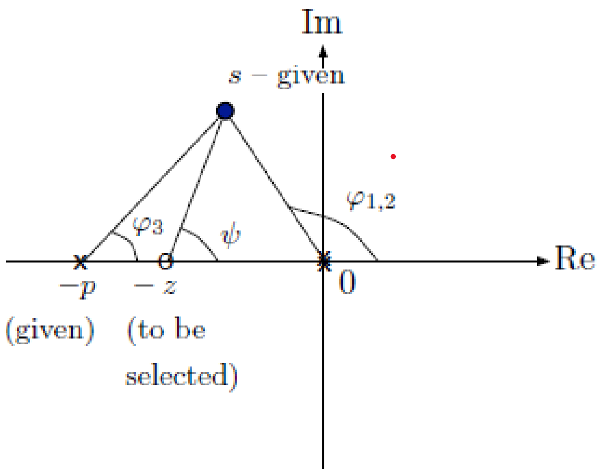


Must have

$$\underbrace{\psi}_{\substack{\text{angle from}\\ s \text{ to zero}}} - \sum_i \underbrace{\varphi_i}_{\substack{\text{angles from}\\ s \text{ to poles}}} = 180^\circ$$

So, we want $\psi = 180^\circ + \sum_i \varphi_i$

Pole Placement using RL



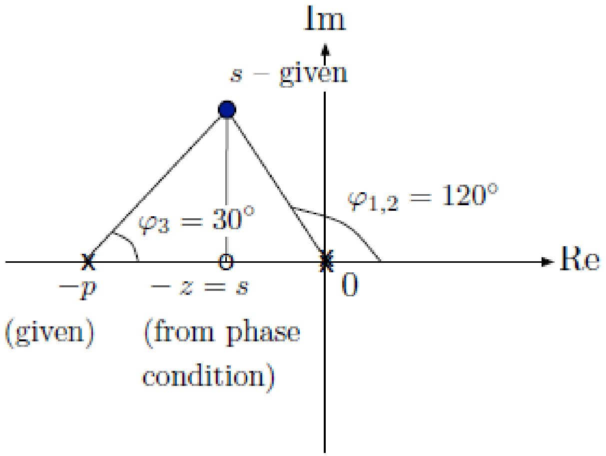
Suppose
$\varphi_1 = \varphi_2 = 120°$,
$\varphi_3 = 30°$.

