Voltage Multiplier

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March 8, 2025

1 Voltage Clamper

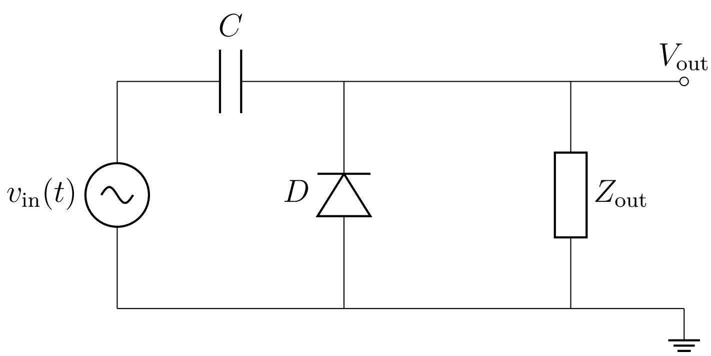


Fig. 1: Voltage Clamper

Adopt the differential equation used in fast transient RC circuit analysis, or from the Kirchhoff's law for voltage, we could write down the system of differential equations for the circuit in fig. 1 as

$$v_{\text{in}}(t) = v_{R_{\text{in}}}(t) + v_{C_1}(t) + v_{\text{out}}(t), \tag{1.1}$$

where $v_{\text{out}}(t) = v_D(t)$ is the output voltage. The current through the capacitor is $i_C(t) = C\frac{\mathrm{d}v_C(t)}{\mathrm{d}t}$, and the current through the diode is $i_D(t) = I_S\left(e^{\frac{-v_D(t)}{V_T}} - 1\right)$ from Shockley's diode equation, which is very small reverse saturation current for $v_D < 0$. Substitute $v_{R_{\text{in}}}(t) = R_{\text{in}} i_C(t)$, we have

$$v_{\text{in}}(t) = R_{\text{in}} C\frac{\mathrm{d}v_C(t)}{\mathrm{d}t} + v_C(t) + v_{\text{out}}(t). \tag{1.2}$$

To solve this equation, assume $v_D = 0$, the diode acts like a switch (open or short circuit) to control the direction of current flow through the capacitor.

- $v_{\text{in}}(t) > v_C(t)$, the diode is off, $i_C(t)$ flows right, treat as a discharging process;
- $v_{\text{in}}(t) < v_C(t)$, the diode is on, $i_C(t)$ flows left, treat as a charging process.

The difference between discharging and charging processes is not only the direction of current flow, the most important thing is there is almost no current through the output when the diode is on. That's because diode will take account of most current flow (like short circuit), making output impedance nearly open circuited. That will make the differential equation differently, or piecewise.

$$\begin{cases} v_{\text{in}}(t) = (R_{\text{in}} + R_{\text{out}})Cv'_C(t) + v_C(t), & v_{\text{in}}(t) > v_C(t), \\ v_{\text{in}}(t) = R_{\text{in}}Cv'_C(t) + v_C(t), & v_{\text{in}}(t) < v_C(t). \end{cases} \tag{1.3}$$

Organize the above equations to only leave first-order in the left of the formulas, we have

$$Cv'_C(t) = \begin{cases} \dfrac{1}{R_{\text{in}} + R_{\text{out}}} \left(v_{\text{in}}(t) - v_C(t)\right), & v_{\text{in}}(t) > v_C(t), \\ \dfrac{1}{R_{\text{in}}} \left(v_{\text{in}}(t) - v_C(t)\right), & v_{\text{in}}(t) < v_C(t). \end{cases} \tag{1.4}$$

Put this into Mathematica using `NDSolve` to solve the differential equation, with initial conditions $v_C(0) = 0.$, we could get the voltage across the capacitor $v_C(t)$, and the output voltage $v_{\text{out}}(t) = v_D(t)$. In the experiment, we set $v_{\text{in}}(t) = V_0 \sin \omega t$, $V_0 = 5\text{V}$, $\omega = 2\pi f$, $f = 1\text{kHz}$, $R_{\text{in}} = 50\Omega$, $R_{\text{out}} = 47\text{k}\Omega$, $C = 1\mu\text{F}$. Here are some simulation results:

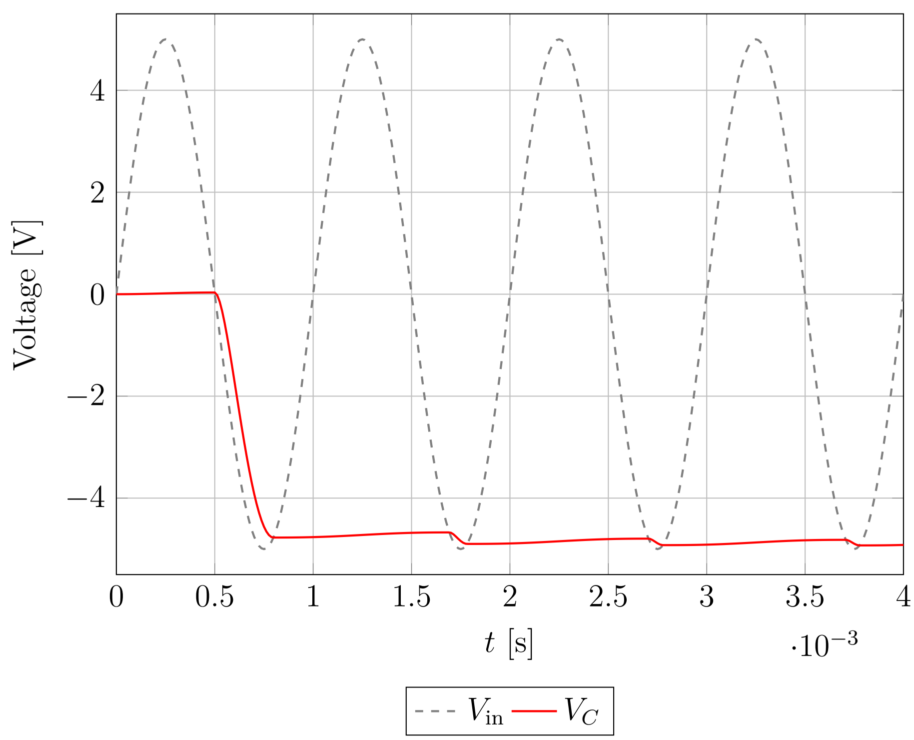


Fig. 2: Capacitor Voltage in Clamper circuit

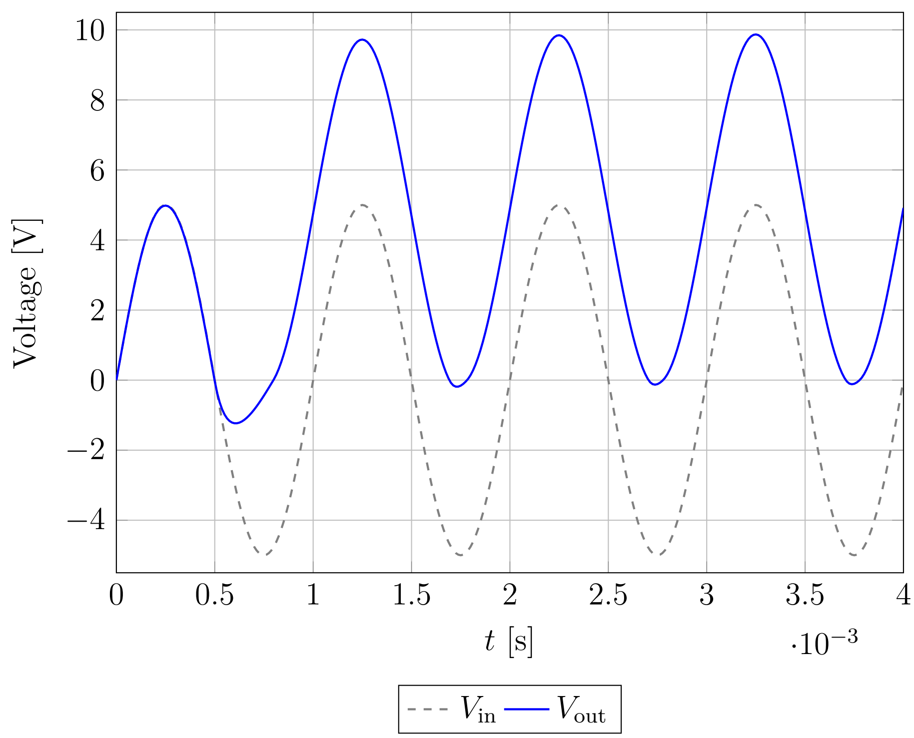


Fig. 3: Output Voltage in Clamper circuit

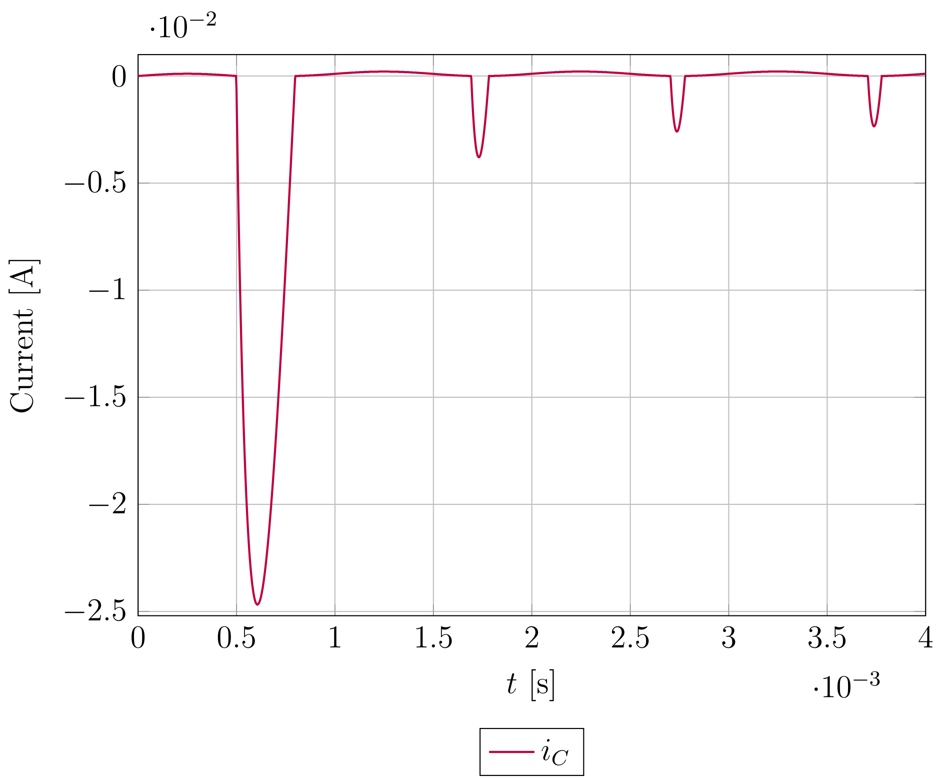


Fig. 4: Current through capacitor in Clamper circuit

Use Shockley's diode equation to calculate the current through the diode, we have

$$i_D(t) = I_S \left(e^{\frac{v_D(t)}{mV_T}} - 1 \right), \tag{1.5}$$

where I_S is the reverse saturation current, V_T is the thermal voltage, $V_T = \frac{k_B T}{q}$. m is the ideality factor, which is usually between 1 and 2. According to 1N4004 Diode datasheet[1], we could get several data points from