1 NP-completeness of minACS

Now we show that the minACS problem is NP-complete. To do this, we prove a problem related to minACS is NP-Complete, and we reduce that problem to minACS.

Lemma 1. *For a given graph G, a vertex q, and an integer k, if there exists a clique C such that $q \in C$ and $|C| = k + 1$, then C is the smallest solution of minACS.*

Problem 1. (MCC). For a graph $G(V, E)$ and $q \in V$, find the maximum clique that contains q.

Lemma 2. MCC *is NP-Complete.*

Proof. It is clear that MCC belongs to NP. We prove it by reducing Maximal Clique (MC), a well-known NP-complete problem, to MCC. For any graph $G(V, E)$, we construct a new graph $G'(V', E')$ by adding a vertex v_0 and connecting v_0 to all vertices in G, that is, $V' = V \cup \{v_0\}$ and $E' = E \cup \{(v_i, v_0)|v_i \in V\}$. Thus, an MCC in G' is an MC in G.

Theorem 1. *minACS is NP-complete.*

Proof. It is easy to see that minACS belongs to NP. Now we show that minACS is NP-complete by reducing MCC to it. Let $G(V, E)$ be a graph, and let v_0 be the query vertex. Consider the decision problem that corresponds to the optimization problem of MCC: Determine whether G has a clique of at least size k that contains v_0. We can construct the following decision problem for minACS. Determine whether a solution $H \subseteq V$ exists such that $|H| = k$ and it satisfies the minACS conditions: i) $v_0 \in H$; ii) $G[H]$ is connected; and iii)$\delta(G[H]) \geq k - 1$. If H is the answer, then apparently $G[H]$ is a clique, as any node in H has $degree \geq k - 1$.