We want $\psi = 180° + \sum_i \varphi_i$

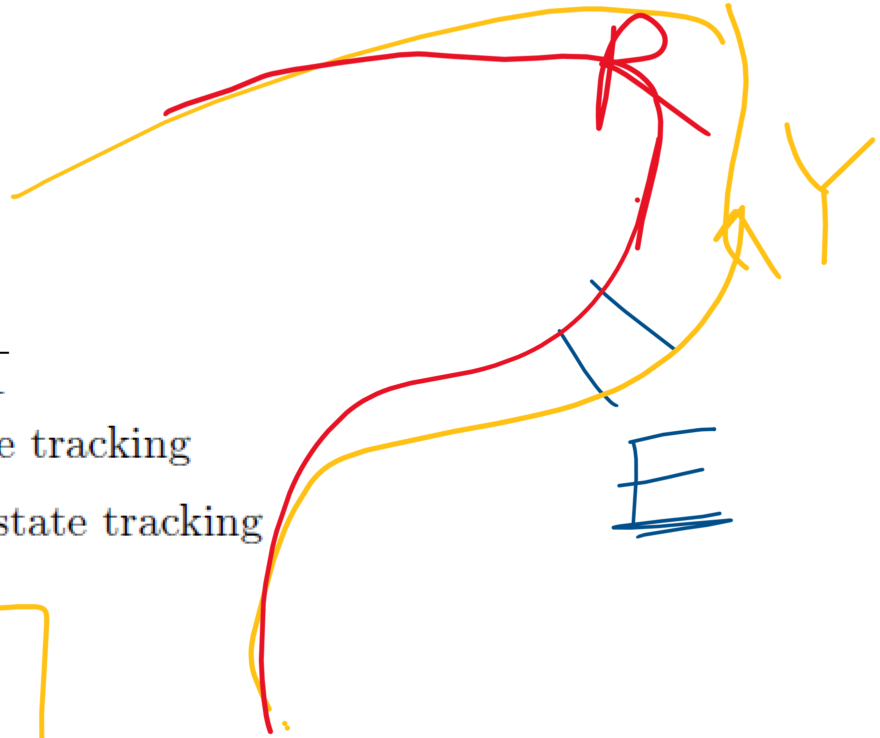
Must have

$$\begin{aligned} \psi &= 180° + 120° + 120° + 30° \\ &= 450° \\ &= 90° \mod 360° \end{aligned}$$

Thus, we should have $z = -s$



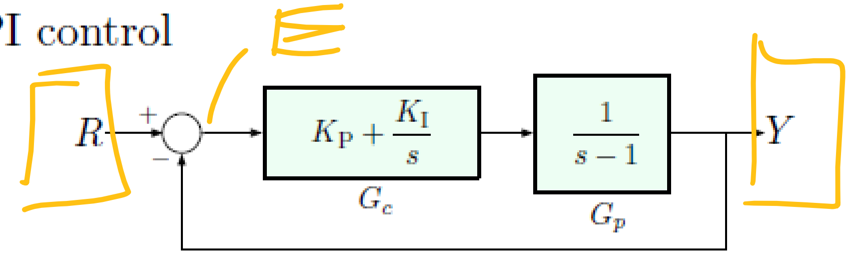
Control Design using RL



Case study: plant transfer function $G_p(s) = \dfrac{1}{s-1}$

Control objective: stability and constant reference tracking

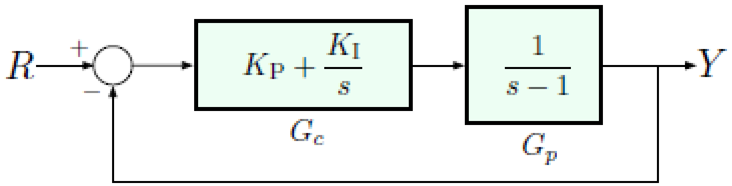
In earlier lectures, we saw that for perfect steady-state tracking we need PI control



Closed-loop poles are determined by:

$$1 + \left(K_P + \frac{K_I}{s}\right)\left(\frac{1}{s-1}\right) = 0$$

$$L(s) = \frac{s+1}{s(s-1)}$$



Characteristic equation: $$1 + \underbrace{\left(K_P + \frac{K_I}{s}\right)}_{G_c(s)} \underbrace{\left(\frac{1}{s-1}\right)}_{G_p(s)} = 0$$

To use the RL method, we need to convert it into the Evans form $1 + KL(s) = 0$, where $L(s) = \frac{b(s)}{a(s)} = \frac{s^m + b_1 s^{m-1} + \dots}{s^n + a_1 s^{n-1} + \dots}$

$$1 + \left(K_P + \frac{K_I}{s}\right)\frac{1}{s-1} = 1 + \frac{K_P s + K_I}{s}\frac{1}{s-1}$$

$$= 1 + K_P \frac{s + K_I/K_P}{s(s-1)}$$

$$\Longrightarrow \quad K = K_P, \quad \boxed{L(s) = \frac{s + K_I/K_P}{s(s-1)}} \qquad \text{(assume } K_I/K_P \text{ fixed, } = 1)$$

Root Locus for PI Compensation

$$L(s) = \frac{s+1}{s(s-1)}$$

Rule A: 2 branches

Rule B: branches start at $p_1 = 0, p_2 = 1$ (RHP!!)

