

Question 1

$$R1 = D^c$$

$$R2 = B.C$$

$$\begin{aligned} R3 &= (R1 + A)^c \\ &= (D^c + A)^c \end{aligned}$$

Using De Morgan's law: $(X.Y)^c = X^c + Y^c$

$$\begin{aligned} &= (D^c)^c.A^c \\ &= D.A^c \end{aligned}$$

$$R4 = R1 + R2 = D^c + B.C$$

$$\begin{aligned} R5 &= (R1.R3)^c \\ &= (D^c.(D.A^c))^c \end{aligned}$$

Using Associative law: $X.(Y.Z) = (X.Y).Z$

$$\begin{aligned} &= ((D^c.D).A)^c \\ &= (0.A^c)^c \\ &= (0)^c \\ &= 1 \end{aligned}$$

$$\begin{aligned} X &= R4.R5 = 1.(D^c + B.C) \\ &= D^c + B.C \end{aligned}$$

$$\begin{aligned} Y &= R3.R4 \\ &= (D.A^c).(D^c + B.C) \\ &= D.A^c.D^c + D.A^c.B.C \end{aligned}$$

Using Cumulative law : $X + Y = Y + X$

$$= D^c.D.A^c + D.A^c.B.C$$

Using Associative law: $X.(Y.Z) = (X.Y).Z$

$$\begin{aligned} &= (D^c.D).A^c + D.A^c.B.C \\ &= 0.A^c + D.A^c.B.C \\ &= 0 + D.A^c.B.C \\ &= D.A^c.B.C \end{aligned}$$

A	B	C	D	R1	R2	R3	R4	R5	X	Y
0	0	0	0	1	0	0	1	1	1	0
0	0	0	1	0	0	1	0	1	0	0
0	0	1	0	1	0	0	1	1	1	0
0	0	1	1	0	0	1	0	1	0	0
0	1	0	0	1	0	0	1	1	1	0
0	1	0	1	0	0	1	0	1	0	0
0	1	1	0	1	1	0	1	1	1	0
0	1	1	1	0	1	1	1	1	1	1
1	0	0	0	1	0	0	1	1	1	0
1	0	0	1	0	0	0	0	1	0	0
1	0	1	0	1	0	0	1	1	1	0
1	0	1	1	0	0	0	0	1	0	0
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	0	0	0	1	0	0
1	1	1	0	1	1	0	1	1	1	0
1	1	1	1	0	1	0	1	1	1	0

Question 2

$$R1 = C + D$$

$$R2 = B^c$$

$$R3 = C.D$$

$$R4 = R1^c$$

$$= (C + D)^c$$

$$\text{Using De Morgan's law: } (X.Y)^c = X^c + Y^c$$

$$= C^c.D^c$$

$$R5 = B.R4$$

$$= B.C^c.D^c$$

$$R6 = R1.R2$$

$$= (C + D).B^c$$

$$\text{Using Distributive law: } X.(Y + Z) = X.Y + X.Z$$

$$= C.B^c + D.B^c$$

$$R7 = B.R1$$

$$= B.(C + D)$$

$$\text{Using Distributive law: } X.(Y + Z) = X.Y + X.Z$$

$$= B.C + B.D$$

$$W = A + R7$$

$$= A + B.C + B.D$$

$$X = R5 + R6$$

$$= B.C^c.D^c + C.B^c + D.B^c$$

$$Y = R3 + R4$$

$$= C.D + C^c.D^c$$

$$Z = D^c$$

A	B	C	D	R1	R2	R3	R4	R5	R6	R7	W	X	Y	Z
0	0	0	0	0	1	0	1	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	1	0	0	1	0	0
0	0	1	0	1	1	0	0	0	1	0	0	1	0	1
0	0	1	1	1	1	1	0	0	1	0	0	1	1	0
0	1	0	0	0	0	0	1	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0	0	0	1	1	0	0	0
0	1	1	0	1	0	0	0	0	0	1	1	0	0	1
0	1	1	1	1	0	1	0	0	0	1	1	0	1	0
1	0	0	0	0	1	0	1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0	0	1	0	1	1	0	0
1	0	1	0	1	1	0	0	0	1	0	1	1	0	1
1	0	1	1	1	1	1	0	0	1	0	1	1	1	0
1	1	0	0	0	0	0	1	1	0	0	1	1	1	1
1	1	0	1	1	0	0	0	0	0	1	1	0	0	0
1	1	1	0	1	0	0	0	0	0	1	1	0	0	1
1	1	1	1	1	0	1	0	0	0	1	1	0	1	0

Question 3

$$R1 = A^c$$

$$R2 = C^c$$

$$R3 = D^c$$

$$R4 = E^c$$

$$R5 = F^c$$

$$R6 = A.R2.E$$

$$= A.C^c.E$$

$$R7 = A.R2.F$$

$$= A.C^c.F$$

$$R8 = A.R3.E$$

$$= A.D^c.E$$

$$R9 = A.R3.F$$

$$= A.D^c.F$$

$$R10 = D.C.D.R4.R5$$

$$= B.C.D.E^c.F^c$$

$$R11 = R1.B.C.D$$

$$= A^c.B.C.D$$

$$R12 = A.B.E$$

$$R13 = A.B.F$$

$$R14 = B.C.E$$

$$R15 = B.C.F$$

$$G = R6 + R7 + R8 + R9 + R10$$

$$= A.C^c.E + A.C^c.F + A.D^c.E + A.D^c.F + B.C.D.E^c.F^c$$

$$H = R11 + R12 + R13 + R14 + R15$$

$$= A^c.B.C.D + A.B.E + A.B.F + B.C.E + B.C.F$$