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Generalization of Malus' law and spatial coherence relations for linear polarizers and non-uniform polarizers

JIA XU,[1] GREG GBUR,[2] AND TACO D. VISSER[1,3,4,*]

[1] Department of Physics and Astronomy, Vrije Universiteit, Amsterdam, NL-1081HV, The Netherlands
[2] Department of Physics and Optical Science, University of North Carolina at Charlotte, Charlotte, North Carolina 28223, USA
[3] The Institute of Optics, University of Rochester, Rochester, New York 14627, USA
[4] Shandong Normal University, Jinan, 250358, China
*Corresponding author: t.d.visser@vu.nl

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We study the transmission of partially polarized, partially coherent beams through linear polarizers and polarization elements that are non-uniform. An expression for the transmitted intensity, which reproduces Malus' law for special cases, is derived, as are formulas for the transformation of spatial coherence properties.

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Introduction. The works of Étienne-Louis Malus [1] were foundational for the development of physical optics in the early nineteenth century. This is especially true for his discovery of the law that now bears his name [2], which states that the transmission efficiency of horizontally polarized light through a polarizer with transmission angle α equals $\cos^2(\alpha)$. An immediate consequence of Malus' law is that the transmission for unpolarized light is 50% for any orientation of the transmission axis. A question naturally arises: What is the transmission for beams that are partially polarized and spatially partially coherent? Also, how are the incident beam's coherence properties affected? Our aim is to answer these questions, not just for uniform linear polarizers, but also for non-uniform polarizing devices. The latter can be used to generate vector beams [3] that are now commonly applied in, e.g., tight focusing [4] and single molecule imaging [5]. We employ a generalized matrix formalism [6–8], which we briefly review and cast in a different but otherwise equivalent format. Our results provide insight into the relation between partial coherence and the state of polarization of electromagnetic beams.

Theory. Consider a linear system with input plane $z' = 0$ (Fig. 1). Positions in this plane are denoted $\boldsymbol{\rho}' = (x', y')$. A normally incident beam-like field $\mathbf{E}^{(\mathrm{i})}$ at frequency ω produces an electric field $\mathbf{E}^{(\mathrm{o})}$ in the output plane of the form

$$E_x^{(\mathrm{o})}(\boldsymbol{\rho},\omega) = \int_{z'=0} \left[G_{xx}(\boldsymbol{\rho}',\boldsymbol{\rho},\omega)\,E_x^{(\mathrm{i})}(\boldsymbol{\rho}',\omega) + G_{xy}(\boldsymbol{\rho}',\boldsymbol{\rho},\omega)\,E_y^{(\mathrm{i})}(\boldsymbol{\rho}',\omega)\right] d^2\rho', \tag{1}$$

$$E_y^{(\mathrm{o})}(\boldsymbol{\rho},\omega) = \int_{z'=0} \left[G_{yx}(\boldsymbol{\rho}',\boldsymbol{\rho},\omega)\,E_x^{(\mathrm{i})}(\boldsymbol{\rho}',\omega) + G_{yy}(\boldsymbol{\rho}',\boldsymbol{\rho},\omega)\,E_y^{(\mathrm{i})}(\boldsymbol{\rho}',\omega)\right] d^2\rho', \tag{2}$$

where $G_{ij}(\boldsymbol{\rho}',\boldsymbol{\rho},\omega)$, with $i,j \in \{x,y\}$, are the elements of the Green tensor that characterizes the linear system.

Assuming beam-like propagation through a thin system, these elements are proportional to a Dirac delta function, i.e.,

$$G_{ij}(\boldsymbol{\rho}',\boldsymbol{\rho},\omega) = G_{ij}(\boldsymbol{\rho}',\omega)\delta^2(\boldsymbol{\rho}'-\boldsymbol{\rho}), \tag{3}$$

with $G_{ij}(\boldsymbol{\rho}',\omega) \in \mathbb{C}$. Substitution from Eq. (3) into Eqs. (1) and (2) leads to

$$E_x^{(\mathrm{o})}(\boldsymbol{\rho},\omega) = G_{xx}(\boldsymbol{\rho},\omega)\,E_x^{(\mathrm{i})}(\boldsymbol{\rho},\omega) + G_{xy}(\boldsymbol{\rho},\omega)\,E_y^{(\mathrm{i})}(\boldsymbol{\rho},\omega), \tag{4}$$

