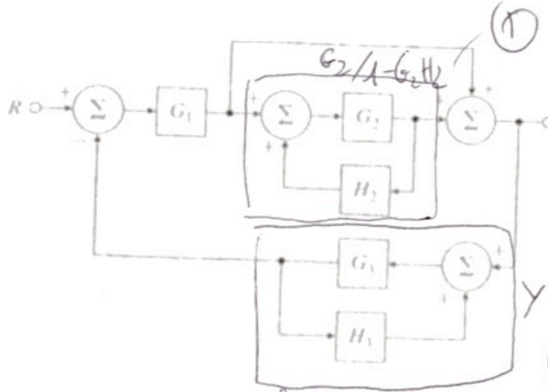
Reading: FPE (Franklin, Powell, Emami-Naeini, 6th or 7th edition), Sections Sections 3.1 and 3.2. Sections 3.3–3.6.

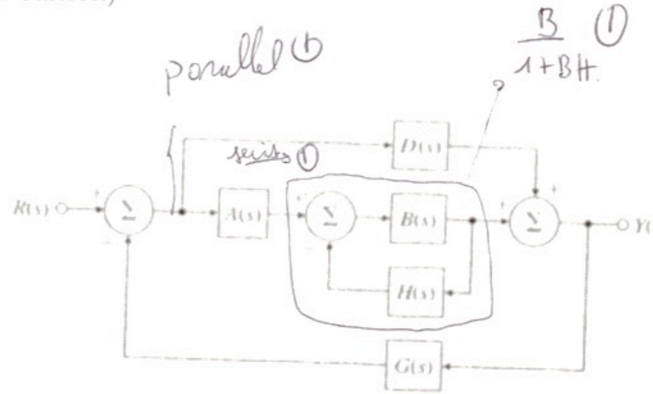
Problems:

1. Using techniques for block diagram reduction discussed in class, find the transfer functions of the systems shown below (p156 from the textbook, 3rd edition)



(a)

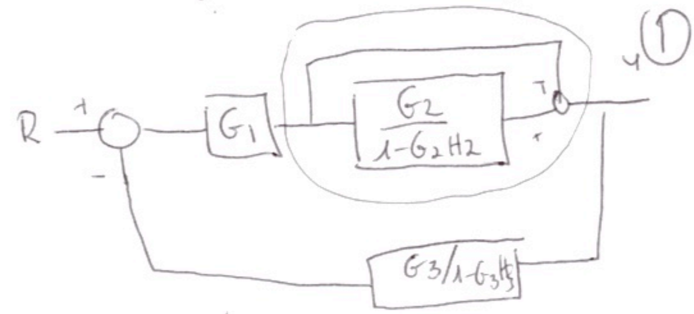
feedback w/ $-H_3$

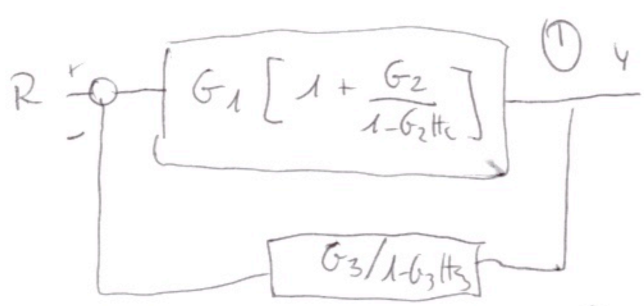


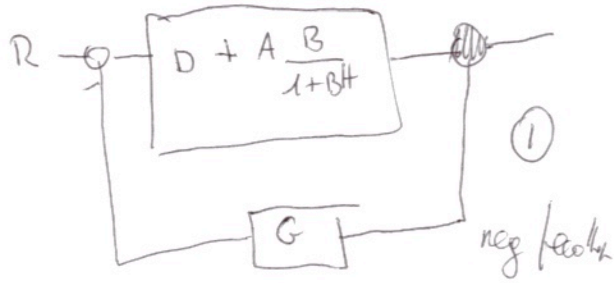
(b)

