ECE 486: Control Systems Homework 6

Question 1

Consider the system given by $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}u$ and output $y = \mathbf{Cx} + Du$

i) Write down an expression for the transfer function of this system (2 Points)

ii) Give an expression of the zeros of the system transfer function (1 Points)

iii) Give two expressions of the poles of the system transfer function (2 Points)

Question 2

Consider the control system shown

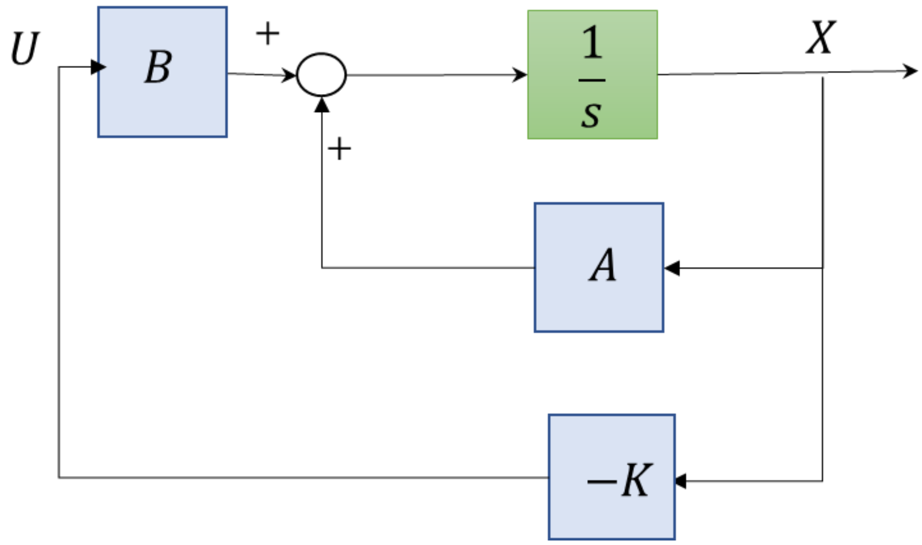

Figure 1

The plant is given by $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}u$

where $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Choosing the desired closed-loop poles at $s = -2 \pm j4$, $s = -10$ and do the following

i) Obtain the controllability matrix $\mathbf{M}$ (4 Points)

ii) Comment on the controllability of the system. (2 Points)

iii) Explain how you could obtain the state-feedback gain matrix K. (9 Points)

Solution

Question 1

i) $G(s) = C(Is - A)^{-1}B + D$

ii) $Z = \text{root of } \det\begin{pmatrix} Is - A & -B \\ C & D \end{pmatrix} = 0$

iii) $P = \det(Is - A)$ or $eig(A)$

Question 2

i) $\mathbf{M} = [B| \quad AB| \quad A^2B] = \begin{bmatrix} 0 & 0 & 1 \\ 0| & 1\ | & -6 \\ 1 & -6 & -5+36 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 31 \end{bmatrix}$

ii) $\text{Rank}(\boldsymbol{M}) = 3$ controllable

iii)
characteristic polynomial $\det(Is - A) = s^3 + 6s^2 + 5s + 1$
set as $s^3 + a_1s^2 + a_2s + a_3$ i.e. $a_1 = 6,\ a_2 = 5,\ a_3 = 1$
desired characteristic polynomial equation:

$$(s + 2 - j4)(s + 2 + j4)(s + 10) = s^3 + 14s^2 + 60s + 200$$

Letting characteristic equation be $s^3 + \alpha_1s^2 + \alpha_2s + \alpha_3$, we have
$\alpha_1 = 14,\ \alpha_2 = 60,\ \alpha_3 = 200$,
$k = [\alpha_1 - a_1|\ \alpha_2 - a_2|\alpha_3 - a_3]T^{-1}$; $T = I$ since already in CCF

$$k = [200 - 1| \quad 60 - 5| \quad 14 - 6] = [199 \quad 55 \quad 8]$$