$$E_y^{(\mathrm{o})}(\boldsymbol{\rho},\omega) = G_{yx}(\boldsymbol{\rho},\omega)\,E_x^{(\mathrm{i})}(\boldsymbol{\rho},\omega) + G_{yy}(\boldsymbol{\rho},\omega)\,E_y^{(\mathrm{i})}(\boldsymbol{\rho},\omega), \tag{5}$$

which can be jointly expressed as

$$\mathbf{E}^{(\mathrm{o})}(\boldsymbol{\rho},\omega) = \mathbf{G}(\boldsymbol{\rho},\omega)\mathbf{E}^{(\mathrm{i})}(\boldsymbol{\rho},\omega), \tag{6}$$

where

$$\mathbf{G}(\boldsymbol{\rho},\omega) = \begin{pmatrix} G_{xx}(\boldsymbol{\rho},\omega) & G_{xy}(\boldsymbol{\rho},\omega) \\ G_{yx}(\boldsymbol{\rho},\omega) & G_{yy}(\boldsymbol{\rho},\omega) \end{pmatrix} \tag{7}$$

and

$$\mathbf{E}(\boldsymbol{\rho},\omega) = \left(E_x(\boldsymbol{\rho},\omega),\, E_y(\boldsymbol{\rho},\omega)\right)^{\mathrm{T}}. \tag{8}$$

A partially coherent electromagnetic beam may be described by a cross-spectral density (CSD) matrix [9]:

$$\mathbf{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = \begin{pmatrix} \langle E_x^*(\boldsymbol{\rho}_1,\omega)\,E_x(\boldsymbol{\rho}_2,\omega)\rangle & \langle E_x^*(\boldsymbol{\rho}_1,\omega)\,E_y(\boldsymbol{\rho}_2,\omega)\rangle \\ \langle E_y^*(\boldsymbol{\rho}_1,\omega)\,E_x(\boldsymbol{\rho}_2,\omega)\rangle & \langle E_y^*(\boldsymbol{\rho}_1,\omega)\,E_y(\boldsymbol{\rho}_2,\omega)\rangle \end{pmatrix}, \tag{9}$$

where the angular brackets indicate an ensemble average. We can express the transformation of the CSD matrix by the linear

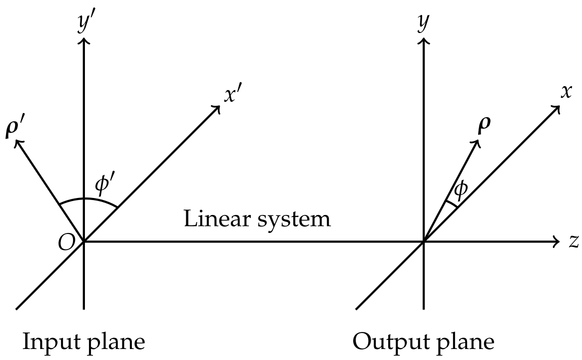

Fig. 1. Illustrating the notation. Positions and azimuthal angles in the input plane are denoted $\boldsymbol{\rho}'$ and ϕ', and those in the output plane are denoted $\boldsymbol{\rho}$ and ϕ.

system, while no longer displaying the ω dependence, by writing

$$E_i^{(\mathrm{o})}(\boldsymbol{\rho}) = \sum_{m=x,y} G_{im}(\boldsymbol{\rho}) E_m^{(\mathrm{i})}(\boldsymbol{\rho}), \tag{10}$$

from which it follows that the CSD matrix of the beam in the output plane equals

$$\begin{aligned} W_{ij}^{(\mathrm{o})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) &= \left\langle \sum_{m=x,y} G_{im}^{*}(\boldsymbol{\rho}_1) E_m^{(\mathrm{i})*}(\boldsymbol{\rho}_1) \right. \\ &\quad \left. \times \sum_{p=x,y} G_{jp}(\boldsymbol{\rho}_2) E_p^{(\mathrm{i})}(\boldsymbol{\rho}_2) \right\rangle \\ &= \sum_{m,p} G_{im}^{*}(\boldsymbol{\rho}_1) G_{jp}(\boldsymbol{\rho}_2) W_{mp}^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2). \end{aligned} \tag{11}$$