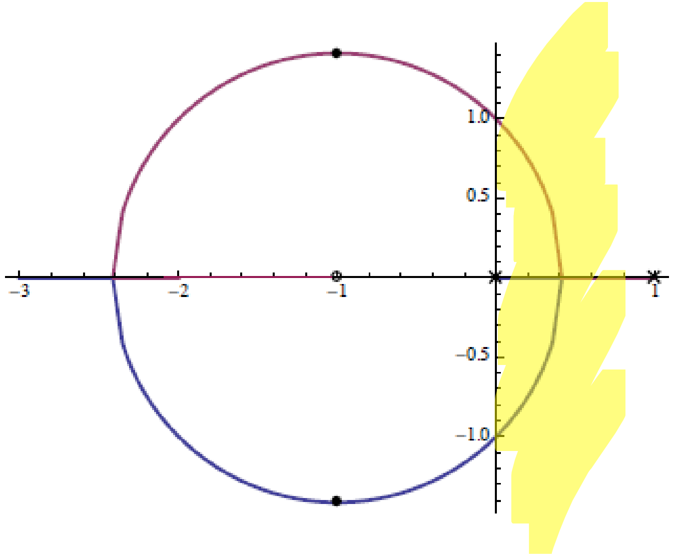
Rule C: branches end at $z_1 = -1, \pm\infty$

Rule D: real locus $= [0,1], (-\infty, -1]$

Rule E: asymptote at 180°

Rule F: $j\omega$-crossings:

$$a(s) + Kb(s) = 0$$
$$s(s-1) + K(s+1) = 0$$
$$s^2 + (K-1)s + K = 0$$
$$K_{\text{critical}} = 1 \implies \omega_0 = 1$$



- The system is stable for $K > 1$ (from Routh-Hurwitz)
- For very large K, we get a completely damped system, with *negative real poles*
- Perfect steady-state tracking of constant references:

$$\frac{E}{R} = \frac{1}{1 + G_c G_p}$$
$$= \frac{s(s-1)}{s(s-1) + K(s+1)}$$

DC gain$(R \to E) = 0$ (for $K > 1$)

- However: $1/s$ is not a stable element.

Approximate PI via Dynamic Compensation

PI control achieves the objective of stabilization and perfect steady-state tracking of constant references; however, just as with PD earlier, we want a *stable controller*.

Here's an idea:

$$\text{replace } K\frac{s+1}{s} \quad \text{by} \quad K\frac{s+1}{s+p}, \text{ where } p \text{ is small}$$

More generally, if $z = K_{\mathrm{I}}/K_{\mathrm{P}}$, then

$$\text{replace } K\frac{s+z}{s} \quad \text{by} \quad K\frac{s+z}{s+p}, \text{ where } p < z$$

This is lag compensation (or lag control)!

We use lag controllers as dynamic compensators for approximate PI control.

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We use lag controllers as dynamic compensators for approximate PI control.

Approximate PI via Lag Compensation

$$G_c(s) = K\frac{s+z}{s+p}, \quad p < z \qquad\qquad G_p(s) = \frac{1}{s-1}$$

How good is this controller?

Tracking a constant reference: assuming closed-loop stability, the FVT gives

$$e(\infty) = \left.\frac{1}{1+G_c(s)G_p(s)}\right|_{s=0} = \left.\frac{1}{1+K\frac{s+z}{(s+p)(s-1)}}\right|_{s=0} = \frac{1}{1-\frac{Kz}{p}}$$

Check for stability: no RHP poles for $\dfrac{1}{1+G_c(s)G_p(s)}$

$$(s+p)(s-1) + K(s+z) = 0$$
$$s^2 + (K+p-1)s + Kz - p = 0$$

Conditions for stability: $K > 1-p, \;\; Kz > p$

Approximate PI via Lag Compensation

Tracking a constant reference: if the stability conditions

$$K > 1 - p, \quad Kz > p$$

are satisfied, then the steady-state error is

$$e(\infty) = \frac{1}{1 - \frac{Kz}{p}}$$

— this will be close to zero (and negative) if $\frac{Kz}{p}$ is large.

