一、$X\sim P(\lambda)$. $E(X)=\lambda$ 未知.

$$P\{X=x\}=\frac{\lambda^x}{x!}e^{-\lambda}$$

$$\therefore L(\lambda)=\frac{\lambda^{x_1}}{x_1!}e^{-\lambda}\cdots\frac{\lambda^{x_n}}{x_n!}e^{-\lambda}=\frac{\lambda^{\sum_{i=1}^n x_i}}{x_1!\cdots x_n!}e^{-n\lambda}$$

$$\ln L(\lambda)=\sum_{i=1}^n x_i\ln\lambda-n\lambda-\sum_{i=1}^n\ln(x_i!)$$

$$\frac{d\ln L(\lambda)}{d\lambda}=\frac{\sum x_i}{\lambda}-n=0\Rightarrow\lambda=\frac{1}{n}\sum_{i=1}^n x_i=\bar{x}$$

$$\therefore\hat{\lambda}=\bar{X}.$$

二、X：某星球和天文台间距离（测量）

X_i：第i次测得距离 $E(X_i)=d$. $D(X_i)=2^2$.

设需n次. $\bar{X}=\frac{1}{n}\sum_{i=1}^n(X_i)$

由中心极限定理可知. $\bar{X}$ 近似服从 $N(d,2^2/n)$.

$$\therefore P\{|\bar{X}-d|\le 0.5\}\ge 95\%.$$

$$P\left\{\left|\frac{\bar{X}-d}{\frac{2}{\sqrt{n}}}\right|\le\frac{0.5}{\frac{2}{\sqrt{n}}}\right\}\approx 2\Phi(0.25\sqrt{n})-1\ge 95\%$$

$$\Phi(0.25\sqrt{n})\ge 0.975=\Phi(1.96).$$

$$0.25\sqrt{n}\ge 1.96\Longrightarrow n\ge 61.5$$

$n=62$

三、X：大马哈鱼重. $X\sim N(\mu,0.3^2)$. $(\hat{\mu}_1,\hat{\mu}_2)$

$1-\alpha=95\%$，$\alpha=0.05$

对μ的区间估计. 误差为0.2.

即 $2Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\le 0.2\Rightarrow n.$

区间长度.

Ⅲ. $\alpha=0.05=P\{\bar{X}-10\le c\mid \mu=10\}$

$\mu=10$ 时 $X\sim N(10,3^2)$ $\bar{X}\sim N(10,\frac{3^2}{16})$.

$\therefore P\{\bar{X}-10\le c\}=P\{\frac{\bar{X}-10}{\frac{3}{4}}\le\frac{c}{\frac{3}{4}}\}=\Phi(\frac{c}{\frac{3}{4}})=\alpha=0.05$.

$Z_{0.05}=1.65\Rightarrow \Phi(1.65)=0.95$ $\Phi(-1.65)=0.05$.

$\therefore \frac{c}{\frac{3}{4}}=-1.65\Rightarrow c=-1.65\times\frac{3}{4}=-1.2375$

$\beta=P\{\bar{X}-10>c\mid \mu=8\}$

$\mu=8$ $\bar{X}\sim N(8,\frac{3^2}{16})$.

$\therefore \beta=P\{\bar{X}-10>c\}=P\{\frac{\bar{X}-8}{\frac{3}{4}}>\frac{c+2}{\frac{3}{4}}\}$

$=1-\Phi(\frac{c+2}{\frac{3}{4}})=1-\Phi(\frac{-1.2375+2}{\frac{3}{4}})=1-\Phi(1.02)$.