ECE 486: Control Systems Homework 4

Question 1

Consider the feedback system in Figure 1.

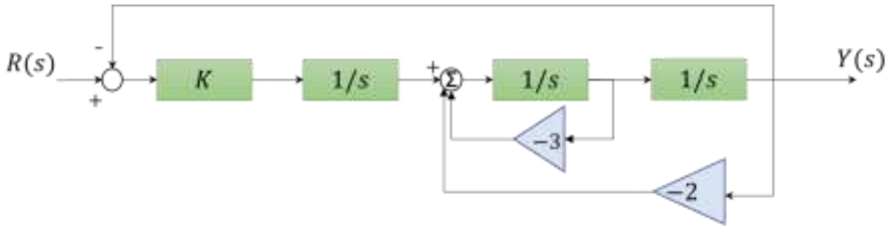

Figure 1

a) Write down the closed loop transfer function of Figure 1 in terms of K. *(3 Points)*

b) Write down the characteristic polynomial of the closed-loop transfer function in terms of K. *(2 Points)*

c) Find the range of K for closed loop system being stable *(6 Points)*

d) Show that the system can be expressed in Evan's form as in Figure 2.

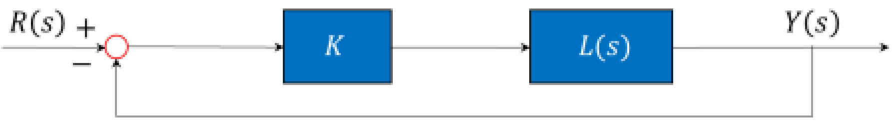

Figure 2

d) Write down the poles of $L(s)$ *(3 Points)*

e) Sketch the root locus *(6 Points)*

Solution

a) Root Locus shows the path of s for the characteristic equation with varying K

$H_{CL}(s) = \frac{Y(s)}{R(s)} = \frac{K}{s(s^2+3s+2)+K}$

b) Characteristic Equation: $s^3 + 3s^2 + 2s + K = 0$

c) Using Routh-Hurwitz Stability Analysis

Characteristic polynomial equation $s^3 + 3s^2 + 2s + K = 0$

The necessary condition is that K >0.

Routh Array

$$\begin{array}{c|c|c|}
s^3 & 1 & 2 \\
s^2 & 3 & K \\
s^1 & \frac{6-K}{2} & 0 \\
s^0 & K &
\end{array}$$

The necessary and sufficient condition for stability is $0 < K < 6$

d) In Evan's form: $H_{CL}(s) = \frac{KL(s)}{1+KL(s)}$, where $L(s) = \frac{1}{(s^3+3s^2+2s)} = \frac{1}{s(s+1)(s+2)}$

e)

Rule A: <u># branch</u>= deg(s^3) = 3

Rule B: <u>Start Points</u> at open loop poles (from (d))

$s(s+1)(s+2) = 0 \Rightarrow s = 0, -1, -2$

Rule C: <u>End Points</u> at open loop zeros

Zeros at $\pm\infty$

Rule D: <u>Real Locus</u>:

$\angle L(s) = \angle\frac{b}{a} = 180^{o}$, try a few points to validate that $s \leq -2$ and $-1 \leq s \leq 0$, if s is real (i.e. on horizontal axis).

Rule E: <u>Asymptotes</u>

Branches near ∞ have phase

$$\angle s = \frac{180(2l+1)}{n-m}, l = 0,1, \ldots n-m-1$$

$\angle s = \frac{180(1)}{3} = 60^o, l = 0$

$\angle s = \frac{180(3)}{3} = 180^o, l = 1$

$\angle s = \frac{180(5)}{3} = 300^o \ or -60^o \ , l = 2$

Rule F: <u>$j\omega$-crossings</u>

Using Routh-Hurwitz Stability Analysis

Characteristic polynomial equation $s^3 + 3s^2 + 2s + K = 0$

The necessary condition is that K >0.

Routh Array

$$\begin{array}{c|c|c|}
s^3 & 1 & 2 \\
s^2 & 3 & K \\
s^1 & \dfrac{6-K}{3} & 0 \\
s^0 & K &
\end{array}$$

Therefore $0 < K < 6$ implying K having a critical value of 6.

$$(j\omega)^3 + 3(j\omega)^2 + 2j\omega + 6 = 0$$

At $j\omega$-crossing, real part equal zero, $\omega = \pm\sqrt{2}$