$$\frac{G_3}{1-G_3H_3}$$

// connection







neg feedback

$$\Rightarrow \frac{D + A\frac{B}{1+BH}}{1+G\left[D + \frac{AB}{1+BH}\right]} \quad (1)$$

feedback

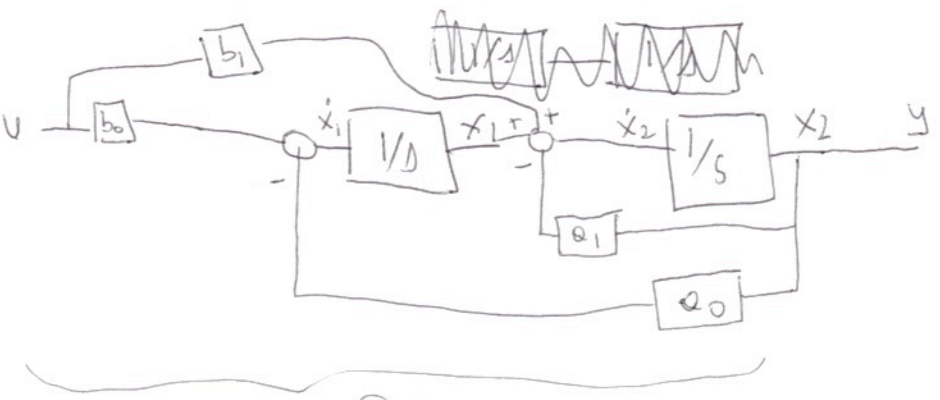
$$\Rightarrow \frac{G_1\left[1+\frac{G_2}{1-G_2H_2}\right]}{1+G_1\left[1+\frac{G_2}{1-G_2H_2}\right]\frac{G_3}{1-G_3H_3}} \quad (1)$$

2. Consider the following state-space model (so-called "observer canonical form"):

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}0 & -a_0\\ 1 & -a_1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}b_0\\ b_1\end{pmatrix}u, \qquad y = \begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}.$$

Build an all-integrator diagram for this system.

2nd order → 2 integrators. Create signals $\dot{x}_1$ & $\dot{x}_2$ to feed into integrators



5

3. Consider the plant with transfer function $L(s) = \dfrac{1}{s^2 + 2s + K}$ where K is a positive parameter you can tune.

a) Consider the settling time spec $t_s \le 4$. Give some value (or range of values) of K for which the system meets this spec. Justify your choice.

b) Consider the rise time spec $t_r \le 1$. Give some value (or range of values) of K for which the system meets this spec.

c) Consider the overshoot spec $M_p \le 0.1$. Give some value (or range of values) of K for which the system meets this spec. Justify your choice.

Prototype: $\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ ← scale factor $\Rightarrow$ $2\zeta\omega_n = 2$, $\omega_n^2 = K$ $\Rightarrow$ $\omega_n = K^{1/2}$, $\zeta = K^{-1/2}$

2

* $y(\infty) = 1/K$

a) $t_s \simeq 3/\sigma$ with $\sigma = \zeta\omega_n$ ($\Rightarrow \sigma = 1$)

$3/1 \le 4$. All values of $K > 0$ are ok! 1

b) $t_r \simeq 1.8/\omega_n = 1,8/\sqrt{K}$ $\Rightarrow t_r \le 1 \iff 1.8 \le \sqrt{K} \rightarrow K \ge 1.8^2$ 1

c) $M_p \le .1 \Rightarrow -\dfrac{\pi K^{-1/2}}{\sqrt{1-K^{-1}}} \le \log .1 \Rightarrow \dfrac{-\pi}{\sqrt{K-1}} \le -2.3 \Rightarrow \sqrt{K-1} \le \pi/2.3 \Rightarrow K \le 1 + \left[\dfrac{\pi}{2.3}\right]^2$ 1