**Instructions**

1. Do not start writing until you are instructed to do so.
2. Do not continue to write when you are told to stop.
3. You are not allowed to communicate with one another during the quiz.
4. The quiz is closed-book, closed-notes. You may bring one (double-sided) sheet of notes with any necessary formulas. A calculator will NOT be necessary NOR helpful.
5. Answer in the answer-sheet and submit both question- and answer-sheets before the end of the quiz.
6. Write your name and student number clearly in all the sheets.
7. Answer all questions. There are 4 questions with sub-questions.

**Question 1 (12 Points)**

a) Consider a water tank of cross-sectional area *A* with volume of water flowing in at a rate of $q_{in}$ and volume flowing out at a rate of $q_{out}$. Taking gravitational constant to be *g*, surface pressure $P_a$ and flow resistance at the outlet $R = \frac{g}{q_{out}} y(t)$

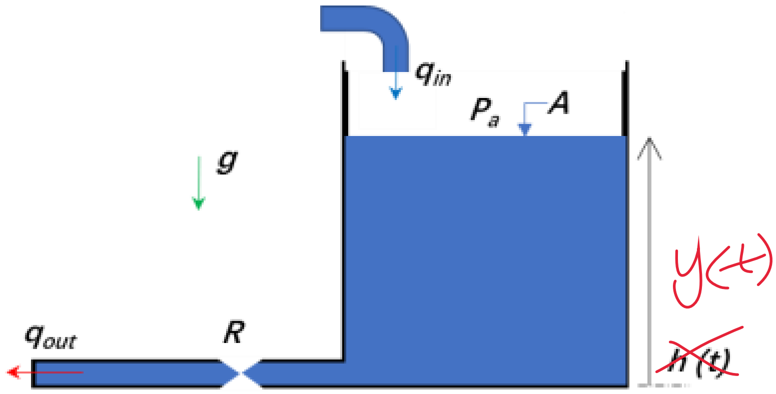


**Figure 1**

Show that the dynamics of the system can be expressed an ordinary differential equation:

$$A\dot{y}(t) + \frac{g}{R} y(t) = q_{in}$$

*(3 Points)*

*** Hint the rate of change of water volume in the tank is the same as the net volume flow*

b) How are the system input $q_{in}(t)$ and the output $y(t)$ related via the convolution operation? *(1 Points)*

c) If at *t*=0, the height *y(0)=5* and $q_{in} = 0$ (the inlet to the tank is switched off), obtain the system response i.e. the variation of height, ~~*h(t)*~~ y(t) over time assuming *A*=1. *(3 Points)*

d) Write down τ, the time constant (i.e. time taken for the tank to drain to a height of 1/e of its initial height) in terms of e, *A*, *R* and *g* if $q_{in}$=0. *(2 Points)*

e) Using Laplace transformation, find the transfer function that relates the input and output in the s-domain assuming zero-initial condition. *(3 Points)*

**Question 2  (6 Points)**

A dynamic system with no input is governed by the equation:

$\ddot{x} = 0.5(x^2 - 1)\dot{x} + 1.5x$

a. Choosing state variables $(x_1\ x_2) = (x\ \dot{x})$, write down its non-linear state-space model for the system. *(2 Points)*

b. Derive the linearized state-space model at the equilibrium point. *(4 Points)*

**Question 3 (12 Points)**

A dynamic system can be represented by the following block diagram:

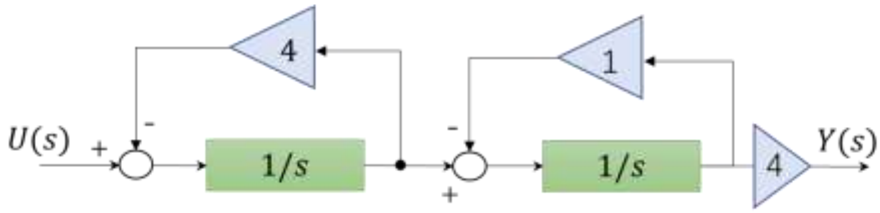


**Figure 2**

a) Show that the transfer function of the system could be expressed in

$$H(s) = \frac{Y(s)}{U(s)} = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

Also, write down the value for $\omega_n$ and $\zeta$. *(4 Points)*

b) Use Routh-Hurwitz stability analysis to check if system in (a) is stable. *(4 Points)*

c) A specification on 5% settling time $t_s < 0.5$ is required for the 2nd order system

i) Sketch the region of pole locations on the complex plane to meet the spec.

ii) Explain whether the system in (a) meets the requirement. *(4 Points)*

You may use the following formula for TD specifications for the system:

$$H(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2} = \frac{\sigma^2 + {\omega_d}^2}{(s+\sigma) + {\omega_d}^2}$$

$$t_r \approx \frac{1.8}{\omega_n}$$

$$t_p = \frac{\pi}{\omega_d}$$

5% settling time: $t_s \approx \frac{3}{\sigma} = \frac{3}{\zeta\omega_n}$

% Overshoot $M_n = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$

**Question 4 (10 Points)**

a) State the key purpose of incorporating integral control in a PID controller *(1 Points)*

b) The sensor of a control system is subject to a lot of noise from the working environment which term in the PID control is likely to worsen the effect of noise. Explain. *(1 Points)*

c) A control system is implemented as represented by the block diagram.

