ECE 486: Control Systems Homework 5

1. Consider the closed loop system in Figure 1.

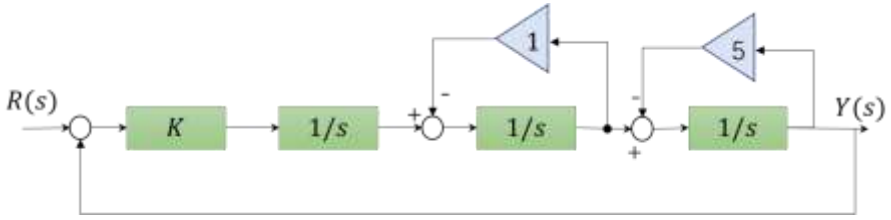


Figure 1

a) When K = 10, the bode plot is given by Figure 2. Indicate the frequency values where there are changes in the magnitude slope. *(2 Points)*
b) Given the Gain Margin (GM)=+8 dB, Phase Margin (PM)=+21°, on the bode plot on Figure 2, label (I) GM and (II) PM. *(4 Points)*
c) Suppose we look at K=100.
   I. Sketch the new Bode diagram *(4 Points)*
   II. Write down the new GM *(2 Points)*
   III. Show graphically on your sketch, how you would measure the new PM. *(2 Points)*
d) Comment on the choice of K value. *(2 Points)*

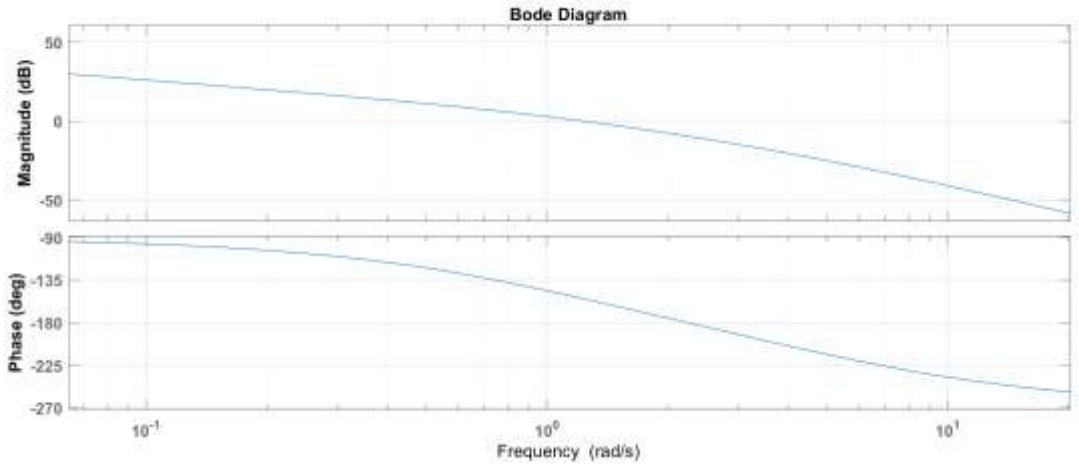


Figure 2

2. Consider unity feedback closed loop control system with open loop transfer function $K\boldsymbol{G}(\boldsymbol{s}) = \frac{\boldsymbol{K}}{\boldsymbol{s-1}}$.

Determine the critical value of K for stability using the Nyquist stability criterion. *(4 Points)*

Solution

a)

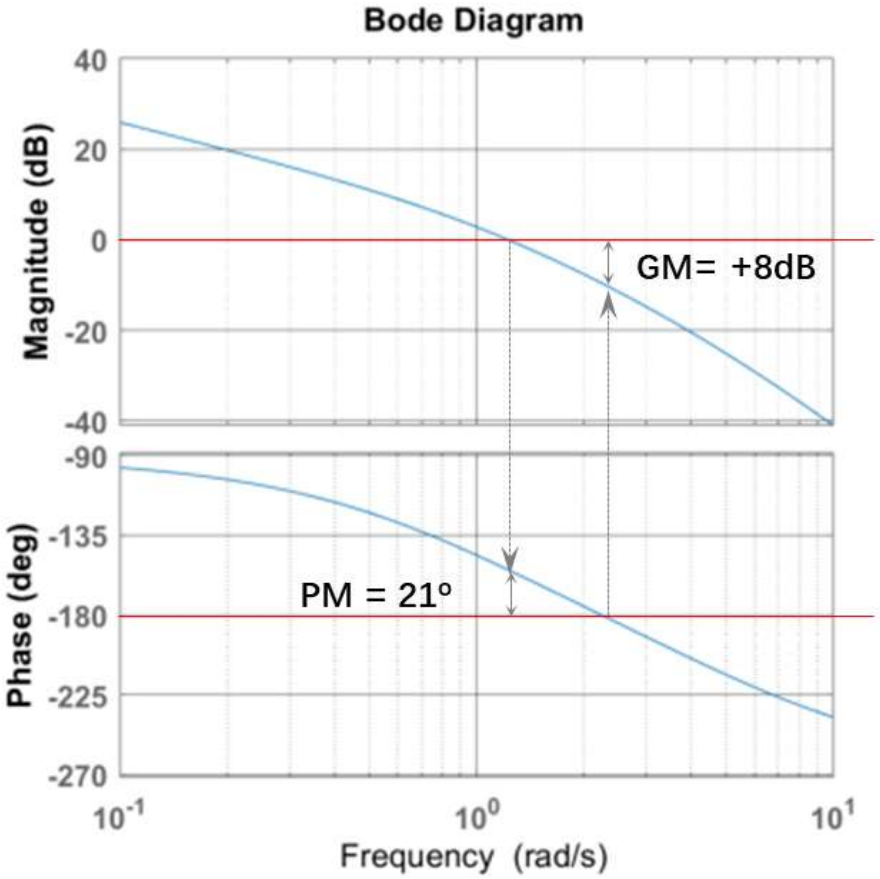
$$H(s) = \frac{10}{s(s+1)(s+5)} = \frac{2}{s(s+1)(\frac{s}{5}+1)}$$

