Name: Student ID:

Term Test 2

Instructions

1. Do not start writing until you are instructed to do so.
2. Do not continue to write when you are told to stop.
3. You are not allowed to communicate with one another during the quiz.
4. The quiz is closed-book, closed-notes. You may bring two sheets of notes (each double-sided) with any necessary formulas. A calculator will NOT be necessary NOR helpful.
5. Answer in the answer-sheet and submit both question- and answer-sheet before the end of the quiz.
6. Write your name and student number clearly in all sheets.
7. There are 2 questions (40 points in total) with sub-questions

(Please do NOT turn over until told to do so)

Question 1

Consider the system illustrated below

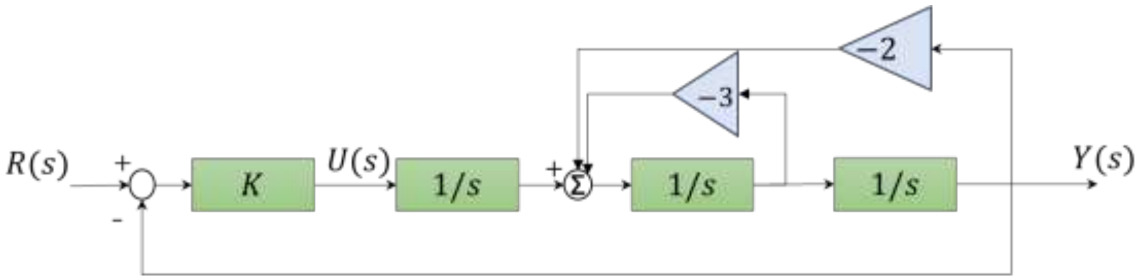


Figure 1a

i) Write down an expression for the transfer function $L(s) = \frac{Y(s)}{U(s)}$ in terms of s *(3 Points)*

ii) Write down the closed-loop transfer function $H_{CL}(s) = \frac{Y(s)}{R(s)}$ in terms of s *(3 Points)*

iii) Show that the characteristic polynomial of the system can be expressed as

$$s^3 + 3s^2 + 2s + K = 0$$

(*1 Points*)

iv) Write down the poles of the transfer function $L(s)$. (*1 Points*)

v) Find the necessary and sufficient condition for K to ensure stability given the Routh's array as follows:

$$\begin{array}{c|c|c|}
s^3 & 1 & 2 \\
s^2 & 3 & K \\
s^1 & 6-K & 0 \\
s^0 & K &
\end{array}$$

(4 *Points*)

vi) Obtain the value of ω at where the root-locus intercept the imaginary axis i.e. $j\omega - crossing$. (*2 Points*)

vii) Label (A)-(F) with the appropriate values showing how you derive your answers.

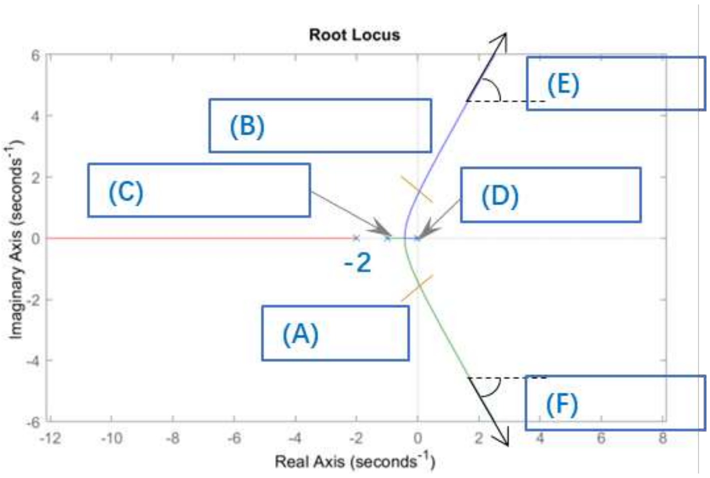


Figure 1b

(*6 Points*)

Name: Student ID:

Solution Q1

i)

$$L(s) = \frac{1}{s(s^2+3s+2)} = \frac{1}{s(s+2)(s+1)}$$

ii)

closed loop transfer function:

$$H_{CL}(s) = \frac{KL(s)}{1+KL(s)} = \frac{K}{s(s^2+3s+2)+K} = \frac{K}{s^3+3s^2+2s+K}$$

iii) Characteristic polynomial is when $1 + KL(s) = 0$,

$$1 + \frac{K}{s(s+2)(s+1)} = 0$$

$$s^3 + 3s^2 + 2s + K = 0$$

iv)

open loop poles: $\alpha_1 = -2,\ \alpha_2 = -1,\ \alpha_3 = 0$

v) Using Routh-Hurwitz Stability Analysis

Characteristic polynomial equation $s^3 + 3s^2 + 2s + K = 0$

The necessary condition is that K >0.

