A modular abstract interpreter for Julia
MPRI internship report

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General context

Julia[BEK14, 1] is a dynamically typed, JIT compiled, programming language implementation with an emphasis on generic programming through a powerful and ubiquitous symmetric, dynamic multiple dispatch mechanism. It achieves superior run-time performance by relying heavily on static analysis and aggressive code specialization. It is meant to provide a tool for scientific computations that is both familiar and fast enough for this demanding usage. The Julia compiler also provides a range of metaprogramming and code generation techniques to the advanced user.

Efforts into optimizing dynamic languages date back to Common Lisp tradition[BAP86], and more recently the sizable effort into making various JavaScript engines reach high performance[GAE09]. Julia has the interesting advantage of having been designed from the ground up to be a good analysis target and, while being dynamic, avoids pathological behavior patterns that are very hard to infer statically.

Studied problem

Most optimizations in the Julia compiler are done by the LLVM[Lat02] library in the backend. LLVM is a widely used, production-grade compiler and generally generates excellent machine code. However, its intermediate representation operates at an abstraction level comparable to that of C.

Some program optimizations would be better performed on the much higher level semantic that Julia code exhibits earlier in the compilation pipeline. The backend has to deal with additional complexity due to the integration between the generated code and the various run-time systems: dynamic memory allocation, garbage collection, dynamic dispatch call-sites, ...

At the Julia level, all those operations are implicit and the allowed behaviors are much more restricted: it has no interior pointers, no stack escape, memory operations cannot trap, type-punning is forbidden, the list goes on. It makes some analysis easier, especially those modelling memory, such as alias and shape analysis.

Proposed contributions

As the main project of this internship, an extensible static analysis library for Julia was started. Dubbed the *Green Fairy*, it is written itself in Julia. The exposed interface is that of an abstract interpreter, providing an intuitive model for program behavior inference.

It uses a spatially sparse representation of the interpreter state enabling scalability across large programs. The main value domain is an extensible generic implementation of the reduced product construction, making it reasonably easy to extend it with new models.

It also features a work-in-progress interprocedural flow-sensitive shape analysis, based on static shape graphs[SMR98]. The shape analysis uses a sparse representation as well. It is interprocedural but context-insensitive. It tracks bounded allocation lifetimes across loop iterations precisely, opening up the possibility of interesting static garbage collection optimizations.

Arguments supporting their validity

The implementation is available as an ongoing work at [2]. It is already capable of applying both value-level (currently constant propagation and type inference) and shape analysis to a significant body of Julia code. On this body of code, it matches or exceeds the precisoin of Julia's current type inference pass. It succeeds in demonstrating our ideas on a *non-toy* real world language, including its more exotic and idiosyncratic features. We hope to integrate our work into the main compiler in the coming year.

Future work

Besides tying up the various technical loose ends and improving the non-algorithmic bottlenecks of the implementation, we have several goals for the future of this project.

A consequent amount of technical work has to be done to make the later stages of the compiler more modular. Today, there is no straightforward way of taking advantage of new static information during compilation. In particular, we plan to implement optimizations using the shape analysis to provide precise no-escape information that will allow memory reuse and stack allocation, greatly reducing the load on the garbage collector and the dynamic allocator.

In the longer term, the natural extension to this work is to devise an intuitive interface that makes it possible for users to extend the abstract state of the interpreter with domain-specific informations. We hypothesize that there is a large class of optimizations that are hard-to-impossible to teach a compiler using only knowledge of the language's semantic, but are fairly easy to formulate once more information is given about specific data structures and idioms in use.

Those points are developed further along in the report.

Julia

Julia has a dynamically-typed semantic. The execution model is that of a typical imperative language : impure and call by value. It is entierly built around the concept of generic functions.

Type system In Julia, simple types are organized into a single-inheritence subtyping tree. They are optionally parametrized by arbitrary values, including other types. The type lattice is the closure of simple types under finite unions and bounded existentials. Although parametrized types are invariant, one can construct covariant types using existentials : $\exists \mathrm{T} \leq \mathrm{Number}, \mathrm{Vector}\{\mathrm{T}\}$ can be thought of the covariant version of $\mathrm{Vector}\{\mathrm{Number}\}$.

Generic functions A function in Julia is in fact a collection of methods, partially ordered by their signature types as a sub-lattice of the type lattice[BEK14]. The dispatch mechanism is dynamic – albeit statically inferred in many cases – and is symmetric in all the arguments: the most specific method will be called at run-time depending on the argument types. It is ensured that for any argument type there exists a well defined least upper bound in the sub-lattice. More details about Julia's generic functions can be found in [BEK14, Bez15].

Generic functions have interesting repercussions on analysis strategies. Since virtually every function call is indirect, not knowing the argument types can lead to a combinatorial blowup of code to explore in an interprocedural analysis, leading to widening and more precision loss. It is therefore often worth to trade performance for precision as long as it can keep types fully known[Bez15].

Memory model At the specification level, Julia objects are tagged memory cells living in an infinite heap. Each has a finite set of numbered mutable fields, possibly pointing to other objects. The language relies on automatic memory management to fit in a physical finite heap. The current implementation uses a generational tracing garbage collector.

Compiler The main analysis performed on Julia code by the compiler is an interprocedural flow-sensitive type inference. It is followed by an inlining pass and several ad-hoc peephole optimization. The last step is to generate LLVM intermediate representation while using the infered types to remove as much of the dynamic semantic as possible. The approach of using a dynamic semantic complemented by type inference has the advantage that the inference pass can be customized without breaking user code. Heuristics are added transparently as certain language idioms become popular and require special treatment for a precise analysis.