The transformation of the CSD matrix can be cast in a compact form by considering $\mathbf{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ as a column vector rather than a 2×2 matrix, i.e.,

$$\mathbf{W}^{(\mathrm{o})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \begin{pmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{pmatrix} \begin{pmatrix} W_{xx}^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \\ W_{xy}^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \\ W_{yx}^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \\ W_{yy}^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \end{pmatrix}, \tag{12}$$

where the transfer matrix $\mathbf{T}$ of the system is given by the tensor product

$$\begin{aligned} \mathbf{T}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) &= \mathbf{G}(\boldsymbol{\rho}_1) \otimes \mathbf{G}(\boldsymbol{\rho}_2) \\ &\equiv \begin{pmatrix} G_{xx}^{*}(\boldsymbol{\rho}_1)\mathbf{G}(\boldsymbol{\rho}_2) & G_{xy}^{*}(\boldsymbol{\rho}_1)\mathbf{G}(\boldsymbol{\rho}_2) \\ G_{yx}^{*}(\boldsymbol{\rho}_1)\mathbf{G}(\boldsymbol{\rho}_2) & G_{yy}^{*}(\boldsymbol{\rho}_1)\mathbf{G}(\boldsymbol{\rho}_2) \end{pmatrix}. \end{aligned} \tag{13}$$

The effect of the linear system can thus be written as

$$\mathbf{W}^{(\mathrm{o})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \mathbf{T}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \mathbf{W}^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2). \tag{14}$$

For a cascaded system in which the beam passes, with negligible propagation effects, through a series of elements with transfer matrices $\mathbf{T}_1$, $\mathbf{T}_2, \ldots, \mathbf{T}_N$, this generalizes to

$$\mathbf{W}^{(\mathrm{o})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \mathbf{T}_N(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)\mathbf{T}_{N-1}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \ldots \mathbf{T}_1(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)\mathbf{W}^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2). \tag{15}$$

Unlike the Mueller matrix formalism, this generalized approach can handle spatially partially coherent beams.

Generalizing Malus' law. Consider the Green tensor, from here on called the Jones matrix, for a uniform linear polarizer fixed at an angle α to the x axis [10]:

$$\mathbf{J}_{\mathrm{lin}\,\alpha} = \begin{pmatrix} \cos^2(\alpha) & \cos(\alpha)\sin(\alpha) \\ \cos(\alpha)\sin(\alpha) & \sin^2(\alpha) \end{pmatrix}. \tag{16}$$

On making use of Eq. (18) in Eq. (14), we obtain a symmetric transfer matrix of the form

$$\mathbf{T}_{\mathrm{lin}\,\alpha} = \begin{pmatrix} \mathrm{C}^4 & \mathrm{C}^3\mathrm{S} & \mathrm{C}^3\mathrm{S} & \mathrm{C}^2\mathrm{S}^2 \\ \mathrm{C}^3\mathrm{S} & \mathrm{C}^2\mathrm{S}^2 & \mathrm{C}^2\mathrm{S}^2 & \mathrm{C}\mathrm{S}^3 \\ \mathrm{C}^3\mathrm{S} & \mathrm{C}^2\mathrm{S}^2 & \mathrm{C}^2\mathrm{S}^2 & \mathrm{C}\mathrm{S}^3 \\ \mathrm{C}^2\mathrm{S}^2 & \mathrm{C}\mathrm{S}^3 & \mathrm{C}\mathrm{S}^3 & \mathrm{S}^4 \end{pmatrix}, \tag{17}$$

where C and S stand for $\cos(\alpha)$ and $\sin(\alpha)$, respectively.

We assume a partially polarized incident beam of the Gaussian–Schell model type [9], with a spectral density that does not vary over the area of the linear polarizer. In such a case, the CSD matrix elements are of the form

$$\begin{aligned} W_{xx}^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) &= \exp\left[-(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2/(2\delta_{xx}^2)\right], \\ W_{yy}^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) &= \exp\left[-(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2/(2\delta_{yy}^2)\right], \\ W_{xy}^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) &= B_{xy}\exp\left[-(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2/(2\delta_{xy}^2)\right], \\ W_{yx}^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) &= W_{xy}^{(\mathrm{i})*}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2), \end{aligned} \tag{18}$$

where δ_{ij} is one of three independent coherence radii (since $\delta_{xy} = \delta_{yx}$) [11] and B_{xy}, with $|B_{xy}| \leq 1$, denotes the correlation coefficient between the two Cartesian components of the electric field.