I-V curve under $T = 298.15\text{K}$, do a linear fit of $(V_D, \ln(i_D))$ to get $m = 2.2615$, $I_S = 7.33098 \times 10^{-8}\text{A}$.

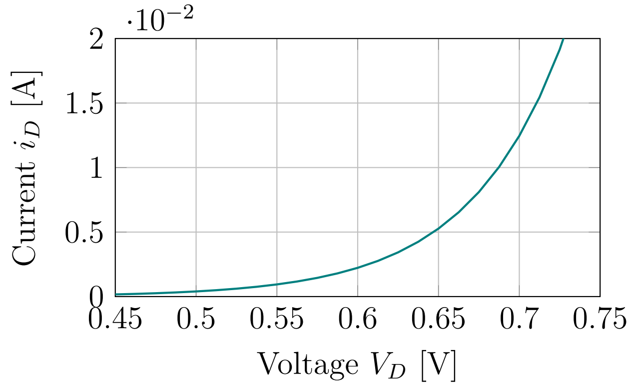
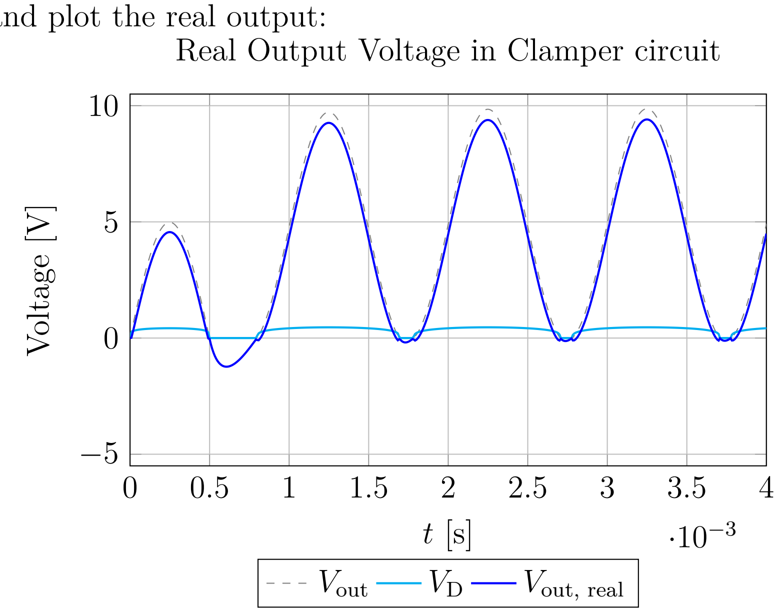


Fig. 5: I-V Curve of Diode

We could use the current from fig. 4 to determine real value of $v_D(t)$ by inverse of Shockley's diode equation and plot the real output:



And the average value of the real output voltage could be calculated after the circuit is stabilized, using midpoint rule of integration, we find the average value is $\langle V_{\text{out, real}} \rangle = 4.50\text{V}$, which is close to the measured value of 4.34V.