The project started in this internship aims to address the limitation that the only static information computed on high level Julia code today is in the type lattice.

Green Fairy

The Green Fairy provides a generic framework that computes the fixed point of user-provided abstractions over a Julia program. The library handles the control flow semantics and presents itself as an extensible interpreter : users enrich the abstract machine's state with custom abstractions. We will assume that the reader is familiar with the abstract interpretation[CC77] formalism.

We provide a set of generic basic blocks to construct those state extensions. The most common one is a non-relational variable analysis over any value domain. For example, instantiating this variable state domain over the value lattice of constants builds the algorithm known as Sparse Conditional constant propagation[WZ91]. It can be used jointly with the supplied parametric reduced product[CCM11] implementation, allowing for cooperative behavior between different domains.

We will start by giving two simple examples of value domains implementations.

Interval example

Follows an implementation of a classical example of abstract domain: the (closed) numeric interval.

```
immutable Interval{T} <: Lattice
    lo :: T
    hi :: T
end

bot{T}(::Type{Interval{T}}) = Interval(typemax(T),typemin(T))
top{T}(::Type{Interval{T}}) = Interval(typemin(T),typemax(T))
isbot(x::Interval) = x.hi < x.lo

<=(x::Interval,y::Interval) = isbot(x) || y.lo <= x.lo && x.hi <= y.hi
join{T}(x::Interval{T},y::Interval{T}) = Interval(min(x.lo,y.lo), max(x.hi,y.hi))

function meet_ext{T}(x::Interval{T}, c::Const)
    !istop(c) && isa(c.v,T) || return x
    x.lo <= c.v <= x.hi && return Interval(c.v, c.v)
    bot(Interval{T})
end
```

This domain is parametric over any ordered numeric types having an upper and lower bound. Defining a method for `isbot` is necessary since there are multiple representation of $\perp$.

The `meet_ext` function is the product domain reduction function. In that case we express that the knowledge that a value is constant can reduce an interval to a single point or $\perp$ depending on the particular value. We are free to add other method definitions to this function to increase precision using information from other domains than `Const`. This modelization of the reduced product relies on multiple dispatch to easily provide extension points: the user can seamlessly provide reduction functions for any pair of domains, possibly defined by someone else.

Describing transfer functions is also straightforward, as demonstrated by this simple example implementing the transfer for the addition on the interval lattice:

```
function eval_call{T,V}(::Type{Interval{T}}, f::V, args::Vector{V})
    if f <= Const(Base.(+))
        intervals = map(v -> convert(Interval{T}, v), args)
        lbs, ubs = map(i -> i.lo, intervals), map(i -> i.hi, intervals)
        return Interval(sum(lbs), sum(ubs))
    end
    top(Interval{T})
end
```

We can see that the arguments can be provided in different domain than that of intervals, requiring a `convert` call. This gives access to the full product to a transfer function, making it possible to use information from other domains in the computation.

Finite disjunction

We give another example that demonstrates the construction of a *finite disjunction* domain that is parametric over any other domain and can thus serve as a composable building block. It can be used to enhance precision while keeping the implementation simple and decoupled. For the sake of simplicity, the implementation shown here is very naive: the widening does $O(n^3)$ comparisons and transfer is exponential in the number of arguments. Nonetheless, nothing prevents swapping this implementation for a more efficient one while keeping the genericity of the interface and its independence over the underlying domain.

```
immutable FiniteSum{T <: Lattice,n}
    values :: NTuple{n,T}
end
top{T,n}(::Type{FiniteSum{T,n}}) = FiniteSum(ntuple(i->top(T),n))
istop(v::FiniteSum) = any(istop, v.values)
bot{T,n}(::Type{FiniteSum{T,n}}) = FiniteSum(ntuple(i->bot(T),n))
isbot(v::FiniteSum) = all(isbot, v.values)
<={T,n}(v1::FiniteSum{T,n}, v2::FiniteSum{T,n}) =
     all(a -> exists(b -> a <= b, v2.values), v1.values)

# other domains can opt-in to provide more precise costs
join_cost(v1,v2) = v1 <= v2 || v2 <= v1 ? 0.0 : 1.0
function FiniteSum{T,n}(values::Vector{T})
    while length(values) > n
        costs = [join_cost(a,b) for a in values, b in values]
        idx_a, idx_b = ind2sub(costs, indmin(costs))
        push!(values, join(values[idx_a], values[idx_b]))
        deleteat!(values, idx_a, idx_b)
    end
    FiniteSum{T,n}(tuple(values...))
end
join{T,n}(v1::FiniteSum{T,n},v2::FiniteSum{T,n}) =
    FiniteSum{T,n}([v1.values..., v2.values...])
```

We can then define generic transfer and reduction functions that delegates the computation.

```
function eval_call{T,n,V}(::Type{FiniteSum{T,n}}, f::V, args::Vector{V})
    args = [convert(FiniteSum{T,n},args[i]).values[j] for i in eachindex(args), j=1:n]
    eval_with(acc, i) =
        i > length(args) ? eval_call(T,f,acc) :
            reduce(vcat, [eval_with([acc..., args[i,j]],i+1) for j=1:n])
    FiniteSum(eval_with([],1))
end
meet_ext{T,n}(v1::FiniteSum{T,n}, v2::Lattice) =
    FiniteSum(map(v -> meet_ext(v, v2), v1.values))
meet_ext{T,n}(v1::FiniteSum{T,n}, v2::FiniteSum{T,n}) =
    FiniteSum(map(v -> meet_ext(v, v2), v1.values))
meet_ext{T,n}(v1::Lattice, v2::FiniteSum{T,n}) =
    reduce(join, map(v -> meet_ext(v1, v), v2.values))
```

Here, we show that this finite disjunction can specify the reduced product reduction for both $T_1 \times (\sqcup^n T_2) \to T_1$ and $(\sqcup^n T_2) \times T_1 \to \sqcup^n T_2$ with no particular knowledge of T_1 or T_2.

