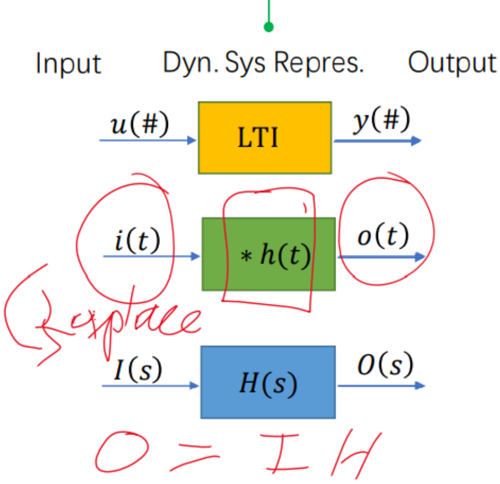
System Representation



Configuration form

Equations of Motion
$$\begin{cases}\ddot{q}_1 = f_1(q_1,\dots q_n,\dot{q}_1\dots \dot{q}_n,t)\\ \ddot{q}_1 = f_2(q_1,\dots q_n,\dot{q}_1\dots \dot{q}_n,t)\\ \dots\\ \ddot{q}_n = f_n(q_1,\dots q_n,\dot{q}_1\dots \dot{q}_n,t)\end{cases}$$

Initial Conditions
$$\begin{cases}q_1(0) = q_{1_0},\dots,q_n(0) = q_{n_0}\\ \dot{q}_1(0) = \dot{q}_{1_0},\dots,\dot{q}_n(0) = \dot{q}_{n_0}\end{cases}$$

ODE:
$$\frac{d^n y}{dt^n}+a_{n-1}\frac{d^{n-1}y}{dt^{n-1}}+\dots+a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} = b_m\frac{d^m x}{dt^m}+b_{m-1}\frac{d^{m-1}x}{dt^{m-1}}+b_o x(t)$$

ICs $\frac{d^k y}{dt^k} = y_k$

Transfer Function:
$$\frac{Y(s)}{X(s)} = \frac{b_m s^m + b_{m-1}s^{m-1}+\dots+b_o}{s^n+a_{n-1}s^{n-1}+\dots+a_o}$$

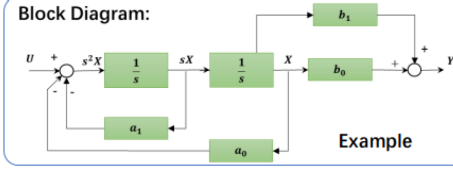
ICs= 0

State space model:

State Equation $\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}0 & 1\\ -a_0 & -a_1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}+\begin{pmatrix}0\\ 1\end{pmatrix}u$

Output Equation $y = (b_0 \quad b_1)\begin{pmatrix}x_1\\ x_2\end{pmatrix}$

Block Diagram:



Example

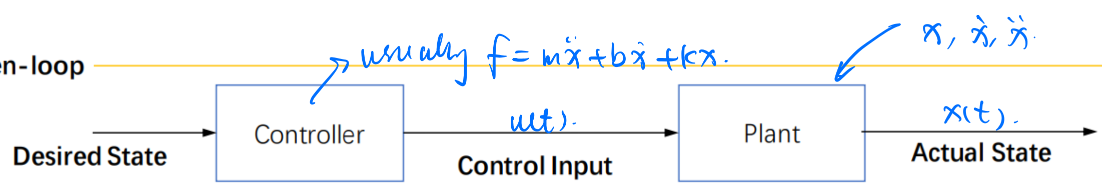
Control Systems:

- achieve target output.
- generate inputs
- in a dynamic environment.
- specified performance criteria

Benefits:

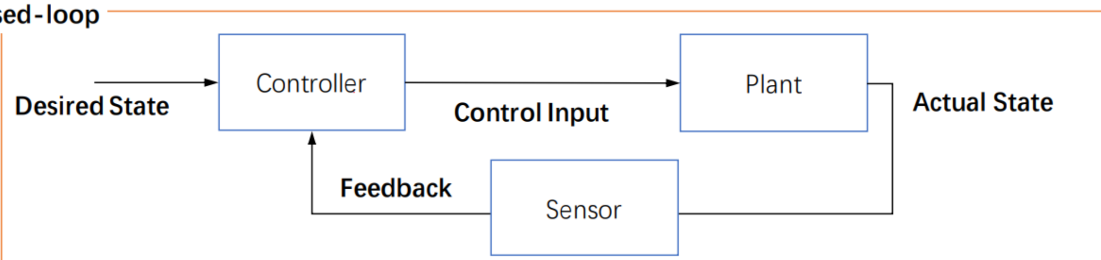
- Power Amplification
- Motion Scaling
- Remote Operation
- Ease of Input
- Compensation of Disturbance

Open-loop



Through preestablished model, generate the command that will achieve the desired state.

