Instructions

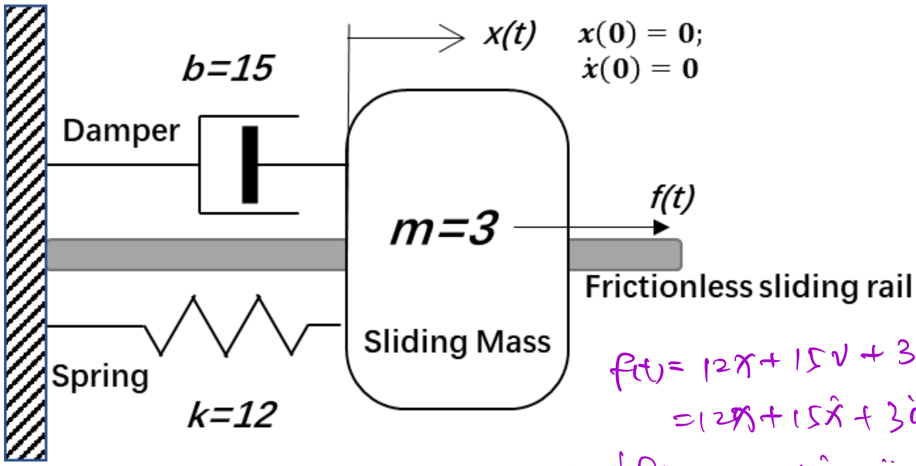
1. Do not start writing until you are instructed to do so.
2. Do not continue to write when you are told to stop.
3. You are not allowed to communicate with one another during the quiz.
4. The quiz is closed-book, closed-notes. You may bring one (double-sided) sheet of notes with any necessary formulas. A calculator will NOT be necessary NOR helpful.
5. Answer in the answer-sheet and submit both question- and answer-sheets before the end of the quiz.
6. Write your name and student number clearly in the all sheets.
7. Answer all questions. There are 2 questions (25 points each) with sub-questions.

Question 1

1. linear: u_1 ...
2. TI: impulse time w/ after-~~signal~~ venture shape.

a) Explain what a linear time invariant system is. *(2 Points)*

b) A sliding mass-spring-damper system is shown in Figure 1. Please state assumptions when necessary while answering the questions.



$f(t) = 12x + 15v + 3a$
$= 12x + 15\dot{x} + 3\ddot{x}$
$\Rightarrow \frac{1}{3}f(t) = 4x + 5\dot{x} + \ddot{x} = u(t),$
$u(t) = \frac{1}{3}f.$

Figure 1

Using physical laws including Newton's law and free-body diagrams, show that the dynamic system can be represented by a second order differential equation

$$\ddot{x}(t) + 5\,\dot{x}(t) + 4x(t) = u(t)$$

(5 Points)

c) Show the state-space representation of this system.

$\dot{x}_2 + 5x_2 + 4x_1 = u.$
$\Rightarrow \dot{x}_2 = -5x_2 - 4x_1 + u$

(4 Points)

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}x_2\\ -5x_2 - 4x_1 + u\end{bmatrix} = \begin{bmatrix}0 & 1\\ -4 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u.$$

d) Obtain the transfer function *H(s)* as shown:

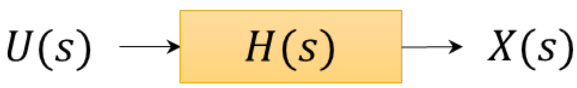
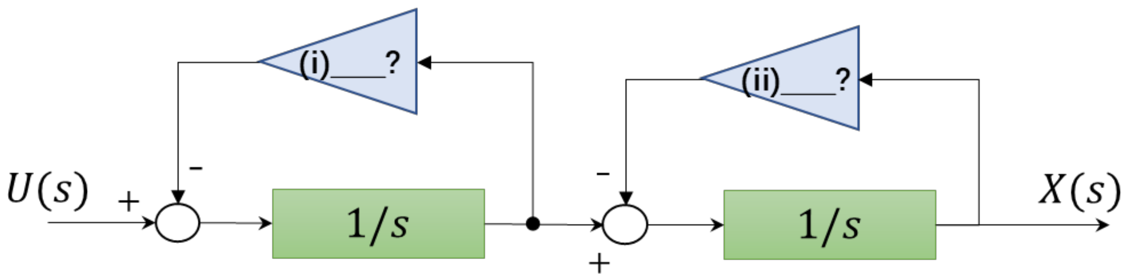


Figure 2

(4 points)

e) Write down the poles of *H(s)* *(2 points)*

f) Fill in the values in (i) and (ii)



(2 points)

g) Verify that the following block diagram is also an equivalent representation.