Those two examples illustrate the benefits of the pervasive use of ad-hoc polymorphism and multiple dispatch in Julia: it allows for the construction of reusable abstractions.

Several important opt-in features for value domains and their transfer functions were not described here for the sake of brevity. This includes fused join-comparisons, widening, participation in the dispatch machinery, the ability to control the propagation of values over function call boundaries (context sensitivity), as well as state-altering actions such as exception raising or heap operations.

Intermediate Representation

Our interpreter operates on a low-level dialect of Julia source code. We will give here a simplified description of this internal representation.

A function is a control graph composed of extended basic blocks (EBB). Their defining property is to have a single control flow entry point as their head. Each one contains a linear succession of branches and variable assignments to literals and function calls.

Some function calls have built-in semantic. We will mostly discuss `setfield!(x,f,y)` and `getfield(x,f)`, noted respectively $x.f = y$ and $x.f$. As their name indicate they describe field access and mutation on Julia objects.

Our implementation deals with most of the current Julia semantic. We will leave out some of those features because while they require additional work, they are not directly relevant to this report. This includes

- Exception handling. Even though special care was taken to handle it with complete precision (assuming of course precise throw-site information).
- Closure capture.
- Function calls with variable number of arguments, or “splatting”.

- Arrays of references. We treat them as an object with a single weakly updated field.

Analysis sparsity

The Green Fairy is oriented towards building flow-sensitive analysis, that is, it provides information dependent on the program point. The advantage of flow sensitivity is in the precision gain, although it comes with a performance price compared to the more approximative flow-insensitive analysis. There are different techniques to lower the cost of flow sensitivity.

A simple way to implement a flow-sensitive computation is to store a dense mapping of program points to interpreter state. It has the benefit of being oblivious to the structure of said state. However, as the state or the analyzed program gets larger, there is a growing cost associated to the copying and comparison of those state atoms. In the vast majority of cases, the states at two nearby program points are highly redundant and taking advantage of this redundancy can lead to consequent improvements in analysis performance. Exploiting this so-called *spatial* sparsity of the state space is an important design point of our implementation.

There is another related kind of sparsity, that is sometimes called *temporal*, where the analyzer benefits from knowledge of state dependencies to avoid recomputing transfer operators that only use unchanged subset of the state. It allows the interpreter to make results from computations flow directly into use sites. In the current implementation we do not use temporal sparsity, mainly due to a combination of time constraints and soundness concerns. However, care was taken to ensure the feasibility of this optimization in the future. In the literature, the word *sparse* without qualification usually refers to temporal sparsity but we will mostly discuss spatial sparsity here.

Non-relational

We implemented a sparse representation of non-relational state that accommodates any user-defined domain for forward analysis in a transparent manner. This representation *does not* require use/def sites to be identified ahead of time, unlike the Static Single Assignment (SSA) transform or other techniques relying on pre-analysis[OHL12]. Furthermore, we present an extension of this idea to a relational domain, namely, the static shape graph heap model as introduced in [SMR98].

Static single assignment form The classical example of sparsity is the *SSA* transform. If a program is in SSA form, any forward variable analysis can easily be both spatially and temporally sparse. Taking advantage of the SSA transform is a two phase process. A pre-analysis identifies dataflow dependencies of variable uses over definitions. Once done, those dependencies are made explicit in the source code by renaming variables and inserting explicit join operations (φ instructions). It enables the use of a flow-insensitive analysis while keeping the precision of a flow-sensitive one.

Dominator tree An important notion in the SSA transform is that of domination: we say that a program point *a dominates* another point b if any path in the control flow graph from the root to b has to go through a. The domination relation is a partial order on program points and is moreover a tree (every initial segment is well ordered). It is usually called the dominator tree. In our case, since every path to a program point has to go through the head of its containing EBB – and all the points between the head and itself – we will say that a program point dominates an EBB if it dominates its head. Given an EBB, the program point being its parent in the dominator tree is called the *immediate dominator*.

The dominance frontier of a program point a, noted $\mathrm{DF}(a)$, is the set of EBB heads that, while not being dominated by a, have predecessors that are. $\mathrm{DF}(a)$ is the set of nodes that one has to pass through in order to, starting from a, leave its domination. Following the usual convention, we note $\mathrm{IDF}(a) = \mathrm{lfp}_a \, \mathrm{DF}$ the iterated dominance frontier of a.

If a variable v has two definitions at a and b respectively, the SSA form disambiguates between them by inserting φ nodes in a subset of $\mathrm{IDF}(a) \cap \mathrm{IDF}(b)$. This set is an upper bound, because some of those φ may end up without any use. Computing the minimal φ placement is harder and requires a form of liveness analysis. The advantage of the iterated dominance frontier is that $\mathrm{IDF}(a)$ only depends on a and can be cached independently of the variable considered.

Sparse map We take a slightly different approach from the usual SSA transform: we compute the same information but *on the fly*. We assume that use sites of variables cannot be statically determined in advance without computing transfer functions. It allows new use and definition sites to be discovered at an already processed program point in later iterations. The main reason for this choice is to allow for other quantities than variable values to profit from the sparse infrastructure. For example, we use it in our implementation to track thrown exceptions: knowing precisely if an instruction can or may throw an exception, as well as properties about the hypothetical exception, requires computing the transfer function.

