## AML\_HW1\_Questions

September 23, 2024

## 0.1 Homework 1: Applied Machine Learning

This assignment covers contents of the first three lectures.

The emphasis for this assignment would be on the following: 1. Data Visualization and Analysis 2. Linear Models for Regression and Classification 3. Support Vector Machines

```
[159]: import warnings

def fxn():
    warnings.warn("deprecated", DeprecationWarning)

with warnings.catch_warnings():
    warnings.simplefilter("ignore")
    fxn()
```

```
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
from numpy.linalg import inv
%matplotlib inline
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler, OneHotEncoder, OrdinalEncoder
from sklearn.metrics import r2_score
from sklearn.svm import LinearSVC, SVC
```

## 0.2 Part 1: Data Visualization and Analysis

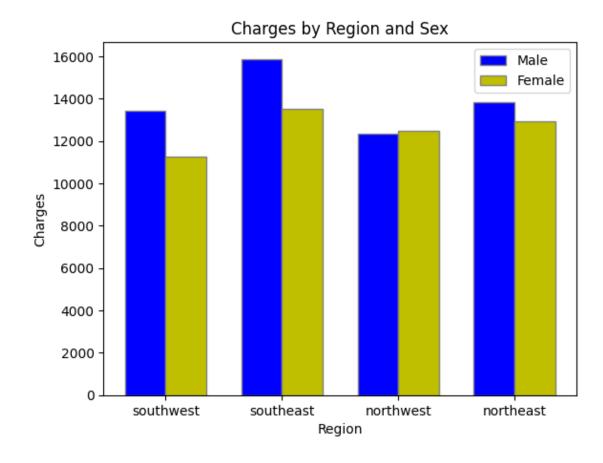
Understanding data characteristics and patterns is crucial for building effective models. In this part, we will visualize and analyze the insurance.csv dataset.

Note: Remember to label plot axes while plotting.

```
[161]: # Load the dataset
insurance_df = pd.read_csv('insurance.csv')
```

1.1 Create a bar chart to compare the average insurance charges by sex and region.

```
[162]: regions = insurance_df['region'].unique()
     mean_charges_male = [insurance_df[(insurance_df['region'] == region) &__
      mean_charges_female = [insurance_df[(insurance_df['region'] == region) &__
      bar_width = 0.35
     r1 = np.arange(len(regions))
     r2 = [x + bar_width for x in r1]
     plt.bar(r1, mean_charges_male, color='b', width=bar_width, edgecolor='grey', u
      →label='Male')
     plt.bar(r2, mean_charges_female, color='y', width=bar_width, edgecolor='grey', u
      ⇔label='Female')
     plt.title('Charges by Region and Sex')
     plt.xlabel('Region')
     plt.ylabel('Charges')
     plt.xticks([r + bar_width/2 for r in range(len(regions))], regions)
     plt.legend()
     plt.show()
```



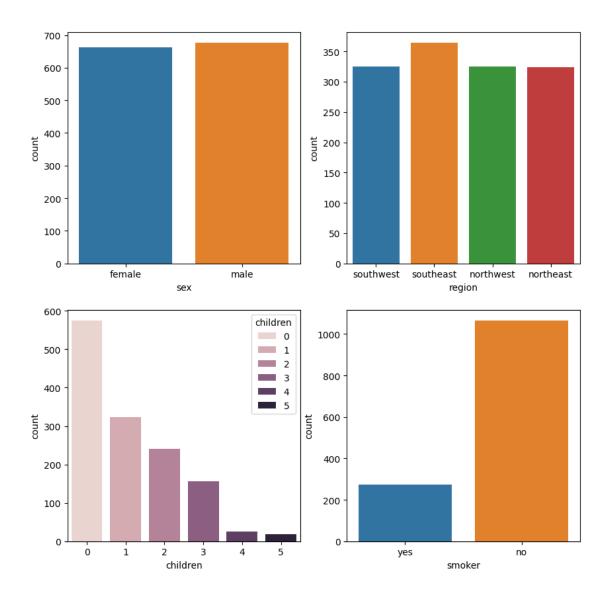
The plot shows the average charges varies by region but it shows that the charges for female-identified individuals are lower than for male identified individuals in all regions.