Closed-loop

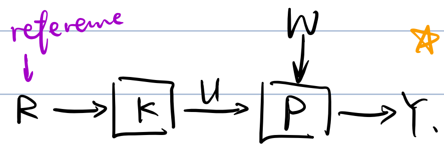


Through sensors, we are able to **feedback** the measurement to produce the command that will minimize the error between desired and actual targeted profile.

reliability & accuracy.

Architectures.

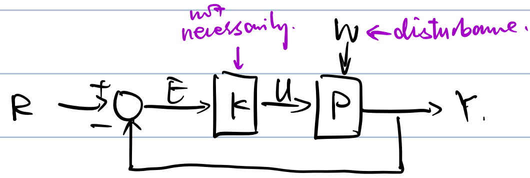
open:



☆ does not destablised.

$H_{ol} = \frac{Y}{R} = KP$ ← does not introduce new poles.

closed:



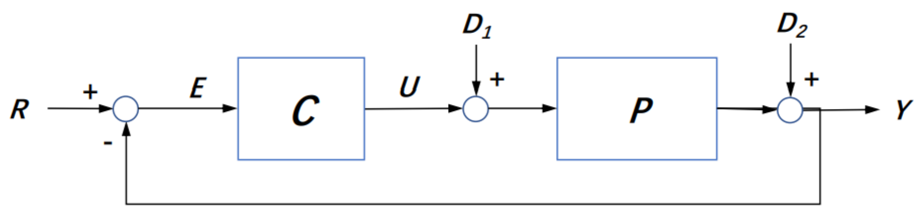
☆ may destablise; only way to stablise an unstable plant.

$H_{cl} = \frac{Y}{R} = \frac{KP}{1+KP}$.

↑ may introduce new poles.

Benefit of feedback control:

1. reduces steady-state error to disturbances.
2. reduces steady-state sensitivity to model uncertainty (parameter variations)
3. improve time response.



Systems		Variables	
C: Controller	R: Reference	U: Input	D_1: Disturbance 1
P: Plant	E: Error	Y: Output	D_2: Disturbance 2

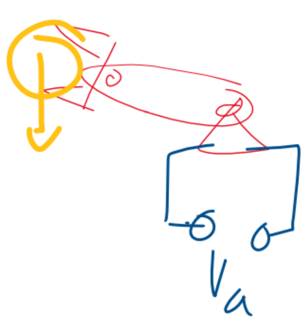
$$\begin{cases} Y = D_2 + P(U + D_1) \\ U = CE \\ E = R - Y. \end{cases}$$

$$\Rightarrow Y = D_2 + PCR - PCY + PD_1$$

$$= \frac{PC}{1+PC} R + \frac{P}{1+PC} D_1 + \frac{1}{1+PC} D_2.$$

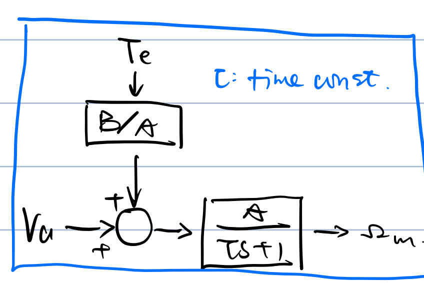
CJ: $Y \rightsquigarrow \frac{PC}{1+PC} R = R.$

DC Motor.



Va: input voltage.

Te: load/disturbance torque.



Output: $\omega_m \rightarrow$ angular speed of the motor.

system gain.

Transfer function: $\Omega_m = \frac{A}{\tau s+1} V_a + \frac{B}{\tau s+1} T_e$

Disturbance Rejection:

Goal: maintain $\omega_m = \omega_{ref}$ in steady-state in the presence of constant disturbance.

Open:

$$\Omega_m = \frac{A}{\tau s+1} V_a = \frac{K_{ol}A}{\tau s+1}\Omega_{ref}.$$

Want DC gain $=1 \longrightarrow$ set $K_{ol} = \frac{1}{A}$.

$t\to\infty$: $\omega_m(\infty) = \frac{1}{A}A\,\omega_{ref} = \omega_{ref}$ $\quad (T_e = 0)$.