[1] https://cdn.sparkfun.com/assets/learn_tutorials/4/2/3/1N4004.pdf

2 Voltage Doubler

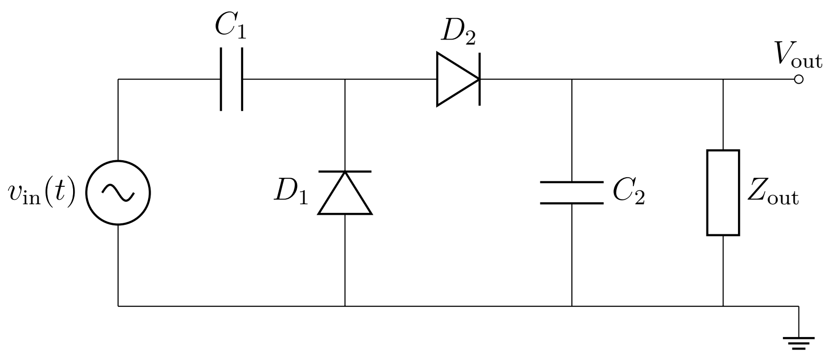


Fig. 6: Voltage Doubler

For a voltage doubler, we could write down the system of differential equations for the circuit in fig. 6 as

$$\begin{cases} v_{in} = i_{C_1} R_{in} + v_{C_1} + v_{C_2}, \\ v_{C_2} = (i_{C_1} - i_{C_2})\, R_{out}\,. \end{cases} \tag{2.1}$$

Substitute $i_{C_1} = C_1 \dfrac{\mathrm{d}v_{C_1}(t)}{\mathrm{d}t}$ and $i_{C_2} = C_2 \dfrac{\mathrm{d}v_{C_2}(t)}{\mathrm{d}t}$, as well as $v_{in} = V_0 \sin \omega t$ into the above equations,

$$\begin{cases} v_{in} = R_{in}\, C_1 v'_{C_1}(t) + v_{C_1}(t) + v_{C_2}(t), \\ v_{C_2} = R_{out}\, \left(C_1 v'_{C_1}(t) - C_2 v'_{C_2}(t) \right). \end{cases} \tag{2.2}$$

Express linear differential equations with left variables $(v'_{C_1}(t), v'_{C_2}(t))$ and right variables $(v_{C_1}(t), v_{C_2}(t))$, which will made numerical solution easier, we have

$$\begin{cases} C_1 v'_{C_1}(t) = \dfrac{1}{R_{in}} \left(v_{in}(t) - v_{C_1}(t) - v_{C_2}(t) \right), \\ C_2 v'_{C_2}(t) = \dfrac{1}{R_{in}} \left(v_{in}(t) - v_{C_1}(t) - v_{C_2}(t) \right) - \dfrac{v_{C_2}(t)}{R_{out}}. \end{cases} \tag{2.3}$$

with initial conditions $v_{C_1}(0) = v_{C_2}(0) = 0$. Assume $v_{D_1} = v_{D_2} = 0$ for both diodes, the diodes will control the flow of current like a switch. ($V_{out,1} = v_{in} - v_{C_1}$)

1. D_1 condition:
   - $v_{in} > v_{C_1}$, D_1 is off, i_{C_1} flows right, treat as a discharging process;
   - $v_{in} < v_{C_1}$, D_1 is on, i_{C_1} flows left, treat as a charging process.
2. D_2 condition:
   - $v_{out,1} > v_{C_2}$, D_2 is on, i_{C_2} flows downwards, treat as a charging process;
   - $v_{out,1} < v_{C_2}$, D_2 is off, i_{C_2} flows upwards, treat as a discharging process.

Let's use this to rewrite the system of differential equations using piecewise functions, combined with eq. (2.3) to organize the equations, we have 4 kinds of situations

$$\begin{cases} C_1 v'_{C_1}(t) = \begin{cases} \frac{1}{R_{\text{in}}}\left(v_{\text{in}}(t) - v_{C_1}(t) - v_{C_2}(t)\right), & v_{\text{in}}(t) > v_{C_1}(t), \\ \frac{1}{R_{\text{in}}}\left(v_{\text{in}}(t) - v_{C_1}(t)\right), & v_{\text{in}}(t) < v_{C_1}(t), \end{cases} \\ C_2 v'_{C_2}(t) = \begin{cases} \frac{1}{R_{\text{in}}}\left(v_{\text{in}}(t) - v_{C_1}(t) - v_{C_2}(t)\right) - \frac{v_{C_2}(t)}{R_{\text{out}}}, & v_{\text{out},1}(t) > v_{C_2}(t), \\ -\frac{v_{C_2}(t)}{R_{\text{out}}}, & v_{\text{out},1}(t) < v_{C_2}(t). \end{cases} \end{cases} \tag{2.4}$$