Essentially, we are providing a generic sparse map implementation from an arbitrary set of names to any value domain. It can be used to represent any non-relational internal interpreter state of the user's choice.

It is hence inefficient in our case to maintain the renaming property. Instead, when encountering an use, the interpreter walks the dominator tree upward to find the nearest definition. This lookup is done following the tree spine, giving it an O(dominator tree depth) worst case complexity. In practice, the dominator tree is shallow and wide.

As for the implementation, we do not materialize φ instructions: they are kept out of band as a single compound summary information for each EBB head. They can be efficiently dynamically updated when required by the discovery of new definitions. We compute the dominator tree and frontiers using the algorithm described in [CDH01].

We have not yet witnessed this process to be a bottleneck. Would it become the case, there are several ideas that could be implemented to lower its cost. The most obvious one being use-forwarding. As an aside, the choice of EBBs for the program representation offsets the time spent doing this lookup in practice by making common dominator trees shallower. It is not a worst-case improvement but a welcomed average constant factor gain.

Relational

Although [OHL12] discusses relational domains, it only describes a solution for packed variable domains. It is precisely the class of relational domains that can be reframed as non-relational over a different set of names. The sparse map discussed in the previous section can be used as a basis to implement packed relational analysis.

This structure cannot be readily used for general relational domains, however we can extract an important insight from the non-relational case. We can think of the φ node living at an EBB's head as a summary of the state difference on any path between its immediate dominator and itself. It is less apparent in the case of a non-relational domain, since this difference is usually represented as an element of the domain itself, i.e., a small name-value mapping that only refers to changed names.

[good example + explanation of a generic translation]

Shape analysis

It should be noted that, compared to the state of the art in shape analysis, especially in safety analysis, what we implemented and present here is much less precise. In particular, the interprocedural component is its weaker part since it is not context sensitive. We are however seeing promising performance results and are hopeful that we can include this analysis as part of a production compiler, which is not typically possible with more precise shape analysis.

We are also more interested in bounded heap structures and local invariants for optimization, whereas typical shape analysis try to infer global facts on unbounded data structures. In other words, we aim to fill the need for more precise escaping and aliasing information in an optimizing compiler, rather than match competitive shape analysis.

Alias analysis

For a moment, we will ignore object fields and summary objects to concentrate on variable aliasing and its sparse structure. We will introduce those back later. One of the fundamental insight of [SMR98] is in the scheme used to describe abstract heap locations. The abstract address of an object is given by the exact set of stack variables that refers to it. It means that we can represent the abstract heap as a collection of set of variable names. The main advantage of this approach compared to the other usual choices, such as allocation site segregation, is that transfer functions operating on objects referenced by variables can *always* perform strong updates. It is due to the fact that each one of those abstract addresses can refer to at most a single object in any given concrete heap. Another very important point about this choice is that it provides a canonical way to compare two abstract heaps : there is no ambiguity on the mapping between the nodes, avoiding any kind of graph isomorphism problems.

In this framework there is no explicit representation of the may/must alias sets. We can construct them in the following way :

$$\begin{aligned}
&p(H,x) = \{o \in H \mid x \in o\}\\
&\text{may-alias}(H,x) = \{y \mid p(y) \cap p(x) \neq \emptyset\}\\
&\text{must-alias}(H,x) = \{y \mid p(x) = p(y)\}
\end{aligned}$$

We refer to [SMR98] for the formal deduction rules and computation recipes in this abstract domain. We will focus here on exposing a way to implement a similar state domain with a sparse representation. The main inspiration comes from [NL09]: they develop a sparse alias analysis for local variables that we use as a basis for our shape domain. The authors make the following observation.

Assuming that the analyzed code is in SSA form, we can order the set of program variable under the domination relation of their definition. That being done, it is clear that when transferring the state over a non-dominating edge, we can trim all heap objects of variables whose definition stopped dominating the current control point. In fact, we can always remove any dead variables from our abstract heap, and non-dominating definitions are always dead thanks to the fundamental property of SSA.

To be slightly more precise, here is a sketch of the transfer function for variable assignment and φ nodes with two predecessors.

$$T[x=y](o) = \begin{cases} o \cup \{x\} & \text{if } y \in o \\ o & \text{otherwise} \end{cases} \qquad T[x=y](H) = \Big\{\ T[x=y](o) \mid o \in H\ \Big\}$$

$$T[\tilde{x}=\tilde{y}] = T[x_1=y_1] \circ \cdots \circ T[x_n=y_n]$$

$$T[\varphi(\tilde{x}=\tilde{y},\tilde{z})](H_l,H_r) = \big(T[\tilde{x}=\tilde{y}](H_l) \cup T[\tilde{x}=\tilde{z}](H_r)\big) \cap 2^{\text{dom}(\tilde{x})}$$

where $\text{dom}(\tilde{x})$ is the set of variable having their definition in a dominating position over the φ node, including itself. It is important to notice that we don't need to remove x from any object when assigning to x since the definition was non-dominating before applying T.

As we alluded to in the previous section, one can look at the restriction to dominating definitions as a *summarization* of the state between the φ's EBB head and its immediate dominator.

From this formulation, we can deduce a sparse format for this simplified shape graph : there is a consequent amount of sharing between subsequent states. In fact, we can see from the transfer functions that they only need to manipulate the "most recently" defined variables in each object. In [NL09], this sharing is exploited by representing heap objects as sorted singly-linked list with hash-consing. The elimination of redundancy is thus implicitly handled by the data structure.