The spectral degree of polarization (DOP) equals [9]

$$P(\boldsymbol{\rho}) = \sqrt{1 - \frac{4\,\mathrm{Det}\,\mathbf{W}(\boldsymbol{\rho}, \boldsymbol{\rho})}{[\mathrm{Tr}\,\mathbf{W}(\boldsymbol{\rho}, \boldsymbol{\rho})]^2}}, \tag{19}$$

where Det and Tr indicate the determinant and trace, respectively. It is seen that for this beam the DOP $P^{(\mathrm{i})}(\boldsymbol{\rho}) = |B_{xy}|$. On using Eq. (18) in Eq. (14), it follows that the output beam has

$$\begin{aligned} W_{xx}^{(\mathrm{o})}(\boldsymbol{\rho}, \boldsymbol{\rho}) &= \mathrm{C}^2 + \mathrm{C}^3\mathrm{S}[B_{xy} + B_{xy}^{*}], \\ W_{yy}^{(\mathrm{o})}(\boldsymbol{\rho}, \boldsymbol{\rho}) &= \mathrm{S}^2 + \mathrm{C}\mathrm{S}^3[B_{xy} + B_{xy}^{*}]. \end{aligned} \tag{20}$$

The spectral density $S(\boldsymbol{\rho})$ is given by the trace of the CSD matrix [9] at two coincident points, namely

$$S(\boldsymbol{\rho}) = \mathrm{Tr}\,\mathbf{W}(\boldsymbol{\rho}, \boldsymbol{\rho}). \tag{21}$$

The transmission efficiency T is, therefore,

$$T = \frac{\mathrm{Tr}\,\mathbf{W}^{(\mathrm{o})}(\boldsymbol{\rho}, \boldsymbol{\rho})}{\mathrm{Tr}\,\mathbf{W}^{(\mathrm{i})}(\boldsymbol{\rho}, \boldsymbol{\rho})} = \frac{1}{2} + \frac{1}{2}\sin(2\alpha)\Re(B_{xy}), \tag{22}$$

where $\Re$ denotes the real part. Notice that the transmission depends on both the orientation of the polarizer and the DOP of the beam. This is illustrated in Fig. 2.

From the generalized transmission expression [Eq. (22)], we retrieve the two classical results of Malus for unpolarized light and fully polarized light as two special cases. First, when $\Re(B_{xy}) = \pm 1$, the field is linearly polarized under an angle $\pm\pi/4$. Equation (22) then reduces to Malus' law, i.e.,

$$T = \cos^2(\alpha \mp \pi/4). \tag{23}$$

Second, when $\Re(B_{xy}) = 0$, the incident field is completely unpolarized and Eq. (22) becomes

$$T = 0.5. \tag{24}$$

It is readily verified that the output beam is fully polarized, i.e., $P^{(\mathrm{o})}(\boldsymbol{\rho}) = 1$, as expected. We next turn our attention to the

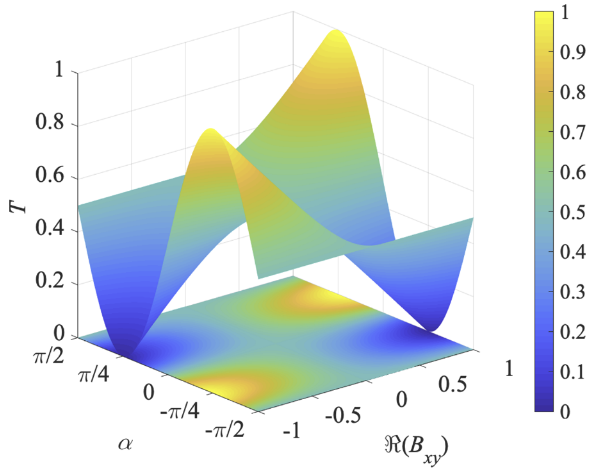

Fig. 2. Transmission efficiency T of a linear polarizer, for an incident beam that is partially polarized.

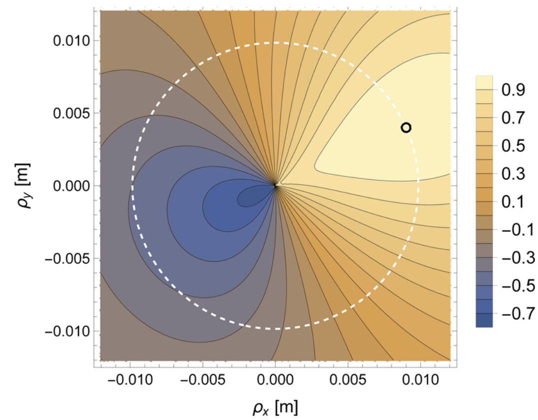

Fig. 3. Contours of a spectral DOC of radial vector beam. In this example, $\boldsymbol{\rho}_2$ is kept fixed at (9, 4) mm (indicated by the black circle), while $\boldsymbol{\rho}_1$ is varied. Coherence radius $\delta_{xx} = 15$ mm.