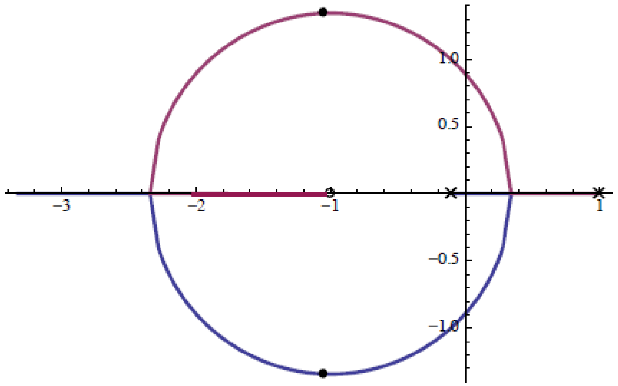
Lag compensation *does not* give perfect tracking (indeed, it does not change system type), but we can get as good a tracking as we want by playing with K, z, p. On the other hand, unlike PI, lag compensation gives a stable controller.

Effect of Lag Compensation Root Locus

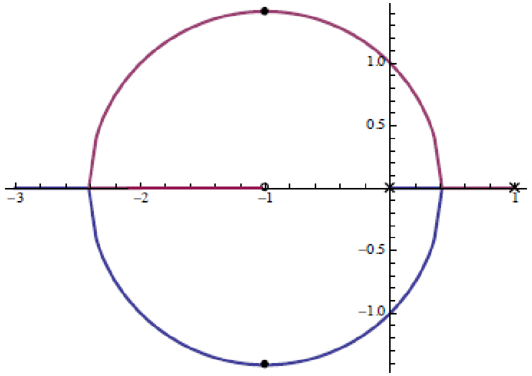
$$L(s) = \frac{s+1}{(s+p)(s-1)}$$

Intuition: By choosing p very close to zero, we can make the root locus arbitrarily close to PI root locus (stable for large enough K). Let's check:

Try $p = 0.1$



Compare to PI root locus:



What do we see? Compared to PD vs. lead, there is no qualitative change in the shape of RL, since we are not changing #(poles) or #(zeros).

More Pole Placement

As before, we can choose z_{lag} for a fixed p_{lag} (or vice versa) based on desired pole locations.

The procedure is exactly the same as the one we used with lead. (In fact, depending on the pole locations, we may end up with either lead or lag.)

Main technique: select parameters to satisfy the phase condition (points on RL must be such that $\angle L(s) = 180°$).

Caveat: may not always be possible!

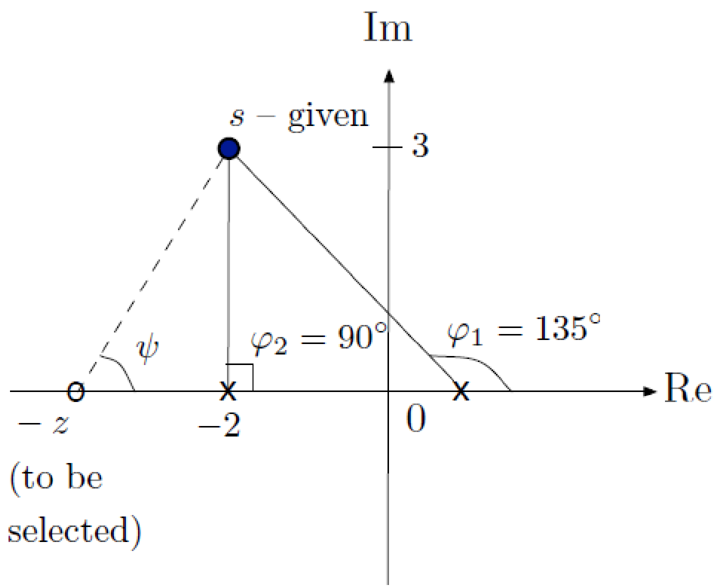
Pole Placement Via RL

Let $G_p(s) = \dfrac{1}{s-1}, \qquad G_c(s) = K\dfrac{s+z}{s+p}$

Problem: given $p = 2$, find K and z to place poles at $-2 \pm 3j$.

