Question 1

a) 11001100_2 $= 1 * 2^7 + 1 * 2^6 + 0 * 2^5 + 0 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 0 * 2^0$

$= 128 + 64 + 0 + 0 + 8 + 4 + 0 + 0$

$= 204_{10}$

b) 11001100_3 $= 1 * 3^7 + 1 * 3^6 + 0 * 3^5 + 0 * 3^4 + 1 * 3^3 + 1 * 3^2 + 0 * 3^1 + 0 * 3^0$

$= 2187 + 729 + 0 + 0 + 27 + 9 + 0 + 0$

$= 2952_{10}$

c) 11001100_4 $= 1 * 4^7 + 1 * 4^6 + 0 * 4^5 + 0 * 4^4 + 1 * 4^3 + 1 * 4^2 + 0 * 4^1 + 0 * 4^0$

$= 16384 + 4096 + 0 + 0 + 64 + 16 + 0 + 0$

$= 20560_{10}$

Question 2

a) 10000_{10} =

$10000 / 2 = 5000$	Remainder: 0
$5000 / 2 = 2500$	Remainder: 0
$2500 / 2 = 1250$	Remainder: 0
$1250 / 2 = 625$	Remainder: 0
$625 / 2 = 312$	Remainder: 1
$312 / 2 = 156$	Remainder: 0
$156 / 2 = 78$	Remainder: 0
$78 / 2 = 39$	Remainder: 0
$39 / 2 = 19$	Remainder: 1
$19 / 2 = 9$	Remainder: 1
$9 / 2 = 4$	Remainder: 1
$4 / 2 = 2$	Remainder: 0
$2 / 2 = 1$	Remainder: 0
$1 / 2 = 0$	Remainder: 1

= 10011100010000_2

b) $FEDC.BA_{10}$ = $1111\ 1110\ 1101\ 1100\ .1011\ 1010_2$

= $001\ 111\ 111\ 011\ 011\ 100.101\ 110\ 100_2$

= 177334.564_8

c) 12345.67_8 = $001\ 010\ 011\ 100\ 101.110\ 111_2$

= $0001\ 0100\ 1110\ 0101.1101\ 1100_2$

= $14E5.DC_{10}$

Question 3

a) $1001_S = 19684_{10}$ $\quad\quad\quad$ $1001_S = 1 * s^3 + 1 * s^0 = s^3 + 1$

$S^3 + 1 = 19684_{10}$

$S^3 = 19684_{10} = \sqrt[3]{19684} = 27$

Therefore, S is equal to **27.**

b) $1011_t = 4931_{10}$

$1 * t^3 + 1 * t * 1 * t^0 = 4931_{10}$

$t^3 + t + 1 = 4931_{10}$

$t^3 + t - 4930 = 0$

Let $f(t) = t^3 + t - 4930$. Then $f(17) = (17)^3 + (17) - 4930 = 0$. This means one value of t will be 17. Now through long division…

$$
\begin{array}{r|l}
 & \underline{t^2 + 17t + 290} \\
(t - 17) & t^3 + 0t^2 + t - 4930 \\
 & \underline{t3 - 17t^2} \\
 & \quad 17t^2 + t \\
 & \quad \underline{17t^2 - 289t} \\
 & \quad\quad 290t - 4930 \\
 & \quad\quad \underline{290t - 4930} \\
 & \quad\quad\quad 0
\end{array}
$$

So now we have $t^2 + 17t + 290$. To see if there is t to make this equation equal to 0, sub it in to $b^2 - 4ac$

$17^2 - 4(1)(290) = -871$

Since the result of this was negative, that means there isn't any t to make this equation equal to 0. Therefore only solution for t is **17.**

Question 4

a) -9876_{10}

9876/2 = 4938	Remainder: 0
4938/2 = 2469	Remainder: 0
2469/2 = 1234	Remainder: 1
1234/2 = 617	Remainder: 0
617/2 = 308	Remainder: 1
308/2 = 154	Remainder: 0
154/2 = 77	Remainder: 0
77/2 = 38	Remainder: 1
38/2 = 19	Remainder: 0
19/2 = 9	Remainder: 1
9/2 = 4	Remainder: 1
4/2 = 2	Remainder: 0
2/2 = 1	Remainder: 0
1/2 = 0	Remainder: 1

