Zeros on the Transient Response.

$H_1(s) = \frac{1}{s^2+2\zeta s+1}$ $(\omega_n = 1)$.

$\downarrow$ (zero at $s=-a$).

$H_2(s) = \frac{\frac{s}{a}+1}{s^2+2\zeta s+1} = H_1(s) + \frac{1}{a}\cdot\frac{s}{s^2+2\zeta s+1}$

$= H_1(s) + \frac{1}{a} H_d(s)$, where $H_d = sH_1$.

Step response: $H_2 = \frac{Y_2}{U} \Rightarrow Y_2 = UH_2 = \frac{1}{s}H_2 = \frac{1}{s}H_1(s) + \frac{1}{a}\cdot \frac{sH_1(s)}{s}$ $Y_1(s)$.

$= Y_1(s) + \frac{1}{a}\cdot s\,Y_1(s)$.

$\Rightarrow y_2(t) = y_1(t) + \boxed{\frac{1}{a}\dot{y}_1(t)}$ (assuming $a>0$.)

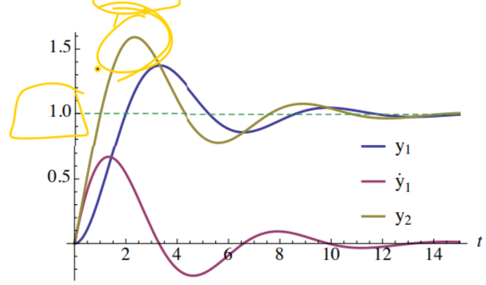
Effect of a LHP Zero.

① increased overshoot.

② little influence on settling time

③ $a \to \infty$ yields less significant effect.

$y_2(t) = y_1(t) + \frac{1}{a}\dot{y}_1(t)$ where $y_1(t)$ = original step response



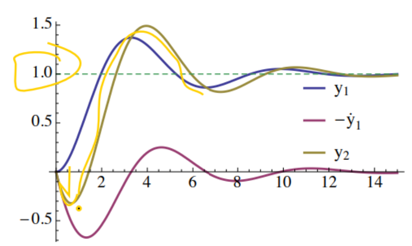
Effect of a RHP Zero.

① delays the response.

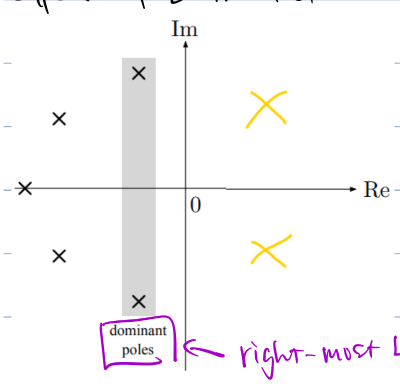
② creates an undershoot. (when a is small enough)

$H_1(s) = \frac{1}{s^2+2\zeta s+1} \xrightarrow{\text{add zero at } s=a} H_2(s) = H_1(s) - \frac{1}{a}\cdot sH_1(s)$

$y_2(t) = y_1(t) - \frac{1}{a}\cdot\dot{y}_1(t)$



Effects of Extra Poles.



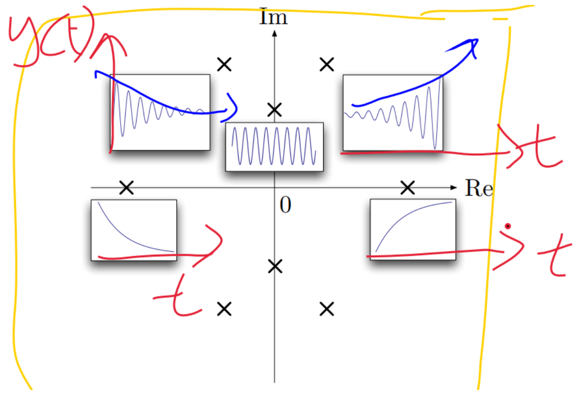
right-most LHP.

☆ extra LHP poles, → significant change real parts 5× that of dominant LHP poles.

$y = \sum C_k e^{-\boxed{\lambda_k} t}$.

$Re(\text{pole}) = \lambda_k$.

Effect of Pole Locations. (dominant poles).



① RHS → unstable.

② LHS → stable.

③ Im axis :

$$H(s) = \frac{w^2}{s^2+w^2} = \frac{Y}{X}.$$

I. i. impulse response :

$$Y = \frac{w^2}{s^2+w^2} \Rightarrow y = w\sin(wt).$$

ii. step response :

$$Y = \frac{w^2}{s(s^2+w^2)} \Rightarrow y = 1-\cos(wt)$$

$\Big\}$ unstable if $w \neq 0$.

II. $w=0$: $H(s)=0$, poles at the origin. →
- impulse : $Y(s) = \frac{1}{s} \Rightarrow y(t) = 1(t)$. stable.
- step $Y(s) = \frac{1}{s^2} \Rightarrow y(t) = t$. unit ramp 单位斜坡.

Stability.

Definition: BIBO (bounded-input, bounded-output).

① Every bounded input $u(t)$ results in a bounded output $y(t)$, regardless of initial conditions.

② $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$ (absolutely integrable).

③ all poles of $H(s)$ in OLHP.

Routh-Hurwitz Criterion: check if $p(s) = s^n + a_1 s^{n-1} + \cdots + a_n$ is strictly stable.

Routh arrays

$$\begin{array}{lcccccc}
s^n : & 1 & a_2 & a_4 & a_6 & \cdots \\
s^{n-1} : & a_1 & a_3 & a_5 & a_7 & \cdots \\
s^{n-2} : & b_1 & b_2 & b_3 & \cdots & \\
\vdots & & & & &
\end{array}$$

always 1.

where
$$b_1 = -\frac{1}{a_1} \cdot \begin{vmatrix} 1 & a_2 \\ a_1 & a_3 \end{vmatrix}$$

$$b_2 = -\frac{1}{a_1} \cdot \begin{vmatrix} 1 & a_4 \\ a_1 & a_5 \end{vmatrix}$$

$$b_3 = -\frac{1}{a_1} \cdot \begin{vmatrix} 1 & a_6 \\ a_1 & a_7 \end{vmatrix}$$

$\vdots$

s^{n-3} : c_1 c_2 c_3 $\cdots$

$$c_1 = -\frac{1}{b_1} \cdot \begin{vmatrix} a_1 & a_3 \\ b_1 & b_2 \end{vmatrix}$$

$$c_2 = -\frac{1}{b_1} \cdot \begin{vmatrix} a_1 & a_5 \\ b_1 & b_3 \end{vmatrix}$$

$\vdots$

★ ① p is stable iff all entries in the first column are positive. (with a necessary cond. : $a_1, \ldots, a_n > 0$).

② otherwise # of RHP poles = # of sign changes in 1st column.

$- \to +$
or $+ \to -$

For lower order

2nd: $s^2 + a_1 s + a_2$ is stable iff $a_1, a_2 > 0$

3rd: $s^3 + a_1 s^2 + a_2 s + a_3$ is stable iff $\begin{cases} a_1, a_2, a_3 > 0 \\ a_1 a_2 > a_3. \end{cases}$

Routh-Hurwitz as a Design Tool.

$$H = \frac{Y}{R} = \frac{\text{Fwd gain}}{1 + \text{loop gain}} = \frac{\cdots}{\text{polynomial}}$$ check if it's stable.