Routh Array

s^3	1	2
s^2	3	K
s^1	$6-K$	0
s^0	K	

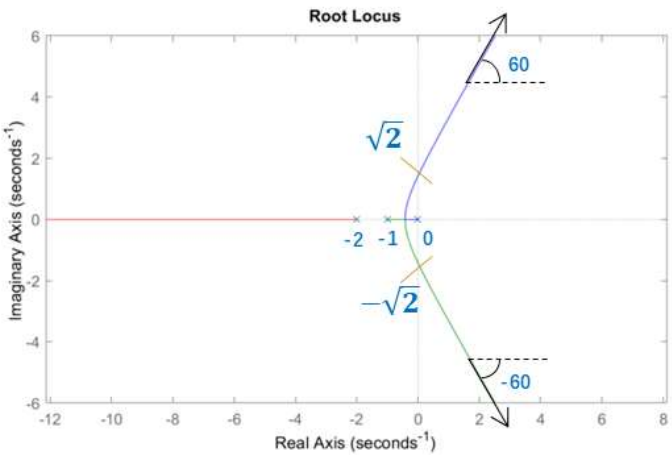
Therefore $0 < K < 6$ is a necessary and sufficient condition

vi) From (v), K has a critical value of 6. Substituting in the characteristic polynomial

$$(j\omega)^3 + 3(j\omega)^2 + 2j\omega + 6 = 0$$

At $j\omega$-crossing, real part equal zero, $\omega = \pm\sqrt{2}$

vii)



Question 2

Consider the closed loop system in the following Figure 2a.

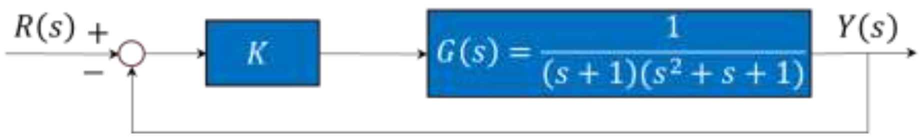


Figure 2a

Figure 2b shows the Bode plot of the *KG(s)* when *K*=1

i) Fill in the values for (a)~(d) *(4Points)*

ii) Indicate the gain margin and phase margin graphically in Figure 2b *(4Points)*