$9876_{10} = 10011010010100_2$

$+9876_{10} = 010011010010100_2$

$-9876_{10} = 101100101101100_2$

b) -98.7654310

98/2 = 49	Remainder: 0	0.7654310*2 =1.530862
49/2 = 24	Remainder: 1	0.530862*2 = 1.061724
24/2 = 12	Remainder: 0	0.061724*2 = 0.123448
12/2 = 6	Remainder: 0	0.123448*2 = 0.246896
6/2 = 3	Remainder: 0	0.246896*2 = 0.493792
3/2 = 1	Remainder: 1	0.493792*2 = 0.987584
1/2 = 0	Remainder: 1	0.987584*2 = 1.975168
		0.975168*2 = 1.950336
		0.950336*2 = 1.900672
		0.900672*2 = 1.801344

0.801344*2 = 1.602688

From this far of calculation, we can see that…

98.78654310 = 1100010.1100001111 and further more. However, if we want value to be 10 binary fraction, then we round using truncation at 10th decimal. So we round the 1 at the end.

98.78654310 = 1100010.110000111_2

+98.78654310 = 01100010.110000111_2

-98.78654310 = 10011101.001111001_2

Question 5

a) $1010.1010_2 = 1 * 2^3 + 1 * 2^1 + 1 * 2^{-1} + 1 * 2^{-3}$

$= 8 + 2 + 0.5 + 0.125$

$= 10.625$

b) $1010.1010_2 = -010.1010_2$ $\quad 010_2 = 2_{10}$

$0.1010_2 = 0.625_2$ (from last question)

$010.1010_2 = 2.625_{10}$

$1010.1010_2 = -2.625_{10}$

c) 1010.1010_2 is negative number. So…

$1010.1010_2 = -0101.0110_2$ $\quad 0101_2 = 5_{10}$ $\quad 0.0110_2 = 2^{-2} + 2^{-3} = 0.375_{10}$

$0101.0110_2 = 5.375_{10}$

Question 6

a) 1010 1010 + 1111 1111

```
 111111
 10101010
+11111111
110101001
```

There is a carry out to be ignored.

Usually, there can be overflow in this kind of situation. However,

$1010\ 1010 = -0101\ 0110 = -(2^6 + 2^4 + 2^2 + 2^1) = -87$

$1111\ 1111 = -0000\ 0001 = -1$

$1010\ 1001 = -0101\ 0111 = -(2^6 + 2^4 + 2^2 + 2^1 + 1) = -88$

Since $-87_{10} + -1_{10} = -88_{10}$, and $1010\ 1001_2 + 1111\ 1111_2 = 1010\ 1001_2$ which is -88_{10}, it resulted negative result. So no overflow occurred.

b) 0101 1111 + 0111 0101

```
 1111111
 01011111
+01110101
 11010100
```

Usually, there can be overflow in this situation, and indeed, there have been overflow occurred. Two positive numbers should result positive number (should result 0 at most significant bit). However, it results negative. So there is overflow occurring.

c) 1111 0101 + 0101 0101

```
 111 1 1
 11110101
+01010101
101001010
```

There is a carry out to be ignored.

There can be no overflow occurring.

Question 7

a) -1234.785_{10}

- Convert -1234.875 into fixed binary

1234/2 = 617	Remainder: 0	0.875 * 2 = 1.75
617/2 = 308	Remainder: 1	0.75 * 2 = 1.5
308/2 = 154	Remainder: 0	0.5 * 2 = 1.
154/2 = 77	Remainder: 0	
77/2 = 38	Remainder: 1	
38/2 = 19	Remainder: 0	
19/2 = 9	Remainder: 1	
9/2 = 4	Remainder: 1	
4/2 = 2	Remainder: 0	
2/2 = 1	Remainder: 0	
1/2 = 0	Remainder: 1	