1.2 Plot a small multiple of bar charts to visualize the data distribution for the following categorical variables: 1. sex 2. region 3. children 4. smoker

Make subplots in the same graph

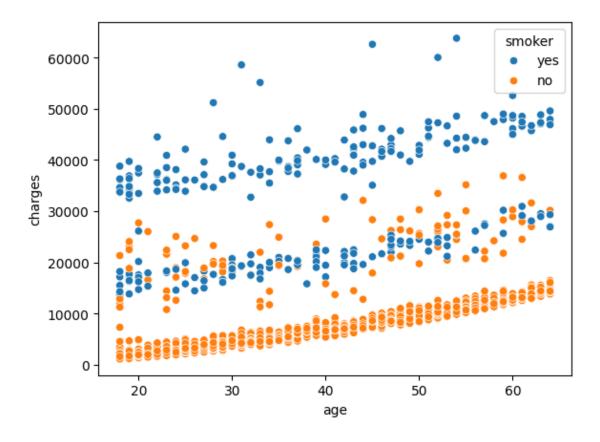
```
[163]: variables = ["sex", 'region', 'children', 'smoker']
fig, ax = plt.subplots(2,2,figsize=(10,10))

for var, subplot in zip(variables, ax.flatten()):
    sns.countplot(x=var, data=insurance_df, ax=subplot,hue = var)
```



1.3 Compare the insurance charges by age and smoker. Create a Scatter plot for age vs insurance charges categorize them by smoker type.

```
[164]: sns.scatterplot(x='age',y='charges',data=insurance_df,hue='smoker') plt.show()
```



## 0.3 Part 2: Linear Models for Regression and Classification

In this section, we will be implementing three linear models linear regression, logistic regression, and SVM.

### 0.3.1 2.1 Linear Regression

We will now proceed with splitting the dataset and implementing linear regression to predict insurance charges.

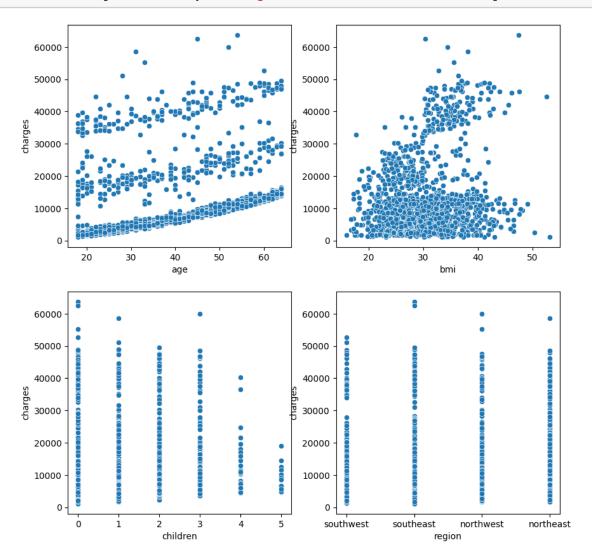
```
[165]: # Split the dataset into features and labels
insurance_X = insurance_df.drop(columns=['charges'])
insurance_y = insurance_df['charges']
```

2.1.1 Plot relationships between features (age, bmi, children, region) and the target variable charges as a small multiple of scatter plots. 1. age 2. bmi 3. children 4. region

Make sure to label the axes.

```
[166]: variables = ["age",'bmi','children','region']
fig, ax = plt.subplots(2,2,figsize=(10,10))

for var, subplot in zip(variables, ax.flatten()):
```



## 2.1.2 From the visualizations above, do you think linear regression is a good model for this problem? Why and/or why not? Please explain.

Linear regression could be a good model if we take all the categorical data into consideration - as some of the features shows linear relationship within certain clusters. If these cluster can be modeled using categorical features, then linear regression could work. Linear regression is good because we want to predict the final charges, which is a numerical value. However, it is important to note that these variables do not show a straightforward continuous relationship with the target variables.

### 0.3.2 Data Preprocessing

Before we can fit a linear regression model, several pre-processing steps should be applied to the dataset:

- 1. Encode categorical features appropriately (e.g., sex, smoker, region).
- 2. Check for multicollinearity by analyzing the correlation matrix and removing any highly collinear features.
- 3. Split the dataset into training (60%), validation (20%), and test (20%) sets.
- 4. Standardize the feature matrices (X\_train, X\_val, and X\_test) to have zero mean and unit variance. Ensure that the standardization parameters (mean, variance) are learned from X train and then applied to all sets to avoid information leakage.
- 5. Add a column of ones to X\_train, X\_val, and X\_test for learning the bias term in the linear model.
- 2.1.3 Encode the categorical variables of the Insurance dataset.