effect on the degree of spatial coherence (DOC) produced by a linear polarizer. The DOC is a measure of visibility in Young's interference experiment, and equals [9]

$$\eta(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \equiv \frac{\mathrm{Tr}\,\mathbf{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)}{\sqrt{\mathrm{Tr}\,\mathbf{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1)}\sqrt{\mathrm{Tr}\,\mathbf{W}(\boldsymbol{\rho}_2, \boldsymbol{\rho}_2)}}. \tag{25}$$

From Eq. (14), we find that a polarizer produces an output CSD matrix with, in general, four non-zero elements; hence

$$\eta^{(\mathrm{o})}_{\mathrm{lin}\,\alpha}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \frac{\mathrm{C}^2 W^{(\mathrm{i})}_{xx} + \mathrm{CS}[W^{(\mathrm{i})}_{xy} + W^{(\mathrm{i})}_{yx}] + \mathrm{S}^2 W^{(\mathrm{i})}_{yy}}{2\mathrm{CS}\,\Re(B_{xy})}, \tag{26}$$

with the matrix elements evaluated at $(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$. It is seen that the polarizer transforms the spatial degree of coherence in a complicated way. Although, according to Eq. (25), the DOC of the input beam does not depend on the off diagonal CSD elements (and hence not on the correlation coefficient B_{xy}), the spatial coherence of the output beam $\eta^{(\mathrm{o})}_{\mathrm{lin}\,\alpha}$ clearly does.

Vector beams. It is instructive to compare these results with the action of a non-uniform polarizer that performs a local rotation of the electric field based on the azimuthal angle ϕ. Such a device with Jones matrix,

$$\mathbf{J}(\phi) = \begin{pmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{pmatrix}, \tag{27}$$

acting on an incident field that is horizontally or vertically polarized, produces a radial or azimuthal vector beam, respectively. In both cases, the transmission is seen to be unity. The matrix defined in Eq. (27) is typically an effective matrix describing the action of a series of elements [3]. Consider an incident field that is x-polarized, meaning that only the element $W^{(\mathrm{i})}_{xx}$ is non-zero. On using Eq. (27) in Eq. (13) for the $\mathbf{T}$ matrix, we obtain expressions for the CSD elements of the radially polarized output beam, from which it follows that

$$\eta^{(\mathrm{o})}_{\mathrm{rad}}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \cos(\phi_1 - \phi_2)\,\eta^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2). \tag{28}$$

In words, the non-uniform polarizer transforms a linear correlation for E_x into a correlation for the radial field that is of the same form multiplied by a geometrical factor. Specifically, opposite points $\boldsymbol{\rho}_1 = -\boldsymbol{\rho}_2$ are anti-correlated. Equation (28) is a generalization of a similar result for spatially fully coherent light reported by Brown *et al.* [12]. In Fig. 3, the spatial degree of coherence $\eta^{(\mathrm{o})}_{\mathrm{rad}}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ is shown for a fixed reference point $\boldsymbol{\rho}_2$ (black circle). The dashed white circle indicates points at the same distance from the beam axis. The coherence is mirror symmetric about the line through the beam axis and $\boldsymbol{\rho}_2$. A second line perpendicular to this, and also passing through the beam axis, separates the regions where $\eta^{(\mathrm{o})}_{\mathrm{rad}}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ has an opposite sign. This means that, for all points $\boldsymbol{\rho}$ on this line, $\eta^{(\mathrm{o})}_{\mathrm{rad}}(\boldsymbol{\rho}, \boldsymbol{\rho}_2) = 0$, and hence $(\boldsymbol{\rho}, \boldsymbol{\rho}_2)$ are coherence singularities [13,14]. Such singularities are not present in the incident beam, but are created by the polarizing device.

For an input with vertical polarization, which generates an azimuthally polarized beam, the same relation between $\eta^{(\mathrm{o})}_{\mathrm{az}}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$, the DOC in the output plane, and $\eta^{(\mathrm{i})}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ is found. For both input polarizations it is readily found that $\mathrm{Det}\,\mathbf{W}^{(\mathrm{o})}(\boldsymbol{\rho}, \boldsymbol{\rho}) = 0$, and hence $P^{(\mathrm{o})}(\boldsymbol{\rho}) = 1$.

Conclusions. We have studied the transmission properties of linear polarizers and non-uniform polarizers. For the first type, a generalization of Malus' law, which is also valid for partially polarized beams, was derived. Both devices were found to affect the spatial coherence properties of a beam in a non-trivial way. In particular, coherence singularities may be created. In all cases, the output beam is fully polarized but spatially partially coherent.

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