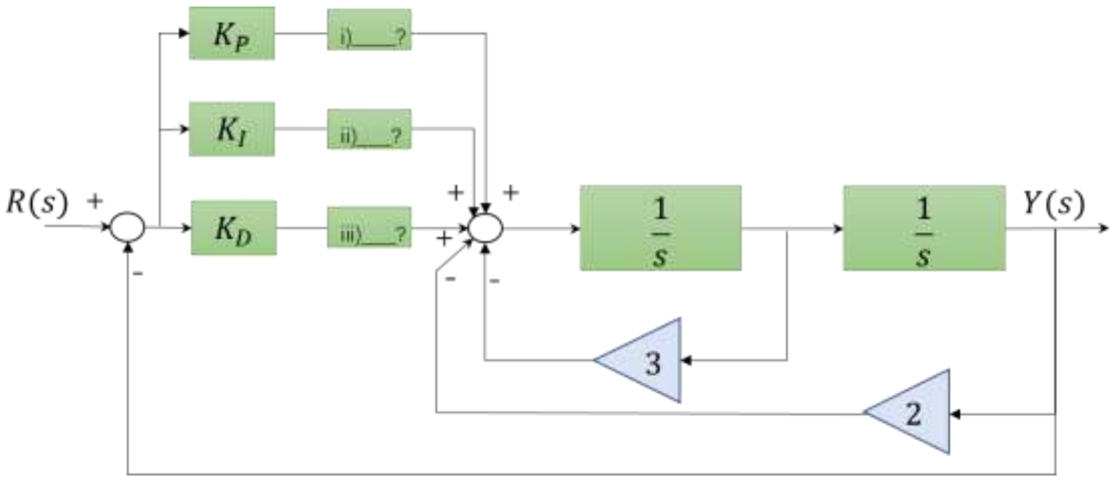


**Figure 3**

Fill in the blocks (i)-(iii) *(3 Points)*

iv) Write down the closed-loop transfer function. *(5 Points)*

## Table of Laplace Transforms

| Number | $F(s)$ | $f(t), t \geq 0$ |
|---|---|---|
| 1 | $1$ | $\delta(t)$ |
| 2 | $\frac{1}{s}$ | $1(t)$ |
| 3 | $\frac{1}{s^2}$ | $t$ |
| 4 | $\frac{2!}{s^3}$ | $t^2$ |
| 5 | $\frac{3!}{s^4}$ | $t^3$ |
| 6 | $\frac{m!}{s^{m+1}}$ | $t^m$ |
| 7 | $\frac{1}{(s+a)}$ | $e^{-at}$ |
| 8 | $\frac{1}{(s+a)^2}$ | $te^{-at}$ |
| 9 | $\frac{1}{(s+a)^3}$ | $\frac{1}{2!}t^2e^{-at}$ |
| 10 | $\frac{1}{(s+a)^m}$ | $\frac{1}{(m-1)!}t^{m-1}e^{-at}$ |
| 11 | $\frac{a}{s(s+a)}$ | $1-e^{-at}$ |
| 12 | $\frac{a}{s^2(s+a)}$ | $\frac{1}{a}(at-1+e^{-at})$ |
| 13 | $\frac{b-a}{(s+a)(s+b)}$ | $e^{-at}-e^{-bt}$ |
| 14 | $\frac{s}{(s+a)^2}$ | $(1-at)e^{-at}$ |
| 15 | $\frac{a^2}{s(s+a)^2}$ | $1-e^{-at}(1+at)$ |
| 16 | $\frac{(b-a)s}{(s+a)(s+b)}$ | $be^{-bt}-ae^{-at}$ |
| 17 | $\frac{a}{(s^2+a^2)}$ | $\sin at$ |
| 18 | $\frac{s}{(s^2+a^2)}$ | $\cos at$ |
| 19 | $\frac{s+a}{(s+a)^2+b^2}$ | $e^{-at}\cos bt$ |
| 20 | $\frac{b}{(s+a)^2+b^2}$ | $e^{-at}\sin bt$ |
| 21 | $\frac{a^2+b^2}{s[(s+a)^2+b^2]}$ | $1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$ |