For $T_e \neq 0$: $\Omega_m = \frac{A}{\tau s+1}\cdot\frac{1}{A}\Omega_{ref} + \underbrace{\frac{B}{\tau s+1}}_{\text{DC gain} = B.} T_e$

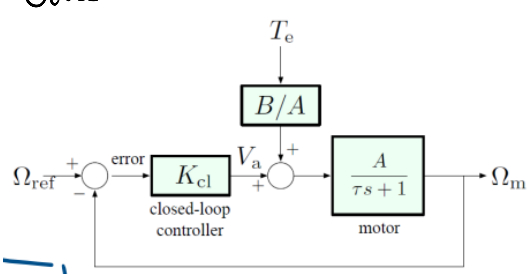
$$\Rightarrow \omega_m(\infty) = \underbrace{\omega_{ref}}_{\text{step input}} + \boxed{\underbrace{B\,T_e}_{\text{step input}}}$$

poor disturbance rejection. No control over B.

steady-state

good reference-tracking

Closed:



$$V_a = K_{cl}E = K_{cl}(\Omega_{ref} - \Omega_m).$$

$$\Omega_m = \frac{A}{\tau s+1}K_{cl}(\Omega_{ref} - \Omega_m) + \frac{B}{\tau s+1}T_e.$$

$$\Rightarrow \Omega_m = \underbrace{\frac{AK_{cl}}{\tau s+1+AK_{cl}}}_{\text{DC gain: } \frac{AK_{cl}}{1+AK_{cl}}}\Omega_{ref} + \underbrace{\frac{B}{\tau s+1+AK_{cl}}}_{\text{DC gain: } \frac{B}{1+AK_{cl}}}T_e.$$

$$\omega_m = \frac{AK_{cl}}{1+AK_{cl}}\omega_{ref} + \frac{B}{1+AK_{cl}}T_e.$$

small ($\to 0$) for large K_{cl}. better disturbance rejection.

as $K_{cl}\to\infty$: $\frac{AK_{cl}}{1+AK_{cl}} \approx 1$.

good with high gain (worse than OL).

Sensitivity

Open-Loop:

$$T_{ol} = K_{ol}A = \frac{1}{A}A = 1.$$

As $A \to A+\delta A$, $T_{ol} \to K_{ol}(A+\delta A) = \underbrace{\frac{1}{A}}_{\text{design choice}}(A+\delta A) = \underbrace{1}_{T_{ol}} + \underbrace{\frac{\delta A}{A}}_{\delta T_{ol}}.$$

Sensitivity: $S_{ol} = \dfrac{\delta T_{ol}/T_{ol}}{\delta A_{ol}/A_{ol}} = \dfrac{\delta T/T}{\delta A/A} = 1.$

i.e. 1% error in A causes a 1% error in T_{ol}.

Closed-loop: Nomial: $T_{cl} = \dfrac{AK_{cl}}{1+AK_{cl}}$.

Perturbed: $A \to A + \delta A$, $T_{cl} \to T_{cl} + \delta T_{cl}$.

By Taylor Expansion: $T(A+\delta A) = T(A) + \dfrac{dT}{dA}(A)\,\delta A + \cdots$.

$$\Rightarrow \frac{dT_{cl}}{dA} = \frac{K_{cl}}{1+AK_{cl}} - \frac{AK_{cl}^2}{(1+AK_{cl})^2} = \frac{K_{cl}}{(1+AK_{cl})^2}$$

$$\delta T_{cl} = \frac{K_{cl}}{(1+AK_{cl})^2}\,\delta A.$$

$$\Rightarrow T_{cl} = \frac{AK_{cl}}{1+AK_{cl}}.$$

Therefore $$\delta T_{cl}/T_{cl} = \frac{\frac{K_{cl}}{(1+AK_{cl})^2}\,\delta A}{\frac{AK_{cl}}{1+AK_{cl}}} = \frac{1}{1+AK_{cl}}\,\delta A/A$$

$$\Rightarrow S_{cl} = \frac{\delta T_{cl}/T_{cl}}{\delta A/A} = \frac{1}{1+AK_{cl}} \quad (\ll 1 \text{ for large } K_{cl})$$

Time Response.

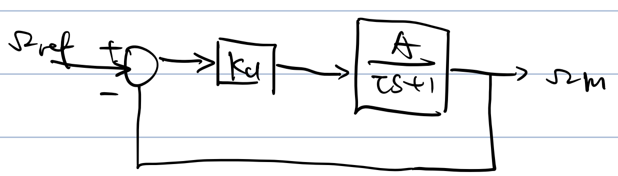
Open: $\Omega_m = \dfrac{AK_{cl}}{\tau s+1}\,\Omega_{ref}$. (poles at $s = -\frac{1}{\tau}$ $\Rightarrow$ transient response: $e^{-t/\tau}$).

τ: time it takes the system response to decay to $\frac{1}{e}$ of the starting value.

☆ $\tau\uparrow$, faster convergence to steady-state.

not affected by K_{cl}.

Close:



$\Omega_m = \dfrac{AK_{cl}}{\tau s+1+AK_{cl}}\,\Omega_{ref}$

(pole: $-\frac{1}{\tau}(1+AK_{cl})$)

Transient Response: $e^{-\frac{1+AK_{cl}}{\tau}t}$,

time constant: $\dfrac{\tau}{1+AK_{cl}}$.