It has the benefit of keeping the implementation close to the mathematical structure, but forces the algorithm to keep the linked list heads of *all* live heap objects at all control points, defeating some of the sparsity. We take an alternate route of representing the tree explicitly with pointers in both directions. At the cost of additional lookups along its spine – much like in the non-relational case – we can cut down the storage at each control point to the set of modified objects. Doing this, we also avoid the need for hash-consing.

Alias tree Our encoding of variable aliasing takes the form of a forest of doubly-linked trees. Each node is labeled by a variable definition and several nodes can have the same label. Every tree respects the dominance ordering of variable definitions. An initial segment $x_1 \dots x_n$ of this forest represents the heap object $\{x_1, \dots, x_n\}$ at the control point where the definition x_n is.

Given a program point a and a variable definition x dominating a, we can characterize all heap objects x points to when the interpreter is at a. They are initial segments of the forest, they contain x, all their elements dominate a and they are maximal for those properties.

We can see from this description that additional work is needed to materialize the point-to set of a definition x at a control point further away from the definition point.

High level $T[x = y]$ and $T[\varphi]$ for sparse variable alias analysis on the alias tree

```
function MATERIALIZE($H$, $a$, $x$)
    $d = $ FINDDEF($x$, $H$, $a$)
    $R = \{$nodes of $H$ labeled by $d\}$
    while $R$ changed
        for $o \in R$
            $C = $ children($o$ in $H$) $\cap$ dom($a$)
            if $C \neq \emptyset$
                $R = (R - \{o\}) \cup C$
            end
        end
    end
    return $R$
end

function TRANSFER[$x = y$]($H$, $a$)
    $R = $ MATERIALIZE($H$, $a$, $y$)
    for $r \in R$
        $H, o = $ ADDNODE($H$, def[$a : x$])
        $H = $ ADDEDGE($H$, $o \to n$)
    end
    return $H$
end

function RESTRICT($H$, $\tilde{y}$, $n$, $a$, $b$)
    for $i \in [1, n]$
        $R_i = $ MATERIALIZE($H$, $a$, $y_i$)
    end
    while $R$ changed
        for $i \in [1, n]$
            $U = R_i \cap \big(\mathrm{dom}(b) \cup \mathrm{roots}(H)\big)$
            $V = \{\mathrm{parent}(d) \mid d \in R_i - U\}$
            $R_i = U \cup V$
        end
    end
    return $R$
end

function TRANSFER[$\varphi(\tilde{x} = \tilde{y}, \tilde{z})$]($H_{\mathrm{idom}}$, $a$, $H_l$, $a_l$, $H_r$, $a_r$, $n$)
    $H = H_{\mathrm{idom}}$
    $R^l = $ RESTRICT($H_l$, $\tilde{y}$, $n$, $a_l$, idom($a$))
    $R^r = $ RESTRICT($H_r$, $\tilde{z}$, $n$, $a_r$, idom($a$))
    for $d \in \cup_i R^l_i \cup R^r_i$
        $V = \big\{ i \in [1, n] \mid d \in R^r_i \cup R^l_i \big\}$
        $H, n = $ ADDNODE($H$, $\varphi[a : \tilde{x}_V]$)
        if $d \in$ dom(idom($a$))
            $H = $ ADDEDGE($H$, $d \to n$)
        end
    end
    return $H$
end
```

Generation tracking

A traditional escape analysis is not capable of proving memory reuse opportunities in a loop with cross iteration dependencies. For example, in the following loop, every object allocated in the loop body survives for exactly 3 iterations.

```
while ?
    use(x,y,z)
    z = y
    y = x
    x = new()
end
```

We can take advantage of this information to allocate 3 memory cells and reuse them in a round robin fashion, instead of relying on the garbage collector at runtime. This optimization improves performance by removing automatic collection overhead, amortizing allocation costs, and improving CPU cache behavior.

Although this example might seem contrived, this situation often arises when using heap objects (such as linear tensors) as accumulators inside loops. It is a common pattern in iterative numeric algorithms.

To compute this information, we add to each heap object a "generation number" that represents the maximum number of younger and live objects from the same allocation site. This counter lives in the lattice $\{\bot, 0, \leq 1, \leq 2, \ldots, \leq \infty\}$ with widening to avoid infinite increasing chains. When an alias tree edge crosses its own allocation site, this counter is increased.

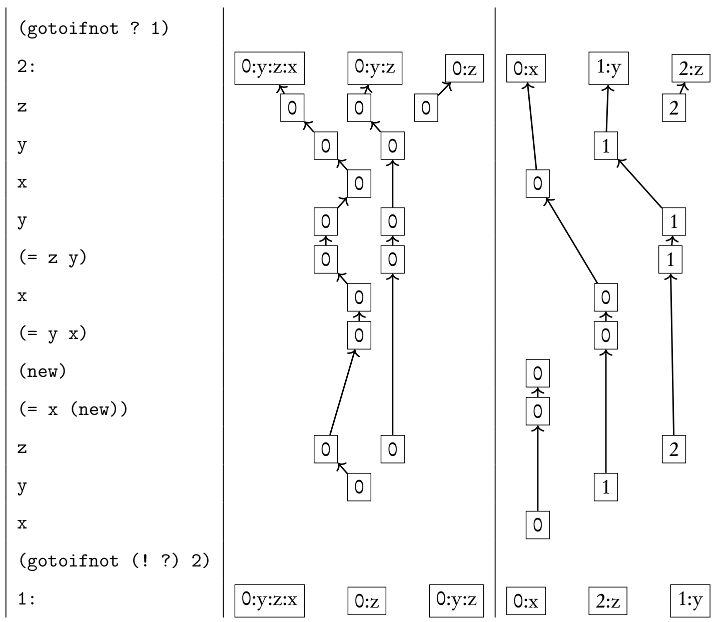
In order to preserve the property that the heap can always be materialized by looking up the dominator tree, we also compute a generation delta between an EBB's head and its immediate dominator for each allocation site. We apply this delta when we restrict alias edges across dominating control edges.