Or more precisely, in the definition of function, the current through C_1 is influced by the current through C_2, we could do a little modification to the first equation to make it more close to reality (put condition from current C_2 in the first equation):

$$\begin{cases} C_1 v'_{C_1}(t) = \begin{cases} \begin{cases} \frac{1}{R_{\text{in}}}\left(v_{\text{in}}(t) - v_{C_1}(t) - v_{C_2}(t)\right), & v_{\text{in}}(t) > v_{C_1}(t) \text{ and } v_{\text{out},1}(t) > v_{C_2}(t), \\ \frac{1}{R_{\text{in}} + R_{\text{out}}}\left(v_{\text{in}}(t) - v_{C_1}(t) - v_{C_2}(t)\right), & v_{\text{in}}(t) > v_{C_1}(t) \text{ and } v_{\text{out},1}(t) < v_{C_2}(t), \end{cases} \\ \frac{1}{R_{\text{in}}}\left(v_{\text{in}}(t) - v_{C_1}(t)\right), & v_{\text{in}}(t) < v_{C_1}(t), \end{cases} \\ C_2 v'_{C_2}(t) = \begin{cases} \frac{1}{R_{\text{in}}}\left(v_{\text{in}}(t) - v_{C_1}(t) - v_{C_2}(t)\right) - \frac{v_{C_2}(t)}{R_{\text{out}}}, & v_{\text{out},1}(t) > v_{C_2}(t), \\ -\frac{v_{C_2}(t)}{R_{\text{out}}}, & v_{\text{out},1}(t) < v_{C_2}(t). \end{cases} \end{cases} \tag{2.5}$$

where $v_{out,1} = v_{\text{in}} - v_{C_1}$, and initial conditions $v_{C_1}(0) = v_{C_2}(0) = 0$. Similarly, put this into Mathematica using `NDSolve` to solve the differential equation, we could get the voltage across the capacitors $v_{C_1}(t)$ and $v_{C_2}(t)$, and the output voltage $v_{\text{out}}(t) = v_{C_2}(t)$.

