1a.

The optimal solution is to minimize the effect of the human choice. In particular, we can select where to place our blockade and the side the human picks is the one which will give the player the minimum value of fruits. This means we want to “maximize” this minimum value. In addition, we lose a portion of valid locations after every blockade we place.

Therefore, the recurrence equation is -

OPT(i, j) = min(sum[i:j-k] + OPT(i, j – k), sum[j – k:j] + OPT(j – k, j)) for k from i to j – 1 s.t. we get the maximum possible value of the minimum of the two sides.

To clairfy, a value of k = n means there is a blockade between an index n and n + 1. We take the minimum of the two values because the humans will take the maximum. We also define some special cases-

If i == j, then return 0.

sum[n:n] = p(n)

1b.

Correctness-

Humans will always pick the choice which minimizes the ape’s fruits, which means we want to minimize the impact of what the human does (maximizing this minimum value). This is because there are a limited amount of fruits each round, and whichever side is worse is left to the apes, which means we want to make both sides as close as possible in amount of fruits. The humans make the choice that minimizes the total amount of fruits collected, which means we need to examine the optimal solution in future rounds as well (simulating human actions). Therefore, the recurrence equation returns the optimal solution.

Time Complexity-

The optimal solution has two parameters- the start and end index. And each element of the n x n array takes log(n) time to compute. This is because we can do a “binary search”-like algorithm by examining which side of blockade k is larger when minimizing the difference (divide by 2). In particular, we know that by moving the blockade left we increase the fruits on the right side and moving the blockade right will increase the fruits on the left side. In addition, the sum between ranges of sections can be pre-computed in O(n^2) time. Therefore, the total runtime is O(n^2*log(n))