$$H(j\omega) = 2(j\omega)^{-1}(j\omega+1)^{-1}(\frac{j\omega}{5}+1)^{-1}$$

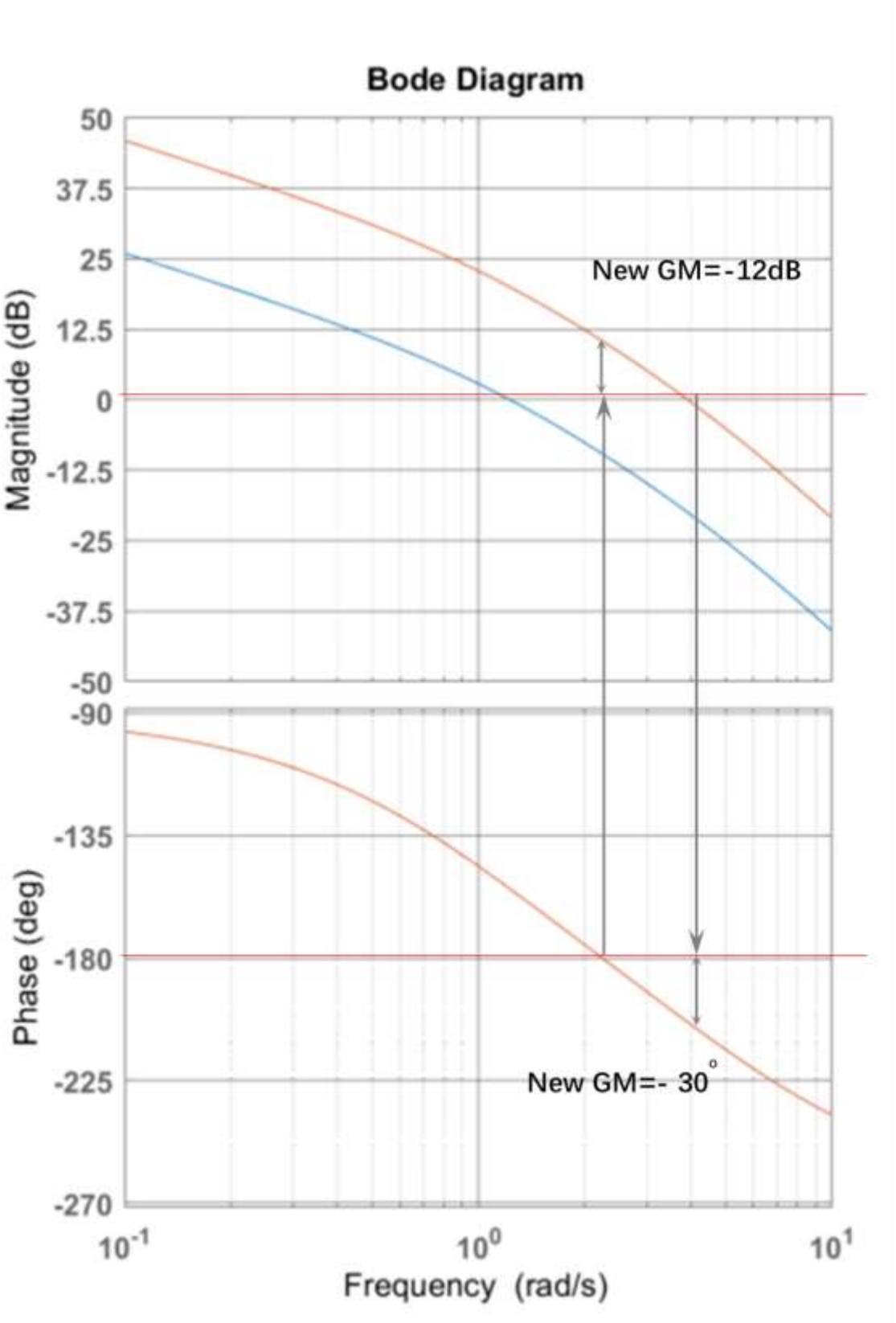
The slope changes as 1 and 5

*** Do not worry about the final answer for this question. The original plot was not clear enough to read off directly. You may show your labels on the graph or obtain the answer mathematically, both procedures are accepted.

b) (i) (ii)



b)



d) Both PM and GM become negative suggesting that K=100 is not stable but K=10 is.

2.

For $\frac{K}{j\omega-1}$, the polar plot is a circle with radius *K*/2, center $-K/2$ on the negative real axis.

As ω increases from $-\infty$ to ∞ the locus of $G(j\omega)$ makes a counterclockwise rotation. In this system, *P*=1 because there is one pole of *G(s)* in the RHP. For the closed-loop system to be stable, *Z* must be equal to zero. There for *N* = *Z*-*P* must be equal to -1, or there must be one counterclockwise encirclement of the -1+*j*0 point for stability. The system is unstable if there is no encirclement of this point. Therefore for stability, K must be greater than unity, and *K*=1 gives the stability limit. The following figure shows (a) the polar plot and (b) stable vs unstable plot of K greater than and smaller than unity, respectively.