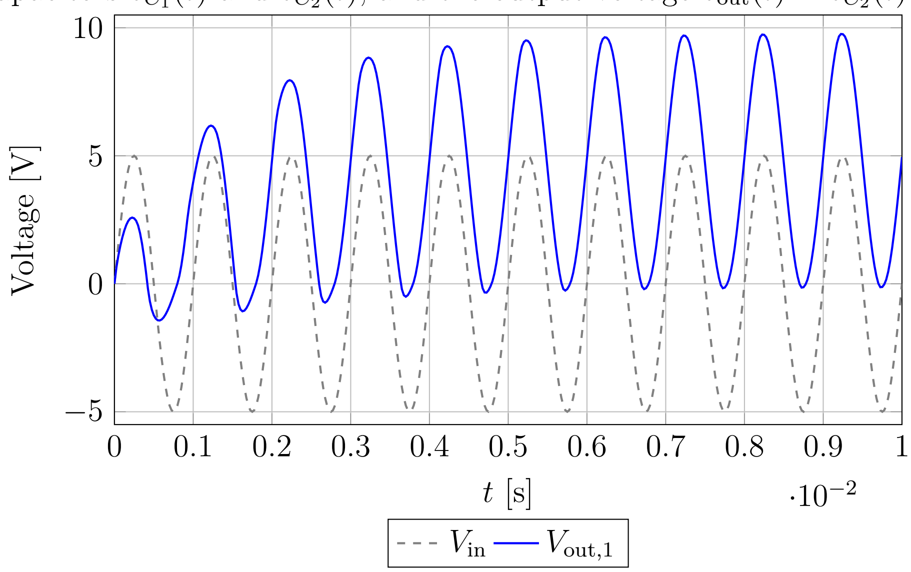


Fig. 7: Output Voltage in stage 1 (not final output) in Doubler circuit

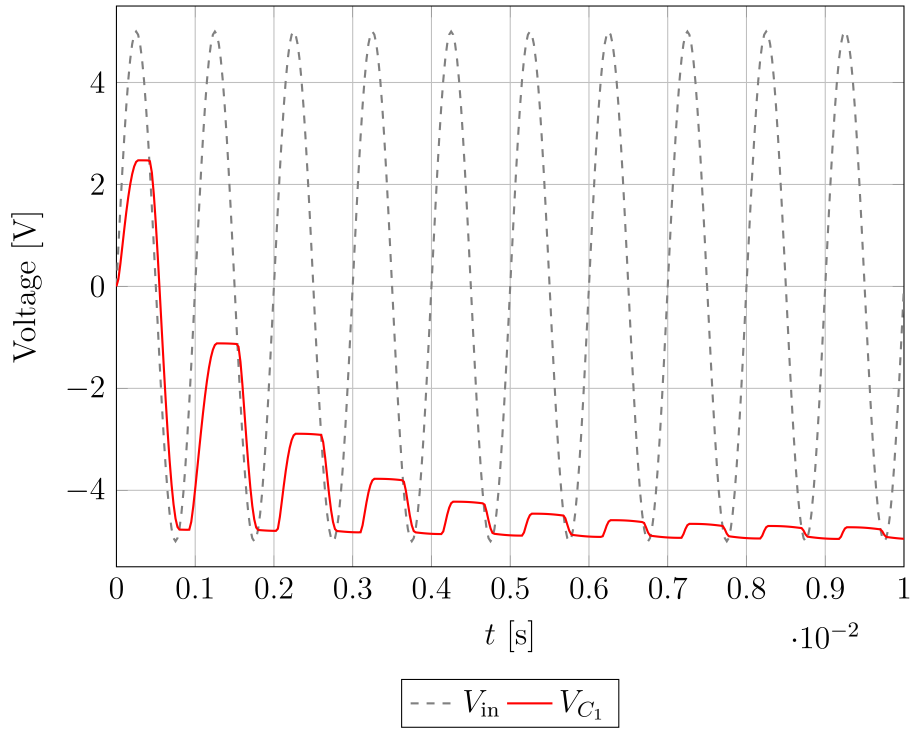


Fig. 8: Capacitor 1 Voltage in Doubler circuit

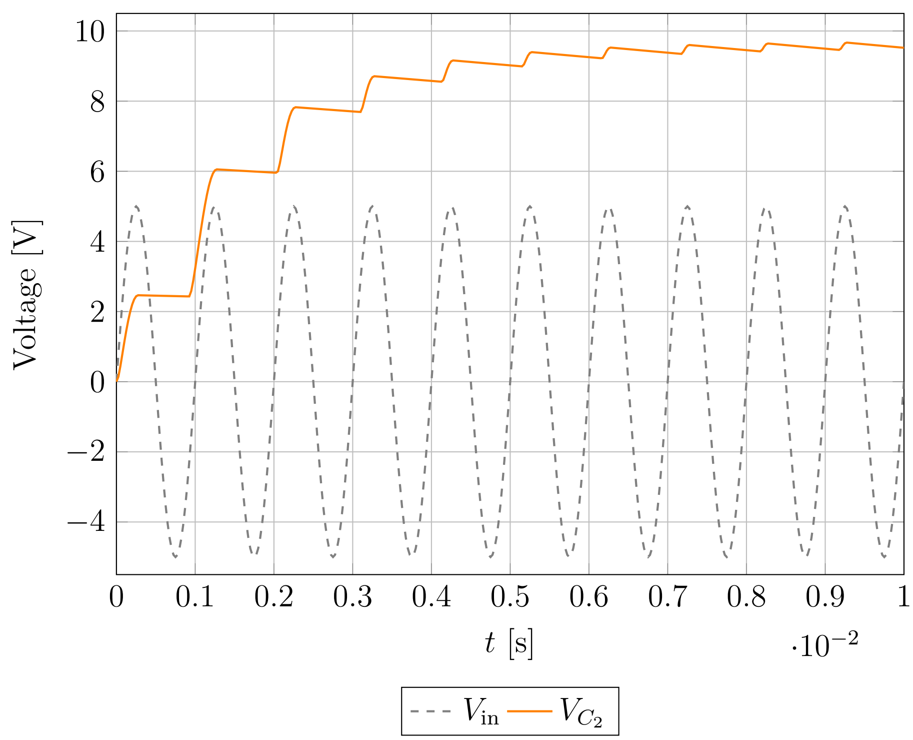


Fig. 9: Capacitor 2 Voltage in Doubler circuit