Figure 1 is an example illustrating the result of the generation count and alias analysis for such a loop. The displayed trees are automatically generated by the interpreter.

Object fields

The full abstract heap model with object field is a graph where nodes are labelled with sets of variable definitions and edges with field indices. We will describe most of the abstract semantic assuming a single field name for simplicity, but the extension to a finite number field names is straightforward. Adding object fields also implies the presence of summary nodes. They are necessary to avoid infinite increasing chain in the lattice of shape graphs. A summary node is a node that no local variable points to, and may represent several concrete objects. There are a lot of different techniques and heuristics on when to collapse empty nodes into summary nodes. To preserve a fast implementation, we enforce the property that there can be no edge between two summary nodes: we always collapse them together. This limits the language of heap path we can describe outside our local stack frame to regular expressions of the form $(f_1 + \cdots + f_n)^*$ where the f_i are field names.

```
x = y = z = new()
while ?
    z; y; x
    z = y
    y = x
    x = new()
    z; y; x
end
```



```
(gotoifnot ? 1)
2:
z
y
x
y
(= z y)
x
(= y x)
(new)
(= x (new))
z
y
x
(gotoifnot (! ?) 2)
1:
```

Figure 1: Example of alias forest with generation counting
From left to right: a simple loop with allocation dependencies crossing 3 iterations, its linearized body's Julia IR, and the two alias trees corresponding to the two allocation sites. We expanded the 3 variables two times to make the aliasing and the generation increases more evident. Nodes with explicit variable names are part of an EBB's head summary (our "φ" nodes).

We can define the ordering on this lattice in the following way. We will say that $H \leq H'$ iff H can be obtained from H' by a succession of moves among those possibilities:

- Remove a node and all its inbound and outbound edges
- Remove an edge
- Split a summary node into two summary nodes and duplicate all edges accordingly (Figure 2)
- Demote a summary node to a normal node

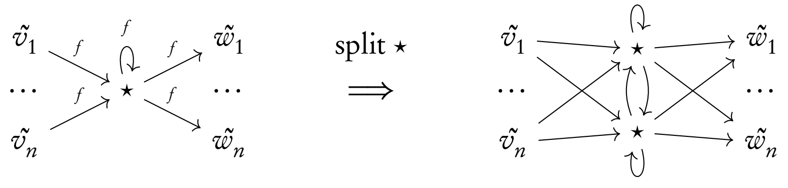


Figure 2: Summary node splitting

Most transfer operators on the abstract heap are straightforward. Variable assignments do not change from the simpler model exposed in the last section. Field assignments are always strong except when the source is a summary node.

We can improve on the naive field access transfer by taking into account incompatible objects. Assuming no summary nodes, we can describe field access by

$$E(x,f)=\left\{x\tilde{u}\to\tilde{v}\in H_{\text{edges}}\right\}\qquad N(x,f)=\{\text{dest}(e)\mid e\in E(x,f)\}$$
$$P(y,x,f,\tilde{u})=\{\tilde{v}\in N(x,f)\mid \tilde{v}\cap\tilde{u}=\emptyset\}$$
$$T[y=x.f](H)_{\text{nodes}}=\left(H_{\text{nodes}}-N(x,f)\right)\cup\bigcup_{\tilde{v}\in N(x,f)}P(y,x,f,\tilde{v})\cup\{\tilde{v}y\}$$
$$T[y=x.f](H)_{\text{edges}}=\left(H_{\text{edges}}-E(x,f)\right)\cup\{\text{src}(e)\to\text{dest}(e)y\mid e\in E(x,f)\}$$

The graph representation of this operation is shown in Figure 3.

If there is an f-edge from x to some summary node, then $T[y=x.f]$ does a split where the duplicate node is a normal node containing only y. This way, we keep only memory not referenced by the stack summarized.

$$\begin{tikzcd}
y\tilde{w}_1 & & \tilde{v}_1 & T[y=x.f] & \tilde{w}_1 & & y\tilde{v}_1 & \tilde{v}_1 \\
\cdots & x\tilde{u} \arrow[ur, "f"] \arrow[dr, "f"'] & \cdots & \Longrightarrow & \cdots & x\tilde{u} \arrow[ur, "f"] \arrow[dr, "f"'] & \cdots & \cdots \\
y\tilde{w}_m & & \tilde{v}_n & & \tilde{w}_m & & y\tilde{v}_n & \tilde{v}_n
\end{tikzcd}$$

$$\begin{tikzcd}
 & zw & T[y=x.f] & & zwy & \\
x \arrow[ur] \arrow[r] \arrow[dr] & w & \Longrightarrow & x \arrow[ur] \arrow[r] \arrow[dr] & wy & w \\
 & z & & & zy & z
\end{tikzcd}$$

Figure 3: Transfer function for field access
The first graph shows the general case. The second is an example demonstrating incompatible nodes.