2.1.4 Plot the correlation matrix, and check if there is high correlation between the given numerical features (Threshold  $\geq = 0.8$ ). If yes, drop one from each pair of highly correlated features from the dataframe. It is fine if you do not find any highly correlated features. Why could this be necessary before proceeding further?

```
[168]: insurance_corr = insurance_X.corr()

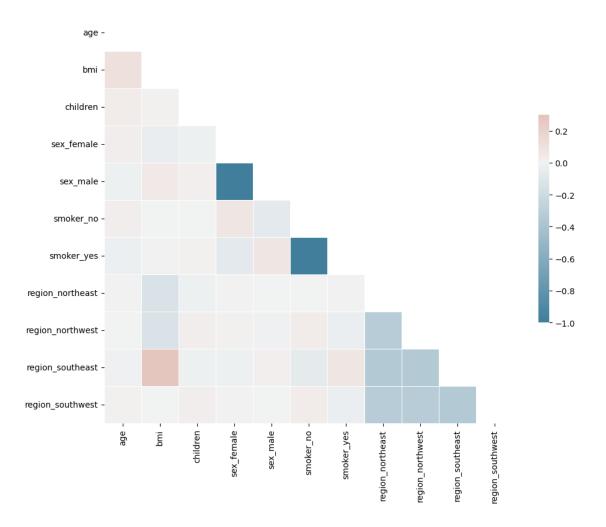
mask = np.triu(np.ones_like(insurance_corr, dtype=bool))

f, ax = plt.subplots(figsize=(11, 9))

cmap = sns.diverging_palette(230, 20, as_cmap=True)

sns.heatmap(insurance_corr, mask=mask, cmap=cmap, vmax=.3, center=0, square=True, linewidths=.5, cbar_kws={"shrink": .5})
```

[168]: <Axes: >



```
[169]: insurance_X.drop(columns=['sex_female','smoker_no','region_northeast'],⊔

inplace=True)
```

We want to avoid multicolinearity because we are seeking a close formed solution, and you can not find the inverse of the X matrix if it is not full rank.

## 2.1.5 Split the dataset into training (60%), validation (20%), and test (20%) sets.

```
insurance_X_dev, insurance_X_test, insurance_y_dev, insurance_y_test =
train_test_split(insurance_X, insurance_y, test_size=0.2, random_state=0)
insurance_X_train, insurance_X_val, insurance_y_train, insurance_y_val =
train_test_split(insurance_X_dev, insurance_y_dev, test_size=0.25,__
random_state=0)
```

#### 2.1.6 Standardize the columns in the feature matrices.

At the end of this pre-processing, you should have the following vectors and matrices:

- insurance\_X\_train: Training set feature matrix.
- insurance\_X\_val: Validation set feature matrix.
- insurance\_X\_test: Test set feature matrix.
- insurance\_y\_train: Training set labels (insurance charges).
- insurance\_y\_val: Validation set labels.
- insurance\_y\_test: Test set labels.

#### 0.3.3 Implement Linear Regression

Now that the data is preprocessed, we can implement a linear regression model, specifically Ridge Regression, which incorporates L2 regularization.

Given a feature matrix ( X ), a label vector ( y ), and a weight vector ( w ), the hypothesis function for linear regression is:

$$y = Xw$$

The objective is to find the optimal weight vector (w) that minimizes the following loss function:

$$\min_{w} \|Xw - y\|_2^2 + \alpha \|w\|_2^2$$

Where:  $-\|Xw-y\|_2^2$  penalizes predictions that differ from actual labels.  $-\alpha \|w\|_2^2$  is the regularization term, helping reduce overfitting by penalizing large weights.  $-\alpha$  is the regularization parameter.

The closed-form solution for Ridge Regression is given by the Normal Equations:

$$w = (X^T X + \alpha I)^{-1} X^T y$$

### 2.1.7 Implement a LinearRegression class with train and predict methods

We will now implement a custom LinearRegression class with L2 regularization (Ridge Regression).

Note: You may NOT use sklearn for this implementation. You may, however, use np.linalg.solve to find the closed-form solution. It is highly recommended that you vectorize your code.

```
[173]: class LinearRegression():
           111
           Linear regression model with L2-regularization (i.e. ridge regression).
           Attributes
           ____
           alpha: regularization parameter
           w: (n \ x \ 1) weight vector
           def __init__(self, alpha=0):
               self.alpha = alpha
               self.w = None
           def train(self, X, y):
               '''Trains model using ridge regression closed-form solution.
               Parameters:
               X : (m \times n) feature matrix
               y: (m \ x \ 1) label vector
               if type(y) == pd.Series:
                   y = y.values
               if type(X) == pd.DataFrame:
                   X = X.values
               self.w = np.linalg.solve(X.T @ X + self.alpha * np.eye(X.shape[1]), X.T__
        →@ y)
           def predict(self, X):
               '''Predicts on X using trained model.
               Parameters:
               X : (m \times n) feature matrix
               Returns:
               y_pred: (m x 1) prediction vector
               y_predict = X @ self.w
```

```
return y_predict
```