Desired characteristic polynomial:

$$(s+2)^2 + 9 = s^2 + 4s + 13, \qquad \text{damping ratio } \zeta = \frac{2}{\sqrt{13}} \approx 0.555$$

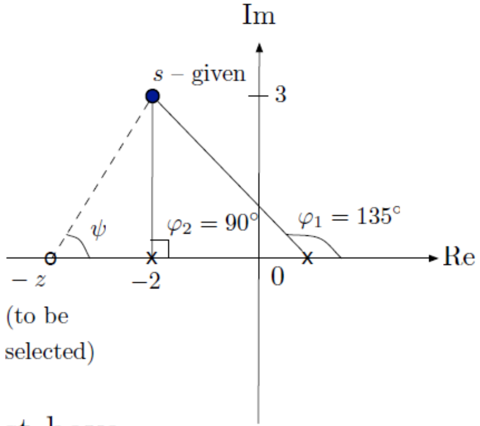


Must have

$$\underbrace{\psi}_{\substack{\text{angle from} \\ s \text{ to zero}}} - \sum_i \underbrace{\varphi_i}_{\substack{\text{angles from} \\ s \text{ to poles}}} = 180°$$

So, we want $\psi = 180° + \sum_i \varphi_i$

Pole Placement Via RL



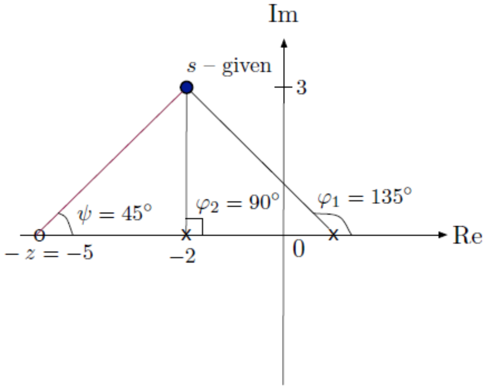
We have
$\varphi_1 = 135°,$
$\varphi_2 = 90°$

We want $\psi = 180° + \sum_i \varphi_i$

Must have

$$\begin{aligned}\psi &= 180° + 135° + 90° \\ &= 405° \\ &= 45° \mod 360°\end{aligned}$$

Thus, we should have $z = -5$



Pole Placement Via RL

Let $\quad G_p(s) = \dfrac{1}{s-1}, \qquad G_c(s) = K\dfrac{s+z}{s+p}$

Problem: given $p = 2$, find z to place poles at $-2 \pm 3j$.

Solution:

- we already found that we need $z = 5$
- resulting characteristic polynomial:

$$
\begin{gathered}
(s-1)(s+2) + K(s+5) \\
s^2 + (K+1)s + 5K - 2
\end{gathered}
$$

- compare against desired characteristic polynomial:

$$
s^2 + 4s + 13 \qquad \Longrightarrow \qquad K+1 = 4,\ 5K - 2 = 13
$$

so we need $K = 3$

- compute s.s. tracking error: $\left|\dfrac{1}{1-\frac{Kz}{p}}\right| = \dfrac{1}{6.5} \approx 15\%$

Story So Far

PD control:

- provides stability, allows to shape transient response specs
- replace noncausal D-controller Ks with a causal, stable lead controller $K\dfrac{s+z}{s+p}$, where $p > z$
- this introduces a zero in LHP (at $-z$), pulls the root locus into LHP
- shape of RL differs depending on how large p is

PI control:

- provides stability and perfect steady-state tracking of constant references
- replace unstable I-controller K/s with a stable lag controller $K\dfrac{s+z}{s+p}$, where $p < z$
- this does not change the shape of RL compared to PI

What about PID Control

Obvious solution — combine lead *and* lag compensation.