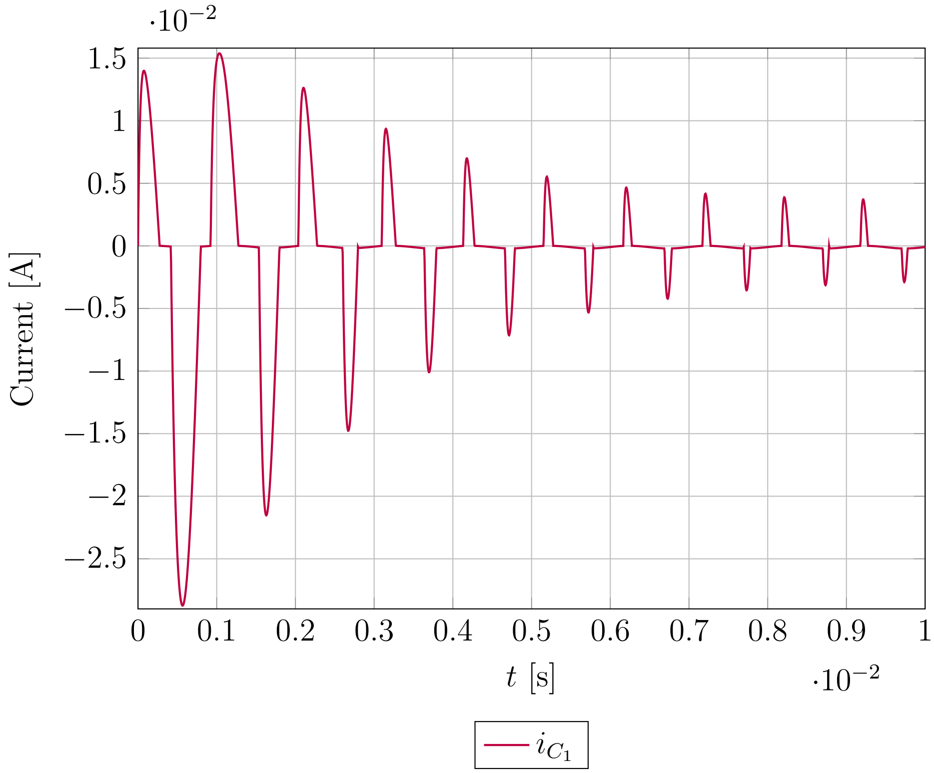


Fig. 10: Current through capacitor 1 in Doubler circuit

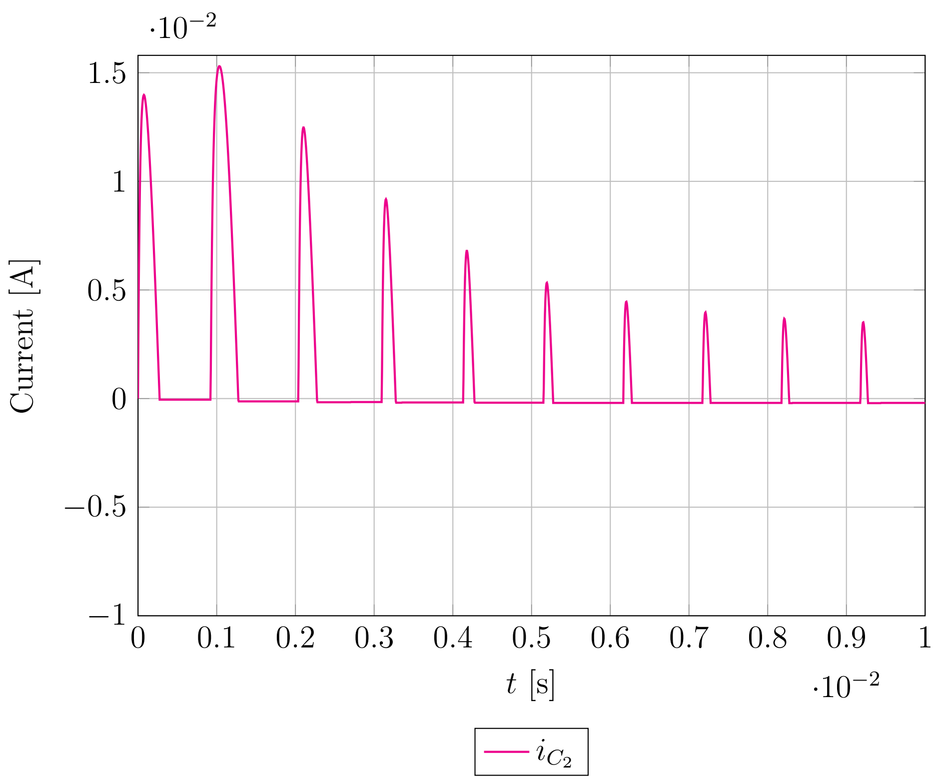


Fig. 11: Current through capacitor 2 in Doubler circuit

3 Voltage Multiplier

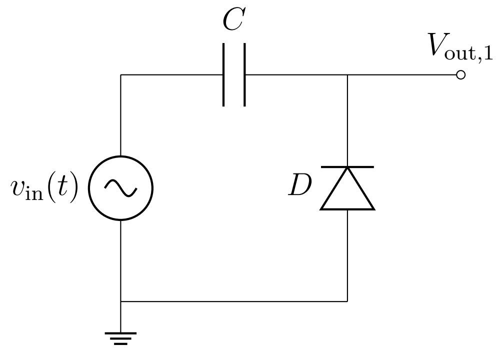


Fig. 12: Voltage Clamper