### 2.1.8 Train, Evaluate, and Interpret Linear Regression Model

Train a linear regression model ( $\alpha = 0$ ) on the insurance dataset. Make predictions and report the  $R^2$  score on the training, validation, and test sets. Report the first 3 and last 3 predictions on the test set, along with the actual labels.

```
[175]: model = LinearRegression()
model.train(insurance_X_train, insurance_y_train)
y_pred = model.predict(insurance_X_test)
get_report(y_pred, insurance_y_test)
```

```
[175]:
                   Prediction
                                     Actual
       Position
       1
                 11351.075203
                                9724.53000
       2
                  9700.882215
                                8547.69130
       3
                 38235.614552 45702.02235
       266
                 16352.480222 20709.02034
                 32989.621066 40932.42950
       267
       268
                  9542.694645
                                9500.57305
```

```
[176]: model.train(insurance_X_train, insurance_y_train)
    y_train_pred = model.predict(insurance_X_train)
    r2_train = r2_score(insurance_y_train, y_train_pred)
    print(f"R^2 on training set is: {r2_train:.4f}")

    y_val_pred = model.predict(insurance_X_val)
    r2_val = r2_score(insurance_y_val, y_val_pred)
    print(f"R^2 on validation set is: {r2_val:.4f}")
```

```
y_test_pred = model.predict(insurance_X_test)
r2_test = r2_score(insurance_y_test, y_test_pred)
print(f"R^2 on test set is: {r2_test:.4f}")
```

```
R^2 on training set is: 0.7410 R^2 on validation set is: 0.7242 R^2 on test set is: 0.7998
```

2.1.9 Use the mean of the training labels (insurance\_y\_train) as the prediction for all instances. Report the  $R^2$  on the training, validation, and test sets using this baseline.

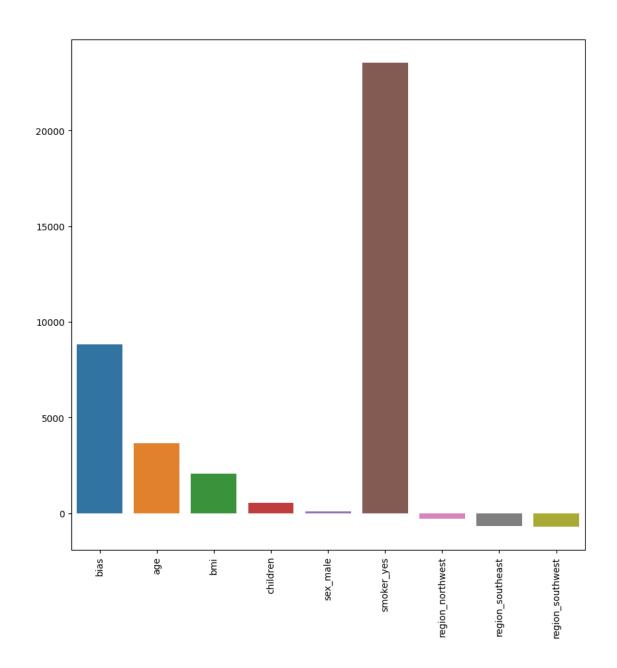
This is a common baseline used in regression problems and tells you if your model is any good. Your linear regression  $R^2$  should be much higher than these baseline  $R^2$ .

Baseline R^2: Training = 0.0000, Validation = -0.0008, Test = -0.0004

2.1.10 Interpret your model trained on the insurance dataset using a bar chart of the model weights. Make sure to label the bars (x-axis) and don't forget the bias term!

```
[178]: weights = model.w
names = ['bias'] + list(columns)

fig = plt.figure(figsize=(10,10))
ax = sns.barplot(x=names,y = weights, hue = names)
ax.tick_params(axis='x', rotation=90)
```



# 2.1.11 According to your model, which features are the greatest contributors to insurance charges?

Smoker status and the age are the greatest contributor to insurance charges

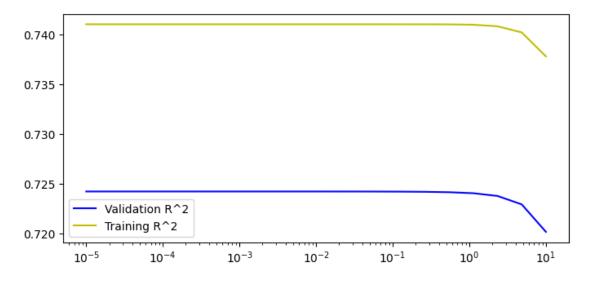
## **0.3.4** Hyperparameter Tuning $(\alpha)$

Now, let's tune the  $\alpha$  regularization parameter for ridge regression on the insurance dataset.