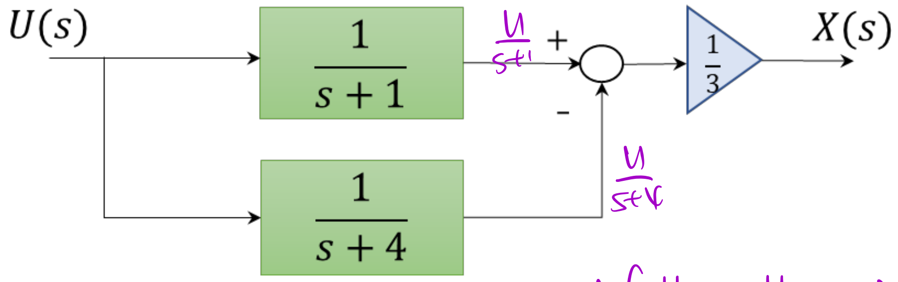


Figure 3

$\frac{1}{3}\left(\frac{u}{s+1} - \frac{u}{s+4}\right) = X.$

(2 points)

$\frac{X}{U} = \frac{1}{3}\cdot\frac{3}{(s+1)(s+4)} = \frac{1}{s^2+5s+4}$

h) Find the system response *x(t)* in time domain given a unit step input *u(t) =1(t)* assuming zero-ICs using the following Laplace transformations: $\mathcal{L}\{1(t)\} = \frac{1}{s}$; $\mathcal{L}\left\{\frac{1}{b-a}\left(e^{-at} - e^{-bt}\right)\right\} = \frac{1}{(s+a)(s+b)}$

(4 points)

$X = \frac{1}{3}\left(\frac{1}{s+1} - \frac{1}{s+4}\right) U.$

$= \frac{1}{3s}\left(\frac{1}{s+1} - \frac{1}{s+4}\right) = \frac{1}{3}\left(\frac{1}{(s+0)(s+1)} - \frac{1}{(s+0)(s+4)}\right) = \cdots$

Question 2

a) Give an advantage and disadvantage of feedback in a control system. *(2 Points)*

b) State the key purpose of incorporating integral control in a PID controller *(1 Points)* reduce steady-state error.

c) The sensor of a control system is subject to a lot of noise from the working environment which term in the PID control is likely to worsen the effect of noise. D term. *(1 Points)*

d) A control system is implemented as represented by the block diagram.

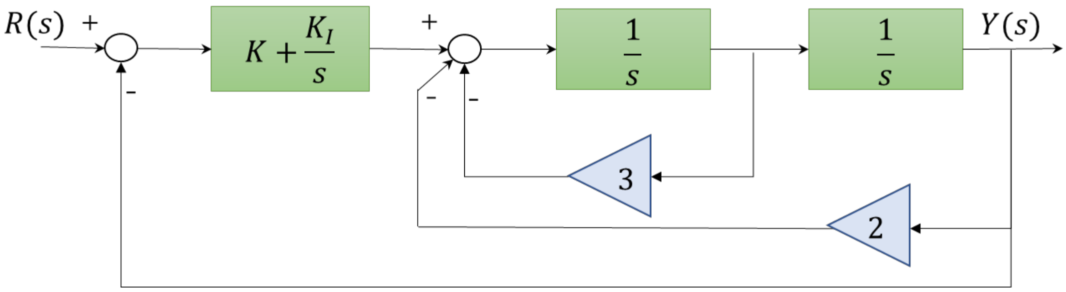


Figure 4

i. Write down the closed-loop transfer function. *(4 Points)*

ii. Construct the Routh array in terms of K and K_I. *(6 Points)*

e) A second order system with a closed loop transfer function

$$H(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

has poles within the shaded region of the complex plane to meet the transient response specification of percentage overshoot M_p as shown in the figure.

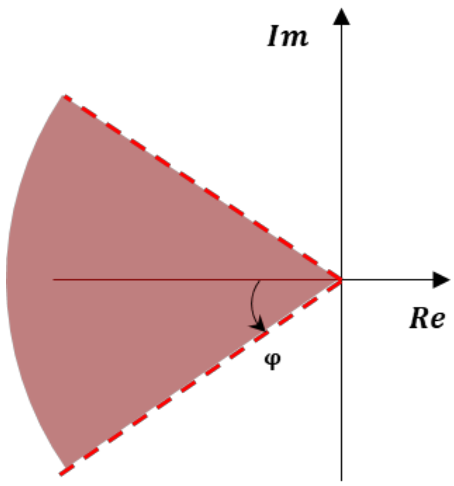


Figure 5

i. State the value of φ given that the damping ratio $\zeta = \frac{1}{\sqrt{2}} = 0.7071$

ii. Sketch on a complex plane, with labels, for the region where the 5% settling time $t_s < 1.2$

iii. Comment about the value of ω_n

iv. Sketch on a complex plane, with labels, the region that will satisfy the all the above requirements.

v. The closed-loop transfer function of a system is

$$H(s) = \frac{25}{s^2 + 6s + 25}$$

Comment on whether the specifications are satisfied.

** Note that $cos(45^o) = \frac{1}{\sqrt{2}}$. You may leave your expressions in their irrational form WITHOUT evaluating the numerical value. State any assumption you made. *(10 Points)*

You may use the following formula for TD specifications for the system:

$$H(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2} = \frac{\sigma^2 + {\omega_d}^2}{(s+\sigma) + {\omega_d}^2}$$

$$t_r \approx \frac{1.8}{\omega_n}$$

$$t_p = \frac{\pi}{\omega_d}$$

5% settling time: $t_s \approx \frac{3}{\sigma} = \frac{3}{\zeta\omega_n}$

% Overshoot $M_n = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$

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