iii) Sketch the new Bode plot if *K*=0.1 on Figure 2b. *(2Points)*

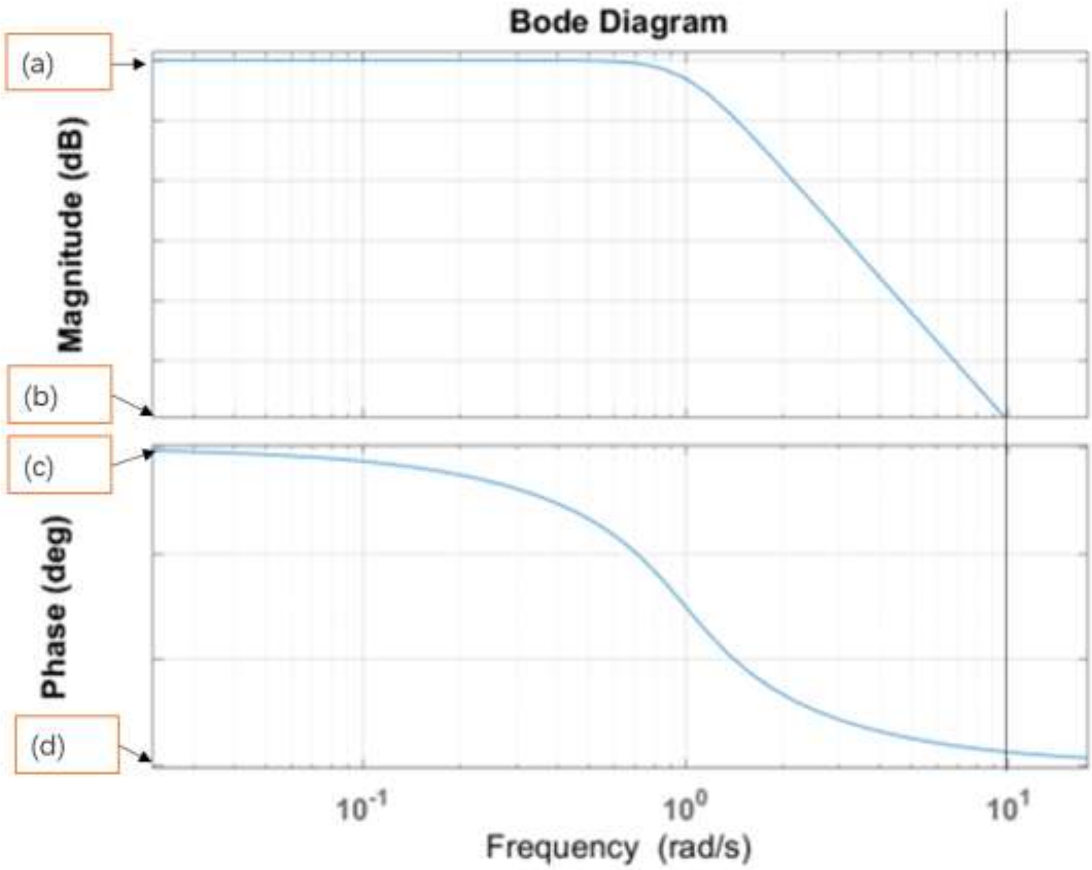


Figure 2b

iv) Fill in the coordinates (a)~(d) in the Nyquist Plot of *G(s)* given in Figure 2c.
You may use the following information: *(4Points)*

$$\omega = \frac{1}{\sqrt{2}} \Longrightarrow |G(j\omega)| = 0.95, \angle G(j\omega) = -90^o$$

$$\omega = \sqrt{2} \Longrightarrow |G(j\omega)| = \frac{1}{3}, \angle G(j\omega) = -180^o$$

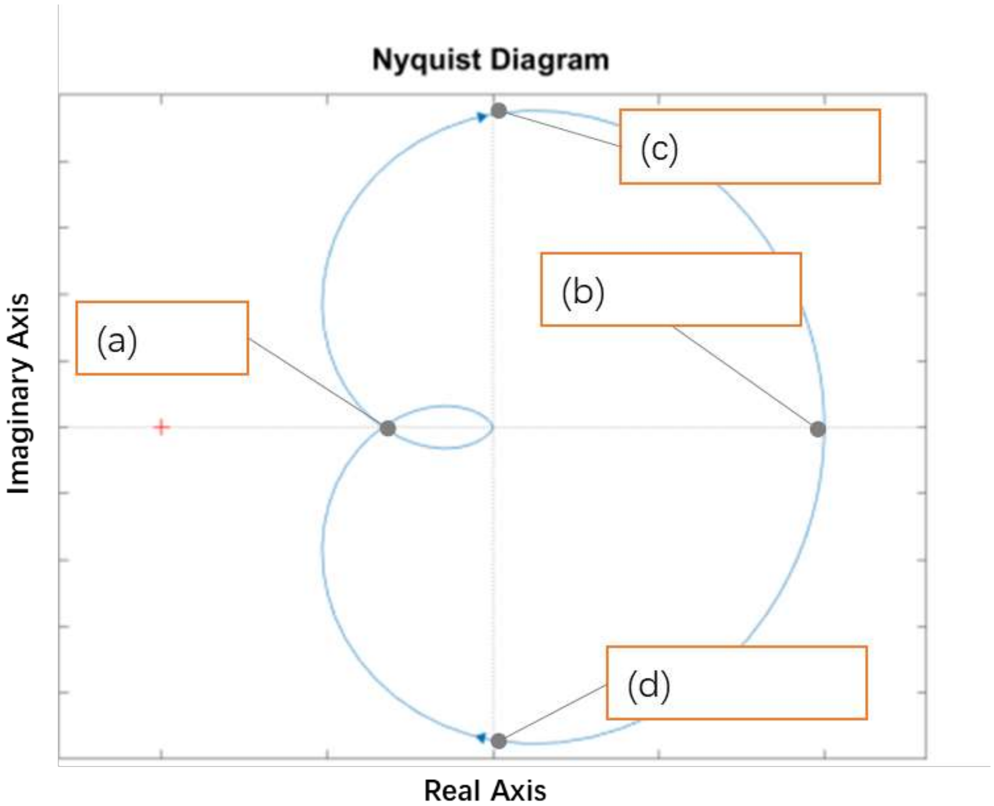


Figure 2c

v) Indicate on Figure 2c the points associated with $\omega \to 0$ and $\omega \to \infty$ *(2 Points)*

vi) Using Nyquist stability criterion, determine all values of *K* that stabilize the closed-loop system. *(4Points)*