1234.875 = 10011010010.111

+1234.875 = 010011010010.111

-1234.875 = 101100101101.001

- Normalized 101100101101.001 = 1.01100101101001 * 2^{11}
- The sign bit is 1 because the number is negative
- The biased exponent is true exponent plus 127. That is …

$11 + 127 = 138_{10} = 10001010_2$

138/2 = 69	Remainder: 0
69/2 = 34	Remainder: 1
34/2 = 17	Remainder: 0
17/2 = 8	Remainder: 1
8/2 = 4	Remainder: 0
4/2 = 2	Remainder: 0
2/2 = 1	Remainder: 0
1/2 = 0	Remainder: 1

- The Significand is 011 0010 1101 0010 0000 0000
  - Leading 1 is stripped
  - The significand expanded to 23 bits.
- So final number is 1100 0101 0011 0010 1101 0010 0000 0000_2, or $C532D200_{16}$.

b) +7654.3

- Convert 7654.3 into fixed binary

7654/2 = 3827	Remainder: 0	0.3*2 = 0.6
3827/2 = 1913	Remainder: 1	0.6*2 = 1.2
1913/2 = 956	Remainder: 1	0.2*2 = 0.4
956/2 = 478	Remainder: 0	0.4*2 = 0.8
478/2 = 239	Remainder: 0	0.8*2 = 1.6
239/2 = 119	Remainder: 1	0.6*2 = 1.2
119/2 = 59	Remainder: 1	0.2*2 = 0.4
59/2 = 29	Remainder: 1	0.4*2 = 0.8
29/2 = 14	Remainder: 1	0.8*2 = 1.6
14/2 = 7	Remainder: 0	…
7/2 = 3	Remainder: 1	Will keep repeating
3/2 = 1	Remainder: 1	
1/2 = 0	Remainder: 1	

7654.3 = $1110111001110.010011001\ldots_2$

- Normalized $1110111100110.010011001\ldots_2$ = 1.110 1111 0011 0010 0110 0110 * 2^{12}
- Sign bit is 0 because number is positive
- The biased exponent is true exponent + 127. That is…

12 + 127 = 139		139/2 = 69	Remainder: 1
	= 1000 1011	69/2 = 34	Remainder: 1
		34/2 = 17	Remainder: 0
		17/2 = 8	Remainder: 1
		8/2 = 4	Remainder: 0
		4/2 = 2	Remainder: 0
		2/2 = 1	Remainder: 0
		1/2 = 0	Remainder: 1

- The Significand is 110 1111 0011 0010 0110 0110
  - Leading 1 is stripped
  - Significand truncated to 23 bits.
- So final number is $0100\ 0101\ 1110\ 1111\ 0011\ 0010\ 0110\ 0110_2$ or $45EF3266_{16}$

Question 8

a) $FEDCBA98_{16}$ = $1111\ 1110\ 1101\ 1100\ 1011\ 1010\ 1001\ 1000_2$

S = 1

E = 1111 1101

F = 101 1100 1011 1010 1001 1000

- Number is negative since sign number is 1.
- Subtract 127 from biased exponents to get true exponent.

$1111\ 1101_2 - 0111\ 1111_2 = 0111\ 1110_2$

$= 126_{10}$

Which is 1 less than 127_{10}

$$\begin{array}{r} 111\ 111 \\ 1111\ 1101 \\ \underline{-0111\ 1111} \\ 0111\ 1110 \end{array}$$

- Fractional significand is .101 1100 1011 1010 1001 1000.
- With leading one, it gives 1.101 1100 1011 1010 1001 1000
- The final number is $-1.101\ 1100\ 1011\ 1010\ 1001\ 1000_2 * 2^{126}$
- In decimal, it will be…

1 = 1   0.101 1100 1011 1010 1001 1000

$= 2^{-1} + 2^{-3} + 2^{-4} + 2^{-5} + 2^{-8} + 2^{-10} + 2^{-11} + 2^{-12} + 2^{-14} +$

$+ 2^{-16} + 2^{-19} + 2^{-20}$

$= 0.72444438934$

$\underline{-1.072444438934 * 2^{126}}$

b) $89ABCDEF_{16}$ = 1000 1001 1010 1011 1100 1101 1110 1111

S = 1

E = 0001 0011

F = 010 1011 1100 1101 1110 1111

- Number is negative since sign number is 1.
- Subtract 127 from biased exponents to get true exponent.