Implementation Object fields are additional edges in the alias tree, labelled with a field index. Given an initial segment $x_1\ldots x_n$ of the forest, we define the possible values of its kth field to be the set of destination of k-edges originating at x_p where p is the largest index such that x_p has at least one k-edge. Given this description, translating the abstract transfer into an implementation over the forest is straightforward.

When summarizing the alias tree, we collect fields set during the summarized segment of code and copy those in the EBB head. This is also the only operation that can introduce new empty nodes. We promote them into summary nodes and merge connected ones.

Context sensitivity

All the preceding discussion only describes an intraprocedural setting. Making a heap analysis scale well interproceduraly is challenging. The most straightforward way of introducing context sensitivity is through *heap cloning*. Although it has been demonstrated that heap cloning can scale very well[LLA07], this result is only in the context of a flow-insensitive analysis.

We make the choice not to introduce any context sensitive information while computing the fixpoint of the abstract heap over a function. In the situation we presented until now, we would have to treat function arguments – and objects reachable from them – conservatively as a single summary node. We instead introduce a new type of node that tries to keep more information by being less pessimistic about aliasing. Of course, inside the callee, we still can't answer the question whether those outside objects alias each other or not. However, by keeping them separated we allow the caller to regain some information by eliminating impossible nodes and perform a form of "post-facto context sensitivity" using call site information.

This new type of node, that we will call *context* nodes, do not have the guarantee not to alias each other. Unlike summary nodes however, each is still mapped to at most a single concrete object. We also add a new type of edge that we will call *weak* edges to mitigate the loss of precision from conservative alias information. Weak edges can only originate from a context node. We will note context nodes by prefixing their content with γ. We include context nodes in the abstract semantic by adding the following rule to the set of $H \leq H'$ moves previously described. Given two context nodes they can be contracted into a single (context) node.

To preserve soundness, field assignment on context nodes must be relaxed. $T[x.f = y]$ creates a strong edge between any $\gamma x \ldots$ node and $y \ldots$ node, as well as a weak edge between any $\gamma \ldots$ and $y \ldots$ node. Creating a strong edge removes any weak edge sharing the same label and origin.

Field access transfer is unchanged by the addition of context nodes: it must follow weak edges conservatively in addition to strong edges.

The point of this construction is to setup a context insensitive model of the caller's heap: an infinite tree of γ nodes that assumes no aliasing. One can see it as the *free* heap. Each node is uniquely identified by the field path that leads to it from the function argument.

When a function call returns into the caller, it must first contract pairs of γ nodes to match the shape of its pre-call heap. The caller also removes every weak edge from the callee's heap. This establishes a mapping between caller objects reachable from passed arguments and γ nodes from the callee return heap. The caller can then apply field mutation to its own heap following the edges of the callee.

As for the implementation, obviously, the infinite tree is not materialized: it is constructed lazily. This means that weak edges must be kept separatly to be retroactively applied to materialized nodes.

Improvements Several improvements to this scheme are possible that we did not implement as of today. In particular, one can keep a partial ordering of strong edges recording temporality of the field mutation (ties are possible when joining two heaps together). When doing a contraction, the caller can trim outbound edges strictly older than another edge with the same label. Another possible improvement is to use information from other domains, such as types, to avoid adding weak edges to objects that can't possibly alias the mutated object.

Clearly, this is not as precise as a context sensitive analysis. However, it allows us to analyze every function independently of its call site. Moreover, it keeps strong precision for functions that access a statically bounded portion of the heap and do not mutate caller objects. For mutating functions, this model is still able to recover context information in simple cases.

It might be worth exploring demand driven techniques that fall back to limited forms of heap cloning when they detect catastrophic precision losses. We don't have enough experience exercising our model to come up with a good set of heuristics yet. Especially since it depends on what type of optimization are clients to the analysis.

Future work

The current implementation of the Green Fairy is still a prototype in many regards. We will discuss longer term objectives and projects here, keeping in mind that the immediate goal is to test and improve the library to reach a reasonable confidence of soundness on any Julia program.

Backward analysis Our implementation only provides facilities for forward analysis. Several fundamental analysis, such as variable liveness, necessitate backward propagation. We wish to extend the fixpoint computation to support those use case, including the generalization of our sparse structure to backward dataflow (similar to the Static Single Information form).

Generic sparsification We hypothesize that a general transform from dense to sparse state structure underlies the particular one presented in this report and included in our implementation. Although we do not hope for a fully mechanized way to do such a conversion, it is likely that we could provide a set of generic reusable component to make the implementation as well as the soundness proof of those state domains easier. As more use cases appear we will try to develop a general framework, both theoretical and implementation-wise, for this process.

User provided optimizations We briefly hinted at domain-specific optimizations in the introduction. Our long term plan is to provide hooks in the compiler and the analyzer to allow for user defined analysis and optimization. Moreover, those should be able to easily build upon the existing infrastructure. Although the abstract interpreter model is a good fit in that regard, we have not yet found a fully satisfactory equivalent for the optimization side.

As a good example of the envisioned usefulness of this kind of modularity, consider the development of a high-level interface to a BLAS library. This example is particularly relevant since the linear algebra routines are in fact a sizable chunk of the Julia standard library. BLAS is used as their back end in the common case floating point numeric types.

BLAS is easy to use under a simple model where the output of the routines is freshly allocated memory containing the result. This avoids aliasing bugs, however it is inefficient because it does not take advantage of the specialized BLAS functions that can output the result in place, reusing the memory of an argument. Reasoning that the two are equivalent if one of the argument (and all its aliases) are dead after the BLAS call is almost impossible to the compiler because it requires understanding of the routine, of which the core is often hand written assembly. However, the rules of BLAS aliasing are very regular at a high level and someone writing a wrapper for the library could easily, given the right interface, integrate those rules directly as a compiler extension.