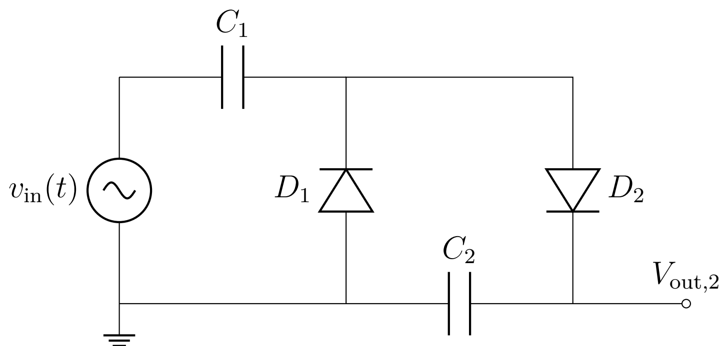


Fig. 13: Voltage Doubler

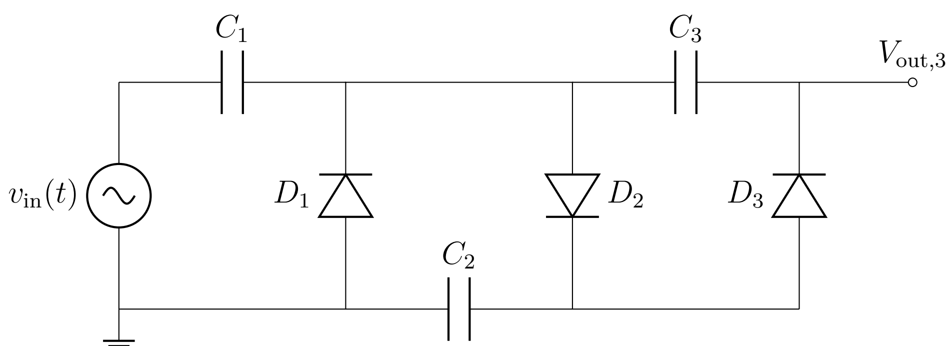


Fig. 14: Voltage Tripler

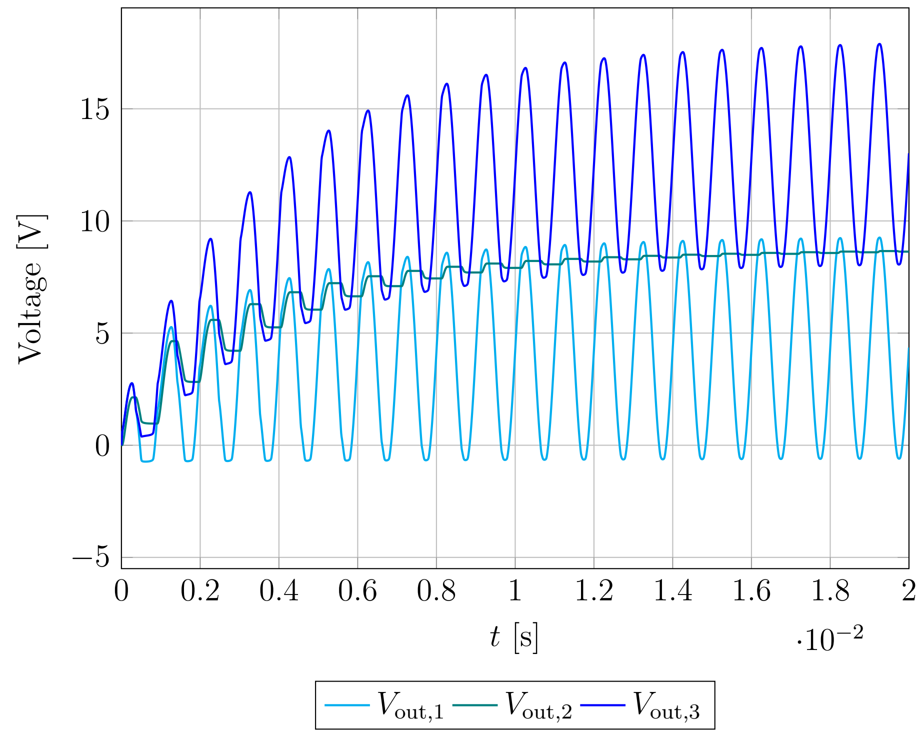


Fig. 15: Tripler Voltage Output

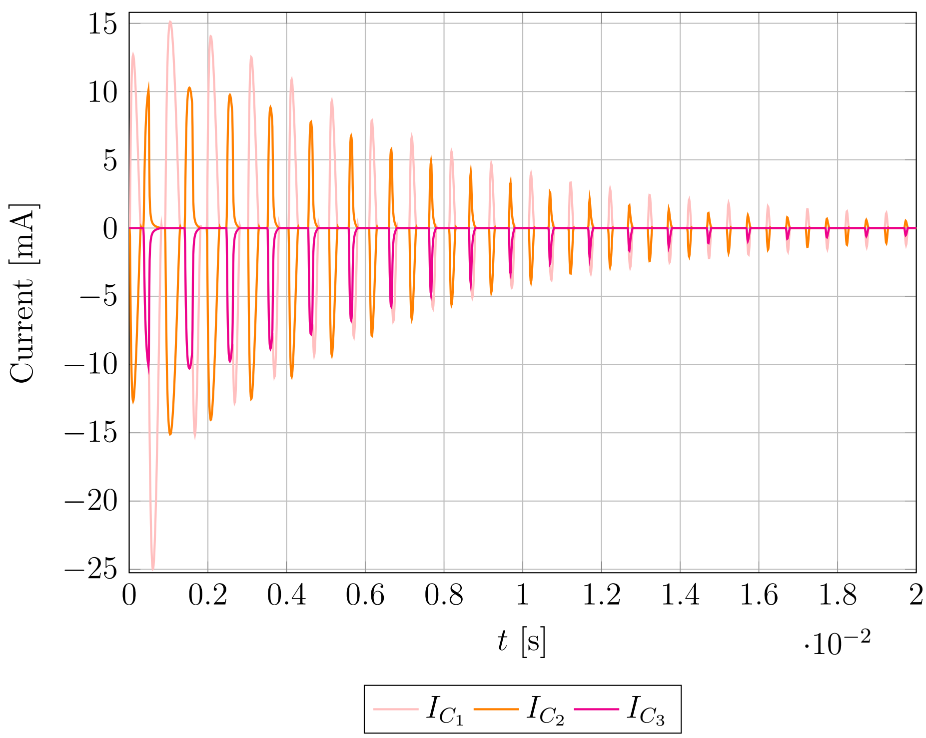


Fig. 16: Tripler Current through capacitors