2.1.12 Sweep out values for  $\alpha$  using alphas = np.logspace(-5, 1, 20). Perform a grid search over these  $\alpha$  values, recording the training and validation  $R^2$  for each  $\alpha$ . Plot the results with a log scale for  $\alpha$ . A simple grid search is fine, no need for k-fold cross

validation. Plot the training and validation  $R^2$  as a function of  $\alpha$  on a single figure. Make sure to label the axes and the training and validation  $R^2$  curves. Use a log scale for the x-axis.\*\*

```
[179]: ### Code here
       alphas= np.logspace(-5,1,20)
       val scores = []
       train_scores = []
       for alpha in alphas:
           model = LinearRegression(alpha=alpha)
           model.train(insurance X train, insurance y train)
           y_pred_train = model.predict(insurance_X_train)
           y_pred_val = model.predict(insurance_X_val)
           validation_r2 = r2_score(insurance_y_val, y_pred_val)
           train_r2 = r2_score(insurance_y_train, y_pred_train)
           val_scores.append(validation_r2)
           train_scores.append(train_r2)
       fig = plt.figure(figsize=(8,8))
       ax = fig.add_subplot(2, 1, 1)
       plt.plot(alphas, val_scores, label='Validation R^2',color = 'b')
       plt.plot(alphas, train_scores, label='Training R^2',color='y')
       ax.set_xscale('log')
       plt.legend()
       plt.show()
```



2.1.13 Explain your plot above. How do training and validation  $R^2$  behave with increasing  $\alpha$ ?

The model's performance decreases while increasing alpha

### 0.3.5 2.2 Logistic Regression

#### 2.2.1 Load the dataset, the dataset to be used is loan\_data.csv

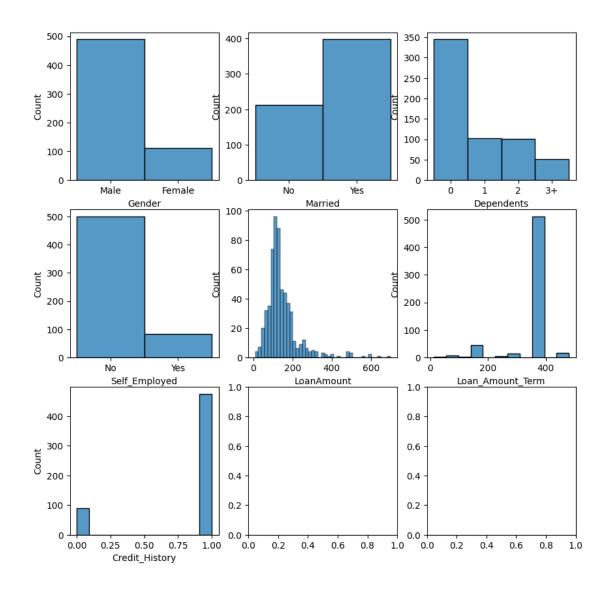
```
[180]: loan_data_df = pd.read_csv('loan_data.csv')
[181]: loan_data_df = loan_data_df.drop(columns=['Loan_ID'])
```

## 2.2.2 Are there any missing values in the dataset? If so, what is the best way to deal with it and why?

There are 21.82% rows with missing values.

There are a lot of missing data - about 22% rows have them. We can't simply drop the rows because that will greatly reduce our dataset size. For the columns with missing values, I want to examine the distribution of the data. If the distribution centers around some mode, I will use the mode to backfill the missing data.

```
[183]: columns_with_missing_values = loan_data_df.columns[loan_data_df.isnull().any()]
fig, ax = plt.subplots(3,3,figsize=(10,10))
for var, subplot in zip(columns_with_missing_values, ax.flatten()):
    sns.histplot(x=var, data=loan_data_df, ax=subplot)
```



```
[184]: columns_to_back_fill = columns_to_back_fill = columns_to_back_fill: for column in columns_to_back_fill: loan_data_df[column] = loan_data_df[column].fillna(loan_data_df[column].comode()[0])

loan_data_df['LoanAmount'] = loan_data_df[column].fillna(loan_data_df[column].comodian())

print("Now there are %.2f%% rows with missing values. We will drop these rows." column colu
```

```
loan_data_df = loan_data_df.dropna(axis=0)
```

Now there are 0.00% rows with missing values. We will drop these rows.