Solution Q2

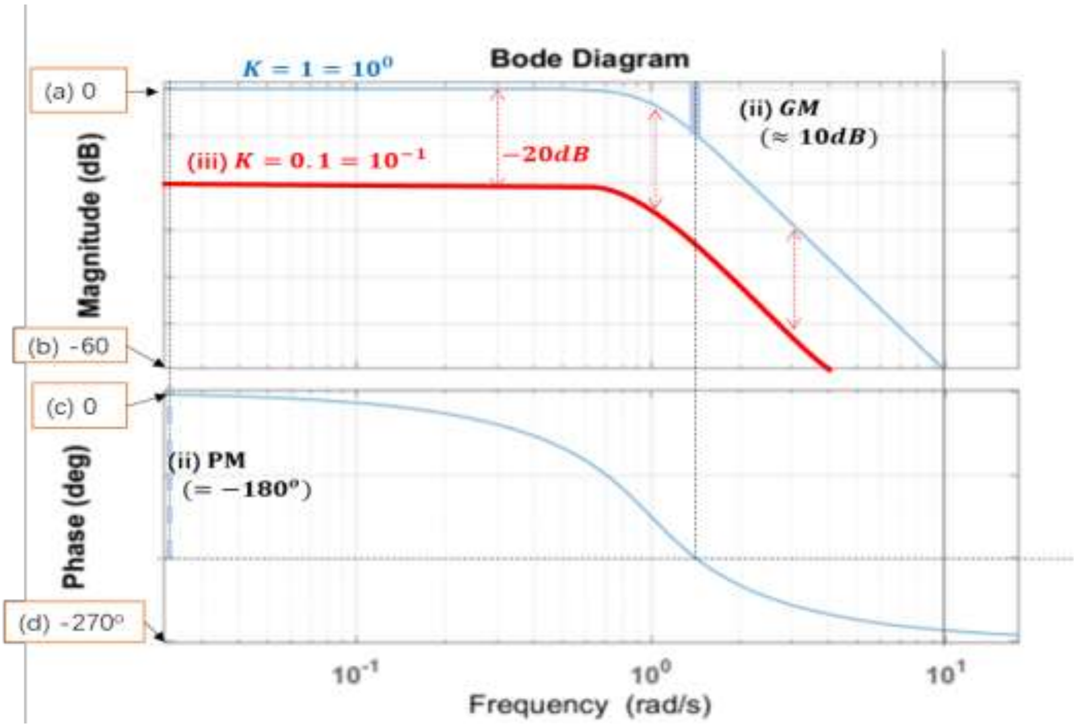
i)

(a) As $\omega \to 0$, $|G(s)| \to 1$, $M = 20\log(|G(s)|) = 0$

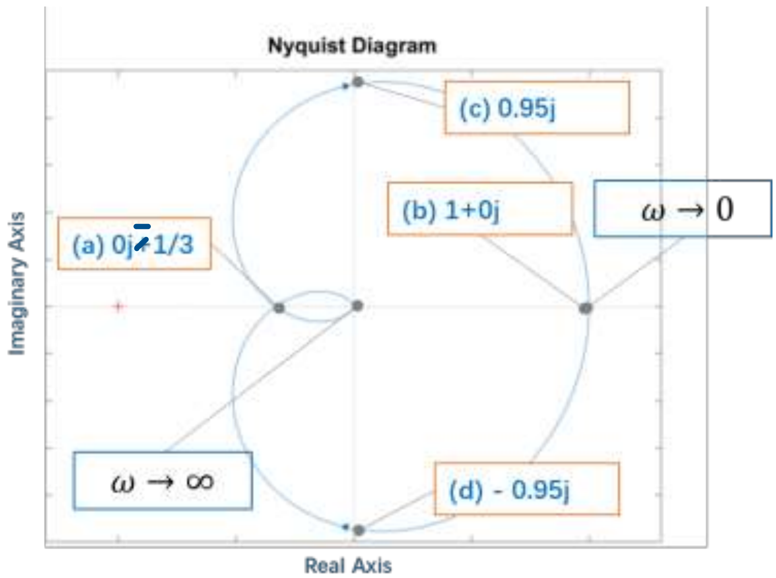
(b) slope down by 3 i.e., -60 dB per decade

(c) As $\omega \to 0$, $G(s) = \text{Real value}$, $\varphi = 0$

(d) As $\omega \to \infty$, $\varphi = 3(-90^o) = -270^o$



(iv),(v)



vi) Since there is no open-loop pole *Z*=0, *P* must also be 0 in order to not have RHP roots of 1+*KG(s)*. according to the argument principle *N*=*Z*-*P*.

Hence $-\frac{1}{K} < -\frac{1}{3}$ or $-\frac{1}{K} > 1$. Therefore $0 < K < 3$ or $-1 < K < 0$

i.e., $-1 < K < 3$