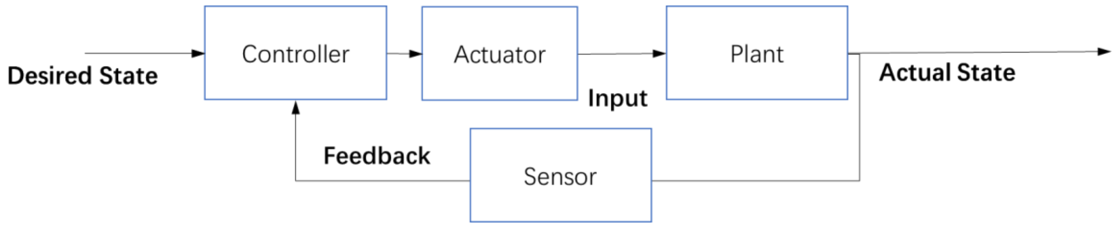
ECE 486: Control Systems Assignment # 00

Issued: Oct 01 Due: Oct 08, 2023

Question 1 (6 points)

Give an example of a closed loop control system. Using your example, explain the following terms associated with the control system represented by Figure 1:

a) Plant
b) Sensors
c) Actuator
d) Desired State
e) Actual State
f) Feedback



Question 2 (9 points)

Given $z = \frac{1}{j}\left(\frac{1-j}{2+2j} - \frac{1+j}{2-2j}\right)$

a) Write z in the form $\alpha + \beta j$
b) Sketch z in the complex plane
c) Obtain the inverse of z in polar form
d) Given $x^3 = -8$, find the complex values of x that satisfy the equation.

Question 3 (5 points)

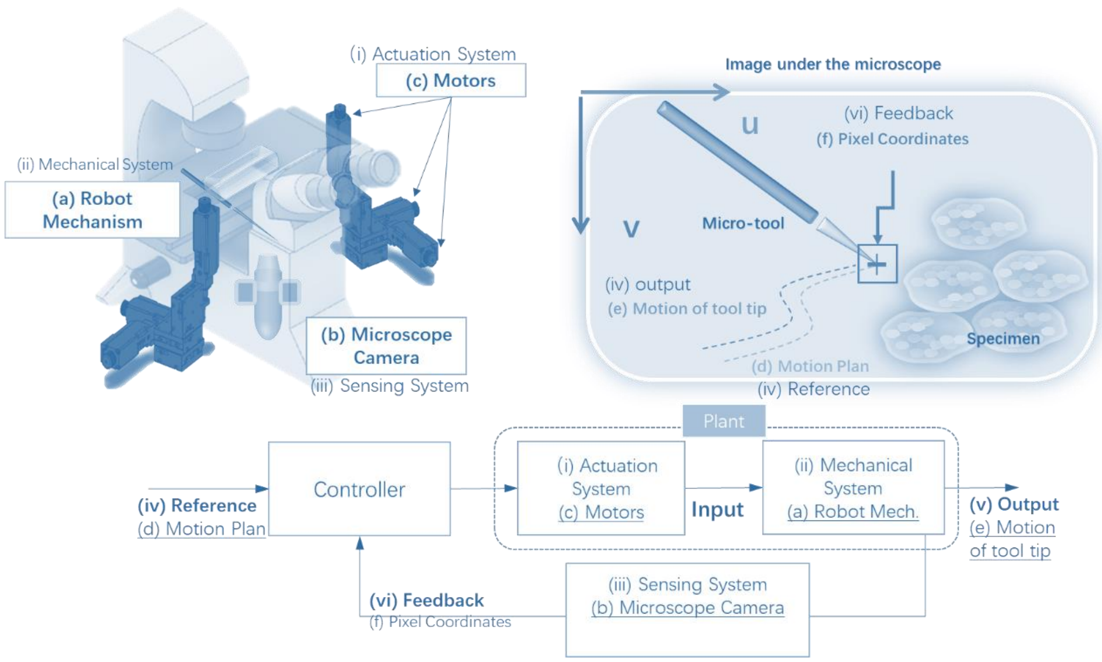
Consider the following differential equation:

$\ddot{x}(t) + 5\dot{x}(t) + 2x(t) = 0$. Find all values of λ such that $x(t) = e^{\lambda t}$ satisfies the above differential equation.

Suggested Solution

Q1:As discussed in lecture, any other example similar to the following is acceptable.

The microscope image-guided robotic system is an example of a control system with the **robot mechanism actuated by motors to produce motion of the tool tip,** which becomes an **output measured by a microscope camera sensor as feedback in pixel coordinates** to a controller that, **based on a motion plan as the reference,** in turn generates an actuator input to the motors to drive the robot mechanism based on the feedback and a motion plan as the reference.



Q2:

Given $z = \frac{1}{j}\left(\frac{1-j}{2+2j} - \frac{1+j}{2-2j}\right)$

a) Write z in the form $\alpha + \beta j$

b) Sketch z in the complex plane

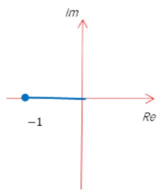
c) Obtain the inverse of z in polar form

d) Given $x^3 = -8$, find the complex values of x that satisfy the equation.

a)

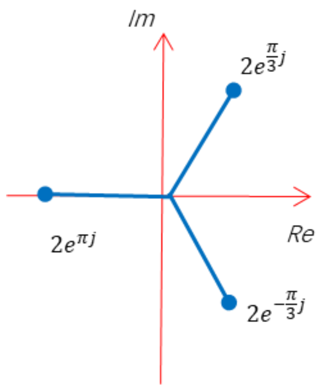
$$z = \frac{(1-j)(2-2j) - (1+j)(2+2j)}{j(2+2j)(2-2j)} = \frac{-4j-4j}{j(4+4)} - 1$$

b)



c) $z^{-1} = e^{j(-\pi)} = e^{j\pi}$

d) $x_1 = 2e^{j\pi} = -2;\ x_2 = 2e^{j\frac{\pi}{3}};\ x_3 = 2e^{j\frac{5\pi}{3}}$



Q3:

Let $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 2x(t) = 0$ be Eq. 1.

$$x(t) = e^{\lambda t} \Rightarrow \frac{dx}{dt} = \lambda e^{\lambda t} \Rightarrow \frac{d^2x}{dt^2} = \lambda^2 e^{\lambda t}$$

If $x(t)$ satisfies Eq. 1, then

$$\lambda^2 e^{\lambda t} + 5\lambda e^{\lambda t} + 2e^{\lambda t} = 0 \text{ for all } t.$$

$\Rightarrow (\lambda^2 + 5\lambda + 2)e^{\lambda t} = 0$ for all t

$$\lambda = \frac{-5 \pm \sqrt{5^2 - 4(2)}}{2} = \frac{-5 \pm \sqrt{17}}{2}$$