一、

① 设X：微信群里发言人数 $X \sim B(10, 0.3)$

$$P\{X>5\} = \sum_{i=6}^{10} C_{10}^{i}\, 0.3^{i}\, 0.7^{10-i} = 0.05$$

② 设Y：…… $Y \sim B(100, 0.03)$ $E(Y)=3$ $D(Y)=2.91$

Y 近似 $\sim N(3, 2.91)$

$$P\{Y>5\} = \cancel{\sum_{i=6}^{100} C_{100}^{i}\, 0.03^{i}\, 0.97^{100-i} = 0.08} = P\left\{\frac{Y-3}{\sqrt{2.91}} > \frac{5-3}{\sqrt{2.91}}\right\} = \cdots$$

（泊松分布近似也可，但要查表） Y 近似服从 $P(\lambda)$ $\lambda = np = 3$

③ Z：100人微信群两次发言间隔时间

$Z \sim Exp(3)$

$\therefore E(Z) = \frac{1}{\lambda} = \frac{1}{3}$ Z 的无记忆性

④ $P\{Z \geq 1 \mid Z \geq \frac{20}{60}\} = P\{Z \geq \frac{2}{3}\} = e^{-3\cdot\frac{2}{3}} = e^{-2}$

二、 X：某商品每周的需求量 $X \sim N(50, 10^2)$

$$\bar{X} = \frac{X_1+X_2+X_3+X_4}{4} \sim N(50, \frac{10^2}{4})$$

① $P\{\bar{X} > 50\} = P\left\{\frac{\bar{X}-50}{\frac{10}{2}} > \frac{50-50}{\frac{10}{2}}\right\} = 1-\Phi(0) = 0.5$

② 设应配库存n台，则

$$P\left\{\sum_{i=1}^{4} X_i \leq n\right\} \geq 0.98$$

而 $\sum_{i=1}^{4} X_i \sim N(50\times4, 4\times10^2)$

$$\therefore P\left\{\sum_{i=1}^{4} X_i \leq n\right\} = P\left\{\frac{\sum X_i - 200}{2\times10} \leq \frac{n-200}{2\times10}\right\}$$

$$= \Phi\left(\frac{n-200}{20}\right) \geq 0.98 = \Phi(2)$$

$$\frac{n-200}{20} \geq 2 \implies n \geq 200+40 = 240$$

$\therefore$ n至少取240

三. X: 样本中赞成某位候选人的人数

$X \sim B(200, 0.45)$.

ⓐ $E(X) = np = 200 \times 0.45 = 90$ $D(X) = np(1-p) = 49.5$.

ⓑ $P\{X > 100\} = ?$

由中心极限定理可知. X 近似服从 $N(90, 49.5)$.

$$P\{X > 100\} = P\{X \geq 101\} \approx P\{X > 100.5\}.$$

$$= P\left\{\frac{X-90}{\sqrt{49.5}} > \frac{100.5-90}{\sqrt{49.5}}\right\} \approx 1 - \Phi\left(\frac{10.5}{\sqrt{49.5}}\right).$$

四. X: 所花时间 $X \sim N(50, 6^2)$.

$X_1 \cdots X_{10}$. 则 $\bar{X} \sim N(50, \frac{6^2}{10})$ $\frac{10-1}{6^2} S^2 \sim \chi^2(9)$.

$$P\{S^2 < 76\} = P\left\{\frac{9}{6^2} S^2 < \frac{9}{6^2} \times 76\right\}.$$

$$= P\left\{\frac{9}{6^2} S^2 < 19\right\} = 1 - 0.025 = 0.975.$$

而 $\chi^2_{0.025}(9) = 19$

五. $\left(\bar{x} - t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}},\ \bar{x} + t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}\right)$.