Another simple example that was brought to our attention: an user is in the process of writing an access layer for a remote database. The interface provided is able to use arbitrary julia functions for, e.g., predication. As an optimization, it would be useful to provide a way to overapproximate the set of datum that the predicate will load at run-time to perform prefetching. This analysis has no purpose outside of this particular use case. However, reimplementing all Julia semantics is wasteful considering that the database specific part of the analysis is relatively small.

As we work towards making the interpreter more accessible, the need for good tooling becomes unavoidable.

Abstract debugging We would like to provide a common infrastructure aimed at debugging abstract domains. In its simplest form, this system would open the possibility for a domain to provide small run-time guards checking that the state conforms to what was inferred. The analyzer would then, in "verification" mode, insert those guards in the code before running it, along with the necessary information to trace back the exact transfer function that computed an unsound result. This would obviously not replace a formal proof of soundness but would help weed out implementation errors by large scale testing on the consequent amount of freely available Julia source.

Visualization As a way to help implementation work, a graphical visualization of the interpreter was hastily put together using the `Escher.jl` library[3]. The usefulness of this tool exceeded our – admittedly low – expectations. A screen capture is provided in the appendix. We believe that interactive feedback is a key aspect of allowing users with no particular background in program analysis to develop an understanding of the – in fact quite natural – underlying model. It would be interesting to explore the design space of such tools.

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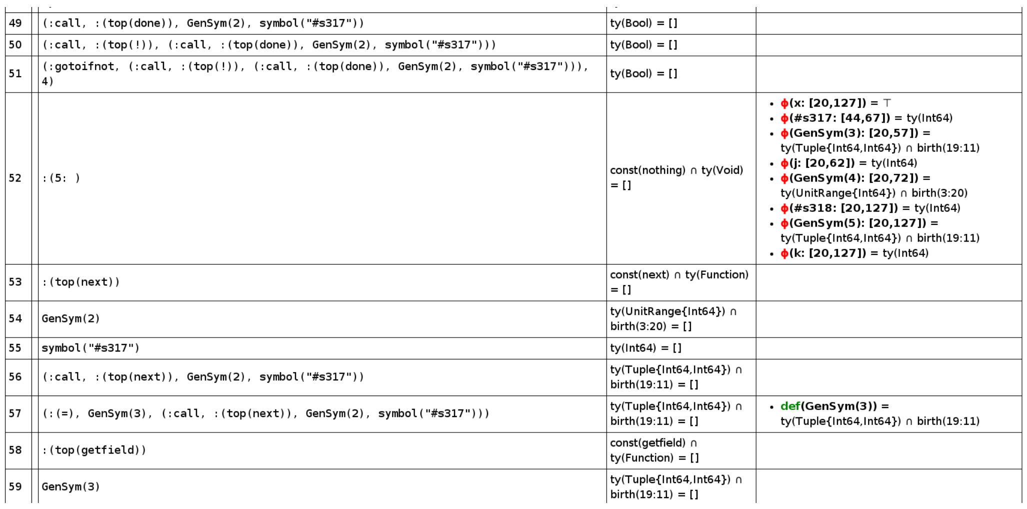
[2] `https://github.com/carnaval/green-fairy`

[3] `https://github.com/shashi/Escher.jl`

Appendix



49		`(:call, :(top(done)), GenSym(2), symbol("#s317"))`	ty(Bool) = []	
50		`(:call, :(top(!)), (:call, :(top(done)), GenSym(2), symbol("#s317")))`	ty(Bool) = []	
51		`(:gotoifnot, (:call, :(top(!)), (:call, :(top(done)), GenSym(2), symbol("#s317"))), 4)`	ty(Bool) = []	
52		`:(5: )`	const(nothing) ∩ ty(Void) = []	• **ϕ(x: [20,127])** = ⊤ • **ϕ(#s317: [44,67])** = ty(Int64) • **ϕ(GenSym(3): [20,57])** = ty(Tuple{Int64,Int64}) ∩ birth(19:11) • **ϕ(j: [20,62])** = ty(Int64) • **ϕ(GenSym(4): [20,72])** = ty(UnitRange{Int64}) ∩ birth(3:20) • **ϕ(#s318: [20,127])** = ty(Int64) • **ϕ(GenSym(5): [20,127])** = ty(Tuple{Int64,Int64}) ∩ birth(19:11) • **ϕ(k: [20,127])** = ty(Int64)
53		`:(top(next))`	const(next) ∩ ty(Function) = []	
54		`GenSym(2)`	ty(UnitRange{Int64}) ∩ birth(3:20) = []	
55		`symbol("#s317")`	ty(Int64) = []	
56		`(:call, :(top(next)), GenSym(2), symbol("#s317"))`	ty(Tuple{Int64,Int64}) ∩ birth(19:11) = []	
57		`(:(=), GenSym(3), (:call, :(top(next)), GenSym(2), symbol("#s317")))`	ty(Tuple{Int64,Int64}) ∩ birth(19:11) = []	• **def(GenSym(3))** = ty(Tuple{Int64,Int64}) ∩ birth(19:11)
58		`:(top(getfield))`	const(getfield) ∩ ty(Function) = []	
59		`GenSym(3)`	ty(Tuple{Int64,Int64}) ∩ birth(19:11) = []	

Figure 4: Visualizing the analysis

The tool allows for step-by-step or live iteration and proved quite useful in debugging the implementation.