2.2.3 Encode the categorical variables.

```
[185]: ohe_features=["Education",'Gender','Married','Dependents','Self_Employed','Property_Area','Cre
loan_data_X = loan_data_df.drop(columns=['Loan_Status'])
loan_data_X = pd.get_dummies(loan_data_X, columns=ohe_features,drop_first=True)
loan_data_y = loan_data_df['Loan_Status']
features = loan_data_X.columns
```

2.2.4 Do you think that the distribution of labels is balanced? Why/why not? Hint: Find the probability of the different categories.

the probability of loan status being yes is 0.69% and being no is 0.31%, which is very loopsised to yes

the probability of loan status being yes is 0.69% and being no is 0.31%, which is very loopsised to yes

2.2.5 Plot the correlation matrix (first separate features and Y variable), and check if there is high correlation between the given numerical features (Threshold >=0.9). If yes, drop those highly correlated features from the dataframe.

```
[187]: loan_corr = loan_data_X.corr()

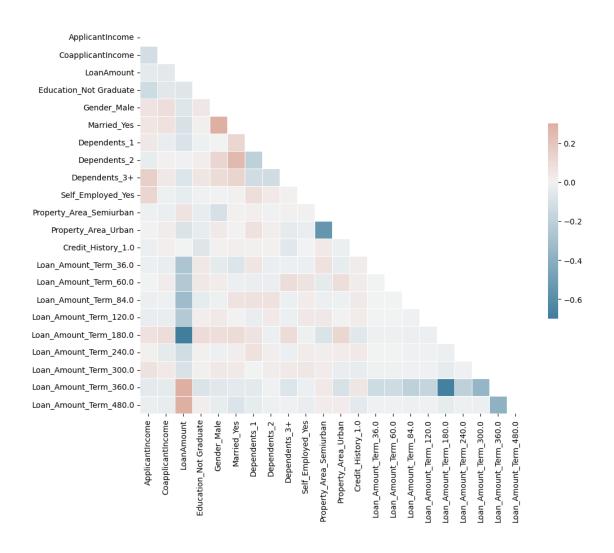
mask = np.triu(np.ones_like(loan_corr, dtype=bool))

f, ax = plt.subplots(figsize=(11, 9))

cmap = sns.diverging_palette(230, 20, as_cmap=True)

sns.heatmap(loan_corr, mask=mask, cmap=cmap, vmax=.3, center=0, square=True, linewidths=.5, cbar_kws={"shrink": .5})
```

[187]: <Axes: >



## 2.2.6 Apply the following pre-processing steps:

- 1. Convert the label from a Pandas series to a Numpy (m x 1) vector. If you don't do this, it may cause problems when implementing the logistic regression model.
- 2. Split the dataset into training (60%), validation (20%), and test (20%) sets.
- 3. Standardize the columns in the feature matrices. To avoid information leakage, learn the standardization parameters from training, and then apply training, validation and test dataset.
- 4. Add a column of ones to the feature matrices of train, validation and test dataset. This is a common trick so that we can learn a coefficient for the bias term of a linear model.

#### 0.3.6 Implement Logisitc Regression

We will now implement logistic regression with L2 regularization. Given an  $(m \times n)$  feature matrix X, an  $(m \times 1)$  label vector y, and an  $(n \times 1)$  weight vector w, the hypothesis function for logistic regression is:

$$y = \sigma(Xw)$$

where  $\sigma(x) = \frac{1}{1+e^{-x}}$ , i.e. the sigmoid function. This function scales the prediction to be a probability between 0 and 1, and can then be thresholded to get a discrete class prediction.