$0001\ 0011_2 - 0111\ 1111_2$

$= -0110\ 1100$

$= -(2^6 + 2^5 + 2^3 + 2^2)$

$= -108$

$$\begin{array}{r} 0001\ 0011 \\ \underline{-0111\ 1111} \end{array} \text{ to } \begin{array}{r} 0111\ 1111 \\ \underline{-0001\ 0011} \\ 0110\ 1100 \end{array}$$

Then

$$\begin{array}{r} 0001\ 0011 \\ \underline{-0111\ 1111} \\ -0110\ 1100 \end{array}$$

- Fractional significand is .010 1011 1100 1101 1110 1111
- With leading one, it is 1.010 1011 1100 1101 1110 1111
- The final number is 1.010 1011 1100 1101 1110 1111 * 2^{-108}
- In decimal it will be…

1 = 1   0.010 1011 1100 1101 1110 1111

$= 2^{-2} + 2^{-4} + 2^{-6} + 2^{-7} + 2^{-8} + 2^{-9} + 2^{-12} + 2^{-13} + 2^{-15}$

$+ 2^{-16} + 2^{-17} + 2^{-18} + 2^{-20} + 2^{-21} + 2^{-22} + 2^{-23}$

$= 0.34222209453$

$\underline{1.34222209453 * 2^{-108}}$

Question 9

a) $FEDCBA98_{16}$ + $89ABCDEF_{16}$

$FEDCBA98_{16}$ = $1111\ 1110\ 1101\ 1100\ 1011\ 1010\ 1001\ 1000_2$

$89ABCDEF_{16}$ = $1000\ 1001\ 1010\ 1011\ 1100\ 1101\ 1110\ 1111_2$

```
 11111 1111 1111 1111 1111 1111  111
  1111 1110 1101 1100 1011 1010 1001 1000
+ 1000 1001 1010 1011 1100 1101 1110 1111
------------------------------------------
 11000 1000 1000 1000 1000 1000 1000 0111
```

- There was no carry out.
- Overflow cannot occur.
- So final number is $1000\ 1000\ 1000\ 1000\ 1000\ 1000\ 1000\ 0111_2$ which is 88888887_{16}

b) $00FCD6EB_{16}$ + $80FCD6EA_{16}$

$00FCD6EB_{16}$ = 0000 0000 1111 1100 1101 0110 1110 1101

$80FCD6EA_{16}$ = 1000 0000 1111 1100 1101 0110 1110 1010

```
          1 1111 1  1 1 1  11 1 11 1
  0000 0000 1111 1100 1101 0110 1110 1101
+ 1000 0000 1111 1100 1101 0110 1110 1010
------------------------------------------
  1000 0001 1111 1001 1010 1101 1101 0111
```

- There was no carry out
- Overflow could occur in this situation. However, it did not occur.
- So final number is $1000\ 0001\ 1111\ 1001\ 1010\ 1101\ 1101\ 0111_2$ which is $81F9ADD7_{16}$.

c) $00FCD6EB_{16}$ + $09ABCDEF_{16}$

$00FCD6EB_{16}$ = 0000 0000 1111 1100 1101 0110 1110 1011

$09ABCDEF_{16}$ = 0000 1001 1010 1011 1100 1101 1110 1111

```
        11 1111 1111 1 11 1111 11 1 111
  0000 0000 1111 1100 1101 0110 1110 1011
+ 0000 1001 1010 1011 1100 1101 1110 1111
  ---------------------------------------
  0000 1010 1010 1000 1010 0100 1101 1010
```

- There was no carry out.
- Overflow cannot occur.
- So final number is $0000\ 1010\ 1010\ 1000\ 1010\ 0100\ 1101\ 1010_2$ which is $0AA8A4DA_{16}$.