We will develop this further in homework and later in the course using frequency-response design methods — which are the subject of several lectures, starting with today's.

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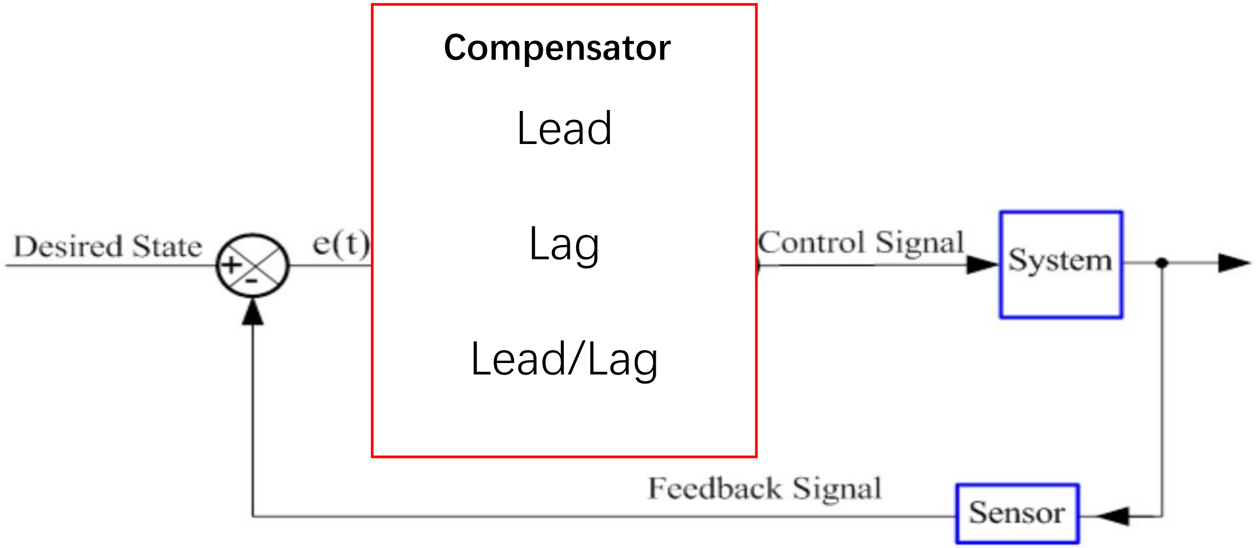
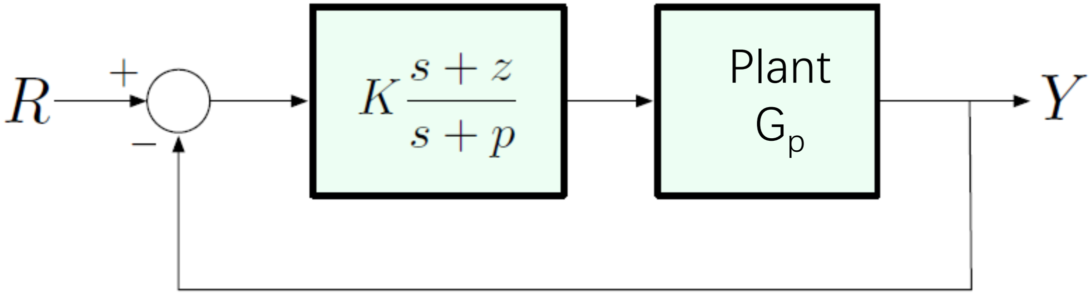
PID: Combine Lead and Lag



Dynamic Compensation

Objectives: stabilize the system and satisfy given time response specs using a *stable*, *causal* controller.



Characteristic equation:

$$1 + K \cdot \frac{s+z}{s+p} \cdot G_p = 1 + KL(s) = 0$$