Just as with linear regression, our objective in logistic regression is to learn the weights w which best fit the data. For L2-regularized logistic regression, we find an optimal w to minimize the following loss function:

$$\min_{w} \ -y^T \ \log(\sigma(Xw)) \ - \ (\mathbf{1} - y)^T \ \log(\mathbf{1} - \sigma(Xw)) \ + \ \alpha \|w\|_2^2$$

Unlike linear regression, however, logistic regression has no closed-form solution for the optimal w. So, we will use gradient descent to find the optimal w. The (n x 1) gradient vector g for the loss function above is:

$$g = X^T \Big( \sigma(Xw) - y \Big) + 2\alpha w$$

Below is pseudocode for gradient descent to find the optimal w. You should first initialize w (e.g. to a (n x 1) zero vector). Then, for some number of epochs t, you should update w with w - g \$, where  $\eta$  is the learning rate and g is the gradient. You can learn more about gradient descent here.

$$w = \mathbf{0}$$
 for  $i = 1, 2, ..., t$  \$ w = w - g \$

A LogisticRegression class with five methods: train, predict, calculate\_loss, calculate\_gradient, and calculate\_sigmoid has been implemented for you below.

```
[189]: class LogisticRegression():
           Logistic regression model with L2 regularization.
           Attributes
           alpha: regularization parameter
           t: number of epochs to run gradient descent
           eta: learning rate for gradient descent
           w: (n \ x \ 1) \ weight \ vector
           111
           def __init__(self, alpha=0, t=100, eta=1e-3):
               self.alpha = alpha
               self.t = t
               self.eta = eta
               self.w = None
           def train(self, X, y):
               '''Trains logistic regression model using gradient descent
               (sets w to its optimal value).
               Parameters
               ____
               X : (m \times n) feature matrix
               y: (m x 1) label vector
               Returns
               losses: (t x 1) vector of losses at each epoch of gradient descent
               loss = list()
               self.w = np.zeros((X.shape[1],1))
               for i in range(self.t):
                   self.w = self.w - (self.eta * self.calculate_gradient(X, y))
                   loss.append(self.calculate_loss(X, y))
               return loss
           def predict(self, X):
               '''Predicts on X using trained model. Make sure to threshold
               the predicted probability to return a 0 or 1 prediction.
               Parameters
```

```
X : (m \times n) feature matrix
      Returns
      y_pred: (m x 1) 0/1 prediction vector
      y_pred = self.calculate_sigmoid(X.dot(self.w))
      y_pred[y_pred >= 0.5] = 1
      y_pred[y_pred < 0.5] = 0
      return y_pred
  def calculate_loss(self, X, y):
       '''Calculates the logistic regression loss using X, y, w,
       and alpha. Useful as a helper function for train().
      Parameters
      X : (m \ x \ n) \ feature \ matrix
      y: (m \ x \ 1) label vector
      Returns
       loss: (scalar) logistic regression loss
      loss= -y.T.dot(np.log(self.calculate_sigmoid(X.dot(self.w)))) - (1-y).T.
→dot(np.log(1-self.calculate_sigmoid(X.dot(self.w)))) + self.alpha*np.linalg.
⇒norm(self.w, ord=2)**2
      return loss.item()
  def calculate_gradient(self, X, y):
       '''Calculates the gradient of the logistic regression loss
       using X, y, w, and alpha. Useful as a helper function
      for train().
      Parameters
      X : (m \times n) feature matrix
      y: (m \ x \ 1) label vector
      Returns
      gradient: (n x 1) gradient vector for logistic regression loss
      return X.T.dot(self.calculate_sigmoid( X.dot(self.w)) - y) + 2*self.
⇒alpha*self.w
```

```
def calculate_sigmoid(self, x):
    '''Calculates the sigmoid function on each element in vector x.
    Useful as a helper function for predict(), calculate_loss(),
    and calculate_gradient().

Parameters
------
x: (m x 1) vector

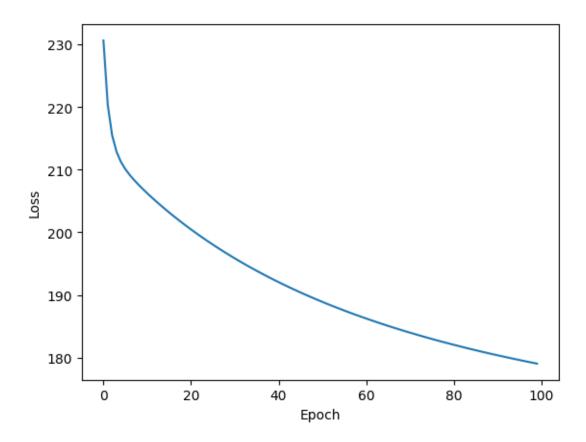
Returns
------
sigmoid_x: (m x 1) vector of sigmoid on each element in x
'''
return (1)/(1 + np.exp(-x.astype('float')))
```

