Q1

state-space

Let's consider a real string problem.

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

Here we can figure out that, real ~~world~~ world machanical problem is easy to trans form into state space, and it is easy to ~~converts~~ check controllability and design ~~plo~~ pole replacement.

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Date Question 2:

$$G(s) = C(sI-A)^{-1}B + D$$

$$= [1\ 1\ 0]\left(\begin{bmatrix} s & -1 & \\ & s & -1 \\ 1 & 1 & s+2 \end{bmatrix}\right)^{-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

~~$\det(sI-A) = s^2(s+2)$~~

$$= \frac{[1\ 1\ 0]}{s^3+2s^2+s+1}\begin{bmatrix} (s+1)^2 & (s+2) & 1 \\ -1 & s(s+2) & s \\ -s & -(s+1) & s^2 \end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$= \frac{1}{s^3+2s^2+s+1}[s^2+2s \quad s^2+3s+2 \quad s+1]\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$= \frac{s+1}{s^3+2s^2+s+1}$$

Question. 3

$$\dot{x} = \begin{pmatrix} 0 & 0 & a_3 \\ 1 & 0 & a_2 \\ 0 & 1 & a_1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

$$y = (0 \quad 0 \quad 1)x$$

~~Actually~~

~~$\mathcal{C}(\hat{A}, B) = \begin{pmatrix}0\\0\\1\end{pmatrix}\begin{pmatrix} 0 & 0 & a_3 \\ 1 & 0 & a_2 \\ 0 & 1 & a_1 \end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}$~~

(1) $AB = \begin{pmatrix} a_3 \\ a_2 \\ a_1 \end{pmatrix}$

$$A^2B = \begin{pmatrix} 0 & 0 & a_3 \\ 1 & 0 & a_2 \\ 0 & 1 & a_1 \end{pmatrix}\begin{pmatrix} a_3 \\ a_2 \\ a_1 \end{pmatrix}$$

$$= \begin{pmatrix} a_3 a_1 \\ a_3 + a_1 a_2 \\ a_2 + a_1^2 \end{pmatrix}$$

$$\Rightarrow \mathcal{C}(A, B) = \begin{pmatrix} 0 & a_3 & a_3 a_1 \\ 0 & a_2 & a_3 + a_1 a_2 \\ 1 & a_1 & a_2 + a_1^2 \end{pmatrix}$$

$$\det(\mathcal{C}(A, B)) = a_3(a_3 + a_1 a_2) - a_1 a_2 a_3$$

$$= a_3^2$$

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$\Rightarrow$ Iff $a_3=0$, the system is non controllable.
else it is completely controllable.

(2) For $C=(0,0,1)$

$$\Rightarrow \quad CA = (0,0,1)\begin{bmatrix} 0 & 0 & a_3 \\ 1 & 0 & a_2 \\ 0 & 1 & a_1 \end{bmatrix}$$

$$= (0,\ 1,\ a_1)$$

$$CA^2 = (0,1,a_1)\begin{bmatrix} 0 & 0 & a_3 \\ 1 & 0 & a_2 \\ 0 & 1 & a_1 \end{bmatrix}$$

$$= (1,\ a_1,\ a_2+a_1^2)$$

$$\Rightarrow O(A,C) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & a_1 \\ 1 & a_1 & a_2+a_1^2 \end{bmatrix}$$

$$\det(O(A,C)) = -1 \neq 0$$

$\Rightarrow$ the system totally observable.
there's no such set of $(a_1\ a_2\ a_3)$