1 Externality

Kevin is a graduate student in economics and is endowed with a dollar amount y_A per day. Francisco is not an economist and is endowed with dollar amount y_B per day. It costs Kevin \$4 an hour to produce economics research. Let x denote the number of hours per day that Kevin devotes to research. Kevin's research gives Francisco trading signals, and thus Francisco also has a preference over Kevin's research. Kevin's and Francisco's utilities are u_A, u_B, respectively:

$$u_A(x, y_A) = \begin{cases} y_A + 8x - x^2/2 & x \leq 8 \\ y_A + 32 & o.w. \end{cases}$$
$$u_B(x, y_B) = \begin{cases} y_B + 12x - x^2/2 & x \leq 12 \\ y_B + 72 & o.w. \end{cases}$$

1. What are the marginal rates of substitution between Francisco and Kevin?
2. How much will Kevin produce if he doesn't know that Francisco exists.
3. Suppose that Kevin produces the amount of research in 2 and Francisco has found a way to consume Kevin's research by downloading it for free from ArXiv. Kevin has no way to prevent Francisco's free-riding. How much consumer surplus do Francisco and Kevin obtain?
4. Pareto efficient allocations are characterized by the relation, "sum of marginal rates of substitution equals marginal cost of production." What is the Pareto efficient volume of research for Kevin to produce? What is the total surplus at this level of research?
5. Now suppose that Kevin has found a way to protect his research from Francisco's free-loading. From now on, Francisco will have to pay p dollars for every unit of research he downloads. If Kevin chooses the price p that maximizes his "profit from Francisco" (revenue from Francisco, minus cost of production), what price will he charge and how much research will he produce? What will be Kevin and Francisco's consumer surpluses?

2 Spicy Cobb–Douglas

Suppose there are two goods x, y and utility

$$u(x, y) = \begin{cases} (x - \overline{a})^{\alpha}(y - \overline{b})^{1-\alpha} & x > \overline{a}, y > \overline{b} \\ -\infty & o.w. \end{cases}$$

with $\overline{a}, \overline{b} > 0$. Let w denote wealth and assume $p_x\overline{a} + p_y\overline{b} < w$.

1. Derive the Marshallian demand for x, y.

2. Derive the Hicksian demand functions for x and y from the cost minimization problem.

3. Show that the own-price Slutsky equation for good x holds for this utility function.

4. Using your Slutsky decomposition from the previous part, can you say which demand function, Marshallian or Hicksian, is more sensitive to price changes? Is this true for all goods and all utility functions (briefly)?

3 Vertical Monopolies

There is a unit mass of identical price-taking consumers. Each consumer has income y and consumes widgets x and some numéraire good x_0, with utility function

$$u(x, x_0) = x_0 + \frac{1}{\sigma + 1} x^{\sigma+1}, \quad -1 < \sigma < 0$$

The price of x is p and the price of x_0 is 1.

1. Assume that y is large so that x_0, x have positive demand. Derive the Marshallian demand for x as a function of p. Given p, derive the consumer surplus in good x.

2. What's the elasticity of demand?

3. Assume the consumers purchase positive amounts of x. The consumers purchase x from a monopolist retailer, who sets the price p. Suppose the retailer faces constant marginal cost c_R, solve for the profit-maximizing p as a function of c_R. Derive the retailer's profit π_R.

4. Assume the monopolist purchases x from a monopolistic wholesaler, who sets c_R and faces constant, exogenous marginal cost c_W. Solve for the profit-maximizing c_W. Derive the wholesaler's profit π_W and the sum of the wholesaler and retailer's profits $\pi_W + \pi_R$.

5. What is the total surplus $\text{CS} + \pi_W + \pi_R$?

6. Suppose the retailer and wholesaler merge into a monopoly that charges p_{merged} to the consumers and takes c_W marginal cost. Find the optimal p_{merged} and total surplus.

7. Suppose $\sigma = -1/2$. Should the total-surplus-maximizing Federal Trade Commission let the retailer and wholesaler merge?

8. (Bonus) Prove that the total surplus under the merged firm is larger for all $\sigma \in (-1, 0)$. [Hint: Use a version of Bernoulli's inequality: $(1+t)^r \geq rt$ for $t > -1$, $r \leq 0$]

9. True/False/Uncertain: Mergers always hurt the consumer.