## 2.2.7 Plot Loss over Epoch and Search the space randomly to find best hyperparameters.

- i) Using your implementation above, train a logistic regression model (alpha=0, t=100, eta=1e-3) on the loan training data. Plot the training loss over epochs. Make sure to label your axes. You should see the loss decreasing and start to converge.
- ii) Using alpha between (0,1), eta between (0, 0.001) and t between (0, 100), find the best hyperparameters for LogisticRegression. You can randomly search the space 20 times to find the best hyperparameters.
- iii) Compare accuracy on the test dataset for both the scenarios.

```
[190]: ### Code here
model = LogisticRegression(alpha=0, t=100, eta=1e-3)
epoch = np.array(list(range(100)))
loan_X_train= loan_X_train.astype('float')
loss = model.train(loan_X_train, loan_y_train)

fig = sns.lineplot(x=epoch, y=loss)
fig.set(xlabel='Epoch', ylabel='Loss')
plt.show()
```



```
[191]: val_scores = []
alpha_random = np.random.sample(20)
eta_random = np.random.sample(20)*0.001
t_random = np.random.randint(0,100,20)

for alpha in alpha_random:
    for eta in eta_random:
        for t in t_random:
            model = LogisticRegression(alpha=alpha, t= int(t), eta=eta)
            model.train(loan_X_train, loan_y_train)
            y_pred = model.predict(loan_X_val)
            val_score = r2_score(loan_y_val, y_pred)
            val_scores.append((val_score, (alpha, eta, t)))
```

[192]: best\_params = max(val\_scores, key=lambda x: x[0])[1]

print(f"Best hyperparameters: alpha={best\_params[0]}, eta={best\_params[1]},

ot={best\_params[2]}")

Best hyperparameters: alpha=0.5478893422258532, eta=0.0009858761830776905, t=81

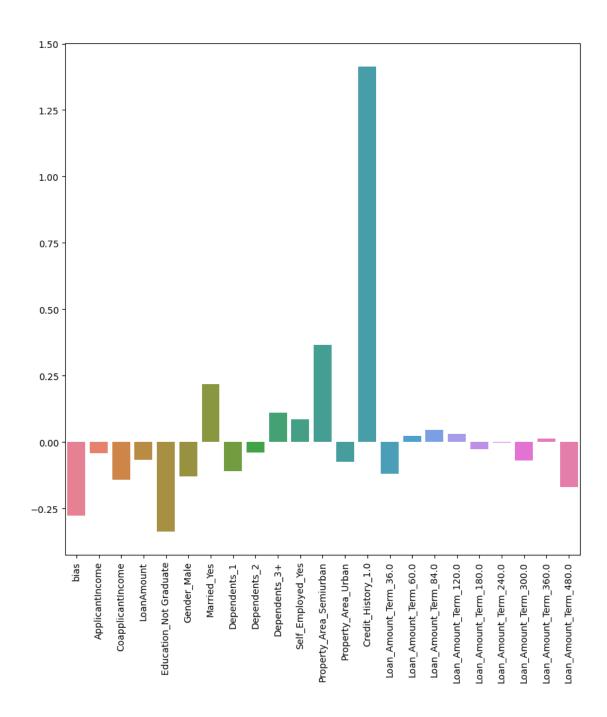
Scenario 1 R^2: 0.006 and Scenario 2 R^2: 0.006

### 0.3.7 Feature Importance

2.2.8 Interpret your trained model using a bar chart of the model weights. Make sure to label the bars (x-axis) and don't forget the bias term!

```
[194]: weights = model_scenario_2.w.reshape(-1)
feature_names = ['bias'] + list(features)

fig = plt.figure(figsize=(10,10))
ax = sns.barplot(x=feature_names,y = weights, hue = feature_names)
ax.tick_params(axis='x', rotation=90)
```



No credit history severly negatively impacts the loan decision. Aside from credit history, other factors can impact loan decisions include property area. Semi-urban property owners are more likely to obtain a positive loan decision. Loan applicants with "graduate" education label are more likely to obtain a positive loan decision

## 0.3.8 2.3 Support Vector Machines

In this part, we will be using support vector machines for classification on the loan dataset.

#### 0.3.9 Train Primal SVM

2.3.1 Train a primal SVM (with default parameters) on the loan dataset. Make predictions and report the accuracy on the training, validation, and test sets.

Primal SVM  $R^2$  for training data: 0.161, for validation data 0.062, for test data 0.130

## 0.3.10 Train Dual SVM

2.3.2 Train a dual SVM (with default parameters) on the heart disease dataset. Make predictions and report the accuracy on the training, validation, and test sets.

Dual SVM  $R^2$  for training data: 0.135, for validation data 0.062, for test data 0.130