Full-State Feedback with observer.

☆ system has initial condition.

System: $\begin{cases} \dot{x} = Ax + Bu. \\ y = Cx. \end{cases}$

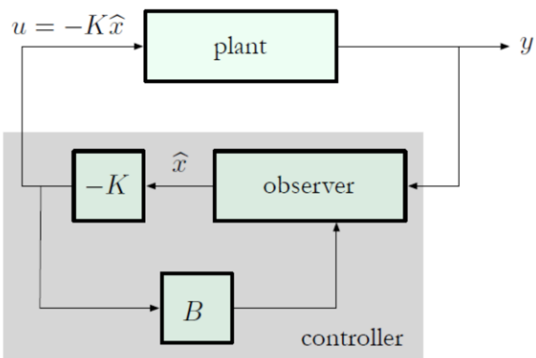
Observer: $\dot{\hat{x}} = (A-LC)\hat{x} + Ly + Bu.$

Error: $\dot{e} = (A-LC)e.$ $\qquad$ $\hat{x}$: estimated state feedback.

Controller: $u = -K\hat{x}.$

$\Rightarrow$ overall O-C system:
$$\begin{aligned} \dot{\hat{x}} &= (A-LC)\hat{x} + Ly + B(-K\hat{x}) \\ &= (A-LC-BK)\hat{x} + Ly \\ u &= -K\hat{x} \end{aligned}$$

dynamic output feedback.



Dynamic Output Feedback

$$\boxed{\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & -BK \\ LC & A-LC-BK \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}.}$$

transform to $\begin{pmatrix} x \\ e \end{pmatrix}$:

$$\begin{pmatrix} x \\ e \end{pmatrix} = \begin{pmatrix} x \\ x-\hat{x} \end{pmatrix} = \underbrace{\begin{pmatrix} I & 0 \\ I & -I \end{pmatrix}}_{T.} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

$$\begin{cases} \dot{x} = Ax - BK\hat{x} = (A-BK)x + BK(x-\hat{x}) = (A-BK)x + BKe \\ \dot{e} = (A-LC)e. \end{cases}$$

$$\Rightarrow \boxed{\begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} A-BK & BK \\ 0 & A-LC \end{pmatrix} \begin{pmatrix} x \\ e \end{pmatrix}.}$$

CL characteristic polynomial: $= \det \begin{pmatrix} Is-A+BK & -BK \\ 0 & Is-A+LC \end{pmatrix}$

$= \det(Is-A+BK) \cdot \det(Is-A+LC).$

Separation Principle:

CL eigenvalues are $\{$Controller poles$\} \cup \{$Observer poles$\}$

if the system is controllable and observable.